Regulatory Capacity in a Game of Asymmetric Regulation

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Abstract

In a model of asymmetric regulation, a firm can comply with two regulatory targets, and a regulator can audit the firm for compliance. Inspection by the regulator is imperfect, and it assesses the firm's compliance with the targets with different success probabilities. The firm fully complies only if compliance costs are low. Otherwise, the firm always prioritizes the requirement that is easier to enforce. Expanding regulatory capacity positively affects compliance with the easy-to-enforce target; however, a higher capacity can harm compliance with the hard-to-enforce target.

Keywords: agency resources, asymmetric enforcement, compliance, multi-tasking, regulation

JEL Codes: H32; K20; L51

1   Introduction

Regulation is often multidimensional, requiring firms to comply with multiple targets or tasks. When some of these tasks are easier to verify than others, then the enforceability of the regulation is asymmetric. The literature is full of such examples, ranging from the Environmental Protection Agency's regulation of environmental pollutants (Viscusi et al., 2005) to the EU's Digital Markets Act (Colangelo and Martínez, 2025).[1] In practice, asymmetry in enforceability means that firms and regulators will struggle to assess compliance with some of the requirements more than with others. Analogous to a multi-task setting in incentive contracting (e.g., Holmstrom and Milgrom, 1991), where agents allocate more effort towards the highly incentivized and easily measured (or verifiable) tasks,[2] in a regulatory setting, we expect firms to neglect the hard-to-enforce task. This outcome is particularly prevalent when agencies and regulators have limited resources and prioritize the easy-to-enforce tasks.

In this note, we demonstrate that, in a setting where regulatory targets have asymmetric enforceability, regulatory capacity alone is not sufficient to enhance compliance with hard-to-enforce targets. On the contrary, expanding regulatory capacity may induce firms to comply with the easy-to-enforce ones at the expense of the harder ones—higher capacity of regulators may lower compliance.

In Section 2, we propose a simple framework for studying the compliance of multidimensional regulation with asymmetrically enforceable requirements. We build on the game-theory literature on audits and tax avoidance (Dresher, 1962; Fellingham and Newman, 1985; Graetz et al., 1986) and model this interaction as a simultaneous-move game: The firm chooses which requirements to comply with, and the regulator chooses which requirements, if any, to audit. Audits are assumed to be imperfect (non-compliance might go undetected) but error-free (compliance is always identified). Our framework is closest to Heyes (1994), which models the thoroughness of inspection of a single requirement as an endogenous choice. In our note, we assume the asymmetric success probabilities of detecting non-compliance with multiple requirements as given (e.g., Macho-Stadler and Perez-Castrillo, 2006), and focus on the firm's and regulator's choices of which requirements to comply with and audit.[3]

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[1]Conventional pollutants are generally viewed as simple to enforce, whereas the enforcement with respect to toxic chemicals (because of the needs for specific testing) is more difficult (Viscusi et al., 2005, pp. 775–6). The Digital Markets Act required Apple to open up its ecosystem to third-party app stores while not engaging in self-preferencing when competing for users' attention. The existence of third-party app stores, however, does not imply users will be able to use them without hurdles (Colangelo and Martínez, 2025).

[2]For empirical evidence, see Hong et al. (2018), Knutsson and Tyrefors (2022), or de Janvry et al. (2023).

[3]Our assumption of asymmetric (and strictly positive) success probabilities is unlike the analysis in Laffont and Tirole (1993, chs. 3.8, 4) where multi-dimensional effort is not verifiable at all.

In Section 3, we derive equilibrium outcomes for regulators with low capacity (that can enforce only one of the requirements and, therefore, have to prioritize) and regulators with high capacity (that can enforce one or both requirements). Our equilibrium analysis demonstrates that the asymmetry in enforceability leads to (weak) undercompliance with the hard-to-enforce requirements regardless of the regulator's capacity.

In Section 4, we establish that higher capacity is effective in ensuring compliance with the easy-to-enforce requirement, even in the presence of imperfect audits. However, expanding regulatory capacity can have a detrimental impact on compliance with the hard-to-enforce requirement.[4]

2   A Model of Asymmetric Regulation

A firm acts subject to two distinct regulatory targets, $j = a$ and $j = b$. Compliance costs are c per target. The firm's compliance choice is an action profile $(a, b) \in \{0, 1\} \times \{0, 1\}$ where $j = 1$ indicates compliance and $j = 0$ non-compliance. A regulator enforces these targets by auditing the firm to assess compliance with either, neither, or both targets. If it finds non-compliance, it can challenge and fine the firm.

The firm's unchallenged value of its action is $v > 0$. Given its compliance choices, its unchallenged payoffs are $v - ca - cb$. If the regulator finds non-compliance and fines the firm, the firm's payoffs are normalized to zero. Given Assumption 1, compliance never leads to negative payoffs:

Assumption 1. $0 < c < \frac{v}{2}$

Non-compliance generates a social loss of $-\gamma < 0$. The regulator's objective is to minimize this social loss. Its payoffs from an unchallenged non-compliant policy are $-\gamma$, and the payoffs from a challenged or compliant policy are zero. We assume that inspecting a target takes time, and that regulators differ in their *capacity*. A *high-capacity* regulator has sufficient resources to inspect both targets. A *low-capacity* regulator can inspect at most one target.

We further assume that audits are imperfect in that the inspection for target j leads to the discovery of its state (either $j = 1$ or $j = 0$) with probability $\pi_j < 1$. To capture

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[4]Our results parallel the classic findings (Bailey and Coleman, 1971; Sweeney, 1981) that lagged adjustments of regulation (e.g., by low-capacity regulators), can have positive effects on technology adoption in otherwise suboptimal rate-of-return regulation (Averch and Johnson, 1962; Sherman, 1992)—with more dynamic adjustments of regulation implementable only by high-capacity regulators.

Table 1: Normal-Form Representation of the Compliance-Inspection Game

Payoffs: (firm, regulator)		**Regulator's strategy ($\to q$)**			
		i_0	i_a	i_b	$i_{a,b}$
Firm's strategy ($\to p$)	$(0,0)$	$\begin{pmatrix} v, \\ -\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_a)v, \\ -(1-\pi_a)\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_b)v, \\ -(1-\pi_b)\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_a)(1-\pi_b)v, \\ -(1-\pi_a)(1-\pi_b)\gamma \end{pmatrix}$
	$(a,0)$	$\begin{pmatrix} v-c, \\ -\gamma \end{pmatrix}$	$\begin{pmatrix} v-c, \\ -\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_b)(v-c), \\ -(1-\pi_b)\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_b)(v-c), \\ -(1-\pi_b)\gamma \end{pmatrix}$
	$(0,b)$	$\begin{pmatrix} v-c \\ -\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_a)(v-c), \\ -(1-\pi_a)\gamma \end{pmatrix}$	$\begin{pmatrix} v-c, \\ -\gamma \end{pmatrix}$	$\begin{pmatrix} (1-\pi_a)(v-c), \\ -(1-\pi_a)\gamma \end{pmatrix}$
	(a,b)	$\begin{pmatrix} v-2c, \\ 0 \end{pmatrix}$	$\begin{pmatrix} v-2c, \\ 0 \end{pmatrix}$	$\begin{pmatrix} v-2c, \\ 0 \end{pmatrix}$	$\begin{pmatrix} v-2c, \\ 0 \end{pmatrix}$

asymmetric enforceability, we assume (without loss of generality) that inspecting a has a higher chance of discovering its actual state, $\pi_a > \pi_b$. Given Assumption 2, for all feasible values c, the firm does not always strictly prefer not to comply with either of the targets (that is, action profile $(0,0)$ is never dominant):

Assumption 2. $\frac{1}{2} < \pi_b < \pi_a < 1$.

We summarize the firm's and regulator's pure strategies and respective payoffs in Table 1. The firm chooses *full non-compliance* $(a = 0, b = 0)$, *partial compliance* $((a = 0, b = 1) \vee (a = 1, b = 0))$, or *full compliance* $(a = 1, b = 1)$. To ease notation, we refer to $j = 1$ as j, and $j = 0$ as 0, for $j \in \{a, b\}$. A regulator chooses *no inspection* (i_0), *partial inspection* (i_j), or *full inspection* $(i_{a,b})$; the last option is not available for a low-capacity regulator.

3 Equilibria for Both Regulator Types

From Table 1, it is immediate to see that $i_{a,b}$ is the **high-capacity regulator**'s dominant strategy. The following proposition summarizes the Nash equilibria in this scenario:

Proposition 1 (High-Capacity Regulator). *Let $c_h = \frac{\pi_b}{1+\pi_b}v$. If $c < c_h$, the equilibrium is $(i_{a,b}, (a,b))$. If $c \geq c_h$, the equilibrium is $(i_{a,b}, (a,0))$.*

Proof. We solve the game by iterated dominance. Full inspection $i_{a,b}$ is the regulator's (weakly) dominant strategy. Firm strategies $(0,0)$ and $(0,b)$ are dominated by $(a,0)$ in the

4

resulting reduced game. Firm strategy $(a, 0)$ dominating $(0, b)$ follows from Assumption 2 where $\pi_a > \pi_b$ implies $(1 - \pi_b)(v - c) > (1 - \pi_a)(v - c)$. Firm strategy $(a, 0)$ dominating $(0, 0)$, follows from $(1 - \pi_b)(v - c) > (1 - \pi_a)(1 - \pi_b)v$ if, and only if, $c < \pi_a v$, which holds under Assumptions 1 $(0 < c < \frac{v}{2})$ and 2 $(\pi_a > \pi_b > \frac{1}{2})$. Lastly,

$$v - 2c > (1 - \pi_b)(v - c) \iff c < \frac{\pi_b}{1 + \pi_b}v = c_h. \qquad \square$$

Because full inspection $i_{a,b}$ is a dominant strategy, the firm always prefers compliance with the easy-to-enforce target, $(a, 0)$, to compliance with the hard-to-enforce target, $(0, b)$. When choosing partial compliance, the firm opts for the target for which non-compliance is easier to detect. Moreover, by Assumption 2, the firm either chooses full compliance (a, b) or non-compliance in b (so that $(a, 0)$). Because $\pi_a > \pi_b$, partial compliance with a dominates not complying at all under the assumption that compliance never leads to negative payoffs for the firm. Moreover, if compliance costs c are small, full compliance is cheap, and the benefits outweigh the costs. Alternatively, when c is high, the firm prefers not to comply with b, hoping that its non-compliance will go undetected. All else equal, as the success probability π_b of the hard-to-enforce target increases, the firm strictly prefers to comply with both targets for more values of c.

For a **low-capacity regulator**, a full audit with $i_{a,b}$ is not feasible. It always chooses i_a or i_b. It is straightforward to see that a pure-strategy equilibrium cannot exist. Suppose the regulator selects to inspect the easy-to-enforce target, i_a, with probability one. Then, the firm's best response is to choose $(a, 0)$. The regulator is then unable to challenge the non-complying firm and instead wants to deviate to i_b, inducing the firm to respond with $(0, b)$. And so forth. In equilibrium, the regulator will always play a mixed strategy, choosing i_a and i_b with strictly positive probabilities.

To construct the resulting equilibria, we find the firm's probabilities for partial compliance $\Pr(a = 1, b = 0) =: p_a$ and $\Pr(a = 0, b = 1) =: p_b$ and no compliance $\Pr(a = 0, b = 0) = 1 - p_a - p_b$ that make the regulator indifferent between inspecting i_a and i_b:

$$p_a = \frac{\pi_a - (1 - p_b)\pi_b}{\pi_a}; \tag{1}$$

$$p_b \in \left[0, \frac{\pi_b}{\pi_a + \pi_b}\right]; \tag{2}$$

$$1 - p_a - p_b = (1 - p_b)\frac{\pi_b}{\pi_a} - p_b. \tag{3}$$

Note that he firm plays $(a, 0)$ with strictly positive probability (because $p_a > 0$).

The low-capacity regulator has only two undominated strategies, i_a and i_b. We use $q_{a|b}$ and $q_{a|0}$ to denote the regulator's probabilities of playing i_a that make the firm indifferent between $(a,0)$ and $(0,b)$ or $(a,0)$ and $(0,0)$, respectively:

$$q_{a|b} = \frac{\pi_b}{\pi_a + \pi_b}; \tag{4}$$

$$q_{a|0} = \frac{(1 - \pi_b)\,c}{\pi_a v - \pi_b c}. \tag{5}$$

The following proposition summarizes the Nash equilibria in this scenario of the low-capacity regulator:

Proposition 2 (Low-Capacity Regulator). *Let*

$$\underline{c}_l := \frac{\pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a} v < \frac{\pi_b \pi_a}{\pi_b + \pi_a - \pi_b \pi_a} v =: \overline{c}_l.$$

Then:

   i. *if $\frac{1}{2} < \pi_b < \pi_a < 1$ and $0 < k < \underline{c}_l$, there is a continuum of payoff-equivalent equilibria: the regulator plays i_a with probability $q_{a|b} \in \left[\underline{q}^b, \overline{q}^b\right]$ and i_b with the remaining probability, and the firm complies with both targets with probability one;*

   ii. *if $\frac{1}{2} < \pi_b < \min\left(\pi_a, \frac{\pi_a}{3\pi_a - 1}\right) \leq \frac{2}{3}$ and $\underline{c}_l < c < \overline{c}_l$, in the unique equilibrium the regulator mixes following $q_{a|b}$, and the firm mixes between $(a,0)$ and $(0,b)$ with $p_a = \frac{\pi_a}{\pi_a + \pi_b}$ and $p_b = 1 - p_a$;*

   iii. *if $\frac{1}{2} < \pi_b < \min\left(\pi_a, \frac{\pi_a}{3\pi_a - 1}\right) \leq \frac{2}{3}$ and $\overline{c}_l < c < \frac{v}{2}$, in the unique equilibrium the regulator mixes following $q_{a|0}$, and the firm mixes between $(a,0)$ and $(0,0)$ with $p_a = 1 - \frac{\pi_b}{\pi_a}$ and $p_b = 0$;*

   iv. *if $\frac{\pi_a}{3\pi_a - 1} < \pi_b < \pi_a < 1$ and $\underline{c}_l < c < \frac{v}{2}$, in the unique equilibrium the regulator mixes following $q_{a|b}$, and the firm mixes between $(a,0)$ and $(0,b)$ with $p_a = \frac{\pi_a}{\pi_a + \pi_b}$ and $p_b = 1 - p_a$.*

Proof. The proof of the proposition and the closed-form solutions for $\underline{q}^b$ and $\overline{q}^b$ can be found in Appendix A. $\qquad\square$

The game has either a unique equilibrium for high compliance costs $c \geq \underline{c}_l$ in which the firm is always more likely to comply with the easy-to-enforce target $(p_a > p_b)$; or a continuum of payoff-equivalent equilibria for low compliance costs $c < \underline{c}_l$ in which the firm

fully complies with both targets. This multiplicity arises because the cheaper it is to comply, the easier it is for the regulator to induce the firm to play (a, b).[5]

4   Improving the Regulator's Funding

In equilibrium, the firm never focuses on the harder-to-enforce target b more than on target a (because $p_a \geq p_b$). That is, even if the regulator has the capacity to inspect both targets, the firm either complies fully (with action profile (a, b)) or ignores b altogether (Proposition 1). In fact, compliance with b can decrease if a regulator receives the capacity to conduct more thorough inspections. While a low-capacity regulator does not always inspect for a, a high-capacity regulator does. When compliance costs are high, anticipated enforcement of a shifts the firm's attention towards the easy-to-enforce target a—because not complying with a is more likely to lead to a challenge—at the detriment of b. Formally, after an increase in regulatory capacity, the firm would stop mixing between $(a, 0)$ and $(0, b)$, and instead play $(a, 0)$ with probability one. We summarize our results in Proposition 3.

Proposition 3 (Increasing Regulator's Capacity). *When the regulator's capacity increases, equilibrium compliance with the easy-to-enforce target a increases if $c > \underline{c}_l$; compliance with the hard-to-enforce target b increases if $\underline{c}_l < c < c_h$, but decreases if $c_h < c < \min\{v/2, \overline{c}_l\}$.*

Proof. The proof follows immediately from Propositions 1 and 2 if $0 < \underline{c}_l < c_h < \overline{c}_l < \frac{v}{2}$ holds. The outer conditions are satisfied under Assumptions 1 and 2. It is then sufficient to show that:

$$\underline{c}_l = \frac{\pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a} < \frac{\pi_b}{1 + \pi_b} = c_h \iff 1 > \frac{\pi_a + \pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a},$$

and:

$$c_h = \frac{\pi_b}{1 + \pi_b} < \frac{\pi_b \pi_a}{\pi_b + \pi_a - \pi_b \pi_a} = \overline{c}_l \iff \pi_a > 1 - \pi_a.$$

Both hold under Assumption 2. □

In Figure 1, we depict the equilibrium effects of improving a regulator's resources. In the panels on the left-hand side, we plot the firm's compliance (captured by its strategy p_a) with the easy-to-enforce target a (in blue) for the high-capacity regulator (solid lines) and the low-capacity regulator (dashed lines). The pictures show that expanding the regulator's

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[5]Proposition 2 accounts for all equilibria except for knife-edge scenarios in which the agents are indifferent between two of the above equilibria. In the Appendix, we demonstrate that no mixed-strategy equilibria exist except those characterized in Proposition 2.

Figure 1: Effect of Expanding Capacity on Compliance

(a) Low asymmetry case ($\pi_a = 0.75$, $\pi_b = 0.65$)

(b) High asymmetry case ($\pi_a = 0.75$, $\pi_b = 0.55$)
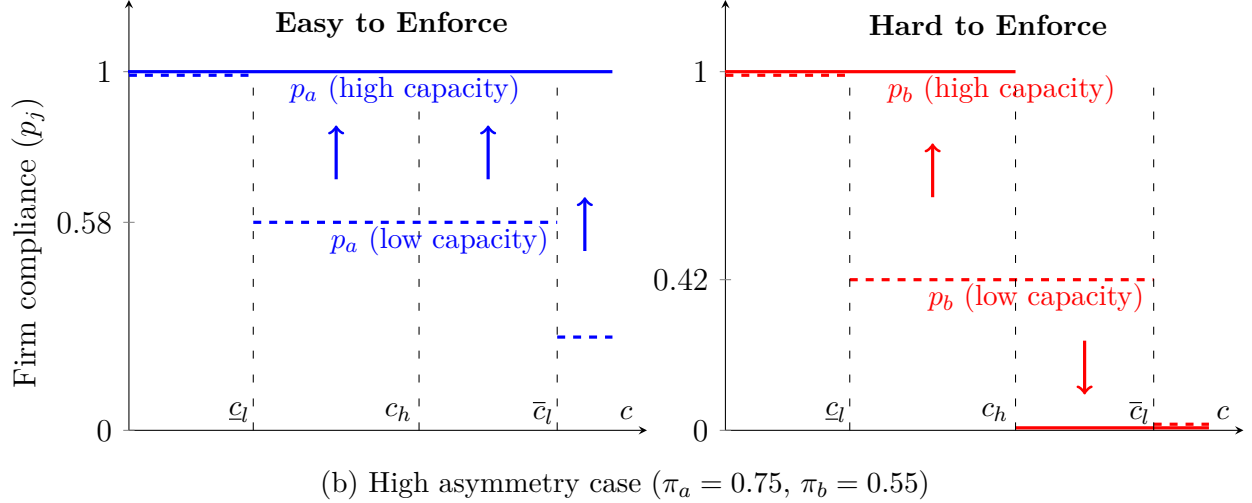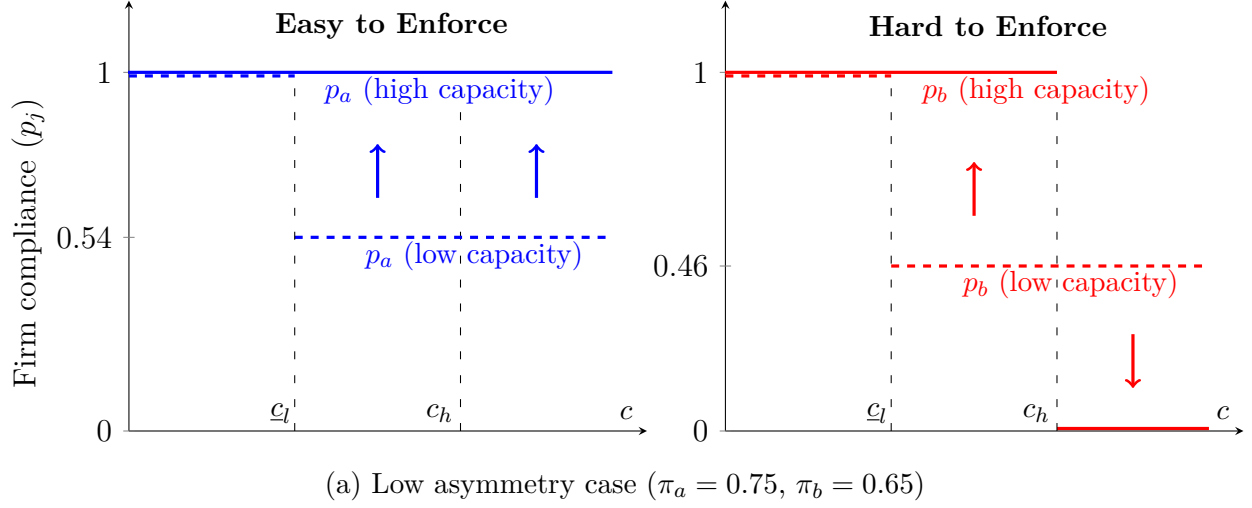
Notes: Equilibrium probability of compliance p with the easy-to-enforce target a (blue) and hard-to-enforce target b (red), with a low capacity (dashed) and high capacity (solid) regulator. Arrows indicate the direction of compliance when the regulator increases its enforcement capacity.

capacity has an unambiguously positive effect on the firm's compliance with a. In the panels on the right-hand side, we plot the firm's compliance with the hard-to-enforce target b (in red). First, if the firm was already fully complying or not complying at all with b (for low or high compliance costs), increasing the regulator's capacity does not affect compliance in equilibrium. For intermediate values of compliance costs, when under a low-capacity regulator, the firm randomizes between both targets for partial compliance (Proposition 2(ii) and (iv)), increasing the regulator's capacity lowers compliance with the hard-to-enforce target b.

The result in Proposition 3 (and Figure 1) generates a striking insight for regulatory de-

sign. When targets have asymmetric enforceability, regulatory capacity alone is not sufficient to enhance compliance with hard-to-enforce targets. On the contrary, expanding regulatory capacity may induce firms to comply with the easier-to-enforce targets at the expense of the harder ones.

5 Concluding Remarks

The model highlights three factors that drive equilibrium behavior in a setting of multi-dimensional (and asymmetric) regulation. The firm's cost of compliance, the regulator's (asymmetric) success probabilities, and the regulator's capacity. While the firm generally complies more with targets that are easier to enforce (in response to anticipating more enforcement by the regulator), the interplay of these factors determines if, and when, we can expect to see more or less compliance with the hard-to-enforce regulatory target (with weaker anticipated enforcement). We demonstrate that expanding regulatory capacity may indeed lower compliance when it induces firms to prioritize easy-to-enforce targets over more challenging ones. Improving enforceability by reducing ambiguity, rather than expanding agencies' capacity, is likely more effective in increasing compliance across targets.

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A   Proof of Proposition 2

We look for mixed strategy equilibria. The regulator is indifferent between i_a and i_b if the firm plays $(a, 0)$ with probability p_a; $(0, b)$ with probability p_b; and $(0, 0)$ with probability $1 - p_a - p_b$. These probabilities satisfy:

$$(1 - p_a - p_b)(1 - \pi_a)(-\gamma) - p_a \gamma - p_b(1 - \pi_a)\gamma$$
$$= (1 - p_a - p_b)(1 - \pi_b)\ (-\gamma) - p_a(1 - \pi_b)\gamma - p_b \gamma.$$

10

Rearranging yields:

$$p_b \in \left[0, \frac{\pi_b}{\pi_a + \pi_b}\right], \quad p_a = \frac{\pi_a - (1 - p_b)\,\pi_b}{\pi_a} > 0, \quad \text{and} \quad 1 - p_a - p_b = (1 - p_b)\frac{\pi_b}{\pi_a} - p_b.$$

The firm is indifferent between $(a,0)$ and $(0,b)$ or $(a,0)$ and $(0,0)$ if i_a is played with probabilities $q_{a|b}$ or $q_{a|0}$. These probabilities solve:

$$q_{a|b}\,(v - c) + \left(1 - q_{a|b}\right)(1 - \pi_b)\,(v - c) = q_{a|b}\,(1 - \pi_a)\,(v - c) + \left(1 - q_{a|b}\right)(v - c)$$

and

$$q_{a|0}\,(v - c) + \left(1 - q_{a|0}\right)(1 - \pi_b)\,(v - c) = q_{a|0}\,(1 - \pi_a)\,v + \left(1 - q_{a|0}\right)(1 - \pi_b)\,v$$

so that

$$q_{a|b} = \frac{\pi_b}{\pi_a + \pi_b}, \quad q_{a|0} = \frac{(1 - \pi_b)\,k}{\pi_a c - \pi_b c}.$$

We check for profitable deviations. In the equilibrium with the firm mixing between $(a,0)$ and $(0,b)$, it holds:

$$
\begin{aligned}
E[(a,0)]|_{q_{a|b}} = E[(0,b)]|_{q_{a|b}} &= \frac{\pi_b}{\pi_a + \pi_b}(v - c) + \left(1 - \frac{\pi_b}{\pi_a + \pi_b}\right)(1 - \pi_b)\,(v - c) \\
&= \frac{(v - c)[\pi_a(1 - \pi_b) + \pi_b]}{\pi_a + \pi_b} \\
E[(0,0)]|_{q_{a|b}} &= \frac{\pi_b}{\pi_a + \pi_b}(1 - \pi_a)\,v + \left(1 - \frac{\pi_b}{\pi_a + \pi_b}\right)(1 - \pi_b)\,v \\
&= \frac{v[\pi_a + \pi_b - 2\pi_a\pi_b]}{\pi_a + \pi_b}
\end{aligned}
$$

By direct comparison, $E[(a,0)]|_{q_{a|b}} = E[(0,b)]|_{q_{a|b}} > \max\{E[(0,0)]|_{q_{a|b}}, v - 2c\}$ if, and only if,

$$
\begin{aligned}
\frac{1}{2} < \pi_b < \min\left(\pi_a, \frac{\pi_a}{3\pi_a - 1}\right) &\le \frac{2}{3} \;\wedge\; c < \frac{\pi_b \pi_a}{\pi_b + \pi_a - \pi_b \pi_a}v, \\
\frac{\pi_a}{3\pi_a - 1} < \pi_b < \pi_a < 1 &\;\wedge\; c < \frac{v}{2}, \\
c < \frac{\pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a}v. &
\end{aligned}
$$

In the equilibrium in which the firm mixes between $(d,0)$ and $(0,0)$, it holds:

$$
\begin{aligned}
E[(a,0)]|_{q_{a|0}} = E[(0,0)]|_{q_{a|0}} &= \frac{(1 - \pi_b)\,c}{\pi_a v - \pi_b c}(1 - \pi_a)\,v + \left(1 - \frac{(1 - \pi_b)\,c}{\pi_a v - \pi_b c}\right)(1 - \pi_b)\,v \\
&= \frac{(v - c)v\pi_a(1 - \pi_b)}{\pi_a v - \pi_b c},
\end{aligned}
$$

11

and

$$E[(0,b)]|_{q_{a|0}} = \frac{(1-\pi_b)\,c}{\pi_a v - \pi_b c}\,(1-\pi_a)\,(v-c) + \left(1 - \frac{(1-\pi_b)\,c}{\pi_a v - \pi_b c}\right)(v-c)$$

$$= \frac{(v-c)[c\pi_a - v\pi_b + c\pi_b(1-\pi_a)]}{\pi_a v - \pi_b c}.$$

Again, by direct comparison, $E[(a,0)]|_{q_{a|0}} = E[(0,0)]|_{q_{a|0}} > \max\{E[(0,b)]|_{q_{a|0}}, v - 2c\}$ if, and only if,

$$\frac{1}{2} < \pi_b < \min\left(\pi_a, \frac{\pi_a}{3\pi_a - 1}\right) \le \frac{2}{3} \quad \wedge \quad c > \frac{\pi_b \pi_a}{\pi_b + \pi_a - \pi_b \pi_a} v.$$

No other equilibria can exist for $c > \underline{c}_l = \frac{\pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a} v$.

For $c < \underline{c}_l$, the firm never plays $(0,0)$ with positive probability. We compare (a,b) and mixing between $(a,0)$, $(0,b)$. First, (a,b) dominates for $c < \underline{c}_l = \frac{\pi_b \pi_a}{\pi_b + \pi_a + \pi_b \pi_a} v$, and infinite payoff equivalent equilibria exist for $c < \underline{c}_l$. In these equilibria, the firm plays (a,b) with probability one; the regulator mixes between i_a and i_b. Recall that:

$$E[(a,0)]|_{q_{a|b}} = q_{a|b}\,(v-c) + \left(1 - q_{a|b}\right)(1 - \pi_b)\,(v-c)$$

$$E[(0,b)]|_{q_{a|b}} = q_{a|b}\,(1 - \pi_a)\,(v-c) + \left(1 - q_{a|b}\right)(v-c)$$

We are interested in values $q_{a|b}$ that make the firm weakly better off selecting (a,b) than $(a,0)$ or $(0,b)$. The former case arises when $v - 2c > q_{a|b}\,(v-c) + \left(1 - q_{a|b}\right)(1 - \pi_b)\,(v-c)$, which is equivalent to:

$$q_{a|b} \le \bar{q}^b = \frac{v[\pi_a(3 - \pi_b) + \pi_b]\pi_b - c(3 - \pi_b)(\pi_a + \pi_b + \pi_a \pi_b)}{v[\pi_a(3 - \pi_b) + \pi_b]\pi_b - c(2 - \pi_b)(\pi_a + \pi_b + \pi_a \pi_b)}$$

The latter arises when $v - 2c \ge_{a|b} (1 - \pi_a)\,(v-c) + \left(1 - q_{a|b}\right)(v-c)$, which is equivalent to:

$$q_{a|b} \ge \underline{q}^b = \frac{c}{(v-c)\pi_a}.$$

For all $c \in (0, \underline{c}_l)$, any strategy in which the regulator plays i_a with probability $q_{a|b} \in [\underline{q}^b, \bar{q}^b]$ induces the firm to play (a,b) with probability one.

No other equilibria exist. Suppose a mixed-strategy equilibrium exists that involves the firm playing (a,b) with some positive probability different from one. Then, the regulator will optimally play the best response to the other action with probability one, to which the firm's best response is something different from (a,b).

12