Discussion Paper Series – CRC TR 224

Discussion Paper No. 704
Project B 02

Entry Deterrence with Public Signals: Revisiting the Chain-Store Paradox

Francesc Dilmé[1]
Aaron Kolb[2]

October 2025

[1] University of Bonn. Email: fdilme@uni-bonn.de.
[2] Kelley School of Business, Indiana University, Email: kolba@iu.edu.

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

Entry Deterrence with Public Signals: Revisiting the Chain-Store Paradox

Francesc Dilmé and Aaron Kolb*

September 24, 2025

Abstract

We revisit the classic chain-store paradox by introducing a novel element: the arrival of exogenous, public signals about the incumbent's private type over time. As the horizon lengthens, two opposing forces come into play. On one hand, standard reputational incentives grow stronger; on the other, the increasing availability of information makes it more difficult to sustain a reputation. We show that full deterrence can still emerge as the horizon grows arbitrarily long, though not always, and we provide a complete characterization of the conditions under which it arises.

Selten (1978) observed that deterrence games can exhibit implausible equilibrium behavior. He illustrated this with the chain-store game in Figure 1. In the one-period version of this game, an entrant chooses to stay out or enter. If it enters, a monopolist chooses to fight or acquiesce. In the unique subgame-perfect Nash

*Dilmé: University of Bonn, fdilme@uni-bonn.de. Kolb: Kelley School of Business, Indiana University, kolba@iu.edu. We thank Sarah Auster, Avi Lichtig, and Stephan Lauermann for their helpful comments, and the participants at the seminars at the University of Bonn, Indiana University, and the University of Wisconsin. This work was funded by a grant from the European Research Council (ERC 949465) and the Philpott Fellowship at Indiana University. Dilmé gratefully acknowledges the support from the German Research Foundation (DFG) through CRC TR 224 (Project B02) and under Germany's Excellence Strategy – EXC 2126/1–390838866 and EXC 2047–390685813.

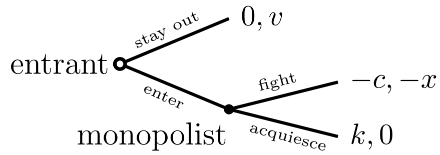

Figure 1: Selten (1978) chain store game.

equilibrium of any finite repetition of this game—where the monopolist faces a new entrant in each period—all entrants enter at all histories. Selten argued this prediction was implausible, asserting that if entrants observed multiple fights by the monopolist in previous periods, they would adjust their behavior.

Kreps and Wilson (1982) and Milgrom and Roberts (1982) addressed Selten's chain-store paradox by introducing a small uncertainty about the monopolist's rationality. Their results are obtained in a model where monopolist is either normal, with the payoffs specified in Figure 1, or tough, in which case it always fights. This alteration reverses Selten's prediction in long games: even with a small probability of a tough monopolist, the unique subgame-perfect equilibrium features full deterrence for an arbitrarily large number of initial periods as the time horizon lengthens. Intuitively, when the game's end is distant, the value from deterring future entry is high, so the normal monopolist fights current entry to convince future entrants that it is tough. The anticipation of such behavior deters current entrants even when the probability of the monopolist being tough is low.

We provide a new bridge between the extreme predictions from these two settings by taking the models of Kreps and Wilson (1982) and Milgrom and Roberts (1982) and introducing a public signal about the monopolist's type that arrives at the end of each period. We study perfect Bayesian equilibria and show that there is a unique equilibrium outcome. We derive conditions under which full deterrence arises as the time horizon lengthens.

Our first result establishes that all equilibria feature cutoff strategies based on the posterior probability of the monopolist being tough (its reputation): There is a unique sequence of cutoffs $\phi_t^\dagger$ such that in any equilibrium, the period-t entrant enters if the monopolist's reputation is below $\phi_t^\dagger$ and stays out above $\phi_t^\dagger$. We show that deterrence is higher, i.e. $\phi_t^\dagger$ is lower, in early periods of the game. The monopolist's continuation value is therefore monotone increasing in its reputation level and in the number of periods remaining.

While the value of reputation increases with the horizon length, the signal's existence attenuates this growth. Indeed, as shown in Kreps and Wilson (1982) and Milgrom and Roberts (1982), without a signal, the value of reputation increases significantly in the number of periods until the deadline, eventually leading to full deterrence — the Stackelberg outcome. The signal introduces a competing force: the flow of information about the monopolist's type reduces the monopolist's incentive to build reputation in the first place. By standard properties of learning, the normal monopolist's reputation becomes arbitrarily close to zero with arbitrarily high probability at sufficiently late dates.

We show that this second force need not cause deterrence to unravel. In fact, there can still be asymptotic full deterrence; that is, for any fixed t, $\phi_t^\dagger$ can converge to 0 as the horizon lengthens to infinity. We show that this is the case if the signal is not too informative in a precise sense described below. Hence, despite the downward drift in the monopolist's reputation, its reputational incentives might remain sufficiently strong to deter entry even when its reputation is initially low. However, if the signal is sufficiently informative, there is no asymptotic full deterrence. We provide a sharp characterization in terms of the cost of fighting, x: there exists a threshold $\overline{x} > 0$ such that asymptotic full deterrence arises if and only if $x \leq \overline{x}$.

Related literature. Since the work of Kreps and Wilson (1982) and Milgrom and Roberts (1982), an extensive literature has been devoted to studying reputation in entry deterrence, reviewed in Wilson (1992). More broadly, reputation building by long-lived players has been studied in numerous settings, most saliently in infinitely repeated games (see Mailath and Samuelson, 2006, for a review).

Our paper examines implications of exogenous news arrival on deterrence.[1] The closest papers to ours in this regard are Wiseman (2009) and Hu (2014). They study exogenous learning in a simultaneous-move, infinite-horizon game, and provide lower bounds for the payoff of the long-lived player. Our focus is different: we fully characterize the unique equilibrium outcome of the finite repetition of the classical chain-store game and determine the conditions for asymptotic full deterrence. The finiteness of the horizon is crucial to studying the backward buildup of the value of reputation from the deadline.[2]

1 Model

There is a monopolist and a sequence of short-lived entrants. In every period $t \in \{0, 1, 2, ..., T\}$, the stage game depicted in Figure 1 is played. The t-entrant chooses to enter (E) or stay out (O); a_t denotes this choice. If the t-entrant enters, the monopolist chooses whether to fight (F) or acquiesce (A); d_t denotes this choice.

The monopolist has a private persistent type $\theta \in \{L, H\}$, where H is chosen with probability $\phi_0 \in (0, 1)$. We model type H as a behavioral "tough" type that always fights and type L as a rational "normal" type with payoffs defined below (we will often just use "monopolist" to refer to the normal monopolist). At the end of each period, an exogenous public signal $s_t \in \mathbb{R}$ arrives, which is distributed i.i.d. conditional on θ and with distribution

$$s_t \sim F_\theta(\cdot) \ .$$

In all periods t, the stage game payoffs are as follows. If the t-entrant stays out, it receives a payoff of 0 and the monopolist obtains a benefit of $v > 0$. If the t-entrant enters and the monopolist fights, the t-entrant's payoff is $-c < 0$ and the monopolist's

[1]Other aspects of deterrence that have been studied include the role of endogenous types (Pitchik, 1993) or nonconstant types (Aoyagi, 1996 and Wiseman, 2008).

[2]Other work has focused on studying the role of imperfect observation of actions on reputation (e.g., Cripps et al., 2004, Faingold and Sannikov, 2011, or Dilmé, 2025).

payoff is $-x<0$. If the t-entrant enters and the monopolist acquiesces, the t-entrant's payoff is $k>0$ and the monopolist's payoff is 0. The monopolist maximizes the expected discounted sum of the stage game payoffs with discount factor $\delta \in (0,1)$. Throughout, we focus on the case where $x<\frac{\delta}{1-\delta}\,v$, as otherwise fighting is a strictly dominated strategy in any period for any T.

We impose the following assumption on the signal:

Assumption 1. The signal distributions have strictly positive densities $f_\theta(\cdot)$ on $\mathbb{R}$, and the log-likelihood ratio, $\log \frac{f_H}{f_L}$, is strictly increasing with range $\mathbb{R}$.

This assumption ensures that each period, from any interior belief, the posterior belief based on the signal is continuously distributed with full support on $[0,1]$ under either type.

1.1 Strategies and equilibrium

Histories. For $t \geq 0$, let $o_t \in O := \{\mathrm{O}, (\mathrm{E},\mathrm{F}), (\mathrm{E},\mathrm{A})\}$ denote the outcome of period t (not including the signal). For $t>0$, define $h^t = (o_0, s_0, \ldots, o_{t-1}, s_{t-1})$ as the history of all past actions and signals before period t, and set $h^0 = \emptyset$. For $t>0$, define $H^t = (O \times \mathbb{R})^t$, $H^0 = \{\emptyset\}$, and $H = \cup_{t=0}^T H^t$.

Strategies. An *entrant's strategy* is a function $\alpha : H \to [0,1]$ mapping histories to entry probabilities. Similarly, a *monopolist's strategy* is a function $q : H \to [0,1]$ mapping histories to fight probabilities conditional on entry.[3]

Monopolist's continuation payoffs. Fix a strategy profile (α, q) and a history $h^t \in H$. The *monopolist's continuation value*, $V(h^t; \alpha, q)$, is defined recursively by

$$V(h^T; \alpha, q) = (1-\alpha(h^T))\, v + \alpha(h^T)\, q(h^T)\, (-x)$$

[3]Since the monopolist only moves in period t if an entry occurs, we define q only as a function of h^t, without explicitly appending "A" to the history. Our equilibrium concept will require sequential rationality for the monopolist after entry.

in the final period and[4]

$$
\begin{aligned}
V(h^t;\alpha,q) = &(1-\alpha(h^t))\,(v+\delta\,\mathbb{E}^L_{s_t}[V(h^t,\mathrm{O},s_t;\alpha,q)]) \\
&+\alpha(h^t)\,q(h^t)\,(\delta\,\mathbb{E}^L_{s_t}[V(h^t,\mathrm{E},\mathrm{F},s_t;\alpha,q)]-x) \\
&+\alpha(h^t)\,(1-q(h^t))\,\delta\,\mathbb{E}^L_{s_t}[V(h^t,\mathrm{E},\mathrm{A},s_t;\alpha,q)]
\end{aligned}
$$

for $t<T$, where $\mathbb{E}^L_{s_t}$ indicates expectation with respect to the signal s_t under the type L. The terms on the right side correspond to the monopolist's continuation payoff when the t-entrant stays out (first term), when the t-entrant enters and the monopolist fights (second term), and when the t-entrant enters and the monopolist acquiesces (third term).

Entrant's payoff. The t-entrant's expected payoff from entering is $-c$ if the monopolist is tough and

$$q(h^t)\,(-c)+(1-q(h^t))\,k$$

if the monopolist is normal. The payoff from staying out is 0.

Perfect Bayesian equilibria

Note that all history is public except for the realization of the type. Therefore, when defining perfect Bayesian equilibria, it is enough to focus on beliefs over the type, as each information set of each entrant contains only two histories.

Definition 1.1. A *perfect Bayesian equilibrium* consists of a strategy profile (α,q) and a belief system $\phi: H\to[0,1]$ with $\phi(\emptyset)=\phi_0$ such that, for all $h^t\in H$:

1. Entrant optimality: $\alpha(h^t)=1$ and $\alpha(h^t)=0$ when

$$\phi(h^t)(-c)+(1-\phi(h^t))\left(q(h^t)\,(-c)+(1-q(h^t))\,k\right)$$

[4] Because there is no risk of confusion and to simplify notation, we use $V(h^t,\mathrm{E},d,s_t;\alpha,q)$ to denote $V((h^t,((\mathrm{E},d),s_t));\alpha,q)$, where $(h^t,((\mathrm{E},d),s_t))$ is the history entering period $t+1$ and $d\in\{\mathrm{F},\mathrm{A}\}$.

is positive and negative, respectively.

2. Monopolist optimality: $q(h^t) = 1$ and $q(h^t) = 0$ when

$$\delta\, \mathbb{E}^L_{s_t}[V(h^t, \mathrm{E}, \mathrm{F}, s_t; \alpha, q)] - x - \delta\, \mathbb{E}^L_{s_t}[V(h^t, \mathrm{E}, \mathrm{A}, s_t; \alpha, q)]$$

is positive and negative, respectively.

3. Bayes consistency: For all (o_t, s_t) we have that $\phi(h^t, o_t, s_t) = \hat{\phi}_{s_t}(\phi(h^t, o_t))$, where

$$\hat{\phi}_s(\phi) := \frac{\phi\, f_H(s)}{\phi\, f_H(s) + (1-\phi)\, f_L(s)},$$

where $\phi(h^t, o_t)$ is obtained through Bayes' rule whenever possible, that is,

(a) $\phi(h^t, \mathrm{O}) = \phi(h^t)$,

(b) $\phi(h^t, \mathrm{E}, \mathrm{F}) = \frac{\phi(h^t)}{\phi(h^t) + q(h^t)\,(1-\phi(h^t))}$ if $\phi(h^t) \neq 0$ or $q(h^t) \neq 0$,

(c) $\phi(h^t, \mathrm{E}, \mathrm{A}) = 0$ if $\phi(h^t) \neq 1$ and $q(h^t) \neq 1$.[5]

Property 1 of Definition 1.1 states that the t-entrant enters if the expected payoff from entering is positive and stays out otherwise. Property 2 says that the monopolist fights if the increase in discounted expected continuation value from fighting exceeds the fight cost, x, and acquiesces otherwise. Property 3 says that beliefs update within a period first based on actions (resulting in $\hat{\phi}$) and then based on the public signal. If the entrant stays out, then beliefs update only from the signal. Bayes' rule always applies except in two situations: (i) the monopolist acquiesces when $\phi(h^t) = 1$ or $q(h^t) = 1$, and (ii) the monopolist fights when both $\phi(h^t) = 0$ and $q(h^t) = 0$.

[5] In part 3, we abuse notation by extending the definition of $\phi(\cdot)$ to histories that include the action(s) in period t but not the signal.

2 Equilibrium analysis

In this section, we will characterize equilibrium behavior, which will be unique up to zero-probability events.

2.1 Last period

It is instructive to first analyze behavior in the last period. Fix an equilibrium (α, q, ϕ).

To characterize the t-entrant's behavior, define

$$G(\hat{\phi}, \hat{q}) := \hat{\phi}\,(-c) + (1-\hat{\phi})(\hat{q}\,(-c) + (1-\hat{q})\,k) \tag{1}$$

for all $\hat{\phi}, \hat{q} \in [0,1]$; this is the t-entrant's expected payoff of entering when the monopolist's reputation is $\hat{\phi}$ and the conjectured probability of fight is $\hat{q}$. From the entrant's optimality condition, the entrant enters for sure if $G(\phi(h^T), q(h^T)) > 0$, and it stays out for sure if $G(\phi(h^T), q(h^T)) < 0$. Note that $G(\hat{\phi}, \hat{q})$ is decreasing in $\hat{\phi}$ and $\hat{q}$.

In the last period, the monopolist acquiesces against any entry. Thus, the entrant enters if $\phi(h^T) < \overline{\phi}$ and stays out if $\phi(h^T) > \overline{\phi}$, where

$$\overline{\phi} := \frac{k}{c+k}$$

is the unique belief such that $G(\overline{\phi}, 0) = 0$.

It follows that, in any equilibrium, the strategies at time T are uniquely pinned down by the posterior $\phi(h^T)$ except if $\phi(h^T) = \overline{\phi}$. This implies that the monopolist's continuation value satisfies

$$V(h^T) \begin{cases} = 0 & \text{if } \phi(h^T) < \overline{\phi}, \\ \in [0, v] & \text{if } \phi(h^T) = \overline{\phi}, \\ = v & \text{if } \phi(h^T) > \overline{\phi}. \end{cases} \tag{2}$$

2.2 Continuation at extreme beliefs

We now provide a result that will significantly ease our analysis. It establishes that, if the monopolist loses its reputation (i.e., $\phi(h^t)=0$ at some history h^t) or if it perfectly convinces the entrants about its type being tough (i.e., $\phi(h^t)=1$), then the continuation play is equal to the unique continuation play of the game when the monopolist's type is common knowledge.

Proposition 2.1. *If the monopolist has no reputation (i.e., $\phi(h^t)=0$), its continuation payoff is 0 (i.e., $V(h^t)=0$): in all remaining periods, the entrant enters and the monopolist acquiesces. If the monopolist is instead perceived to be tough for sure (i.e., $\phi(h^t)=1$), no entrant enters in the future and the monopolist's payoff is $V(h^t)=\frac{1-\delta^{T-t+1}}{1-\delta}\,v$.*

2.3 Existence and uniqueness

Strategies of the following form will play an important role in our analysis.

Definition 2.1. An equilibrium (α, q, ϕ) is *in cutoff strategies* with cutoffs $(\phi_t^{\shield}, \phi_t^{\dagger})_{t=0}^{T}$ if, for all $h^t \in H$, we have

$$q(h^t)=\begin{cases} 1 & \text{if } \phi(h^t)\geq \phi_t^{\shield}, \\ \frac{\phi(h^t)(1-\phi_t^{\shield})}{\phi_t^{\shield}(1-\phi(h^t))} & \text{if } \phi(h^t)<\phi_t^{\shield}, \end{cases} \tag{3}$$

whenever $\phi(h^t)\neq 1$, and

$$\alpha(h^t)=\begin{cases} 0 & \text{if } \phi(h^t)>\phi_t^{\dagger}, \\ 1 & \text{if } \phi(h^t)<\phi_t^{\dagger}. \end{cases} \tag{4}$$

Note that Definition 2.1 does not impose any requirement on the monopolist's strategy when $\phi(h^t)=1$ or on the entrants' strategy when $\phi(h^t)=\phi_t^{\dagger}$. As we shall see, such flexibility will allow us to establish that all equilibria are in cutoff strategies.

Note also that because no entrant enters if it believes that the monopolist is tough for sure, the value of $q(h^t)$ when $\phi(h^t) = 1$ is irrelevant for the equilibrium outcome. The value of $\alpha(h^t)$ when $\phi(h^t) = \phi_t^\dagger$ will only be relevant at time 0 (in the non-generic case where $\phi_0 = \phi_0^\dagger$) because the probability that $\phi(h^t) = \phi_t^\dagger$ for some $t > 0$ will be 0 in all equilibria.

In Section 2.1 we established that, in all equilibria, $q(h^T)$ and $\alpha(h^T)$ are of the form (3) and (4) with cutoffs $\phi_T^\heartsuit = 1$ and $\phi_T^\dagger = \overline{\phi}$. The following lemma characterizes each t-entrant's best reply in an equilibrium in cutoff strategies.

Lemma 2.1. *Suppose that in period t, the monopolist's strategy has the form* (3) *for some $\phi_t^\heartsuit > 0$. Then the t-entrant's best response has the form* (4)*, where*

$$\phi_t^\dagger = \overline{\phi}\, \phi_t^\heartsuit. \tag{5}$$

Lemma 2.1 follows from the requirement that the t-entrant is indifferent when $\phi(h^t) = \phi_t^\dagger$. This is equivalent to the condition $G(\phi^\dagger, q(h^t)) = 0$ (where G is defined in (1) and $q(h^t)$ satisfies (3)). Naturally, $\phi_t^\dagger$ is increasing in $\phi_t^\heartsuit$, since the probability of the monopolist fighting is weakly decreasing in $\phi_t^\heartsuit$.

Proposition 2.2. *An equilibrium exists. There is a sequence $(\phi_t^\heartsuit, \phi_t^\dagger)_{t=0}^T$ (unique and independent of ϕ_0) such that for all $\phi_0 \in [0, 1]$, all equilibria are in cutoff strategies with cutoffs $(\phi_t^\heartsuit, \phi_t^\dagger)_{t=0}^T$.*

While Proposition 2.2 establishes the uniqueness of the cutoffs of the equilibrium strategies, it does not, in general, guarantee that the continuation equilibrium outcome after each history is unique. The reason is that cutoff strategies do not specify the behavior of entrants when they are indifferent (i.e., when $\phi(h^t) = \phi_t^\dagger$).

Nonetheless, the equilibrium outcome (i.e., the ex ante distribution over terminal histories) is generically unique. To see this, observe that for all $t > 0$, the probability that $\phi(h^t) = \phi_t^\dagger$ is zero. This follows from the fact that $\phi_t^\dagger \in (0, 1)$ for all t and Assumption 1.[6] This yields the following result.

[6]Note that for any h^{t-1} and o_t, either $\phi(h^{t-1}, o_t) \in \{0, 1\}$ (and then the continuation play is

Corollary 2.1. *The equilibrium outcome is unique unless* $\phi_0^\dagger = \phi_0$.

Equilibrium construction

The proof of Proposition 2.2 recursively shows that there is a unique function $V_t : [0,1] \to [0, \frac{1}{1-\delta}\, v]$ with the properties that (i) in any equilibrium, $V(h^t) = V_t(\phi(h^t))$ for all histories h^t such that $\phi(h^t) \neq \phi_t^\dagger$, and (ii) $V_t(\phi_t^\dagger) = 0$.[7] Such a function is the unique solution to

$$V_t(\phi) = \mathbb{I}_{\phi > \phi_t^\dagger} \left(v + \delta\, \mathbb{E}_{s_t}^L [V_{t+1}(\hat{\phi}_{s_t}(\phi))]\right), \tag{6}$$

where V_T is fully determined by the right side of (2) with $V_T(\overline{\phi}) = 0$ (as $\phi_T^\dagger = \overline{\phi}$). From Proposition 2.1, we have that $V_t(0) = 0$ and $V_t(1) = \frac{1-\delta^{T-t+1}}{1-\delta}\, v$ for all t.

If $\delta\, V_{t+1}(1) \leq x$, the monopolist acquiesces with certainty after entry in period t, because there is no posterior belief after fighting for which the expected continuation value would offset the cost of fighting; in this case, the monopolist's strategy satisfies (3) with $\phi_t^\heartsuit = 1$. This occurs, for example, in periods sufficiently close to the deadline. Otherwise, $\phi_t^\heartsuit$ is uniquely pinned down by

$$\delta\, \mathbb{E}_s^L [V_{t+1}(\hat{\phi}_s(\phi_t^\heartsuit))] = x. \tag{7}$$

Indeed, $q(h^t) \in (0,1)$ whenever $\phi(h^t) \in (0, \phi_t^\heartsuit)$, and so the monopolist is indifferent between acquiescing and fighting. In this case, the posterior after a fight is $\phi_t^\heartsuit$. Since by Proposition 2.1 the monopolist obtains 0 by acquiescing, the continuation value after fighting (left side of (7)) must be equal to the cost of fighting (right side of (7)). This occurs, for example, in early periods if the deadline is long enough.[8]

uniquely pinned down by Proposition 2.1), or $\phi(h^{t-1}, o_t) \in (0,1)$ (and then $\phi(h^{t-1}, o_t, s_t)$ is continuously distributed on $[0,1]$ by Assumption 1). That $\phi_t^\dagger \leq \overline{\phi} < 1$ for all t follows from equation (5). That $\phi_t^\dagger > 0$ for all t is easily derived from the arguments in the proof of Proposition 2.2.

[7]It is convenient to choose V_t so that $V_t(\phi(h^t))$ coincides with the monopolist's continuation value in an equilibrium where entrants enter whenever they are indifferent in doing so (which always exists).

[8]Because for any given t, $\delta\, V_{t+1}(1)$ converges to $\frac{\delta}{1-\delta}\, v > x$ as $T \to \infty$, the condition that

Based on this logic, equations (5)-(7) fully determine the sequence $(V_t, \phi_t^{\dagger}, \phi_t^{\heartsuit})$, which is solved from T backwards.

2.4 Monotonicity of the continuation value

The following proposition establishes monotonicity of the main equilibrium objects.

Proposition 2.3. *For all t, $V_t(\phi)$ is increasing in ϕ, and for all $\phi \in (0,1)$, $V_t(\phi)$ is decreasing in t. Moreover, $\phi_t^{\dagger}$ and $\phi_t^{\heartsuit}$ are increasing in t.*

The proposition first establishes that the monopolist's value function is increasing in its reputation level. This is intuitive: the monopolist gets v until entry and 0 thereafter, and from a higher reputation, entry occurs later.

The proposition also establishes that the value function is monotone in t. Intuitively, near the deadline, there is less time remaining for the monopolist to benefit from its reputation through a reduced probability of entry, and hence the continuation value weakly decreases at each belief. Hence, if the monopolist still mixes at any beliefs, to preserve its indifference, it must induce a higher posterior belief when it fights, as demonstrated by (7). Through (3), this implies a reduced probability of fighting. Hence, there is less deterrence near the deadline: the entrant's threshold increases in time via (5).

2.5 Example

We now present an example of the equilibrium behavior for a particular signal structure. Suppose that, for each $\theta \in \{L=0, H=1\}$, each period's signal s is normally distributed with mean θ and variance σ^2. Figure 2 shows the continuation value functions at $t \in \{0, 10, 20\}$ and the cutoff sequences $(\phi_t^{\heartsuit}, \phi_t^{\dagger})_{t=0}^{T}$ for a numerical example where the horizon is $T = 20$.

$\delta V_{t+1}(1) > x$ will hold if T is high enough. Note that this condition and (7) imply that $\phi_0^{\heartsuit} < 1$ and $\phi_0^{\dagger} < \overline{\phi}$ for sufficiently large T.

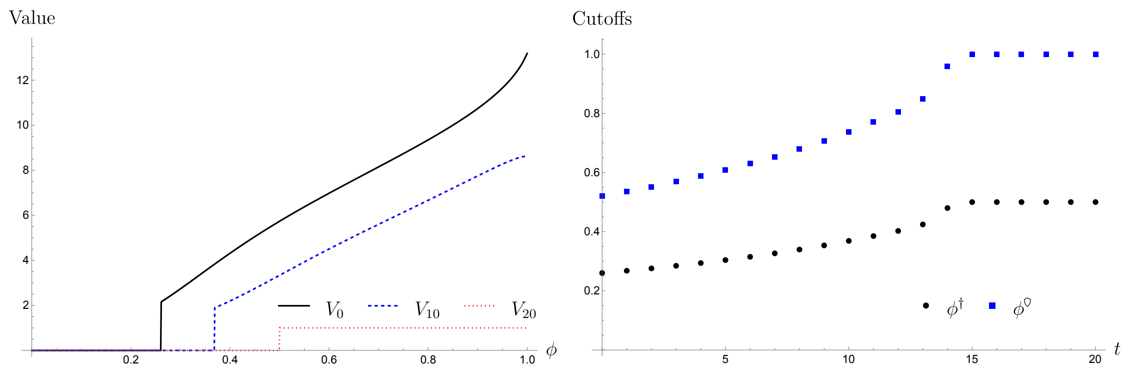

Figure 2: Equilibrium objects for $(k, c, \delta, v, x, \sigma, T) = (1, 1, .95, 1, 5, 2, 20)$.

3 Long-horizon limit

In this section, we present the main results of the paper. We examine the limit as $T \to \infty$ to understand whether long horizons still ensure deterrence in early periods, as they do in the model without exogenous news arrival.

Observe that an implication of Proposition 2.3 is that $\phi_0^\dagger$ and $\phi_0^\heartsuit$ decrease as T grows, where we use subscript T for the time horizon when appropriate. Let $\phi_*^\dagger$ and $\phi_*^\heartsuit$ denote the limits of $\phi_0^\dagger$ and $\phi_0^\heartsuit$ as $T \to \infty$, respectively, and let V_* denote the pointwise limit of V_0 as $T \to \infty$, where again the subscript T denotes the horizon length.[9] We say that there is *asymptotic full deterrence* if $\phi_*^\dagger = 0$. Instead, we say that there is *asymptotic partial deterrence* if $\phi_*^\dagger > 0$.

Note that the result in the models of Kreps and Wilson (1982) and Milgrom and Roberts (1982), where there is no exogenous signal, no entrant enters for an arbitrarily large number of initial periods as the time horizon lengthens for any prior belief, that is, there is asymptotic full deterrence.[10] We will show that with a signal, there is asymptotic partial deterrence for some parameter values: in this case, if the prior belief on the type being tough is low enough, the first entrant enters independently of the length of the horizon.

[9] These limits exist because $\phi_0^\dagger$, $\phi_0^\heartsuit$, and V_0 are monotone in T by Proposition 2.3 and bounded.

[10] Recall that we are assuming that $x < \frac{\delta}{1-\delta} v$. Recall also that when $x \geq \frac{\delta}{1-\delta} v$, fighting is a strictly dominated strategy regardless of the presence of a signal.

3.1 Limit equilibrium objects

We now provide asymptotic analogues of the equilibrium conditions (5)-(7) at $t = 0$ (or any fixed t) by taking $T \to \infty$, and later show they characterize the limit equilibrium objects. These are given by the system of equations

$$V(\phi) = \mathbb{I}_{\phi > \phi^\dagger} \left(v + \delta \, \mathbb{E}_s^L [V(\hat{\phi}_s(\phi))] \right) , \tag{8}$$

$$\phi^\dagger = \overline{\phi} \, \phi^\heartsuit , \tag{9}$$

$$\delta \, \mathbb{E}_s^L [V(\hat{\phi}_s(\phi^\heartsuit))] = x . \tag{10}$$

It is not difficult to see that equations (8)-(10) correspond to the equilibrium conditions of stationary equilibria in cutoff strategies with interior cutoffs $(\phi^\dagger_\infty, \phi^\heartsuit_\infty)_{t=0}^\infty$ in a model with infinite horizon. The following result relates solutions to equations (8)-(10) with the asymptotic equilibrium objects as $T \to \infty$.

Proposition 3.1. *If* $\phi^\dagger_* > 0$ *then* $(V_*, \phi^\dagger_*, \phi^\heartsuit_*)$ *is the unique solution to* (8)-(10). *If, instead,* $\phi^\dagger_* = 0$, *then there is no solution to* (8)-(10).

Proposition 3.1 provides a direct method for characterizing the long-horizon limits of finite horizon equilibria: either there is no solution to (8)-(10), in which case there is asymptotic full deterrence, or there is a unique solution and asymptotic partial deterrence. Below, we show that these two possibilities are not vacuous.

3.2 Conditions for asymptotic full deterrence

The following result establishes that asymptotic full deterrence is obtained if and only if the cost of fighting is below a threshold $\overline{x}$. That is, independently of the signal distribution or the discount rate, asymptotic full or partial deterrence can be obtained depending on the value of x.

Proposition 3.2. *There is some threshold* $\overline{x} \in (0, \frac{\delta}{1-\delta} v)$ *(independent of* x*) such that there is asymptotic full deterrence if and only if* $x \leq \overline{x}$.

Proposition 3.2 is intuitive: asymptotic full deterrence occurs only if the cost of fighting is low enough. Importantly, the threshold cost $\overline{x}$ is positive. That is, independently of the rest of the parameters, asymptotic full deterrence arises if the cost of fighting is low enough, and there is asymptotic partial deterrence if the cost of fighting is high enough. The threshold is strictly lower than the maximum potential benefit from fighting, $\frac{\delta}{1-\delta}\,v$.

Using the conditions (8)-(10) that apply in the asymptotic partial deterrence case, the threshold $\overline{x}$ is obtained as follows. First, we change variables using the log-likelihood ratio $z \equiv \log(\phi/(1-\phi))$ instead of the belief ϕ. Let $V(z;z^\dagger)$ denote the solution to[11]

$$V(z;z^\dagger) = \mathbb{I}_{z>z^\dagger}\left(v+\delta\,\mathbb{E}_s^L\left[V\left(z+\log\tfrac{f_H(s)}{f_L(s)};z^\dagger\right)\right]\right)\ , \tag{11}$$

which is analogous to (8). By changing variables to $\hat{z} \equiv z-z^\dagger$, we have

$$V(\hat{z};0) = \mathbb{I}_{\hat{z}>0}\left(v+\delta\,\mathbb{E}_s^L\left[V\left(\hat{z}+\log\tfrac{f_H(s)}{f_L(s)};0\right)\right]\right)\ . \tag{12}$$

Note that $V(\hat{z};0)$ does not depend on $z^\dagger$, and corresponds to the monopolist's payoff when $\hat{z}_0 = \hat{z}$ and it obtains v in every period until the first period τ where $\hat{z}_\tau \leq 0$ and zero thereafter. By standard arguments, $V(\hat{z};0)$ is strictly increasing on $[0,\infty)$.

The indifference conditions for the entrants and the monopolist ((9) and (10)) imply that

$$x = \delta\,\mathbb{E}_s^L\left[V\left(\Delta(\phi^\dagger)+\log\tfrac{f_H(s)}{f_L(s)};0\right)\right]\ , \tag{13}$$

where, returning to using beliefs instead of log-likelihood ratios,

$$\Delta(\phi^\dagger) := \log\left(\frac{\phi^\dagger/\overline{\phi}}{1-\phi^\dagger/\overline{\phi}}\right) - \log\left(\frac{\phi^\dagger}{1-\phi^\dagger}\right) = -\log\left(\frac{\overline{\phi}-\phi^\dagger}{1-\phi^\dagger}\right)\ . \tag{14}$$

[11] Note that if $z(h^t)\in\mathbb{R}$ is the log-likelihood ratio at time t and the t-entrant does not enter, then the updated log-likelihood ratio after signal s_t is $z(h^t)+\log(f_H(s_t)/f_L(s_t))$ by Bayes' rule.

There exists a solution $(V, \phi^\dagger, \phi^\circ)$ to (8)-(10) with $\phi^\dagger > 0$ if and only if there is a value $\phi^\dagger \in (0, \overline{\phi})$ such that $\Delta(\phi^\dagger)$ satisfies equation (13). Note that $\Delta(\cdot)$ is continuous and strictly increasing, tends to $+\infty$ as $\phi^\dagger \to \overline{\phi}$, and tends to $-\log(\overline{\phi}) > 0$ as $\phi^\dagger \to 0$. We then conclude that there is asymptotic partial deterrence if and only if $x > \overline{x}$, where

$$\overline{x} := \delta\, \mathbb{E}_s^L \left[V\left(-\log(\overline{\phi}) + \log \tfrac{f_H(s)}{f_L(s)}; 0 \right) \right] = \delta\, \mathbb{E}_\tau^L \left[\tfrac{1-\delta^{\tau-1}}{1-\delta}\, v \right], \tag{15}$$

where τ is the first time $\hat{z}_t$ reaches 0 when $\hat{z}_0 = -\log(\overline{\phi})$ under the signal. Note that because the right side of (15) is bounded above by $\frac{\delta}{1-\delta}\, v$, we have that $\overline{x} \in (0, \frac{\delta}{1-\delta}\, v)$.

Equation (15) proves to be extremely useful, as it immediately allows us to analyze the comparative statics of $\overline{x}$ with respect to the input parameters of the model. Intuitively, full deterrence is more likely to occur when the monopolist's value from deterrence is higher (higher δ or v) or when it is easier to deter entrants (higher c or lower k).

Corollary 3.1. *The threshold $\overline{x}$ is increasing in δ, v, and c, and decreasing in k.*

We explore the relationship between signal informativeness and deterrence in Section 3.3.

Patient monopolist limit

A remarkable property of our analysis is that the threshold $\overline{x}$ stays finite and strictly positive in the limit $\delta \to 1$. That is, even when the monopolist is fully patient ($\delta = 1$)—so the potential value of reputation grows unbounded as the horizon lengthens—there is a wide range of parameters where the monopolist cannot fully deter entries.

To see why, assume $\delta = 1$. The above arguments continue to hold: in this case, $V_*(\hat{z}; 0)$ is the expected number of periods until the log-likelihood ratio reaches 0 if its initial value is $\hat{z}$. Because the log-likelihood ratio has a negative drift under type L, this is a finite value independently of $\hat{z}$ (but this value increases toward infinity as $\hat{z} \to \infty$). In particular, this implies that $\overline{x}$ in (15) remains bounded as $\delta \to 1$.

3.3 More informative signal

Our results thus far establish that in the presence of exogenous news arrival, there can still be full deterrence in the long-horizon limit, but there need not be. In this section, we explore how the informativeness of the signal affects deterrence and the monopolist's payoff for long horizons.

To motivate our approach for studying informativeness below, let us comment on alternative approaches. First, Blackwell dominance is not strong enough to guarantee lower $\overline{x}$. We can see this by examining the last term in (15), which is equal to the expected discounted time it takes for the log-likelihood $\hat{z}_t$ to first cross 0 from $\hat{z}_0 = -\log(\overline{\phi}) > 0$. The key is that the realizations s satisfying that $\log \frac{f_H(s)}{f_L(s)} > \log(\overline{\phi})$ can be garbled to increase the probability that $\hat{z}_1 \equiv -\log(\overline{\phi}) + \frac{f_H(s_0)}{f_L(s_0)}$ is lower than 0. Then, if δ is small enough, such garblings make the last term in (15) decrease, lowering the value of $\overline{x}$, thereby shrinking the set of parameters where asymptotic full deterrence occurs. In other words, making the signal less informative may change asymptotic full deterrence into asymptotic partial deterrence.

A second approach would be the following. For a fixed signal structure f, one would look for signal structures g satisfying that the distribution of the likelihood ratio under g conditional on L first-order stochastically dominates the distribution of the likelihood ratio under f conditional on L. It is easy to see that, in this case, the last term in (15) would be higher under g than under f. Nevertheless, such a condition is vacuous: under Assumption 1, the law of total probability ensures that only $g = f$ would satisfy this condition.[12]

To summarize, Blackwell dominance is too weak, while first order stochastic dominance is too strong. To proceed, instead of defining a partial order over signal structures, we take an asymptotic approach. Given a signal structure with $s \sim f_\theta$ for $\theta \in \{H, L\}$, define the likelihood ratio $\Lambda_f(s) := \frac{f_H(s)}{f_L(s)}$. We say that a sequence of

[12] Indeed, note that for $h \in \{f, g\}$, $\int_{\mathbb{R}} \frac{h_H(s)}{h_L(s)} h_L(s)\, \mathrm{d}s = \int_{\mathbb{R}} h_H(s)\, \mathrm{d}s = 1$, whereas the first order stochastic dominance condition implies $\mathbb{E}^{L,g}[\frac{g_H(s)}{g_L(s)}] > \mathbb{E}^{L,f}[\frac{f_H(s)}{f_L(s)}]$.

signal structures with densities f_θ^n satisfying Assumption 1 is *asymptotically revealing* if $\Lambda_{f^n}(s) \xrightarrow{p} 0$ under $\theta = L$, where $\xrightarrow{p}$ denotes convergence in probability. We say that it is *asymptotically uninformative* if $\Lambda_{f^n}(s) \xrightarrow{p} 1$ under $\theta = L$.[13] We use $\phi_*^{\dagger n}$ to denote the value of $\phi_*^\dagger$ which depends on n through f_θ^n.

Proposition 3.3. *Let f_θ^n be a sequence of signal structures. If f_θ^n is asymptotically revealing, then: $\phi_*^{\dagger n} \to \overline{\phi}$ and $V_*^n(\phi) \to \mathbb{I}_{\phi \geq \overline{\phi}}\, v$ for all $\phi \in [0,1]$. If f_θ^n is asymptotically uninformative, then for large n, $\phi_*^{\dagger n} = 0$ and $V_*^n(\phi) = \mathbb{I}_{\phi > 0}\, \frac{1}{1-\delta}\, v$ for all $\phi \in [0,1]$.*

An immediate implication of Proposition 3.3 is that making the signals sufficiently informative (i.e., going sufficiently far along an asymptotically revealing sequence) hurts the normal monopolist, while making the signals sufficiently uninformative helps. Intuitively, when the monopolist is normal and signals become very informative, the entrant learns quickly about the monopolist from exogenous news. This makes a high reputation more transient for the monopolist, reducing the monopolist's continuation value, leading to entry at lower beliefs and a lower probability of fighting. In the limit, the normal monopolist's payoff converges to the payoff in the static game. In contrast, as signals become very uninformative, these forces act in the opposite direction, and the monopolist is better off.

The proposition also implies that if the signal is not *too* informative (i.e., it occurs sufficiently far along an asymptotically uninformative sequence), there is still asymptotic full deterrence. This follows from the convergence of the t-entrant's cutoff to $\phi_*^{\dagger n}$ as $T \to \infty$, which is 0 for sufficiently large n.

[13]For example, returning to the specification in Section 2.5, let σ_n be a sequence of strictly positive real numbers and define f_θ^n to be the PDF of $N(\theta, \sigma_n)$. The sequence f_θ^n is asymptotically revealing if $\sigma_n \to 0$, and it is asymptotically uninformative if $\sigma_n \to \infty$. Note that for general f_θ^n, the definition of asymptotically revealing implies that $\Lambda_{f^n}(s) \xrightarrow{p} \infty$ under $\theta = H$; likewise, the definition of asymptotically uninformative implies $\Lambda_{f^n}(s) \xrightarrow{p} 1$ under $\theta = H$.

4 Discussion

While we have considered a behavioral tough type who fights with certainty for simplicity, our results can be extended in multiple ways. A first possibility would consider a tough type who fights with a probability less than, but close to, 1. It is not difficult to see that the arguments above would continue to hold with an appropriate adjustment in the definition of $\overline{\phi}$. Another possibility would be to let the tough type be a payoff type instead of a behavioral type. In line with Kreps and Wilson (1982), this behavioral type would prefer fighting to acquiescing after entry but still prefers deterrence to entry. In the Online Appendix, we show that our predictions remain equilibrium predictions with a payoff type.

A Proofs

Proof of Proposition 2.1. Consider an arbitrary PBE (α, q, ϕ). We first show by induction that if $\phi(h^t) = 0$ for some $h^t \in H$, then (i) $V(h^t) = 0$, (ii) $\alpha(h^t) = 1$, and (iii) $q(h^t) = 0$. Clearly, (i)-(iii) hold in the last period: we have $q(h^T) = 0$, so if $\phi(h^T) = 0$, then $\alpha(h^T) = 1$ and $V(h^T) = 0$. Now suppose (i)-(iii) hold for all $t \geq \hat{T}+1$ for some $\hat{T} \geq 0$. We show that they also hold for $t = \hat{T}$. To rule out $q(h^{\hat{T}}) > 0$, note that this would imply that $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}, s_{\hat{T}}) = 0$ for all $s_{\hat{T}}$ by Bayes' rule, and therefore $V(h^{\hat{T}}, \mathrm{E}, \mathrm{F}, s_{\hat{T}}; \alpha, q) = 0$ by the induction hypothesis. But then the monopolist's optimality condition would imply $q(h^{\hat{T}}) = 0$, a contradiction. Hence, $q(h^{\hat{T}}) = 0$, and so $\alpha(h^{\hat{T}}) = 1$, so the monopolist's continuation payoff in period $\hat{T}$ is 0 as desired. This completes the induction argument. Moreover, note that if $\phi(h^t) = 0$, then since $q(h^t) = 0$, Bayes' rule implies the monopolist's reputation stays at zero and (i)-(iii) apply for the remainder of the game.

Next, consider $\phi(h^t) = 1$. In this case, the entrants stay out every period and the posterior stays equal to 1. So, the monopolist earns v in all periods $t, \ldots, T$, for a discounted value of $V(h^t) = \frac{1-\delta^{T-t+1}}{1-\delta} v$. □

Proof of Lemma 2.1. Fix a history h^t and a monopolist strategy of the form (3) with fixed $\phi_t^{\heartsuit} > 0$. The t-entrant's expected payoff from entering is $G(\phi(h^t), q(h^t))$. Using (3) and simplifying yields

$$G(\phi(h^t), q(h^t)) = (\phi(h^t)/\phi_t^{\heartsuit})(-c) + (1 - \phi(h^t)/\phi_t^{\heartsuit})k \ .$$

This is strictly decreasing in $\phi(h^t)$ on $[0, \phi_t^{\heartsuit}]$ and vanishes when $\phi(h^t) = \frac{k}{c+k}\,\phi_t^{\heartsuit} = \overline{\phi}\,\phi_t^{\heartsuit}$. Thus, any best reply of the entrant satisfies (4) with $\phi_t^{\dagger} = \overline{\phi}\,\phi_t^{\heartsuit}$. □

Proof of Proposition 2.2. We use an induction argument to derive the sequence of cutoffs referred to in the proposition and show that any equilibrium must be in cutoff strategies with these cutoffs. We then establish existence.

Induction hypothesis for arbitrary $\hat{T} \leq T$: For all $t \geq \hat{T}$, there is a unique threshold $\phi_t^{\dagger} \in (0, \overline{\phi}]$ and a function $V_t : [0,1] \to \mathbb{R}_+$ such that in all equilibria, (i) the strategies at time t satisfy equations (3), (4), and (5); (ii) $V(h^t) = V_t(\phi(h^t))$ whenever $\phi(h^t) \neq \phi_t^{\dagger}$; and (iii) $V_t(\phi_t^{\dagger}) = 0$. Moreover, (iv) V_t is weakly increasing, $V_t(\phi) = 0$ for all $\phi \in [0, \phi_t^{\dagger})$, and $V_t(\phi) > 0$ for all $\phi \in (\phi_t^{\dagger}, 1]$. Without loss, we define $V_t(\phi_t^{\dagger}) = 0$.

Base Case: $\hat{T} = T$. The result holds for $\hat{T} = T$ as explained in Section 2.1.

Induction Step: Proof for $\hat{T} \in \{0, \ldots, T-1\}$. Assume that the induction hypothesis holds for $\hat{T}+1$. For all $t = \hat{T}, \ldots, T-1$, define

$$W_t(\phi) := \mathbb{E}_{s_t}^{L}[\delta\, V_{t+1}(\hat{\phi}_{s_t}(\phi))]$$

for all $\phi \in [0,1]$ and $W_T \equiv 0$. Because V_{t+1} is increasing and non-constant by the induction hypothesis and the signal distribution is continuous with a log-likelihood ratio that is strictly increasing and has full support on $\mathbb{R}$, we have that W_t is strictly increasing and continuous.

Note that for any history $h^{\hat{T}}$, the monopolist's expected utility from fighting is

$$-x + W_{\hat{T}}(\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F})). \tag{16}$$

There are two cases:

- **Case 1:** $W_{\hat{T}}(\phi) < x$ **for all** $\phi \in [0,1]$**.** In this case, for any history $h^{\hat{T}}$, (16) is strictly negative, whereas the monopolist can obtain at least 0 by acquiescing. Hence, the monopolist's optimality condition implies that $q(h^{\hat{T}}) = 0$ for all $h^{\hat{T}}$. Then by the same logic as in Section 2.1, it follows that $\alpha(h^{\hat{T}}) = 1$ if $\phi(h^{\hat{T}}) < \overline{\phi}$ and $\alpha(h^{\hat{T}}) = 0$ if $\phi(h^{\hat{T}}) > \overline{\phi}$. Hence, in this case, (3) and (4) are satisfied at $t = \hat{T}$ with $\phi^{\dagger}_{\hat{T}} = \overline{\phi}$ and $\phi^{\shield}_{\hat{T}} = 1$. We define

$$V_{\hat{T}}(\phi) := \begin{cases} 0 & \text{if } \phi \le \overline{\phi} \\ v + W_{\hat{T}}(\phi) & \text{if } \phi > \overline{\phi}. \end{cases}$$

- **Case 2:** $W_{\hat{T}}(\phi) \ge x$ **for some** $\phi \in [0,1]$**.** Because $W_{\hat{T}}$ is strictly increasing and tends to 0 as $\phi \to 0$ by the dominated convergence theorem, there is exactly one $\phi^{\shield}_{\hat{T}} \in (0,1]$ for which $W_{\hat{T}}(\phi^{\shield}_{\hat{T}}) = x$.

  We argue that the monopolist plays according to (3) in period $\hat{T}$ with cutoff $\phi^{\shield}_{\hat{T}}$, and we characterize belief updating. Assume $\phi(h^{\hat{T}}) < 1$ (as (3) is only relevant in this case). First, if $\phi(h^{\hat{T}}) \ge \phi^{\shield}_{\hat{T}}$, it must be that the monopolist fights with probability 1, as in (3). Toward a contradiction, assume the contrary. Bayes' rule would imply that $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{A}) = 0$, which we have shown implies the payoff from acquiescing would be 0. Also, Bayes' rule would imply $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}) > \phi(h^{\hat{T}}) \ge \phi^{\shield}_{\hat{T}}$, so (16) would be strictly positive. Thus, fighting would be strictly optimal, a contradiction.

  Second, if $0 < \phi(h^{\hat{T}}) < \phi^{\shield}_{\hat{T}}$, a similar contradiction arises if $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}) > \phi^{\shield}_{\hat{T}}$; and if $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}) < \phi^{\shield}_{\hat{T}}$, we would have $q(h^{\hat{T}}) = 0$, but then $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}) = 1 > \phi(h^{\hat{T}})$, again a contradiction. Hence, it must be that $\phi(h^{\hat{T}}, \mathrm{E}, \mathrm{F}) = \phi^{\shield}_{\hat{T}}$, and Bayes' rule implies q has the form in (3). Third, if $\phi(h^{\hat{T}})=0$, Proposition 2.1 already establishes that $q(h^{\hat{T}})=0$, consistent with (3).

Given $\phi^{\circ}_{\hat{T}}$, by Lemma 2.1, the entrant plays according to (4) with $\phi^{\dagger}_{\hat{T}}=\overline{\phi}\,\phi^{\circ}_{\hat{T}}$. In this case, define

$$V_{\hat{T}}(\phi):=\begin{cases}0 & \text{if } \phi\leq\phi^{\dagger}_{\hat{T}},\\ v+W_{\hat{T}}(\phi) & \text{if } \phi>\phi^{\dagger}_{\hat{T}}.\end{cases}$$

We now establish properties (ii) and (iv) of the induction hypothesis for $\hat{T}$. Fix any history $h^{\hat{T}}$. If $\phi(h^{\hat{T}})<\phi^{\dagger}_{\hat{T}}$, then entry occurs for sure, and since $\phi^{\dagger}_{\hat{T}}<\phi^{\circ}_{\hat{T}}$, the monopolist is willing to concede as $q(h^{\hat{T}})<1$, so the monopolist's continuation payoff is equal to its payoff when it acquiesces, which is 0; thus $V(h^{\hat{T}})=0$ if $\phi(h^{\hat{T}})<\phi^{\dagger}_{\hat{T}}$. Also, if $\phi(h^{\hat{T}})>\phi^{\dagger}_{\hat{T}}$, we have

$$V(h^{\hat{T}})=v+\delta\,\mathbb{E}^{L}_{s_{\hat{T}}}[V(h^{\hat{T}},\mathrm{O},s_{\hat{T}})]=v+\delta\,\mathbb{E}^{L}_{s_{\hat{T}}}[V_{\hat{T}+1}(\hat{\phi}_{s_{\hat{T}}}(\phi(h^{\hat{T}})))]\;=v+W_{\hat{T}}(\phi(h^{\hat{T}})),$$

where the second equality holds by the induction hypothesis for $\hat{T}+1$. In summary,

$$V(h^{\hat{T}})=\begin{cases}0 & \text{if } \phi(h^{\hat{T}})<\phi^{\dagger}_{\hat{T}},\\ v+W_{\hat{T}}(\phi(h^{\hat{T}})) & \text{if } \phi(h^{\hat{T}})>\phi^{\dagger}_{\hat{T}},\end{cases}$$

It is then clear that, for either Case 1 or Case 2 above, $V_{\hat{T}}$ is the unique function satisfying $V(h^{\hat{T}})=V_{\hat{T}}(\phi(h^{\hat{T}}))$ for all $h^{\hat{T}}\in H^{\hat{T}}$ such that $\phi(h^{\hat{T}})\neq\phi^{\circ}_{\hat{T}}$ and $V_{\hat{T}}(\phi^{\dagger}_{t})=0$. Moreover, $V_{\hat{T}}$ is weakly increasing, and it satisfies $V_{\hat{T}}(\phi)=0$ for all $\phi\in[0,\phi^{\dagger}_{\hat{T}}]$ and $V_{\hat{T}}(\phi)>0$ for all $\phi\in(\phi^{\dagger}_{\hat{T}},1]$. This completes the induction argument.

Existence: The previous argument establishes that equilibria are in cutoff strategies. Furthermore, the sequence of cutoffs $(\phi^{\circ}_{t},\phi^{\dagger}_{t})_{t=0}^{T}$ pins down those strategies as a function of beliefs except at histories where $\phi(h^{t})=\phi^{\dagger}_{t}$ or $\phi(h^{t})=1$. To establish existence, we need only to specify the belief system and to specify the strategies and verify their optimality at $\phi(h^t)=\phi_t^\dagger$ or $\phi(h^t)=1$. Whenever $\phi(h^t)=\phi_t^\dagger$, set $\alpha(h^t)=1$; since the entrant is indifferent at its cutoff, this is optimal. When $\phi(h^t)=1$, set $q(h^t)=0$ if $T-t<\tilde{t}$ and $q(h^t)=1$ if $T-t\geq\tilde{t}$, where $\tilde{t}>0$ is defined as the unique solution to $\delta\frac{1-\delta^{\tilde{t}}}{1-\delta}v=x$. We define the belief system recursively going forward in time from $\phi(\emptyset)=\phi_0$. Given any $\phi(h^t)$, set $\phi(h^t,\mathrm{E},\mathrm{A})=0$; note that this is implied by the Bayes' Consistency property when $\phi(h^t)\neq 1$ or $q(h^t)\neq 1$. Set $\phi(h^t,\mathrm{E},\mathrm{F})$ according to the Bayes' Consistency property when $q(h^t)\neq 0$ or $\phi(h^t)\neq 0$, and set $\phi(h^t,\mathrm{E},\mathrm{F})=0$ when $(\phi(h^t),q(h^t))=(0,0)$. The monopolist's strategy is optimal by construction whenever $\phi(h^t)\neq 1$. When $\phi(h^t)=1$, note that under the given belief system, the monopolist's payoff from acquiescing is 0, but if it fights, it pays a cost $-x$ and continues with a belief of 1 in period $t+1$, for a discounted continuation payoff of $\delta\,\frac{1-\delta^{T-(t+1)+1}}{1-\delta}\,v$ by Proposition 2.1. Hence, fighting is optimal if and only if $T-t\geq\tilde{t}$. □

Proof of Corollary 2.1. The proof follows directly from Proposition 2.2 and the argument in the main text. □

Proof of Proposition 2.3. In this proof, we will use the functions $\{V_t,W_t:[0,1]\to\mathbb{R}|t=0,...,T\}$ defined and used in the proof of Proposition 2.2. Recall that all V_t satisfy that $V_t(\phi)=0$ for all $\phi\leq\phi_t^\dagger$, $V_t(\phi)>0$ for all $\phi>\phi_t^\dagger$, and V_t is weakly increasing. Recall also that all W_t are strictly increasing and continuous, and satisfy that if $\phi_t^{\heartsuit}\in(0,1)$, then $W_t(\phi_t^{\heartsuit})=x$. We will use the following equality, which is valid for all $t\in\{0,...,T-1\}$ and $\phi\in[0,1]$,

$$W_t(\phi)=\mathbb{E}_{s_t}^L\left[\mathbb{I}_{\hat{\phi}_{s_t}(\phi)>\phi_{t+1}^\dagger}\,\delta\left(v+W_{t+1}(\hat{\phi}_{s_t}(\phi))\right)\right]\;.$$

We proceed by induction.

Induction hypothesis for $\hat{T}$: For all $t\in\{\hat{T},...,T-1\}$, we have (i) $V_t(\phi)\geq V_{t+1}(\phi)$ and $W_t(\phi)>W_{t+1}(\phi)$ for all $\phi\in(0,1)$, (ii) $\phi_t^{\heartsuit}\leq\phi_{t+1}^{\heartsuit}$ and $\phi_t^\dagger\leq\phi_{t+1}^\dagger$.

Part 1. We prove the result for $\hat{T}=T-1$. First, observe that $\phi_{T-1}^{\heartsuit}\leq 1=\phi_T^{\heartsuit}$, which implies $\phi_{T-1}^\dagger=\overline{\phi}\,\phi_{T-1}^{\heartsuit}\leq\overline{\phi}=\phi_T^\dagger$. Also, because $V_T(\phi)$ is positive for all $\phi>\overline{\phi}$, we have that $W_{T-1}(\phi) > 0 = W_T(\phi)$ for all $\phi > 0$. Finally, note that for $\phi \leq \phi^{\dagger}_{T}$, $V_T(\phi) = 0 \leq V_{T-1}(\phi)$ and for $\phi > \phi^{\dagger}_{T} \geq \phi^{\dagger}_{T-1}$, we have $V_T(\phi) = v + \delta\, W_T(\phi) \leq v + \delta\, W_{T-1}(\phi) = V_{T-1}(\phi)$.

Part 2. Assume the induction hypothesis holds for $\hat{T}+1$ where $\hat{T}+1 \leq T-1$. This implies that for all $\phi > 0$

$$\begin{aligned} W_{\hat{T}}(\phi) &= \mathbb{E}^{L}_{s_t}\left[\mathbb{I}_{\hat{\phi}_{s_{\hat{T}}}(\phi) > \phi^{\dagger}_{\hat{T}+1}}\ \delta(v + W_{\hat{T}+1}(\hat{\phi}_{s_{\hat{T}}}(\phi)))\right] \\ &> \mathbb{E}^{L}_{s_t}\left[\mathbb{I}_{\hat{\phi}_{s_{\hat{T}}}(\phi) > \phi^{\dagger}_{\hat{T}+2}}\ \delta(v + W_{\hat{T}+2}(\hat{\phi}_{s_{\hat{T}}}(\phi)))\right] \\ &= W_{\hat{T}+1}(\phi)\ . \end{aligned}$$

The inequality holds for two reasons: $\phi^{\dagger}_{\hat{T}+2} \geq \phi^{\dagger}_{\hat{T}+1}$ (which shrinks the range where the integrand is non-zero) and, by assumption, $W_{\hat{T}+2}(\hat{\phi}_{s_{\hat{T}}}(\phi)) < W_{\hat{T}+1}(\hat{\phi}_{s_{\hat{T}}}(\phi))$.

Since $W_{\hat{T}}(\phi) > W_{\hat{T}+1}(\phi)$ for all $\phi \in [0,1]$, we have that $\phi^{\shield}_{\hat{T}+1} \geq \phi^{\shield}_{\hat{T}}$ and so $\phi^{\dagger}_{\hat{T}+1} \geq \phi^{\dagger}_{\hat{T}}$.[14] It is also clear that $V_{\hat{T}}(\phi) \geq V_{\hat{T}+1}(\phi)$ for all $\phi \in [0,1]$. □

Proof of Propositions 3.1 and 3.2. We divide the proof into parts.

Part 0. We first formalize several preliminary results from the discussion following Proposition 3.2. First, we note that there is a unique function $V(\cdot;0)$ satisfying equation (12).[15] Indeed, the functional

$$\check{V} \mapsto \left(\hat{z} \mapsto \mathbb{I}_{\hat{z}>0}\left(v + \delta\, \mathbb{E}^{L}_{s}\left[\check{V}\left(\hat{z} + \log \tfrac{f_H(s)}{f_L(s)}\right)\right]\right)\right)$$

is a contraction in the space of functions from $\mathbb{R}$ to $[0, \frac{1}{1-\delta}\, v]$ equipped with the sup norm. Note that $V(\hat{z};0)$ is equal to the payoff obtained by the monopolist if (i) the log-likelihood of the initial belief is $\hat{z}$, (ii) the posterior evolves only according to the signal, and (iii) the monopolist obtains v each period until the first period where

[14] Recall that, for all $t < T$, (i) if $W_t(1) > x$ then $W_t(\phi^{\shield}_t) = x$, (ii) if $W_t(1) \leq x$ then $\phi^{\shield}_t = 1$, and (iii) $\phi^{\dagger}_t = \overline{\phi}\, \phi^{\shield}_t$.

[15] For ease of exposition, we do not introduce equation (12) until Section 3.2, but there is no circularity in our arguments.

$\hat{z} \leq 0$, and obtains zero afterwards.

Also, for the reasons provided in the main text, there is some $\phi^\dagger > 0$ satisfying equations (13) and (14) if and only if $x > \overline{x}$, where $\overline{x}$ is defined in equation (15). Also, if $x > \overline{x}$, $\phi^\dagger$ is unique, as the right hand side of (13) is strictly increasing in $\phi^\dagger$.

Part 1. We now consider the equilibrium behavior as T grows. Let $(\phi^\dagger_{T,t})_{t=0}^T$ denote the entrants' thresholds in the model with deadline T. Note that $\phi^\dagger_{T,t} = \phi^\dagger_{T-t,0}$ for all $T \in \mathbb{Z}_+$ and $t \leq T$. Note also that $\phi^\dagger_{T,t}$ is increasing in t. For a fixed T, note that

$$V_{T,0}(\phi_0) = \frac{\mathbb{E}^L_{\tau_{T,0}}[1-\delta^{\tau_{T,0}}|\phi_0]}{1-\delta}\, v \ ,$$

where $\tau_{T,0}$ is the first time t' such that $\phi(h^{t'}) \leq \phi^\dagger_{T,t'}$. Because $\phi^\dagger_{T,0}$ is decreasing in T, it converges to some $\phi^\dagger_* \geq 0$ as $T \to \infty$.

Part 2. We analyze the case where $\phi^\dagger_* > 0$. In this case, as $T \to \infty$, using that by Assumption 1 the signal and posterior distributions are continuous, we have that for all $\phi_0 \in [0,1]$, $V_{T,0}(\phi_0)$ converges to[16]

$$\frac{\mathbb{E}^L_{\tau_*}[1-\delta^{\tau_*}|\phi_0]}{1-\delta}\, v,$$

where τ_* is defined as the first time t' such that $\phi_{t'} \leq \phi^\dagger_*$ (and $\tau_* = +\infty$ if $\phi_{t'} > \phi^\dagger_*$ for all t'); note that by definition, this equals $V(z_0 - z^\dagger_*; 0)$ defined in the main body. And since $\lim_{T\to\infty} V_{T,0}(\phi_0) = V_*(\phi_0)$ for all $\phi_0 \in [0,1]$ by definition, we have $V_*(\phi_0) = V(z_0 - z^\dagger_*; 0)$ for all ϕ_0 (where $z_0 = \log(\frac{\phi_0}{1-\phi_0})$).

By footnote 8, $\phi^{\heartsuit}_{T,0} < 1$ for all sufficiently large T, and the following indifference condition must hold:

$$x = \delta\, \mathbb{E}^L_s[V_{T-1,0}(\hat{\phi}_s(\phi^{\heartsuit}_{T,0}))].$$

[16]Note that, because $\phi^\dagger_{T,0}$ is decreasing in T and each $V_T(\phi) = 0$ for all $\phi \leq \phi^\dagger_{T,0}$, we have that $\lim_{T\to\infty} V_{T,0}(\phi) = 0$ for all $\phi \leq \phi^\dagger_*$.

Taking $T \to \infty$ yields $x = \delta\,\mathbb{E}_s^L[V_*(\hat{\phi}_s(\phi_*^{\heartsuit}))]$. Also, after a log-likelihood transformation, $\hat{\phi}_s(\phi_*^{\heartsuit})$ becomes $z_*^{\heartsuit} + \log\frac{f_H(s)}{f_L(s)}$, so $V_*(\hat{\phi}_s(\phi_*^{\heartsuit})) = V\big(z_*^{\heartsuit} + \log\frac{f_H(s)}{f_L(s)} - z_*^{\dagger}; 0\big)$, which in turn equals $V\big(\Delta(\phi_*^{\dagger}) + \log\frac{f_H(s)}{f_L(s)}; 0\big)$.

Putting these together, we have

$$x = \delta\,\mathbb{E}_s^L\left[V\left(\Delta(\phi_*^{\dagger}) + \log\tfrac{f_H(s)}{f_L(s)}; 0\right)\right] ,$$

which is (13) with $\phi_*^{\dagger}$ playing the role of $\phi^{\dagger}$. Because $\phi^{\dagger}$ denotes the unique solution to (13), we have $\phi_*^{\dagger} = \phi^{\dagger}$, and $(V_*, \phi_*^{\dagger}, \phi_*^{\heartsuit})$ is the unique solution to (8)-(10). Also, recall that a solution with $\phi^{\dagger} > 0$ exists if and only if $x > \overline{x}$.

Part 3. We now analyze the remaining case $\phi_*^{\dagger} = 0$. Note that $\phi^{\dagger} = 0$ cannot be part of a solution to (8)-(10).[17] Toward a contradiction, suppose there is a solution with $\phi^{\dagger} > 0 = \phi_*^{\dagger}$. Then, moving to log-likelihood space, there must be some T such that $z_{T,0}^{\dagger} < z^{\dagger} \leq z_{T-1,0}^{\dagger}$. It is clear that $V_{T-1,0}(z) < V(z; z^{\dagger})$ for all $z > z^{\dagger}$.[18] Then,

$$x = \delta\,\mathbb{E}_s^L\left[V_{T-1,0}\left(z_{T,0}^{\heartsuit} + \log\tfrac{f_H(s)}{f_L(s)}\right)\right] < \delta\,\mathbb{E}_s^L\left[V\left(z_{T,0}^{\heartsuit} + \log\tfrac{f_H(s)}{f_L(s)}, z^{\dagger}\right)\right] .$$

Because the right side of the inequality is increasing in $z_{T,0}^{\heartsuit}$ and is equal to x when $z_{T,0}^{\heartsuit} = z^{\heartsuit}$, we have $z_{T,0}^{\heartsuit} > z^{\heartsuit}$, which implies $z_{T,0}^{\dagger} > z^{\dagger}$. This contradicts that $z_{T,0}^{\dagger} < z^{\dagger}$.

We conclude that (when $\phi_*^{\dagger} = 0$) there is no solution to (8)-(10). Hence, it must be that $x \geq \overline{x}$. In this case, we have $V_{T,0}(\phi) \to \mathbb{I}_{\phi > 0}\,\frac{1}{1-\delta}\,v$ pointwise as $T \to \infty$. □

Proof of Corollary 3.1. Recall the last expression in (15). The result with respect to v is then trivial, and the result for δ is also immediate by differentiation. For c and k, observe that from (15), $\overline{x}$ is increasing in τ. Now τ is increasing in $-\log(\overline{\phi})$, which in turn is increasing in c and decreasing in k, proving the results for c and k. □

Proof of Proposition 3.3. Fix a sequence of signal structures f_θ^n. For each n, let $(V_*^n, \phi^{\dagger n}_*, \phi^{\heartsuit n}_*)$ limits of $(V_0, \phi_0^{\dagger}, \phi_0^{\heartsuit})$ as $T \to \infty$ under signal structure f_θ^n. Recall from

[17]If there were a solution to this system with $\phi^{\heartsuit} = 0$, but then $\hat{\phi}_s(\phi^{\heartsuit}) = 0$. Since $V(0) = 0$ from (8), the left hand side of (10) would also be 0, which cannot equal x.

[18]Indeed, because $z_{T-1,t}^{\dagger}$ is increasing in t, we have that $z_{T-1,t}^{\dagger} > z_*^{\dagger}$ for all $t = 0, ..., T-1$.

Section 3.2 that if $\phi^{\dagger n}_* > 0$, then in the space of log-likelihood beliefs,

$$x = \delta\, \mathbb{E}^L_s \left[V^n_* \left(\Delta(\phi^{\dagger n}_*) + \log(\Lambda_{f^n}(s)); 0 \right) \right] \ . \tag{17}$$

Part 1. Suppose the sequence f^n_θ is asymptotically revealing. Fix any $\hat{z} > 0$. By definition, for any $\epsilon > 0$, there exists N such that for $n \geq N$, $\Lambda_{f^n}(s) < \epsilon$ with probability at least $1 - \epsilon$ under $\theta = L$. Choose $\epsilon > 0$ sufficiently small that $\Lambda_{f^n}(s) < \epsilon$ implies $\hat{z} + \log(\Lambda_{f^n}(s)) < 0$. Also, note that $V^n_*(\cdot; 0) \in [0, \frac{v}{1-\delta}]$. Hence, for $n \geq N$, $\delta\, \mathbb{E}^L_s \big[V^n_*(\hat{z} + \log(\Lambda_{f^n}(s)); 0) \big] \leq \epsilon\, \delta \frac{v}{1-\delta}$. Therefore, $\delta\, \mathbb{E}^L_s \big[V^n_*(\hat{z} + \log(\Lambda_{f^n}(s)); 0) \big] \to 0$ pointwise in $\hat{z}$ as $n \to \infty$.

This fact has three implications. First, by setting $\hat{z} = -\log(\overline{\phi})$, we get that $\overline{x}^n$ in (15) tends to 0 as $n \to \infty$. Hence, by Proposition 3.2, for all sufficiently large n, we have $\phi^{\dagger n}_* > 0$; there is asymptotic partial deterrence. Second, and moreover, we must have $\phi^{\dagger n}_* \to \overline{\phi}$. Toward a contradiction, if this were not true, then there would exist a subsequence of signal structures along which $\phi^{\dagger n}_*$ is bounded away from $\overline{\phi}$ and therefore $\Delta(\phi^{\dagger n}_*)$ is also bounded above by some $M > 0$. But then setting $\hat{z} = M$ and taking n sufficiently large, we have $\delta\, \mathbb{E}^L_s \big[V^n_*(\hat{z} + \log(\Lambda_{f^n}(s)); 0) \big] < x$, and since V^n_* and Δ are nondecreasing, (17) cannot be satisfied for any $\phi^{\dagger n}_*$ along this subsequence. From this contradiction, we conclude that (along the original sequence) $\phi^{\dagger n}_* \to \overline{\phi}$.

Third, for all fixed $\hat{z}$, we have $V^n_*(\hat{z}; 0) \to \mathbb{I}_{\hat{z}>0}(v + 0) = \mathbb{I}_{\hat{z}>0}\, v$ by (12). Using this fact, we now prove the claim that $V^n_*(\phi) \to \mathbb{I}_{\phi \geq \overline{\phi}}\, v$. Fix any $z \geq \overline{z}$ and any $\epsilon > 0$. For sufficiently large, n, we have $\overline{z} - \epsilon < z^{\dagger n}_* < \overline{z} \leq z$, where the first inequality follows from $z^{\dagger n}_* \to \overline{z}$, and where $z^{\dagger n}_* < \overline{z}$ follows from footnote 8. That is, for all n and any finite horizon, $z^{\dagger n}_0 < \overline{z}$, and since $z^{\dagger n}_0$ is decreasing in T, we have $z^{\dagger n}_* = \lim_{T \to \infty} z^{\dagger n}_0 < \overline{z}$. Thus, since $V^n_*(\cdot; 0)$ is weakly increasing, $V^n_*(z; z^{\dagger n}_*) = V^n_*(z - z^{\dagger n}_*; 0) \leq V^n_*(z - (\overline{z} - \epsilon); 0) \to v$. Also, since $z \geq \overline{z} > z^{\dagger n}_*$, we have $V^n_*(z; z^{\dagger n}_*) \geq v$ by (11) under signal structure f^n_θ. Combining these facts gives $V^n_*(z; z^{\dagger n}_*) \to v$. And for any $z < \overline{z}$ and $\epsilon < \overline{z} - z$, for sufficiently large n, $z^{\dagger n}_* > \overline{z} - \epsilon > z$. Thus, for $z < \overline{z}$ and sufficiently large n, $V^n_*(z; z^{\dagger n}_*) = V^n_*(z - z^{\dagger n}_*; 0) = 0$.

Part 2. Next, suppose the sequence f_θ^n is asymptotically uninformative. Clearly $V_*^n(\phi)=0$ when $\phi=0$; we now show that $V_*^n(\phi)\to\frac{v}{1-\delta}$ for all $\phi>0$. To that end, we first prove that $\phi^{\dagger n}_*=0$ for all sufficiently large n; equivalently, we show that $z^{\dagger n}_*=-\infty$ for all sufficiently large n. By way of contradiction, suppose not, and pass to a subsequence with $z^{\dagger n}_*>-\infty$ for all n. Recall the change of variables from z to $\hat{z}=z-z^{\dagger n}_*$ so that $V_*^n(\hat{z};0)=V_*^n(z;z^{\dagger n}_*)$. Now fix any value $\hat{z}>0$, and recall that $V_*^n(\hat{z};0)$ solves

$$V_*^n(\hat{z};0)=\mathbb{I}_{\hat{z}>0}\left(v+\delta\,\mathbb{E}_s^L[V_*^n(\hat{z}+\log(\Lambda_{f^n}(s));0)]\right). \tag{18}$$

Since $V_*^n(\cdot;0)$ is continuous on $(0,\infty)$, for any $\epsilon>0$, there exists $\eta>0$ such that $|V_*^n(y;0)-V_*^n(\hat{z};0)|\le\epsilon$ for all $y\in(\hat{z}-\eta,\hat{z}+\eta)$. In turn, there exists N such that $n\ge N$ implies $|\log(\Lambda_{f^n}(s))|<\eta$ with probability at least $1-\eta$ by the continuous mapping theorem and the fact that $\Lambda_{f^n}(s)\xrightarrow{p}1$. Thus, choosing $\epsilon<\frac{2v}{1-\delta}$, we have for large n

$$\left|V_*^n(\hat{z};0)-\mathbb{E}_s^L[V_*^n(\hat{z}+\log(\Lambda_{f^n}(s));0)]\right|\le(1-\eta)\epsilon+\eta\,\frac{2v}{1-\delta},$$

which is in $o(\epsilon)$ for sufficiently small η. Thus, for any $\hat{z}>0$,

$$V_*^n(\hat{z};0)=v+\delta\,V_*^n(\hat{z};0)+o(\epsilon). \tag{19}$$

This implies that for any $\hat{z}>0$, we have $V_*^n(\hat{z};0)\to\frac{v}{1-\delta}$.

Then from (15) and (18), with $-\log(\overline{\phi})$ playing the role of $\hat{z}$, it follows that

$$\overline{x}^n:=\delta\,\mathbb{E}_s^L\left[V_*^n\left(-\log(\overline{\phi})+\log(\Lambda_{f^n}(s));0\right)\right]=V_*^n(-\log(\overline{\phi});0)-v\to\frac{\delta\,v}{1-\delta}>x\ ,$$

which implies $\phi^{\dagger n}_*=0$ by Proposition 3.2, a contradiction. We conclude that, indeed, $\phi^{\dagger n}_*=0$ for all sufficiently large n. This implies that for all $\phi>0$, for all sufficiently large n, along the equilibrium path, there is no entry and the monopolist's reputation never hits zero, so $V_*^n(\phi)=\frac{v}{1-\delta}$. □

References

AOYAGI, M. (1996): "Reputation and entry deterrence under short-run ownership of a firm," *Journal of Economic Theory*, 69, 411–430.

CRIPPS, M. W., G. J. MAILATH, AND L. SAMUELSON (2004): "Imperfect monitoring and impermanent reputations," *Econometrica*, 72, 407–432.

DILMÉ, F. (2025): "Repeated trade with imperfect information about previous transactions," *Theoretical Economics*, 20, 209–254.

FAINGOLD, E. AND Y. SANNIKOV (2011): "Reputation in continuous-time games," *Econometrica*, 79, 773–876.

HU, J. (2014): "Reputation in the presence of noisy exogenous learning," *Journal of Economic Theory*, 153, 64–73.

KREPS, D. M. AND R. WILSON (1982): "Reputation and imperfect information," *Journal of Economic Theory*, 27, 253–279.

MAILATH, G. J. AND L. SAMUELSON (2006): *Repeated games and reputations: long-run relationships*, Oxford University Press.

MILGROM, P. AND J. ROBERTS (1982): "Predation, reputation, and entry deterrence," *Journal of Economic Theory*, 27, 280–312.

PITCHIK, C. (1993): "Commitment, reputation, and entry deterrence," *Games and Economic Behavior*, 5, 268–287.

SELTEN, R. (1978): "The chain store paradox," *Theory and Decision*, 9, 127–159.

WILSON, R. (1992): "Chapter 10 Strategic models of entry deterrence," Elsevier, vol. 1 of *Handbook of Game Theory with Economic Applications*, 305–329.

WISEMAN, T. (2008): "Reputation and impermanent types," *Games and Economic Behavior*, 62, 190–210.

——— (2009): "Reputation and exogenous private learning," *Journal of Economic Theory*, 144, 1352–1357.

B Online Appendix

B.1 Extension: payoff type

In this section, we extend our results to the case where the tough monopolist is a payoff type instead of an action type. Following Milgrom and Roberts (1982), we now assume that the tough monopolist is strategic. In each given period, the stage payoff of the tough monopolist coincides with that of the normal monopolist except if the entrant enters and the monopolist fights, in which case the tough monopolist's payoff is $x_H \in (0, v)$ instead of $-x$. Hence, like the L-monopolist, the tough monopolist prefers that the entrant does not enter, but unlike the L-monopolist, the tough monopolist myopically prefers fighting to acquiescing after the entrant enters.[19] The discount rate of the tough monopolist is $\delta \in (0,1)$.

Strategies are defined as in our base model — on the same set of histories with same set of feasible actions at each history. We now differentiate between q_L (the strategy of the normal monopolist) and q_H (the strategy of the tough monopolist). A perfect Bayesian equilibrium is defined as in Definition 1.1 except for two modifications. The first is the additional optimality condition for the H-monopolist: $q_H(h^t) = 1$ and $q_H(h^t) = 0$ when

$$\delta\, \mathbb{E}^H_{s_t}[V^H(h^t, \mathrm{E}, \mathrm{F}, s_t; \alpha, q)] - x_H - \delta\, \mathbb{E}^H_{s_t}[V^H(h^t, \mathrm{E}, \mathrm{A}, s_t; \alpha, q)]$$

is positive and negative, respectively. The second is that the Bayes consistency properties (b) and (c) are now

(b) $\phi(h^t, \mathrm{E}, \mathrm{F}) = \frac{\phi(h^t) q_H(h^t)}{\phi(h^t) q_H(h^t) + (1-\phi(h^t)) q_L(h^t)}$ if $\phi(h^t) q_H(h^t) + (1-\phi(h^t)) q_L(h^t) \neq 0$

(c) $\phi(h^t, \mathrm{E}, \mathrm{A}) = \frac{\phi(h^t)(1-q_H(h^t))}{\phi(h^t)(1-q_H(h^t)) + (1-\phi(h^t))(1-q_L(h^t))}$ if $\phi(h^t)(1-q_H(h^t)) + (1-\phi(h^t))(1-q_L(h^t)) \neq 0$.

[19] Note that this does not immediately imply fighting is always optimal; if the continuation payoff after acquiescing is higher than that after fighting, the total payoffs must be compared.

The following result demonstrates how our equilibrium outcome survives in the model with a strategic tough monopolist.

Proposition B.1. *For all $\phi_0 \neq \phi_0^\dagger$, there exists an equilibrium with the same outcome as that of the model with a behavioral tough monopolist. Moreover, if x_H is sufficiently close to v, this is the unique equilibrium outcome.*

Proof. For the existence claim, consider any strategy profile and belief system of the original model supporting the unique outcome, and augment it with $q_H(h^t) = 1$ for all h^t. Additionally, specify that (i) the belief immediately after acquiescing is 0, and (ii) if the belief ever reaches 0, it remains 0 for the rest of the game. Note that these properties are consistent with the Bayes Consistency requirements of the equilibrium concept. We need only show that the tough monopolist's strategy is indeed optimal. Since the strategies and beliefs are Markovian, there exists V_t^H such that $V^H(h^t) = V_t^H(\phi(h^t))$ for all h^t. As before, define $W_t^H(\phi) := \mathbb{E}_{s_t}^H[\delta\, V_{t+1}^H(\hat{\phi}_{s_t}(\phi))]$ for all ϕ, where $V_{T+1}^H(\phi) := 0$. Then the IC condition is

$$x_H + W_t^H(\phi(h^t, \mathrm{E}, \mathrm{F})) \geq 0 + W_t^H(\phi(h^t, \mathrm{E}, \mathrm{A})). \tag{20}$$

This is trivial in period T, so assume $t < T$. The left side is at least $x_H(1 + \delta + \cdots + \delta^{T-t})$ by fighting against all entries in the remaining periods. The right side is at most $x_H(\delta + \cdots + \delta^{T-t})$, since $\phi(h^t, \mathrm{E}, \mathrm{A}) = 0$ and $\phi(h^{t'}) = 0$ for all $t' > t$, and therefore the entrants enter in all remaining periods regardless of the monopolist's actions. Hence, the IC condition holds.

For uniqueness of outcome, note that for any $\phi \in [0,1]$, we have $\delta \frac{1-\delta^{T-t}}{1-\delta} x_H \leq W_t^H(\phi) \leq \delta \frac{1-\delta^{T-t}}{1-\delta}\, v$. This follows from the fact that v is the maximum feasible payoff in any period, and that, as argued above, the tough monopolist can guarantee at least x_H every period by fighting against all entries. Hence, the left side of (20) is at least $x_H + \delta \frac{1-\delta^{T-t}}{1-\delta} x_H$, while the right side is at most $\delta \frac{1-\delta^{T-t}}{1-\delta}\, v$. For x_H sufficiently close to v, the former is strictly larger, so the tough monopolist strictly prefers to fight every period. □