Discussion Paper Series – CRC TR 224

Discussion Paper No. 794
Project B 03

Utility Maximization Under Endogenous Uncertainty

Ayush Gupta[1]

November 2026

[1]University of Mannheim, Department of Economics; ayush.gupta@uni-mannheim.de

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Utility Maximization Under Endogenous Uncertainty

Ayush Gupta*

20 October 2026

Please click here for the latest version

Abstract

This paper studies decision problems where the decision maker's choice of action affects the probability distribution of a payoff relevant random variable. We establish sufficient conditions for the existence of an expected utility maximizing action in such settings. The main requirement is a mild continuity condition on the family of possible distributions. We also show that this condition is a minimal requirement. Our result does not require common assumptions such as the monotone likelihood ratio property (MLRP) or the convexity of distribution functions condition (CDFC). It can therefore be used to prove the existence of an optimal action in many settings where existing results do not apply, including an important class of problems where the support of the random variable depends on the decision maker's choice and the density functions are not pointwise continuous.

*University of Mannheim, Department of Economics; ayush.gupta@uni-mannheim.de

I am indebted to Mark Kon, Alex Chan, and Jawwad Noor for many helpful discussions and suggestions. I also thank the participants at ESEM 2026 for helpful comments. Support by the German Research Foundation (DFG) through CRC TR 224 (Project B03) is gratefully acknowledged.

1 Introduction

The existence of an expected utility maximizing action is a minimal requirement for any model of decision-making under uncertainty. When the uncertainty facing the decision maker can be represented by a random variable with a fixed probability distribution, such an optimal action exists under mild continuity and compactness conditions. However, the relevant random variable need not have a fixed distribution. If the distribution depends on her choice of action, we say that the decision maker faces endogenous uncertainty. Such endogenous uncertainty is salient in many economic settings. For example, consider a firm that must choose how much to invest in R&D. If the level of investment affects the probability of a technological breakthrough, the distribution of the relevant random variable (R&D outcome) depends on the firm's action. Similarly, a worker's level of effort may affect the probability of her being promoted; an individual's choice of lifestyle may affect the distribution of her future health status.

In settings with endogenous uncertainty, existing results require strong assumptions such as the monotone likelihood ratio property (MLRP) and the convexity of distribution functions condition (CDFC). The existence of an optimal action therefore cannot be verified in a broad class of economic problems. In particular, when the support of the random variable depends on the decision maker's choice, existing results are often not applicable. This paper establishes a more general existence result that addresses these concerns. Our result shows that it is sufficient for the family of probability distributions to satisfy a mild continuity condition. We also show that this is a minimal requirement in the sense that an optimal action need not exist whenever the condition is violated.

To illustrate, consider a firm choosing among R&D projects. Choosing a more speculative project may increase the probabilities of both a technological breakthrough and complete failure, potentially violating CDFC and MLRP. If the value of a breakthrough also varies across projects, the support of the outcome distribution may depend on the firm's choice and the densities need not be pointwise continuous. Our result can nevertheless establish the existence of an optimal project.

1.1 Related Literature

This paper builds on the classic literature that studies the conditions under which an expected utility maximizing action exists. For settings without endogenous uncertainty, Jordan (1977) provided a very general result for continuous and bounded utility functions. Even when the utility function is unbounded, Kennan (1981) showed that mild continuity and compactness conditions are sufficient for the existence of an optimal action.

This paper is also related to the principal-agent literature, where a worker's effort affects the distribution of wages (Holmström, 1979). The first-order approach used in this literature typically requires additional assumptions such as MLRP and CDFC (Rogerson, 1985).

Endogenous uncertainty is salient in many other literatures as well. For example, Hansen and Sargent (2001), Paciello and Wiederholt (2014), and Fajgelbaum et al. (2017) all study environments in which the decision maker's choice of action affects the uncertainty she faces.

2 Notation and Assumptions

A is the set of actions available to the decision maker.

X is the set of possible realizations of a random variable. $\mathcal{F}$ is the Borel σ-algebra on X.

The probability measure on X depends on the decision maker's choice of action. The measure is given by m_a when the decision maker chooses $a \in A$. Note that we do not require the support of the random variable to be the same for every choice of action.

$u\colon A \times X \to \mathbb{R}$ is the decision maker's utility function. The expected utility of a is:

$$v(a) = \int_X u(a, x)\, dm_a(x)$$

Notice that changing a has two distinct effects on expected utility: a direct effect through the utility function and an indirect effect through the probability measure. This is the key difference between

our setting and a setting without endogenous uncertainty – the probability measure is fixed when there is no endogenous uncertainty.

Our existence result requires the following assumptions:

Assumption 1. A is a compact and first countable topological space.

Assumption 2. u is continuous on $A \times X$.

Assumption 3. $\overline{u}(x) \equiv \sup_{a \in A} |u(a,x)|$ is integrable with respect to all m_a.

Assumption 4. Let $g\colon X \to \mathbb{R}$ be any continuous function that is integrable with respect to all m_a. Then $a_n \to a$ implies:

$$\int_X g(x)\, dm_{a_n}(x) \;\to\; \int_X g(x)\, dm_a(x) \tag{1}$$

Assumption 1 and Assumption 2 are ubiquitous in the literature. Note that only the set of actions is assumed to be compact; X may be any arbitrary topological space. In particular, we allow the support of the random variable to be unbounded.

Assumption 3 is a generalization of Assumption A4' in Kennan (1981). It ensures that expected utility is always finite, without requiring the utility function to be bounded.[1] Intuitively, Assumption 3 allows utility to be arbitrarily high (or low) for some realizations of the random variable, as long as the probability of such realizations is sufficiently low.

Assumption 4 formalizes our main restriction on the family of probability measures. It requires that the expectation of any continuous and integrable function of the random variable is continuous with respect to the measures. We stress that Assumption 4 does not require the probability of individual elements of X to be continuous. Pointwise discontinuities are allowed as long as probability is shifting between elements that are "close" to each other.[2]

Notice that Assumption 4 is stronger than weak convergence of measures. That is because weak convergence requires (1) to hold for continuous and bounded functions only. Assumption 4 requires (1) to hold for all continuous and integrable functions.[3] The stronger condition is needed because

[1] The St. Petersburg paradox is an example of a decision problem ruled out by Assumption 3.

[2] Since X need not be a metric space, the word "close" is used informally to mean close in the topology of X.

[3] Assumption 4 is generally stronger than convergence with respect to the Wasserstein metric for the same reason.

we allow for unbounded utility functions.

If the utility function is bounded or X is compact, Assumption 3 can be dropped and Assumption 4 relaxed to weak convergence. However, restricting attention to these settings excludes many economic problems of interest. For example, the utility function is unbounded and X is not compact in the standard portfolio choice problem (Merton, 1969). We therefore retain the more general formulation.

3 Results

The decision maker wants to maximize her expected utility. Her optimization problem can be written as:

$$\max_a v(a) = \max_a \int_X u(a, x)\, dm_a(x)$$

Our main result says that:

Theorem 1. The decision maker's expected utility maximization problem has at least one solution under Assumptions 1 - 4.

Proof. A is compact (Assumption 1). So a maximizer exists if v is upper semi-continuous.

A is also first countable (Assumption 1). So v is upper semi-continuous if:

$$\limsup_n \int_X u(a_n, x)\, dm_{a_n}(x) \leq \int_X u(a, x)\, dm_a(x) \tag{2}$$

whenever $a_n \to a$. This proof will show that (2) holds under Assumptions 1 - 4. Note that the present approach is necessary because the utility function and the measure both depend on n. This dual dependence prevents us from using standard convergence theorems to move the limit inside the integral.

We start by defining:

$$u_m(x) = \sup_{k \geq m} u(a_k, x)$$

$$A_m = \{a\} \cup \{a_k \colon k \geq m\}$$

A_m is a compact set because any open cover of A_m will include an open set O such that $a \in O$. $a_k \to a$ so all but finitely many elements of A_m will lie in O. The remaining elements can then be covered by a finite subset of the open cover.

Then:

$$\begin{aligned} u_m(x) &= \sup_{a \in \{a_k \colon k \geq m\}} u(a, x) \\ &= \sup_{a \in A_m} u(a, x) \\ &= \max_{a \in A_m} u(a, x) \end{aligned}$$

The second equality comes from the fact that u is continuous (Assumption 2). So adding a limit point does not change the supremum. The third equality comes from the fact that a continuous function on a compact set attains its supremum.

$u_m(x)$ is thus the maximum of a continuous function over a compact set. So Berge's Maximum Theorem says that $u_m(x)$ is continuous on X.

Next note that $u(a_n, x) \leq u_m(x)$ if $m \leq n$. So $\limsup_n u(a_n, x) \leq u_m(x)$ for every m. Thus for every m:

$$\limsup_n \int_X u(a_n, x)\, dm_{a_n}(x) \leq \limsup_n \int_X u_m(x)\, dm_{a_n}(x)$$

In particular:

$$\limsup_n \int_X u(a_n, x)\, dm_{a_n}(x) \leq \inf_m \limsup_n \int_X u_m(x)\, dm_{a_n}(x)$$

Now recall that $\overline{u}(x) \equiv \sup_{a \in A} |u(a, x)|$. So for every m:

$$|u_m(x)| \leq \overline{u}(x)$$

Therefore u_m is integrable because $\overline{u}$ is integrable. Thus by Assumption 4:

$$\begin{aligned}\inf_m \limsup_n \int_X u_m(x)\, dm_{a_n}(x) &= \inf_m \int_X u_m(x)\, dm_a(x) \\ &= \lim_{m\to\infty} \int_X u_m(x)\, dm_a(x)\end{aligned}$$

The last equality comes from the fact that u_m is decreasing in m. We can now apply the Dominated Convergence Theorem to get:

$$\begin{aligned}\lim_{m\to\infty} \int_X u_m(x)\, dm_a(x) &= \int_X \lim_{m\to\infty} u_m(x)\, dm_a(x) \\ &= \int_X \lim_{m\to\infty} \sup_{k\geq m} u(a_k, x)\, dm_a(x) \\ &= \int_X \limsup_{k\to\infty} u(a_k, x)\, dm_a(x) \\ &= \int_X u(a, x)\, dm_a(x)\end{aligned}$$

We have thus shown that v is upper semi-continuous. Therefore the decision maker's maximization problem has at least one solution.

□

Remark 1. A symmetric argument can be used to show that v is also lower semi-continuous. The expected utility function is therefore continuous under Assumptions 1 - 4.

Corollary 1. Suppose Assumption 1 holds. Whenever Assumption 4 fails, there exists a utility function such that Assumption 2 and Assumption 3 are satisfied but expected utility is not upper semi-continuous.

Proof. Fix A such that Assumption 1 is satisfied.

Fix X and a family of probability measures $\{m_a \colon a \in A\}$ such that Assumption 4 is violated.

Then there exists a continuous function $g \colon X \to \mathbb{R}$ such that it is integrable with respect to all m_a

and for some $a_n \to a$:

$$\int_X g(x)\, dm_{a_n}(x) \not\to \int_X g(x)\, dm_a(x)$$

Then at least one of the following inequalities holds:

$$\limsup_n \int_X g(x)\, dm_{a_n}(x) > \int_X g(x)\, dm_a(x) \tag{3}$$

$$\liminf_n \int_X g(x)\, dm_{a_n}(x) < \int_X g(x)\, dm_a(x) \tag{4}$$

Notice that (4) implies:

$$\limsup_n \int_X -g(x)\, dm_{a_n}(x) > \int_X -g(x)\, dm_a(x)$$

Since $-g$ is also continuous and integrable, it is without loss of generality to assume (3).

Now define $u(a, x) = g(x)$.

Assumption 2 is trivial because g is continuous on X.

Assumption 3 is satisfied because $\sup_{a \in A} |u(a, x)| = |g(x)|$ and $g(x)$ is integrable.

Recall that:

$$v(a) = \int_X u(a, x)\, dm_a(x) = \int_X g(x)\, dm_a(x)$$

So (3) implies that for some $a_n \to a$:

$$\limsup_n v(a_n) > v(a)$$

We thus obtain a decision problem that satisfies Assumptions 1 - 3 but v is not upper semi-continuous because Assumption 4 is violated.

□

The proof shows that, whenever Assumption 4 is violated, there exists a utility function such that expected utility is not upper semi-continuous. It is clear that an optimal action need not exist without upper semi-continuity. Assumption 4 is therefore a minimal condition – it cannot be weakened without imposing additional restrictions on the decision problem.

Note that Assumption 4 is a condition on the family of probability measures. It does not require the measures to admit density functions with respect to a common reference measure. Even when such densities exist, Assumption 4 is not equivalent to a pointwise continuity condition on those densities.

Formally, suppose X is equipped with a σ-finite measure μ such that every m_a is absolutely continuous with respect to μ. Then every m_a admits a density function f_a with respect to μ. This family of densities is pointwise continuous if for every $x \in X$:

$$f_{a_n}(x) \to f_a(x)$$

whenever $a_n \to a$. We stress that pointwise continuity of the density functions is not necessary for Assumption 4. Section 4 provides an explicit example of a decision problem where the densities are not pointwise continuous but Assumption 4 is satisfied. Intuitively, pointwise continuity is most likely to fail when the support of the random variable depends on the choice of action.

4 Example

The decision maker is a physician.

$a \in [0, 1]$ is the dosage of an experimental cancer drug.

$x \in [-1, 1]$ is the patient's clinical outcome. $x = 1$ represents complete recovery while $x = -1$ represents death due to severe toxicity.

Let $U[I]$ denote the uniform probability measure on the closed interval I. Then the probability measure on X is:

$$m_a = U\left[-\frac{1+a}{2}, \frac{1+a}{2}\right]$$

Note that the support of x depends on the choice of a. Higher dosages allow for better clinical outcomes but also allow for worse outcomes.

The utility function is given by $u(a,x) = b(x) - c(a)$. $b(x)$ represents the value of the realized outcome and $c(a)$ captures the cost of the dosage. $c(a)$ can be interpreted as the disutility from the known side effects of the drug. Assume that b and c are both continuous functions.

Every probability measure in this example admits a density function with respect to the standard Lebesgue measure:

$$f_a(x) = \frac{1}{1+a}\,\mathbb{1}\left(|x| \leq \frac{1+a}{2}\right)$$

However, this family of densities is not pointwise continuous.[4]

To see that, fix any $x \in \left[-1, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right]$. Let $a_n \to a$ be any sequence such that $a_n < a$ for every n and $\frac{1+a}{2} = |x|$. Then:

$$|x| > \frac{1+a_n}{2} \implies f_{a_n}(x) = 0 \not\to \frac{1}{1+a} = f_a(x)$$

The densities are thus discontinuous at $\frac{1+a}{2} = |x|$.

It is also clear that this example violates CDFC. That is because $F_a(x) = \frac{1}{2} + \frac{x}{1+a}$ when $|x| \leq \frac{1+a}{2}$.

Differentiating twice with respect to a gives $F_a''(x) = \frac{2x}{(1+a)^3}$. Notice that $F_a''(x) < 0$ if $x < 0$.

The distribution function is thus concave, rather than convex, for all $x \in \left[-\frac{1}{2}, 0\right)$.

MLRP requires that:

$$f_{a'}(x_H)\, f_a(x_L) \geq f_{a'}(x_L)\, f_a(x_H)$$

when $a' > a$ and $x_H > x_L$. Now fix:

[4]The density functions are not unique, but there is no choice of densities that makes $a \mapsto f_a$ pointwise continuous.

$$-\frac{1+a'}{2} < x_L < -\frac{1+a}{2} < x_H < \frac{1+a}{2} < \frac{1+a'}{2}$$

So x_H is in the support of both a and a' but x_L is only in the support of a'. Then:

$$f_{a'}(x_H)\ f_a(x_L) = 0$$
$$f_{a'}(x_L)\ f_a(x_H) = \frac{1}{(1+a)(1+a')} > 0$$

MLRP is thus violated in this example.

Nevertheless, Theorem 1 says that an expected utility maximizing action exists.

It is obvious that Assumption 1 and Assumption 2 are satisfied.

The utility function is continuous and A and X are both compact sets. So $\overline{u}$ is also bounded and hence integrable. Assumption 3 is therefore satisfied.

To see that Assumption 4 is satisfied, note that for any continuous and integrable function g:

$$\int_X g(x)\ dm_a(x) = \frac{1}{1+a} \int_{-\frac{1+a}{2}}^{\frac{1+a}{2}} g(x)\ dx$$

The Fundamental Theorem of Calculus says that $\int_{-\frac{1+a}{2}}^{\frac{1+a}{2}} g(x)\ dx$ is a continuous function of a because g is continuous.

$\frac{1}{1+a}$ is also continuous so Assumption 4 is satisfied in this example.

Theorem 1 then ensures that an expected utility maximizing action exists.

5 Conclusion

This paper establishes sufficient conditions for the existence of an expected utility maximizing action under endogenous uncertainty. The main requirement is a continuity condition on the family of probability measures rather than pointwise continuity of densities. Our result can therefore be applied even when the support of the random variable varies with the action and common

assumptions such as MLRP and CDFC fail.

References

Fajgelbaum, Pablo D., Edouard Schaal, and Mathieu Taschereau-Dumouchel, "Uncertainty traps," *The Quarterly Journal of Economics*, 2017, *132* (4), 1641–1692.

Hansen, Lars Peter and Thomas J Sargent, "Robust control and model uncertainty," *American Economic Review*, 2001, *91* (2), 60–66.

Holmström, Bengt, "Moral hazard and observability," *The Bell Journal of Economics*, 1979, *10* (1), 74–91.

Jordan, James S., "The continuity of optimal dynamic decision rules," *Econometrica*, 1977, *45* (6), 1365–1376.

Kennan, John, "The existence of expected utility maximizing decisions when utility is unbounded," *Econometrica*, 1981, *49* (1), 215–218.

Merton, Robert C., "Lifetime portfolio selection under uncertainty: The continuous-time case," *The Review of Economics and Statistics*, 1969, *51* (3), 247–257.

Paciello, Luigi and Mirko Wiederholt, "Exogenous information, endogenous information, and optimal monetary policy," *The Review of Economic Studies*, 2014, *81* (1), 356–388.

Rogerson, William P, "The first-order approach to principal-agent problems," *Econometrica*, 1985, *53* (6), 1357–1367.