

Discussion Paper Series – CRC TR 224

Discussion Paper No. 785  
Project C 02

# The Power of Persistence

## How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes

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October 2026

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)  
through CRC TR 224 is gratefully acknowledged.

# THE POWER OF PERSISTENCE

## How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes

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**Abstract:** Does a more persistent demand shock always generate a larger output response? Conventional wisdom says yes: forward-looking agents front-load spending in anticipation of higher lifetime resources, and the more lasting the shock, the stronger the expansion. This paper shows that this logic fails in New Keynesian models with an active monetary policy. The interplay between persistence and monetary policy stance governs a fundamental trade-off between two opposing forces: a *permanent income* channel that amplifies output, and a *real interest rate* channel that dampens it. Their balance generates three monetary policy regimes. In the empirically relevant intermediate regime, the output multiplier is hump-shaped in persistence.

**Keywords:** Demand Shocks, Monetary Policy, New Keynesian Model, Persistence.

**JEL Code:** E32.

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*First Version:* January 2024. We would like to thank in no specific order Klaus Adam, Florin Bilbiie, Jordi Galí, Mishel Ghassibe, Christian Hellwig, Bjarne Horst, Matthias Meier, Oliver Pfäuti, and Joseph Stiglitz for helpful discussions. Collard and Fève acknowledge funding from the French National Research Agency (ANR) under the Investments for the Future (Investissements d'Avenir) program, grant ANR-17-EURE-0010. Collard and Fève also acknowledge funding from the French National Research Agency (ANR) under the project "ANR-23-CE26-0011-01". Wangner thanks the UniCredit & Universities foundation, INET, the Hewlett Foundation and the German Research Foundation through CRC-TR-224 (project C02) for financial support.

## 1. Introduction

The macroeconomic literature that aims at identifying the main sources of fluctuations consistently reports that aggregate demand shocks are key drivers of the business cycle. In the structural vector autoregressive (SVAR) literature, [Blanchard and Quah \(1989\)](#), among others, show that demand shocks account for almost all output volatility at very short horizons, [Galí \(1992\)](#) estimates that IS shocks explain about a third of short-run output growth, and [Christiano et al. \(2005\)](#) find that monetary policy shocks account for roughly 15-20% of GDP growth at one year (about 40% after two years). Quantitative Dynamic Stochastic General Equilibrium (DSGE) models deliver a very similar message—overall, demand shocks explain more than 50% of output fluctuations in [Smets and Wouters \(2007\)](#) and [Bilbiie et al. \(2023\)](#), while [Christiano et al. \(2010\)](#) put the short-run contribution of monetary policy shocks around 15-20%. Yet, despite this consensus, a critical dimension of their transmission remains largely unexplored: *how does the persistence of a demand shock shape its macroeconomic impact?*

This question is not merely theoretical. Empirically, the persistence of demand shocks varies considerably across episodes—from short-lived disruptions such as transitory confidence shocks or temporary monetary policy interventions, to longer-lasting forces such as sustained shifts in aggregate demand or persistent shifts in policy targets. Theoretically, the common wisdom is that greater persistence amplifies the impact response of aggregates. Intuitively, more persistent shocks raise the present value of lifetime resources and induce forward-looking agents to front-load private spending, prompting a greater response of macroeconomic variables.

**Main Contribution.** This paper challenges this conventional wisdom. Our main point of departure is that this reasoning implicitly holds the real interest rate fixed (see [Hall, 1978](#)) and is likely to fail in general equilibrium. In this paper, we analyze how, in the New Keynesian (NK) framework, a persistent demand shock that shifts the dynamic IS (DIS)<sup>1</sup> equation affects short run multipliers, in particular that of output. Indeed, in this class of models featuring an active monetary policy rule, higher persistence, by reshaping expected inflation dynamics, triggers a stronger response of the real interest rate that dampens fluctuations. We make two main contributions.

First, we analytically show that the relationship between shock persistence and the short-run output multiplier is driven by the interplay between two channels that move in opposite direction: a *permanent-income* (PI) channel and a *real-interest-rate* (RIR) channel. The presence of these two forces leads to the existence of three distinct regimes that depend on the stance of systematic monetary policy. In particular, we show that, in the intermediate regime, there exists a non monotonous relationship between persistence and the output multiplier.

Second, we assess, through various quantitative experiments, the empirical relevance of our analytical findings in a medium-scale DSGE model that matches key features of the transmission of demand shocks in the U.S. economy. We show that the intermediate hump-shaped regime is indeed the empirically relevant case. This finding is robust across a population of models and thus challenges the conventional wisdom.

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<sup>1</sup>This class of so-called DIS shocks captures, among others, standard monetary policy shocks, bond premium shocks, capital tax shocks, or shocks to liquidity preferences.

**Analytical Insights.** We consider a textbook three equation version of the Representative Agent New Keynesian (RANK) model (see, e.g. [Woodford, 2003](#), [Galí, 2015](#)) with a standard monetary policy rule that reacts to expected inflation ([Clarida et al., 2000](#)) and obtain the analytical response of aggregates to a DIS shock. We then decompose the output impact multiplier into the sum of the PI and the RIR channels. The PI channel encapsulates the expected discounted effect of demand shocks on agents income, is independent of monetary policy and is strictly increasing in the degree of persistence. The RIR channel captures the expected discounted sum of the reaction of the real interest rate, reflecting the interaction between persistence, systematic monetary policy, and nominal rigidities. As persistence rises, the PI channel strengthens because the shock resembles a shift in the path of private spending rather than a temporary fluctuation (see [Hall, 1978](#)). At the same time, greater persistence strengthens the RIR channel because inflationary pressure becomes longer-lived; once monetary policy systematically leans against expected inflation, agents anticipate a tighter real interest rate path, and that anticipated tightening operates as an *automatic stabilizer* whose force increases with persistence.

Systematic monetary policy shapes the interaction between these two forces and yields a clean three-regime classification. When inflation stabilization is accommodative, the RIR channel is muted and higher persistence unambiguously amplifies the impact output multiplier. When systematic monetary policy is sufficiently aggressive, the RIR channel dominates and higher persistence reduces the impact effect. Between these two cases lies a broad intermediate region in which the impact multiplier is hump-shaped in persistence: as persistence rises from low levels, the PI channel initially dominates and the multiplier increases; beyond a threshold, the anticipated real interest rate response becomes strong enough that additional persistence reduces the impact effect. The implication is counterintuitive and sharp: a relatively transitory demand shock can generate a larger contemporaneous output expansion than an almost permanent one—not because the shock is smaller, but because the expected policy response differs sharply once inflation is anticipated to remain elevated for longer.

This regime classification is robust to natural extensions of the baseline monetary policy rule. Rules that react to current rather than expected inflation, include the output gap, or feature interest-rate smoothing preserve the same PI/RIR trade-off; these modifications primarily shift the boundaries between regimes by dampening or amplifying the RIR channel, without altering the underlying logic. The non-monotonicity is also symptomatic of demand-type shocks specifically: for cost-push shocks, persistence does not operate through the same front-loaded expenditure motive, and output responses decline monotonically. However, the hump-shaped relationship between persistence and the multiplier is shifted to the inflation rate. Finally, because both channels operate through expected paths, the same logic extends to cumulative multipliers: greater persistence can raise cumulative output effects when policy is accommodative, but lower them when policy is aggressive, as the prolonged inflation impulse induces a prolonged real-rate offset that compresses activity over time.

**Quantitative Exploration.** We then show that the regime classification does not hinge on the RANK framework or on particular model ingredients, but instead arises as a general implication of the underlying New Keynesian structure—both in more general analytical setups as well as in more quantitative environments.

To this end, we develop a calibrated medium-scale DSGE model that captures key features of the transmission of demand shocks and extends our analytical results to a richer framework. Specifically, the model features capital accumulation with adjustment costs, heterogeneous households (savers and hand-to-mouth households) that face a precautionary saving motive from the interaction of idiosyncratic income risk and cyclical consumption and income inequality, a departure from full-information expectation formation, a working-capital channel, as well as price and wage nominal rigidities with respective indexation.<sup>2</sup>

A calibrated version of the model gives rise to the same three regimes, with the intermediate regime being empirically salient: for a wide range of central bank inflation-response coefficients consistent with standard estimates, the impact multiplier is hump-shaped in persistence, reaching its maximum for persistent but not permanent demand shocks. The PI/RIR trade-off is therefore not a theoretical curiosity; it is the relevant case under policy rules commonly used to describe postwar monetary policy.

Sensitivity exercises reveal how standard model ingredients shift the balance between PI and RIR channels and thereby move the economy across regimes. Departures from full-information expectations are pivotal: imposing rational expectations strengthens the inflation-expectations channel and the associated policy-induced real-rate offset, shifting the economy toward earlier RIR dominance and lowering the persistence at which the multiplier peaks. The configuration of nominal rigidities matters most quantitatively. When wages are flexible but prices are sticky, the economy enters the RIR-dominant region at much lower persistence because inflation and policy respond more sharply to the demand shock. When prices are flexible but wages are sticky, the monetary policy response can offset the demand shock even on impact, so that persistent demand expansions become comparatively weak and, in some parameterizations, contractionary. On the contrary, reducing the share of hand-to-mouth consumers strengthens forward-looking spending responses and shifts the economy toward the PI side: multipliers rise and the persistence that maximizes them increases. Importantly, cyclical income inequality leaves the regime characterization almost unaffected as induced variations in the PI and the RIR channel quantitatively offset each other in the presence of sticky prices and sticky wages.

We also extend the framework to study monetary–fiscal interactions. The cyclicity of public spending changes the effective monetary regime: more countercyclical spending strengthens inflation and triggers a tighter expected real interest rate response under a given rule, while procyclical spending does the opposite.

Finally, a Monte Carlo experiment draws widely parameters related to household preferences, technology, nominal rigidities, fiscal and monetary policy coefficients,

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<sup>2</sup>These features were introduced in the literature to account for several prominent empirical puzzles, including the lack-of-amplification puzzle, the forward-guidance puzzle, the missing-inflation puzzle, and the hump-shaped response of inflation and prices.

household heterogeneity, and expectation formation to evaluate the robustness of our findings across a broad population of models. Three results emerge. First, the three-regime structure is pervasive: a substantial share of draws produces hump-shaped multipliers rather than this arising only for a thin parameter set. Second, the intermediate regime aligns with empirically plausible monetary policy behavior: inflation-response estimates commonly reported in the Taylor-rule literature typically fall in the range where both PI and RIR forces matter. Third, the persistence that maximizes the multiplier is often higher than empirical estimates of demand-shock persistence, implying that many empirically estimated models operate on the upward-sloping part of the multiplier–persistence relationship. The same pattern appears for consumption, investment, hours worked, and asset prices, indicating a general transmission logic rather than a mechanism specific to a single margin.

Taken together, the analytical and quantitative results highlight the central role of the persistence of demand shocks for both transmission mechanisms and policy design: output multipliers cannot be inferred from persistence alone, nor summarized by more or less aggressive systematic monetary policy in isolation. What matters is the interaction between persistence, the systematic policy rule agents anticipate, and the structural features that determine whether PI incentives or RIR offsets dominate as shocks become longer-lived. From this perspective, persistence is a policy-relevant object.

**Related Literature.** This paper contributes to three strands of the literature.

First, our analytical analysis is rooted in the canonical RANK framework of [Woodford \(2003\)](#) and [Galí \(2015\)](#). However, the regime characterization and the PI/RIR logic apply naturally to a larger class of tractable New Keynesian models that feature household heterogeneity through cyclical income inequality ([Bilbiie, 2008](#), [Broer et al., 2020](#), [Bilbiie, 2020](#), [Debortoli and Galí, 2024](#)) or cyclical idiosyncratic income risk ([Werning, 2015](#), [Acharya and Dogra, 2020](#), [Ravn and Sterk, 2020](#), [Bilbiie, 2025](#)), a departure from the full information rational expectation assumption ([Angeletos and Lian, 2018](#), [Farhi and Werning, 2019](#), [Gabaix, 2020](#), [Pfäuti and Seyrich, 2023](#)), or other wedges relevant for intertemporal substitution ([Campbell et al., 2017](#), [Michaillat and Saez, 2021](#), [Del Negro et al., 2023](#), [Beaudry and Portier, 2018](#), [Beaudry et al., 2024](#)).<sup>3</sup> Our quantitative analysis operationalizes many of these ingredients in a medium-scale DSGE model in the tradition of [Christiano et al. \(2005\)](#), [Smets and Wouters \(2003, 2007\)](#), [Galí et al. \(2007\)](#) and [Bilbiie et al. \(2022\)](#). The paper thus provides a unified interpretation of why popular model extensions can produce different multipliers even when hit by the same type of demand shock.

Second, our paper contributes through the PI/RIR decomposition to an ongoing debate over whether NK transmission meaningfully operates through the real interest rate channel. Several contributions argue that the observed negative co-movement between output and real interest rate responses may be fragile and could instead be a result of specific parameterizations rather than structural properties ([Barsky et al., 2007](#), [Rupert](#)

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<sup>3</sup>To the extent that tractable models can approximate dynamics of quantitative incomplete market models, our results also apply in the HANK framework (see, among many others, [McKay et al. \(2016\)](#), [Kaplan et al. \(2018\)](#), [Auclert \(2019\)](#), [Bayer et al. \(2019\)](#)).

and Šustek, 2019, Brault and Khan, 2022). Our results reconcile these views by showing that the RIR channel is endogenous: its strength depends jointly on shock persistence and on how aggressively monetary policy stabilizes inflation. Disagreements about the RIR channel can therefore reflect differences in beliefs about the persistence of demand forces and in the prevailing policy regime.

Finally, our analysis relates to work that documents time-variation in systematic monetary policy and policy regime changes. Empirically, Istrefi (2018) constructs a measure of systematic monetary policy preferences, classifying 130 FOMC members serving during 1960-2015 into hawks, doves, and switchers based on newspaper and financial media coverage. Her analysis shows that the composition of hawks and doves varies significantly at the FOMC level, featuring slow- and fast-switching hawkish and dovish regimes.<sup>4</sup> Structurally, Bianchi (2013) documents recurrent shifts between dovish and hawkish monetary-policy behavior and shows that beliefs about future regime changes are central for interpreting postwar U.S. dynamics. Bianchi and Ilut (2017) extends this perspective to the joint monetary–fiscal policy mix. We complement this literature by clarifying a distinct transmission mechanism: regime shifts need not merely rescale multipliers; they can change how multipliers vary with shock persistence by altering the balance between PI incentives and policy-induced movements in expected real rates.<sup>5</sup>

**Paper Outline.** The paper proceeds as follows. Section 2 develops the analytical results in the RANK model and establishes the PI/RIR decomposition. Section 3 discusses extensions to alternative monetary policy rules, cumulative multipliers, alternative propagation decompositions, and other shocks. Section 4 presents the quantitative DSGE analysis, decomposes the role of key mechanisms, studies the interaction between monetary and fiscal policy, and assesses the empirical plausibility through a Monte Carlo experiment. The final section concludes. Proofs and additional results are provided in the online appendix.

## 2. Demand Shock Persistence in the RANK Model

It is commonly accepted that, in rational expectations forward-looking models in which agents value the future positively, more persistent shocks have larger effects on macroeconomic outcomes, particularly output. This occurs because agents front-load their reactions based on the expected discounted future effects of the shock, with greater persistence increasing its present value and triggering a stronger response. This section shows that this logic fails to hold in New Keynesian models. Starting with the textbook discrete time sticky price RANK economy (see e.g. Woodford, 2003, Galí, 2015), we highlight two opposing propagation channels that shape the relationship between shock persistence and output—a permanent income channel and a real interest rate channel. We

<sup>4</sup>Cieslak et al. (2023), Hack et al. (2024), or Granziera et al. (2025) document similar time variations.

<sup>5</sup>This perspective also connects to Bianchi et al. (2022) and Bianchi and Melosi (2013), who show that beliefs about regime duration shape real rates and business-cycle outcomes. We do not model learning explicitly, but our results clarify why such belief revisions matter for business-cycle outcomes: when agents place more weight on the persistent component of demand (or on a longer-lived policy stance), systematic policy determines whether persistence is amplified into real activity or offset by anticipated tightening.

then identify three monetary policy regimes that navigate this trade-off and derive, in closed form, a persistence threshold that maximizes the short-run output stimulus as a weighted combination of both channels.

## 2.1 The Economic Environment

We consider a closed economy whose equilibrium dynamics are summarized by the following (log-)linear equations:

$$y_t = \mathbb{E}_t[y_{t+1}] - (i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}) + \xi_t, \quad (1)$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa_y y_t, \quad (2)$$

$$i_t = \bar{r} + \phi_\pi \mathbb{E}_t[\pi_{t+1}], \quad (3)$$

$$\xi_t = \rho \xi_{t-1} + \varepsilon_{\xi,t}, \quad (4)$$

where  $y_t$ ,  $\pi_t$  and  $i_t$  denote respectively output, the inflation rate and the nominal interest rate.  $\xi_t$  is a demand shock.  $\mathbb{E}_t[\cdot]$  is the full information rational expectation operator conditional on the information set available at period  $t$ .

Equation (1) specifies the DIS equation and captures intertemporal consumption-saving decisions, that fundamentally depend on the real interest rate gap  $r_t - \bar{r} \equiv i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}$ , where  $\bar{r} > 0$  is the natural rate of interest.<sup>6</sup> Equation (2) defines the NKPC, which describes the price-setting behavior of intermediary firms. The sensitivity of current inflation to expected future inflation is governed by the discount factor  $\beta \in (0, 1)$ , while a marginal cost channel links current inflation to output via  $\kappa_y > 0$ . The exact micro-foundations of the NKPC are immaterial for our main results, although, in some instances, we will find it convenient to refer explicitly to specific micro-foundations (e.g. Rotemberg, 1982, Calvo, 1983) to put perspective on some results. Equation (3) describes the behavior of a Central Bank, which follows a simple interest rate rule à la Taylor (1993) and adjusts the nominal interest rate to stabilize expected inflation.<sup>7</sup> Finally, Equation (4) specifies the stochastic process for the *exogenous DIS-demand shock*  $\xi_t$ , modeled as an autoregressive process of order one, AR(1). We consider, in line with the empirical evidence, a positive persistence parameter  $\rho \in [0, 1)$ .<sup>8</sup>

Bullard and Mitra (2002) show that the equilibrium of model (1)–(4) is stable if and only if  $1 < \phi_\pi < \phi_\pi^{\max} \equiv 1 + \frac{2(1+\beta)}{\kappa_y}$  — i.e. the *Taylor principle* holds, but the Central Bank

<sup>6</sup>Implicit in this specification is that household's preferences are represented by a time-separable logarithmic utility function and output is used for consumption purposes only. Without technology shocks, the natural rate,  $r_t^n$ , equals the discount rate,  $\bar{r} \equiv -\ln \beta > 0$ , where  $\beta \in (0, 1)$  is the household's discount factor.

<sup>7</sup>We follow the suggestion of an anonymous referee and specify the Taylor rule in terms of expected inflation (see, e.g., Clarida et al. (2000)) mainly for tractability purposes as it greatly simplifies the algebra and allows to recover some important results as limit cases. We will later consider a Taylor rule featuring both contemporaneous inflation and output gaps.

<sup>8</sup>A rationale for this shock can be found in Fisher (2015). An alternative is to follow Smets and Wouters (2007) who interpret  $e^{-\xi_t} R_t$  as the effective nominal return on bonds, resulting in a nominal bond premium of  $(e^{-\xi_t} - 1)R_t$  or a real bond premium of  $(e^{-\xi_t} - 1)R_t/\pi_{t+1}$ , where  $\ln R_t = i_t$ . In that case,  $\xi_t$  is referred to as a bond premium shock.

is not too aggressive. Under the previous condition the solution dynamics take the form

$$y_t = \mathcal{M}_y(\rho)\xi_t, \quad \pi_t = \mathcal{M}_\pi(\rho)\xi_t, \quad i_t = \bar{r} + \mathcal{M}_i(\rho)\xi_t, \quad \text{and} \quad r_t = \bar{r} + \mathcal{M}_r(\rho)\xi_t,$$

where the multipliers  $\mathcal{M}_x(\rho)$ ,  $x \in \{y, \pi, i, r\}$  are given in the following proposition.

**PROPOSITION 1.** *If  $1 < \phi_\pi < \phi_\pi^{\max}$ , the locally determinate solution of the model yields the following impact output multiplier:*

$$\mathcal{M}_y(\rho) = \frac{1}{1 - \rho + \rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}} > 0.$$

*In addition, impact multipliers for inflation, the nominal and real interest rate are:*

$$\mathcal{M}_\pi(\rho) = \frac{\kappa_y}{1 - \beta\rho}\mathcal{M}_y(\rho), \quad \mathcal{M}_i(\rho) = \phi_\pi\rho\mathcal{M}_\pi(\rho), \quad \text{and} \quad \mathcal{M}_r(\rho) = (\phi_\pi - 1)\rho\mathcal{M}_\pi(\rho).$$

The impact multipliers are such that the proposed decision rules solve system (1)-(3) using (4). In the sequel, particular attention will be devoted to the behavior of the output multiplier,  $\mathcal{M}_y(\rho)$ —the impact effect of a DIS-demand shock on output—as the degree of persistence of the shock,  $\rho$ , varies.

A key insight from our analysis is that the short-run output response reflects the combination of two forces that operate in opposite directions: a *permanent income (PI) channel* and a *real interest rate (RIR) channel*. To make this decomposition explicit, evaluate the dynamic IS equation (1) at the general equilibrium solution:

$$\mathcal{M}_y(\rho) = \rho\mathcal{M}_y(\rho) - \mathcal{M}_r(\rho) + 1.$$

Rearranging term yields

$$\mathcal{M}_y(\rho) = \frac{1}{1 - \rho} - \frac{\mathcal{M}_r(\rho)}{1 - \rho} = \underbrace{\overline{\mathcal{M}}_y(\rho)}_{\text{PI Channel}} - \underbrace{\overline{\mathcal{M}}_r(\rho)}_{\text{RIR Channel}}. \quad (5)$$

The PI channel,  $\overline{\mathcal{M}}_y(\rho)$ , captures the expected discounted effect of the demand shock on agents income. It is entirely independent of monetary policy and is strictly increasing in the degree of persistence.

The PI channel can simply be isolated by considering a monetary policy that offsets expected inflation one-for-one, *i.e.*  $\phi_\pi \rightarrow 1$ . This policy nests the fixed real interest case, *de facto* eliminating the RIR channel. In that case, amplification only occurs through permanent income motives yielding the multiplier

$$\lim_{\phi_\pi \rightarrow 1} \mathcal{M}_y(\rho) = \frac{1}{1 - \rho},$$

which increases monotonically with persistence.

With active monetary policy,  $\phi_\pi > 1$ , the RIR channel becomes operative because the real rate reacts to expected inflation,  $r_t - \bar{r} = (\phi_\pi - 1)\mathbb{E}_t[\pi_{t+1}]$ . The RIR channel,

$\overline{\mathcal{M}}_r(\rho)$ , captures the expected discounted sum of the reaction of the real interest rate. This term reflects the interaction between persistence, systematic monetary policy, and nominal rigidities. Whenever  $\mathcal{M}_r(\rho)$  increases with  $\rho$ , the RIR channel works against the PI channel. Proposition 2 shows that in the baseline RANK economy, this monotonicity property holds in so far as the Taylor principle is satisfied.

**PROPOSITION 2.** *Under the Taylor principle, the real interest rate increases monotonically in the persistence of a DIS-demand shock, i.e.,  $\mathcal{M}'_r(\rho) > 0$ .*

A key implication of the NKPC is that this policy feedback strengthens disproportionately with persistence as the long-run effect of a given increase in current and expected future marginal costs on inflation is scaled by the factor  $(1 - \beta\rho)^{-1}$ , which is increasing and convex in  $\rho$ . Intuitively, higher persistence raises not only current inflation but also expected future inflation, thereby amplifying the response of the real rate. The RIR channel therefore depends fundamentally on expectations.

It is useful to make this expectation layer explicit with two counterfactual exercises. First, consider a setting in which firms hold their inflation expectations fixed—as if the Phillips curve were static (i.e.  $\pi_t = \kappa_y y_t$ ), implying that expected inflation only matters through its effect on monetary policy.<sup>9</sup> The output multiplier,  $\mathcal{M}_y^{\text{no } \mathbb{E}[\pi]}$ , then becomes

$$\mathcal{M}_y^{\text{no } \mathbb{E}[\pi]}(\rho) = \frac{1}{1 - \rho + \rho(\phi_\pi - 1)\kappa_y}.$$

In this case, the sign of the change in the multiplier as  $\rho$  increases is independent from  $\rho$ , but relates to the degree of reaction of the Central Bank towards inflation. The multiplier is monotonically increasing in persistence when monetary policy is not too aggressive (if  $\phi_\pi < 1 + \kappa_y^{-1}$ ) and monotonically decreasing otherwise. Turning the expected inflation channel back on affects this result as future inflation expectations in the NKPC affects current inflation, making the multiplier non monotonic in  $\rho$ . Otherwise stated, the forward-looking component of inflation complicates the relationship between persistence and the response of output.

Consider now a setting in which expected demand is held fixed in the DIS equation, implying that shocks do not affect current demand through expected future income but only through their effect on the real interest rate (i.e.  $y_t = -(r_t - \bar{r}) + \xi_t$ ).<sup>10</sup> This yields the counterfactual multiplier,  $\mathcal{M}_y^{\text{no } \mathbb{E}[y]}(\rho)$ :

$$\mathcal{M}_y^{\text{no } \mathbb{E}[y]}(\rho) = \frac{1}{1 + \rho(\phi_\pi - 1) \frac{\kappa_y}{1 - \beta\rho}},$$

In this case, the multiplier is strictly decreasing in  $\rho$  as soon as monetary policy is active, as it is solely driven by the response of the real interest rate. Moreover, the decrease is

<sup>9</sup>A static NKPC can be derived under Rotemberg pricing with adjustment costs relative to the previous period average price index; see, for instance, Bilbiie (2025).

<sup>10</sup>A static DIS equation can be derived under cognitive discounting with a complete anchoring of expectations to the perceived steady state reference level; see, for instance, Gabai (2020).

faster at higher persistence precisely because the real-rate channel intensifies as persistence raises the expected path of inflation and through the policy rule (*i.e.*  $(1 - \beta\rho)^{-1}$  rises with  $\rho$ ). Just like in the previous case, bringing back the forward looking component complicates the relationship as the DIS equation then encompasses all the future effects of a change in current monetary policy.

These observations will lead to a compact analytical characterization of how persistence affects the equilibrium multiplier in the next section. Notably, our decomposition is not specific to the RANK economy but reflects a generic feature of the Euler and NKPC equations governing the dynamics in NK models. Whenever household spending responds to a DIS shock, the impact on output necessarily combines a PI channel and a RIR channel. All analytical results that follow—both in the baseline RANK model and in the more general frameworks studied below—ultimately arise from the interaction between these two forces. It will turn out that systematic monetary policy determines their relative strength and, therefore, whether persistence amplifies or dampens demand shocks.

## 2.2 Main Results

The following proposition shows that, even in the baseline RANK model, more persistent demand shocks do not necessarily trigger a larger response of output. This depends on the aggressiveness of monetary authorities towards inflation.<sup>11</sup>

**PROPOSITION 3.** *There exist two cut-off values,  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  of the Central Bank's degree of reaction to the inflation gap*

$$\underline{\phi}_\pi \equiv 1 + (1 - \beta)^2 \kappa_y^{-1} \quad \text{and} \quad \bar{\phi}_\pi \equiv 1 + \kappa_y^{-1}, \quad \text{with} \quad \phi_\pi^{\max} > \bar{\phi}_\pi > \underline{\phi}_\pi > 1,$$

*such that the impact multiplier of output to a DIS-demand shock satisfies:*

- (a) *If  $\phi_\pi \leq \underline{\phi}_\pi$ ,  $\mathcal{M}_y$  increases monotonically in  $\rho$ .*
- (b) *If  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ ,  $\mathcal{M}_y$  is hump-shaped in  $\rho$ , *i.e.*,*

$$\exists \rho^* \equiv \beta^{-1} \left( 1 - \sqrt{\kappa_y (\phi_\pi - 1)} \right) \quad \text{such that} \quad \mathcal{M}'_y(\rho) \gtrless 0 \text{ if } \rho \lesseqgtr \rho^*.$$

- (c) *If  $\phi_\pi \geq \bar{\phi}_\pi$ ,  $\mathcal{M}_y$  decreases monotonically in  $\rho$ .*

The proposition shows that when policy is highly accommodative ( $1 < \phi_\pi \leq \underline{\phi}_\pi$ ), the nominal interest rate reacts only weakly to inflation expectations. As a result, the

<sup>11</sup>It should be clear to the reader that changing the level of the persistence parameter also influences the volatility of the shock. However, in this paper, we focus on linear models or linearized versions of models. In such cases, the model's solution and the transmission mechanisms—the focus of this paper—are not affected by second-order moments related to shock volatility. Accordingly, neither the form nor the level of multipliers is affected by the volatility of the shocks. Therefore, because we are mainly interested in the mechanisms, to preserve clarity and keep the analysis simple, we hold the volatility of the shock innovation constant when varying persistence, even if this increases the overall volatility of the shock.

real interest rate barely reacts to a demand shock, effectively neutralizing the RIR channel. In this environment, the output response is dominated by the PI channel: as the persistence of the demand shock,  $\rho$ , rises, households anticipate a larger and longer-lasting increase in lifetime income. The forward looking Euler equation then implies a stronger rise in current consumption and aggregate output. Formally, the output multiplier  $\mathcal{M}_y(\rho)$  increases monotonically with  $\rho$ , as in Part (a) of the Proposition.

By contrast, when monetary policy is very aggressive ( $\phi_\pi^{\max} > \phi_\pi \geq \bar{\phi}_\pi$ ), the nominal rate reacts strongly to expected inflation. As the persistence of the demand shock rises, the higher expected path of nominal rates translates into a tighter real rate path, depressing current demand. The output multiplier therefore decreases monotonically with  $\rho$ , as in Part (c).

The intermediate regime,  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , arises when neither channel clearly dominates. At low levels of persistence, the PI channel prevails, and increasing  $\rho$  raises  $\mathcal{M}_y(\rho)$ . Beyond a critical level of persistence, however, the RIR channel becomes sufficiently strong to offset and eventually dominate the PI effect, producing a hump-shaped pattern in  $\mathcal{M}_y(\rho)$ . The turning point,  $\rho^*$ , thus represents the degree of persistence at which the two channels exactly offset.

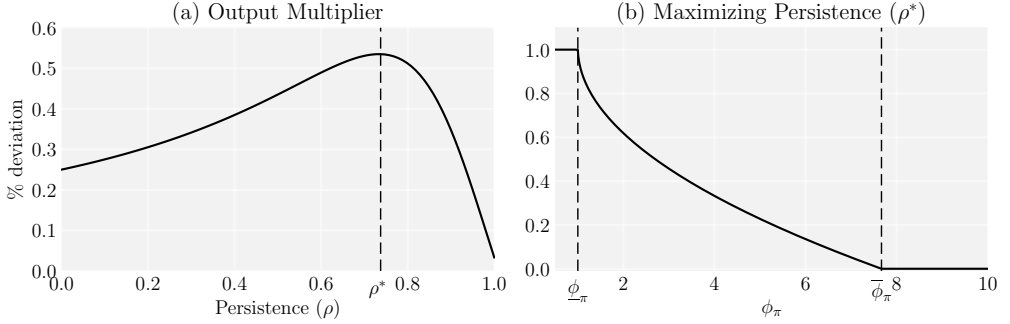
Panel (a) of Figure 1 illustrates the relationship between the output impact multiplier and the persistence of the DIS-demand shock. The pattern exhibits the non-monotonic behavior predicted by the theory: for low values of  $\rho$ , greater persistence amplifies the response of output to the demand shock via the PI channel. However, beyond the threshold  $\rho^* = 0.733$ , the RIR channel becomes dominant, and  $\mathcal{M}_y(\rho)$  begins to decline. This turning point marks the point at which the real interest rate adjustment—monetary policy’s counteracting force—overtakes the forward-looking consumption response. Notably, a transitory disturbance ( $\rho = 0$ ) leads to a stronger immediate output expansion,  $\mathcal{M}_y(0) = 0.25$ , than a nearly permanent shock ( $\rho \rightarrow 1$ ), for which  $\lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) = 0.03$ .<sup>12</sup>

Panel (b) explores how the location of  $\rho^*$  depends on the stance of monetary policy. When monetary policy is insufficiently aggressive ( $1 < \phi_\pi \leq \underline{\phi}_\pi$ ), the output multiplier peaks at  $\rho \rightarrow 1$ , indicating the strict dominance of the PI channel. Conversely, under highly aggressive monetary policy ( $\phi_\pi^{\max} > \phi_\pi \geq \bar{\phi}_\pi$ ), the multiplier is maximized at  $\rho = 0$ , unless one assumes anti-persistent shocks—an empirically irrelevant case. In this regime, the RIR channel takes over the PI effect. Between these policy thresholds, a trade-off emerges, yielding an interior solution where  $\rho^*$  lies strictly between 0 and 1. A higher degree of aggressiveness towards inflation,  $\phi_\pi$ , steepens the response of nominal and real rates to inflation expectations, thereby amplifying the RIR channel and reducing  $\rho^*$ .

The structure of the NKPC, summarized by the parameters  $\beta$  and  $\kappa_y$ , plays a critical role in determining how strongly the RIR channel responds to monetary policy. A steeper NKPC (larger  $\kappa_y$ ) strengthens the sensitivity of inflation to output, magnifying the real

<sup>12</sup>The asymmetry of the output response is a generic feature of the RANK model. Specifically, we show that  $\mathcal{M}_y(0) > \lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho)$  applies whenever  $\phi_\pi > 1 + (1 - \beta)/\kappa_y$ , see Corollary 2 in Section A of the Online Appendix.

FIGURE 1. Impact Output Multiplier and Persistence



Note: The impact multiplier is expressed in percentage deviation from steady state. This figure is obtained assuming  $\beta = 0.99$ ,  $\kappa_y = 0.15$ ,  $\phi_\pi = 1.5$ . Given these values, we have  $\underline{\phi}_\pi = 1.0007$  and  $\bar{\phi}_\pi = 7.666$ . We consider a shock of 25 basis points.

interest rate response and dampening  $\mathcal{M}_y(\rho)$  for any given  $\phi_\pi$ . In contrast, a higher discount factor  $\beta$  increases the degree of forward-looking behavior: while this strengthens the RIR channel by making future inflation more salient in the Euler equation, it simultaneously flattens the NKPC under a [Calvo \(1983\)](#) pricing structure, partially mitigating that same effect.<sup>13</sup> Hence, the slope of the NKPC determines whether higher  $\beta$  ultimately amplifies or attenuates the influence of persistence on  $\mathcal{M}_y(\rho)$ .

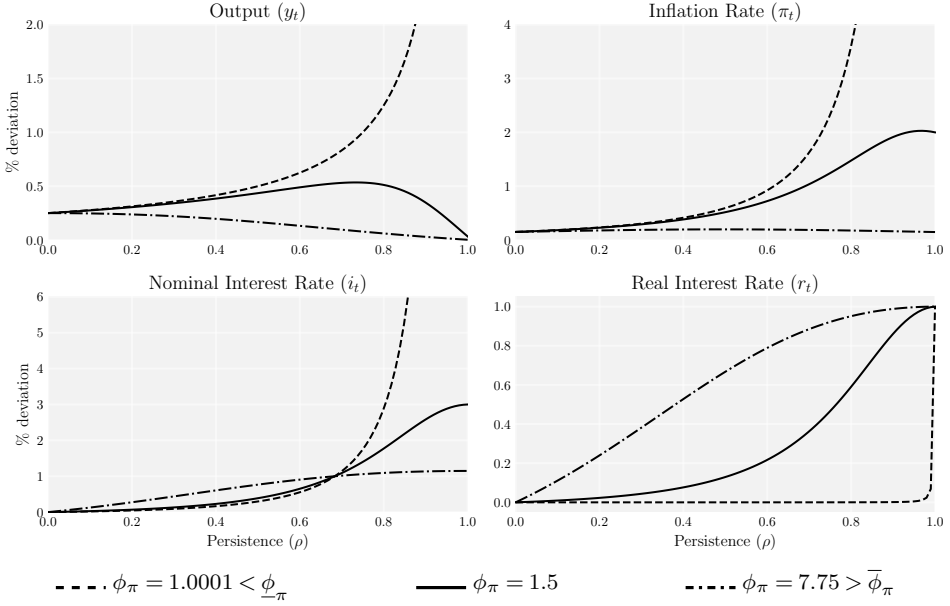
To clarify how the monetary policy stance governs this trade-off, Figure 2 plots the impact effects on output, inflation, and real and nominal interest rates for three policy regimes: a highly accommodative rule ( $\phi_\pi = 1.0001 < \underline{\phi}_\pi$ ), a benchmark Taylor rule ( $\phi_\pi = 1.5$ ), and an aggressive anti-inflation stance ( $\phi_\pi = 7.75 > \bar{\phi}_\pi$ ). Consistent with Proposition 3, the output multiplier increases monotonically with persistence when systematic monetary policy is weakly responsive to inflation and declines monotonically when it is very aggressive. In the intermediate regime, however, the hump-shaped profile reappears, reflecting the shift in dominance from the PI channel at low  $\rho$  to the RIR channel at high  $\rho$ .

Both nominal and real interest rates exhibit monotonically increasing responses as persistence rises, as shown in the lower panels. Indeed, while policy rates mechanically scale with the expected duration of the shock, output and inflation are shaped by the interaction between forward-looking consumption smoothing and monetary stabilization. In particular, Proposition 4 shows that the logic governing the output multiplier extends directly to inflation once the thresholds and turning point are adjusted appropriately. In other words, the inflation multiplier inherits the same regime structure and monotonicity/hump shape properties with respect to persistence as the output multiplier, but with cutoffs shifted by the slope and forward-looking nature of the NKPC.<sup>14</sup>

<sup>13</sup>In the case of time-dependent pricing,  $\kappa_y = \Theta \frac{(1-\theta)(1-\beta\theta)}{\theta}$ , where  $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)$ . In addition,  $\varphi > 0$  is the inverse Frisch labor supply elasticity,  $\alpha \in [0, 1]$  is the degree of decreasing returns to scale,  $\epsilon > 1$  is the demand elasticity and  $\theta \in (0, 1)$  is the probability of not resetting the price in a given period.

<sup>14</sup>However, it should be noted that this finding depends fundamentally on the structure of the monetary policy rule. Non-monotone behavior of inflation arises under forward-looking feedback rules, whereas

FIGURE 2. Impact Multipliers and Persistence



Note: The impact multiplier is expressed in percentage deviation from steady state. This figure is obtained assuming  $\beta = 0.99$  and  $\kappa_y = 0.15$ . Given these values, we have  $\bar{\phi}_\pi = 1.0007$  and  $\bar{\phi}_\pi = 7.666$ . We consider a shock of 25 basis points.

**PROPOSITION 4.** *The results of Proposition 3 extend to the impact multiplier of inflation once the thresholds and turning point are adjusted as*

$$\underline{\phi}_\pi^\pi = 1 + \frac{1 - \beta}{\kappa_y}, \quad \bar{\phi}_\pi^\pi = 1 + \frac{1 + \beta}{\kappa_y}, \quad \text{and} \quad \rho^* = \frac{1 + \beta - \kappa_y(\phi_\pi - 1)}{2\beta}.$$

The reason can be directly seen from the NKPC evaluated at the GE solution:

$$\mathcal{M}_\pi(\rho) = \frac{\kappa_y}{1 - \beta\rho} \mathcal{M}_y(\rho).$$

This expression makes transparent how inflation inherits both the marginal-cost response and the forward-looking structure of the NKPC. The numerator,  $\kappa_y \mathcal{M}_y(\rho)$ , captures the contemporaneous effect of the demand shock on real marginal costs. The denominator,  $1 - \beta\rho$ , reflects how persistence strengthens the link between expected future inflation and current inflation: when  $\rho$  rises, the shock induces more persistent future marginal-cost movements, raising expected inflation and therefore amplifying today's price adjustment through the NKPC.

Using the previous PI/RIR decomposition, the inflation multiplier rewrites

$$\mathcal{M}_\pi(\rho) = \frac{\kappa_y \bar{\mathcal{M}}_y(\rho)}{1 - \beta\rho} - \frac{\kappa_y \bar{\mathcal{M}}_r(\rho)}{1 - \beta\rho}.$$

---

monotone behavior obtains under contemporaneous feedback rules. See the proof of Proposition 4 in On-line Appendix A for further details.

The first term —originating from the PI channel of aggregate demand— describes how inflation responds when firms adjust prices purely in response to the stronger aggregate demand they face and the associated higher marginal costs, absent any movement in real interest rates. This component is, as the PI channel, strictly increasing in the degree of persistence of the shock, for  $\rho \in [0, 1)$ . The higher is the persistence, the larger and the more persistent is the increase in aggregate demand. This leads firms to raise their level of employment, calling for an increase in the equilibrium real wage and the marginal cost, and eventually yielding higher prices.

The second term relates to the monetary policy stance. Through the RIR channel, higher persistence raises the real interest rate, provided the Taylor principle holds (see Proposition 2), which dampens marginal costs and therefore inflation. This term therefore operates in the opposite direction to the first channel.

Proposition 4 follows from comparing the relative potency of these two forces. When monetary policy is very accommodative ( $1 < \phi_\pi \leq \underline{\phi}_\pi^\pi$ ), the weak response of interest rates induces inflation to rise more strongly with persistence. When policy is very aggressive instead ( $\phi_\pi^{\max} > \phi_\pi \geq \overline{\phi}_\pi^\pi$ ), the rise in the real interest rate causes the inflation multiplier to fall with persistence. In the intermediate region, neither force dominates for all  $\rho$ : at low persistence the PI channel dominates, while at high persistence the RIR channel prevails, generating a hump-shaped profile analogous to that observed for the output multiplier.

### 3. Discussion and Extensions

This section provides a broader perspective on our main findings. First, we qualify the extent to which the results of Proposition 3 depend on the type of monetary policy rule adopted by the Central Bank. We then discuss how the properties of the impact multiplier extend to cumulative multipliers. In addition, we emphasize their connection to the demand-side nature of the shocks. Specifically, we examine the dynamics that arise in response to monetary policy shocks and cost push shocks. Last, we show how our PI/RIR decomposition relates to alternative decompositions proposed in the literature.

#### 3.1 The Role of Monetary Policy

The previous section highlighted the interaction between a simple monetary policy rule involving solely the objective of expected inflation stabilization and the persistence of DIS-demand shocks in shaping their impact effect on output dynamics. This section considers rules that extend the benchmark monetary policy rule along three dimensions. First, we jointly allow for expected inflation and output feedback. Second, we study standard contemporaneous feedback rules. Finally, we investigate the consequences of monetary rules with interest rate smoothing for our main results.

PROPOSITION 5. *Under local determinacy, the following results apply:*

(a) Assume the Central Bank reacts to the one-step ahead expected inflation and output gap, i.e.  $i_t = \bar{r} + \phi_\pi \mathbb{E}_t [\pi_{t+1}] + \phi_y \mathbb{E}_t [y_{t+1}]$ , with  $\phi_\pi > 1$ .<sup>15</sup>

(i) If  $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , the results of Proposition 3 remain valid once the thresholds and turning point are adjusted for  $\phi_y$ :

$$\underline{\phi}_\pi \equiv 1 + \frac{(1 - \phi_y)(1 - \beta)^2}{\kappa_y}, \bar{\phi}_\pi \equiv 1 + \frac{1 - \phi_y}{\kappa_y} \text{ and } \rho^* = \beta^{-1} \left( 1 - \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y} \right).$$

(ii) If  $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , only the upper regime survives and the output multiplier decreases monotonically with  $\rho$ .

(b) Assume the Central Bank reacts to both the current inflation rate and the current output gap, i.e.  $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t$ . Then, the results of Proposition 3 remain valid once the thresholds and turning point are adjusted as:

$$\underline{\phi}_\pi \equiv \beta^{-1} \left( 1 + \frac{(1 - \beta)^2}{\kappa_y} \right), \bar{\phi}_\pi \equiv \beta^{-1} \left( 1 + \frac{1}{\kappa_y} \right) \text{ and } \rho^* \equiv \beta^{-1} \left( 1 - \sqrt{(\phi_\pi \beta - 1) \kappa_y} \right).$$

Part (a) reveals that when monetary policy reacts to one-step-ahead expectations of inflation and the output gap, the latter plays a significant role in determining how persistence shapes the short-run output response to a DIS-demand shock. Three monetary policy regimes still obtain as long as the Central Bank's output gap response remains moderate, i.e.,  $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ . Within the hump-shaped regime, the  $\rho^*$  decreases with  $\phi_y$  as stronger output stabilization weakens the PI channel.<sup>16</sup> When  $\phi_y$  becomes sufficiently large, i.e.  $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , the hump-shaped regime disappears and output declines monotonically with  $\rho$ . In this case, the RIR channel dominates strictly the PI channel.

Part (b) of Proposition 5 shows that our main results are still obtained when monetary authorities stabilize current inflation and current output. Note that, not too surprisingly, the monetary policy thresholds  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  only differ from those obtained in a rule involving expected output and inflation by the scaling factor  $\beta^{-1}$  pertaining to the discount factor. In addition, the presence of output stabilization concerns does not alter the monetary policy thresholds  $(\underline{\phi}_\pi, \bar{\phi}_\pi)$  or  $\rho^*$ , as their expression and properties are totally independent from  $\phi_y$ . This obtains from the fact that  $\phi_y$  affects symmetrically the PI and the RIR channels in the DIS equation. However, a more aggressive output stabilization policy—a larger  $\phi_y$ —reduces nevertheless the size of the impact multipliers of output and inflation.

Up to this point, and in order to keep the analytical characterization fully tractable, we abstracted from interest rate smoothing. We now consider a Central Bank that aims

<sup>15</sup>We provide a full statement of the proposition including the case  $\phi_\pi > 0$  in Section A of the Online Appendix.

<sup>16</sup>Technically, the expected output feedback effectively introduces discounting in the DIS equation (see Section 4.2).

at targeting expected inflation while smoothing the nominal interest rate, implying a policy rule of the form

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)(\bar{r} + \phi_\pi \mathbb{E}_t[\pi_{t+1}]),$$

where  $\phi_i \in [0, 1)$  denotes the degree of interest rate smoothing. The following proposition generalizes our previous findings.

**PROPOSITION 6.** *Provided that the equilibrium is locally determinate, let  $\mu_i \equiv \mu(\phi_\pi, \phi_i, \beta, \kappa_y) \in (0, \phi_i)$  denote the stable root of the equilibrium dynamics. Then the model yields the same regime structure as in Proposition 3, where the cut-off values  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  uniquely solve*

$$\underline{\phi}_\pi = \frac{1}{\Psi(\underline{\phi}_\pi)} \left( 1 + \frac{(1 - \tilde{\beta}(\underline{\phi}_\pi))^2}{\kappa_y} \right), \quad \text{and} \quad \bar{\phi}_\pi = \frac{1}{\Psi(\bar{\phi}_\pi)} \left( 1 + \frac{1}{\kappa_y} \right).$$

In addition, the turning point is characterized by

$$\rho^* = \tilde{\beta}(\phi_\pi)^{-1} \left( 1 - \sqrt{\kappa_y(\Psi(\phi_\pi)\phi_\pi - 1)} \right),$$

where  $\tilde{\beta}(\phi_\pi) \equiv \beta\mu_i/\phi_i$  and  $\Psi(\phi_\pi) \equiv \frac{1 - \beta\mu_i}{(1 - \beta\mu_i)(1 - \mu_i) - \kappa_y\mu_i} \frac{(1 - \phi_i)\mu_i}{\phi_i}$ .

This proposition therefore establishes that our main results, obtained with the baseline Taylor rule, are robust to the inclusion of interest rate smoothing. Importantly, the stable root  $\mu_i$  depends on the structural parameters of the model but remains independent of the persistence of the shock. Mathematically, the characteristic polynomial is of third order and lacks trivial roots. While a closed-form solution for  $\mu_i$  exists, its complexity precludes a full analytical investigation. However, the next corollary examines how interest rate smoothing reshapes the boundaries of monetary policy regimes and the location of the turning point  $\rho^*$  in the neighborhood of  $\phi_i = 0$ .<sup>17</sup>

**COROLLARY 1.** *In the neighborhood of  $\phi_i = 0$ , the following comparative statics obtain:*

$$\frac{d\underline{\phi}_\pi}{d\phi_i} > 0, \quad \frac{d\bar{\phi}_\pi}{d\phi_i} > 0, \quad \text{and} \quad \frac{d\rho^*}{d\phi_i} > 0.$$

Corollary 1 has clear policy implications. As the Central Bank smooths interest rate movements more, the thresholds  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  shift upward, and so does the turning point  $\rho^*$ . Interest rate smoothing dampens the response of nominal and real interest rates, weakening the RIR channel and increasing the relative importance of the PI channel. As a result, offsetting persistent demand shocks requires a more aggressive inflation response. The maximal output effect of the shock is therefore attained at higher levels of shock persistence.

Recently, two alternative classes of monetary rules have received particular attention: *real interest rate rules* and *robust real rate rules*. Discussing these rules further

<sup>17</sup>Furthermore, we will provide numerical illustrations in Section 4.

highlights that our conclusions regarding the hump-shaped behavior of persistent DIS shocks depend fundamentally on how the monetary stance balances the PI and RIR channels.

On the one hand, *real interest rate rules* of the form  $i_t = \bar{r} + \mathbb{E}_t[\pi_{t+1}] + \phi_y y_t$ , with  $\phi_y \geq 0$ , specify the nominal interest rate as a function of expected inflation and contemporaneous output (see, among others, [Woodford, 2011](#), [Debortoli and Galí, 2024](#), [Fornaro and Wolf, 2023](#), [Angeletos et al., 2024b,a](#)). Under this specification, the sign of the output feedback parameter ( $\phi_y$ ) captures the cyclicity of the real interest rate. Such rules effectively eliminate the RIR channel as the real interest rate is independent of the NKPC. As such, only the monotonous regime survives. Otherwise stated, our trade-off is only active when the Central Bank is aggressive enough towards (expected) inflation.

On the other hand, *robust real rate rules* (see [Holden, 2024](#)) of the form  $i_t = r_t + \phi_\pi \pi_t$ , with  $\phi_\pi > 1$ , specify the nominal interest rate as a function of the real interest rate and contemporaneous inflation feedback. These rules are particularly compelling because they remain robust to key features of modern monetary theory, such as household heterogeneity or a departure from strict rationality, while ensuring equilibrium stability. This robustness arises because inflation dynamics obtain from the monetary policy rule and the Fisher equation as opposed to the interplay between the dynamic IS equation and the NKPC.<sup>18</sup> In other words, this class of rules effectively eliminates both the PI and RIR channels. As a result, they represent an extreme case compared to the dynamics discussed in Proposition 5, bypassing the tension between these two channels.<sup>19</sup>

### 3.2 Cumulative Multipliers

It can be more informative to examine cumulative multipliers rather than impact or period-by-period multipliers, as cumulative measures provide a fuller account of how policy interventions propagate through the economy over time. The following proposition shows that, for a maximal planning horizon  $h \leq \bar{h}(\phi_\pi)$ , the cumulative multiplier inherits the same properties as the impact multiplier in the intermediate range of monetary policy aggressiveness.<sup>20</sup>

PROPOSITION 7. *Let the cumulative output multiplier at horizon  $h \in \mathbb{N}$  be defined as*

$$C_y(\rho, h) = \mathcal{M}_y(\rho) \sum_{i=0}^h \rho^i.$$

<sup>18</sup>Specifically, under robust real rate rules, DIS demand shocks are entirely absorbed by adjustments in the nominal interest rate, leaving output and inflation completely unaffected. Conversely, expansionary monetary policy shocks result in expansions of both output and inflation. In this case, the output response is non-monotonic in persistence, depending on whether  $\phi_\pi$  is greater or less than the inverse of the sensitivity of current to expected inflation within the NKPC.

<sup>19</sup>In a related vein, [Rupert and Šustek \(2019\)](#) question whether monetary policy shocks propagate via the RIR channel in a broad class of NK models with physical capital. Compared to their analysis, our findings emphasize that the presence of a RIR channel critically hinges on the feedback rule adopted by the Central Bank.

<sup>20</sup>Online Appendix A provides a complete statement of the proposition for any horizon  $h \in \mathbb{R}_+$  and any degree of aggressiveness toward inflation  $1 < \phi_\pi < \phi_\pi^{\max}$ .

Moreover, let  $\rho^*$  be the level of persistence maximizing  $\mathcal{M}_y(\rho)$  and  $\tilde{\rho}(h)$  the level of persistence maximizing  $C_y(\rho, h)$ . Consider the intermediate regime  $\underline{\phi}_\pi < \phi_\pi \leq \bar{\phi}_\pi$  and define the horizon threshold

$$\bar{h}(\phi_\pi) \equiv \lceil \bar{h}_c(\phi_\pi) \rceil - 1 \in \mathbb{N}, \quad \text{with} \quad \bar{h}_c(\phi_\pi) \equiv \frac{2}{1-\beta} \left( \frac{\phi_\pi - \underline{\phi}_\pi}{\phi_\pi - 1} \right),$$

where  $\lceil h_c(\phi_\pi) \rceil$  denotes the integer part of  $h_c(\phi_\pi)$ . If  $1 \leq h \leq \bar{h}(\phi_\pi)$  applies, there exists at least one global maximizer  $\tilde{\rho}(h) \in (\rho^*, 1)$ , satisfying  $\frac{\partial C_y(\rho, h)}{\partial \rho} \Big|_{\rho=\tilde{\rho}(h)} = 0$ .

Proposition 7 characterizes how the persistence parameter  $\tilde{\rho}(h)$  that maximizes the cumulative output effect,  $C_y(\rho, h)$ , depends on monetary policy coefficient  $\phi_\pi$  and on horizon  $h$ .

To understand the proposition, we first describe the case in which monetary policy is only weakly aggressive towards inflation, i.e.,  $\phi \leq \underline{\phi}_\pi$ . In this case, the PI channel strictly dominates and both the impact multiplier  $\mathcal{M}_y(\rho)$  and the geometric-sum  $\sum_{i=0}^h \rho^i$  favor greater persistence, so  $C_y(\rho, h)$  is strictly increasing in  $\rho$  and the global maximizer is  $\tilde{\rho}(h) \rightarrow 1$ , that is maximal persistence.

In the intermediate regime  $\underline{\phi}_\pi < \phi_\pi \leq \bar{\phi}_\pi$ , the optimal  $\rho$  depends on the horizon: for short horizons,  $h \leq \bar{h}(\phi_\pi)$ , the cumulative multiplier  $C_y(\rho, h)$  is locally decreasing as  $\rho \rightarrow 1$ , so  $\rho \rightarrow 1$  cannot be optimal and the maximizer is interior, satisfying the first-order condition and lying strictly above the persistence  $\rho^*$  that maximizes the impact multiplier  $\mathcal{M}_y$ . The main reason is that over a short horizon, the trade-off between the RIR and the PI channel is sufficiently potent. Intuitively, the maximal horizon over which a tradeoff between the PI and RIR channels emerges increases with the firms' degree of forward-lookingness—that is, with  $\beta$ . In addition, lower nominal rigidities, reflected in a larger slope of the NKPC ( $\kappa_y$ ), or a more aggressive monetary policy (a higher  $\phi_\pi$  relative to  $\underline{\phi}_\pi$ ) expand this horizon, as they improve the potency of the RIR channel.<sup>21</sup>

Finally, when monetary policy is very aggressive,  $\phi_\pi > \bar{\phi}_\pi$ , the impact multiplier itself is decreasing in  $\rho$ , so persistence reduces the impact response; nonetheless, for short horizons and not-too-large  $\phi_\pi$  an interior maximizer can still exist, while for long horizons  $\rho \rightarrow 1$  again becomes a boundary local maximizer and may or may not be globally optimal depending on whether interior peaks dominate.

How "short" is the maximal planning horizon  $\bar{h}(\phi_\pi)$  that induces a trade-off in the intermediate regime? To put things into perspective, consider the quarterly parametrization we used to generate Figure 1. The maximal horizon is roughly 200 quarters. This is far beyond the time horizon typically relevant for business cycle stabilization. In other

<sup>21</sup> For long horizons instead,  $h \geq \bar{h}(\phi_\pi) + 1$ , the geometric sum places a sufficiently high weight on persistence so that  $\rho \rightarrow 1$  is a boundary local maximizer; however, global optimality need not be guaranteed because the interaction between  $\mathcal{M}_y(\rho)$ , which still features an active trade-off between the PI and the RIR channels, and the geometric-sum term can generate interior peaks. Accordingly, either  $C_y(\rho, h)$  is non-decreasing and  $\rho \rightarrow 1$  is the global maximizer, or  $C_y(\rho, h)$  has interior local maxima and the global maximizer is determined by a level comparison between  $\lim_{\rho \rightarrow 1} C_y(\rho, h)$  and the values attained at those interior peaks.

words, the interplay between the PI and RIR channels, and the resulting tradeoff, remains quantitatively meaningful even when attention shifts from impact to cumulative multipliers.

### 3.3 Relation To Alternative Decompositions

The literature has proposed several ways of decomposing the transmission of demand disturbances into *direct* and *indirect* components. In the HANK tradition, [Kaplan et al. \(2018\)](#) and [Bilbiie \(2020\)](#) define *direct effects* as the response of consumption to a change in the real interest rate, holding fixed the general-equilibrium adjustments that operate through household income and prices—the *indirect effects*. By contrast, [Maliar and Naubert \(2025\)](#) defines direct effects with respect to the innovation to the monetary shock itself. Under this alternative convention, the portion of the real-rate response that is induced by the central bank’s systematic feedback rule is allocated to the indirect component.<sup>22</sup>

In what follows, we relate our PI/RIR decomposition to this direct–indirect approach. Because our regime characterization hinges on the endogenous interest-rate response, we adopt the definition in [Maliar and Naubert \(2025\)](#): the direct effect is the partial derivative of the consumption response to the persistent innovation of the DIS shock, holding household income and the real interest rate fixed. The indirect effect is then the residual, *i.e.*, the gap between the impact output multiplier and the direct effect. Under this convention, our PI channel can be expressed as:<sup>23</sup>

$$\text{PI} = \mathcal{M}_y^D + \mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1} = \frac{\beta}{1 - \beta\rho} + \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)}.$$

This expression makes transparent that the PI channel splits into a direct component,  $\mathcal{M}_y^D$ , and an indirect component evaluated at a constant real interest rate,  $\mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1}$ . The direct term is the New Keynesian analogue of the permanent-income mechanism in [Hall’s \(1978\)](#) canonical consumption–savings model under a constant real rate: the shock changes intertemporal resources and therefore desired consumption on impact. The indirect term instead captures the familiar New Keynesian cross amplification: stronger aggregate demand raises production and labor income, which feeds back into aggregate demand. In this sense, the PI channel inherits exactly the same direct–indirect decomposition as [Bilbiie’s \(2020\)](#) impact output multiplier for an AR(1) disturbance to the real interest rate.

In a similar spirit, the RIR channel admits the decomposition

$$\text{RIR} = -\mathcal{M}_y^D + \mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1} = -\frac{(1 - \beta) - \beta\rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}}{(1 - \beta\rho)\left[1 - \rho + \rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}\right]} + \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)}.$$

<sup>22</sup>[Kaplan et al. \(2018\)](#) note (footnote 22, p. 714) that, for standard persistence and Taylor-rule feedback coefficients, the quantitative difference between the two conventions is small in their baseline model.

<sup>23</sup>For a detailed derivation, we refer the reader to Online Appendix B.

Thus, the RIR channel is the sum of two indirect terms. The first term,  $-\mathcal{M}_y^{\text{ID}}$ , aggregates all general-equilibrium propagation forces that are orthogonal to the DIS shock itself—most notably, the interaction between the endogenous real-rate response and New Keynesian cross feedback. The second term,  $\mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1}$ , is the same constant-real-rate indirect effect as above. The strength of the first component is increasing in  $\phi_\pi$ , and the overall RIR channel vanishes as  $\phi_\pi \rightarrow 1$ , consistent with the disappearance of endogenous real-rate movements in that limit. Moreover, for sufficiently aggressive inflation feedback, the total indirect effect becomes (weakly) negative:  $\phi_\pi \geq \phi_\pi^{\text{ID}}$ , where  $\phi_\pi^{\text{ID}}$  denotes the sign-switch threshold (for an analytical characterization under a contemporaneous monetary feedback rule, see [Maliar and Naubert \(2025\)](#)). Our contribution is distinct from this observation: in our setup  $\phi_\pi^{\text{ID}} > \phi_\pi$ , so that negative indirect effects are necessary and sufficient for our regime characterization.

### 3.4 Other Shocks

So far, our main focus has been placed on DIS–demand shocks. This section investigates the extent with which our results extend to other usual shocks in the literature.

**3.4.1 Monetary Policy Shocks:** All preceding results naturally apply to the case of a standard monetary policy shock—a shock to the Taylor rule. We opt to focus on the DIS-demand shock for the sake of clarity of the exposition only. The output and inflation multipliers associated with monetary policy shocks are identical to those obtained with the DIS-demand shock. They however differ for the interest rates.<sup>24</sup> It is a well-known feature of NK models that the nominal interest rate increases on impact if the persistence of the expansionary monetary policy shock is sufficiently high. However, our findings on monetary regimes and  $\rho^*$  are independent of this observation, as they are driven by the response of the real interest rate, not the response of the nominal interest rate. This distinction is critical because the  $\rho^*$  generically differs from the threshold at which the nominal interest rate response changes sign and so-called "neo-Fisherian" effects can occur in response to monetary policy shocks.<sup>25</sup>

**3.4.2 Cost-Push Shocks:** We now consider a version of the model (1)–(4) in which we replace the DIS-demand shock with a cost-push shock, such that the NKPC rewrites

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa_y y_t + \nu_t,$$

where  $\nu_t$  is the cost push shock, which follows an exogenous AR(1) process with persistence  $\rho_\nu \in [0, 1)$ . In that case, the following proposition obtains.

<sup>24</sup>The response of the interest rate to a DIS shock is scaled up compared to the response to an expansionary monetary policy shock of the same size.

<sup>25</sup>The persistence threshold beyond which neo-Fisherian effects occur is implicitly characterized by  $\rho[1 + \kappa_y/(1 - \beta\rho)] > 1$ , see e.g. [Galí \(2015\)](#), [Garín et al. \(2018\)](#) or [Bilbiie \(2022\)](#).

PROPOSITION 8. *Under local determinacy, the impact multipliers of inflation,  $\mathcal{M}_\pi(\rho_\nu)$ , and output,  $\mathcal{M}_y(\rho_\nu)$  to a cost-push shock are given by*

$$\mathcal{M}_\pi(\rho_\nu) = \frac{1}{1 - \beta\rho_\nu + \rho_\nu(\phi_\pi - 1)\frac{\kappa_y}{1 - \rho_\nu}}, \quad \text{and} \quad \mathcal{M}_y(\rho_\nu) = -\frac{(\phi_\pi - 1)\rho_\nu}{1 - \rho_\nu}\mathcal{M}_\pi(\rho_\nu).$$

*The following statements apply:*

(a) *The output impact multiplier decreases monotonically with  $\rho_\nu$ .*

(b) *The inflation impact multiplier satisfies:*

(b.1) *If  $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$ ,  $\mathcal{M}_\pi(\rho_\nu)$  is humped-shaped in  $\rho_\nu$ , i.e.,*

$$\exists \rho_\nu^* \equiv 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} \quad \text{such that} \quad \mathcal{M}'_\pi(\rho_\nu) \gtrless 0 \text{ for } \rho_\nu \lesseqgtr \rho_\nu^*.$$

(b.2) *If  $\phi_\pi^{max} \geq \phi_\pi \geq 1 + \frac{\beta}{\kappa_y}$ ,  $\mathcal{M}_\pi(\rho_\nu)$  decreases monotonically in  $\rho_\nu$ .*

The first statement of Proposition 8 establishes that, unlike the DIS-demand shock, a cost-push shock does not lead to the emergence of three monetary policy regimes or a hump-shaped relationship between shock persistence and the output impact multiplier. Instead, the output impact multiplier decreases monotonically with the persistence of the shock. The focus shifts to the inflation impact multiplier, where non-monotonic behavior arises. A decomposition similar to (5) applied on the NKPC helps shedding light on the forces at work. Evaluating the NKPC at the equilibrium yields

$$\mathcal{M}_\pi(\rho_\nu) = \frac{1}{1 - \beta\rho_\nu} + \frac{\kappa_y\mathcal{M}_y(\rho_\nu)}{1 - \beta\rho_\nu}.$$

As with a DIS-demand shock, monetary policy balances two opposing channels. The first term corresponds to the present value of future expected effects of the cost-push shock, keeping marginal costs constant. Hence, the more persistent the shock, the greater its effect on inflation. The second term corresponds to the present value of expected marginal cost variations induced by the cost-push shock. A positive cost-push shock puts upward pressure on prices, which depresses aggregate demand, production, and hence labor demand—as witnessed by the presence of the output multiplier in the second term. This puts downward pressure on wages and so on the marginal cost. Accordingly, this latter effect goes in the opposite direction to the former effect, creating the hump-shaped relationship between persistence and the response of inflation. When monetary policy becomes extremely aggressive, the Central Bank is hiking the interest rate to such an extent that the recession it triggers makes the second effect fully dominate the determination of the inflation multiplier.

Note that the properties displayed in Proposition 8 will be shared by the impact response of output to a shock affecting both the left hand side and the right hand side of the DIS equation, hence having both a contemporaneous and a future effect like technology, preference, or government spending shocks.

## 4. Quantitative Insights

We now turn to a medium-scale DSGE model that brings together several mechanisms that have been advanced to address distinct empirical puzzles pertaining to the transmission of demand shocks. This will allow for a quantitative analysis of the properties we laid out in the previous section.

Specifically, the model extends the standard NK model with physical capital accumulation by: (i) household heterogeneity via limited access to asset markets (savers versus hand-to-mouth) in the presence of income risk, (ii) behavioral expectations in the form of cognitive discounting, (iii) a working-capital cost channel, through which firms finance the wage bill at the policy rate, (iv) nominal wage rigidities and (v) price and wage indexation. These ingredients address, respectively, the lack-of-amplification puzzle, the forward-guidance puzzle, the missing-inflation puzzle, and the hump-shaped inflation/price puzzle. Our framework is best understood as a hybrid version of the seminal models developed in [Christiano et al. \(2005\)](#), [Gabaix \(2020\)](#), and [Bilbiie et al. \(2022\)](#).<sup>26</sup>

### 4.1 Environment

The economy is populated by a unit mass of infinitely-lived households, a continuum of firms in the final and intermediate good sectors, a financial intermediary, a government that levies taxes and a Central Bank.

**Households.** Households are indexed by  $i \in [0, 1]$ . Their preferences are defined by

$$\mathcal{U}(C_t^i, L_t^i) = \frac{C_t^{i1-\sigma}}{1-\sigma} - v \frac{L_t^{i1+\varphi}}{1+\varphi},$$

where  $\sigma^{-1} > 0$  is the intertemporal elasticity of substitution,  $\varphi^{-1} > 0$  is the Frisch elasticity, and  $v > 0$  scales disutility from work. Households discount the future at rate  $\beta \in (0, 1)$  and supply differentiated labor, with wages set by unions.

As in [Bilbiie et al. \(2022\)](#), households differ in their access to financial markets. At each date, they are either *savers* ( $S$ ) or *hand-to-mouth* ( $H$ ). Savers participate in asset markets: they hold one-period liquid nominal bonds, illiquid physical capital, and illiquid equity claims on intermediate firms. Hand-to-mouth households only have access to the liquid bond and face a binding non-negativity constraint so they cannot borrow. Households switch types according to a Markov process: a saver becomes hand-to-mouth with probability  $1 - s$ , while a hand-to-mouth household becomes a saver with probability  $1 - h$ . In the stationary distribution, a fraction  $\lambda = (1 - s)/(2 - h - s)$  are hand-to-mouth.

Within each group, households pool resources and are identical at the beginning of the period. Savers enter period  $t$  with capital  $K_t^S$  paying after-tax return  $(1 - \tau^K)R_t^K$ , equity  $\omega_t^S$  priced at  $q_t$  and paying after-tax dividends  $(1 - \tau^D)D_t$  and bonds  $B_t^S$  paying

<sup>26</sup>For expositional purposes, we keep the description of the model concise and refer the interested reader to Section C.1 of the Online Appendix for details and derivations.

the nominal gross return  $R_{t-1}$  scaled by the DIS shock  $e^{-\xi_{t-1}}$ , where  $\xi_t$  follows a stationary AR(1) process. In addition,  $\pi_t \equiv P_t/P_{t-1}$  denotes price inflation. Together with labor income  $\int_0^1 W_t(l) L_t^S(l) dl$  and after-tax transfers from the financial intermediary  $(1 - \tau^F) F_t^S$ , these resources finance consumption, investment, next-period portfolios, and lump-sum taxes. The saver budget constraint is

$$C_t^S + b_{t+1}^S + q_t \frac{\omega_{t+1}^S}{1 - \lambda} + I_t^S = \int_0^1 W_t(l) L_t^S(l) dl + \frac{e^{-\xi_{t-1}} R_{t-1}}{\pi_t} \frac{B_t^S}{1 - \lambda} + (1 - \tau^K) R_t^K K_t^S + \left( q_t + (1 - \tau^D) D_t \right) \frac{\omega_t^S}{1 - \lambda} + (1 - \tau^F) F_t^S - T_t^S, \quad (6)$$

where  $b_{t+1}^S \geq 0$  is saver bond demand and  $B_t^S \equiv (1 - \lambda) s b_t^S + \lambda(1 - h) b_t^H$  denotes aggregate bond holdings of households entering period  $t$  as savers.

Capital is owned by savers and accumulates as

$$K_{t+1}^S = (1 - \delta) K_t^S + \Phi \left( \frac{I_t^S}{K_t^S} \right) K_t^S, \quad (7)$$

where  $\delta \in (0, 1)$  is the constant depreciation rate;  $\Phi(\cdot)$  captures capital adjustment costs with  $\Phi'(\cdot) > 0$ ,  $\Phi''(\delta) = -1/\eta \leq 0$ ,  $\Phi(\delta) = \delta$ ,  $\Phi'(\delta) = 1$  implying that costs are absent in steady state. Aggregate capital and investment are  $K_t = (1 - \lambda) K_t^S$  and  $I_t = (1 - \lambda) I_t^S$ . Savers choose consumption and savings into the liquid bond as well as illiquid physical capital to maximize expected discounted utility subject to (6), (7), the law of motion of aggregate bond holdings and savers' borrowing constraint.

Hand-to-mouth households face an analogous but simpler problem: they only invest into the liquid bond but cannot borrow ( $b_{t+1}^H \geq 0$ ). Their budget constraint is

$$C_t^H + b_{t+1}^H = \int_0^1 W_t(l) L_t^H(l) dl + \frac{e^{-\xi_{t-1}} R_{t-1}}{\pi_t} \frac{B_t^H}{\lambda} - T_t^H, \quad (8)$$

where  $B_t^H \equiv \lambda h b_t^H + (1 - \lambda)(1 - s) b_t^S$  are bond holdings of all households entering period  $t$  as hand-to-mouth. Hand-to-mouth households choose consumption and saving to maximize expected discounted utility subject to (8), the law of motion of aggregate bond holdings and their borrowing constraint.

Finally, we assume that hours worked are uniformly distributed across household types in the steady state, i.e.,  $L^S = L^H = L$ . In the spirit of [Werning \(2015\)](#) or [Patterson \(2023\)](#), they are allowed to fluctuate disproportionately off the steady state, i.e.,  $\hat{L}_t^H = \chi \hat{L}_t$ , where  $0 \leq \chi < \bar{\chi}$  and a hat denotes the relative deviation from steady state. This, *de facto*, sets earnings incidence across households and affects the cyclical of income inequalities.

**Behavioral Expectation Formation.** As in [Gabaix \(2020\)](#), we introduce cognitive discounting on the effects of aggregate shocks. For any variable  $X_t$ , let  $\bar{X}_t$  denote a perceived reference level for this variable in period  $t$  (e.g., its steady state). Behavioral expectations are then defined by

$$\tilde{\mathbb{E}}_t[X_{t+1}] = \bar{X}_t + \mu \mathbb{E}_t[X_{t+1} - \bar{X}_t], \quad (9)$$

where  $\mathbb{E}_t[\cdot]$  is the full-information rational expectation (FIRE) operator. When  $\mu = 1$ , expectations coincide with FIRE; when  $\mu = 0$ , expectations are fully anchored to the reference level. We allow for decision-specific attention parameters  $\mu_b$ ,  $\mu_p$ , and  $\mu_w$  for bond choices, price setting, and wage setting.

**Firms: Technology, Cost Channel, and Nominal Price Rigidities.** A continuum of competitive final good firms combines intermediate inputs  $X_t(j)$  into the final good  $Y_t$  using the [Dixit and Stiglitz \(1977\)](#) aggregator with elasticity  $\epsilon_p > 1$ . Intermediate good firms are monopolistic competitors. Firm  $j$  rents capital  $K_t(j)$  and hires a CES aggregate of differentiated labor, with labor demand elasticity  $\epsilon_w > 1$ , to produce

$$X_t(j) = AK_t(j)^\alpha L_t(j)^{1-\alpha}, \quad \text{with } A > 0 \text{ and } \alpha \in (0, 1).$$

Firms face a working-capital constraint: they must finance the nominal wage bill at the policy rate through a financial intermediary *before* production takes place. This generates a cost channel linking monetary policy to marginal costs and inflation. The intermediary has access to Central Bank funds at no costs and rebates profits as transfers to savers ( $F_t^S$ ). Finally, prices are sticky as in [Calvo \(1983\)](#): each period, a fraction  $1 - \theta_p$  reoptimizes prices, while the remaining  $\theta_p$  index prices to past inflation with indexation parameter  $\chi_p \in [0, 1]$ . Reoptimizing firms choose a common reset price trading off current markups against the expected continuation value of the price, using behavioral expectations with attention  $\mu_p$ .

**Nominal Wage Rigidities and Indexation.** Following [Colciago \(2011\)](#), each labor type  $l \in [0, 1]$  is represented by a union that sets the nominal wage  $W_t(l)$  for its members (households are uniformly distributed across unions). Wages are sticky à la Calvo: in each period, a fraction  $1 - \theta_w$  of unions resets wages, while the remaining  $\theta_w$  index wages to past inflation at rate  $\chi_w \in [0, 1]$ . A re-optimizing union internalizes that its chosen wage may remain in place for multiple periods and sets it to maximize the expected utilitarian welfare of its members, subject to the induced labor demand schedule and both household budget constraints. Expectations in wage setting are governed by attention  $\mu_w$ . In symmetric equilibrium, all adjusting unions choose the same wage<sup>27</sup> and the aggregate wage index follows the standard Calvo recursion.

**Fiscal and Monetary Authorities.** The fiscal authority collects taxes on capital and dividend income as well as financial intermediary profits and rebates revenues via lump-sum transfers to both household groups, balancing its budget each period:

$$\lambda T_t^H + (1 - \lambda)T_t^S + \tau^D D_t + \tau^K R_t^K K_t + \tau^F F_t = 0.$$

Monetary policy follows a contemporaneous Taylor rule:

$$R_t = R_{t-1}^{\phi_i} \left( R(\pi_t/\pi)^{\phi_\pi} (Y_t/Y)^{\phi_y} \right)^{1-\phi_i}, \quad \text{with } \phi_\pi > 1, \phi_y \geq 0, \text{ and } \phi_i \in [0, 1),$$

where  $R$  and  $\pi$  are steady-state values and  $\phi_i$  is the degree of interest rate smoothing.

<sup>27</sup>Unions do not internalize the effects of earnings incidence when setting the wage.

**Benchmark Parametrization.** We calibrate the model to match key features of the U.S. economy at a quarterly frequency; Table C.1 in the Online Appendix reports all baseline parameters. Preference parameters follow standard values:  $\beta = 0.99$  (4% annual real rate), log utility ( $\sigma = 1.00$ ) as in [Christiano et al. \(2005\)](#), and inverse Frisch elasticity  $\varphi = 1.00$  ([Chetty et al., 2013](#)). We set the labor disutility parameter to  $v = 5.00$  to match a steady-state work share of one-third. Household heterogeneity follows [Bilbiie et al. \(2022\)](#): savers become hand-to-mouth with probability  $1 - s = 0.02$  per quarter ( $s = 0.98$ ). The hand-to-mouth share is  $\lambda = 0.30$ , consistent with [Kaplan et al. \(2014\)](#) (SCF 1989–2010), implying  $h = 0.95$ . Consistent with evidence in [Guvenen et al. \(2017\)](#), [Broer et al. \(2022\)](#) and [Patterson \(2023\)](#), hours worked at the bottom of the income distribution are assumed to react about 50% more than aggregate hours over the business cycle, leading to  $\chi = 1.5$ . Savers accumulate capital with depreciation  $\delta = 0.025$  per quarter ([Christiano and Eichenbaum, 1992](#)). The steady state second derivative of adjustment costs,  $\Phi''(\cdot)$ , is set to 0.5 (close to [Galí et al. 2007](#)). Following [Gabaix \(2020\)](#) and [Pfäuti and Seyrich \(2023\)](#), we set cognitive discounting to  $\mu_b = \mu_p = \mu_w = 0.85$  for consumption-saving and price/wage setting. On the production side, we normalize  $A = 1$  and set  $\alpha = 0.33$ . Markups are 20% in prices and wages ( $\epsilon_p = \epsilon_w = 6$ ), consistent with [Christiano et al. \(2005\)](#) and [Galí \(2015\)](#). Nominal rigidities follow [Galí \(2015\)](#):  $\theta_p = 0.65$  and  $\theta_w = 0.75$ , with partial indexation  $\chi_p = \chi_w = 0.30$  (close to [Smets and Wouters 2007](#)). Fiscal parameters are set to the statutory U.S. tax rates:  $\tau^K = \tau^D = \tau^F = 0.20$ . We choose a constant transfer  $T_t^S = T^S$  so that steady-state consumption is equalized across household types ( $C^S = C^H$ ). As a benchmark, monetary policy follows a basic Taylor rule with  $\phi_i = 0$ ,  $\phi_\pi = 1.50$  and  $\phi_y = 0.125$ . Under this calibration, the equilibrium is locally determinate<sup>28</sup> and income and consumption inequalities are countercyclical (see, among others, [Coibion et al. \(2017\)](#) or [Ampudia et al. \(2018\)](#)).

When assessing the role of household heterogeneity, it will prove useful to vary the degree of cyclical income inequality in the subsequent sections. In particular, we will study versions of the model with acyclical and procyclical income inequality. Acyclical income inequality is achieved by setting the tax rates to the share of hand-to-mouth households ( $\tau^K = \tau^D = \tau^F = \lambda$ ) and the earning incidence to unity ( $\chi = 1$ ). Instead, procyclical income inequality can be obtained by setting the capital income tax and the tax on intermediaries profits to zero, while assuming a uniform earnings incidence.

## 4.2 Quantitative Drivers of Monetary Policy-Persistence Regimes

This section attempts to shed light on the various forces determining the relative importance of the PI and RIR channels, and thereby the monetary regime thresholds and the  $\rho^*$ . To structure our discussion, it will prove useful to consider a stripped down version of our quantitative model that eliminates the working capital channel, abstracts from capital accumulation ( $\alpha = 0$ ), sticky wages ( $\theta_w = 1$ ), price indexation ( $\chi_p = 0$ ) and simplifies the monetary policy rule ( $\phi_i = \phi_y = 0$ ). In this case, the model reduces to an extended

<sup>28</sup>Section C.3 in the Online Appendix reports the local determinacy regions and the Impulse Responses Functions to a positive DIS shock of the main variables of the model.

version of the RANK model laid out in the previous section:

$$y_t = \zeta_f \mathbb{E}_t [y_{t+1}] - \zeta_r (i_t - \mathbb{E}_t [\pi_{t+1}] - \bar{r}) + \zeta_r \xi_t, \quad (10)$$

$$\pi_t = \beta_f \mathbb{E}_t [\pi_{t+1}] + \kappa_y y_t, \quad (11)$$

$$i_t = \bar{r} + \phi_\pi \pi_t \quad (12)$$

This formulation generalizes the benchmark RANK model in two key dimensions. First, the dynamic IS equation (10) introduces two new elasticities,  $\zeta_f$  and  $\zeta_r$ . The parameter  $\zeta_f > 0$  captures the sensitivity of current demand to expected future income. When  $\zeta_f > 1$ , the economy exhibits a compounded DIS equation, while  $\zeta_f < 1$  implies a discounted DIS equation. The parameter  $\zeta_r > 0$  governs the responsiveness of demand to the real interest rate. The elasticity of current inflation to future inflation,  $\beta_f \in [0, 1)$ , is allowed to differ from the household discount factor  $\beta$  inside the NKPC (11). All in all, we consider the parameters  $(\zeta_f, \zeta_r, \beta_f, \kappa_y)$  as *sufficient statistics*, i.e., their exact structural composition is irrelevant for the derivation of the following analytical results. Under local determinacy, the following proposition holds.<sup>29</sup>

**PROPOSITION 9.** *Let us denote  $\tilde{\kappa}_y = (\zeta_r/\zeta_f) \cdot \kappa_y$ , then Proposition 3 holds in the general model provided the monetary thresholds and  $\rho^*$  are adjusted as*

$$\underline{\phi}_\pi = \frac{1}{\beta_f} \left( 1 + \frac{(1 - \beta_f)^2}{\tilde{\kappa}_y} \right), \quad \bar{\phi}_\pi = \frac{1}{\beta_f} \left( 1 + \frac{1}{\tilde{\kappa}_y} \right) \quad \text{and} \quad \rho^* = \frac{1}{\beta_f} \left( 1 - \sqrt{(\phi_\pi \beta_f - 1) \tilde{\kappa}_y} \right).$$

Proposition 9 generalizes Proposition 3 with respect to the monetary policy regime thresholds  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$ , as well as the output-maximizing persistence  $\rho^*$ . Notably, the proposition establishes that their formal expressions share an identical analytical structure when  $\beta$  is replaced by  $\beta_f$  and  $\kappa_y$  is scaled by  $\zeta_r/\zeta_f$ . If  $\zeta_r$  increases relative to  $\zeta_f$ , the RIR channel gains in importance relative to the PI channel. As such, regime thresholds and output-maximizing persistence fall. In other words, the results obtained in Proposition 3 are not the outcome of particular assumptions placed on the primitives of the model, or on the micro-foundations underlying it, but are fundamentally related to the generic structure of the NK model. Note that, similarly, Proposition 5 generalizes in the same manner as  $\kappa_y$  is replaced by  $\tilde{\kappa}_y$ .

Interestingly, the parameter  $\tilde{\kappa}_y$  determines the two monetary regime thresholds in an asymmetric way, mechanically leaving  $\underline{\phi}_\pi$  almost unaffected by its variations as its effect is scaled by  $(1 - \beta_f)^2$ . This will, to a large extent, explain the asymmetry we will obtain when varying structural parameters of the quantitative model in Table 1.

To unpack the mechanisms driving the emergence of distinct monetary policy regimes and the role of persistence in shaping output multipliers in our full-fledged quantitative model, we conduct a sequence of sensitivity experiments switching off and on key specific features of the model susceptible to affect the regime thresholds,  $\rho^*$ ,

<sup>29</sup>See Section E3 of the Online Appendix for a detailed analysis.

and the associated output impact multiplier. The results, summarized in Table 1, highlight the robustness of the three-regime structure and provide insight into the structural forces behind it. Because the monetary regime thresholds are primarily functions of the reduced-form objects  $(\zeta_f, \zeta_r, \beta_f, \kappa_y)$ , Table 1 can be read as mapping structural features of the quantitative model—*i.e.* expectation formation, intertemporal substitution, nominal rigidities and household heterogeneity—into shifts of the medium-scale counterparts  $\underline{\phi}_\pi$ ,  $\overline{\phi}_\pi$ , and the implied peak-persistence level  $\rho^*$ .<sup>30</sup> Other mechanisms, such as working capital or capital adjustment costs, which are important to shape the overall dynamics of the model but play a milder role in monetary regimes will be considered in Section 4.4.

Given our benchmark calibration, the intermediate regime occurs for policy parameters ranging from 1.34 to 3.22, which includes standard values of estimated degrees of aggressiveness towards inflation for the US. The  $\rho^*$  is 0.85, which lies within the range of values estimated for the degree of persistence of DIS shocks in the recent literature (see, among others, Fève et al., 2022, Bilbiie et al., 2023, Bayer et al., 2024).

TABLE 1. Monetary Policy Regimes, Impact Multipliers and  $\rho^*$

| MODEL VARIANT                     | PARAMETERS                                     | REGIME                 |                       | IMPACT IRF |                   |
|-----------------------------------|------------------------------------------------|------------------------|-----------------------|------------|-------------------|
|                                   |                                                | $\underline{\phi}_\pi$ | $\overline{\phi}_\pi$ | $\rho^*$   | $\mathcal{M}_y^*$ |
| <b>Benchmark Model</b>            |                                                | 1.34                   | 3.22                  | 0.85       | 0.44              |
| <b>Expectations Formation</b>     |                                                |                        |                       |            |                   |
| Rational Expectations             | $\mu_{b,p,w} = 1.00$                           | –                      | 3.15                  | 0.71       | 0.44              |
| <b>Intertemporal Substitution</b> |                                                |                        |                       |            |                   |
| Higher EIS                        | $\sigma = 0.33$                                | 1.33                   | 2.59                  | 0.76       | 0.55              |
| <b>Nominal Rigidities</b>         |                                                |                        |                       |            |                   |
| No Price/Wage Indexation          | $\chi_{p,w} = 0.00$                            | 1.26                   | 3.41                  | 0.83       | 0.43              |
| No Wage Indexation                | $\chi_w = 0.00$                                | 1.34                   | 3.37                  | 0.88       | 0.47              |
| No Price Indexation               | $\chi_p = 0.00$                                | 1.25                   | 3.23                  | 0.80       | 0.41              |
| Flexible Wages (only)             | $\theta_w = 0.00$                              | 1.20                   | 1.69                  | 0.23       | 0.15              |
| Flexible Prices (only)            | $\theta_p = 0.00$                              | 0.74                   | 0.90                  | 0.00       | -0.29             |
| <b>Households Heterogeneity</b>   |                                                |                        |                       |            |                   |
| Less HtM Households               | $\lambda = 0.01$                               | 1.47                   | 3.36                  | 0.94       | 0.60              |
| Acyclical Inc. inequality         | $\tau^K = \tau^D = \tau^F = \lambda, \chi = 1$ | 1.32                   | 3.35                  | 0.86       | 0.40              |
| Procyclical Inc. inequality       | $\tau^K = \tau^F = 0, \chi = 1$                | 1.23                   | 3.39                  | 0.86       | 0.37              |

<sup>30</sup>In Section E3 of the Online Appendix, we shed further light on this mapping and provide an analytical characterization of the relationship between some structural mechanisms considered below and our main objects of interest  $\underline{\phi}_\pi$ ,  $\overline{\phi}_\pi$ , and  $\rho^*$ .

**Behavioral Frictions and Cognitive Biases:** Behavioral expectations significantly reduce the inflation expectation channel in the NKPC—a drop in  $\beta_f$  in the analytical version laid out above—and reduce the PI channel in the DIS equation—a drop in  $\zeta_f$ . Therefore, shifting to rational expectations has two direct consequences. First, it substantially strengthens the PI channel in the DIS equation. Second, it amplifies the role of inflation expectations in the NKPC, thereby strengthening the RIR channel—the same response of monetary policy exerts a larger effect on consumption and output. In our parameterization, the latter effect dominates, hence requiring a less aggressive monetary policy stance to stabilize inflation. Thus, the lower threshold  $\underline{\phi}_\pi$  decreases so much that the lower monetary policy regime effectively disappears. The upper threshold decreases (from 3.22 to 3.15). The relatively stronger RIR channel then yields a drop in  $\rho^*$  from 0.86 in the benchmark model to 0.71.

Other departures from the FIRE benchmark, such as [Angeletos and Lian \(2018\)](#), theoretically complement the previous findings by exploring the role of incomplete information and higher-order beliefs. In such a framework, the dynamics are akin to those generated by discounted Euler equations, strengthening the link between cognitive frictions and monetary non-neutrality.

**Household Preferences and Consumption Smoothing:** Household preferences play a central role in determining the responsiveness of consumption to interest rates and inflation. A natural starting point is the standard constant relative risk aversion (CRRA) framework, where the elasticity of inter-temporal substitution (EIS) affects both the elasticity of the real interest rate in the DIS equation and the slope of the NKPC. In our benchmark calibration, we opted for a unit intertemporal elasticity of substitution. However, recent studies, such as [Jakobsen et al. \(2020\)](#) and [Holm et al. \(2024\)](#), suggest a higher EIS in the range of 2–3, *i.e.*,  $\sigma \in [0.33, 0.50]$ , while [Heathcote et al. \(2010\)](#) specifies  $\sigma = 1.50$  (EIS of 0.67). To assess the importance of EIS, we follow [Jakobsen et al. \(2020\)](#) and [Holm et al. \(2024\)](#) and increase the IES to 3. This variation in the EIS strengthens the RIR channel in the DIS equation—an increase in  $\zeta_r$  in the analytical version—and reduces the sensitivity of the NKPC to output— $\kappa_y$  in the analytical model, thus strengthening the role of inflation expectations. Both effects play in opposite direction, but the effect of  $\zeta_r$  dominates quantitatively. Consequently, *ceteris paribus*, the monetary policy regime thresholds  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  decrease. However, this decline is partially offset by the monetary policy cost channel in the NKPC, which increases the sensitivity of the NKPC to output in equilibrium and plays in the opposite direction. Under our parameterization, this results in a slight decrease in the lower threshold  $\underline{\phi}_\pi$  (from 1.34 to 1.33), while the upper threshold,  $\bar{\phi}_\pi$ , decreases substantially (from 3.22 to 2.59). This narrows the range of values over which the output multiplier exhibits a hump-shaped relationship with respect to persistence. The dominance of the RIR channel leads to a lower  $\rho^*$ , 0.76, together with a larger impact multiplier, 0.55.

**Nominal Rigidities:** We now assess the role of price and wage indexation, which act as an endogenous source of persistence in inflation and wage dynamics. The results reported in the lower panel of Table 1 suggest that, while it affects the exact location of the monetary regime,  $\rho^*$  and the multiplier, indexation—be it in wages and/or prices—does

not play an important quantitative role for any of these indicators. The main reason for this result lies in the fact that, while indexation may create some propagation of nominal rigidities, it does not fundamentally affect the transmission of monetary policy on impact.

In contrast, the combined presence of nominal price and wage rigidity plays a significant role. When nominal wages are flexible and prices are sticky, real wages—and consequently, firms’ marginal costs—become more sensitive to changes in demand. In this context, monetary policy gains greater effectiveness. As a result, the upper regime threshold decreases sharply, dropping from 3.22 to 1.69, while the lower threshold falls from 1.34 to 1.2. This reduction is accompanied by a lower  $\rho^*$  (0.23).

In the case of flexible prices and sticky wages, the inflation rate becomes significantly more responsive compared to the previous scenario. Since the central bank targets price inflation—and not wage inflation—the RIR channel is much stronger than in an economy characterized by flexible wages and sticky prices. As a result, both the upper and lower monetary policy thresholds drop sharply. Specifically, the lower threshold decreases from 1.34 to 0.75, while the upper threshold drops from 3.22 to 0.90. This sharp reduction is accompanied by a collapse of  $\rho^*$  to 0.00, as the RIR channel overwhelmingly dominates the PI channel in the DIS equation. Furthermore, since wages are rigid and prices adjust freely, the demand shock leads to a drop in the real wage and hence the income of hand to mouth agents which puts downward pressure on aggregate demand. For our calibration, this leads to a pronounced recessionary effect on output.

**Household Heterogeneity:** Empirical evidence (see, *e.g.*, [Campbell and Mankiw, 1989, 1991](#)) indicates that consumption–saving behavior departs from the predictions of the standard Euler equation. Models with household heterogeneity help rationalize these deviations by highlighting cyclical movements in income inequality and income risk. To clarify how heterogeneity shapes monetary regimes and the output-maximizing degree of policy persistence, we begin with economic intuition from analytical heterogeneous-agent models before turning to our quantitative results.

Consider first the two-agent New Keynesian (TANK) framework of [Bilbiie \(2008, 2020\)](#). In these economies, cyclical income inequality between savers and hand-to-mouth households substantially reshapes the transmission of demand shocks. Countercyclical income inequality, by raising the real-interest-rate elasticity  $\zeta_r$ , amplifies the effects of monetary policy, whereas procyclical inequality dampens them. Notably, as cyclical inequality approaches critical thresholds,  $\rho^*$  tends toward zero, underscoring the destabilizing potential of income inequalities.

Beyond cyclical inequality, cyclical income risk (which we abstract from in our model) also affects consumption–saving decisions and, in turn, aggregate dynamics. In the pseudo–representative-agent New Keynesian (PRANK) model of [Acharya and Dogra \(2020\)](#), for instance, procyclical income risk implies discounted DIS behavior ( $\zeta_f < 1$ ), whereas countercyclical income risk generates compounding ( $\zeta_r > 1$ ).<sup>31</sup> In frameworks

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<sup>31</sup>Importantly, contemporaneous income risk has limited bite because it scales the PI and RIR channels in broadly similar ways. By contrast, expectations about future income risk materially shift their *relative* strength and therefore shape dynamic responses. In particular, anticipated countercyclical (procyclical) income risk raises (lowers)  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ .

that combine cyclical inequality and income risk, such as the tractable heterogeneous-agent New Keynesian (THANK) model (see, e.g., [Bilbiie, 2025](#)) or our quantitative model, cyclical income inequality operates jointly through the PI and RIR channels. In an analytical variant of the model, one can show that countercyclical inequality strengthens the RIR channel relative to the PI channel when the share of hand-to-mouth households  $\lambda$  and the persistence of saver status  $s$  satisfy  $\lambda + s > 1$ . Specifically, countercyclical (resp. procyclical) inequality then lowers (resp. raises)  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ .

Introducing hand-to-mouth households in our quantitative model attenuates both the PI and RIR channels, since a larger share of households responds little to intertemporal incentives. In general equilibrium, however, this conclusion can be overturned. In particular, [Werning \(2015\)](#) and [Bilbiie \(2025\)](#) show that when income risk and inequality are countercyclical, the induced redistribution and income-composition effects can move the economy away from the “*as if*” representative-agent benchmark and strengthen both channels. Accordingly, eliminating hand-to-mouth households weakens both channels. In our framework, and for our calibration, permanent-income effects regain prominence and monetary policy must react more aggressively to achieve the same degree of stabilization. Quantitatively, this shifts both regime thresholds upward, to  $\underline{\phi}_\pi = 1.47$  and  $\bar{\phi}_\pi = 3.36$ . Consistent with the stronger PI channel,  $\rho^*$  increases and the peak multiplier rises by nearly 50 percent (0.6 versus 0.4 in the benchmark).

Under our parameterization of earnings incidence ( $\chi$ ), income inequality is countercyclical, implying a strong RIR channel ( $\zeta_r \geq \zeta_f$ ). Moving inequality from countercyclical to acyclical and then to procyclical should therefore tilt transmission away from the RIR channel and toward the PI channel—calling for a more aggressive policy rule (higher  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$ ) and delivering a larger  $\rho^*$ . In the benchmark calibration, however, the cyclicity of inequality has a quantitatively negligible effect on these objects.

This near-neutrality should not be interpreted as a general irrelevance of heterogeneity. Rather, it is specific to the combination of our calibration and the form of nominal rigidities. In particular, shutting down nominal wage rigidities revives the forces emphasized above. In the flexible-wage version of the model, making inequality acyclical (resp. procyclical) raises  $\rho^*$  by about one-third (resp. two-thirds) and reduces the peak multiplier by about 13 percent (resp. 20 percent). With flexible wages and countercyclical inequality, a positive demand shock raises the real wage and marginal cost; through the NKPC this increases inflation and induces a stronger policy response, reinforcing the RIR channel. As inequality becomes less countercyclical—and eventually procyclical—this RIR amplification fades and transmission shifts toward the PI channel. By contrast, when nominal wages are sticky, real-wage and marginal-cost responses to a demand shock are muted. Monetary policy then needs to respond less aggressively, weakening the RIR channel to begin with and leaving less scope for the cyclicity of inequality to matter.

Finally, a number of hybrid models that combine household heterogeneity with deviations from FIRE (see, e.g., [Pfäuti and Seyrich, 2023](#), [Meichtry, 2023](#), [Gallegos, 2024](#)) reveal how market incompleteness and cognitive frictions, sticky or dispersed information jointly influence monetary policy transmission. Specifically, these elements amplify

the RIR channel while dampening the PI channel. Consequently, they lead to a reduction in  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$  relative to a representative agent model.

**Other Mechanisms:** Other mechanisms not included in our quantitative model are equally likely to affect the determination of monetary regimes and the  $\rho^*$ . For example, recent studies (see, e.g., [Del Negro et al., 2023](#)) incorporate perpetual youth structures (e.g., [Blanchard, 1985](#), [Yaari, 1965](#)) into medium-scale DSGE models, primarily to address the forward guidance puzzle. In these frameworks, stochastic death probabilities act on three key margins. First, they lead to discounted DIS equations ( $\zeta_f < 1$ ), reflecting households' shorter effective planning horizons. Second, they reduce the sensitivity of current inflation to inflation expectations ( $\beta_f < \beta$ ). Third, they steepen the slope of the NKPC. These findings align with insights from [Eggertsson et al. \(2019\)](#), which highlight how life-cycle effects reduce the persistence of expectations, thereby weakening the efficacy of forward guidance policies. Demographic considerations in these models generally increase  $\underline{\phi}_\pi$ , while their effects on  $\bar{\phi}_\pi$  and  $\rho^*$  depend heavily on the degree of price stickiness. Likewise, borrowing constraints and default risk offer another mechanism through which household heterogeneity affects the relative potency of the PI and RIR channels.

Recent extensions of CRRA preferences introduce wealth dependence, reflecting the idea that relative wealth influences utility. For instance, [Michaillat and Saez \(2021\)](#) addresses anomalies observed at the zero lower bound by incorporating wealth in the utility function into a RANK framework. In this setup, a discounted DIS equation emerges when the marginal utility of wealth is positive, effectively dampening the PI channel. Hence, just like a higher EIS, wealth in the utility function decreases  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ .

Models such as [Beaudry and Portier \(2018\)](#) introduce information asymmetries between borrowers and lenders, resulting in borrowing costs that depend on debt levels. More specifically, borrowing constraints cause  $\zeta_f < 1$  and bound  $\zeta_r$  from above—strictly below the EIS. While these dynamics alter household consumption behavior, they leave monetary policy thresholds and the  $\rho^*$  essentially unchanged compared to standard CRRA models, as the ratio  $\zeta_f/\zeta_r$  is independent of the sensitivity of borrowing rates to debt.

Overall, the analysis shows that systematic monetary-policy persistence is governed by the relative strength of the PI and RIR propagation channels—strengths that are endogenous to the models core frictions: household preferences and heterogeneity (via cyclical income inequality and income risk), behavioral and life-cycle forces, borrowing constraints, nominal rigidities, and expectation formation. Through this lens, mechanisms introduced to mitigate the forward-guidance puzzle typically do so by effectively discounting the DIS equation and thereby weakening the PI channel, while mechanisms designed to restore amplification in RANK-style environments work primarily by strengthening the RIR channel. Either way, regime boundaries shift inward—lowering  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ —and approaches that tackle the “Catch-22” by operating on both channels compress the regime space further, pushing down thresholds and the  $\rho^*$ .

### 4.3 Monetary and Fiscal Policy Interactions

So far the analysis has shown that the interaction between monetary policy and the persistence of dynamic IS shocks crucially determines the short-run output response. We now explore how this relationship is affected by an active *fiscal policy*, and in particular, by the *size of government* (see, e.g. Galí, 1994), the *degree of countercyclicality* of its policy (see, e.g. Leeper et al., 2010, Fève et al., 2013) and the financing of government expenditures.

We first consider a version of our quantitative model in which we shut down household heterogeneity ( $\lambda = 0$ ,  $s = 1$ ), consider a log-utility function, ignore capital accumulation ( $\alpha = 0$ ), dispense from the working capital assumption, nominal wage rigidity ( $\theta_w = 0$ ), and price indexation ( $\chi_p = 0$ ), simplify the monetary policy rule ( $\phi_i = \phi_y = 0$ ) and shut down behavioral expectations, effectively going back to our baseline RANK model. We consider a government that adopts a government spending policy of the type:

$$G_t = s_g Y (Y_t/Y)^{\gamma_g}, \quad (13)$$

where  $\gamma_g \in \mathbb{R}$  controls the cyclicity of the policy and  $s_g = G/Y$  denotes the share of government spending in the deterministic steady state. The flow of public expenditures is financed through lump-sum taxation. The DIS equation then becomes:

$$y_t = \mathbb{E}_t[y_{t+1}] - \Gamma_g(i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}) + \Gamma_g \xi_t,$$

where  $\Gamma_g \equiv (1 - s_g)/(1 - \gamma_g s_g) \in [0, 1]$  scales the sensitivity of private demand to the real interest rate in the DIS equation and therefore governs the strength of the RIR channel identified above (note that  $\Gamma_g$  plays a role analogous to  $\zeta_r$  in Proposition 9). Let us first consider the case of acyclical public spending, *i.e.*  $\gamma_g = 0$ . In that case,  $\Gamma_g$  reduces to  $1 - s_g$ . The mere presence of government spending ( $s_g > 0$ ) reduces the share of aggregate demand that is subject to intertemporal substitution. As a result, a given movement in the real interest rate translates mechanically into a smaller response of output. Put differently, holding everything else equal, a higher fiscal share makes the economy less interest-rate sensitive. This dampening is reinforced on the pricing side because the NKPC slope increases when  $s_g$  is larger, so that prices absorb a larger fraction of demand disturbances and quantity adjustments are correspondingly muted.<sup>32</sup>

Let us not consider the case of state-contingent government spending, *i.e.*  $\gamma_g \neq 0$ . In that case,  $\Gamma_g = (1 - s_g)/(1 - \gamma_g s_g)$  captures how cyclicity further reshapes intertemporal transmission. When spending is countercyclical ( $\gamma_g < 0$ ), government purchases rise in downturns and fall in booms, stabilizing aggregate demand directly; equivalently, the effective wedge  $\Gamma_g$  declines relative to the acyclical case, so that the DIS equation becomes less responsive to movements in the real interest rate. By contrast, procyclical policy ( $\gamma_g > 0$ ) amplifies demand fluctuations and raises  $\Gamma_g$  relative to  $1 - s_g$ . In either case, cyclicity changes how much stabilization must be delivered by monetary policy; and, by operating through  $\Gamma_g$ , it directly weakens (or strengthens) the RIR channel

<sup>32</sup>In that case, we have  $\kappa_g = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \left( \Gamma_g^{-1} + (1-\alpha+\varphi)/\alpha \right)$ , implying that  $\partial(\Gamma_g \kappa_g)/\partial s_g \geq 0$  for  $\gamma_g \geq 1$  and  $\partial(\Gamma_g \kappa_g)/\partial \gamma_g > 0$ .

depending on whether fiscal policy is countercyclical (or procyclical). This simple modification leads to the following result.

**PROPOSITION 10.** *The results of Proposition 3 remain valid once the thresholds and turning point are adjusted for the fiscal share,  $s_g$ , and the degree of countercyclicality,  $\gamma_g$ , as:*

$$\underline{\phi}_\pi = \frac{1}{\beta} \left( 1 + \frac{(1-\beta)^2}{\Gamma_g \kappa_g} \right), \quad \bar{\phi}_\pi = \frac{1}{\beta} \left( 1 + \frac{1}{\Gamma_g \kappa_g} \right) \quad \text{and} \quad \rho^* = \frac{1}{\beta} \left( 1 - \sqrt{\Gamma_g \kappa_g (\beta \phi_\pi - 1)} \right).$$

Consider first the case where the government simply finances a constant flow of government spending—i.e.,  $\gamma_g = 0$ ,  $s_g > 0$  and  $\Gamma_g = 1 - s_g$ . In this case, as in Galí (1994), government size acts as an automatic stabilizer. A larger government share  $s_g$  attenuates the RIR channel, thereby amplifying the relative strength of the permanent income channel. As a result, monetary policy must be more aggressive—that is, characterized by a higher  $\phi_\pi$ —to achieve the same degree of stabilization. Both policy thresholds,  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$ , therefore rise with  $s_g$ . However, the upper bound increases faster than the lower bound, hence broadening the range of monetary policy configurations for which the intermediate, hump-shaped regime arises—that is, the range where  $0 < \rho^* < 1$ . In such economies, moderate monetary activism interacts with fiscal cushioning to generate a rich, non-monotonic relationship between persistence and output effects.

When fiscal policy is countercyclical—i.e.,  $\gamma_g < 0$ ,  $s_g > 0$ —the same logic carries through once one considers  $\Gamma_g = (1 - s_g)/(1 - \gamma_g s_g)$ . Countercyclical spending amplifies the insulating role of government policy, further weakening the RIR channel and requiring an even stronger monetary response to discipline demand. In this sense, fiscal activism not only shifts the level of the thresholds, but also reshapes the terrain on which monetary policy operates.

Table 2 further explores the interaction between fiscal and monetary policies in our quantitative model. In order to better highlight the role of redistribution in shaping the monetary regimes, we consider a version of our baseline model in which we shutdown the cyclicity of income inequalities. This corresponds to our benchmark economy in the Table. In this case, the monetary policy thresholds are respectively given by  $\underline{\phi}_\pi = 1.32$  and  $\bar{\phi}_\pi = 3.35$ —the acyclical income inequality case reported in Table 1. We then assume that the government spends resources following rule (13). These expenditures are financed either by lump sum taxation— $G_t = \tau_t$ —or by issuing public debt. In the latter case, debt evolves as  $R_t^{-1} B_{t+1} = B_t + G_t - \tau_t$ .

Following Galí et al. (2007), the lump-sum tax,  $\tau_t$ , relates to public debt and government spending as  $\hat{\tau}_t = \phi_b \hat{b}_t + \phi_g \hat{g}_t$ , where  $\hat{\tau}_t = (\tau_t - \tau)/Y$ ,  $\hat{g}_t = (G_t - G)/Y$  and  $\hat{b}_t = (B_t/P_{t-1} - B/P)/Y$ .<sup>33</sup> In both cases, following Bilbiie et al. (2022) and Ferriere and Navarro (2024), we retain a progressive element in the tax system. Specifically, we allow hand-to-mouth households to contribute less than their population share by assuming  $\lambda \tau_t^h = \lambda \bar{\tau}^h + \gamma_h (\tau_t - \tau)$ . When  $\gamma_h = \lambda$ , the tax system is non-progressive; when  $\gamma_h < \lambda$ ,

<sup>33</sup>We borrow directly  $\phi_b = 0.30$  and  $\phi_g = 0.12$  from Galí et al. (2007). Furthermore, in what follows we will consider an economy in which, in the steady state, there is no public debt ( $B = 0$ ) and the tax burden,  $\tau$  is equally shared by households ( $\tau^H = \tau^S = \tau$ ).

(resp.  $\gamma_h > \lambda$ ) hand-to-mouth agents contribute a smaller (resp. larger) share of marginal tax adjustments making the tax system progressive (resp. regressive). In the sequel, we will consider the two extreme cases where  $\gamma_h = 0$  where only savers contribute to financing the tax—a fully progressive system— and  $\gamma_h = 1$ —a fully regressive system.

TABLE 2. Monetary Policy Regimes and Fiscal Policy

| MODEL VARIANT     | PARAMETERS |            | TAX FINANCED           |                  | DEBT FINANCED          |                  |
|-------------------|------------|------------|------------------------|------------------|------------------------|------------------|
|                   | $\gamma_g$ | $\gamma_h$ | $\underline{\phi}_\pi$ | $\bar{\phi}_\pi$ | $\underline{\phi}_\pi$ | $\bar{\phi}_\pi$ |
| Benchmark Model   | –          | –          | 1.32                   | 3.35             | 1.32                   | 3.35             |
| Acyclical G       | 0.0        | $\lambda$  | 1.28                   | 3.49             | 1.28                   | 3.49             |
| Procyclical G     | 1.0        | $\lambda$  | 1.34                   | 3.32             | 1.34                   | 3.32             |
| – Progressive     | 1.0        | 0.0        | 1.35                   | 3.23             | 1.35                   | 3.23             |
| – Regressive      | 1.0        | 1.0        | 1.28                   | 3.51             | 1.28                   | 3.32             |
| Countercyclical G | -0.6       | $\lambda$  | 1.25                   | 3.59             | 1.25                   | 3.62             |
| – Progressive     | -0.6       | 0.0        | 1.24                   | 3.63             | 1.24                   | 3.63             |
| – Regressive      | -0.6       | 1.0        | 1.28                   | 3.48             | 1.29                   | 3.59             |

We start by exploring the effect of introducing an acyclical government spending in the model,  $G_t = G = s_g Y$  where we set the government share to its empirical counterpart ( $s_g = 0.2$ ). By mechanically absorbing part of the increase in demand, the mere presence of government purchases dampens the response of output, which mutes the induced inflationary pressure and weakens the endogenous propagation of the DIS shock. This stabilizing effect re-maps the monetary-policy regime boundaries. The upper threshold  $\bar{\phi}_\pi$  rises from 3.35 to 3.49 as by stabilizing inflation and expected inflation, fiscal absorption compresses movements in expected real rates; entering the RIR-dominant region therefore requires a more aggressive Taylor coefficient, pushing up  $\bar{\phi}_\pi$  (see also Proposition 10 and footnote 32). Symmetrically, in presence of nominal wage rigidities, by cushioning the increase in output (and therefore weakening the PI component of the DIS shock), government spending makes it harder for the PI channel to dominate unless monetary policy is sufficiently accommodative—hence the lower threshold  $\underline{\phi}_\pi$  falls from 1.32.<sup>34</sup> These forces strengthen when fiscal policy leans against the cycle, in which case we set  $\gamma_g = -0.6$  (see Fève et al. (2013)). Under countercyclical spending, the thresholds shift further— $\underline{\phi}_\pi$  drops to 1.25 and  $\bar{\phi}_\pi$  increases to 3.6—because fiscal contractions in booms (and expansions in slumps) directly offset the demand component of the shock, further weakening the PI channel and requiring a higher  $\phi_\pi$  for RIR effects to dominate. Procyclical spending ( $G_t = s_g Y_t$ , see Galí (1994)) works in the opposite direction: by reinforcing the output response, it restores inflationary pressure and compresses the RIR-dominant region, raising  $\underline{\phi}_\pi$  and lowering  $\bar{\phi}_\pi$  (to 1.34 and 3.32).

<sup>34</sup>Note that absent nominal wage rigidities, we are essentially back to Proposition 10, and the asymmetry between the variation in the upper and lower threshold disappears, both increase with the government share.

In presence of heterogeneous households, financing of government spending and tax incidence become first-order because they determine who absorbs the fiscal adjustment on impact. Two patterns stand out in Table 2. First, debt versus tax financing matters precisely when fiscal policy moves systematically with the cycle (most clearly under countercyclical spending) and when the marginal tax burden falls on hand-to-mouth households. Debt financing smooths taxes over time, muting contemporaneous disposable-income movements for hand-to-mouth households; consequently, the regime boundaries are close across financing modes in many rows, but begin to diverge when countercyclicality and regressive incidence interact (e.g., the larger rise in  $\bar{\phi}_\pi$  under debt-financed, regressive countercyclical policy).

Second, tax progressivity mainly matters under tax financing because it governs the contemporaneous redistribution of the fiscal adjustment. Under countercyclical spending, a positive DIS shock raises output while  $G_t$  and (under tax finance)  $\tau_t$  fall. If the tax system is regressive ( $\gamma_h = 1$ ), hand-to-mouth households receive a larger tax cut on impact, which translates one-for-one into higher consumption. This strengthens aggregate demand and inflation: the economy requires a more aggressive monetary response to prevent inflation from becoming dominant, raising  $\underline{\phi}_\pi$  (from 1.25 to 1.28) and reducing the degree of aggressiveness needed for RIR effects to take over (lowering  $\bar{\phi}_\pi$  from 3.59 to 3.48 under tax finance). Under a fully progressive system ( $\gamma_h = 0$ ), the marginal tax adjustment bypasses hand-to-mouth households; the contemporaneous consumption boost is smaller, inflationary pressure is weaker, and the RIR-dominant region expands (lower  $\underline{\phi}_\pi$  and higher  $\bar{\phi}_\pi$ ). Under procyclical spending the logic reverses because the fiscal response reinforces the boom; shifting marginal taxes toward hand-to-mouth agents then tempers, rather than amplifies, the demand impulse.

Overall, Table 2 shows that fiscal stabilization operates primarily by tilting the balance between the PI and RIR channels, while tax incidence matters through the marginal propensity to consume of the agents who bear the fiscal adjustment. Debt finance attenuates these redistribution-driven channels by spreading the adjustment intertemporally, leaving progressivity most consequential under contemporaneous tax finance.

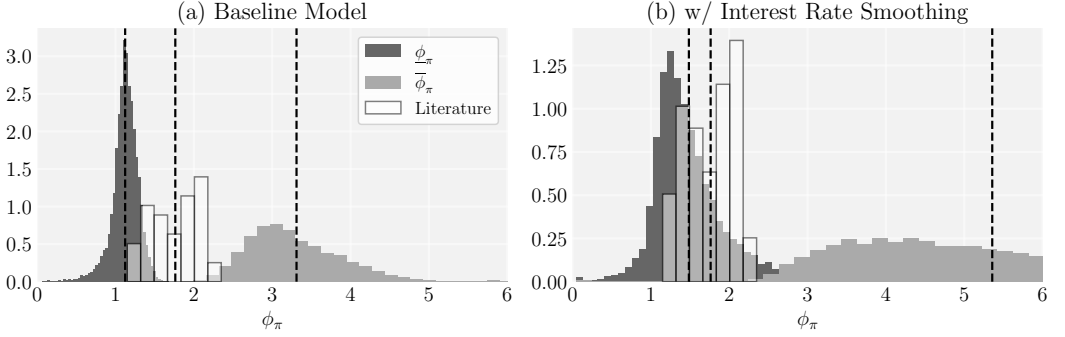
#### 4.4 Monetary Policy Regimes in a Population of Models: A Monte Carlo Assessment

As informative as it is about the key forces driving the results, the preceding analysis does not inform about the empirical relevance of the three monetary policy regimes, which is ultimately a quantitative question. To evaluate this, we conduct a Monte Carlo simulation with 10,000 draws from uniform distributions over key structural parameters—*i.e.* preferences, heterogeneity, taxation, adjustment costs, nominal rigidities (in both prices and wages), monetary policy and expectation formation mechanisms—hence assessing a population of models. The full set of parameter ranges is reported in Online Appendix C.5. For each draw, we compute the two thresholds defining the low, intermediate, and high monetary policy regimes,  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$ , as well as the persistence  $\rho^*$  that maximizes the output multiplier.

Panel (a) of Figure 3 reports the distribution of the two thresholds across the Monte Carlo samples (gray histograms) in our baseline version (without interest rate smooth-

ing), alongside the empirical distribution of  $\phi_\pi$  coefficients drawn from the literature.<sup>35</sup> The average values of the simulated thresholds,  $\underline{\phi}_\pi = 1.12$  and  $\bar{\phi}_\pi = 3.31$ , bracket the literatures average estimated monetary policy reaction coefficient,  $\phi_\pi = 1.76$ . This suggests that, in most estimated models, monetary policy operates within an intermediate regime, where both the PI and RIR channels are simultaneously active.

FIGURE 3. Monetary Regimes: Distributions across Population of Models



*Note:* The vertical dashed lines in Panel (a) correspond to the average values of  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and the empirical average of  $\phi_\pi$  reported in the literature.

The economic implication is twofold. First, the intermediate regime is not a knife-edge case: the distributions of  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  are clearly separated, with no overlap, underscoring that this regime occupies a wide and empirically relevant region of parameter space. Second, the figure indicates that most of the empirical estimates of  $\phi_\pi$  fall within this intermediate region. As such, the trade-off between stimulating current demand (via the RIR channel) and anchoring expectations (via the PI channel) is not only conceptually relevant but quantitatively central in monetary transmission as captured by standard models. Interestingly, while some estimates lie near or below the lower threshold  $\underline{\phi}_\pi$ , corresponding to models dominated by the PI channel, only very few lie above the upper threshold  $\bar{\phi}_\pi$ . This suggests that regimes in which only the RIR channel drives the multiplier are almost not represented in the empirical literature, reinforcing the robustness of the intermediate case.<sup>36</sup>

Panel (b) of the Figure replicates the same exercise for an economy in which the central bank smooths the nominal interest rate. We obtain very similar results. Two observations are worth reporting. First, as suggested by Proposition 6 and Corollary 1, the distributions of  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$  shift towards the right, with now an average of 1.48 and 5.36 respectively (versus 1.12 and 3.31 in our baseline version). Second, there is more overlap

<sup>35</sup>The complete list of articles is available in Section E of the online appendix. It shall be clear to the reader that all these models however hinge on possibly different mechanisms and are not necessarily estimated on the same datasets or sample.

<sup>36</sup>The same results are obtained when we allow for deviations from the working capital assumption, see Section D in the Online Appendix.

between the distributions of the lower threshold and distribution of the estimated monetary policy reaction coefficient, indicating that, in presence of interest rate smoothing, the PI regime is more prevalent. It shall however be noted that the literatures average estimated monetary policy reaction coefficient,  $\phi_\pi = 1.76$ , is well below the average of the lower threshold, indicating that the intermediate regime survives as a robust outcome.

Interestingly, these findings shed further light on results obtained by [Bianchi \(2013\)](#)<sup>37</sup> who estimated monetary rules allowing for regime switches between hawkish and dovish policies. At the mode, the reaction to inflation in the dovish regime is estimated to be 1.1, which, in our case, places it in the PI regime while its value in the hawkish regime, 2.6, places it in the intermediate regime.

FIGURE 4. Distribution of  $\rho^*$  across Population of Models: Interest Rate Smoothing

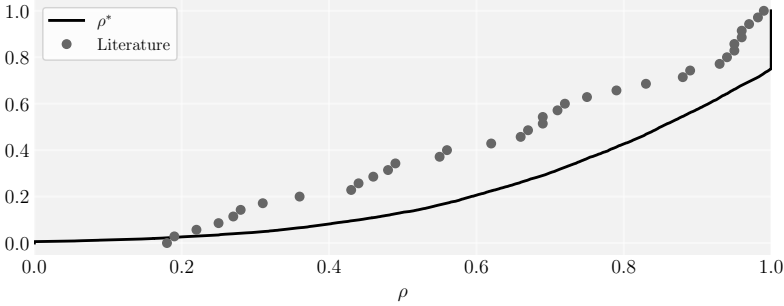


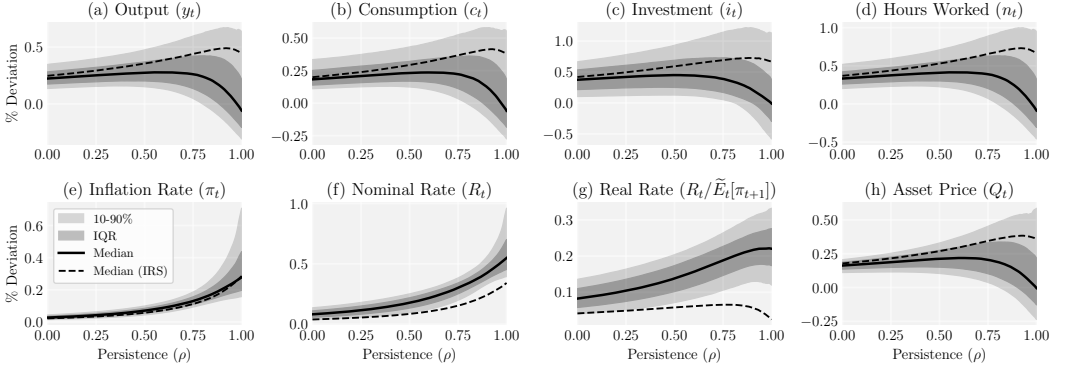
Figure 4 reports the empirical cumulative distribution function of  $\rho^*$  across the population of models with interest rate smoothing, alongside point estimates of DIS shock persistence obtained from the empirical literature. The graph indicates that estimated models all estimate persistence parameters of DIS shocks significantly below the  $\rho^*$  (an average of 0.64 vs 0.83), and, hence confirming previous results, mostly lie in the intermediate monetary regime. This indicates that estimated models lie on the upward sloping part of the locus mapping persistence to output multiplier, which implies that, following a marginal increase in persistence, the central bank still has the ability to increase its aggressiveness towards inflation and achieve the same level of output stabilization.

Finally, Figure 5 explores how the short-run multipliers vary with the persistence of DIS demand shocks across our representative population of 10,000 of models. The figure plots the median impact response of key macroeconomic variables to a standardized 25 basis point DIS shock as a function of its persistence,  $\rho \in [0, 1]$ . In addition, we display the interquartile range (dark gray) and the 10-90% percentiles (light gray), highlighting the dispersion in the population of models.

The results reveal a robust hump-shaped relationship between the persistence of the shock and the impact multiplier across all major aggregates —output, consumption, investment, hours worked, and asset prices (Tobin's  $Q$ ). The multiplier initially increases

<sup>37</sup>See also [Bianchi and Ilut \(2017\)](#) in the case of an interaction between fiscal and monetary policies.

FIGURE 5. Impact Multipliers across Population of Models



Note: Impact IRFs are expressed as percentage deviations from steady state. The nominal interest rate, inflation, and real interest rate responses are reported in annualized terms. IRS denotes interest rate smoothing, IQR is the interquartile range.

with  $\rho$ , as the PI channel dominates, and reaches a peak at intermediate persistence levels, and subsequently declines as the RIR channel gains traction. This non-monotonic response persists even in the presence of backward-looking mechanisms such as capital accumulation, adjustment costs, nominal indexation (prices and wages), and cognitive discounting, demonstrating the resilience of the theoretical mechanism derived in Section 2. Likewise, the figure also reveals that this hump-shaped pattern also extends to economies where the monetary authorities smooth the evolution of the interest rate (dashed line). In this latter case though, the peak of the hump is reached for more persistent DIS shocks—*i.e.* the  $\rho^*$  is shifted towards 1, indicating, not surprisingly, that interest rate smoothing weakens the RIR channel by spreading the response of monetary authorities over time.

Importantly, the hump-shaped pattern is not merely a feature of the median or average response. It holds across the distribution of parameterizations. At every decile, the output multiplier displays a peak at intermediate levels of persistence. This indicates that the non-monotonicity is a generic property of this class of models, rather than an artifact of a particular calibration. The multiplier regain a monotone shape only in the top percentiles—when monetary policy is dovish enough to place the economy in the dominant PI regime.

The policy implication of this analysis is sharp: multipliers are not pinned down solely by how aggressively the Central Bank targets inflation, but by where policy lies in the regime space given the economies underlying frictions. Consequently, empirically plausible adjustments in the stance of policy can induce first-order changes in amplification, and the relationship between persistence and policy potency is generally non-linear—intermediate persistence can generate disproportionate effects. At the same time, overly aggressive inflation stabilization may blunt transmission by neutralizing the intertemporal trade-off, whereas economies with a strong RIR channel or sufficiently flexible prices leave less room for demand-side stabilization and make a mis-calibrated stimulus more likely to be self-defeating.

## 5. Conclusion

A common wisdom in macroeconomics holds that more persistent demand shocks generate larger output responses: forward-looking households, anticipating a sustained rise in lifetime resources, front-load spending and amplify the initial expansion. This paper shows that this breaks down in standard New Keynesian settings with an active monetary policy rule because persistence is not simply a scaling device for multipliers—it governs the balance between two opposing forces.

On the one hand, a permanent income channel implies that more persistent shocks raise the present value of resources and stimulate current demand. On the other hand, a real interest rate channel works in the opposite direction: greater persistence makes inflationary pressures longer lasting, increases expected inflation, and—under an active Taylor rule—leads agents to anticipate a tighter real interest rate path that restrains demand. The interaction between these forces depends critically on the central bank's systematic response to inflation. When policy is sufficiently accommodative, the permanent-income channel dominates and multipliers increase with persistence. When policy is sufficiently aggressive, the real interest rate channel dominates and multipliers decline. Between these extremes lies an empirically relevant intermediate region, where the output multiplier is hump-shaped in persistence. Then, a transitory demand shock can generate a larger contemporaneous expansion than a near-permanent one—not because it is stronger, but because it triggers a fundamentally different expected policy response once inflation is expected to remain elevated. This three-regime characterization, derived analytically in a textbook RANK model, proves robust across alternative policy rules, horizons, and a broad class of medium-scale DSGE models—highlighting that persistence reshapes, rather than simply scales, the transmission of demand shocks.

Taken together, these results underscore that understanding whether demand pressures are transitory or persistent is not merely a forecasting question – it is central to the design and calibration of the policy response itself.

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# The Power of Persistence

## How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes

ONLINE APPENDIX  
(NOT FOR PUBLICATION)

### APPENDIX A: Proofs of Propositions

*Proof Proposition 1.* Substituting the Taylor rule into the DIS equation (1) results in

$$y_t = \mathbb{E}_t [y_{t+1}] - (\phi_\pi - 1) \mathbb{E}_t [\pi_{t+1}] + \xi_t .$$

We guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \xi_t$  and  $\pi_t = \mathcal{M}_\pi \xi_t$ , such that the previous equation together with the NKPC can be rewritten as

$$\begin{aligned} (1 - \rho) \mathcal{M}_y &= -(\phi_\pi - 1) \rho \mathcal{M}_\pi + 1 , \\ (1 - \beta \rho) \mathcal{M}_\pi &= \kappa_y \mathcal{M}_y . \end{aligned}$$

Substituting the latter into the former equation and rearranging yields

$$\mathcal{M}_y = \frac{1 - \beta \rho}{(1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)} > 0 .$$

Impact multipliers for inflation, the nominal interest rate, and the real interest rate are

$$\begin{aligned} \mathcal{M}_\pi &= \frac{\kappa_y}{(1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)} > 0 , \\ \mathcal{M}_i &= \frac{\rho \phi_\pi \kappa_y}{(1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)} \geq 0 , \\ \mathcal{M}_r &= \frac{(\phi_\pi - 1) \rho \kappa_y}{(1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)} \geq 0 , \end{aligned}$$

where the sign of  $\mathcal{M}_r$  follows from the Taylor principle  $\phi_\pi > 1$ . □

*Proof Proposition 2.* Taking the derivative of  $\mathcal{M}_r$  with respect to  $\rho$  yields

$$\frac{\partial \mathcal{M}_r}{\partial \rho} = \frac{\kappa_y (\phi_\pi - 1) (1 - \beta \rho^2)}{[(1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)]^2} .$$

Defining  $D \equiv (1 - \rho)(1 - \beta \rho) + \rho \kappa_y (\phi_\pi - 1)$ , the previous expression simplifies to

$$D^2 \frac{\partial \mathcal{M}_r}{\partial \rho} = \kappa_y (\phi_\pi - 1) (1 - \beta \rho^2) ,$$

which, given  $\beta \in (0, 1)$  and  $|\rho| < 1$ , is positive as long as the Taylor principle is satisfied ( $\phi_\pi > 1$ ). □

*Proof Proposition 3.* Taking the derivative of  $\mathcal{M}_y$  with respect to  $\rho$  yields

$$\frac{\partial \mathcal{M}_y}{\partial \rho} = \frac{-\beta[(1-\rho)(1-\beta\rho) + \rho\kappa_y(\phi_\pi - 1)] + (1-\beta\rho)[(1-\beta\rho) + \beta(1-\rho) - \kappa_y(\phi_\pi - 1)]}{[(1-\rho)(1-\beta\rho) + \rho\kappa_y(\phi_\pi - 1)]^2}.$$

Defining  $D \equiv (1-\rho)(1-\beta\rho) + \rho\kappa_y(\phi_\pi - 1)$ , the previous expression simplifies to

$$D^2 \frac{\partial \mathcal{M}_y}{\partial \rho} = (1-\beta\rho)^2 - \kappa_y(\phi_\pi - 1).$$

As a result, the sign of the previous expression is determined by a second order polynomial in  $\rho$ , i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = \text{sgn}(a\rho^2 + b\rho + c),$$

where the auxiliary parameters are given by

$$a \equiv \beta^2, \quad b \equiv -2\beta, \quad c \equiv 1 - \kappa_y(\phi_\pi - 1).$$

The corresponding roots are

$$\rho^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a},$$

where the discriminant  $\Delta \equiv b^2 - 4ac$  is given by

$$\Delta = 4\beta^2 - 4\beta^2[1 - \kappa_y(\phi_\pi - 1)] = 4\beta^2\kappa_y(\phi_\pi - 1).$$

Under Local determinacy, we necessarily have  $\Delta > 0$ , such that the polynomial admits two real-valued roots that are given by

$$\rho^{+,-} = \frac{2\beta \pm \sqrt{4\beta^2\kappa_y(\phi_\pi - 1)}}{2\beta^2} = \beta^{-1} \left( 1 \pm \sqrt{\kappa_y(\phi_\pi - 1)} \right).$$

Since  $\beta \in (0, 1)$  and  $\phi_\pi > 1$ , the only feasible candidate on  $\rho \in (0, 1)$  is

$$\rho^* \equiv \rho^- = \beta^{-1} \left( 1 - \sqrt{\kappa_y(\phi_\pi - 1)} \right).$$

Note that  $\rho^* \in (0, 1)$  if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , where we have

$$\underline{\phi}_\pi \equiv 1 + (1-\beta)^2\kappa_y^{-1}, \quad \text{and} \quad \bar{\phi}_\pi \equiv 1 + \kappa_y^{-1}.$$

As  $f(\rho) \equiv a\rho^2 + b\rho + c$  is a strictly convex function in  $\rho$ , statement (b) of Proposition 3 follows.

To show statement (a) of Proposition 3, notice that  $\frac{\partial \mathcal{M}_y}{\partial \rho}$  is strictly decreasing in  $\phi_\pi$ . Hence, substituting in the lower bound  $\underline{\phi}_\pi$ , we obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)\bigg|_{\phi_\pi = \underline{\phi}_\pi} = \beta^2\rho^2 - 2\beta\rho + 1 - \kappa_y(\underline{\phi}_\pi - 1) = (1-\beta\rho)^2 - (1-\beta)^2 > 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  for all  $\phi_\pi \leq \underline{\phi}_\pi$ .

Finally, to show statement (c) we analogously substitute in for the upper bound  $\bar{\phi}_\pi$  to obtain

$$\text{sgn} \left( \frac{\partial \mathcal{M}_y}{\partial \rho} \right) \Big|_{\phi_\pi = \bar{\phi}_\pi} = (1 - \beta\rho)^2 - 1 \leq 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . Notice that the previous inequality is strict for  $\rho > 0$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$  for all  $\phi_\pi \geq \bar{\phi}_\pi$ .  $\square$

**COROLLARY 2.** *We have*

$$\mathcal{M}_y(0) \lesseqgtr \lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) \iff \phi_\pi \lesseqgtr 1 + \frac{1 - \beta}{\kappa_y}.$$

*Proof Corollary 2.* Direct calculation yields  $\mathcal{M}_y(0) = 1$  and  $\lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) = \frac{1 - \beta}{\kappa_y(\phi_\pi - 1)}$  such that

$$\mathcal{M}_y(0) \lesseqgtr \lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) \iff \frac{\kappa_y(\phi_\pi - 1)}{1 - \beta} \lesseqgtr 1,$$

and the result then follows.  $\square$

*Proof Proposition 4.* Direct calculation of  $\frac{\partial \mathcal{M}_\pi}{\partial \rho}$  yields

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho} = \frac{\kappa_y}{D^2} [1 + \beta - \kappa_y(\phi_\pi - 1) - 2\beta\rho],$$

where  $D \equiv (1 - \beta\rho)(1 - \rho) + \kappa_y\rho(\phi_\pi - 1)$ . Hence, we obtain:

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho} \gtrless 0 \iff \rho \lesseqgtr \rho^*, \text{ with } \rho^* = \frac{1 + \beta - \kappa_y(\phi_\pi - 1)}{2\beta}.$$

Then,  $\rho^* \in (0, 1)$  as long as  $\underline{\phi}_\pi^\pi < \phi_\pi < \bar{\phi}_\pi^\pi$ , with  $\underline{\phi}_\pi^\pi = 1 + \frac{1 - \beta}{\kappa_y}$  and  $\bar{\phi}_\pi^\pi = 1 + \frac{1 + \beta}{\kappa_y}$ . Note that

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho} \Big|_{\phi_\pi = \underline{\phi}_\pi^\pi} = 2\beta(1 - \rho) \geq 0,$$

implying that  $\frac{\partial \mathcal{M}_\pi}{\partial \rho} \geq 0$  for  $\phi_\pi < \underline{\phi}_\pi^\pi$  and  $\rho^* \rightarrow 1$ . Similarly, we have

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho} \Big|_{\phi_\pi = \bar{\phi}_\pi^\pi} = -2\beta\rho \leq 0,$$

implying that  $\frac{\partial \mathcal{M}_\pi}{\partial \rho} \leq 0$  for  $\phi_\pi > \bar{\phi}_\pi^\pi$  and  $\rho^* = 0$ . Finally, note that the inflation impact multiplier under a contemporaneous rule is given by:

$$\mathcal{M}_\pi = \frac{\kappa_y}{(1 - \rho)(1 - \beta\rho) + (\phi_\pi - \rho)\kappa_y} > 0,$$

which is strictly increasing in  $\rho$ .  $\square$

We offer a full statement of the Proposition 5. Specifically, under a forward-looking Taylor rule, three sub-cases arise depending on the aggressiveness of inflation and output feedback, i.e., the following proposition complements statement (a) in the main text by two sub-cases, (a.2)  $\phi_\pi = 1$  and (a.3)  $0 \leq \phi_\pi < 1$ .

**Proposition 5 (Full statement)** *The following results apply.*

(a) *Assume the monetary authorities react to the one-step ahead expected inflation and output gap, i.e.  $i_t = \bar{r} + \phi_\pi \mathbb{E}_t[\pi_{t+1}] + \phi_y \mathbb{E}_t[y_{t+1}]$ . The subsequent case distinction applies.*

(a.1) *Consider the case  $\phi_\pi > 1$ :*

- *If  $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , there exist two regime thresholds given by  $\underline{\phi}_\pi \equiv 1 + (1 - \phi_y)(1 - \beta)^2 \kappa_y^{-1}$  and  $\bar{\phi}_\pi \equiv 1 + (1 - \phi_y) \kappa_y^{-1}$  such that the relationship between the output impact multiplier and the degree of persistence (i) is increasing if  $\phi_\pi \leq \underline{\phi}_\pi$ , (ii) is decreasing if  $\phi_\pi \geq \bar{\phi}_\pi$ , and (iii) displays a hump-shaped pattern if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ . In this latter case, the degree of persistence that maximizes the output multiplier is given by  $\rho^* = \beta^{-1} \left(1 - \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y}\right)$ .*
- *If  $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , only the upper regime survives and the output multiplier decreases monotonically with  $\rho$ .*

(a.2) *Consider the case  $\phi_\pi = 1$ :*

- *If  $0 < \phi_y < 1$ , the output multiplier increases monotonically in  $\rho$ .*
- *If  $\phi_y = 1$ , the output multiplier is constant as  $\rho$  is varied.*
- *If  $1 < \phi_y < 2$ , the output multiplier decreases monotonically in  $\rho$ .*

(a.3) *Consider the case  $0 \leq \phi_\pi < 1$ :*

- *If  $\frac{1 - \phi_\pi}{1 - \beta} \kappa_y < \phi_y \leq \min\{1, 1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$  applies, the output multiplier increases monotonically in  $\rho$ .*
- *If  $\max\{1, \frac{1 - \phi_\pi}{1 - \beta} \kappa_y\} < \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ , there exist two regime thresholds given by  $\underline{\phi}_\pi \equiv 1 + (1 - \phi_y) \kappa_y^{-1}$  and  $\bar{\phi}_\pi \equiv 1 + (1 - \phi_y)(1 - \beta)^2 \kappa_y^{-1}$  such that the relationship between the output impact multiplier and the degree of persistence (i) is increasing if  $\phi_\pi \leq \underline{\phi}_\pi$ , (ii) is decreasing if  $\phi_\pi \geq \bar{\phi}_\pi$ , and (iii) displays a u-shaped pattern if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ . In this latter case, the degree of persistence that minimizes the output multiplier is given by  $\rho^* = \beta^{-1} \left(1 - \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y}\right)$ .*

(b) *Assume the Central Bank reacts to both the current inflation rate and the current output gap, i.e.  $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t$ . Then, the results of Proposition 3 remain valid once the thresholds and turning point are adjusted as:*

$$\underline{\phi}_\pi \equiv \beta^{-1} \left(1 + \frac{(1 - \beta)^2}{\kappa_y}\right), \bar{\phi}_\pi \equiv \beta^{-1} \left(1 + \frac{1}{\kappa_y}\right) \text{ and } \rho^* \equiv \beta^{-1} \left(1 - \sqrt{(\phi_\pi \beta - 1) \kappa_y}\right).$$

*Additional Discussion.* To begin with, consider the case (a.2), in which the RIR channel is eliminated as  $\phi_\pi = 1$ . There are three sub-cases. First, if  $0 < \phi_y < 1$ , the output multiplier monotonically increases in persistence. This is the case as the income expectation channel is positive. Second, if  $\phi_y = 1$  applies, the income expectation channel and the output feedback in the monetary policy rule offset each other, implying that output is constant in persistence. Third, if  $1 < \phi_y < 2$ , the output multiplier monotonically decreases in persistence. This is the case as the income expectation channel is negative. Notice that fixing the output feedback  $\phi_y = 1$  is pedagogical as it allows us to isolate and link the output response to the magnitude of inflation feedback  $\phi_\pi$  in a more transparent manner. Specifically, if  $\phi_\pi < 1$  holds, output increases monotonically in  $\rho$  as the real interest rate falls in  $\rho$ . Moreover, if additionally  $\phi_\pi = 1$  holds, the overall income expectation and the real interest channel are both eliminated such that output is constant. Finally, if  $\phi_\pi > 1$  applies, output falls in  $\rho$  as the real interest rate increases in  $\rho$ .

Second, consider the case (a.3), in which inflation feedback is moderate, i.e.,  $0 \leq \phi_\pi < 1$ . If additionally output feedback is moderate, i.e.,  $\frac{1-\phi_\pi}{1-\beta}\kappa_y < \phi_y < \min\{1, 1 + \beta^{-1}, 2 - \frac{\phi_\pi-1}{1-\beta}\kappa_y\}$ , output increases monotonically in  $\rho$ . This is the case, as the income expectation channel and the real interest rate channel move in the same direction. In contrast, if output feedback is sufficiently strong, i.e.,  $\max\{1, \frac{1-\phi_\pi}{1-\beta}\kappa_y\} < \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi-1}{1-\beta}\kappa_y\}$ , the income expectation channel flips sign and we obtain three regimes: output increases monotonically in  $\rho$  for  $\phi_\pi < \underline{\phi}_\pi$ , is u-shaped in  $\rho$  for  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , and decreases monotonically in  $\rho$  for  $\phi_\pi > \bar{\phi}_\pi$ . The u-shaped regime arises from two opposing channels: the income expectation channel now decreases output for higher persistence values, while the real interest rate channel increases output for higher persistence values. As such, the real interest rate channel dominates the income expectation channel for  $\phi_\pi < \underline{\phi}_\pi$ , whereas the income expectation channel dominates the real interest channel for  $\phi_\pi > \bar{\phi}_\pi$ .

*Proof Proposition 5 (Full Statement).* We prove each part sequentially.

**Part (a):** We proceed in three steps. First, we provide determinacy conditions. Second, we derive the output impact multiplier. Third, we study all cases individually.

*Local Determinacy.* The following lemma from [Bullard and Mitra \(2002\)](#) states conditions of local determinacy.

LEMMA 1 ([BULLARD AND MITRA \(2002\)](#)). *Under interest rate rules with forward expectations, the necessary and sufficient conditions for a rational expectations equilibrium to be unique are*

$$\phi_y < 1 + \beta^{-1}, \quad (\text{A.1})$$

$$\kappa_y(\phi_\pi - 1) + (1 + \beta)\phi_y < 2(1 + \beta), \quad (\text{A.2})$$

$$\kappa_y(\phi_\pi - 1) + (1 - \beta)\phi_y > 0. \quad (\text{A.3})$$

*Impact Multiplier.* Substituting the forward-looking Taylor rule into the DIS equation results in

$$y_t = (1 - \phi_y)\mathbb{E}_t[y_{t+1}] - (\phi_\pi - 1)\mathbb{E}_t[\pi_{t+1}] + \xi_t.$$

As before, we guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \xi_t$  and  $\pi_t = \mathcal{M}_\pi \xi_t$ , such that the previous equation together with the NKPC can be rewritten as

$$\begin{aligned} [1 - (1 - \phi_y)\rho] \mathcal{M}_y &= -(\phi_\pi - 1)\rho \mathcal{M}_\pi + 1, \\ (1 - \beta\rho) \mathcal{M}_\pi &= \kappa_y \mathcal{M}_y. \end{aligned}$$

Substituting the latter into the former equation gives us

$$[1 - (1 - \phi_y)\rho] \mathcal{M}_y = -(\phi_\pi - 1) \frac{\kappa_y \rho}{1 - \beta\rho} \mathcal{M}_y + 1,$$

which finally yields

$$\mathcal{M}_y = \frac{1 - \beta\rho}{[1 - (1 - \phi_y)\rho](1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho} > 0.$$

The impact output multiplier is strictly positive as the denominator,  $\mathcal{M}_y^d$ , can be rewritten as follows

$$\begin{aligned} \mathcal{M}_y^d &= (1 - \rho)(1 - \beta\rho) + \rho[\phi_y(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)] \\ &= (1 - \rho)(1 - \beta\rho) + \rho[\phi_y(1 - \beta) + \kappa_y(\phi_\pi - 1) + \phi_y\beta(1 - \rho)] > 0, \end{aligned}$$

where the strict inequality is due to the determinacy condition  $\phi_y(1 - \beta) + \kappa_y(\phi_\pi - 1) > 0$ .

*Comparative Statics.* Defining  $x \equiv [1 - (1 - \phi_y)\rho](1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho$  and taking the derivative of  $\mathcal{M}_y$  with respect to  $\rho$  yields

$$x^2 \frac{\partial \mathcal{M}_y}{\partial \rho} = (1 - \phi_y)(1 - \beta\rho)^2 - \kappa_y(\phi_\pi - 1).$$

As a result, the sign of the previous expression is determined by the sign of the second order polynomial in  $\rho$ , i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2 + b\rho + c,$$

with  $a \equiv (1 - \phi_y)\beta^2$ ,  $b \equiv -2(1 - \phi_y)\beta$ , and  $c \equiv 1 - \phi_y - \kappa_y(\phi_\pi - 1)$ .

The corresponding roots are

$$\rho^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a},$$

where the discriminant is  $\Delta \equiv b^2 - 4ac = 4\beta^2\kappa_y(\phi_\pi - 1)(1 - \phi_y)$ .

Then 3 cases must be considered:

1. First, consider the case  $\phi_\pi > 1$ . In this case (A.1)-(A.3) imply that the model is locally determinate iff  $0 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$ . To begin with, consider the sub-case in which  $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$  such that  $\Delta > 0$ ,  $a > 0$ , and  $b < 0$ . As such, the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \frac{2(1 - \phi_y)\beta \pm \sqrt{4\beta^2 \kappa_y (\phi_\pi - 1)(1 - \phi_y)}}{2(1 - \phi_y)\beta^2} = \beta^{-1} \left( 1 \pm \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y} \right).$$

Evidently,  $\rho^+ > 1$  as  $\beta \in (0, 1)$  and  $\phi_\pi > 1$  such that the only feasible candidate on  $\rho \in (0, 1)$  is

$$\rho^* \equiv \rho^- = \beta^{-1} \left( 1 - \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y} \right).$$

Notice that  $\rho^* \in (0, 1)$  if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , where we have

$$\underline{\phi}_\pi \equiv 1 + (1 - \phi_y)(1 - \beta)^2 \kappa_y^{-1}, \quad \text{and} \quad \bar{\phi}_\pi \equiv 1 + (1 - \phi_y) \kappa_y^{-1}.$$

As  $f(\rho) \equiv a\rho^2 + b\rho + c$  is a strictly *convex* function in  $\rho$ , the first part of statement (a.1) follows. As before, note that  $\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$  is strictly decreasing in  $\phi_\pi$ . Hence, substituting in the lower bound  $\underline{\phi}_\pi$ , we obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi} = \text{sgn}\left((1 - \phi_y)[(1 - \beta\rho)^2 - (1 - \beta)^2]\right) > 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  for all  $\phi_\pi \leq \underline{\phi}_\pi$ . Similarly, we substitute in for the upper bound  $\bar{\phi}_\pi$  to obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \bar{\phi}_\pi} = \text{sgn}\left((1 - \phi_y)\beta\rho(\beta\rho - 2)\right) \leq 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . Notice that the previous inequality is strict for  $\rho > 0$ . As a result,  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0 \forall \phi_\pi \geq \bar{\phi}_\pi$ . This completes the derivation of the first part of statement (a.1).

To show the second part of statement (b.1), consider the sub-case in which  $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1 + \beta} \kappa_y\}$  such that  $\Delta \leq 0$ ,  $a \leq 0$ ,  $b \geq 0$ , and  $c < 0$ . As such, the second order polynomial is strictly negative on  $\rho \in [0, 1)$ .

2. Second, consider the case  $\phi_\pi = 1$ . In this case (A.1)-(A.3) imply that the model is locally determinate iff  $0 < \phi_y < 2$ . Three sub-cases arise: first, if  $0 < \phi_y < 1$ , we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ ; second, if  $\phi_y = 1$ , we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} = 0$ ; and third, if  $1 < \phi_y < 2$ , we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$ .
3. Finally, consider the case  $0 \leq \phi_\pi < 1$ . In this case (A.1)-(A.3) imply that the model is locally determinate iff  $\frac{1 - \phi_\pi}{1 - \beta} \kappa_y^{-1} < \phi_y < \min\{1 + \beta^{-1}, 2 + \frac{1 - \phi_\pi}{1 + \beta} \kappa_y\}$ . There arise, in turn, two sub-cases. First, if additionally  $\frac{1 - \phi_\pi}{1 - \beta} \kappa_y^{-1} < \phi_y \leq \min\{1, 1 + \beta^{-1}, 2 +$

$\frac{1-\phi_\pi}{1+\beta} \kappa_y\}$  applies, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ . Second, if additionally  $\max\{1, \frac{1-\phi_\pi}{1-\beta} \kappa_y^{-1}\} < \phi_y \leq \min\{1 + \beta^{-1}, 2 + \frac{1-\phi_\pi}{1+\beta} \kappa_y\}$  applies, we have  $\Delta > 0$ ,  $a < 0$ ,  $b > 0$ , and  $f(\rho) \equiv \text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2 + b\rho + c$  is strictly *concave* and has two real-valued roots that are given by

$$\rho^{+,-} = \beta^{-1} \left( 1 \pm \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y} \right).$$

Evidently,  $\rho^+ > 1$  as  $\beta \in (0, 1)$ ,  $0 \leq \phi_\pi < 1$ , and  $\phi_y > 1$  such that the only feasible candidate on  $\rho \in (0, 1)$  is

$$\rho^* \equiv \rho^- = \beta^{-1} \left( 1 - \sqrt{\frac{\phi_\pi - 1}{1 - \phi_y} \kappa_y} \right).$$

Note that  $\rho^* \in (0, 1)$  if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , where we have

$$\underline{\phi}_\pi \equiv 1 + (1 - \phi_y) \kappa_y^{-1}, \quad \text{and} \quad \bar{\phi}_\pi \equiv 1 + (1 - \phi_y)(1 - \beta)^2 \kappa_y^{-1}.$$

As  $f(\rho)$  is a strictly *concave* function in  $\rho$ , the second part of statement (a.3) follows. As before, note that  $\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$  is strictly decreasing in  $\phi_\pi$ . Substituting in the lower bound  $\underline{\phi}_\pi$ , we thus get

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi} = \text{sgn}((1 - \phi_y)\beta\rho(\beta\rho - 2)) \geq 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . Note that the previous inequality is strict for  $\rho > 0$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  for all  $\phi_\pi \leq \underline{\phi}_\pi$ . Similarly, we substitute in for the upper bound  $\bar{\phi}_\pi$  and finally obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \bar{\phi}_\pi} = \text{sgn}((1 - \phi_y)((1 - \beta\rho)^2 - (1 - \beta)^2)) < 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$  for all  $\phi_\pi \geq \bar{\phi}_\pi$ .

**Part (b):** Substituting the Taylor rule into the DIS equation results in

$$(1 + \phi_y)y_t = \mathbb{E}_t[y_{t+1}] + \mathbb{E}_t[\pi_{t+1}] - \phi_\pi \pi_t + \xi_t.$$

We guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \xi_t$  and  $\pi_t = \mathcal{M}_\pi \xi_t$ , such that the previous equation together with the NKPC can be rewritten as

$$\begin{aligned} (1 + \phi_y - \rho)\mathcal{M}_y &= -(\phi_\pi - \rho)\mathcal{M}_\pi + 1, \\ (1 - \beta\rho)\mathcal{M}_\pi &= \kappa_y \mathcal{M}_y. \end{aligned}$$

Substituting the latter into the former equation gives us

$$(1 + \phi_y - \rho) \mathcal{M}_y = -(\phi_\pi - \rho) \frac{\kappa_y}{1 - \beta\rho} \mathcal{M}_y + 1 ,$$

which finally yields

$$\mathcal{M}_y = \frac{1 - \beta\rho}{(1 + \phi_y - \rho)(1 - \beta\rho) + (\phi_\pi - \rho)\kappa_y} > 0 .$$

Taking the derivative of  $\mathcal{M}_y$  with respect to  $\rho$  yields

$$\frac{\partial \mathcal{M}_y}{\partial \rho} = \frac{-\beta [(1 + \phi_y - \rho)(1 - \beta\rho) + (\phi_\pi - \rho)\kappa_y] + (1 - \beta\rho) [(1 - \beta\rho) + \beta(1 + \phi_y - \rho) + \kappa_y]}{[(1 + \phi_y - \rho)(1 - \beta\rho) + (\phi_\pi - \rho)\kappa_y]^2} .$$

Defining  $x \equiv (1 + \phi_y - \rho)(1 - \beta\rho) + (\phi_\pi - \rho)\kappa_y$ , the previous expression simplifies to

$$x^2 \frac{\partial \mathcal{M}_y}{\partial \rho} = (1 - \beta\rho)^2 + (1 - \beta\phi_\pi)\kappa_y .$$

As a result, the sign of the previous expression is determined by that of a second order polynomial in  $\rho$ , i.e.,

$$\text{sgn} \left( \frac{\partial \mathcal{M}_y}{\partial \rho} \right) = a\rho^2 + b\rho + c ,$$

with  $a \equiv \beta^2$ ,  $b \equiv -2\beta$ , and  $c \equiv 1 + (1 - \beta\phi_\pi)\kappa_y$ .

The corresponding roots are

$$\rho^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a} ,$$

where the discriminant is  $\Delta \equiv b^2 - 4ac = 4\beta^2(\beta\phi_\pi - 1)\kappa_y$ .

Consequently, the following case distinction applies. First,  $\phi_\pi < \beta^{-1}$  implies  $\Delta < 0$  and the second order polynomial has two distinct complex roots. Second,  $\phi_\pi = \beta^{-1}$  implies  $\Delta = 0$  and the second order polynomial has a unique real root that is given by  $-\frac{b}{2a} = \beta^{-1} > 1$ . Third,  $\phi_\pi > \beta^{-1}$  implies that the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \frac{2\beta \pm \sqrt{4\beta^2(\beta\phi_\pi - 1)\kappa_y}}{2\beta^2} = \beta^{-1} \left( 1 \pm \sqrt{(\beta\phi_\pi - 1)\kappa_y} \right) .$$

Evidently,  $\rho^+ > 1$  as  $\beta \in (0, 1)$  and  $\phi_\pi > \beta^{-1}$  such that the only feasible candidate on  $\rho \in (0, 1)$  is

$$\rho^* \equiv \rho^- = \beta^{-1} \left( 1 - \sqrt{(\beta\phi_\pi - 1)\kappa_y} \right) .$$

Notice that  $\rho^* \in (0, 1)$  if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , where we have

$$\underline{\phi}_\pi \equiv \beta^{-1} (1 + (1 - \beta)^2 \kappa_y^{-1}) , \quad \text{and} \quad \bar{\phi}_\pi \equiv \beta^{-1} (1 + \kappa_y^{-1}) .$$

As  $f(\rho) \equiv a\rho^2 + b\rho + c$  is a strictly convex function in  $\rho$ , statement (b) of Proposition 5 follows.

Notice that  $\frac{\partial \mathcal{M}_y}{\partial \rho}$  is strictly decreasing in  $\phi_\pi$ . Hence, substituting in the lower bound  $\underline{\phi}_\pi$ , we obtain

$$\text{sgn} \left( \frac{\partial \mathcal{M}_y}{\partial \rho} \right) \Big|_{\phi_\pi = \underline{\phi}_\pi} = \text{sgn} \left( (1 - \beta\rho)^2 - (1 - \beta)^2 \right) > 0 ,$$

where the last inequality applies as  $\rho \in [0, 1)$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  for all  $\phi_\pi \leq \underline{\phi}_\pi$ .

Finally, substituting in for the upper bound  $\bar{\phi}_\pi$  we obtain

$$\text{sgn} \left( \frac{\partial \mathcal{M}_y}{\partial \rho} \right) \Big|_{\phi_\pi = \bar{\phi}_\pi} = \text{sgn} (\beta\rho(\beta\rho - 2)) \leq 0 ,$$

where the last inequality applies as  $\rho \in [0, 1)$ . Notice that the previous inequality is strict for  $\rho > 0$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$  for all  $\phi_\pi \geq \bar{\phi}_\pi$ .  $\square$

*Proof Proposition 6.* The system now features an additional state variable,  $i_{t-1}$ , such that we make the following guess (ignoring constant terms)

$$\begin{aligned} i_t &= \mu_i i_{t-1} + \mathcal{M}_i \xi_t , \\ y_t &= \mu_y i_{t-1} + \mathcal{M}_y \xi_t , \\ \pi_t &= \mu_\pi i_{t-1} + \mathcal{M}_\pi \xi_t . \end{aligned} \tag{A.4}$$

Given the linearity of equilibrium dynamics, the deterministic and the stochastic parts of the model can be solved separately. Under local determinacy,  $\mu_i \in [0, 1)$ , corresponds to the stable root of the  $3^{rd}$  order characteristic polynomial of the dynamic system. Then, making use of the guess in the equilibrium,  $\mu_y$  and  $\mu_\pi$  satisfy

$$\begin{aligned} \mu_y &= \mu_i \mu_y - \mu_i + \mu_i \mu_\pi , \\ \mu_\pi &= \kappa_y \mu_y + \beta \mu_i \mu_\pi , \end{aligned}$$

which admits the solution

$$\begin{aligned} \mu_i &= \phi_i + (1 - \phi_i) \phi_\pi \mu_\pi \mu_i , \\ \mu_y &= - \frac{\mu_i (1 - \beta \mu_i)}{(1 - \mu_i)(1 - \beta \mu_i) - \kappa_y \mu_i} , \\ \mu_\pi &= - \frac{\kappa_y \mu_i}{(1 - \mu_i)(1 - \beta \mu_i) - \kappa_y \mu_i} . \end{aligned}$$

The multipliers associated with the DIS shock,  $\mathcal{M}_i$ ,  $\mathcal{M}_y$ , and  $\mathcal{M}_\pi$ , satisfy

$$\begin{aligned} \mathcal{M}_y &= \mu_y \mathcal{M}_i + \rho \mathcal{M}_y - \mathcal{M}_i + \mu_\pi \mathcal{M}_i + \rho \mathcal{M}_\pi + 1 , \\ \mathcal{M}_\pi &= \kappa \mathcal{M}_y + \beta (\mu_\pi \mathcal{M}_i + \rho \mathcal{M}_\pi) , \\ \mathcal{M}_i &= (1 - \phi_i) \phi_\pi (\mu_\pi \mathcal{M}_i + \rho \mathcal{M}_\pi) . \end{aligned} \tag{A.5}$$

After simple, although tedious, algebra, the output multiplier obtains as

$$\mathcal{M}_y = \left[ 1 - \rho + \left( -\mu_y \frac{1 - \phi_i}{\phi_i} \phi_\pi - 1 \right) \frac{\kappa_y \rho}{1 - \beta \rho \mu_i / \phi_i} \right]^{-1}.$$

Then using the definition of  $\mu_y$  and denoting

$$\Psi = -\mu_y \frac{1 - \phi_i}{\phi_i} = \frac{\mu_i (1 - \beta \mu_i)}{(1 - \mu_i)(1 - \beta \mu_i) - \kappa_y \mu_i} \frac{1 - \phi_i}{\phi_i},$$

as well as  $\tilde{\beta} \equiv \beta \mu_i / \phi_i$  and  $\tilde{\phi}_\pi = \Psi \phi_\pi$ , this rewrites

$$\mathcal{M}_y = \frac{1}{1 - \rho + \left( \tilde{\phi}_\pi - 1 \right) \frac{\kappa_y \rho}{1 - \tilde{\beta} \rho}},$$

which has exactly the same form as the multiplier obtained in Proposition 3. Therefore, the same results apply and we have

$$\tilde{\phi}_\pi = 1 + (1 - \tilde{\beta})^2 \kappa_y^{-1} \quad \text{and} \quad \bar{\phi}_\pi = 1 + \kappa_y^{-1}, \quad (\text{A.6})$$

such that using the definition of  $\tilde{\phi}_\pi$  yields

$$\underline{\phi}_\pi = \Psi^{-1} \left( 1 + (1 - \tilde{\beta})^2 \kappa_y^{-1} \right) \quad \text{and} \quad \bar{\phi}_\pi = \Psi^{-1} \left( 1 + \kappa_y^{-1} \right).$$

Finally, using the same argument, we have

$$\rho^* = \tilde{\beta}^{-1} \left( 1 - \sqrt{\kappa_y (\Psi \phi_\pi - 1)} \right).$$

At this stage, there however remains an open question: That of the existence and uniqueness of the solutions to system (A.6). To address this question it first proves useful to go back to the characteristic polynomial of the system. Plugging the solution for  $\mu_\pi$  computed above in the equation for  $\mu_i$  and rearranging, the characteristic polynomial obtains as

$$P(x) = (x - \phi_i) [(1 - x)(1 - \beta x) - \kappa_y x] + (1 - \phi_i) \phi_\pi \kappa_y x^2.$$

LEMMA 2. *Under local determinacy, the stable root of  $P(\cdot)$  satisfies  $0 < \mu_i < \phi_i$ .*

*Proof Lemma 2.* Note that  $P(0) = -\phi_i$  and  $P(\phi_i) = (1 - \phi_i) \phi_\pi \kappa_y \phi_i^2 > 0$ , hence the polynomial has –under the assumption of local stability– a single root between 0 and  $\phi_i$ .  $\square$

Now, define

$$D(x) \equiv (1 - x)(1 - \beta x) - \kappa_y x.$$

such that  $P(x) = (x - \phi_i)D(x) + (1 - \phi_i) \phi_\pi \kappa_y x^2$ .

In the sequel, we will make explicit the fact that  $\Psi$  and  $\tilde{\beta}$  are functions of  $\phi_\pi$ . The proof of existence and uniqueness will proceed by eliminating  $\phi_\pi$  and  $\Psi(\phi_\pi)$  in favor of the root  $\mu_i$ . Then each fixed-point condition from system (A.6) becomes a one-variable polynomial equation in  $\mu_i$  for which proving existence of  $(0, \phi_i)$  is simpler.

**Step 1: Eliminate  $\phi_\pi$  and  $\Psi$  in favor of  $\mu_i$ .** Let  $\mu_i$  be the stable root of the characteristic polynomial so that  $P(\mu_i) = 0$ . We then have

$$(\mu_i - \phi_i)D(\mu_i) + (1 - \phi_i)\phi_\pi\kappa_y\mu_i^2 = 0.$$

Since  $\mu_i < \phi_i$ , we can rearrange to

$$\phi_\pi = \frac{(\phi_i - \mu_i) D(\mu_i)}{(1 - \phi_i)\kappa_y\mu_i^2}. \quad (\text{A.7})$$

Moreover, by definition

$$\Psi(\phi_\pi) = \frac{1 - \beta\mu_i}{(1 - \beta\mu_i)(1 - \mu_i) - \kappa_y\mu_i} \cdot \frac{(1 - \phi_i)\mu_i}{\phi_i} = \frac{1 - \beta\mu_i}{D(\mu_i)} \cdot \frac{(1 - \phi_i)\mu_i}{\phi_i},$$

so

$$\frac{1}{\Psi(\phi_\pi)} = \frac{D(\mu_i)}{1 - \beta\mu_i} \cdot \frac{\phi_i}{(1 - \phi_i)\mu_i}. \quad (\text{A.8})$$

At any fixed point we must have  $\phi_\pi > 0$ , which forces  $\mu_i > 0$  and  $D(\mu_i) > 0$  by (A.7).

**Step 2: Find  $\bar{\phi}_\pi$ .** Consider the fixed point equation

$$\phi_\pi = \frac{1}{\Psi(\phi_\pi)} \left(1 + \frac{1}{\kappa_y}\right). \quad (\text{A.9})$$

Substituting (A.7)–(A.8) and canceling the common factor  $D(\mu_i)/\mu_i(1 - \phi_i) > 0$  yields

$$\frac{\phi_i - \mu_i}{\kappa_y\mu_i} = \left(1 + \frac{1}{\kappa_y}\right) \frac{\phi_i}{1 - \beta\mu_i} \iff (\phi_i - \mu_i)(1 - \beta\mu_i) = (\kappa_y + 1)\phi_i\mu_i,$$

i.e.  $\mu_i$  solves the quadratic equation

$$\bar{Q}(x) \equiv \beta x^2 - [1 + \phi_i(\beta + \kappa_y + 1)]x + \phi_i = 0. \quad (\text{A.10})$$

Note that  $\bar{Q}''(x) = 2\beta > 0$ , so  $\bar{Q}(x)$  is strictly convex. Furthermore,

$$\bar{Q}(0) = \phi_i > 0, \quad \text{and} \quad \bar{Q}(\phi_i) = \phi_i^2(\beta - (\beta + \kappa_y + 1)) = -\phi_i^2(\kappa_y + 1) < 0.$$

Hence, by continuity there exists a root in  $(0, \phi_i)$ , and by strict convexity there is exactly one such root; call it  $\bar{\mu}_i \in (0, \phi_i)$ . Define

$$\bar{\phi}_\pi \equiv \frac{(\phi_i - \bar{\mu}_i) D(\bar{\mu}_i)}{(1 - \phi_i)\kappa_y\bar{\mu}_i^2},$$

which is well-defined and positive. Any  $\phi_\pi$  satisfying the fixed point (A.9) condition must have  $\mu_i$  solving (A.10), hence  $\mu(\phi_\pi) = \bar{\mu}_i$ , and then equation (A.7) forces  $\phi_\pi = \bar{\phi}_\pi$ . Thus, equation (A.9) admits a unique fixed point.

**Step 3: Find  $\underline{\phi}_\pi$ .** Consider the fixed point equation

$$\phi_\pi = \frac{1}{\Psi(\phi_\pi)} \left( 1 + \frac{(1 - \tilde{\beta}(\phi_\pi))^2}{\kappa_y} \right), \quad \text{where} \quad \tilde{\beta}(\phi_\pi) = \frac{\beta \mu_i(\phi_\pi)}{\phi_i}. \quad (\text{A.11})$$

Using (A.7)–(A.8), canceling  $D(\mu_i)/\mu_i(1 - \phi_i) > 0$  and rearranging gives

$$(\phi_i - \mu_i)(1 - \beta \mu_i) = \phi_i \mu_i \left( \kappa_y + \left( 1 - \frac{\beta \mu_i}{\phi_i} \right)^2 \right).$$

Let us denote  $r \equiv \mu_i/\phi_i \in (0, 1)$  (from Lemma 2) and rewrite the previous equation as

$$\underline{Q}(r) \equiv 1 - [1 + \phi_i(\beta + \kappa_y + 1)]r + 3\beta\phi_i r^2 - \beta^2\phi_i r^3 = 0. \quad (\text{A.12})$$

We have

$$\underline{Q}(0) = 1 > 0 \quad \text{and} \quad \underline{Q}(1) = -\phi_i [(1 - \beta)^2 + \kappa_y] < 0.$$

Thus, by continuity, there exists at least one root  $r^* \in (0, 1)$ .

For uniqueness, we compute

$$\underline{Q}''(r) = 6\beta\phi_i(1 - \beta r).$$

Since  $r \in (0, 1)$  and  $\beta \in (0, 1)$ , we have  $1 - \beta r > 0$ , so  $\underline{Q}''(r) > 0$  on  $(0, 1)$  and hence  $\underline{Q}$  is strictly convex on  $(0, 1)$ . Since  $\underline{Q}(0) > 0$  and  $\underline{Q}(1) < 0$ , and given that a strictly convex function can cross the horizontal axis at most once on an interval, the root  $r^* \in (0, 1)$  is unique. Let  $\underline{\mu}_i \equiv \phi_i r^* \in (0, \phi_i)$  and define

$$\underline{\phi}_\pi \equiv \frac{(\phi_i - \underline{\mu}_i)D(\underline{\mu}_i)}{(1 - \phi_i)\kappa_y \underline{\mu}_i^2}.$$

As before, any fixed point  $\phi_\pi$  for equation (A.11) forces  $r = \mu_i(\phi_\pi)/\phi_i$  to satisfy (A.12), hence  $r = r^*$  and  $\mu_i(\phi_\pi) = \underline{\mu}_i$ , which in turn forces  $\phi_\pi = \underline{\phi}_\pi$  by (A.7). Therefore, the underlying equation admits a unique fixed point.  $\square$

*Proof Corollary 1.* The system of restrictions (A.4) (see above) implies

$$\mu_i = \frac{\phi_i}{1 - (1 - \phi_i)\phi_\pi \mu_\pi}.$$

The solution coefficients pertaining to the stochastic part,  $\mathcal{M}_i$ ,  $\mathcal{M}_y$  and  $\mathcal{M}_\pi$ , satisfy system (A.5). From the last restriction of the system we have

$$\mathcal{M}_i = \frac{(1 - \phi_i)\phi_\pi\rho}{1 - (1 - \phi_i)\phi_\pi\mu_\pi}\mathcal{M}_\pi = \Lambda\phi_\pi\rho\mathcal{M}_\pi \quad \text{with} \quad \Lambda \equiv \frac{1 - \phi_i}{1 - (1 - \phi_i)\phi_\pi\mu_\pi}.$$

For future references, it is useful to note that

$$\frac{\partial\Lambda}{\partial\phi_i} = \frac{(1 - \phi_i)^2\phi_\pi\frac{\partial\mu_\pi}{\partial\phi_i} - 1}{[1 - (1 - \phi_i)\phi_\pi\mu_\pi]^2}.$$

Plugging the preceding result in the NKPC (second restriction), we get

$$\mathcal{M}_\pi = \kappa_y\mathcal{M}_y + \beta\rho(\mu_\pi\Lambda\phi_\pi + 1)\mathcal{M}_\pi \iff \mathcal{M}_\pi = \frac{\kappa_y}{1 - \beta\rho\Theta}\mathcal{M}_y, \quad \text{with} \quad \Theta \equiv 1 + \Lambda\phi_\pi\mu_\pi.$$

Making use of this result in the DIS equation (first restriction) yields

$$\mathcal{M}_y = \frac{1}{1 - \rho + \rho(\Psi\phi_\pi - 1)}\frac{\kappa_y}{1 - \beta\Theta\rho}, \quad \text{with} \quad \Psi = \Lambda(1 - \mu_y - \mu_\pi).$$

Defining  $\tilde{\beta} = \Theta\beta$  and  $\tilde{\phi}_\pi = \Psi\phi_\pi$ , as established in Proposition 6,  $\mathcal{M}_y$  has the same form as in Proposition 3

$$\mathcal{M}_y = \frac{1}{1 - \rho + \rho(\tilde{\phi}_\pi - 1)}\frac{\kappa_y}{1 - \tilde{\beta}\rho},$$

which implies that

$$\tilde{\phi}_\pi = 1 + (1 - \tilde{\beta})^2\kappa_y^{-1}, \quad \text{and} \quad \tilde{\phi}_\pi = 1 + \kappa_y^{-1},$$

such that using the definition of  $\tilde{\phi}_\pi$  yields

$$\phi_\pi = \Psi^{-1}\left(1 + (1 - \tilde{\beta})^2\kappa_y^{-1}\right), \quad \text{and} \quad \bar{\phi}_\pi = \Psi^{-1}\left(1 + \kappa_y^{-1}\right).$$

Finally, using the same argument, we have

$$\rho^* = \tilde{\beta}^{-1}\left(1 - \sqrt{\kappa_y(\Psi\phi_\pi - 1)}\right).$$

We now investigate the comparative statics of  $\phi_\pi$ ,  $\bar{\phi}_\pi$  and  $\rho^*$  in a neighborhood of  $\phi_i = 0$ . Note that as soon as  $\phi_i = 0$ , we have  $\mu_i = \mu_y = \mu_\pi = 0$ , implying that  $\Lambda = \Theta = \Psi = 1$ . Taking a first order Taylor expansion of the system of restrictions pertaining to the deterministic part of the model around  $\phi_i = 0$  yields

$$\begin{aligned} d\mu_i &= d\phi_i, \\ d\mu_y &= -d\mu_i = -d\phi_i, \\ d\mu_\pi &= \kappa_y d\mu_y = -\kappa_y d\phi_i. \end{aligned}$$

Note that, around  $\phi_i = 0$ ,  $\partial\Lambda/\partial\phi_i$  reduces to

$$\frac{\partial\Lambda}{\partial\phi_i} = -(1 + \phi_\pi \kappa_y) .$$

Then,  $\Theta$  can be approximated as

$$d\Theta = \phi_\pi \mu_\pi d\Lambda + \Lambda \phi_\pi d\mu_\pi = -\phi_\pi \kappa_y d\phi_i .$$

Likewise,  $\Psi$  approximates as

$$d\Psi = \frac{\partial\Lambda}{\partial\phi_i} d\phi_i - \Lambda \frac{\partial\mu_y}{\partial\phi_i} d\phi_i - \Lambda \frac{\partial\mu_\pi}{\partial\phi_i} d\phi_i = -\kappa_y (\phi_\pi - 1) d\phi_i .$$

Then, note that since  $\bar{\phi}_\pi$  is a decreasing function of  $\Psi$ , under the Taylor principle,  $\bar{\phi}_\pi$  increases with  $\phi_i$  in a neighborhood of  $\phi_i = 0$ . Likewise,  $\rho^*$  is decreasing both in  $\Theta$  and  $\Psi$ , implying that  $\rho^*$  increases with  $\phi_i$  in a neighborhood of  $\phi_i = 0$ . Finally, in the neighborhood of  $\phi_i = 0$ ,  $\underline{\phi}_\pi$  approximates as

$$d\underline{\phi}_\pi = [(\phi_\pi - 1)(\kappa_y + (1 - \beta)^2) + 2(1 - \beta)\beta\phi_\pi] d\phi_i ,$$

such that, under the Taylor principle,  $\underline{\phi}_\pi$  increases with  $\phi_i$  in the neighborhood of  $\phi_i = 0$ . Hence, we have:

$$\frac{d\underline{\phi}_\pi}{d\phi_i} > 0 , \quad \frac{d\bar{\phi}_\pi}{d\phi_i} > 0 , \quad \frac{d\rho^*}{d\phi_i} > 0 .$$

□

Like for Proposition 5, we offer a more general version of proposition 7 in this section.

**Proposition 7 (Extended Version)** *Let the cumulative output multiplier at horizon  $h \in \mathbb{N}$  be defined as*

$$C_y(\rho, h) = \mathcal{M}_y(\rho) \sum_{i=0}^h \rho^i .$$

*Let  $\rho^* \in \operatorname{argmax}_{\rho \in [0,1)} \mathcal{M}_y(\rho)$  denote the level of persistence maximizing the impact output multiplier, and  $\tilde{\rho}(h)$  the level of persistence maximizing  $C_y(\rho, h)$ . Define the horizon threshold*

$$\bar{h}(\phi_\pi) \equiv \lceil \bar{h}_c(\phi_\pi) \rceil - 1 \in \mathbb{N}, \quad \text{where} \quad \bar{h}_c(\phi_\pi) \equiv \frac{2}{1 - \beta} \left( \frac{\phi_\pi - \underline{\phi}_\pi}{\phi_\pi - 1} \right), \quad (\text{A.13})$$

*Then, the global maximizer(s) of  $C_y(\rho, h)$  can be characterized as follows.*

- (i) *If  $1 < \phi_\pi \leq \underline{\phi}_\pi$ , then  $C_y(\rho, h)$  is strictly increasing and  $\operatorname{argmax}_{\rho \in [0,1)} C_y(\rho, h) \rightarrow \{1\}$ .*

(ii) If  $\underline{\phi}_\pi < \phi_\pi \leq \bar{\phi}_\pi$ , then the following holds:

(ii.a) If  $1 \leq h \leq \bar{h}(\phi_\pi)$  then there exists at least one global maximizer  $\tilde{\rho}(h) \in (\rho^*, 1)$  satisfying  $\frac{\partial C_y(\rho, h)}{\partial \rho} \Big|_{\rho=\tilde{\rho}(h)} = 0$ .

(ii.b) If  $h \geq \bar{h}(\phi_\pi) + 1$  then  $\rho \rightarrow 1$  is a boundary local maximizer in the sense that  $\lim_{\rho \uparrow 1} \frac{\partial C_y(\rho, h)}{\partial \rho} \geq 0$ . Moreover, exactly one of the following two sub-cases holds:

1.  $C_y(\rho, h)$  is non-decreasing,  $\frac{\partial C_y(\rho, h)}{\partial \rho} \geq 0 \forall \rho \in [0, 1)$ , in which case

$$\operatorname{argmax}_{\rho \in [0, 1)} C_y(\rho, h) \rightarrow \{1\}.$$

2. There exists  $\rho_0 \in (0, 1)$  such that  $\frac{\partial C_y(\rho, h)}{\partial \rho} \Big|_{\rho=\rho_0} < 0$ . Then there exist at least two distinct interior solutions, and at least one interior local maximizer. Let  $\mathcal{L}_h \subset (0, 1)$  denote the (nonempty) set of interior local maximizers of  $C_y(\rho, h)$ .<sup>1</sup> Then every global maximizer belongs to  $\{1\} \cup \mathcal{L}_h$  and

$$\operatorname{argmax}_{\rho \in [0, 1)} C_y(\rho, h) \rightarrow \{1\} \iff \lim_{\rho \rightarrow 1} C_y(\rho, h) \geq \max_{\rho \in \mathcal{L}_h} C_y(\rho, h).$$

In addition, every interior maximizer satisfies  $\tilde{\rho}(h) > \rho^*$ .

(iii) If  $\bar{\phi}_\pi < \phi_\pi < \phi_\pi^{\max}$ , then the following holds:<sup>2</sup>

(iii.a) If  $\phi_\pi < 1 + \frac{2}{\kappa_y}$  and  $1 \leq h \leq \bar{h}(\phi_\pi)$ , then there exists at least one global maximizer  $\tilde{\rho}(h) \in (0, 1)$  satisfying  $\frac{\partial C_y(\rho, h)}{\partial \rho} \Big|_{\rho=\tilde{\rho}(h)} = 0$ .

(iii.b) If  $h \geq \bar{h}(\phi_\pi) + 1$ , then  $\lim_{\rho \uparrow 1} \frac{\partial C_y(\rho, h)}{\partial \rho} \geq 0$ , so  $\rho \rightarrow 1$  is a boundary local maximizer. If, in addition,  $\phi_\pi < 1 + \frac{2}{\kappa_y}$ , then  $\rho = 0$  is not a maximizer (since  $\lim_{\rho \downarrow 0} \frac{\partial C_y}{\partial \rho} > 0$ ). As in (ii.b), the global maximizer is either  $\rho \rightarrow 1$  or an interior local maximizer, and 1 is a global maximizer if and only if its level dominates all interior local maximizers.

*Proof Proposition 7 (Extended version).* Fix the horizon  $h \in \mathbb{N}$  and define

$$\Theta \equiv \kappa_y(\phi_\pi - 1), \quad D(\rho) \equiv (1 - \rho)(1 - \beta\rho) + \rho\Theta, \quad S_h(\rho) \equiv \sum_{i=0}^h \rho^i.$$

Then  $C_y(\rho, h) = \mathcal{M}_y(\rho)S_h(\rho)$  with  $\mathcal{M}_y(\rho) = (1 - \beta\rho)/D(\rho)$ . Since, under local determinacy,  $\Theta > 0$ , we have  $D(\rho) > 0$  for all  $\rho \in [0, 1]$  and hence  $\mathcal{M}_y(\rho)$  and  $C_y(\rho, h)$  are well-defined and continuous on  $[0, 1]$ . Moreover,  $\lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) = (1 - \beta)/\Theta$  and  $S_h(\rho) \rightarrow$

<sup>1</sup>That is,  $\rho \in \mathcal{L}_h$  iff  $\frac{\partial C_y(\rho, h)}{\partial \rho} = 0$  and  $\frac{\partial C_y(\rho, h)}{\partial \rho}$  changes sign from + to - at  $\rho$ .

<sup>2</sup>Strictly speaking, there emerges another sub-case for  $\phi_\pi > 1 + \frac{2}{\kappa_y}$ . In this case, we have  $C'(0, h) < 0$  and, depending on the value  $h$ ,  $C'_y(1^-, h) \gtrless 0$ . This requires, as in case (ii.b) or (iii.b), the comparison of boundary values with the ones of a potential interior maximizer.

$h + 1$  as  $\rho \uparrow 1$ , so  $C_y(\rho, h)$  has a continuous extension to  $[0, 1)$  by  $\lim_{\rho \rightarrow 1} C_y(\rho, h) = (h + 1) \lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho)$ . Note also that  $C_y(\rho, h)$  is differentiable on  $[0, 1)$ . Furthermore, from Proposition 3, we have  $\frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} \gtrless 0$  if  $\rho \lesseqgtr \rho^*$ .

The first order condition characterizing a maximal cumulative multiplier at horizon  $h$  on  $\rho \in [0, 1)$  is given by:

$$\frac{\partial C_y(\rho, h)}{\partial \rho} = \frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} S_h(\rho) + \mathcal{M}_y(\rho) \frac{\partial S_h(\rho)}{\partial \rho}, \quad \frac{\partial S_h(\rho)}{\partial \rho} = \sum_{i=1}^h i \rho^{i-1} > 0.$$

At  $\rho = 0$ ,  $S_h(0) = 1$ ,  $\frac{\partial S_h(\rho)}{\partial \rho} \big|_{\rho=0} = 1$ ,  $\mathcal{M}_y(0) = 1$ , and  $\frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} \big|_{\rho=0} = 1 - \Theta$ , we have

$$C'(0, h) \equiv \frac{\partial C_y(\rho, h)}{\partial \rho} \big|_{\rho=0} = 2 - \Theta. \quad (\text{A.14})$$

As  $\rho \rightarrow 1$ ,  $S_h(\rho) \rightarrow h + 1$  and  $\frac{\partial S_h(\rho)}{\partial \rho} \rightarrow h(h + 1)/2$ , while  $\lim_{\rho \rightarrow 1} \mathcal{M}_y(\rho) = (1 - \beta)/\Theta$  and  $\lim_{\rho \rightarrow 1} \left( \frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} \right) = ((1 - \beta)^2 - \Theta)/\Theta^2$ , we have

$$\text{sgn}(C'_y(1^-, h)) = \text{sgn} \left( (1 - \beta)^2 - \Theta + \frac{\Theta(1 - \beta)h}{2} \right), \quad (\text{A.15})$$

where we have defined  $C'(1^-, h) \equiv \lim_{\rho \uparrow 1} \frac{\partial C_y(\rho, h)}{\partial \rho}$ . Therefore, it follows that

$$C'_y(1^-, h) \begin{cases} < 0 & \iff 1 \leq h \leq \bar{h}(\phi_\pi), \\ \geq 0 & \iff h \geq \bar{h}(\phi_\pi) + 1, \end{cases} \quad (\text{A.16})$$

where  $h(\phi_\pi) = \lceil h_c(\phi_\pi) \rceil - 1$ , with

$$h_c(\phi_\pi) = \frac{2}{1 - \beta} \left( \frac{\kappa_y(\phi_\pi - 1) - (1 - \beta)^2}{\kappa_y(\phi_\pi - 1)} \right) = \frac{2}{1 - \beta} \left( \frac{\phi_\pi - \phi_\pi}{\phi_\pi - 1} \right).$$

Because  $C_y(\rho, h)$  is continuous and differentiable on  $[0, 1)$ , we have the following two results:

- (R1) If  $C'(0, h) > 0$  and  $C'_y(1^-, h) < 0$ , then neither endpoint can be a global maximizer. As a result, at least one global maximizer lies in  $(0, 1)$  and must satisfy  $\frac{\partial C_y(\bar{\rho}(h), h)}{\partial \rho} = 0$ .
- (R2) If  $C'(0, h) > 0$  and  $C'_y(1^-, h) \geq 0$ , then either  $\frac{\partial C_y}{\partial \rho} \geq 0$  on  $[0, 1)$  (so  $C_y$  is non-decreasing and  $\rho \rightarrow 1$  is the unique global maximizer), or else there exists  $\rho_0 \in (0, 1)$  with  $\frac{\partial C_y(\rho_0, h)}{\partial \rho} < 0$ , in which case continuity of  $\frac{\partial C_y(\rho, h)}{\partial \rho}$  implies there are at least two distinct roots of  $\frac{\partial C_y(\rho, h)}{\partial \rho}$  in  $(0, 1)$  and at least one interior local maximizer.

**Proof of (i):** If  $\phi_\pi \leq \underline{\phi}_\pi$ , then  $\Theta \leq (1 - \beta)^2$ , hence  $\sqrt{\Theta} \leq 1 - \beta$  and  $\rho^* = (1 - \sqrt{\Theta})/\beta \geq 1$ . By Proposition 3,  $\frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} > 0$  for all  $\rho \in [0, 1)$ . Since also  $S_h(\rho) > 0$  and  $\frac{\partial S_h}{\partial \rho} > 0$  on  $[0, 1)$ , we have  $\frac{\partial C_y(\rho, h)}{\partial \rho} > 0$  on  $[0, 1)$ , so  $C_y(\rho, h)$  is strictly increasing and uniquely maximized at  $\tilde{\rho} \rightarrow 1$ .

**Proof of (ii.a):** Under  $\underline{\phi}_\pi < \phi_\pi \leq \bar{\phi}_\pi$  we have  $\Theta \leq 1 < 2$ , so Equation (A.14) implies  $C'_y(0, h) > 0$ . If  $1 \leq h \leq \bar{h}(\phi_\pi)$ , then Equation (A.16) implies  $C'_y(1^-, h) < 0$ . Thus (R1) yields existence of at least one interior global maximizer satisfying the first-order condition. It remains to prove  $\tilde{\rho}(h) > \rho^*$ . If  $\rho^* > 0$ , then by Proposition 3 one has  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  on  $[0, \rho^*]$ , hence for  $\rho \in [0, \rho^*]$ ,

$$\frac{\partial C_y(\rho, h)}{\partial \rho} = \frac{\partial \mathcal{M}_y(\rho)}{\partial \rho} S_h(\rho) + \mathcal{M}_y(\rho) \frac{\partial S_h(\rho)}{\partial \rho} > 0.$$

Therefore  $C_y(\rho, h)$  is strictly increasing on  $[0, \rho^*]$ , so no maximizer can lie in  $[0, \rho^*]$ . If  $\rho^* = 0$  (i.e.  $\Theta = 1$ ), then  $C'_y(0, h) = 2 - \Theta = 1 > 0$  excludes  $\tilde{\rho} = 0$ , so any interior maximizer satisfies  $\tilde{\rho}(h) > 0 = \rho^*$ . Finally  $C'_y(1^-, h) < 0$  excludes  $\tilde{\rho} \rightarrow 1$ , so  $\tilde{\rho}(h) < 1$ .

**Proof of (ii.b):** If  $h \geq \bar{h}(\phi_\pi) + 1$ , then by Equation (A.16) one has  $C'_y(1^-, h) \geq 0$ , hence  $\rho \rightarrow 1$  is a boundary local maximizer. Since  $\Theta \leq 1 < 2$ , Equation (A.14) implies  $C'_y(0, h) > 0$ , so  $\rho = 0$  cannot be a global maximizer. Applying (R2) yields the dichotomy (1)–(2). In sub-case (2), the characterization of global maximizers follows because any global maximizer must be either the boundary point  $\rho \rightarrow 1$  or an interior local maximizer. Moreover, the criterion

$$\operatorname{argmax}_{\rho \in [0, 1)} C_y(\rho, h) \rightarrow \{1\} \iff \lim_{\rho \rightarrow 1} C_y(\rho, h) \geq \max_{\rho \in \mathcal{L}_h} C_y(\rho, h)$$

is immediate from the definition of  $\mathcal{L}_h$  and continuity of  $C_y$ . Finally, the same argument as in (ii.a) shows that  $\frac{\partial C_y(\rho, h)}{\partial \rho} > 0$  on  $[0, \rho^*)$  when  $\rho^* > 0$ , and  $C'_y(0, h) > 0$  when  $\rho^* = 0$ ; hence no maximizer can lie in  $[0, \rho^*]$  and every interior maximizer satisfies  $\tilde{\rho}(h) > \rho^*$ .

**Proof of (iii):** If  $\phi_\pi > \bar{\phi}_\pi$ , then  $\Theta > 1$  and  $\rho^* = 0$ . If  $\Theta < 2$  and  $1 \leq h \leq \bar{h}(\phi_\pi)$ , then Equation (A.14) gives  $C'_y(0, h) > 0$  and Equation (A.16) gives  $C'_y(1^-, h) < 0$ , so (R1) yields an interior global maximizer satisfying the first-order condition, proving (iii.a). If instead  $h \geq \bar{h}(\phi_\pi) + 1$ , then Equation (A.16) gives  $C'_y(1^-, h) \geq 0$ , hence  $\rho \rightarrow 1$  is a boundary local maximizer. If additionally  $\Theta < 2$ , then (A.14) implies  $C'_y(0, h) > 0$  so  $\rho = 0$  is excluded as a global maximizer, and the same reasoning as in (ii.b) applies: the global maximizer is either  $\rho \rightarrow 1$  or an interior local maximizer, and 1 is globally optimal if and only if it dominates all interior local maximizers.  $\square$

*Proof Proposition 8.* First of all, we guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \nu_t$  and  $\pi_t = \mathcal{M}_\pi \nu_t$ , such that the DIS and NKPC can be rewritten as

$$\begin{aligned} (1 - \rho_\nu) \mathcal{M}_y &= -(\phi_\pi - 1) \rho_\nu \mathcal{M}_\pi, \\ (1 - \beta \rho_\nu) \mathcal{M}_\pi &= \kappa_y \mathcal{M}_y + 1. \end{aligned}$$

Substituting the second equation into the first and rearranging yields

$$\mathcal{M}_y = \frac{-\rho_\nu(\phi_\pi - 1)}{(1 - \rho_\nu)(1 - \beta\rho_\nu) + \rho_\nu\kappa_y(\phi_\pi - 1)} < 0.$$

The impact multiplier for inflation then follows

$$\mathcal{M}_\pi = \frac{(1 - \rho_\nu)}{(1 - \rho_\nu)(1 - \beta\rho_\nu) + \rho_\nu\kappa_y(\phi_\pi - 1)} > 0.$$

Differentiating  $\mathcal{M}_y$  with respect to  $\rho_\nu$  yields

$$\frac{\partial \mathcal{M}_y}{\partial \rho_\nu} = -\frac{(\phi_\pi - 1)[(1 - \beta\rho_\nu)(1 - \rho_\nu) + \rho_\nu(1 - \beta\rho_\nu) + \rho_\nu\beta(1 - \rho_\nu)]}{[(1 - \rho_\nu)(1 - \beta\rho_\nu) + \rho_\nu\kappa_y(\phi_\pi - 1)]^2} < 0,$$

as long as the Taylor principle holds. This establishes part (a) of the proposition.

Differentiating  $\mathcal{M}_\pi$  with respect to  $\rho_\nu$  yields

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu} = \frac{\beta(1 - \rho_\nu)^2 - \kappa_y(\phi_\pi - 1)}{[(1 - \rho_\nu)(1 - \beta\rho_\nu) + \rho_\nu\kappa_y(\phi_\pi - 1)]^2},$$

which sign depends on the sign of the numerator. The latter is a second order polynomial in  $\rho_\nu$ ,  $P(\rho_\nu)$ , given by

$$P(\rho_\nu) = \beta\rho_\nu^2 - 2\beta\rho_\nu + \beta - \kappa_y(\phi_\pi - 1),$$

which, as long as the Taylor principle holds, possesses the two real roots

$$\rho_\nu^{+,-} = 1 \pm \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}}.$$

Since the Taylor principle holds,  $\rho_\nu^+ > 1$ , so the only admissible root is

$$\rho_\nu^* = 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}}.$$

Since  $\rho_\nu \in [0, 1)$ , this solution lies in the  $(0, 1)$  interval as long as  $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$ . Since  $\beta > 0$ , the polynomial is convex and positive (resp. negative) below (resp. above)  $\rho_\nu^*$ . Accordingly, as long as  $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$ ,  $\mathcal{M}_\pi$  is increasing (resp. decreasing) for all  $\rho_\nu < \rho_\nu^*$  (resp.  $\rho_\nu > \rho_\nu^*$ ). For any  $\phi_\pi$  above  $1 + \frac{\beta}{\kappa_y}$ , the multiplier is decreasing in  $\rho_\nu$ . This establishes part (b) of the proposition.  $\square$

**PROPOSITION 9.** We proceed in three steps.

*Determinacy.* For a derivation of the determinacy condition, see, for instance, Proposition 3 in [Gabaix \(2020\)](#), respectively Proposition 1 in [Bilbiie \(2025\)](#). Directly applying

their technics yields

$$\phi_\pi > \tilde{\phi} \equiv \max \left( 1 + \frac{(1 - \beta_f)(\zeta_f - 1)}{\zeta_r \kappa_y}, \frac{\zeta_f \beta_f - 1}{\zeta_r \kappa_y} \right).$$

*Multipliers.* Substituting the Taylor rule  $i_t = \bar{r} + \phi_\pi \pi_t$  into the DIS equation results in

$$y_t = \zeta_f \mathbb{E}_t [y_{t+1}] - \zeta_r \mathbb{E}_t [\phi_\pi \pi_t - \pi_{t+1}] + \zeta_r \xi_t.$$

We guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \xi_t$  and  $\pi_t = \mathcal{M}_\pi \xi_t$ , such that the previous equation together with the NKPC yield the following restrictions on the solution

$$\begin{aligned} (1 - \rho \zeta_f) \mathcal{M}_y &= -\zeta_r (\phi_\pi - \rho) \mathcal{M}_\pi + \zeta_r, \\ (1 - \beta_f \rho) \mathcal{M}_\pi &= \kappa_y \mathcal{M}_y. \end{aligned}$$

Substituting the latter into the former equation and rearranging yields

$$\mathcal{M}_y = \frac{\zeta_r (1 - \beta_f \rho)}{(1 - \beta_f \rho) (1 - \rho \zeta_f) + (\phi_\pi - \rho) \zeta_r \kappa_y}.$$

*Sign.* To determine the sign of the multipliers, we derive a reduced form system of output and inflation dynamics of the following form

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \mathbb{E}_t [y_{t+1}] \\ \mathbb{E}_t [\pi_{t+1}] \end{pmatrix} + \mathbf{B}_T \xi_t,$$

where

$$\begin{aligned} \mathbf{A}_T &\equiv \frac{1}{1 + \phi_\pi \zeta_r \kappa_y} \begin{pmatrix} \zeta_f & \zeta_r (1 - \phi_\pi \beta_f) \\ \zeta_f \kappa_y & \beta_f + \zeta_r \kappa_y \end{pmatrix}, \\ \mathbf{B}_T &\equiv \begin{pmatrix} \zeta_r \\ \zeta_r \kappa_y \end{pmatrix}. \end{aligned}$$

Using the fact that  $\mathbb{E}_t [y_{t+1}] = \rho y_t$  and  $\mathbb{E}_t [\pi_{t+1}] = \rho \pi_t$  hold on the equilibrium path, we can write

$$\begin{aligned} \begin{pmatrix} y_t \\ \pi_t \end{pmatrix} &= (\mathbf{I}_2 - \rho \mathbf{A}_T)^{-1} \mathbf{B}_T \xi_t \\ &= \frac{1}{(1 - \rho A_{11})(1 - \rho A_{22}) - \rho^2 A_{12} A_{21}} \begin{pmatrix} 1 - \rho A_{22} & \rho A_{12} \\ \rho A_{21} & 1 - \rho A_{11} \end{pmatrix} \mathbf{B}_T \xi_t \\ &= \frac{1}{(1 - \rho A_{11})(1 - \rho A_{22}) - \rho^2 A_{12} A_{21}} \begin{pmatrix} (1 - \rho A_{22}) B_{11} + \rho A_{12} B_{21} \\ \rho A_{21} B_{11} + (1 - \rho A_{11}) B_{21} \end{pmatrix} \xi_t. \end{aligned}$$

Notice that we have

$$\begin{aligned}
(1 - \rho A_{22})B_{11} + \rho A_{12}B_{21} &= \left(1 - \rho \frac{\beta_f + \zeta_r \kappa_y}{1 + \phi_\pi \zeta_r \kappa_y}\right) \frac{\zeta_r}{1 + \phi_\pi \zeta_r \kappa_y} \\
&\quad + \rho \frac{\zeta_r(1 - \phi_\pi \beta_f)}{1 + \phi_\pi \zeta_r \kappa_y} \frac{\zeta_r \kappa_y}{1 + \phi_\pi \zeta_r \kappa_y} \\
&= \frac{\zeta_r(1 - \rho \beta_f)}{1 + \phi_\pi \zeta_r \kappa_y} > 0,
\end{aligned}$$

where the strict inequality follows under the assumption  $1 > \rho \beta_f$  and  $\phi_\pi > 0$ . As a result, this implies that  $\text{sgn}(\mathcal{M}_y) = \text{sgn}(\det(\mathbf{I}_2 - \rho \mathbf{A}_T))$ . One can show that

$$\begin{aligned}
A_{11}A_{22} - A_{12}A_{21} &= \frac{\zeta_f \beta_f}{1 + \phi_\pi \zeta_r \kappa_y} > 0, \\
A_{11} + A_{22} &= \frac{\zeta_f + \beta_f + \zeta_r \kappa_y}{1 + \phi_\pi \zeta_r \kappa_y} > 0.
\end{aligned}$$

Moreover, we can rewrite

$$\begin{aligned}
\det(\mathbf{I}_2 - \rho \mathbf{A}_T) &= (1 - \rho A_{11})(1 - \rho A_{22}) - \rho^2 A_{12}A_{21} \\
&= 1 - \rho(A_{11} + A_{22}) + \rho^2(A_{11}A_{22} - A_{12}A_{21}).
\end{aligned}$$

As  $A_{11} + A_{22} > 0$  applies, we can write

$$\begin{aligned}
\det(\mathbf{I}_2 - \rho \mathbf{A}_T) &= 1 - \rho(A_{11} + A_{22}) + \rho^2(A_{11}A_{22} - A_{12}A_{21}) \\
&> 1 - \rho(1 + A_{11}A_{22} - A_{12}A_{21}) + \rho^2(A_{11}A_{22} - A_{12}A_{21}) \\
&= (1 - \rho)[1 - \rho(A_{11}A_{22} - A_{12}A_{21})] \\
&> 0,
\end{aligned}$$

where the first strict inequality is due to the determinacy condition  $A_{11} + A_{22} < 1 + A_{11}A_{22} - A_{12}A_{21}$  and the second strictly inequality is due to the determinacy condition  $A_{11}A_{22} - A_{12}A_{21} < 1$ . As a result, this implies

$$\mathcal{M}_y = \frac{\zeta_r(1 - \rho \beta_f)}{(1 - \beta_f \rho)(1 - \rho \zeta_f) + (\phi_\pi - \rho)\zeta_r \kappa_y} > 0.$$

Taking the derivative of  $\mathcal{M}_y$  with respect to  $\rho$  yields

$$\frac{x^2}{\zeta_r} \frac{\partial \mathcal{M}_y}{\partial \rho} = \zeta_f(1 - \beta_f \rho)^2 + \zeta_r \kappa_y(1 - \beta_f \phi_\pi),$$

where  $x \equiv (1 - \beta_f \rho)(1 - \zeta_f \rho) + (\phi_\pi - \rho)\zeta_r \kappa_y$ . As a result, the sign of the previous expression is determined by a second order polynomial in  $\rho$ , with discriminant

$$\Delta = 4\zeta_f^2 \beta_f^2 - 4\zeta_f \beta_f^2 [\zeta_f + (1 - \beta_f \phi_\pi)\zeta_r \kappa_y] = 4\zeta_f \zeta_r \beta_f^2 (\beta_f \phi_\pi - 1) \kappa_y.$$

Consequently, the following case distinction applies. If  $\phi_\pi < \beta_f^{-1}$  then  $\Delta < 0$  and the second order polynomial has two distinct complex roots. If,  $\phi_\pi = \beta_f^{-1}$ , then  $\Delta = 0$  and the second order polynomial has a unique real root that is given by  $\beta_f^{-1} > 1$ . Finally, if  $\phi_\pi > \beta_f^{-1}$  then the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \beta_f^{-1} \left( 1 \pm \sqrt{(\beta_f \phi_\pi - 1) \frac{\zeta_r}{\zeta_f} \kappa_y} \right).$$

Evidently,  $\rho^+ > 1$  as  $\beta_f \in (0, 1)$  and  $\phi_\pi > \beta_f^{-1}$  such that the only feasible candidate on  $\rho \in [0, 1)$  is

$$\rho^* \equiv \rho^- = \beta_f^{-1} \left( 1 \pm \sqrt{(\beta_f \phi_\pi - 1) \frac{\zeta_r}{\zeta_f} \kappa_y} \right).$$

Note that  $\rho^* \in (0, 1)$  if  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ , where we have

$$\underline{\phi}_\pi \equiv \beta_f^{-1} \left( 1 + (1 - \beta_f)^2 \frac{\zeta_f}{\zeta_r} \kappa_y^{-1} \right), \quad \text{and} \quad \bar{\phi}_\pi \equiv \beta_f^{-1} \left( 1 + \frac{\zeta_f}{\zeta_r} \kappa_y^{-1} \right).$$

Since  $\text{sgn} \left( \frac{\partial \mathcal{M}_y}{\partial \rho} \right)$  is strictly decreasing in  $\phi_\pi$ , substituting in the lower bound  $\underline{\phi}_\pi$ , we obtain

$$\left. \frac{\partial \mathcal{M}_y}{\partial \rho} \right|_{\phi_\pi = \underline{\phi}_\pi} \propto \zeta_f [(1 - \beta_f \rho)^2 - (1 - \beta_f)^2] > 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$  for all  $\phi_\pi \leq \underline{\phi}_\pi$ .

Similarly substitute in for the upper bound  $\bar{\phi}_\pi$  to obtain

$$\left. \frac{\partial \mathcal{M}_y}{\partial \rho} \right|_{\phi_\pi = \bar{\phi}_\pi} \propto \zeta_f \beta_f \rho [\beta_f \rho - 2] \leq 0,$$

where the last inequality applies as  $\rho \in [0, 1)$ . Notice that the previous inequality is strict for  $\rho > 0$ . As a result, we have  $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$  for all  $\phi_\pi \geq \bar{\phi}_\pi$ .  $\square$

**PROPOSITION 10.** We guess and verify that the solution takes the form  $y_t = \mathcal{M}_y \xi_t$  and  $\pi_t = \mathcal{M}_\pi \xi_t$ . Plugging the guess in the DIS and NKPC equation and defining  $\tilde{\kappa} \equiv \kappa_g \Gamma_g$ , we obtain:

$$\mathcal{M}_y = \frac{\Gamma_g}{1 - \rho + \frac{\tilde{\kappa}}{1 - \beta_\rho} (\phi_\pi - \rho)}.$$

Given that  $\Gamma_g$  is multiplicative of  $\left[ 1 - \rho + \frac{\tilde{\kappa}}{1 - \beta_\rho} (\phi_\pi - \rho) \right]^{-1}$ , all the results pertaining to  $\phi_\pi$ ,  $\bar{\phi}_\pi$  and  $\rho^*$  are the same as in statement (b) of Proposition 5 in which we replace  $\kappa_y$  by  $\tilde{\kappa}$ .  $\square$

## APPENDIX B: Relation To Alternative Decompositions

Following [Bilbiie \(2020\)](#) and dropping constant terms, the consumption function of the RANK model is given by:

$$c_t = \beta \mathbb{E}_t [c_{t+1}] + (1 - \beta)y_t - \beta r_t, \quad (\text{B.1})$$

which simplifies under market clearing ( $c_t = y_t$ ) to the standard DIS equation:

$$c_t = \mathbb{E}_t [c_{t+1}] - r_t.$$

### Decomposition into Direct and Indirect Effects

Currently, there are two approaches in the literature to decompose the propagation of monetary shocks into direct and indirect effects.

*Decomposition via Real Interest Rate Response* [Kaplan et al. \(2018\)](#) and [Bilbiie \(2020\)](#) define direct effects with respect to the change in the real interest rate of the liquid bond,  $r_t$ , keeping household income  $y_t$  fixed. Assuming that the real interest rate follows an AR(1) process with persistence  $\rho$  yields the following decomposition:

$$\begin{aligned} \text{Multiplier: } \mathcal{M}_y &= \frac{1}{1 - \rho}, \\ \text{Direct Effect: } \mathcal{M}_y^D &= \frac{\beta}{1 - \beta\rho}, \\ \text{Indirect Effect: } \mathcal{M}_y^{\text{ID}} &= \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)}. \end{aligned}$$

*Decomposition via Shock Innovation* [Maliar and Naubert \(2025\)](#) instead define the direct effect as the partial derivative of the consumption response with respect to the persistent contemporaneous innovation of the DIS shock, holding household income and the real interest rate fixed. Substituting in the monetary rule into (B.1) yields the following Keynesian consumption function:

$$c_t = \beta \mathbb{E}_t [c_{t+1}] + (1 - \beta)y_t - \beta((\phi_\pi - 1)\mathbb{E}_t [\pi_{t+1}] + \epsilon_t).$$

In this case, we obtain:

$$\begin{aligned} \text{Multiplier: } \mathcal{M}_y &= \frac{1}{1 - \rho + \rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}}, \\ \text{Direct Effect: } \mathcal{M}_y^D &= \frac{\beta}{1 - \beta\rho}, \\ \text{Indirect Effect: } \mathcal{M}_y^{\text{ID}} &= \frac{1 - \beta - \beta\rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}}{(1 - \beta\rho)\left[1 - \rho + \rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}\right]}. \end{aligned}$$

There are two points worth mentioning: (i) if  $\phi_\pi$  is sufficiently large, then indirect effects may become negative, precisely due to the endogenous real interest rate response - note that this only applies for  $\rho > 0$ ; (ii) if we shut down the endogenous real interest rate response ( $\phi_\pi \rightarrow 1$ ), we recover the previous decomposition via the real interest rate response, highlighting that the total indirect effect is driven by indirect income effects and indirect real interest rate effects.

We now compare the previous decomposition to the one proposed in our paper. The PI channel can be written as:

$$\text{PI} = \frac{1}{1 - \rho} = \mathcal{M}_y^D + \mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1}.$$

Finally, the RIR channel can be written as:

$$\begin{aligned} \text{RIR} &= \frac{\beta\rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho} - (1 - \beta)}{(1 - \beta\rho)\left[1 - \rho + \rho(\phi_\pi - 1)\frac{\kappa_y}{1 - \beta\rho}\right]} + \frac{1 - \beta}{(1 - \rho)(1 - \beta\rho)} \\ &= -\mathcal{M}_y^{\text{ID}} + \mathcal{M}_y^{\text{ID}}|_{\phi_\pi \rightarrow 1}. \end{aligned}$$

## APPENDIX C: Quantitative Modeling Details

This section provides the reader with a more detailed exposition of the model, sometimes repeating statements already made in the main text for expositional and clarity purposes.

### C.1 The Quantitative Model

The economy is populated by infinitely-lived heterogeneous households, final and intermediate good firms, a financial intermediary, a government, and a central bank. Throughout, we depart from the classical full information rational expectation (FIRE) assumption and introduce cognitive discounting in the DIS and NKPC equations of the model.

**C.1.1 Households** The economy features a unit mass of infinitely-lived households, indexed by  $i \in [0, 1]$ , with preferences over consumption  $C_t^i$  and labor  $L_t^i$  represented by the following utility function:

$$\mathcal{U}(C_t^i, L_t^i) = \frac{C_t^{i1-\sigma}}{1-\sigma} - v \frac{L_t^{i1+\varphi}}{1+\varphi},$$

where  $\sigma^{-1}$  is the inter-temporal elasticity of substitution (EIS),  $v > 0$  is a labor disutility parameter, and  $\varphi^{-1}$  is the Frisch elasticity. Households discount the future at the constant rate  $\beta \in (0, 1)$ . Finally, households supply a variety of differentiated labor input  $L_t^i(l)$ . Importantly, wage-setting decisions are made by labor type specific unions indexed by  $l \in (0, 1)$ , detailed below.

While household share the same preferences, they are heterogeneous with respect to their access to financial markets. Households are subject to idiosyncratic shocks that make them switch between two states — saver,  $S$ , and hand-to-mouth,  $H$ , households. Savers participate in asset markets on which they hold a portfolio consisting of three types of assets: (i) liquid bonds, (ii) shares of illiquid real stocks of intermediary good firms, and (iii) illiquid physical capital. Hand-to-mouth households cannot participate in asset markets and insure against income risk by holding liquid bonds. Transitions between states are modeled as a Markov process with probabilities  $\mathbb{P}(H_{t+1} | S_t) = 1 - s$  and  $\mathbb{P}(S_{t+1} | H_t) = 1 - h$ , where  $s, h \in (0, 1)$  govern persistence. The stationary equilibrium ensures the unconditional mass of hand-to-mouth households is  $\lambda = \frac{1-s}{2-h-s}$ .<sup>3</sup>

At the beginning of period  $t$ , all households of a given group ( $S$  or  $H$ ) pool resources, hence fully insuring each other and observe the aggregate shock prior to making their consumption and saving decisions. At the end of the period, they observe their period  $t + 1$  state on the asset market, and pool their liquid assets only with their peers. For instance, let  $B_{t+1}^S$  (resp.  $B_{t+1}^H$ ) denote the aggregate real bond holdings by all savers (resp. hand-to-mouth households) at the beginning of period  $t + 1$ .  $B_{t+1}^S$  comprises all the individual liquid assets holdings,  $b_{t+1}^S$  of the  $(1 - \lambda) \times s$  period  $t$  savers that remain savers

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<sup>3</sup>In this setup, transition probabilities also represent the proportion of households switching states.

in  $t + 1$ , to which add all the individual liquid assets holdings,  $b_{t+1}^H$  of the  $\lambda \times (1 - h)$  period  $t$  hand-to-mouth households that become savers in  $t + 1$ . Accordingly, we have

$$B_{t+1}^S = (1 - \lambda)sb_{t+1}^S + \lambda(1 - h)b_{t+1}^H. \quad (C.1)$$

Similarly,  $B_{t+1}^H$  comprises all the individual liquid assets holdings,  $b_{t+1}^h$  of the  $\lambda \times h$  period  $t$  hand-to-mouth households that remain hand-to-mouth in  $t + 1$ , to which add all the individual liquid assets holdings,  $b_{t+1}^s$  of the  $(1 - \lambda) \times (1 - s)$  period  $t$  savers that become hand-to-mouth households in  $t + 1$ ,

$$B_{t+1}^H = \lambda hb_{t+1}^H + (1 - \lambda)(1 - s)b_{t+1}^S. \quad (C.2)$$

Finally, the following assumptions on asset holdings are placed on households' behavior as an equilibrium selection device on financial markets.

ASSUMPTION 1.

(A1.a) *There is perfect insurance across households in the same state, but not across states.*

(A1.b) *Stocks and physical capital are illiquid and cannot be transferred across states.*

(A1.c) *Liquid bond holdings are weakly positive and the positivity constraint of hand-to-mouth households binds in each period.*

(A1.d) *No bonds are traded in equilibrium.*

(A1.a)–(A1.d) jointly apply and ensure a tractable representation of the bond market equilibrium. Under (A1.a)–(A1.b), all savers (resp. hand-to-mouths) make equivalent consumption and saving decisions. (A1.c) guarantees that  $H$  households are strictly hand-to-mouth, while (A1.d) imposes a zero liquidity limit (see e.g. [Krusell et al., 2011](#)).

Physical capital is illiquid, cannot be transferred across states and accumulates as

$$K_{t+1}^S = (1 - \delta)K_t^S + \Phi\left(\frac{I_t^S}{K_t^S}\right)K_t^S, \quad (C.3)$$

where  $\delta \in (0, 1]$  denotes the constant capital depreciation rate and  $\Phi(\cdot)$  describes how investment in final goods is turned into capital goods. This technology is increasing with investment intensity ( $\Phi'(\cdot) > 0$ ) and exhibits decreasing returns to scale ( $\Phi''(\cdot) \leq 0$ ), a manifestation of capital adjustment costs. We further impose  $\Phi(\delta) = \delta$  and  $\Phi'(\delta) = 1$  such that these costs are not operative in the deterministic steady state. Note that, since only savers have access to capital markets, we have  $K_t = (1 - \lambda)K_t^S$  and  $I_t = (1 - \lambda)I_t^S$ .

*Savers:* Saver households enter period  $t$  with the bond holdings,  $B_t^S/(1 - \lambda)$ , carried from period  $t - 1$  and which yield a real return  $\exp(-\xi_{t-1})R_{t-1}/\pi_t$ , where  $R_{t-1}$  is the gross nominal interest rate,  $\pi_t$  is the inflation rate between  $t - 1$  and  $t$ , and  $\xi_{t-1}$  is our DIS demand shock.<sup>4</sup> The shock is assumed to follow an exogenous AR(1) process with

<sup>4</sup>Formally,  $\xi_t$  is the risk premium shock in [Smets and Wouters \(2007\)](#), but we will stick to the DIS shock labeling so as to insure consistency with the main part of the paper.

persistence  $\rho \in [0, 1)$ . On top of these liquid assets, savers also carry physical capital  $K_t^S$  that yield an after tax return  $(1 - \tau^K)R_t^K$ , where  $\tau^K \in [0, 1)$  denotes the constant capital income tax and  $R_t^K$  is the real return on capital. Finally, their capital income also comprises  $\omega_t^S/(1 - \lambda)$  stocks each yielding an after tax dividend  $(1 - \tau^D)D_t$ , with  $\tau^D \in [0, 1)$ , and that resell at price  $q_t$ . The savers also receive a (after-tax) transfer  $(1 - \tau^F)F_t^S$  from a financial intermediary and pay a lump sum tax  $T_t^S$  to the government. Finally, they supply a variety of differentiated labor inputs  $L_t^S(l)$  at a nominal wage  $W_t(l)$ , which generates a labor income  $Y_t^S = \int_0^1 W_t(l)L_t^S(l)dl/P_t$ . These revenues finance consumption and investment expenditures,  $C_t^S$  and  $I_t^S$ , and savers' financial investment in the form of liquid bonds,  $b_{t+1}^S$  and stocks,  $\omega_{t+1}^S$ , so as to maximize their intertemporal utility. Accordingly, the optimal plan of a saver solves the Bellman equation

$$V^S(B_t^S, \omega_t^S, K_t^S) = \max_{\{C_t^S, I_t^S, b_{t+1}^S, \omega_{t+1}^S\}} \mathcal{U}(C_t^S, L_t^S) + \beta \tilde{\mathbb{E}}_t \left[ V^S(B_{t+1}^S, \omega_{t+1}^S, K_{t+1}^S) + \frac{\lambda}{1 - \lambda} V^H(B_{t+1}^H) \right]$$

subject to

$$\begin{aligned} C_t^S + b_{t+1}^S + q_t \frac{\omega_{t+1}^S}{1 - \lambda} + I_t^S &= Y_t^S + \frac{e^{-\xi_{t-1}} R_{t-1}}{\pi_t} \frac{B_t^S}{1 - \lambda} + \left( q_t + (1 - \tau^D)D_t \right) \frac{\omega_t^S}{1 - \lambda} \\ &+ (1 - \tau^K)R_t^K K_t^S + (1 - \tau^F)F_t^S - T_t^S, \end{aligned} \quad (\text{C.4})$$

$$b_{t+1}^S \geq 0 \quad \text{and} \quad (\text{C.1}) - (\text{C.3}),$$

where the value function accounts for the possibility that, in period  $t + 1$ , the saver can become a hand-to-mouth household.<sup>5</sup>  $\tilde{\mathbb{E}}_t$  denotes the expectation operator that may deviate from the full information rational expectation operator as discussed below. The optimal behavior is then characterized by the set of first order conditions (FOCs):

$$\begin{aligned} (C_t^S)^{-\sigma} &= \lambda_t^s, \\ \lambda_t^s &= \beta e^{-\xi_t} R_t^r \tilde{\mathbb{E}}_t^b \left[ s \lambda_{t+1}^s + (1 - s) \lambda_{t+1}^h \right], \\ q_t \lambda_t^s &= \beta \tilde{\mathbb{E}}_t^k \left[ \lambda_{t+1}^s \left( (1 - \tau_k) R_{t+1}^k + q_{t+1} \left( 1 - \delta + \Phi \left( \frac{I_{t+1}^s}{K_{t+1}^s} \right) - \Phi' \left( \frac{I_{t+1}^s}{K_{t+1}^s} \right) \frac{I_{t+1}^s}{K_{t+1}^s} \right) \right) \right], \\ 1 &= q_t \Phi' \left( \frac{I_t^s}{K_t^s} \right), \end{aligned}$$

where  $R_t^r$  is given by  $R_t^r = R_t/\mathbb{E}_t[\pi_{t+1}]$ , and where we make explicit use of Assumption A1.c which insures that the borrowing constraint never binds for savers.

*Hand-to-mouth:* The problem of hand-to-mouth households is similar to that of savers, to the notable exception that they do not have access to capital markets, implying that they cannot accumulate neither capital nor stocks and receive transfers from the

<sup>5</sup>The next period hand-to-mouth value function is scaled by  $\lambda/(1 - \lambda)$  as the state variable is written in terms of total island bond holdings. As such, the relative transition probability  $\lambda/(1 - \lambda)$  maps into the corresponding individual transition probabilities  $(s, 1 - s)$ .

government ( $T_t^H \leq 0$ ). Just like savers, hand-to-mouth households decide consumption expenditures,  $C_t^H$ , their liquid bond holdings,  $b_{t+1}^H$ , so as to maximize their intertemporal utility. Accordingly, the optimal plan of a saver solves the Bellman equation

$$V^H(B_t^H) = \max_{\{C_t^H, b_{t+1}^H\}} \mathcal{U}(C_t^H, L_t^H) + \beta \tilde{\mathbb{E}}_t \left[ V^H(B_{t+1}^H) + \frac{1-\lambda}{\lambda} V^S(B_{t+1}^S, \omega_{t+1}^S, K_{t+1}^S) \right]$$

subject to

$$C_t^H + b_{t+1}^H = Y_t^H + \frac{e^{-\xi_{t-1}} R_{t-1}}{\pi_t} \frac{B_t^H}{\lambda} - T_t^H, \quad (\text{C.5})$$

$$b_{t+1}^H \geq 0 \quad \text{and} \quad (\text{C.1}) - (\text{C.2}),$$

where  $Y_t^H \equiv \int_0^1 W_t(l) L_t^H(l) dl / P_t$  denotes real labor income of hand-to-mouth agents. The optimal behavior of the household is then given by

$$(C_t^h)^{-\sigma} = \lambda_t^h, \\ C_t^h = \frac{W_t}{\Delta_t^w} L_t^h - T_t^h,$$

where  $\Delta_t^w$  is a distribution term originating from the Calvo wage setting (see below).

Finally, we assume that hours worked are uniformly distributed across household types in the steady state, *i.e.*,  $L^S = L^H = L$ . In the spirit of [Werning \(2015\)](#) or [Patterson \(2023\)](#), they are allowed to fluctuate disproportionately off the steady state, *i.e.*,  $\hat{L}_t^H = \chi \hat{L}_t$ ,<sup>6</sup> where  $0 \leq \chi < \bar{\chi}$ . This, *de facto*, sets earnings incidence across households and affects the cyclicity of income inequalities.

Aggregate consumption and labor are given by:

$$C_t = \lambda C_t^h + (1-\lambda) C_t^s, \\ L_t = \lambda L_t^h + (1-\lambda) L_t^s.$$

**C.1.2 Expectation Formation:** We follow [Gabaix \(2014, 2020\)](#) and introduce cognitive discounting on the effects of aggregate shocks.<sup>7</sup> Consider a random variable  $X_t$  and let  $\bar{X}_t$  be the default value that agents assign to this variable in a given period  $t$ .<sup>8</sup> Let  $\hat{X}_{t+1} \equiv X_{t+1} - \bar{X}_t$  be the deviation of the future realization  $X_{t+1}$  from the default value as of period  $t$ . The behavioral expectation operator,  $\tilde{\mathbb{E}}_t[\cdot]$ , is then defined as

$$\tilde{\mathbb{E}}_t[X_{t+1}] = \tilde{\mathbb{E}}_t[\bar{X}_t + \hat{X}_{t+1}] = \bar{X}_t + \mu \mathbb{E}_t[\hat{X}_{t+1}], \quad (\text{C.6})$$

<sup>6</sup>A hat denotes relative deviations from the steady state.

<sup>7</sup>Notice that saver households have positive wealth holdings due to the two asset structure, even under the zero liquidity limit. To ensure comparability of the cognitive discounting across a larger class of models, we differ from [Gabaix \(2020\)](#) by assuming that the behavioral expectation operator (C.6) is defined with respect to the state comprising only the aggregate shock, not individual wealth. In mathematical terms, we assume  $\tilde{\mathbb{E}}_t[X_{t+1}^S(\mathcal{S}_{t+1}, K_{t+1}^S)] = \tilde{\mathbb{E}}_t[X_{t+1}^S(\mathcal{S}_{t+1}, 0)]$  for some exogenous aggregate state vector  $\mathcal{S}_t$ . This assumption is innocuous in the sense that it maps into the same counterparts of the *sufficient statistics*  $(\zeta_f, \beta_f)$  and only changes their values without affecting their expression.

<sup>8</sup>For example, agents may have in mind the steady value of  $X_t$  as a default value, in which case  $\bar{X}_t = \bar{X}$ .

where  $\mathbb{E}_t$  is the standard conditional expectation operator used under FIRE and  $\mu \in [0, 1]$  is the cognitive discounting parameter. If  $\mu = 1$  then  $\tilde{\mathbb{E}}_t[X_{t+1}] = \mathbb{E}_t[X_{t+1}]$  and expectations are rational. In contrast, if  $\mu = 0$  then expectations are fully anchored to the default value  $\bar{X}_t$ . Subsequently, we follow [Gabaix \(2020\)](#) and assume that households correctly perceive the contemporaneous real interest rate on bond holdings and physical capital. We denote the cognitive discounting parameter of the self-insurance equation by  $\mu_b$  and the one of the physical capital investment equation accordingly by  $\mu_k$ .<sup>9</sup>

**C.1.3 Firms** The economy comprises a continuum  $j \in [0, 1]$  of intermediary and a continuum of completely identical final good firms.

*Final Good Sector:* The final good sector is populated by a continuum of identical firms that act in a competitive environment. They purchase intermediate goods,  $X_t(j)$ , at price  $P_t(j)$  on a competitive market and produce a homogeneous good,  $Y_t$ , that is sold for consumption and investment purposes at a price  $P_t$  and produced with the constant return technology

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \quad \text{with } \epsilon_p > 1.$$

Profit maximization leads to the constant elasticity demand for intermediate good  $j$

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t,$$

where  $P_t = \left( \int_0^1 P_t(j)^{1 - \epsilon_p} dj \right)^{\frac{1}{1 - \epsilon_p}}$  denotes the aggregate price level.

*Intermediate Good Sector:* Each intermediate good  $j$  is produced by means of capital,  $K_t(j)$ , and labor,  $L_t(j)$ , according to a constant returns to scale technology described by the production function

$$Y_t(j) = AK_t(j)^\alpha L_t(j)^{1 - \alpha}, \quad \text{with } A > 0 \text{ and } \alpha \in (0, 1).$$

The labor input used by firm  $j$  is a composite of differentiated labor types and defined by  $L_t(j) = \left( \int_0^1 L_t(j, l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ .

Intermediary good firms rent capital and labor in perfectly competitive factor markets. However, the firm is subject to working capital and workers must be paid ahead of production. Consequently, every intermediate firm  $j$  borrows its nominal wage bill  $W_t L_t(j) = \int_0^1 W_t(l) L_t(j, l) dl$  from a financial intermediary and repays the loan at the end of period at an interest rate  $R_t^\psi$ . When  $\psi = 1$  we have the working capital assumption, when  $\psi = 0$  the firm is not affected by working capital. Intermediate values shall be interpreted as a weak form of working capital. For simplicity, we assume that the financial

<sup>9</sup>Heterogeneity in cognitive discount factors is meant to capture a combination of heterogeneous attention to specific variables and a uniform cognitive discounting of the state, leading to a *term structure of attention*, see [Gabaix \(2020\)](#), Section V.A.

intermediary has access to an inexhaustible credit line with the central bank at no interest payments, and that it redistributes its aggregate profits at the end of period to savers —i.e.,  $F_t^S = (R_t^\psi - 1)W_t N_t$ . The optimal firm behavior implies

$$MC_t(j) = MC_t = \frac{(R_t^k)^\alpha (w_t R_t^\psi)^{1-\alpha}}{A\alpha^\alpha (1-\alpha)^{1-\alpha}},$$

$$R_t^k K_t(j) = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t} R_t^\psi N_t(j),$$

where  $MC_t(j)$  denotes the real marginal cost of firm  $j$ , identical across all firms.

Intermediary goods firms behave under monopolistic competition, i.e., firm  $j$  has local monopoly power and sets its price as in Calvo (1983). For instance, only a fraction  $1 - \theta_p$  can adjust their price in each period. The remaining firms cannot re-optimize their price and instead partially adjust their prices to past inflation according to  $P_{t+\tau|t}(j) = P_{t+\tau-1|t}(j) \Xi_{t+\tau-1}^{\chi_p}$ , where  $\Xi_t = \pi_{t-1} \Xi_{t-1} \forall t \geq 1$  with  $\Xi_0 = 1$ , and  $\chi_p \in [0, 1]$  reflects the degree of indexation to past inflation. The firms that are allowed to re-adjust their price do so by maximizing their discounted expected profits

$$\max_{P_t^*} \tilde{\mathbb{E}}^P \left[ \sum_{\tau=0}^{\infty} \theta_p^\tau \Psi_{t,t+\tau} Y_{t+\tau|\tau}(j) \left( \frac{P_t^* \Xi_{t+\tau-1}^{\chi_p}}{P_{t+\tau}} - MC_{t+\tau} \right) \right]$$

$$\text{subject to } Y_{t+\tau|\tau}(j) = \left( \frac{P_t^* \Xi_{t+\tau-1}^{\chi_p}}{P_{t+\tau}} \right)^{-\epsilon_p} Y_{t+\tau},$$

where  $\Psi_{t,t+\tau} \equiv \beta^\tau (C_{t+\tau}^S / C_t^S)^{-\sigma}$  is the savers' stochastic discount factor and  $\tilde{\mathbb{E}}^P[\cdot]$  denotes the behavioral expectation operator of intermediate firms with cognitive discounting parameter  $\mu_p$ . Finally, aggregating across firms and using the Poisson process associated with the Calvo fairy, the aggregate price index satisfies

$$P_t^{1-\epsilon_p} = \theta_p (P_{t-1} \pi_{t-1}^{\chi_p})^{1-\epsilon_p} + (1 - \theta_p) (P_t^*)^{1-\epsilon_p}.$$

The optimal price setting behavior is summarized by the following set of FOCs:

$$\Omega_{\pi,t}^n = \lambda_t^s MC_t Y_t + \beta \theta_p \tilde{\mathbb{E}}^P \left[ \left( \frac{\pi_t^{\chi_\pi}}{\pi_{t+1}} \right)^{-\epsilon_p} \Omega_{\pi,t+1}^n \right],$$

$$\Omega_{\pi,t}^d = \lambda_t^s \pi_t^* Y_t + \beta \theta_p \tilde{\mathbb{E}}^\pi \left[ \left( \frac{\pi_t^{\chi_\pi}}{\pi_{t+1}} \right)^{1-\epsilon_p} \left( \frac{\pi_t^*}{\pi_{t+1}^*} \right) \Omega_{\pi,t+1}^d \right],$$

$$\varepsilon_p \Omega_{\pi,t}^n = (\varepsilon_p - 1) \Omega_{\pi,t}^d,$$

$$\Delta_t^\pi = \theta_\pi \left( \frac{\pi_{t-1}^{\chi_\pi}}{\pi_t} \right)^{-\varepsilon_p} \Delta_{t-1}^\pi + (1 - \theta_\pi) (\pi_t^*)^{-\varepsilon_p},$$

where  $\pi_t = P_t / P_{t-1}$  denotes the inflation rate and  $\pi_t^*$  is the optimal price change of a firms who revise its price setting decision.  $\Delta_t^\pi$  denotes the distribution of prices across contracts.

**C.1.4 Nominal Wage Rigidities** Just like prices, nominal wages are subject to nominal rigidities. Following [Colciago \(2011\)](#), we assume that each variety (or skill) of labor,  $l \in [0, 1]$ , supplied by households is represented by a labor union. Each skill-specific union aggregates labor across households and sets nominal wages subject to [Calvo \(1983\)](#)–type nominal rigidities. In any given period, only a fraction  $1 - \theta_w$  of labor skill-specific unions can re-optimize their wages. The remaining fraction adjusts wages based on past price inflation, such that  $W_{t+\tau|t}(l) = W_{t+\tau-1|t}(l) \Xi_{t+\tau-1}^{\chi_w}$ , where  $\chi_w \in [0, 1]$  reflects the degree of wage indexation to inflation, and  $\Xi_t$  was defined earlier. When allowed to re-optimize, the union sets wages to maximize the utilitarian expected welfare of its members – both savers and hand-to-mouth workers – subject to the aggregate demand for labor of type  $l$ .<sup>10</sup> The union's optimization problem is:<sup>11</sup>

$$\begin{aligned} \max_{W_t^*(l)} \quad & \tilde{\mathbb{E}}_t^w \left[ \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau \left( \lambda \mathcal{U}(C_{t+\tau}^H, L_{t+\tau}^H) + (1 - \lambda) \mathcal{U}(C_{t+\tau}^S, L_{t+\tau}^S) \right) \right] \\ \text{subject to} \quad & L_{t+\tau}(l) = \left( \frac{W_t^*(l) \Xi_{t+\tau-1}}{W_{t+\tau}} \right)^{-\epsilon_w} L_{t+\tau}, \quad (\text{C.4}) \ \& \ (\text{C.5}). \end{aligned}$$

where  $\tilde{\mathbb{E}}_t^w[\cdot]$  denotes the behavioral expectation operator of unions with a cognitive discounting factor  $\mu_w$ , and  $W_t$  is the aggregate wage given by  $W_t \equiv \left( \int_0^1 W_t(l)^{1-\epsilon_w} dl \right)^{\frac{1}{1-\epsilon_w}}$ . In a symmetric equilibrium where all unions set the same wage,  $W_t^*(l) = W_t^*$ , and applying the Calvo fairy, nominal wage dynamics simplify to:

$$W_t^{1-\epsilon_w} = \theta_w (W_{t-1} \pi_{t-1}^{\chi_w})^{1-\epsilon_w} + (1 - \theta_w) (W_t^*)^{1-\epsilon_w}.$$

The optimal wage setting behavior is then characterized by

$$\begin{aligned} \Omega_{w,t}^n &= v \left( \frac{w_t}{w_t^*} \right)^{\varepsilon_w(1+\varphi)} \left( \frac{L_t}{\Delta_t^w} \right)^{1+\varphi} + \beta \theta_w \tilde{\mathbb{E}}_t^w \left[ \left( \frac{\pi_t^{\chi_w} w_t^*}{\pi_{t+1} w_{t+1}^*} \right)^{-\varepsilon_w(1+\varphi)} \Omega_{w,t+1}^n \right], \\ \Omega_{w,t}^d &= \lambda_t w_t^{*1-\varepsilon_w} w_t^{\varepsilon_w} \frac{L_t}{\Delta_t^w} + \beta \theta_w \tilde{\mathbb{E}}_t^w \left[ \left( \frac{\pi_t^{\chi_w} w_t^*}{\pi_{t+1} w_{t+1}^*} \right)^{1-\varepsilon_w} \Omega_{w,t+1}^d \right], \\ \varepsilon_w \Omega_{w,t}^n &= (\varepsilon_w - 1) \Omega_{w,t}^d, \\ \Delta_t^w &= \theta_w \left( \frac{\pi_t^{\chi_w}}{\pi_t^w} \right)^{-\varepsilon_w} \Delta_{t-1}^w + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{-\varepsilon_w}, \\ \pi_t^w &= \frac{w_t}{w_{t-1}} \pi_t, \end{aligned}$$

<sup>10</sup>The aggregate demand for labor of type  $l$  is given by  $L_t(l) = \int_0^1 L_t(j, l) dj$ . Using the fact that  $L_t(j, l) = (W_t(l)/W_t)^{-\epsilon_w} L_t(j)$ , we have  $L_t(l) = (W_t(l)/W_t)^{-\epsilon_w} \int_0^1 L_t(j) dj = (W_t(l)/W_t)^{-\epsilon_w} L_t$ .

<sup>11</sup>As stated in the main text, unions do not internalize the potentially heterogeneous earnings incidence when setting wages. [Gaillard and Wangner \(2022\)](#) and [Gerke et al. \(2023\)](#) show that a cyclical earnings incidence may steepen the NKPC as unions provide insurance against cyclical inequality by limiting fluctuations in aggregate hours worked.

where  $\pi_t^w$  denotes wage inflation,  $w_t = W_t/P_t$  is the real wage,  $w_t^*$  is the optimal real wage and  $\Delta_t^w$  is the wage distribution term.

**C.1.5 Policy Authorities** The fiscal authority redistributes tax revenues across households, while maintaining a balanced budget without resorting to debt issuance:

$$\lambda T_t^H + (1 - \lambda)T_t^S + \tau^D D_t + \tau^K R_t^K K_t + \tau^F F_t = 0 .$$

In addition, the monetary authority follows a simple Taylor rule of the form:

$$R_t = R_{t-1}^{\phi_i} \left[ R(\pi_t/\pi)^{\phi_\pi} (Y_t/Y)^{\phi_y} \right]^{1-\phi_i} ,$$

where  $\phi_\pi > 1$ ,  $\phi_y > 0$  and  $\phi_i \in [0, 1)$ .

**Equilibrium Conditions:** For completeness, we subsequently list all equilibrium conditions denoted in real terms.

*Households.* The household conditions are as follows:

$$(C_t^s)^{-\sigma} = \lambda_t^s ,$$

$$\lambda_t^s = \beta e^{-\xi_t} R_t^r \tilde{\mathbb{E}}_t^b \left[ s \lambda_{t+1}^s + (1 - s) \lambda_{t+1}^h \right] ,$$

$$R_t = R_t^r \mathbb{E}_t[\pi_{t+1}] ,$$

$$K_{t+1}^s = (1 - \delta) K_t^s + \Phi \left( \frac{I_{t+1}^s}{K_{t+1}^s} \right) K_t^s ,$$

$$q_t \lambda_t^s = \beta \tilde{\mathbb{E}}_t^k \left[ \lambda_{t+1}^s \left( (1 - \tau_k) R_{t+1}^k + q_{t+1} \left( 1 - \delta + \Phi \left( \frac{I_{t+1}^s}{K_{t+1}^s} \right) - \Phi' \left( \frac{I_{t+1}^s}{K_{t+1}^s} \right) \frac{I_{t+1}^s}{K_{t+1}^s} \right) \right) \right] ,$$

$$1 = q_t \Phi' \left( \frac{I_t^s}{K_t^s} \right) ,$$

$$(C_t^h)^{-\sigma} = \lambda_t^h ,$$

$$C_t^h = \frac{w_t}{\Delta_t^w} L_t^h - T_t^h ,$$

$$L_t^h = L + \chi(L_t - L) .$$

*Firms.* The firm conditions are as follows:

$$\Delta_t^\pi Y_t = K_t^\alpha (L_t/\Delta_t^w)^{1-\alpha} ,$$

$$MC_t = \frac{R_t^{k\alpha} (w_t R_t^\psi)^{1-\alpha}}{A \alpha^\alpha (1 - \alpha)^{1-\alpha}} ,$$

$$R_t^k K_t = \frac{\alpha}{1 - \alpha} \frac{w_t}{\Delta_t^w} R_t^\psi L_t ,$$

$$D_t = Y_t (1 - MC_t) .$$

*Government.* The government conditions are as follows:

$$T_t^s = \bar{T}^s ,$$

$$0 = \lambda T_t^h + (1 - \lambda) T_t^s + \tau^D D_t + \tau^K R_t^K K_t + \tau^F F_t .$$

*Price Setting.* The price setting conditions are as follows:

$$\Omega_{\pi,t}^n = \lambda_t^s M C_t Y_t + \beta \theta_\pi \mathbb{E}^p \left[ \left( \frac{\pi_t^{\chi_\pi}}{\pi_{t+1}} \right)^{-\varepsilon_p} \Omega_{\pi,t+1}^n \right] ,$$

$$\Omega_{\pi,t}^d = \lambda_t^s \pi_t^* Y_t + \beta \theta_\pi \mathbb{E}^p \left[ \left( \frac{\pi_t^{\chi_\pi}}{\pi_{t+1}} \right)^{1-\varepsilon_p} \left( \frac{\pi_t^*}{\pi_{t+1}^*} \right) \Omega_{\pi,t+1}^d \right] ,$$

$$\varepsilon_p \Omega_{\pi,t}^n = (\varepsilon_p - 1) \Omega_{\pi,t}^d ,$$

$$1 = \theta_\pi \left( \frac{\pi_t^{\chi_\pi - 1}}{\pi_t} \right)^{1-\varepsilon_p} + (1 - \theta_\pi) \pi_t^{*1-\varepsilon_p} ,$$

$$\Delta_t^\pi = \theta_\pi \left( \frac{\pi_t^{\chi_\pi - 1}}{\pi_t} \right)^{-\varepsilon_p} \Delta_{t-1}^\pi + (1 - \theta_\pi) \pi_t^{*- \varepsilon_p} .$$

*Wage Setting.* The wage setting conditions are as follows:

$$\Omega_{w,t}^n = v \left( \frac{w_t}{w_t^*} \right)^{\varepsilon_w(1+\varphi)} \left( \frac{L_t}{\Delta_t^w} \right)^{1+\varphi} + \beta \theta_w \tilde{\mathbb{E}}_t^w \left[ \left( \frac{\pi_t^{\chi_w}}{\pi_{t+1}} \frac{w_t^*}{w_{t+1}^*} \right)^{-\varepsilon_w(1+\varphi)} \Omega_{w,t+1}^n \right] ,$$

$$\Omega_{w,t}^d = \lambda_t (w_t^*)^{1-\varepsilon_w} w_t^{\varepsilon_w} \frac{L_t}{\Delta_t^w} + \beta \theta_w \tilde{\mathbb{E}}_t^w \left[ \left( \frac{\pi_t^{\chi_w}}{\pi_{t+1}} \frac{w_t^*}{w_{t+1}^*} \right)^{1-\varepsilon_w} \Omega_{w,t+1}^d \right] ,$$

$$\varepsilon_w \Omega_{w,t}^n = (\varepsilon_w - 1) \Omega_{w,t}^d ,$$

$$1 = \theta_w \left( \frac{\pi_t^{\chi_w - 1}}{\pi_t} \right)^{1-\varepsilon_w} + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{1-\varepsilon_w} ,$$

$$\Delta_t^w = \theta_w \left( \frac{\pi_t^{\chi_w - 1}}{\pi_t} \right)^{-\varepsilon_w} \Delta_{t-1}^w + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{-\varepsilon_w} ,$$

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t .$$

*Aggregation and Policy.* The aggregation and policy conditions are as follows:

$$Y_t = C_t + I_t ,$$

$$C_t = \lambda C_t^h + (1 - \lambda) C_t^s ,$$

$$L_t = \lambda L_t^h + (1 - \lambda) L_t^s ,$$

$$I_t = (1 - \lambda) I_t^s ,$$

$$K_t = (1 - \lambda)K_t^s ,$$

$$\lambda_t = \lambda \lambda_t^h + (1 - \lambda) \lambda_t^s ,$$

$$R_t = R_{t-1}^{\phi_i} \left[ R(\pi_t/\pi)^{\phi_\pi} (Y_t/Y)^{\phi_y} \right]^{1-\phi_i} .$$

**Log-linear Version:** We now log-linearize the preceding set of equations around its deterministic steady state, which, after tedious (but simple) algebra and manipulation of equations, yields the following system:

$$\begin{aligned} & -\sigma \widehat{C}_t^s - \widehat{\lambda}_t^s = 0 , \\ & -\sigma \widehat{C}_t^h - \widehat{\lambda}_t^h = 0 , \\ & \widehat{C}_t - \lambda \widehat{C}_t^h - (1 - \lambda) \widehat{C}_t^s = 0 , \\ & \widehat{L}_t^h - \chi \widehat{L}_t = 0 , \\ & \widehat{L}_t - \lambda \widehat{L}_t^h - (1 - \lambda) \widehat{L}_t^s = 0 , \\ & \widehat{K}_{t+1} - \delta \widehat{I}_t - (1 - \delta) \widehat{K}_t = 0 , \\ & \widehat{Y}_t - \alpha \widehat{K}_t - (1 - \alpha) \widehat{L}_t = 0 , \\ & \widehat{Y}_t - s_c \widehat{C}_t - (1 - s_c) \widehat{I}_t = 0 , \\ & \widehat{R}_t^k + \widehat{K}_t - \widehat{w}_t - \widehat{L}_t - \psi \widehat{R}_t = 0 , \\ & \widehat{MC}_t - \alpha \widehat{R}_t^k - (1 - \alpha) \widehat{w}_t - \psi(1 - \alpha) \widehat{R}_t = 0 , \\ & \widehat{\pi}_t^w - \widehat{w}_t - \widehat{\pi}_t + \widehat{w}_{t-1} = 0 , \\ & \widehat{D}_t - \widehat{Y}_t + (\varepsilon_p - 1) \widehat{MC}_t = 0 , \\ & \widehat{T}_t^s = 0 , \\ & \frac{T^h}{Y} \widehat{T}_t^h + \frac{\tau^D}{\varepsilon_p \lambda} \widehat{D}_t + \frac{\tau^K}{\lambda} \frac{R^k K}{Y} (\widehat{R}_t^k + \widehat{K}_t) + \frac{\tau^F}{\lambda} \frac{F}{Y} \widehat{F}_t = 0 , \\ & \widehat{\lambda}_t^s - \widehat{R}_t + \widehat{\xi}_t - \mu_b \mathbb{E}_t[\widehat{\pi}_{t+1} + s \widehat{\lambda}_{t+1}^s + (1 - s) \widehat{\lambda}_{t+1}^h] = 0 , \\ & \widehat{\lambda}_t^s + \widehat{q}_t - \mathbb{E}_t[(1 - \beta(1 - \delta)) \widehat{R}_{t+1}^k + \widehat{\lambda}_{t+1}^s + \beta \widehat{q}_{t+1}] = 0 , \\ & \widehat{q}_t - \frac{1}{\eta} (\widehat{I}_t^s - \widehat{K}_t^s) = 0 , \\ & \theta_p [1 + \beta \chi_p (1 - (1 - \mu_p) \theta_p)] \widehat{\pi}_t - (1 - \beta \theta_p) (1 - \theta_p) \widehat{MC}_t - \chi_p \theta_p \widehat{\pi}_{t-1} - \beta \mu_p \theta_p \mathbb{E}_t[\widehat{\pi}_{t+1}] = 0 , \\ & (1 - \theta_w) \widehat{w}_t^* - (1 - \theta_w) \widehat{w}_t - \theta_w \widehat{\pi}_t^w + \theta_w \chi_w \widehat{\pi}_{t-1} = 0 , \\ & (1 + \varphi \epsilon_w) \widehat{w}_t^* - (1 - \beta \theta_w) \varphi \epsilon_w \widehat{w}_t - (1 - \beta \theta_w) \varphi \widehat{L}_t + (1 - \beta \theta_w) \widehat{\lambda}_t^h + (1 - \beta \theta_w) (1 - \lambda) \widehat{\lambda}_t^s \\ & + \beta \theta_w \chi_w (1 + \epsilon_w \varphi) \widehat{\pi}_t - \beta \theta_w \mu_w (1 + \epsilon_w \varphi) \mathbb{E}_t[\widehat{w}_{t+1}^* + \widehat{\pi}_{t+1}] = 0 , \\ & \widehat{R}_t - \phi_i \widehat{R}_{t-1} - (1 - \phi_i) (\phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t) = 0 . \end{aligned}$$

## C.2 Benchmark Parametrization

We parametrize the model to match some key features of the US economy. Table C.1 summarizes the baseline parameters used in the calibration.

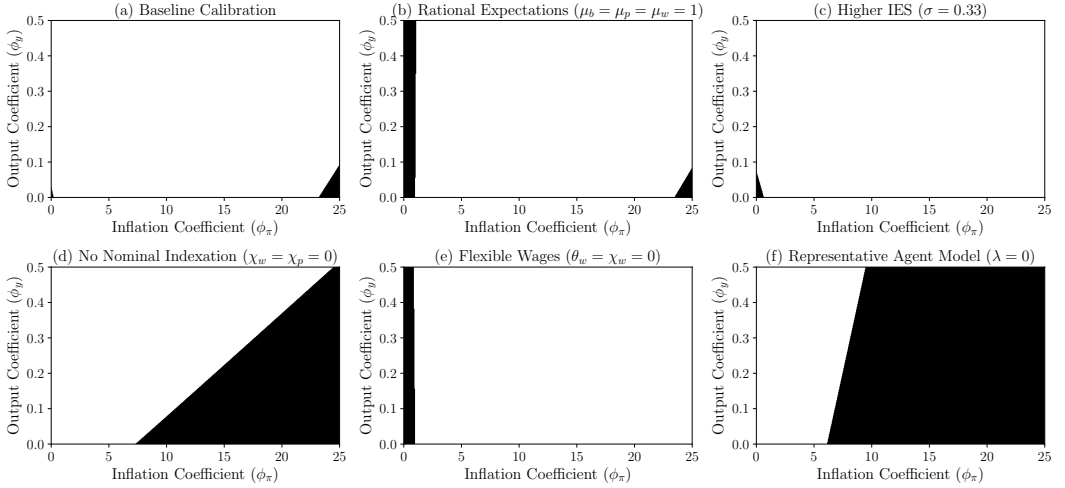
TABLE C.1. Parameters in the Benchmark US Economy

| PARAMETER                                   | SYMBOL       | VALUE | SOURCE/TARGET                                          |
|---------------------------------------------|--------------|-------|--------------------------------------------------------|
| <i>Household Preferences</i>                |              |       |                                                        |
| Discount factor                             | $\beta$      | 0.99  | annual nominal interest rate of 4%                     |
| Inverse intertemporal elasticity            | $\sigma$     | 1.00  | logarithmic utility                                    |
| Labor disutility                            | $v$          | 5.00  | calibrated to match 1/3 time at work                   |
| Frisch elasticity                           | $\varphi$    | 1.00  | Meta-analysis <a href="#">Chetty et al. (2013)</a>     |
| <i>Household Heterogeneity</i>              |              |       |                                                        |
| Idiosyncratic risk                          | $s$          | 0.98  | <a href="#">Bilbiie et al. (2022)</a>                  |
| Hand-to-mouth share                         | $\lambda$    | 0.30  | <a href="#">Kaplan et al. (2014)</a>                   |
| Earnings incidence                          | $\chi$       | 1.50  | Match cov(MPC,Earning)                                 |
| <i>Expectation Formation</i>                |              |       |                                                        |
| Cognitive discounting HH's bond DIS         | $\mu_b$      | 0.85  | <a href="#">Gabaix (2020)</a>                          |
| Cognitive discounting HH's capital DIS      | $\mu_k$      | 1.00  | benchmark value                                        |
| Cognitive discounting firm's price setting  | $\mu_p$      | 0.85  | <a href="#">Gabaix (2020)</a>                          |
| Cognitive discounting unions's wage setting | $\mu_w$      | 0.85  | <a href="#">Gabaix (2020)</a>                          |
| <i>Intermediate Good Production</i>         |              |       |                                                        |
| TFP                                         | $A$          | 1.00  | benchmark value                                        |
| Capital share                               | $\alpha$     | 0.33  | US capital income share                                |
| <i>Working Capital</i>                      |              |       |                                                        |
| Loan interest rate elasticity               | $\psi$       | 1.00  | benchmark value, <a href="#">Beaudry et al. (2024)</a> |
| <i>Investment</i>                           |              |       |                                                        |
| Depreciation                                | $\delta$     | 0.025 | <a href="#">Christiano and Eichenbaum (1992)</a>       |
| Adjustment Costs                            | $\eta$       | 2.000 | benchmark value                                        |
| <i>Price and Wage Setting</i>               |              |       |                                                        |
| Demand elasticity for intermediates         | $\epsilon_p$ | 6.00  | price markup of 20%                                    |
| Demand elasticity for labor                 | $\epsilon_w$ | 6.00  | wage markup of 20%                                     |
| Calvo price reset probability               | $\theta_p$   | 0.65  | three quarter average price duration                   |
| Calvo wage reset probability                | $\theta_w$   | 0.75  | annual average wage duration                           |
| Price indexation                            | $\chi_p$     | 0.30  | <a href="#">Smets and Wouters (2007)</a>               |
| Wage indexation                             | $\chi_w$     | 0.30  | <a href="#">Smets and Wouters (2007)</a>               |
| <i>Government Policy</i>                    |              |       |                                                        |
| Capital income tax                          | $\tau^K$     | 0.20  | US marginal long-term capital gains tax                |
| Dividend tax                                | $\tau^D$     | 0.20  | US federal corporate income tax                        |
| Intermediary dividend tax                   | $\tau^F$     | 0.20  | US federal corporate income tax                        |
| <i>Systematic Monetary Policy</i>           |              |       |                                                        |
| Interest rate smoothing                     | $\phi_i$     | 0.00  | benchmark value                                        |
| Taylor rule inflation coefficient           | $\phi_\pi$   | 1.50  | benchmark value                                        |
| Taylor rule output coefficient              | $\phi_y$     | 0.125 | benchmark value                                        |

### C.3 Local Determinacy

Our model features a locally determinate equilibrium under the benchmark calibration. Recall that the systematic monetary policy parameters ( $\phi_\pi, \phi_y$ ) have a dual role in our analysis. On the one hand, they ensure a determinate equilibrium. On the other hand, they are crucial in determining the monetary policy regimes. Out of this reason, we show the determinacy region for the benchmark economy and various model variants in Figure C.1.

FIGURE C.1. Regions of Local Determinacy and Indeterminacy



Note: The grey shaded area denotes the model implied indeterminacy region, whereas the white region denotes the determinacy region. Each model variant is obtained by changing only the parameter value denoted inside the sub-caption, while keeping the baseline values of Table C.1 otherwise unchanged.

To begin with, let us consider the determinacy properties of the benchmark economy in Panel (a). Due to behavioral expectation formation in form of cognitive discounting, the benchmark model features a discounted bond DIS equation such that its dynamics are locally determinate even under an interest rate peg, i.e.,  $\phi_\pi = \phi_y = 0$  (see [Gabaix \(2020\)](#)). Importantly, the model is indeterminate if  $\phi_\pi$  is sufficiently high and, simultaneously,  $\phi_y$  too low in the presence of a cost channel of monetary policy (see [Beaudry et al. \(2024\)](#)).

Moreover, in Panel (b) we show that the departure from rational expectations crucially alters the (in)determinacy regions. Under rational expectations (i.e.,  $\mu_b = \mu_p = \mu_w = 1.0$ ) idiosyncratic income risk leads to a compounded bond DIS equation and hence requires a more aggressive Taylor feedback to inflation for determinacy ([Acharya and Dogra, 2020](#), [Bilbiie et al., 2022](#), [Bilbiie, 2025](#)), i.e.,  $\phi_\pi$  needs to be strictly positive.

In Panel (c) we plot the determinacy region under a larger elasticity of intertemporal substitution ( $\sigma = 0.33$ ). Two observations stand out: on the one hand, the indeterminacy region due to the cost channel of monetary policy disappears (or occurs at a much

higher value for  $\phi_\pi$ ); on the other hand, the behavioral expectation formation is not sufficient anymore to ensure determinacy under an interest rate peg.

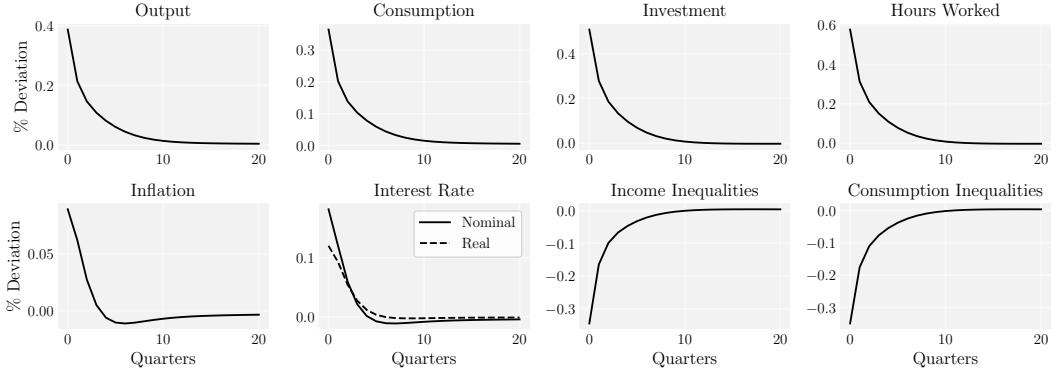
We also study the impact of two modeling choices related to the price and wage setting mechanisms. First, we simultaneously eliminate the partial price and wage indexation ( $\chi_p = \chi_w = 0$ ). As such, the determinacy region becomes much smaller. This is the case since contemporaneous and forward-looking drivers of inflation gain weight in the NKPC, which increases the nominal interest rate response and, hence, the active forces of the cost channel. Second, we study a model variant with flexible wages ( $\theta_w = \chi_w = 0$ ). In this case, behavioral expectations are no longer sufficient to restore determinacy under an interest rate peg. The reasoning is as follows: the real wage response and the aggregate demand complementarity between saver and hand-to-mouth households are much more pronounced under flexible wages such that the monetary authority must fight inflation in a sufficiently aggressive manner (see [Colciago \(2011\)](#)). As a byproduct, the indeterminacy of the cost channel disappears as well (or occurs at a much higher value for  $\phi_\pi$ ).

Finally, we eliminate household heterogeneity by setting the hand-to-mouth share to zero in Panel (f). In this case the determinacy region shrinks substantially.

### C.4 Impulse Response Functions

Under the benchmark calibration, the equilibrium is locally determinate. Figure C.2 reports the impulse response functions (IRF's) of the main variables of the model to a positive DIS shock.

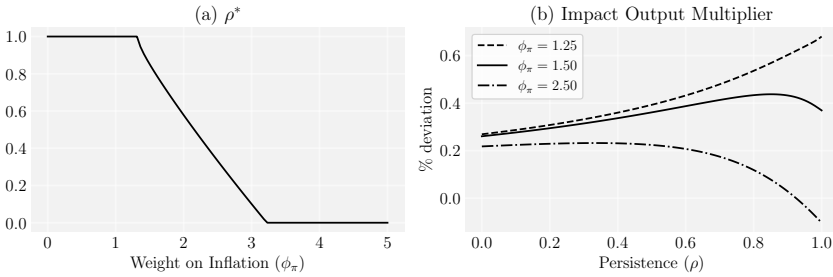
FIGURE C.2. Impulse Response Functions to a 25-basis point DIS shock



Note: Income and Consumption inequalities are measured respectively by  $y_t^S / y_t^H$  and  $c_t^S / c_t^H$ .

In addition, Figure C.3 shows the output-maximizing persistence as well as the comparative statics of the impact output multiplier under the three monetary regimes.

FIGURE C.3. Output Impact Multipliers and the  $\rho^*$



Note: Impact multipliers are expressed in terms of % deviation from steady state following a 25-basis points DIS shock.

## C.5 Monte-Carlo Experiment

TABLE C.2. Support of Uniform Distributions

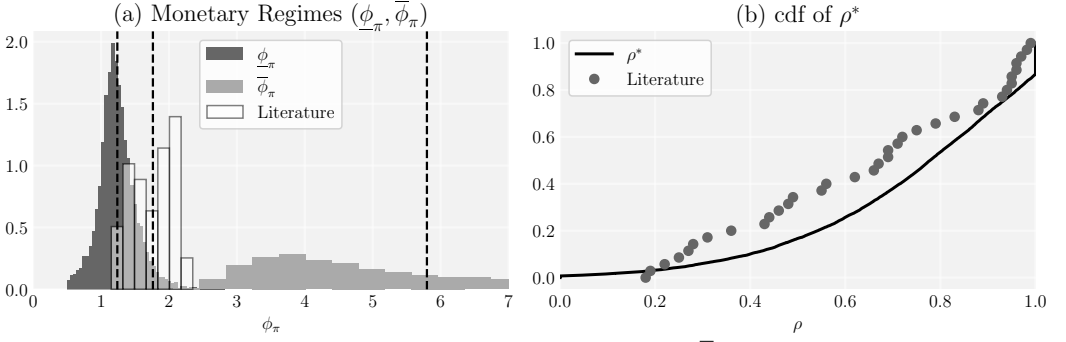
| Parameter                    |                                        | Lower Bound | Upper Bound |
|------------------------------|----------------------------------------|-------------|-------------|
| <b>Preferences</b>           |                                        |             |             |
| $\sigma$                     | Inverse intertemporal elasticity       | 0.30        | 3.00        |
| $\varphi$                    | Inverse Frisch elasticity              | 0.50        | 2.00        |
| <b>Heterogeneity</b>         |                                        |             |             |
| $\lambda$                    | fraction of hand-to-mouth households   | 0.01        | 0.40        |
| $s$                          | Idiosyncratic risk                     | 0.95        | 1.00        |
| $\chi$                       | Earnings incidence                     | 1.00        | 2.00        |
| <b>Taxation</b>              |                                        |             |             |
| $\tau_K$                     | Capital income tax                     | 0.00        | 0.35        |
| $\tau_D$                     | Dividend tax                           | 0.00        | 0.35        |
| $\tau_F$                     | Intermediary dividend tax              | 0.00        | 0.35        |
| <b>Technology</b>            |                                        |             |             |
| $\eta$                       | Investment/Capital elasticity          | 0.10        | 5.00        |
| <b>Price Setting</b>         |                                        |             |             |
| $\epsilon_p$                 | Demand elasticity for intermediates    | 4.00        | 8.00        |
| $\epsilon_w$                 | Demand elasticity for labor            | 4.00        | 8.00        |
| $\chi_p$                     | Price indexation                       | 0.00        | 0.50        |
| $\chi_w$                     | Wage indexation                        | 0.00        | 0.50        |
| $\theta_p$                   | Calvo parameter price setting          | 0.60        | 0.70        |
| $\theta_w$                   | Calvo parameter wage setting           | 0.70        | 0.80        |
| <b>Monetary Policy</b>       |                                        |             |             |
| $\phi_i$                     | Interest rate smoothing* (when active) | 0.00        | 0.85        |
| $\phi_\pi$                   | Taylor rule inflation coefficient      | 1.00        | 3.00        |
| $\phi_y$                     | Taylor rule output coefficient         | 0.00        | 0.25        |
| <b>Cognitive Discounting</b> |                                        |             |             |
| $\mu_s$                      | Consumption-saving decision            | 0.80        | 1.00        |
| $\mu_\pi$                    | Price setting                          | 0.80        | 1.00        |
| $\mu_w$                      | Wage setting                           | 0.80        | 1.00        |

Note: \* Not implemented in our benchmark experiment.

## APPENDIX D: Milder Working Capital Assumption

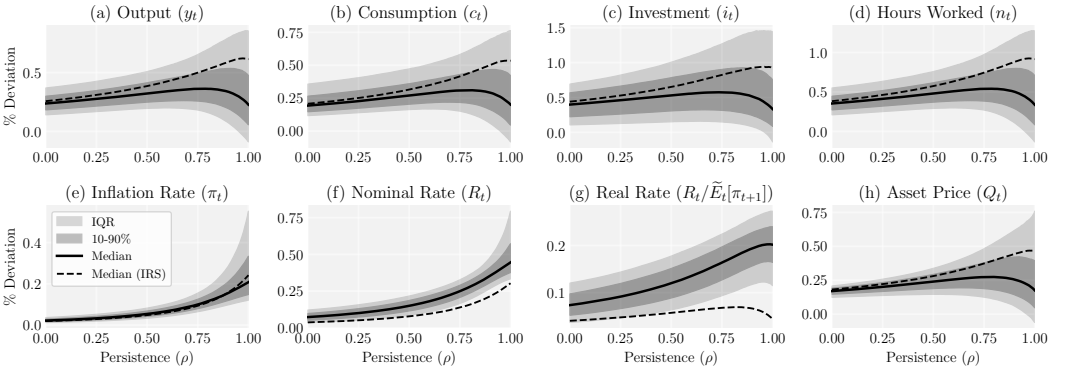
Following [Christiano et al. \(2005\)](#), [Ravenna and Walsh \(2006\)](#), [Beaudry et al. \(2024\)](#), our benchmark model makes the working capital assumption which introduces a direct link between interest rates and firms marginal costs, thereby steepening the NKPC. As shown in [Figures D.1 and D.2](#), weakening this assumption by considering that firms need to repay  $R_t^\psi W_t N_t$  and drawing uniformly from  $\psi \in [0, 1]$  does not fundamentally affect the results.

FIGURE D.1. Monetary Regimes across Simulations (Drawing Working Capital)



Note: The vertical dashed lines correspond to the average values of  $\phi_\pi, \bar{\phi}_\pi$ , and the empirical average of  $\phi_\pi$  reported in the literature.

FIGURE D.2. Impact Multipliers across Population of Models (Drawing Working Capital)



Note: Impact IRFs are expressed as percentage deviations from steady state. The nominal interest rate, inflation, and real interest rate responses are reported in annualized terms. IRS denotes interest rate smoothing, IQR is the interquartile range.

## APPENDIX E: List of Papers Estimating Persistence of DIS Shocks

This section lists the papers we used to obtain estimates of the persistence of the DIS shock and the monetary policy parameter reported in Figure 4 of the paper. Some of these papers consider variants of their model and report several estimates of these parameters. Our Figure therefore provides a population of estimated parameters corresponding to a population of models.

1. BAYER, CHRISTIAN, BENJAMIN BORN, AND RALPH LUETTICKE, (2024): "Shocks, Frictions, and Inequality in US Business Cycles", *American Economic Review*, 114(5), pp. 1211-1247.
2. BILBIE, FLORIN O., GIORGIO PRIMICERI, AND ANDREA TAMBALOTTI, (2023): "Inequality and Business Cycles," NBER Working Papers 31729.
3. BOIVIN, JEAN, AND MARC GIANNONI, (2006): "DSGE Models in a Data-Rich Environment," NBER Technical Working Papers 12772.
4. CAI, MICHAEL, DEL NEGRO, MARC, GIANNONI, MARC P., GUPTA, ABHI, LI, PEARL, AND MOSZKOWSKI, ERICA, (2019): "DSGE Forecasts of the Lost Recovery," *International Journal of Forecasting*, 35(4), pp. 1770-1789.
5. DEL NEGRO, MARCO, MARC P. GIANNONI, AND FRANK SCHORFHEIDE, (2015): "Inflation in the Great Recession and New Keynesian Models," *American Economic Journal: Macroeconomics*, 7(1), pp. 168-196.
6. FARIA E CASTRO, MIGUEL, (2025): "The St. Louis Fed DSGE Model", Federal Reserve Bank of St. Louis Working Paper 2024-014.
7. FÈVE, PATRICK, ALBAN MOURA, AND OLIVIER PIERRARD, (2022): "The Fall in Shadow Banking and the Slow U.S. Recovery," *Journal of Economic Dynamics and Control*, vol. 139, pp. 1044-1064.
8. FORNERON, JEAN-JACQUES, AND ZHONGJUN QU, (2025): "Fitting Dynamically Misspecified Models: An Optimal Transportation Approach", mimeo Boston University.
9. GALÍ, JORDI, FRANK SMETS, AND RAFAEL WOUTERS, (2012): "Unemployment in an Estimated New Keynesian Model," *NBER Macroeconomics Annual*, vol. 26, pp. 329 - 360.
10. GILCHRIST, SIMON, ALBERTO ORTIZ, AND ZAKRAJEK, EGON, (2009): "Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model", mimeo Boston University.
11. GOMES, SANDRA, ISKREV, NIKOLAY, AND MENDICINO, CATERINA, (2017): "Monetary Policy Shocks: We Got News!," *Journal of Economic Dynamics and Control*, 74(C), pp. 108-128.

12. GÖRTZ, CHRISTOPH, JOHN D. TSOUKALAS, AND FRANCESCO ZANETTI, (2022): "News Shocks under Financial Frictions," *American Economic Journal: Macroeconomics*, 14(4), pp. 210-243.
13. KOLASA, MARCIN, MICHA RUBASZEK AND SKRZYPCZYSKI, PAWE, (2009): "Putting the New Keynesian DSGE Model to the Real-Time Forecasting Test," Working Paper Series 1110, European Central Bank.
14. MILANI, FABIO, AND ASHISH RAJBHANDARI, (2020): "Observed Expectations, News Shocks, and the Business Cycle," *Research in Economics*, 74(2), pp. 95-118.
15. MILANI, FABIO, AND JOHN TREADWELL, (2012): "The Effects of Monetary Policy News and Surprises," *Journal of Money, Credit and Banking*, 44(8), p. 1667-1692
16. SAHUC, JEAN-GUILLAUME, AND FRANK SMETS, (2008): "Differences in Interest Rate Policy at the ECB and the Fed: An Investigation with a Medium-Scale DSGE Model," *Journal of Money, Credit and Banking*, 40(2-3), pp. 505-521.
17. SMETS, FRANK AND RAFAEL WOUTERS, (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, A 97(3), pp. 586-606

## APPENDIX F: Additional Results

### F.1 Prevalence of the Intermediate Regime

The prevalence of the intermediate regime can be simply measured by the size of the intermediate regime,  $\Delta_\pi \equiv \bar{\phi}_\pi - \underline{\phi}_\pi = \beta(2 - \beta)/\kappa_y$ , which depends fundamentally on the discount factor,  $\beta$ , and the slope of the NKPC,  $\kappa_y$ , that is inversely related to the degree of price stickiness.

**COROLLARY 3.** *The size of the hump-shaped region,  $\Delta_\pi$ , satisfies the following comparative statics:*

1.  $\frac{\partial \Delta_\pi}{\partial \kappa_y} = -\frac{\beta(2 - \beta)}{\kappa_y^2} < 0$ ,
2.  $\frac{\partial \Delta_\pi}{\partial \beta} = \frac{2(1 - \beta)\kappa_y - \beta(2 - \beta)\frac{\partial \kappa_y}{\partial \beta}}{\kappa_y^2} > 0$ .

The first part of Corollary 3 indicates that a flatter NKPC – identically greater price stickiness – makes the intermediate monetary policy regime unambiguously more prevalent. The second part of the corollary calls for more discussion. Let us first consider state dependent pricing à la Rotemberg (1982), in this case the slope of the NKPC is independent from  $\beta$  and  $\partial \kappa_y / \partial \beta = 0$ . Accordingly, the size of the intermediate region increases with the discount factor. The Calvo (1983) case is slightly more intricate as the slope of the NKPC depends, negatively, on the discount factor, reinforcing the expansion of the intermediate regime.

*Proof Corollary 3.* To begin with, it follows from the regime thresholds of Proposition 3 that

$$\Delta_\pi \equiv \bar{\phi}_\pi - \underline{\phi}_\pi = (1 + \kappa_y^{-1}) - (1 + (1 - \beta)^2 \kappa_y^{-1}) = \frac{\beta(2 - \beta)}{\kappa_y}.$$

As a result, we obtain

$$\frac{\partial \Delta_\pi}{\partial \kappa_y} = -\frac{\beta(2 - \beta)}{\kappa_y^2} < 0, \quad \text{and} \quad \frac{\partial \Delta_\pi}{\partial \beta} = \frac{2(1 - \beta)\kappa_y - \beta(2 - \beta)\frac{\partial \kappa_y}{\partial \beta}}{\kappa_y^2}.$$

In the case of Rotemberg pricing, we have  $\frac{\partial \kappa_y}{\partial \beta} = 0$ . Contrary, in the case of Calvo pricing, we obtain  $\frac{\partial \kappa_y}{\partial \beta} = -\kappa_y \frac{\theta}{1 - \beta\theta} < 0$  and the sign of  $\frac{\partial \Delta_\pi}{\partial \beta}$  follows.  $\square$

### F.2 Technology Shocks

Finally, we consider a version of the RANK model in which we replace the DIS-demand shock with a technology shock

$$\hat{y}_t = \mathbb{E}_t [\hat{y}_{t+1}] - (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n), \quad (\text{E1})$$

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa_y \hat{y}_t, \quad (\text{F.2})$$

$$i_t = \bar{r} + \phi_\pi \mathbb{E}_t [\pi_{t+1}], \quad (\text{F.3})$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \quad (\text{F.4})$$

where  $\hat{y}_t \equiv y_t - y_t^n$  denotes the output gap between actual output  $y_t$  and the level of output,  $y_t^n$ , attained in the flexible price allocation. Moreover,  $r_t^n \equiv \bar{r} + \mathbb{E}_t [\Delta a_{t+1}]$  denotes the natural interest rate, where  $a_t$  is a technology shock with corresponding AR(1) persistence  $\rho_a \in [0, 1)$ . In that case, the following proposition obtains.

**PROPOSITION 11.** *Under the Taylor principle, the impact multipliers of output,  $\mathcal{M}_y(\rho_a)$ , and inflation,  $\mathcal{M}_\pi(\rho_a)$  to a technology shock are given by:*

$$\mathcal{M}_y(\rho_a) = \frac{\kappa_y(\phi_\pi - 1)\rho_a}{(1 - \beta\rho_a)(1 - \rho_a) + \kappa_y(\phi_\pi - 1)\rho_a}, \quad \text{and} \quad \mathcal{M}_\pi(\rho_a) = \frac{-\kappa_y(1 - \rho_a)}{(1 - \beta\rho_a)(1 - \rho_a) + \kappa_y(\phi_\pi - 1)\rho_a}.$$

(a) *The output impact multiplier increases monotonically with  $\rho_a$ .*

(b) *The inflation impact multiplier satisfies:*

(b.1) *If  $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$ ,  $\mathcal{M}_\pi(\rho_a)$  is U-shaped in  $\rho_a$ , i.e.*

$$\exists \rho_a^* \equiv 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} \quad \text{such that} \quad \mathcal{M}'_\pi(\rho_a) \lesseqgtr 0 \quad \text{for} \quad \rho_a \lesseqgtr \rho_a^*.$$

(b.2) *If  $\phi_\pi > 1 + \frac{\beta}{\kappa_y}$ ,  $\mathcal{M}_\pi(\rho_a)$  increases monotonically in  $\rho_a$ .*

The first statement of Proposition 11 establishes that, unlike the DIS-demand shock, a technology shock does not lead to the emergence of three monetary policy regimes or a hump-shaped relationship between shock persistence and the output impact multiplier. Instead, the output impact multiplier increases monotonically with the persistence of the shock. The focus shifts again to the inflation impact multiplier, where non-monotonic behavior arises. As with a DIS-demand shock, monetary policy balances two opposing channels. A positive technology shock reduces the output gap, lowering inflation. Since the output gap exhibits similar comparative statics to output in response to shock persistence, the *output gap channel* increases inflation with persistence. Conversely, with forward-looking inflation ( $\beta > 0$ ), a more persistent technology shock amplifies the drop in inflation via an *inflation expectation channel*. If monetary authorities adopt a moderately aggressive stance toward inflation (Part b.1 of the proposition), a U-shaped relationship in the inflation impact multiplier emerges. However, under sufficiently aggressive policy (Part b.2 of the proposition), the inflation expectation channel weakens, and inflation increases monotonically with shock persistence, driven by stronger feedback effects on the output gap.

*Proof of Proposition 11.* Subsequently, we provide derivations with a contemporaneous Taylor rule that reacts to expected inflation, i.e.,  $i_t = \bar{r} + \phi_\pi \mathbb{E}_t [\pi_{t+1}]$ . Using as a solution

guess  $\hat{y}_t = \widehat{\mathcal{M}}_y a_t$  and  $\pi_t = \mathcal{M}_\pi a_t$ , and using the fact that  $\hat{y}_t = y_t - y_t^n = y_t - a_t$ , implying  $\mathcal{M}_y = 1 + \widehat{\mathcal{M}}_y$ , the impact multiplier for output and inflation are given by

$$\mathcal{M}_y = 1 - \frac{(1 - \rho_a)(1 - \beta\rho_a)}{(1 - \rho_a)(1 - \beta\rho_a) + \kappa_y(\phi_\pi - 1)\rho_a} = \frac{\kappa_y(\phi_\pi - 1)\rho_a}{(1 - \rho_a)(1 - \beta\rho_a) + \kappa_y(\phi_\pi - 1)\rho_a} ,$$

$$\mathcal{M}_\pi = - \frac{(1 - \rho_a)\kappa_y}{(1 - \rho_a)(1 - \beta\rho_a) + \kappa_y(\phi_\pi - 1)\rho_a} .$$

To begin with, we show Statement (a). Defining the auxiliary variable  $x \equiv (1 - \rho_a)(1 - \beta\rho_a) + \kappa_y(\phi_\pi - 1)\rho_a$  and taking the comparative statics with respect to the persistence  $\rho_a$  yields

$$x^2 \frac{\partial \mathcal{M}_y}{\partial \rho_a} = \kappa_y(\phi_\pi - 1) [(1 - \rho_a)(1 - \beta\rho_a) + \kappa_y(\phi_\pi - 1)\rho_a] \\ + \kappa_y(\phi_\pi - 1)\rho_a [\beta(1 - \rho_a) + (1 - \beta\rho_a) - \kappa_y(\phi_\pi - 1)] .$$

Collecting terms and simplifying the previous expression we get

$$x^2 \frac{\partial \mathcal{M}_y}{\partial \rho_a} = \kappa_y(\phi_\pi - 1) (1 - \beta\rho_a^2) .$$

Recall that the equilibrium is locally determinate if  $\phi_\pi > 1$  such that we have

$$x^2 \frac{\partial \mathcal{M}_y}{\partial \rho_a} > 0 \quad \forall \rho_a \in [0, 1) ,$$

which shows that the impact output multiplier monotonically increases in  $\rho_a$ .

To prove Statement (b), the derivative of  $\mathcal{M}_\pi$  with respect to  $\rho_a$  yields

$$x^2 \frac{\partial \mathcal{M}_\pi}{\partial \rho_a} = \kappa_y (\kappa_y(\phi_\pi - 1) - \beta(1 - \rho_a)^2) .$$

As a result, the sign of the previous expression is determined by a second order polynomial in  $\rho_a$ , which admits as roots

$$\rho_a^{+,-} = 1 \pm \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} .$$

Evidently,  $\rho_a^+ > 1$  holds by the determinacy condition, i.e., the only feasible candidate on  $\rho_a \in [0, 1)$  is

$$\rho_a^* \equiv \rho_a^- = 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} .$$

Note that  $\rho_a^* \in (0, 1)$  if  $0 < \kappa_y(\phi_\pi - 1) < \beta$ , which is equivalent to  $1 < \phi_\pi < \bar{\phi}_\pi$ , where

$$\bar{\phi}_\pi \equiv 1 + \frac{\beta}{\kappa_y} .$$

Since  $\kappa_y(\phi_\pi - 1) - \beta(1 - \rho_a)^2$  is a strictly concave function in  $\rho_a$ , statement (b.1) of Proposition 11 follows.

To show statement (b.2), recall that

$$\text{sgn} \left( \frac{\partial \mathcal{M}_\pi}{\partial \rho_a} \right) = \text{sgn} (\kappa_y(\phi_\pi - 1) - \beta(1 - \rho_a)^2) .$$

Evaluating the previous expression at  $\phi_\pi = \bar{\phi}_\pi$  yields

$$\text{sgn} \left( \frac{\partial \mathcal{M}_\pi}{\partial \rho_a} \right) \Big|_{\phi_\pi = \bar{\phi}_\pi} = \beta(1 - (1 - \rho_a)^2) \geq 0 ,$$

where the last inequality applies as  $\rho_a \in [0, 1)$  and is strict for  $\rho_a > 0$ . As a result, we have  $\frac{\partial \mathcal{M}_\pi}{\partial \rho_a} > 0$  for all parameter values  $\kappa_y(\phi_\pi - 1) \geq \beta$ , identically  $\phi_\pi > \bar{\phi}_\pi$ .  $\square$

### F.3 Towards a Structural Interpretation

In this section, we provide formal statements and proofs for the sensitivity of monetary regime thresholds and  $\rho$ -max in case specific micro-foundations act on multiple sufficient statistics at the same time. We summarize our findings in Proposition 12 and refer to the quantitative section of the paper for a discussion of the driving forces at play in generating these result.

PROPOSITION 12. *The following statements apply.*

- (a) *Household Preferences: a higher elasticity of inter-temporal substitution or wealth in the utility decrease  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ .*
- (b) *Household Heterogeneity: counter-cyclical (respectively, pro-cyclical) income risk increases (respectively, decreases)  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ . Moreover, in a THANK economy, counter-cyclical (respectively, pro-cyclical) income inequality decreases (respectively, increases)  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$  if  $\lambda + s > 1$ , where  $\lambda$  is the share of hand-to-mouth households and  $s$  the probability to stay a saver household.*
- (c) *Behavioral Frictions: a departure from the full information rational expectation assumption via cognitive discounting increases  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$ .*
- (d) *Life-Cycle: life-cycle dynamics with stochastic death increase  $\underline{\phi}_\pi$  unambiguously, while it increases  $\bar{\phi}_\pi$  and  $\rho^*$  if  $1 \geq \frac{\theta\beta_f}{1-\theta\beta_f} \frac{\zeta_f}{\kappa_y}$ , i.e., prices are sufficiently flexible.*
- (e) *Household Debt and Default: information asymmetries leading to interest rate spreads between borrowers and lenders leave  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$  unaffected if  $\kappa_r = 0$  and increases all of them if  $\kappa_r > 0$ .*
- (f) *Long-Run Interest Rate: a higher steady state real interest rate decreases (respectively, increases)  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$  if  $\zeta_f + \zeta_r\kappa_y - \beta_f \geq (\beta_f - \kappa_r)\zeta_r$  (respectively,  $\zeta_f + \zeta_r\kappa_y - \beta_f < (\beta_f - \kappa_r)\zeta_r$ ).*

*Proof Proposition 12*

PROOF. Subsequently, we prove all statements in the corresponding order. To do so, we make use of an auxiliary Lemma, which pertains to a New Keynesian model with a cost channel of monetary policy and the following equilibrium dynamics:

$$\begin{aligned} y_t &= \zeta_f \mathbb{E}_t [y_{t+1}] - \zeta_r (i_t - \mathbb{E}_t [\pi_{t+1}] - \bar{r}) + \zeta_r \xi_t, \\ \pi_t &= \beta_f \mathbb{E}_t [\pi_{t+1}] + \kappa_y y_t + \kappa_r (i_t - \mathbb{E}_t [\pi_{t+1}] - \bar{r}) - \kappa_r \xi_t, \\ i_t &= \bar{r} + \phi_\pi \pi_t + \phi_y y_t, \\ \xi_t &= \rho \xi_{t-1} + \varepsilon_{\xi,t}. \end{aligned}$$

This representation introduces a cost channel of monetary policy into the NKPC that links inflation to the real interest rate, with corresponding elasticity  $\kappa_r > 0$ . Relative to the main text, we scale up the demand shock in the DIS and NKPC equations by  $\zeta_r$ , respectively by  $\kappa_r$ , to ensure that output and inflation dynamics coincide under a bond premium shock with the ones under a standard monetary policy shock. This particular assumption is innocuous for our subsequent results.

LEMMA 3. *Consider a forward-looking and upward-sloping NKPC, i.e.,  $\beta_f > 0$  and  $\kappa_y > 0$ . Moreover, suppose  $\min\{\beta_f, \zeta_r \kappa_y\} > \kappa_r$ .<sup>12</sup> There exist two cut-off values,  $\underline{\phi}_\pi$  and  $\bar{\phi}_\pi$ , of the Central Bank's degree of reaction to the inflation gap*

$$\underline{\phi}_\pi^g \equiv \beta_f^{-1} \left( 1 + (1 - \beta_f)^2 \frac{(\beta_f - \kappa_r) \zeta_f}{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r} \right) \quad \text{and} \quad \bar{\phi}_\pi^g \equiv \beta_f^{-1} \left( 1 + \frac{(\beta_f - \kappa_r) \zeta_f}{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r} \right),$$

with  $\bar{\phi}_\pi > \underline{\phi}_\pi > 1$  such that the impact multiplier of output satisfies:

- (a) If  $\phi_\pi \leq \underline{\phi}_\pi$ ,  $\mathcal{M}_y$  increases monotonically in  $\rho$ .
- (b) If  $\underline{\phi}_\pi < \phi_\pi < \bar{\phi}_\pi$ ,  $\mathcal{M}_y$  is hump-shaped in  $\rho$ , i.e.,

$$\exists \rho^* \equiv \beta_f^{-1} \left( 1 - \sqrt{(\phi_\pi \beta_f - 1) \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r}{(\beta_f - \kappa_r) \zeta_f}} \right) \text{ such that } \mathcal{M}'_y(\rho) \gtrless 0 \text{ if } \rho \lesseqgtr \rho^*.$$

- (c) If  $\phi_\pi \geq \bar{\phi}_\pi$ ,  $\mathcal{M}_y$  decreases monotonically in  $\rho$ .

The proof is omitted, but follows the same logic as in our benchmark economy.

Statement (a). Following Chapter 3 in Galí (2015) the RANK model with an arbitrary elasticity of inter-temporal substitution is characterized by

$$\zeta_f = 1, \quad \zeta_r = \sigma^{-1}, \quad \beta_f = \beta, \quad \kappa_y = \bar{\kappa} \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right), \quad \text{and} \quad \kappa_r = 0.$$

<sup>12</sup>The previous assumption is known as *Patmann condition*; for a discussion, see Beaudry et al. (2024).

In this case, we can define  $\Xi \equiv \frac{1}{\zeta_r \kappa_y}$  such that we have

$$\underline{\phi}_\pi = \beta^{-1} (1 + (1 - \beta)^2 \Xi) , \quad \bar{\phi}_\pi = \beta^{-1} (1 + \Xi) , \quad \text{and} \quad \rho^* = \beta^{-1} \left( 1 - \sqrt{(\phi_\pi \beta - 1) \Xi^{-1}} \right) .$$

The first part of Statement (a) follows immediately from  $\frac{\partial \Xi}{\partial \sigma} = \frac{1}{(\zeta_r \kappa_y)^2} \frac{\bar{\kappa}}{\sigma^2} \frac{\varphi + \alpha}{1 - \alpha} > 0$ . Following Online Appendix C in [Michaillat and Saez \(2021\)](#), the WUNK model with logarithmic utility over consumption is, in turn, characterized by

$$\zeta_f = \frac{\beta}{\beta + u'(0)y^n} , \quad \zeta_r = 1 , \quad \beta_f = \beta , \quad \kappa_y = \bar{\kappa} \left( 1 + \frac{\varphi + \alpha}{1 - \alpha} \right) , \quad \text{and} \quad \kappa_r = 0 ,$$

where we work with Calvo price setting rather than Rotemberg adjustment costs. In this case, we can define  $\Xi \equiv \frac{\zeta_f}{\kappa_y}$  such that we have

$$\underline{\phi}_\pi = \beta^{-1} (1 + (1 - \beta)^2 \Xi) , \quad \bar{\phi}_\pi = \beta^{-1} (1 + \Xi) , \quad \text{and} \quad \rho^* = \beta^{-1} \left( 1 - \sqrt{(\phi_\pi \beta - 1) \Xi^{-1}} \right) .$$

The second part of Statement (a) follows from  $\frac{\partial \Xi}{\partial u'(0)} = -\frac{1}{\kappa_y} \frac{\beta}{(\beta + u'(0)y^n)^2} y^n < 0$ .

Statement (b). Following Section 2 in [Bilbiie \(2025\)](#) the tractable HANK economy based on a spender-saver dichotomy with cyclical income inequality is characterized by

$$\zeta_f = 1 + (\chi - 1) \frac{1-s}{1-\lambda\chi} , \quad \zeta_r = \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi} , \quad \beta_f = \beta , \quad \kappa_y = \bar{\kappa} \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) , \quad \text{and} \quad \kappa_r = 0 ,$$

where the measure of cyclical inequality,  $\chi$ , is given by  $\chi = 1 + \varphi(1 - \frac{\tau^D}{\lambda})$ . In this case, we can define  $\Xi \equiv \frac{\zeta_f}{\zeta_r \kappa_y}$  such that we have

$$\underline{\phi}_\pi = \beta^{-1} (1 + (1 - \beta)^2 \Xi) , \quad \bar{\phi}_\pi = \beta^{-1} (1 + \Xi) , \quad \text{and} \quad \rho^* = \beta^{-1} \left( 1 - \sqrt{(\phi_\pi \beta - 1) \Xi^{-1}} \right) .$$

The following comparative statics apply

$$\begin{aligned} \frac{\partial \Xi}{\partial \chi} &= \frac{\sigma}{\kappa_y} \frac{1 - s - \lambda}{1 - \lambda} , \\ \frac{\partial \Xi}{\partial s} &= \frac{\sigma}{\kappa_y} \frac{1 - \chi}{1 - \lambda} , \\ \frac{\partial \Xi}{\partial \lambda} &= \frac{\sigma}{\kappa_y} \frac{(1 - \chi)s + (1 - \lambda)(1 - \lambda - s) \frac{\partial \chi}{\partial \lambda}}{(1 - \lambda)^2} , \end{aligned}$$

As a result,  $\underline{\phi}_\pi$ ,  $\bar{\phi}_\pi$ , and  $\rho^*$  decrease in  $\chi$  if  $s + \lambda > 1$ , and decrease in  $s$  if  $\chi > 1$ . Moreover, they decrease in  $\lambda$  if  $(1 - \chi)s + (1 - \lambda)(1 - \lambda - s) \frac{\partial \chi}{\partial \lambda} < 0$ , which is satisfied if cyclical inequality is sufficiently countercyclical, i.e.,  $\chi$  large enough. Regarding cyclical income risk, we follow [Bilbiie \(2025\)](#) in making the following case distinction. First, we consider the case where the probability to stay in the saver state depends on the *current* aggregate

state of the economy, i.e.,  $s(Y_t)$ . In this case, one arrives at the following representation

$$\tilde{\zeta}_f = \frac{1}{1-\eta} \left[ 1 + (\chi - 1) \frac{1-\tilde{s}}{1-\lambda\chi} \right], \quad \tilde{\zeta}_r = \frac{1}{1-\eta} \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi}, \quad \tilde{s} = \frac{s}{s+(1-s)\Gamma^\sigma}, \quad \eta = \frac{s_y Y}{1-s} (1 - \Gamma^{-\sigma}) (1 - \tilde{s}) \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi}.$$

where  $\Gamma = \frac{c^S}{c^H}$  is a measure of steady state consumption inequality. Notice that  $\beta_f, \kappa_y$ , and  $\kappa_r$  take the same values as before. It is straightforward to see that pro- or counter-cyclical risk, i.e.,  $\eta \neq 0$  leaves  $\frac{\tilde{\zeta}_f}{\tilde{\zeta}_r}$  unchanged, i.e.,  $\frac{\tilde{\zeta}_f}{\tilde{\zeta}_r} = \frac{\zeta_f}{\zeta_r}$ . As a result, the comparative statics of  $\phi_\pi, \bar{\phi}_\pi$ , and  $\rho^*$  do not depend on the cyclicity of income risk and are unaffected otherwise up to replacing  $s$  by  $\tilde{s}$ . Second, we consider the case where the probability to stay in the saver state depends on the *future* aggregate state of the economy, i.e.,  $s(Y_{t+1})$ . In this case, one arrives at the following representation

$$\tilde{\zeta}_f = 1 + (\chi - 1) \frac{1-\tilde{s}}{1-\lambda\chi} + \eta, \quad \tilde{\zeta}_r = \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi}, \quad \tilde{s} = \frac{s}{s+(1-s)\Gamma^\sigma}, \quad \eta = \frac{s_y Y}{1-s} (1 - \Gamma^{-\sigma}) (1 - \tilde{s}) \frac{1}{\sigma} \frac{1-\lambda}{1-\lambda\chi}.$$

Finally, defining  $\Xi \equiv \frac{\tilde{\zeta}_f}{\tilde{\zeta}_r \kappa_y}$  we arrive at the following comparative statics

$$\begin{aligned} \frac{\partial \Xi}{\partial \chi} &= \frac{\sigma}{\kappa_y} \frac{1 - \tilde{s} - \lambda}{1 - \lambda}, \\ \frac{\partial \Xi}{\partial s} &= \frac{1}{\kappa_y} \frac{1}{(s + (1-s)\Gamma^\sigma)^2} \left[ s_y Y (\Gamma^\sigma - 1)^2 + \sigma \frac{1-\chi}{1-\lambda} \Gamma^\sigma \right], \\ \frac{\partial \Xi}{\partial s_y} &= \frac{1}{\kappa_y} \frac{Y}{1-s} (1 - \Gamma^{-\sigma}) (1 - \tilde{s}), \\ \frac{\partial \Xi}{\partial \lambda} &= \frac{\sigma}{\kappa_y} \frac{(1-\chi)\tilde{s} + (1-\lambda)(1-\lambda-\tilde{s})}{(1-\lambda)^2} \frac{\partial \chi}{\partial \lambda}, \end{aligned}$$

from where the results in the proposition follow. Notice that the PRANK economy of [Acharya and Dogra \(2020\)](#) leads to  $\zeta_f = 1 - \frac{\sigma}{2} \left( \frac{R-1}{R} \right)^2 \frac{\partial \sigma^2(y^*)}{\partial y}$  such that the comparative statics are qualitatively similar to the ones with respect to  $s_y$  when  $s$  depends on the future aggregate state, i.e., procyclical risk reduces  $\tilde{\zeta}_f$  and  $\Xi$ .

Statement (c). Following Proposition 2 in [Gabaix \(2020\)](#) the behavioral RANK model is characterized by

$$\zeta_f = \bar{m} \quad \text{and} \quad \beta_f = \bar{m} \left( \theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}} (1-\theta) \right).$$

Notice that we have

$$\frac{\partial \beta_f}{\partial \bar{m}} > 0, \quad \text{and} \quad \frac{\partial \beta_f}{\partial \bar{m}} \frac{\bar{m}}{\beta_f} = 1 + \frac{\beta\theta\bar{m}}{1-\beta\theta\bar{m}} \frac{(1-\beta\theta)(1-\theta)}{\theta(1-\beta\theta\bar{m}) + (1-\beta\theta)(1-\theta)} \geq 1.$$

In this case, we can define  $\Xi \equiv \frac{\zeta_f}{\zeta_r \kappa_y}$ , with  $\frac{\partial \Xi}{\partial \bar{m}} = \frac{1}{\zeta_r \kappa_y}$  such that we have

$$\phi_\pi = \beta_f^{-1} (1 + (1-\beta_f)^2 \Xi), \quad \bar{\phi}_\pi = \beta_f^{-1} (1 + \Xi), \quad \text{and} \quad \rho^* = \beta_f^{-1} (1 - \sqrt{(\phi_\pi \beta_f - 1) \Xi^{-1}}).$$

From there, we obtain

$$\begin{aligned}\frac{\partial \phi_\pi}{\partial \bar{m}} &= \left[ -\beta_f^{-2} (1 + (1 - \beta_f)^2 \Xi) - 2\beta_f^{-1} (1 - \beta_f) \Xi \right] \frac{\partial \beta_f}{\partial \bar{m}} + \beta_f^{-1} (1 - \beta_f)^2 \frac{\Xi}{\bar{m}} \\ &= \left[ -\beta_f^{-2} - 2\beta_f^{-1} (1 - \beta_f) \Xi \right] \frac{\partial \beta_f}{\partial \bar{m}} + \beta_f^{-1} (1 - \beta_f)^2 \frac{\Xi}{\bar{m}} \left[ 1 - \frac{\partial \beta_f}{\partial \bar{m}} \frac{\bar{m}}{\beta_f} \right] \\ &< 0.\end{aligned}$$

Moreover, we have

$$\frac{\partial \bar{\phi}_\pi}{\partial \bar{m}} = -\beta_f^{-2} (1 + \Xi) \frac{\partial \beta_f}{\partial \bar{m}} + \beta_f^{-1} \frac{\Xi}{\bar{m}} = -\beta_f^{-2} \frac{\partial \beta_f}{\partial \bar{m}} + \beta_f^{-1} \frac{\Xi}{\bar{m}} \left[ 1 - \frac{\partial \beta_f}{\partial \bar{m}} \frac{\bar{m}}{\beta_f} \right] < 0.$$

Finally, we show that

$$\begin{aligned}\frac{\partial \rho^*}{\partial \bar{m}} &= \left[ -\beta_f^{-2} \left( 1 - [(\phi_\pi \beta_f - 1) \Xi^{-1}]^{\frac{1}{2}} \right) - \frac{\beta_f^{-1}}{2} [(\phi_\pi \beta_f - 1) \Xi^{-1}]^{-\frac{1}{2}} \phi_\pi \Xi^{-1} \right] \frac{\partial \beta_f}{\partial \bar{m}} \\ &\quad + \frac{\beta_f^{-1}}{2} [(\phi_\pi \beta_f - 1) \Xi^{-1}]^{-\frac{1}{2}} (\phi_\pi \beta_f - 1) \frac{\Xi^{-1}}{\bar{m}} \\ &= -\beta_f^{-2} \left( 1 - [(\phi_\pi \beta_f - 1) \Xi^{-1}]^{\frac{1}{2}} \right) \frac{\partial \beta_f}{\partial \bar{m}} - \frac{\beta_f^{-1}}{2} [(\phi_\pi \beta_f - 1) \Xi^{-1}]^{-\frac{1}{2}} \Xi^{-1} \left[ \phi_\pi \frac{\partial \beta_f}{\partial \bar{m}} - \frac{\phi_\pi \beta_f - 1}{\bar{m}} \right] \\ &< 0,\end{aligned}$$

where the last inequality follows by the observation that

$$\phi_\pi \frac{\partial \beta_f}{\partial \bar{m}} - \frac{\phi_\pi \beta_f - 1}{\bar{m}} = \frac{1}{\bar{m}} + \frac{\phi_\pi \beta_f}{\bar{m}} \left[ \frac{\partial \beta_f}{\partial \bar{m}} \frac{\bar{m}}{\beta_f} - 1 \right] > 0.$$

This completes the proof of Statement (c).

**Statement (d).** Following [Del Negro et al. \(2023\)](#) the NK model with a perpetual youth structure and no habit persistence admits the following representation

$$\zeta_f = \left( 1 + \frac{p}{1-p} \frac{1-\beta(1-p)}{1+\varphi} \frac{s}{c} \right)^{-1}, \quad \zeta_r = 1, \quad \beta_f = \beta \zeta_f, \quad \kappa_y = \bar{\kappa} \beta_f \left( 1 + \frac{\varphi+\alpha}{1-\alpha} \right), \quad \text{and} \quad \kappa_r = 0,$$

where  $p \in [0, 1)$  denotes the probability to die and  $s/c$  is the wealth to consumption steady state ratio. In this case, we can define  $\Xi \equiv \frac{\zeta_f}{\kappa_y}$  such that we have

$$\phi_\pi = \beta_f^{-1} (1 + (1 - \beta_f)^2 \Xi), \quad \bar{\phi}_\pi = \beta_f^{-1} (1 + \Xi), \quad \text{and} \quad \rho^* = \beta_f^{-1} \left( 1 - \sqrt{(\phi_\pi \beta_f - 1) \Xi^{-1}} \right).$$

Notice that the following auxiliary results apply:

$$\begin{aligned}\frac{\partial \zeta_f}{\partial p} &= -\zeta_f^2 \frac{s}{c} \frac{1}{1+\varphi} \frac{1-\beta(1-p)^2}{(1-p)^2}, \quad \frac{\partial \beta_f}{\partial p} = \beta \frac{\partial \zeta_f}{\partial p}, \\ \frac{\partial \kappa_y}{\partial p} &= -\kappa_y \frac{\beta \theta}{1 - \beta \theta \zeta_f} \frac{\partial \zeta_f}{\partial p}, \quad \frac{\partial \Xi}{\partial p} = \frac{1}{\kappa_y (1 - \beta \theta \zeta_f)} \frac{\partial \zeta_f}{\partial p}.\end{aligned}$$

Using the previous auxiliary results, we obtain

$$\begin{aligned}
\frac{\partial \phi}{\partial p} &= \left[ -\beta_f^{-2} (1 + (1 - \beta_f)^2 \Xi) - 2\beta_f^{-1} (1 - \beta_f) \Xi \right] \beta \frac{\partial \zeta_f}{\partial p} + \beta_f^{-1} (1 - \beta_f)^2 \frac{1}{\kappa_y (1 - \beta \theta \zeta_f)} \frac{\partial \zeta_f}{\partial p} \\
&= \left[ -\beta_f^{-2} - 2\beta_f^{-1} (1 - \beta_f) \Xi \right] \beta \frac{\partial \zeta_f}{\partial p} + \beta_f^{-1} (1 - \beta_f)^2 \frac{\theta \beta_f}{1 - \theta \beta_f} \frac{1}{\kappa_y} \frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-2} \beta \frac{\partial \zeta_f}{\partial p} + \left[ \frac{1 - \beta_f}{\beta_f} \frac{\theta \beta_f}{1 - \theta \beta_f} - 2 \right] \frac{1 - \beta_f}{\kappa_y} \frac{\partial \zeta_f}{\partial p} \\
&> 0,
\end{aligned}$$

where the terminal strict inequality holds as  $\theta(1 + \beta_f) \leq 2$  due to  $\theta \leq 1$  and  $\beta_f < \beta < 1$ . In a similar vein, we obtain for the upper regime threshold

$$\frac{\partial \bar{\phi}_\pi}{\partial p} = -\beta_f^{-2} (1 + \Xi) \beta \frac{\partial \zeta_f}{\partial p} + \beta_f^{-1} \frac{1}{\kappa_y (1 - \beta \theta \zeta_f)} \frac{\partial \zeta_f}{\partial p}.$$

The previous expression can be rewritten by substituting in for  $\Xi$  as

$$\begin{aligned}
\frac{\partial \bar{\phi}_\pi}{\partial p} &= -\beta_f^{-2} \beta \frac{\partial \zeta_f}{\partial p} + \frac{\beta_f^{-1}}{\kappa_y} \left[ \frac{1}{1 - \beta \theta \zeta_f} - \frac{\beta \zeta_f}{\beta_f} \right] \frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-2} \beta \frac{\partial \zeta_f}{\partial p} + \frac{\beta_f^{-1}}{\kappa_y} \frac{\theta \beta_f}{1 - \beta \theta \zeta_f} \frac{\partial \zeta_f}{\partial p}.
\end{aligned}$$

Finally, the former expression can be simplified to

$$\frac{\partial \bar{\phi}_\pi}{\partial p} = -\beta_f^{-1} \frac{\partial \zeta_f}{\partial p} \left[ \frac{\beta}{\beta_f} - \frac{1}{\kappa_y} \frac{\theta \beta_f}{1 - \beta \theta \zeta_f} \right] = -\frac{1}{\beta_f \zeta_f} \frac{\partial \zeta_f}{\partial p} \left[ 1 - \frac{\zeta_f}{\kappa_y} \frac{\theta \beta_f}{1 - \theta \beta_f} \right].$$

As a result, we obtain that  $\frac{\partial \bar{\phi}_\pi}{\partial p} \geq 0$  if  $1 \geq \frac{\zeta_f}{\kappa_y} \frac{\theta \beta_f}{1 - \theta \beta_f}$ . To conclude the proof of statement (d), we finally study the comparative statics of the output-maximizing persistence

$$\begin{aligned}
\frac{\partial \rho^*}{\partial p} &= -\beta_f^{-1} \rho^* \beta \frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2} \left[ (\phi_\pi \beta_f - 1) \frac{\kappa_y}{\zeta_f} \right]^{-\frac{1}{2}} \left\{ \phi_\pi \beta \frac{\kappa_y}{\zeta_f} \frac{\partial \zeta_f}{\partial p} + (\phi_\pi \beta_f - 1) \frac{\frac{\partial \kappa_y}{\partial p} \zeta_f - \frac{\partial \zeta_f}{\partial p} \kappa_y}{\zeta_f^2} \right\} \\
&= -\beta_f^{-1} \rho^* \beta \frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2} \left[ (\phi_\pi \beta_f - 1) \frac{\kappa_y}{\zeta_f} \right]^{-\frac{1}{2}} \left\{ \phi_\pi \beta \frac{\kappa_y}{\zeta_f} \frac{\partial \zeta_f}{\partial p} - (\phi_\pi \beta_f - 1) \frac{\kappa_y}{1 - \theta \beta_f} \frac{\partial \zeta_f}{\partial p} \frac{1}{\zeta_f^2} \right\} \\
&= -\beta_f^{-1} \rho^* \beta \frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2} \left[ (\phi_\pi \beta_f - 1) \frac{\kappa_y}{\zeta_f} \right]^{-\frac{1}{2}} \left\{ \phi_\pi \beta - (\phi_\pi \beta_f - 1) \frac{1}{1 - \theta \beta_f} \frac{1}{\zeta_f} \right\} \frac{\kappa_y}{\zeta_f} \frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-1} \rho^* \beta \frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2} \left[ (\phi_\pi \beta_f - 1) \frac{\kappa_y}{\zeta_f} \right]^{-\frac{1}{2}} \left\{ \phi_\pi \beta_f - (\phi_\pi \beta_f - 1) \frac{1}{1 - \theta \beta_f} \right\} \frac{\kappa_y}{\zeta_f^2} \frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-1} \rho^* \beta \frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2} \left[ (\phi_\pi \beta_f - 1) \frac{\kappa_y}{\zeta_f} \right]^{-\frac{1}{2}} \left\{ 1 - (\phi_\pi \beta_f - 1) \frac{\theta \beta_f}{1 - \theta \beta_f} \right\} \frac{\kappa_y}{\zeta_f^2} \frac{\partial \zeta_f}{\partial p}.
\end{aligned}$$

Notice that the sign of the previous expression is positive if  $1 \geq (\phi_\pi \beta_f - 1) \frac{\theta \beta_f}{1 - \theta \beta_f}$ . Substituting in for the upper bound  $\bar{\phi}_\pi = \beta_f^{-1} \left(1 + \frac{\zeta_f}{\kappa_y}\right)$  yields the sufficient condition  $1 \geq \frac{\theta \beta_f}{1 - \theta \beta_f} \frac{\zeta_f}{\kappa_y}$ .

Statement (e). Following [Beaudry and Portier \(2018\)](#) the NK model with an upward sloping interest rate schedule admits the following representation

$$\zeta_f = \frac{\sigma}{\sigma + \epsilon_p}, \quad \zeta_r = \frac{1}{\sigma + \epsilon_p}, \quad \beta_f = \beta, \quad \kappa_y = \bar{\kappa} \left(1 + \frac{\varphi + \alpha}{1 - \alpha}\right), \quad \text{and} \quad \kappa_r \geq 0,$$

where  $\epsilon_p \in [0, \infty)$  captures the increase in borrowing costs depending on the level of debt. Notice that  $\kappa_r$  does not depend on  $\epsilon_p$ . In this case, we can define  $\Xi \equiv \frac{(\beta_f - \kappa_r) \zeta_f}{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r}$  such that we have

$$\underline{\phi}_\pi = \beta_f^{-1} (1 + (1 - \beta_f)^2 \Xi), \quad \bar{\phi}_\pi = \beta_f^{-1} (1 + \Xi), \quad \text{and} \quad \rho^* = \beta_f^{-1} \left(1 - \sqrt{(\phi_\pi \beta_f - 1) \Xi^{-1}}\right).$$

Notice that we have

$$\frac{\partial \zeta_f}{\partial \epsilon_p} = -\frac{1}{\sigma + \epsilon_p} \zeta_f, \quad \text{and} \quad \frac{\partial \zeta_r}{\partial \epsilon_p} = -\frac{1}{\sigma + \epsilon_p} \zeta_r.$$

As such, we obtain

$$\begin{aligned} \frac{\partial \Xi}{\partial \epsilon_p} &= \frac{(\beta_f - \kappa_r) \frac{\partial \zeta_f}{\partial \epsilon_p} [\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r] - (\beta_f - \kappa_r) \zeta_f \left[ \beta_f \kappa_y \frac{\partial \zeta_r}{\partial \epsilon_p} + \frac{\partial \zeta_f}{\partial \epsilon_p} \kappa_r \right]}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \\ &= (\beta_f - \kappa_r) \beta_f \frac{\frac{\partial \zeta_f}{\partial \epsilon_p} [\zeta_r \kappa_y - \kappa_r] - \frac{\partial \zeta_r}{\partial \epsilon_p} \zeta_f \kappa_y}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2}. \end{aligned}$$

Substituting in the expression for  $\frac{\partial \zeta_f}{\partial \epsilon_p}$  and  $\frac{\partial \zeta_r}{\partial \epsilon_p}$ , we finally obtain

$$\frac{\partial \Xi}{\partial \epsilon_p} = \frac{(\beta_f - \kappa_r) \beta_f}{\sigma + \epsilon_p} \frac{\zeta_f \kappa_r}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \geq 0,$$

from which the results stated in the main text directly follow.

Statement (f). Following [Beaudry et al. \(2024\)](#) the NK model with a cost channel of monetary policy admits the following representation

$$\zeta_f = \frac{\sigma}{\sigma + \epsilon_p}, \quad \zeta_r = \frac{1}{\sigma + \epsilon_p}, \quad \beta_f = \beta, \quad \kappa_y = \bar{\kappa} \left( \frac{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right), \quad \text{and} \quad \kappa_r = \bar{\kappa} \left( \frac{\frac{\beta}{b} \frac{1+i}{1+\pi}}{\frac{1}{a} \frac{W}{P} + \frac{\beta}{b} \frac{1+i}{1+\pi}} \right),$$

where  $\epsilon_p \in [0, \infty)$  captures the increase in borrowing costs depending on the level of debt,  $W/P$  is the real wage,  $a$  and  $b$  capture the degree of substitutability between input goods within the Leontief technology,  $1 + i$  is the gross nominal rate, and  $1 + \pi$  gross

price inflation. Notice that  $\bar{\kappa} = \frac{(1-\theta)(1-\beta\theta)}{\theta} > 0$  does not depend on the nominal interest rate. In this case, we can define  $\Xi \equiv \frac{(\beta_f - \kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}$  such that we have

$$\underline{\phi}_\pi = \beta_f^{-1} (1 + (1 - \beta_f)^2 \Xi), \quad \bar{\phi}_\pi = \beta_f^{-1} (1 + \Xi), \quad \text{and} \quad \rho^* = \beta_f^{-1} \left( 1 - \sqrt{(\phi_\pi \beta_f - 1) \Xi^{-1}} \right).$$

Subsequently, we define  $r \equiv \frac{1+i}{1+\pi}$ . Moreover, notice that the following relations hold

$$\frac{\partial \kappa_y}{\partial r} = -\bar{\kappa} \frac{\frac{\beta}{ab} \frac{W}{P}}{\left[ \frac{1}{a} \frac{W}{P} + \frac{\beta}{b} r \right]^2}, \quad \text{and} \quad \frac{\partial \kappa_r}{\partial r} = \bar{\kappa} \frac{\frac{\beta}{ab} \frac{W}{P}}{\left[ \frac{1}{a} \frac{W}{P} + \frac{\beta}{b} r \right]^2}.$$

As a result, we obtain

$$\begin{aligned} \frac{\partial \Xi}{\partial r} &= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r} [\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r] - (\beta_f - \kappa_r) \zeta_f \left[ \beta_f \zeta_r \frac{\partial \kappa_y}{\partial r} + (\zeta_f - \beta_f) \frac{\partial \kappa_r}{\partial r} \right]}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \\ &= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r} [\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r + (\beta_f - \kappa_r)(\zeta_f - \beta_f)] - (\beta_f - \kappa_r) \zeta_f \beta_f \zeta_r \frac{\partial \kappa_y}{\partial r}}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \\ &= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r} [\beta_f \zeta_r \kappa_y + \beta_f (\zeta_f - \beta_f)] - (\beta_f - \kappa_r) \zeta_f \beta_f \zeta_r \frac{\partial \kappa_y}{\partial r}}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \\ &= -\zeta_f \beta_f \frac{\frac{\partial \kappa_r}{\partial r} [\zeta_r \kappa_y + (\zeta_f - \beta_f)] + (\beta_f - \kappa_r) \zeta_r \frac{\partial \kappa_y}{\partial r}}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \\ &= -\frac{\zeta_f \beta_f}{[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f) \kappa_r]^2} \frac{\kappa \frac{\beta}{ab} \frac{W}{P}}{\left[ \frac{1}{a} \frac{W}{P} + \frac{\beta}{b} r \right]^2} \{ \zeta_r \kappa_y + \zeta_f - \beta_f - (\beta_f - \kappa_r) \zeta_r \}. \end{aligned}$$

As a result, we obtain that  $\frac{\partial \Xi}{\partial r} \leq 0$  if  $\zeta_r \kappa_y + \zeta_f - \beta_f \geq (\beta_f - \kappa_r) \zeta_r$ . This completes the proof of the statement in the main text.  $\square$