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Central Bank Reserves and the Balance Sheet of Banks

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Central Bank Reserves and the Balance Sheet of Banks*

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Abstract

We introduce a tractable model of the two-tier monetary system with heterogeneous agents and incomplete markets. We use this model to characterize the dynamics of bank lending under general fiscal and monetary policy and derive the welfare optimal level of Central Bank reserves and the optimal interest rate on reserves. We also identify a new risk channel of monetary policy. In the model, banks have a dual role as loan providers and money creators, and cannot fully diversify credit risk. Central Bank reserves are used to settle interbank claims and serve as a safe asset, thereby buffering risks for banks. We show how the Central Bank and the Treasury can implement any desired allocation by setting interest rates, issuing a particular amount of reserves, and imposing taxes, and show that uncoordinated policy responses to shocks by the Central Bank alone may cause sub-optimal outcomes and significant instability.

Keywords: Central Bank reserves, interest rate on reserves, liquidity requirements, monetary system, incomplete markets

JEL Classification: E42, E43, E45, E50

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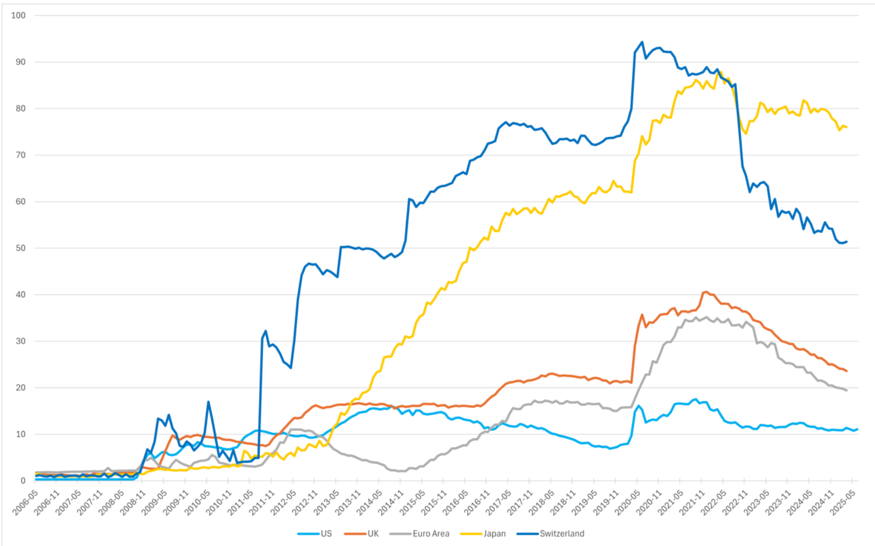
1 Introduction

Since the Global Financial Crisis (GFC) of 2007/2008, Central Bank reserves of the banking systems in the major currency areas have been rising strongly, albeit to different degrees and not always monotonically. Expansive monetary policies, in the form of exceptional liquidity assistance, large-scale asset purchases (so-called "Quantitative Easing") or massive interventions on foreign exchange markets, led to large reserve balances of banks at Central Banks. The measures adopted by Central Banks during the Covid-19 pandemic have caused a further increase in the level of Central Bank reserves, but more recently we have seen downward adjustments. The pattern is shown in Figure 1 for some of the major Central Banks.

Figure 1: Total reserve balances relative to annual GDP (in percent)



Note: Total reserve balances are the reserves maintained by private banks (domestic and foreign) at the Federal Reserve Bank (Fed), the Bank of England (BoE), the European Central Bank (ECB), the Bank of Japan (BoJ) and the Swiss National Bank (SNB), respectively.

Sources: Fed, BoE, ECB, BoJ, SNB; as of 15/09/2025.

Consider, for instance, the US. Until 2008, total reserves were small, because US banks were allowed to incur significant daily overdrafts. However, after the GFC, intraday liquidity requirements were implemented and the volume of reserves increased to roughly 2.8 trillion in 2014 (see Copeland et al. (2021)). During the Covid pandemic, the amount of reserves increased even further (see d'Avernas et al. (2023)).

Banks can use Central Bank reserves in two ways. First, they can use them to settle interbank liabilities that arise, for instance, at an individual bank when deposit outflows occur, when other banks reduce their lending to this bank, or when depositors convert their deposits into cash. Second, banks can use reserves as an investment. In particular, since Central Banks started to pay interest on reserves –as the Fed did in 2008, for instance –, banks can use these reserves as an interest-bearing safe asset. There are several suggestions to reduce interest rate payments on reserve (e.g. Tucker (2022), van Lerven and Caddick (2022) or abolish them altogether (De Grauwe and Ji (2023))) to avoid a decline of the remittances by Central Banks to the government budget. To examine such proposals, one needs a model that involves large Central Bank reserves and incorporates the feedback effects of such policies on the government budget and the macroeconomic equilibrium.

In this paper, we construct an analytically tractable dynamic model of the two-tier monetary system, characterized by a hierarchy of Central Bank, commercial banks, and the public. We explicitly model the creation of deposits by banks, the dominant form of money in today's economies (see e.g. Faure and Gersbach (2021)). The model involves heterogeneous agents, namely bankers and households. Each banker owns a bank that faces idiosyncratic shocks from its investments. Financial markets are incomplete because the profits of individual banks are privately observable and, therefore, their idiosyncratic risk cannot be fully diversified. Hence, bank equity cannot be traded. For the same reason, taxes or subsidies cannot be conditional on profits.

Moreover, we assume that all transactions in the economy must use money[1], in the form of bank deposits for households or reserves for banks. Banks have a dual role: granting loans for real investment and managing households' deposits. Central Bank reserves are used to settle interbank claims and serve as a safe asset. Prices are flexible.

We show that the Central Bank and the Treasury together can implement any desired feasible allocation by setting interest rates, issuing a particular amount of reserves, providing remittances, and imposing wealth taxes, respectively. Even if prices are flexible and interest rate policies do not affect real allocations, reserve policies have real effects.

We assume a specific form of financial friction, namely that the returns on the investments of banks are not publicly observable. From this, we characterize optimal allocations under incentive compatibility constraints. We show that these constrained optimal allocations can be decentralized by the instruments of the Central Bank and the Treasury mentioned above. This allows us to characterize the welfare optimal reserve-to-GDP ratio and study how it is related to structural parameters of the model such as volatility of idiosyncratic shocks, productivity or patience. This provides a direct interpretation of Figure 1 in terms

[1]In order to motivate this assumption, one can invoke standard justifications based on spatial frictions in trading going back to Lucas (1980).

of economic fundamentals.

Furthermore, we demonstrate the general instability of stationary allocations and show how partial policy responses to shocks may cause significant deviations from a desired allocation. We show that neither the Central Bank alone nor the Treasury alone is able to stabilize the economy when it experiences volatility shocks. If, for instance, only the Central Bank attempts to stabilize volatility shocks by its interest rate and reserve policy, the economy may diverge strongly from the desired allocation, and the banking system may even collapse. These results open the door for a broader discussion about the interplay between fiscal and monetary policy in heterogeneous-agent models, reminiscent of the Fiscal Theory of the Price Level (see Cochrane (2022)).

Related Literature

Our work contributes to a rapidly growing literature on optimal Central Bank policy and monetary-fiscal interaction. An important concern of the recent debate has been the optimal size of the Central Bank balance sheet. Low reserve balances can lead to liquidity shortages, causing inefficiencies in the money markets, and threatening financial stability.[2] The famous 2019 repo rate hike in the US, causing a significant disruption of repo markets, occurred at reserve levels that were almost 30 times higher than before 2008. The main reason was the new liquidity requirements introduced by the Fed after the financial crisis of 2008/2009 and subsequent changes in bank holding companies in handling and settling payments as well as in repo markets, together with the Fed's balance sheet normalization before September 2019 (see e.g. Afonso et al. (2011) and Correa et al. (2020)).

Because of these concerns, ample liquidity requirements are advocated. For instance, Gagnon and Sack (2020) wrote: "The minimum level of reserves is conceptually murky, impossible to estimate, and likely to vary over time. The best approach is to steer well clear of it, especially since maintaining a high level of reserves as a buffer has no meaningful cost."

However, there are also concerns about abundant reserves, the most obvious being potential inflation pressure (see Gersbach (2021)) and interest expenses, as discussed above. Other concerns are political economy risks (see Cavallo et al. (2019)) or crowding out other forms of intermediation (see Covas and Nelson (2019) and Plosser (2018)). We provide a welfare perspective on the desired level of Central Bank reserves by focusing on optimal levels of reserves as a safe asset for banks, coupled with concerns about liquidity provision.

[2] See, e.g. Bush et al. (2019), Hamilton (1996), McAndrews and Potter (2002), Bech and Garratt (2003), Ashcraft and Duffie (2007), Bech (2008), Ashcraft et al. (2011), Ihrig (2019), Afonso et al. (2011), Afonso and Shin (2011), and Yang (2020).

We also show that one can stop paying interest on reserves if this measure is coupled with suitable reserve requirements and adjustments of taxes. If only the first two measures are undertaken, the economy becomes unstable and may strongly divert from a desired allocation. This can provide a perspective on the suggestions to reduce interest rate payments on reserve (e.g. Tucker (2022), van Lerven and Caddick (2022) or abolish them (De Grauwe and Ji (2023))).

We work with a flexible price framework as this allows to identify the real effect of money as a safe asset for banks in the form of Central Bank reserves. It is thus complementary to Di Tella (2020) who identifies in a flexible price framework the real effect of money as a safe store of value for agents when downturns increase idiosyncratic risks and the risk premium, and thus make investments in capital less attractive.

Finally, our paper is complementary to the work on public debt as a safe asset in the presence of idiosyncratic risks and incomplete markets which has recently been significantly extended and applied to fiscal theory of the price level (Brunnermeier et al. (2021)) and to the possibility/impossibility of governments running Ponzi schemes (Brunnermeier et al. (2022) and Reis (2021)). We study how Central Bank reserves can serve as a safe asset in the context of a two tier monetary system operated by the Central Bank and commercial banks.

Outline

The remainder of this paper is organized as follows. Section 2 presents the model and Section 3 characterizes the optimal individual decisions within the model. Section 4 characterizes the macroeconomic equilibrium for given monetary and fiscal policies when liquidity requirements are not binding. Appendix A deals with the case of binding liquidity requirements. Section 5 shows how the monetary system can implement constrained optimal allocations and determines the welfare optimal reserve-to-GDP ratio. Section 6 shows, however, that these constrained optimal allocations are locally instable. Section 6.2 studies external shocks to the system and examines what happens when either the Central Bank or the Treasury alone attempts to stabilize volatility shocks.

2 The Model

We consider a continuous-time, infinite-horizon model that describes how banks create money in a fractional reserve system controlled by a Central Bank. The model has two main parts: on the real side, a simple AK production structure with incomplete financial markets,[3] and a monetary system in which banks create money and manage deposits while Central Bank reserves serve as settlement balances and safe assets for banks. There is one physical good that can be consumed or invested, and four types of actors: households, banks, and two government agents, the Central Bank and the Treasury.

There are two types of money: bank deposits for households and reserves for banks. Deposits have two roles. First, they serve as a medium of exchange.[4] All transactions in the physical good are settled in money. Second, they are the unique store of value for households.[5] Reserves are the liquid store of value of the fractional reserve banking system and only available to banks. There are no other assets, and the consumption/investment good cannot be stored.

The private sector consists of a continuum of competitive banks indexed by $l \in [0, 1]$ and a representative household representing a large number of households who live off their initial endowment and only save and consume. At each time $t > 0$, the representative household has savings H_t (in the form of bank deposits) and has a consumption flow c_t^H. We do not model firms explicitly, who stay in the background of the model. They borrow from banks, produce by using risky constant-to-scale production technologies, and sell their output to households. To simplify the model, we assume that firms are passive and banks can extract all the surplus from their loans. This amounts to assuming that banks operate all physical investments directly and create the necessary amounts of deposits in the process. At time t, bank l therefore has a_t^l units of the physical good,[6] the monetary value of which we denote by $\ell_t^l = p_t a_t^l$, where p_t is the price of the good at time t. We denote the inflation rate by $\pi_t = \frac{\dot{p}_t}{p_t}$.[7]. From this investment, bank l receives an instantaneous random output

$$[\mu dt + \sigma dz_t^l] a_t^l$$

where $\mu > 0$ is the average productivity of capital net of depreciation, $\sigma \geq 0$ is the volatility of the instantaneous return on capital, and z_t^l is a bank-specific Brownian motion, independent

[3] A broader analysis of the real side of the economy is provided in Gersbach et al. (2024).

[4] We abstract from the use of cash as a medium of exchange, which is of lesser importance in modern financial systems. The possibility to switch from bank deposits to banknotes would add an extra layer of complexity to the model that can neglect here.

[5] In Section 7, we discuss how to model bonds as an alternative store of value, together with transactions costs in the spirit of Reis and Tenreyro (2022), to motivate the coexistence of two safe assets.

[6] Throughout, we denote individual bank quantities by lower case and aggregate quantities by upper case letters.

[7] We take p_0 as given. For ways to determine p_0, depending on whether the Central Bank backs the Treasury or not, see Benigno (2020) and Benigno and Nistico (2020).

across banks. The bank's (nominal) revenue flow is

$$\begin{aligned} dy_t^l &= p_t[\mu dt + \sigma dz_t^l]a_t^l \\ &= [\mu dt + \sigma dz_t^l]\ell_t^l \end{aligned} \tag{1}$$

In addition to the two assumptions that firms are passive and that money is required for exchange, the third main assumption of our model is that the individual productivity shock dz_t^l is privately observed by bank l. This implies that banks cannot freely issue securities or sign contracts that are contingent on their revenues. Thus, financial markets are incomplete. Moreover, government transfers (taxes or subsidies) cannot be conditional on banks' individual profits, nor their consumption. However, the balance sheets of banks and households are publicly observable.

In the aggregate, individual productivity shocks are eliminated by the Law of Large Numbers. Hence, aggregate nominal output at time t (net of depreciation) is

$$Y_t dt = \int_0^1 dy_t^l dl = \mu L_t dt, \tag{2}$$

where

$$L_t = \int_0^1 \ell_t^l dl$$

is the aggregate nominal investment of the banking system.

Banks allocate their output flow between consumption (dividends), investment, and (positive or negative) sales of the good. Goods are purchased by households (using their deposits), other banks, and by the government (using its deposit account at the Central Bank).[8] Banks have the privilege to create bank deposits at any point in time, subject to a regulatory liquidity constraint, and these deposits show up as liabilities on their balance sheets. In exchange, they are obliged to take deposits in the payment process and must hold Central Bank reserves against their deposit holdings.[9]

[8] In the U.S., this account is called the Treasury General Account (TGA). For a broader discussion, see Duffie (2026).

[9] The precise sequencing in the intra-period lending and product markets with a full set of actors would be as follows. First, banks grant loans to the firms who bank with them and create the same amount of bank deposits and bank loans. Then, firms buy goods from the households, which they invest in physical production. The resulting monetary transfer involves shifting deposits from the firm's bank to the household's bank. Then firms receive payments from households, which are credited to the deposit accounts of firms (typically involving other banks than in the previous transaction). Finally, firms pay back their loans (including interest) by transferring these deposits to the banks, and each bank cancels the corresponding deposits. In our simplification, the first step is dropped and the second replaced by banks buying goods from households by creating the corresponding deposits, and then investing the goods directly. The last step is then a pure intra-bank activity, involving the netting of accounts.

At time t, then, bank l has equity e_t^l and deposits m_t^l, and invests ℓ_t^l by buying or selling goods from or to other banks and adjusting its deposit base in the process[10]. In addition to the (nominal) assets ℓ_t^l, bank l holds reserves r_t^l. The balance sheet of bank l at time t is therefore

$$\begin{array}{c|c} \text{Assets} & \text{Liabilities} \\ \hline r_t^l & m_t^l \\ \ell_t^l & e_t^l \end{array} \tag{3}$$

Central Bank reserves are required for banks to fulfill their obligations in the payment process. In particular, if bank deposits at a bank l are transferred to another bank l' when households or firms make payments, the Central Bank reserves at bank l are reduced by the corresponding amount and credited to bank l'. This settles the interbank claim between l and l'.

We do not explicitly model how banks compete for deposits. We just assume that all banks pay their depositors the same interest rate i_t^D and that households are therefore indifferent about where to deposit their savings. For convenience, we assume that these deposits are allocated proportionally to the size of the banks, so that the deposit-to-equity ratio is the same across banks:[11]

$$\forall(l,t), \qquad \frac{m_t^l}{e_t^l} \equiv h_t. \tag{4}$$

If we denote aggregate bank deposits by M_t and aggregate bank equity by E_t, (3) implies the following aggregate nominal bank balance-sheet identity at each point in time:[12]

$$L_t + R_t = E_t + M_t. \tag{5}$$

Each bank l is owned by a banker who, at each time t, consumes c_t^l out of its profits. At each time t, households and bankers maximize the expectation of their discounted utility

$$\int_t^\infty e^{-\rho(s-t)} \log c_s^k ds, \quad k = l, H,$$

where $\rho > 0$ is the discount rate and the expectation is taken over the evolution of the shock processes $(z_t^l)_{l\in[0,1]}$ and the associated filtration.[13]

[10]We express all variables in the bank balance sheet in nominal terms.

[11]This assumption is made for simplicity, but it is clearly reasonable, as household deposits are proportional to the households' real consumption and banks' real production is proportional to their balance sheet size. As discussed further in Section 4, we do not include a further stage in the process, where banks actively compete to re-allocate these deposits.

[12]Interbank deposits wash out in the aggregate.

[13]As there is no aggregate uncertainty, households will not bear risk in equilibrium and therefore consume a riskless consumption stream.

At date 0, the economy has a total initial endowment of goods A_0, held by bankers and households. The representative household owns A_0^H of them. Each bank $l \in [0, 1]$ has an initial endowment of a_t^l. The aggregate real endowment of bankers thus is

$$A_t^B = \int_0^1 a_t^l dl.$$

where $A_0 = A_0^H + A_0^B$.

There are two government agents, whose policies are not necessarily coordinated. The Central Bank controls the aggregate supply of reserves R_t and sets the interest rate on reserves, denoted by i_t^R. Throughout the paper, we assume that $i_t^R \geq 0$.

For $t > 0$, aggregate Central Bank reserves evolve according to

$$\dot{R}_t = i_t^R R_t + S_t \tag{6}$$

where S_t are remittances of the Central Bank to the Treasury (or transfers from the Treasury to the Central Bank if S_t is negative). The Central Bank engineers these remittances by creating deposits in the Treasury's account. If these deposits are transferred to an account at a private bank for buying goods, banks obtain them as reserves and the Treasury can use bank deposits to buy goods.

The Treasury, acting as fiscal authority, can redistribute wealth between households and banks by taxation and subsidies. In addition to taxes, the Treasury obtains remittances from the Central Bank and finances exogenous government expenditures. We assume that the Treasury does not issue debt. At each date t, banks are taxed a fraction τ_t^B of their equity, and the representative household is taxed a fraction τ_t^H of its wealth. τ_t^B and τ_t^H can be negative, in which case they represent subsidies.[14] For $t > 0$, aggregate net tax revenue is

$$T_t = \tau_t^H H_t + \tau_t^B E_t \tag{7}$$

Since the government does not issue debt, its budget constraint is

$$\nu Y_t = T_t + S_t = \tau_t^H H_t + \tau_t^B E_t + S_t \tag{8}$$

where νY_t is the flow of government expenditure and $0 < \nu < 1$ is fixed. All transactions between the public and the Treasury are executed with bank deposits.

[14] The specific form of taxes and transfers is assumed in the present paper, but it relies on the explicit contract-theoretic analysis in our companion paper Biais et al. (2024). Note that individual profits and consumption of banks cannot be observed and therefore not be taxed. In Section 5, we sketch the corresponding argument and thus provide a microfoundation for the policy instruments used in the model. This foundation also allows to address questions of welfare and Pareto optimality.

To initialize the monetary economy, the Central Bank credits the Treasury General Account with an initial remittance S_0, which become $R_0 = S_0$ aggregate reserves when the Treasury makes initial deposits at banks and households. Banks then create overall deposits $p_0 A_0^H$ to buy goods from households. In this way, all initial physical endowments are converted into productive capital held by banks with aggregate nominal value $L_0 = p_0 A_0$.[15] The date-0 aggregate bank balance sheet of the banking sector therefore satisfies $L_0 + R_0 = E_0 + M_0$.

The final ingredient of the model is banking regulation. We do not endogenize such restrictions in this paper, but rather take them as given, as our focus is on the monetary side of banking and not on its safety and soundness.[16] We will focus on liquidity regulation, as this is particularly relevant for our theory of fractional reserve banking, and assume that each bank l is subject to the reserve requirement

$$\hat{\eta} m_t^l \leq r_t^l \tag{9}$$

for all t, with $0 < \hat{\eta} < 1$. We interpret this reserve requirement broadly as a liquidity constraint on banks' reserve holdings.[17]

3 Individual Decisions

3.1 Households

After the initial round of lump sum redistribution by the government, the representative household has initial (nominal) net worth H_0 invested in bank deposits $M_0 = H_0$. These deposits receive interest and are used to pay taxes and consumption. The household chooses a consumption path C_t^H to maximize

$$\int_0^\infty e^{-\rho t} \log C_t^H \, dt$$

[15] We do not (need to) specify which banks get how many reserves. In addition to distributing reserves, the Treasury can also redistribute the nominal value of initial endowments between banks and households by means of lump sum transfers and taxes. If L^k, $k = H, B$, denote initial lump sum transfers to households and banks, respectively, with negative values corresponding to taxes, then the date-0 Treasury budget is $L^H + L^B = S_0$. The resulting household date-0 wealth is $H_0 = p_0 A_0^H + L^H$, and the aggregate bank date-0 equity is $E_0 = p_0 A_0^B + L^B$. Individual date-0 bank equity, e_0^l, depends on their initial endowments and the Treasury's initial transfer rule.

[16] See Freixas and Rochet (2023) for a broad treatment.

[17] In most jurisdictions, in the wake of Quantitative Easing, traditional reserve requirements do not play a significant role anymore, but liquidity requirements such as the HQLA or LCR requirements under Basel III have become significant instead (see Copeland et al. (2021) and d'Avernas et al. (2023)). Furthermore, in the U.S. the Fed introduced a series of new liquidity requirements after the Great Financial Crisis to discourage daylight overdrafts on banks' reserve accounts at the Fed.

subject to

$$\dot{H}_t = H_t(i_t^D - \tau_t^H) - p_t C_t^H \tag{10}$$

The problem has a well-known simple solution: for all $t \in [0, \infty)$,

$$C_t^H = \frac{\rho H_t}{p_t}. \tag{11}$$

3.2 The Banks' Problem

The problem of a bank l is more complex, as it faces idiosyncratic risk, is subject to a liquidity requirement, and is part of the payment process. When managing its balance sheet (3) using the investment technology (1), its flow of funds is given by

$$(\mu dt + \sigma dz_t^l)\ell_t^l + \pi_t \ell_t^l dt + \left(i_t^R r_t^l - i_t^D m_t^l\right)dt = (p_t c_t^l + \tau_t^B e_t^l)dt + de_t^l \tag{12}$$

where the left-hand side represents instantaneous profits and the right-hand side consumption (dividends), taxes/subsidies, and the change in equity as a residual. Note that the left-hand side reflects two sources of value creation from bank lending: first, the nominal value of the real returns from investing, (1), and second, the nominal appreciation of the bank's capital stock, $\pi_t \ell_t^l dt$.

Given its initial equity e_0^l after the Treasury's lump sum interventions, the bank chooses a policy to maximize

$$\mathbb{E} \int_0^\infty e^{-\rho t} \log c_t^l dt$$

subject to the balance sheet constraint

$$\ell_t^l + r_t^l = m_t^l + e_t^l, \tag{13}$$

the liquidity requirement (9), and the law of motion of equity, given by (12) as

$$de_t^l = (\mu + \pi_t)\ell_t^l dt + \sigma \ell_t^l dz_t^l + \left(i_t^R r_t^l - i_t^D m_t^l\right)dt - (p_t c_t^l + \tau_t^B e_t^l)dt$$

Eliminating r_t^l by (13) and using assumption (4) that the deposit-to-equity ratios h_t are the same across banks, the law of motion becomes

$$de_t^l = \ell_t^l[(\mu + \pi_t - i_t^R)dt + \sigma dz_t^l] + e_t^l[i_t^R + (i_t^R - i_t^D)h_t - \tau_t^B]dt - p_t c_t^l dt \tag{14}$$

and the liquidity requirement (9) becomes

$$\ell_t^l \leq ((1 - \hat{\eta})h_t + 1)e_t^l. \tag{15}$$

As noted in Section 2, assumption (4) implies that the interest income is proportional to the bank's equity.

3.3 Linearity, Homogeneity, and Aggregation

Our analysis builds on two structural properties that simplify things considerably and lead to a low-dimensional aggregate problem in the next section. Before moving on, it is worth highlighting them.

The first property is the scale invariance of the individual bank's problem, which yields a value function of the form

$$V(t,e) = V(t,1) + \frac{1}{\rho}\log e.$$

This holds because the law of motion (14) is homogeneous of degree one in consumption and the relevant balance sheet variables, which in turn relies on logarithmic utility, the linear production structure, and the assumption (4) of identical bank deposit-to-equity ratios h_t.

The homogeneity of (14) and (4) also imply the second important property, namely the linearity of individual decision rules in equity e_t^l, which will allow a simple aggregation in the next section, preserving linearity in the aggregate.

3.4 The Banks' Optimum

The Hamilton-Jacobi-Bellman equation of the bank's problem can be written as

$$\frac{\partial V}{\partial t} + \sup_{c,\ell}\Big(\log c + \frac{\partial V}{\partial e}[\ell(\mu+\pi_t - i_t^R) - p_t c_t + (i_t^R + (i_t^R - i_t^D)h_t - \tau_t^B)e] + \frac{\partial^2 V}{\partial e^2}\frac{\sigma^2\ell^2}{2}\Big) = 0$$

under the liquidity requirement (15).

By the structure of the value function discussed above, we have

$$\frac{\partial V}{\partial e} = \frac{1}{\rho e} \text{ and } \frac{\partial^2 V}{\partial e^2} = -\frac{1}{\rho e^2}.$$

Since (15) is linear in e_t^l, the first-order conditions are

$$c_t^l = \frac{\rho e_t^l}{p_t} \tag{16}$$

$$\ell_t^l = \begin{cases} \frac{\mu+\pi_t - i_t^R}{\sigma^2} e_t^l & \text{if } (1-\hat{\eta})h_t + 1 \geq \frac{\mu+\pi_t-i_t^R}{\sigma^2} \\ (1-\hat{\eta})h_t e_t^l + e_t^l & \text{if } (1-\hat{\eta})h_t + 1 \leq \frac{\mu+\pi_t-i_t^R}{\sigma^2} \end{cases} \tag{17}$$

where we must have $i_t^R \leq \mu + \pi_t$ for a solution to exist.[18] Hence, at the optimum, the loan-to-equity ratio

$$x_t \equiv \frac{l_t^l}{e_t^l} = \min\left(\frac{\mu+\pi_t - i_t^R}{\sigma^2}, 1 + (1-\hat{\eta})h_t\right) \tag{18}$$

[18]This will be shown to hold later. See Proposition 2.

is the same between banks, and the reserve constraint binds for each bank if and only if

$$(1-\hat{\eta})h_t \leq \frac{\mu+\pi_t-i_t^R}{\sigma^2}-1$$

i.e. iff banks do not have enough deposits relative to their equity.

In the solution just derived, banks use the interbank market to redistribute their liquidity according to their desired equity and lending positions.[19] In general, the liquidity requirement would imply that their optimal deposit positions depend on their individual equity positions, which are random and heterogeneous and thus cannot be aggregated easily. Under our assumption that the bank deposit-to-equity ratio is constant across banks, the problem disappears because this assumption implies that the liquidity requirement either binds for all banks or for none.

Wrapping up, bank lending is proportional to their equity

$$\ell_t^l = x_t e_t^l \tag{19}$$

with x_t given by (18), and we can write the liquidity requirement (15) as

$$h_t \geq \eta(x_t - 1) \tag{20}$$

where we have set

$$\eta \equiv \frac{1}{1-\hat{\eta}} > 1. \tag{21}$$

Finally, the law of motion (14) of individual bank equity becomes

$$\frac{de_t^l}{e_t^l} = \Big[(\mu+\pi_t-i_t^R)x_t + i_t^R - \tau_t^B - \rho + (i_t^R - i_t^D)h_t\Big]dt + \sigma x_t dz_t^l \tag{22}$$

4 Macroeconomic Equilibrium

4.1 Equilibrium Concept and Law of Motion

We now aggregate the results of the last section and investigate equilibrium. By (10) and (11), at the individual optimum, household wealth H_t follows the aggregate law of motion

$$\dot{H}_t = \left(i_t^D - \tau_t^H - \rho\right) H_t. \tag{23}$$

The individual balance sheets of banks follow random trajectories driven by (22). Yet, due to the Law of Large Numbers, the aggregate balance sheet of the banking sector is

[19] Each deposit transaction in the goods markets is exactly offset be a transfer of reserves. As a result, banks will trade among themselves to obtain their desired reserve-deposit ratio.

deterministic:

Assets	Liabilities
L_t	M_t
R_t	E_t

Using the individual expressions (18) and (19), the aggregates M_t and L_t are deterministic and satisfy

$$M_t = h_t E_t \tag{24}$$

$$L_t = x_t E_t \tag{25}$$

From the individual law of motion of equity (22) we obtain by aggregation

$$\dot{E}_t = \Big[(\mu + \pi_t - i_t^R)x_t + i_t^R - \tau_t^B - \rho + (i_t^R - i_t^D)h_t\Big]E_t \tag{26}$$

The dynamics of reserves are given by their supply (6) and the government budget constraint (8), where we can make taxes explicit and use (2) to obtain

$$\dot{R}_t = i_t^R R_t + \nu\mu L_t - \tau_t^H H_t - \tau_t^B E_t \tag{27}$$

Using (24) and (25) in the aggregate bank balance sheet identity (5) yields

$$R_t = (1 - x_t + h_t)E_t. \tag{28}$$

If $H_t = M_t$ (the deposit market clears), then (28) is the economy's aggregate market clearing condition at time t.

Taken together, we therefore have six necessary equations for equilibrium:

- the aggregate dynamics for H_t, E_t, and R_t: (23), (26), (27),
- the aggregate first-order condition for investment: (25),
- market clearing of the deposit market: $H_t = M_t$ inserted in (24),
- the aggregate bank balance sheet identity (28).

We can now define equilibrium with fiscal and monetary policy. These policies consist of initial values and dynamic policy instruments as follows:

- (P1) aggregate lump sum transfers L^H and L^B to households and banks, respectively, at date 0, with negative values corresponding to taxes, financed by issuing Central Bank reserves $R_0 = L^H + L^B$ and a lump sum remittance $S_0 = R_0$,
- (P2) instantaneous linear taxes on wealth (again, subsidies if negative) at rates τ_t^H for households and τ_t^B for banks,

- (P3) a differentiable path of reserves $R_t > 0$,
- (P4) a continuous path of interest rates on reserves i_t^R.

Remember that the initial lump sum transfers fix the households' and banks' aggregate initial wealth at $H_0 = p_0 A_0^H + L^H$ and $E_0 = p_0 A_0^B + L^B$. The above list (P1)-(P4) treats Central Bank reserves and the interest on reserves as policy variables, as is the case in practice. Given these four elements, the path of remittances from the Central Bank to the Treasury, (S_t), is determined by (6) and $S_0 = R_0$.[20] We take the reserve constraint (9) as exogenous and given, as it belongs neither to monetary nor fiscal policy.

Given policy, an equilibrium is a collection of aggregate balance sheet variables (L_t, H_t, E_t), prices $(p_t)_{t=0}^{\infty}$, and deposit interest rates $(i_t^D)_{t=0}^{\infty}$ such that banks and households act optimally, the budget of the treasury is balanced, the deposit market clears, and the aggregate bank balance sheet constraint (28) holds at each point in time.[21] It can easily be seen that the above six conditions (23) - (28), together with the definition of x_t in (18), are not only necessary, but sufficient for equilibrium.

By (22), bank profits and equity depend on the interest rate margin $i_t^R - i_t^D$. As explained in Section 2, we assume that banks take deposit rates as given and do not compete for deposits, which makes their interest income $(i_t^R - i_t^D)h_t$ exogenous. This is consistent with the argument made by Drechsler et al. (2017) and others about the lack of competitiveness of banking markets: if $i_t^D < i_t^R$, banks do not necessarily pass the interest earned on reserves on to their depositors.[22] To keep the analysis sufficiently general, we simplify the expressions by denoting the interest margin (identical across banks) by

$$\delta_t \equiv i_t^R - i_t^D. \tag{29}$$

We now derive a useful characterization of equilibrium dynamics, which will also yield an existence proof. Differentiating (24) with $M_t = H_t$ yields

$$\frac{\dot{H}_t}{H_t} = \frac{\dot{h}_t}{h_t} + \frac{\dot{E}_t}{E_t}$$

Together with (23) and (26), this is equivalent to

$$\frac{\dot{h}_t}{h_t} = \tau_t^B - \tau_t^H - \delta_t(1 + h_t) - (\mu + \pi_t - i_t^R)x_t. \tag{30}$$

[20] In fact, given the interest rate trajectory (i_t^R), (S_t) and (R_t) are equivalent instruments – one implies the other. In practice, the formal policy decision often concerns S_t and R_t adjusts accordingly.

[21] This definition is a simplification, as it ignores consumption and individual balance sheet decisions of banks, which are random and heterogeneous. But our analysis of Section 3 shows that for given individual initial conditions in (P1) the stochastic evolution of these trajectories is uniquely determined (in probability) and yields the aggregate outcomes (24), (25), and (26).

[22] The problem has been widely noted. For recent studies, see, e.g., Egan et al. (2025), Yankov (2024), and Argyle et al. (2026).

Hence, in equilibrium, the bank deposit-equity ratio h_t follows a first-order ordinary differential equation that depends on the banks' (common) loan-to-equity ratio. After substituting x_t from (18), we obtain the following non-linear ODE:

$$\dot{h}_t = \Big[\tau_t^B - \tau_t^H - \delta_t(1+h_t) - (\mu + \pi_t - i_t^R)\min\Big(1 + (1-\hat{\eta})h_t, \frac{\mu + \pi_t - i_t^R}{\sigma^2}\Big)\Big]h_t \tag{31}$$

The dynamics of h_t in (31) depend on the endogenous trajectories of δ_t and π_t, which also influence x_t and thus indirectly again h_t. Hence, the determination of the fundamental aggregate balance sheet ratios (x_t, h_t) is more complex. We therefore consider the cases where the reserve constraint binds and where it is slack separately. In the latter case, the balance sheet ratios and the inflation rate can be determined independently of each other. In the latter case, they are connected by the reserve constraint and must be solved for jointly.

In the rest of this section, we consider the case of non-binding reserve constraints as a lead example. The case of a binding requirement is presented in Appendix A.

4.2 Non-binding Reserve Constraints

Assume that there is an equilibrium where the reserve constraint (9) is never binding. In this case, we can characterize the equilibrium dynamics (x_t, h_t, π_t, i_t^D) resulting from the policy choices (P1)-(P4) constructively as follows.

If the reserve requirement (20) is slack, (18) links inflation, x_t, and i_t^R directly by

$$\pi_t = \sigma^2 x_t - \mu + i_t^R \tag{32}$$

which implies the following simple expression for the ODE (30):

$$\frac{\dot{h}_t}{h_t} = \tau_t^B - \tau_t^H - \delta_t(1+h_t) - \sigma^2 x_t^2.$$

Furthermore, differentiating (25) yields

$$\dot{L}_t = \dot{x}_t E_t + x_t \dot{E}_t \tag{33}$$

Differentiating (5) and re-writing the resulting identity $\dot{R}_t = \dot{H}_t + \dot{E}_t - \dot{L}_t$, using (23), (27), and (33) we obtain the differential equation

$$i_t^R R_t + \nu\mu L_t - \tau_t^H H_t - \tau_t^B E_t = (i_t^R + \delta_t - \tau_t^H - \rho)H_t - (x_t - 1)\dot{E}_t - \dot{x}_t E_t,$$

which by (26) is equivalent to

$$\dot{x}_t = \Big((\mu + \pi_t - i_t^R)x_t - \rho\Big)(1 - x_t) + (\tau_t^B - \nu\mu)x_t - (x_t\delta_t + \rho)h_t. \tag{34}$$

Using (32), we can collect these observations in the following lemma.[23]

[23]Note that (34) has not used (32). We can therefore use it for the analysis of the constrained case in the Appendix.

Lemma 1. *Suppose that there exists an equilibrium in which the reserve constraint is not binding on an interval* $[0, T)$, $T \leq \infty$. *Then, on* $[0, T)$ *the aggregate balance sheet ratios* (x_t, h_t) *satisfy the following system of ODEs:*

$$\dot{h}_t = \left(\tau_t^B - \tau_t^H - \delta_t(1+h_t) - \sigma^2 x_t^2\right) h_t \tag{35}$$

$$\dot{x}_t = (\sigma^2 x_t^2 - \rho)(1-x_t) + (\tau_t^B - \nu\mu)x_t - (x_t\delta_t + \rho)h_t \tag{36}$$

Note that the system (35)-(36) depends on the interest rate spread δ_t, but not on the interest rates i_t^R and i_t^D independently. Its initial conditions are given by initial policy (P1) and after the initial sales of goods by households as

$$h_0 = \frac{H_0}{E_0} = \frac{p_0 A_0^H + L^H}{p_0 A_0^B + L^B} \tag{37}$$

$$x_0 = \frac{L_0}{E_0} = \frac{p_0 A_0}{p_0 A_0^B + L^B} \tag{38}$$

We can now reverse the argument and construct the aggregate equilibrium allocation from the reserve policies (P3)-(P4) and (x_t, h_t). Consider any continuous path of interest rate spreads $(\delta_t)_{t\geq 0}$. By the standard theory of differential equations (e.g., Hirsch and Smale (1974)), there is a $\hat{T} \leq T$ such that the system (35)-(36) has a unique solution (x_t, h_t) on $[0, \hat{T})$ (which depends on the path (δ_t)).

To obtain an equilibrium allocation, we first use the aggregate market clearing condition (28) to obtain aggregate equity as

$$E_t = \frac{R_t}{1 + h_t - x_t} \tag{39}$$

Note that $E_t > 0$ by the reserve requirement, which implies that $1 + h_t - x_t > 0$. We then obtain H_t and L_t by (24) and (25). It is straightforward but lengthy to show that *if* the resulting aggregate balance sheet (L_t, R_t, H_t, E_t) satisfies the dynamic government budget constraint (27) it also satisfies the two remaining dynamic equilibrium conditions, 26) and (23). In fact, the dynamics of E_t as defined in (39) are given by

$$(1 + h_t - x_t)\dot{E}_t + (\dot{h}_t - \dot{x}_t)E_t = \dot{R}_t,$$

and using (35) and (36), one can show that this then is consistent with (26). A similar calculation using the definition of h_t and 26) shows that (27) implies (23).

In a final step we must therefore show that the aggregate balance sheet constructed above satisfies (27). Inserting (24), (25), and (39) into (27) yields

$$\begin{aligned} \dot{R}_t &= i_t^R R_t + \nu\mu x_t E_t - \tau_t^H h_t E_t - \tau_t^B E_t \\ &= \left[i_t^R + \frac{\nu\mu x_t - \tau_t^H h_t - \tau_t^B}{1 + h_t - x_t} \right] R_t \end{aligned} \tag{40}$$

Note that (40) depends directly on i_t^R and indirectly (via (x_t, h_t)) on δ_t, that is on i_t^D. This implies that, if the Central Bank chooses a policy path (R_t, i_t^R) satisfying (40), together with an "intended" deposit rate path (i_t^D), then $\delta_t = i_t^R - i_t^D$ will induce a solution to the dynamic system (35)-(36) that, following the above steps, yields the allocation of consumption, investment, and savings constructed above as an equilibrium. Inflation is then given by (32) and can directly be targeted by interest rate policy (i_t^R). Such monetary policy is possible, as for any initial condition $R_0 > 0$, the differential equation (40) has a unique solution,

$$R_t = R_0 \exp \int_0^t \Big(i_s^R + \frac{\nu \mu x_s - \tau_s^H h_s - \tau_s^B}{1 + h_s - x_s} \Big) ds \tag{41}$$

As (40) also shows, there will not necessarily be a general equilibrium for *any* monetary policy. Alternatively, we argue that the monetary policy variables R_t and i_t^R and the tax variables τ_t^B, τ_t^H must be sufficiently coordinated to make the government budget constraint (27) hold and the market settle on a specific deposit interest rate path (i_t^D) in general equilibrium.

As a consequence, if the trajectory (x_t, h_t) with which we have started out stays in the interior of $\mathbb{R}_+^2$ and satisfies the strict reserve constraint $h_t > \eta(x_t - 1)$ for all t, it is an equilibrium trajectory and the resulting allocation is an equilibrium allocation. Remember that the reserve constraint implies that all banks hold a positive amount of Central Bank reserves at all times. Without reserves, the monetary system would not be able to operate. We can summarize our analysis as follows.

Proposition 1. *Suppose that p_0 is fixed and that policy is fully coordinated on (P1) - (P4) by $S_0 = R_0 = L^H + L^B$ and (41), and let (δ_t) be an arbitrary continuous path of interest rate spreads. If the resulting trajectory of the system (35)–(36) with initial value given by (37) and (38) stays in the interior of $\mathbb{R}_+^2$ and satisfies $h_t > \eta(x_t - 1)$ for all t, then it is an equilibrium trajectory, and (H_t, L_t, E_t, π_t) are given by (24), (25), (39), and (32), respectively. The deposit market clears at the interest rate $i_t^D = i_t^R - \delta_t$.*

Two features of the above equilibrium are particularly noteworthy. First, it is well possible that trajectories (x_t, h_t) generated by the above procedure do not stay in the interior of $\mathbb{R}_+^2$. In this case, they don't define equilibria – we discuss this further in Section 6 below.[24] Second, the deposit rates (i_t^D) are part of a rational expectations equilibrium. The policy maker chooses a coordinated policy $L^B, L^H, (\tau_t^H, \tau_t^B), R_0, (R_t, i_t^R)$ anticipating that the deposit market will clear at the interest rate path (i_t^D), and the path (i_t^D) induces individual decisions that are optimal given this policy and clear the deposit market.

[24]This problem does not arise with stationary solutions, which we discuss in detail in Section 5 below.

As a natural benchmark and in order to simplify the analysis, we now consider the equilibrium with $i_t^D = i_t^R \equiv i_t$, where the banks' interest rate margin is zero.[25] By Proposition 1, this equilibrium exists under the appropriate policy coordination. The dynamics of the aggregate balance sheet variables in Lemma 1 then simplify to

$$\dot{h}_t = (\tau_t^B - \tau_t^H - \sigma^2 x_t^2)h_t \tag{42}$$

$$\dot{x}_t = (\sigma^2 x_t^2 - \rho)(1 - x_t) + (\tau_t^B - \nu\mu)x_t - \rho h_t. \tag{43}$$

This system has a number of interesting properties and implications. First, the system (42) and (43), and hence the basic bank balance sheet ratios (x_t, h_t), do not depend on the monetary policy rate i^R, only on the fiscal policy variable (τ^B, τ^H).

Second, equations (42) and (43) show that any differentiable trajectory (x_t, h_t) in the interior of $\mathbb{R}_+^2$ and strictly above the reserve constraint $h_t = \eta(x_t - 1)$ can be implemented as a solution to the dynamic system by a suitable choice of the two tax rates (τ_t^B, τ_t^H). Indeed, solving (42) and (43) for these tax rates gives

$$\tau_t^B = \nu\mu + \sigma^2 x_t(x_t - 1) + \rho\left(\frac{1 + h_t}{x_t} - 1\right) + \frac{\dot{x}_t}{x_t}, \tag{44}$$

$$\tau_t^H = \tau_t^B - \sigma^2 x_t^2 - \frac{\dot{h}_t}{h_t} \tag{45}$$

The resulting tax rates do not depend on monetary policy. [26]

Third, this observation also shows that equilibria always exist, because each point (x, h) in the interior of $\mathbb{R}_+^2$ and strictly above the reserve constraint $h_t = \eta(x_t - 1)$ is a stationary trajectory, for which (44) and (45) with $\dot{x}_t = \dot{h}_t = 0$ provide an implementation through stationary tax rates.

Fourth, the inflation equation (32) shows that the Central Bank can control inflation 1:1 through interest on reserves. This is particularly important as i_t^R also influences the real allocation through (41). More generally, (32) establishes the structural determinants of inflation. Ceteris paribus, lower volatility of idiosyncratic risks or lower bank loan-to-equity ratios lead to lower inflation, if the Central Bank does not react by adjusting the interest on reserves.

[25] As discussed earlier, if $i_t^D < i_t^R$, banks do not pass the interest earned on reserves on to their depositors, as they do not actively compete for deposits, which is consistent with empirical evidence. A classic alternative assumption, Bertrand price competition for deposits would rule out deposit rates $i_t^D < i_t^R$ by the usual undercutting argument. Since deposits have an economic value and reserves are necessary to operate them, the case $i_t^D \geq i_t^R$ requires more attention.

[26] Comparing (42) and (43) to the general law of motion (35) and (36) shows that the above implementation depends on the equilibrium selection. In equilibria with $\delta_t \neq 0$, the deposit interest rate spread will matter.

Fifth, the Central Bank's reserve-to-GDP ratio at date t is

$$\Delta_t \equiv \frac{R_t}{\mu L_t} = \frac{1+h_t-x_t}{\mu x_t}. \tag{46}$$

As discussed in the introduction, equation (46) describes one of the most debated economic developments in recent years. Figure 1 documents that in most countries Δ_t has increased strongly after the Great Financial Crisis and has changed frequently and differently in different countries ever since. In Section 5 we show how our model can shed light on this phenomenon.

Next, from (33), the economy's real equilibrium growth rate can be computed as

$$\begin{aligned} g_t = \frac{\dot{L}_t}{L_t} - \pi_t &= \frac{\dot{x}_t}{x_t} + \frac{\dot{E}_t}{E_t} - \pi_t \\ &= (1-\nu)\mu - \rho\frac{1+h_t}{x_t} \end{aligned} \tag{47}$$

Note, in particular, that real growth depends negatively on the government spending coefficient ν. Combining (46) and (47) shows that the reserve-to-GDP ratio and the real growth rate are negatively linearly correlated. But (46) and (47) make it clear that there is no causality in this relation, they predict how the two variables are driven apart by the same forces.

Finally, consider Central Bank remittances, the central tool of engineering monetary policy through reserve balances. By (6), the share of remittances over total Central Bank reserves is

$$s_t = \frac{S_t}{R_t} = \frac{\dot{R}_t}{R_t} - i_t^R$$

Remember that $s_0 = 1$: the initial stock of reserves is fully transmitted to the economy by remittances to the Treasury. By (24), (25), and (28),

$$s_t = \left[\nu\mu x_t - \tau_t^H h_t - \tau_t^B\right]\frac{1}{1+h_t-x_t}. \tag{48}$$

Summarizing these and other qualitative results, we can state:

Proposition 2. *Assume that the equilibrium trajectory (x_t, h_t) stays in the interior of $\mathbb{R}^2_+$, satisfies $h_t > \eta(x_t - 1)$ for all t, and consider the case $i_t^D = i_t^R = i_t$. Then, in equilibrium,*

1. *the real interest rate is a decreasing function of the loan-to-equity ratio of banks:*

$$i_t - \pi_t = \mu - \sigma^2 x_t,$$

2. *the share of remittances over total reserves is*

$$s_t = \left[\nu\mu x_t - \tau_t^H h_t - \tau_t^B\right]\frac{1}{1+h_t-x_t},$$

3. output growth is a linearly decreasing function of the reserve-to-GDP ratio:

$$g_t = (1-\nu)\mu - \rho - \rho\mu\Delta_t,$$

4. the money multiplier (the ratio of deposits to Central Bank reserves),

$$\frac{h_t}{1+h_t-x_t},$$

is determined by h_t and x_t and thus depends on monetary and fiscal policy variables.

5. monetary policy is neutral with respect to interest rates, but not with respect to reserves.

In the case $i_t^D = i_t^R$, since Central Bank interest rate policy does not affect the determinants of (x_t, h_t) of real growth, we have an instance of monetary policy neutrality with respect to interest rates. Central Bank interest rate policy directly affects inflation through (32), but not real growth. But there is a second channel of monetary policy that operates via the level of reserves in the system. As defined in (P1) in the definition of policy above, the Central Bank must endow banks with reserves and needs to transfer (or receive) a particular amount of remittances to the treasury to ensure that the equilibrium supported by (x_t, h_t) exists. Hence, as evidenced by the initial condition $S_0 = R_0$ and the optimal dynamic remittance policy (48), reserve policy is not neutral.[27]

5 Welfare and Optimal Policy

5.1 Welfare Maximization

Having characterized equilibrium allocations, given monetary and fiscal policy, we now explore optimal policy choices. We evaluate optimal policy by adopting a simple utilitarian welfare function, in which the welfare weights of households and bankers can differ:

$$W = \alpha V^B + (1-\alpha)V^H \tag{49}$$

where $0 < \alpha < 1$ is the weight put on bankers' utilities, V^B the aggregate utility of bankers and V^H the utility of the representative household.

To characterize W along competitive allocations (i.e., those that arise from the individual decisions of Section 3) note that, given the stationarity of returns and exponential discounting, W is maximum for stationary allocations, where $\frac{E_t}{p_t}$ grows at a constant rate

[27]The main effect is through initial transfers, i.e., the determination of levels. As for reserve flows between banks that settle deposit flows, remittances to the treasury create the same amount of bank deposits at private banks.

g, while $x_t \equiv x$ and $h_t \equiv h$ are constant. For such a stationary allocation, welfare is easily computed. Indeed, household consumption $C_t^H = \frac{\rho h L_t}{x p_t}$ grows at rate g, so that

$$\log C_t^H = \log \frac{\rho h L_0}{x p_0} + gt.$$

Similarly, the consumption flow of an individual banker $c_t^\ell = \rho e_t^\ell$ grows on average at rate g but is subject to idiosyncratic shocks of volatility σx. Thus

$$\log c_t^\ell = \log \frac{\rho L_0}{x p_0} + (g - \frac{\sigma^2 x^2}{2})t + \sigma x z_t^\ell.$$

By aggregation, the idiosyncratic shocks wash out and we obtain:

$$\rho W = \int_0^\infty \rho e^{-\rho t} [\log \frac{\rho L_0}{x p_0} + (g - \alpha \frac{\sigma^2 x^2}{2})t + (1-\alpha)\log h] dt.$$

Since $g = \mu(1-\nu) - \rho \frac{1+h}{x}$ by (47), the integral can be written in simple terms:

$$\rho W = \log \frac{\rho L_0}{x p_0} + \frac{1}{\rho}[\mu(1-\nu) - \alpha \frac{\sigma^2 x^2}{2}] + (1-\alpha)\log h - \frac{1+h}{x}.$$

This expression is easily maximized with respect to h and x. More generally, we obtain:

Proposition 3. *Consider any competitive allocation (x_t, h_t) that does not necessarily satisfy the reserve requirement (9).*

(i) If the allocation is welfare optimal it is stationary: $x_t \equiv x^*, h_t \equiv h^*$, and

$$h^* = (1-\alpha)x^* \tag{50}$$

$$\frac{\sigma^2}{\rho} x^{*3} + x^* = \frac{1}{\alpha}. \tag{51}$$

(ii) Welfare optimal allocations satisfy $h^* > x^* - 1$. Hence, $R_t^* > 0$ for all t.

(iii) The welfare maximizing reserve-to-GDP ratio is

$$\Delta^* = \frac{1 - \alpha x^*}{\mu x^*}. \tag{52}$$

(iv) If $h^* > \eta(x^* - 1)$, the allocation can be implemented as an equilibrium with $i^D = i^R$ by the monetary system through suitable monetary and fiscal policy variables (R_t, τ^B, τ^H) and the corresponding remittances. The allocation can be implemented by any reserve rate $i^R \geq 0$.

The proof of the first three parts of Proposition 3 is in Appendix B and builds on the basic insights from our companion paper Biais et al. (2024).[28] There we model the

[28]The final part is proved more generally in the next subsection for any stationary allocation.

market incompleteness problem explicitly as the result of informational frictions in a multi-agent Principal-Agent problem in the financing of firms. In our context, this means that banks/firms generate non-verifiable revenues, and a share of these revenues can be secretly diverted for private consumption. The general optimal mechanism identified in Biais et al. (2024) shows that the marginal returns of investment must be constant (which corresponds to the linear interest in the present model), the marginal redistribution between agents must be constant (which corresponds to the linear taxation here), and the instantaneous consumption ratio of the equity holder to the capital stock of a firm must be independent of history and this ratio must be the same for all firms. This is exactly what the implementation and equilibrium in the model of the present paper deliver.

Proposition 3 allows us to characterize the optimal level of Central Bank reserves. It is immediate from (52) that Δ^* increases when the productivity of capital μ decreases. Now (51) shows that x^* is a decreasing function of $\frac{\sigma^2}{\rho}$. Thus Δ^* is an increasing function of this ratio. Everything else fixed, an increase of the uncertainty in the economy, captured by a higher variance of idiosyncratic shocks requires a higher reserve-to-GDP ratio. Arguably, these two effects have been particularly important after the Global Financial Crisis of 2007/08. An interpretation of the cross-country data in our introductory Figure 1 in the light of Proposition 3 then is that, with some delays, the Central Banks all initially reacted the same way to the GFC, while their later policies were heterogeneous depending on the local evolution of σ and μ.

The proposition also illustrates the welfare impact of Central Bank reserves. In our model, the size of reserves has three effects: a balance sheet effect for banks, a risk-reduction effect, and a growth effect. The balance sheet effect increases aggregate wealth in the economy and the risk-reduction effect benefits bankers directly, through lower risk exposure and an increase of the real interest rate. These effects compensate for the decrease in growth due to higher equilibrium consumption. Moreover, by continuously intervening to keep the economy on a steady state growth path, fiscal policy redistributes the gains from Central Bank activity across bankers and households.[29]

At first glance, it is surprising that we obtain constrained efficiency with incomplete markets, given the well-known result by Geanakoplos and Polemarchakis (1986) that competitive equilibria with incomplete markets are generically constrained inefficient: equilibrium allocations typically leave scope for Pareto improvements, subject to market structure

[29] As noted in Section 2, our formal model is a simplification of a more complex model with banks and firms. More generally, the first two effects just mentioned benefit banks and firms differently, depending on the market structure. In our case, where banks have all the market power, they only benefit banks, and importantly, they spill over from the bank/firm sector to the household sector through the growth effect and ongoing redistribution.

constraints.

In our model, however, policymakers have enough instruments to span the relevant state-contingent market space, as their fiscal–monetary toolkit includes $(\tau_H, \tau_B, i_t^R, R_t)$ and relies on optimal coordination between monetary and fiscal authorities. These two features circumvent the inefficiency result of Geanakoplos and Polemarchakis (1986).

5.2 Stationary Allocations and Inflation Targets

Proposition 3 has shown that welfare optimal allocations are stationary. We therefore now consider stationary points of the law of motion more generally and show how to implement them as stationary equilibrium allocations. This will also yield a proof of statement (iv) of Proposition 3.

By (32), (x, h) also influences inflation. In our model, this is of little meaning, because inflation does not impact welfare directly and prices are flexible. However, as we will see, implementing (x, h) in an equilibrium with $i^D = i^R$ does not pin down the reserve rate i^R. We therefore have an additional degree of freedom that we can use to target inflation. By (32), this is sufficient.[30] We continue to consider the case where the reserve constraint does not bind and delegate the corresponding analysis for the constrained case to Appendix A.

Proposition 4. *Let $(x, h) >> 0$ and $h > \eta(x-1)$. Then the following policy implements the allocation (x, h, π) as a stationary equilibrium with $i^D = i^R$:*

$$
\begin{aligned}
\tau^B &= \nu\mu + \sigma^2 x(x-1) + \rho\Big(\frac{h+1}{x} - 1\Big) \\
\tau^H &= \tau^B - \sigma^2 x^2 \\
i^R &= \mu + \pi - \sigma^2 x \\
E_0 &= \frac{1}{x} p_0 A_0 = p_0 A_0^B + L^B \\
H_0 &= \frac{h}{x} p_0 A_0 = p_0 A_0^H + L^H \\
R_0 &= S_0 = H_0 + E_0 - L_0 = \Big(\frac{h+1}{x} - 1\Big) p_0 A_0
\end{aligned}
$$

Proof. The expressions for τ^B, τ^H, i^R follow directly from the general expressions (44),(45), and (32). For the initial conditions, remember that initial lending L_0 is given by the total initial stock of physical goods, A_0. The steady-state condition $x = \frac{L_0}{E_0}$ then implies $E_0 = \frac{p_0 A_0}{x}$ and the rest.

□

[30] One can motivate such an inflation target directly by adding costs of inflation to our model, in the spirit of Rotemberg (1982).

Note that the Central Bank can freely target inflation with the interest rate i^R, although i^R is needed to set reserves appropriately in the economy's aggregate balance sheet through (41). The reason is that this use of i^R does not change the real interest rate, and, as (47) shows, the economy's investment and consumption only depend on the real rate. Hence, interest rate policy is fully neutral.

One potential difficulty with this implementation result is that it requires specific lump sum transfers at date $t = 0$ so that the economy starts at the desired values of the deposit-to-equity ratio $h_0 = h$ and the loan-to-equity ratio $x_0 = x$. These lump sum transfers may be unrealistic. A more natural question may be whether the economy can converge to the desired values (x^*, h^*) when it starts from arbitrary initial values (x_0, h_0). This is examined in the next section.

6 Instability and Uncoordinated Policies

This section studies whether the optimal allocation (x^*, h^*), or more generally any stationary allocation $(\overline{x}, \overline{h})$ is locally stable when either lump sum transfers cannot be implemented, fiscal and monetary policies are not coordinated, or when there are volatility shocks. For simplicity, we continue to focus on equilibria with assume $i_t^D = i_t^R$, which leads to the law of motion (42)-(43). All arguments hold qualitatively for the general case, but equilibrium values must be adjusted to reflect the different tax incidence.

6.1 Local Stability of Optimal Allocations

Suppose that taxes (τ^H, τ^B) are set to implement $(\overline{x}, \overline{h})$ as in Proposition 4, but that lump sum transfers at $t = 0$ are not available. The law of motion for (x_t, h_t) is

$$\begin{pmatrix} \dot{x}_t \\ \dot{h}_t \end{pmatrix} = f(x_t, h_t) = \begin{pmatrix} f_x(x_t, h_t) \\ f_h(x_t, h_t) \end{pmatrix}$$

where

$$f_x(x, h) = -\sigma^2 x^3 + \sigma^2 x^2 + \rho \frac{\overline{h} + 1}{\overline{x}} + \sigma^2 \overline{x}(\overline{x} - 1) - \rho(h + 1) \quad (53)$$

$$f_h(x, h) = \sigma^2(\overline{x}^2 - x^2)h \quad (54)$$

The system has exactly one stationary point $(\overline{x}, \overline{h})$ in $\mathbb{R}^2_{++}$. It can be linearized around $(\overline{x}, \overline{h})$:

$$\begin{pmatrix} \dot{x}_t \\ \dot{h}_t \end{pmatrix} \approx Df(\overline{x}, \overline{h}) \begin{pmatrix} x_t - \overline{x} \\ h_t - \overline{h} \end{pmatrix} \quad (55)$$

where

$$Df(\overline{x}, \overline{h}) = \begin{pmatrix} -3\sigma^2\overline{x}^2 + 2\sigma^2\overline{x} & -\rho \\ -2\sigma^2\overline{h}\overline{x} & 0 \end{pmatrix} \quad (56)$$

Proposition 5. *The stationary point* $(\overline{x}, \overline{h})$ *is locally unstable.*

Proof. The eigenvalues λ_1, λ_2 of $Df(\overline{x}, \overline{h})$ are given by the zeroes of the quadratic polynomial

$$P_\lambda = \lambda^2 + A\lambda + B$$

where

$$\begin{aligned} A &= 3\sigma^2\overline{x}^2 - 2\sigma^2\overline{x} \\ B &= -2\rho\sigma^2\overline{x}\overline{h}. \end{aligned}$$

Since $\overline{x} > 0, \overline{h} > 0$ we have $B < 0$, so the two eigenvalues have opposite signs. Therefore $(\overline{x}, \overline{h})$ is a saddle point and is locally unstable. □

To illustrate what can happen, Figures 2 and 3 display the phase diagram of the dynamic system with no government spending, $\nu = 0$, and $\tau^B = \tau^H = 0$. Hence, only two parameters matter: ρ and σ^2. All trajectories have to lie above the line $h_t = x_t - 1$, drawn in orange, which corresponds to the case of zero reserves. There are two possible stationary states: $(1, 0)$ and $(\frac{\sqrt{\rho}}{\sigma}, 0)$. The figures show two flow diagrams: Figure 2 for $(\rho, \sigma) = (0.02, 0.30)$ $(\rho < \sigma^2)$, and Figure 3 for $(\rho, \sigma) = (0.03, 0.1)$ $(\rho > \sigma^2)$. There exists a critical line running through the stationary point to the left of the two stationary points on the x-axis, which separates the quadrant into two parts. The critical line is a strictly increasing function, derived numerically.

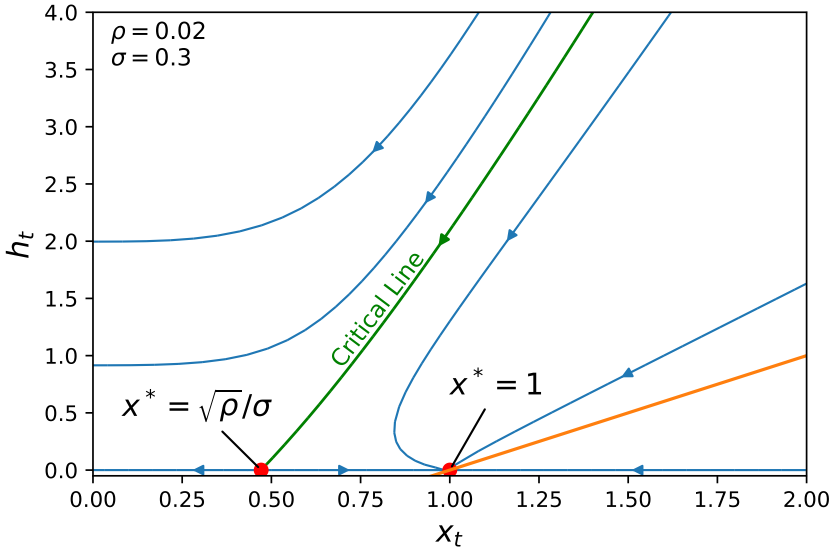


Figure 2: Partial Stability of Banking

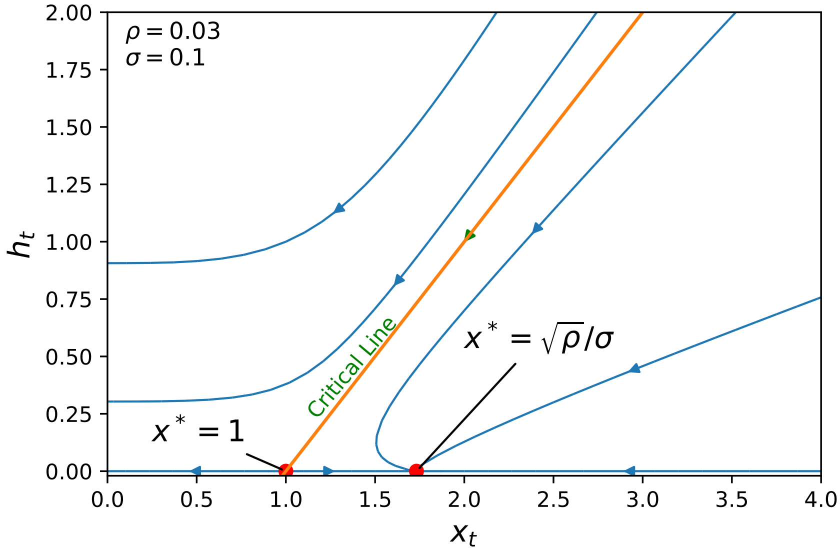


Figure 3: Instability of Banking. The stationary point $(\sqrt{\rho}/\sigma, 0)$ is to the right of $(1, 0)$.

Note that for $\rho > \sigma^2$, every path will reach $x = 0$ in finite time and thus there is no stable point in the region $h > x - 1$ and $x > 0, h > 0$. In the case $\rho < \sigma^2$, this also happens if the starting point is above the critical line. If it is below, the trajectory will converge to the x-axis, without ever reaching it in finite time.

The reason why x hits zero is that the economy enters a path of negative growth, since reserves have boosted the wealth of bankers so much that consumption of households and bankers exceeds the return on capital. Since the real interest rate continuously increases, banks invest less and less in real resources and comparatively more in reserves, in line with the declining capital stock. When households and bankers continue to consume, based on their wealth, which is higher than the ever smaller capital stock, the capital stock is wiped out and x becomes zero.

The case where the trajectory hits the h-axis in finite time cannot represent an equilibrium if agents are forward looking. If it were an equilibrium agents would be short-sighted and would not anticipate a banking collapse. Section 6.2 discusses what happens in such cases.

6.2 Shocks and the Real Effects of Reserves

Section 5.2 has shown how perfect coordination between the Central Bank and the Treasury can implement any desired aggregate balance sheet + inflation target (x_t, h_t, π_t). We now investigate whether such coordination is needed. We illustrate the problem by assuming that the stationary target (x, h, π) has been implemented according to Proposition 4 and that the economy experiences an unexpected volatility shock such that σ increases to $\tilde{\sigma}$. We continue to assume that $h > \eta(x-1)$ and that the equilibrium has $i^D = i^R$.

By Proposition 4, the only way to meet the inflation target without changing x is to adjust i^R. Hence, the Central Bank must decrease its interest rate to

$$\tilde{i}^R = \mu + \pi - \tilde{\sigma}^2 x.$$

The question is whether any one of the government actors can stabilize the aggregate balance sheet variables when acting in isolation.

6.2.1 What can the Treasury do alone?

Suppose that the Central Bank does not change its reserve policy R and leaves remittances at $S = \dot{R} - i^R R$ as given by (6). Can the fiscal authorities stabilize the economy and keep the allocation at (x, h) after the σ shock?

Proposition 6. *There exist no tax rates $\tilde{\tau}^H$ and $\tilde{\tau}^B$ that can implement the stationary allocation (x, h), when the Central Bank does not adjust its reserve policy and the Treasury keeps a balanced budget.*

Proof. The post-shock dynamics of the system are given by (42)-(43),

$$\begin{aligned}
\dot{h}_t &= (\hat{\tau}^B - \hat{\tau}^H - \tilde{\sigma}^2 x_t^2) h_t \\
\dot{x}_t &= (\tilde{\sigma}^2 x_t^2 - \rho)(1 - x_t) + (\hat{\tau}^B - \nu\mu) x_t - \rho h_t
\end{aligned}$$

If the Treasury wants the allocation to stay at (x, h) after the shock, Proposition 4 implies that it must adjust its tax rates to

$$\begin{aligned}
\tilde{\tau}^B &= \nu\mu + \tilde{\sigma}^2 x(x-1) + \rho\left(\frac{1+h}{x} - 1\right) \\
\tilde{\tau}^H &= \tilde{\tau}^B - \tilde{\sigma}^2 x^2.
\end{aligned}$$

Suppose that the budget of the treasury remains balanced. Hence, at any point in time, (8) implies

$$S + T = \nu\mu L.$$

This implies

$$s + \tilde{\tau}^H h + \tilde{\tau}^B - \nu\mu x = 0$$

with s given by:

$$\begin{aligned} s &= \nu\mu x - \tau^H h - \tau^B \\ &= \nu\mu x + (\rho + g - \mu)(h+1) + \sigma^2 x(h+1) - \sigma^2 x^2 \end{aligned}$$

In order for (x, h) to be a stationary solution, we therefore must have

$$\tilde{\tau}^H h + \tilde{\tau}^B = \tau^H h + \tau^B.$$

This implies

$$\begin{aligned} & (\tilde{\sigma}^2 - \sigma^2)\left[(1+h)x(x-1) - hx^2\right] = 0 \\ \Leftrightarrow \quad & (\tilde{\sigma}^2 - \sigma^2)[x(x-1-h)] = 0. \end{aligned} \tag{57}$$

Since $(x, h) >> 0$, (57) implies $x = 1 + h$, in contradiction to the liquidity requirement (20).

□

Proposition 6 shows that reserve policies that use remittances and pay interest on reserves cannot be neutralized by the Treasury.

6.2.2 What can the Central Bank do alone?

Now suppose that the fiscal authority remains passive and keeps the tax rates at the pre-shock levels τ^H, τ^B according to Proposition 4. Can the Central Bank stabilize the economy through adjustments of the remittances and thus keep the allocation at (x, h)?

Proposition 7. *There exist no remittance policy $\tilde{S}$ that can implement the stationary allocation (x, h) if the Treasury does not react to the shock. For unchanged individual behaviour, the economy converges either to a state in which all wealth is concentrated in the banking sector or the banking system collapses.*

The result is a consequence of Proposition (5). Since the coordinated policy package that guarantees the stationary solution (x, h) for a particular σ is uniquely determined, a shock to volatility will necessarily move the economy away from (x, h) if taxes are not adjusted. This is because the Central Bank has to adjust remittances to satisfy the government budget constraint. Hence, there are two objectives and only one instrument. As shown in Section 6.1 either the banking system will collapse, or in the case when $\rho < \tilde{\sigma}^2$ and depending on the initial conditions, the economy converges to a state in which bankers own all wealth in the economy (not reached in finite time).

We stress that Proposition 7 has been derived under the assumption that individual behaviour does not change in anticipation of what happens. In the case where the economy converges to a state with $h = 0$ (bankers own all wealth), individual behaviour remains optimal. If the banking system collapses, this is not the case. The Proposition then shows that equilibrium will fail to exist.

7 Conclusion

In this paper, we have provided a model in which banks create money, but cannot fully diversify credit risk, and financial markets are incomplete. In such an environment, Central Bank reserves have real effects not only in their role in settling interbank claims but also as a safe asset. Next to the basic results that we have studied, such as existence, stability, policy coordination, or optimality, the model allows us to address many policy issues that we are exploring in ongoing and future work. We sketch some of these extensions and applications in these final remarks.

First, a central feature of the dynamic system governing the loan-equity ratio x_t and the deposit-equity ratio h_t is that the stationary allocations (x, h) implemented by a given fiscal-monetary policy package are generically *locally unstable* when taxes (τ_H, τ_B) are held fixed. As shown in Section 6.1, the dynamic system governing the economy is locally unstable and will possible lead to collapse after a small perturbation. This property calls for coordination between fiscal and monetary authorities.

A further implication of our analysis concerns price-level determination. The need for ongoing coordination between monetary and fiscal authorities suggests a parallel with the Fiscal Theory of the Price Level (Cochrane (2022)). In the FTPL, the price level adjusts to ensure consistency between the real value of government liabilities and the expected path of fiscal surpluses. In our framework, central bank reserves and bank-created deposits constitute the key nominal liabilities, and their real value depends jointly on reserve policy, bank balance-sheet dynamics, and fiscal policy. Consequently, the aggregate quantity of reserves allocated to banks, together with bank money creation and fiscal policy, may form part of a theory of equilibrium price-level determination. This perspective points toward what might be termed a Banking Theory of the Price Level, in which reserves and bank-created money play a role analogous to government debt in the FTPL. A formal analysis is left for future research.

Our results may also shed light on the potential and limitations of the so-called Modern Monetary Theory, one interpretation of which is that some government expenditures can be financed by money creation without causing inflation if they are accompanied by appropriate tax policies. Indeed, in our model, the real value of Central Bank reserves – and thus the real value of remittances – can be continuously increasing, which means that part of government expenditure can be financed by money creation. However, there are limits to such non-inflationary monetary financing of government expenditures. First, the share that can be financed through money creation is limited. Moreover, as the growth rate is strictly decreasing in $\nu\mu$, increasing government expenditures may render financing of government expenditures by money creation infeasible altogether.

Next, a core feature of our base model is interest-rate neutrality, which arises under three assumptions:

1. The reserve constraint is slack: banks hold reserves voluntarily.
2. Bank competition is perfect (leading to $i^D = i^R$).
3. Fiscal policy ensures interior solutions for households' and bankers' wealth.

Under these assumptions, the Euler conditions for households and bankers imply that nominal interest rates affect only the common safe return. This mechanism is similar to the classical Modigliani–Miller neutrality in corporate finance. If monetary policy shifts all nominal yields symmetrically, but not relative returns, banks' portfolio choices remain unchanged. However, our general Proposition 1, which deals with the general dynamic system (35)-(36), points to the monetary policy consequences of imperfectly competitive banking markets, even in the presence of flexible prices. The resulting non-neutral monetary dynamics are an important subject of future research.

In a similar vein, the level of remittances impacts fiscal policy. When reserves are large, a change in i_t^R can produce substantial fiscal flows between the Central Bank and the Treasury. If fiscal policy does not offset those changes, as in our basic model, interest rate policies become non-neutral. Also, adding bonds and portfolio decisions, as outlined below, will, in general, introduce elements of non-neutrality in interest rate policies.

An important shortcoming of the analysis in this paper is the missing bond market. The current model gives the government only access to one financial instrument, namely reserves. Including government bonds in the model would make public financing choices richer and create a further coordination problem between the Central Bank and the fiscal authorities. Adding bonds would also yield more realistic portfolio decisions, which we are currently exploring – between bonds and bank deposits on the household side, and between bonds and reserves on the bank side. This extension is particularly useful for a broader analysis of policy tools, such as quantitative easing and the role of the Central Bank's balance sheet in general. In a further extension, adding risky long-term private debt as a possible investment by the Central Bank would allow for addressing how the combination with interest-bearing reserves may create losses for the Central Bank and potentially even negative remittances and the need for fiscal backing, as examined, for instance, in Benigno and Nistico (2020) and Del Negro and Sims (2015).

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Appendix A: Binding Reserve Constraints

This appendix presents the solutions for Sections 4.2 and 6 when the reserve constraint binds in equilibrium. In the main text, we have considered the case where the reserve constraint *never* binds, here we consider the case where it *always* binds. The more general case where the constraint sometimes binds and sometimes not can be solved similarly, but is a bit more complex.

A.1 Binding Reserve Constraint: Existence and Characterization

This subsection complements the analysis of Section 4.2 and proves the counterpart of Proposition 1 for the case where the reserve constraint always binds.

In this case, (30) is

$$\frac{\dot{h}_t}{h_t} = \tau_t^B - \tau_t^H + \delta_t + i_t^R - \mu - \pi_t + \left[\delta_t - (1-\hat{\eta})(\mu + \pi_t - i_t^R)\right] h_t \tag{58}$$

and $\dot{x}_t$ is given by

$$\dot{h}_t = \eta \dot{x}_t. \tag{59}$$

Hence, the (x_t, h_t) trajectory stays on the constraint by construction (the law of motion for (x_t, h_t) is essentially one-dimensional).

In order to characterize equilibrium, we can now proceed similarly to the unconstrained case in the main text, by first characterizing equilibrium trajectories (x_t, h_t) and then constructing equilibrium allocations from such trajectories.

Let the fiscal policy (τ_t^H, τ_t^B) and initial quantities in (P1) be given. Take any path of interest rate differentials δ_t and inflation rates π_t. With a starting value (x_0, h_0) such that $h_0 = \eta(x_0 - 1)$, a trajectory that follows the general law of motion (30) and (34) will not stay on the constraint. We therefore need to make sure that the general law of motion takes the form (58) and (59). This is obvious for (30) and (58). For $\dot{x}_t$, note that (34) can be written as

$$\dot{x}_t = \left(\delta_t \eta - (\mu + \pi_t - i_t^R)\right) x_t (x_t - 1) + (\tau_t^B + \rho - \nu\mu - \rho\eta) x_t + (\eta - 1)\rho. \tag{60}$$

On the other hand, using the relation (58), the dynamics of h_t yield a necessary equation for $\dot{x}$, which must hold under the binding reserve constraint. The equation is given by

$$\dot{x}_t = \left(\delta_t \eta - (\mu + \pi_t - i_t^R)\right) x_t (x_t - 1) + (\tau_t^B - \tau_t^H - (\eta - 1)\delta_t) x_t + \tau_t^H - \tau_t^B + (\eta - 1)\delta_t. \tag{61}$$

Comparing (60) and (61) shows that both are identical iff

$$\delta_t = \rho + \frac{\nu\mu - \tau_t^H}{\eta - 1} + \frac{\eta(\nu\mu - \tau_t^B)}{(\eta - 1)h_t}. \tag{62}$$

Now, as in the unconstrained case, using the reserve constraint and the reserve policy (P3) in the market clearing condition (28) yields aggregate equity as

$$E_t = \frac{R_t}{\hat{\eta} h_t} \tag{63}$$

from which we get H_t and L_t by (24) and (25).

For equilibrium, we now must satisfy the compatibility condition (62) and the equity dynamics (26), for which we have two free variables, δ_t and π_t. One can show that under mild conditions this is possible, but we now cannot assume that $\delta_t = 0$, as in the unconstrained case. As (62) shows, δ_t will generically be different from 0.[31]

We can summarize our findings as follows.

Proposition 8. *Suppose that p_0 is fixed and that policy is given by (P1)-(P4). If the deposit rates (i_t^D) and the inflation rates (π_t) satisfy the compatibility condition (62) and the equity dynamics (26), and if the initial values (x_0, h_0) satisfy $h_0 = \eta(x_0 - 1)$ and $h_0 \leq \eta\left(\frac{\mu + \pi_0 - i_0^R}{\sigma^2} - 1\right)$ for all t, then the law of motion (30) and (34) yields an equilibrium trajectory that satisfies the reserve constraint with equality and stays in the interior of $\mathbb{R}_+^2$ for all t. In general, the equilibrium deposit rate i_t^D is different from i_t^R. Moreover, the inflation rate π_t is given by (26) and is higher than it would be in the unconstrained case in (32).*

The final statement of the proposition immediately follows from the constraint condition in (18).[32]

In the case of binding reserve constraints of Proposition 8, interest setting by Central Banks may affect real allocations, as can be seen from the law of motion of x_t in (34). In this case, the reserve rate directly affects bank lending, as deposit fluctuations cannot be offset by adjusting reserves at the individual bank level.

Using the compatibility condition (62), we can make the equilibrium deposit rate of Proposition 8 explicit:

Proposition 9. *In the equilibrium identified in Proposition 8, the equilibrium deposit rate is uniquely given by*

$$i_t^D = i_t^R + \rho + \frac{\nu\mu - \tau_t^H}{\eta - 1} + \frac{\eta(\nu\mu - \tau_t^B)}{(\eta - 1)h_t}. \tag{64}$$

[31] As in the unconstrained case, the remaining equilibrium conditions (23 and (27 will be slack.

[32] Proposition 8 is stronger than Proposition 1 in the sense that it shows existence. The reason is that in the case considered here, the trajectory (x_t, h_t) will automatically stay in the interior of $\mathbb{R}_+^2$. Hence, the qualification in Proposition 1, which is substantial as the analysis in Section 6 shows, does not apply.

A.2 Binding Constraint: Boundary Case and Local Instability

For the binding instability analysis below, we record the boundary case in which bank l has a liquidity requirement at the optimal choice of a_t^l and the constraint coincides with the individual optimality condition in (17).[33] Thus this subsection focuses on the boundary case in which the liquidity requirement binds at the bank's optimal lending choice, so that $h_t = \eta(x_t - 1)$ and $\sigma^2 x_t = \mu + \pi_t - i_t^R$. In this case, the law of motion is given by (58) and (59) and becomes

$$\dot{h}_t = \big((\tau_t^B - \tau_t^H) + \delta_t(1 + h_t) - \sigma^2 x_t^2)\big) h_t \tag{65}$$

$$\dot{x}_t = (\sigma^2 x_t^2 - \rho)(1 - x_t) + (\tau_t^B - \nu\mu)x_t - (\rho + \delta_t x_t)h_t \tag{66}$$

We observe that the dynamical system differs in both equations from the zero-spread non-binding system (42)-(43). When $i_t^D > i_t^R$, it is intuitive that h is increasing more and x is increasing more, or declining less, than in the case when $i_t^D = i_t^R$. Inflation is then given by

$$\pi_t = i_t^R - \mu + \sigma^2 x_t. \tag{67}$$

Note that law of motion again is effectively one-dimensional, as we have

$$\eta \dot{x}_t = \dot{h}_t. \tag{68}$$

which is ensured by the equilibrium deposit interest rate.

We next assume that the reserve constraint is binding at time t, i.e. $h_t = \eta(x_t - 1)$, and the reserve constraint is fixed at η, and where the policy is set such that the system is kept on the reserve constraint in equilibrium. However, in any small admissible neighborhood of (x_t, h_t) with $h > \eta(x - 1)$, the reserve constraint is slack and the policy does not take off-constraint points back to the constraint. Hence, for admissible off-constraint perturbations, the local instability argument follows from the one in Section 6.1 in the main text.

As an illustration, let us focus on the case when the reserve constraint is binding at the optimal choice of banks. Then, the dynamics of the system is given by (65)-(66) and the reserve constraint $h_t = \eta(x_t - 1)$ which pins down the equilibrium value of the deposit interest rate.

Let $(\overline{x}, \overline{h}) \in (0, \infty)^2$ be a stationary point of the dynamical system. We obtain:

[33] There are many other constellations that can be of interest and are left for future research.

Proposition 10. *Any stationary point of the above dynamical system (65)-(66) at which the reserve requirement is binding is locally unstable relative to admissible off-constraint perturbations.*

The reason for the instability is that, in the neighborhood of a stationary point $(\overline{x}, \overline{h})$, the reserve constraint is not binding for nearby values satisfying $h > \eta(x - 1)$. On this admissible side of the boundary, the dynamics are governed by the unconstrained ones in Section 6.1 in the main text, so the binding path cannot be locally stable unless the off-constraint dynamics also return perturbations to the boundary. The argument is therefore a slack-side instability argument, applied to admissible off-constraint perturbations rather than to perturbations restricted to the binding boundary.

Appendix B: The Welfare Optimum

To perform a welfare analysis based on the trajectories (x_t, h_t), we use Gersbach et al. (2024) to express the intertemporal utility of the representative household and the individual expected utilities of bankers.[34]

As the representative household always consumes a fraction ρ of its wealth, its intertemporal utility (from date 0) is given by:

$$V^H = \int_0^\infty e^{-\rho t} \log \left[\rho \frac{H_t}{p_t}\right] dt = \int_0^\infty e^{-\rho t} \log \frac{\rho h_t L_t}{p_t x_t} dt.$$

Integrating partially and using (47), we obtain an expression that solely depends on the state variables (x_t, h_t):

$$\int_0^\infty \rho e^{-\rho t} \log \frac{L_t}{p_t} dt = \log \frac{L_0}{p_0} + \int_0^\infty e^{-\rho t} \left[\mu - \nu\mu - \rho \frac{1+h_t}{x_t}\right] dt.$$

By rearranging terms, we have:

$$V^H = U + \int_0^\infty e^{-\rho t} \left[\log \frac{h_t}{x_t} - \frac{1+h_t}{x_t}\right] dt$$

with U being a constant, depending on initial endowments and price level. In a similar way, the aggregate expected utility of bankers is[35]

$$V^B = U - \int_0^\infty e^{-\rho t} \left[\log x_t + \frac{1+h_t}{x_t} + \frac{\sigma^2 x_t^2}{2\rho}\right] dt. \quad (69)$$

As assumed in (49), the social planner maximizes the following objective:

$$W = \alpha V^B + (1-\alpha) V^H,$$

where $0 < \alpha < 1$ is the weight put by the government on bankers' utilities. Hence,

$$W = U + \int_0^\infty e^{-\rho t} \left[(1-\alpha)\log h_t - \frac{h_t+1}{x_t} - \log x_t - \alpha \frac{\sigma^2 x_t^2}{2\rho}\right] dt. \quad (70)$$

The maximum of W is obtained by the pointwise maximization of the integrand in (70). For each t, this integrand attains its maximum for $(x_t, h_t) = (x^*, h^*)$ independently of t, where x^* and h^* are uniquely given by the first-order conditions (50) and (51) in Proposition 3, (i), and can easily be seen to be sufficient. For any $0 < \alpha < 1$, (51) has a unique positive

[34] Remember that household wealth and consumption is risk-free, while that of individual equity holders of banks is not.

[35] This involves Itô's Lemma for the individual banker. The σ^2 term is the Itô term, which then appears in the expectation in V^B.

solution x^*. Equation (50) then determines h^*. In other words, there is a unique welfare maximum corresponding to a stationary point of the (x_t, h_t)-dynamics.

Moreover, it can easily be shown that $1 + h^* - x^* > 0$, which implies that Central Bank reserves are strictly positive. The final statement of Proposition 3 has been proved for general stationary allocations in 4.[36]

[36] If the unrestricted optimum in (50)-(51) violates the liquidity requirement, it is not difficult to show that the Pareto optimum is again stationary and lies on the line $h = \eta(x - 1)$.