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Recommendation Power and Competition

Martin Peitz[1]
Anton Sobolev[2]

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[1] Corresponding author. Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany, E-Mail: Martin.Peitz@gmail.com
[2] Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany, E-Mail: anton.sobolev@uni-mannheim.de.

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Abstract

A firm may decide to make total-surplus-reducing purchase recommendations in response to consumer heterogeneity in an experience good setting. First, we show under which conditions the firm chooses to make such biased recommendations in a monopoly setting. Second, we propose a duopoly model with differentiated products in which single-product firms compete in uniform prices and recommendation policies. We provide conditions under which both firms choose to bias their recommendations, whereas the bias would be absent if products were more differentiated or one of the two products were withdrawn from the market.

Keywords: recommendation bias, recommender system, competition

JEL-classification: L12, L13, L15, D21, D42, D43, M37

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†Corresponding author. Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; E-Mail: Martin.Peitz@gmail.com; also affiliated with CEPR and CESifo.

‡Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; E-Mail: anton.sobolev@uni-mannheim.de.

1. Introduction

Recommender systems are an integral part of digital platforms and online retailers. By guiding users' choices, such systems can stimulate consumption (Aridor et al., 2022; Donnelly et al., 2024). While such systems ostensibly help consumers make more informed purchasing decisions, firms can design them in ways that limit informativeness. When recommendations are shaped by commercial interests, they may reduce consumer surplus (*CS bias*) and/or total surplus (*TS bias*). Understanding how information-design incentives interact with pricing is central to assessing the competitive and welfare effects of recommender systems.

This paper presents a simple framework with ex ante heterogeneous consumers, where a monopolist or two competing firms simultaneously choose a uniform price and the informativeness of their recommendation policies. Each firm controls the informativeness of its recommendations – captured by a bias parameter β – which determines how frequently consumers with valuations below marginal cost receive a purchase recommendation. A more informative system (lower β) better matches products to consumers but reduces the purchasing frequency, while a more biased one (higher β) increases the frequency at the cost of reduced targeting precision. Firms thus face a fundamental trade-off between *recommendation frequency* and *informativeness*.

We first analyze a monopoly benchmark with two consumer groups to isolate this trade-off. This setting is closely related to but different from Peitz and Sobolev (2025, forthcoming). When prices are flexible, the monopolist may deliberately degrade the informativeness of its recommender system to expand its sales volume, even though doing so lowers allocative efficiency. The monopolist chooses fully informative, partially informative, or fully uninformative recommendations. A recommendation bias may serve as a profit-maximizing substitute for price discrimination, which we assume firms cannot implement directly.

We then extend the analysis to a duopoly with horizontally differentiated products, where recommendation bias and price are strategic variables. In equilibrium, duopolists may either set low prices and sell to both user groups or adopt a limited-sales strategy focusing on one consumer group. In the former case, the equilibrium may feature biased recommendations, showing that such biases can also arise under competition.

As competition intensifies, the equilibrium may shift from a configuration in which both firms serve only one consumer group with fully informative recommendations to one in which they serve both groups and make biased recommendations. Moreover, the market environment may be such that a monopolist would choose not to bias recommendations, whereas after the entry of a rival, the duopoly equilibrium entails biased recommendations. Both comparative-statics results demonstrate that the recommendation bias may become stronger as competition becomes more intense.

European policymakers have begun to address recommendation bias explicitly, combining transparency, user-empowerment, and non-discrimination rules; see Peitz (2025) for a broader overview of governance choices and regulatory tools. Two EU regulations are of particular relevance in our context.

The *Platform-to-Business Regulation* (P2B) (EU 2019/1150) targets potential bias in platform intermediation. Article 5(1) requires that *"providers of online intermediation services shall set out in their terms and conditions the main parameters determining ranking and the reasons for the relative importance of those main parameters as opposed to other parameters."* Article 7(1) further obliges providers to disclose *"any differentiated treatment which they give, or might give, in relation to goods or services offered ... by, on the one hand, either that provider itself or any business users which that provider controls and, on the other hand, other business users."* These transparency obligations do not prohibit CS- or TS-bias but make differential treatment more observable to users.

The *Digital Services Act* extends transparency requirements to recommender systems targeting consumers directly. According to Article 27(1), *"providers of online platforms that use recommender systems shall set out in their terms and conditions, in plain and intelligible language, the main parameters used in their recommender systems, as well as any options for the recipients of the service to modify or influence those main parameters."* Very Large Online Platforms must also offer users a non-profiling option (Article 38). These provisions aim to mitigate hidden bias by enabling user choice and oversight rather than by constraining the recommendation policy.[1]

[1] The Digital Markets Act addresses a more structural form of bias—self-preferencing by large "gatekeeper" platforms. Article 6(5) stipulates that *"the gatekeeper shall not treat more favourably, in ranking and related indexing and crawling, services and products offered by the gatekeeper itself than similar services or products*

These regulations aim to enhance transparency and accountability and set the starting point for our analysis, namely that consumers understand firms' recommendation policies. An economic explanation is therefore required to identify the forces that generate biased recommendations even in such information environments.

The remainder of this article examines the market forces that generate recommendation bias in monopoly and competitive environments. Section 2 introduces a basic mechanism that can give rise to biased recommendations in a monopoly setting when prices are exogenous. Section 3 presents the formal model with endogenous prices, where recommendation bias can emerge from consumer heterogeneity. Section 4 analyzes a monopoly environment, and Section 5 extends the analysis to competition between firms offering differentiated products, highlighting the interaction between recommendation bias and pricing. Section 6 concludes and discusses two extensions. All proofs are relegated to Appendix A; an additional appendix complements the exposition in Section 6.

2. Biased recommendations: Definitions, first observations, and related literature

Before turning to the formal model, we clarify the notion of recommendation bias and illustrate its economic logic in a simple monopoly setting. This helps build intuition for the model explored in later sections. We also explain how our approach connects to the literature.

Definitions We use two complementary definitions of *bias in recommendations*:

1. **Consumer-surplus bias** (CS-bias): a recommendation policy is biased if it *reduces expected consumer surplus* relative to a feasible alternative.

2. **Total-surplus bias** (TS-bias): a policy is biased if it *reduces total surplus* (consumer

of a third party." This rule directly targets a prominent mechanism for bias in digital markets. Together with the anti-circumvention clause (Article 13), the DMA's prohibition of price-parity clauses (Article 5(3)) prevents gatekeepers from using algorithmic ranking to penalize sellers that set lower off-platform prices after parity clauses are banned; see Franck and Peitz (2024). Biased recommendations have also featured in several abuse-of-dominance investigations both within and outside the EU.

plus seller and recommender surplus) relative to a feasible alternative.[2]

Both CS-bias and TS-bias can be *short-term* – capturing short-term effects due to current price and purchase decisions – or *long-term* – reflecting future data use, market structure changes, or investment incentives. In this paper, we take a short-term perspective with endogenous prices.

First observations Established online retailers also act as recommenders, and we focus throughout on such integrated firms. The arguably simplest way to formalize a CS- and TS-biased recommendation policy is through the following monopoly example.

Suppose that consumers are risk-neutral and have valuations v_h or v_l, with respective probabilities λ and $1-\lambda$. Hence, the unconditional expected valuation is $E[v] = \lambda v_h + (1-\lambda)v_l$. The firm observes a consumer's valuation but the consumer does not. The firm can commit to a recommendation policy and may recommend the product even when this yields a negative surplus. If the price p exceeds v_l, such a recommendation is CS-biased. If, in addition, the seller faces constant marginal cost $c \in (v_l, p)$, the same recommendation is also TS-decreasing. When the price is exogenous, a monopoly seller maximizes profits by deciding on its recommendation frequency.

To have any trade at all, the price must satisfy $p \in [0, v_h]$. The seller recommends the product with probability $\beta \in [0, 1]$ to a consumer with valuation v_l, and with probability 1 when the valuation is v_h. If $p \leq E[v]$, the seller always recommends ($\beta = 1$). The more interesting case arises when $p \in (E[v], v_h)$, because informative recommendations are then essential for trade to occur. The seller's problem is

$$\max_{\beta \in [0,1]} (p - c)\big(\lambda + (1-\lambda)\beta\big)$$

subject to the participation constraint of the consumer who receives a recommendation:

$$\frac{\lambda v_h + (1-\lambda)\beta v_l}{\lambda + (1-\lambda)\beta} \geq p.$$

[2]These benchmarks can diverge: a policy may increase total surplus (e.g., via better matching) yet reduce consumer surplus because of higher prices.

In the profit maximum, this constraint is binding and determines the optimal β. For expositional clarity, set $\lambda = 1/2$ in the remainder of this article. Then the binding constraint reduces to

$$\frac{v_h + \beta v_l}{1+\beta} = p,$$

which implies the profit-maximizing recommendation probability

$$\beta = \frac{v_h - p}{p - v_l}.$$

Hence, the seller introduces a bias whenever $p \in (E[v], v_h)$: consumers with low valuations are occasionally recommended to buy. This simple structure captures the essence of CS- and TS-biased recommendations under exogenous pricing.

When the price is instead endogenous, the mechanism behind biased recommendations is less obvious. A monopolist facing homogeneous consumers has no incentive to bias recommendations, as it can optimally adjust the price. When consumers are heterogeneous in tastes or information, however, the monopolist may have an incentive to distort recommendations. Peitz and Sobolev (2025) study a monopolist facing two consumer segments differing by the information available to consumers: one perfectly informed about its valuation and another relying on the intermediary's recommendation. The recommendation bias then reflects the difference between monopoly prices under informed and uninformed demand.

As shown by Peitz and Sobolev (forthcoming), when recommendations are directed at audiences with different tastes, sellers may inflate recommendations for some consumers with low valuations in order to compress dispersion in expected valuations among consumers who decide to purchase. This mechanism generates CS-bias and, whenever inefficient trades are induced, TS-bias as well. An implication of their analysis is that mandates for fully informative recommendations can backfire.

Our analysis below closely connects to the analysis in Peitz and Sobolev (forthcoming). We look at a different model with heterogeneous consumers that lends itself to study competition in a meaningful and tractable way.

Related literature Our paper contributes to the literature on information design and biased intermediation. In our model, firms strategically manage consumer expectations by controlling the informativeness of their recommendations. In Lewis and Sappinton (1994), a profit-maximizing monopolist either perfectly reveals or completely conceals information. Rayo and Segal (2010, Section VIII.C) show that when a monopolist chooses both price and how much information to disclose about product value, the profit-maximizing disclosure policy is fully revealing. By contrast, in our framework firms may optimally choose an intermediate level of informativeness, inducing some consumers to buy even when expected gains from trade are negative. Formally, our setting is an instance of Bayesian persuasion (Kamenica and Gentzkow, 2011), in which the firm commits to a recommendation policy that shapes consumers' posterior beliefs. In our model, biased recommendations are a substitute for price discrimination.[3] In a single-consumer–single-firm framework, Roesler and Szentes (2017) and Kartik and Zhong (2024) characterize the set of allocations attainable under monopoly pricing for arbitrary information structures. When production costs may exceed consumer valuations, Roesler and Szentes (2017, online appendix) show that inefficient trade can occur with positive probability in the consumer-optimal, but not in the firm-optimal mechanism – the latter finding is in contrast to the one in our stream of articles.

When the recommender and the seller are distinct, additional sources of bias may arise. One natural mechanism is differential remuneration: the recommender may earn product-specific royalties or margins, inducing a bias toward the more profitable product. Bourreau and Gaudin (2022) analyze a streaming platform that intermediates between content providers and consumers, charging a subscription fee and paying per-stream royalties. When royalties differ, the platform maximizes profit by biasing recommendations toward lower-royalty content. Lee (2021) analyzes a mechanism design problem of a platform that monetizes only on the seller side and shows that the platform may bias its recommendations. Relatedly, biased recommendations can arise when intermediaries internalize price effects (e.g., Armstrong and Zhou, 2011; Hagiu and Jullien, 2011; de Cornière and Taylor, 2019).

A particular instance of biased intermediation is *self-preferencing*, which can occur when

[3]Bergemann, Brooks, and Morris (2015)) study a vertical chain in which an intermediary reveals information about consumer characteristics to sellers, enabling personalized pricing.

a firm both operates as a recommender and sells its own products in addition to third-party sellers and describes a situation in which consumers are steered to own products (de Cornière and Taylor, 2019). Recent work includes Zennyo (2022) and Aridor and Gonçalves (2022). Our mechanism can also be applied to such settings (see the discussion in Peitz and Sobolev, forthcoming).

Our work complements this literature by linking recommendation bias to price-discrimination motives. Extending our earlier results, we show that this mechanism persists beyond monopoly and operates also in competitive environments.

3. The model

Consider a differentiated-product market with two single-product firms A and B.

Consumers There is a unit mass of risk-neutral consumers, each demanding at most one unit in the market. Products are experience goods: each product yields gross utility v_h or v_l, depending on the match between the product and the consumer. A consumer's gross utility v_i is drawn from $\{v_l, v_h\}$ with equal probability 1/2. Thus, without further information, the ex-ante expected gross value of any product is $(v_h + v_l)/2$.

There are two groups of consumers. A fraction α are *contestable consumers*, who view the two products as substitutes and purchase the one yielding the higher expected net utility. Contestable consumers are uniformly distributed on $[0, 1]$. A consumer of type x incurs transport cost tx when buying product A and $t(1 - x)$ when buying product B, as in the standard Hotelling model of horizontal product differentiation. A contestable consumer is characterized by a triple (v_A, v_B, x). The joint distribution on (v_A, v_B) is assumed to be independent of x.

The remaining fraction $1 - \alpha$ are *captive consumers*, each considering only one of the two products. Among captives, half are captive to A and half to B. A captive consumer purchases if their expected net utility exceeds a reservation level $r_0 > 0$. Denote $v_m \equiv v_h - r_0$.

Consumers decide which, if any, of the two firms to visit. When visiting a firm, they receive a personalized recommendation according to the firm's recommendation policy. This policy allows consumers to update their beliefs about the valuation they will obtain if they purchase the product. If a consumer's valuation is v_h, they always receive a recommendation, whereas if their valuation is v_l they receive a recommendation with probability β (which may depend on whether they belong to the group of contestable or the group of captive consumers). Thus, if the valuation is v_l, they do not receive a purchase recommendation with probability $1 - \beta$ and, in this case, consumers learn that they have valuation v_l. When the product is recommended, the conditional expected gross value of a recommended product is

$$r(\beta) \equiv \frac{v_h + \beta v_l}{1 + \beta}.$$

Firms Each firm $i \in \{A, B\}$ sets a price p_i and a recommendation policy β_i^k, where $k \in \{\text{ca}, \text{co}\}$ distinguishes between captive and contestable consumers. This is the probability that a consumer with valuation v_l receives a recommendation.[4] Each unit sold yields margin $p_i - c$, where c denotes constant marginal cost, identical across firms.

Admissible parameters We make the following assumption on the set of admissible parameters: $v_h - t > c + 2t > v_m > c > v_l$. Since $v_h - t > v_m$ (or equivalently, $t < r_0$), then under uniform prices and type-independent recommendation policies, all contestable consumers enjoy a higher expected net surplus than captive consumers and thus can be considered to be of the ex-ante high type. The assumption $c + 2t > v_m$ implies that this ordering is preserved under competition.[5] In our analysis, we will clarify where these conditions play out.

Recommendations to captive consumers Since $v_h > c > v_l - r_0$, a captive consumer with low valuation v_l prefers not to buy even at marginal cost c. For $p_i \geq v_l$, consumers purchase only if they receive a recommendation. A consumer who is captive to firm i buys product i if they receive a purchase recommendation and $r(\beta_i^{\text{ca}}) - p_i \geq r_0$.

With fully informative recommendation for captives, $\beta_i^{ca} = 0$, this boils down to $v_m \geq p_i$. It will become clear in the analysis that under our parameter restriction (in particular,

[4] One can show that firms always recommend their product when the realized valuation is high (i.e. v_h).
[5] In Section 6 we analyze what happens if the reverse inequality $c + 2t < v_m$ holds.

$c + 2t > v_m$), firms never recommend to captive consumers with low realized valuation v_l; that is, $\beta_i^{\text{ca}} = 0$. In other words, firms do not bias recommendations directed at their own captive base.

Information structure and timing At the beginning of the game, consumers know whether they are captive or contestable. In addition, captives know which firm they are captive of, while contestable consumers know their location x.[6] Consumers do not know whether a product delivers v_h or v_l. Firms know whether a consumer is contestable or captive and the recommendation policy is such that recommendations can be conditioned on the this and the consumer's ex-post valuation v_i; however, recommendations must be independent of consumer location x if the consumer is of the contestable type. Because firms always set $\beta_i^{\text{ca}} = 0$, we remove the choice of β_i^{ca} from the game and simplify notation by using β_i instead of β_i^{co}.

The game proceeds as follows:

1. Firms simultaneously choose uniform prices p_i and recommendation policies for contestable consumers, β_i, $i \in \{A, B\}$.

2. Consumers observe all prices and recommendation policies (but do not yet receive recommendations) and decide which firm to visit. Visiting is costless, but each consumer can visit only one firm.

3. Consumers visiting firm i receive personalized purchase recommendations and then decide whether to buy.

We solve for *Perfect Bayesian Nash equilibrium (PBNE)*, in which each firm's pricing and recommendation strategy maximizes expected profit given beliefs consistent with observed recommendation policies and rational consumer behavior. We restrict our attention to symmetric equilibria.

The timing assumption that firms can commit to their information design captures the idea that recommender systems form part of a firm's observable market conduct. Such com-

[6] A richer but computationally more demanding variant would feature two groups of consumers, each distributed on a Hotelling line. If one group faced an outside option that is sufficiently attractive, only part of that group would be served in equilibrium, effectively making them captive.

mitment can be justified by transparency and disclosure requirements that render steering strategies observable, as reflected in EU regulation discussed in the introduction. Put differently, a firm's recommendation policy can be interpreted as a *contractible information policy* that consumers and regulators can monitor.

The parameter β_i captures the degree of *recommendation bias* or, equivalently, the lack of informativeness of firm i's recommender system. When $\beta_i = 0$, recommendations are fully informative: consumers receive a recommendation only when the realized valuation is high (v_h). When $\beta_i = 1$, recommendations are entirely uninformative, as both high- and low-valuation products are recommended with equal probability. Intermediate values $\beta_i \in (0, 1)$ represent partially informative recommendation policies that lie between these two extremes. A higher β_i thus corresponds to a stronger recommendation bias – potentially expanding sales but reducing informativeness and thereby allocative efficiency.

4. Recommendation bias in monopoly

We begin with a monopoly setting to isolate the fundamental tradeoff between the informativeness and reach of recommendations before introducing strategic interaction. Only firm A is active, and we omit firm subscripts throughout. The monopolist faces the same consumer environment as in the duopoly model – with ex-ante high and low types corresponding, respectively, to the contestable and captive types in the competitive setting – and operates under the same information structure as described above.

If $\alpha = 1$, the monopolist sets its price equal to $v_h - t$ and its recommendation policy $\beta = 0$; all consumers buy the good. The monopoly profit is $v_h - t - c$. For $\alpha \in (0, 1)$, the monopolist may still decide to serve only the ex-ante high-type consumers. Conditional on doing so, as in the case $\alpha = 1$, the monopolist sets $p^M = v_h - t$ and $\beta = 0$. The monopoly profit with this limited-sales strategy is $\frac{\alpha}{2}(v_h - t - c)$.

Alternatively, the monopolist sets a lower price to also cater to the ex-ante low-type consumers. To do so, it has to set its price $p \leq v_m$. Ex-ante low-type consumers always receive fully informative recommendations and, at $p = v_m$, buy with probability 1/2. Thus, demand of ex-ante low-type consumers is $\frac{1-\alpha}{2} \cdot \frac{1}{2} = \frac{1-\alpha}{4}$. An ex-ante high-type consumer located at x buys the good if and only if they receive a purchase recommendation and

$$\frac{v_h + \beta v_l}{1+\beta} - p - tx \geq 0.$$

A contestable consumer receives a purchasing recommendation with probability $\frac{1+\beta}{2}$. Hence, for $p \leq v_m$, the profit of the monopolist is

$$\begin{aligned}\pi(p,\beta) &= \left(\alpha \frac{1+\beta}{2t} \min\left\{\frac{v_h+\beta v_l}{1+\beta} - p, t\right\} + \frac{1-\alpha}{4}\right)(p-c) \\ &= \left(\frac{\alpha}{2t} \min\left\{v_h - p - \beta(p - v_l), (1+\beta)t\right\} + \frac{1-\alpha}{4}\right)(p-c).\end{aligned}$$

Lemma 1. *The monopolist's profit-maximizing price is either $p^M = v_m$ or $p^M = v_h - t$.*

Suppose that the profit-maximizing price is $p^M = v_m$. We call this the inclusive-pricing strategy because the price is set sufficiently low to sell also to ex-ante low-type consumers. In this case, the profit is given by

$$\pi(v_m,\beta) = \left(\frac{\alpha}{2t} \min\left\{v_h - v_m - \beta(v_m - v_l), (1+\beta)t\right\} + \frac{1-\alpha}{4}\right)(v_m - c).$$

If $(v_h - v_m) - (v_m - v_l) \geq 2t$, then the profit-maximizing recommendation policy is $\beta = 1$. If instead $(v_h - v_m) - (v_m - v_l) < 2t$, it is $\beta = (v_h - v_m - t)/(v_m - v_l + t)$. A higher recommendation bias β increases the share of ex-ante high-type consumers who receive a recommendation and buy but reduces the informativeness of the signal they receive. The monopolist sets its bias to satisfy the participation constraint of the ex-ante high-type consumer with $t = 1$. For sufficiently large v_h this constraint becomes slack at $\beta = 1$.

When competition from the outside option is strong for ex-ante high-type consumers (small t) or ex-ante high-type consumers are numerous (large α), there is an interior optimum $\beta \in (0,1)$. Conversely, when competition from the outside option is weak on average (large t) or there are few ex-ante high-type consumers (small α), the firm finds it profitable to blur information completely ($\beta^* = 1$).

The resulting profit is

$$\pi^M = \begin{cases} \frac{1+3\alpha}{4}(v_m - c), & \text{for} \quad v_h - v_m - t \geq v_m - v_l + t, \\ \left(\frac{\alpha}{2}\frac{v_h - v_l}{v_m - v_l + t} + \frac{1-\alpha}{4}\right)(v_m - c), & \text{for} \quad 0 < v_h - v_m - t < v_m - v_l + t. \end{cases}$$

The monopolist implements an inclusive-pricing strategy if this profit exceeds the profit under the limited-sales strategy, $\frac{\alpha}{2}(v_h - t - c)$. Settings the two equations equal, defines the critical value of α:

$$\alpha_M \equiv \begin{cases} \frac{v_m - c}{2(v_h - t - v_m) - (v_m - c)}, & \text{for} \quad v_h - t - v_m \geq v_m - v_l + t, \\ \frac{v_m - c}{2(v_h - t - c) - 2\frac{v_h - v_l}{v_m - v_l + t}(v_m - c) + (v_m - c)}, & \text{for} \quad 0 < v_h - t - v_m < v_m - v_l + t. \end{cases}$$

We have thus shown the following result:

Proposition 1. *If $\alpha > \alpha^M$, then the monopolist induces the outcome with limited sales: $p^M = v_h - t$ and $\beta = 0$. If $\alpha < \alpha^M$, then the monopolist induces the outcome with inclusive pricing: $p^M = v_m$ and the recommendation policy $\beta = 1$ in case $(v_h - v_m) - (v_m - v_l) \geq 2t$ and $\beta = (v_h - v_m - t)/(v_m - v_l + t)$ in the opposite case $(v_h - v_m) - (v_m - v_l) < 2t$.*

The monopoly benchmark illustrates how biased recommendations emerge from a tradeoff between recommendation frequency and informativeness. At the inclusive price $p = v_m$, a monopolist facing heterogeneous consumers may intentionally reduce the precision of its recommender system to broaden demand among ex-ante high-type consumers, even at the cost of allocative efficiency.

5. Recommendation bias under competition

We now turn to the case of two competing firms to examine how strategic interaction shapes the incentives to bias recommendations. Competition introduces an additional layer of complexity: each firm's choice of recommendation policy affects not only its own demand but also the rival's, as prices and perceived product valuations jointly determine market shares. As preliminaries, we consider the case in which all consumers are contestable ($\alpha = 1$). We then turn to the analysis of a mix of contestable and captive consumers.

Only contested consumers ($\alpha = 1$) We assume that v_h is high enough so that the market is fully covered. A contestable consumer of type x purchases product A if $r(\beta_A) - p_A - tx \geq r(\beta_B) - p_B - t(1-x)$.

The profit of firm A setting β_A and p_A when the rival's strategy is (β_B, p_B) is given by

$$\pi_A(p_A, \beta_A; p_B, \beta_B) = \frac{1+\beta_A}{2}(p_A - c)\hat{x}(p_A, \beta_A, p_B, \beta_B),$$

where

$$\hat{x}(p_A, \beta_A; p_B, \beta_B) \equiv \frac{1}{2} + \frac{\beta_A - \beta_B}{4t}v_l + \frac{1+\beta_B}{4t}p_B - \frac{1+\beta_A}{4t}p_A$$

represents the location of the indifferent consumer. Because the market is fully covered in equilibrium, there exists such an interior indifferent consumer in equilibrium.

The first-order condition of the firm A's profit maximization problem with respect to p_A is given by

$$-\frac{1+\beta_A}{4t}(p_A - c) + \hat{x}(p_A, \beta_A, p_B, \beta_B) = 0,$$

implying that

$$\begin{aligned} p_A &= \frac{c}{2} + \frac{t}{1+\beta_A} + \frac{\beta_A - \beta_B}{2(1+\beta_A)}v_l + \frac{1+\beta_B}{2(1+\beta_A)}p_B \\ &= \frac{c}{2} + \frac{t}{1+\beta_A} + \frac{v_l}{2} + \frac{1+\beta_B}{2(1+\beta_A)}(p_B - v_l) \end{aligned}$$

is the profit-maxizing price when firm A sets β_A given that the rival has chosen (p_B, β_B). The profit function of firm A becomes

$$\begin{aligned} \frac{(1+\beta_A)^2}{8t}(p_A - c)^2 &= \frac{(1+\beta_A)^2}{8t}\left(-\frac{1}{2}(c - v_l) + \frac{t}{1+\beta_A} + \frac{1+\beta_B}{2(1+\beta_A)}(p_B - v_l)\right)^2 \\ &= \frac{1}{8t}\left(-\frac{1}{2}(c - v_l)(1+\beta_A) + t + \frac{1+\beta_B}{2}(p_B - v_l)\right)^2, \end{aligned}$$

implying that for any $\beta_B \in [0,1]$, $p_B \geq c$, firm A's profit is decreasing in β_A. It follows that for any $\beta_A > 0$, firm A finds it profitable to slightly decrease β_A and increase p_A to keep $\hat{x}$

unchanged, given any strategy (p_B, β_B) of the rival. Therefore, there is a unique equilibrium, in which both firms set fully informative recommendations policies; that is, $\beta_A^* = \beta_B^* = 0$. The equilibrium prices solve the following system of equations

$$p_i^* = \frac{c}{2} + t + \frac{p_j^*}{2},$$

for $i, j \in \{A, B\}$, $j \neq i$. This implies that $p_A^* = p_B^* = c + 2t$ in equilibrium. The equilibrium profit of each firm is $\frac{1}{2} \times 2t \times \frac{1}{2} = \frac{t}{2}$. The parameter restriction $c + 2t > v_m$ implies that the equilibrium price for $\alpha = 1$ is larger than the equilibrium price for $\alpha = 0$.

Adding captive consumers We now reintroduce captive consumers while preserving the Hotelling structure for the contestable segment. A fraction $1 - \alpha$ of consumers have an outside option $r_0 > 0$. Within this group, half are captive to firm A and half to firm B, so that each firm faces a captive mass of $(1-\alpha)/2$, with half of them drawing the high valuation v_h. Hence, the effective mass of captives receiving a purchase recommendation is $(1 - \alpha)/4$ per firm.

As stated in the following lemma, there does not exist an equilibrium in which one or both firms set price strictly below v_m.

Lemma 2. *In equilibrium, both firms set prices* $p_A^*, p_B^* \geq v_m$.

The presence of captive customers therefore places a lower bound on potential equilibrium prices, which parallels the monopoly setting, in which v_m marks the threshold below which further price cuts cease to be profitable.

Limited-sales equilibrium Suppose that in equilibrium both firms set prices strictly higher than v_m and thus captive consumers are not served. Then, both firms set $\beta_A^* = \beta_B^* = 0$ and $p_A^* = p_B^* = c + 2t$. The equilibrium profits of each firm is given by $\frac{\alpha}{2} \times 2t \times \frac{1}{2} = \frac{\alpha t}{2}$.

In Appendix A, we establish that the most profitable deviation from this equilibrium candidate is to set the price $p_A' = v_m$ and optimally adjust the recommendation bias β_A. Therefore, to show that this configuration constitutes an equilibrium, it is sufficient to verify that deviations to any pair $(p_A', \beta_A) = (v_m, \beta_A)$ are not profitable.

We characterize the corresponding β_A that maximizes the deviation profits when $p'_A = v_m$. Note that the consumer located at $x = 1$ does not purchase from firm A for any $\beta_A \in [0, 1]$, since $\hat{x}(v_m, \beta_A; c + 2t, 0) \leq \hat{x}(v_m, 0; c + 2t, 0) = 1 - \frac{v_m - c}{4t} < 1 - \frac{2t}{4t} < 1$. This implies that if there is an indifferent consumer at some β_A, then their location is strictly below $x = 1$. Thus, firm A chooses $\beta_A \in [0, 1]$ to maximize the expected demand from the contested consumers, which is given by

$$\begin{aligned}
\alpha\frac{1+\beta_A}{2}\max\{\hat{x}(v_m, \beta_A; c+2t, 0), 0\} &= \alpha\frac{1+\beta_A}{2}\max\left\{\left(1 - \frac{\beta_A}{4t}(v_m - v_l) - \frac{v_m - c}{4t}\right), 0\right\} \\
&= \alpha\frac{1+\beta_A}{2}\frac{v_m - v_l}{4t}\max\left\{\left(\frac{4t}{v_m - v_l} - \frac{v_m - c}{v_m - v_l} - \beta_A\right), 0\right\}.
\end{aligned}$$

This function is strictly concave in β_A on the interval where it is strictly positive, and it attains its maximum at

$$\beta'_A = \begin{cases} 0, & \text{if } \ \frac{1}{2}\left(v_m - \frac{c+v_l}{2}\right) \geq t > \frac{1}{2}(v_m - c), \\ \frac{4t-(v_m-c)}{2(v_m-v_l)} - \frac{1}{2}, & \text{if } \ v_m - \frac{c+3v_l}{4} \geq t > \frac{1}{2}\left(v_m - \frac{c+v_l}{2}\right), \\ 1, & \text{if } \ t > v_m - \frac{c+3v_l}{4}. \end{cases} \tag{1}$$

There are three possible *bias regimes for the deviating firm*, corresponding to the optimal choice of β'_A in the deviation to $p'_A = v_m$. If t is high and satisfies $t > v_m - (c + 3v_l)/4$, then the deviating profit is maximized with maximal bias, $\beta'_A = 1$. If instead t is low and satisfies $t < (v_m - (c - v_l)/2)/2$, then the optimal deviation involves no bias, that is, $\beta'_A = 0$. For intermediate values of t, the deviation to price v_m is optimally combined with an interior bias.

The respective profits of firm A from the deviation (v_m, β'_A), is given by

$$\pi'_A \equiv \pi_A(v_m, \beta'_A; c + 2t, 0) = \left(\alpha\frac{1+\beta'_A}{2}\hat{x}(v_m, \beta'_A; c+2t, 0) + \frac{1-\alpha}{4}\right)(v_m - c),$$

which can be written as

$$\pi'_A = \begin{cases} \left(\frac{\alpha}{2}\left(1 - \frac{v_m - c}{4t}\right) + \frac{1-\alpha}{4}\right)(v_m - c), & \text{if} \quad \frac{1}{2}\left(v_m - \frac{c+v_l}{2}\right) \geq t > \frac{1}{2}(v_m - c), \\ \frac{1}{2}\left(\alpha \frac{(t+(c-v_l)/4))^2}{t(v_m - v_l)} + \frac{1-\alpha}{2}\right)(v_m - c), & \text{if} \quad v_m - \frac{c+3v_l}{4} \geq t > \frac{1}{2}\left(v_m - \frac{c+v_l}{2}\right), \\ \left(\alpha \frac{4t-(v_m-c)-(v_m-v_l)}{4t} + \frac{1-\alpha}{4}\right)(v_m - c), & \text{if} \quad t > v_m - \frac{c+3v_l}{4}, \end{cases}$$

This deviation is profitable if and only if $\pi'_A > \alpha t/2$.

We now fully characterize the set of parameters under which the equilibrium with limited sales exists. Define $y_c \equiv (v_m - c)/t$, $y_l \equiv (v_m - v_l)/t$. Note that the assumption $c + 2t > v_m$, implies that $y_c \in (0, 2)$ and moreover, since $c > v_l$, we also have that $y_l > y_c$. For all $y_c \in (0, 2)$, we also define

$$\alpha_{LS} = \begin{cases} 1/\left(\frac{2}{y_c} + \frac{y_c}{2} - 1\right) & \text{if} \quad y_l > 4 - y_c, \\ 1/\left(1 + \frac{2}{y_c} - \frac{2\left(1 + \frac{y_l}{4} - \frac{y_c}{4}\right)^2}{y_l}\right), & \text{if} \quad 4 - y_c \geq y_l > \max\{(4 - y_c)/3, y_c\} \\ 1/\left(\frac{2}{y_c} + y_c + y_l - 3\right), & \text{if} \quad (4 - y_c)/3 \geq y_l > y_c. \end{cases}$$

The following proposition establishes the result.

Proposition 2. *There exists an equilibrium with limited sales if and only if* $\alpha \geq \alpha_{LS} \in (0, 1)$*. In this equilibrium, both firms set* $\beta^*_A = \beta^*_B = 0$ *and* $p^*_A = p^*_B = c + 2t$*, and each earns profits of* $\alpha t/2$*.*

The best deviation to also serve captive consumers varies monotonically with the transport-cost parameter t. For fixed (v_m, c, v_l), the optimal deviation bias $\beta'_A = [\,4t - (v_m - v_l) - (v_m - c)\,]/[\,2(v_m - v_l)\,]$ is increasing in t. As t rises – corresponding to less intense competition – the deviating firm traverses the three bias regimes: for $t \leq (2v_m - c - v_l)/4$, the optimal deviation is unbiased ($\beta'_A = 0$); for $(2v_m - c - v_l)/4 < t < v_m - (c + 3v_l)/4$, the optimal bias is interior ($\beta'_A \in (0, 1)$); and for $t \geq v_m - (c + 3v_l)/4$, the deviating firm adopts full bias ($\beta'_A = 1$).

The parameter α does not affect the location of these regime boundaries but determines whether the limited-sales outcome can be sustained as an equilibrium. A higher share of contestable consumers (larger α) raises the candidate equilibrium profit $\alpha t/2$ and makes the deviation relatively less attractive, expanding the parameter region in which the limited-sales configuration constitutes an equilibrium outcome.

Inclusive-pricing equilibrium We now consider whether both firms can sustain inclusive pricing in equilibrium. Then, firms set the same price $p_A = p_B = v_m$ and compete only in the informativeness of their recommendation policies β_i directed at contestable consumers.

Firm A's profit is

$$\pi_A(v_m, \beta_A; v_m, \beta_B) = (v_m - c)\left[\alpha\,\frac{1+\beta_A}{2}\,\hat{x}(v_m, \beta_A; v_m, \beta_B) + \frac{1-\alpha}{4}\right],$$

where the indifferent contestable consumer satisfies

$$\hat{x}(v_m, \beta_A, v_m, \beta_B) = \frac{1}{2} + \frac{\beta_B - \beta_A}{4t}\,(v_m - v_l).$$

A reduction in β_A makes recommendations more informative and shifts the marginal consumer toward firm B and therefore increases firm A's market share.

Given firm B's equilibrium strategy, the profit function of firm A setting β_A is given by

$$\pi_A(v_m, \beta_A; v_m, \beta_B^*) = \left[\alpha\frac{1+\beta_A}{2}\left(\frac{1}{2} + (\beta_B^* - \beta_A)\frac{v_m - v_l}{4t}\right) + \frac{1-\alpha}{4}\right](v_m - c).$$

Firm A's best response is to set $\beta_A^{BR} = 0$ if $2 < (1-\beta_B^*)\frac{v_m - v_l}{t}$, $\beta_A^{BR} = 1$ if $2 > (3-\beta_B^*)\frac{v_m - v_l}{t}$, and

$$\beta_A^{BR} = \frac{t}{v_m - v_l} - \frac{1}{2}(1 - \beta_B^*)$$

otherwise.

Lemma 3. *In the candidate inclusive-pricing equilibrium with $p_A = p_B = v_m$, both firms choose the recommendation bias*

$$\beta^* = \begin{cases} 0, & \text{if} \quad t \leq \frac{v_m - v_l}{2}, \\ \frac{2t}{v_m - v_l} - 1, & \text{if} \quad \frac{v_m - v_l}{2} < t < v_m - v_l, \\ 1, & \text{if} \quad t \geq v_m - v_l. \end{cases}$$

At the symmetric bias β^*, each firm's profit is

$$(v_m - c)\left[\alpha\,\frac{1+\beta^*}{4} + \frac{1-\alpha}{4}\right] = \frac{v_m - c}{4}\,(1+\alpha\beta^*).$$

We observe that when competition is intense (t small), firms maintain fully informative recommendations ($\beta^* = 0$); as differentiation increases, the bias rises continuously until recommendations become entirely uninformative ($\beta^* = 1$) for sufficiently large t. Put differently, as competition becomes more intense the bias decreases and eventually vanishes under inclusive pricing. We note that, in monopoly, the corresponding situation in which the firm serves both consumer groups and does not bias recommendations cannot be profit-maximizing.

Next, we establish the parameter set for which an inclusive-pricing equilibrium exists. Define $y_l \equiv (v_m - v_l)/t$ and $y_c \equiv (v_m - c)/t$. Our assumptions on the parameters imply that $y_l > y_c$ and $y_c \in (0, 2)$. With this change of variables, we have that $\beta^* = 0$ if $y_l > 2$; $\beta^* = \frac{2}{y_l} - 1$ if $2 \geq y_l \geq 1$; and $\beta^* = 1$ if $y_l < 1$. The resulting equilibrium profits are given by $\pi^* = \frac{1}{4} y_c t$ if $y_l > 2$; $\pi^* = \left(\frac{\alpha}{2y_l} + \frac{1-\alpha}{4}\right) y_c t$ if $2 \geq y_l \geq 1$; and $\frac{1+\alpha}{4} y_c t$ if $y_l < 1$. For all $y_c \in (0, 2)$ and $y_l > y_c$, we define

$$\alpha_{IP} = \begin{cases} 8y_c / \left(2 + y_c\right)^2, & \text{if} \quad y_l > 2, \\ 8y_c / \left(8y_c + (4 + y_c - y_l)^2 - \frac{16 y_c}{y_l}\right), & \text{if} \quad 2 \geq y_l \geq 1 \\ 8y_c / \left((2 + y_c + y_l)^2 - 8y_c\right), & \text{if} \quad y_l < 1. \end{cases}$$

In the following proposition, we fully characterize the set of parameters under which the equilibrium with inclusive pricing exists; we recall that we assume that $v_m > c + 2t$, which is equivalent to $y_c = (v_m - c)/t \in (0, 2)$.

Proposition 3. *There exists an equilibrium with inclusive pricing if and only if $\alpha \leq \alpha_{IP}$, where $\alpha_{IP} \in (0, 1)$. In this equilibrium, firms set $\beta_A^* = \beta_B^* = \beta^*$ according to Lemma 3 and $p_A^* = p_B^* = v_m$, and each earns profits of π^*.*

For this parameter set, no profitable deviation to $(p_A > v_m, \beta_A = 0)$ exists. The inclusive-pricing equilibrium represents the competitive analogue of the monopolist's low-price strategy. At prices $p_A = p_B = v_m$, firms choose how informative to make their recommendations.

Because $v_l < v_m$, a higher bias β reduces each firm's contested market share by lowering its relative appeal (market share effect). However, it raises the frequency of positive recommendations (recommendation–frequency effect) and thus increases sales for contestable consumers $x < \hat{x}$.

The equilibrium exists whenever the profits from charging a higher price with truthful recommendations are dominated by the profit in the equilibrium candidate. As the share of captive consumers decreases, this condition may fail, prompting firms to raise prices and focus exclusively on contestable consumers.

In the inclusive-pricing equilibrium, the recommendation bias increases with the degree of horizontal differentiation t: when competition is intense (small t), both firms maintain fully informative recommendations; as competition weakens, they gradually inflate recommendations; and for sufficiently weak competition ($t \geq v_m - v_l$), recommendations become fully uninformative.

The full picture We now provide the full characterization of symmetric equilibria. Denote the equilibrium profits of the limited-sales equilibrium by $\pi^*_{LS}(\alpha)$ and of the inclusive-pricing equilibrium by $\pi^*_{IP}(\alpha)$.

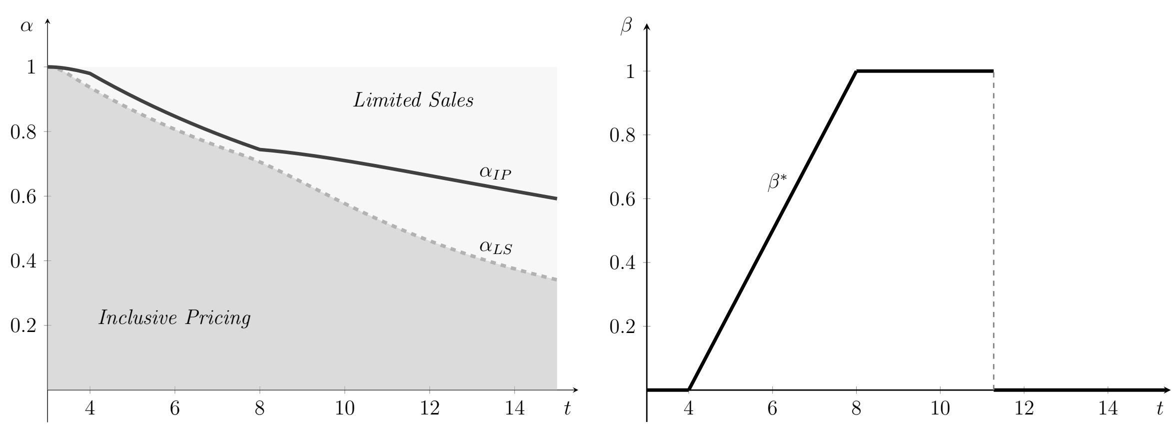
Proposition 4. *The inequality $\alpha_{LS} < \alpha_{IP}$ always holds. Hence, for $\alpha < \alpha_{LS}$, the unique symmetric equilibrium is the inclusive-pricing equilibrium; for $\alpha > \alpha_{IP}$, the unique symmetric equilibrium is the limited-sales equilibrium; for $\alpha \in [\alpha_{LS}, \alpha_{IP}]$, there are two symmetric equilibria, the inclusive-pricing and the limited-sales equilibrium, where the latter profit-dominates the former; that is, $\pi^*_{LS}(\alpha) > \pi^*_{IP}(\alpha)$ for all $\alpha \in [\alpha_{LS}, \alpha_{IP}]$.*

Thus, in the range of α with multiple equilibria, the limited-sales equilibrium is the equilibrium that is preferred by the firms.

The left-hand panel of Figure 1 illustrates our findings in terms of the degree of product differentiation t and the fraction of contestable consumers α. For all combinations (α, t) to the northeast of the dashed line α_{LS}, the limited-sales equilibrium exists, whereas for all combinations (α, t) to the southwest of the solid line α_{IP}, the inclusive-pricing equilibrium exists. In the region between the two lines, both equilibria coexist. The light-gray area marks the parameter space in which the limited-sales equilibrium is either the only or the profit-dominant equilibrium, while the dark-gray area marks the remaining region associated with the inclusive-pricing equilibrium.

A larger fraction of contestable consumers α favors the emergence of the limited-sales equilibrium. More interesting are the comparative statics with respect to the degree of product differentiation t. When product differentiation is low (t small), the equilibrium is the inclusive-pricing equilibrium without any recommendation bias. As t increases beyond a threshold, firms chooses a recommendation bias $\beta \in (0, 1)$ and once t exceeds a further threshold, recommendations become completely uninformative in this equilibrium. However, at a sufficiently large t the inclusive-pricing equilibrium may no longer exist or be profit-dominated by limited sales.

In the right-hand panel of Figure 1 we report the bias as a function of t for $\alpha = 0.5$. Given the other parameter values, the parameter t must be at least 3 to satisfy that $c+2t > v_m$. The figure shows that, using profit-dominance as the equilibrium selection criterion, $\beta^* = 0$ for small t, then β^* increases linearly with t until reaching 1. Beyond that point, over a range of values of t, recommendations remain completely uninformative until a threshold is reached at which β^* drops back to 0. Recommendations are thus fully informative for sufficiently small t in the inclusive-pricing equilibrium and again fully informative for sufficiently large t, when the limited-sales equilibrium prevails, whereas the bias is strictly positive for intermediate values of t. We therefore obtain the following result:



(a) α_{LS} and α_{IP} (b) β^* at $\alpha = 0.5$

Figure 1: The equilibrium outcomes for $v_m = 10$, $v_l = 2$, $c = 4$, and $t > 3$.

Remark 1. *Consider an increase in the degree of product differentiation from t_0 to t_1, holding all other parameters fixed – corresponding to a reduction in competition. For some parameter values, this raises the equilibrium recommendation bias, whereas for others, it induces firms to cease biasing recommendations altogether.*

A similar finding obtains when comparing monopoly and duopoly. For parameter values under which duopoly gives inclusive pricing with $\beta^* \in (0, 1]$, a monopolist facing sufficiently high v_h implements the limited-sales outcome with $\beta = 0$. Hence, the entry of a competitor may increase recommendation bias.

6. Discussion and conclusion

We have studied how firms strategically design the informativeness of their recommendation policy when consumers differ in ex ante valuation and product fit – consumers observe the latter only after purchase. A firm can inflate its recommendations by recommending the product with probability $\beta > 0$ even to consumers with a bad match, thereby trading off recommendation frequency against informativeness.

In this framework, the recommendation bias β and price p jointly determine consumers' purchase decisions and, through them, market outcomes. Our analysis isolates how these two instruments interact under monopoly and duopoly market structures.

In the monopoly benchmark, the firm implements one of two policies. In some environments, the monopolist sets a high price and targets only ex ante high-type consumers through fully informative recommendations – what we term a *limited-sales* policy. In others, the monopolist lowers its price to broaden its appeal and optimally introduces a bias toward consumers of the ex-ante high type – an *inclusive-pricing* policy.

Extending the model to a symmetric duopoly reveals that the same mechanism persists under competition: firms may jointly sustain limited sales or adopt an inclusive-pricing strategy accompanied by uninformative, partially informative, or fully informative recommendations. Unlike in the monopoly model, inclusive pricing under competition is compatible with unbiased recommendations. Nevertheless, recommendation bias is not an artifact of monopoly power but a robust feature that extends to competitive environments. More intense competition or the entry of a rival may even induce a recommendation bias that would otherwise be absent.

In two related settings, we further explore the emergence of recommendation bias.

Intermediation Our framework naturally applies to electronic retailers that both set retail prices and decide whether to recommend a product. It does not directly capture digital platforms, which instead determine which products to list and whether to recommend them to particular consumers, but typically do not set retail prices.

To extend the model to such settings with intermediation, consider a vertical chain consisting of a listed seller and a platform. Suppose that the platform extracts an exogenous share of the seller's profit, does not charge consumers directly, and that sellers can reach consumers only through the platform. In this case, the incentives of the platform and the seller are perfectly aligned. If the platform and the seller make their decisions simultaneously, the characterization of the inclusive-pricing equilibrium in duopoly remains identical to that in the baseline model but can be sustained for a larger set of parameters. The reason is that, with intermediation, the seller takes the platform's recommendation policy β as given, so a deviation to a higher price is not accompanied by an adjustment in the platform's strategy, which makes such deviations less profitable. The same logic applies in monopoly. Hence, our main findings are robust to separating an integrated vertical chain into a seller and a platform that operates as a recommender.

Inclusive pricing for higher-value captives In the main part, we derived our results under the parameter restriction $c+2t > v_m$. In the opposite case, $c+2t < v_m$, serving captives only would lead to a higher price than serving contestable consumers only. We explicitly have to account for the possibility that firms may inflate recommendations to captives; that is, $\beta_i^{ca} > 0$. Different from our previous findings, firms may choose to do so in equilibrium, while they always provide unbiased recommendations to contestable consumers.

For $\alpha \in (0,1)$, the symmetric equilibrium price p^* lies strictly between the equilibrium price set for contestables only and the captives' value (i.e. $c + 2t < p^* < v_m$) and, given this price, the equilibrium bias $\beta^{ca*} \in (0,1)$ satisfies the participation constraint of captive consumers. In a numerical example with parameter values $(v_h, v_m, v_l, c, t, \alpha) =$

$(12, 10, 3, 6, 1.5, 0.8)$, we obtain that $p^* \approx 9.47$ and $\beta^{ca*} \approx 0.06$. Hence, strictly more than one half of a firm's captives receive positive recommendations instead of exactly one half under unbiased recommendations. The resulting profit per firm, about 0.88, exceeds the gain from any deviation by firm i, including the deviation to $p_i' = v_m$ and $\beta_i^{\text{ca}'} = 0$. Derivations and further details are provided in Appendix B.

This numerical example illustrates that firms may strategically bias recommendations that they make to their captive consumer base. The incentive to do so arises from the same trade-off that governs the inclusive-pricing equilibrium in the main text but now operates with respect to captive consumers. A bias directed at captives (that is, ex-ante low-type consumers in the monopoly setting) would never occur under monopoly provided that $v_h - t > v_m > c > v_l$, which we assumed throughout.

Appendix

A. Relegated Proofs

Proof of Lemma 1. As argued in the main text, a monopolist selling to ex-ante high-type consumers only sets its price $p = v_h - t$. Thus it does not set its price neither in $(v_m, v_h - t)$ not in $(v_h - t, \infty)$. To establish the lemma, it remains to be shown that the monopolist never sets its price such that $p < v_m$. By contradiction, suppose that this is the case. Then, $\beta = 0$ cannot be optimal as $v_h - p > v_h - v_m > t$ and the firm can slightly increase β. Thus, $\beta > 0$. If $v_h - p - \beta(p - v_l) < (1 + \beta)t$, then the firm can slightly increase its price and earn higher profits. If instead $v_h - p - \beta(p - v_l) > (1 + \beta)t$, then the seller can again slightly increase its price and earn higher profits. Suppose that $v_h - p - \beta(p - v_l) = (1 + \beta)t$. Consider a deviation to a slightly higher price and a recommendation policy with slightly lower β, such that $v_h - p - \beta(p - v_l) = (1 + \beta)t$ continues to hold. The adjusted recommendation policy is characterized by $\hat{\beta}(p) \equiv \frac{v_h - v_l}{p - v_l + t} - 1$. Then, the profit of the monopolist is given by

$$\pi(p, \hat{\beta}(p)) \equiv \left(\frac{\alpha}{2t}(1 + \beta)t + \frac{1 - \alpha}{4}\right)(p - c) = \left(\frac{\alpha}{2t}\frac{(v_h - v_l)t}{p - v_l + t} + \frac{1 - \alpha}{4}\right)(p - c).$$

Taking the derivative with respect to p, we obtain

$$\begin{aligned} \frac{d\pi}{dp} &= \frac{\alpha}{2}\frac{v_h - v_l}{p - v_l + t} + \frac{1 - \alpha}{4} - \frac{\alpha}{2}\frac{v_h - v_l}{(p - v_l + t)^2}(p - c) \\ &= \frac{\alpha}{2}\frac{v_h - v_l}{(p - v_l + t)^2}(c - v_l + t) + \frac{1 - \alpha}{4} > 0, \end{aligned}$$

implying that such deviation is profitable. Thus, we have that the monopolist either sets price $p^M = v_m$ and serves ex-ante high-type consumers only or it sets price $v_h - t$ and serves also ex-ante low-type consumers. □

Lemma 4. *Suppose that firms set prices $p_k \geq c$ and recommendation policies β_k, $k \in \{A, B\}$, such that there exists an indifferent consumer $\hat{x} \in [0, 1]$ in the contested segment. For some firm i, let $\beta_i > 0$. Then, if $p_i > v_m$ and $\hat{x} > 0$, there exists $p_i' > p_i$ such that firm i can profitably deviate to $(p_i', 0)$. If instead $p_i < v_m$, then firm i has a profitable deviation to either*

$(p_i', 0)$ *for some* $p_i' \in (p_i, v_m]$ *or to* (v_m, β_i') *for some* $\beta_i' < \beta_i$.

Proof. Firms set prices $p_k \geq c$ and recommendation policies β_k, $k \in \{A, B\}$, such that there exists an indifferent consumer $\hat{x} \in [0, 1]$ in the contested segment. For some firm i, let $\beta_i > 0$.

We must have that

$$\hat{x} = \hat{x}(p_A, \beta_A; p_B, \beta_B) = \frac{1}{2} + \frac{\beta_A - \beta_B}{4t} v_l + \frac{1 + \beta_B}{4t} p_B - \frac{1 + \beta_A}{4t} p_A.$$

Without loss of generality, suppose that for firm A, $\beta_A > 0$ and $p_A \neq v_m$.

First, we will show that if $p_A < v_m$, firm A has a profitable deviation to $(p_A', 0)$ for some $p_A' \in (p_i, v_m]$ or to (v_m, β_A') for some $\beta_A' < \beta_A$. Second, we will show that, if $p_A > v_m$ and $\hat{x} > 0$, there exists $p_A' > p_A$ such that firm A can profitably deviate to $(p_A', 0)$.

If $p_A < v_m$ and $\hat{x} = 0$, then the result of the lemma holds trivially, as firm A can strictly increase its profit from captive consumers by setting $p_A' = v_m$ and keeping the recommendation policy the same. In what follows, we consider the case in which $\hat{x} > 0$.

Consider a deviation by firm A to a recommendation policy β_A' with β_A' slightly less than β_A, and a price p_A', which is slightly above p_A, while remaining weakly below v_m whenever $p_A < v_m$, such that $(1+\beta_A')(p_A' - v_l) = (1+\beta_A)(p_A - v_l)$. This implies that the location of the indifferent consumer in the contested segment does not change – that is, $\hat{x}(p_A', \beta_A'; p_B, \beta_B) = \hat{x}(p_A, \beta_A; p_B, \beta_B)$. The profit after the deviation is

$$\begin{aligned}
\pi_A(p_A', \beta_A'; p_B, \beta_B) &= (p_A' - c)\left[\alpha \frac{1 + \beta_A'}{2} \hat{x}(p_A', \beta_A'; p_B, \beta_B) + \frac{1 - \alpha}{4} \mathbb{I}_{p_A' \leq v_m}\right] \\
&= (p_A' - c)\left[\alpha \frac{1 + \beta_A'}{2} \hat{x}(p_A, \beta_A; p_B, \beta_B) + \frac{1 - \alpha}{4} \mathbb{I}_{p_A \leq v_m}\right] \\
&> \alpha \frac{(1 + \beta_A')(p_A' - c)}{2} \hat{x}(p_A, \beta_A; p_B, \beta_B) + \frac{1 - \alpha}{4} \mathbb{I}_{p_A \leq v_m}(p_A - c) \\
&> \alpha \frac{(1 + \beta_A)(p_A - c)}{2} \hat{x}(p_A, \beta_A; p_B, \beta_B) + \frac{1 - \alpha}{4} \mathbb{I}_{p_A \leq v_m}(p_A - c) \\
&= \pi_A(p_A, \beta_A; p_B, \beta_B),
\end{aligned}$$

leading to the fourth line follows from the fact that

$$\begin{aligned}(1+\beta'_A)(p'_A-c) &= (1+\beta'_A)(p'_A-v_l)-(1+\beta'_A)(c-v_l)\\ &= (1+\beta_A)(p_A-v_l)-(1+\beta'_A)(c-v_l)\\ &> (1+\beta_A)(p_A-v_l)-(1+\beta_A)(c-v_l)\\ &= (1+\beta_A)(p_A-c).\end{aligned}$$

Thus, we established that firm A can strictly increase its profit by deviating to a slightly more informative recommendation policy β'_A combined with a higher price p'_A (that remains weakly below v_m whenever $p_A < v_m$). Therefore, firm A can profitably apply this local deviation to move along the iso-$(1+\beta'_A)(p'_A - v_l)$ curve, thereby increasing its profit until it reaches the boundary – that is, $\beta'_A = 0$ or $p'_A = v_m$ (the latter being possible only when $p_A < v_m$).

The case in which $\beta_B > 0$ and $p_B < v_m$ is analogous and follows trivially. □

Proof of Lemma 2. Assume, by contradiction, that one firm – say firm A – sets price $p^*_A < v_m$ in equilibrium. If all contested consumers buy from firm B in equilibrium, then firm A can profitably deviate to price v_m and earn higher profits from its captives. If instead all contested consumers buy from firm A, then there must exist an indifferent consumer in the contested market located at 1 (as otherwise, firm A could slightly increase its price). Hence, there exists an indifferent consumer located at some $\hat{x}^* = \hat{x}(p^*_A, \beta^*_A; p^*_B, \beta^*_B) > 0$.

Consider firm A setting a price p_A in a neighborhood of p^*_A and β_A in a neighborhood of β^*_A (if $\hat{x}^* = 1$, we restrict attention to $p_A > p^*_A$; if, in addition, $\beta^*_A < 1$ we also restrict attention to $\beta_A > \beta^*_A$). Given the rival's choice $(p^*_B, \beta^*_B$, firm A's profit is

$$\pi_A(p_A, \beta_A; p^*_B, \beta^*_B) = (p_A - c)\left[\alpha \frac{1+\beta_A}{2}\hat{x}(p_A, \beta_A; p^*_B, \beta^*_B) + \frac{1-\alpha}{4}\right],$$

where

$$\hat{x}(p_A, \beta_A; p_B, \beta_B) \equiv \frac{1}{2} + \frac{\beta_A - \beta_B}{4t}v_l + \frac{1+\beta_B}{4t}p_B - \frac{1+\beta_A}{4t}p_A.$$

The partial derivative of firm A's profit with respect to its own price at $p_A = p^*_A$ (we take the right-hand derivative in the case of $\hat{x}^* = 1$) is given by

$$\left.\frac{\partial \pi_A}{\partial p_A}\right|_{p_A = p_A^*} = \alpha \frac{1+\beta_A^*}{2} \hat{x}(p_A^*, \beta_A^*; p_B^*, \beta_B^*) + \frac{1-\alpha}{4} - \alpha \frac{(1+\beta_A^*)^2}{8t}(p_A^* - c).$$

In profit maximum, we must have that $\frac{\partial \pi_A}{\partial p_A}|_{p_A = p_A^*} = 0$ if $\hat{x}^* < 1$ and $\frac{d\pi_A}{dp_A}|_{p_A = p_A^*} \leq 0$ if $\hat{x}^* = 1$. This implies that

$$\begin{aligned} p_A^* &= \frac{c}{2} + \frac{t}{1+\beta_A^*} + \frac{\beta_A^* - \beta_B^*}{2(1+\beta_A^*)} v_l + \frac{1+\beta_B^*}{2(1+\beta_A^*)} p_B^* + \frac{t(1-\alpha)}{\alpha(1+\beta_A^*)^2} \\ &= \frac{c}{2} + \frac{t}{1+\beta_A^*} + \frac{v_l}{2} + \frac{1+\beta_B^*}{2(1+\beta_A^*)}(p_B^* - v_l) + \frac{t(1-\alpha)}{\alpha(1+\beta_A^*)^2}, \end{aligned}$$

if $\hat{x}^* < 1$ and

$$p_A^* \geq \frac{c}{2} + \frac{t}{1+\beta_A^*} + \frac{v_l}{2} + \frac{1+\beta_B^*}{2(1+\beta_A^*)}(p_B^* - v_l) + \frac{t(1-\alpha)}{\alpha(1+\beta_A^*)^2},$$

if $\hat{x}^* = 1$.

We next show that we must have that $\beta_A^* > 0$ if p_A^* were strictly less than v_m. We show this by contradiction assuming that $\beta_A^* = 0$. Then, the first-order condition of firm A's maximization problem implies that

$$p_A^* \geq \frac{c}{2} + t + \frac{v_l}{2} + \frac{1+\beta_B^*}{2}(p_B^* - v_l) + \frac{t(1-\alpha)}{\alpha}.$$

If $p_B^* \geq v_m$, it follows from the inequality above that $p_A^* > \frac{c}{2} + t + \frac{v_l}{2} + \frac{v_m - v_l}{2} \geq v_m$, which is a contradiction. Thus, we must have that $p_B^* < v_m$. This implies that, in equilibrium, the indifferent consumer must be in the interior – that is, $\hat{x}^* \in (0, 1)$. Therefore, the first-order condition for firm B's profit-maximization problem with respect to its own price must be satisfied with equality. This yields

$$\alpha \frac{1+\beta_B^*}{2} \hat{x}(p_B^*, \beta_B^*; p_A^*, 0) + \frac{1-\alpha}{4} = \alpha \frac{(1+\beta_B^*)^2}{8t}(p_B^* - c).$$

Using this condition, we note that

$$\begin{aligned}\left.\frac{\partial \pi_B}{\partial \beta_B}\right|_{\beta_B=\beta_B^*} &= \hat{x}(p_B^*, \beta_B^*; p_A^*, 0) - \frac{1+\beta_B^*}{4t}(p_B^* - v_l) \\ &= \frac{1+\beta_B^*}{4t}(p_B^* - c) - \frac{1-\alpha}{2\alpha(1+\beta_B^*)} - \frac{1+\beta_B^*}{4t}(p_B^* - v_l) \\ &= -\frac{(1+\beta_B^*)}{4t}(c - v_l) - \frac{1-\alpha}{2\alpha(1+\beta_B^*)} < 0,\end{aligned}$$

implying that firm B has incentives to reduce β_B and adjust its price accordingly. Hence, in equilibrium, we would have that $\beta_B^* = 0$ as well. The resulting system of equations for the equilibrium prices is given by

$$p_i^* = \frac{c}{2} + t + \frac{p_j^*}{2} + \frac{t(1-\alpha)}{\alpha}, \quad i, j \in \{A, B\}, j \neq i.$$

The unique solution of this system is $p_A^* = p_B^* = c + 2t + \frac{t(1-\alpha)}{\alpha}$, which is strictly greater than v_m, a contradiction. Therefore, we must have that $\beta_A^* > 0$.

Given that $\beta_A^* > 0$, consider a deviation by firm A to a recommendation policy β'_A with β'_A slightly less than β_A^*, and a price $p'_A < v_m$, which is slightly above p_A^*, such that $(1+\beta'_A)(p'_A - v_l) = (1+\beta_A^*)(p_A^* - v_l)$. This implies that the location of the indifferent consumer does not change – that is, $\hat{x}(p'_A, \beta'_A; p_B, \beta_B) = \hat{x}(p_A^*, \beta_A^*; p_B, \beta_B)$. The profit after the deviation is

$$\begin{aligned}\pi_A(p'_A, \beta'_A; p_B^*, \beta_B^*) &= (p'_A - c)\left[\alpha\frac{1+\beta'_A}{2}\hat{x}(p'_A, \beta'_A; p_B^*, \beta_B^*) + \frac{1-\alpha}{4}\right] \\ &= (p'_A - c)\left[\alpha\frac{1+\beta'_A}{2}\hat{x}(p_A^*, \beta_A^*; p_B^*, \beta_B^*) + \frac{1-\alpha}{4}\right] \\ &> \alpha\frac{(1+\beta'_A)(p'_A - c)}{2}\hat{x}(p_A^*, \beta_A^*; p_B^*, \beta_B^*) + \frac{1-\alpha}{4}(p_A^* - c) \\ &> \alpha\frac{(1+\beta_A^*)(p_A^* - c)}{2}\hat{x}(p_A^*, \beta_A^*; p_B^*, \beta_B^*) + \frac{1-\alpha}{4}(p_A^* - c) \\ &= \pi_A(p_A^*, \beta_A^*; p_B^*, \beta_B^*),\end{aligned}$$

where the inequality leading to the fourth line follows from the fact that

$$\begin{aligned}(1+\beta_A')(p_A'-c) &= (1+\beta_A')(p_A'-v_l)-(1+\beta_A')(c-v_l)\\ &= (1+\beta_A^*)(p_A^*-v_l)-(1+\beta_A')(c-v_l)\\ &> (1+\beta_A^*)(p_A^*-v_l)-(1+\beta_A^*)(c-v_l)\\ &= (1+\beta_A^*)(p_A^*-c).\end{aligned}$$

Thus, we established that firm A has incentives to deviate to a slightly more informative recommendation policy combined with a higher price. This is a contradiction. We conclude that, in any equilibrium, prices must satisfy $p_A^*, p_B^* \geq v_m$.

□

Proof of Proposition 2. Consider a deviation of firm A from the equilibrium strategy $(p_A^*, \beta_A^*) = (c+2t, 0)$ to some recommendation policy $\beta_A > 0$ and a price $p_A \neq v_m$. Clearly, if firm A deviates to $p_A > v_m$, then such a deviation is unprofitable (this follows from the analysis of the case of $\alpha = 1$). In the following, we characterize the most profitable deviation of firm A conditional on $p_A \leq v_m$. Suppose that $p_A < v_m$. If all contested consumers purchase from firm B, then a deviation to price v_m would be more profitable for firm A. Conversely, if all contested consumers purchase from firm A, then firm A can raise its price until the consumer located at $x = 1$ becomes indifferent. This implies that there exists an indifferent consumer located at

$$\begin{aligned}\hat{x}(p_A, \beta_A; c+2t, 0) &= \frac{1}{2} + \frac{\beta_A}{4t}v_l + \frac{c+2t-(1+\beta_A)p_A}{4t}\\ &= 1 - \frac{\beta_A}{4t}(p_A - v_l) - \frac{p_A - c}{4t},\end{aligned}$$

and, moreover, we have that $\hat{x}(p_A, \beta_A; c+2t, 0) \in (0, 1)$. The profit of firm A from such a deviation is given by:

$$\pi_A(p_A, \beta_A; c+2t, 0) = \left[\alpha\frac{1+\beta_A}{2}\hat{x}(p_A, \beta_A; c+2t, 0) + \frac{1-\alpha}{4}\right](p_A - c).$$

If $\beta_A = 0$, then firm A's profits from the contested consumers increase in price up to $c+2t$, implying that firm A can earn strictly higher profits by keeping $\beta_A = 0$ and deviating to v_m rather than to any lower price. If $\beta_A > 0$, then, given that there exists an indifferent consumer located at $\hat{x}(p_A, \beta_A; c+2t, 0)$, it follows from Lemma 4 that firm A has a profitable deviation to either $(p'_A, 0)$ for some $p'_A \in (p_A, v_m]$ or to (v_m, β'_A) for some $\beta'_A < \beta_A$. Since the former deviation with $p'_A < v_m$ and $\beta'_A = 0$ was shown to be unprofitable, we conclude that, among all possible deviations to $p_A \leq v_m$, firm A's most profitable deviation from the limited-sales equilibrium is to set $p_A = v_m$.

In the main text, we showed that the most profitable deviation to $p_A = v_m$ is combined with recommendation policy β'_A given by equation (1).

Using the notation $y_c \equiv (v_m - c)/t$ and $y_l \equiv (v_m - v_l)/t$, firm A's profits from a deviation to (v_m, β'_A) can be rewritten as follows:

$$\pi'_A = \begin{cases} \left(\frac{\alpha}{2}\left(1 - \frac{y_c}{4}\right) + \frac{1-\alpha}{4}\right) y_c t, & \text{if} \quad y_l > 4 - y_c, \\ \frac{1}{2}\left(\alpha \frac{(t+(c-v_l)/4))^2}{t(v_m - v_l)} + \frac{1-\alpha}{2}\right) y_c t, & \text{if} \quad 4 - y_c \geq y_l > \max\{(4-y_c)/3, y_c\} \\ \left(\alpha\left(1 - \frac{y_c}{4} - \frac{y_l}{4}\right) + \frac{1-\alpha}{4}\right) y_c t, & \text{if} \quad (4-y_c)/3 \geq y_l > y_c, \end{cases}$$

where $y_c \in (0, 2)$. The following picture represents the region of parameters (y_c, y_l) in the case of $\beta'_A = 0$ (Case 1), $\beta'_A \in (0, 1)$ (Case 2) and $\beta'_A = 1$ (Case 3).

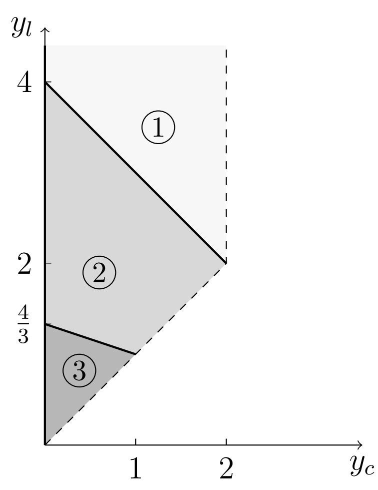


Figure 2: The region of parameters (y_c, y_l) corresponding to the three different cases.

Next, we characterize the parameters, in which the limited-sales equilibrium occurs. First, suppose that $y_l + y_c > 4$. Then, firm A's most profitable deviation in which captives are also served is $p_A = v_m$ and $\beta'_A = 0$. This deviation is unprofitable if and only if

$$\left(\frac{\alpha}{2}\left(1 - \frac{y_c}{4}\right) + \frac{1-\alpha}{4}\right) y_c \leq \frac{\alpha}{2},$$

which is equivalent to

$$\alpha \geq \alpha_1 \equiv \frac{1}{\frac{2}{y_c} + \frac{y_c}{2} - 1} = \frac{2y_c}{2y_c + (2-y_c)^2}.$$

Note that since $y_c \in (0, 2)$, we have that the denominator of α_1 is strictly greater than 1, implying that $\alpha_1 \in (0, 1)$. Thus, we obtain that if $y_c + y_l > 4$, there is an equilibrium with limited sales whenever $\alpha \geq \alpha_1$.

Second, suppose that $4 - y_c \geq y_l > \max\left\{\frac{4}{3} - \frac{y_c}{3}, y_c\right\}$. In this case, we have that $\beta'_A = \frac{2}{y_l} - \frac{y_c}{2y_l} - \frac{1}{2}$, implying that $\hat{x}(v_m, \beta'^{*}_A; c+2t, 0) = \frac{y_l}{4}\left(\frac{2}{y_l} - \frac{y_c}{2y_l} + \frac{1}{2}\right) = \frac{1}{2} - \frac{y_c}{8} + \frac{y_l}{8}$. Firm A does not have profitable deviations from the outcome with limited sales if and only if $\pi'_A \leq \alpha t/2$, which is equivalent to

$$\left(\alpha\frac{\left(1 + \frac{y_l}{4} - \frac{y_c}{4}\right)^2}{y_l} + \frac{1-\alpha}{2}\right) y_c \leq \alpha.$$

This condition holds true for all α satisfying

$$\alpha \geq \alpha_2 \equiv \frac{1}{1 + \frac{2}{y_c} - \frac{2\left(1+\frac{y_l}{4} - \frac{y_c}{4}\right)^2}{y_l}}.$$

We show that $\alpha_2 \in (0, 1)$. To show that $\alpha_2 < 1$, it is sufficient to show that

$$\begin{aligned}\phi &\equiv \frac{1}{2} y_c y_l \left(1 - \frac{1}{\alpha_2}\right) \\ &= y_c + (y_l - y_c) - y_c\left(1 + \frac{y_l - y_c}{4}\right)^2\end{aligned}$$

is strictly positive. Observe that the second derivative of ϕ with respect to $y_l - y_c$ is $-\frac{y_c}{8}$, which is negative, implying that the function ϕ is strictly concave in $y_l - y_c$. It follows that, within the considered parameter region, ϕ has an infimum either at $y_l - y_c = \max\left\{\frac{4}{3}(1-y_c), 0\right\}$ (recall that y_l is strictly greater than $\max\left\{\frac{4}{3} - \frac{y_c}{3}, y_c\right\}$) or at the maximal feasible value of

$y_l - y_c$ within the considered region, which equals $4 - 2y_c$. It follows that, for any $y_c \in (0, 1]$, we have

$$
\begin{aligned}
\phi &> \min \left\{ \frac{4}{3} - \frac{y_c}{3} - y_c \left(\frac{4}{3} - \frac{y_c}{3} \right)^2, 4 - y_c - \frac{y_c}{4} (4 - y_c)^2 \right\} \\
&= \min \left\{ \frac{1}{3} \left(\frac{4}{3} - \frac{y_c}{3} \right) (1 - y_c)(3 - y_c), \frac{1}{4} (4 - y_c)(2 - y_c)^2) \right\} \\
&\geq 0.
\end{aligned}
$$

If instead $y_c \in (1, 2)$, we have that

$$
\phi > \min \left\{ 0, \frac{1}{4} (4 - y_c)(2 - y_c)^2) \right\} = 0.
$$

We showed that ϕ is positive and therefore $\alpha_2 < 1$. Next, we show that $\alpha_2 > 0$. The sign of the denominator of α_2 is determined by the sign of $y_l y_c + 2\phi$, which is positive as $\phi > 0$ and $y_c, y_l > 0$. It immediately follows that $\alpha_2 > 0$.

It remains to consider the third case, in which y_l and y_c satisfy $\frac{4}{3} - \frac{y_c}{3} \geq y_l$. We showed that in this parameter region $\beta_A'^{*} = 1$. Firm A does not find it profitable to deviate to $(v_m, 1)$ if and only if

$$
\left(\alpha \left(1 - \frac{y_c + y_l}{4} \right) + \frac{1 - \alpha}{4} \right) y_c < \frac{\alpha}{2}.
$$

This condition holds for all α satisfying

$$
\alpha \geq \alpha_3 \equiv \frac{1}{\frac{2}{y_c} + y_c + y_l - 3}.
$$

Since the denominator of α_3 is strictly in increasing in y_l on $(y_c, (4 - y_c)/3]$, we have that

$$
\begin{aligned}
\frac{2}{y_c} + y_c + y_l - 3 &> \frac{2}{y_c} + 2y_c - 3 \\
&\geq 2\sqrt{\frac{2}{y_c} \times 2y_c} - 3 = 1,
\end{aligned}
$$

implying that $\alpha_3 \in (0, 1)$.

We established that $\alpha_{LS} \in (0, 1)$ and the equilibrium with limited sales exists for all

$\alpha \geq \alpha_{LS}$. $\square$

Proof of Lemma 3. In a symmetric equilibrium $\beta_A = \beta_B = \beta$, so from the best-response condition, $\beta = (\beta - 1)/2 + t/(v_m - v_l)$, which solves to $\beta^* = -1 + 2t/(v_m - v_l)$. This interior solution applies for $\frac{v_m - v_l}{2} < t < v_m - v_l$; otherwise, the corner values $\beta^* \in \{0, 1\}$ obtain. $\square$

Proof of Proposition 3. We begin by showing that a deviation of firm A to a price $p_A < v_m$ combined with any recommendation policy $\beta_A \in [0, 1]$ is unprofitable (the proof for firm B follows analogously). Suppose, towards a contradiction, that such a deviation yields strictly higher profits for firm A. This implies that, under the considered deviation, some contested consumers must buy from firm A. If all contested consumers purchase from firm A, then it can raise its price either up to v_m (in which case a contradiction arises) or until the consumer located at $x = 1$ becomes indifferent. This implies that there exists an indifferent consumer located at $\hat{x}(p_A, \beta_A; v_m, \beta^*) \in (0, 1)$. Thus, by Lemma 4, there exists a deviation to either $(p'_A, 0)$ for some $p'_A \in (p_A, v_m]$ or to (v_m, β'_A) for some $\beta'_A < \beta_A$ that yields even greater profits for firm A. Clearly, a deviation to $p'_A = v_m$ and some $\beta'_A \in [0, 1]$ cannot be profitable. Then, it must be that a deviation to some $p'_A < v_m$ and $\beta'_A = 0$ is profitable.

Since this deviation is profitable, we must have that $\hat{x}' = \hat{x}(p'_A, 0; v_m, \beta^*) > 0$. The sign of the derivative of $\pi_A(p''_A, 0; v_m, \beta^*)$ with respect to price p''_A at $p''_A = p'_A$ (we take the right-hand derivative in case $\hat{x}' = 1$) is determined by the sign of

$$\frac{c}{2} + t + \frac{v_l}{2} + \frac{1 + \beta^*}{2}(v_m - v_l) + \frac{t(1 - \alpha)}{\alpha} - p'_A$$

and is positive, as $p'_A < v_m$ and $c + 2t > v_m$. Thus, firm A can reach even higher profits by increasing its price up to v_m. This is a contradiction, since firm A chooses optimally β^* when both firms set price v_m and $\beta^*_B = \beta^*$ and cannot derive strictly higher profits from deviation to (v_m, β'_A).

In the following, we consider upward price deviations of firm A under which it no longer serves its captives. If such an upward deviation to $p_A > v_m$ and some β_A is profitable, then it must be that firm A serves some of the contested consumers. Clearly, if all consumers buy from firm A after the deviation and the consumer located at $x = 1$ strictly prefers firm A, then firm A can further increase its profit by raising its price until this consumer becomes indifferent. Thus, we can restrict attention to deviations for which there exists an indifferent consumer $\hat{x} = \hat{x}(p_A, \beta_A; v_m \beta^*) > 0$. By Lemma 4, firm A can then increase its profit by choosing some $p'_A > v_m$ and setting $\beta'_A = 0$. The profit from this deviation is given by

$$\begin{aligned}\pi_A(p'_A, 0; v_m, \beta^*) &= \frac{\alpha}{2}\hat{x}(p'_A, 0; v_m, \beta^*)(p'_A - c)\\ &= \frac{\alpha}{2}\left(\frac{1}{2} + (1+\beta^*)\frac{v_m - v_l}{4t} - \frac{p'_A - v_l}{4t}\right)(p'_A - c).\end{aligned}$$

The first-order condition implies that the deviation profits are maximized at price

$$p'^*_A = \frac{c}{2} + t + \frac{v_m}{2} + \frac{\beta^*}{2}(v_m - v_l).$$

It is easy to see that p'^*_A lies above v_m, since $c + 2t > v_m$. The resulting maximal profits from maximal deviation is given

$$\pi'^*_A \equiv \frac{\alpha}{8t}(p'^*_A - c)^2 = \frac{\alpha}{8}t\left(1 + \frac{y_c}{2} + \beta^*\frac{y_l}{2}\right)^2.$$

Plugging the expression for β^*, we obtain

$$\pi'^*_A = \begin{cases} \frac{\alpha}{8}\left(1 + \frac{y_c}{2}\right)^2 t, & \text{if} \quad y_l > 2,\\ \frac{\alpha}{8}\left(2 + \frac{y_c}{2} - \frac{y_l}{2}\right)^2 t, & \text{if} \quad 2 \geq y_l \geq 1,\\ \frac{\alpha}{8}\left(1 + \frac{y_c}{2} + \frac{y_l}{2}\right)^2 t, & \text{if} \quad y_l < 1. \end{cases}$$

This upward price deviation is unprofitable if and only if $\pi^*_A \geq \pi'^*_A$.

First, suppose that $y_l > 2$, then there is an equilibrium with inclusive pricing if

$$\frac{1}{4}y_c t \geq \frac{\alpha}{8}\left(1 + \frac{y_c}{2}\right)^2 t,$$

or, equivalently,

$$\alpha \leq \alpha'_1 \equiv \frac{8y_c}{(2+y_c)^2}.$$

Since $(2+y_c)^2 - 8y_c = (2-y_c)^2 > 0$, we have that $\alpha'_1 \in (0,1)$ for any $y_c \in (0,2)$. Next, suppose that $y_l \in [1,2]$ and $y_l > y_c$. In this region of parameters, the inclusive-pricing equilibrium

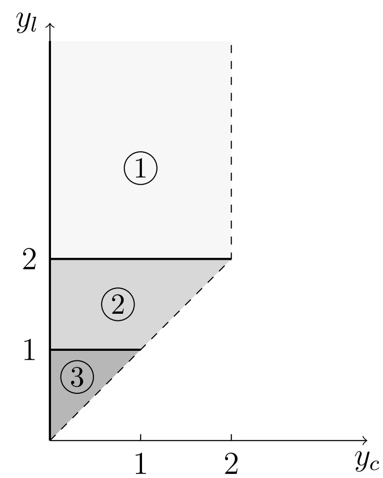


Figure 3: The region of parameters (y_c, y_l) corresponding to the three different cases.

exists if

$$\left(\frac{\alpha}{2y_l} + \frac{1-\alpha}{4}\right) y_c t \geq \frac{\alpha}{8}\left(2 + \frac{y_c}{2} - \frac{y_l}{2}\right)^2 t$$

or, equivalently,

$$\alpha \leq \alpha_2' \equiv \frac{8y_c}{8y_c + \frac{h}{y_l}},$$

where

$$h(y_l) \equiv y_l(4 + y_c - y_l)^2 - 16y_c.$$

Note that $h' = (4+y_c-y_l)(4+y_c-3y_l)$ and it equals zero in the considered parameter range only when $4+y_c-3y_l^* = 0$. It follows that that the critical point of this function is given by $y_l^* = (4+y_c)/3$ (note that $y_l^* \in (1,2)$ for any $y_c \in (0,2)$). Since $h''(y_l^*) = -3(4+y_c-y_l^*) = -6y_l^* < 0$, we have that function y_l^* is a local maximum, implying that function h attains its minimum on the boundary of the feasible interval. First, suppose that $y_c \in (0,1)$. Then, $h(y_l) \geq \min\{h(1), h(2)\}$ for any $y_l \in [1,2]$. Since $h(1) = (3+y_c)^2 - 16y_c = (1-y_c)(9-y_c) > 0$ for any $y_c \in (0,1)$ and $h(2) = 2((2+y_c)^2 - 8y_c) = 2(2-y_c)^2 > 0$, we obtain that $h(y_l) > 0$ for any $y_c \in (0,1)$ and $y_l \in [1,2]$. If instead $y_c \in [1,2)$, we have that $h(y_l) > h(2) > h(y_c) = 0$ for any $y_l \in [y_c, 2]$. Therefore, $h(y_l) > 0$ in the considered parameter range, implying that $\alpha_2' \in (0,1)$.

Finally, consider the region in which $y_l < 1$. There exists an equilibrium with inclusive pricing if

$$\frac{1+\alpha}{4}y_c t \geq \frac{\alpha}{8}\left(1+\frac{y_c}{2}+\frac{y_l}{2}\right)^2 t,$$

or, equivalently,

$$\alpha \leq \alpha_3' \equiv \frac{8y_c}{(2+y_c+y_l)^2-8y_c} = \frac{8y_c}{8y_c+((2+y_c+y_l)^2-16y_c)}.$$

Note that since $(2+y_c+y_l)^2-16y_c > (2+y_c+y_c)^2-16y_c = (2-y_c)^2 > 0$ for any $y_c \in (0,2)$, we have that $\alpha_3' \in (0,1)$.

We established that $\alpha_{IP} \in (0,1)$ and the equilibrium with limited sales exists for all $\alpha \leq \alpha_{IP}$. □

Proof of Proposition 4. Suppose that $y_l > 4-y_c$. Then,

$$\alpha_{LS}-\alpha_{IP} = \frac{2y_c}{2y_c+(2-y_c)^2} - \frac{8y_c}{(2+y_c)^2}.$$

Note that

$$\begin{aligned}4(2y_c+(2-y_c)^2)-(2+y_c)^2 &= 3(2-y_c)^2+8y_c-\left((2+y_c)^2-(2-y_c)^2\right)\\ &= 3(2-y_c)^2 > 0,\end{aligned}$$

implying that the denominator of α_{LS} is strictly greater than the denominator of α_{IP} for any $y_c \in (0,2)$. Thus, we have that $\alpha_{LS} > \alpha_{IP}$ for any $y_l > 4-y_c$ and $y_c \in (0,2)$. If $y_l \in (2, 4-y_c]$, we have that

$$\alpha_{LS}-\alpha_{IP} = \frac{y_c}{2+y_c-\frac{2y_c\left(1+\frac{y_l}{4}-\frac{y_c}{4}\right)^2}{y_l}} - \frac{y_c}{\frac{1}{8}(2+y_c)^2},$$

The derivative of the denominator of α_{LS} with respect to y_l is given by

$$-2y_c\left(\frac{\left(1+\frac{y_l}{4}-\frac{y_c}{4}\right)\frac{y_l}{2}-\left(1+\frac{y_l}{4}-\frac{y_c}{4}\right)^2}{y_l^2}\right) = -2\frac{y_c}{y_l^2}\left(1+\frac{y_l}{4}-\frac{y_c}{4}\right)\left(\frac{y_c}{4}+\frac{y_l}{4}-1\right) \geq 0,$$

implying that the denominator of α_{LS} increases in y_l on $(2, 4-y_c]$ and, therefore, is bounded below by $2 + y_c - y_c \left(\frac{6-y_c}{4}\right)^2$. Moreover, we have that

$$\frac{1}{\alpha_{LS}} - \frac{1}{\alpha_{IP}} \geq 2 + y_c - y_c \left(\frac{6-y_c}{4}\right)^2 - \frac{(2+y_c)^2}{8} = \frac{1}{8}(2-y_c)^2 \left(3 - \frac{y_c}{2}\right) > 0$$

for every $y_c \in (0, 2)$. It follows that $\alpha_{LS} < \alpha_{IP}$.

If instead $y_l \in (\max\{(4-y_c)/3, y_c\}, 2]$, we have that

$$\alpha_{LS} - \alpha_{IP} = \frac{1}{1 + \frac{2}{y_c} - \frac{(4+y_l-y_c)^2}{8y_l}} - \frac{1}{1 - \frac{2}{y_l} + \frac{(4+y_c-y_l)^2}{8y_c}}.$$

The difference in the denominators of α_{LS} and α_{IP} is given by

$$\begin{aligned}
\frac{1}{\alpha_{LS}} - \frac{1}{\alpha_{IP}} &= \frac{2}{y_c} - \frac{(4+y_l-y_c)^2}{8y_l} + \frac{2}{y_l} - \frac{(4+y_c-y_l)^2}{8y_c} \\
&= \frac{1}{8y_c}(16 - (4+y_c-y_l)^2) + \frac{1}{8y_l}(16 - (4+y_l-y_c)^2) \\
&= \frac{1}{8y_c}(8+y_c-y_l)(y_l-y_c) + \frac{1}{8y_l}(8+y_l-y_c)(y_c-y_l).
\end{aligned}$$

Simplifying further, we obtain,

$$\begin{aligned}
\frac{1}{\alpha_{LS}} - \frac{1}{\alpha_{IP}} &= \frac{y_l-y_c}{8y_cy_l}(y_l(8+y_c-y_l) - y_c(8+y_l-y_c)) \\
&= \frac{(y_l-y_c)^2(8-y_l-y_c)}{8y_cy_l} \\
&> 0,
\end{aligned}$$

for any $y_l \in (\max\{(4-y_c)/3, y_c\}, 2]$. Therefore, in the considered parameter range, we again have that $\alpha_{LS} < \alpha_{IP}$.

Next, we consider the case, in which $y_l \in [1, (4-y_c)/3]$. Note that in this parameter range, we must have that $y_c(0, 1)$. The difference between α_{LS} and α_{IP} can be represented as

$$\alpha_{LS} - \alpha_{IP} = \frac{1}{\frac{2}{y_c} + y_c + y_l - 3} - \frac{1}{1 - \frac{2}{y_l} + \frac{(4+y_c-y_l)^2}{8y_c}}.$$

The difference in the denominators of α_{LS} and α_{IP} is given by

$$\begin{aligned}\frac{1}{\alpha_{LS}} - \frac{1}{\alpha_{IP}} &= \frac{2}{y_c} + y_c + y_l - 4 + \frac{2}{y_l} - \frac{(4 + y_c - y_l)^2}{8y_c} \\ &= \frac{2 + y_c^2}{y_c} - 2 + \frac{2 + y_l^2}{y_l} - 2 - \frac{(4 + y_c - y_l)^2}{8y_c} \\ &= \frac{h(y_l)}{8y_c y_l},\end{aligned}$$

where

$$h(y_l) \equiv 8y_l \left(1 + (1 - y_c)^2\right) + 8y_c \left(1 + (1 - y_l)^2\right) - y_l(4 + y_c - y_l)^2.$$

Note that

$$h'(y_l) = 8\left(1 + (1 - y_c)^2\right) - 16y_c(1 - y_l) - (4 + y_c - y_l)^2 + 2y_l(4 + y_c - y_l)$$

and

$$\begin{aligned}h''(y_l) &= 16y_c + 2(4 + y_c - y_l) + 2(4 + y_c - y_l) - 2y_l \\ &= 20y_c + 8 - 6y_l > 20y_c + 8 - 6\frac{4 - y_c}{3} \\ &= 22y_c > 0.\end{aligned}$$

Plugging in $y_l = 1$ into the expression for $h'(y_l)$, we find that

$$\begin{aligned}h'(1) &= 8\left(1 + (1 - y_c)^2\right) - (3 + y_c)^2 + 2(3 + y_c - y_l) \\ &= 8\left(1 + (1 - y_c)^2\right) - (3 + y_c)(1 + y_c) \\ &> 8 - 4 \times 2 = 0,\end{aligned}$$

where we used that the expression in line leading to the final inequality strictly decreases in y_c. Thus, we showed that $h'(y_l) > h'(1) > 0$, implying that function $h(\cdot)$ strictly increases in

y_l on $[1,(4-y_c)/3]$ for any $y_c \in (0,1)$. Therefore,

$$\begin{aligned} h(y_l) > h(1) &= 8\left(1+(1-y_c)^2\right) + 8y_c - (3+y_c)^2 \\ &= 8(1-y_c)^2 - 1 + 2y_c - y_c^2 \\ &= 7(1-y_c)^2 > 0. \end{aligned}$$

Therefore, the denominator of α_{LS} is strictly greater than the denominator of α_{IP}, implying that $\alpha_{IP} > \alpha_{LS}$ in the considered region.

It remains to consider the case of $y_l < 1$. In this region of parameters, we have

$$\alpha_{LS} - \alpha_{IP} = \frac{1}{\frac{2}{y_c} + y_c + y_l - 3} - \frac{1}{\frac{(2+y_c+y_l)^2}{8y_c} - 1}.$$

Note that the difference in the denominators of α_{LS} and α_{IP} is given by

$$\begin{aligned} \frac{1}{\alpha_{LS}} - \frac{1}{\alpha_{IP}} &= \frac{2}{y_c} + y_c + y_l - 2 - \frac{(2+y_c+y_l)^2}{8y_c} \\ &= \frac{(16-(2+y_c+y_l)^2) + 8(y_c+y_l-2)y_c}{8y_c} \\ &= \frac{(6+y_c+y_l)(2-y_c-y_l) + 8(y_c+y_l-2)y_c}{8y_c} \\ &= \frac{(2-y_c-y_l)(6-7y_c+y_l)}{8y_c}. \end{aligned}$$

Since $y_l < 1$ and $y_c < y_l$, we have that $y_c < 1$ and thus $y_c + y_l < 2$. Moreover, $6 - 7y_c + y_l > 6(1-y_l) > 0$. Thus, we showed that the denominator of α_{LS} is strictly greater than the denominator of α_{IP} for any $y_l \in (y_c, 1)$, implying that $\alpha_{LS} < \alpha_{IP}$.

We conclude that $\alpha_{LS} < \alpha_{IP}$ for any $y_c \in (0,2)$ and $y_l > y_c$.

It remains to be shown that $\alpha \in [\alpha_{LS}, \alpha_{IP}]$. By Proposition 3, firm A does not find it profitable to deviate to (v_m, β^*), where β^* is the equilibrium recommendation policy in the inclusive-pricing equilibrium (that exists by Proposition 2). This implies that

$$\pi_A(c+2t, 0; c+2t, 0) \geq \pi_A(v_m, \beta^*; c+2t, 0).$$

First, suppose that $y_l > 2$, implying that $\beta^* = 0$. Then, there exists an indifferent consumer $\hat{x}(v_m, 0; c+2t, 0) = 1 - \frac{v_m - c}{4t} = 1 - \frac{y_c}{4} \in (1/2, 1)$. It follows that $\hat{x}(v_m, 0; c+2t, 0) > 1/2 = \hat{x}(v_m, 0; v_m, 0)$, implying that $\pi_A(v_m, 0; c+2t, 0) > \pi_A(v_m, 0; v_m, 0)$.

Second, consider the region of parameters satisfying $y_l \in [1, 2]$. In this region we have that $\beta^* = 2/y_l - 1$. Note that $\hat{x}(v_m, \beta^*; c+2t, 0) = \frac{c+2t-v_l}{4t} = \frac{1}{2} + \frac{c-v_l}{4t} \in (1/2, 1)$, since $\frac{c-v_l}{4t} = \frac{y_l - y_c}{4} \in (0, 1/2)$. Given that $\hat{x}(v_m, \beta^*; v_m, \beta^*) = 1/2 < \hat{x}(v_m, \beta^*; c+2t, 0)$, we obtain that $\pi_A(v_m, \beta^*; c+2t, 0) > \pi_A(v_m, \beta^*; v_m, \beta^*)$.

Finally, consider the case where $y_l < 1$, implying that in the inclusive-pricing equilibrium firms set recommendation policy with $\beta^* = 1$. Then, $\hat{x}(v_m, 1; c+2t, 0) = \frac{1}{2} + \frac{c+2t-v_l}{4t} - \frac{v_m - v_l}{2t}$. Note that $\frac{c+2t-v_l}{4t} - \frac{v_m - v_l}{2t} = \frac{1}{2} - \frac{y_l + y_c}{4} \in (0, 1/2)$ for any $y_c < y_l < 1$. This implies that $\hat{x}(v_m, 1; c+2t, 0) \in (1/2, 1)$. Given that $\hat{x}(v_m, 1; v_m, 1) = 1/2 < \hat{x}(v_m, \beta^*; c+2t, 0)$, we obtain that $\pi_A(v_m, 1; c+2t, 0) > \pi_A(v_m, 1; v_m, 1)$.

We conclude that for all parameters

$$\pi^*_{LS}(\alpha) = \pi_A(c+2t, 0; c+2t, 0) > \pi_A(v_m, \beta^*; v_m, \beta^*) = \pi^*_{IP}(\alpha). \qquad \square$$

B. Inclusive pricing for higher-value captives

In this appendix, we spell out the details of inclusive pricing for higher-value captives such that $c+2t < v_m$ (maintaining that $v_h - t > v_m > c > v_l$). We will show that recommendations to contestable consumers do not have any bias, whereas firms may inflate recommendations to their *own* captive consumers in equilibrium.

Consider a symmetric equilibrium candidate, in which both firms set prices $p_A = p_B = p^* < v_m$, employ the recommendation policies β^{co} and β^{ca} for contestable and captives consumers, respectively. In this equilibrium, the demand from contested consumers (who receive purchasing recommendations) is evenly split between the firms, and the captives consumers are also served (if instead the captives were not served, then either firm could profitably deviate to $\beta^{ca} = 0$ and additionally serve captives at the equilibrium price p^*).

Consider a local deviation of firm A to $(p, \beta^{co}_A, \beta^{ca}_A)$ such that $p < v_m$ and the captives consumers continue to buy, i.e., $r(\beta^{ca}) - p_i \geq r_0 = v_h - v_m$. Then, the profit of firm A is given by

$$\pi_A(p,\beta_A^{co},\beta_A^{ca};p^*,\beta^{co},\beta^{ca}) = (p_A-c)\left[\alpha\frac{1+\beta_A^{co}}{2}\hat{x}(p,\beta_A^{co};p^*,\beta^{co}) + \frac{1-\alpha}{2}\frac{1+\beta^{ca}}{2}\right],$$

where

$$\hat{x}(p_A,\beta_A;p_B,\beta_B) \equiv \frac{1}{2} + \frac{1+\beta_B}{4t}(p_B-v_l) - \frac{1+\beta_A}{4t}(p_A-v_l).$$

Since the IC constraint of captives must be binding, we have that

$$\beta_A^{ca}(p_A) = \frac{v_m-p_A}{p_A+(v_h-v_m)-v_l}.$$

Therefore, the deviation profit can be written as

$$\begin{aligned}\pi_A(p,\beta_A^{co},\beta_A^{ca};p^*,\beta^{co*},\beta^{ca*}) = {} & \alpha\frac{1+\beta_A^{co}}{2}\hat{x}(p,\beta_A^{co};p^*,\beta^{co*})(p_A-c)\\ & + \frac{1-\alpha}{2}\frac{v_h-v_l}{2}\left[1-\frac{(v_h-v_m)+(c-v_l)}{p_A+(v_h-v_m)-v_l}\right].\end{aligned}$$

Since the profits from captive consumers are locally increasing in p_A (with firm A adjusting β_A^{ca} accordingly), and following the same steps as in the proof of Lemma 4, we must have that $\beta^{co*}=0$ in equilibrium. Otherwise, firm A could deviate by choosing a lower β_A^{co} than β^{co*} and setting a price p_A above p^* in such a way that the indifferent consumer remains unchanged, wich increases profits from contestable consumers. Moreover, with $\beta_A^{co}>0$, since $p_A<v_m$, firm A can obtain a higher deviation profit also from its captive consumers by adjusting β_A^{ca} in response to the price increase.

In response to firm B's equilibrium strategy, firm A always sets $\beta_A^{co}=0$ to maximize its profits. Taking this into account, the deviation profit of firm A is

$$\frac{\alpha}{2}\left(\frac{1}{2}+\frac{p^*-p_A}{4t}\right)(p_A-c) + \frac{1-\alpha}{2}\frac{v_h-v_l}{2}\left[1-\frac{(v_h-v_m)+(c-v_l)}{p_A+(v_h-v_m)-v_l}\right].$$

Evaluatig the first-order condition of profit maximization at $p_A=p^*$ gives

$$\frac{\alpha}{4} - \frac{\alpha}{2}\frac{p^*-c}{4t} + \frac{1-\alpha}{2}\frac{v_h-v_l}{2}\frac{(v_h-v_m)+(c-v_l)}{(p^*+(v_h-v_m)-v_l)^2} = 0,$$

or equivalently,

$$p^* = c + 2t + \frac{1-\alpha}{\alpha}\frac{2t(v_h - v_l)((v_h - v_m) + (c - v_l))}{(p^* + (v_h - v_m) - v_l)^2}.$$

We obtain that we must have that $p^*(\alpha) > c + 2t$ in the candidate equilibrium for $\alpha < 1$. We observe that $\lim_{\alpha \to 1} p^*(\alpha) = c + 2t$.

We provide a numerical example to illustrate the equilibrium featuring a recommendation bias for captives. Take parameter values $v_h = 12, v_m = 10, v_l = 3, c = 6, t = 1.5, \alpha = 0.8$, which satisfy $v_h > v_m > c > v_l$ and $c + 2t = 9 < v_m = 10$. Here $v_h - v_m = 2$ and $v_h - v_m - v_l = -1$. Then, the equation determining the equilibrium price is given by

$$p^* = 6 + \frac{33.75}{(p^* - 1)^2}.$$

This equation has a unique root of $p^* \approx 9.4704 > c + 2t = 9$. The captive-side bias at p^* is

$$\beta^{ca*} = \frac{v_m - p^*}{p^* + (v_h - v_m) - v_l} = \frac{10 - 9.4704}{9.4704 - 1} \approx 0.0625 \in (0, 1),$$

which is interior. The recommendation probability to captives is $(1 + \beta^{ca*})/2 \approx 0.531$.

Equilibrium demand and profits follow directly. With unbiased recommendations to contestable consumers, the symmetric indifferent location is $\hat{x} = \frac{1}{2}$, so each firm's expected contestable demand is $\alpha \cdot \frac{1}{2} \cdot \hat{x} = \frac{\alpha}{4} = 0.20$. Each firm faces a captive mass $(1 - \alpha)/2 = 0.1$ and sells to a fraction $(1 + \beta^{ca*})/2 \approx 0.531$ of its captives (instead of 1/2 under unbiased recommendations). Thus, $D_A^{\text{ca}*} = 0.1 \times 0.531 \approx 0.0531$, and $D_A^* = 0.2 + 0.0531 \approx 0.2531$. The price-cost margin is $p^* - c \approx 3.4704$, yielding profit $\pi_A^* = \pi_B^* \approx 3.4704 \times 0.2531 \approx 0.8784$.

We note that a non-local deviation to $p'_A = v_m$ and $\beta^{\text{ca}'} = 0$ would yield demand $\frac{0.8}{2}\left(\frac{1}{2} + \frac{9.4704 - 10}{4 \times 1.5}\right) \approx 0.165$ for contestables and thus a deviation profit of $\pi'_A = (10 - 6)\left(0.165 + \frac{0.2}{4}\right) \approx 0.86$. This illustrates that firms do not find it profitable to deviate to a sufficiently high price that would extract the full surplus from their captive consumers.

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