Discussion Paper Series – CRC TR 224

Discussion Paper No. 700
Projects B03 and B04

Designing Vertical Differentiation with Information

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August 2025

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

Designing Vertical Differentiation with Information*

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July 29, 2025

Abstract

We study information design in a vertically differentiated market. Two firms offer products of ex-ante unknown qualities. A third party designs a system to publicly disclose information. More precise information guides consumers toward their preferred product but increases expected product differentiation, allowing firms to raise prices. Full disclosure of the product ranking alone suffices to maximize industry profits. Consumer surplus is maximized, however, whenever no information about the product ranking is disclosed, as the benefit of competitive pricing always dominates the loss from suboptimal choices. The provision of public information on product quality becomes questionable.

JEL Classification Numbers: D43, D82, L13, L15.
Keywords: Information Design, Vertical Product Differentiation, Quality Rankings, Competition

*We are grateful to comments we have received from Volker Nocke and Nicolas Schutz. Anton Sobolev and Konrad Stahl gratefully acknowledge support by the Deutsche Forschungsgemeinschaft through CRC TR 224 (projects B03 and B04). The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada.

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1 Introduction

In many markets, consumers would agree on the ranking of products if they had full information about their qualities. However, they often need to rely on third parties for information about the products they consider purchasing. When pricing their goods, sellers incorporate the impact of this third-party information on consumers' relative product perceptions and thus market demand. In such a situation, it is not a priori clear who benefits from public information provision.

We are looking at markets in which the informing third party is a certifier, a platform on which the sellers are active, or a regulatory institution. They may provide signals via different means, e.g., direct product comparison, a search ranking or product ratings, informing both sides of the market about the relative quality of the products offered.[1] Indeed, widely accessed third-party evaluations such as Consumer Reports in the United States or reports by the German Stiftung Warentest focus largely on the comparative evaluation of relative qualities, instead of a descriptive comparison of products along dimensions evaluated differently across consumers.

A trade-off arises because information simultaneously allows consumers to make better decisions and allows firms to condition their prices on the revealed information, which may increase prices. Our first key insight is that the relevant property of information structures is their informational content about the ranking of product qualities. Specifically, when signals are uninformative, consumers rank products according to their prior beliefs, and firms compete based on the perceived, ex ante vertical differentiation. If signals are informative about the actual product ranking, however, consumers will perceive firms as more differentiated. The increase in differentiation relaxes competition, thereby raising expected prices. While this harms consumers, the information also allows heterogeneous consumers to choose their preferred product more frequently.

We show that this trade-off has an unequivocal resolution under rather mild assumptions. The gain from more intense competition always dominates the loss in consumer surplus due to a suboptimal allocation: Not revealing any information on the quality ranking of products beyond the consumers' prior belief maximizes consumer surplus. Hence, consumers prefer not to become informed when firms can react to the information by charging prices reflecting the revealed ranking. This holds even if they expect product qualities to differ substantially, e.g., under negative correlation of product qualities, where information could a priori be considered most valuable. Consumer-induced public certification of product quality in vertically differentiated markets becomes questionable.

In contrast, firms always prefer public information that reveals the true quality ranking of their products. Differentiation due to a revealed product ranking benefits them in

[1] In most cases, such as on online platforms, firms can easily obtain access to information even if it is primarily targeted at consumers.

two ways. First, a better allocation of consumers to their preferred product increases the surplus that can potentially be extracted. Second, the relaxed competition from increased differentiation facilitates this extraction. The latter effect is sufficiently strong so that even a firm believed to offer relatively lower-quality products makes higher profits compared to any rank-preserving information structure.

Since consumers and producers value information strictly opposite to each other, the partial welfare outcome, which is comprised of consumer and producer surplus, depends on their respective magnitudes, as well as the welfare weights associated with them. We show that maximizing unweighed welfare requires the provision of the products' rank order by quality. Yet, by current welfare standards employed in the regulatory and competition policy context, consumer surplus outweighs producer surplus. Under such standards, no information provision about relative product qualities can be the welfare optimum.

We derive these results by considering a duopoly of specialized firms that offer a product of differentiated quality from a finite set of possible quality levels, and consumers with preferences that involve a taste parameter distributed log-concavely. We allow for asymmetries as well as arbitrary correlation in the distribution of product qualities. A third party, such as a commercial supplier of information, a platform, or a regulator, offers arbitrary experiments ranging from full information to being completely uninformative.

In our baseline analysis, we proceed under the assumption that all consumers purchase, i.e., that the market is fully covered. However, we show that our main insights extend to the case where consumers may decide against purchase. This is despite the fact that under an extensive margin, the firms can no longer extract the rents from inelastically demanding consumers, but are faced with outside competition. To keep the analysis in this extension tractable and focused, we restrict attention to a state space with two asymmetric qualities and a uniformly distributed consumer preference parameter. In this setup, rank revelation—which coincides with full information revelation in the asymmetric two-state case—maximizes industry profits, while the consumer-optimal information structure maximizes information revelation conditional on preserving the prior ranking between the two products.[2]

Below, we embed our argument into the extant literature. The model is specified in Section 2. In Section 3, we analyze the model. We allow for the possibility that consumers decide against purchase in Section 4, and offer concluding remarks in Section 5.

Related Literature We contribute to a recent and growing literature on Bayesian persuasion and information design; see Bergemann and Morris (2019), Bergemann and Bonatti (2019), and Kamenica (2019) for surveys. We relate to the literature on the degree of information provision, including e.g., Hopenhayn and Saeedi (2023), and to

[2]Relative to the baseline result, the latter modification is because information revelation subject to this condition keeps prices low while maximizing market participation, which increases consumer surplus.

quality disclosure by a certifier as surveyed by Dranove and Jin (2010).

Closely related to our work are Armstrong and Zhou (2022) and Roesler and Szentes (2017). The latter show that the consumer-optimal signal structure in a monopoly involves posterior distributions featuring partial learning. We find no learning beyond initial product rankings to be the consumer-optimal signal structure, as it stifles competition—an effect absent in their monopoly model. As Armstrong and Zhou (2022), we focus on a competitive duopoly, and, in line with earlier studies such as Anderson and Renault (2009), on a central provider of information rather than decentralized information provision, e.g., by firms.

While the key trade-off—more information results in less competition and higher prices, but also allows consumers to choose the "right" alternative more frequently—is similar to Armstrong and Zhou (2022) and Biglaiser et al. (2025), we consider vertical instead of horizontal differentiation and allow for ex ante asymmetric firms. In line with idiosyncratic consumer preferences, they consider personalized provision of information, while we focus on public information provision with common product rankings across consumers. The latter appears more suitable when information is about common, objective product components.

We show that these changes to the setting are economically relevant. The firm-optimal information policy reveals the ranking between product qualities. Consumers remain heterogeneous even after product qualities are perfectly revealed, allowing both firms to retain positive market shares. Thus, in contrast to Armstrong and Zhou (2022), the firm-optimal policy does not induce allocative efficiency. Moreover, the consumer-optimal policy never discloses any information about the quality ranking beyond the prior. This also holds in the empirically relevant situation in which there is an extensive margin. These findings are related in spirit to those in Lewis and Sappington (1994), who consider a monopolistic supplier.

Hopenhayn and Saeedi (2023) primarily focus on the effect of information as mitigating adverse selection, by which low-quality firms crowd out high-quality firms' sales by pooling with them when consumers do not know the difference. Their attribution of the surplus generated by information to consumers vs. producers varies with concavity vs. convexity of the market supply function.[3] Adverse selection does not figure in our setup.

Finally, quality certification—an early example is Albano and Lizzeri (2001), a recent one Zapechelnyuk (2020)—may be focused primarily on the incentives to affect the suppliers' provision of higher quality. This is not the issue at stake here. We take quality as given and focus on the consequences of a third party informing both buyers and suppliers about quality differences.

[3]The finding that information may hurt consumers when the supply function is sufficiently convex is also present in Schlee (1996).

2 Model

There are two firms, indexed by $i \in \{1, 2\}$, each producing a product of different quality, and competing in prices. The quality of firm i's product is $v_i \in \mathcal{V} \equiv \{v^1, \ldots, v^n\}$, where $v^n > v^{n-1} > \ldots > v^1 \geq 0$. We assume that the production cost is independent of quality and normalized to zero. The state of the market, defined by the pair of product qualities, is denoted by $\omega \in \Omega \equiv \{(v_1, v_2)|v_i \in \mathcal{V}\}$. Let $\lambda_{k\ell} \in [0, 1)$ be the common prior probability that the quality of firm 1 is v^k and the quality of firm 2 is v^ℓ, where $k, \ell = 1, \ldots, n$. There is initial uncertainty about the ranking of the firms' qualities: without loss of generality, we assume that the prior satisfies $\mathbb{P}[v_1 > v_2] \in (0, 1)$.

There is a unit mass of consumers, each demanding one unit of the good. Consumers differ in their willingness to pay for quality θ, which is distributed according to the CDF $F : [\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} \geq 0$. We assume that it admits density $f(\theta) > 0$, which is twice differentiable and log-concave on $[\underline{\theta}, \overline{\theta}]$. To ensure an interior solution of the pricing subgame, we additionally assume that $\underline{\theta} < 1/f(\underline{\theta})$.

A consumer's valuation from purchasing a good of quality v at price p, given her willingness to pay for quality θ, is $\theta v - p$. For now, we assume that the outside option $\underline{v}$ of consumers is sufficiently low (e.g., $\underline{v} = -\infty$), ensuring that the market is fully covered. In Section 4, we discuss the implications of relaxing this assumption.

We assume that there is an information designer who chooses an experiment that generates a public signal about the firms' qualities. We separately analyze two distinct objectives of the information designer: *(i)* the maximization of total industry profit, and *(ii)* the maximization of consumer surplus.[4] Given the finite state space, we can restrict attention to finite signal structures without loss of generality. Formally, an experiment $\mathcal{E}$ is represented by a finite set of signals Σ and a collection of conditional probabilities $\{s(\cdot|\omega)\}_{\omega \in \Omega}$ over Σ, where $s(\sigma|\omega)$ is the probability that signal $\sigma \in \Sigma$ realizes conditional on state $\omega \in \Omega$.

The timing of the game unfolds as follows. The information designer commits to an experiment. Then, the public signal is realized according to the experiment and the latent underlying state. In response to the signal, firms simultaneously set prices p_1 and p_2. Consumers, observing both the signal and the prices, make their purchase decisions. We characterize the subgame perfect equilibria of the game.

3 Analysis

We begin by characterizing equilibria in the pricing subgame for a given experiment and a realized public signal about the firms' qualities. Then, using backward induction, we

[4]When the information designer's objective is a convex combination of *(i)* and *(ii)*, the results follow directly from the analysis of the boundary cases, as we show that consumer surplus can be represented as an affine function of industry profits.

characterize the necessary and sufficient conditions for an experiment to maximize total industry profits and consumer surplus, respectively.

3.1 Price Setting Stage

Consider the subgame in which the information designer has chosen an experiment $\mathcal{E} = \{s(\sigma|\omega)\}_{(\sigma,\omega)}$ and a public signal $\sigma \in \Sigma$ has realized. Define $\Delta V_{i,j}(\sigma)$ as the expected quality difference between firm i and firm j conditional on signal σ, i.e.,

$$\Delta V_{i,j}(\sigma) \equiv \mathbb{E}[v_i - v_j|\sigma].$$

If $\Delta V_{i,j}(\sigma) = 0$, then firms compete à la Bertrand and charge zero prices, resulting in zero profits. Otherwise, suppose, without loss of generality, that firm i has a strictly higher expected quality conditional on signal σ than firm j, i.e., $\Delta V_{i,j}(\sigma) > 0$. Let there exist a consumer of type $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ who is indifferent between purchasing from firm i and firm j when the firms set prices p_i and p_j, respectively. Then, the indifference condition for this consumer is given by

$$\hat{\theta}\mathbb{E}[v_i|\sigma] - p_i = \hat{\theta}\mathbb{E}[v_j|\sigma] - p_j, \tag{1}$$

which implies that

$$\hat{\theta} = \frac{p_i - p_j}{\Delta V_{i,j}(\sigma)}. \tag{2}$$

Consumers with $\theta \geq \hat{\theta}$ buy from firm i, and the remaining consumers with $\theta < \hat{\theta}$ buy from firm j. The resulting firms' profit functions are:

$$\pi_i(p_i, p_j|\sigma) = \left(1 - F\left(\frac{p_i - p_j}{\Delta V_{i,j}(\sigma)}\right)\right) p_i, \tag{3}$$

$$\pi_j(p_j, p_i|\sigma) = F\left(\frac{p_i - p_j}{\Delta V_{i,j}(\sigma)}\right) p_j. \tag{4}$$

If the indifference condition (1) is solved for some $\theta > \overline{\theta}$, then firm i makes zero profits as all profits accrue to firm j. If instead it is solved for some $\theta < \underline{\theta}$, then all consumers buy from firm i, and firm j earns zero profits.

Since the density function f is log-concave, it follows that the firms' demand functions $F(\cdot)$ and $1-F(\cdot)$ are log-concave (by Theorems 1 and 2 in Bagnoli and Bergstrom, 2005). Consequently, the profit functions given in equations (3) and (4) are also log-concave, and therefore quasi-concave. Thus, a solution to the system of first-order conditions together with equation (2) determines an equilibrium of the pricing subgame. The following lemma establishes, for any given experiment $\mathcal{E}$ and realized signal σ, the existence of a unique equilibrium of the pricing subgame and characterizes the equilibrium prices.

Lemma 1. *Consider an experiment $\mathcal{E} = \{s(\sigma|\omega)\}_{(\sigma,\omega)}$ and a realized public signal $\sigma \in \Sigma$.*

If $\Delta V_{i,j}(\sigma) \geq 0$, then in the unique equilibrium of the pricing subgame, firms set prices

$$p_i^*(\sigma) = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})} \Delta V_{i,j}(\sigma) \quad \textit{and} \quad p_j^*(\sigma) = \frac{F(\hat{\theta})}{f(\hat{\theta})} \Delta V_{i,j}(\sigma),$$

where $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ uniquely solves

$$\hat{\theta} = \frac{1 - 2F(\hat{\theta})}{f(\hat{\theta})}.$$

Consumers with $\theta \in [\hat{\theta}, \overline{\theta}]$ buy from firm i, and the remaining consumers buy from firm j. Furthermore, $\hat{\theta}$ is independent of the signal σ.

The proof of Lemma 1 can be found in the Appendix. To see why the type of the indifferent consumer, $\hat{\theta}$, is invariant to the expected quality difference, define prices normalized by the expected quality gap, $p_i' = p_i/\Delta V_{i,j}(\sigma)$ and $p_j' = p_j/\Delta V_{i,j}(\sigma)$, as the new strategic variables of the firms. Then, the profit functions of firms i and j can be rewritten as $\Delta V_{i,j}(\sigma)(1 - F(p_i' - p_j'))p_i'$ and $\Delta V_{i,j}(\sigma)F(p_i' - p_j')p_j'$, respectively. Since $\Delta V_{i,j}(\sigma)$ enters the profit functions only as a multiplicative constant, the equilibrium normalized prices p_i' and p_j' do not depend on the expected quality gap. Therefore, $\hat{\theta} = p_i' - p_j'$ is independent of $\Delta V_{i,j}(\sigma)$ in equilibrium, and consequently also independent of the signal σ.

Given that the equilibrium normalized prices do not depend on the expected quality difference, the equilibrium prices, $p_i^*(\sigma)$ and $p_j^*(\sigma)$, increase linearly in $\Delta V_{i,j}(\sigma)$. The same applies to the equilibrium profits, which are given by:

$$\pi_i^*(\sigma) \equiv \frac{(1 - F(\hat{\theta}))^2}{f(\hat{\theta})} \Delta V_{i,j}(\sigma) \quad \text{and} \quad \pi_j^*(\sigma) \equiv \frac{F^2(\hat{\theta})}{f(\hat{\theta})} \Delta V_{i,j}(\sigma).$$

Intuitively, firms benefit from the reduced competitive pressure associated with increased quality differentiation. Indeed, the demand of the firm with higher expected quality becomes less elastic.[5] Furthermore, as price competition under vertical differentiation features strategic complementarity, the price increase of firm i triggers a strategic response of firm j to raise its price as well. Thus, even the firm with lower expected quality benefits from an increasing expected quality difference, $\Delta V_{i,j}(\sigma)$.

The resulting total industry profit is given by

$$\Pi(\sigma) \equiv \pi_1^*(\sigma) + \pi_2^*(\sigma) = \Psi|\Delta V_{1,2}(\sigma)|, \tag{5}$$

[5]The absolute value of the price elasticity of demand of firm i is $|\varepsilon_i| = \frac{f\left(\frac{p_i - p_j}{\Delta V_{i,j}(\sigma)}\right)}{1 - F\left(\frac{p_i - p_j}{\Delta V_{i,j}(\sigma)}\right)} \frac{p_i}{\Delta V_{i,j}(\sigma)}$. Both components are positive for the firm perceived to be of higher quality. As the hazard rate function, $f/(1-F)$, increases in its argument (and thus decreases in $\Delta V_{i,j}(\sigma)$), while the factor $p_i/V_{i,j}(\sigma)$ decreases as well, the absolute value of the elasticity decreases in $\Delta V_{i,j}(\sigma)$.

where

$$\Psi \equiv \frac{(1-F(\hat{\theta}))^2 + F^2(\hat{\theta})}{f(\hat{\theta})}. \tag{6}$$

Consumer surplus for a given signal σ can be written as follows:

$$CS(\sigma) \equiv \overline{T}\mathbb{E}[v_i|\sigma] + \underline{T}\mathbb{E}[v_j|\sigma] - \Pi(\sigma), \tag{7}$$

where

$$\overline{T} \equiv \int_{\hat{\theta}}^{\overline{\theta}} \theta dF(\theta) \quad \text{and} \quad \underline{T} \equiv \int_{\underline{\theta}}^{\hat{\theta}} \theta dF(\theta).$$

Let us define the ex-ante total industry profit and the ex-ante consumer surplus for a given experiment $\mathcal{E}$ as

$$\Pi^{\mathcal{E}} \equiv \sum_{\omega\in\Omega}\sum_{\sigma\in\Sigma} \lambda_\omega s(\sigma|\omega)\Pi(\sigma) \quad \text{and} \quad CS^{\mathcal{E}} \equiv \sum_{\omega\in\Omega}\sum_{\sigma\in\Sigma} \lambda_\omega s(\sigma|\omega)CS(\sigma).$$

In the following section, we provide separate characterizations of the experiments that maximize ex-ante total industry profit, $\Pi^{\mathcal{E}}$, and ex-ante consumer surplus, $CS^{\mathcal{E}}$.

3.2 Optimal Information Structures

Towards characterizing which information structures are most beneficial for firms and consumers, respectively, we introduce two classes of experiments, namely *rank-revealing* and *rank-preserving* ones.

A *rank-revealing* experiment does not pool states in which firm qualities differ in their order; that is, it does not pool states in which $v_i > v_j$ with states in which $v_i < v_j$. Let $\mathbb{S}$ be the set of all rank-revealing experiments. Formally, we define rank-revealing experiments as follows.

Definition 1 (Rank-revealing signals and experiments)**.** *A signal σ derived from an experiment $\mathcal{E} = \{s(\cdot|\omega)\}_{\omega\in\Omega}$ is rank-revealing, that is, $\mathcal{E} \in \mathbb{S}$, if and only if for every $k > \ell$, we have that $s\left(\sigma|\left(v^k, v^\ell\right)\right) > 0$ implies that $s\left(\sigma|\left(v^h, v^m\right)\right) = 0$ for every $h < m$. An experiment $\mathcal{E}$ is rank-revealing if and only if all signals σ sent with positive probability are rank-revealing.*

Note that the fully informative experiment belongs to $\mathbb{S}$, implying that $\mathbb{S}$ is non-empty. Next, we introduce experiments that are *rank-preserving.* They never overturn the ranking of firms' prior expected qualities. Put differently, if firm i is ex ante perceived to be of strictly higher expected quality than firm j, there cannot be a signal realization $\sigma \in \Sigma$ such that $\mathbb{E}[v_i|\sigma] < \mathbb{E}[v_j|\sigma]$. Let $\mathbb{H}$ be the set of all rank-preserving experiments. Formally, we define rank-preserving experiments as follows.

Definition 2 (Rank-preserving experiments)**.** *An experiment* $\mathcal{E} = \{s(\cdot|\omega)\}_{\omega\in\Omega}$ *is rank-preserving, that is,* $\mathcal{E} \in \mathbb{H}$*, if and only if* $\mathbb{E}[v_i] > \mathbb{E}[v_j]$ *implies that* $\mathbb{E}[v_i|\sigma] \geq \mathbb{E}[v_j|\sigma]$ *for every signal* $\sigma \in \Sigma$*.*

Note that the uninformative experiment belongs to $\mathbb{H}$ by construction, implying that $\mathbb{H}$ is non-empty as well.

Industry profit-maximizing information design. For any rank-revealing experiment, we have by construction that, for all σ,

$$|\Delta V_{1,2}(\sigma)| = |\mathbb{E}[v_1 - v_2|\sigma]| = \mathbb{E}[|v_1 - v_2|\,|\sigma]. \tag{8}$$

This implies that the expected industry profit under a rank-revealing experiment $\mathcal{E} \in \mathbb{S}$ is given by:

$$\begin{aligned}
\Pi^{\mathcal{E}} &= \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma\in\Sigma} s(\sigma|\omega)\Psi|\Delta V_{1,2}(\sigma)| && (9)\\
&= \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma\in\Sigma} s(\sigma|\omega)\Psi\mathbb{E}\left[|v_1 - v_2|\,|\sigma\right] && (10)\\
&= \Psi\mathbb{E}\left|v_1 - v_2\right|, && (11)
\end{aligned}$$

where the last step follows from the law of iterated expectations.

We will next establish that any non-rank-revealing experiment attains lower industry profits. Towards this, consider any experiment $\mathcal{E}' \notin \mathbb{S}$, implying that there exists a signal σ' that does not fully reveal firms' quality ranking.[6] Applying Jensen's inequality, we obtain

$$|\Delta V_{1,2}(\sigma')| = |\mathbb{E}[v_1 - v_2|\sigma']| < \mathbb{E}[|v_1 - v_2|\,|\sigma'], \tag{12}$$

where the strict inequality stems from the fact that there are at least two nonzero terms with opposing signs in the summation over states. Note that (12) holds for any signal σ' that is non-revealing, while any revealing signal σ satisfies (8). Given the existence of at least one non-revealing signal realization, we obtain that

$$\begin{aligned}
\Pi^{\mathcal{E}'} &= \Psi \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma'\in\Sigma'} s(\sigma|\omega')|\Delta V_{1,2}(\sigma')| && (13)\\
&< \Psi \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma'\in\Sigma'} s(\sigma|\omega')\mathbb{E}\left[|v_1 - v_2|\,|\sigma'\right] && (14)\\
&= \Psi\mathbb{E}\left|v_1 - v_2\right| = \Pi^{\mathcal{E}}, && (15)
\end{aligned}$$

[6] Formally, this implies the existence of k', ℓ', h', m', such that $s'\left(\sigma'|\left(v^{k'}, v^{\ell'}\right)\right) > 0$ and $s'\left(\sigma'|\left(v^{h'}, v^{m'}\right)\right) > 0$ hold simultaneously with $v^{k'} > v^{\ell'}$ and $v^{h'} < v^{m'}$.

which holds for any $\mathcal{E} \in \mathbb{S}$. Given the non-emptiness of $\mathbb{S}$, we can conclude the following result about industry-profit maximizing information structures.

Proposition 1. *An experiment $\mathcal{E}$ maximizes expected industry profits if and only if it is rank-revealing, i.e., $\mathcal{E} \in \mathbb{S}$.*

Consumer surplus-maximizing information design. Consider a signal σ and suppose that $\Delta V_{i,j}(\sigma) \geq 0$. Then, consumer surplus given in equation (7) can be rewritten as follows:

$$CS(\sigma) = \frac{\overline{T} + \underline{T}}{2} \left(\mathbb{E}[v_1|\sigma] + \mathbb{E}[v_2|\sigma]\right) + \frac{\overline{T} - \underline{T}}{2} \left|\mathbb{E}[v_1|\sigma] - \mathbb{E}[v_2|\sigma]\right| - \Psi|\Delta V_{1,2}(\sigma)| \tag{16}$$

$$= \frac{\overline{T} + \underline{T}}{2} \mathbb{E}[v_1 + v_2|\sigma] + \left(\frac{\overline{T} - \underline{T}}{2}\right) |\Delta V_{1,2}(\sigma)| - \Psi|\Delta V_{1,2}(\sigma)| \tag{17}$$

This expression allows us to intuitively disentangle the components affecting consumer surplus. The first term represents the total expected gross consumer surplus if consumers were randomly allocated across the two products in a uniform fashion. The second term represents the change in gross surplus—relative to the random allocation—due to an allocation based on the difference in expected qualities. Consumers with a high willingness-to-pay for quality (represented by $\overline{T} = (1 - F(\hat{\theta}))\mathbb{E}[\theta|\theta \geq \hat{\theta}]$) gain from more frequently purchasing the high-quality product, while consumers with a low willingness-to-pay for quality (represented by $\underline{T} = F(\hat{\theta})\mathbb{E}[\theta|\theta \leq \hat{\theta}]$) lose, as they purchase the high-quality good less frequently. Finally, the third term captures the total expenditures of consumers.

Based on this formulation of interim consumer surplus given σ, we can express ex-ante consumer surplus as

$$CS^{\mathcal{E}} = \frac{\overline{T} + \underline{T}}{2} \mathbb{E}[v_1 + v_2] + \left(\frac{\overline{T} - \underline{T}}{2\Psi} - 1\right) \Pi^{\mathcal{E}}. \tag{18}$$

Thus, the effect of information on consumer surplus is directly related to how information affects total industry profits. Whether consumers' incentives regarding information are aligned with, or are opposed to firms' incentives depends on whether the sign of the factor multiplying profits in (18) is positive or negative.

As $\Psi > 0$, this sign is equivalent to that of $\frac{\overline{T} - \underline{T}}{2} - \Psi$, which captures the marginal effects of increased perceived quality differentiation in (17). Specifically, $\frac{\overline{T} - \underline{T}}{2}$ is the effect on consumers' gross surplus resulting from a marginal change in expected quality difference, while Ψ is the effect on industry profits—here equivalent to consumers' total expenditures—resulting from the same marginal change.

To see this, note that we can rewrite

$$\frac{\overline{T}-\underline{T}}{2}-\Psi=(1-F(\hat{\theta}))\left(\frac{\mathbb{E}[\theta|\theta\geq\hat{\theta}]}{2}-\frac{1-F(\hat{\theta})}{f(\hat{\theta})}\right) \tag{19}$$

$$-F(\hat{\theta})\left(\frac{\mathbb{E}[\theta|\theta\leq\hat{\theta}]}{2}+\frac{F(\hat{\theta})}{f(\hat{\theta})}\right). \tag{20}$$

In (19), $(1-F(\hat{\theta}))$ is the mass of consumers purchasing the product perceived to be of higher quality. Because $\hat{\theta}$ remains constant following an increase in perceived quality differentiation, the shares of consumers allocated to firms are not impacted by any increase in this. Relative to the baseline surplus from the uniformly random allocation in the first component of (17), the gain in gross consumer surplus is thus one half—the share of consumers re-allocated relative to the random allocation—times the expected willingness-to-pay for quality of the consumers purchasing the product with higher perceived quality, $\mathbb{E}[\theta|\theta\geq\hat{\theta}]$. However, all consumers who purchase the high-quality product suffer the higher price, which increases by $\frac{1-F(\hat{\theta})}{f(\hat{\theta})}$ times the increase in differentiation.

Analogously, in (20), $F(\hat{\theta})$ is the mass of consumers purchasing the product perceived to be of lower quality, while $\mathbb{E}[\theta|\theta\leq\hat{\theta}]$ is the loss in gross consumer surplus for those consumers reallocated relative to the random allocation, and $\frac{F(\hat{\theta})}{f(\hat{\theta})}$ is the price increase which affects all consumers purchasing the product perceived to be of lower quality.

The following lemma shows that, following increased quality differentiation, the firms can always extract strictly more than the consumers can gain in terms of gross surplus. This implies that consumers' and firms' incentives regarding information provision are opposed.

Lemma 2. *It holds that* $\left(\overline{T}-\underline{T}\right)/2<\Psi$*.*

It directly follows from (18) and Lemma 2 that an experiment maximizes consumer surplus if and only if it minimizes industry profits. We next establish that any rank-preserving experiment minimizes industry profits. Towards this, consider a rank-preserving experiment $\mathcal{E}$. As the order of posteriors never changes, all $\Delta V_{1,2}(\sigma)$ have the same sign. It therefore holds for the expected industry profit that

$$\Pi^{\mathcal{E}}=\Psi\sum_{\omega\in\Omega}\lambda_\omega\sum_{\sigma\in\Sigma}s(\sigma|\omega)\left|\Delta V_{1,2}(\sigma)\right| \tag{21}$$

$$=\Psi\left|\sum_{\omega\in\Omega}\lambda_\omega\sum_{\sigma\in\Sigma}s(\sigma|\omega)\Delta V_{1,2}(\sigma)\right| \tag{22}$$

$$=\Psi\left|\mathbb{E}[v_1-v_2]\right|. \tag{23}$$

But for any experiment $\mathcal{E}'\notin\mathbb{H}$, there exists a signal σ' that changes the expected quality ranking. By Jensen's inequality, we therefore have that

$$\Pi^{\mathcal{E}'} = \Psi \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma'\in\Sigma'} s(\sigma|\omega') \left|\Delta V_{1,2}(\sigma')\right| \tag{24}$$

$$= \Psi \sum_{\omega\in\Omega} \lambda_\omega \sum_{\sigma'\in\Sigma'} s(\sigma|\omega') \left|\mathbb{E}[v_1 - v_2|\sigma']\right| \tag{25}$$

$$> \Psi \left|\mathbb{E}[v_1 - v_2]\right| \tag{26}$$

$$= \Pi^{\mathcal{E}}, \tag{27}$$

where $\mathcal{E} \in \mathbb{H}$. As rank-preserving experiments minimize expected industry profits, they maximize expected consumer surplus. We summarize this in the following proposition.

Proposition 2. *An experiment $\mathcal{E}$ maximizes expected consumer surplus if and only if it is rank-preserving, i.e., $\mathcal{E} \in \mathbb{H}$.*

4 Allowing for an Extensive Margin

At the outset, one might expect that our results are weakened if not nullified by allowing for an extensive margin: it limits the extractive power of the firms, by having them compete against an elastic outside option rather than only against each other. In this section, we show that this is *not* the case. To keep the analysis tractable and focused, we restrict attention to perfectly negatively correlated qualities and uniformly distributed consumer types.

Let $\mathcal{V} = \{v^l, v^h\}$ with $v^l < v^h$ and $\Omega = \{(v^l, v^h), (v^h, v^l)\}$. Denote by λ the probability that $\omega = (v^h, v^l)$, i.e., that firm 1 is of high quality. Further, we assume that $\theta \sim U : [0, 1]$. The value of the outside option is normalized to zero, i.e., $\underline{v} = 0$.

Within this framework, the information-design problem is characterized by choosing a distribution over a single posterior belief μ—the belief that firm one is the high-quality firm—subject to Bayes plausibility. Thus, given a posterior belief μ, consumers expect firm 1 (2) to be of higher quality if $\mu > (<)1/2$.

For any belief μ, an equilibrium in the pricing game exists and is unique (see Benassi et al., 2019, for a general treatment of the pricing game). The following lemma characterizes the unique equilibrium for a given belief μ.

Lemma 3. *For any belief $\mu \in [0, 1]$, the unique equilibrium has prices*

$$p_1^*(\mu) = \begin{cases} 2(v^h - v^l)(2\mu - 1)\frac{\mu v^h + (1-\mu)v^l}{(5\mu-1)v^h + (4-5\mu)v^l}, & \text{if } \mu \geq 1/2 \\ (v^h - v^l)(1 - 2\mu)\frac{(1-\mu)v^h + \mu v^l}{(4-5\mu)v^h + (5\mu-1)v^l}, & \text{if } \mu < 1/2 \end{cases} \tag{28}$$

$$p_2^*(\mu) = \begin{cases} (v^h - v^l)(2\mu - 1)\frac{(1-\mu)v^h + \mu v^l}{(5\mu-1)v^h + (4-5\mu)v^l}, & \text{if } \mu \geq 1/2 \\ 2(v^h - v^l)(1 - 2\mu)\frac{\mu v^h + (1-\mu)v^l}{(4-5\mu)v^h + (5\mu-1)v^l}, & \text{if } \mu < 1/2. \end{cases} \tag{29}$$

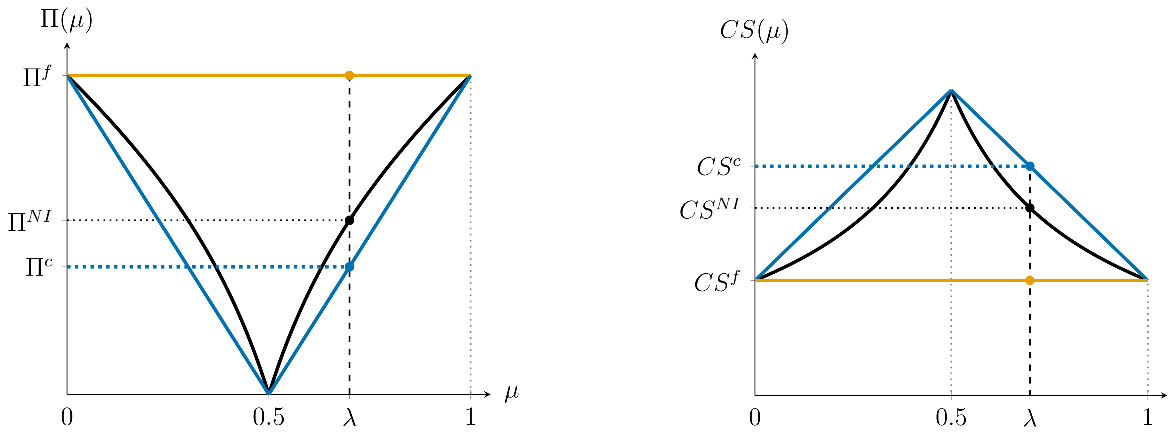

Figure 1: Industry profits and consumer surplus with an extensive margin.

The left panel depicts the industry profits $\Pi(\mu)$ in black. The orange line depicts the concavification of $\Pi(\mu)$. The blue line corresponds to the convexification of $\Pi(\mu)$. The right panel depicts the consumer surplus $CS(\mu)$. The blue line depicts the concavification $CS(\mu)$. The orange line depicts the convexification of $CS(\mu)$.
The graphs highlight three policies and their corresponding payoffs. (i) The orange points are associated with the industry-profit maximizing, fully informative policy (inducing Π^f, CS^f); (ii) the blue points are associated with the consumer-optimal maximally-informative, rank-preserving policy (inducing Π^c, CS^c); and (iii) the black points are associated with no information revelation (inducing Π^{NI}, CS^{NI}).

Industry Profits Consider the total industry profits conditional on a posterior μ as $\Pi(\mu) = \pi_1^*(\mu) + \pi_2^*(\mu)$, where $\pi_i^*(\mu)$ are the profits induced by the equilibrium pricing strategies defined in Lemma 3. We derive the explicit expression for the corresponding industry profits in Appendix C. Here, we only observe that the industry profits are necessarily symmetric around $\mu = 1/2$ (as the model is symmetric in the firm identity). Moreover, industry profits are strictly increasing towards extreme beliefs—that is, strictly decreasing in μ for $\mu \in [0, 1/2)$ and strictly increasing in μ for $\mu \in (1/2, 1]$. Thus, the industry profit function has two (global) maxima at $\mu = 0$ and $\mu = 1$. It follows immediately that industry profits are maximized under full information revelation.

Proposition 3. *The perfectly informative experiment, which is the only rank-revealing experiment, uniquely maximizes industry profits when there is an extensive margin.*

As in the previous section, in which market demand was completely inelastic so that all consumers purchased, industry profits were maximized for rank-revealing information structures. In the specific case with perfectly negatively correlated binary qualities, the only rank-revealing information structure is full information revelation. The intuition follows again from the differentiation incentive of the firms. The more precisely informed the consumers are, the more differentiated they perceive the firms' products, and hence, the less competitive the market becomes.

Consumer Surplus As before, we can express consumer surplus as being composed of three components: the gross surplus of consumers purchasing the perceived low-quality product, the gross surplus of consumers purchasing the perceived high-quality product, and the industry profits, which need to be subtracted from the first two components.

Building on the equilibrium prices in Lemma 3, we can obtain the consumer surplus function, which also has to be symmetric around $\mu = 1/2$. We provide the expression in Appendix C. We note here that consumer surplus strictly decreases as the belief approaches the extremes. Thus, it has a global maximum at $\mu = 1/2$ and two global minima at $\mu = 0$ and $\mu = 1$. It follows that consumer surplus is minimized under the fully informative structure that maximizes industry profits. It turns out that the consumer surplus function is strictly convex in the belief within each subinterval $\mu \in [0, 1/2)$ and $\mu \in (1/2, 1]$. Therefore, the consumer-optimal information structure is such that it is (i) rank-preserving, and (ii) the most informative rank-preserving information structure.

If firm 1 is the a priori higher-quality firm (i.e., if $\lambda > 1/2$), this implies that consumers never believe that firm 2 is the higher-quality firm after any signal realization, and that the consumer-optimal experiment reveals firm 1 to be the high-quality firm sometimes and otherwise makes consumers indifferent between the two firms.

Proposition 4. *The unique consumer-optimal information structure is the most informative rank-preserving information structure when there is an extensive margin.*

There are two forces behind this result. First, as in the preceding analysis without the extensive margin, consumers benefit from not learning the ranking of the firms due to more intense price competition. Second, the consumers nevertheless benefit from some information revelation, as it affects participation in the market. Without an extensive margin, all consumers purchase under any rank-preserving information structure. With an intensive margin, however, the purchasing decision becomes relevant, and with that marginal information. Hence, among those information structures that lead to intense competition, more information benefits consumers by encouraging efficient selection of consumers into purchasing. Indeed, the existence of an outside option limits the rent-extraction ability of firms when their market power increases due to differentiation. The firms now have to balance inframarginal gains from higher prices against marginal losses due to consumers' non-participation. This new effect limits the losses in consumer surplus as firms become more differentiated, causing the consumer surplus function to become convex. This convexity, in turn, introduces the marginal benefit of information for consumers by limiting losses after a firm is perfectly revealed to be of high quality, but retaining the benefits of fierce competition after the uninformative signal is observed.

Note that due to the strict local concavity of industry profits in the rank-preserving regions, the consumer-optimal information structure minimizes industry profits. Similarly, that consumer surplus is decreasing toward extreme beliefs implies that the profit-maximizing, perfectly revealing information structure minimizes consumer surplus. Thus, we retain perfectly opposing incentives regarding information between firms and consumers in the model with an extensive margin.

5 Concluding Remarks

We analyze a model in which duopolistic sellers offer vertically differentiated products to heterogeneous buyers. A third party provides public information about firms' product qualities. This information helps consumers make better decisions about which product to purchase, but also allows firms to adjust their prices to updated consumer beliefs. The latter is particularly important, as information may relax competition by increasing perceived product differentiation. We establish that the primary criterion determining the welfare consequences of information structures is their content regarding product rankings, rather than their informational content regarding quality levels or quality differences.

Our main finding is that buyer-optimal and seller-optimal information structures are opposite to each other. Sellers prefer the quality ranking of their products to be disclosed, while buyers prefer the prior ranking to be preserved even if it is ex post incorrect. These results obtain in a fully covered market and when an extensive margin is considered. In the latter case, the consumer-optimal information structure is such that it is maximally informative subject to preserving the prior ranking.

Our results have implications for the design of third-party public information, such as search rankings and ratings on platform markets, or information disseminated by consumer organizations, certifiers, or regulators. In all, they raise questions about consumer-induced certification of product qualities or rankings.

Appendix

A Proof of Lemma 1

Assume for a contradiction that there is no indifferent consumer in equilibrium. Then, a firm serving the entire market can slightly increase its price and achieve greater profits, leading to a contradiction. Therefore, in any equilibrium, there exists an indifferent consumer.

Suppose that firms set prices p_i and p_j and there is an indifferent consumer $\hat{\theta} = (p_i - p_j)/\Delta V_{i,j}(\sigma) \in [\underline{\theta}, \overline{\theta}]$. In the main text we established that the profit functions given in equations (3) and (4) are quasi-concave and therefore it is sufficient to consider the system of the first-order conditions given by

$$0 = 1 - F(\hat{\theta}) - \frac{f(\hat{\theta})p_i}{\Delta V_{i,j}(\sigma)}$$
$$0 = F(\hat{\theta}) - \frac{f(\hat{\theta})p_j}{\Delta V_{i,j}(\sigma)}.$$

Solving the system with respect to p_i and p_j, we obtain

$$p_i = \frac{1 - F(\hat{\theta})}{f(\hat{\theta})}\Delta V_{i,j}(\sigma) \quad \text{and} \quad p_j = \frac{F(\hat{\theta})}{f(\hat{\theta})}\Delta V_{i,j}(\sigma).$$

Plugging the expressions for p_i and p_j into $\hat{\theta} = (p_i - p_j)/\Delta V_{i,j}(\sigma)$, we find that the type of the indifferent consumer, $\hat{\theta}$, satisfies the following equation

$$\hat{\theta} - \frac{1 - 2F(\hat{\theta})}{f(\hat{\theta})} = 0. \tag{30}$$

As f is a log-concave function, $(1 - F)/f$ is a non-increasing function and F/f is a non-decreasing function. Thus, the expression in equation (30) is non-decreasing in $\hat{\theta}$. Since $\underline{\theta} < 1/f(\underline{\theta})$, the intermediate value theorem guarantees the existence of a unique $\hat{\theta} \in (\underline{\theta}, \overline{\theta})$ that solves equation (30).

B Proof of Lemma 2

Plugging in the expressions for $\overline{T}, \underline{T}, \Psi$, we have:

$$\overline{T} - \underline{T} - 2\Psi = \int_{\hat{\theta}}^{\overline{\theta}} \theta dF(\theta) - \int_{\underline{\theta}}^{\hat{\theta}} \theta dF(\theta) - 2\frac{(1 - F(\hat{\theta}))^2 + F^2(\hat{\theta})}{f(\hat{\theta})}. \tag{31}$$

Integrating by parts the first two integrals, we obtain:

$$\overline{T} - \underline{T} - 2\Psi = \hat{\theta} - \hat{\theta}F(\hat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} (1 - F(\theta))d\theta$$
$$- \left(\hat{\theta}F(\hat{\theta}) - \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta \right) - 2\frac{(1 - F(\hat{\theta}))^2 + F^2(\hat{\theta})}{f(\hat{\theta})} \tag{32}$$
$$= \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} (1 - F(\theta))d\theta + \hat{\theta}(1 - 2F(\hat{\theta})) - 2\frac{(1 - F(\hat{\theta}))^2 + F^2(\hat{\theta})}{f(\hat{\theta})}. \tag{33}$$

Plugging in $\hat{\theta} = (1 - 2F(\hat{\theta}))/f(\hat{\theta})$ into the third term, we obtain:

$$\overline{T} - \underline{T} - 2\Psi = \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} (1 - F(\theta))d\theta + \frac{(1 - 2F(\hat{\theta}))^2}{f(\hat{\theta})} - 2\frac{(1 - F(\hat{\theta}))^2 + F^2(\hat{\theta})}{f(\hat{\theta})} \tag{34}$$
$$= \int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} (1 - F(\theta))d\theta - \frac{1}{f(\hat{\theta})}. \tag{35}$$

By Theorem 1 and Theorem 3 of Bagnoli and Bergstrom (2005), log-concavity of the density function $f(\theta)$ implies that the functions $G(\theta) \equiv \int_{\underline{\theta}}^{\theta} F(t)dt$ and $H(\theta) \equiv \int_{\underline{\theta}}^{\theta}(1 - F(t))dt$ are also log-concave on $[\underline{\theta}, \overline{\theta}]$. By the definition of log-concavity, we have that $GG'' - (G')^2 \leq 0$ and $HH'' - (H')^2 \leq 0$, implying that

$$\int_{\underline{\theta}}^{\hat{\theta}} F(\theta)d\theta \leq \frac{F^2(\hat{\theta})}{f(\hat{\theta})}, \tag{36}$$
$$\int_{\hat{\theta}}^{\overline{\theta}} (1 - F(\theta))d\theta \leq \frac{(1 - F(\hat{\theta}))^2}{f(\hat{\theta})}. \tag{37}$$

Applying these inequalities, we finally have that

$$\overline{T} - \underline{T} - 2\Psi \leq \frac{1}{f(\hat{\theta})} \Big((1 - F(\hat{\theta}))^2 + F^2(\hat{\theta}) - 1 \Big) \tag{38}$$
$$= -\frac{2}{f(\hat{\theta})}(1 - F(\hat{\theta}))F(\hat{\theta}) \tag{39}$$
$$< 0, \tag{40}$$

where we used the fact that $\theta \in (\underline{\theta}, \overline{\theta})$ to obtain the last strict inequality.

C Proof of Lemma 3

Suppose that $\mu > 1/2$. Then, firm 1 is the high-quality firm in expectation. With an extensive margin, there will be two marginal consumers, $\hat{\theta}_h$ and $\hat{\theta}_l$, both in $[0,1]$. Existence and uniqueness of equilibrium follow from Benassi et al. (2019). We conjecture that the equilibrium is such that $\hat{\theta}_h > \hat{\theta}_l \geq 0$, and verify it in the following. The marginal consumers define the firms' demand functions:

$$q_1(\hat{\theta}_h) = 1 - \hat{\theta}_h \tag{41}$$

$$q_2(\hat{\theta}_h, \hat{\theta}_l) = \hat{\theta}_h - \hat{\theta}_l. \tag{42}$$

Further, the marginal consumers are implicitly defined by the indifference conditions

$$\hat{\theta}_h(\mu v^h + (1-\mu)v^l) - p_1 = \hat{\theta}_h((1-\mu)v^h + \mu v^l) - p_2 \tag{43}$$

$$\hat{\theta}_l((1-\mu)v^h + \mu v^l) - p_2 = 0. \tag{44}$$

From here, we obtain the explicit expressions for the marginal consumers

$$\hat{\theta}_h = \frac{p_1 - p_2}{(2\mu - 1)(v^h - v^l)} \tag{45}$$

$$\hat{\theta}_l = \frac{p_2}{(1-\mu)v^h + \mu v^l} \tag{46}$$

and the corresponding profit functions

$$\pi_1(p_1, p_2; \mu) = \left(1 - \frac{p_1 - p_2}{(2\mu - 1)(v^h - v^l)}\right) p_1 \tag{47}$$

$$\pi_2(p_1, p_2; \mu) = \left(\frac{p_1 - p_2}{(2\mu - 1)(v^h - v^l)} - \frac{p_2}{(1-\mu)v^h + \mu v^l}\right) p_2. \tag{48}$$

The profit functions π_i are strictly concave in their own price p_i for any p_{-i}. Hence, first-order conditions are sufficient to find the unique best reply functions:

$$p_1^{BR}(p_2) = \frac{p_2 + (2\mu - 1)(v^h - v^l)}{2} \tag{49}$$

$$p_2^{BR}(p_1) = p_1 \frac{(1-\mu)v^h + \mu v^l}{2(\mu v^h + (1-\mu)v^l)}. \tag{50}$$

Solving this system of equations yields the prices stated in Lemma 3. By equilibrium uniqueness, the analogous argument for $\mu < 1/2$, and the standard Bertrand price competition logic for $\mu = 1/2$, the characterization follows.

For later reference, we obtain as equilibrium profits:

$$\pi_1^*(\mu) = \begin{cases} 4(2\mu-1)(v^h-v^l)\left(\frac{\mu v^h+(1-\mu)v^l}{(5\mu-1)v^h+(4-5\mu)v^l}\right)^2, & \text{if } \mu \geq 1/2 \\ (1-2\mu)(v^h-v^l)\frac{(\mu v^h+(1-\mu)v^l)((1-\mu)v^h+\mu v^l)}{\left((4-5\mu)v^h+(5\mu-1)v^l\right)^2}, & \text{if } \mu < 1/2 \end{cases}$$

$$\pi_2^*(\mu) = \begin{cases} (2\mu-1)(v^h-v^l)\frac{(\mu v^h+(1-\mu)v^l)((1-\mu)v^h+\mu v^l)}{\left((5\mu-1)v^h+(4-5\mu)v^l\right)^2}, & \text{if } \mu \geq 1/2 \\ 4(1-2\mu)(v^h-v^l)\left(\frac{(1-\mu)v^h+\mu v^l}{(4-5\mu)v^h+(5\mu-1)v^l}\right)^2, & \text{if } \mu < 1/2 \end{cases}$$

$$\Pi(\mu) = \begin{cases} (v^h-v^l)(2\mu-1)(\mu v^h+(1-\mu)v^l)\frac{v^h+v^l+3(\mu v^h+(1-\mu)v^l))}{\left((1-5\mu)v^h+(5\mu-4)v^l\right)^2}, & \text{if } \mu \geq 1/2 \\ (v^h-v^l)(1-2\mu)((1-\mu)v^h+\mu v^l)\frac{v^h+v^l+3((1-\mu)v^h+\mu v^l))}{\left((5\mu-4)v^h+(1-5\mu)v^l\right)^2}, & \text{if } \mu < 1/2 \end{cases} \tag{51}$$

For consumer surplus, we obtain in equilibrium

$$\begin{aligned} CS(\mu) = &\begin{cases} (1-\hat{\theta}_h)\frac{1+\hat{\theta}_h}{2}\left(\mu v^h+(1-\mu)v^l\right)+(\hat{\theta}_h-\hat{\theta}_l)\frac{\hat{\theta}_h+\hat{\theta}_l}{2}\left(\mu v^l+(1-\mu)v^h\right), & \text{if } \mu \geq 1/2 \\ (1-\hat{\theta}_h)\frac{1+\hat{\theta}_h}{2}\left((1-\mu)v^h+\mu v^l\right)+(\hat{\theta}_h-\hat{\theta}_l)\frac{\hat{\theta}_h+\hat{\theta}_l}{2}\left((1-\mu)v^l+\mu v^h\right), & \text{if } \mu < 1/2 \end{cases} \\ &- \Pi(\mu) \\ = &\begin{cases} \frac{(\mu v^h+(1-\mu)v^l)^2((5-\mu)v^h+(4+\mu)v^l)}{2\left(v^h(1-5\mu)+(5\mu-4)v^l\right)^2}, & \text{if } \mu \geq 1/2 \\ \frac{((1-\mu)v^h+\mu v^l)^2((4+\mu)v^h+(5-\mu)v^l)}{2\left(v^h(5\mu-4)+(1-5\mu)v^l\right)^2}, & \text{if } \mu < 1/2. \end{cases} \end{aligned} \tag{52}$$

D Proof of Proposition 3

We first show that industry profits are increasing and strictly concave in μ for $\mu \in (1/2, 1]$. Recall that the industry profits in this case are (see (51))

$$\Pi(\mu) = (v^h-v^l)(2\mu-1)(\mu v^h+(1-\mu)v^l)\frac{v^h+v^l+3(\mu v^h+(1-\mu)v^l))}{((1-5\mu)v^h+(5\mu-4)v^l)^2}. \tag{53}$$

For our purposes, we can ignore the factor (v^h-v^l). For convenience, denote $r := v^l/v^h \in (0,1)$, $A(\mu) := r+(1-r)\mu$ and $d(\mu) := 1-4r-5(1-r)\mu$. Factoring out $(v^h)^2$ in the numerator and denominator, we obtain

$$\frac{\Pi(\mu)}{v^h-v^l} = \frac{(2\mu-1)(A(\mu))(1+r+3(A(\mu))))}{d(\mu)^2}. \tag{54}$$

Note that $d'(\mu) = -5(1-r) < 0$ as well as $d(\mu = 1/2) = -\frac{3}{2}(1+r) < 0$. Hence, the denominator is strictly positive and the function is smooth for all $\mu \in (1/2, 1]$.

Defining $B(\mu) := 1+4r+3(1-r)\mu$ and $C(\mu) := 1+7r+6(1-r)\mu$, straightforward computations deliver for the first and second derivatives

$$\left(\frac{\Pi(\mu)}{v^h - v^l}\right)' = \frac{2d(\mu)A(\mu)B(\mu) + (1-r)(2\mu - 1)(d(\mu)C(\mu) + 10A(\mu)B(\mu))}{d(\mu)^3} \tag{55}$$

$$\left(\frac{\Pi(\mu)}{v^h - v^l}\right)'' = -\frac{2(1-r)(1+r)^2\,(17(1-r)\mu + 11 + 28r)}{d(\mu)^4} < 0, \tag{56}$$

where the sign of the second derivative follows directly by noting that all terms in the numerator are strictly positive and that the denominator has an even power. Thus, industry profits are strictly concave in the belief μ for all $\mu \in (1/2, 1]$.

Given the concavity of industry profits, we only need to show that the first derivative at $\mu = 1$ is positive to conclude that industry profits are strictly increasing in μ for $\mu \in (1/2, 1]$.

Note the following evaluations

$$A(\mu = 1) = 1, \quad B(\mu = 1) = 4 + r, \quad C(\mu = 1) = 7 + r, \quad d(\mu = 1) = -4 + r,$$

and conclude that for $\mu = 1$ all terms in both the numerator and the denominator are finite and non-zero. Evaluating the overall expression at $\mu = 1$ then yields

$$\left.\left(\frac{\Pi(\mu)}{v^h - v^l}\right)'\right|_{\mu=1} = \frac{20 - r(1 - r(10 + r))}{(4-r)^3} > 0. \tag{57}$$

Hence, we can conclude due to concavity that industry profits are increasing in μ for $\mu \in (1/2, 1]$.

The analogous reasoning by symmetry yields that the industry profits are strictly decreasing and concave in μ for all $\mu \in [0, 1/2)$. A simple computation shows that the left- and right-limit at $\mu = 1/2$ coincide. Thus, $\Pi(\mu)$ is continuous on its entire domain, and we obtain $\Pi(\mu = 1/2) = 0$.

These properties imply that industry profits have to global maxima at $\mu = 0$ and $\mu = 1$ and a global minimum at $\mu = 1/2$. Thus, the concavification of industry profits is the horizontal line segment connecting $(0, \Pi(0))$ and $(1, \Pi(1))$. It follows from the standard concavification argument, as in e.g., Kamenica and Gentzkow (2011); Aumann et al. (1995) that the optimal policy is fully revealing and induces the posterior beliefs $\mu = 0$ and $\mu = 1$ (with probability $(1-\lambda)$ and λ, respectively).

E Proof of Proposition 4

We again derive properties of the consumer surplus function $CS(\mu)$ from (52) for the case of $\mu \in (1/2, 1]$ and the analogous properties for the case $\mu \in [0, 1/2)$ follow by symmetry. Recall $A(\mu) = r + (1-r)\mu$ and $d(\mu) = 1 - 4r - 5(1-r)\mu$ and let $E(\mu) := 5 + 4r - (1-r)\mu$.

Then, we obtain[7]

$$CS(\mu) = \frac{A(\mu)^2 E(\mu)}{2d(\mu)^2} v^h \tag{58}$$

$$CS'(\mu) = \frac{(1-r)A(\mu)\big[d(\mu)(2E(\mu) - A(\mu)) + 10A(\mu)E(\mu)\big]}{2d(\mu)^3} v^h \tag{59}$$

$$= \frac{(1-r)(\mu + r - \mu r)P(\mu, r)}{2\,d(\mu)^3} v^h < 0, \text{ where} \tag{60}$$

$$P(\mu, r) = 10(1 + r + \mu r(1-\mu)) + 5\mu^2(1 + r^2) + 7r(1 - \mu r) + 12r^2 - 3\mu > 0 \tag{61}$$

$$CS''(\mu) = \frac{(1-r)^2(1+r)^2\big[47(\mu(1-r) + r) + 5(1+r)\big]}{d(\mu)^4} v^h > 0, \tag{62}$$

where the sign of the inequality in the third line follows from $d(\mu) < 0$ and the odd power in the denominator.

Consumer surplus is continuous at $\mu = 1/2$, as the left- and right-limits at $\mu = 1/2$ coincide. Hence, it follows that consumer surplus is strictly increasing and strictly convex in μ for all $\mu \in [0, 1/2)$ and strictly decreasing and strictly convex in μ for all $\mu \in (1/2, 1]$. Further, consumer surplus attains a global maximum at $\mu = 1/2$ and two global minima at $\mu = 0$ and $\mu = 1/2$. Hence, its concavification is given by the two line segments from $(0, CS(0))$ to $(1/2, CS(1/2))$ and $(1/2, CS(1/2))$ to $(1, CS(1))$. Inspecting the concavification at $\mu = \lambda$ for $\lambda > 1/2$ pins down the maximal consumer surplus, which is supported by an information structure that induces the posterior $\mu = 1/2$ and $\mu = 1$ subject to Bayes' plausibility.[8] Hence, the optimal experiment is the most informative rank-preserving information structure. In particular, if firm 1 is of high quality, it is revealed as such with probability $\frac{2\lambda-1}{\lambda}$ and pooled with firm 2 with probability $\frac{1-\lambda}{\lambda}$. Firm 2 is never recommended as the higher-quality firm.

References

Albano, G. L. and A. Lizzeri (2001): "Strategic certification and provision of quality," *International economic review*, 42, 267–283.

Anderson, S. P. and R. Renault (2009): "Comparative advertising: disclosing horizontal match information," *The RAND Journal of Economics*, 40, 558–581.

Armstrong, M. and J. Zhou (2022): "Consumer information and the limits to competition," *American Economic Review*, 112, 534–577.

[7]Note that on their respective supports $\mu \in (1/2, 1]$, $d(\mu)$ is strictly negative, and that $A(\mu)$ and $E(\mu)$ are strictly positive, implying that consumer surplus is strictly positive and smooth on its support.

[8]The case $\lambda < 1/2$ is analogous.

Aumann, R. J., M. Maschler, and R. E. Stearns (1995): *Repeated games with incomplete information*, MIT press.

Bagnoli, M. and T. Bergstrom (2005): "Log-concave probability and its applications," *Economic theory*, 26, 445–469.

Benassi, C., A. Chirco, and C. Colombo (2019): "Vertical differentiation beyond the uniform distribution," *Journal of Economics*, 126, 221–248.

Bergemann, D. and A. Bonatti (2019): "Markets for information: An introduction," *Annual Review of Economics*, 11, 85–107.

Bergemann, D. and S. Morris (2019): "Information design: A unified perspective," *Journal of Economic Literature*, 57, 44–95.

Biglaiser, G., J. Gu, and F. Li (2025): "Information Acquisition and Product Differentiation Perception," *American Economic Journal: Microeconomics*, 17, 1–34.

Dranove, D. and G. Z. Jin (2010): "Quality disclosure and certification: Theory and practice," *Journal of economic literature*, 48, 935–963.

Hopenhayn, H. and M. Saeedi (2023): "Optimal information disclosure and market outcomes," *Theoretical Economics*, 18, 1317–1344.

Kamenica, E. (2019): "Bayesian persuasion and information design," *Annual Review of Economics*, 11, 249–272.

Kamenica, E. and M. Gentzkow (2011): "Bayesian persuasion," *American Economic Review*, 101, 2590–2615.

Lewis, T. R. and D. E. Sappington (1994): "Supplying information to facilitate price discrimination," *International Economic Review*, 309–327.

Roesler, A.-K. and B. Szentes (2017): "Buyer-optimal learning and monopoly pricing," *American Economic Review*, 107, 2072–2080.

Schlee, E. E. (1996): "The value of information about product quality," *The RAND Journal of Economics*, 803–815.

Zapechelnyuk, A. (2020): "Optimal quality certification," *American Economic Review: Insights*, 2, 161–176.