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Propagation of Export Shocks: The Great Recession in Japan*

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Abstract

This study analyzes the Japanese economy during the Great Recession period (2007-2009). The Japanese GDP dropped significantly during this period, despite limited exposure to the US housing market, and exports also declined sharply. Motivated by this fact, we construct a multi-sector, multi-region small open economy model. Each region has a representative consumer, and regions and sectors are linked through inter-regional input-output tables and consumers' final demand. We measure the export shocks in each region-sector using trade statistics. Using our model, we quantitatively evaluate how the decline in export demand propagates throughout the country. We find that export shocks account for a significant portion of the GDP decline in many regions. To inspect the mechanism, we conduct counterfactual exercises in which we examine the change in GDP resulting from an export shock in a specific industry-region. The propagation is decomposed within and across regions, as well as within and across sectors.

Keywords: Great Recession, export demand, inter-regional input-output table, multi-sector model

JEL Classification: D57, E32, F41, F44, R15

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1 Introduction

The Great Recession, which spanned the period from 2007 to 2009, began with the collapse of the US housing market. It was not only the largest post-war recession in the United States at that time, but it also had a global impact. Japan's real GDP fell by 8.8% from the first quarter of 2008 to the first quarter of 2009. This decline was substantially larger even than the decline of real GDP in the United States during the same period, which was 3.3%. The drop in Japanese exports was even more significant: during the same period, the real value of Japanese exports fell by 36.1%. Given this collapse of exports, a large fraction of which were exports to the United States, it is natural to deduce that some of the decline in Japanese GDP was caused by a reduction in export demand, which arrived in Japan as an exogenous shock.

This event is a rare natural experiment where (i) macroeconomic shocks arrive at a large economy without much anticipation; (ii) shocks ("the impulse") are identifiable at the regional and industry level through the customs-level trade data; and (iii) the regional and sectoral links ("the propagation") can be traced through inter-regional input-output (IRIO) tables. Thus, a detailed examination of this event provides important insight into the macroeconomic propagation mechanism of exogenous shocks. The Japanese government constructs detailed IRIO tables every five years, which are an essential element in our analysis. It is not common to have such tables—the United States, for example, does not have such comprehensive information on region-industry level linkages. This study also provides a theoretical framework that allows us to conduct counterfactual experiments to examine the propagation process in detail.

We analyze the effect of this large export decline in Japan using a multi-sector, multi-region small open economy model. Shocks to exports from a particular region and industry propagate to other regions and industries through two channels. First, the reduction in demand in one region and industry decreases the demand for intermediate goods from another region and industry. Second, the decline in production reduces consumers' income through lower wages and profits. The reduction of consumption demand from another region also acts as a propagation mechanism.

We treat the export demand shock as an exogenous shock from the viewpoint of the Japanese economy. Bems et al. (2010) show that the large decline in durable demand was a major cause of the 2008-2009 trade collapse. A more recent paper by Miyamoto and Nguyen (2024) finds a similar result. Eaton et al. (2016) estimate a multicountry general equilibrium model and argue that the trade collapse during the Great Recession is mainly caused by the shift in spending away from tradable sectors.[1]

Our contributions are threefold. First, we develop a small open economy framework where export demand shocks drive the comovement of output, consumption, and labor input. Second, we quantitatively evaluate our dynamic general equilibrium model in the real business cycle tradition. We make progress in this research agenda by explicitly measuring shocks and tracing their propagation across sectors and regions. Third, our use of customs data to construct export demand shocks and the application of the IRIO matrix in the analysis of business cycle propagation are also novel.

Our study advances the real business cycle research program, in the tradition of Kydland and

[1] Some studies examine the supply-side factors. For example, Amiti and Weinstein (2011) analyzes how trade finance affected the decline in exports and finds that trade finance can explain less than half of the export decline in Japan during this period. Therefore, treating the export decline observed in the data as primarily caused by a decline in exogenous foreign demand is a reasonable approximation.

Prescott (1982) and Long and Plosser (1983). Since Frisch (1933), macroeconomists have analyzed the business cycle through the lens of shocks and their propagation. Since the 1980s, this approach has given rise to vector autoregressions on one side and the real business cycle approach on the other. Skeptics of real business cycle theory, such as Summers (1986) in response to Prescott (1986), have criticized the difficulty in interpreting the "shocks," particularly the technology shocks that were dominant in the early contributions. Cochrane (1994) also emphasizes this difficulty. We consider shocks that are particularly relevant to the Japanese economy during the Great Recession: export demand shocks. The progress we make here is that we can identify the "impulse" in two dimensions (exports from particular regions and industries) and can trace out the propagation process through (i) the IRIO matrix and (ii) production, consumption, and labor supply decisions in the model.

Many recent studies consider the propagation and amplification of shocks through the IO network (Atalay, 2017; Vom Lehn and Winberry, 2022; Liu and Tsyvinski, 2024). The classic studies include Long and Plosser (1983), Horvath (2000), Foerster et al. (2011), and Acemoglu et al. (2012). This literature primarily focuses on how productivity shocks in one industry propagate across different industries and impact the aggregate economy. This study advances the literature by considering the propagation of shocks across regions. In particular, our quantitative and analytical results speak to the nonlinear effects of production network (Baqaee and Farhi, 2019; Dew-Becker, 2023), regional multiplier effects (Nakamura and Steinsson, 2014; Flynn et al., 2024), and a gravity model of inter-regional transactions of tradable intermediate goods (Allen and Arkolakis, 2025).

Some recent papers also study the propagation of trade shocks. Huneeus (2020) analyzes the Chilean economy during the Great Recession using a firm-to-firm production network model. Dhyne et al. (2022) considers the Belgian economy. Similarly to our analysis, both papers treat the export shock as an exogenous decline in foreign demand. Their focus is on the firm-to-firm network, whereas the current paper highlights the propagation across industries and regions.

In the field of international trade, an emerging literature analyzes the global propagation of shocks through the international input-output network. Examples include Huo et al. (2023), Ho et al. (2024), and Boeckelmann et al. (2024). Our paper is analogous in the sense that we emphasize comovement across regions through the input-output network. However, these papers are unable to analyze the demand shock from outside the set of regions they model, given that there is no "outside" in their global-economy models. Our model is unique in that the demand shock comes outside the set of regions, and we can measure the impulse using the customs data.

A study with a motivation closely related to our paper is Caliendo et al. (2018). They analyze the propagation of sectoral productivity shocks across regions in the US economy. The difference between this study and Caliendo et al. (2018) is threefold. First, the model's characteristics differ. Their model is a closed-economy model with perfect competition based on Eaton and Kortum (2002). Our model is a dynamic small open economy model that features monopolistic competition, and the monopolistic competition structure allows us to analyze the effect of price rigidity. Second, their analysis focuses on productivity shocks, whereas we consider foreign demand shocks. Our foreign demand shocks are very large in size, compared to typical productivity shocks, and the shocks are directly measurable through the trade data. Third, their regional analysis is based on the Commodity Flow Survey and is limited to the manufacturing sector. Our inter-regional input-output matrix includes all sectors. The inclusion of the service sector is important, given its size and the significance of the inter-regional service trade. Recent studies emphasize that service trade is significant even in the international context (Han et al., 2025). The tradability of services is expected to be even higher within a country.

The remainder of this paper is organized as follows. Section 2 provides an overview of the main macro facts about the Great Recession in Japan. Section 3 describes the model. Section 4 presents analytical results in a special case to illustrate the basic mechanism of export-demand-led output fluctuations. Section 5 computes the model and calibrates it to the data. Section 6 simulates the model with the export series. Section 7 conducts counterfactual experiments and decomposes various channels of the change in regional GDP. Section 8 presents a simplified model to examine various factors of propagation and analyzes the model's nonlinearity. Section 9 concludes.

2 Overview of the Great Recession in Japan

In this section, we present the general time-series pattern of various statistics from Japan during the Great Recession period. We present statistics for the country as a whole, by region, and by industry. Our data primarily comes from public sources, including the Ministry of Economy, Trade and Industry (METI), the Research Institute of Economy, Trade and Industry (RIETI), the Ministry of Health, Labour and Welfare (MHLW), the Ministry of Finance (MoF), and the Statistics Bureau of Japan. METI has provided the inter-regional input-output tables (IRIO) until 2005, constructed for 9 regions and x number of industries, where $x \in \{12, 29, 53\}$.[2] We reconstruct an IRIO with 26 industries and 9 regions using the tables from 2005 so that the industry classification aligns with the Prefectural Account.[3] We use the JIP database from the RIETI to compute the technology parameter later. The Labour Force Survey (LFS) from the Statistics Bureau and the Monthly Labour Survey (MLS) conducted by the MHLW are used to calibrate parameters related to labor supply for each region.[4] Finally, to construct regional and sectoral export data, we use the Trade Statistics of Japan (TSJ), constructed by the MoF.

2.1 Time series of GDP and export

Figure 1 plots the time series of the quarterly real GDP (seasonally adjusted, in billions of 2015 yen), adjusted to annual values. One can see the sharp drop in 2008. From the first quarter of 2008 to the first quarter of 2009, the decline in real GDP was approximately 8.8%. Figure 2 draws the time series of the exports of goods and services (seasonally adjusted, in billions of 2015 yen).[5] The decline of export from the first quarter of 2008 to the first quarter of 2009 was larger than that of GDP, amounting to 36.1%.

Figure 3 plots the export time series separately across industries. All industries experienced a decline in exports during the Great Recession, and the decline was particularly sharp for the transportation equipment (TE) industry. The TE industry includes automobile industry, and this outcome largely reflects the decline in automobile demand in the United States.

[2] The construction method of IRIO is detailed in Arai (2020) and references therein. Transactions in manufacturing sectors are based on the Commodity Distribution Survey. Interregional transactions in services are reported when estimation is feasible. Some service sectors, such as construction, are categorically considered non-tradable. Factors considered include margins in retail and financial sectors, service consumption by tourists and commuters, and back-office and managerial services provided internally within multi-regional firms.

[3] The Prefectural Account is the prefectural version of the GDP statistics. See, https://www.esri.cao.go.jp/jp/sna/sonota/kenmin/kenmin_top.html (only the Japanese version is available).

[4] See, https://www.stat.go.jp/english/index.html.

[5] Both series are taken from https://www.esri.cao.go.jp/jp/sna/data/data_list/sokuhou/files/2022/qe221_2/tables/gaku-jk2212.csv.

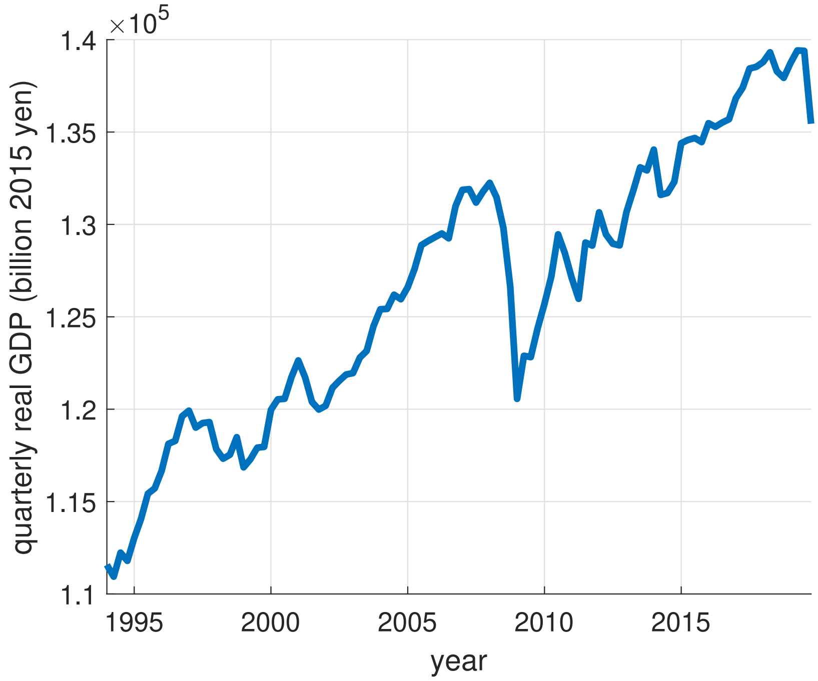

Figure 1: Time series of Japanese GDP

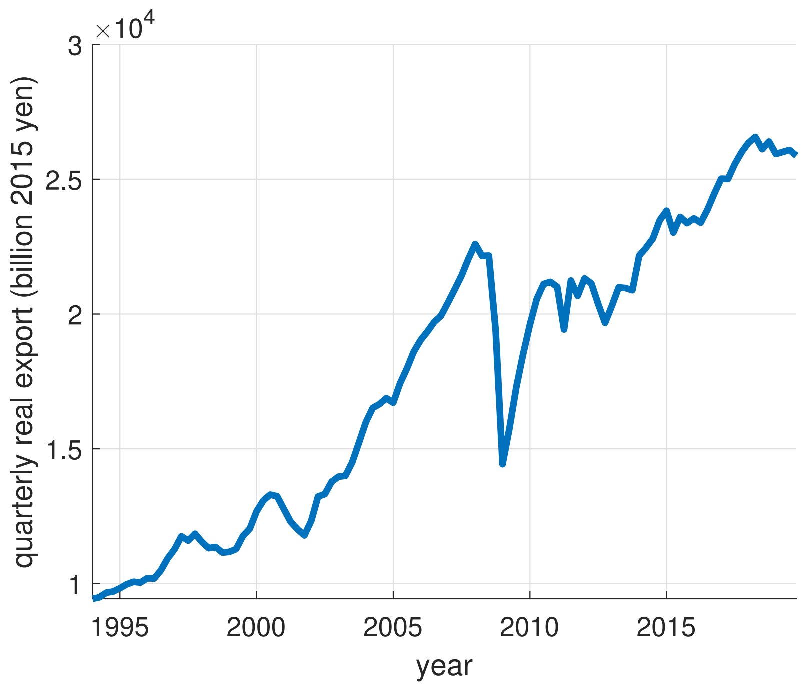

Figure 2: Time series of Japanese exports

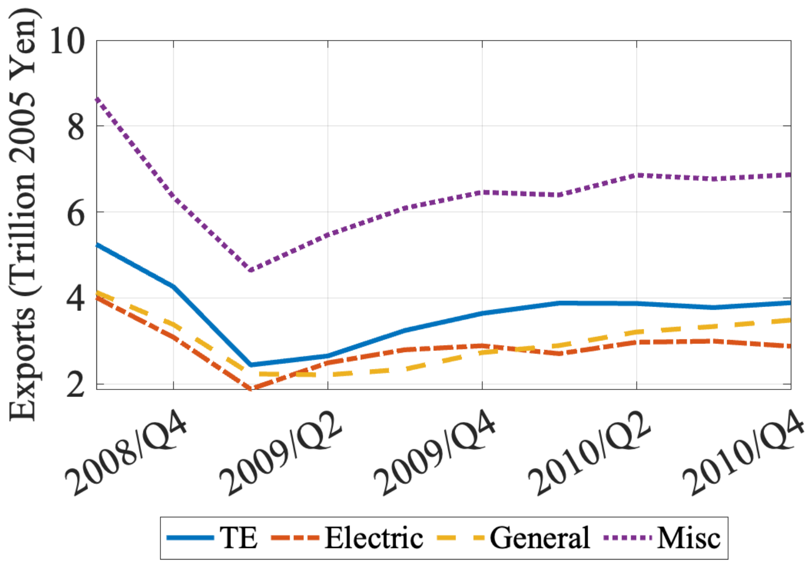

Figure 3: Real sectoral exports: 2008Q3−2010Q4. "TE," "General," "Electric," and "Misc" refer to Transportation Equipment, General Machinery, Electric Machinery, and Miscellaneous (comprising the rest of the 23 industries).

2.2 Regional heterogeneity

A key aspect of our analysis is the explicit consideration of regional linkages. Figure 4 describes how we divide Japan into nine regions: Hokkaidō, Tōhoku, Kantō, Chūbu, Kansai,[6] Chūgoku, Shikoku, Kyūshū, and Okinawa. The division is mainly motivated by the availability of the inter-regional IO matrices. The precise mapping of prefectures into regions is listed in Appendix A. In Figure 4, a thicker color indicates a larger value of regional GDP. Kantō, including the Tōkyō area, is the largest economic region among the nine. Kansai includes the Ōsaka area, which is the second-largest economic region, and Chūbu includes Nagoya, the third-largest economic region. Chūbu also includes the headquarters of Toyota, the largest automaker and auto exporter.

Figure 5 plots the time series of the regional real GDP. All regions except for Okinawa experienced a significant decline in GDP during the Great Recession. We can also observe a considerable heterogeneity across regions in terms of the magnitude of the decline.

Figure 6 draws the export series from 2008Q3 to 2010Q4 for each region as a fraction of 2008Q3 GDP in that region. Drawn from the TSJ, the details of the export data construction are presented in Appendix B. The time series reveals considerable heterogeneity across regions, both in the composition of industries and the magnitude of the drop in exports during the Great Recession. The most severe shock occurred in the transportation equipment sector in the Chūbu region: its export value declined by 62.8% in the first quarter of 2009.

[6]Kansai corresponds to "Kinki" in the inter-regional input-output dataset in Japanese. We follow METI's English expression for the region.

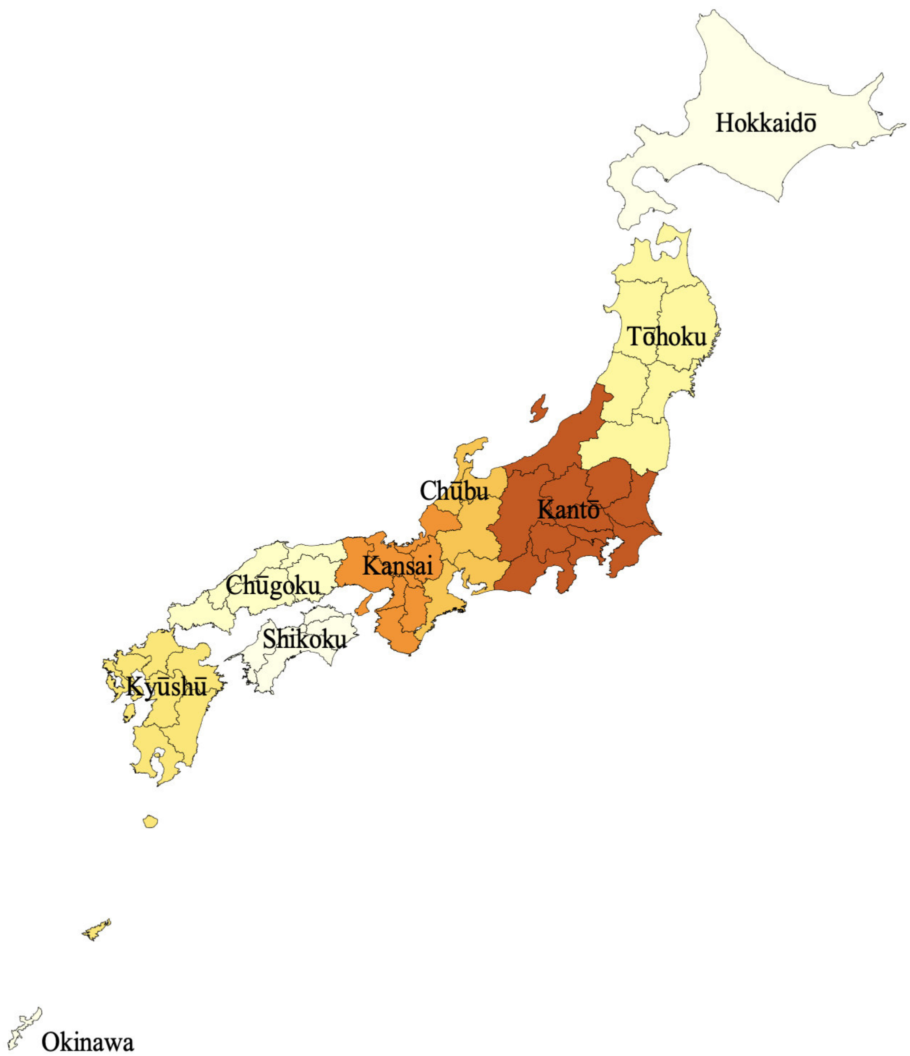

Figure 4: Division of Japanese Regions

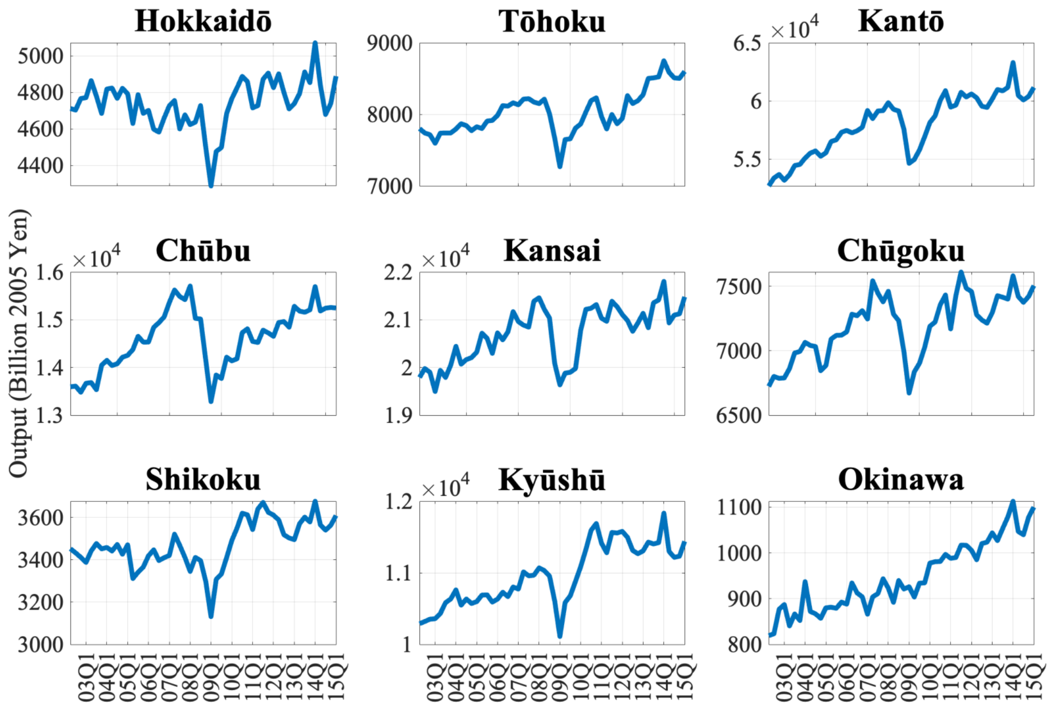

Figure 5: Regional GDP: 2002Q4−2015Q1

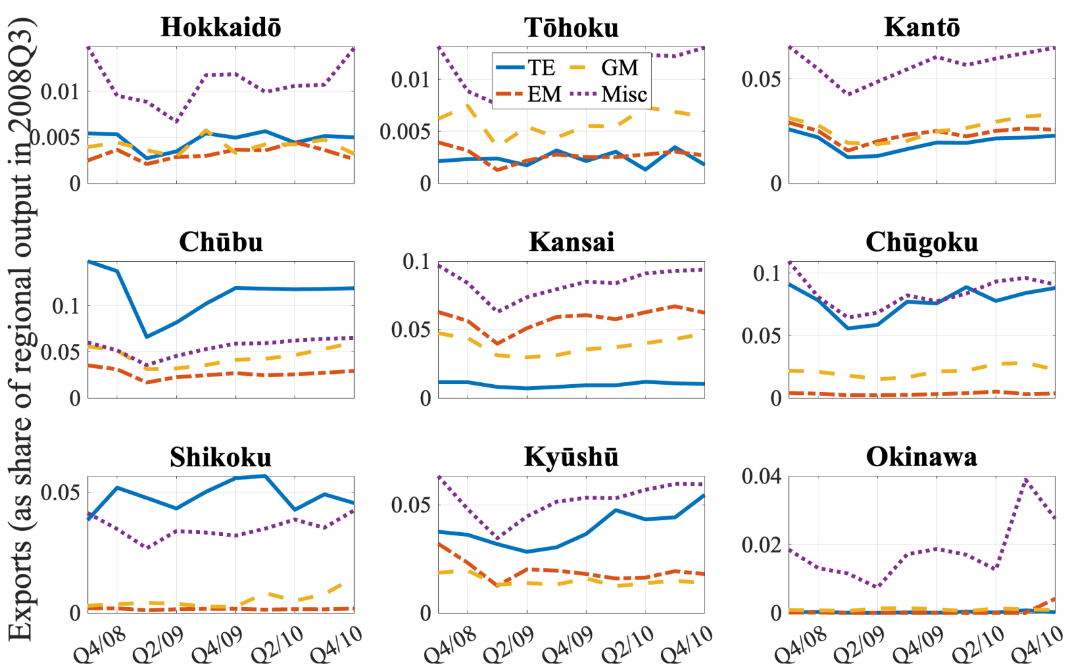

Figure 6: Regional exports: 2008Q3−2010Q4. "TE", "GM", "EM", and "Misc" refer to Transportation Equipment, General Machinery, Electric Machinery, and Miscellaneous (comprising the rest of the 23 industries).

2.3 Gravity effects in inter-regional transactions

Because the IRIO data contains a geographical dimension, the tables reveal a gravity structure of inter-regional trade in intermediate demands for manufacturing and services. This section investigates the quantitative impact of distance on inter-regional transactions. This exercise is not only of independent interest, as empirical studies on inter-regional gravity structures are scarce, but also an important counterpart to the structural model in the next section, as our comparative static exercise reveals the transmission of shocks across regions. Note that the gravity results in this section and the transmission of shocks in the structural model do not have a direct one-to-one correspondence, especially because the reduced-form regression does not focus on particular industries where the shocks arrive.

Below, we see that a straightforward OLS estimation reveals that geographical distance has a significant impact on inter-regional demand for intermediate manufacturing. We define $DISTANCE(i, j)$ as the geographic distance between the regional offices of the Bureau of Economy, Trade and Industry in regions i and j.[7] We regress the $\log_{10}(T(si, hj))$ variable—the common logarithm of the transaction value of sector s in region i sold to sector h in region j—on this distance measure, controlling for fixed effects for the sectoral pair (s, h), home i and destination j regions, and an indicator for $i = j$ to isolate the home bias in intermediate demand for any sectoral pair. We split the sample based on whether the originating sector s is non-services (classification codes 1 to 16 in Table 7) or services (17 to 26). We also include a regression for intermediate demand from j aggregated across h, represented by M_j.

In addition, we estimate the gravity equations when the demand for si (sector s in region i) comes from consumption C_j and investment X_j of region j. In these regressions, the sample size is smaller than the one for intermediate demand from hj, since we lose the dimension of the destination sector h.

Table 1 shows the estimation results. The benchmark estimate for $T(si, hj)$, which includes fixed effects for sectoral pairs (s, h), indicates that distance has a negative effect on intermediate demand. Increasing the distance by 1,000 km decreases the log transaction by 0.35 for non-service sectors, a magnitude comparable to the one reported in Allen and Arkolakis (2025) for US manufacturing goods. Interestingly, when the sample is restricted to service sectors, the estimate remains similar: -0.36. Note that our regression accounts for a home bias effect, $i = j$. Thus, the coefficient reflects the impact of distance on only tradable goods and services. The home bias effect is substantial, estimated at 2.01 for services, while 0.84 for non-services. These estimates suggest that home demand in services far exceeds demand from outside by a factor of 10^2, while the gap between home and outside demands for non-services is less than a factor of 10. Therefore, the data imply that the flow of intermediate demand follows a geographic pattern, with neighboring regions being more closely interconnected, while home bias dominates demand in the service sectors.

The estimates using the aggregated intermediate (M_j), consumption (C_j), and investment (X_j) demand also reveal an adverse impact of distance and a strong home bias for services. The negative impact of distance on M_j is more pronounced than on (hj), suggesting that the heterogeneity of sector pair (s, h) explains some of the negative effects seen in aggregate M_j. Comparing the aggregate effects of M_j, C_j, and X_j, we find that the distance's influence on consumption demand is less significant than on intermediate demand. Additionally, we observe that the home bias in services is more pronounced in consumption demand than in investment demand. In the following

[7] See https://www.meti.go.jp/english/network/regionalbureau.html for details.

Originating sector s	$\log_{10} T(si, hj)$ non	$\log_{10} T(si, hj)$ service	$\log_{10} T(si, M_j)$ non	$\log_{10} T(si, M_j)$ service	$\log_{10} T(si, C_j)$ non	$\log_{10} T(si, C_j)$ service	$\log_{10} T(si, X_j)$ non	$\log_{10} T(si, X_j)$ service
Distance (1000km)	−0.35	−0.36	−0.69	−0.53	−0.51	−0.45	−0.31	−0.21
	(0.010)	(0.011)	(0.040)	(0.052)	(0.042)	(0.058)	(0.050)	(0.068)
Home ($i = j$)	0.84	2.01	0.93	2.20	0.93	2.17	0.53	1.26
	(0.019)	(0.019)	(0.072)	(0.093)	(0.077)	(0.103)	(0.091)	(0.121)
FE i and j	✓	✓	✓	✓	✓	✓	✓	✓
FE $s \times h$	✓	✓						
FE s			✓	✓	✓	✓	✓	✓
N. obs	33696	21060	1296	810	1296	810	1296	810

Table 1: Estimates of the gravity equation. All estimated coefficients are statistically significant at the 1% level.

analysis, we will use this information to interpret our numerical results on regional propagations.

3 Model

Our model is a natural extension of a small open economy real business cycle model to a multi-sector, multi-region setting. The economy consists of I regions. In each region, S industries operate. An industry is indexed by (s, i), where $s \in [0, S]$ and $i \in [0, I]$. In the model description, we treat s and i as real numbers, although they are integers in the quantitative exercise.

We adopt monopolistic competition as the market structure. One could alternatively assume perfect competition for each good produced in each region, and the propagation mechanism would work similarly. We utilize this formulation partly because this type of formulation is common in international trade models. More importantly, this formulation enables us to analyze the situation where prices are sticky.

We assume each industry (s, i) is monopolistic, and only one firm produces product (s, i). The production of a good requires capital, labor, and intermediate goods. As we will detail later, we assume that capital and labor inputs are not mobile across regions; that is, industries in region i have to use the capital and labor supplied in region i. Intermediate goods are mobile; that is, a firm in region i can use intermediate inputs from any region.

As in the standard real business cycle model, there is one representative consumer in each region. The representative consumer maximizes the discounted sum of instantaneous utility over an infinite horizon, making consumption and saving decisions, as well as labor supply decisions. Firms make static production decisions, hiring labor, renting capital stock, and purchasing intermediate goods. Each firm's goods are used for consumption, investment, export, and the production of intermediate goods.

3.1 Representative consumer

The representative consumer in region i solves the utility-maximization problem

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left[\frac{(C_{i,t})^{1-\sigma_c} - 1}{1 - \sigma_c} - \chi \frac{(N_{i,t})^{1+\zeta}}{1+\zeta} \right] \tag{1}$$

subject to

$$\int_0^S \int_0^I p_{sj,t} c^i_{sj,t} dj ds + \int_0^S p_{sf,t} c^i_{sf,t} ds + \int_0^S \int_0^I p_{sj,t} x^i_{sj,t} dj ds + \int_0^S p_{sf,t} x^i_{sf,t} ds \\ \leq \int_0^S w_{si,t} n_{si,t} ds + r_{i,t} K_{i,t} + \Pi_{i,t} + B_{i,t} \tag{2}$$

and

$$K_{i,t+1} = (1-\delta) K_{i,t} + X_{i,t},$$

where

$$C_{i,t} = \left[\int_0^S \int_0^I (\xi^i_{sjc})^{\frac{1}{\sigma}} (c^i_{sj,t})^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\xi^i_{sfc})^{\frac{1}{\sigma}} (c^i_{sf,t})^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}},$$

$$X_{i,t} = \left[\int_0^S \int_0^I (\xi^i_{sjx})^{\frac{1}{\sigma}} (x^i_{sj,t})^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\xi^i_{sfx})^{\frac{1}{\sigma}} (x^i_{sf,t})^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}},$$

and

$$N_{i,t} = \left[\int_0^S (n_{si,t})^{\frac{\tau+1}{\tau}} ds \right]^{\frac{\tau}{\tau+1}}.$$

Here, $C_{i,t}$ is the consumption of composite goods in period t. The price of good (s, j) in period t is represented as $p_{sj,t}$, and it is assumed to be common across destination regions i. The prices of the imported goods are assumed to be exogenously given at $p_{sf,t}$. The notation c^i_{sj} represents the consumption of good s from region j by agent i, while ξ^i_{sj} is a parameter. The consumption goods with c^i_{sf} are imported goods, where f represents "foreign." The imported goods have a S variety, which is common across regions and exogenously given. The variable N_i is the composite labor supply. The consumer supplies labor n_{si} for the production of good s. The labor market is perfectly competitive, and the wage rate is w_{si} for industry (s, i). The variable $\Pi_{i,t}$ is the profit from the firms in region i. $B_{i,t}$ denotes an international transfer exogenous to the households in i. The variable $X_{i,t}$ is the investment of composite goods by consumer i in period t. The notation x^i_{sj} represents the investment of good s from region j by region i in period t and ξ^i_{sjx} is a time-invariant parameter. $K_{i,t}$ is the capital stock in region i and period t, which is augmented via the region-specific investment good $X_{i,t}$ and depreciates at rate δ every period.

For the consumption of the goods, the consumer allocates consumption across goods by solving the expenditure-minimization problem each period t:

$$\min_{\{c^i_{sj,t}\}_{sj}} \int_0^S \int_0^I p_{sj,t} c^i_{sj,t} dj ds + \int_0^S p_{sf,t} c^i_{sf,t} ds$$

subject to

$$C_{i,t} = \left[\int_0^S \int_0^I (\xi^i_{sjc})^{\frac{1}{\sigma}} (c^i_{sj,t})^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\xi^i_{sfc})^{\frac{1}{\sigma}} (c^i_{sf,t})^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}.$$

The solution of the optimization implies the demand for domestic goods

$$c^i_{sj,t} = \left(\frac{p_{sj,t}}{P^c_{i,t}} \right)^{-\sigma} \xi^i_{sjc} C_{i,t},$$

and for imported foreign goods

$$c^i_{sf,t} = \left(\frac{p_{sf,t}}{P^c_{i,t}} \right)^{-\sigma} \xi^i_{sfc} C_{i,t}, \tag{3}$$

where the price index $P^c_{i,t}$ is written as

$$P^c_{i,t} \equiv \left[\int_0^S \int_0^I \xi^i_{sjc}(p_{si,t})^{1-\sigma} dids + \int_0^S \xi^i_{sfc}(p_{sf,t})^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}}. \quad (4)$$

Similarly, the household's minimization of investment costs yields

$$x^i_{sj,t} = \left(\frac{p_{sj,t}}{P^x_{i,t}}\right)^{-\sigma} \xi^i_{sjx} X_{i,t}$$

for $j \in \{[0, I], f\}$, where the price index $P^x_{i,t}$ is given as

$$P^x_{i,t} \equiv \left[\int_0^S \int_0^I \xi^i_{sjx}(p_{sj,t})^{1-\sigma} djds + \int_0^S \xi^i_{sfx}(p_{sf,t})^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}}.$$

The consumer's budget constraint can now be rewritten as

$$P^c_{i,t} C_{i,t} + P^x_{i,t} X_{i,t} \leq \int_0^S w_{si,t} n_{si,t} ds + r_{i,t} K_{i,t} + \Pi_{i,t} + B_{i,t}.$$

The consumer's intertemporal optimization implies the Euler equation and the labor supply relationship

$$\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{-\sigma_c} = \frac{1}{1+\rho}\left(1 + \frac{r_{i,t+1}}{P^x_{i,t+1}} - \delta\right) \quad (5)$$

and

$$\frac{w_{si,t}}{P^c_{i,t}} = \chi (C_{i,t})^{\sigma_c} (N_{i,t})^{\zeta} \left(\frac{n_{si,t}}{N_{i,t}}\right)^{\frac{1}{\tau}}. \quad (6)$$

3.2 Production

In region i, good h is produced by the production function

$$y_{hi,t} = A_{hi}(M_{hi,t})^{\alpha}(N_{hi,t})^{\beta}(K_{hi,t})^{1-\alpha-\beta}, \quad (7)$$

where

$$M_{hi,t} = \left[\int_0^S \int_0^I (\gamma^{hi}_{sj})^{\frac{1}{\sigma}} (m^{hi}_{sj,t})^{\frac{\sigma-1}{\sigma}} djds + \int_0^S (\gamma^{hi}_{sf})^{\frac{1}{\sigma}} (m^{hi}_{sf,t})^{\frac{\sigma-1}{\sigma}} ds\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, $m^{hi}_{sj,t}$ is intermediate good s from region j (and $m^{hi}_{sf,t}$ imported good s) used in production of good h in region i and period t, and γ^{hi}_{sj} is a parameter.

The demand function for intermediate goods is

$$m^{hi}_{sj,t} = \left(\frac{p_{sj,t}}{P^m_{hi,t}}\right)^{-\sigma} \gamma^{hi}_{sj} M_{hi,t},$$

for $j \in \{[0, I], f\}$, where

$$P^m_{hi,t} \equiv \left[\int_0^S \int_0^I \gamma^{hi}_{sj}(p_{sj,t})^{1-\sigma} djds + \int_0^S \gamma^{hi}_{sf}(p_{sf,t})^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}}. \quad (8)$$

Thus, the total demand for good (s, j) in period t is, by adding the consumption demand, investment demand, and intermediate good demand,

$$\begin{aligned} y_{sj,t} &= \int_0^I (c^i_{sj,t} + x^i_{sj,t})di + \int_0^S \int_0^I m^{hi}_{sj,t} didh + y^f_{sj,t} \\ &= (p_{sj,t})^{-\sigma} \left(\int_0^I \left((P^c_{i,t})^\sigma \xi^i_{sjc} C_{i,t} + (P^x_{i,t})^\sigma \xi^i_{sjx} X_{i,t} \right) di + \int_0^S \int_0^I (P^m_{hi,t})^\sigma \gamma^{hi}_{sj} M_{hi,t} didh \right) + y^f_{sj,t}, \end{aligned} \tag{9}$$

where $y^f_{sj,t}$ represents the foreign (export) demand in period t. Assume that foreign demand takes the form

$$y^f_{sj,t} = \omega^f_{sj,t}(p_{sj,t})^{-\sigma}(\bar{P}_t)^\sigma, \tag{10}$$

that is, foreign demand has the same price elasticity as domestic demand. $\bar{P}$ is the price level in the foreign country.

The demand of good (s, j) can be expressed as $(p_{sj,t})^{-\sigma} D_{sj,t}$ where $D_{sj,t}$ is a demand shifter:

$$D_{sj,t} \equiv \left(\int_0^I \left((P^c_{i,t})^\sigma \xi^i_{sjc} C_{i,t} + (P^x_{i,t})^\sigma \xi^i_{sjx} X_{i,t} \right) di + \int_0^S \int_0^I (P^m_{hi,t})^\sigma \gamma^{hi}_{sj} M_{hi,t} didh \right) + \omega^f_{sj,t}(\bar{P}_t)^\sigma. \tag{11}$$

We analyze the firm's problem in two steps. First, the firm chooses the combination of inputs to minimize the unit cost:

$$\min_{M_{sj,t}, N_{sj,t}, K_{sj,t}} \; P^m_{sj,t} M_{sj,t} + w_{sj,t} N_{sj,t} + r_{j,t} K_{sj,t}$$

subject to

$$1 = A_{sj}(M_{sj,t})^\alpha (N_{sj,t})^\beta (K_{sj,t})^{1-\alpha-\beta}.$$

The solution yields the unit cost $\lambda_{sj,t}$:

$$\lambda_{sj,t} = \frac{(P^m_{sj,t})^\alpha (w_{sj,t})^\beta (r_{j,t})^{1-\alpha-\beta}}{A_{sj} \alpha^\alpha \beta^\beta (1-\alpha-\beta)^{1-\alpha-\beta}} \tag{12}$$

and the derived factor demand for unit output:

$$M_{sj,t} = \frac{\alpha}{P^m_{sj,t}} \lambda_{sj,t},$$

$$N_{sj,t} = \frac{\beta}{w_{sj,t}} \lambda_{sj,t},$$

and

$$K_{sj,t} = \frac{1-\alpha-\beta}{r_{j,t}} \lambda_{sj,t}.$$

Second, the firm maximizes profit:

$$\max_{p_{sj,t}} \; (p_{sj,t} - \lambda_{sj,t})(p_{sj,t})^{-\sigma} D_{sj,t}. \tag{13}$$

The result is the standard constant markup rule:

$$p_{sj,t} = \frac{\sigma}{\sigma - 1} \lambda_{sj,t}. \tag{14}$$

Thus the production of good (s, j) is

$$y_{sj,t} = \left(\frac{\sigma}{\sigma - 1}\lambda_{sj,t}\right)^{-\sigma} D_{sj,t}. \tag{15}$$

The derived factor demand can, therefore, be computed from:

$$M_{sj,t} = \frac{\alpha}{P^m_{sj,t}}\lambda_{sj,t}y_{sj,t}, \tag{16}$$

$$N_{sj,t} = \frac{\beta}{w_{sj,t}}\lambda_{sj,t}y_{sj,t}, \tag{17}$$

and

$$K_{sj,t} = \frac{1 - \alpha - \beta}{r_{j,t}}\lambda_{sj,t}y_{sj,t}. \tag{18}$$

3.3 Trade balance

We allow for trade imbalances at both the national and regional levels. Trade imbalances are possible in our model only when international transfers exist. We denote by $B_{i,t}$ an (exogenous) international transfer to households in the region i at time t. Then, the international trade account in our model is written as

$$\int_0^I B_{i,t}di = \int_0^S \int_0^I p_{sf,t}\left(c^i_{sf,t} + x^i_{sf,t} + \int_0^S m^{hi}_{sf,t}dh\right) dids - \int_0^S \int_0^I p_{si,t}y^f_{si,t}dids. \tag{19}$$

The first term is the value of imports, where $p_{sf,t}$ is the price of import good s, which is exogenous from the small open economy assumption, and $(c^i_{sf,t}, x^i_{sf,t}, m^{hi}_{sf,t})$ are the consumption, investment, and industry h's intermediate demand by region i of import good s. The second term on the right-hand side is the value of export, where $p_{si,t}$ is the price of good s from region i and $y^f_{si,t}$ is the export demand of good s from region i.

3.4 Equilibrium

We treat the foreign-good prices $p_{sf,t}$ for all s, t as exogenous. Given the goods prices (both domestic and foreign), factor prices, and profit income, consumers (in region i, at time t) demand consumption goods $(c^i_{sj,t}, c^i_{sf,t})$ for all j and investment goods $(x^i_{sj,t}, x^i_{sf,t})$ for all j as the result of optimization (1).

Firms (in region j, at time t) face demand from consumers and foreign countries as a function of $p_{sj,t}$. Given factor prices, the firm sets the price of its good to maximize profit (13).

The good market clears for each good in each region. The total demand for good s in region i is expressed as (9), with export demand (10), and the total supply for good h in region i is expressed as (7). The market-clearing conditions for labor and capital require:

$$N_{si,t} = n_{si,t},$$

and

$$\int_0^S K_{si,t}ds = K_{i,t}$$

for all (s, i) and i. The profit income in equilibrium has to satisfy

$$\Pi_{i,t} = \int_0^S (p_{si,t} - \lambda_{si,t}) y_{si,t} ds. \tag{20}$$

Note that once the goods markets and the factor markets clear, the trade balance condition (19) is automatically satisfied. Later in the quantitative analysis, we will set $B_{i,t}$ as a value that fits the data. This fact can be seen from aggregating the budget constraint for the consumer (2) and imposing the market-clearing conditions.

4 Basic mechanism: an analysis of a simple static model

In this section, we present an analysis of a special case of the model to obtain an intuition on how foreign demand shock affects the aggregate economy. We will make three main points. First, a foreign demand shock results in an increase in output and domestic prices (relative to the foreign prices). Second, there is a multiplier effect. Third, the change in prices attenuates the effect of shocks.

For analytical tractability, we abstract from saving and capital accumulation and analyze a static model. The full static model is described in Appendix F. In this section, we further make a simplifying assumption that all sectors and regions are homogeneous, that is, goods from all sector-regions enter the CES aggregators symmetrically: $\xi^i_{sj} = \xi^i_{sf} = \gamma^{hi}_{sj} = 1, \forall h, i, j, s$. We let the import weight for intermediate goods be $\gamma^{hi}_{sf} = \gamma_f, \forall h, i, s$. In addition, regions are assumed symmetric in productivity: $A^{hi} = A, \forall h, i$.

To simplify the expression, let us set $S = I = 1$ and $A = (\alpha^\alpha(1-\alpha)^{1-\alpha})^{-1}$. In this symmetric environment, the consumer's demand for domestic and foreign goods can be written as

$$c^i_{sj} = c = \left(\frac{p}{P}\right)^{-\sigma} C$$

and

$$c^i_{sf} = c_f = \left(\frac{p_f}{P}\right)^{-\sigma} C,$$

where the price index is

$$P = (SIp^{1-\sigma} + Sp_f^{1-\sigma})^{1/(1-\sigma)} = (p^{1-\sigma} + p_f^{1-\sigma})^{1/(1-\sigma)}$$

and the aggregate consumption is

$$C = (SIc^{(\sigma-1)/\sigma} + Sc_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} = (c^{(\sigma-1)/\sigma} + c_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}.$$

The intermediate-good demand for domestic goods is

$$m = \left(\frac{p}{P_m}\right)^{-\sigma} M$$

and that for import goods is

$$m_f = \left(\frac{p_f}{P_m}\right)^{-\sigma} \gamma_f M,$$

where

$$P_m = (p^{1-\sigma} + \gamma_f p_f^{1-\sigma})^{1/(1-\sigma)}.$$

Thus, $M = (m^{(\sigma-1)/\sigma} + \gamma_f^{1/\sigma} m_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$. We assume the foreign demand as $y^f = p^{-\sigma}\omega^f$, where ω^f is the foreign demand parameter. The total demand for a good is

$$y = p^{-\sigma}(IP^\sigma C + SIP_m^\sigma M + \omega^f) = p^{-\sigma}(P^\sigma C + P_m^\sigma M + \omega^f).$$

The optimal pricing of the monopolist follows the markup rule:

$$p = \frac{\sigma}{\sigma - 1}\lambda,$$

where λ is the unit cost

$$\lambda = P_m^\alpha w^{1-\alpha}.$$

Because the (perfectly competitive) final-good sector's production function is $y = AM^\alpha N^{1-\alpha}$, the factor demand of the final-good firms are

$$M = \frac{\alpha \lambda y}{P_m}$$

and

$$N = \frac{(1-\alpha)\lambda y}{w}.$$

Hence, $M/N = (\alpha/(1-\alpha))(w/P_m)$.

The labor supply function is, from the household's optimization,

$$\frac{w}{P} = \chi C^{\sigma_c} N^\zeta.$$

We obtain the following result.

Proposition 1 *Suppose $\sigma_c \leq 1$. An equilibrium exists uniquely for each $\omega^f > 0$, and the equilibrium p/P decreases continuously to 1 when ω^f decreases to 0. Equilibrium consumption C, output y, real wages w/P, and the price p of domestically produced intermediate goods relative to imported goods are strictly increasing functions of ω^f.*

Proof: See Appendix D.

In Appendix D, we show that equilibrium (p, y) is determined by the two equations

$$y = \frac{p^{-\sigma}\omega^f}{1 - (\mu_c(p/P)^{1-\sigma} + (1-\mu_c)(p/P_m)^{1-\sigma})}$$

and

$$y^{\sigma_c+\zeta} = \left(\frac{p}{P}\right)^{1-\sigma_c} \left(\frac{p}{P_m}\right)^{\frac{\alpha(1+\zeta)}{1-\alpha}} \frac{((\sigma-1)/\sigma)^{\frac{1+\alpha\zeta}{1-\alpha}}}{\chi \xi_c^{\sigma_c}(1-\alpha)^\zeta}$$

with definitions of P and P_m, where $\mu_c \equiv 1 - \alpha(\sigma-1)/\sigma$ turns out to be the share of consumption expenditure in output sales $\mu_c = PC/py$.

The first equation shows that an exogenous increase in export revenue $p^{1-\sigma}\omega^f$ pushes up sales py with a multiplier effect. The term $1 - \left(\mu_c(p/P)^{1-\sigma} + (1-\mu_c)(p/P_m)^{1-\sigma}\right)$ reduces to $(1 - (p/P)^{1-\sigma})\mu_c$ when $\gamma_f = 0$. Instead, if $\gamma_f = 1$, this term is $1 - (p/P)^{1-\sigma}$. Hence, there is a "leakage" of the multiplier effect in the presence of intermediate imports. The increase of demand by ω^f also raises domestic price p and dampens the multiplier in the equation. This secondary effect, however,

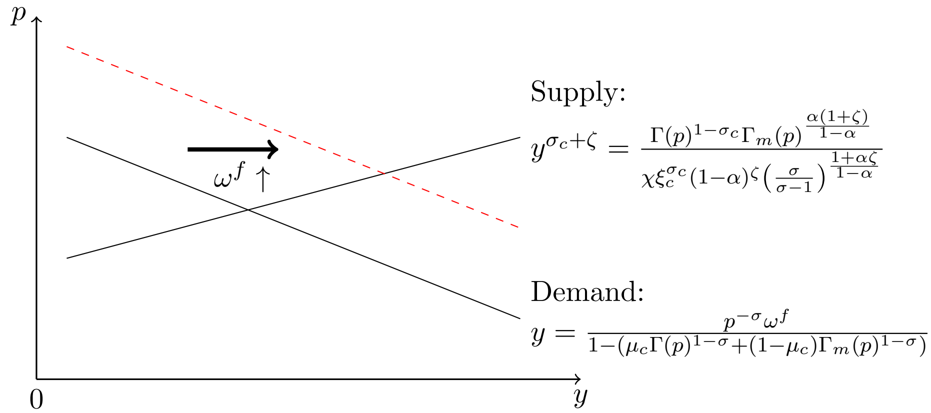

Figure 7: Demand and supply for domestically produced goods. ($\Gamma(p)$ and $\Gamma_m(p)$ are increasing functions and satisfy $\Gamma(p) = p/P$ and $\Gamma_m(p) = p/P_m$ in equilibrium.)

does not overturn the primary effect. The first equation can be drawn as a downward-sloping curve on the (y, p) plane. The second equation is definitely upward-sloping if $\sigma_c < 1$, and it may be upward-sloping under $\sigma_c > 1$ if the effect of intermediate import $\gamma_f > 0$ is sufficiently strong. Only the first equation shifts with ω^f, and we can see that the result on y in Proposition 1 comes from the slope of this second equation.

The mechanism is reminiscent of the demand externality effect in a monopolistically competitive economy with international trade, as in Matsuyama (1992), although our model lacks the increasing returns to scale feature. An increase in foreign demand increases the price of domestically produced intermediate goods and wages. If $\sigma_c = 1$ and $\gamma_f = 0$, the equilibrium employment does not change because the income and substitution effects cancel out, but the income is increased, leading to greater consumption. The total value added $(1 - \alpha + \alpha/\sigma)py$ is spent on domestic goods $p^{1-\sigma}C$ and import $p_f^{1-\sigma}C$. C increases proportionally to revenue py and less than proportionally to ω^f. Hence, the import share of household consumption increases.

In the next section, we move back to the original dynamic model and quantitatively examine the effect of the export shock during the Great Recession period. We will see that the basic mechanisms we described here are also at work in the more complex quantitative model.

5 Computation and calibration

We quantify the model based on the Japanese economy. First, we provide a brief outline of the computational method. Then, we describe how we calibrate the baseline economy.

5.1 Computation

Here, we outline the computation of our quantitative model. The detail of the computational procedure is described in Appendix G.

First, we compute the model's steady state. We assume the initial steady state is the Japanese economy in 2008Q3 and compute the model's steady state with constant parameter values so that the model fits the data in 2008Q3. This steady state serves as the initial point of our experiment.

We assume that the export shock arrives as an unanticipated change in $\omega_{sj,t}^f$ for $t = 1$ to $t = T$, where $t = 1$ is 2008Q4 and $t = T$ is 2010Q4. That is, until 2008Q3 ($t = 0$), the economic agents believe that the economy will be in the (initial) steady state forever. At the beginning of $t = 1$, the new information of future $\omega^f_{sj,t}$, $t = 1, ...$ arrives. As we will see in the next section, we will set $\omega^f_{sj,t}$ from $t = 1$ to $t = T$ so that the realized export values during that period match the corresponding data from Japan for all (s, j, t). We assume $\omega^f_{sj,t}$ stays at the constant value (the 2010Q4 value) after $t = T$. Therefore, the economy will eventually settle in the new steady state with $\omega^f_{sj,T}$.

We assume that, at time $t = 1$, all future time series of $\omega^f_{sj,t}$ starting from $t = 2008Q4$ are revealed to the economic agents. Therefore, after $t = 1$, the economy follows a perfect-foresight transition dynamics. This type of experiment is often referred to as an "MIT shock" in recent literature.

5.2 Calibration

We assume that one period is equivalent to one quarter. The initial baseline economy is in a steady state, with the 2008Q3 outcome. We set the baseline parameter values to ensure that the equilibrium outcome aligns with the data statistics of the Japanese economy at that time.

The consumption share parameters $\{\xi^i_{sjc}\}_{i,sj}$ are calibrated so that the consumption expenditure share of sj, which represents good s produced in region j, by the region i consumer matches the data in the baseline economy. The consumption shares are taken from the inter-regional input-output table in 2005 (IRIO2005), which is the closest time period before 2008Q3. Parameters governing the demand for the sj production by foreign countries, $\{\omega^f_{sj}\}_{sj}$, are set so that the GDP share of export goods sj matches the data computed in IRIO2005.[8]

In the baseline economy, we assume that the parameter governing the wage elasticity of labor supply choice in each industry, τ, is equal to 1, as in Horvath (2000). The inverse of Frisch elasticity of overall labor supply, ζ, is set to 2.5 based on the empirical estimate by Kuroda and Yamamoto (2008).[9] The labor disutility parameter χ_i is calibrated to replicate the regional variation of the employed population in 2008Q3 taken from the LFS, conducted by the MHLW.[10] Note that the variation in the employed population reflects those in the labor force (or working-age population) and employment rate.[11] The time discount rate ρ is set to 0.01. As a benchmark, we consider the case of $\sigma_c \to 1$, that is, a log utility.

The investment share parameters $\{\xi^i_{sjx}\}_{i,sj}$ are calibrated so that the investment expenditure share of each sj in region i in the benchmark economy matches the data taken from IRIO2005. The parameter governing the elasticity of substitution, σ, is set to 5.0 in the baseline economy. The parameters governing the cost share of each intermediate good sj for the producer of good h in region i, $\{\gamma^{hi}_{sj}\}_{hi,sj}$, are set so that those in the benchmark match the data counterparts in IRIO2005. The factor-neutral productivity for each industry sj, A_{sj}, is given by the product of the industry- and region-specific productivity parameters; that is, $A_{sj} = A_s \times A_j$, where A_s stands for the industry-specific productivity while A_j stands for the region-specific productivity. First, we map the industry classification in the JIP Database to ours and compute the industrial TFP

[8] As a result, the export-to-GDP ratio and the share of sj in the total export match the data.

[9] Kuroda and Yamamoto (2008) estimate the Frisch elasticity in Japan and report that the elasticity on the extensive and intensive margins combined ranges between 0.2 and 0.7 for males. $\zeta = 2.5$ implies the Frisch elasticity of 0.4.

[10] See, https://www.stat.go.jp/english/index.html.

[11] Although it is better to incorporate the variation of working hours per labor force, there are no reliable data disaggregating working hours into each region.

Parameter	Description	Value	Target/Source
Preference			
ρ	time discount rate	0.01	Assumed
σ_c	curvature	1.0	Assumed
χ_i	disutility of labor supply	Table 8	LFS (2008)
ζ	inverse of Frisch elasticity	2.5	Kuroda and Yamamoto (2008)
τ	elasticity of substitution (labor)	1.0	Benchmark in Horvath (2000)
$\{\xi^i_{sjc}\}_{i,sj}, \{\xi^i_{sfc}\}_{i,s}$	weight on consumption goods	Figure 19a	IRIO (2005)
$\{\omega^f_{sj}\}_{sj}$	weight on export goods	Figure 19d	IRIO (2005)
Technology			
$\{A_{sj}\}_{sj}$	factor neutral productivity	Tables 7 and 8	JIP (2005), MLS (2008)
$\{\alpha_s\}_s$	cost share of intermediate goods	Table 7	JIP (2005)
$\{\beta_s\}_s$	labor share	Table 7	JIP (2005)
σ	elasticity of substitution	5.0	Assumed
$\{\gamma^{hi}_{sj}\}_{hi,sj}, \{\gamma^{hi}_{sf}\}_{hi,s}$	weight on intermediate goods	Figure 19c	IRIO (2005)
$\{\xi^i_{sjx}\}_{i,sj}, \{\xi^i_{sfx}\}_{i,s}$	weight on investment goods	Figure 19b	IRIO (2005)
δ	capital depreciation rate	0.015	Assumed

Table 2: Summary of the parameter values, their source/reference, and data for setting targets.

and cost share of intermediate goods for each industry s ($\{\alpha_s\}_s$). Given all other parameters, the region-specific productivity A_s is pinned down so that the regional variation of the average wage rate in the benchmark replicates the data counterpart computed using the MLS.

We determine the level of international transfers to each region in the initial economy, $\{B_{i,0}\}_{i\in I}$, to match the national net exports in 2008Q3, which amounted to 4.32% of national GDP. We also internally determine the weight parameters for imported goods, $\{\xi^i_{sfc}\}_{i,s}$, $\{\xi^i_{sfx}\}_{i,s}$, and $\{\gamma^{hi}_{sf}\}_{hi,s}$, based on strategies similar to those used to pin down the weight parameters for domestically produced goods. Further details on the calibration procedure are provided in Appendix C.

Table 2 summarizes the parameter values. The values of A_j and $\{\alpha_s, \beta_s\}_s$ are summarized in Table 7 in Appendix C. The regional parameters A_i and χ_i are summarized in Table 8 in Appendix C. The parameters $\{\xi^i_{sjc}\}_{i,sj}$ $\{\xi^i_{sjx}\}_{i,sj}$ $\{\gamma^{hi}_{sj}\}_{hi,sj}$, and $\{\omega^f_{sj}\}_{sj}$ are too numerous to be summarized in a table and are represented as heatmaps in Figure 19 in Appendix C.

6 Simulating the model with the Great Recession export shocks

The primary purpose of building our quantitative model is to analyze the propagation of the export shocks through the Great Recession episode. As we mentioned earlier, we set the export demand parameter $\omega^f_{sj,t}$ to ensure that the resulting export time series aligns with the data. More precisely, we simulate the export shocks to industry si (i.e., industry s in region i) in period t by changing

$y^f_{si,t}$ for the following to hold in equilibrium:

$$\frac{y^f_{si,t}}{y^f_{si,t=0}} = \frac{\text{real export of } si \text{ in } t \text{ in data}}{\text{real export of } si \text{ in } t=0 \text{ in data}},$$

where reference period $t = 0$ corresponds to 2008Q3.

A variable of primary interest is domestic final demand, comprised of consumption and investment. The *real* consumption is computed excluding the imported goods consumption. Formally, the real consumption for region i in period t, $\bar{C}_{i,t}$, is defined as

$$\bar{C}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} c^i_{sj,t} dj ds,$$

where $p_{sj,t=0}$ is the goods price produced in industry sj in reference period $t = 0$. Similarly, the real investment for region i in period t, $\bar{X}_{i,t}$, is defined as

$$\bar{X}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} x^i_{sj,t} dj ds.$$

The national real consumption and investment in period t, $\bar{C}_{Japan,t}$ and $\bar{X}_{Japan,t}$, are then defined as

$$\bar{C}_{Japan,t} = \int_0^I \bar{C}_{i,t} di$$

and

$$\bar{X}_{Japan,t} = \int_0^I \bar{X}_{i,t} di.$$

6.1 National level response

Figure 8 draws the time series of GDP, exports, consumption, and investment at the national level from the model and the data. Consumption data includes only private consumption and excludes government consumption. Similar construction applies to investment. By construction, the model's export values match the data exactly.

The consumption and investment time series are not our targets, and thus our procedure does not guarantee that these would fit the data. The model implies a slightly larger decline in consumption than observed in the data. The recovery is also slower in the model. One potential explanation for the discrepancy is that we do not capture the increased government spending observed in the data during the periods.[12] The increased government spending would have increased household disposable income and mitigated the consumption decline to some extent. Thus, abstracting the government spending in the model would lead to a more significant decrease in (private) consumption than observed in the data.

The response of investment is significantly smaller in the model. This outcome suggests that the movement of investment during this period was primarily driven by factors outside the scope of this model. Another possible reason is that our assumption of perfect foresight attenuates the response of investment. That is, because the economic agents know that the recovery of exports is relatively quick, investment (which would respond more to a permanent shock) does not adjust

[12] For example, the sum of government consumption and investment increased by 12% in the first quarter of 2009 compared with the third quarter of 2008, corresponding to 3.3% of the domestic demand in the first quarter of 2008.

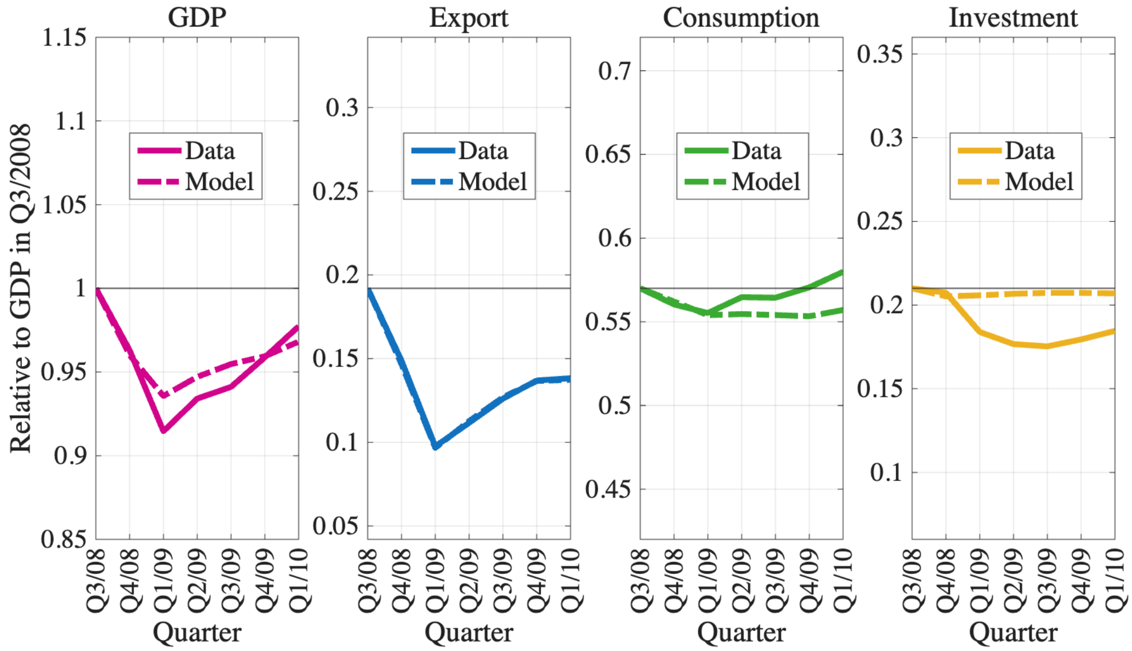

Figure 8: National responses. Each series is expressed relative to GDP in 2008Q3. "Data" plots the fluctuations of HP-filtered variables. Consumption data includes only private consumption and excludes government consumption. Similar construction applies to investment.

much in the model. In reality, the duration of the export shock was uncertain, and it is possible that firms perceived the shock to be potentially more persistent.

The overall response of the output is comparable to the data. The model response was smaller, reflecting the large discrepancy in the investment response. The persistent decline in consumption and the smaller response of investment offset each other, bringing the model's GDP recovery close to the data. In the next section, we examine the propagation of the export shock across regions in greater detail.

6.2 Responses at the regional level

Figure 9 compares the regional demand for domestic final goods between the model and data. The model explains the data especially well for regions like Chūbu, Chūgoku, and Kyūshū, which saw a large decline in exports. In contrast, for regions such as Hokkaidō, Tōhoku, and Shikoku, where exports are either a small part of regional GDP or the export declines were not significant, the model poorly explains the regional GDP decline.

The model's varying explanatory power across regions is illustrated in Figure 10, which displays the decline in regional GDP from 2008Q3 to 2009Q1 in the data (vertical axis) compared to the model's predictions (horizontal axis). The size of the markers indicates the export share of GDP for each region. We can see that the scatter plots lie below the 45-degree line. The model, in general, tends to underestimate the decline in GDP. We observe that the model nearly accounts for all GDP declines in regions with high export shares, such as Chūbu, Kyūshū, and Chūgoku. In contrast, the model explains a relatively smaller fraction of the decline for the regions with a small export share. However, the existence of regions such as Tōhoku, where the model decline is substantial despite the small share of export, indicates the presence of inter-regional propagation of shocks.

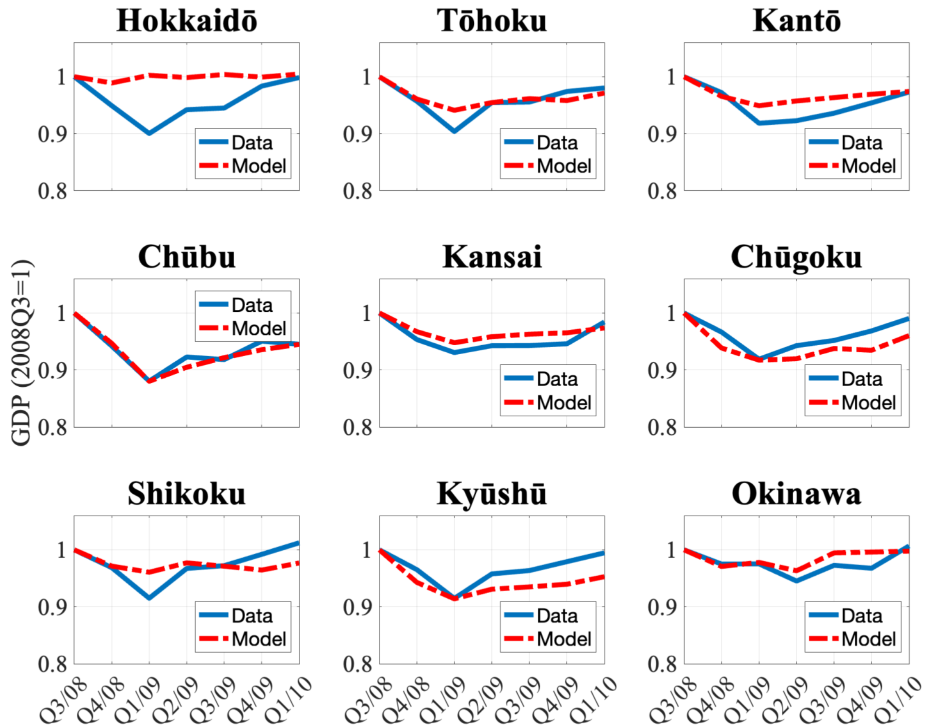

Figure 9: Regional responses

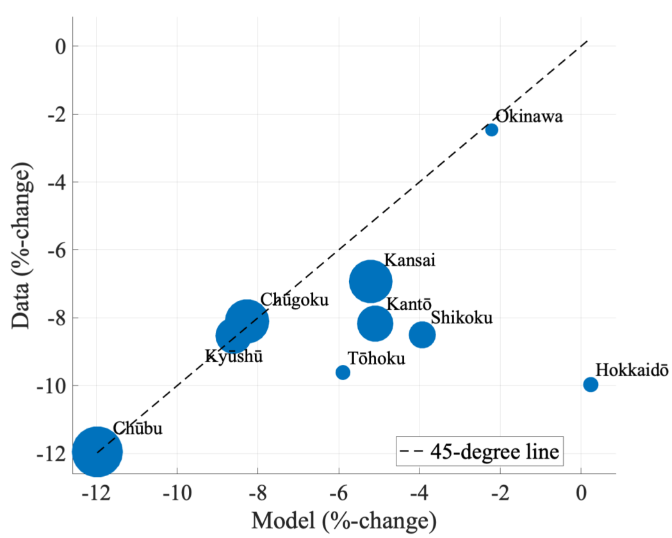

Figure 10: Regional GDP declines in model predictions and data. The size of the circle shows the export share of regional GDP.

Among the regions where the model decline is small, Shikoku, Hokkaidō, and Tōhoku represent small portions of the national GDP (2.7%, 3.8%, and 6.1%, respectively, in the 2005 IRIO). Their movement is relatively inconsequential in the aggregate dynamics of GDP.

Kantō and Kansai are two major economic regions contributing 44.5% and 16.5% to the GDP, respectively. The underestimation of GDP declines in Kantō and Kansai is likely the result of omitted factors other than the export shock. In the annual report (the Japanese Economy and Public Finance 2008), the Japanese Cabinet Office highlighted the decline in capital formation due to lower expected growth and the decrease in housing construction as significant shocks to the Japanese economy in 2008, second only to the export shock. Our model does not incorporate such medium- to long-term trends related to capital formation and housing. Furthermore, fiscal stabilization efforts following the export shock likely had different effects across regions. Due to data limitations, it is challenging to attribute fiscal expenses to specific regions using our current information, and therefore, the investigation of this mechanism is left for future research.

Overall, Figure 10 demonstrates that our model effectively captures regional differences in responses to export shocks, particularly for relatively large regions. This outcome accounts for the model's ability to explain the national response shown in Figure 8 (an 8 percent drop in GDP from 2008Q3 to 2009Q1, of which 6 percentage points are explained by the model), especially regarding the decline in GDP and consumption early in the Great Recession.

7 Counterfactual experiments

Given that the model can account for a substantial part of the national decline in output and the regional heterogeneity in responses, it is of interest to examine the mechanism at work. In particular, the mechanism for the inter-regional and inter-sectoral propagation of shocks is a unique feature of this model that deserves special attention.

In the following, we run counterfactual experiments to investigate the mechanism of propagation in different depths. In particular, we run a controlled experiment by feeding the model only the (permanent) shock on $y^f_{sj,t}$ to one region and industry sj for some t (2009Q1) and computing the new steady state, keeping $y^f_{hi,t}$ of the other regions and industries $hi(\neq sj)$ constant. Here, we consider a negative export shock to the transportation equipment industry in the Chūbu region. We chose this industry and region because (i) decline of the automobile export is one of the most important feature of this recession and (ii) the headquarter of Toyota, the largest auto producer and exporter, is located in the Chūbu region.

7.1 Decomposition

To see how the export demand shock in a region affects other regions, we conduct a decomposition analysis. Our decomposition is based on different demand components. First, note that our dynamic model comprises four demand factors: domestic consumption demand, domestic investment demand, domestic intermediate-good demand, and foreign demand. In the following equation, the first term on the right-hand side is the domestic consumption demand, the second term is the investment demand, the third term is the domestic intermediate-good demand, and the fourth term is the foreign demand for good s produced in region j.

$$y_{sj,t} = \int_0^I (c^i_{sj,t} + x^i_{sj,t})di + \int_0^S \int_0^I m^{hi}_{sj,t} didh + y^f_{sj,t}.$$

The domestic consumption demand is represented as

$$c^i_{sj,t} = \left(\frac{p_{sj,t}}{P^c_{i,t}}\right)^{-\sigma} \xi^i_{sjc} C_{i,t}. \tag{21}$$

The domestic investment demand is represented as

$$x^i_{sj,t} = \left(\frac{p_{sj,t}}{P^x_{i,t}}\right)^{-\sigma} \xi^i_{sjx} X_{i,t}. \tag{22}$$

The domestic intermediate-good demand from industry h in region i is

$$m^{hi}_{sj,t} = \left(\frac{p_{sj,t}}{P^m_{hi,t}}\right)^{-\sigma} \gamma^{hi}_{sj} M_{hi,t}. \tag{23}$$

Given this background, we compute two economies. The first is the baseline economy without any shocks. That is, this economy stays at the 2008Q3 steady state. The second is the economy with export shock in 2009Q1, but with only one industry and one region (transportation equipment industry in the Chūbu region). Then, we maintain the shock value constant at the 2009Q1 level and compute the new steady state. The comparison of these two provides the overall changes of the (real) sales in each region given this particular shock, where the real sales for region i is formulated as follows:

$$\bar{Y}_{i,t} = \int_0^S p_{si,t=0} y_{si,t} ds.$$

Note that we use the price at $t = 0$ to create the real variable. Our decomposition exercise involves two steps. In the first step, we decompose the sales change in each region into the following five factors separately. The first factor is the effect of *prices*. These are $p_{sj,t}$ for all s and j (21), (22), and (23) and price indices $P^c_{i,t}$, $P^x_{i,t}$ and $P^m_{hi,t}$ in (21), (22), and (23), which affect the demand for goods produced in region j. Note that the foreign price $\bar{P}$ is fixed because of the small open economy assumption. The second to fifth factors are $C_{i,t}$, $X_{i,t}$, $M_{hi,t}$, and $y^f_{sj,t}$. The first step reveals through which factor a region's sales is affected, but is silent about through which region. The second step then decomposes the contribution of each factor into the regions from where those effects originate.

Figure 11 plots the decomposition result. This figure is intended to provide insight into how the region- and industry-specific export shock propagates. The triangle dot is the effect of this particular shock on the total sales of each region. The colored bars represent the decomposition. They are labeled as consumption C, investment X, intermediate goods M, export y^f, and prices.

The first main takeaway from the figure is that the effect of this shock on Chūbu itself is substantially larger than the other regions that are not directly hit by the shock. At the same time, the propagation to the other regions is not negligible.

The second takeaway is that, although the shock was on exports, the other demand components were also affected. For the intermediate goods, this outcome implies that the intra-regional network effect is important. For consumption, the changes in wages and profit affects consumption demand. Because the demand for goods from Chūbu drops, the price also drops for these goods, mitigating the output effect of shocks.

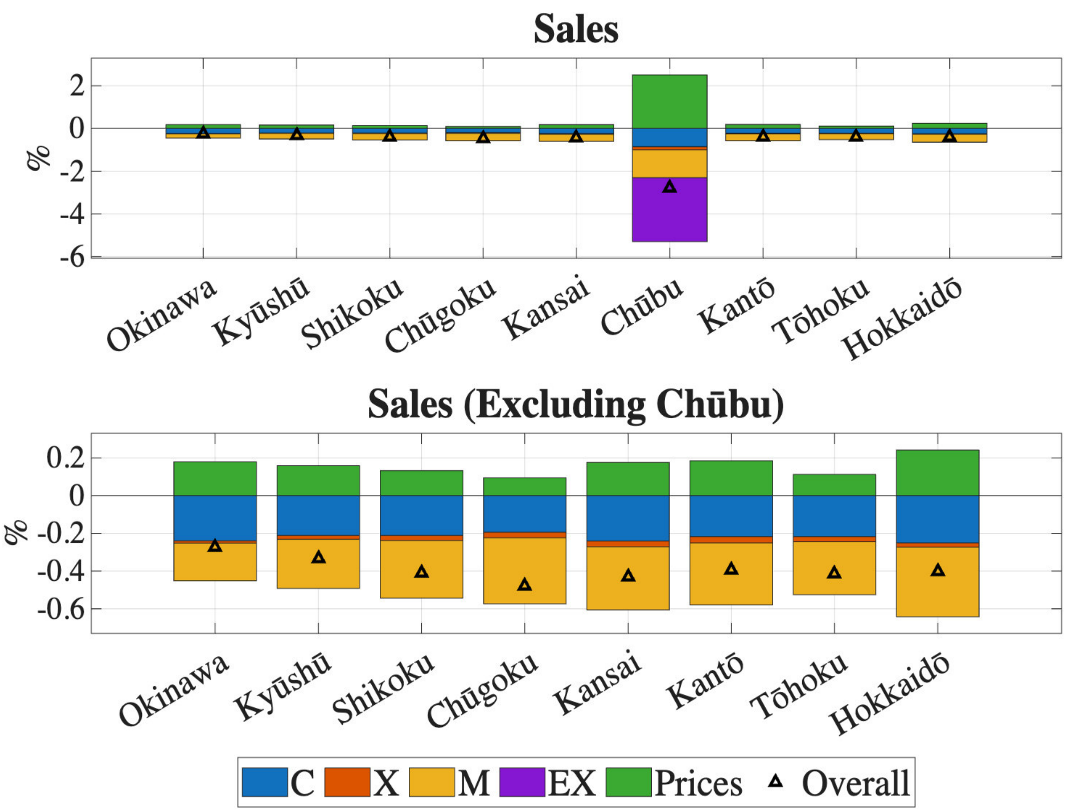

Figure 11: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

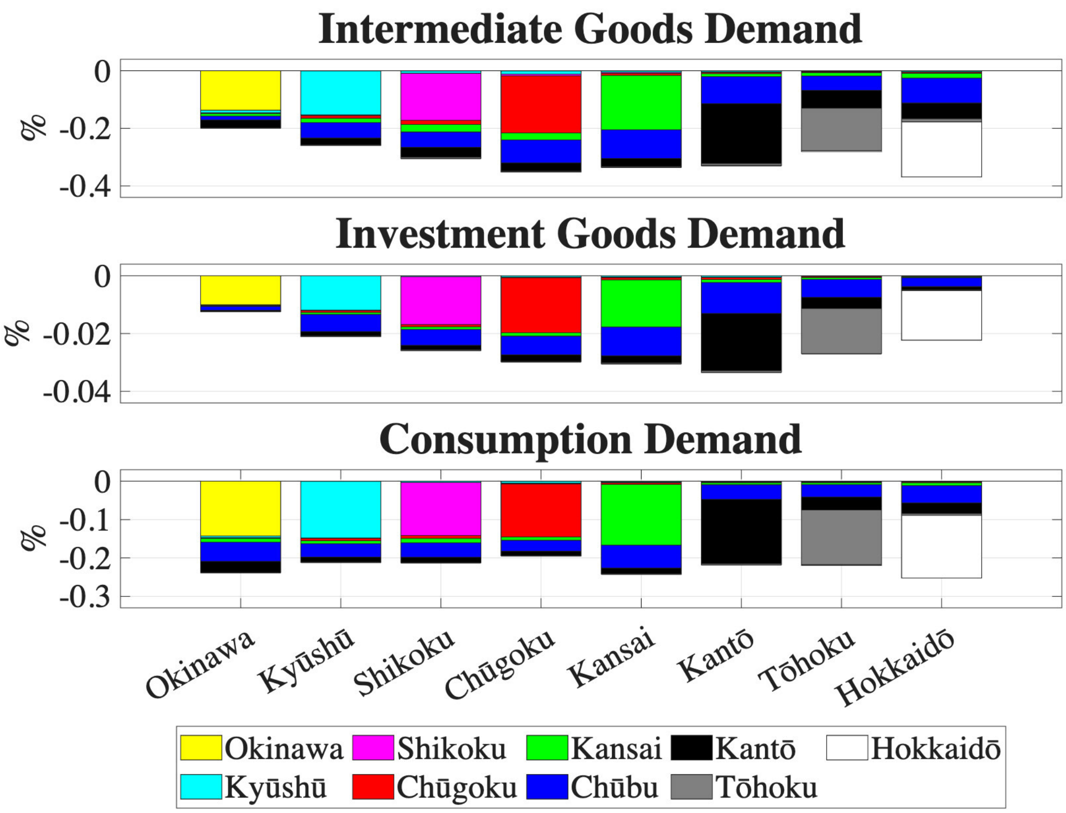

Figure 12: Changes in demand for intermediate, investment, and consumption goods with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

To focus on the inter-regional propagation, the bottom panel of Figure 11 plots the same objects as the top panel, but excluding the Chūbu region. The triangle dot is the overall effect. All regions experience a reduction in overall sales from this shock to the Chūbu region transportation equipment industry.

There are several takeaways on the propagation. First, geography matters—the closer the region is to Chūbu, the more significant its overall decline tends to be. As we can see from the map in Figure 4, the Chūbu region is located approximately in the middle of Japan. Figure 11 lists the regions from west (Okinawa) to east (Hokkaidō). Thus, in the figure, the closer to the edges, the farther away from where the shock hits. The black triangle exhibits a pattern that the decline is the largest in the middle. This result is consistent with the gravity pattern of IRIO in Section 2.3.

Second, the negative effects are largely due to consumption and intermediate-goods demand. The effect of investment goods is small. The novel finding here is the importance of consumption demand in the propagation process. In the macro-network literature, the focus has chiefly been on the propagation through intermediate input. Our results show that the consumption linkage is quantitatively as important.

Third, the price effect, which attenuates the negative demand effect, is quantitatively significant. As in the case of the Chūbu region, the price effect (along the demand curve) mitigates the effect of demand decline. This result underscores the importance of modeling the general equilibrium properly. Section 7.2 below examines the impact of the price flexibility in more detail.

In the second step of the decomposition, we examine each component—intermediate input, investment, and consumption—separately and decompose the change into their regional contents. Figure 12 graphs the contributions of each region in accounting for the column region's decline in intermediate input, investment, and consumption demand, respectively.

Two components stand out as quantitatively important in accounting for the demand decline in all three graphs. First, the decline of demand from Chūbu, where the shock hits, is substantial. The shock to Chūbu reduces demand for intermediate goods through the input-output network. It reduces the demand for consumption and investment goods from other regions because of the decline in the Chūbu consumer's income. This cross-regional demand propagation, in turn, reduces income in other regions. Second, this decline in income in regions other than Chūbu reduces the demand for the goods from the own region. This secondary effect turns out to be quantitatively more important than the direct effect from the Chūbu region.

At the top panel of Figure 12, we observe a mild gravity pattern. Recall that Chūbu is located in the middle of Japan (see Figure 4) between Kansai and Kantō, and regions are ordered from west to east. By focusing on the effect from Chūbu, which is represented by blue, we notice that negative effects are strong in the central regions and weaker toward the edges, except for Hokkaidō. Once again, this pattern aligns with the gravity structure identified in the IRIO matrix (Section 2.3). The overall demand decline, including the effect from regions other than Chūbu, follows a similar pattern. We see a more prominent gravity pattern for investment goods (middle panel of Figure 12) and no gravity pattern for consumption goods (bottom panel of Figure 12). The heterogeneity of spatial propagation across different channels is a novel finding of this paper.

7.2 The role of price flexibility

The model above is in the tradition of the real business cycle model in that all prices are flexible. In the decomposition, we see that price flexibility does indeed play a role: the effect of price change mitigates the output decline associated with negative demand shocks. In this section, to further investigate how the prices affect the propagation, we make the opposite assumption: fix all prices at the level of $t = 0$.

Figure 13 repeats Figure 11 for fixed prices. Comparing these two figures, we can see the main difference for the Chūbu region is the absence of the price effect, which strengthens the negative effect on output. The composition of each component is almost identical.

Figure 14 plots the results corresponding to Figure 12 earlier. These figures look almost identical to those for flexible prices, except that the scale of the negative effects is larger than in the flexible price setup.

7.3 Regional multiplier

Finally, we calculate regional multipliers using our calibrated model. Nakamura and Steinsson (2014) estimated the multiplier effects of military procurement spending on regional GDP to range from 1.4 to 1.9 and 2.5 to 2.8, depending on the estimation methods and data.

Our model considers foreign export demand as the sole source of shocks. Therefore, if a region experiences no export shock, the only shocks affecting it are the intermediate and final demands from other regions. When the region is small enough, we can neglect the national equilibrium effect, where the region's response to other regions' demand causes ripple effects through the region's demand for others. Under this small-economy assumption, we estimate the multiplier effect as the

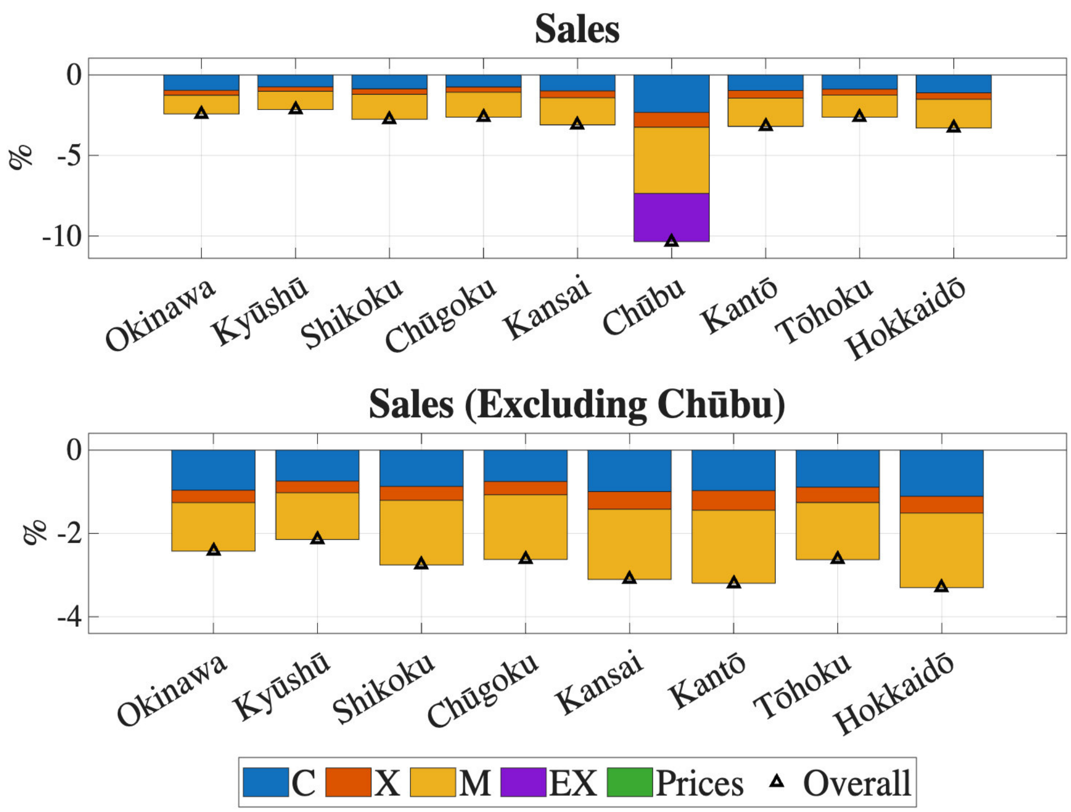

Figure 13: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

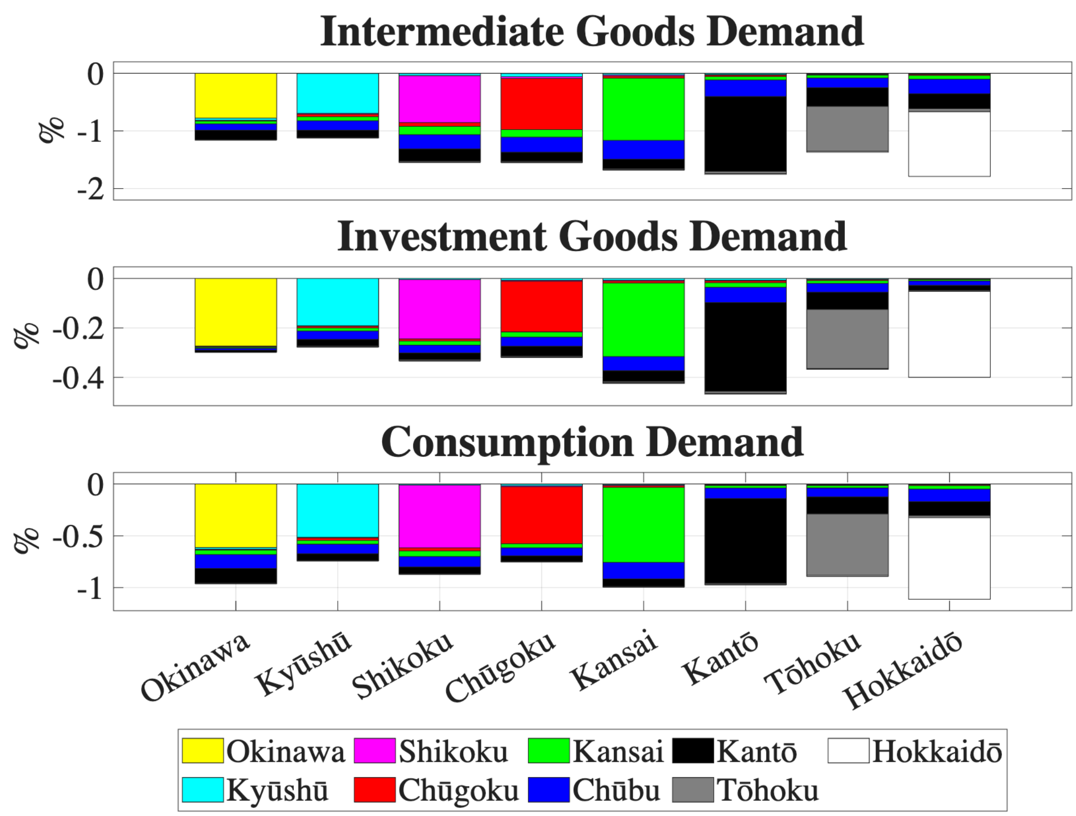

Figure 14: Changes in demand for intermediate, investment, and consumption goods with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

	Okinawa	Kyūshū	Shikoku	Chūgoku	Kansai	Kantō	Tōhoku	Hokkaidō
Multiplier	2.46	2.06	2.31	2.05	2.10	2.55	2.54	2.31

Table 3: Regional multipliers estimated from a counterfactual experiment when $y^f_{\text{Chūbu, TE}}$, the export demand for transportation equipment industry in Chūbu, incurrs an exogenous -10% shock.

ratio of the region's GDP increase to the increase in demand from other regions for the region's output.

To estimate the regional multiplier, we use our counterfactual experiment where Chūbu's transportation equipment (TE) export demand quantity ($y^f_{\text{Chūbu, TE}}$) experiences an exogenous 10% decline. We calculate equilibrium changes in region i's GDP and in intermediate and final demands for i from all other regions. The results are shown in Table 3. The estimated regional multipliers, excluding Chūbu, Kantō, and Kansai, range from 2.05 (Chūgoku) to 2.54 (Tōhoku), whereas the regional multipliers for Kantō and Kansai are 2.55 and 2.10, respectively. We note that these estimates align with those reported in Nakamura and Steinsson (2014).

Our model offers a structural explanation for the multiplier effect. In the model, an increase in demand from other regions raises labor demand and output by local firms, resulting in higher equilibrium wages and increased hours worked. The rise in labor income then boosts regional demand, particularly in the service sector, which primarily relies on local consumers. Notable exceptions include tourism, inter-regional distribution margins, and intra-firm trade of managerial services, all of which are explicitly accounted for in our IRIO tables.

The share of the non-service sector varies across regions from 12.0% in Okinawa to 48.9% in Chūbu with an average of 32.2%. Even within tradable sectors, 51.3% of demand comes from the home region. This strong home bias is also reflected in our gravity estimates (Section 2.3). Therefore, the home bias in goods demand underpins the regional multiplier effect in our model. Additionally, this bias helps explain the nonlinear effects we observe (Section 8.2). When the shock is small, the first-order effect of inter-regional demand is mitigated by price responses: a wage decrease in Chūbu enhances the demand in other regions. When the shock is large, wages in other regions decline nonlinearly, which can lead to a regional multiplier effect driven by home-biased consumption demand.

8 The static model, once again

Finally, we reassess the static model, which we utilized for an analytical solution in Section 4, to further explore the propagation of shocks in the model. Compared to Section 4, we do not impose symmetry across sectors and regions; thus, the model can still be evaluated quantitatively. This section makes two novel contributions.

First, we analytically derive the explicit formula for the decomposition we conducted in Section 7. The formula explicitly connects the decomposition to fundamental forces in the model. Because the simplifying assumptions are minimal, we can still evaluate the outcome of the counterfactual experiment quantitatively.

Second, we utilize the analytical formula, which represents the approximated local responses, to examine the importance of nonlinearity in analyzing responses to large shocks. The comparison here is between the local (small-shock) response of the analytical formula and the full static model (presented in Appendix F). We find that the nonlinearity of the model is quantitatively important, and thus the local approximation may yield misleading results in the cases of large shocks.

8.1 First-order effects of export shocks in a static model with $\sigma_c = 1$ and $\gamma^{\cdot}_{\cdot f} = 0$

We start from the static version of our quantitative model in Section 3. The details of the static model are presented in Appendix F. The only differences from the model in Section 3 are that we remove the consumption and saving decision from the consumer's decision and do not have capital stock as a factor of production.

Here, we impose minimal simplifying assumptions to the static model: $\sigma_c = 1$ and $\gamma^i_{sf} = 0$, that is, labor supply is inelastic, and imports are not used as intermediate inputs. With these assumptions, we obtain an analytical expression for the comparative statics of ω^f below, without sacrificing heterogeneity across regions and sectors. That is, we maintain the full input-output structure $(\xi^i_{sj}, \gamma^{hi}_{sj})$. Below, we only present the main result and defer detailed derivations to Appendix E.

First, we set up the new notations that we use below. We denote the international transfer per regional GDP as $b^i \equiv B_i/Y^i$. Let $q_{si} \equiv (p_{si})^{\sigma-1}$, $Q^{si} \equiv (P^{si})^{\sigma-1}$, $Q^i \equiv (P^i)^{\sigma-1}$, $Y^{si} \equiv p_{si}y_{si}$, and $Y^i \equiv \sum_{s=1}^{S} Y^{si}$. In the static equilibrium, labor supply in region i is $\hat{N}^i$, which is a constant up to b^i. Y^{si} is proportional to $\hat{N}^i(w^i_s)^{1+\tau}$. Let $\boldsymbol{q}$ be a SI vector of the matrix $[q_{si}]$. Similarly define $\boldsymbol{Q}, \boldsymbol{Y}, \boldsymbol{\omega}^f$. Also, let $\boldsymbol{b}_I$ be a length I vector $(b_i)_i$. Let $\operatorname{diag}(qY)$ denote a diagonal matrix with value $q_{si}Y^{si}$ in the $(S(i-1)+s, S(i-1)+s)$-th element. Similarly, $\operatorname{diag}(Q)$ and $\operatorname{diag}(\omega^f)$ denote a diagonal matrix with Q^{si} and ω^f_{si}, respectively. Finally, Γ^0 denotes a $SI \times I$ matrix containing $\hat{\gamma}^{0j}_{si}Q^{0j}Y^{0j}$ in $(S(i-1)+s, j)$.

With these notations, we obtain an analytical expression for an equilibrium response of prices, $d\ln \boldsymbol{q}$, as a linear function of exogenous changes in export demand and international transfer, $d\ln \boldsymbol{\omega^f}$ and $d\boldsymbol{b}_I$. The formula is derived in Appendix E.

The analytical comparative statics helps analyze decomposed effects:

$$d\ln \boldsymbol{Y} = \operatorname{diag}(qY)^{-1}\Bigg[\underbrace{\Gamma^0 \operatorname{diag}\left(\frac{1}{1-\alpha(\sigma-1)/\sigma + b_I}\right) d\boldsymbol{b}_I + \operatorname{diag}(\omega^f)(\bar{P})^{\sigma} d\ln \boldsymbol{\omega^f}}_{\text{direct exogenous effects}} + \underbrace{G d\ln \boldsymbol{Y}}_{M \text{ effect}} + \underbrace{\Gamma^0 d\ln \boldsymbol{Y}^0}_{C \text{ effect}} + \underbrace{G d\ln \boldsymbol{Q} + \Gamma^0 d\ln \boldsymbol{Q}^0 - \operatorname{diag}(qY) d\ln \boldsymbol{q}}_{\text{Price effects}}\Bigg] \tag{24}$$

The second line of (24) shows the propagation effects of export shocks through intermediate goods demand M, consumption goods demand C, and price adjustment effects. The M effect corresponds to the traditional input-output propagations of demand quantity, whereas the price effects incorporate substitution channels. The C effect signifies the general equilibrium effect, where changes in output affect final demand through regional income.

The question we ask below is: can we use this local decomposition formula for analyzing the propagation of shocks, as we did in Section 7? The answer is twofold. First, for small shocks, the formula provides a quantitative outcome that is similar to the full model. Second, for large shocks, such as the export shock analyzed in this paper, scaling the local response deviates significantly from the full nonlinear solution.

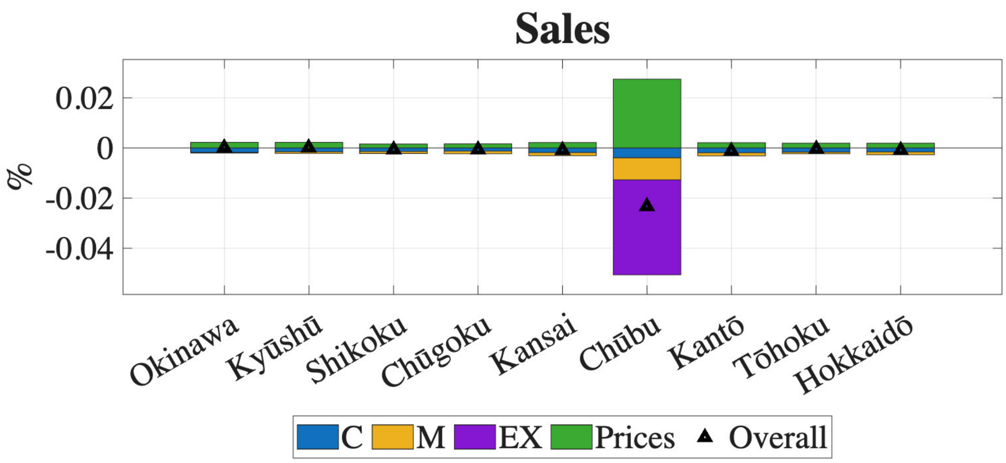

Figure 15: Analytical decomposition of an export demand shock on regional outputs. One percent negative shock hits the Transportation Equipment sector in the Chūbu region.

For the first point, Figure 15 illustrates a decomposition exercise where the export demand for transportation equipment in the Chūbu region drops by one percent.[13] Here, we impose a small shock to express the local approximation. Because we impose $\gamma^i_{sf} = 0$ in this exercise, we recalibrate the static model presented in Appendix F accordingly and use the resulting parameters to implement the decomposition in (24).

The decomposition shows three features that we already saw in Section 7 (Figure 11). First, the region affected by the export shock (i.e., Chūbu) experiences a significant decline in output, while the shock also negatively impacts the output of other regions. Second, the effects of propagation through consumption are as important as those through intermediate demand. Third, the price effects are substantial and largely counteract the demand propagation effects in regions outside Chūbu. Thus, the locally-approximated decomposition provides a reasonable approximation of the full model for small shocks.

For the second point, Figure 16 computes the same decomposition calculated numerically using a static model that does not rely on the analytical formula of the first-order effect. We use the recalibrated version of the model with $\gamma^i_{sf} = 0$ to ensure comparability with the results based on (24). Here, we impose a big shock: the magnitude of the reduction in the transportation equipment sector in the Chūbu region is 62.8%, matching the observed value in 2009Q1.

We find that (i) an extrapolation of the local effect (that is, multiplying the result in Figure 15 by 62.8) would underestimate the propagation, (ii) this underestimation is larger for non-Chūbu regions than for Chūbu region, and (iii) the mitigating effects of price adjustments outside Chūbu are only evident in the local approximation (Figure 15) and not in the full models (Figures 11 and 16).

The above experiment indicates the presence of important nonlinearity embedded in the model. In the next section, we further explore this difference between the local (linear) approximation and

[13] We assume that the regional international transfer b^i is zero in the initial equilibrium and shifts one-to-one with changes in the region's net exports.

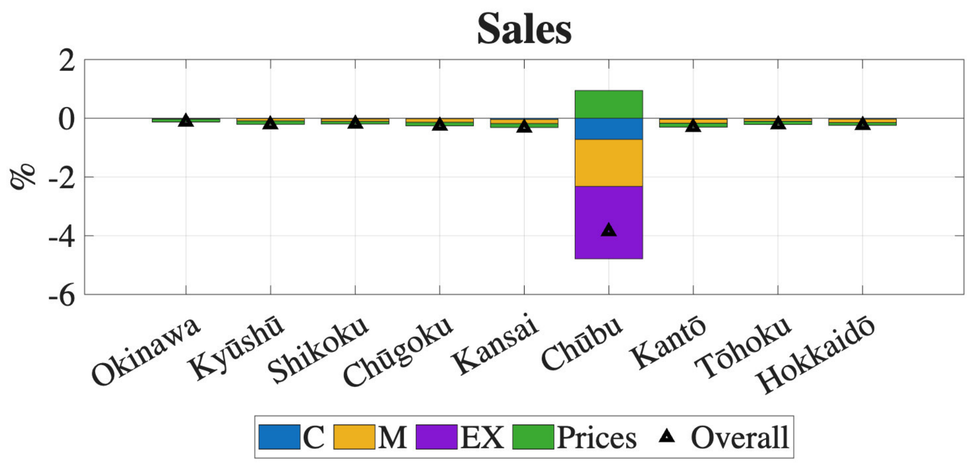

Figure 16: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

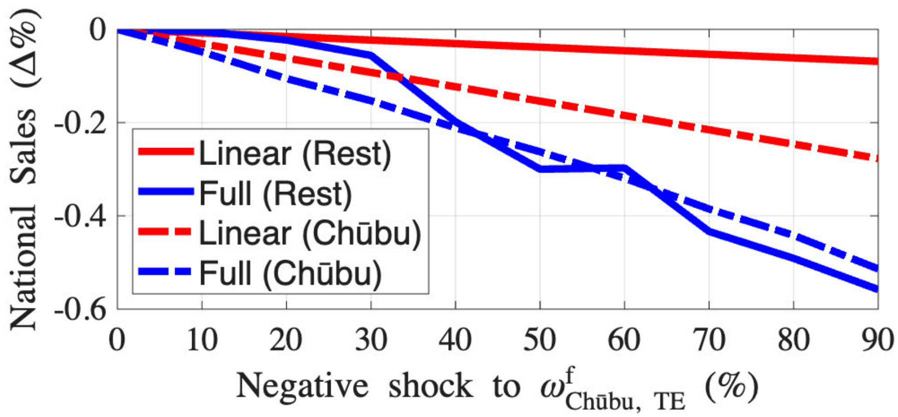

Figure 17: Comparison of the first-order effects with the full model for various shock sizes

the full nonlinear solution.

8.2 Nonlinearity

To further investigate the nonlinearity of the model, we compare the first-order effects with those observed in the full model with various magnitudes of shocks. Figure 17 shows the responses of sales in Chūbu and the rest of Japan, respectively, to an export shock in $\omega^{f}_{\text{Chūbu, TE}}$ for various magnitudes. We use the static model in Appendix F for the full model. We find that the full model, which accounts for the nonlinear effects, predicts a larger propagation effect of the export shock on aggregate sales.

Figure 17 shows that nonlinearity is significant. The gap between the full and first-order effects widens—both in difference and ratio—as the shock size increases, as clearly shown by the responses of the rest of Japan. These responses are similar for small shocks but start to diverge at around 20%; at 50%, similar to the scale of the 2008 crisis, the full model's response is -0.37%, while the first-order effect remains only −0.04%. Figure 17 indicates that the nonlinear effect is more prominent in the rest of Japan than in Chūbu. The nearly linear response of the full model for Chūbu is understandable, since the shock directly influences TE and its supplier industries within Chūbu.

Inspections of the model outcome show that the nonlinear effects reflect the nonlinear responses of wages. With our setting of $\sigma_c = 1$, the regional labor supply response is rigid. Therefore, a negative shock to inter-regional demand shifts labor demand down and reduces equilibrium wages, with the response determined by the elasticity of labor substitution across industries, τ. Low labor income from the tradable goods sector then spreads to the non-tradable goods sector in the region, causing a regional multiplier effect, as discussed in Section 7.3.

9 Conclusion

This study constructs a multi-region, multi-sector model to analyze the propagation of export shocks in Japan during the Great Recession period. Our model features monopolistic competition, inter-regional IO linkage, and a representative consumer in each region.

We measure export shocks in each region using trade statistics. The inter-regional input-output matrix, unique to Japan, enables us to analyze the propagation of shocks through the input-output network. Our model also features the explicit treatment of consumers, whose final goods demand is affected by shocks through the effect on income.

Calibrating the model to 2008Q3, we examine how the model outcome with export shocks performs compared with the data. We find that the model with flexible prices can replicate close to half of the output decline and the entire consumption decline at the macro level. At the regional level, the export shock can be particularly seen to have a large impact on output in regions where exports account for a large portion of regional GDP.

We run several counterfactual experiments to examine the propagation of shocks across regions and industries. In the main experiment, we feed the model an export shock that hits only a particular industry (transportation equipment) in a specific region (Chūbu). We find that a shock to one region and industry propagates to the other regions through the consumption demand and IO linkages. The effect is especially powerful for geographically closer regions. The secondary effect of the decline in own consumption caused by the income drop is also important. The decline in prices attenuates the negative effects.

We also conduct an experiment with fixed prices, and we find that both within- and across-region output declines are significantly larger, with no mitigating factors. The analytical version of our model helps understand various channels through which the output (sales) is affected. The comparison between the local approximation of the simple model and the full (nonlinear) solution indicates that there is an important nonlinearity in the model's propagation mechanism, and the local linear approximation would underestimate the magnitude of propagation, especially for large shocks.

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Appendix

A Mapping prefectures into regions

Table 4 below describes the correspondence between prefectures and the regions we use in the study.

Regions	Prefectures
Hokkaidō	Hokkaidō
Tōhoku	Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima
Kantō	Ibaraki, Tochigi, Gunma, Saitama, Chiba, Tōkyō,
	Kanagawa, Niigata, Yamanashi, Nagano, Shizuoka
Chūbu	Toyama, Ishikawa, Gifu, Aichi, Mie
Kansai	Fukui, Shiga, Kyōto, Ōsaka, Hyōgo, Nara, Wakayama
Chūgoku	Tottori, Shimane, Okayama, Hiroshima, Yamaguchi
Shikoku	Tokushima, Kagawa, Ehime, Kōchi
Kyūshū	Fukuoka, Saga, Nagasaki, Kumamoto, Ōita, Miyazaki, Kagoshima
Okinawa	Okinawa

Table 4: Region classification in our model. This classification is based on the inter-regional input-output (2005) provided by the Ministry of Economy, Trade and Industry (METI).

B Details of the data construction

We use the Trade Statistics of Japan (TSJ), constructed by the Ministry of Finance (MoF), to create a quarterly series of exports for each industry, sj.[14] The TSJ monthly reports the values of 28 goods exported at ten customs, where the ten customs can be further broken down into 166 offices. We map each office to our region classification and each good to our industry classification, and aggregate the raw data to construct the quarterly export series for each region-industry.

We also construct the export series of automobiles for each region using public data.[15] First, the Japan Automobile Manufacturers Association provides data recording monthly production and export of each automobile category (e.g., standard-sized car, bus, truck, etc.) for each carmaker.[16] Second, for most carmakers, we can count how many (and which category of) cars are produced in which establishment by checking their website or online documents. These two sets of information reveal how many (and which category of) cars are produced and exported from each region. Third, we can compute the prices of each car category using the Current Survey of Production conducted by the METI,[17] which enables us to construct the export value series of automobiles for each region.

[14] See, https://www.customs.go.jp/toukei/info/index_e.htm.
[15] Note that the Auto is included by the Transportation Equipment in our industry classification.
[16] See, https://www.jama.or.jp/english/.
[17] See, https://www.meti.go.jp/english/statistics/tyo/seidou/index.html.

C More on Calibration

(i) Static model

#	Industry (our classification)	JIP(2008)	A_s	α_s
1	Agriculture, Forestry, Fisheries	1-6	1.000	0.71
2	Mining and Quarrying of Stone and Gravel	7	0.497	0.72
3	Food and Beverage	8-14	2.596	0.79
4	Textile Mill Products	5	2.078	0.66
5	Pulp, Paper, and Paper Products	18	1.631	0.82
6	Chemical Products	23-29	1.975	0.86
7	Petroleum and Coal Products	30,31	3.149	0.98
8	Ceramic, Stone, and Cray Products	32-35	0.813	0.66
9	Iron and Steel	36,37	3.412	0.90
10	Non-Ferrous Metals	38,39	1.271	0.83
11	Fabricated Metal Products	40,41	2.154	0.63
12	General-Purpose Machinery	42-45	2.879	0.72
13	Electrical Machinery	46-53	3.165	0.75
14	Transportation Equipment	54-56	5.876	0.82
15	Information and Communication Electronics Equipment	57	1.188	0.67
16	Miscellaneous Manufacturing Products	16,17,19-22,58,59	2.092	0.69
17	Construction	60,61,72	15.465	0.77
18	Electricity, Gas, Heat Supply and Water	62-66	4.330	0.81
19	Whole Sale and Retail Trade	67,68	18.296	0.50
20	Finance, Insurance, and Real Estate	69-71	8.289	0.50
21	Transportation	73-77	4.350	0.59
22	Information and Communication	78,79, 90-93	4.308	0.67
23	Education, Medical, Health Care, and Welfare	80-84,98-107	7.578	0.39
24	Services for Businesses	85-88,	5.754	0.60
25	Services for Consumers	89,94-97	4.802	0.57
26	Others	108	1.389	0.98

Table 5: Industry classification for our model, its correspondence with the JIP Database, their factor-neutral productivity (with agriculture=1), and intermediate goods share. Those parameters are computed based on the JIP Database 2008 (https://www.rieti.go.jp/en/database/JIP2008/index.html), provided by the RIETI. We map the industry classification in the JIP Database to ours and compute the industrial TFP and intermediate shares.

	Okinawa	Kyushu	Shikoku	Chugoku	Kansai	Chubu	Kanto	Tohoku	Hokkaido
A_i	0.978	1.077	1.172	1.253	1.306	1.348	1.377	1.030	1.000
χ_i	8.8e-7	4.7e-11	1.2e-13	2.5e-14	3.8e-12	3.5e-19	6.5e-9	1.4e-13	1.2e-10

Table 6: Parameter values for the regional TFP, disutility of labor, and weight on import goods.

Tables 5 and 6 list parameter values for calibration of the static model. Figures 18a, 18b, and 18c are heatmaps of other parameter values.

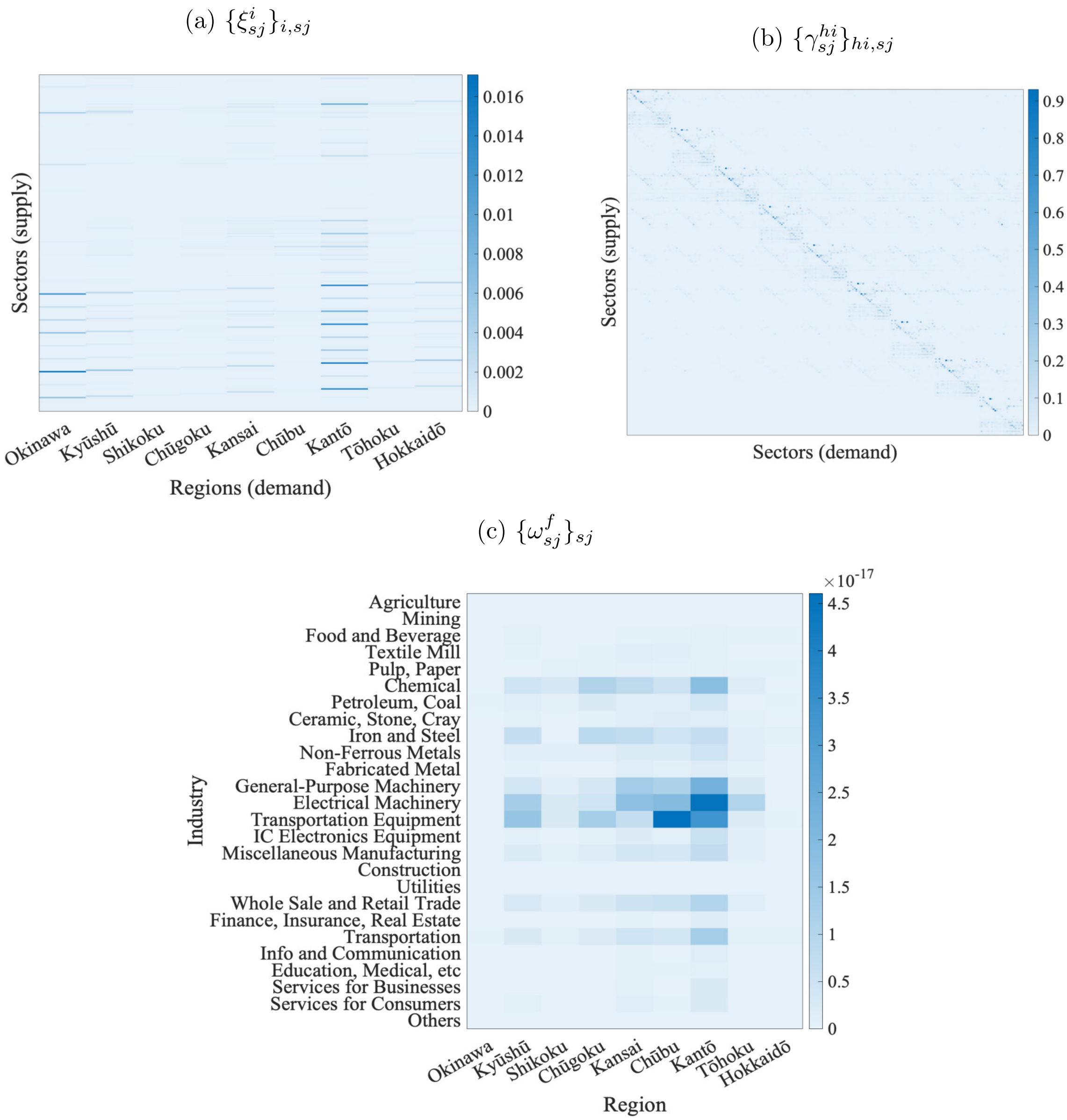

Figure 18: Values for the weight parameters

(ii) Dynamic model

#	Industry (our classification)	JIP(2008)	A_s	α_s	β_s
1	Agriculture, Forestry, Fisheries	1-6	1.000	0.51	0.21
2	Mining and Quarrying of Stone and Gravel	7	0.497	0.63	0.24
3	Food and Beverage	8-14	2.596	0.72	0.19
4	Textile Mill Products	5	2.078	0.59	0.30
5	Pulp, Paper, and Paper Products	18	1.631	0.74	0.16
6	Chemical Products	23-29	1.975	0.77	0.13
7	Petroleum and Coal Products	30,31	3.149	0.94	0.02
8	Ceramic, Stone, and Cray Products	32-35	0.813	0.58	0.29
9	Iron and Steel	36,37	3.412	0.82	0.09
10	Non-Ferrous Metals	38,39	1.271	0.76	0.16
11	Fabricated Metal Products	40,41	2.154	0.59	0.35
12	General-Purpose Machinery	42-45	2.879	0.65	0.25
13	Electrical Machinery	46-53	3.165	0.66	0.22
14	Transportation Equipment	54-56	5.876	0.75	0.16
15	Information and Communication Electronics Equipment	57	1.188	0.55	0.28
16	Miscellaneous Manufacturing Products	16,17,19-22,58,59	2.092	0.63	0.28
17	Construction	60,61,72	15.465	0.44	0.22
18	Electricity, Gas, Heat Supply and Water	62-66	4.330	0.54	0.14
19	Whole Sale and Retail Trade	67,68	18.296	0.46	0.45
20	Finance, Insurance, and Real Estate	69-71	8.289	0.42	0.40
21	Transportation	73-77	4.350	0.47	0.34
22	Information and Communication	78,79, 90-93	4.308	0.56	0.29
23	Education, Medical, Health Care, and Welfare	80-84,98-107	7.578	0.33	0.52
24	Services for Businesses	85-88,	5.754	0.46	0.34
25	Services for Consumers	89,94-97	4.802	0.50	0.38
26	Others	108	1.389	0.98	0.02

Table 7: Industry classification for our model, its correspondence with the JIP Database, their factor-neutral productivity (with agriculture=1), and intermediate goods share. These parameters are computed based on the JIP Database 2008 (https://www.rieti.go.jp/en/database/JIP2008/index.html), provided by the RIETI. We map the industry classification in the JIP Database to ours and compute the industrial TFP and intermediate shares.

	Okinawa	Kyūshū	Shikoku	Chūgoku	Kansai	Chūbu	Kantō	Tōhoku	Hokkaidō
A_i	0.978	1.077	1.172	1.253	1.306	1.348	1.377	1.030	1.000
χ_i	35212.8	12.8	855.2	89.1	2.2	9.1	8.2	39.6	262.3

Table 8: Parameter values for the regional TFP and disutility of labor.

Tables 7 and 8 list parameter values for calibration of the baseline dynamic model. Figures 19a, 19b, 19c, and 19d are heatmaps of other parameter values.

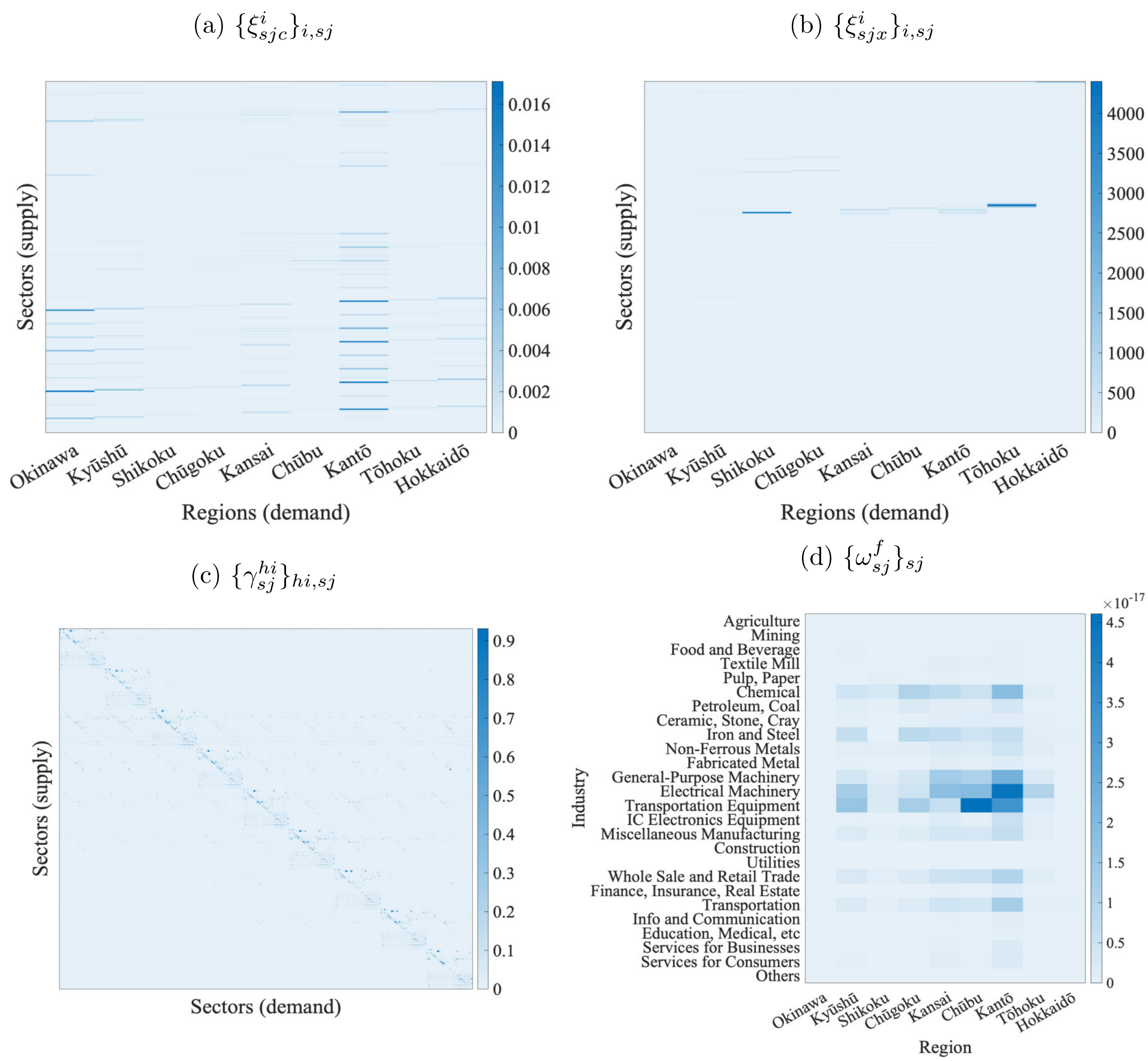

Figure 19: Values for the weight parameters.

Details on the calibration procedure: In determining the distribution of transfers across regions, we assume that the share of region i in the transfers corresponds to its share in national GDP:

$$\frac{B_{i,0}}{\int_0^I B_{j,0}\ dj} = \frac{GDP_{i,0}}{\int_0^I GDP_{j,0}\ dj},$$

where $GDP_{j,0}$ denotes the regional GDP of region j in the initial economy. Given that the calibration of $\{\omega_{si}^f\}_{si}$ ensures that the national export-to-GDP ratio in the model matches its data counterpart, replicating net exports implies that the import-to-GDP ratio also matches the data.

Although it would be ideal to determine the weight parameters for imported goods ($\{\xi_{sfc}^i\}_{i,s}$, $\{\xi_{sfx}^i\}_{i,s}$, and $\{\gamma_{sf}^{hi}\}_{hi,s}$) based on the same procedure as for their domestic counterparts (i.e., the strategy used to pin down $\{\xi_{sj}^i\}_{i,sj}$ and $\{\gamma_{sj}^{hi}\}_{hi,sj}$), the corresponding target moments are not available due to data limitations. In particular, there is no available information on (i) the share of goods $s \in S$ that region i imports from foreign countries, and (ii) how such goods are allocated across consumption, investment, and intermediate use. Therefore, we assume that each region i allocates imported goods in the same proportion as domestically produced ones. Specifically, given the total imports of region i, the share of good s from foreign countries allocated to consumption corresponds to the share of domestically produced good s in total domestic consumption. The same assumption applies to investment and intermediate use. We formalize these assumptions below.

Let IM_i be the total import of region i:

$$IM_i = \int_0^S p_{sf,t}\left(c_{sf,t}^i + x_{sf,t}^i + \int_0^S m_{sf,t}^{hi} dh\right) ds.$$

Likewise, let DD_i denote the total domestic demand of region i:

$$DD_i = \int_0^I \int_0^S p_{sj,t}\left(c_{sj,t}^i + x_{sj,t}^i + \int_0^S m_{sj,t}^{hi} dh\right) dsdj.$$

Our assumption means that

$$\frac{p_{sf,t}c_{sf,t}^i}{IM_i} = \frac{\int_0^I p_{sj,t}c_{sj,t}^i dj}{DD_i} \quad \forall (i,s) \in I \times S$$

and

$$\frac{p_{sf,t}x_{sf,t}^i}{IM_i} = \frac{\int_0^I p_{sj,t}x_{sj,t}^i dj}{DD_i} \quad \forall (i,s) \in I \times S.$$

Finally, let M_i^d and M_i^f denote region i's total expenditures on intermediate goods produced in Japan and in foreign countries:

$$M_i^d = \int_0^I \int_0^S p_{sj,t} \int_0^S m_{sj,t}^{hi} dhdsdj$$

and

$$M_i^f = \int_0^S p_{sf,t} \int_0^S m_{sf,t}^{hi} dhds.$$

Then, our assumptions mean that the intermediate good demand from sector hi for imported good s satisfies the following condition:

$$\frac{p_{sf,t}m_{sf,t}^{hi}}{M_i^f} = \frac{\int_0^I p_{sj,t}m_{sj,t}^{hi} dj}{M_i^d}.$$

Online Appendix

D Proof of Proposition 1

Now we set $S = I = 1$ and $A = (\alpha^\alpha(1-\alpha)^{1-\alpha})^{-1}$. Then, $\lambda = P_m^\alpha w^{1-\alpha}$ hold. In equilibrium, N and M are linear in y as

$$N = (1-\alpha)(\lambda/w)y = (1-\alpha)\left(\frac{\sigma}{\sigma-1}\frac{P_m}{p}\right)^{\alpha/(1-\alpha)} y$$

and

$$M = \alpha(\lambda/P_m)y = \alpha\left(\frac{\sigma}{\sigma-1}\frac{P_m}{p}\right)^{-1} y. \tag{25}$$

(25) implies $py - P_m M = \mu_c py$, where $\mu_c \equiv 1 - \alpha(\sigma-1)/\sigma$. Also, we have $PC = py - P_m M$ from the household budget constraint.

The following equations determine the equilibrium:

$$\frac{p}{P} = \left(1 + \left(\frac{p}{p_f}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} =: \Gamma(p), \tag{26}$$

$$\frac{p}{P_m} = \left(1 + \gamma_f\left(\frac{p}{p_f}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} =: \Gamma_m(p), \tag{27}$$

$$y = p^{-\sigma}\left(P^\sigma C + P_m^\sigma M + \omega^f\right), \tag{28}$$

$$PC = \mu_c py, \tag{29}$$

and

$$\frac{w}{P} = \chi C^{\sigma_c} N^\zeta. \tag{30}$$

Then, the demand for domestic goods (28) is modified as

$$\left(1 - \left(\mu_c\Gamma(p)^{1-\sigma} + (1-\mu_c)\Gamma_m(p)^{1-\sigma}\right)\right) p^\sigma y = \omega^f. \tag{31}$$

The left-hand side is increasing in p, and thus, this equation can be drawn as a downward-sloping demand curve. More importantly, the demand curve shifts rightward with ω^f.

Note that (26) and (27) imply $p/P > 1$ and $p/P_m \geq 1$ (equality holds when $\gamma_f = 0$). Then, $1/(1-\mu_c\Gamma(p)^{1-\sigma}-(1-\mu_c)\Gamma_m(p)^{1-\sigma})$ works like a "multiplier" of export sales $p^{1-\sigma}\omega^f$ to consumption expenditure PC, and it is greater when the domestic goods price p relative to CPI P is smaller.

If no imports are used as intermediate inputs, i.e. $\gamma_f = 0$, we have $\Gamma_m(p) = 1$ and (31) reduces to $\omega^f = (1-\Gamma(p)^{1-\sigma})\mu_c p^\sigma y$. If the import use in the intermediate sector is symmetric to the consumption sector, i.e. $\gamma_f = 1$, we have $\Gamma_m(p) = \Gamma$ and (31) reduces to $\omega^f = (1-\Gamma(p)^{1-\sigma})p^\sigma y$.

The supply function of domestic goods is derived from (29) and (30). We use

$$\frac{w}{P} = \left(\frac{\sigma-1}{\sigma}\left(\frac{p}{P}\right)^{1-\alpha}\left(\frac{p}{P_m}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}.$$

Then, we obtain

$$\chi\xi_c^{\sigma_c}(1-\alpha)^\zeta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1+\alpha\zeta}{1-\alpha}} y^{\sigma_c+\zeta} = \left(\frac{p}{P}\right)^{1-\sigma_c}\left(\frac{p}{P_m}\right)^{\frac{\alpha(1+\zeta)}{1-\alpha}} = \Gamma(p)^{1-\sigma_c}\Gamma_m(p)^{\frac{\alpha(1+\zeta)}{1-\alpha}}. \tag{32}$$

The supply function is upward sloping if the wealth effect is not too strong, that is, $\sigma_c < 1$. Even if the wealth effect is substantial ($\sigma_c > 1$), (The upward sloping supply function obtains if the effects through intermediate import $\gamma_f > 0$ are strong enough.)

If $\sigma_c \leq 1$, the right-hand side of (32) is finite and continuously increasing in p. In (31), as p travels from 0 to $+\infty$ for a fixed y, the left-hand side of the equation continuously and strictly decreases from $+\infty$ to 0. Hence, the solution p that satisfies demand (31) and supply (32) exists uniquely. Moreover, since the right-hand side of (31) is strictly increasing in ω^f, we obtain that $dp/d\omega^f > 0$ for $\sigma_c \leq 1$.

From (26), $d(p/P)/dp > 0$. Thus, $d(p/P)/d\omega^f \geq 0$ for $\sigma_c \leq 1$. Then, (32) implies $dy/d\omega^f \geq 0$ for $\sigma_c \leq 1$, where equality holds if $\sigma_c = 1$ and $\gamma_f = 0$. Moreover, since $C \propto (p/P)y$, we obtain $dC/d\omega^f > 0$.

Finally, as $\omega^f \searrow 0$, we obtain $p/P \to 1$ and $p \to 0$. □

E Analytical results on the first-order effect of an export shock

(i) Static model of Appendix F with $\sigma_c = 1$

Using the labor demand function, we have labor income $w_s^i N^{si} = (1-\alpha)\lambda^{si} y_{si} = ((1-\alpha)(\sigma-1)/\sigma)Y^{si}$, where we define the revenue of si as $Y^{si} \equiv p_{si}y_{si}$. Letting regional total revenue $Y^i \equiv \sum_s Y^{si}$, regional profits are written as $\Pi^i = (1-(\sigma-1)/\sigma)Y^i$. The international transfer associated with the trade deficit is B^i. We write its ratio to regional GDP as $b^i \equiv B^i/Y^i$. From the household budget constraint we have,

$$P^i C^i = E^i = \sum_s (w_s^j N^{sj}) + \Pi^i + B^i = (1-\alpha(\sigma-1)/\sigma + b^i)Y^i.$$

Aggregating the labor demand function, we have $\sum_s N^{si} w_s^i = ((1-\alpha)(\sigma-1)/\sigma)Y^i$. Using the labor supply function, we obtain

$$\frac{\sum_s n_s^i w_s^i}{P^i} = \chi_i (C^i)^{\sigma_c} (N^i)^{\zeta-1/\tau} \sum_s (n_s^i)^{(1+\tau)/\tau} = \chi_i (C^i)^{\sigma_c} (N^i)^{\zeta+1}.$$

In equilibrium, $n_s^i = N^{si}$. Thus, combining the above equations gives

$$((1-\alpha)(\sigma-1)/\sigma)\frac{Y^i}{P^i} = \chi_i \left((1-\alpha(\sigma-1)/\sigma + b^i)\frac{Y^i}{P^i}\right)^{\sigma_c} (N^i)^{\zeta+1}.$$

Hence, when $\sigma_c = 1$, N^i is determined as $N^i = \bar{N}^i = \left(\frac{(1-\alpha)(\sigma-1)/\sigma}{\chi_i(1-\alpha(\sigma-1)/\sigma+b^i)}\right)^{\frac{1}{\zeta+1}}$ that is a constant independent of prices.

From the production function and the labor supply, we obtain

$$\begin{aligned} y_{si} &= A^{si}(M^{si})^\alpha (N^{si})^{1-\alpha} = A^{si}\left(\frac{\alpha}{1-\alpha}\frac{w_s^i}{P^{si}}\right)^\alpha N^{si} = A^{si}\left(\frac{\alpha}{1-\alpha}\frac{w_s^i}{P^{si}}\right)^\alpha \left(\frac{w_s^i}{P^i C^i \chi_i (\bar{N}^i)^{\zeta-1/\tau}}\right)^\tau \\ &= A^{si}\left(\frac{\alpha}{1-\alpha}\frac{w_s^i}{P^{si}}\right)^\alpha \left(\frac{w_s^i}{(1-\alpha(\sigma-1)/\sigma+b^i)\chi_i Y^i (\bar{N}^i)^{\zeta-1/\tau}}\right)^\tau. \end{aligned}$$

Also, w_s^i is a function of (P^{si}, p_{si}), since

$$\frac{\sigma-1}{\sigma}p_{si} = \lambda^{si} = \frac{(P^{si})^\alpha (w_s^i)^{1-\alpha}}{A^{si}\alpha^\alpha(1-\alpha)^{1-\alpha}}. \tag{33}$$

Multiplying p_{si} and y_{si}, we have

$$\begin{aligned} Y^{si} = p_{si}y_{si} &= \frac{\sigma/(\sigma-1)}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^\alpha \frac{(w_s^i)^{1+\tau}}{\left((1-\alpha(\sigma-1)/\sigma+b^i)\chi_i Y^i (\bar{N}^i)^{\zeta-1/\tau}\right)^\tau} \\ &= \frac{(\hat{N}^i w_s^i)^{1+\tau}}{(Y^i)^\tau} \end{aligned} \tag{34}$$

where

$$\hat{N}^i \equiv \left(\frac{\sigma/((\sigma-1)(1-\alpha))}{\left((1-\alpha(\sigma-1)/\sigma+b^i)\chi_i(\bar{N}^i)^{\zeta-1/\tau}\right)^\tau}\right)^{1/(1+\tau)} = \left(\frac{(\sigma/((\sigma-1)(1-\alpha)))^\zeta}{(1-\alpha(\sigma-1)/\sigma+b^i)\chi_i}\right)^{1/(1+\zeta)}.$$

Thus, we obtain Y^i as a function of $(w^i_s)_{s,i}$:

$$Y^i = \hat{N}^i \left(\sum_s (w^i_s)^{1+\tau} \right)^{1/(1+\tau)}. \tag{35}$$

The demand function for good sj implies

$$\begin{aligned} p^\sigma_{sj} y_{sj} &= \sum_i (P^i)^\sigma \xi^i_{sj} C^i + \sum_{h,i} (P^{hi})^\sigma \gamma^{hi}_{sj} M^{hi} + \omega^f_{sj} (\bar{P})^\sigma \\ &= \sum_i \left(1 - \alpha \frac{\sigma - 1}{\sigma} + b^i \right) (P^i)^{\sigma-1} \xi^i_{sj} Y^i + \alpha \frac{\sigma - 1}{\sigma} \sum_{h,i} (P^{hi})^{\sigma-1} \gamma^{hi}_{sj} p_{hi} y_{hi} + \omega^f_{sj} (\bar{P})^\sigma, \end{aligned} \tag{36}$$

using $P^i C^i = (1 - \alpha(\sigma - 1)/\sigma + b^i) Y^i$ and $P^{sj} M^{sj} = \alpha \lambda^{sj} y_{sj} = \alpha \frac{\sigma-1}{\sigma} p_{sj} y_{sj}$.

Equations (33), (34), (35), and (36) determine $(w^i_s, Y^i, P^{si}, p_{si})_{s,i}$. (33) defines w^i_s as a function of (p_{si}, P^{si}). (35) determines Y^i as an aggregation of $(w^i_s)_s$. Substituting y_{si} out by using (34), the left-hand side of (36) is $p^{\sigma-1}_{sj}$ and the right-hand size involves $(P^i, p_{hi})_{h,i}$. Moreover, the prices of consumption and intermediate composites (P^i, P^{hi}) satisfy

$$(P^i)^{1-\sigma} = \sum_{s,j} \xi^i_{sj} (p_{sj})^{1-\sigma} + S \xi^i_f (p_f)^{1-\sigma}$$

and

$$(P^{hi})^{1-\sigma} = \sum_{s,j} \gamma^{hi}_{sj} (p_{sj})^{1-\sigma} + \sum_s \gamma^{hi}_{sf} (p_{sf})^{1-\sigma}.$$

Thus, the modified (36) can be solved for an equilibrium price vector.

(ii) Comparative statics

We have a system of equations (36) that involves only a vector of $(p_{sj})_{s,j}$. This formulation allows us the comparative statics of $(p_{sj})_{s,j}$ when ω^f_{sj} is perturbed.

For the ease of exposition, we denote the final goods sector by $s = 0$ and write $Y^{0i} \equiv Y^i$. We introduce a new weight matrix $\hat{\Gamma}$ whose elements are defined as

$$\hat{\gamma}^{0i}_{sj} = \left(1 - \alpha \frac{\sigma - 1}{\sigma} + b^i \right) \xi^i_{sj}$$

and

$$\hat{\gamma}^{hi}_{sj} = \alpha \frac{\sigma - 1}{\sigma} \gamma^{hi}_{sj}.$$

Using (33), (34), (35), and (36), we have the following system for $(p_{si}, P^{si}, Y^{si}, w^i_s)_{si}$:

$$p^{\sigma-1}_{si} Y^{si} = \sum_{h=0}^{S} \sum_{j=1}^{I} \hat{\gamma}^{hj}_{si} (P^{hj})^{\sigma-1} Y^{hj} + \omega^f_{si} (\bar{P})^\sigma. \tag{37}$$

$$Y^{si} = \hat{N}^i \frac{(w^i_s)^{1+\tau}}{\left(\sum_{h=1}^{S} (w^i_h)^{1+\tau} \right)^{\tau/(1+\tau)}} \quad \text{for } s = 1, 2, \ldots, S, \tag{38}$$

$$Y^{0i} = \hat{N}^i \left(\sum_{h=1}^{S} (w^i_h)^{1+\tau} \right)^{1/(1+\tau)},$$

$$(P^{si})^{1-\sigma} = \sum_{h=1}^{S}\sum_{j=1}^{I} \gamma_{hj}^{si}(p_{hj})^{1-\sigma} + \sum_{h=1}^{S} \gamma_{hf}^{si}(p_{hf})^{1-\sigma} \quad \text{for } s = 1, 2, \ldots, S, \tag{39}$$

$$(P^{0i})^{1-\sigma} = \sum_{h=1}^{S}\sum_{j=1}^{I} \xi_{sj}^{i}(p_{hj})^{1-\sigma} + S\xi_f^i(p_f)^{1-\sigma},$$

and

$$w_s^i = \left(\frac{\sigma-1}{\sigma}\alpha^\alpha(1-\alpha)^{1-\alpha}A^{si}p_{si}(P^{si})^{-\alpha}\right)^{1/(1-\alpha)} \quad \text{for } s = 1, 2, \ldots, S. \tag{40}$$

Consumption is determined by $C^i = (1 - \alpha(\sigma-1)/\sigma + b^i)Y^{0i}/P^{0i}$.

These equations allow for the analytical expression of elasticity $d\ln p_{hj}/d\ln \omega_{si}^f$. To simplify the expression, we change variables as $q_{si} \equiv p_{si}^{\sigma-1}$ and $Q^{si} \equiv (P^{si})^{\sigma-1}$. From (39), we have $d\ln Q^{si}/d\ln q_{hj} = Q^{si}\gamma_{hj}^{si}/q_{hj}$. With the new notation (q_{si}, Q^{si}), (37) is written as $q_{si}Y^{si} = \sum_{h=0}^{S}\sum_{j=1}^{I}\hat{\gamma}_{si}^{hj}Q^{hj}Y^{hj} + \omega_{si}^f(\bar{P})^\sigma$. By log-differentiation, we obtain:

$$q_{si}Y^{si}(d\ln q_{si} + d\ln Y^{si}) = \sum_{h=1}^{S}\sum_{j=1}^{I}\hat{\gamma}_{si}^{hj}Q^{hj}Y^{hj}(d\ln Q^{hj} + d\ln Y^{hj}) + dD_{si}^c + \omega_{si}^f(\bar{P})^\sigma d\ln\omega_{si}^f, \tag{41}$$

where $D_{si}^c \equiv \sum_{j=1}^{I}\hat{\gamma}_{si}^{0j}Q^{0j}Y^{0j}$ corresponds to consumption demand.

Note that (38), (40), and the definition of Q imply:

$$d\ln Y^{si} = (1+\tau)d\ln w_s^i - \tau\sum_{s'}\frac{(w_{s'}^i)^{1+\tau}d\ln w_{s'}^i}{\sum_{s''}(w_{s''}^i)^{1+\tau}} + d\ln\hat{N}^i,$$

$$d\ln\hat{N}^i = \frac{-db^i}{(1+\zeta)(1-\alpha(\sigma-1)/\sigma + b^i)}$$

$$d\ln w_s^i = \frac{d\ln p_{si} - \alpha d\ln P^{si}}{1-\alpha} = \frac{d\ln q_{si} - \alpha\sum_{s',i'}\frac{\gamma_{s'i'}^{si}Q^{si}}{q_{s'i'}}d\ln q_{s'i'}}{(\sigma-1)(1-\alpha)},$$

and

$$d\ln Q^{si} = Q^{si}\sum_{s',i'}(\gamma_{s'i'}^{si}/q_{s'i'})d\ln q_{s'i'}.$$

Vectorize $S\times I$ matrices into $SI\times 1$ vectors and denote $\boldsymbol{q}$, $\boldsymbol{w}$, $\boldsymbol{Y}$, $\boldsymbol{Q}$, $\boldsymbol{\omega}^f$, and $\boldsymbol{D^c}$. Also vectorize $\mathbf{1}_{S\times 1}\cdot(\hat{N}^i)_i$ and $\mathbf{1}_{S\times 1}\cdot(b^i)_i$ and denote them by $\hat{\boldsymbol{N}}$ and $\boldsymbol{b}$ (i.e., each $\hat{N}^i$ (b^i) is duplicated for S times and stacked in a column). Let $\mathrm{diag}\,(qY)$ denote a diagonal matrix with value $q_{si}Y^{si}$ in $(S(i-1)+s, S(i-1)+s)$. Similarly, $\mathrm{diag}\,(Q)$ and $\mathrm{diag}\,(\omega^f)$ denote a diagonal matrix with Q^{si} and ω_{si}^f, respectively.

Let W_I denote an $I\times SI$ matrix with value $(w_s^i)^{1+\tau}/\sum_{s'}(w_{s'}^i)^{1+\tau}$ in element $(i, S(i-1)+s)$. Let W denote an $SI\times SI$ matrix in which row vector $W_I(i, S(i-1)+s)$ for fixed i is duplicated in rows from $S(i-1)+1$ to Si. Moreover, set G_w as a $SI\times SI$ matrix with $\gamma_{s'i'}^{si}/q_{s'i'}$ in $(S(i-1)+s, S(i'-1)+s')$.

Then, the above equations are written as follows.

$$d\ln\boldsymbol{Y} = ((1+\tau)I_{SI} - \tau W)d\ln\boldsymbol{w} + d\ln\hat{\boldsymbol{N}},$$

$$d\ln\boldsymbol{w} = \frac{(I_{SI} - \alpha\,\mathrm{diag}\,(Q)G_w)d\ln\boldsymbol{q}}{(\sigma-1)(1-\alpha)},$$

and

$$d\ln \boldsymbol{Q} = \mathrm{diag}\,(Q)G_w d\ln \boldsymbol{q},$$

where I_{SI} denotes an identity matrix of size SI. In short,

$$d\ln \boldsymbol{Y} = \frac{((1+\tau)I_{SI} - \tau W)(I_{SI} - \alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)} d\ln \boldsymbol{q} + d\ln \hat{\boldsymbol{N}}.$$

Furthermore, define G as a $SI \times SI$ matrix with $\hat{\gamma}_{si}^{s'i'} Q^{s'i'} Y^{s'i'}$ in $(S(i-1)+s, S(i'-1)+s')$. Then, (41) is written as

$$\mathrm{diag}\,(qY)(d\ln \boldsymbol{q} + d\ln \boldsymbol{Y}) = G(d\ln \boldsymbol{Q} + d\ln \boldsymbol{Y}) + d\boldsymbol{D^c} + \mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f}. \tag{42}$$

This equation leads to a decomposition equation.

$$d\ln \boldsymbol{Y} = \mathrm{diag}\,(qY)^{-1}\left[G d\ln \boldsymbol{Y} + (G d\ln \boldsymbol{Q} - \mathrm{diag}\,(qY) d\ln \boldsymbol{q}) + d\boldsymbol{D^c} + \mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f}\right]. \tag{43}$$

Response of consumption The consumption term is $D_{si}^c = \sum_{j=1}^I \hat{\gamma}_{si}^{0j} Q^{0j} Y^{0j}$. Let $\boldsymbol{Y}^0$ be a vector $(Y^{0i})_i$, $\boldsymbol{Q}^0$ a vector $(Q^{0i})_i$, and Γ^0 an $SI \times I$ matrix containing $\hat{\gamma}_{si}^{0j} Q^{0j} Y^{0j}$ in $(S(i-1)+s, j)$. Note that $\hat{\gamma}_{sj}^{0i}$ includes b^i. Let $\boldsymbol{b}_I$ denote a vector $(b^1, b^2, \ldots, b^I)'$. Also, let $\hat{\boldsymbol{N}}_I$ denote a vector $(\hat{N}^1, \hat{N}^2, \ldots, \hat{N}^I)'$. Then, we can write

$$d\boldsymbol{D^c} = \Gamma^0 \left(d\ln \boldsymbol{Q}^0 + d\ln \boldsymbol{Y}^0 + \mathrm{diag}\left(\frac{1}{1-\alpha(\sigma-1)/\sigma + b_I}\right) d\boldsymbol{b}_I \right)$$

Plugging into (43), we obtain a decomposition equation:

$$\begin{aligned} d\ln \boldsymbol{Y} = \mathrm{diag}\,(qY)^{-1} \big[& G d\ln \boldsymbol{Y} + (G d\ln \boldsymbol{Q} - \mathrm{diag}\,(qY) d\ln \boldsymbol{q} + \Gamma^0 d\ln \boldsymbol{Q}^0) \\ & + \Gamma^0 d\ln \boldsymbol{Y}^0 + \Gamma^0\,\mathrm{diag}\left(\frac{1}{1-\alpha(\sigma-1)/\sigma + b_I}\right) d\boldsymbol{b}_I + \mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f} \big] \end{aligned}$$

We further proceed with

$$Y^{0i} = \hat{N}^i \left(\sum_{s=1}^S (w_s^i)^{1+\tau} \right)^{1/(1+\tau)},$$

$$(Q^{0j})^{-1} = \sum_{s=1}^S \sum_{i=1}^I \xi_{si}^j q_{si}^{-1} + S\xi_f^j (p_f)^{1-\sigma}.$$

From these, we have

$$\begin{aligned} d\ln Y^{0i} = & \frac{\sum_s (w_s^i)^{1+\tau} d\ln w_s^i}{\sum_{s'} (w_{s'}^i)^{1+\tau}} + d\ln \hat{N}_I \\ d\ln Q^{0j} = & Q^{0j} \sum_{s,i} \left(\xi_{si}^j q_{si}^{-1} d\ln q_{si} \right). \end{aligned}$$

Then,

$$d\ln \boldsymbol{Y}^0 = W_I d\ln \boldsymbol{w} + d\ln \hat{N}_I = W_I \frac{I_{SI} - \alpha\,\mathrm{diag}\,(Q)G_w}{(\sigma-1)(1-\alpha)} d\ln \boldsymbol{q} + d\ln \hat{N}_I.$$

Also, let matrix $\Xi_{I\times SI}$ contain $Q^{0j}\xi^j_{si}q^{-1}_{si}$ in $(j, S(i-1)+s)$. Then we obtain

$$d\ln \boldsymbol{Q}^0 + d\ln \boldsymbol{Y}^0 = \left(\frac{W_I(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)} + \Xi\right) d\ln \boldsymbol{q} + d\ln \hat{N}_I.$$

Using (42),

$$\begin{aligned}
&\mathrm{diag}\,(qY)\left(I_{SI} + \frac{((1+\tau)I_{IS}-\tau W)(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)}\right) d\ln \boldsymbol{q} + \mathrm{diag}\,(qY) d\ln \hat{\boldsymbol{N}}\\
&= G\left(\mathrm{diag}\,(Q)G_w + \frac{((1+\tau)I_{IS}-\tau W)(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)}\right) d\ln \boldsymbol{q} + G d\ln \hat{\boldsymbol{N}}\\
&+ d\boldsymbol{D^c} + \mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f}
\end{aligned}$$

or,

$$\begin{aligned}
d\ln \boldsymbol{q} =& \left[\mathrm{diag}\,(qY) - G\,\mathrm{diag}\,(Q)G_w + (\mathrm{diag}\,(qY)-G)\,\frac{((1+\tau)I_{IS}-\tau W)(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)}\right]^{-1}\\
&\cdot\left(d\boldsymbol{D^c} + \mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f} + (G-\mathrm{diag}\,(qY))d\ln \hat{\boldsymbol{N}}\right). \qquad (44)
\end{aligned}$$

Construct a $SI\times SI$ matrix G^0, whose $(S(i-1)+1)$-th column equals the i-th column of Γ^0 and other columns are zeros. Substituting out $d\boldsymbol{D^c}$ in (44) yields

$$\begin{aligned}
d\ln \boldsymbol{q} =& \Bigg[\mathrm{diag}\,(qY) - G\,\mathrm{diag}\,(Q)G_w\\
&+ (\mathrm{diag}\,(qY)-G)\,\frac{((1+\tau)I_{IS}-\tau W)(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)}\\
&- \Gamma^0\left(\frac{W_I(I_{SI}-\alpha\,\mathrm{diag}\,(Q)G_w)}{(\sigma-1)(1-\alpha)} + \Xi\right)\Bigg]^{-1}\\
&\cdot\left[\mathrm{diag}\,(\omega^f)(\bar{P})^\sigma d\ln \boldsymbol{\omega^f} + \left(\frac{G-\mathrm{diag}\,(qY)}{1+\zeta} + G^0\right)\mathrm{diag}\left(\frac{-1}{1-\alpha(\sigma-1)/\sigma+b}\right)d\boldsymbol{b}\right].
\end{aligned}$$

The real output is $\bar{Y}^i \equiv \sum_s \bar{p}_{si}y_{si} = \sum_s Y_{si}(\bar{q}_{si}/q_{si})^{1/(\sigma-1)}$. Hence, $d\ln \bar{Y}^i = \sum_s (Y_{si}/\bar{Y}^i)(d\ln Y_{si} - (1/(\sigma-1))d\ln q_{si})$.

F Static model

The baseline model presented in the main text (Section 3) is a dynamic model that incorporates investment. In this section, we consider a static model to contrast with the results in the main text. Even though some portions are straightforward modifications of the baseline model, we allow for some overlap with the main text for the sake of a self-contained exposition of the static model.

(i) Model setting

The setting is identical to the baseline model, except for the absence of capital stock and investment. Consider a small open economy with I regions. In each region, there are S industries. Thus an industry is indexed by (s, i), where $s \in [0, S]$ and $i \in [0, I]$. Each industry (s, i) is monopolistically competitive; that is, only one firm produces in industry (s, i). The production of a good requires labor and intermediate goods as inputs. Product (s, i) is used for consumption, intermediate goods for production, and export. Each region i has a representative consumer who owns the firm in region i, supplies labor for the firms in region i, and consumes both domestic goods and imported goods.

(A) Representative consumer

The representative consumer in region i maximizes utility

$$U^i = \frac{(C^i)^{1-\sigma_c} - 1}{1-\sigma_c} - \chi_i \frac{(N^i)^{1+\zeta}}{1+\zeta}$$

subject to

$$P^i C^i \leq E^i \equiv \int_0^S w_s^i n_s^i ds + \Pi^i + B^i, \tag{45}$$

where

$$C^i = \left[\int_0^S \int_0^I (\xi_{sj}^i)^{\frac{1}{\sigma}} (c_{sj}^i)^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\xi_{sf}^i)^{\frac{1}{\sigma}} (c_{sf}^i)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}$$

and

$$N^i = \left[\int_0^S (n_s^i)^{\frac{\tau+1}{\tau}} ds \right]^{\frac{\tau}{\tau+1}}.$$

The notations are the same as our baseline formulation. The variable E^i represents the expenditure of consumer i.

The consumer's optimization implies the labor supply relationship

$$\frac{w_s^i}{P^i} = \chi_i (C^i)^{\sigma_c} (N^i)^{\zeta} \left(\frac{n_s^i}{N^i} \right)^{\frac{1}{\tau}}. \tag{46}$$

For the consumption of the goods, the consumer allocates consumption across goods by solving the expenditure-minimization problem

$$\min_{c_{sj}^i, c_{sf}^i} \quad \int_0^S \int_0^I p_{sj} c_{sj}^i d_j ds + \int_0^S p_f c_{sf}^i ds$$

subject to

$$C^i = \left[\int_0^S \int_0^I (\xi_{sj}^i)^{\frac{1}{\sigma}} (c_{sj}^i)^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\xi_f^i)^{\frac{1}{\sigma}} (c_{sf}^i)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}}.$$

Here, p_{sj} is the price of good (s,j), which is common across regions. The prices of the imported goods are assumed to be common at p_f. The solution of the optimization implies the demand for domestic goods

$$c^i_{sj} = \left(\frac{p_{sj}}{P^i}\right)^{-\sigma} \xi^i_{sj} C^i,$$

and for imported foreign goods

$$c^i_{sf} = c^i_f = \left(\frac{p_f}{P^i}\right)^{-\sigma} \xi^i_f C^i,$$

where the price index is written as

$$P^i \equiv \left[\int_0^S \int_0^I \xi^i_{sj} (p_{sj})^{1-\sigma} dj ds + S\xi^i_f (p_f)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{47}$$

(B) Production

In region i, good h is produced by the production function

$$y_{hi} = A^{hi}(M^{hi})^{\alpha}(N^{hi})^{1-\alpha},$$

where

$$M^{hi} = \left[\int_0^S \int_0^I (\gamma^{hi}_{sj})^{\frac{1}{\sigma}} (m^{hi}_{sj})^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^S (\gamma^{hi}_{sf})^{\frac{1}{\sigma}} (m^{hi}_{sf})^{\frac{\sigma-1}{\sigma}} ds\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, m^{hi}_{sj} is intermediate good s from region j used in production of good h in region i and γ^{hi}_{sj} is a parameter. Similarly, m^{hi}_{sf} is imported intermediate good s used in production of good h in region i and γ^{hi}_{sf} is a parameter.

The demand function for intermediate goods is

$$m^{hi}_{sj} = \left(\frac{p_{sj}}{P^{hi}}\right)^{-\sigma} \gamma^{hi}_{sj} M^{hi} \quad \text{for } j \in \{[0,1], f\},$$

where

$$P^{hi} \equiv \left[\int_0^S \int_0^I \gamma^{hi}_{sj} (p_{sj})^{1-\sigma} dj ds + \int_0^S \gamma^{hi}_{sf} (p_{sf})^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}}. \tag{48}$$

Thus, the total demand for good (s,j) is, by adding the consumption demand and the intermediate good demand,

$$\begin{aligned} y_{sj} &= \int_0^I c^i_{sj} di + \int_0^S \int_0^I m^{hi}_{sj} di dh + y^f_{sj} \\ &= (p_{sj})^{-\sigma} \left(\int_0^I (P^i)^{\sigma} \xi^i_{sj} C^i di + \int_0^S \int_0^I (P^{hi})^{\sigma} \gamma^{hi}_{sj} M^{hi} di dh\right) + y^f_{sj}, \end{aligned}$$

where y^f_{sj} represents the foreign (export) demand. Assume that the foreign demand takes the form

$$y^f_{sj} = \omega^f_{sj} (p_{sj})^{-\sigma} (\bar{P})^{\sigma},$$

that is, foreign demand has the same price elasticity as domestic demand. $\bar{P}$ is the price level in the foreign country.

Let

$$D_{sj} \equiv \left(\int_0^I (P^i)^{\sigma} \xi^i_{sj} C^i + \int_0^S \int_0^I (P^{hi})^{\sigma} \gamma^{hi}_{sj} M^{hi} di dh + \omega^f_{sj} (\bar{P})^{\sigma}\right) \tag{49}$$

so that the demand of good (s,j) can be expressed as $(p_{sj})^{-\sigma} D_{sj}$ We analyze the firm's problem in two steps. First, the firm chooses the combination of inputs to minimize the unit cost:

$$\min_{M^{sj},N^{sj}} \quad P^{sj} M^{sj} + w_s^j N^{sj}$$

subject to

$$1 = A^{sj} (M^{sj})^{\alpha} (N^{sj})^{1-\alpha}.$$

The solution yields the unit cost λ^{sj}:

$$\lambda^{sj} = \frac{(P^{sj})^{\alpha} (w_s^j)^{1-\alpha}}{A^{sj} \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{50}$$

and the derived factor demand for unit output:

$$M^{sj,1} = \frac{\alpha}{P^{sj}} \lambda^{sj},$$

$$N^{sj,1} = \frac{1-\alpha}{w_s^j} \lambda^{sj}.$$

Second, the firm maximizes profit:

$$\max_{p_{sj}} \ (p_{sj} - \lambda^{sj})(p_{sj})^{-\sigma} D_{sj}.$$

The result is the standard constant markup rule:

$$p_{sj} = \frac{\sigma}{\sigma - 1} \lambda^{sj}. \tag{51}$$

Thus the production of good (s,j) is

$$y_{sj} = \left(\frac{\sigma}{\sigma - 1} \lambda^{sj} \right)^{-\sigma} D_{sj}. \tag{52}$$

The derived factor demand can, therefore, be computed from:

$$M^{sj} = \frac{\alpha}{P^{sj}} \lambda^{sj} y_{sj} \tag{53}$$

and

$$N^{sj} = \frac{1-\alpha}{w_s^j} \lambda^{sj} y_{sj}. \tag{54}$$

(C) Trade balance

As in the baseline model, we allow for trade imbalance. At the national level, the international account is

$$\int_0^I B^i di = \int_0^S \int_0^I p_{sf} \left(c_{sf}^i + \int_0^S m_{sf}^{hi} dh \right) dids - \int_0^S \int_0^I p_{si} y_{si}^f dids.$$

(D) Equilibrium

The labor market equilibrium requires

$$N^{sj} = n_s^i$$

for all (s,i). The total profit income is

$$\Pi^i = \int_0^S (p_{si} - \lambda^{si}) y_{si} ds. \tag{55}$$

From these two pieces of information and the budget constraint, we can compute the equilibrium value of C^i.

(ii) Computation and calibration

(A) Computation

The equilibrium of the model economy is computed with the following steps. Note that this model is static, and therefore, we can compute it period-by-period.

1. Normalize the foreign price level $\bar{P} = 1$. Normalize the import price $p_f = 1$.

2. Guess w_s^i for all (s, i) and p_{si} for all (s, i). Then, we can compute price indices P^i and P^{si} from (47) and (48). The unit cost λ^{si} can be computed from (50), and then p_{si} can be verified using the markup formula (51). Thus, for a given (w_s^i), we can obtain p_{si} that is consistent with this (w_s^i) from this routine.

3. Further guess D_{si} for all (s, i). Then y_{sj}, M^{sj}, N^{sj} can be computed by (52), (53), (54). Π^i can be computed from (55). Then $E^i = \sum_{s=1}^{S} w_s^i n_s^i + \Pi^i + B^i$ can be computed. Budget constraint (45) can be used to compute C^i, and the information on M^{sj} and C^i can be used in (49) to check whether the initial guess on D_{si} was correct.

4. Finally, we check (w_s^i) using (46).

(B) Calibration

Calibration is similar to the baseline model. We start from the economy in 2008Q3, that is, just before the export shock hits. The consumption share parameters $\{\xi_{sj}^i\}_{i,sj}$ are calibrated so that the consumption expenditure share of good (s, j), which represents good s produced in region j, by the region i consumer matches the data in the baseline economy. The consumption shares are taken from the inter-regional input-output table for 2005 (IRIO2005), which is the closest time period before 2008Q3. Similarly, we set the target for $\{\xi_i^f\}_i$ as the GDP share of export goods produced in region i. Parameters governing the demand for the (s, j) production by foreign countries, $\{\omega_{sj}^f\}_{sj}$, are set so that the GDP share of export goods (s, j) matches the data computed in IRIO2005.[18]

We assume that the parameter governing the wage elasticity of labor supply choice in each industry, τ, is equal to 1, as in Horvath (2000). The inverse of Frisch elasticity of overall labor supply, ζ, is set to 2.5 based on Kuroda and Yamamoto (2008).[19] The labor disutility parameter χ_i is calibrated to replicate the regional variation of the employed population in 2008Q3 taken from the Labour Force Survey (LFS), conducted by the Ministry of Health, Labour and Welfare (MHLW).[20] Note that the variation in the employed population reflects that of the labor force (or working-age population) and employment rate.[21] As a benchmark, we consider the case of $\sigma_c \to 1$.

The parameter governing the elasticity of substitution, σ, is assumed to be 5. The parameters governing the cost share of each intermediate good (s, j) for the producer of good h at region i, $\{\gamma_{sj}^{hi}\}_{hi,sj}$, are set so that those in the benchmark match the data counterparts in IRIO2005. The factor-neutral productivity for each industry (s, j), A_{sj}, is given by the product of the industry-

[18] As a result, the export-to-GDP ratio and the share of good (s, j) in the total export match the data.

[19] Kuroda and Yamamoto (2008) estimate the Frisch elasticity in Japan and report that the elasticity on the extensive and intensive margins combined ranges between 0.2 and 0.7 for males. $\zeta = 2.5$ implies the Frisch elasticity of 0.4.

[20] See, https://www.stat.go.jp/english/index.html.

[21] Although it is better to incorporate the variation of working hours per labor force, there are no reliable data disaggregating working hours into each region.

Parameter	Description	Value	Target/Source
Preference			
σ_c	curvature	1.0	Assumed
χ_i	disutility of labor supply	Table 6	LFS (2008)
ζ	inverse of Frisch elasticity	2.5	Kuroda and Yamamoto (2008)
τ	elasticity of substitution (labor)	1.0	Benchmark in Horvath (2000)
$\{\xi^i_{sj}\}_{i,sj}$	weight on consumption goods	Figure 18a	IRIO (2005)
$\{\omega^f_{sj}\}_{sj}$	weight on export goods	Figure 18c	IRIO (2005)
Technology			
$\{A_{sj}\}_{sj}$	factor neutral productivity	Tables 5 and 6	JIP (2005), MLS (2008)
$\{\alpha_s\}_s$	cost share of intermediate goods	Table 5	JIP (2005)
σ	elasticity of substitution	5.0	Assumed
$\{\gamma^{hi}_{sj}\}_{hi,sj}$	weight on intermediate goods	Figure 18b	IRIO (2005)

Table 9: Summary of the parameter values, their source/reference, and data for setting targets.

and region-specific productivity parameters; that is, $A_{sj} = A_s \times A_j$, where A_s stands for the industry-specific productivity while A_j stands for the region-specific productivity. First, we map the industry classification in the JIP Database to ours and compute the industrial TFP and cost share of intermediate goods for each industry s ($\{\alpha_s\}_s$). Given all other parameters, the region-specific productivity A_s is pinned down so that the regional variation of the average wage rate in the benchmark replicates the data counterpart computed using the Monthly Labour Survey (MLS).

Table 9 summarizes the parameter values. The values of A_j and $\{\alpha_s\}_s$ are summarized in Table 5 in Appendix C. The regional parameters A_i, χ_i, and ξ^i_f are summarized in Table 6 in Appendix C. The parameters $\{\xi^i_{sj}\}_{i,sj}$ $\{\gamma^{hi}_{sj}\}_{hi,sj}$, and $\{\omega^f_{sj}\}_{sj}$ are too numerous to be summarized in a table and are represented as heatmaps in Figure 18 in Appendix C.

(iii) Simulating the model with the Great Recession export shocks

We repeat the same experiments as in the main text. Below, we set the time series of y^f_{sj} so that the time path of the export value replicates the regional export data.

We simulate the export shocks to industry si (i.e., industry s in region i) in period t by changing $y^f_{si,t}$ for the following to hold in equilibrium:

$$\frac{y^f_{si,t}}{y^f_{si,t=0}} = \frac{\text{real export of } si \text{ in } t \text{ in data}}{\text{real export of } si \text{ in } t=0 \text{ in data}},$$

where the reference period $t = 0$ corresponds to the third quarter of 2008 (2008Q3).

A variable of our primary interest is domestic (final) demand, equivalent to consumption in this static model, at the national and regional levels. The real consumption is computed excluding the consumption of imported goods. Formally, the real consumption for region i in period t, $\bar{C}_{i,t}$, is defined as

$$\bar{C}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} c^i_{sj,t} dj ds,$$

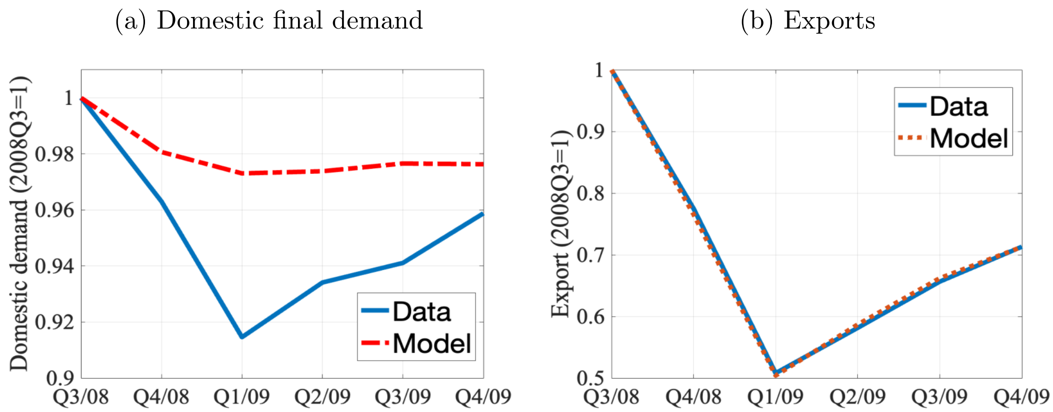

Figure 20: National responses. "Data" plots the fluctuations of HP-filtered variables, normalizing 2008Q3's value as 1.

where $p_{sj,t=0}$ is the goods price produced in industry sj in reference period $t = 0$. The national (real) consumption in period t, $\bar{C}_{Japan,t}$, is then defined as

$$\bar{C}_{Japan,t} = \int_0^I \bar{C}_{i,t} di.$$

(A) National level response

Figure 20 draws the domestic final demand and exports at the national level. By construction, the model's export values exactly match the data. The model accounts for 63.2% of the decline in consumption in 2009Q1 and 19.5% of the decline in average consumption from 2008Q4 to 2009Q4. The demand decline in the static model is more modest than in the dynamic model because the static model does not capture investment. According to the data, investment experienced a substantial decline, which contributed significantly to the decline in domestic demand and GDP.

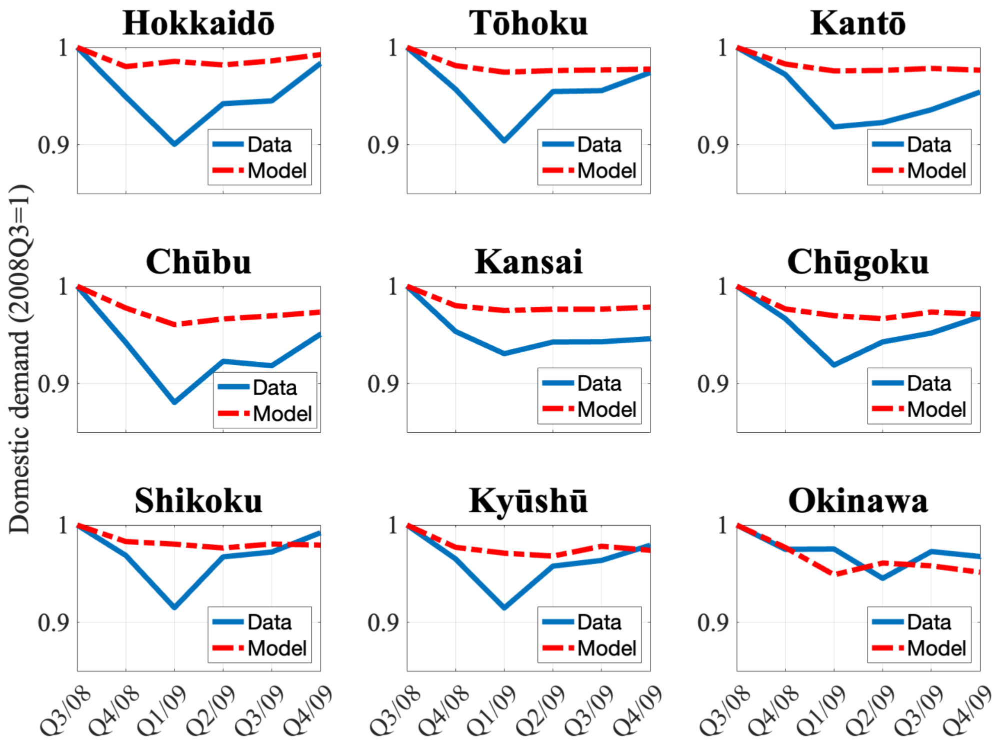

Figure 21: Regional responses

(B) Responses at the regional level

Figure 21 compares the regional demand for domestic final goods between the model and data. The model explains the data particularly well for regions such as Chūbu, Kantō, and Kansai, which experienced a large decline in exports. In contrast, for regions such as Shikoku and Okinawa, where the export shocks were not significant, the model performed poorly in explaining the regional decline in GDP.

(iv) Counterfactual experiments

In this section, we conduct a controlled experiment by feeding the model only the shock on y^f_{sj} for one region and industry, while keeping y^f_{sj} for the other regions and industries constant. Here, as in the main text, we consider a negative export shock to the transportation equipment (TE) industry in Chūbu.

(A) Decomposition

To see how the export demand shock in a region affects other regions, we conduct a decomposition analysis. Our decomposition is based on different demand components. In the following equation, the first term on the right-hand side represents domestic consumption demand, the second term represents domestic intermediate-good demand, and the third term represents foreign demand for goods s produced in region j.

$$y_{sj} = \int_0^I c_{sj}^i di + \int_0^S \int_0^I m_{sj}^{hi} didh + y_{sj}^f$$

The domestic consumption demand is represented as

$$c_{sj}^i = \xi_{sj}^i \left(\frac{p_{sj}}{P^i}\right)^{-\sigma} C^i, \tag{56}$$

and the domestic intermediate-good demand from industry h in region i is

$$m_{sj}^{hi} = \gamma_{sj}^{hi} \left(\frac{p_{sj}}{P^{hi}}\right)^{-\sigma} M^{hi}. \tag{57}$$

Given this background, we compute two economies. The first is the baseline economy (2008Q3) without any shocks. The second is the economy, which experienced an export shock in 2009Q1, hitting only one industry and one region. Here, as we mentioned above, we chose the transportation equipment industry in the Chūbu region. The comparison of these two provides the overall changes of the (real) output in each region given this particular shock, where the real output for region i is formulated as follows:

$$\bar{Y}_{i,t} = \int_0^S p_{si,t=0} y_{si,t} ds.$$

Our decomposition exercise involves two steps. In the first step, we decompose the output change in each region into several demand factors. More specifically, we consider the following three factors separately in decomposing the output change in region i. That is, we change only one of these factors in equations (56) and (57). The first set of factors represents the effect of *prices*. These are $p_{sj,t}$ for all s and j in (56) and (57), and price indices $P_{i,t}^c$, $P_{i,t}^x$ and $P_{hi,t}^m$ in (56) and (57). Note that the foreign price $\bar{P}$ is fixed because of the small open economy assumption. The second and third factors are $C_{i,t}$ and $M_{hi,t}$. The first step reveals through which factor a region's output is affected, but is silent about through which region. The second step then decomposes the contribution of each factor into the regions from whence those effects originate.

Figure 22 plots the decomposition result for the first step for regions other than Chūbu. The overall effect, indicated as triangles, can be positive or negative. The closer the region to Chūbu, the more significant its overall decline tends to be. The price changes lead to greater output for each region, reflecting the decline in the price of domestic goods relative to imported goods.

Figures 23 and 24 indicate the contributions of each region in accounting for the column region's decline in consumption and intermediate goods demand, respectively. The decline in each demand component for each region is largely attributed to a decline in Chūbu's demand for that region. The decline in a region's demand for its own goods and services also accounts for the decline, which is particularly important in accounting for consumption decline.

(B) The role of price flexibility

Once again, we consider a situation where all prices are fixed at the level of $t = 0$.

Figures 25 to 27 plot the results comparable to those for the flexible price benchmark. Now, the outputs of all other regions move negatively. The lack of price effect implies that consumption demand and intermediate-good demand directly affect the output of other regions.

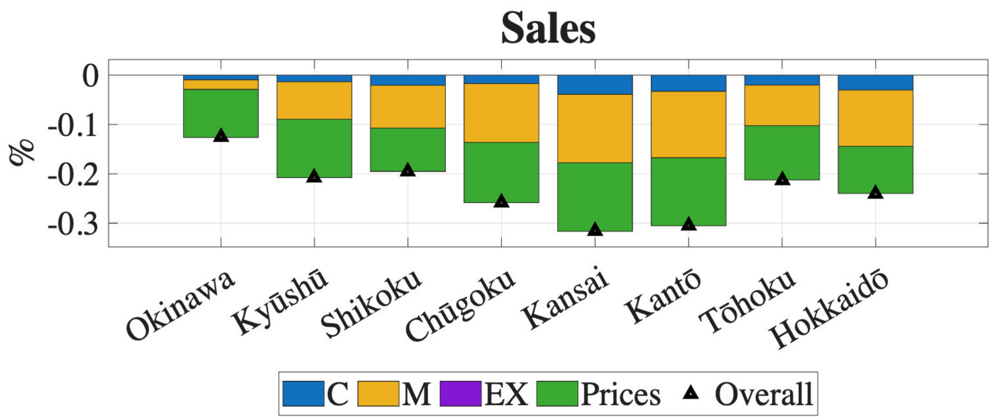

Figure 22: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

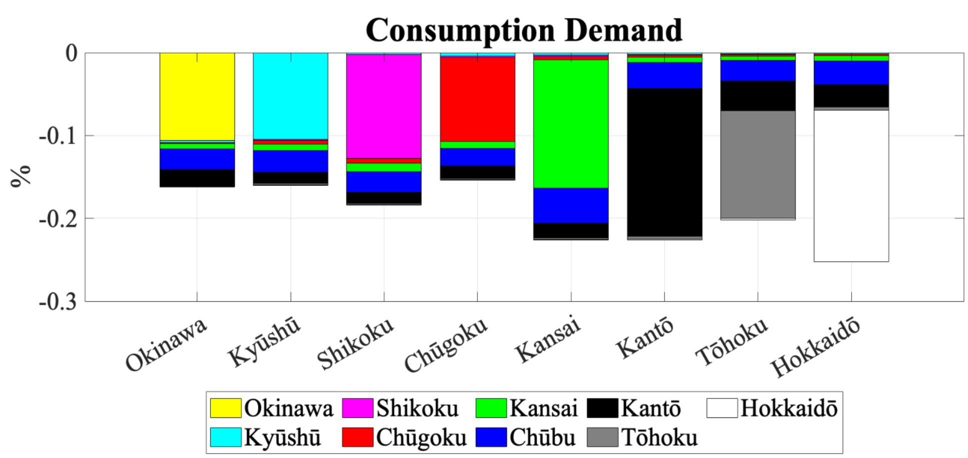

Figure 23: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

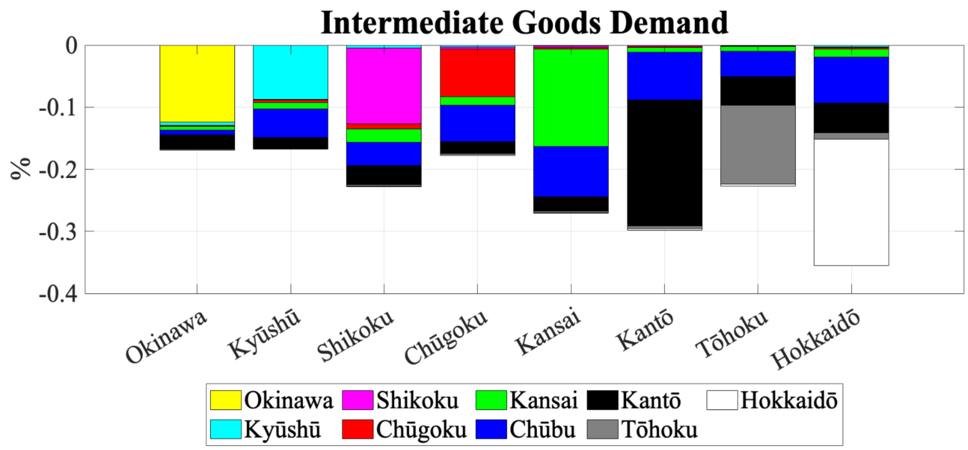

Figure 24: Changes in intermediate goods demand with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

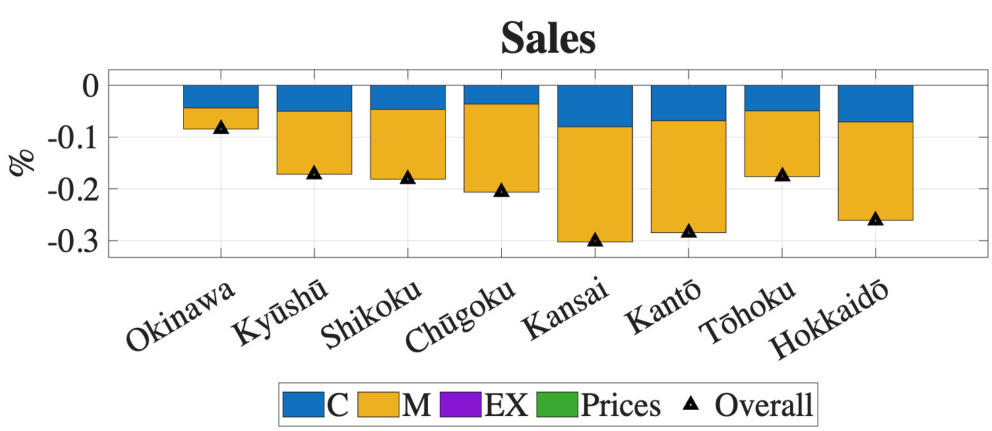

Figure 25: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

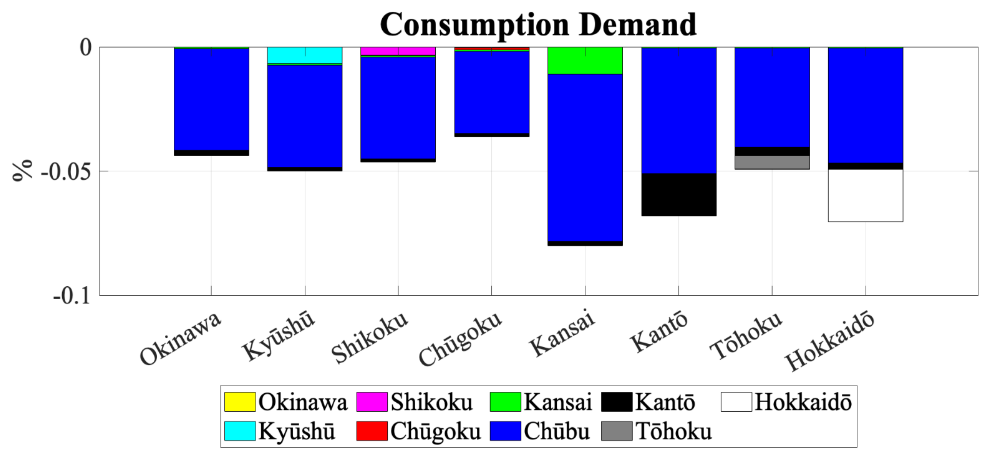

Figure 26: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

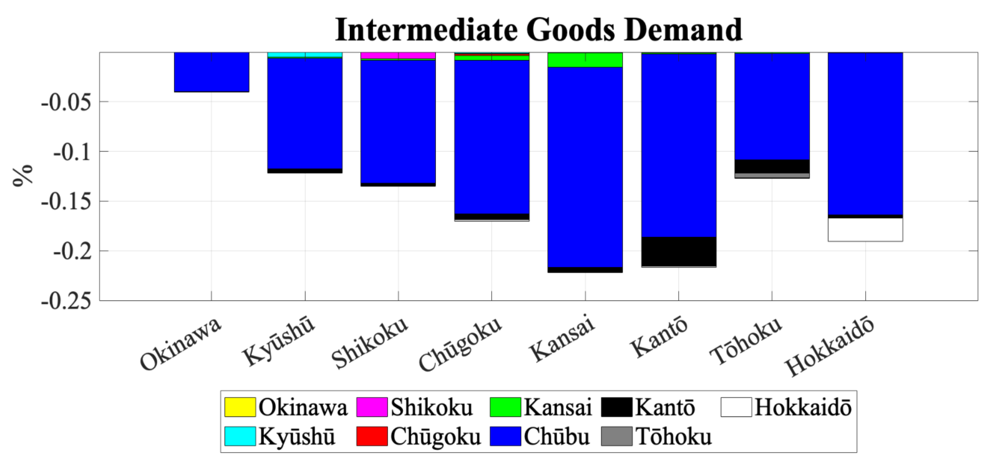

Figure 27: Changes in intermediate goods demand with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

G Computation of the dynamic model

This section details the computation of our main quantitative model.

Steady state: The steady state of the model economy is computed by the following steps.

1. Normalize the foreign price level $\bar{P} = 1$. Normalize the import prices $p_{sf} = 1$.

2. Guess w_{si} for all (s, i), p_{si} for all (s, i), and r_i for all i. Then we can compute price indices P_i^c and P_{si}^m from (4) and (8). The unit cost λ_{si} can be computed from (12), and then p_{si} can be verified using the markup formula (14). Thus, for a given (w_{si}, r_t), we can obtain p_{si} that is consistent with this (w_{si}, r_i) from this routine.

3. Using the computed prices p_{si}, we define the price indices for investment goods P_i^x for each i. Given the guessed nominal interest rates r_i, we check if the implied real interest rate r_i/P_i^x is equal to $\rho + \delta$ for each i and, if not, update r_i and return to step 2.

4. Further guess D_{si} for all (s, i). Then y_{sj}, M_{sj}, N_{sj}, K_i can be computed by (15), (16), (17), (18). Π_i can be computed from (20). Then the expenditure $E_i = \sum_{s=1}^{S} w_{si} n_{si} + \Pi_i + B^i$ can be computed. In the steady state, $X_i = \delta K_i$ has to hold. Then, the budget constraint (2) can be used to compute C_i, and the information on M_{sj} and C_i can be used in (11) to check whether the initial guess on D_{si} was correct.

5. Finally, we check w_{si}. w_{si} can be checked using (6). c_{sf}^i is given by (3), which is computed using P_i^c from step 2 and C_i from step 4.

Transition dynamics: Consider the time path of new export parameter values from $t = 1$ (i.e., 2008Q4 in our exercise) onward, and the economy reaches the final steady state at $t = T$. The final period T is set at 2010Q4.

1. Compute the final steady state at period T.

2. Guess sequences of nominal interest rates for each region, $\{\tilde{r}_{i,t}\}_{i,t=1,\dots,T-1}$.

3. Given $\{\tilde{r}_{i,t}\}_{i,t=1,\dots,T-1}$ and $\{K_{i,T}\}_i$, implement the following subroutine for each t, backward from $t = T - 1$. The algorithm of this subroutine is the same as that used to solve the steady state except for two points: (1) we do not update the guess on nominal interest rates in this subroutine, and (2) the investment $X_{i,t}$ is given by $K_{i,t+1} - (1 - \delta)K_{i,t}$, not $\delta K_{i,t}$.

   - Normalize the foreign price level $\bar{P} = 1$. Normalize the import prices $p_{sf} = 1$.
   - Guess w_{si} for all (s, i) and p_{si} for all (s, i). Then we can compute price indices P_i^c and P_{si}^m from (4) and (8). The unit cost λ_{si} can be computed from (12), and then p_{si} can be verified using the markup formula (14). Thus, for a given (w_{si}, r_t), we can obtain p_{si} that is consistent with this (w_{si}, r_i) from this routine.
   - Further guess D_{si} for all (s, i). Then y_{sj}, M_{sj}, N_{sj}, K_i can be computed by (15), (16), (17), (18). Π_i can be computed from (20). Then $E_i = \sum_{s=1}^{S} w_{si} n_{si} + \Pi_i + B^i$ can be computed. $X_{i,t}$ is given by $K_{i,t+1} - (1 - \delta)K_{i,t}$. Then, the budget constraint (2) can be used to compute C_i, and the information on M_{sj} and C_i can be used in (11) to check whether the initial guess on D_{si} was correct.

- Finally, we check w_{si}. w_{si} can be checked using (6). c^i_{sf} is given by (3), which is computed using P^c_i from step 2 and C_i from step 4.

4. Check if the implied allocations satisfy the following conditions for each t. If not, update the guesses on $r_{i,t}$ and return to step 3:

   - $t = 2, ..., T-1$: Check if the Euler equation (5) are satisfied.
   - $t = 1$: Check if capital markets clear.
     - The capital supply is fixed to its initial steady state level for each region.
     - The demand is determined by producers' optimal conditions given the nominal interest rates.