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Horizontally Differentiated Internal and External Labor Markets: An Exploration, Part 1

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Horizontally Differentiated Labor Markets: An Exploration, Part 1*

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Abstract

A worker's productivity is only imperfectly revealed when employed to perform a particular task: the match could be imperfect, and her productivity dependent on that of other workers in the team she is working in. We embed both elements of informational imperfection in combination, within a parsimonious parametric model that involves horizontally differentiated skills, as well as firms varying in size and structure of tasks from specialized to diversified. The typical worker's productivity is revealed only via that of her team, the firm. The team's productivity can be improved upon by frictionless re-matching within the internal labor market. Any misalignment between the distribution of skills and tasks can be removed only by external labor market actions constrained by informational frictions.

There are two parts. In part 1, we analyze internal market equilibrium, but assume for the external market that firms can realize their desired demand. In part 2, we realistically consider labor supply in the external market as furnished by separations, and structured by signals involving the typical worker's employment history.

We demonstrate generic opposite effects involving internal and external market activities. In the internal market, constrained efficient re-matching yields the average productivity of firms to *increase* in size and diversity of structure. Beyond that, the cumulative distributions of firms by productivity are stochastically dominant across firms by increasing degree of diversification.

While with the satisfaction of desired demand in the external market the firms improve in expected productivity, the strength of improvement *decreases* in firm size and diversity of structure. Productivity gains via re-matching in the internal market may be overturned by the losses due to imprecise matches of hires from the external market: The more internal re-matching leads to leftward skewness in the distribution of firms towards higher productivity, the less informative is the specification of desired demand (and, as we show in part 2: in the supply) in the external market.

JEL-classification: J21, J23, J62, L23.
Key words: Internal Labor Markets, Horizontal Differentiation of Labor

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1 Introduction

In the labor market, the productivity of a worker with given skills is often unknown before she is employed in a particular task, and only partially revealed to her or the firm after she has performed in that task for some time. Indeed, the worker's performance varies not only across tasks, but in the interaction with other workers within the firm. Sometimes, the productivity of a team can be isolated; but often, only the aggregate outcome of all matches within the firm is revealed via the firm's operational result. Experimentation via re-matching workers by skills to tasks may improve on that productivity, but is constrained by the current stock of hires.[1]

When active in the external labor market to relax that constraint, the firm may typically try to fill a vacated task by hiring a worker that optimally matches that task. By doing so, however, the above informational imperfections may hinder the maximization of the firm's productivity. Rather than pursuing *task-specific* optimality, the firm should anticipate productivity-increasing re-matching before the hire, i.e., pursue *firm-specific* optimality. To yield efficiency, the challenge in the firm's external labor market activity is then to minimize the mismatch between the *distribution of workers by skills*, and the *distribution of tasks* requiring particular skills.

On the labor demand side, the challenge is that the observed team, or firm productivity may be realized with different match patterns, hindering the precise inference of the worker skill in need. On the labor supply side, the challenge is to infer the precise skill of the typical worker from a noisy signal provided by her employment history. It should be an immediate consequence of intensified internal labor market activity resulting in improvements in the quality of matches, that, after a separation, the precision of information improves on both the demand for the vacated task, and the supply generated by the separation. That, in turn, should result in improvements in match quality due to external labor market activity. We show that this is *not* the case.

At any rate, we analyze these challenges within a stylized parametric model involving a purely horizontally differentiated labor market akin to the horizontally differentiated product market that is a central workhorse of industrial economics. Similarly to the context there, the key feature here is that all workers are equally productive if matched into the task which they perform best. But skills and tasks differ, resulting in match patterns, and thus productivity that systematically differs across heterogeneous firms.

Our labor market is decomposed into many firm-specific internal markets and one external market.[2] Firms are differentiated by size, i.e., number of tasks, and structure, i.e., composition of tasks, from specialized to differentiated. Firms draw workers from the external market in number corresponding to their number of tasks. To isolate the effects of internal labor market activity, we assume here that the first draw is from an undifferentiated pool of workers. After matched to the tasks specific to the firm, the workers' productivity is revealed only collectively

[1] A typical example: large consulting firms, such as McKinsey, tend not to hire for particular tasks, but to re-match new hires with different tasks in order to find out about optimal matches.

[2] Firm-internal markets could also be subdivided by teams of workers and corresponding tasks.

by the firm's productivity. The wage payment consequently reflects that aggregate productivity and cannot be individualized.Re-matching of workers to tasks is considered frictionless. The intensity of internal labor market activity varies with the diversity of tasks in the firm, however. While efficient, allocation decisions in the internal market are naturally constrained by the workers in the firm being limited to the current draw. This constraint is relaxed by resorting to the external market.

To isolate the effects of information that emerges from internal labor market activity on the external market, we assume that a randomly selected worker leaves each firm and joins the external market. Each firm then demands one worker in the external market to fill the vacated slot. Hiring from the external market is impaired by improved, but potentially still incomplete information about the skills required to maximize the firm's productivity, and about the skills offered by formerly employed workers that eventually constitute the dominant portion of supply available in the external market. To isolate the information effect at the demand side, we assume in this part of the paper that the typical firm's desired demand is perfectly satisfied.

In all, we use several assumptions that are not standard in the theoretical labor market literature. First, we consider indivisible agents: discrete firm sizes and compositions by tasks, together with exclusive matching: each task can be occupied by one worker only. Second, the match quality, or productivity of a worker in a particular task is only indirectly revealed via the productivity of the entire team –here the firm. Internal re-matching my locally maximize that productivity. The revelation of productivity at the firm rather than the individual level necessitates payment of firm-specific uniform wages. Third, we summarize releases and quits by looking at random separations. Fourth, we (temporarily) assume that the firms' desired demands are satisfied without constraints.

The use of the first assumption, while realistic, requires the employment of a parametric model. By focusing on three purely horizontally differentiated tasks and skills, our market is differentiated beyond a binomial distribution of match quality. This invites a critically much richer information environment than heretofore, but substantially complicates the analysis of labor market activities. At any rate, the exclusion of vertical differentiation allows us to isolate interesting effects of imperfect information involving the horizontally differentiated labor market.

With the second assumption, we emphasize that an individual worker's productivity, and thus her remuneration not only depends on her individual skills, but also on the match quality of her co-workers, her team –an externality that is often suppressed in the labor market literature. Furthermore, we ignore the possibility that information on individual productivity is generated within internal labor market activities. This reduced-form setup allows us to focus on the information generated by the constrained efficient match, a critical signal from the typical worker's employment history transferred to the external labor market.

Random separations –our third assumption highlighted here– proxy situations in which workers separate from their firm for exogenous reasons, e.g., retirement, or outmigration; furthermore, the realistic situation in which the revelation of quits vs. dismissals is often subject to strategic bias, thus uninformative. With our fourth assumption, we isolate the demand side effect of limited information on external labor market activity. Any constraints on the supply side would smudge this effect. In the second part of our paper, we will complement the demand side by the supply generated from the very separations considered here, including the information imperfections generic to the setup we are working in.

By our main result, the allocation effect generated by frictionless re-matching in the internal market and the resulting information effect work in *opposite directions*: While re-matching increases the firms' productivity, the resulting left-skewed distribution of the firms by productivity smudges the information created by improved matches, to the extent that the above productivity increases may be more than counteracted by lower productivity resulting from external market activity.

In more detail: in the temporary equilibrium after optimal re-matching in all internal labor markets, firm-specific productivity and thus wage payments averaged over all realized productivity levels increase in firm size at a decreasing rate; furthermore, at given size, average equilibrium wages increase with increasing diversification in the firm structure. Indeed, the distributions of firms by productivity can be strictly ordered by the 1st order stochastic dominance criterion.

By the nature of the uncertainty considered here and the inferences that can be drawn from the matching decisions, it remains unclear which pattern of matches by skills into tasks has generated the observed productivity. Thus, uncertainty remains on both sides of the market: on the demand side about the desired skill, and on the supply side on the skill offered by the typical worker; this even when characteristics of the most recent employment are revealed.

We show that nevertheless, the skill information provided to the external labor market improves on both the demand and the supply side. In contrast to what one might expect, however, an intensification of internal labor market activity that increases the aggregate productivity of matches does not increase, but decrease the quality of skill information provided from the internal to the external market. And this up to the point that the productivity advantage enjoyed from internal re-matching activity by firms with a diversified structure of tasks is more than compensated by the disadvantage generated by the lower quality of information.

The micro-detail we derive is revealing here: while intensified internal market activity increases aggregate productivity with increasing firm size and diversification in the firm's task structure, any loss of a then more productively matched worker due to separation increases the loss in the typical firm's productivity. At the same time, however, the information about the desired demand replacing any loss diminishes with firm size and diversification. In all, the increase in productivity to be expected even when the external labor market supplies workers with known skills, becomes smaller with increasing firm size and diversification. We show by example that this may result in a switch in the rank order of firms by productivity, relative to the rank order generated by internal market activity.

To the best we know, we are the first to account for the effects of internal labor market activity on the productivity of the firm, and for the information generated to the external market on a critical element of the typical worker's employment history –her last job. In passing, that information generates a wedge between these workers and workers newly entering the labor market, contributing to the empirically observed discrimination against the latter. We will analyze these effects by comparing the effect on firm productivity of hires based on the signaled information, against the productivity generated from uninformed hires.

Central to our approach is that after optimal internal re-matching constrained by the draws from the external market, to yield efficiency, the firm's activities in the external market should, as indicated before, be geared to minimize the mismatch between the *distribution of workers by skills*, and the *distribution of tasks*. This may imply that the skills sought by the firm in the external market do not necessarily correspond to the vacated tasks.

We arguably are also first to account for the hysteresis effects on the formation of labor demand generated by the remaining stock of workers when a task is vacated, and correspondingly the effects of the typical worker's employment history on the labor supply offered in the external market; all of those are influenced by internal market activities. Note that the more information available on released workers, the more the external market becomes differentiated on the supply side not only horizontally, but also vertically. Vertical differentiation is generated by increasing variation in the specificity of information on the typical worker's previous match quality –and with it, on her skills. As we will detail in the second part of our paper, a high previous wage indicates high productivity, and thus a good match in the worker's previous employment.[3]

The remainder of this paper is structured as follows: In the next section, we report on the (sparse) literature we relate to. In Section 2 we specify the model, and in Section 3 we report on the results as based on optimal demand. We conclude with Section 5. In Part 2 of this paper, we close the model by deriving the supply structures generated from random separation, and solving for the equilibrium as well as characterizing it.

2 Related Literature

In the voluminous search and matching literature, the labor market, particularly the worker's search for an appropriate job, and the matching of workers by skill to occupations is taken as a guiding example. Many frictions may arise within this process. The characteristics of job offers are not fully transparent, requiring costly search by the worker to identify the job opening that potentially fits her skills; and in a particular match, the firm and/or worker only learns over time about the quality of fit between skill and skill required in a particular match. Models of undi-

[3] As we consider purely horizontal differentiation, a very low wage is also informative, namely of a bad match, by which one predicts more productive employment in another task. This is a deliberately created artifact to isolate the effects of purely horizontal product differentiation.

rected or directed search characterize the effect of search frictions and matching imperfections, and with them explain important empirical labor market regularities. The classical survey is by Mortensen and Pissarides (1999). Lauermann and Wolinsky (2016) is one of many more recent contributions. In a recent survey, Mueller and Spinnewijn (2022) relate expectations data to theory building. We focus on generic two-sided imperfect information, i.e., the fact that the productivity of a worker is conditional not only on the match with a particular task, but also its/her positioning within a team, and revealed only indirectly by the revelation of the team's –and in the extreme– the firm's productivity.

Almost all matching models involve the vertical differentiation of labor. Analyses involving horizontal differentiation of labor and tasks; and the firm's internal labor market activities and their relationship to its external ones are sparse. To the best of our knowledge, only Li and Tian (2013), Papageorgiou (2018), and Pastorino (2015) model an internal labor market, and only in Li and Tian (2013) the firm takes an active role in this. Team productivity is nowhere an issue. Furthermore, the external labor market remains undifferentiated, as any worker returning to that market is identical to a newly entering worker. This is not so in our model. Here the worker carries with her information about her previous employment relationship, with obvious implications on the firms' selection of workers. Accounting for the worker's previous employment history strikes us as an important aspect for human capital allocation and for the evaluation of the efficiency of the external labor market –also in the interplay with the internal labor market that potentially importantly contributes to its structure.

As to more detail, in Li and Tian (2013) the firm can create up to two occupations with differing skill requirements. By observing match qualities after an initial random assignment, the firm can potentially improve on those by re-assigning workers. Their matching quality is binary, so match quality is perfectly identifiable. We consider imperfectly productive matches as well, together with the revelation of only team productivity. By this, the same outcome may be generated by different match patterns. Furthermore, it yields an empirically relevant potential misalignment between the firm's and the typical worker's objectives: while the firm could match a worker to a task leading to her maximal *individual* productivity, it may prefer to arrange workers to achieve maximal *firm* productivity. With this we also highlight the team aspect involved in a firm's production process. Furthermore, we include generic noise on the demand side, and noisy information about previous employment on the supply side, both influenced by internal labor market activity. This yields our external labor market to be differentiated in a critical way, as influenced by the internal market.

Papageorgiou (2018)'s model is essentially single-agent –and essentially, single-firm. The worker takes the only active role in moving between occupations within that firm. This way she sequentially learns her match-quality. By assumption the occupations sought by the worker are available at any time, and thus supplied fully elastically –which (realistically) is not the case in our model. The worker's aim is to find the match maximizing her productivity –and with it, her *individual* wage. In his model, maximizing the worker's productivity is also in the firm's interest. Thus, the firm's and the worker's interests are co-aligned. In our model, the occupations offered by the typical firm are limited. The worker's best match occupation may be filled with another worker, which introduces yet another realistic friction. Furthermore, the firm seeks to match workers to occupations in order to maximize the firm's, rather than the individual worker's productivity.

While Papageorgiou (2018) focuses on the internal labor market and ignores essential features of the external one, Papageorgiou (2014) focuses on the external labor market, but ignores the internal labor one. When the worker has learned that she is unproductive in one (single occupation) firm, she may direct her search towards other firms with occupations that likely require different skills. The model is rather similar to Papageorgiou (2018), and with this shares the differences to our model. In particular, the worker learns perfectly her match quality; the supply side of the external labor market remains unstructured; and firms remain inactive in the hiring process. (Note that within our model, hires from an unstructured external labor market corresponds to hiring from a random distribution of skills.) Inasmuch relevant within our analysis, we replicate the stylized facts presented in Papageorgiou (2014) and Papageorgiou (2018) within a very different, but arguably more realistic modeling framework.

Pastorino (2015) considers learning in a vertically differentiated labor market, about workers' ability on jobs at different hierarchical levels (see also Pastorino (2024)). High-ability workers generate higher expected output in each job than low-ability worker. Furthermore, the high-ability worker perform best in high-ranking, and low-ability workers in low-ranking jobs. Jobs are differentiated not only according to how important high worker quality is for productivity but also how informative success or failure are about the worker ability level. Based on the information on the workers' performance in particular jobs, workers are reallocated to the jobs available in the economy.

With her model, the author captures several empirically established features of careers in firms such as wage increases at promotion and that worker migration across firms based on wage differentials. Our approach differs mainly by our horizontal differentiation, by our firm orientation distinguishing between firms by size and structure, by our team productivity and remuneration approach. With our focus on horizontal differentiation, learning takes place not about workers' ability levels like in Pastorino (2015) but solely about his/her best-suited task. Horizontal differentiation affects also the structure of demands in that firms recruit for an optimal distribution of workers rather than trying to find the best worker for a given vacancy.

A crucial feature of our paper is that only the outcome of a group of workers is observed by the firm. Beginning with Holmstrom (1982), the focus of the sizable literature on teams has been on the provision of effort incentives in settings when team members' individual contribution to performance is difficult to ascertain. Incentives may extend to helping or harming team members (e.g., Itoh (1991), Auriol et al. (2002), Chalioti (2016)). These papers typically assume that the allocation of workers to the teams is exogeneously given. In contrast, our concern is the dynamic reallocation of individuals to tasks within and across firms.

Team incentives have been analyzed in more general frameworks, for example, for the acquisition of skills (Bar-Isaac and Hörner (2014)), the allocation of authority (Aghion and Tirole (1997), Onuchic and Ramos (2023), and team composition (Jeon (1996), Mello and Ruckes (2006), Bar-Isaac (2007)).

Tate and Yang (2015) document the active re-matching of workers to jobs within firms. In their sample, diversified firms are especially likely to reallocate workers to lines of business in different industries within the firm in response to changing opportunities. Their evidence suggests that diversified firms use internal labor markets to allocate human capital more efficiently than the external labor market does: workers in diversified firms not only are on average more productive than in focused firms but also tend to be more highly sought-after in the external labor market. The transferability of human capital across industries within diversified firms may even facilitate diversifying acquisitions (Tate and Yang (2024)).

Their empirical results reflect the significant role of diversified firms for labor allocation in our model as well as the attractiveness of workers of diversified workers in the external market. However, our results are more nuanced as we show that workers of moderately diversified firms tend to be even more attractive to other employers than those of highly diversified firms.

We contribute conceptually by transferring and adapting the horizontal differentiation model known from the IO literature to the labor market.

3 Model

We model an island economy consisting of a large but finite number of risk neutral firms and a correspondingly large finite pool of workers. Firms, producing with labor as the only factor, vary exogenously by number and structure of tasks, and workers differ exogenously by skills. All differentiations are symmetric. A uniform distribution of tasks corresponds to a uniform distribution of skills, so that, if the economy were frictionless, the market would clear by having each task perfectly matched with a worker exhibiting the corresponding skill. We ignore the output market.

Only firms are active, workers remain passive. The typical firm acts rationally, i.e., it knows the model. All labor market activities are costless. The market is perfectly competitive.

We limit firm size s to maximally $s = 3$ and call these *large* firms, and *passim* extend our analysis to involve *medium-sized* two task firms, and *small* one task firms. There are three types of tasks $T, T \in \{A, B, C\}$. For instance, in a large firm that thus involves $s = 3$ tasks, T_n denotes task T at position $n \in \{1, 2, 3\}$. Our economy thus involves altogether 10 types of large firms, with structures $AAA, AAB, AAC, BBA, BBB, BBC, CCA, CCB, CCC, ABC$; 6 types of medium-sized firms with structures AA, AB, AC, BB, BC, CC; and 3 types of small firms with structures A, B, C.

The typical firm of size s needs s tasks to be filled with one worker each. Different tasks require workers with different skills to be matched with the tasks in a maximally productive mode. There are 3 horizontally differentiated skills. A worker is characterized by skill $t \in \{a,b,c\}$. *Ex ante* that skill is unknown to both workers and firms. It is revealed only indirectly via the productivity of the team involving all matches in the firm in which she is employed.

Workers with different skills are imperfect substitutes in fulfilling a given task. The degree of substitution decreases clockwise as in a circular Vickrey-Salop-model. A match is denoted by $(T,t+i), i \in \{0,1,2\}$. The productivity $P(T,t+i)$ of a match, observable only to the modeler, is quantified as follows: $P(T,t) = 1, P(T,t+mod\{1\}) = 2/3$, and $P(t+mod\{2\}) = 0$.[4] We call these perfect, imperfect, and bad matches, respectively. Figure 1 further illustrates the notation.[5]

Examples of the types of skills we have in mind are intellectual vs. communicative vs. manual skills. They may be productive independently of a particular educational background, and only in a particular working environment. Applied to our substitution framework, one could imagine that commmunicative skills are an imperfect substitute for intellectual ones and a zero substitute for manual ones; intellectual skills an imperfect substitute for manual and a zero substitute for communicative skills; and manual skills an imperfect substitute for communicative but a zero substitute for intellectual skills.

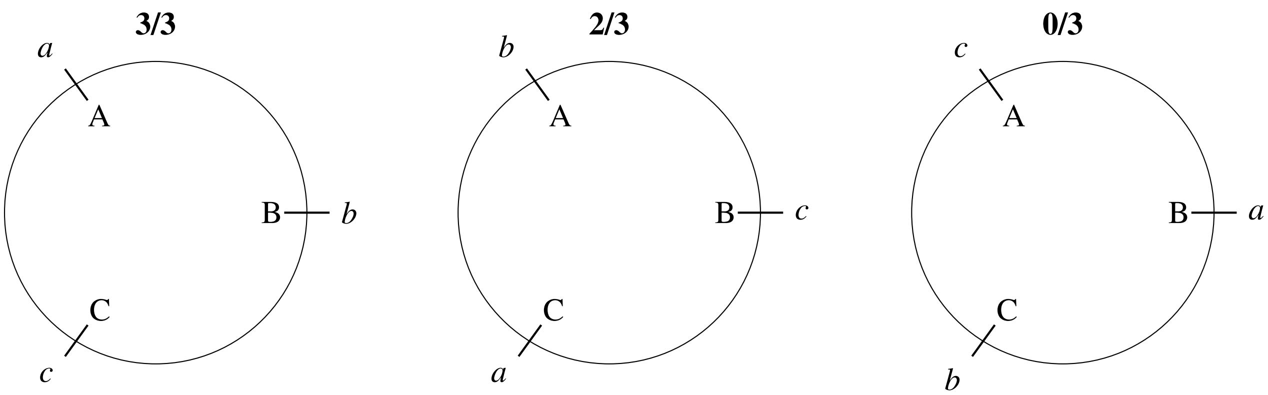

Figure 1: Productivity of Individual Matches

Prominently in our analysis figure variations in internal labor market activity and their effects, that are due to variations in firm structure. We therefore need to develop a language to describe these. Variations are obviously maximal in large firms. To begin, in a large firm structure, we call a task ***rare*** if it is contained only once, and ***common*** if otherwise.[6] Furthermore, as hires in the external market are uncertain w.r.t. skills, we organize firms by structure in view of their potential to productively accommodate any skill. We distinguish between fully and partially diversified, as well as partially and fully specialized firms. By this, a firm is called

[4]In more detail, let $T = B$. Then $(T,t) = (B,b)$ with productivity $P(B,b) = 1; (T,t+1) = (B,c)$ with productivity $P(B,c) = 2/3$; and $(T,t+2) = (B,a)$ with productivity $P(B,a) = 0$.

[5]Setting productivity to zero for bad matches is just a normalization.

[6]Thus, in a firm with structure *AAB*, *B* is a rare, and *A* a common task.

- ***fully diversified****, or FD-firm if it can match all available skills perfectly*
- ***partially diversified****, or PD-firm if one of the skills can be matched only imperfectly, but in a common task*
- ***partially specialized****, or PS-firm if one of the skills can be matched only imperfectly, but in a rare task*
- ***fully specialized****, or FS-firm if it can match one skill only badly,*[7]

A firm's technology is additive. With match qualities known to the modeler, the productivity of a large firm, that is characterized by a triple of arbitrary matches, is specified by

$$P(\boldsymbol{T}, \boldsymbol{t}) = 1/3 \sum_{n=1}^{3} P(T_n, t + i_n) \in [0, 1] \tag{1}$$

if all tasks are matched with a worker, and 0 otherwise. In this specification we assume away economies of scale, in order to isolate the efficiency improving effect of larger internal labor markets in larger firms that involve a more differentiated structure of tasks. We assume the firm's productivity to shrink to zero, however, if one or more tasks remain unserved.

A final simplification of the presentation: When the firm structure under scrutiny is clear, we denote a match pattern simply by the sequence of the workers' skills. By example, for an *AAC*-firm, the alternative match patterns involving two perfect matches and one imperfect match are denoted by *aaa*, *abc*, and *bac*. With small modifications indicated below at the appropriate places, the specifications translate into smaller firm sizes.

A any rate, to begin, the typical firm hires from the external market the number of workers corresponding to its number of vacant tasks, offering a wage conditioned on its maximal productivity realized after internal re-matching. In equilibrium all firms pay a wage that is proportional to their productivity. Since only team productivity is revealed, the wage is uniform across all employees.

Uniformity of wages appears to be an extreme assumption, but is consistent with our (initial) information structure. Furthermore, it can be motivated by the following sequence of actions: The firm hires a worker, offering the conditional wage contract. Then, re-matching takes place. Based on the resulting productivity signal, the firm offers a corresponding uniform wage. Since productivity jumps to zero whenever one worker refuses to accept the offer, all workers have identical outside options, i.e., identical bargaining power. The bargaining outcome yields, e.g., the typical worker's Shapley value. Empirically, firm-specifica wages are supported by the observation that adding a firm specific productivity component to any Mincerian wage equation typically contributes significantly to wage determination.

[7] Hence *ABC* is fully diversified; *AAC*, *BBA*, or *CCB* are partially diversified; *AAB*, *BBC*, or *CCA* are partially specialized; *AAA*, *BBB* or *CCC* are fully specialized. While these definitions are specific to the limited size and structural composition of firms discussed in this model, they should indicate that within the context developed here, differences in the potential to productively accommodate skills may figure importantly.

We consider two types of equilibria: An internal markets (IM-)equilibrium, and an external markets (EM-)equilibrium. In an IM-equilibrium, all firm maximize their productivity via frictionless internal re-matching, e.g., by trial and error, constrained by the set of workers as drawn from the external market, and pay a wage corresponding to the resulting productivity. In an EM-equilibrium, all firms optimally hire from the external market to fill a randomly vacated task. That external market is supplied by both newcomers without, and workers with employment history.

For expositional purposes, however, we consider in this paper special versions of the two equilibria. The market opens by all firms hiring one worker each for *all* its tasks from an unstructured market (a market involving newcomers only), and operate their internal markets via re-matches to maximize their productivity, subject to the constraint that workers are taken from the set of initial hires. Let I denote the set of workers hired by a firm and I^k the set of potential matches with tasks involving structure k. Pursuing frictionless internal re-matching on this basis involves, for large firms,

$$max_{i\in\{I^k\}} 1/3 \sum_{n=1}^{3} P(T_n^k, t+i_n), k \in \{FD, PD, PS, FS\}. \quad (2)$$

This corresponds to selecting i from the set I drawn randomly, such that the sum of the "distances" between task and skill is minimized:[8]

$$min_{i\in\{I^k\}} \sum_{n=1}^{3} (i_n|k). \quad (3)$$

We remain agnostic about the micro-structure of that re-matching process. However, we concretely assume that the re-matching process yields a signal to the firm perfectly indicating a productivity-maximizing match. If more than one such match exists, the signal indicates each of them with equal probability. The re-matching process is beneficial to both the firm and the individual worker –in spite of the possibility that a worker may be re-matched from a perfect to an imperfect match to increase the firm's productivity. In an IM-equilibrium, no firm is incentivized to change its pattern of matches. The equilibrium is characterized by densities $\phi_s^k(t^*)$ of firms of given structure and size, by productivity realized in IM-equilibrium. The effect of internal re-matching is expressed by changes in these densities. We compare these densities by moments. Towards that, let $\Phi_s^k(t^*)$ denote the corresponding cumulative distribution functions.

In the concept of EM-equilibrium employed in this paper, the desired demand generated to fill the vacated slot is assumed to be satisfied optimally and without uncertainty, rather than satisfied from a market composed of newcomers, dismissals, and quits. With this assumption, we tease out the information effect on the demand generated from the internal market. We reserve to the second paper closing the model with a more realistically composed supply side.

[8]This formulation corresponds to the concept that the firm seeks to minimize the difference between the distribution of its tasks and the distribution of its workers' skills.

As to more detail on the external market activities, let a worker vacate task n in a typical large firm type k with productivity P^k realized before the worker has vacated that task. The firm with structure k, knowing both the vacated task and its productivity realized before vacation, infers from this the set of workers $\{I^k|n\}$ remaining in the firm, and the skill $t+i$ of the worker to be hired. That hire is called ***firm-optimal***. With a firm-optimal hire the firm maximizes its productivity $\Pi^k(\boldsymbol{T,t}|n)$, that is conditional upon optimally re-matching workers internally. The desired skill may, or may not perfectly match the vacated task. If it does, so $i^* = 0|n$, we call the hire ***task-optimal***.[9]

Finally, we introduce an index, called ***precision*** that indicates the information generated by market activities on the typical worker's skills. It is specified as the inverse of Shannon's entropy measure. At the level of individual productivity j realized in the IM-equilibrium, it is defined as

$$\pi_j^k \equiv 1/E_j^k \in [1,\infty), j \in \{1...,10\} \tag{4}$$

where $E_j^k \equiv \phi_j^k \cdot \{pr_j^k \cdot |ln(pr_j^k)|\}$ is the entropy,[10] $pr_j^k = 1/3\sum_{n=1}^{3}\sum_{i=0}^{2} pr(t+i_n^*|T_n^k,)$, the probability, and ϕ_j^k = frequency by which productivity j materializes in structure k.

The average precision involving firm structure k is simply

$$\pi^k \equiv \frac{1}{\sum_{j=1}^{10} E_j^k}. \tag{5}$$

4 Results

We will address first the internal, and then the external market activities; and within the respective subsections focus first variations across the largest firms in our economy involving the richest structures, and then on variations in firm size.

4.1 Equilibrium re-matching in the internal labor market

We demonstrate and characterize the productivity effects of internal labor market activity by comparing them to the productivity achieved with random matching, i.e., before re-matching activities. That productivity naturally depends on the mode of selecting workers from the external market, which is random as well. The effect is the larger, the more intense the internal labor market activity, which depends on firm size and structure. The productivity of individual matches tends to be, but is not necessarily increased by internal market operations. If it is increased, it also increases the precision by which worker skills can be predicted from the task performed by the worker.

[9] With this general set-up we are at explicit variance with the labor market literature –in particular Papageorgiou (2014) and Papageorgiou (2018), in which hires are always considered task-optimal, no matter the decision maker firm or worker.

[10] By continuity it is defined $0 \cdot ln(0) = 0$.

4.1.1 Variations across structures of large firms

In Table 1 we specify the match patterns for our guiding examples that characterize the four task structures, by productivity levels that arise first from random matches, and then from optimal internal re-matching.[11] By example, under the possible patterns arising in the FD-firm with task structure *ABC*, the 4th column indicates four alternative random assignments that yield firm productivity .67. None of these assignments survives optima-l re-matching. Rematching of *abb*,*aac* and *cbc* yields productivity .78; re-matching of *bca* yields productivity 1.0. The match patterns in brackets under the relevant columns indicate all possible draws and initial random matches for which optimal re-matching yields that productivity.

As not infrequently observed in real life, the first three of the indicated patterns including two perfect matches and one bad one are not constrained optimal, but can be improved upon by re-matching so that two workers are imperfectly matched. Hence task-optimal matches are not necessarily firm-optimal. Overall, the distribution of matches that is (almost) symmetric under random matching becomes rightward skewed under re-matching.

In Table 2 we document the densities by productivity that arise in internal market equilibrium for large firms. The standard of comparison, the density from random matches, is shown in the first relevant row. It is identical across all firm structures. The overall mean productivity is .56. The densities below are organized by decreasing diversity of firm structure. By example, while random matching yields productivity .89 with probability 3/27, re-matching in FD-firms increases that probability to 9/27, in PD-firms to 7/27, and in PS-firms to 6/27. In FS-firms, it falls back to the probability involving random matching, as identical tasks do not allow for productivity increasing re-matching. The overall average productivity realized after re-matching shifts upwards by 50 percent for FD-firms, and monotonically decreases with decreasing diversity, while the range of the densities increases. It follows immediately that the relative importance of external labor market activity towards obtaining efficient matches decreases with increasing diversity.

We summarize our first insights in

Result 1: *In internal market equilibrium involving large firms,*

(i.) *the productivity realized with internal re-matching strictly increases in all firms with diversified task structure*

(ii.) *realized productivity increases with increasing diversity, i.e.,* $P^{FS} < P^{PS} < P^{PD} < P^{FD}$.

Proof: Inspection of Table 2.

[11] *AAC* also stands for *BBA* and *CCB*; *AAB* for *BBC* and *CCA*; *AAA* for *BBB* and *CCC*.

Table 1: Large firms: match patterns with random and with re-match

FD: ***ABC***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	abc	aba acc bbc	aca bba bcc	abb aac cbc bca	aaa acb cba ccc bac bbb	cca baa bcb	aab cbb cac	bab ccb caa	cab
	Re-Match	abc (acb) (bac) (bca) (cab) (cba)	aba (aab) (baa) acc (cac) (cca) bbc (cbb) (bcb)	aca (aac) (caa) bba (abb) (bab) bcc (ccb) (cbc)		aaa bbb ccc				
PD: ***AAC***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	aac	aaa abc bac	aba baa bbc	aab acc cac bba	abb bab aca caa bcc cbc	cba bca bbb	acb cab ccc	cca cbb bcb	ccb
	Re-Match	aac (aca) (caa)	aaa abc bac (acb) (cab) (cba) (bca)	aba baa (aab) bbc (cbb) (bcb)	acc cac (cca) bba (abb) (bab)	cbc bcc (ccb)	bbb	ccc		
PS: ***AAB***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match	aab	aac abb bab	abc bac bbb	aaa acb cab bbc	aba baa acc cac bcb cbb	bba bcc cbc	aca caa ccb	bca cba ccc	cca
	Re-Match	aab (aba) (baa)	aac (aca) (caa) abb bab (bba)	abc bac (acb) (cab) (cba) (bca) bbb	aaa bbc (cbb) (bcb)	acc cac (cca)	cbc bcc (ccb)		ccc	
FS: ***AAA***	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
	Random Match =Re-Match	aaa	aab aba baa	abb bab bba	aac aca caa bbb	abc acb bac bca cab cba	bbc bcb cbb	acc cac cca	ccb cbc bcc	ccc

Beyond these results, we can show that the distributions by productivity, and thus, of firm wages structure obey a strict order according to the 1st order stochastic dominance criterion:

Table 2: Large firms: productivity with random and re-matching

	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
No re-match	Frequency	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	Avg. prod.				.56					
FD-firm										
Re-match	Frequency	6/27	9/27	9/27		3/27				
	Avg. prod.				.84					
PD-firm										
Re-match	Frequency	3/27	7/27	6/27	6/27	3/27	1/27	1/27		
	Avg. prod.				.75					
PS-firm										
Re-match	Frequency	3/27	6/27	7/27	4/27	3/27	3/27		1/27	
	Avg. prod.				.73					
FS-firm										
Re-match	Frequency	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	Avg. prod.				.56					

Result 2: *In internal market equilibrium,*

(i.) *the cumulative productivity distributions of large firms dominate first order stochastically, if fully diversified, those of the less diversified firms, i. e.* $\phi_3^{FD}(t^*) \leq \phi_3^{FD}(t^*) \leq \phi_3^{FD}(t^*) \leq \phi_3^{FD}(t^*)$, *with at least one strict inequality*

(ii.) *stochastic dominance is strict between fully and partially diversified large firms,* $\Phi^{FD}(t^*) < \Phi^{PD}(t^*)$, *as well as between partially and fully specialized firms,* $\Phi^{PS}(t^*) < \Phi^{FS}(t^*)$, *and weak between partially diversified and partially specialized firms.*

Proof: Inspection of Table 2.

Result 2 is illustrated in Figure 2. Both Results 1 and 2 reflect the fact that firms with a more diverse task structure are better able to productively accommodate workers with unknown skills.

Table 3: Large firms: productivity gains from internal market activity relative to random assignment

	Common tasks		Rare tasks	
Prod.	PD-firms	PS-firms	PD-firms	PS-firms
.89	-.03	.03	.06	-.06
.78	-.04	.02	.04	-.08
.67	-.09	.08	.16	-.17
.56	-.23	-.06	.44	.12
.44	.23	-.11	-.44	.23
.33	-.33		.67	
.22		-.22		.45

A comparison of the microstructure of internal market equilibrium involving firms with an

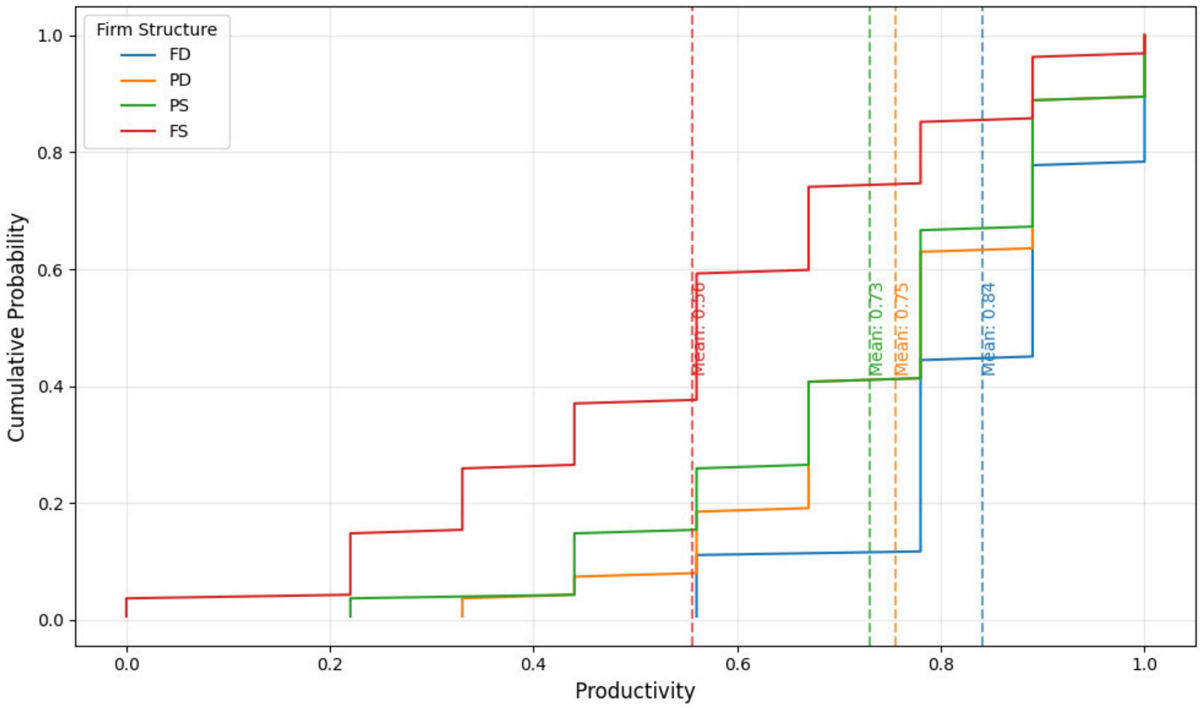

Figure 2: Internal market equilibrium: cdf's involving productivity, by structure large firms

asymmetric task structure, i.e., PD- and PS-firms, is particularly intriguing. Table 3 informs separately for common and rare tasks about the changes in the productivity of individual matches due to internal re-matching. It is constructed by inferring from Table 1 the changes due to re-matching in the frequency, by which productivity levels arise.

Several patterns are striking: As to common tasks, productivity changes in PS-firms strictly dominate those in PD-firms. Indeed, above the mean productivity of .56, productivity changes in the former are positive, while in the latter they are negative. As to rare tasks, exactly the opposite holds. We summarize these regularities in

Result 3: *Consider internal market equilibrium in large PD- and PS-firms.*

(i.) In PD-firms, re-matches into common tasks contribute (almost always) ***less*** *to firm productivity than those into rare tasks.*[12]

(ii.) In PS-firms, re-matches into common tasks above mean realized productivity levels contribute ***more*** *to firm productivity than those into rare tasks.*

(iii.) The changes across firm structures in the contributions to firm productivity are above mean realized productivity levels (almost) exactly opposite.[13]

Proof: Inspection of Table 3.

These results can be explained by the combination of random draws and re-matching: random draws imply that the probability that good matches are drawn several times for the common tasks is lower than that for the rare task. By our very definitions of firm structures, PD-firms are

[12]The outlier is .44, a productivity level involving identical skills that do not allow for re-matching.
[13]Modulo rounding errors.

more flexible in productively accommodating workers with unknown skills than PS-firms –yet, PS-firms can substitute away more easily from rare to common tasks. That the differences concentrate on the above mean productivity levels should not be of concern, as internal re-matching pushes weights into this part of the distribution (see Table 1).

Let us finally look, in Table 4, at the information generated from rematching in internal markets. For all structures, the level of precision is approximately u-shaped across realized productivity levels. The precision is infinite when workers are both perfectly well or perfectly badly matched. The productivity level at which the latter is the case decreases with decreasing diversification in firm structure. The precision is minimal close to mean productivity. In this region the number of alternative match patterns that generate the observed productivity is maximal, with the consequence that the inference firms and workers can draw from match patterns about skills is minimal.

In terms of total precision, the firm structures are ranked exactly opposite to the ranking by productivity realized after re-matching. Observe finally that while the information generated from matches is minimal for fully specialized firms with precision 1.33, and close to minimal for the fully diversified firms with precision 1.82. We summarize in

Result 4: *In internal market equilibrium involving large firms,*

(i.) *the information (precision) realized with internal re-matching is approximately u-shaped in productivity for every task structure;*

(ii.) *zero re-matching generates minimal precision*

(iii.) *precision decreases with increasing diversity, so* $\pi^{PS} > \pi^{PD} > \pi^{FD}$.

4.1.2 Variation across firm sizes

Here we perform the same analysis for the smaller firms, as done above for large ones. The medium sized firm hires randomly two workers to be employed in two tasks, with $T_n \in \{A,B,C\}, n = 1,2$. The small number of tasks allows the distinction only between diversified (D) and specialized (S) firms. Our guiding examples are AB and AA, respectively.[14] Matches and firm productivity before, and after re-matching are presented in Table 5 for a diversified and a specialized firm. The resulting frequencies are shown in Table 6. The average productivity before re-matching stays as in the large firms. With re-matching, it increases in D-firms to .74, but stays strictly below the productivity of large FD-firms. As before, the internal market remains inoperative for the specialized firms.

Finally, turning to the small firms involving just a single employee, the typical firm, endowed with task $T \in \{A,B,C\}$, exhibits productivity $P \in \{.00, .67, 1.0\}$, with an average realized productivity of .56 as before. The actual productivity obviously fully reveals the match quality.

[14] Here, AB stands also for AC and BC; AA stands also for BB and CC.

Table 4: Large firms: Information generated from internal re-matching

	Productivity	1.00	.89	.78	.67	.56	.44	.33	.22	.00
No re-match	Frequency	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	π_j	∞	14.29	14.29	6.67	4.17	14.29	14.29	14.29	∞
	π				1.33					
FD-firm										
Re-match	Frequency	6/27	9/27	9/27		3/27				
	π_j^{FD}	∞	4.76	4.76		8.33				
	π^{FD}				1.82					
PD-firm										
Re-match	Frequency	3/27	7/27	6/27	6/27	3/27	1/27	1/27		
	π_j^{PD}	∞	6.67	7.69	4.76	20.00	∞	∞		
	π^{PD}				1.85					
PS-firm										
Re-match	Frequency	3/27	6/27	7/27	4/27	3/27	3/27		1/27	
	π_j^{PS}	∞	7.69	6.67	12.50	20.00	20.00		∞	
	π^{PS}				2.13					
FS-firm										
Re-match	Frequency	1/27	3/27	3/27	4/27	6/27	3/27	3/27	3/27	1/27
	π_j^{FS}	∞	14.29	14.29	6.67	4.17	14.29	14.29	14.29	∞
	π^{FS}				1.33					

Table 5: Medium-sized firms: match patterns with random and with re-match

D: ***AB***		1.00	.83	.67	.50	.33	.00
	Random Match	ab	ac	bc	aa	ba	ca
			bb		cb	cc	
	Re-Match	ab	ac	bc	aa	cc	
		(ba)	(ca)	(cb)			
			bb				
S: ***AA***		1.00	.83	.67	.50	.33	.00
	Random Match	aa	ab	bb	ac	bc	cc
	=Re-Match		ba		ca	cb	

Table 6: Medium-sized firms: productivity with random and internal re-matching

	Productivity	1.00	.83	.67	.50	.33	.00
No re-match	Frequency	1/9	2/9	1/9	2/9	2/9	1/9
	Avg. prod.			.56			
D-firm							
Re-match	Frequency	2/9	3/9	2/9	1/9	1/9	
	Avg. prod.			.74			
S- firm							
Re-match	Frequency	1/9	2/9	1/9	2/9	2/9	1/9
	Avg. prod.			.56			

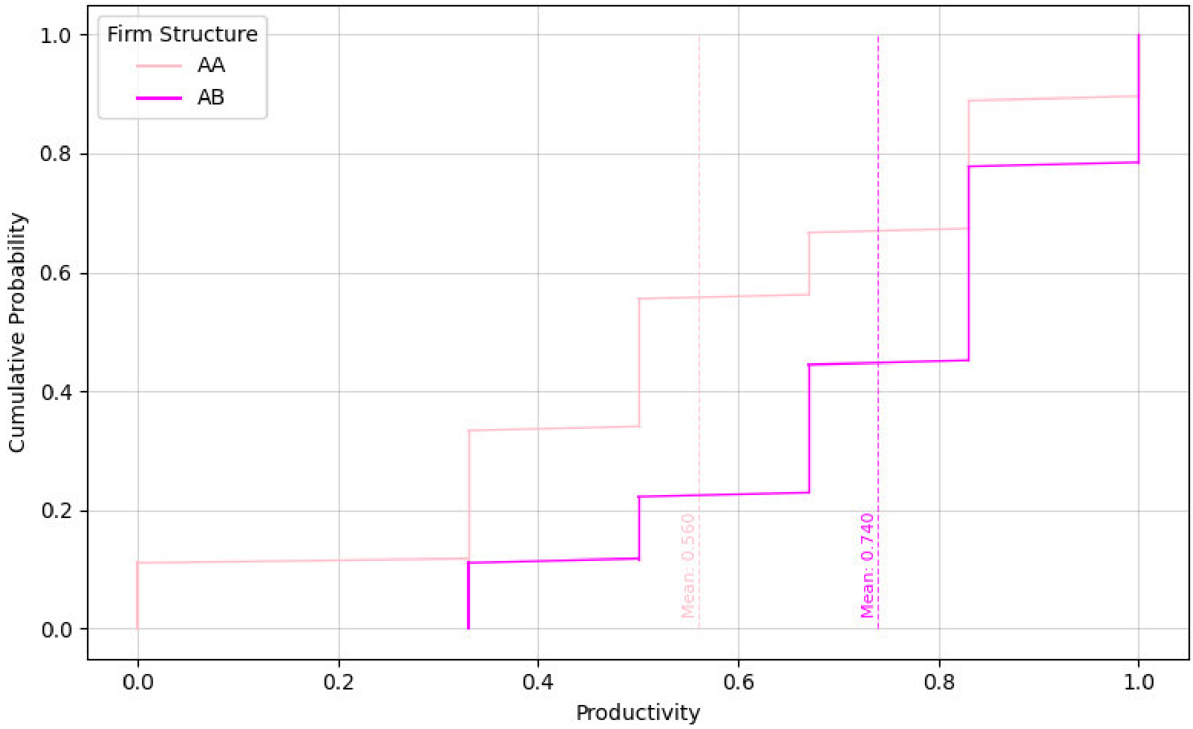

Figure 3: Internal market equilibrium: cdf's involving productivity, by structure medium-sized firms

We compare firm size outcomes in

Result 5: *In internal market equilibrium,*

(i.) *the cumulative productivity distribution of diversified medium-sized firms dominates first order stochastically that of specialized firms, i. e.* $\phi_s^D(t^*) < \phi_2^S(t^*)$.

(ii.) *the average firm productivity of diversified firms increases at decreasing rate with increasing firm size and increasing diversification, i.e.,* $P < P^D < P^{FD}$.

(iii.) *the cumulative productivity distribution of large diversified firms dominates first order stochastically that of the medium sized firms, and in turn that of medium sized that of small firms net of a small numbers effect, i. e.* $\phi_3^{FD}(t^*) < \phi_2^D(t^*) \leq \phi_1(t^*)$.

Proof: Comparison of Tables 2 and 6. See also Figures 3 and 4.

Not unsurprisingly, inspection of Tables 4 and 7, together with the immediate observation that precision is perfect int the small firm case, reveals that the information (precision) after rematching continues to increase with decreasing firm size.

4.2 External market with satisfied desired demand

We wish to isolate the effect of internal market activity on the typical firm's activity in the external market –and this separately on the specification of the firm's demand, and on the supply

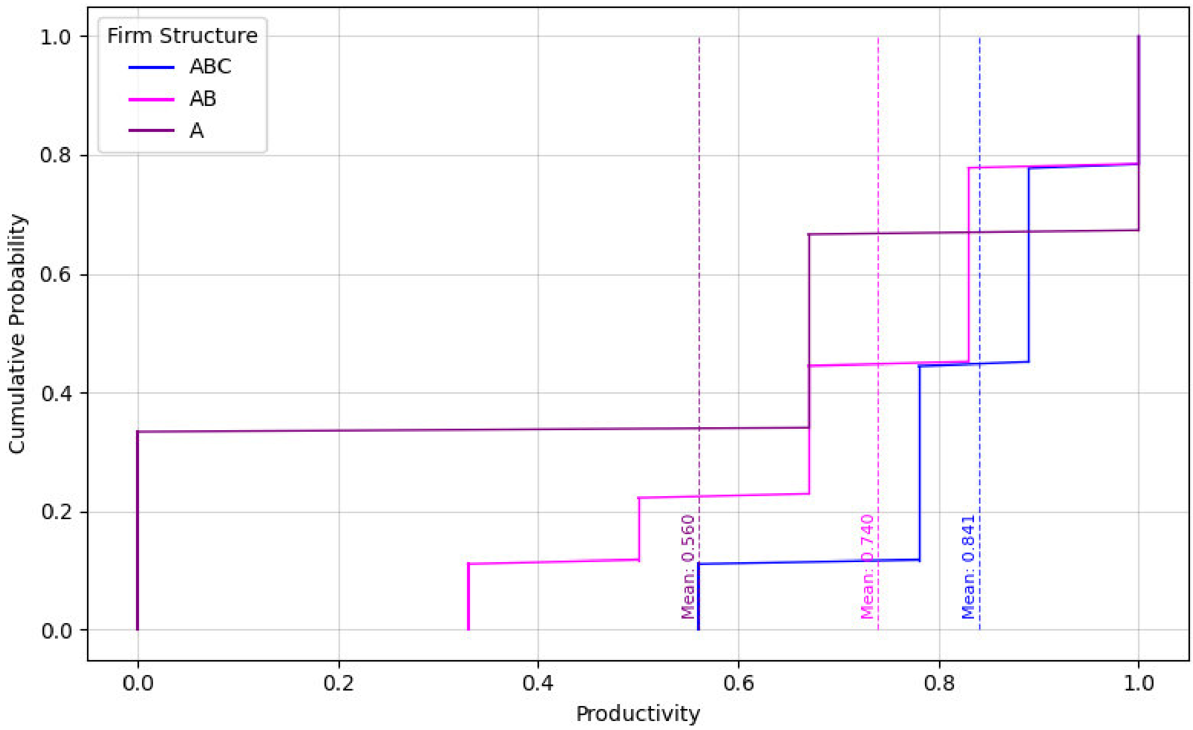

Figure 4: Internal market equilibrium: cdf's involving productivity, by firm size

Table 7: Medium-sized firms: information generated from re-matching

	Productivity	1.00	.83	.67	.50	.33	.00
No re-match	Frequency	1/9	2/9	1/9	2/9	2/9	1/9
	π_j	∞	4.76	∞	4.76	4.76	∞
	π			1.56			
D-firm							
Re-match	Frequency	2/9	3/9	2/9	1/9	1/9	
	π_j^D	∞	4.76	∞	∞	∞	
	π^D			4.76			
S- firm							
Re-match	Frequency	1/9	2/9	1/9	2/9	2/9	1/9
	π_j^S	∞	4.76	∞	4.76	4.76	∞
	π^S			1.56			

generated from releases to the external market. Towards the former, we do this by looking at market outcome under the assumption that every firm satisfies its desired demand to fill a randomly vacated task.[15] We quantify the effect by the difference between the typical firm's expected productivity when hiring a worker with the preferred skill into the vacated task, relative to the productivity before the task was vacated. That difference depends on the quality of the match before its vacation, and the precision by which the demand can be specified. The more productive the previous match, the larger the loss that needs to be covered with the hire. The higher the precision by which the demand for a skill can be specified, the higher the improvement in productivity. Both effects depend on the firm's size and structure.

The firm specifies its firm-optimal demand by selecting the worker whose skill, combined with the skills of the retained workers, maximizes the firm's expected productivity after internal re-matching –or else, chooses a match pattern such that the difference in the distribution of its tasks and the distribution of the skills available to it is minimized. Since the firm can typically realize the same productivity with different matches, the retained workers' skills can be inferred only imperfectly and with it, the firm's demand for the skill complementing these. To determine the hire's optimal skill and the wage offered to her (and with it, to the retained workers), we specify, in principle, a vector involving the firm's expected productivity conditional on the hire's skill. The elements of this vector reflect the firm's willingness to pay for the given skill, i.e., its expected productivity.

In this paper, we analyze and compare, across firm structures, demands that if satisfied maximize the typical firm's expected productivity, i.e., the maximal element in that vector. As discussed above, this allows us to isolate the information effect on the demand side of the external market. In our follow-up paper, the satisfaction of that demand is constrained the supply generated from the random separations: There, corresponding to the demand, the individual worker's supply is characterized by a vector of probabilities that reflect the typical worker's skill. The probabilities are formed from the information available from the most recent facet of the worker's employment history, i.e., the wage previously earned, the size and structure of the previously employing firm, and the task she was last employed in.

4.2.1 Large firms

To illustrate the derivation of the firm's demand, consider the fully diversified FD-firm, exemplified by ABC. Before the worker leaves the firm, the firm observes its productivity P^{ABC}, and thereafter the task T_n vacated by the worker. Knowing it structure, it infers the remaining workers' skills from these two items. By observing, e.g., previous productivity 1.0 and vacated task $T_1 = A$, the firm infers that with probability 1 the retained workers exhibit skills b and c. Thus, the firm's best hire involves skill a.[16]

[15] By this assumption, the typical firm is served without uncertainty what it demands.

[16] See the first row of Table 14 in the Appendix. Its two utmost left columns indicate the firm's productivity level and the vacated task; the next rightward column the alternative skill tuples of the retained workers, and in brackets

The three leftward columns of Table 8 involve FD-firms, exemplified by a firm with structure *ABC*. They contain the productivity expected when a worker with skill t complements the pool of remaining workers.[17] Not unexpectedly the typical firm's expected average productivity generated from the best hire decreases monotonically with decreasing productivity realized before separation, while the difference between realized and expected maximal productivity increases. The decrease reflects the decreasing productivity loss due to the separation, and the increase the increase in the opportunity to make up for it with the new hire.

The three rightward columns of Table 8 involve PD-firms exemplified by a firm with structure *AAC*. The productivity effect of hiring a worker with task-specific skill into the common task (here a for A) is larger than a worker hired with task-specific skill into the rare task (here c for C) –with the skill pattern *bbb* as the outlier known from Result 3. By contrast, around and below mean realized productivity, firm-specific choices that deviate from task-specific choices are dominant in terms of productivity, but almost exclusively in the rare task.[18] Finally, when comparing re-hires for vacated common with with those for vacated rare tasks at mean realized productivity P, the productivity effect is larger across all worker skills when the worker has separated from the rare task.[19]

The demands of PS- and FS-firms presented in Table 9 are again structurally different. In contrast to the situation we identified for the PD-firms, the productivity of task-specific hires is higher here into the rare task (here B) as opposed to hires into the common task (here A). The exception is at the lowest realized productivity level (here .22). As for PD-firms, the firm-specific choices dominate the task-specific choices when hires replace separations from the rare task –with the exception of replacements at the two highest realized productivity levels.[20]

We summarize the productivity gains in Table 10. It shows clearly that the gains increase with decreasing realized productivity before separation (indicated in the first column), and this over all firm structures. It also shows that firm-optimal always correspond to task-optimal choices in FD- and FS-firms, and almost always in PD- and PS-firms as long as replacements involve common tasks.[21]

the probabilities by which these occur; finally, the three most rightward columns exhibit the firm's productivity expected when it hires skill t.

[17] In Tables 8 and 9 we condense the results of the more detailed tables in the Appendix.

[18] Here, the firm with the outlier pattern prefers to hire on a vacated common task A a worker with skill optimally matching the rare task C, and this in spite of the fact that it is certain to have employed an optimal match for the common skill. Its realized productivity would otherwise have been higher.

[19] In the right-hand part of Table 8, replacement from the common task A yields lower expected productivity than replacement from the rare task C in every skill slot.

[20] In the *AAB*-firms considered in the leftward three columns of the table, skill a is the preferred hire when task B is vacated –and this hire improves on productivity more than if task A were vacated. The reason here is that in the internal market, arrivals of workers with skill c can only be productively accommodated in task B. A typical second best allocation involves a transfer of a skill b worker into one of the tasks A. Now, if the rare task B is vacated, that b-worker is optimally transferred into task B, inviting the specification of firm-optimal demand a, given the vacated task is B.

[21] The outlier is once again the *bbb*-firm, in which c is the preferred hire no matter the separation.

When they involve rare tasks, however, firm-optimal choices strictly dominate task-optimal choices in 75% of all cases. The reason are internal switches from the common to the rare task (as discussed in footnote 20).

Table 8: Demand typical FD-firms (*ABC*) and PD-firms (*AAC*) conditional on realized productivity *P* and vacated task *T*

P/T	Exp. prod. *ABC*			Exp. prod. *AAC*		
1.0	*a*	*b*	*c*	*a*	*b*	*c*
A	1.0	.78	.89	1.0	.89	.67
B	.89	1.0	.78			
C	.78	.89	1.0	.89	.78	1.0
.89						
A	.93	.78	.81	.94	.83	.67
B	.81	.93	.78			
C	.78	.81	.93	.79	.68	.91
.78						
A	.85	.81	.85	.86	.75	.75
B	.85	.85	.81			
C	.81	.85	.85	.72	.56	.83
.67						
A				.81	.69	.69
B						
C				.83	.67	.72
.56						
A	.74	.74	.74	.78	.67	.44
B	.74	.74	.74			
C	.74	.74	.74	.89	.78	.56
.44						
A				.67	.44	.78
B						
C				.67	.44	.78
.33						
A				.67	.56	.33
B						
C				.67	.56	.33

Table 9: Demand typical PS-firms (AAB) and FS-firms (AAA) conditional on realized productivity P and vacated task T

P/T_n	Exp. prod. AAB			Exp. prod. AAA		
1.0	a	b	c	a	b	c
A	1.0	.89	.78	1.0	.89	.67
B	.67	1.0	.89			
.89						
A	.92	.81	.64	.93	.81	.59
B	.83	.94	.83			
.78						
A	.84	.73	.52	.85	.74	.52
B	.98	.87	.76			
.67						
A	.75	.75	.56	.78	.67	.44
B	.92	.83	.72			
.56						
A	.72	.61	.39	.70	.59	.37
B	.89	.78	.56			
.44						
A	.67	.56	.33	.63	.52	.30
B	.78	.67	.44			
.33						
A				.56	.44	.22
.22						
A	.56	.44	.22	.48	.37	.15
B	.56	.44	.22			
.00						
A				.33	.22	.00

Table 10: Large firms: Productivity gains from external market activity

	Common tasks			Rare tasks		
Prod.	PD-firm	PS-firm	FS-firm	FD-firm	PD-firm	PS-firm
.89	.05	.03	.04	.04	.02	.05
.78	.08	.06	.07	.07	.05	.20$\Leftarrow a$
.67	.14	.08	.11		.16$\Leftarrow a$	.25$\Leftarrow a$
.56	.22	.16	.14	.18	.33$\Leftarrow a$	.33$\Leftarrow a$
.44	.34$\Leftarrow c$	.23	.19		.34	.34$\Leftarrow a$
.33	.34		.23		.34$\Leftarrow a$	
.22		.34	.26			.34$\Leftarrow a$
.00			.33			

Result 6: *With satisfaction of desired demand,*

(i.) expected maximal over realized productivity

- is strictly positive at each but the highest productivity level

- increases with decreasing P.

(ii.) separations from common tasks imply that

- firm-optimal hires are almost always task-optimal

- productivity increases in PD-firms dominate those in PS-firms at all comparable levels of realized productivity

- productivity increases in PD-firms dominate those in FD- and FS-firms at all comparable levels of realized productivity

- productivity increases in FS-firms dominate those in PS-firms above mean realized productivity, and vice versa those at and below mean

(iii.) separations from rare tasks imply that

- in PD-firms and PS-firms, firm-optimal dominate task-optimal hires in 3/4 of all cases - PS-firms outperform both FD- and PD-firms at every productivity level

(iv.) in PS-firms, separations from the rare tasks dominate separations from the common tasks in both PS- and PD-firms at every productivity level

Proof: (i.), (ii.) and (iv.): Inspection of Table 10. (iii.): Calculations combining Tables 2 and 10.

Result 6 (i.) is not obvious because of three features: Internal labor market equilibrium delivers (constrained) best matches; separation was random rather than based on bad matches; and in all but the highest and the lowest realized productivity levels, the typical firm cannot perfectly identify the matches that had generated that productivity level –which implies that it cannot precisely specify its desired demand. Nevertheless, the match pattern as anticipated by the firm dominates in productivity the match lost via random separation.[22]

By part (ii.), when separations take place from common tasks the PD-firms outperform all other firm types. When from rare tasks, however, the pattern changes substantially, as specified in parts (iii.) and (iv.). Here, PS-firms outperform both FD- and PD-firms when we control for realized productivity. Almost all hires that are not task-optimal, and thus associated with re-matching.

As to PD-firms, the match productivity on the rare task is in almost all cases higher than on the common tasks. By Result 3 (i.), a worker separated from a common task has a relatively low match quality –implying that the retained workers have a relatively high one. Thus, hiring task-specifically yields a relatively large productivity gain. By contrast, for the PS-firms, the average match quality is higher in the common tasks than in the rare tasks for high productivity levels, see Result 3 (ii.). For below mean productivity, the realized match quality is higher in the rare task than in the common tasks. Interestingly, on average over all productivity levels the

[22] In the second part of our paper, we will study whether this remains to be the case with frictional supply.

match qualities on the common and rare tasks are identical –which is again reflected in Result 3, here (iii.). That all these outcomes can be explained by the outcomes generated from internal re-matching is related to the fact that with our assumption on the satisfaction of desired demand, we isolate for the uncertainty on the supply side of the external market. This documents the powerful influence of the internal market on external market outcomes.

That desired demands exercised by PD- and PS-firms in the rare tasks tend not to be task-specific is due to several factors. One could have expected that the rare task was on average matched better than the common one, in which case the vacated rare task should be filled with the same skill. Especially in PS-firms, however, re-matching in the internal market arises by substitution from optimal matches in the rare task to suboptimal matches in common tasks, with the outcome clearly reflected in Table 3. This induces a low match quality in the rare task, hence a small productivity loss that can be more than compensated by large productivity gains based the specified hire in the external market.

Against all this, however, works uncertainty about the remaining workers' skills. Therefore, especially in the intermediate firm types, i.e., PD- and PS-firms, the best hire is an optimal match for the common task. Here the PS-structure confers an advantage to the PD-firm, as the former admits substitution of that hire into the rare task. That this pattern concentrates on the firms with asymmetric task structures appears to be generated by the feature that once the firms don't know the pattern by which realized productivity is generated, they have difficulties inferring from separations what they have lost, and thus hedge against uncertainty by demanding a good match for the task represented more broadly in these structures.

Note finally that FD-firms cannot improve on expected productivity via firm-optimal hires relative to task-optimal ones. The reason is again noise in the specification of desired demand: the high realized productivity levels are achieved with equal probability by different match patterns, that reduce the precision by which these firms can specify their desired demand. By contrast, FS-firms can perfectly specify their demand, but improvements in productivity are constrained by the fact that the internal market is inoperative.

Separation from a less well matched task, together with improved specification of desired demand provides an opportunity to improve greatly expected productivity. Result 7 illustrates this opportunity:

Result 7: *With satisfaction of desired demand, PS-firms, when separating from rare tasks, achieve higher expected productivity than FD-firms.*

Proof: Calculations from Table 2 and l.h.s. of Table 9 for PS-firms, as well as Table 2 and l.h.s. of Table 8 for FD-firms. The resulting expected productivity of PS- and FD-firms is given by .92 and .90, respectively.

By this result, the PS-firm, a firm type that was relatively disadvantaged in achieving high productivity via internal re-matching, overturns in maximal productivity expected from external market activity the FD-firm, the firm type that was maximally benefiting from internal re-matching. Figure 5 illustrates that the cdf involving the PS-firms' expected productivity, while

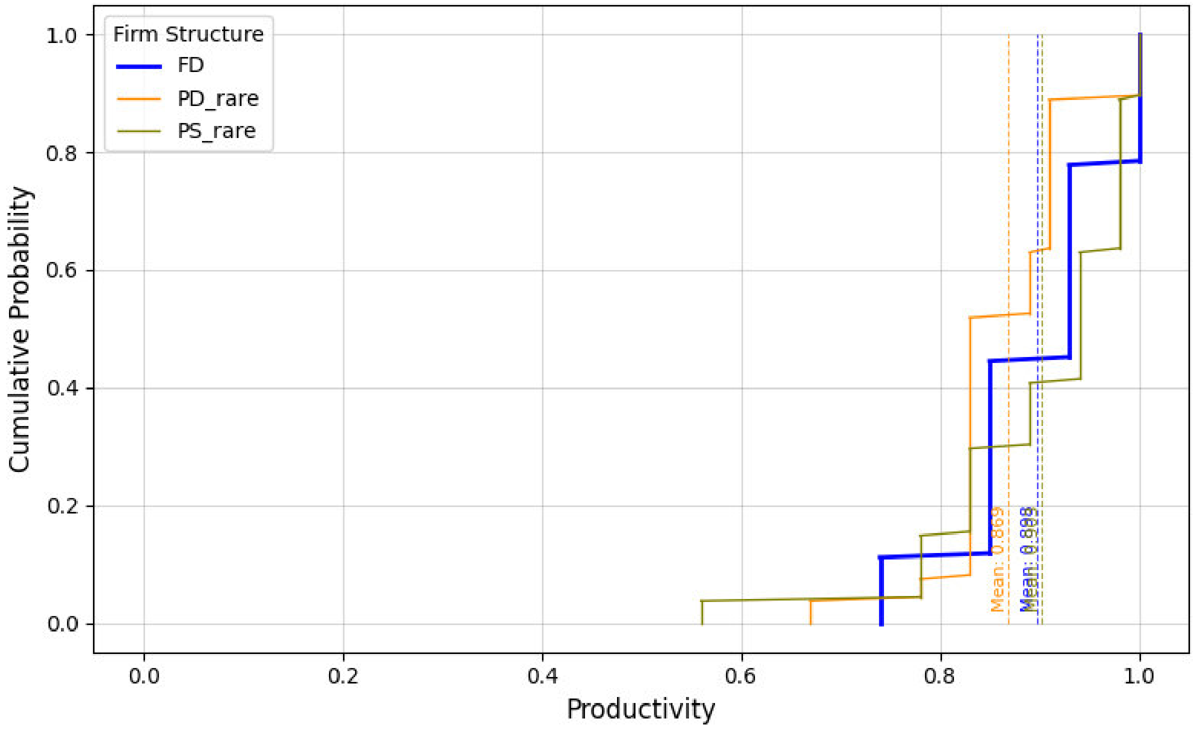

Figure 5: Cumulative distribution functions involving productivity for large firms in hypothetical external market equilibrium, by firm structure and separation on rare task

not stochastically dominating that of the FD-firms, dominates in the middle range of expected productivity. That Figure shows also that due to Result 6 (i.), the variance of the distributions involving expected productivity is substantively reduced by comparison to those characterizing the internal market equilibrium.

With this example we document our foundational insight: We have extracted two counteracting forces influencing the potential to improve on the productivity of firms as differentiated by size and structure. On one hand, internal market activities benefit the large and diversified firms: they achieve high productivity levels at higher frequency than small or specialized firms by internal re-matching. On the other hand, productivity increasing hires in the external market require a precise specification of the firms' desired demand by skill, which necessitates precise knowledge of the firm-internal misalignment between its distribution of tasks and the distribution of its employee's skills. Here the concentration of alternative match patterns at high productivity levels turns into a disadvantage, as it contaminates the precision by which the firm can specify its desired demand.

By Result 7, the second force may even overturn the first, when it comes to a comparison of firms by structure, given size. In particular, by Result 3 (ii.) and (iii.), also illustrated in Figure 2, PS-firms gain minimally from re-matching into common tasks, so that they lose relatively little from separation. FD-firms gain maximally from re-matching, however, so their loss from separation is high. By contrast, by Result 4, the precision generated from internal re-matching is maximal for PS-firms, implying that they can specify their desired demand with minimal noise, while FD-firms suffer minimal precision in the specification of their demand.

4.2.2 Variation across firm sizes

The demand of typical two-worker firms *AB* and *AA* is specified in Table 11. In Table 12 we summarize the gains in expected productivity when demand is optimally satisfied. Finally, in Table 13 we document the demand for the typical smallest, i.e., the one-person firm.

The arrows in Table 12 indicate that in 37.5% of all cases involving D-firms, firm-optimal demands do not correspond to task-optimal ones. By calculations similar to the ones showing Result 7, now involving Tables 6 and 12, we obtain that when separation takes place from the rare task, the expected maximal productivity exceeds with .96 the maximal one observed for large firms. In turn, the maximal productivity expected for small firms is with 1.0 even larger. We summarize in

Result 8: *With satisfaction of desired demand, medium-sized D-firms dominate S-firms in expected productivity at every realized productivity level.*

Proof: See Table 12.

Result 9: *With satisfaction of desired demand, expected maximal productivity, given maximally differentiated firms, decreases in firm size for any realized level below maximal productivity.*

Proof: Calculations from Table 2 and l.h.s. of Table 8 and from Table 6 and l.h.s. of Table 11. Inspection of Table 13 immediately yields maximal expected productivity. The resulting expected productivity of small, medium-sized and large firms is given by 1.0, .91, and .90, respectively.

The intuition for these results follows the intuition specified for Result 7. Roughly speaking, both match pattern alternatives and internal re-matching are constrained by firm size, but because of this, demand estimates are rather precise (for medium-sized firms, see Tables 6 and 7).

Table 11: Demand typical D-firms (*AB*) and S-firms (*AA*) conditional on realized productivity *P* and vacated task *T*

P/T_n	Exp. prod. *AB*			Exp. prod. *AA*		
$P = 1.0$	*a*	*b*	*c*	*a*	*b*	*c*
A	1.0	.83	.67	1.0	.83	.50
B	.50	1.0	.83			
$P = .83$						
A	.89	.72	.44	.92	.75	.42
B	.67	.94	.78			
$P = .67$						
A	.83	.67	.33	.83	.67	.33
B	1.0	.83	.67			
$P = .50$						
A	.50	1.0	.83	.75	.58	.25
B	.50	1.0	.83			
$P = .33$						
A	.83	.67	.33	.67	.50	.17
B	.83	.67	.33			
$P = .00$						
A				.50	.33	.00
B						

Table 12: Medium-sized firms: Productivity gains from external market activity

	Common task		Rare task
Prod.	**A***B*	**AA**	*A***B**
.83	.06	.09	.11
.67	.16	.16	.33⇐ *a*
.50	.50⇐ *b*	.34	.50
.33	.50	.34	.50⇐ *a*

Table 13: Demand A-firms

P/T	Prod. after hiring		
1.0	*a*	*b*	*c*
A	1.0	.67	.00
.67			
A	1.0	.67	.00
.00			
A	1.0	.67	.00

5 Concluding remarks

We have developed a novel parsimonious, yet conceptually rich parametric model of the labor market involving purely horizontally differentiated tasks and skills, to include and compare the performance of firms differentiated by size and structure in internal and external labor market activities under a critical structural uncertainty: Information about workers' skills is revealed only via team productivity.

In this situation, firm performance is shown to be critically influenced by two complementary forces. Increasing firm size and differentiation in the structure of tasks lead naturally to an increase in internal labor market activity, that via re-matching of workers by skill to tasks yields higher productivity. Concomitant to that, however, alternative match patterns that yield the same high productivity level culminate, and blur the specification of the demand for the skills complementing the skill pattern involving the incumbent workers.

The jury is out as to which of the two effects dominates. We have shown just an example in which the latter effect dominates the former. In empirical research, there is plenty of evidence that, e.g., wages (that in our model correspond to productivity) increase in firm size, and this eventually even for equally qualified personnel. In our current model, this pattern can only be generated by the dominance of internal over external labor market activity in generating high productivity, which may materialize in larger firms.

As indicated before, in the second part of our paper we will complete the model by specifying the supply of workers with employment history. It will consist of the very separations that generate the demand analyzed here in the first part of the paper. We will implement an equilibrium concept in which the firms will participate in a series of first-price auctions about the available workers, and show how a sequence of equilibria converges to a stationary one.

The framework developed here, while not elegant, allows for a number of interesting extensions. Examples include the re-specification of the space of workers' skills to include vertical differentiation; the consideration of endogenous quits and dismissals; the explicit consideration of the firm-specific information generated by internal re-matching; or the consideration of demand, or supply shocks via, e.g., the business cycle.

Finally, our results allow for many empirical predictions. Skill information depends critically on firm productivity. Inasmuch firm productivity is reflected in wages (there is ample evidence for this in Mincerian wage estimates), we have an extraordinary base for bringing our predictions to a test.

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Appendix: Additional tabulations

Table 14: Demand typical FD-firms (*ABC*) conditional on realized productivity P and vacated task T

P/T_n	remaining skills (pr's)	Exp. prod. *ABC*		
1.0		*a*	*b*	*c*
A	bc (1)	1.00	.89	.78
B	ac (1)	.89	1.00	.78
C	ab (1)	.78	.89	1.00
.89				
A	ab (1/3), bc (1/3), cc (1/3)	.93	.82	.78
B	aa (1/3), ac (1/3), bc (1/3)	.78	.93	.82
C	ab (1/3), ac (1/3), bb (1/3)	.82	.78	.93
.78				
A	ac (1/3), ab (1/3), cc (1/3)	.85	.85	.82
B	aa (1/3), ab (1/3), bc (1/3)	.82	.85	.85
C	ac (1/3), bb (1/3), bc (1/3)	.85	.82	.85
.56				
A	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74
B	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74
C	aa (1/3), bb (1/3), cc (1/3)	.74	.74	.74

Table 15: Demand typical PD-firms (*AAC*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AAC*		
1.0		*a*	*b*	*c*
A	ac (1)	1.00	.89	.67
C	aa (1)	.89	.67	1.00
.89				
A	aa (1/7), bc (3/7), ac (3/7)	.94	.83	.67
C	aa (1/7), ab (6/7)	.79	.68	.91
.78				
A	ba (1/4), aa (1/4), bc (1/2)	.86	.75	.75
C	ab (1/2), bb (1/2)	.72	.56	.83
.67				
A	cc (1/4), ac (1/4), ba (1/2)	.81	.69	.69
C	ac (1/2), bb (1/2)	.83	.67	.72
.56				
A	cc (1/2), bc (1/2)	.78	.67	.44
C	bc (1)	.89	.78	.56
.44				
A	bb (1)	.67	.44	.78
C	bb (1)	.67	.44	.78
.33				
A	cc (1)	.67	.56	.33
C	cc (1)	.67	.56	.33

Table 16: Demand typical PS-firms (*AAB*) conditional on realized productivity P and vacated task T

P/T_n	remaining skills (pr's)	Exp. prod. *AAB*		
1.0		*a*	*b*	*c*
A	ab (1)	1.00	.89	.78
B	aa (1)	.67	1.00	.89
.89				
A	ac (1/2), bb (1/4), ab (1/4)	.92	.81	.64
B	aa (1/2), ab (1/2)	.83	.94	.83
.78				
A	bc (3/7), ac (3/7), bb (1/7)	.84	.73	.52
B	ab (6/7), bb (1/7)	.98	.87	.76
.67				
A	aa (1/4), bc (3/4)	.75	.75	.56
B	aa (1/4), bb (3/4)	.83	.83	.72
.56				
A	cc (1/2), ac (1/2)	.72	.61	.39
B	ac (1)	.89	.78	.56
.44				
A	bc (1/2), cc (1/2)	.67	.56	.33
B	bc (1)	.78	.67	.44
.22				
A	cc (1)	.56	.44	.22
B	cc (1)	.56	.44	.22

Table 17: Demand typical FS-firms (*AAA*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AAA*		
1.0		*a*	*b*	*c*
A	aa (1)	1.00	.89	.67
.89				
A	ab (2/3), aa (1/3)	.93	.82	.59
.78				
A	bb (1/3), ab (2/3)	.85	.74	.52
.67				
A	ac (1/2), aa (1/4), bb (1/4)	.78	.67	.44
.56				
A	bc (1/3), ac (1/3), ab (1/3)	.70	.59	.37
.44				
A	bc (2/3), bb (1/3)	.63	.52	.30
.33				
A	cc (1/3), ac (2/3)	.56	.44	.22
.22				
A	bc (2/3), cc (1/3)	.48	.37	.15
.00				
A	cc (1)	.33	.22	.00

Table 18: Demand typical D-firms (*AB*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AB*		
1.0		*a*	*b*	*c*
A	b (1)	1.00	.83	.67
B	a (1)	.50	1.00	.83
.83				
A	c (2/3), b (1/3)	.89	.72	.44
B	a (2/3), b (1/3)	.67	.94	.78
.67				
A	c (1)	.83	.67	.33
B	b (1)	1.00	.83	.67
.50				
A	a (1)	.50	1.00	.83
B	a (1)	.50	1.00	.83
.33				
A	c (1)	.83	.67	.33
B	c (1)	.83	.67	.33

Table 19: Demand typical S-firms (*AA*) conditional on realized productivity *P* and vacated task *T*

P/T_n	remaining skills (pr's)	Exp. prod. *AA*		
1.0		*a*	*b*	*c*
A	a (1)	1.00	.83	.50
.83				
A	b (1/2), a (1/2)	.92	.75	.42
.67				
A	b (1)	.83	.67	.33
.50				
A	c (1/2), a (1/2)	.75	.58	.25
.33				
A	c (1/2), b (1/2)	.67	.50	.17
.00				
A	c (1)	.50	.33	.00