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# The Equilibrium Effects of Mortality Risk

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# The equilibrium effects of mortality risk*

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**Abstract**

In this paper, we investigate how mortality risk affects agents' optimal decisions and asset prices within a general equilibrium framework. In our model, risk-averse households facing a stochastic mortality rate allocate their net worth among consumption, risky capital production, and risk-free bonds to maximise intertemporal utility. In this setting, we show that a negative and time-varying correlation exists between mortality and risky asset prices, even when production and mortality risks are mutually independent. The correlation arises because higher mortality rates reduce the incentive to save for the future, leading to increased current consumption and decreased capital investment. As a result, higher mortality lowers the prices of risky capital and raises the risk-free rate in equilibrium. Calibrated simulations suggest that endogenous price effects account for the largest share of welfare gains and losses following sharp changes in mortality, such as the COVID-19 pandemic. (JEL C6 G11 G52)

**Keywords:** equilibrium; mortality risk; portfolio choice; stochastic optimal control

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# 1 Introduction

The worldwide phenomenon of population ageing, driven by decreasing mortality rates, has profoundly reshaped societies in recent decades. The debate over the effects of this demographic transition on financial markets and the broader macroeconomy, however, remains far from settled (see Sudo and Takizuka, 2020, and the references cited therein for a general discussion).[1] Mortality risk – defined as the uncertainty surrounding mortality rates – has emerged as a key factor in these developments, with significant implications for risk management and the sustainability of pension funds (Hari et al., 2008).[2]

Despite its importance, the literature has yet to reach a consensus on how to model the correlation between mortality risk and financial risk, often relying on simplifying assumptions and operating in a partial equilibrium setting (Dhaene et al., 2013). The vast majority of studies (see e.g. Biffis, 2005; Jalen and Mamon, 2009; Luciano et al., 2012; Deelstra et al., 2024) assume that the interest rate and mortality risks are independent. Others (e.g., Menoncin and Regis, 2020) assume an exogenous and constant correlation structure.

Although these assumptions ensure analytical tractability, they often conflict with empirical evidence. Li et al. (2023) provides an example, showing that extreme events can simultaneously impact mortality and financial markets in the short term. Favero et al. (2011) suggests that demographic changes affect asset valuations in the long run, implying that the correlation between mortality risk and asset returns is time-dependent and, critically, endogenous. With this in mind, we develop a dynamic model of a production economy where the correlation between mortality shocks and financial asset prices arises endogenously, and investigate the general equilibrium mechanisms that generate it.

Our economy is populated by a unit mass of households with uncertain lifespans. Households are risk-averse and derive utility from intertemporal consumption until death. The

[1] Different studies on the relationship between mortality and the macroeconomy offer contrasting views. Prettner (2013) argues, for example, that a longer life expectancy could foster economic growth by encouraging precautionary savings, thereby leading to higher investments. In contrast, Bloom et al. (2010) support the hypothesis that ageing slows growth by reducing saving rates and participation in the labour force. Empirical studies have found that both economic and financial developments are negatively related to mortality, due to improved living conditions and health services (Svensson and Krüger, 2012; Blau et al., 2025; Shamsfakhr, 2025).

[2] We use the terms *mortality* and *mortality rate* interchangeably throughout the paper.

death of a household is modelled as the first "jump" time of a Poisson process, as in Biffis (2005). Death events are independent and identically distributed (iid) across the population. Mortality risk is captured by a stochastic jump intensity that is common to all households. Hence, not only is the time of death uncertain, but so is its likelihood. Households use their wealth to purchase risk-free bonds and produce a homogeneous consumption good by investing in risky capital. Capital production is subject to fundamental (productivity) shocks. Importantly, productivity and mortality shocks are *independent* of each other. Markets are incomplete because households cannot ensure against aggregate mortality risk.

In this framework, we demonstrate that mortality risk influences equilibrium asset prices by increasing households' consumption and weakening their precautionary saving motive, thereby reducing demand for long-term investments. As a result of this behaviour, mortality risk generates an endogenous correlation between capital prices and mortality risk – even when the fundamental sources of risk affecting productivity and mortality are independent.

In the second part of the paper, we calibrate the model and numerically investigate the sign and magnitude of its key mechanisms by varying the parameters of the mortality process. To this end, we approximate the global solution using Monte Carlo simulations and conduct a comparative statics analysis.

Our simulations indicate that a higher mortality rate results in a higher optimal consumption rate for households, which in turn decreases the price of risky capital while increasing the risk-free interest rate. This finding aligns with empirical stylised facts showing that the demographic trend of declining mortality since the 1980s has been associated with lower interest rates (Carvalho et al., 2016; Ferrero et al., 2019). Second, we show that the endogenous correlation between capital prices and mortality is negative and becomes weaker (i.e., its absolute value decreases) as the mortality rate rises. The comparative statics also reveal that the negative correlation strengthens with both the long-run average mortality rate and the speed of mean reversion, as these factors increase the optimal consumption rate. Conversely, it weakens with a higher diffusion coefficient, which amplifies the hedging demand for the risky asset.

The final part of the paper demonstrates that accounting for the endogenous correlation between mortality and asset prices has sizeable welfare implications. To this end, we simulate the model's optimal consumption paths in response to a temporary and unanticipated shock that raises aggregate mortality above its long-term average, similar to the surge observed during the COVID-19 pandemic. We compare these paths to those of a benchmark model in which capital prices – and thus the endogenous correlation – are held constant at their long-run average, thereby isolating the feedback effect of mortality risk on households' optimal decisions. We find that the welfare effects of mortality shocks are substantial: a 20-30% instantaneous increase in the current mortality rate leads to welfare losses of approximately 5-10% in consumption-equivalent terms. Furthermore, we demonstrate that endogenous asset pricing effects explain roughly two-thirds of the total welfare losses following the mortality shock.

Our results relate to the works of Prettner and Canning (2014) and Carvalho et al. (2016), who find that increases in longevity tend to lower the interest rate due to higher precautionary savings in anticipation of longer retirement periods. In contrast to these studies, mortality risk in our model affects the prices of *both* risky and risk-free assets, generating time-varying risk-free rates, risk premia, and consumption and investment decisions. In this respect, we complement the work of Bisetti et al. (2017), who analyse the general equilibrium mechanisms linking capital asset returns and mortality, finding that longevity-linked securities significantly affect the risk-return trade-off. In the same spirit, Maurer (2018) propose a general equilibrium overlapping generations model with stochastic birth and death rates, linking the endogenous interest rate to demographic dynamics. Different from our paper, demographic changes in Maurer (2018) influence financial markets due to non-separable preferences, resulting in an increased equity premium when birth rates rise and a decreased one when death rates increase. Endogenous price effects in our model occur under standard CRRA preferences.

**Outline** The paper is structured as follows. Section 2 describes the model and solves the households' stochastic control problem. Section 3 characterises the competitive equilibrium and derives the endogenous correlation between capital returns and the mortality dynamics. Section 4 analyses the equilibrium numerically, providing a comparative statics of the endogenous correlation and the equilibrium interest rates in response to changes in relevant parameters, and assessing the welfare effects of mortality risk. Section 5 concludes.

# 2 Model

## 2.1 Environment

Time $t \in [0, \infty)$ is continuous. A unit mass of homogeneous households populates the economy. Households are risk-averse and have a stochastic lifespan. They utilise their net worth ($n_t$) to invest in risky capital ($k_t$) and risk-free bonds ($b_t$) and derive utility from inter-temporal consumption flows ($c_t$). Households are subject to both capital and mortality uncertainty, modeled through standard independent Brownian motions $W = \{(W_t^{(k)}, W_t^{(\lambda)}), t \geq 0\}$ defined on a complete probability space $(\Omega, \mathcal{F}_t, \mathbb{P})$ equipped with canonical filtration $\mathbb{F}_t := \{\mathcal{F}_t, t \geq 0\}$.

Capital serves as an input in production to generate consumption goods (output), whose price acts as a numéraire for the economy. Capital can be negotiated on the financial market at a competitive price that will be determined in equilibrium. Bonds yield the risk-free rate, which will also be defined in equilibrium.

Next, we discuss each aspect of the model in greater detail.

**Production technology and capital risk** The output flow between time $t$ and $t + dt$ is generated by the following technology:

$$y_t = (A - g_t)\, k_t, \tag{1}$$

where $A > 0$ denotes Total Factor Productivity (TFP) and $g_t$ is the fraction of output spent to generate new capital, as in Di Tella (2017). For a given $g_t$, capital evolves according to the following stochastic differential equation:

$$dk_t = k_t \left( \iota(g_t)dt + \sigma dW_t^{(k)} \right), \quad \text{with } k_0 = k, \tag{2}$$

where $\sigma > 0$ parametrises capital and production volatility.[3] The production function $\iota(g)$ characterises the expected growth rate of capital, and is such that $\iota(0) = 0$, $\iota'(g) \geq 0$, and $\iota''(g) < 0$. Following Brunnermeier and Sannikov (2016), we specify $\iota(g) = \log(1 + g \cdot \kappa)/\kappa$, where $\kappa \geq 0$ parametrizes the adjustment costs.

**Mortality risk and the gain function** Following the stochastic mortality approach (see, e.g., Menoncin and Regis, 2020), we model the random death time of a household as the first arrival time of a Poisson process $N := \{N_t, t \geq 0\}$. The intensity of the process $(\lambda_t)$ is stochastic and identical across households, and follows the Ornstein-Uhlenbeck process:

$$d\lambda_t = \mu_\lambda(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda dW_t^{(\lambda)}, \quad \text{with } \lambda_0 = \lambda, \tag{3}$$

where $\mu_\lambda$ captures the process' half-life, $\bar{\lambda}$ is the mortality long-term average, and $\sigma_\lambda$ captures *mortality risk*. Notably, mortality intensity follows a mean-reverting process and therefore does not capture long-term improvements in households' life expectancy. This assumption is justified by our focus on the uncertainty surrounding mortality within a homogeneous population, rather than on the long-term trend of increasing longevity.

Taking (3) as given, households obtain utility from inter-temporal consumption up to the random death time $\tau := \inf\{t \geq 0 : N_t = 1\}$, discounting the future at the subjective rate $\rho > 0$. The following gain function gives their lifetime utility:

$$\mathbb{E}_0\left[\int_0^\tau e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt\right], \tag{4}$$

where $\gamma > 0$ parametrises relative risk aversion and $\mathbb{E}_0[\cdot]$ represents the expected value given the information at time zero. Using standard results on Poisson processes, the time-$t$ survival

[3] As discussed in Wälde (2011), $W_t^{(k)}$ can be equivalently interpreted as a TFP shock.

probability of the agent can be expressed as follows:[4]

$$\mathbb{P}_0(t \leq \tau) = \mathbb{E}_0\left[e^{-\int_0^t \lambda_s ds}\right]. \tag{5}$$

By applying the law of iterated expectations and using (5), the gain function (4) can be conveniently rewritten as

$$\mathbb{E}_0\left[\int_0^\infty e^{-\left(\rho t+\int_0^t \lambda_s ds\right)} \frac{c_t^{1-\gamma}}{1-\gamma} dt\right]. \tag{6}$$

To ensure stationarity, we assume that when a household dies, it leaves no bequest and a new household is born in its place. As customary, the net worth of deceased households is evenly redistributed across the entire newborn population.

**Markets** Households trade capital continuously at the price $p_t$. Markets are incomplete because households cannot trade claims contingent on mortality shocks.

Since $W$ describes the whole (aggregate) uncertainty in the economy, we conjecture that $p_t$ has the following law of motion:

$$\frac{dp_t}{p_t} = \mu_t^{(p)} dt + \sigma_t^{(p,\lambda)} dW_t^{(\lambda)} + \sigma_t^{(p,k)} dW_t^{(k)}, \quad \text{with } p_0 = p. \tag{7}$$

The unknown drift and diffusion terms of this process are $\mathbb{F}_t$-measurable functions that will be determined in equilibrium. They represent, respectively, the expected increment in the capital price ($\mu_t^{(p)}$), and its sensitivity to mortality ($\sigma_t^{(p,\lambda)}$) and productivity ($\sigma_t^{(p,k)}$) shocks. Given the price process (7), investing net worth in capital yields total returns

$$dR_t = \underbrace{\frac{A-g_t}{p_t}}_{\text{Dividend yield}} dt + \underbrace{\frac{d\left(k_t p_t\right)}{k_t p_t}}_{\text{Capital gain}}$$

[4] Alternatively, as long as $\lambda_t$ follows an affine process, it is possible to write the expectation in (5) as

$$\mathbb{E}_t\left[\mathbb{I}_{t\leq\tau}\right] = e^{\alpha(t)-\beta(t)\lambda_t},$$

where $\alpha(t)$ and $\beta(t)$ solve the following Riccati ordinary differential equations system:

$$\begin{cases} \alpha(t) = \left(\bar{\lambda} - \frac{\sigma^2}{2\mu_\lambda}\right)\left[\beta(t) - t\right] - \frac{\sigma^2}{4\mu_\lambda}\beta(t)^2 \\ \beta(t) = \frac{1}{\mu_\lambda}\left(1 - e^{-\mu_\lambda t}\right) \end{cases}$$

per unit of capital. Applying Ito's lemma to the capital-gain term of this equation yields the following expression:

$$dR_t := \underbrace{\left(\frac{A - g_t}{p_t} + \iota(g_t) + \mu_t^{(p)} + \sigma\sigma_t^{(p,k)}\right)}_{:=\mu_t^{(R)}(g_t)} dt + \left(\sigma + \sigma_t^{(p,k)}\right) dW_t^{(k)} + \sigma_t^{(p,\lambda)} dW_t^{(\lambda)}. \tag{8}$$

**Remark 1.** ***(Endogenous capital-mortality risk correlation)*** *Even if productivity and mortality shocks are independent, returns on capital correlate* endogenously *with mortality risk through the diffusion term $\sigma_t^{(p,\lambda)}$. Therefore, households can partially hedge their exposure to mortality risk by adjusting their capital holdings.*

## 2.2 Households' optimisation problem

Differentiating the balance sheet constraint $n_t = k_t p_t + b_t$, we obtain the dynamics of households' net worth:

$$\frac{dn_t}{n_t} + \frac{c_t}{n_t} dt = \theta_t dR_t + (1 - \theta_t)\, r_t dt, \tag{9}$$

with $n_0 = n$, where $\theta_t := k_t p_t / n_t$ denotes the value of capital holdings as a share of net worth, and $r_t$ is the risk-free rate. Households influence the dynamics described in (9) by choosing their consumption ($c_t$), asset holdings ($\theta_t$), and capital re-investments ($g_t$). The controls $a := \{(c_t, \theta_t, g_t)\}$ are progressively measurable processes (with respect to $\mathbb{F}_t$) valued within the admissible set

$$\mathcal{A}(n_t, \lambda_t) := \{a : \Omega \times [0, \infty) \to \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}\ \}.$$

Formally, households maximize (4) subject to (3) and (9). Their value function is given by

$$J(t, n, \lambda) := \sup_{a_t \in \mathcal{A}} \mathbb{E}_t \left[ \int_t^\infty e^{-\rho(s-t) - \int_t^s \lambda_u du} \frac{c_s^{1-\gamma}}{1-\gamma} ds \right]. \tag{10}$$

Following standard arguments (see, e.g., Yong and Zhou, 1999; Oksendal, 2013), it is possible to show that $J(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) Equation:

$$(\rho+\lambda)J(t,n,\lambda)=\sup_{a\in\mathcal{A}}\left\{\frac{c^{1-\gamma}}{1-\gamma}+\frac{\partial J(t,n,\lambda)}{\partial t}+\mathcal{L}^{a}J(t,n,\lambda)\right\}, \tag{11}$$

with transversality condition $\lim_{t\to\infty}\mathbb{E}_0\left[e^{-\rho t}J(t,n,\lambda)\right]<\infty$, where $\mathcal{L}^a$ is the infinitesimal generator operator associated to the controlled process $x:=\{(n_t,\lambda_t),t\geq 0\}$ defined as

$$\mathcal{L}^{a}J(\cdot):=b(x;a)D_xJ(\cdot)+\frac{1}{2}tr(\xi(x;a)\xi'(x;a))D_x^2J(\cdot))$$

in which $b(x,a)$ and $\xi(x,a)$ denote the (controlled) drift and the diffusion of $x$, respectively. As we show in Appendix A.1, the household's problem can be solved in quasi-closed form.

**Proposition 1.** ***(Optimal control)*** *The optimal controls that solve Problem* (10) *are:*

$$c_t^*=F(t,\lambda)^{-\frac{1}{\gamma}}n_t, \tag{12}$$

$$g_t^*=\frac{p_t-1}{\kappa} \tag{13}$$

$$\theta_t^*=\frac{1}{\gamma}\left[\frac{\mu_t^{(R)}-r_t}{\left(\sigma+\sigma_t^{(p,k)}\right)^2+\left(\sigma_t^{(p,\lambda)}\right)^2}+\frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)}\frac{\sigma_t^{(p,\lambda)}\sigma_\lambda}{\left(\sigma+\sigma_t^{(p,k)}\right)^2+\left(\sigma_t^{(p,\lambda)}\right)^2}\right], \tag{14}$$

*where*

$$F(t,\lambda)=\widehat{\mathbb{E}}_t\left[\left(\int_t^\infty e^{-\frac{1}{\gamma}\int_t^s\bar{\rho}_u du}ds\right)^\gamma\right]. \tag{15}$$

*with*

$$\bar{\rho}_t=\lambda_t+\rho+(\gamma-1)\mu_t^{(R)}-\gamma(\gamma-1)\frac{\left(\sigma+\sigma_t^{(p,k)}\right)^2+\left(\sigma_t^{(p,\lambda)}\right)^2}{2}>0, \tag{16}$$

*and the expectation in* (15) *is taken under the probability measure*

$$\widehat{dW}_t^{(\lambda)}=(\gamma-1)\,\sigma_t^{(p,\lambda)}dt+dW_t^{(\lambda)}. \tag{17}$$

*Moreover, the household's value function has the following representation:*

$$J(t,n,\lambda)=F(t,\lambda)\frac{n^{1-\gamma}}{1-\gamma}. \tag{18}$$

The optimal consumption level in (12) is a linear function of the net worth ($n_t$), and a non-linear function of the mortality rate via $F(t,\lambda)^{-1/\gamma}$. The non-linear function $F(\cdot)$ can be interpreted as a risk- and preference-adjusted discount factor. As intuition suggests, a higher level of $\lambda$ leads to a higher consumption rate through the first term of (16). Notably, a larger impact of mortality risk on capital prices (i.e., a larger $\sigma_t^{(p,\lambda)}$) reduces the consumption rate by strengthening households' precautionary motive through the last term of (16). Similar to the $q$ Theory (Hayashi, 1982), the optimal capital reinvestment share, $g_t^*$, is positively related to the price of capital and negatively related to the adjustment costs ($\kappa$).

Consistent with the standard dynamic portfolio literature, the optimal capital control consists of two components: the speculative portfolio and the hedging portfolio. The speculative portfolio, captured in the first term of (14), is proportional to the Sharpe Ratio, as in Merton (1969). Accordingly, optimal capital holdings increase with the excess return ($\mu_t^{(R)} - r_t$) and decreases in the relative risk aversion ($\gamma$) and volatility ($(\sigma + \sigma_t^{(p,k)})^2 + (\sigma_t^{(p,\lambda)})^2$). The hedging portfolio, captured by the second term in (14), appears because the mortality rate is stochastic and correlated with capital returns, as in Menoncin and Regis (2020). Accordingly, as highlighted in Remark 1, households adjust their capital holdings to (partially) hedge against mortality risk. The magnitude of the hedging portfolio depends on the product between the semi-elasticity of the value function to the mortality rate $\lambda$ ($\partial F/\partial\lambda \times 1/F$) and the "beta" between mortality intensity and capital returns:[5]

$$\beta_t := \frac{Cov(dR_t, d\lambda_t)}{Var(dR_t)} = \frac{\sigma_t^{(p,\lambda)}\sigma_\lambda}{\left(\sigma + \sigma_t^{(p,k)}\right)^2 + \left(\sigma_t^{(p,\lambda)}\right)^2}. \tag{19}$$

Unlike in previous related studies, (19) in our model is time-varying and is determined endogenously in equilibrium.

Despite the semi-closed form characterisation of the optimal control, the sign and actual magnitude of the semi-elasticity and beta terms cannot be derived analytically. However, one can explore the sign of the semi-elasticity term in a "benchmark" model in which $\lambda$ is constant and $F(\lambda)$ can be solved analytically. In fact, when $\bar{\rho}$ is constant, (15) can be

[5] Differentiating (18) with respect to $\lambda$, it is straightforward to obtain $\frac{\partial J(t,n,\lambda)}{\partial\lambda}\frac{1}{J(t,n,\lambda)} = \frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)}$. Given (3) and (8), (19) follows by Ito's calculus rules.

simplified as

$$F(\lambda) = \left(\frac{\gamma}{\bar{\rho}(\lambda)}\right)^{\gamma}.$$

As a result, under the assumptions that $\gamma > 0$, we can derive that

$$\frac{\partial F(\lambda)}{\partial \lambda}\frac{1}{F(\lambda)} = -\frac{\gamma}{\bar{\rho}(\lambda)} < 0. \tag{20}$$

As the optimal consumption rate is a *decreasing* function of $F$ (see (12)), (20) reveals that, when the likelihood of death is higher, households find it optimal to consume more (save less). With this benchmark in mind, in the next section we derive the capital price dynamics (7) in the stochastic model and show that its volatility, and thus the sign and magnitude of the beta term in (14), is entirely determined in equilibrium by the semi-elasticity of $F(\cdot)$.

# 3 Equilibrium

In this section, we characterise the economy's competitive equilibrium. Then, we derive the equilibrium price of capital and show that its correlation with mortality risk depends on the slope of the endogenous function $F(\cdot)$.

**Definition 1.** ***(Competitive equilibrium)*** *A competitive equilibrium is a map from histories of shocks $W$ to $\mathbb{F}-$adapted stochastic (price) processes $\{(p_t, r_t), t \geq 0\}$ such that: (i) households solve the optimal control problem in* (10)*; (ii) capital and consumption goods markets clear.*

The first market-clearing condition requires that households' risky investments equal the value of aggregate capital:

$$n_t \theta_t^* = p_t k_t. \tag{21}$$

Since households are homogeneous and bonds are in zero net supply, (21) implies that[6]

$$k_t p_t = n_t \longleftrightarrow \theta_t^* = 1, \forall t \in [0, \infty). \tag{22}$$

Substituting the optimal portfolio (14) into (22), yields the equilibrium risk-free rate:

$$r_t = \mu_t^{(R)} + \frac{\partial F(t,\lambda)}{\partial \lambda} \frac{\sigma_t^{(p,\lambda)} \sigma_\lambda}{F(t,\lambda)} - \gamma \left[ \left( \sigma + \sigma_t^{(p,k)} \right)^2 + \left( \sigma_t^{(p,\lambda)} \right)^2 \right]. \tag{23}$$

The second market-clearing condition requires that aggregate consumption equals aggregate output after reinvestments:

$$\left( A - g_t^* \right) k_t = c_t^*. \tag{24}$$

By substituting (12) and (13) into (24) and imposing (22), the equilibrium price of capital can be expressed as

$$p_t = \frac{1 + \kappa A}{1 + \kappa F(t,\lambda)^{-\frac{1}{\gamma}}}. \tag{25}$$

This equation shows that capital prices depend on $\lambda$ solely through the optimal consumption rate $c_t^*/n_t = F(t,\lambda)^{-1/\gamma}$, to which it is inversely related.

## 3.1 Capital price and mortality correlation

We can now characterise the dynamics of $p_t$, consistently with our initial guess. Since the optimal control described in Proposition 1 is Markovian (i.e., it does not depend on time but only on the current state of $\lambda$ and $n$), we seek a process that preserves this property. Accordingly, we can apply Ito's lemma to (25) and match the resulting coefficient with (7) to verify that:

$$p_t \mu_t^{(p)} = \frac{\partial p_t}{\partial \lambda} \mu_\lambda (\bar{\lambda} - \lambda_t) + \frac{1}{2} \frac{\partial^2 p_t}{\partial \lambda^2} \sigma_\lambda^2, \tag{26}$$

[6] This condition is standard in all representative-agent general equilibrium models of asset pricing. Although outside the scope of this paper, the model could be generalized to yield an endogenous level of leverage without losing analytical tractability–either by introducing a second type of household with different risk aversion as in Dumas (1989), or by assuming a non-zero exogenous supply of bonds, (whether stochastic or deterministic) that is proportional to total output.

$$p_t\sigma_t^{(p,\lambda)} = \frac{\partial p_t}{\partial \lambda}\sigma_\lambda, \tag{27}$$

$$\sigma_t^{(p,k)} = 0. \tag{28}$$

Eq. (28) implies that the sensitivity of the total return on capital investments to a production shock is equal to that of the capital stock. The reason is that households' optimal consumption rate (see (12)) is a linear function of net worth, and net worth is linear in $k_t$ (see (21)).

Equipped with (26)-(28), one can show that the endogenous correlation between capital returns ($dR_t$) and mortality shocks ($d\lambda_t$) equals

$$Corr(dR_t, d\lambda_t) = \underbrace{\gamma\frac{\frac{\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}{1+\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}}{\sqrt{\sigma^2 + \sigma_\lambda^2\left(\gamma\frac{\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}{1+\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}\frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)}\right)^2}}}_{>0}\cdot\frac{\partial F(t,\lambda)}{\partial \lambda}\frac{1}{F(t,\lambda)}. \tag{29}$$

Since the first term in (29) is always positive, in the following proposition, we can identify the sign of the endogenous correlation between capital returns and mortality increments.

**Proposition 2.** ***(Capital price and mortality correlation)*** *The sign of the endogenous correlation between capital returns and mortality shocks is the sign of the semi-elasticity of the value function with respect to* $\lambda$*.*

By matching the result of Proposition 2 with the optimal portfolio in (14), we obtain the following lemma.

**Lemma 1.** ***(Beta)*** *The sign of the hedging portfolio–the product between the semi-elasticity of the value function wrt to* $\lambda$ *and the beta between increments in mortality and capital returns–is always positive:*

$$\beta_t\cdot\frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)} = \frac{\sigma_\lambda\gamma\frac{\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}{1+\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}}{\sigma^2 + \sigma_\lambda^2\left(\gamma\frac{\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}{1+\kappa F(t,\lambda)^{-\frac{1}{\gamma}}}\frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)}\right)^2}\left(\frac{\partial F(t,\lambda)}{\partial\lambda}\frac{1}{F(t,\lambda)}\right)^2.$$

To further study the competitive equilibrium, we parametrise the model and explore its

| Parameter | Meaning | Value | Source/Target |
|---|---|---|---|
| | Estimated parameters | | |
| $\bar{\lambda}$ | Mortality (long-run avg.) | 0.0133 | StMF data, age 65-74 |
| $\mu_\lambda$ | Mortality (half life) | 0.0834 | StMF data, age 65-74 |
| $\sigma_\lambda$ | Mortality (volatility) | 0.0007 | StMF data, age 65-74 |
| | Externally set parameters | | |
| $\gamma$ | Risk aversion | 5.0 | Standard |
| $\rho$ | Subjective discounting | 0.01 | Standard |
| | Internally set parameters | | |
| $\kappa$ | Capital adjustment cost | 18.0 | Avg market-mortality correlation |
| $A$ | Total factor productivity | 0.5 | Consumption-to-GDP |
| $\sigma$ | Stock market volatility | 0.24 | Risk-free rate |

Table 1: **Parameters**

dynamics numerically via simulations in the next section, since an analytical characterisation is not feasible.

# 4 Numerical analysis

In this section, we parametrise the model and approximate its solution numerically via simulations. Second, we investigate how changes in the mortality intensity process parameters affect the correlation between asset prices and mortality risk in equilibrium. Third, we assess the importance of the endogenous correlation mechanism by measuring its impact on households' welfare following a temporary increase in the mortality rate.

## 4.1 Parametrisation

To select the parameters of the mortality intensity process (3), we use data from the Short-term Mortality Fluctuations (StMF) dataset, which publishes weekly annualised death rates per age group.[7] We consider the Italian population (both genders) and the age class 65-74, as they represent the bulk of investors and wealth owners.[8] Formally, we assume that

[7] Details on the dataset can be found at https://www.mortality.org/Data/STMF

[8] See https://www.empower.com/the-currency/life/average-net-worth-by-age for details on this aspect.

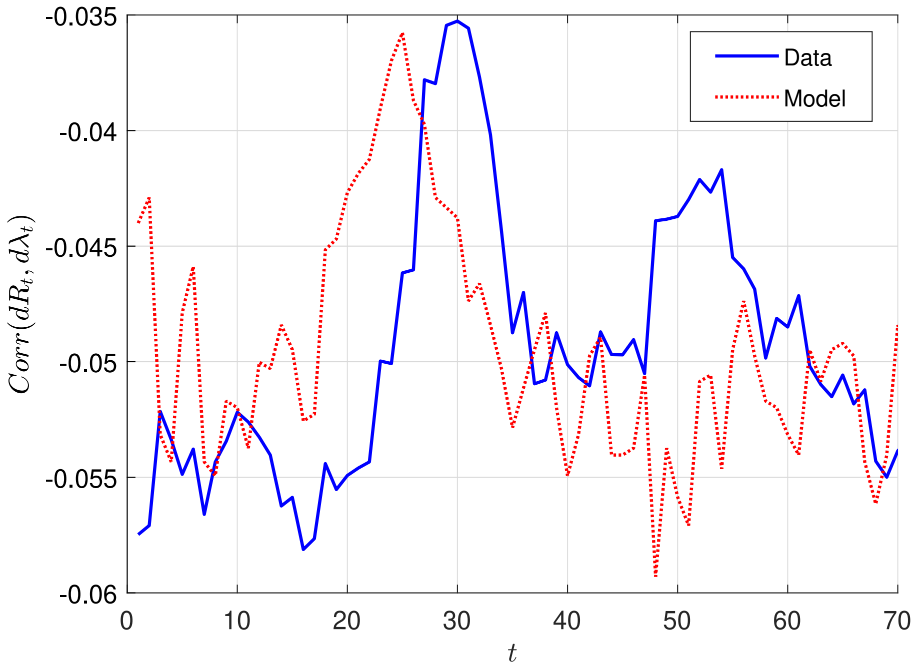


Figure 1: **Stock market-mortality correlation: model vs data** Comparison between the empirical correlation between stock returns and mortality (estimated using a 500-week rolling window) and the model-implied correlation, derived from the model using the same time series of $\lambda_t$.

the (annualised) death rate at time $t$ is a proxy for $\lambda_t$ and estimate the following regression:

$$\lambda_{t+1} = \mu_\lambda \bar{\lambda} + (1 - \mu_\lambda) \times \lambda_t + \epsilon_t,$$

where $\epsilon_t \sim N(0, \sigma_\lambda^2)$. We obtain $\bar{\lambda} = 0.0133$, $\mu_\lambda = 0.0834$, and $\sigma_\lambda = 0.0007$.

We set the risk aversion parameter $\gamma = 5$ and the subjective discount rate $\rho = 0.01$, which are standard values used in the literature. The remaining parameters, namely TFP ($A$), capital adjustment cost ($\kappa$), and volatility ($\sigma$), are set "internally" so that: (i) the consumption rate lies between 70% and 80% of GDP, as in most OECD countries; and (ii) the endogenous correlation in (29) equals its long-term average (i.e., $\lambda_t = \bar{\lambda}$) as observed in the data; (iii) the risk-free interest rate is about 2% at $\lambda_t = \bar{\lambda}$.

To find the long-term average correlation level of Point (ii), we use our mortality data and the S&P 500 returns to compute

$$\mathbb{E}_0 \left[ Corr(d \ln(S\&P500), d(\text{Death rate}_t) \right] \approx -0.05.$$

For point (iii), we obtain $\sigma = 0.24$, which is roughly in line with the S&P500 index volatility. The values of all parameters are summarised in Table 1.

To assess the quality of the model fit, Figure 1 compares the realised correlation in the data, computed using a 500-week rolling window (solid blue line), with that implied by the calibrated model, calculated using the same time series of $\lambda_t$ (dotted red line). The parametrisation yields a long-run real interest rate of about 1.5%.

## 4.2 Equilibrium effects of mortality risk

To begin the analysis, we study the behavior of the equilibrium quantities under the baseline parameterisation as functions of $\lambda$. Details of the numerical approximation algorithm are provided in Appendix A.2. For this purpose, Figure 2 displays the equilibrium levels of households' optimal consumption, the risk-free interest rate, the capital price, and the stock-mortality correlation. The shaded blue region illustrates the stationary density function of $\lambda$. The shaded blue areas represent the stationary density of $\lambda$.

These simulations entail the following four results. First, consumption rates increase with the mortality rate (Panel (a)). This outcome is intuitive: anticipating a shorter life span, the household reduces its level of precautionary savings. Second, the price of capital decreases with mortality (Panel (c)). When the household has a shorter expected lifespan, it invests less and, consequently, demands less exposure to risky assets. Notably, the inverse relationship between consumption and risky capital prices can be readily inferred by examining Eq. (25). By reviewing the optimal investment rate in Eq. (13), one can verify that it also decreases with $\lambda$. Third, the risk-free interest rate increases with $\lambda$ (Panel (b)). This occurs because, with higher mortality, both the demand for risky investments and the demand for hedging decrease. As a result, in equilibrium, asset prices fall while the risk-free rate rises with the mortality rate. This aligns with the stylised fact that the natural level of interest rates has decreased in recent decades, alongside mortality rates (see Carvalho et al., 2016, for instance). Fourth, the endogenous correlation between stock market returns and mortality is negative and increasing (i.e., decreasing in absolute value) with $\lambda$ (Panel (d)). This occurs because, when mortality is higher, the sensitivity of the consumption rate to mortality shocks and capital prices decreases (as shown by the concave relationships in Panels (a) and (c)).

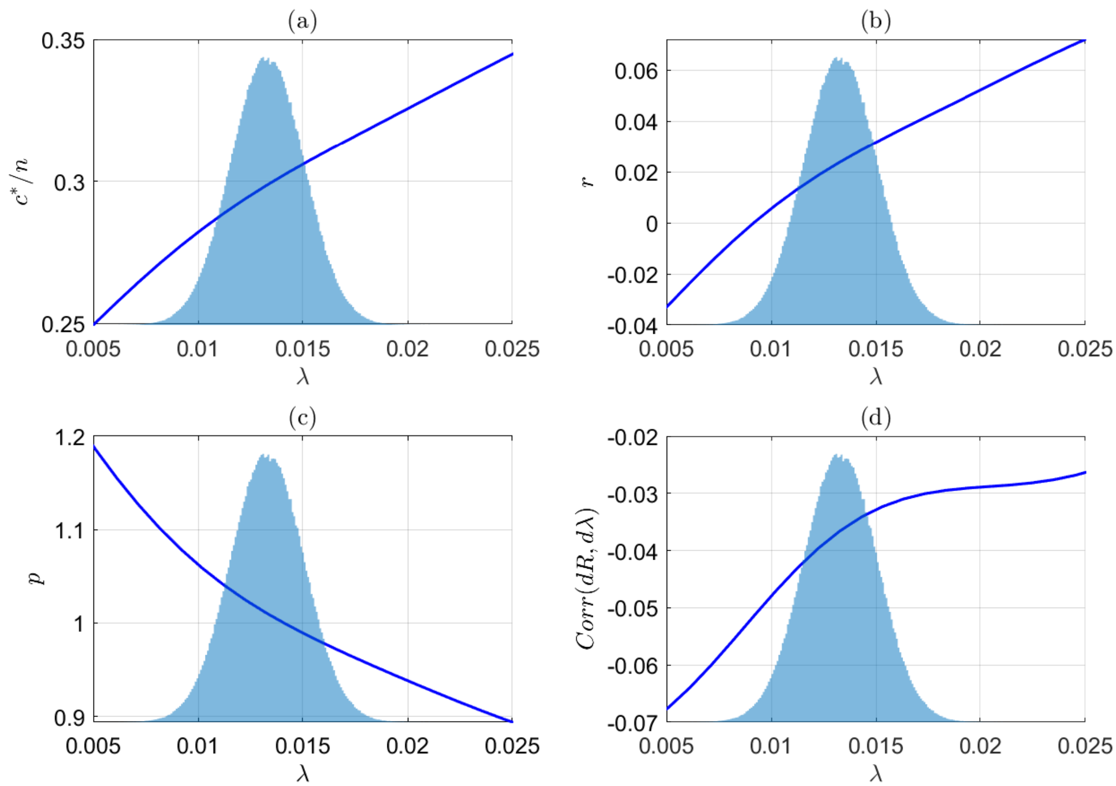


Figure 2: **Numerical solution of the model** The blue lines depict the equilibrium levels of optimal consumption, capital price, risk-free interest rate, and the endogenous stock-mortality correlation across different levels of mortality $\lambda$. The shaded blue area represents the stationary density function of $\lambda$.

As a consequence, the volatility of capital prices declines (see Eq. (27)), as does the absolute value of the correlation.

### 4.3 Comparative statics

Next, we analyse how changes in the parameters of the mortality intensity process (3), which characterise mortality risk, affect equilibrium outcomes.

**Long-run mortality** Figure 3 compares the equilibrium quantities discussed in Section 4.2 under the baseline parametrization (solid blue line) with those under a scenario in which the long-run average mortality rate ($\bar{\lambda}$) is set to 2.5 times its original value (red dotted line). The blue and red shaded areas represent the stationary distributions of $\lambda$ under the low and high $\bar{\lambda}$ scenarios, respectively. The analysis simulates the effect of a hypothetical permanent increase in the long-run mortality rate.

Holding everything else constant, a higher value of $\bar{\lambda}$ shifts the probability density function of $\lambda_t$ to the right. Nevertheless, the behaviour of the equilibrium quantities across different $\lambda$ levels, as described in Section 4.2, remains essentially unchanged by this parameter shift. The following aspects warrant particular attention.

A higher long-run mortality reduces the magnitude of the negative correlation between mortality and capital prices across all levels of $\lambda$ (Panel (d)), as consumption and prices become less sensitive to changes in $\lambda$. This is evident from the flatter slopes of the optimal consumption rate (Panel (a)) and capital price (Panel (c)), noting that the magnitude of the correlation function is pinned down by the slope of $F(\cdot)$ (see (29)). Since the optimal consumption rate and capital prices are negatively related (see (25)), the flattening of $c_t^*/n = F(t,\lambda)^{-1/\gamma}$ and thus of $p_t$ is associated with an increase (decrease) of the former (latter) for lower $\lambda$ levels and a decrease (increase) for higher levels relative to the baseline case. The opposite occurs for the risk-free rate (Panel (b)) because risk-free bonds serve as a hedge against aggregate capital risk.

**Mortality half-life and volatility** The two parts of Figure 4 present the results of a similar comparative statics analysis for the half-life ($\mu_\lambda$) – that is, the speed of mean reversion

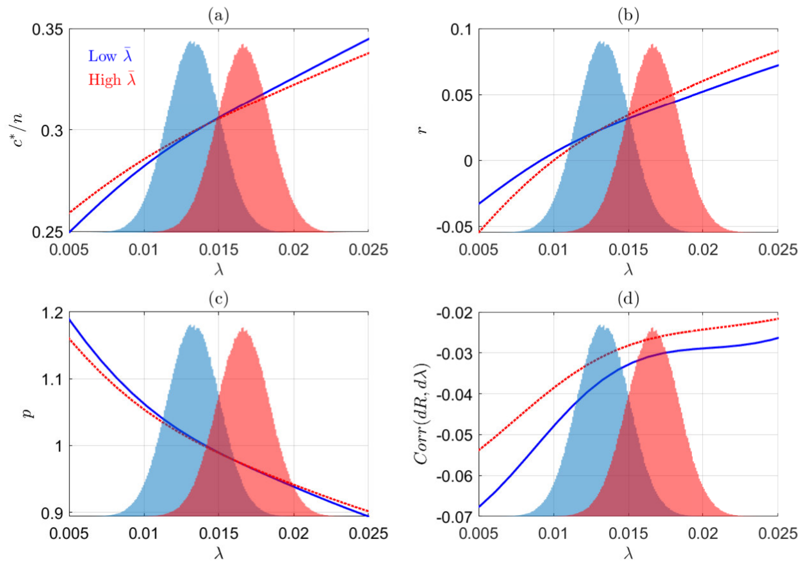


Figure 3: **Comparative statics: mortality long-run average** Optimal consumption, capital price, risk-free interest rate, and the endogenous stock-mortality correlation across different levels of mortality $\lambda$ for different levels of the long-run mortality intensity $\bar{\lambda}$.

of the mortality process (Part I) – and its volatility ($\sigma_\lambda$) (Part II).

A lower half-life (i.e., a higher mean-reversion parameter) reduces uncertainty about future mortality. As a result, households' precautionary savings motive weakens, leading to higher consumption rates (Panel I.a). Consequently, both the price of capital and the risk-free rate decline (Panels I.b and I.c). Hence, the correlation between capital price and mortality becomes less negative, as capital price volatility decreases (Panel I.d).

Opposite and symmetric patterns emerge when the volatility of the mortality rate process (i.e., mortality risk) increases, as higher mortality risk strengthens households' precautionary savings motive. From a quantitative standpoint, however, the effects on the consumption rate and the price of capital are limited, due to the relatively small value of $\sigma_\lambda$. More pronounced effects appear in the endogenous correlation between mortality and capital prices (Panel II,d), which becomes smaller in magnitude and steeper as the level of $\lambda$ varies. As a result, although overall demand for the risky asset remains nearly unchanged, as reflected in the stability of prices in Panel II.c, the hedging component of this demand becomes more significant.

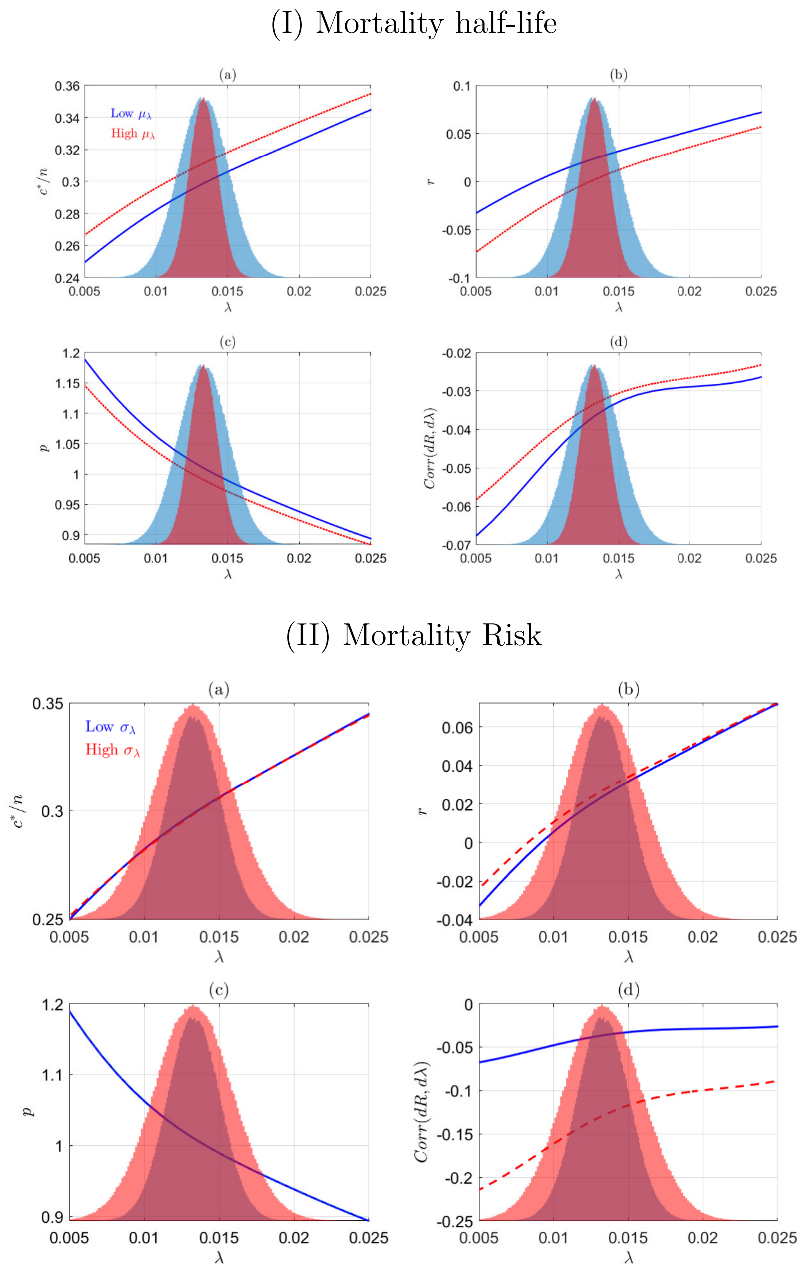


Figure 4: **Comparative statics: mortality half-life and risk** Optimal consumption, capital price, risk-free interest rate, and the endogenous stock-mortality correlation across different levels of mortality $\lambda$ for different levels of the mortality half-life $\mu_\lambda$ (Panel (I)) and mortality risk $\sigma_\lambda$ (Panel (II)) parameters.

| $\zeta(\epsilon; j)$ | $j =$ "adjusted" | $j =$ "not adjusted" |
|---|---|---|
| $\epsilon = +0.005$ | -9.41% | -1.88% |
| $\epsilon = +0.0025$ | -4.90% | -1.22% |
| $\epsilon = -0.005$ | 12.16% | 0.05% |
| $\epsilon = -0.0025$ | 5.46% | 0.02% |

Table 2: **Welfare analysis** The table compares the welfare effects, measured in consumption-equivalent units, of an expected and temporary change in the mortality rate at time $t = 0$, under two pricing scenarios: when prices remain constant ($j =$ "not adjusted") and when they adjust endogenously (($j =$ "adjusted").

## 4.4 Mortality risk and welfare

In this section, we demonstrate that accounting for the endogenous and time-varying correlation between financial asset returns and mortality risk has significant welfare implications. To this end, we use the model to evaluate the impact on households' welfare of a temporary and unexpected increase in the mortality rate $\lambda$ relative to its long-run average $\bar{\lambda}$. We then compare this outcome with the effect of the same shock in a benchmark model in which prices $p_t$ (and therefore $\mu_t^{(R)}$ and $\sigma_t^{(p,\lambda)}$) remain constant.

To assess the effect of temporary changes in the mortality rate on households' welfare, we use the following measure.

**Definition 2.** ***(Welfare measure)*** *The welfare effect of an increase in the mortality rate of size $\epsilon$, expressed in consumption-equivalent terms, is given by the following expression:*

$$\zeta(\epsilon; j) := \left( \frac{F\left(t, n, \bar{\lambda}\right)}{F^j\left(t, n, \bar{\lambda} + \epsilon\right)} \right)^{\frac{1}{\gamma - 1}} - 1, \tag{30}$$

*where $F^j(\cdot)$ denotes the endogenous function in* (15) *when $p_t$ (but not $\lambda_t$) varies over time ($j =$ "adjusted") or remains at its steady-state level ($j =$ "not adjusted").*

Table 2 collects the simulated values of (30) for different (positive and negative) levels of $\epsilon$. As intuition suggests, welfare increases when mortality rates decrease, and vice versa, since agents can expect to enjoy consumption over a longer horizon. More importantly, the simulations reveal that accounting for price adjustments driven by changes in the endogenous correlation has a substantial impact. Specifically, a 30% increase in the initial mortality rate

$\lambda_0$ leads to a welfare loss of approximately 9%, whereas the loss is less than 2% when prices are held constant. This implies that endogenous price adjustments account for roughly 75% of the welfare change. For a 20% increase in $\lambda_0$, the welfare loss is nearly 5% (compared to just 1.2% without price adjustments). The magnitude of the endogenous price effects becomes even more striking when considering reductions in the initial mortality rate. Welfare gains– 12.2% and 5.5% for 30% and 20% shocks, respectively – are more pronounced, as prices and consumption functions are steeper when $\lambda$ is lower (see Figure 2, Panels (a) and (b), respectively). These welfare improvements are largely overlooked when the price of capital is held constant at its initial equilibrium level.

These outcomes can be reconciled with the general equilibrium mechanism described in the previous sections as follows. From Section 4.2 (see in particular the discussion of Figure 2), we know that when $\lambda$ increases (decreases), the price of capital $p_t$ decreases (increases). Consequently, the expected return on capital, $\mu_t^{(R)}$, increases (decreases), while the endogenous volatility $\sigma_t^{(p,\lambda)}$ decreases (increases). These forces affect welfare because $\mu_t^{(R)}$ and $\sigma_t^{(p,\lambda)}$ determine the risk-adjusted discount rate $\bar{\rho}(\lambda)$ in (16) – positively and negatively, respectively – which, in turn, pins down the valuation of future consumption flows negatively and positively.

To illustrate these effects, Figure 5 displays the transition paths (and their average) of 2,500 simulations of $\mu_t^{(R)}$ (panel (a)) and $\sigma_t^{(p,\lambda)}$ (panel (b)) following a 0.005 increment in $\lambda_0$. As $\mu_t^{(R)}$ suddenly increases and $\sigma_t^{(p,\lambda)}$ decreases after the shock, welfare declines because $\bar{\rho}(\lambda)$ falls. Holding prices fixed mitigates these effects, since $\bar{\rho}(\lambda)$ decreases solely due to the instantaneous increase in the mortality rate $\lambda$.

# 5 Conclusions

We have examined how mortality risk influences the optimal consumption and asset allocation decisions of households facing capital and uninsurable production risks, and how these decisions, in turn, affect asset prices in general equilibrium. In particular, we have shown that a negative and time-varying correlation between mortality and asset prices arises in equilibrium, even when production and mortality shocks are mutually independent. This

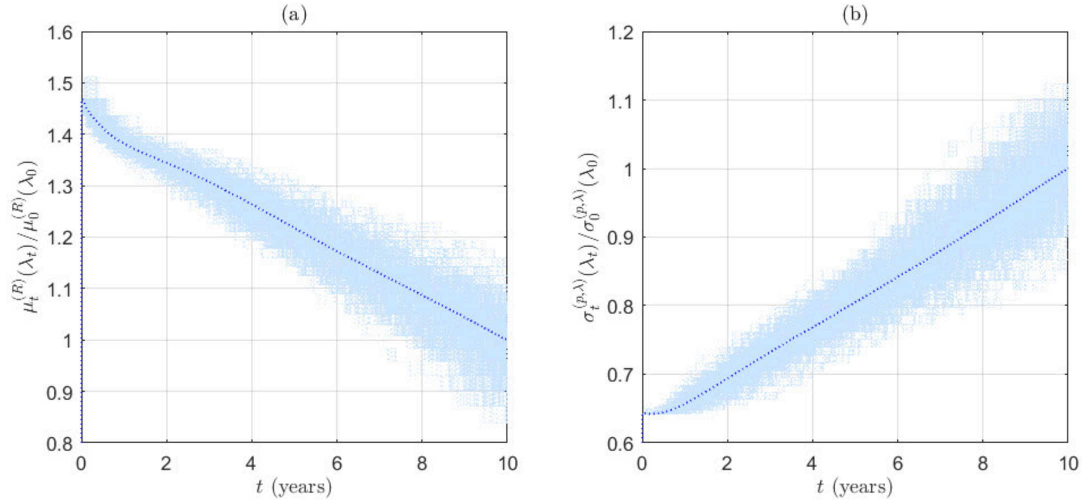


Figure 5: **Endogenous prices and welfare** The figure displays the transition paths (light blue shading) and their average (dotted blue line) from 2,500 simulations of $\mu_t^{(R)}$ and $\sigma_t^{(p,\lambda)}$, normalized by their long-run average values, following a 0.005 increase in $\lambda_0$.

result is driven by the fact that a higher mortality rate leads to increased consumption and, consequently, reduced capital investment.

To evaluate the significance of our findings, we have calibrated the model and computed an approximate global solution to the competitive equilibrium. We then simulated the model to estimate the impact of a temporary increase in the mortality rate on households' welfare, isolating the role of the endogenous price mechanism. This analysis revealed that approximately two-thirds of the welfare losses (or gains) from changes in the aggregate mortality rate can be attributed to this mechanism, underscoring its pivotal role in evaluating the effects of mortality shifts, such as those seen during pandemics.

To maintain tractability and focus, we have modelled households as homogeneous. As a result, our model does not capture potential effects of mortality risk on equilibrium prices and the risk-free rate in settings with heterogeneous agents. We leave this important extension for future research.

# A Appendix

## A.1 Proof of Proposition 1

The proof is divided into three steps: (i) We adopt a guess-and-verify approach to derive the optimal policy; (ii) We characterise the PDE whose solution is the value function; (iii) We provide a Feynman-Kac representation of its unique solution.

**Step (i)** Write the HJB equation in (11) as

$$(\lambda+\rho)\,J=\sup_{a\in\mathcal{A}}\left\{\begin{array}{c}\frac{c^{1-\gamma}}{1-\gamma}+\frac{\partial J}{\partial t}+\frac{\partial J}{\partial n}n\left[r+\kappa\left(\mu^{(R)}-r\right)-\frac{c}{n}\right]+\frac{\partial J}{\partial\lambda}\mu_\lambda(\bar{\lambda}-\lambda)+\\+\frac{1}{2}\left(\frac{\partial^2 J}{\partial n^2}n^2\kappa^2\left(\left(\sigma+\sigma_t^{(p,k)}\right)^2+\left(\sigma_t^{(p,\lambda)}\right)^2\right)+\frac{\partial^2 J}{\partial\lambda^2}\sigma_\lambda^2+2\frac{\partial^2 J}{\partial n\partial\lambda}n\kappa\sigma^{(p\lambda)}\sigma_\lambda\right)\end{array}\right\}, \tag{31}$$

whose FOCs w.r.t. the control $a$ imply that

$$c^{-\gamma}-\frac{\partial J}{\partial n}=0$$

$$\frac{\partial J}{\partial n}n\left(\mu^{(R)}-r\right)+\frac{\partial^2 J}{\partial n^2}n^2\kappa\left(\left(\sigma+\sigma_t^{(p,k)}\right)^2+\left(\sigma_t^{(p,\lambda)}\right)^2\right)+\frac{\partial^2 J}{\partial n\partial\lambda}n\sigma^{(p,\lambda)}\sigma_\lambda=0$$

$$\frac{\partial\iota}{\partial g}-\frac{1}{p}=0.$$

To reduce the dimensionality of the problem, we guess and verify that

$$J(t,\lambda,n)=F(t,\lambda)\frac{n^{1-\gamma}}{1-\gamma}. \tag{32}$$

By substituting this guess function in the FOCs yields the optimal policy as it appears in the main text.

**Step (ii)** Substitute the guess function and the optimal policy in the HJB equation and postulate that $\theta^*=1$ - a condition that will hold in equilibrium - as

$$\bar{\rho}F=\gamma F^{1-\frac{1}{\gamma}}+\frac{\partial F}{\partial t}+\frac{\partial F}{\partial\lambda}\left(\mu_\lambda(\bar{\lambda}-\lambda)+(1-\gamma)\,\sigma^{(p,\lambda)}\sigma_\lambda\right)+\frac{1}{2}\frac{\partial F}{\partial\lambda^2}\sigma_\lambda^2$$

with the following boundary (transversality) condition:

$$\lim_{t \to \infty} F_t < \infty.$$

**Step (iii)** Following Yong and Zhou (1999), the solution of the PDE in Step (ii) has the following Feynman-Kac representation

$$F_t = \widehat{\mathbb{E}}_t \left[ U_t \right]$$

under the probability measure in (17) and the unknown function $U$ solves the following Bernoulli differential equation:

$$\dot{U}_t = \bar{\rho}_t U_t - \gamma U_t^{1-\frac{1}{\gamma}}$$

with the boundary condition $\lim_{t \to \infty} U_t < \infty$. Assuming that $\bar{\rho} > 0$, the unique solution of this equation is

$$U_t = \left[ \int_t^{\infty} e^{-\frac{1}{\gamma} \int_t^s \bar{\rho}_u du} ds \right]^{\gamma}.$$

## A.2 Numerical solution

To compute the value function in (15) and the competitive equilibrium prices, we adopt the following procedure, summarised in Algorithm 1.

**Step 0** Initialize the parameters. Generate an equally-spaced grid $\lambda_{\text{grid}}$ of size $n_{grid}$ spanning the state space of possible values for $\lambda \in [\lambda_{min}, \lambda_{max}]$. Set an initial guess $\underline{f}$ for the function (15).

**Step 1** Use the guess to compute $\underline{p}$ and $\underline{g}$ using (25) and 13. Apply functional interpolation (i.e., Chebychev polynomial) to $\underline{p}$ to approximate $\underline{p}'$ and $\underline{p}''$. Use these approximations to compute $\underline{\mu}^{(p)}$ and $\underline{\sigma}^{(p,\lambda)}$ using (27) and (26). Use these objects and (8) and (16) to approximate $\underline{\bar{\rho}}$ over the grid.

**Step 2** For each grid point, simulate $n_{sim}$ stochastic paths of the process (3) under the probability measure (17) (we adopt an Euler-Maruyama discretisation with size $\Delta t$ and lenth

$T$). At each time step, identify the nearest $\lambda$-grid point to pin down the drift $\mu_\lambda\left(\bar{\lambda}-\lambda_t\right)+(1-\gamma)\,\sigma_t^{(p,\lambda)}\sigma_\lambda\sqrt{\lambda_t}$ and diffusion $\sigma_\lambda\sqrt{\lambda_t}$ along the paths.

Use these simulated values to compute $\bar{\rho}_t$ using (16) along each path. Then, approximate (15) as

$$F(t,\lambda)\approx f(\lambda):=\sum_{i=1}^{n_{\text{sim}}}\frac{\left(\sum_{t=1}^{T}e^{-\frac{1}{\gamma}\sum_{u=1}^{t}\bar{\rho}_s du}\Delta t\right)^{\gamma}}{n_{\text{sim}}} \tag{33}$$

for each point of the grid. Apply a polynomial interpolation to obtain the update $f_1$.

**Step 2** Check whether $\left\|\underline{f}-f_1\right\|_\infty\leq\eta$ for some tolerance level $\eta$. Else, update the guess $\underline{f}\to f_1$ and go back to Step 1.

---

**Algorithm 1**

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**Step 0**
Initialize parameters $\lambda_{\min}$, $\lambda_{\max}$, $\lambda_0$, $\kappa$, $A$, $\gamma$, $\rho$, $\sigma$ $\mu_\lambda$, $\bar{\lambda}$, $\sigma_\lambda$, $n_{\text{sim}}$, $n_{\text{grid}}$, $\eta$
Generate grid $\lambda_{\text{grid}}\leftarrow\text{linspace}(\lambda_{\min},\lambda_{\max},n_{\text{grid}})$
Set initial guess for $F$: $\underline{f}$

**Step 1**
Use the guess to compute $\underline{p}$, $\underline{g}$, $\underline{\iota}$, $\underline{p}'$, $\underline{p}''$, $\underline{\mu}^{(p)}$, and $\underline{\sigma}^{(p,\lambda)}$ for each point of the grid
**for** $m=1$ to $n_{\text{grid}}$ **do**
    $\lambda_0\leftarrow\lambda_{\text{grid}}[m]$
    Initialize $\lambda$ paths: $\lambda_0[m]\leftarrow\lambda_0$
    **for** $i=1$ to $n_{\text{sim}}$ **do**
        **for** $t=2$ to $T$ **do**
            Find the closest index $c$ in $\lambda_{\text{grid}}$ to $\lambda_{i,t-1}[m]$
            Update $\lambda_{i,t}[m]$
            Compute $f_{i,t}[m]$, $p_{i,t}[m]$, $g_{i,t}[m]$, $\iota_{i,t}[m]$, $\mu_{i,t}^{(p)}[m]$, $\sigma^{(p,\lambda)}[m]$
            Compute $\bar{\rho}_{i,t}[m]$ as in (16)
        **end for**
    **end for**
    Compute $f[m]$ as in (15)
**end for**
Fit polynomial $f_1$ from $f$

**Step 2**
**If** $\left\|\underline{f}-f_1\right\|_\infty\leq\eta$, set $F\approx f_1$ **else** $\underline{f}\to f_1$ and go to **Step 1**.

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