

Discussion Paper Series – CRC TR 224

Discussion Paper No. 669
Project B 03

Concentration Indices, Welfare Distortions, and Misallocation in Oligopoly

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October 2026
(First Version : March 2025)

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)
through CRC TR 224 is gratefully acknowledged.

Concentration Indices, Welfare Distortions, and Misallocation in Oligopoly*

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September 29, 2026

Abstract

We derive closed-form expressions for the welfare distortions arising from oligopolistic market power. Our analysis covers competition among multiproduct firms facing constant elasticity of substitution (CES) or multinomial logit demand. The derived formulas rely solely on observed (oligopolistic) firm-level market shares and, under CES demand, the elasticity of substitution. Under CES demand, we fully characterize resource misallocation and extend the analysis to general oligopolistic equilibrium, decomposing welfare losses into within- and between-industry components. Taylor approximations connect these losses to standard concentration indices: within-industry misallocation is proportional to the difference between the cubic Hannah–Kay index and the squared Herfindahl index, while between-industry misallocation is proportional to the variance of the Herfindahl index across industries.

Keywords: Market power, oligopoly, general equilibrium, welfare distortions, misallocation, industry concentration, Herfindahl and Hannah–Kay indices, sufficient statistics.

JEL Codes: L13, D43, D61, E23

1 Introduction

There is a growing empirical literature that reports worrying trends in the evolution of market power and industry concentration. For instance, in an influential paper, De Loecker,

*We thank Michele Bisceglia, Yossi Spiegel, Tommaso Valletti, and seminar and conference audiences at APIOC 2025, CEPR Paris Symposium 2025, EARIE 2025, the Fall 2025 retreat of CRC TR 224, IIOC 2026, Kobe University, MaCCI Annual 2026, and MaCCI Summer Institute 2025 for helpful comments and suggestions. We gratefully acknowledge support from the German Research Foundation (DFG) through CRC TR 224 (Project B03).

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Eeckhout, and Unger (2020) estimate that sales-weighted average markups in the U.S. rose from 21% in 1980 to 61% in 2016.¹ De Loecker and Eeckhout (2018) report comparable figures for the evolution of markups in the global economy. In a similar vein, Barkai (2020) reports that the average four-firm concentration ratio in U.S. NAICS 6-digit industries increased by around 5 percentage points between 1997 and 2012. Koltay, Lorincz, and Valletti (2023) find that, in the five largest European economies, the share of high-concentration industries—those in which the four largest firms account for at least 50% of turnover—grew by more than 60% between 1998 and 2019.²

The question we address in this paper is how much welfare is lost because firms exercise oligopolistic market power, and how much of that loss reflects resources being allocated to the wrong producers or industries. Our results make these quantities recoverable from observed market shares and approximated using concentration measures. Specifically, we derive closed-form expressions for the oligopoly distortions to consumer surplus and aggregate surplus. Moreover, in the spirit of the macroeconomics literature, we provide a resource-based decomposition of the welfare losses from oligopolistic behavior into a within-industry and a between-industry misallocation component. These expressions only rely on observed (oligopolistic) firm-level market shares and one demand parameter. In particular, they require neither detailed information on product-level costs and qualities nor counterfactual market shares under the competitive benchmark. We also relate the expressions to standard concentration indices through Taylor approximations, yielding a role for the Herfindahl index and the cubic Hannah–Kay index in measuring within- and between-industry distortions.

Our baseline framework features quasi-linear preferences, with leisure being the outside good, and a single industry in which firms offer multiple differentiated products and compete in prices. Demand is of either the constant elasticity of substitution (CES) or the multinomial logit (MNL) type. The framework allows for arbitrary firm and product heterogeneity, and, as shown in Nocke and Schutz (2018a, 2025), has several other appealing properties. First, the pricing game is aggregative: each firm’s profit and incentives depend on rivals’ behavior solely through a unidimensional statistic summarizing the intensity of competition. Second, this industry aggregator is also a sufficient statistic for consumer surplus. Third, the type aggregation property holds, so that each firm’s product portfolio can be summarized by a unidimensional statistic—the firm’s type.

We take monopolistic competition as our competitive benchmark. Under this conduct, firms do not internalize their impact on the industry aggregator, and markups are independent of firm size. This is also the limiting outcome when firms become negligible relative to the size of the market. Our comparison thus isolates the additional distortions arising from firms’ oligopolistic market power.

Our first contribution is to express the differences in consumer surplus and aggregate surplus between oligopoly and monopolistic competition in closed form. The formulas re-

¹Consistent with the findings on rising markups, the labor share of GDP has declined in the U.S. and other countries since the early 1980s (see, e.g., Autor, Dorn, Katz, Patterson, and Van Reenen, 2020).

²For critical takes on this literature, see Miller (2025) and Shapiro and Yurukoglu (2026).

quire only the vector of observed (oligopolistic) firm-level market shares and, under CES demand, the elasticity of substitution. The key observation is that each firm’s equilibrium market share pins down its markup and hence the change in its contribution to the industry aggregator if it were to adopt monopolistic competition conduct. These formulas make the welfare comparison operational without requiring the researcher to recover the underlying distribution of product-level costs and qualities.

The formulas clarify both the usefulness and the limitations of concentration as a measure of welfare distortions. A mean-preserving spread of firm-level market shares raises the loss in consumer surplus. The aggregate surplus loss, however, need not increase with concentration. To connect the formulas to standard concentration measures, we consider Taylor approximations around small market shares and around monopolistic competition conduct. At a low order, the distortions are proportional to the Herfindahl index; at higher orders, the cubic Hannah–Kay index also enters (see Hannah and Kay, 1977). These approximations provide a rationale for using standard concentration indices when the full distribution of market shares is unavailable, while making clear that the indices need not capture the exact welfare losses in concentrated industries.

Our second contribution is a resource-based decomposition of the welfare distortion, in the spirit of the macroeconomics literature on misallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). One component measures the welfare loss from the industry employing an inefficient amount of resources; the other measures the loss from inefficiently allocating those resources across products within the industry. In the baseline model, the resource is the representative consumer’s time, allocated between production and leisure (but can equivalently be thought of as any other resource to be allocated between the industry under consideration and a perfectly competitive outside sector). Under CES demand, oligopolistic market power always reduces the resources employed in production relative to monopolistic competition. Under MNL demand, by contrast, both over- and underutilization can arise.

For CES demand, we provide an exact characterization of the within-industry misallocation loss as a function of observed firm-level market shares and the elasticity of substitution. The resource-efficient allocation equalizes relative markups across products, whereas oligopolistic firms with larger market shares charge higher relative markups. Within-industry misallocation therefore vanishes when firms have identical shares. With an outside option, it also vanishes as all but one firm’s shares tend to zero while the remaining firm’s share converges to a value strictly below one, illustrating why greater concentration need not imply greater misallocation. Our approximations show that within-industry misallocation is negligible at a low order and, at a higher order, is proportional to the difference between the cubic Hannah–Kay index and the square of the Herfindahl index. Under MNL demand, market shares alone generally do not identify the resource misallocation loss, because they determine absolute rather than relative markups. Its quantification requires additional product-level information.

Our third contribution extends the sufficient-statistics approach to general oligopolistic equilibrium. We consider a continuum of industries, with CES demand within each industry,

Cobb-Douglas preferences across industries, and a fixed aggregate labor supply. Firms are large within their own industry but negligible relative to the economy, as in Neary (2003), Atkeson and Burstein (2008), and Edmond, Midrigan, and Xu (2023). We derive closed-form expressions for the aggregate welfare loss from oligopolistic behavior and its decomposition into within- and between-industry resource misallocation. The required statistics are the observed firm-level market shares in each industry, the common elasticity of substitution, and industry expenditure shares. Thus, the exact measurement of welfare distortions from observed market shares extends to an economy in which wages and the allocation of resources across industries are endogenous.

General equilibrium changes the nature of the welfare distortion. Aggregate labor supply is fixed, so resources released by one industry must be employed elsewhere. Some industries consequently employ too little labor and others too much relative to the efficient allocation. Moreover, because the elasticity of substitution is common across industries, monopolistic competition equalizes relative markups throughout the economy and is first-best efficient. The welfare loss from oligopoly is therefore entirely due to markup heterogeneity within and across industries. Our exact decomposition separates the gain from efficiently reallocating labor within each industry, holding industry employment fixed, from the additional gain from reallocating labor across industries. The between-industry component vanishes only when the ratio of the wage bill to sales is the same across industries.

Approximating around monopolistic competition conduct allows us again to connect the two components of our welfare decomposition to standard concentration measures. Both components are of second order. The within-industry component is proportional to the expenditure-weighted mean of the difference between the cubic Hannah–Kay index and the squared Herfindahl index. The between-industry component is proportional to the expenditure-weighted variance of the Herfindahl index across industries. Thus, the dispersion of concentration across industries has a distinct role in measuring aggregate misallocation. Even when firms are symmetric within every industry, differences in concentration across industries can generate a welfare loss by distorting the allocation of labor between them.

Related literature. Our paper contributes to the macroeconomics literature on misallocation, pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), and surveyed by Restuccia and Rogerson (2017). This literature studies welfare and productivity losses from frictions that distort the allocation of inputs within and across industries. Markup heterogeneity is one such friction. In a general-equilibrium framework with input-output linkages and distortions represented by wedges, Baqaee and Farhi (2020) study aggregate productivity and allocative efficiency. Particularly related to our analysis, they derive a second-order approximation to the aggregate productivity loss from markups and other wedges around the efficient allocation. Focusing on quantitative models with endogenous markups, Edmond, Midrigan, and Xu (2023) study both monopolistic and oligopolistic competition and decompose the welfare loss from markups into components due to the average markup, factor misallocation, and inefficient entry. Related quantitative work with oligopolistic competition

includes Holmes, Hsu, and Lee (2014), who investigate the effects of heterogeneous markups on allocative efficiency and welfare in a model of international trade, and De Loecker, Eeckhout, and Mongey (2021), who quantify the welfare consequences of changes in market power and market structure in a general-equilibrium model with oligopolistic output markets. In a Cournot model with differentiated products and linear demand, Pellegrino (2025) estimates the deadweight loss from oligopolistic competition and its misallocation component. Our contribution is to derive exact expressions for welfare losses and their resource-misallocation components from observed oligopoly market shares. In general equilibrium, these expressions account for both within- and between-industry misallocation; their approximations identify distinct roles for concentration within industries and its dispersion across industries.

Our work also relates to the literature studying the relationship between industry concentration and market performance. Much of this literature has focused on the homogeneous-goods Cournot model. Cowling and Waterson (1976) show that the Herfindahl index provides a measure of an industry’s average markup and profitability (see Belleflamme and Peitz, 2010, for a textbook treatment). Dansby and Willig (1979) show that the industry performance gradient index, which measures the rate of potential improvement in aggregate surplus from a small variation in the output vector, is proportional to the square root of the Herfindahl index. Assuming that demand has constant curvature, Corchón (2008) shows that the percentage deadweight loss in Cournot oligopoly depends solely on the Herfindahl index, demand curvature, and the market share of the top firm (see also Ritz, 2014). Spiegel (2021) shows that the ratio of producer surplus to consumer surplus is equal to the product of the Herfindahl index and the elasticity of consumer surplus with respect to aggregate output.

Fewer results link concentration indices to industry performance in differentiated-products models. In an oligopoly model with CES preferences and price or quantity competition, Grassi (2017) relates the industry average markup to the Herfindahl index and higher-order Hannah–Kay indices. In Feenstra and Weinstein (2017)’s model with translog preferences, indirect utility depends on the Herfindahl index both directly, through preferences, and indirectly, through endogenous markups. In earlier work, Nocke and Schutz (2023, 2025), we show that the welfare effects of a merger without synergies are approximately proportional to the naively computed, merger-induced change in the Herfindahl index.³ Nocke and Whinston (2022) show theoretically and empirically that the synergies required for a merger not to harm consumers are positively related to the induced change in the Herfindahl index, but not to its level. We contribute exact expressions for the welfare distortions arising from oligopolistic conduct, and establish how their approximations relate to standard concentration measures.

Road map. The remainder of the paper is organized as follows. Section 2 presents the multiproduct-firm oligopoly framework and its equilibrium. Section 3 derives exact expressions for the welfare distortions and relates them to concentration indices. Section 4 develops the resource-based decomposition and characterizes within-industry misallocation. Section 5

³Breinlich, Fadinger, Nocke, and Schutz (2020) use an approximation around monopolistic competition conduct to derive an industry-level gravity equation for trade flows in oligopoly.

extends the analysis to general oligopolistic equilibrium, deriving the aggregate welfare loss and its within- and between-industry components. Section 6 concludes.

2 The Multiproduct-Firm Oligopoly Framework

In this section, we present the multiproduct-firm pricing game with CES and MNL demand of Nocke and Schutz (2025). We describe the model in Section 2.1 and characterize equilibrium behavior using aggregative games techniques in Section 2.2.

2.1 The Model

Consider an economy with a representative agent consuming leisure and horizontally differentiated products. Preferences are quasi-linear—with leisure being the outside good, which we henceforth take as the *numéraire*. The indirect utility function takes the form

$$y + V(\mathbf{p}) = y + V_0 \log \left[H^0 + \sum_{j \in \mathcal{N}} h_j(p_j) \right], \quad (1)$$

where $y > 0$ is income, $p_j > 0$ is the price of product j , $V_0 > 0$ is a market size parameter, and $H^0 \geq 0$ is a baseline-utility parameter. We adopt two alternative specifications for the functions h_j : in the case of MNL demand, these functions take the exponential form $h_j(p_j) = \exp\left(\frac{a_j - p_j}{\lambda}\right)$; in the case of CES demand, they are power functions, $h_j(p_j) = a_j p_j^{1-\sigma}$. The parameter a_j (which must be strictly positive in the CES case) represents the quality of product j ; $\lambda > 0$ and $\sigma > 1$ measure the substitutability of products. Defining the aggregator as

$$H(\mathbf{p}) \equiv H^0 + \sum_{j \in \mathcal{N}} h_j(p_j),$$

consumer surplus can be rewritten as $V(\mathbf{p}) = V_0 \log H(\mathbf{p})$.

The direct utility function associated with these preferences can be obtained by duality. In case $H^0 = 0$, it is given, up to an additive constant, by

$$U(x_0, \mathbf{x}) = x_0 + V_0 \sigma \log \sum_{j \in \mathcal{N}} a_j^{\frac{1}{\sigma}} x_j^{\frac{\sigma-1}{\sigma}}$$

for CES preferences and

$$U(x_0, \mathbf{x}) = \begin{cases} x_0 + \sum_{j \in \mathcal{N}} a_j x_j - V_0 \sum_{j \in \mathcal{N}} \frac{\lambda x_j}{V_0} \log \left(\frac{\lambda x_j}{V_0} \right) & \text{if } \sum_{j \in \mathcal{N}} x_j = \frac{V_0}{\lambda} \\ -\infty & \text{otherwise} \end{cases}$$

for MNL preferences, where x_0 denotes consumption of leisure and x_j consumption of product $j \in \mathcal{N}$. In case $H^0 > 0$, the direct utility functions are more involved and therefore

relegated to Online Appendix F. We assume throughout that the representative consumer's time endowment is sufficiently large, so that leisure remains positive in all the allocations considered.

Applying Roy's identity, we obtain the demand for product i :

$$D_i(\mathbf{p}) = V_0 \frac{-h'_i(p_i)}{H(\mathbf{p})}. \quad (2)$$

This demand system can also be derived from a discrete/continuous choice model, in which each consumer first chooses either one of the products or the outside option (not to be confused with the outside good, leisure), and then decides how many units of the chosen product, if any, to consume (see Anderson, de Palma, and Thisse, 1992; Nocke and Schutz, 2018a). Under this micro-foundation, any given consumer ends up choosing product i with probability h_i/H ; conditional on having chosen product i , a consumer purchases $-h'_i/h_i$ units of that product. Moreover, V_0 is the total mass of consumers, and $\log H^0$ is the mean utility of the outside option. In the remainder of the paper, we normalize V_0 to 1.

The set of firms, $\mathcal{F}$, is a partition of $\mathcal{N}$. That is, each product is sold by one and only one firm. We assume throughout that there are at least two firms. The profit of firm $f \in \mathcal{F}$ is given by

$$\pi^f(\mathbf{p}) = \sum_{j \in f} (p_j - c_j) \frac{-h'_j(p_j)}{H(\mathbf{p})}, \quad (3)$$

where $c_j > 0$ denotes product j 's constant marginal cost of production, incurred in units of labor. Firms compete by simultaneously setting prices.⁴ The solution concept is Nash equilibrium.

The equilibrium characterization below and our results on concentration indices as measures of oligopoly distortions rely on firm-level market shares. We define the market share of firm f as

$$s^f = \sum_{j \in f} \frac{h_j}{H}.$$

In the discrete/continuous choice micro-foundation, this corresponds to the probability that any given consumer chooses one of firm f 's products. Moreover, s^f corresponds to firm f 's output share under MNL demand and to firm f 's revenue share under CES demand. In both cases, the firms' market shares add up to $1 - H^0/H$, where H^0/H is the market share of the outside option.

2.2 Equilibrium Behavior

In this subsection, we summarize the key properties of the equilibrium. We refer the reader to Nocke and Schutz (2018a, 2025) for details and proofs.

⁴In Online Appendix H, we analyze instead the case of quantity competition under CES demand without an outside option.

The equilibrium exists and is unique. Each firm f 's pricing behavior can be summarized by a unidimensional sufficient statistic, $\mu^f > 1$, referred to as firm f 's ι -markup. Specifically, given the ι -markup μ^f , firm f prices each of its products $i \in f$ according to the following inverse elasticity rule:

$$\frac{p_i - c_i}{p_i} = \frac{\mu^f}{p_i h_i''(p_i) / (-h_i'(p_i))}.$$

Under MNL demand, the elasticity term $p_i h_i'' / (-h_i')$ is equal to p_i / λ , and so firm f charges the same absolute markup, $p_i - c_i = \lambda \mu^f$, on every product in its portfolio. Under CES demand, the elasticity term is constant and equal to σ , implying that firm f sets the same Lerner index, $(p_i - c_i) / p_i = \mu^f / \sigma$, on every product $i \in f$. With such a common ι -markup, the profit of firm f is simply given by

$$\pi^f = \alpha \mu^f s^f. \quad (4)$$

The equilibrium ι -markup and market share of firm f satisfy the following condition:

$$\mu^f = \frac{1}{1 - \alpha s^f}, \quad (5)$$

where $\alpha \equiv 1$ under MNL demand and $\alpha \equiv (\sigma - 1) / \sigma$ under CES demand. Equation (5) can be understood as an upward-sloping firm-level supply relation: a firm that has a high market share has significant market power, and therefore sets a high ι -markup.

In equilibrium, μ^f and s^f must also be consistent with the following firm-level demand relation:

$$s^f = \frac{T^f}{H} \psi(\mu^f) \quad \text{with } \psi(\mu) \equiv \begin{cases} (1 - (1 - \alpha)\mu)^{\frac{\alpha}{1-\alpha}} & \text{under CES demand,} \\ e^{-\mu} & \text{under MNL demand,} \end{cases} \quad (6)$$

where $T^f \equiv \sum_{j \in f} h_j(c_j)$ is firm f 's type. According to equation (6), firm f 's market share is a decreasing function of firm f 's markup, with T^f / H acting as a demand shifter. The sufficient statistic T^f summarizes all the relevant information about firm f 's product portfolio. As it is higher if firm f sells more products, with lower marginal costs or with higher qualities, it can be viewed as a measure of how good the firm is. Note from equation (6) that firm f 's contribution to the aggregator when setting an ι -markup of μ^f is equal to $s^f H = T^f \psi(\mu^f)$.

The system of equations (5)–(6) has a unique solution in (μ^f, s^f) , which pins down $m(T^f / H)$ and $S(T^f / H)$, firm f 's *markup fitting-in function* and *market-share fitting-in function*, respectively. Both functions are strictly increasing: a firm that has a higher type or operates in a less competitive environment (lower H) sets a higher ι -markup and receives a larger market share. It can also be shown that firm f 's equilibrium profit satisfies $\pi^f = \mu^f - 1$. We thus obtain the *profit fitting-in function* of firm f :

$$\pi \left(\frac{T^f}{H} \right) \equiv m \left(\frac{T^f}{H} \right) - 1 = \frac{\alpha S(T^f / H)}{1 - \alpha S(T^f / H)}, \quad (7)$$

where we have used equation (5) to obtain the second equality.

Finally, the equilibrium level of the aggregator, H^* , is uniquely pinned down by the condition that market shares add up to one:

$$\frac{H^0}{H} + \sum_{f \in \mathcal{F}} S \left(\frac{T^f}{H} \right) = 1. \quad (8)$$

Summing up: the aggregator level, H^* , is pinned down by equilibrium condition (8). Firm-level equilibrium behavior is then determined by the corresponding fitting-in functions: in equilibrium, firm f sets an ι -markup of $\mu^{f*} = m(T^f/H^*)$, commands a market share of $s^{f*} = S(T^f/H^*)$, and earns a profit of $\pi^{f*} = \pi(T^f/H^*)$. Equilibrium prices can be backed out from the inverse elasticity rule: for every $i \in f$, p_i^* is equal to $c_i + \lambda\mu^{f*}$ under MNL demand, and to $c_i\sigma/(\sigma - \mu^{f*})$ under CES demand.⁵

Monopolistic competition. Monopolistic competition arises under the behavioral assumption that firms believe that they have no impact on the industry aggregator, i.e., that $\partial H/\partial p_i = 0$ for every i . The first-order condition for product $i \in f$ under this behavioral assumption becomes

$$\frac{p_i - c_i}{p_i} = \frac{1}{p_i h_i''(p_i)/(-h_i'(p_i))}.$$

That is, regardless of its market share, each firm f charges an ι -markup of 1 on each of its products. We thus refer to $\mu^f = 1$ as the monopolistic competition ι -markup.

Under fully fledged oligopoly conduct, the outcome in which each firm sets the monopolistic competition ι -markup arises in the limit as firms' market shares tend to zero, i.e., when firms become atomless. Such a limiting outcome can be obtained by infinitely replicating the population of firms, or by making the mean utility of the outside option, $\log H^0$, go to infinity.

Conduct parameter. For some of our approximation results, it is useful to adopt a flexible approach to industry conduct, which bridges the gap between monopolistic competition and fully fledged oligopoly. Specifically, suppose that, when firm f increases the price of its product $i \in f$ by dp_i , it believes that the impact on the aggregator is $\theta(\partial H/\partial p_i)dp_i$, where

⁵A more general version of the model would feature *nested* CES/MNL demand systems, in which the set of products is partitioned into a set of nests. In Nocke and Schutz (2025), we show that, provided each firm owns one or several entire nests of products, the pricing game remains aggregative with consumer surplus being equal to the log of the aggregator, and the common ι -markup and type aggregation properties continue to hold. Moreover, equilibrium behavior is still pinned down by equations (5)–(8), with the corresponding redefinitions of the demand parameter, firm types, and normalized ι -markup described in Appendix B of Nocke and Schutz (2025). As the analysis in the present paper relies only on these equations, this implies that all the results stated below continue to hold under nested CES or MNL demand, provided each firm owns one or several entire nests of products.

$\theta \in [0, 1]$ is the industry conduct parameter.⁶ Note that, for parsimony, the parameter θ is assumed to be identical across firms. The fully fledged oligopoly model, in which firms have correct conjectures, corresponds to $\theta = 1$. Monopolistic competition corresponds to $\theta = 0$.

For a given conduct parameter θ , the analysis proceeds as above. The firm-level supply relation (equation (5)) must be adapted as follows:

$$\mu^f = \frac{1}{1 - \alpha\theta s^f}. \quad (9)$$

The markup and market-share fitting-in functions, $m(T^f/H; \theta)$ and $S(T^f/H; \theta)$, jointly solve equations (6) and (9). The expression for the profit fitting-in function becomes:

$$\pi\left(\frac{T^f}{H}; \theta\right) \equiv \frac{\alpha S(T^f/H; \theta)}{1 - \alpha\theta S(T^f/H; \theta)}. \quad (10)$$

Note that $\pi(x; \theta) = (m(x; \theta) - 1)/\theta$ for $\theta > 0$.

The equilibrium aggregator level, $H^*(\theta)$, continues to be pinned down by market shares having to add up to one:

$$\frac{H^0}{H} + \sum_{f \in \mathcal{F}} S\left(\frac{T^f}{H}; \theta\right) = 1. \quad (11)$$

Let $s^f(\theta) \equiv S(T^f/H^*(\theta); \theta)$ denote firm f 's equilibrium market share. Define also equilibrium consumer surplus as $\text{CS}(\theta) \equiv \log H^*(\theta)$, industry profit as

$$\Pi(\theta) \equiv \alpha \sum_{f \in \mathcal{F}} \frac{s^f(\theta)}{1 - \alpha\theta s^f(\theta)} \quad (\text{see equation (10)}),$$

and aggregate surplus as

$$\text{AS}(\theta) \equiv \text{CS}(\theta) + \Pi(\theta) = \log H^*(\theta) + \alpha \sum_{f \in \mathcal{F}} \frac{s^f(\theta)}{1 - \alpha\theta s^f(\theta)}. \quad (12)$$

The analysis in the remainder of the paper should be understood as being carried out under the assumption that $\theta = 1$, unless explicitly stated otherwise.

3 Market-Share Based Measures of Oligopoly Distortions

In this section, we provide formulas for the loss in consumer surplus and aggregate surplus from oligopolistic market power, $\Delta \text{CS} \equiv \text{CS}(1) - \text{CS}(0)$ and $\Delta \text{AS} \equiv \text{AS}(1) - \text{AS}(0)$. Note

⁶Our treatment of industry conduct under product differentiation is the natural counterpart to the classic approach for homogeneous-products industries, as surveyed in Bresnahan (1989).

that, as a competitive benchmark for the case without oligopolistic market power, we use the equilibrium outcome under monopolistic competition (rather than perfect competition) for the following reason. Consider an infinite replication of the population of firms, so that each firm's limiting size is negligible relative to the size of the market. In our differentiated-goods framework, this results in the monopolistic competition (rather than perfect competition) outcome, as explained in Section 2.2.

We begin by deriving exact formulas for the welfare distortions arising from oligopolistic behavior (Section 3.1). Remarkably, these formulas only require knowledge of the demand-side parameter α and of the vector of observed firm-level market shares (but not of the unobserved, counterfactual market shares under monopolistic competition). In practice, however, researchers may have access only to standard concentration indices. For this reason, we also relate the distortions to combinations of such indices. We do so in Section 3.2 using Taylor approximations both around small market shares and monopolistic competition conduct.

3.1 Oligopoly Distortions: Exact Results

To obtain formulas for the welfare distortions, we first derive expressions for the ratio of the monopolistic competition aggregator to the oligopoly aggregator, and for the market share of the outside option under monopolistic competition. Let $\Sigma(\theta) \equiv \sum_{f \in \mathcal{F}} s^f(\theta)$ denote the equilibrium aggregate share of the inside goods when the conduct parameter is θ . For the ratio of the monopolistic competition aggregator to the oligopoly aggregator, we have:

$$\begin{aligned} \frac{H^*(0)}{H^*(1)} &= \frac{H^0 + \sum_{f \in \mathcal{F}} T^f \psi(1)}{H^*(1)} = 1 - \Sigma(1) + \sum_{f \in \mathcal{F}} \frac{T^f}{H^*(1)} \psi(1) \\ &= 1 - \Sigma(1) + \sum_{f \in \mathcal{F}} s^f(1) \frac{\psi(1)}{\psi[1/(1 - \alpha s^f(1))]}, \end{aligned}$$

where the first equality follows, as firm f 's contribution to the aggregator when setting an ι -markup of μ^f is $T^f \psi(\mu^f)$ and the ι -markup under monopolistic competition is $\mu^f = 1$, and the third equality follows by equations (5) and (6). Taking logs on both sides and making use of the fact that $\text{CS}(\theta) = \log H^*(\theta)$, we obtain $-\Delta \text{CS}$ as a function of the observed (oligopoly) market shares.

Turning to the aggregate share of the inside goods under monopolistic competition, we have:

$$\Sigma(0) = 1 - \frac{H^0}{H^*(1)} \frac{H^*(1)}{H^*(0)} = 1 - \frac{1 - \Sigma(1)}{1 - \Sigma(1) + \sum_{f \in \mathcal{F}} s^f(1) \frac{\psi(1)}{\psi[1/(1 - \alpha s^f(1))]}.$$

Plugging this into

$$\Delta\Pi \equiv \Pi(1) - \Pi(0) = \alpha \left(\sum_{f \in \mathcal{F}} \frac{s^f(1)}{1 - \alpha s^f(1)} - \Sigma(0) \right)$$

(see equation (10)) allows us to express the change in industry profit when going from monopolistic competition to oligopoly as a function of the observed (oligopoly) market shares.

The following proposition gathers these formulas and studies their behavior:⁷

Proposition 1. *The welfare distortions arising from oligopolistic market power can be expressed as functions of observed firm-level market shares under oligopoly as follows:*

$$\begin{aligned} \Delta CS(\mathbf{s}) &= -\log \left[1 + \sum_{f \in \mathcal{F}} s^f (\phi(s^f) - 1) \right], \\ \Delta\Pi(\mathbf{s}) &= \alpha \left[\sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f} - \frac{\sum_{f \in \mathcal{F}} s^f \phi(s^f)}{1 + \sum_{f \in \mathcal{F}} s^f (\phi(s^f) - 1)} \right], \\ \Delta AS(\mathbf{s}) &= \Delta CS(\mathbf{s}) + \Delta\Pi(\mathbf{s}), \end{aligned}$$

where

$$\phi(s^f) \equiv \begin{cases} \left(\frac{1 - \alpha s^f}{1 - s^f} \right)^{\frac{\alpha}{1 - \alpha}} & \text{under CES demand,} \\ \exp \frac{s^f}{1 - s^f} & \text{under MNL demand.} \end{cases}$$

The function ΔCS is strictly negative, coordinatewise strictly decreasing, and strictly Schur-concave, whereas $\Delta\Pi$ is strictly positive, coordinatewise strictly increasing, and strictly Schur-convex. Moreover, $\Delta AS \leq 0$, with equality if and only if demand is MNL, there is no outside option, and firms have identical market shares. In general, ΔAS is neither Schur-concave nor Schur-convex; nor is it globally increasing or decreasing.

Proof. See Appendix A. □

The first part of Proposition 1 shows that the welfare distortions arising from oligopolistic market power can be quantified without knowledge of product-level information, such as the number of products, their marginal costs, and qualities. All that is required are the observed, oligopolistic firm-level market shares, and not the counterfactual market shares under the competitive benchmark of monopolistic competition. (When an outside option is present, implementation also requires its share, $1 - \Sigma$: conventional inside-market shares, which sum to one, must be multiplied by Σ to obtain the shares used in our formulas.)

The second part of the proposition studies how these distortions are affected by changes in market shares (induced by a change in the underlying vector of firm types). According to the proposition, any increase in the vector of firm-level market shares makes ΔCS more

⁷Recall that a function is Schur-concave (resp., Schur-convex) if it is decreasing (resp., increasing) in the mean-preserving spread order.

negative, i.e., raises the distortion to consumer surplus. Moreover, ΔCS becomes more negative as the industry becomes more concentrated—that is, ΔCS is Schur-concave in market shares. The intuition for Schur-concavity can best be seen in the case with an outside option, $H^0 > 0$. From the adding-up condition, $H^0/H^* + \sum_{f \in \mathcal{F}} s^f = 1$, the value of the aggregator under oligopoly can be expressed as a function of the sum of the observed market shares as $H^* = H^0/(1-\Sigma)$. Consider a mean-preserving spread in the vector of market shares, which, in light of the previous sentence, leaves consumer surplus under oligopoly unchanged. The concavity of the market-share fitting-in function $S(\cdot)$, caused by larger firms charging a higher markup, implies that the mean-preserving spread of the market share vector must have been caused by a sum-increasing change in the vector of firm types.⁸ As consumer surplus under monopolistic competition, given by $\log\left(H^0 + \sum_f T^f \psi(1)\right)$, depends only on the sum of those types, this change increases consumer surplus under monopolistic competition, and thus makes ΔCS more negative.

By contrast, the profit gap between oligopoly and monopolistic competition is strictly positive, and strictly increasing in the level of market shares and in their concentration. To see the intuition for Schur-convexity of the profit gap, consider first the case without an outside option, $H^0 = 0$. In that case, industry profit under monopolistic competition is constant (and equal to α). Industry profit under oligopoly, however, strictly increases as concentration increases, by making the market shares of the high-markup firms larger, and by further raising those markups. (The impact through the decrease in the smaller firms' markups is dominated.) It follows that $\Delta \Pi$ increases with industry concentration. In the case with an outside option, $H^0 > 0$, there is a countervailing effect: a mean-preserving spread of market shares under oligopoly raises $\Sigma(0)$, the aggregate share of the inside goods under monopolistic competition (see the previous paragraph), and thus $\Pi(0)$. We show in the Appendix that this effect is outweighed by the effect through $\Pi(1)$.

The impact of oligopolistic behavior on aggregate surplus, ΔAS , is weakly negative, and strictly so unless demand is MNL without an outside option and firms have symmetric market shares. In light of the opposite signs of the effects on consumer surplus and industry profit, the distortion to aggregate surplus is in general neither globally increasing nor decreasing in the vector of market shares. Moreover, and for the same reason, ΔAS is neither globally Schur-concave nor Schur-convex. In the Appendix, we provide examples in which the aggregate-surplus distortion is (i) not increasing, (ii) not decreasing, (iii) not Schur-convex, and (iv) not Schur-concave in the vector of market shares.

Distortions relative to the perfect competition outcome. Although we believe that monopolistic competition is the natural competitive benchmark when products are differentiated, it is also instructive to obtain exact formulas for the welfare distortions arising from oligopolistic market power relative to the perfect competition outcome. Let Δ^{pc} CS and

⁸The concavity of S is stated and proven in Lemma XXVI in the Online Appendix to Nocke and Schutz (2018a).

Δ^{pc} AS denote these welfare distortions.

Let H^{pc} denote the value of the aggregator under perfect competition, i.e., $H^{pc} = H^0 + \sum_{f \in \mathcal{F}} T^f$. We have:

$$\begin{aligned} \frac{H^{pc}}{H^*(1)} &= \frac{H^0 + \sum_{f \in \mathcal{F}} T^f}{H^*(1)} = 1 - \Sigma(1) + \sum_{f \in \mathcal{F}} \frac{T^f}{H^*(1)} \\ &= 1 - \Sigma(1) + \sum_{f \in \mathcal{F}} \frac{s^f(1)}{\psi[1/(1 - \alpha s^f(1))]} \end{aligned}$$

Since firms make zero profit under perfect competition, we immediately obtain:

$$\begin{aligned} \Delta^{pc} \text{CS} &= -\log \left[1 + \sum_{f \in \mathcal{F}} s^f \left(\frac{\phi(s^f)}{\psi(1)} - 1 \right) \right], \\ \Delta^{pc} \text{AS} &= \Delta^{pc} \text{CS} + \alpha \sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f}, \end{aligned}$$

where s^f is firm f 's observed market share in the oligopolistic equilibrium. It can be checked that the monotonicity and convexity properties of the distortions are qualitatively the same as when distortions are measured against the monopolistic competition benchmark.

3.2 Distortions and Concentration Indices

While we have shown that distortions can be written as functions of market shares, the expressions do not involve standard concentration indices. To relate welfare distortions to such standard measures, we undertake a series of Taylor approximations, around small market shares and in the neighborhood of monopolistic competition conduct. As we will see below, these provide additional theoretical underpinnings for the use of the Herfindahl index (HHI) as a concentration measure in practice.

Turning to the definition of those indices, note that if consumers have access to an outside option ($H^0 > 0$), the sum of the choice probabilities for the firms' products, Σ , is strictly less than one. By contrast, in antitrust practice, the outside option is not accounted for in that firms' market shares always add up to one. In the following, we therefore renormalize firms'

market shares when defining concentration indices.⁹ Specifically, the HHI is defined as

$$\text{HHI} \equiv \sum_{f \in \mathcal{F}} \left(\frac{s^f}{\Sigma} \right)^2.$$

As we will see below, our higher-order approximations also give rise to the cubic Hannah–Kay concentration index (CHK), defined as

$$\text{CHK} \equiv \sum_{f \in \mathcal{F}} \left(\frac{s^f}{\Sigma} \right)^3.$$

Depending on the context, we will sometimes think of these indices and Σ as being functions of the parameter θ or the vector of equilibrium market shares $\mathbf{s}$.

Approximations around small market shares. For this set of approximations, we assume that consumers have access to an outside option, so that firms’ (unnormalized) market shares can be made arbitrarily small. We have:

Proposition 2. *In the neighborhood of $\mathbf{s} = \mathbf{0}$, the oligopoly distortions are:¹⁰*

$$\begin{aligned} \Delta \text{CS}(\mathbf{s}) &= -\alpha \Sigma(\mathbf{s})^2 \left(\text{HHI}(\mathbf{s}) + \frac{1}{2}(1 + 2\alpha)\Sigma(\mathbf{s}) \text{CHK}(\mathbf{s}) \right) + o(\|\mathbf{s}\|^3), \\ \Delta \text{AS}(\mathbf{s}) &= -\alpha \Sigma(\mathbf{s})^2 \left(\text{HHI}(\mathbf{s})(1 - \alpha \Sigma(\mathbf{s})) + \frac{1}{2}(1 + 3\alpha)\Sigma(\mathbf{s}) \text{CHK}(\mathbf{s}) \right) + o(\|\mathbf{s}\|^3). \end{aligned}$$

Proof. See Appendix D. □

To understand the expressions for the oligopoly distortions, recall first that $\Sigma(\mathbf{s})$ is close to zero when $\mathbf{s}$ is small. Hence, there is no first-order distortion to consumer surplus and aggregate surplus, whereas the second-order distortion is proportional to the Herfindahl index, $\text{HHI}(\mathbf{s})$. Note that the second-order terms in both approximations are Schur-concave. This is consistent with our result from the previous subsection, that the exact expression for ΔCS is Schur-concave. Although the exact expression for the aggregate surplus distortion

⁹Like any approach based on market shares and concentration measures, ours requires a definition of the relevant market: market shares cannot be computed without first specifying which firms belong to the market. This is a nontrivial issue in applications. In the macroeconomics literature, markets are often identified with industries defined by standard industry classifications, such as NAICS or SIC. While this approach is clearly imperfect—industry classifications need not coincide with economically relevant product markets—it is often the only feasible option when working with economy-wide firm-level data. In antitrust, markets are often delineated based on patterns of demand substitution and, in particular, the hypothetical-monopolist test; see, e.g., Werden (1992) and Davis and Garcés (2010, chap. 4). A recent alternative is provided by Pellegrino (2025). In his linear-demand framework, all publicly listed U.S. firms can in principle be substitutes, with the strength of pairwise substitution determined by the cosine similarity of firms’ product descriptions; a scalar parameter determines the overall weight placed on these common product characteristics in demand.

¹⁰ $o(\cdot)$ is Landau’s little-o notation: $f(x) = o(g(x))$ in the neighborhood of $x = x^0$ if $f(x)/g(x) \xrightarrow{x \rightarrow x^0} 0$.

is not globally Schur-concave (see again the previous subsection), Proposition 2 implies that ΔAS is indeed more negative in more concentrated industries, as long as market shares are not too large.

Finally, note that the oligopoly distortions to consumer surplus and aggregate surplus coincide at the second order, as $\Sigma(\mathbf{s})^3$ is of third order. Put differently, at the second order, there is no difference between industry profit under oligopoly and monopolistic competition. At the third order, the difference is given by

$$\alpha^2 \Sigma(\mathbf{s})^3 \left[\text{HHI}(\mathbf{s}) - \frac{1}{2} \text{CHK}(\mathbf{s}) \right],$$

with the term inside brackets being strictly positive and strictly Schur-convex in $\mathbf{s}$. Thus, the gap between industry profit under oligopoly and monopolistic competition widens as the industry becomes more concentrated.

Approximations around monopolistic competition conduct. The results below hold regardless of whether there is an outside option. For any given conduct parameter θ , exact formulas for the welfare distortions $\Delta CS(\theta) \equiv CS(\theta) - CS(0)$ and $\Delta AS(\theta) \equiv AS(\theta) - AS(0)$ similar to those in Proposition 1 are provided in Appendix E. Approximating these formulas in the neighborhood of monopolistic competition conduct, $\theta = 0$, we obtain:¹¹

$$\Delta CS(\theta) = -\alpha \Sigma(\theta)^2 \text{HHI}(\theta) \theta - \frac{\alpha}{2} \Sigma(\theta)^3 \left[(1 + 2\alpha) \text{CHK}(\theta) - \alpha \Sigma(\theta) \text{HHI}(\theta)^2 \right] \theta^2 + o(\theta^2)$$

and

$$\begin{aligned} \Delta AS(\theta) = & -\alpha \Sigma(\theta)^2 \text{HHI}(\theta) (1 - \alpha \Sigma(\theta)) \theta - \frac{1}{2} \alpha \Sigma(\theta)^3 \times \\ & \left[(1 + 3\alpha - (\alpha + 2\alpha^2) \Sigma(\theta)) \text{CHK}(\theta) - (\alpha + 2\alpha^2(1 - \Sigma(\theta))) \Sigma(\theta) \text{HHI}(\theta)^2 \right] \theta^2 + o(\theta^2). \end{aligned}$$

Proof. See Appendix E. □

The proposition shows that at the first order in θ , the welfare distortions are again proportional to the Herfindahl index for a given Σ . Hence, for sufficiently small θ , $\Delta CS(\theta)$ and $\Delta AS(\theta)$ are both Schur-concave in the vector of observed firm-level market shares (except in the special case of MNL demand without an outside option). At the second order in θ , both the squared Herfindahl index and the cubic Hannah–Kay index appear.

The expression for the aggregate surplus distortion becomes particularly simple in the

¹¹The first-order terms in this proposition can also be obtained from Lemmas 17 and 19 in the Online Appendix to Nocke and Schutz (2025).

case of MNL demand without an outside option:

$$AS(\theta) - AS(0) = -\frac{1}{2}\theta^2 (\text{CHK}(\theta) - \text{HHI}^2(\theta)) + o(\theta^2).$$

Thus, at the first order in θ , there is no distortion. The intuition is that, in the absence of an outside option, the level of the average markup is irrelevant for aggregate surplus, as it is merely a transfer from consumers to firms. Hence, any inefficiencies must be due to markup heterogeneity; these inefficiencies are, however, of second order. At that order, the oligopoly distortion to aggregate surplus is proportional to the difference between the cubic Hannah–Kay index and the square of the Herfindahl index. By the Cauchy-Schwarz inequality, that difference is strictly positive if firms are asymmetric and equal to zero otherwise (see footnote 16 below). Note that, as $\text{CHK} - \text{HHI}^2$ is not Schur-convex in the market share vector, it is generally not true that greater industry concentration results in a larger oligopoly distortion. For example, it is easily seen that $\text{CHK} - \text{HHI}^2$ vanishes not only when firms become perfectly symmetric, but also in the limit as one of the firms’ market share tends to 1 (while those of the others tend to zero).

4 A Resource-Based Decomposition of Oligopoly Distortions

In this section, we study the welfare losses from oligopolistic behavior through the lens of the macroeconomics literature on resource misallocation (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). Under our leading interpretation of the outside good, the only resource in our model is the representative consumer’s time, which is allocated between work (on the different inside goods) and leisure. Alternatively, we could think of a fixed supply of some other resource, which is allocated between production of the inside goods and of an outside good, with the latter being sold by a perfectly competitive industry under constant returns to scale.

We propose a resource-based decomposition of the oligopoly distortion to aggregate surplus:

$$\Delta AS = M^W + M^B.$$

The first term, M^W (which stands for ‘misallocation within’), measures the welfare loss arising from resources being misallocated *within* the same industry. As we will see, this component of the welfare loss is driven by markup heterogeneity. The second term, M^B (which stands for ‘misallocation between’), measures the welfare loss arising from the industry not utilizing the same amount of resources for production as under monopolistic competition. This component is driven by the overall level of the average markup, as we will see below. In our one-industry model with quasi-linear preferences, this term captures the misallocation of the representative consumer’s time *between* work and leisure. Our general oligopolistic equilibrium extension in

Section 5, which features a continuum of industries, explicitly accounts for between-industry resource misallocation.

In Section 4.1, we begin by studying whether oligopolistic market power results in resources being under- or overutilized. We find that underutilization always arises under CES demand, but not necessarily under MNL demand. To set the stage for the analysis of the first term of the decomposition, we then characterize the efficient within-industry resource allocation in Section 4.2. We show that, regardless of the details of the demand system, this allocation obtains when relative markups are equalized across products, independently of who offers which products. In Section 4.3, we provide an exact formula expressing the first term of the decomposition under CES demand as a function of equilibrium firm-level market shares. There, we also discuss why such a formula is generally unavailable in the case of MNL demand. In Section 4.4, we use approximations to relate the welfare loss from within-industry misallocation to standard concentration indices, focusing again on CES demand.

4.1 Over- or Underutilization of Resources

In general, it is unclear whether the industry uses *too few* or *too many* resources relative to the competitive benchmark. On the one hand, equilibrium prices are higher than in the competitive benchmark, implying a lower aggregate demand, and thus fewer resources being used for production. On the other hand, as larger, more efficient firms charge higher equilibrium markups, consumption is distorted towards less efficient firms, which may result in more resources being used.

We now show that, under the assumption of CES demand, oligopolistic market power always results in *too few* resources being used. The amount of resources employed in the equilibrium with conduct parameter θ is equal to

$$\begin{aligned}
C(\theta) &= \sum_{j \in \mathcal{N}} c_j D_j(\mathbf{p}) = \sum_{j \in \mathcal{N}} p_j D_j(\mathbf{p}) - \sum_{j \in \mathcal{N}} (p_j - c_j) D_j(\mathbf{p}) \\
&= \sum_{j \in \mathcal{N}} \frac{-p_j h'_j(p_j)}{H(\mathbf{p})} - \sum_{f \in \mathcal{F}} \pi^f(\mathbf{p}) = (\sigma - 1) \sum_{j \in \mathcal{N}} \frac{h_j(p_j)}{H(\mathbf{p})} - \sum_{f \in \mathcal{F}} \pi^f(\mathbf{p}) \\
&= \frac{\alpha}{1 - \alpha} \Sigma(\theta) - \Pi(\theta),
\end{aligned} \tag{13}$$

where $\Pi(\theta)$ was defined in Section 2.2 as equilibrium industry profit given conduct parameter θ . Note that the strict Schur-convexity of the profit function, holding $\theta > 0$ fixed, implies that a mean-preserving spread of the equilibrium firm-level market shares reduces the amount of resources employed by the industry.

As θ increases from 0 (monopolistic competition conduct) to 1 (fully fledged oligopoly conduct), the industry becomes less competitive and all prices increase. This implies that Σ , the aggregate share of the inside goods, weakly decreases; the decrease is strict if consumers have access to an outside option. Moreover, the equilibrium profit of every firm f increases for two reasons: first, firm f 's rivals raise their prices; second, firm f best-responds to that

price increase. It follows that $C(0) > C(1)$. The following proposition generalizes these insights by showing that $C(\theta)$ is strictly decreasing in θ :

Proposition 4. *Under CES demand, an increase in the conduct parameter θ strictly reduces the amount of resources used for production, $C(\theta)$. In particular, under fully fledged oligopoly ($\theta = 1$), the industry uses strictly fewer resources than under monopolistic competition ($\theta = 0$). That is, an oligopolistic industry underutilizes resources relative to the competitive benchmark.*

Proof. See Appendix B. □

Although the idea that an oligopoly tends to underutilize resources relative to the competitive benchmark may seem intuitive (because oligopolies tend to underproduce), it is easy to construct counterexamples. Indeed, we now show that, under MNL demand, both over- and underutilization can arise. The examples below feature MNL demand with two products, 1 and 2, sold by firms f_1 and f_2 , respectively. With this specification, the total amount of resources used when the conduct parameter is θ is $C(\theta) = c_1 s^{f_1}(\theta) + c_2 s^{f_2}(\theta)$.¹² We denote by $\mu^f(\theta)$ the equilibrium ι -markup of firm f when the conduct parameter is θ .

Example 1 (overutilization of resources under MNL demand): Suppose that there is no outside option ($H^0 = 0$), and marginal costs and qualities satisfy $c_1 < c_2$ and $a_1 = a_2 = a$, respectively. Under monopolistic competition, the two firms charge the same absolute markup, $\mu^{f_1}(0) = \mu^{f_2}(0) = 1$. Instead, when $\theta = 1$, we have that

$$\mu^{f_1}(1) = m\left(\frac{e^{a-c_1}}{H^*(1)}, 1\right) > m\left(\frac{e^{a-c_2}}{H^*(1)}, 1\right) = \mu^{f_2}(1),$$

as the markup fitting-in function is strictly increasing in its first argument. It follows that $s^{f_1}(1) < s^{f_1}(0)$ and $s^{f_2}(1) > s^{f_2}(0)$, so that $C(1) > C(0)$. Thus, this duopoly overutilizes resources relative to the competitive benchmark.

Example 2 (underutilization of resources under MNL demand): Suppose that there is an outside option ($H^0 > 0$), and marginal costs and qualities satisfy $c_1 = c_2 = c$ and $a_1 = a_2 = a$. As firms are symmetric, they charge the same absolute markup in equilibrium, $\mu^{f_1}(\theta) = \mu^{f_2}(\theta)$. As that common markup is strictly higher under oligopoly than under monopolistic competition, $\mu^f(1) > 1 = \mu^f(0)$, we have that $s^f(1) < s^f(0)$, which implies that $C(1) < C(0)$. Thus, this oligopoly underutilizes resources relative to the competitive benchmark.

¹²Without loss of generality, we set the price sensitivity parameter λ equal to 1 to ease notation.

4.2 Efficient Within-Industry Resource Allocation

We now fully characterize the efficient within-industry resource allocation under minimal assumptions on the demand system. Suppose that the representative consumer has quasi-linear preferences that can be represented by the sub-utility function $U(\mathbf{x})$, where $\mathbf{x} = (x_j)_{j \in \mathcal{N}}$ is the output vector and U is concave and continuously differentiable on the interior of its domain. (Note that this differentiability assumption rules out the special case of MNL without an outside option, as the direct sub-utility function is then defined only on the simplex; we will later return to this special case.) Let $\mathbf{p} \equiv \nabla U$ denote the inverse demand function. Each product j has a constant unit cost of production c_j , incurred in units of the resource (which under our leading interpretation is the representative consumer's time). Consider the problem of maximizing aggregate surplus subject to the constraint of using C units of the resource for production:

$$\max_{\mathbf{x} \in \mathbb{R}_+^{\mathcal{N}}} U(\mathbf{x}) - \sum_{j \in \mathcal{N}} c_j x_j \quad \text{s.t.} \quad \sum_{j \in \mathcal{N}} c_j x_j = C. \quad (14)$$

The following proposition has appeared in various forms in the misallocation literature:¹³

Proposition 5. *Any interior solution $\mathbf{x}^*$ to maximization problem (14) must feature a common relative markup on all of the products. That is, $p_i(\mathbf{x}^*)/c_i = p_j(\mathbf{x}^*)/c_j$ for every $i, j \in \mathcal{N}$.*

Proof. Suppose indeed that $\mathbf{x}^*$ is an interior solution to maximization problem (14). Then, for some Lagrange multiplier ζ , $\mathbf{x}^*$ must maximize the Lagrangian

$$U(\mathbf{x}) - \sum_{j \in \mathcal{N}} c_j x_j + \zeta \left(C - \sum_{j \in \mathcal{N}} c_j x_j \right).$$

Hence, for each product i , the first-order condition $\partial U / \partial x_i - (1 + \zeta)c_i = 0$ must hold at $\mathbf{x}^*$. As $\partial U / \partial x_i = p_i(\mathbf{x})$, this implies that $p_i(\mathbf{x}^*)/c_i = 1 + \zeta$ for every i . $\square$

Thus, there is within-industry misallocation whenever relative markups are not equalized across products. Recall that, in the monopolistic competition benchmark, relative markups are indeed equalized across all products within the industry in the case of CES demand—but not in the case of MNL demand (where *absolute* markups are equalized), unless all products have the same marginal costs. That is, from Proposition 5, the competitive benchmark of monopolistic competition induces efficient within-industry resource allocation under CES demand, but in general not under MNL demand (with an outside option).

Efficient within-firm resource allocation. A related question is whether resources are efficiently allocated *within each firm*. The thought experiment here involves choosing firm f 's

¹³For instance, Pellegrino (2025) obtains the same result for the special case in which the sub-utility function U is quadratic, giving rise to linear demand (see his Proposition 5).

output vector so as to maximize aggregate surplus, holding fixed the quantities of rivals, and subject to the constraint that firm f uses C^f units of the resource. The argument used in the proof of Proposition 5 implies that, under the same assumption on the sub-utility function, the within-firm resource-efficient allocation requires a common relative markup across the firm's products. It follows from the equilibrium analysis in Section 2.2 that this condition holds under CES demand, but generically not under MNL demand with an outside option.

MNL demand without an outside option. As mentioned above, Proposition 5 does not cover the special case of MNL demand without an outside option. We provide a complete treatment of this case in Online Appendix G. There, we show that there is a unique output vector solving maximization problem (14), but that this output vector can be implemented by a continuum of price vectors. Specifically, any price vector $\mathbf{p}$ such that $p_j = \tau c_j + K$, where τ is uniquely pinned down by the resource constraint whenever marginal costs are not all equal, and K is any common constant consistent with positive prices, implements the resource-efficient allocation.

In the oligopolistic equilibrium, this condition is generically violated when there are three or more products, giving rise to within-industry misallocation. However, we also find that resources are efficiently allocated within each firm, in contrast to the case with an outside option. In contrast to the oligopolistic equilibrium and the monopolistic competition equilibrium in the presence of an outside option, the monopolistic competition outcome without an outside option is first-best efficient. The reason is that the aggregate quantity consumed is fixed and, as absolute markups are equalized, the choice probabilities are the same as under perfect competition.

4.3 Within-Industry Misallocation: Exact Results

We now turn to the quantification of the welfare loss arising from within-industry misallocation, i.e., term M^W in the resource-based decomposition. We focus on the case of CES demand and express M^W as a function of the equilibrium vector of firm-level market shares and the demand-side parameter α . We also discuss the case of MNL demand but relegate its formal treatment to the Online Appendix.

CES demand without an outside option. The term M^W was defined as the difference between aggregate surplus in the oligopolistic equilibrium (equation (12) with $\theta = 1$) and aggregate surplus in the counterfactual resource-efficient allocation, which maximizes aggregate surplus subject to the constraint of employing the same amount of resources as in the oligopolistic equilibrium. We now derive an expression for the latter term.

In the oligopolistic equilibrium, the amount of resources utilized by the industry can, from equation (13), be written as

$$C = \frac{\alpha}{1 - \alpha} - \Pi(1),$$

where we have used the fact that $\Sigma = 1$ in the absence of an outside option. The resource-efficient allocation features a common relative markup (see Proposition 5), and thus, under CES demand, a common ι -markup $\tilde{\mu}$, the level of which is pinned down by the resource constraint. Industry profit under ι -markup $\tilde{\mu}$ is

$$\tilde{\Pi} = \sum_{j \in \mathcal{N}} (p_j - c_j) D_j(\mathbf{p}) = \sum_{j \in \mathcal{N}} \frac{p_j - c_j}{p_j} \frac{-p_j h'_j(p_j)}{H(\mathbf{p})} = \frac{\sigma - 1}{\sigma} \tilde{\mu} \sum_{j \in \mathcal{N}} \frac{h_j(p_j)}{H(\mathbf{p})} = \alpha \tilde{\mu}, \quad (15)$$

which implies that the amount of resources utilized is

$$\tilde{C} = \sum_{j \in \mathcal{N}} p_j D_j(\mathbf{p}) - \tilde{\Pi} = \frac{\alpha}{1 - \alpha} - \alpha \tilde{\mu}.$$

Hence, the resource constraint in maximization problem (14), $C = \tilde{C}$, boils down to $\tilde{\mu} = \Pi(1)/\alpha$.

Inserting $\tilde{\mu} = \Pi(1)/\alpha$ into equation (15), we obtain that $\tilde{\Pi} = \Pi(1)$, i.e., industry profit in the resource-efficient and equilibrium allocations coincide. The impact of within-industry resource misallocation on aggregate surplus is therefore equal to $M^W \equiv \log H^* - \log \tilde{H}$, where $\tilde{H} \equiv \psi(\tilde{\mu})\mathcal{T}$ is the value of the industry aggregator in the resource-efficient allocation, with $\mathcal{T} \equiv \sum_{f \in \mathcal{F}} T^f$. (Recall that the function ψ was defined in equation (6).)

We can thus express M^W as a function of the equilibrium market shares alone:

$$M^W(\mathbf{s}) = -\log \frac{\mathcal{T}}{H^*} - \log \psi(\tilde{\mu}) = -\log \sum_{f \in \mathcal{F}} \frac{s^f}{\psi(1/(1 - \alpha s^f))} - \log \psi \left(\sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f} \right).$$

CES demand with an outside option. The analysis for the case with an outside option follows the same steps; details can be found in the proof of Proposition 6 below. The resource-efficient allocation features again a common ι -markup $\tilde{\mu}$, which is pinned down by the resource constraint, albeit not in closed form. This markup level, in turn, determines the aggregate market share of the inside goods, $\tilde{\Sigma}$. The welfare loss from within-industry resource misallocation can then be expressed solely as a function of α , the equilibrium aggregate share of the inside goods, Σ , and that share in the resource-efficient allocation, $\tilde{\Sigma}$.

The following proposition collects our results for the cases with and without an outside option:

Proposition 6. *Under CES demand, the impact of within-industry resource misallocation on aggregate surplus can be expressed as a function of observed oligopolistic market shares, as*

$$M^W(\mathbf{s}) = -\log \sum_{f \in \mathcal{F}} \frac{s^f}{\psi(1/(1 - \alpha s^f))} - \log \psi \left(\sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f} \right) \quad (16)$$

if there is no outside option, and as

$$M^W(\mathbf{s}) = \log(1 - \tilde{\Sigma}(\mathbf{s})) - \log(1 - \Sigma(\mathbf{s})) + \frac{\alpha}{1 - \alpha}(\Sigma(\mathbf{s}) - \tilde{\Sigma}(\mathbf{s})) \quad (17)$$

if there is an outside option, where $\tilde{\Sigma}(\mathbf{s})$ is the unique solution to

$$\frac{\alpha}{1 - \alpha} \tilde{\Sigma}^{\frac{1}{\alpha}} \left(\frac{1}{\sum_{f \in \mathcal{F}} \frac{s^f}{\psi(1/(1 - \alpha s^f))}} \frac{1 - \Sigma(\mathbf{s})}{1 - \tilde{\Sigma}} \right)^{\frac{1 - \alpha}{\alpha}} = \frac{\alpha}{1 - \alpha} \Sigma(\mathbf{s}) - \alpha \sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f}.$$

By definition, $M^W(\mathbf{s}) \leq 0$; moreover, this holds with equality if and only if firms have identical market shares.

Proof. See Appendix C. □

Note that $M^W(\mathbf{s})$ vanishes not only when all firms have identical market shares but also in the limit as $\mathbf{s}$ tends to any permutation of the vector $(x, 0, \dots, 0)$, for any arbitrary $x \in (0, 1)$. Hence, M^W is neither Schur-convex nor Schur-concave: higher (or lower) industry concentration does not necessarily result in larger within-industry resource misallocation.

The welfare loss arising from the industry's underutilizing resources in the oligopolistic equilibrium, $M^B(\mathbf{s})$, can be obtained by subtracting $M^W(\mathbf{s})$ from the expression for $\Delta AS(\mathbf{s})$ in Proposition 1.

MNL demand. Recall from the previous subsection that the monopolistic competition outcome features within-industry misallocation if and only if there is an outside option and marginal costs are heterogeneous. This means that, in the presence of an outside option, the resource-based decomposition would require a third term capturing the difference between aggregate surplus in the resource-efficient allocation given the amount of resources used under monopolistic competition, and aggregate surplus under monopolistic competition. As the focus of the paper is on inefficiencies arising from oligopolistic market power, we refrain here from providing such a decomposition for the case of MNL demand with an outside option.

Moreover, as we now argue, even in the absence of an outside option, quantifying the welfare loss from within-industry resource misallocation would necessitate detailed product-level data. To see this, consider the following simple example. Assume $H^0 = 0$ and $\lambda = 1$, and consider two industries, each featuring three single-product firms. In both industries, the market shares of the three firms are given by $(1/2, 1/3, 1/6)$, which results in ι -markups $(2, 3/2, 6/5)$. In the first industry, the vector of marginal costs is $(2, 3/2, 6/5)$, i.e., $c_i = \mu_i$ for every i ; in the second industry, the marginal costs are $(11/5, 6/5, 6/5)$. The product-level qualities in both industries are given by $a_i = c_i + \mu_i + \log s_i$. By construction, the first industry features a common relative markup across products, $p_i/c_i = 2$, and so there is no resource misallocation. In the second industry, however, the absolute markups of products 2 and 3 differ even though they have the same marginal cost. Hence, their prices differ despite

their identical marginal costs, violating the affine-price condition for resource efficiency. The general message is that equilibrium markups depend through the implied market shares on both marginal costs and qualities. Without additional product-level information, such as marginal costs, market shares alone do not pin down the amount of resource misallocation.¹⁴ The fact that, under MNL demand, detailed product-level data are required for the quantification of the welfare loss from resource misallocation runs counter to the sufficient statistics approach pursued in this paper. We therefore refrain from providing such a quantification in the case of MNL demand in the main text. We refer the reader to Online Appendix G for a treatment of these issues.

4.4 Within-Industry Misallocation and Concentration Indices

We now relate the within-industry resource misallocation to standard concentration indices. For the reasons discussed above, we confine attention to the case of CES demand. As in Section 3.2, we rely on Taylor approximations, both around small market shares and around monopolistic competition conduct.

Approximation around small market shares. This approximation requires again assuming that consumers have access to an outside option, $H^0 > 0$. The welfare loss from within-industry resource misallocation is then given by equation (17). Approximating this equation at the third order, we obtain:

Proposition 7. *Under CES demand, in the neighborhood of $\mathbf{s} = \mathbf{0}$, the impact of within-industry resource misallocation on aggregate surplus is:*

$$M^W(\mathbf{s}) = -\frac{1}{2}\alpha\Sigma(\mathbf{s})^3 (\text{CHK}(\mathbf{s}) - \text{HHI}(\mathbf{s})^2) + o(\|\mathbf{s}\|^3).$$

Proof. See Appendix D. □

A first observation is that $M^W(\mathbf{s})$ is negligible at the second order.¹⁵ Thus, at that order, there is no within-industry resource misallocation: the oligopoly distortion to aggregate surplus is entirely driven by resources being underutilized in equilibrium (rather than by resources flowing to the wrong firms). Within-industry misallocation becomes relevant at the third order, where it is proportional to $\text{CHK}(\mathbf{s}) - \text{HHI}^2(\mathbf{s})$. As mentioned in Section 3.2, we have that $\text{CHK}(\mathbf{s}) - \text{HHI}^2(\mathbf{s}) \geq 0$, with the inequality being strict if and only if firms are

¹⁴By contrast, under CES demand, firm-level market shares alone pin down relative markups, which determine whether or not there is resource misallocation, and, as shown above, the associated welfare loss.

¹⁵This holds because $\Sigma(\mathbf{s})^3$ is of order 3, while $\text{HHI}(\mathbf{s})^2$ and $\text{CHK}(\mathbf{s})$ are both of order 0, as

$$\frac{1}{|\mathcal{F}|^2} \leq \text{HHI}(\mathbf{s})^2 \leq 1 \quad \text{and} \quad \frac{1}{|\mathcal{F}|^2} \leq \text{CHK}(\mathbf{s}) \leq 1.$$

asymmetric.¹⁶ Moreover, as $\text{CHK}(\mathbf{s}) - \text{HHI}^2(\mathbf{s})$ is not Schur-convex in the market share vector (see again Section 3.2), it is generally not true that greater industry concentration results in larger misallocation. This is consistent with our observation in the previous subsection, that the welfare loss from within-industry resource misallocation is neither Schur-convex nor Schur-concave. Note that the term $\text{CHK}(\mathbf{s}) - \text{HHI}^2(\mathbf{s})$ corresponds to the variance of normalized firm shares when firms are weighted by those shares. According to the proposition, the welfare loss from within-industry misallocation is proportional to this measure of size heterogeneity.

Approximation around monopolistic competition conduct. We now approximate the welfare loss from within-industry resource misallocation in the neighborhood of monopolistic competition conduct. The following result holds both with and without an outside option:

Proposition 8. *Under CES demand, in the neighborhood of $\theta = 0$, the impact of within-industry resource misallocation on aggregate surplus is*

$$M^W(\theta) = -\frac{1}{2}\alpha\Sigma(\theta)^3 (\text{CHK}(\theta) - \text{HHI}(\theta)^2) \theta^2 + o(\theta^2).$$

Proof. See Appendix E. □

Thus, within-industry resource misallocation has no first-order effect on aggregate surplus. At the second order, the loss is proportional to the difference between the cubic Hannah–Kay index and the squared Herfindahl index, with a coefficient that depends on the aggregate share of the inside goods.

5 Oligopoly Distortions in Multi-Industry General Equilibrium

In this section, we study welfare distortions arising from oligopolistic market power in a general equilibrium framework featuring a continuum of industries, with CES demand in each

¹⁶To see this, define the vectors $\mathbf{x} \equiv ((s^f)^{3/2})_{f \in \mathcal{F}}$ and $\mathbf{y} \equiv ((s^f)^{1/2})_{f \in \mathcal{F}}$. By the Cauchy-Schwarz inequality, we have that $\mathbf{x} \cdot \mathbf{y} \leq \|\mathbf{x}\|_2 \|\mathbf{y}\|_2$, with equality if and only if $\mathbf{x}$ and $\mathbf{y}$ are collinear. Taking squares on both sides, this is equivalent to

$$\left(\sum_{f \in \mathcal{F}} (s^f)^2 \right)^2 \leq \left(\sum_{f \in \mathcal{F}} (s^f)^3 \right) \left(\sum_{f \in \mathcal{F}} s^f \right),$$

with equality if and only if all positive components of $\mathbf{s}$ are identical; at an interior equilibrium, this means all firms have identical shares. Dividing both sides by $\left(\sum_{f \in \mathcal{F}} s^f \right)^4$, the inequality simplifies to $\text{CHK}(\mathbf{s}) \geq \text{HHI}(\mathbf{s})^2$.

industry.¹⁷ One reason for providing such a framework is that it allows us to better connect our work to the macroeconomics literature on misallocation. A second reason is that, in our baseline single-industry model, under CES demand, the industry would necessarily suffer from underutilization of resources. In this richer multi-industry framework, by contrast, some industries overutilize resources while others underutilize them.

We begin by laying out the framework in Section 5.1. We derive the general equilibrium in Section 5.2. Next, in Section 5.3, we relate the welfare loss from oligopolistic behavior to observable, equilibrium firm-level market shares in the different industries. There, we also decompose that welfare loss into a within-industry and a between-industry misallocation component. Finally, in Section 5.5, we relate the overall welfare distortions and the components of the decomposition to within-industry concentration indices and to their dispersion across industries.

5.1 The General Oligopolistic Equilibrium Framework

We consider an economy with a continuum of industries, indexed by $z \in [0, 1]$. The representative consumer's utility function is given by¹⁸

$$U(\mathbf{x}) = \exp \int_{z \in [0,1]} \rho(z) \log \frac{X(z)}{\rho(z)} dz,$$

where

$$X(z) = \left[\sum_{j \in \mathcal{N}(z)} a_j(z)^{1-\alpha} x_j(z)^\alpha \right]^{\frac{1}{\alpha}}$$

is the CES commodity in industry z , $\rho(z)$ is the preference weight on that industry, with the normalization $\int \rho(z) dz = 1$, and $\alpha \equiv (\sigma - 1)/\sigma \in (0, 1)$. The set of differentiated products in industry z is $\mathcal{N}(z)$, with $x_j(z)$ denoting consumption of product j . Thus, the utility function features a Cobb-Douglas upper tier and a CES lower tier (with no outside option).

Maximizing the consumer's utility subject to the budget constraint $\int \sum_{j \in \mathcal{N}(z)} p_j(z) x_j(z) dz \leq y$, we obtain the demand for product i in industry z :

$$D_i(\mathbf{p}, z) = \frac{a_i(z) p_i(z)^{-\sigma}}{P(z)^{1-\sigma}} \rho(z) y,$$

where $P(z) \equiv \left(\sum_{j \in \mathcal{N}(z)} a_j(z) p_j(z)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is the industry-level price index. The industry-

¹⁷An alternative approach to modeling multiple industries would be to maintain the assumption of quasi-linear preferences, but use the nested CES demand system, with the interpretation that nests represent (imperfectly substitutable) industries. We undertake this extension in Online Appendix I.

¹⁸To ensure that all the integrals in this section are well defined, we impose the same regularity assumptions as in Section X in Nocke and Schutz (2018b). Dividing the CES commodity by $\rho(z)$ does not affect preferences but ensures that indirect utility is equal to real income.

level aggregator is $H(\mathbf{p}(z), z) \equiv P(z)^{1-\sigma}$. The associated indirect utility function is

$$V(\mathbf{p}, y) = y \exp \left[- \int_{z \in [0,1]} \rho(z) \log P(z) dz \right],$$

which corresponds to real income. The representative consumer inelastically supplies one unit of labor, the only input into production, and is the owner of all firms in the economy. In the following, we normalize nominal income y , which is the sum of economy-wide profit and the wage bill, to 1.

Each product $j \in \mathcal{N}(z)$ is produced with a constant returns-to-scale technology, requiring $c_j(z)$ units of labor to produce one unit of output. Thus, the constant unit cost of production is $w c_j(z)$, where w is the wage rate. In each industry z , the set of products is partitioned into a set of firms $\mathcal{F}(z)$ containing at least two elements; each firm is assumed to operate in one industry only. The profit of firm f active in industry z is given by

$$\pi^f(\mathbf{p}, w, z) = \sum_{j \in f} (p_j(z) - w c_j(z)) \frac{a_j(z) p_j(z)^{-\sigma}}{H(\mathbf{p}(z), z)} \rho(z). \quad (18)$$

A general oligopolistic equilibrium (GOLE, see Neary, 2003) is a profile of prices $\mathbf{p}^*$ and a wage rate w^* such that: in every industry z , $\mathbf{p}^*(z)$ is a Nash equilibrium of the pricing game with set of players $\mathcal{F}(z)$ and payoff functions (18); the labor market clears.

As in the case of quasi-linear preferences, we will sometimes use a more flexible approach to firm conduct, by parameterizing the extent to which firms internalize their impact on the industry-level aggregator. Monopolistic competition, which remains the competitive benchmark, still corresponds to $\theta = 0$; fully fledged oligopoly to $\theta = 1$.

5.2 Equilibrium Analysis

The equilibrium analysis follows the steps laid out in Section X of Nocke and Schutz (2018b). For a given wage rate w , the pricing game in industry z has a unique equilibrium that can be characterized using the aggregative games approach used in Section 2.2 above. The type of firm $f \in \mathcal{F}(z)$ when the wage rate is w is equal to

$$\sum_{j \in f} a_j(z) (w c_j(z))^{1-\sigma} = w^{1-\sigma} \sum_{j \in f} a_j(z) (c_j(z))^{1-\sigma} \equiv w^{1-\sigma} T^f(z).$$

The equilibrium aggregator level in industry z , $H^*(w, z)$ is the unique solution to

$$\sum_{f \in \mathcal{F}(z)} S \left(\frac{w^{1-\sigma} T^f(z)}{H^*(w, z)} \right) = 1.$$

From the above equation, it immediately follows that $H^*(w, z) = w^{1-\sigma} H^*(1, z) \equiv w^{1-\sigma} H^*(z)$. Moreover, firm f 's equilibrium market share, $s^f(z) \equiv S(T^f(z)/H^*(z))$, is independent of w .

The wage bill in industry z is equal to

$$\rho(z) - \sum_{f \in \mathcal{F}(z)} \pi^f(\mathbf{p}, w, z) = \rho(z) \left(1 - (1 - \alpha) \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} \right).$$

The labor market clearing condition is therefore

$$w^* = 1 - (1 - \alpha) \int \rho(z) \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} dz. \quad (19)$$

Hence, the equilibrium is unique and, given the observed profile of market shares, the equilibrium wage rate is available in closed form.

Summing up:

Proposition 9. *There is a unique general oligopolistic equilibrium. In every industry z , the market share of firm f is given by $s^{f*}(z) = S(T^f(z)/H^*(z))$, where S is the market-share fitting-in function defined in Section 2.2 and $H^*(z)$ is the unique solution to*

$$\sum_{f \in \mathcal{F}(z)} S\left(\frac{T^f(z)}{H^*(z)}\right) = 1.$$

The equilibrium wage rate is given by equation (19).

The approach with flexible conduct follows the same steps. The equilibrium aggregator level is now pinned down as the unique solution to

$$\sum_{f \in \mathcal{F}(z)} S\left(\frac{w^{1-\sigma} T^f(z)}{H^*(w, z; \theta)}; \theta\right) = 1.$$

Moreover, $H^*(w, z; \theta) = w^{1-\sigma} H^*(1, z; \theta) \equiv w^{1-\sigma} H^*(z; \theta)$, and equilibrium market shares $s^{f*}(z; \theta)$ are again independent of w . The equilibrium wage rate continues to be available in closed form:

$$w^*(\theta) = 1 - (1 - \alpha) \int \rho(z) \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z; \theta)}{1 - \alpha s^{f*}(z; \theta)} dz.$$

The monopolistic competition outcome features a wage rate of α .

5.3 Oligopoly Distortions in General Equilibrium

We now derive expressions for the aggregate welfare distortions arising from oligopolistic behavior in our GOLE framework. These distortions are again measured relative to the monopolistic competition benchmark. Note that here, this benchmark is first-best efficient because all firms charge the same relative markup on all products independently of the

industry, implying that relative prices coincide with those under perfect competition.¹⁹

Let $\Delta \log \text{SW}$ denote the change in equilibrium logged social welfare (i.e., indirect utility) when going from monopolistic competition to oligopoly. We have:

$$\begin{aligned}\Delta \log \text{SW} &= \frac{1}{\sigma - 1} \int \rho(z) [\log (w^*(1)^{1-\sigma} H^*(z; 1)) - \log (w^*(0)^{1-\sigma} H^*(z; 0))] dz \\ &= \log w^*(0) - \log w^*(1) + \frac{1}{\sigma - 1} \int \rho(z) [\log H^*(z; 1) - \log H^*(z; 0)] dz.\end{aligned}$$

Using Proposition 1 to express the difference $\log H^*(z; 1) - \log H^*(z; 0)$ as a function of firm-level market shares, and the closed-form expressions for the equilibrium wage rate from the previous subsection, we obtain:

Proposition 10. *In general equilibrium, the loss in logged social welfare arising from oligopolistic behavior is:*

$$\begin{aligned}\Delta \log \text{SW}(\mathbf{s}) &= -\frac{1-\alpha}{\alpha} \int \rho(z) \log \left[\sum_{f \in \mathcal{F}(z)} s^f(z) \phi(s^f(z)) \right] dz \\ &\quad + \log \alpha - \log \left(1 - (1-\alpha) \int \rho(z) \sum_{f \in \mathcal{F}(z)} \frac{s^f(z)}{1 - \alpha s^f(z)} dz \right), \quad (20)\end{aligned}$$

where $s^f(z)$ is the observed oligopolistic market share of firm f in industry z .

The terms in the welfare loss expression are substantially the same as those in Proposition 1 (when $H^0 = 0$, so that $\sum_f s^f = 1$) but they are weighted differently. The term on the first line—which is negative and corresponds to the mean difference between $\log H^*(z; 1)$ and $\log H^*(z; 0)$ —becomes more negative as the industries become more concentrated, for the same reason as in Section 3.1. (As usual, we think of such an increase in concentration as arising from a change in the underlying profile of types, $(T^f(z))$.)

By contrast, the expression on the second line—which is positive and corresponds to the difference in log-wage—becomes more positive as the industries become more concentrated. The intuition is that, as concentration increases, the industries utilize less labor for a given wage rate under oligopoly (as shown in Section 4.1). The equilibrium oligopoly wage rate must therefore decrease for the labor market to clear. If instead of normalizing income to 1 we had chosen labor as our *numéraire*, the term on the second line would correspond to the change in the logarithm of aggregate income, which equals one plus aggregate profit. That change is Schur-convex in market shares, as explained in Section 3.1.

Observe that, globally, $\Delta \log \text{SW}$ is neither increasing nor decreasing in concentration. To see this, suppose that every industry has the same three-firm market-share vector $(x, x, 1 -$

¹⁹The fact that relative markups also coincide across industries follows from our assumption that the elasticity of substitution σ is the same in all industries.

$2x$), where $x \in [1/3, 1/2]$. Increasing x induces a mean-preserving spread of market shares. At $x = 1/3$, all firms have identical shares, so the welfare distortion is zero. For every $x \in (1/3, 1/2)$, however, firms have heterogeneous markups and the welfare distortion is strictly negative. As x tends to $1/2$, the share vector converges to $(1/2, 1/2, 0)$, and the welfare distortion tends to zero again: the two remaining firms have identical markups, and there is no between-industry misallocation because all industries have the same share vector. Thus, greater concentration can either increase or decrease the welfare loss. This illustrates the well-established insight that, in general equilibrium, the level of markups in and of itself does not cause a loss in social welfare (see Lerner, 1934; Neary, 2003). Rather, such a loss is driven by markup heterogeneity within and across industries.

5.4 Resource Misallocation Decomposition in General Equilibrium

In this subsection, we decompose the aggregate welfare distortions arising from oligopolistic behavior into within- and between-industry resource misallocation. In each industry z , the resource-efficient allocation maximizes the value of the CES commodity $X(z)$ subject to the constraint of utilizing the same amount of resources as in the oligopolistic equilibrium. This is equivalent to maximizing the industry's contribution to indirect utility, $\log H(\mathbf{p}(z), z)$, holding fixed the wage rate at its equilibrium level, and subject to the constraint of leaving the industry's wage bill unchanged. The analysis in Section 4.3 implies that the resulting value of the aggregator in industry z , $\tilde{H}(w^*(1), z)$, satisfies

$$\begin{aligned} \log H^*(w^*(1), z) - \log \tilde{H}(w^*(1), z) \\ = -\log \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{\psi(1/(1 - \alpha s^{f*}(z)))} - \log \psi \left(\sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} \right). \end{aligned}$$

Integrating across industries, we obtain the log-welfare loss from within-industry resource misallocation:

$$\begin{aligned} M^W(\mathbf{s}^*) &= \frac{1 - \alpha}{\alpha} \\ &\times \int \rho(z) \left[\log \psi^{-1} \left(\sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} \right) - \log \sum_{f \in \mathcal{F}(z)} s^{f*}(z) \psi^{-1} \left(\frac{1}{1 - \alpha s^{f*}(z)} \right) \right] dz, \end{aligned}$$

where ψ^{-1} denotes the reciprocal (not the inverse) of ψ . The strict convexity of ψ^{-1} implies that $M^W(\mathbf{s}^*) \leq 0$, with the inequality being strict if and only if there is a positive measure of industries featuring heterogeneous firm-level market shares.

Subtracting $M^W(\mathbf{s}^*)$ from $\Delta \log \text{SW}(\mathbf{s}^*)$, we obtain the log-welfare loss from between-

industry resource misallocation:

$$\begin{aligned}
M^B(\mathbf{s}^*) &= \frac{1-\alpha}{\alpha} \int \rho(z) \log \psi \left(\sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} \right) dz \\
&\quad - \log \left[1 - (1-\alpha) \int \rho(z) \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)} dz \right] \\
&= \int \rho(z) \log \left[1 - \frac{1-\alpha}{\alpha} \Pi(\mathbf{s}^*(z)) \right] dz - \log \int \rho(z) \left[1 - \frac{1-\alpha}{\alpha} \Pi(\mathbf{s}^*(z)) \right] dz,
\end{aligned}$$

where $\Pi(\mathbf{s}^*(z))$ was defined earlier as

$$\Pi(\mathbf{s}^*(z)) = \alpha \sum_{f \in \mathcal{F}(z)} \frac{s^{f*}(z)}{1 - \alpha s^{f*}(z)}.$$

The strict concavity of the logarithm implies that $M^B(\mathbf{s}^*) \leq 0$, with the inequality being strict if and only if $z \mapsto \Pi(\mathbf{s}^*(z))$ is not a.e. constant. Put differently, there is resource misallocation across industries whenever the sales-normalized wage bill varies across industries.

Taking stock:

Proposition 11. *In general equilibrium, the loss in log social welfare arising from oligopolistic behavior can be decomposed as $\Delta \log \text{SW}(\mathbf{s}) = M^W(\mathbf{s}) + M^B(\mathbf{s})$, where*

$$\begin{aligned}
M^W(\mathbf{s}) &= \frac{1-\alpha}{\alpha} \\
&\quad \times \int \rho(z) \left[\log \psi^{-1} \left(\sum_{f \in \mathcal{F}(z)} \frac{s^f(z)}{1 - \alpha s^f(z)} \right) - \log \sum_{f \in \mathcal{F}(z)} s^f(z) \psi^{-1} \left(\frac{1}{1 - \alpha s^f(z)} \right) \right] dz \quad (21)
\end{aligned}$$

is the log-welfare loss from within-industry resource misallocation,

$$M^B(\mathbf{s}) = \int \rho(z) \log \left[1 - \frac{1-\alpha}{\alpha} \Pi(\mathbf{s}(z)) \right] dz - \log \int \rho(z) \left[1 - \frac{1-\alpha}{\alpha} \Pi(\mathbf{s}(z)) \right] dz \quad (22)$$

is the log-welfare loss from between-industry resource misallocation, and $\mathbf{s}$ is the profile of observed oligopolistic market shares.

5.5 Distortions and Concentration Indices in General Equilibrium

We now relate the welfare distortions identified in the previous subsections to concentration indices, using approximations in the neighborhood of monopolistic competition conduct. Let $\text{HHI}(z; \theta)$ and $\text{CHK}(z; \theta)$ denote the equilibrium values of the Herfindahl and cubic Hannah–Kay indices in industry z when the conduct parameter is θ . In the Appendix, we obtain

formulas similar to equations (20), (21), and (22) for the overall welfare loss relative to the monopolistic competition benchmark, $\Delta \log \text{SW}(\theta)$, as well as the welfare losses from within- and between-industry resource misallocation $M^W(\theta)$ and $M^B(\theta)$. Approximating these expressions at the second order in θ , we obtain:

Proposition 12. *In general equilibrium, in the neighborhood of $\theta = 0$, the log-welfare loss from within-industry resource misallocation is*

$$M^W(\theta) = -\frac{1}{2}\theta^2(1-\alpha) \int \rho(z) [\text{CHK}(z; \theta) - \text{HHI}(z; \theta)^2] dz + o(\theta^2), \quad (23)$$

while the log-welfare loss from between-industry resource misallocation is

$$M^B(\theta) = -\frac{1}{2}\theta^2(1-\alpha)^2 \left[\int \rho(z) \text{HHI}(z; \theta)^2 dz - \left(\int \rho(z) \text{HHI}(z; \theta) dz \right)^2 \right] + o(\theta^2). \quad (24)$$

The overall log-welfare loss from oligopolistic behavior is $\Delta \log \text{SW}(\theta) = M^W(\theta) + M^B(\theta)$.

Proof. See Appendix E. □

Thus, the welfare loss from within-industry misallocation is proportional to the expenditure-weighted mean of the difference between the cubic Hannah–Kay index and the squared Herfindahl index, as in the case of quasi-linear preferences. Interestingly, the welfare loss from between-industry misallocation is proportional to the variance of the distribution of the Herfindahl index across industries. In the case of quasi-linear preferences, this latter term was of first order in θ ; here it is of second order. The intuition is the following. As θ increases, higher markups reduce each industry’s aggregator, which is a first-order effect. At the same time, the resulting decrease in labor demand pushes down the equilibrium wage. This latter effect, which is also of first order, exactly offsets the former one.

6 Conclusion

This paper develops a sufficient-statistics approach to measuring the welfare costs of oligopolistic market power. In a multiproduct-firm pricing game with CES or MNL demand, we derive closed-form expressions for the distortions to consumer surplus and aggregate surplus relative to monopolistic competition. These formulas only use observed oligopolistic firm-level market shares and (at most) one relevant demand parameter, without requiring detailed information on product-level costs and qualities or counterfactual market shares under the competitive benchmark of monopolistic competition.

Under CES demand, we also characterize the welfare loss from within-industry resource misallocation exactly. This characterization separates the inefficient allocation of resources across products from the industry’s underutilization of resources. The distinction between these components matters for interpreting concentration measures: although a mean-preserving

spread of market shares increases the consumer surplus loss, greater concentration need not imply greater within-industry misallocation or a larger aggregate surplus loss.

The sufficient-statistics approach extends to general oligopolistic equilibrium. With a continuum of CES industries, Cobb-Douglas preferences across industries, and a fixed aggregate labor supply, we obtain closed-form expressions for the aggregate welfare loss and its decomposition into within- and between-industry resource misallocation. These expressions depend on observed firm-level market shares, industry expenditure shares, and the common elasticity of substitution. General equilibrium changes the nature of the distortion: resources released by one industry are employed elsewhere, and welfare losses arise from markup heterogeneity within and across industries. The exact decomposition distinguishes the gains from reallocating resources within industries from the additional gains from reallocating them across industries.

Taylor approximations connect these exact results to standard concentration indices. In general equilibrium, at the second order around monopolistic competition conduct, within-industry misallocation is proportional to the expenditure-weighted mean of the difference between the cubic Hannah–Kay index and the squared Herfindahl index. Between-industry misallocation is proportional to the expenditure-weighted variance of the Herfindahl index across industries. Thus, the distribution of concentration across industries has a distinct role in determining aggregate welfare losses.

Appendix

A Proof of Proposition 1

Preliminaries. For convenience, define

$$h(s) = s(\phi(s) - 1),$$

$$D = D(\mathbf{s}) = 1 + \sum_{f \in \mathcal{F}} h(s^f) = 1 - \Sigma + \sum_{f \in \mathcal{F}} s^f \phi(s^f) = \frac{H^*(0)}{H^*(1)}. \quad (\text{A.1})$$

In particular, $D > 1$ at every nonzero feasible share vector, and

$$\Sigma(0) = 1 - \frac{1 - \Sigma}{D}. \quad (\text{A.2})$$

The formulas in the statement of the proposition can be written as

$$\Delta \Pi = \alpha \left[\sum_{f \in \mathcal{F}} \frac{s^f}{1 - \alpha s^f} - 1 + \frac{1 - \Sigma}{D} \right], \quad (\text{A.3})$$

$$\Delta \text{CS} = -\log D, \quad \Delta \text{AS} = -\log D + \Delta \Pi. \quad (\text{A.4})$$

The following identities follow from the definition of ϕ :

$$\frac{\phi'(s)}{\phi(s)} = \frac{\alpha}{(1-s)(1-\alpha s)}, \quad (\text{A.5})$$

$$h'(s) = \phi(s) \left[1 + \frac{\alpha s}{(1-s)(1-\alpha s)} \right] - 1, \quad (\text{A.6})$$

$$h''(s) = \frac{\alpha \phi(s)(2-s)}{(1-s)^2(1-\alpha s)^2}. \quad (\text{A.7})$$

Hence $h(0) = h'(0) = 0$, while $h'(s) > 0$ for $s > 0$ and $h''(s) > 0$ for $0 \leq s < 1$.

The consumer surplus gap. The monotonicity and Schur-concavity properties of ΔCS follow immediately from the fact that h is strictly increasing and strictly convex.

Monotonicity and positivity of the profit gap. Differentiating (A.3) gives

$$\frac{\partial \Delta \Pi}{\partial s^f} = \alpha \left[\frac{1}{(1-\alpha s^f)^2} - \frac{1}{D} - \frac{1-\Sigma}{D^2} h'(s^f) \right]. \quad (\text{A.8})$$

Fix a coordinate $s = s^f > 0$ and put $d(s) = 1 + h(s) = 1 - s + s\phi(s)$. Since $D \geq d(s)$ and $1 - \Sigma \leq 1 - s$, (A.8) implies

$$\begin{aligned} \frac{1}{\alpha} \frac{\partial \Delta \Pi}{\partial s^f} &\geq \frac{1}{(1-\alpha s)^2} - \frac{1}{d(s)} - \frac{(1-s)h'(s)}{d(s)^2} \\ &= \frac{d(s)^2 - (1-\alpha s)\phi(s)}{(1-\alpha s)^2 d(s)^2}. \end{aligned} \quad (\text{A.9})$$

The equality follows from

$$d(s) + (1-s)h'(s) = \frac{\phi(s)}{1-\alpha s}.$$

To establish that the numerator in (A.9) is positive, observe that

$$\begin{aligned} \frac{d}{ds} \log \frac{d(s)^2}{(1-\alpha s)\phi(s)} &= \frac{2(\phi(s)-1)}{d(s)} \\ &\quad + \frac{\alpha s}{(1-s)(1-\alpha s)} \left(\frac{2\phi(s)}{d(s)} - 1 \right) > 0. \end{aligned}$$

Here $d(s) \leq \phi(s)$, and the log ratio is zero at $s = 0$. Thus $d(s)^2 > (1-\alpha s)\phi(s)$ for $s > 0$, and $\partial \Delta \Pi / \partial s^f > 0$.

At a zero share with some other share positive, (A.8) instead gives

$$\left. \frac{\partial \Delta \Pi}{\partial s^f} \right|_{s^f=0} = \alpha(1 - D^{-1}) > 0.$$

The gradient is zero at the all-zero vector. Integrating the positive marginal effects proves strict coordinatewise monotonicity. Since $\Delta\Pi(\mathbf{0}) = 0$, integrating along the ray from $\mathbf{0}$ to any nonzero feasible vector proves $\Delta\Pi > 0$.

Schur-convexity of the profit gap. We show that $s^f > s^g$ implies $\partial\Delta\Pi/\partial s^f > \partial\Delta\Pi/\partial s^g$. Define the auxiliary functions

$$M(s) = (1 - \alpha s) \left(1 + \frac{\alpha s^2}{1 - s} \right), \quad L(s) = \phi(s)M(s).$$

Equation (A.6) gives $(1 - \alpha s)^2[1 + h'(s)] = L(s)$. Define the rescaled marginal effects

$$\gamma^f \equiv \frac{(1 - \alpha s^f)^2}{\alpha} \frac{\partial\Delta\Pi}{\partial s^f} = 1 - \frac{(1 - \alpha s^f)^2 \sum_k s^k \phi(s^k) + (1 - \Sigma)L(s^f)}{D^2}. \quad (\text{A.10})$$

Note that $\gamma^f > 0$, as shown above.

Fix $x = s^f > y = s^g$ and set

$$\tau = 1 - x - y, \quad \kappa = \alpha(x - y)[2 - \alpha(x + y)] > 0.$$

Subtracting (A.10) for the two firms yields

$$D^2(\gamma^f - \gamma^g) = \kappa \sum_k s^k \phi(s^k) - (1 - \Sigma)[L(x) - L(y)]. \quad (\text{A.11})$$

The function L is strictly increasing on $(0, 1)$, since

$$L'(s) = \frac{\alpha \phi(s)s}{(1 - s)^2} [3 - 2s - 2\alpha s(1 - s)] > 0;$$

the bracket is at least $3 - 4s + 2s^2 = 1 + 2(1 - s)^2 > 0$. Since $1 - \Sigma \leq \tau$ and $\sum_k s^k \phi(s^k) \geq x\phi(x) + y\phi(y)$, the right side of (A.11) is bounded below by

$$\kappa[x\phi(x) + y\phi(y)] - \tau[L(x) - L(y)] = B\phi(y) - A\phi(x), \quad (\text{A.12})$$

where

$$A = \tau M(x) - \kappa x, \quad B = \tau M(y) + \kappa y > 0.$$

Put

$$K = 1 + \frac{(1 - \alpha)xy}{(1 - x)(1 - y)}.$$

Direct expansion gives

$$B - A = \alpha(x - y)K, \quad (\text{A.13})$$

$$K(1-x)(1-\alpha y) - B = y \left[(1-\alpha x)^2 + \frac{x(1-\alpha y)(1-\alpha+\alpha y)}{1-y} \right] \geq 0. \quad (\text{A.14})$$

If $A \leq 0$, (A.12) is strictly positive immediately. If $A > 0$, (A.13)–(A.14) imply

$$\begin{aligned} \log \frac{B}{A} &= -\log \left(1 - \frac{\alpha(x-y)K}{B} \right) \\ &> \frac{\alpha(x-y)K}{B} \geq \frac{\alpha(x-y)}{(1-x)(1-\alpha y)}. \end{aligned} \quad (\text{A.15})$$

For CES, the elementary inequality $\log(1+z) \leq z$ gives

$$\begin{aligned} \log \frac{\phi(x)}{\phi(y)} &= \frac{\alpha}{1-\alpha} \log \left(1 + \frac{(1-\alpha)(x-y)}{(1-x)(1-\alpha y)} \right) \\ &\leq \frac{\alpha(x-y)}{(1-x)(1-\alpha y)}. \end{aligned} \quad (\text{A.16})$$

For MNL, the same bound holds with equality:

$$\log \frac{\phi(x)}{\phi(y)} = \frac{x-y}{(1-x)(1-y)}, \quad \alpha = 1.$$

Combining (A.15) and (A.16) proves $B\phi(y) > A\phi(x)$. Thus $\gamma^f > \gamma^g > 0$. Since $(1-\alpha x)^{-2} > (1-\alpha y)^{-2}$, it follows that

$$\frac{\partial \Delta \Pi}{\partial s^f} = \frac{\alpha \gamma^f}{(1-\alpha x)^2} > \frac{\alpha \gamma^g}{(1-\alpha y)^2} = \frac{\partial \Delta \Pi}{\partial s^g}.$$

Along any nontrivial equalizing transfer from a larger share to a smaller share, the profit gap therefore strictly decreases until the shares meet. Composing such transfers establishes strict Schur-convexity.

Sign of the aggregate surplus gap. The numbers $1 - \Sigma$ and $(s^f)_{f \in \mathcal{F}}$ are nonnegative weights summing to one. Jensen's inequality therefore gives

$$\log D = \log \left[1 - \Sigma + \sum_f s^f \phi(s^f) \right] \geq \sum_f s^f \log \phi(s^f). \quad (\text{A.17})$$

For MNL, $\log \phi(s^f) = \mu^f - 1$. For CES,

$$\begin{aligned} \log \phi(s^f) &= -\frac{\alpha}{1-\alpha} \log \left[1 - \frac{1-\alpha}{\alpha} (\mu^f - 1) \right] \\ &\geq \mu^f - 1, \end{aligned}$$

using $-\log(1 - z) \geq z$ and $0 \leq (1 - \alpha)(\mu^f - 1)/\alpha < 1$.

Using (A.2), the profit gap can also be written as

$$\Delta \Pi = \alpha \sum_f s^f (\mu^f - 1) - \alpha(1 - \Sigma)(1 - D^{-1}).$$

Combining these observations with (A.4) gives

$$\Delta \text{AS} \leq -(1 - \alpha) \sum_f s^f (\mu^f - 1) - \alpha(1 - \Sigma)(1 - D^{-1}) \leq 0. \quad (\text{A.18})$$

Under CES, $1 - \alpha > 0$ and the first sum is strictly positive, so $\Delta \text{AS} < 0$. Under MNL with an outside option, $1 - \Sigma > 0$ and $D > 1$, so the second term is strictly negative.

It remains to consider MNL without an outside option. In this case,

$$\Delta \text{AS} = \sum_f s^f (\mu^f - 1) - \log \left[\sum_f s^f e^{\mu^f - 1} \right]. \quad (\text{A.19})$$

Equality in Jensen's inequality holds exactly when $\mu^f - 1$ is constant across active firms. Since $\mu^f - 1 = s^f / (1 - s^f)$ is strictly increasing in s^f , this is equivalent to equal market shares. Conversely, equal shares make (A.19) zero.

Failure of global monotonicity. Differentiating (A.4) and using (A.8) yields

$$\frac{\partial \Delta \text{AS}}{\partial s^f} = \frac{\alpha}{(1 - \alpha s^f)^2} - \frac{\alpha}{D} - \frac{D + \alpha(1 - \Sigma)}{D^2} h'(s^f). \quad (\text{A.20})$$

At a vector with $s^f = 0$, positive shares for the other firms, and $\Sigma < 1$, equation (A.20) gives

$$\left. \frac{\partial \Delta \text{AS}}{\partial s^f} \right|_{s^f=0} = \alpha(1 - D^{-1}) > 0.$$

By continuity, this remains true for sufficiently small positive s^f , ruling out global coordinate-wise decreasing behavior. Conversely, ΔAS is strictly negative at every positive share vector with $\Sigma < 1$ and tends to zero along the ray toward the origin. Thus global coordinatewise increasing behavior is also impossible.

Failure of a global Schur ordering. To study a mean-preserving spread between two firms initially having the same share r , define

$$F(\varepsilon) = \Delta \text{AS}(\dots, r + \varepsilon, r - \varepsilon, \dots),$$

with all other shares fixed. This is an even analytic function near zero. Along the perturbation, Σ is fixed, $D'(0) = 0$, and $D''(0) = 2h''(r)$. Consequently,

$$\frac{1}{2}F''(0) = \frac{2\alpha^2}{(1-\alpha r)^3} - \frac{D + \alpha(1-\Sigma)}{D^2}h''(r), \quad (\text{A.21})$$

where D and Σ on the right are evaluated at the initial vector. For sufficiently small $\varepsilon > 0$, the perturbed vector is a nontrivial mean-preserving spread of the initial vector.

At the symmetric no-outside-option vector, put $r = 1/n$. We have $\Sigma = 1$ and $D = \phi(r)$. Using (A.7), (A.21) becomes

$$\frac{1}{2}F''(0) = \frac{\alpha}{(1-\alpha r)^3(1-r)^2} [2\alpha(1-r)^2 - (2-r)(1-\alpha r)] < 0.$$

Indeed, the expression in square brackets satisfies

$$\begin{aligned} 2\alpha(1-r)^2 - (2-r)(1-\alpha r) &= -(2-r) + \alpha(2-2r+r^2) \\ &\leq -r(1-r) < 0. \end{aligned}$$

Thus a small mean-preserving spread lowers ΔAS .

Next consider the no-outside-option share vector

$$\mathbf{s} = (1 - (n-1)\eta, \eta, \dots, \eta), \quad \eta > 0, \quad n \geq 3.$$

As $\eta \downarrow 0$, the dominant firm's share tends to one and $D \rightarrow \infty$, for both CES and MNL demand. For a transfer between two small firms, (A.21) gives

$$\frac{1}{2}F''(0) = \frac{2\alpha^2}{(1-\alpha\eta)^3} - \frac{h''(\eta)}{D} \rightarrow 2\alpha^2 > 0,$$

since $h''(\eta) \rightarrow 2\alpha$. For sufficiently small positive η , a small mean-preserving spread therefore raises ΔAS . $\square$

B Proof of Proposition 4

Let us show that $\Sigma(\theta)$ is weakly decreasing in θ while $\Pi(\theta)$ is strictly increasing. We begin by studying the sign of the partial derivatives of the markup and market-share fitting-in functions. Recall from Section 2.2 that $m(x, \theta)$ and $S(x, \theta)$ jointly solve the system

$$\mu = \frac{1}{1 - \theta\alpha s} \quad \text{and} \quad s = x(1 - (1 - \alpha)\mu)^{\frac{\alpha}{1-\alpha}}.$$

Hence,

$$S = x(1 - (1 - \alpha)m)^{\frac{\alpha}{1-\alpha}} = x \left(1 - \frac{1 - \alpha}{1 - \theta\alpha S}\right)^{\frac{\alpha}{1-\alpha}} = x\alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{1 - \theta S}{1 - \theta\alpha S}\right)^{\frac{\alpha}{1-\alpha}}.$$

Taking logs and differentiating yields

$$\frac{dS}{S} = \frac{dx}{x} + \frac{\alpha}{1 - \alpha} \left[\frac{dS}{S} + \frac{d\theta}{\theta} \right] \left(\frac{\alpha\theta S}{1 - \alpha\theta S} - \frac{\theta S}{1 - \theta S} \right).$$

Rearranging terms, we obtain

$$\frac{dS}{S} \frac{1 - \theta S + \alpha(\theta S)^2}{(1 - \theta S)(1 - \alpha\theta S)} = \frac{dx}{x} - \frac{\alpha S}{(1 - \theta S)(1 - \alpha\theta S)} d\theta.$$

The partial derivatives of S are therefore given by

$$\frac{\partial S}{\partial x} = \frac{S}{x} \frac{(1 - \theta S)(1 - \alpha\theta S)}{1 - \theta S + \alpha(\theta S)^2} > 0 \quad \text{and} \quad \frac{\partial S}{\partial \theta} = \frac{-\alpha S^2}{1 - \theta S + \alpha(\theta S)^2} < 0.$$

It follows that

$$\begin{aligned} \frac{\partial m}{\partial \theta} &= \frac{\alpha}{(1 - \alpha\theta S(x, \theta))^2} \left(S(x, \theta) + \theta \frac{\partial S}{\partial \theta} \right) \\ &= \frac{\alpha S(x, \theta)(1 - \theta S(x, \theta))}{(1 - \alpha\theta S(x, \theta))(1 - \theta S(x, \theta) + \alpha(\theta S(x, \theta))^2)} > 0, \end{aligned}$$

while

$$\frac{\partial m}{\partial x} = \frac{\alpha\theta}{(1 - \alpha\theta S(x, \theta))^2} \frac{\partial S}{\partial x} \geq 0,$$

with strict inequality for $\theta > 0$.

Next, we show that $H^*(\theta)$ is strictly decreasing in θ . Totally differentiating equilibrium condition (11), we obtain:

$$-\frac{dH^*}{H^*(\theta)} \left(\frac{H^0}{H^*(\theta)} + \sum_{f \in \mathcal{F}} \frac{T^f}{H^*(\theta)} \frac{\partial S}{\partial x} \left(\frac{T^f}{H^*(\theta)}, \theta \right) \right) + d\theta \sum_{f \in \mathcal{F}} \frac{\partial S}{\partial \theta} \left(\frac{T^f}{H^*(\theta)}, \theta \right) = 0.$$

Hence,

$$\frac{d \log H^*}{d\theta} = \frac{\sum_{f \in \mathcal{F}} \frac{\partial S}{\partial \theta} \left(\frac{T^f}{H^*(\theta)}, \theta \right)}{\frac{H^0}{H^*(\theta)} + \sum_{f \in \mathcal{F}} \frac{T^f}{H^*(\theta)} \frac{\partial S}{\partial x} \left(\frac{T^f}{H^*(\theta)}, \theta \right)} < 0.$$

It follows that $\Sigma(\theta) = 1 - H^0/H^*(\theta)$ is weakly decreasing in θ .

To prove that $\Pi(\theta)$ is strictly increasing, let us show that each firm f 's equilibrium profit, $\pi^f(\theta)$, is strictly increasing. As $\partial m/\partial \theta > 0$ and $\partial m/\partial x \geq 0$ while $dH^*/d\theta < 0$, it follows

that, for every $g \in \mathcal{F}$, the equilibrium ι -markup of firm g , $\mu^g(\theta) = m(T^g/(H^*(\theta)), \theta)$, has a strictly positive derivative. Let

$$\widehat{\pi}^f((\mu^g)_{g \in \mathcal{F}}) \equiv \alpha \mu^f \frac{T^f (1 - (1 - \alpha) \mu^f)^{\frac{\alpha}{1-\alpha}}}{H^0 + \sum_{g \in \mathcal{F}} T^g (1 - (1 - \alpha) \mu^g)^{\frac{\alpha}{1-\alpha}}}$$

be firm f 's profit when firms set the profile of ι -markups $(\mu^g)_{g \in \mathcal{F}}$. Clearly, $\widehat{\pi}^f$ has strictly positive partial derivatives with respect to the ι -markups of rival firms and $\pi^f(\theta) = \widehat{\pi}^f((\mu^g(\theta))_{g \in \mathcal{F}})$. (Recall from equation (4) that, under the ι -markup μ^f , firm f makes a profit of $\alpha \mu^f s^f$.) Moreover, when the profile of ι -markups is equal to $(\mu^g(\theta))_{g \in \mathcal{F}}$, the fraction in the definition of $\widehat{\pi}^f$ is equal to $s^f(\theta)$ and its denominator is equal to $H^*(\theta)$ (see equations (6) and (11)). We have:

$$\begin{aligned} \left. \frac{\partial \widehat{\pi}^f}{\partial \mu^f} \right|_{(\mu^g(\theta))_{g \in \mathcal{F}}} &= \alpha T^f (1 - (1 - \alpha) \mu^f(\theta))^{\frac{\alpha}{1-\alpha}-1} \\ &\quad \times \frac{(1 - \mu^f(\theta)) H^*(\theta) + \alpha \mu^f(\theta) T^f (1 - (1 - \alpha) \mu^f(\theta))^{\frac{\alpha}{1-\alpha}}}{H^*(\theta)^2} \\ &= \frac{\alpha T^f (1 - (1 - \alpha) \mu^f(\theta))^{\frac{\alpha}{1-\alpha}-1}}{H^*(\theta)} [1 - \mu^f(\theta)(1 - \alpha s^f(\theta))] \\ &= \frac{\alpha T^f (1 - (1 - \alpha) \mu^f(\theta))^{\frac{\alpha}{1-\alpha}-1}}{H^*(\theta)} \frac{\alpha s^f(\theta)(1 - \theta)}{1 - \alpha \theta s^f(\theta)}, \end{aligned}$$

where we have used the fact that $\mu^f(\theta)$ and $s^f(\theta)$ satisfy condition (9) to obtain the last line. Note that the final expression is strictly positive unless $\theta = 1$. We can conclude: for every $\theta \in [0, 1)$,

$$\frac{d\pi^f}{d\theta} = \sum_{g \in \mathcal{F}} \left. \frac{\partial \widehat{\pi}^f}{\partial \mu^g} \right|_{(\mu^g(\theta))_{g \in \mathcal{F}}} \frac{d\mu^g}{d\theta} > 0. \quad \square$$

C Proof of Proposition 6

The formula for the case without an outside option was derived in the main text. Suppose now that consumers have access to an outside option, $H^0 > 0$. The amount of resources utilized by the industry in equilibrium now depends on the market share of the inside goods:

$$C = \frac{\alpha}{1 - \alpha} \Sigma(1) - \Pi(1). \quad (\text{C.1})$$

The resource-efficient allocation features again a common ι -markup $\tilde{\mu}$. Under such a common markup, industry profit is

$$\tilde{\Pi} = \alpha \tilde{\mu} \tilde{\Sigma}, \quad (\text{C.2})$$

where

$$\tilde{\Sigma} \equiv \frac{\mathcal{T}\psi(\tilde{\mu})}{H^0 + \mathcal{T}\psi(\tilde{\mu})},$$

is the aggregate share of the inside goods. This results in the following amount of resources being employed

$$\tilde{C} = \frac{\alpha}{1-\alpha}\tilde{\Sigma} - \alpha\tilde{\mu}\tilde{\Sigma} = \frac{\alpha}{1-\alpha}\tilde{\Sigma}(1 - (1-\alpha)\tilde{\mu}). \quad (\text{C.3})$$

For what follows, it is more convenient to work directly with the market share $\tilde{\Sigma}$. We thus invert the relationship between $\tilde{\mu}$ and $\tilde{\Sigma}$ to obtain:

$$\begin{aligned} \tilde{\mu} &= \frac{1}{1-\alpha} \left[1 - \left(\frac{H^0}{\mathcal{T}} \frac{\tilde{\Sigma}}{1-\tilde{\Sigma}} \right)^{\frac{1-\alpha}{\alpha}} \right] \\ &= \frac{1}{1-\alpha} \left[1 - \left(\frac{H^0}{H^*(1)} \frac{1}{\sum_{f \in \mathcal{F}} \frac{T^f}{H^*(1)}} \frac{\tilde{\Sigma}}{1-\tilde{\Sigma}} \right)^{\frac{1-\alpha}{\alpha}} \right] \\ &= \frac{1}{1-\alpha} \left[1 - \left(\frac{1 - \Sigma(1)}{\sum_{f \in \mathcal{F}} \frac{s^f(1)}{\psi(1/(1-\alpha s^f(1)))}} \frac{\tilde{\Sigma}}{1-\tilde{\Sigma}} \right)^{\frac{1-\alpha}{\alpha}} \right], \end{aligned}$$

where the last line follows from equations (5) and (6). Inserting this into equation (C.3) yields

$$\tilde{C} = \frac{\alpha}{1-\alpha} \tilde{\Sigma}^{\frac{1}{\alpha}} \left(\frac{1}{\sum_{f \in \mathcal{F}} \frac{s^f(1)}{\psi(1/(1-\alpha s^f(1)))}} \frac{1 - \Sigma(1)}{1 - \tilde{\Sigma}} \right)^{\frac{1-\alpha}{\alpha}}.$$

While there is generally no closed-form solution for $\tilde{\Sigma}$, the strict monotonicity of the right-hand side of the above equation implies that its value is uniquely pinned down by the resource constraint, $\tilde{C} = C$. We denote this unique solution by $\tilde{\Sigma}(\mathbf{s})$.

Industry profit in the resource-efficient allocation can be rewritten as

$$\tilde{\Pi}(\mathbf{s}) = \frac{\alpha}{1-\alpha} \tilde{\Sigma}(\mathbf{s}) - C = \frac{\alpha}{1-\alpha} (\tilde{\Sigma}(\mathbf{s}) - \Sigma(\mathbf{s})) + \Pi(1), \quad (\text{C.4})$$

where we have used equations (C.2) and (C.3) to obtain the first equality and equation (C.1) to obtain the second. As the market share of the outside option in the resource-efficient allocation is $1 - \tilde{\Sigma}(\mathbf{s}) = H^0/\tilde{H}(\mathbf{s})$, with $\tilde{H}(\mathbf{s})$ denoting the value of the aggregator in that allocation, consumer surplus is given by $\tilde{\text{CS}}(\mathbf{s}) = \log \tilde{H}(\mathbf{s}) = \log H^0 - \log(1 - \tilde{\Sigma}(\mathbf{s}))$. Adding industry profit from equation (C.4), we obtain aggregate surplus in the resource-efficient allocation:

$$\tilde{\text{AS}}(\mathbf{s}) = \log H^0 - \log(1 - \tilde{\Sigma}(\mathbf{s})) + \frac{\alpha}{1-\alpha} (\tilde{\Sigma}(\mathbf{s}) - \Sigma(\mathbf{s})) + \Pi(1).$$

Equilibrium aggregate surplus is equal to $\log H^*(\mathbf{s}) + \Pi(1)$, where $H^*(\mathbf{s}) = H^0/(1 - \Sigma(\mathbf{s}))$ from the adding-up constraint. Subtracting $\widetilde{\text{AS}}(\mathbf{s})$, we obtain the welfare loss from within-industry resource misallocation:

$$M^W(\mathbf{s}) = \log(1 - \widetilde{\Sigma}(\mathbf{s})) - \log(1 - \Sigma(\mathbf{s})) + \frac{\alpha}{1 - \alpha}(\Sigma(\mathbf{s}) - \widetilde{\Sigma}(\mathbf{s})).$$

The fact that $M^W(\mathbf{s})$ is strictly negative if and only if firms have heterogeneous market shares follows by Proposition 5. $\square$

D Approximations Around Small Firms

Write

$$t = \Sigma(\mathbf{s}), \quad Q_2 = \sum_f (s^f)^2 = t^2 \text{HHI}(\mathbf{s}), \quad Q_3 = \sum_f (s^f)^3 = t^3 \text{CHK}(\mathbf{s}).$$

For $t > 0$, $Q_k \leq t^k$. Moreover, t and $\|\mathbf{s}\|$ are equivalent on the nonnegative orthant, so a remainder $O(t^4)$ is $o(\|\mathbf{s}\|^3)$. All remainder bounds below are uniform in the relative shares $\mathbf{s}/t$.

Common expansions. The function ϕ satisfies

$$\phi(x) = 1 + \alpha x + bx^2 + O(x^3), \quad b = \frac{\alpha(1 + 2\alpha)}{2}.$$

Hence, defining

$$B = \sum_f s^f \phi(s^f), \quad P = \sum_f \frac{s^f}{1 - \alpha s^f},$$

we have

$$B = t + \alpha Q_2 + bQ_3 + O(t^4), \quad P = t + \alpha Q_2 + \alpha^2 Q_3 + O(t^4). \quad (\text{D.1})$$

Proof of Proposition 2. The exact formulas of Proposition 1 give

$$D = 1 - t + B = 1 + \alpha Q_2 + bQ_3 + O(t^4), \quad \Delta \text{CS} = -\log D, \quad \Delta \Pi = \alpha \left(P - \frac{B}{D} \right).$$

Since $D - 1 = O(t^2)$,

$$\Delta \text{CS} = -\alpha Q_2 - bQ_3 + O(t^4), \quad \frac{B}{D} = t + \alpha Q_2 + bQ_3 - \alpha t Q_2 + O(t^4).$$

Consequently,

$$\Delta \Pi = \alpha^2 \left(tQ_2 - \frac{Q_3}{2} \right) + O(t^4),$$

and adding this to ΔCS yields

$$\Delta \text{AS} = -\alpha Q_2 + \alpha^2 t Q_2 - \frac{\alpha(1+3\alpha)}{2} Q_3 + O(t^4).$$

Substituting $Q_2 = t^2 \text{HHI}$ and $Q_3 = t^3 \text{CHK}$ proves both formulas. $\square$

Proof of Proposition 7. Demand is now CES, so $0 < \alpha < 1$. Write $\tilde{t} = \tilde{\Sigma}(\mathbf{s})$ and set

$$A = \sum_f \frac{s^f}{\psi(1/(1-\alpha s^f))} = \frac{B}{\psi(1)}, \quad \psi(1) = \alpha^{\alpha/(1-\alpha)}.$$

By equation (C.3), the resource constraint is

$$\frac{\alpha}{1-\alpha} \tilde{t}^{1/\alpha} \left(\frac{1-t}{A(1-\tilde{t})} \right)^{(1-\alpha)/\alpha} = \frac{\alpha}{1-\alpha} t - \alpha P. \quad (\text{D.2})$$

Define

$$u = \frac{B}{t}, \quad v = \frac{t - (1-\alpha)P}{\alpha t}, \quad G(x) = \frac{x}{(1-x)^{1-\alpha}}.$$

Substituting $A = B/\psi(1)$ in (D.2) and raising both sides to the power α gives

$$G(\tilde{t}) = G(t) u^{1-\alpha} v^\alpha. \quad (\text{D.3})$$

By (D.1),

$$u = 1 + \alpha \frac{Q_2}{t} + b \frac{Q_3}{t} + O(t^3), \quad v = 1 - (1-\alpha) \frac{Q_2}{t} - \alpha(1-\alpha) \frac{Q_3}{t} + O(t^3).$$

Expanding their logarithms, the terms in Q_2/t cancel:

$$(1-\alpha) \log u + \alpha \log v = \frac{\alpha(1-\alpha)}{2} \left(\frac{Q_3}{t} - \frac{Q_2^2}{t^2} \right) + O(t^3). \quad (\text{D.4})$$

The right-hand side is $O(t^2)$. Since $G(t) = t + O(t^2)$, exponentiating and using (D.3) therefore gives

$$G(\tilde{t}) - G(t) = \frac{\alpha(1-\alpha)}{2} \left(Q_3 - \frac{Q_2^2}{t} \right) + O(t^4).$$

The same identity implies $\tilde{t} = O(t)$: its right-hand side before subtraction is $O(t)$ and $G(x) \geq x$ on $[0, 1)$. As

$$G'(x) = \frac{1-\alpha x}{(1-x)^{2-\alpha}} = 1 + O(x),$$

the mean value theorem yields

$$\tilde{t} - t = \frac{\alpha(1 - \alpha)}{2} \left(Q_3 - \frac{Q_2^2}{t} \right) + O(t^4). \quad (\text{D.5})$$

Finally, equation (17) implies

$$\begin{aligned} M^W(\mathbf{s}) &= \log(1 - \tilde{t}) - \log(1 - t) + \frac{\alpha}{1 - \alpha}(t - \tilde{t}) \\ &= -\frac{\tilde{t} - t}{1 - \alpha} + O(t^4) \\ &= -\frac{\alpha}{2} \left(Q_3 - \frac{Q_2^2}{t} \right) + O(t^4) \\ &= -\frac{\alpha}{2} t^3 (\text{CHK}(\mathbf{s}) - \text{HHI}(\mathbf{s})^2) + O(t^4). \end{aligned}$$

The bounds $Q_2/t \leq t$ and $Q_3/t \leq t^2$ ensure that the argument applies along arbitrary approaches to the origin, even when relative shares vary. $\square$

E Conduct and Welfare Distortions

This appendix proves Propositions 3, 8, and 12. Throughout, primitives are held fixed as θ varies. All approximations are for $\theta \downarrow 0$. The concentration indices on the right-hand sides are evaluated at the prevailing equilibrium shares.

A common expansion. For a nonnegative share vector $\mathbf{s}$ with $\Sigma = \sum_f s^f \leq 1$, define

$$Q_2 = \sum_f (s^f)^2 = \Sigma^2 \text{HHI}, \quad Q_3 = \sum_f (s^f)^3 = \Sigma^3 \text{CHK},$$

where the equalities involving concentration indices apply when $\Sigma > 0$. Write

$$D_\theta = 1 - \Sigma + \sum_f s^f \phi(\theta s^f), \quad P_\theta = \sum_f \frac{s^f}{1 - \alpha \theta s^f}. \quad (\text{E.1})$$

We have:

Lemma E.1. *For fixed $\alpha \in (0, 1]$, uniformly over all such share vectors,*

$$D_\theta = 1 + \alpha Q_2 \theta + \frac{\alpha(1 + 2\alpha)}{2} Q_3 \theta^2 + O(\theta^3), \quad (\text{E.2})$$

$$\log D_\theta = \alpha Q_2 \theta + \frac{\alpha}{2} [(1 + 2\alpha) Q_3 - \alpha Q_2^2] \theta^2 + O(\theta^3), \quad (\text{E.3})$$

$$P_\theta = \Sigma + \alpha Q_2 \theta + \alpha^2 Q_3 \theta^2 + O(\theta^3). \quad (\text{E.4})$$

The remainder bounds do not depend on the number of firms.

Proof. Under either demand specification,

$$(\log \phi)'(x) = \frac{\alpha}{(1-x)(1-\alpha x)}.$$

Since $\phi(0) = 1$, Taylor's theorem gives

$$\phi(x) = 1 + \alpha x + \frac{\alpha(1+2\alpha)}{2}x^2 + O(x^3), \quad \frac{1}{1-\alpha x} = 1 + \alpha x + \alpha^2 x^2 + O(x^3).$$

On any fixed interval $0 \leq x \leq \bar{\theta} < 1$, both remainders are bounded by a constant times x^3 . Substituting $x = \theta s^f$, multiplying by s^f , and summing bounds each aggregate remainder by a constant times $\theta^3 \sum_f (s^f)^4 \leq \theta^3$. This proves the first and third expansions. Expanding $\log(1+u) = u - u^2/2 + O(u^3)$ proves the second, since $D_\theta - 1 = O(\theta)$ uniformly. $\square$

These uniform bounds allow $\mathbf{s}$ to vary with θ . They also justify integrating the expansions across industries in general equilibrium, since $\int \rho(z) dz = 1$.

Proof of Proposition 3. Evaluate D_θ and P_θ at $\mathbf{s}(\theta)$ and suppress the argument θ of shares and moments. The equilibrium relations $s^f H^*(\theta) = T^f \psi(\mu^f)$ and $\mu^f = (1 - \alpha \theta s^f)^{-1}$ imply

$$\frac{\psi(1)}{\psi(\mu^f)} = \phi(\theta s^f), \quad \frac{H^*(0)}{H^*(\theta)} = 1 - \Sigma + \sum_f s^f \phi(\theta s^f) = D_\theta.$$

Moreover, $1 - \Sigma(\theta) = H^0/H^*(\theta)$ yields

$$\Sigma(0) = 1 - \frac{1 - \Sigma}{D_\theta}.$$

We thus obtain:

$$\Delta \text{CS}(\theta) = -\log D_\theta, \tag{E.5}$$

$$\Delta \text{AS}(\theta) = -\log D_\theta + \alpha \left[P_\theta - 1 + \frac{1 - \Sigma}{D_\theta} \right]. \tag{E.6}$$

By Lemma E.1,

$$D_\theta^{-1} = 1 - \alpha Q_2 \theta + \left[\alpha^2 Q_2^2 - \frac{\alpha(1+2\alpha)}{2} Q_3 \right] \theta^2 + O(\theta^3).$$

Substituting into (E.5)–(E.6) and collecting terms gives

$$\begin{aligned} \Delta \text{CS}(\theta) &= -\alpha Q_2 \theta - \frac{\alpha}{2} [(1+2\alpha) Q_3 - \alpha Q_2^2] \theta^2 + O(\theta^3), \\ \Delta \text{AS}(\theta) &= -\alpha(1 - \alpha \Sigma) Q_2 \theta \end{aligned}$$

$$-\frac{\alpha}{2} \left[(1 + 3\alpha - (\alpha + 2\alpha^2)\Sigma)Q_3 - (\alpha + 2\alpha^2(1 - \Sigma))Q_2^2 \right] \theta^2 + O(\theta^3).$$

Using $Q_2 = \Sigma^2 \text{HHI}$ and $Q_3 = \Sigma^3 \text{CHK}$ proves the result. $\square$

Proof of Proposition 8. Assume CES demand, so that $\alpha \in (0, 1)$, and allow $H^0 \geq 0$. Continue to evaluate all shares and moments at conduct θ , suppressing this argument. Define

$$u_\theta = \frac{D_\theta - 1 + \Sigma}{\Sigma}, \quad R_\theta = \frac{\Sigma - (1 - \alpha)P_\theta}{\alpha\Sigma}. \quad (\text{E.7})$$

By Lemma E.1,

$$u_\theta = 1 + \alpha \frac{Q_2}{\Sigma} \theta + \frac{\alpha(1 + 2\alpha)}{2} \frac{Q_3}{\Sigma} \theta^2 + O(\theta^3), \quad (\text{E.8})$$

$$R_\theta = 1 - (1 - \alpha) \frac{Q_2}{\Sigma} \theta - \alpha(1 - \alpha) \frac{Q_3}{\Sigma} \theta^2 + O(\theta^3). \quad (\text{E.9})$$

These expansions are uniform over share vectors with $0 < \Sigma \leq 1$. Indeed, the proof of the lemma bounds each remainder before division by Σ by a constant times $\theta^3 \sum_f (s^f)^4 \leq \theta^3 \Sigma$. Moreover, $Q_2/\Sigma \leq 1$ and $Q_3/\Sigma \leq 1$, so both u_θ and R_θ are uniformly bounded away from zero for sufficiently small θ . Expanding their logarithms gives

$$\log u_\theta = \alpha \frac{Q_2}{\Sigma} \theta + \left[\frac{\alpha(1 + 2\alpha)}{2} \frac{Q_3}{\Sigma} - \frac{\alpha^2}{2} \frac{Q_2^2}{\Sigma^2} \right] \theta^2 + O(\theta^3), \quad (\text{E.10})$$

$$\log R_\theta = -(1 - \alpha) \frac{Q_2}{\Sigma} \theta - \left[\alpha(1 - \alpha) \frac{Q_3}{\Sigma} + \frac{(1 - \alpha)^2}{2} \frac{Q_2^2}{\Sigma^2} \right] \theta^2 + O(\theta^3). \quad (\text{E.11})$$

Consequently, the first-order terms cancel in the combination

$$(1 - \alpha) \log u_\theta + \alpha \log R_\theta = \frac{\alpha(1 - \alpha)}{2} \left(\frac{Q_3}{\Sigma} - \frac{Q_2^2}{\Sigma^2} \right) \theta^2 + O(\theta^3). \quad (\text{E.12})$$

Without an outside option. Here $\Sigma = 1$ and $u_\theta = D_\theta$. The argument in Section 4.3 applies at each conduct θ : the resource constraint implies that the common ι -markup in the resource-efficient allocation is $\tilde{\mu} = P_\theta$, and industry profits coincide in the two allocations. Hence, since $\psi(P_\theta)/\psi(1) = R_\theta^{\alpha/(1-\alpha)}$, the exact within-industry loss is

$$M^W(\theta) = -\log D_\theta - \frac{\alpha}{1 - \alpha} \log R_\theta. \quad (\text{E.13})$$

Equation (E.12) therefore gives

$$M^W(\theta) = -\frac{\alpha}{2} (Q_3 - Q_2^2) \theta^2 + O(\theta^3).$$

The remainder is uniform over share vectors, as required below for the general-equilibrium

analysis.

With an outside option. Now $H^0 > 0$ and $\Sigma < 1$. Let $\tilde{\Sigma}$ denote the aggregate inside share in the resource-efficient allocation using $C(\theta)$ units of the resource, and define $G(x) = x/(1-x)^{1-\alpha}$. The resource constraint and welfare formula derived in Appendix C continue to hold at conduct θ , with $\mu^f = (1 - \alpha\theta s^f)^{-1}$ and $\Pi(\theta) = \alpha P_\theta$. In the present notation, they give

$$G(\tilde{\Sigma}) = G(\Sigma) u_\theta^{1-\alpha} R_\theta^\alpha, \quad (\text{E.14})$$

$$M^W(\theta) = \log(1 - \tilde{\Sigma}) - \log(1 - \Sigma) + \frac{\alpha}{1 - \alpha}(\Sigma - \tilde{\Sigma}). \quad (\text{E.15})$$

To obtain the first identity from that resource constraint, use

$$\sum_f \frac{s^f}{\psi(\mu^f)} = \frac{\Sigma u_\theta}{\psi(1)}, \quad C(\theta) = \frac{\alpha}{1 - \alpha} \Sigma - \alpha P_\theta = \frac{\alpha^2}{1 - \alpha} \Sigma R_\theta,$$

and raise the constraint to the power α .

By the analysis in Appendix B, $\Sigma(\theta) \rightarrow \Sigma(0) \in (0, 1)$ as $\theta \downarrow 0$. Equation (E.14) likewise implies $\tilde{\Sigma} \rightarrow \Sigma(0)$, since G is strictly increasing. Taking logs in that equation and applying (E.12) gives

$$\log G(\tilde{\Sigma}) - \log G(\Sigma) = \frac{\alpha(1 - \alpha)}{2} \left(\frac{Q_3}{\Sigma} - \frac{Q_2^2}{\Sigma^2} \right) \theta^2 + O(\theta^3).$$

Since $(\log G)'(x) = (1 - \alpha x)/(x(1 - x)) > 0$, the mean value theorem first gives $\tilde{\Sigma} - \Sigma = O(\theta^2)$ and then

$$\tilde{\Sigma} - \Sigma = \frac{\alpha(1 - \alpha)(1 - \Sigma)}{2(1 - \alpha\Sigma)} \left(Q_3 - \frac{Q_2^2}{\Sigma} \right) \theta^2 + O(\theta^3).$$

Substituting into (E.15), we obtain

$$\begin{aligned} M^W(\theta) &= -\frac{1 - \alpha\Sigma}{(1 - \alpha)(1 - \Sigma)}(\tilde{\Sigma} - \Sigma) + O((\tilde{\Sigma} - \Sigma)^2) \\ &= -\frac{\alpha}{2} \left(Q_3 - \frac{Q_2^2}{\Sigma} \right) \theta^2 + O(\theta^3). \end{aligned}$$

The same expression holds when $H^0 = 0$, since then $\Sigma = 1$. Finally, $Q_3 - Q_2^2/\Sigma = \Sigma^3(\text{CHK} - \text{HHI}^2)$ proves the proposition in both cases. $\square$

Proof of Proposition 12. For industry z , let $D_\theta(z)$, $P_\theta(z)$, and $R_\theta(z)$ be defined by (E.1) and (E.7) using its equilibrium shares $s^f(z; \theta)$. Here there is no outside option, so $\Sigma(z; \theta) = 1$ in every industry. Define also

$$\bar{R}_\theta = \int \rho(z) R_\theta(z) dz, \quad Q_k(z) = \sum_{f \in \mathcal{F}(z)} s^f(z; \theta)^k \quad (k = 2, 3).$$

The ratio of an industry's wage bill to sales is

$$\sum_f s^f(z; \theta) [1 - (1 - \alpha) \mu^f(z; \theta)] = 1 - (1 - \alpha) P_\theta(z) = \alpha R_\theta(z).$$

With nominal income and aggregate labor supply normalized to one, labor market clearing gives $w^*(\theta) = \alpha \bar{R}_\theta$. In particular, $w^*(0)/w^*(\theta) = 1/\bar{R}_\theta$. Also $H^*(z; 0)/H^*(z; \theta) = D_\theta(z)$ at a fixed wage. The conduct-dependent counterpart of the exact welfare formula is therefore

$$\Delta \log \text{SW}(\theta) = -\frac{1 - \alpha}{\alpha} \int \rho(z) \log D_\theta(z) dz - \log \bar{R}_\theta. \quad (\text{E.16})$$

Applying (E.13) within each industry at its equilibrium wage bill, and converting log aggregators into log welfare, gives the exact decomposition

$$M^W(\theta) = -\frac{1 - \alpha}{\alpha} \int \rho(z) \log D_\theta(z) dz - \int \rho(z) \log R_\theta(z) dz, \quad (\text{E.17})$$

$$M^B(\theta) = \int \rho(z) \log R_\theta(z) dz - \log \bar{R}_\theta. \quad (\text{E.18})$$

The no-outside-option case of the proof of Proposition 8 and its uniform remainder imply

$$M^W(\theta) = -\frac{1 - \alpha}{2} \theta^2 \int \rho(z) [Q_3(z) - Q_2(z)^2] dz + O(\theta^3).$$

For the between-industry component, write $\bar{Q}_k = \int \rho(z) Q_k(z) dz$. Integrating (E.9) and then expanding the logarithm gives

$$\log \bar{R}_\theta = -(1 - \alpha) \bar{Q}_2 \theta - \left[\alpha(1 - \alpha) \bar{Q}_3 + \frac{(1 - \alpha)^2}{2} \bar{Q}_2^2 \right] \theta^2 + O(\theta^3).$$

Subtracting this from the integral of (E.11) cancels both the first-order terms and the terms involving $\bar{Q}_3$. Hence

$$M^B(\theta) = -\frac{(1 - \alpha)^2}{2} \theta^2 \left[\int \rho(z) Q_2(z)^2 dz - \bar{Q}_2^2 \right] + O(\theta^3).$$

Substituting $Q_2(z) = \text{HHI}(z; \theta)$ and $Q_3(z) = \text{CHK}(z; \theta)$ proves both displayed approximations. Their sum equals (E.16), completing the proof. $\square$

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F Direct Utility Functions

This appendix derives the direct utility functions associated with the indirect utility in Section 2.1 when $H^0 > 0$.

MNL demand. Define

$$q_j \equiv \frac{\lambda x_j}{V_0}, \quad q_0 \equiv 1 - \sum_{j \in \mathcal{N}} q_j.$$

The direct utility function is

$$U(x_0, \mathbf{x}) = \begin{cases} x_0 + \sum_{j \in \mathcal{N}} a_j x_j - V_0 \left[\sum_{j \in \mathcal{N}} q_j \log q_j + q_0 \log \frac{q_0}{H^0} \right], & \text{if } \sum_{j \in \mathcal{N}} x_j \leq \frac{V_0}{\lambda}, \\ -\infty, & \text{otherwise,} \end{cases} \quad (\text{F.1})$$

where $0 \log 0 = 0$. This is the entropy representation of MNL preferences (see Anderson, de Palma, and Thisse, 1992, Chapter 3), with the outside option included as an additional alternative.

CES demand. Let

$$K \equiv V_0(\sigma - 1), \quad A(\mathbf{x}) \equiv \sum_{j \in \mathcal{N}} a_j^{1/\sigma} x_j^\sigma.$$

For each $A \geq 0$, define $t(A) > 0$ as the unique solution to

$$H^0 t(A)^\sigma + A t(A) = K. \quad (\text{F.2})$$

The direct utility function is

$$U(x_0, \mathbf{x}) = x_0 + V_0 \log \frac{K}{t(A(\mathbf{x}))^\sigma} + A(\mathbf{x}) t(A(\mathbf{x})). \quad (\text{F.3})$$

To derive this expression by duality, write $U(x_0, \mathbf{x}) = x_0 + u(\mathbf{x})$ and set

$$u(\mathbf{x}) = \inf_{\mathbf{p} \gg \mathbf{0}} \left\{ V_0 \log \left(H^0 + \sum_{j \in \mathcal{N}} a_j p_j^{1-\sigma} \right) + \sum_{j \in \mathcal{N}} p_j x_j \right\}. \quad (\text{F.4})$$

The expression in braces is strictly convex in the price vector: the log-sum has a positive-definite Hessian, as can be verified directly. For $\mathbf{x} \gg \mathbf{0}$, it diverges as any price approaches

zero or infinity, so its unique minimum satisfies

$$x_j = \frac{K a_j p_j^{-\sigma}}{H}, \quad H = H^0 + \sum_{j \in \mathcal{N}} a_j p_j^{1-\sigma}.$$

Writing $t = (K/H)^{1/\sigma}$ gives

$$p_j = t a_j^{1/\sigma} x_j^{-1/\sigma}, \quad \sum_j p_j x_j = t A, \quad H = H^0 + t^{1-\sigma} A.$$

Combining the last equality with $H = K/t^\sigma$ yields equation (F.2), and substituting into (F.4) yields (F.3). If some $x_j = 0$, the same formula follows by sending the corresponding prices to infinity, or by continuity.

G Resource Misallocation in the MNL Model without an Outside Option

This section provides the results for MNL demand without an outside option referred to in Sections 4.2 and 4.3. Without an outside option, total output is fixed at $1/\lambda$. For any feasible output vector, let $q_j \equiv \lambda x_j$, so that $\mathbf{q}$ belongs to the simplex $\Delta_{\mathcal{N}} \equiv \{\mathbf{q} \geq \mathbf{0} : \sum_j q_j = 1\}$. Aggregate surplus and resource use are, respectively,

$$W(\mathbf{q}) = \frac{1}{\lambda} \sum_{j \in \mathcal{N}} (a_j - c_j) q_j - \sum_{j \in \mathcal{N}} q_j \log q_j, \quad C(\mathbf{q}) = \frac{1}{\lambda} \sum_{j \in \mathcal{N}} c_j q_j, \quad (\text{G.1})$$

where $0 \log 0 = 0$. We write $s_j \equiv \lambda x_j^*$ for equilibrium product shares and retain $s^f \equiv \sum_{j \in f} s_j$ for firm shares. All these shares are strictly positive.

For probability vectors $\mathbf{u}$ and $\mathbf{v}$ on the same finite set, with $\mathbf{v}$ strictly positive, define the Kullback–Leibler divergence as

$$D_{\text{KL}}(\mathbf{u} \parallel \mathbf{v}) \equiv \sum_j u_j \log \frac{u_j}{v_j}.$$

It is nonnegative and vanishes if and only if $\mathbf{u} = \mathbf{v}$. Indeed, writing $t_j = u_j/v_j$, this follows from strict convexity of $t \mapsto t \log t$ and $\sum_j v_j t_j = 1$.

G.1 Within-Industry Resource Efficiency

The resource-efficient allocation at resource level C solves

$$\max_{\mathbf{q} \in \Delta_{\mathcal{N}}} W(\mathbf{q}) \quad \text{subject to} \quad \sum_{j \in \mathcal{N}} c_j q_j = \lambda C. \quad (\text{G.2})$$

Define

$$c_{\min} \equiv \min_j c_j, \quad c_{\max} \equiv \max_j c_j, \quad Z(t) \equiv \sum_{j \in \mathcal{N}} \exp\left(\frac{a_j - tc_j}{\lambda}\right).$$

Proposition G.1. *Problem (G.2) is feasible if and only if $c_{\min}/\lambda \leq C \leq c_{\max}/\lambda$. Whenever it is feasible, it has a unique solution $\tilde{\mathbf{q}}(C)$. If $c_{\min} < \lambda C < c_{\max}$, this solution is strictly positive and satisfies*

$$\tilde{q}_j(C) = \frac{\exp((a_j - \tau c_j)/\lambda)}{Z(\tau)}, \quad \sum_{j \in \mathcal{N}} c_j \frac{\exp((a_j - \tau c_j)/\lambda)}{Z(\tau)} = \lambda C, \quad (\text{G.3})$$

where the second equality uniquely determines $\tau \in \mathbb{R}$. The price vectors implementing this allocation are exactly those of the form

$$p_j = \tau c_j + K \quad (j \in \mathcal{N}), \quad (\text{G.4})$$

where K is any common constant consistent with positive prices.

Proof. The resource constraint specifies a convex combination of the costs, which proves the feasibility claim. The feasible set is compact and convex, and W is continuous and strictly concave on the simplex. Hence a maximizer exists and is unique.

For $t \in \mathbb{R}$, let $q_j(t) = \exp((a_j - tc_j)/\lambda)/Z(t)$. Differentiation yields

$$\frac{d}{dt} \left[\frac{1}{\lambda} \sum_j c_j q_j(t) \right] = -\frac{1}{\lambda^2} \left[\sum_j q_j(t) c_j^2 - \left(\sum_j q_j(t) c_j \right)^2 \right] < 0, \quad (\text{G.5})$$

where strict inequality follows because the costs are not all equal and $\mathbf{q}(t)$ is strictly positive. Resource use converges to $c_{\min}/\lambda$ as $t \rightarrow +\infty$ and to $c_{\max}/\lambda$ as $t \rightarrow -\infty$. Thus there is a unique τ satisfying the resource constraint at any interior resource level.

At $\mathbf{q}(\tau)$, the derivative of the objective is

$$\frac{\partial W}{\partial q_j} = \frac{a_j - c_j}{\lambda} - \log q_j(\tau) - 1 = \frac{(\tau - 1)c_j}{\lambda} + \log Z(\tau) - 1.$$

Consequently, its inner product with any feasible change in shares is zero. Concavity proves that $\mathbf{q}(\tau)$ is the maximizer, and strict concavity proves uniqueness.

Finally, prices of the form (G.4) generate $\mathbf{q}(\tau)$ because the common factor $\exp(-K/\lambda)$ cancels from demand. Conversely, any implementing price vector must satisfy

$$\frac{a_i - p_i - a_j + p_j}{\lambda} = \log \frac{q_i(\tau)}{q_j(\tau)} = \frac{a_i - \tau c_i - a_j + \tau c_j}{\lambda}.$$

Thus $p_i - p_j = \tau(c_i - c_j)$ for every i, j , proving the claim. $\square$

Boundary and equal-cost cases. At $C = c_{\min}/\lambda$, feasibility requires zero output for every product with $c_j > c_{\min}$. Maximizing over the remaining products gives

$$\tilde{q}_j(C) = \begin{cases} \frac{\exp(a_j/\lambda)}{\sum_{k:c_k=c_{\min}} \exp(a_k/\lambda)}, & c_j = c_{\min}, \\ 0, & c_j > c_{\min}. \end{cases}$$

The analogous formula holds at $C = c_{\max}/\lambda$. When costs differ, these boundary allocations arise as limits of (G.3) and cannot be implemented by finite prices for all products. If instead $c_j = \bar{c}$ for every j , the only feasible resource level is $\bar{c}/\lambda$ and the resource constraint is redundant. The unique solution is $\tilde{q}_j = \exp(a_j/\lambda)/\sum_k \exp(a_k/\lambda)$, implemented by any common positive price. In that case, (G.3) gives the same allocation for every τ : the allocation is unique, but the multiplier is not.

Corollary G.1. *The unique first-best allocation, denoted by $\mathbf{q}^{\text{FB}}$, is*

$$q_j^{\text{FB}} = \frac{\exp((a_j - c_j)/\lambda)}{Z(1)}. \quad (\text{G.6})$$

It coincides with the monopolistic competition allocation. At $C^{\text{FB}} \equiv C(\mathbf{q}^{\text{FB}})$, the resource-efficient allocation is therefore implemented by any common absolute markup, $p_j = c_j + K$, consistent with positive prices.

Proof. At (G.6), every component of the gradient of W equals $\log Z(1) - 1$. Strict concavity therefore makes this the unique maximizer over the simplex. This allocation is also the unique maximizer subject to its own resource use. Prices $p_j = c_j + K$ generate these shares; monopolistic competition corresponds to $K = \lambda$. $\square$

G.2 Within-Firm Resource Efficiency

Fix a firm f and hold all rival quantities at their equilibrium levels, $\mathbf{x}_{-f}^*$. Write

$$Q^{f*} \equiv \sum_{j \in f} x_j^* = \frac{s^f}{\lambda}, \quad C^{f*} \equiv \sum_{j \in f} c_j x_j^*.$$

The within-firm problem is

$$\max_{\mathbf{x}_f \geq \mathbf{0}} W(\lambda(\mathbf{x}_f, \mathbf{x}_{-f}^*)) \quad \text{subject to} \quad \sum_{j \in f} x_j = Q^{f*}, \quad \sum_{j \in f} c_j x_j = C^{f*}. \quad (\text{G.7})$$

The first constraint keeps total industry output on the domain of the direct utility function. In particular, fixing rivals' quantities and the firm's resource use does not permit its total output to change.

Proposition G.2. *The unique solution to (G.7) is the equilibrium output vector $\mathbf{x}_f^*$, given by*

$$x_j^* = Q^{f*} \frac{\exp((a_j - c_j)/\lambda)}{T^f}, \quad T^f = \sum_{k \in f} \exp((a_k - c_k)/\lambda), \quad j \in f. \quad (\text{G.8})$$

Thus there is no within-firm resource misallocation. In fact, the same output vector uniquely maximizes aggregate surplus if the resource constraint is omitted while the total-output constraint and rivals' quantities are retained.

Proof. The common absolute markup within firm f implies $p_j^* = c_j + \lambda \mu^f$ for $j \in f$. Hence its equilibrium conditional product shares are

$$r_j^{f*} \equiv \frac{s_j}{s^f} = \frac{\exp((a_j - c_j)/\lambda)}{T^f}.$$

For an alternative with the same total output, let $r_j^f = x_j/Q^{f*}$. These conditional shares sum to one. The part of aggregate surplus that depends on this conditional allocation is

$$s^f \left[\sum_{j \in f} \frac{a_j - c_j}{\lambda} r_j^f - \sum_{j \in f} r_j^f \log r_j^f \right] - s^f \log s^f.$$

Since $\log r_j^{f*} = (a_j - c_j)/\lambda - \log T^f$, we obtain

$$W(\mathbf{s}) - W(\lambda(\mathbf{x}_f, \mathbf{x}_{-f}^*)) = s^f D_{\text{KL}}(\mathbf{r}^f \| \mathbf{r}^{f*}) \geq 0. \quad (\text{G.9})$$

Equality holds only at $\mathbf{r}^f = \mathbf{r}^{f*}$. Thus equilibrium uniquely solves the problem with only total output fixed. As it also uses exactly C^{f*} resources, it uniquely solves (G.7). $\square$

G.3 Welfare Losses and Kullback–Leibler Divergence

Let $C^* \equiv C(\mathbf{s})$ and $\tilde{\mathbf{s}} \equiv \tilde{\mathbf{q}}(C^*)$. Because equilibrium shares are strictly positive, C^* is strictly between the extreme feasible resource levels whenever costs differ. Thus $\tilde{\mathbf{s}}$ is given by (G.3) at a finite τ . If all costs are equal, we set $\tau = 1$ without loss of generality. As in the main text, define

$$M^W \equiv W(\mathbf{s}) - W(\tilde{\mathbf{s}}) \leq 0.$$

Proposition G.3. *The welfare loss from within-industry resource misallocation satisfies*

$$M^W = -D_{\text{KL}}(\mathbf{s} \| \tilde{\mathbf{s}}) = - \sum_{j \in \mathcal{N}} s_j \log \frac{s_j}{\tilde{s}_j}. \quad (\text{G.10})$$

More generally, for any $\mathbf{q} \in \Delta_{\mathcal{N}}$ such that $C(\mathbf{q}) = C^*$,

$$W(\tilde{\mathbf{s}}) - W(\mathbf{q}) = D_{\text{KL}}(\mathbf{q} \| \tilde{\mathbf{s}}). \quad (\text{G.11})$$

In particular, $M^W = 0$ if and only if there exist $\tau, K \in \mathbb{R}$ such that

$$\lambda \mu^{f(j)} = (\tau - 1)c_j + K \quad \text{for every } j \in \mathcal{N}, \quad \mu^f = \frac{1}{1 - s^f}, \quad (\text{G.12})$$

where $f(j)$ denotes the firm owning product j .

Proof. Using $\log \tilde{s}_j = (a_j - \tau c_j)/\lambda - \log Z(\tau)$, for any probability vector $\mathbf{q}$ we have

$$D_{\text{KL}}(\mathbf{q} \parallel \tilde{\mathbf{s}}) = -W(\mathbf{q}) + (\tau - 1)C(\mathbf{q}) + \log Z(\tau). \quad (\text{G.13})$$

At $\mathbf{q} = \tilde{\mathbf{s}}$ the left-hand side is zero, so $W(\tilde{\mathbf{s}}) = (\tau - 1)C^* + \log Z(\tau)$. Substitution proves (G.11) whenever $C(\mathbf{q}) = C^*$, and taking $\mathbf{q} = \mathbf{s}$ proves (G.10).

The divergence vanishes exactly when equilibrium is resource-efficient. By Proposition G.1 and the equal-cost case discussed after its proof, this holds exactly when equilibrium prices $p_j^* = c_j + \lambda \mu^{f(j)}$ have the affine form $\tau c_j + K$. This is (G.12). $\square$

Condition (G.12) concerns price differences, rather than relative markups: adding a common constant to all prices leaves quantities unchanged. If some firm offers two products with different costs, applying this condition to those two products forces $\tau = 1$. In that case, $M^W = 0$ if and only if all firms have the same absolute markup, equivalently the same equilibrium market share.

The resource-based welfare decomposition. The first-best characterization also gives a KL representation of the loss due to using an inefficient amount of resources. Define

$$M^B \equiv W(\tilde{\mathbf{s}}) - W(\mathbf{q}^{\text{FB}}), \quad \Delta \text{AS} \equiv W(\mathbf{s}) - W(\mathbf{q}^{\text{FB}}).$$

This is the decomposition in the main text because first best and monopolistic competition coincide here.

Corollary G.2. *The two components of the aggregate-surplus distortion satisfy*

$$M^W = -D_{\text{KL}}(\mathbf{s} \parallel \tilde{\mathbf{s}}), \quad M^B = -D_{\text{KL}}(\tilde{\mathbf{s}} \parallel \mathbf{q}^{\text{FB}}), \quad (\text{G.14})$$

$$\Delta \text{AS} = -D_{\text{KL}}(\mathbf{s} \parallel \mathbf{q}^{\text{FB}}) = M^W + M^B. \quad (\text{G.15})$$

Equivalently, the following identity holds:

$$D_{\text{KL}}(\mathbf{s} \parallel \mathbf{q}^{\text{FB}}) = D_{\text{KL}}(\mathbf{s} \parallel \tilde{\mathbf{s}}) + D_{\text{KL}}(\tilde{\mathbf{s}} \parallel \mathbf{q}^{\text{FB}}). \quad (\text{G.16})$$

Moreover, $M^B = 0$ if and only if $C^* = C^{\text{FB}}$.

Proof. Setting $\tau = 1$ in the algebra of (G.13) gives, for every probability vector $\mathbf{q}$,

$$W(\mathbf{q}^{\text{FB}}) - W(\mathbf{q}) = D_{\text{KL}}(\mathbf{q} \parallel \mathbf{q}^{\text{FB}}), \quad W(\mathbf{q}^{\text{FB}}) = \log Z(1).$$

Apply this equality to $\mathbf{s}$ and $\tilde{\mathbf{s}}$, and use Proposition G.3. This proves the decomposition and (G.16). Finally, $M^B = 0$ exactly when $\tilde{\mathbf{s}} = \mathbf{q}^{\text{FB}}$, which requires $C^* = C^{\text{FB}}$. Conversely, at that resource level first best is feasible and uniquely optimal in the resource-constrained problem. $\square$

H Quantity Competition under CES Demand

This appendix extends our analysis to quantity competition under CES demand without an outside option. We retain the notation and normalizations of the main text. The equilibrium characterization follows the same steps as under price competition: firm-level supply and demand relations jointly determine the markup and market-share fitting-in functions, and the adding-up constraint pins down the equilibrium aggregator. We then derive exact formulas for welfare distortions and within-industry resource misallocation, and relate these formulas to concentration indices through approximations around monopolistic competition conduct.

H.1 The Model

Demand is CES, $H^0 = 0$, and $V_0 = 1$. There is a finite set of products $\mathcal{N}$, partitioned into a set of firms $\mathcal{F}$, with $|\mathcal{F}| \geq 2$. Product i has quality $a_i > 0$ and constant marginal cost $c_i > 0$, incurred in units of labor. Let $\sigma > 1$ and $\alpha \equiv (\sigma - 1)/\sigma$. Firms simultaneously choose the quantities of their products, taking their rivals' quantities as given.

Up to an additive constant, the representative consumer's utility is

$$U(x_0, \mathbf{x}) = x_0 + \sigma \log Q(\mathbf{x}), \quad Q(\mathbf{x}) \equiv \sum_{j \in \mathcal{N}} a_j^{1/\sigma} x_j^\alpha.$$

Inverse demand is therefore

$$p_i(\mathbf{x}) = \frac{(\sigma - 1)a_i^{1/\sigma} x_i^{-1/\sigma}}{Q(\mathbf{x})}. \quad (\text{H.1})$$

Total expenditure on the industry's products is $\sigma - 1$. Firm f 's profit and revenue share are

$$\pi^f(\mathbf{x}) = \sum_{j \in f} (p_j(\mathbf{x}) - c_j) x_j, \quad s^f = \frac{\sum_{j \in f} p_j x_j}{\sigma - 1} = \frac{\sum_{j \in f} a_j^{1/\sigma} x_j^\alpha}{Q(\mathbf{x})}.$$

In particular, $\sum_f s^f = 1$. At the prices supporting any strictly positive quantity vector, these shares also satisfy

$$s^f = \sum_{j \in f} \frac{a_j p_j^{1-\sigma}}{H(\mathbf{p})}, \quad H(\mathbf{p}) = \sum_{j \in \mathcal{N}} a_j p_j^{1-\sigma}.$$

We continue to use this price aggregator H ; recall that $\text{CS} = \log H$. The quantity aggregator Q is used only to express inverse demand and firms' conjectures below.

H.2 Equilibrium Behavior

Existence and uniqueness of equilibrium, and the common within-firm markup property under CES demand, follow from Nocke and Schutz (2018b); see its treatment of quantity competition in Online Appendix XI, in particular Sections XI.9–XI.10. We express that equilibrium using the ι -markup convention of the present paper:

$$\frac{p_i - c_i}{p_i} = \frac{\mu^f}{\sigma}, \quad i \in f.$$

To derive the relevant supply relation, differentiate (H.1). For $i \in f$, the first-order condition is

$$\begin{aligned} 0 &= p_i - c_i + \sum_{j \in f} x_j \frac{\partial p_j}{\partial x_i} \\ &= p_i - c_i - \frac{p_i}{\sigma} - \frac{\alpha a_i^{1/\sigma} x_i^{\alpha-1}}{Q} \sum_{j \in f} p_j x_j = \alpha p_i (1 - s^f) - c_i. \end{aligned}$$

Thus the common relative markup and the supply relation are

$$\frac{p_i}{c_i} = \frac{1}{\alpha(1 - s^f)}, \quad \mu^f = 1 + (\sigma - 1)s^f. \quad (\text{H.2})$$

A larger market share again induces a higher ι -markup.

The demand relation is unaffected by the mode of competition:

$$s^f = \frac{T^f}{H} \psi(\mu^f). \quad (\text{H.3})$$

Equations (H.2) and (H.3) jointly pin down the quantity-competition versions of the fitting-in functions. We denote them by m_Q and S_Q to distinguish them from their price-competition counterparts. For $z = T^f/H > 0$, they satisfy

$$S_Q(z) = z\alpha^{\sigma-1}[1 - S_Q(z)]^{\sigma-1}, \quad m_Q(z) = 1 + (\sigma - 1)S_Q(z). \quad (\text{H.4})$$

Lemma H.1. *The functions $S_Q : (0, \infty) \rightarrow (0, 1)$ and $m_Q : (0, \infty) \rightarrow (1, \sigma)$ are smooth, strictly increasing, and strictly concave. Their endpoint limits are*

$$S_Q(0+) = 0, \quad S_Q(\infty) = 1, \quad m_Q(0+) = 1, \quad m_Q(\infty) = \sigma.$$

There is a unique $H^* > 0$ solving

$$\sum_{f \in \mathcal{F}} S_Q\left(\frac{T^f}{H^*}\right) = 1. \quad (\text{H.5})$$

Equilibrium shares and ι -markups are $s^{f*} = S_Q(T^f/H^*)$ and $\mu^{f*} = m_Q(T^f/H^*)$.

Proof. The inverse of S_Q is

$$G(s) = \frac{s}{\alpha^{\sigma-1}(1-s)^{\sigma-1}}, \quad 0 < s < 1.$$

It is smooth and strictly increasing from zero to infinity, with

$$G'(s) = \frac{1 + (\sigma - 2)s}{\alpha^{\sigma-1}(1-s)^\sigma} > 0, \quad G''(s) = \frac{(\sigma - 1)[2 + (\sigma - 2)s]}{\alpha^{\sigma-1}(1-s)^{\sigma+1}} > 0.$$

The claimed properties of S_Q and m_Q follow by inversion. In particular,

$$zS'_Q(z) = \frac{S_Q(z)[1 - S_Q(z)]}{1 + (\sigma - 2)S_Q(z)} > 0.$$

The left-hand side of (H.5) is continuous and strictly decreasing in H^* , with limits $|\mathcal{F}| > 1$ at zero and zero at infinity. Hence it crosses one exactly once. $\square$

Equilibrium prices follow from (H.2), and quantities from $x_i = (\sigma - 1)a_i p_i^{-\sigma}/H^*$. The profit fitting-in function is

$$\pi_Q(z) = \alpha m_Q(z) S_Q(z) = \alpha S_Q(z) + \alpha(\sigma - 1) S_Q(z)^2. \quad (\text{H.6})$$

The accounting identity $\pi^f = \alpha \mu^f s^f$ thus continues to hold; the price-competition identity $\pi^f = \mu^f - 1$ does not.

Under monopolistic competition, firms disregard their impact on Q in inverse demand. The first-order condition becomes $c_i = \alpha p_i$, so that $\mu^f = 1$. Writing $\mathcal{T} \equiv \sum_f T^f$, the resulting aggregator and shares are

$$H^*(0) = \alpha^{\sigma-1} \mathcal{T}, \quad s^f(0) = \frac{T^f}{\mathcal{T}}. \quad (\text{H.7})$$

This is the same monopolistic competition benchmark as under price competition. For now, $H^*(1) = H^*$ denotes the quantity-competition equilibrium aggregator.

H.3 Welfare Distortions: Exact Formulas

As in the main text, welfare distortions are signed differences relative to monopolistic competition: $\Delta \text{CS} = \text{CS}(1) - \text{CS}(0)$ and $\Delta \text{AS} = \text{AS}(1) - \text{AS}(0)$, where $\text{AS} = \text{CS} + \Pi$. Let

$$\text{HHI}(\mathbf{s}) \equiv \sum_f (s^f)^2, \quad \text{CHK}(\mathbf{s}) \equiv \sum_f (s^f)^3.$$

No renormalization of shares is needed because there is no outside option.

Proposition H.1. *Let $\mathbf{s}$ be the vector of equilibrium revenue shares under quantity competition. Then*

$$\Delta \text{CS}(\mathbf{s}) = -\log \sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}}, \quad (\text{H.8})$$

$$\Delta \Pi(\mathbf{s}) = \alpha(\sigma - 1) \text{HHI}(\mathbf{s}), \quad (\text{H.9})$$

$$\Delta \text{AS}(\mathbf{s}) = -\log \sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}} + \alpha(\sigma - 1) \text{HHI}(\mathbf{s}). \quad (\text{H.10})$$

Consumer surplus and aggregate surplus are strictly lower under quantity competition than under monopolistic competition, whereas industry profit is strictly higher. The consumer-surplus distortion is strictly Schur-concave in the vector of revenue shares, and the profit difference is strictly Schur-convex.

Proof. By (H.2)–(H.3),

$$\psi(\mu^f) = \alpha^{\sigma-1} (1 - s^f)^{\sigma-1}, \quad \frac{H^*(0)}{H^*(1)} = \sum_f \frac{T^f}{H^*(1)} \psi(1) = \sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}}.$$

Taking logs yields (H.8). Summing (H.6) gives $\Pi(1) = \alpha + \alpha(\sigma - 1) \text{HHI}$, while $\Pi(0) = \alpha$. This proves the other two formulas and the positive profit difference.

Write $h = \text{HHI}(\mathbf{s}) \in (0, 1)$. Jensen's inequality, with weights s^f and the strictly convex function $u \mapsto (1 - u)^{-(\sigma-1)}$, implies

$$\sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}} \geq (1 - h)^{-(\sigma-1)}. \quad (\text{H.11})$$

Consequently, $\Delta \text{CS} \leq (\sigma - 1) \log(1 - h) < 0$ and

$$\Delta \text{AS} \leq (\sigma - 1)[\log(1 - h) + \alpha h] < 0,$$

where the last inequality uses $\log(1 - h) < -h$ and $\alpha < 1$. Finally, $s \mapsto s/(1 - s)^{\sigma-1}$ is strictly convex, as shown by the calculation of G'' above. A mean-preserving spread therefore increases the sum in (H.8) and makes ΔCS strictly more negative. Strict Schur-convexity of $\Delta \Pi$ follows from that of HHI . $\square$

H.4 Within-Industry Misallocation: An Exact Formula

We retain the resource-based decomposition $\Delta \text{AS} = M^W + M^B$. The term M^W is aggregate surplus in equilibrium minus aggregate surplus in the resource-efficient allocation using the same amount of labor. The term M^B is aggregate surplus in that resource-efficient allocation minus aggregate surplus under monopolistic competition. Both components are nonpositive

in the present setting.

Total expenditure is $\sigma - 1$, so the amount of labor employed is

$$C = (\sigma - 1) - \Pi(1) = \alpha(\sigma - 1)(1 - \text{HHI}), \quad C(0) = \alpha(\sigma - 1). \quad (\text{H.12})$$

Quantity competition thus reduces industry employment by the fraction HHI relative to monopolistic competition.

At a fixed resource level $C > 0$, the resource-efficient allocation maximizes $\sigma \log Q(\mathbf{x})$ subject to $\sum_j c_j x_j = C$. Strict concavity of Q and its infinite marginal utility at zero imply a unique, interior solution. Its first-order conditions require a common relative markup p_i/c_i across products, equivalently a common ι -markup $\tilde{\mu}$. Industry profit at this allocation is $\tilde{\Pi} = \alpha\tilde{\mu}$, and the resource constraint implies

$$\tilde{\mu} = \frac{\Pi(1)}{\alpha} = 1 + (\sigma - 1) \text{HHI}, \quad \tilde{H} = \psi(\tilde{\mu})\mathcal{T}. \quad (\text{H.13})$$

Because expenditure and resource use are the same, profits are also the same in the two allocations. Hence $M^W = \log H^*(1) - \log \tilde{H}$.

Proposition H.2. *The within-industry misallocation component under quantity competition is*

$$M^W(\mathbf{s}) = -\log \left[(1 - \text{HHI}(\mathbf{s}))^{\sigma-1} \sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}} \right]. \quad (\text{H.14})$$

It is nonpositive, with equality if and only if all firms have identical shares, equivalently identical types. The resource-underutilization component is

$$M^B(\mathbf{s}) = (\sigma - 1) [\log(1 - \text{HHI}(\mathbf{s})) + \alpha \text{HHI}(\mathbf{s})] < 0. \quad (\text{H.15})$$

Proof. Combining (H.13) with (H.3) yields

$$\begin{aligned} \frac{\tilde{H}}{H^*(1)} &= \psi(1 + (\sigma - 1) \text{HHI}) \sum_f \frac{s^f}{\psi(1 + (\sigma - 1)s^f)} \\ &= (1 - \text{HHI})^{\sigma-1} \sum_f \frac{s^f}{(1 - s^f)^{\sigma-1}}. \end{aligned}$$

This proves (H.14). Inequality (H.11) proves its sign. Equality in Jensen's inequality requires identical shares, as all firms have strictly positive shares. Strict monotonicity of S_Q makes this equivalent to identical types. Subtracting (H.14) from (H.10) gives (H.15). Its sign follows from $\log(1 - h) + \alpha h < 0$ for $0 < h < 1$. $\square$

The common within-firm relative markup implies that resources are efficiently allocated within every firm's portfolio. All within-industry misallocation therefore arises across firms.

Larger firms have higher relative markups. Labor is consequently tilted toward smaller, lower-markup firms relative to the resource-efficient allocation at the same industry employment level.

H.5 Approximations around Monopolistic Competition Conduct

To interpolate between monopolistic competition and quantity competition, suppose that firms perceive their impact on the quantity aggregator Q in (H.1) to be multiplied by a common parameter $\theta \in [0, 1]$. Specifically, when varying x_i , a firm replaces $\partial Q / \partial x_i$ by $\theta \partial Q / \partial x_i$ while retaining the direct effect of x_i on the numerator of its own inverse demand. The cases $\theta = 0$ and $\theta = 1$ correspond to monopolistic competition and Nash quantity competition, respectively.

The perceived first-order condition and supply relation become

$$c_i = \alpha p_i (1 - \theta s^f), \quad \mu^f = 1 + (\sigma - 1) \theta s^f. \quad (\text{H.16})$$

The demand relation (H.3) is unchanged. Consequently,

$$S_Q(z; \theta) = z \alpha^{\sigma-1} [1 - \theta S_Q(z; \theta)]^{\sigma-1}, \quad m_Q(z; \theta) = 1 + (\sigma - 1) \theta S_Q(z; \theta). \quad (\text{H.17})$$

For $\theta > 0$, S_Q is the smooth, strictly increasing and strictly concave inverse of $s \mapsto s / [\alpha^{\sigma-1} (1 - \theta s)^{\sigma-1}]$ on $(0, 1/\theta)$. At $\theta = 0$, $S_Q(z; 0) = \alpha^{\sigma-1} z$ and $m_Q(z; 0) = 1$. As before, $H^*(\theta)$ is the unique solution to

$$\sum_f S_Q(T^f / H^*(\theta); \theta) = 1.$$

The implicit function theorem implies that the equilibrium aggregator and shares are smooth near $\theta = 0$, holding the vector of types fixed. Write $s^f(\theta) = S_Q(T^f / H^*(\theta); \theta)$, and evaluate $\text{HHI}(\theta)$ and $\text{CHK}(\theta)$ at these equilibrium shares.

Exact formulas for arbitrary conduct. The preceding derivations apply with s^f replaced by θs^f in the markup terms, keeping s^f as the revenue weight. Define

$$A(\theta) \equiv \sum_f \frac{s^f(\theta)}{[1 - \theta s^f(\theta)]^{\sigma-1}}.$$

Then $A(\theta) = H^*(0) / H^*(\theta)$ and

$$\begin{aligned} \Delta \text{CS}(\theta) &= -\log A(\theta), \\ \Delta \Pi(\theta) &= \alpha(\sigma - 1) \theta \text{HHI}(\theta), \\ \Delta \text{AS}(\theta) &= -\log A(\theta) + \alpha(\sigma - 1) \theta \text{HHI}(\theta), \\ M^W(\theta) &= -\log A(\theta) - (\sigma - 1) \log[1 - \theta \text{HHI}(\theta)], \end{aligned}$$

$$M^B(\theta) = (\sigma - 1) \{ \log[1 - \theta \text{HHI}(\theta)] + \alpha \theta \text{HHI}(\theta) \}. \quad (\text{H.18})$$

Indeed, $C(\theta) = \alpha(\sigma - 1)[1 - \theta \text{HHI}(\theta)]$, and the common resource-efficient markup is $\tilde{\mu}(\theta) = 1 + (\sigma - 1)\theta \text{HHI}(\theta)$. These formulas give $M^W(0) = M^B(0) = 0$. For every $\theta > 0$, $M^B(\theta) < 0$, and $M^W(\theta) = 0$ precisely when firms have identical shares.

Proposition H.3. *In the neighborhood of $\theta = 0$, the welfare distortions are*

$$\begin{aligned} \Delta \text{CS}(\theta) = & -(\sigma - 1) \text{HHI}(\theta) \theta \\ & - \frac{\sigma - 1}{2} [\sigma \text{CHK}(\theta) - (\sigma - 1) \text{HHI}(\theta)^2] \theta^2 + o(\theta^2), \end{aligned} \quad (\text{H.19})$$

$$\begin{aligned} \Delta \text{AS}(\theta) = & -\alpha \text{HHI}(\theta) \theta \\ & - \frac{\sigma - 1}{2} [\sigma \text{CHK}(\theta) - (\sigma - 1) \text{HHI}(\theta)^2] \theta^2 + o(\theta^2). \end{aligned} \quad (\text{H.20})$$

The components of the resource-based decomposition satisfy

$$M^W(\theta) = -\frac{\sigma(\sigma - 1)}{2} [\text{CHK}(\theta) - \text{HHI}(\theta)^2] \theta^2 + o(\theta^2), \quad (\text{H.21})$$

$$M^B(\theta) = -\alpha \text{HHI}(\theta) \theta - \frac{\sigma - 1}{2} \text{HHI}(\theta)^2 \theta^2 + o(\theta^2). \quad (\text{H.22})$$

All concentration indices on the right-hand sides are evaluated at the equilibrium shares under conduct θ .

Proof. For $s \in [0, 1]$, the Taylor expansion

$$(1 - \theta s)^{-(\sigma-1)} = 1 + (\sigma - 1)\theta s + \frac{\sigma(\sigma - 1)}{2} \theta^2 s^2 + O(\theta^3)$$

has a remainder uniform in s as $\theta \rightarrow 0$. Multiplying by $s^f(\theta)$, summing, and using $\sum_f s^f(\theta) = 1$, we obtain

$$A(\theta) = 1 + (\sigma - 1) \text{HHI}(\theta) \theta + \frac{\sigma(\sigma - 1)}{2} \text{CHK}(\theta) \theta^2 + O(\theta^3).$$

Expanding $-\log A(\theta)$ proves (H.19). Adding the exact profit difference in (H.18), and using $(\sigma - 1)(1 - \alpha) = \alpha$, proves (H.20). Finally,

$$-\log[1 - \theta \text{HHI}(\theta)] = \text{HHI}(\theta) \theta + \frac{1}{2} \text{HHI}(\theta)^2 \theta^2 + O(\theta^3).$$

Substitution in (H.18) gives the two resource components. In particular, the linear terms in M^W cancel, and its quadratic coefficient is $-\sigma(\sigma - 1)(\text{CHK} - \text{HHI}^2)/2$. $\square$

At the first order, the consumer- and aggregate-surplus distortions are proportional to the Herfindahl index. Within-industry misallocation is of second order, so the first-order

aggregate-surplus distortion is entirely due to resource underutilization. At the second order, the within-industry loss is proportional to

$$\text{CHK} - \text{HHI}^2 = \sum_f s^f (s^f - \text{HHI})^2 \geq 0,$$

the revenue-weighted variance of firm-level revenue shares. It vanishes exactly when firms are symmetric. These are the same concentration statistics that arise under price competition, but their coefficients differ. In particular, the coefficient of $\text{CHK} - \text{HHI}^2$ is $\sigma(\sigma - 1)/2 = \alpha/[2(1 - \alpha)^2]$, compared with $\alpha/2$ under price competition.

I Nested CES Demand with Narrow Firms

This appendix extends the price-competition analysis to nested CES demand without an outside option. Firms are narrow: each firm owns products in a single nest and competes with other firms in that nest. Following the notation of Online Appendix I of Nocke and Schutz (2025), we summarize the key equilibrium relations and then derive exact formulas for welfare distortions. We decompose the aggregate-surplus distortion into resource misallocation within nests, resource misallocation across nests, and the distortion between leisure and the inside goods. All three components admit closed-form expressions in observed revenue shares and demand parameters. Finally, we relate them to concentration indices through an approximation around monopolistic competition conduct.

I.1 The Model

Let $\mathcal{L}$ be a finite, nonempty set of nests, each a finite set of products. The nests partition $\mathcal{N}$. Nest $l \in \mathcal{L}$ is partitioned into a set of narrow firms $\mathcal{F}_l$, with $|\mathcal{F}_l| \geq 2$. Each firm belongs to exactly one nest, and $\mathcal{F} = \bigcup_l \mathcal{F}_l$. Product i has quality $a_i > 0$ and constant marginal cost $c_i > 0$, incurred in units of labor. Firms simultaneously choose their prices.

Let $\sigma > 1$ be the within-nest elasticity of substitution and $\beta \in (0, 1]$ the nest exponent. Write $\alpha = (\sigma - 1)/\sigma$. With $V_0 = 1$ and $H^0 = 0$, indirect utility is $y + \log H(\mathbf{p})$, where

$$h_i(p_i) = a_i p_i^{1-\sigma}, \quad H_l = \sum_{i \in l} h_i(p_i), \quad H = \sum_{l \in \mathcal{L}} H_l^\beta. \quad (\text{I.1})$$

For the welfare analysis, it is convenient to introduce the derived quantities

$$E \equiv \beta(\sigma - 1), \quad \varepsilon \equiv 1 + \beta(\sigma - 1), \quad \delta \equiv \frac{\varepsilon - 1}{\varepsilon}.$$

Here E is expenditure on inside goods, and ε is the across-nest elasticity of substitution. The associated price indices are $P_l = H_l^{-1/(\sigma-1)}$ and $P = H^{-1/E}$. Up to an additive constant,

direct utility is

$$U(x_0, \mathbf{x}) = x_0 + E \log X(\mathbf{x}), \quad X = \left(\sum_l X_l^\delta \right)^{1/\delta}, \quad X_l = \left(\sum_{i \in l} a_i^{1/\sigma} x_i^\alpha \right)^{1/\alpha}. \quad (\text{I.2})$$

Roy's identity gives

$$x_i = -\beta \frac{H_l^{\beta-1}}{H} h'_i(p_i) = E \frac{H_l^{\beta-1}}{H} a_i p_i^{-\sigma}, \quad i \in l. \quad (\text{I.3})$$

The nest revenue share s_l , firm f 's conditional revenue share $\tilde{s}^f$, and its industry revenue share s^f are

$$s_l = \frac{H_l^\beta}{H}, \quad \tilde{s}^f = \frac{\sum_{i \in f} h_i(p_i)}{H_l}, \quad s^f = \tilde{s}^f s_l, \quad f \in \mathcal{F}_l. \quad (\text{I.4})$$

Then $\sum_l s_l = 1$ and $\sum_{f \in \mathcal{F}_l} \tilde{s}^f = 1$. Consumer surplus is $\text{CS} = \log H$, and aggregate surplus is $\text{AS} = \text{CS} + \Pi$, with Π total profit. When $\beta = 1$, nesting disappears.

I.2 Equilibrium Behavior

We summarize the equilibrium characterization in Online Appendix I.1 of Nocke and Schutz (2025), specializing it to NCES demand, narrow firms only, and $H^0 = 0$. In particular, equations (40)–(46) and Proposition 12 of that appendix provide the firm- and nest-level fitting-in relations and establish existence and uniqueness.

As in that appendix, firm f 's type and the markup function are

$$T^f = \left(\sum_{i \in f} a_i c_i^{1-\sigma} \right)^\beta, \quad \psi(\mu) = [1 - (1 - \alpha)\mu]^{\alpha/(1-\alpha)}. \quad (\text{I.5})$$

For later use, define

$$\mathcal{T}_l \equiv \sum_{f \in \mathcal{F}_l} (T^f)^{1/\beta} = \sum_{i \in l} a_i c_i^{1-\sigma}, \quad \gamma_l \equiv 1 - \beta + \beta s_l, \quad u_f \equiv \gamma_l \tilde{s}^f. \quad (\text{I.6})$$

Thus $u_f = (1 - \beta)\tilde{s}^f + \beta s^f \in (0, 1)$. Each firm charges a common ι -markup μ^f on all its products. The supply and demand relations are

$$\mu^f = \frac{1}{1 - \alpha \tilde{s}^f (1 - \beta + \beta s_l)} = \frac{1}{1 - \alpha u_f}, \quad \frac{p_i - c_i}{p_i} = \frac{\mu^f}{\sigma}, \quad i \in f, \quad (\text{I.7})$$

$$\tilde{s}^f = \frac{(T^f)^{1/\beta}}{H_l} \psi(\mu^f) = \frac{(T^f)^{1/\beta}}{H_l} r_f^{\sigma-1}, \quad (\text{I.8})$$

where the inverse relative markup and profit are

$$r_f \equiv \frac{c_i}{p_i} = \alpha \frac{1 - u_f}{1 - \alpha u_f}, \quad \pi^f = \alpha \beta \mu^f \tilde{s}^f s_l = E s_l \tilde{s}^f (1 - r_f). \quad (\text{I.9})$$

The solutions of (I.7)–(I.8) define $\tilde{m}((T^f)^{1/\beta}/H_l, s_l)$ and $\tilde{S}((T^f)^{1/\beta}/H_l, s_l)$. For given H , the nest-aggregator fitting-in function $H_l(H)$ solves

$$\sum_{f \in \mathcal{F}_l} \tilde{S} \left(\frac{(T^f)^{1/\beta}}{H_l(H)}, \frac{H_l(H)^\beta}{H} \right) = 1. \quad (\text{I.10})$$

Writing $\Sigma_l(H) = H_l(H)^\beta/H$, the equilibrium industry aggregator H^* is the unique solution of

$$\sum_{l \in \mathcal{L}} \Sigma_l(H^*) = 1. \quad (\text{I.11})$$

Nest shares are $s_l = \Sigma_l(H^*)$, and the firm-level fitting-in functions then determine conditional shares and markups.

Under monopolistic competition, firms disregard their price impact on both H_l and H in the common demand factor $H_l^{\beta-1}/H$. Every product then has $\mu^f = 1$ and $c_i/p_i = \alpha$. Consequently,

$$H_l(0) = \alpha^{\sigma-1} \mathcal{T}_l, \quad H(0) = \alpha^E \sum_l \mathcal{T}_l^\beta, \quad \Pi(0) = E(1 - \alpha), \quad C(0) = E\alpha, \quad (\text{I.12})$$

where C denotes total labor used in producing inside goods. This benchmark allocates any given total amount of labor efficiently, since relative markups are equal across all products. It still underuses labor relative to the first best, whose labor requirement is E .

I.3 Welfare Distortions: Exact Formulas

In this subsection, all shares and markups refer to the oligopolistic equilibrium. For every nest, define

$$R_l = \sum_{f \in \mathcal{F}_l} \tilde{s}^f r_f, \quad A_l = \sum_{f \in \mathcal{F}_l} \tilde{s}^f \left(\frac{\alpha}{r_f} \right)^{\sigma-1}, \quad R = \sum_l s_l R_l, \quad B = \sum_l s_l A_l^\beta. \quad (\text{I.13})$$

Here R_l is nest l 's labor cost per unit of revenue. All of these statistics are explicit functions of observed shares: insert $r_f = \alpha(1 - \gamma_l \tilde{s}^f)/(1 - \alpha \gamma_l \tilde{s}^f)$ from (I.9).

Proposition I.1. *The distortions relative to monopolistic competition are*

$$\Delta \text{CS} = -\log B, \quad \Delta \Pi = E(\alpha - R), \quad \Delta \text{AS} = -\log B + E(\alpha - R). \quad (\text{I.14})$$

Labor use in nest l and in the industry is, respectively,

$$C_l = E s_l R_l, \quad C = ER < E\alpha = C(0). \quad (\text{I.15})$$

Consumer surplus and aggregate surplus are strictly lower than under monopolistic competition, and total profit is strictly higher.

Proof. Equation (I.8) implies

$$A_l = \frac{\alpha^{\sigma-1} \mathcal{T}_l}{H_l} = \frac{H_l(0)}{H_l(1)}, \quad \frac{H(0)}{H(1)} = \sum_l s_l A_l^\beta = B.$$

This proves the consumer-surplus formula. Since revenue is E and labor cost is ER , profit is $E(1 - R)$, proving the remaining identities. Every u_f is positive, so $0 < r_f < \alpha$, $R < \alpha$, and $B > 1$. This proves the claims for consumer surplus, profit, and labor use. The strict aggregate-surplus loss follows from the decomposition and signs established in Proposition I.2 below. $\square$

These are closed-form *measurement* formulas conditional on observed shares. In particular, computing the welfare losses requires no solution of a counterfactual equilibrium. Even the monopolistic-competition shares can be recovered:

$$\nu_l \equiv s_l(0) = \frac{s_l A_l^\beta}{B}, \quad \tilde{s}^f(0) = \frac{\tilde{s}^f(\alpha/r_f)^{\sigma-1}}{A_l}. \quad (\text{I.16})$$

I.4 A Three-Way Resource Decomposition

Let $\mathbf{x}^1$ be the oligopoly allocation. Define $\mathbf{x}^W$ by maximizing X while keeping labor use in every nest fixed at C_l . Define $\mathbf{x}^A$ by maximizing X while keeping only total labor use fixed at $C = \sum_l C_l$. Finally, let $\mathbf{x}^0$ be the monopolistic-competition allocation. For any allocation, write

$$\mathcal{W}(\mathbf{x}) = E \log X(\mathbf{x}) - \sum_i c_i x_i.$$

At demand allocations this equals aggregate surplus up to a common constant: $X = E/P$, $\text{CS} = -E \log P$, and $\Pi = E - C$. The decomposition is

$$\begin{aligned} M^W &= \mathcal{W}(\mathbf{x}^1) - \mathcal{W}(\mathbf{x}^W), \\ M^A &= \mathcal{W}(\mathbf{x}^W) - \mathcal{W}(\mathbf{x}^A), \\ M^L &= \mathcal{W}(\mathbf{x}^A) - \mathcal{W}(\mathbf{x}^0), \quad \Delta \text{AS} = M^W + M^A + M^L. \end{aligned} \quad (\text{I.17})$$

Thus within-nest inefficiencies are removed first, then across-nest inefficiencies, and finally labor use is changed to its benchmark level. This ordering fixes the meaning of the three components.

Lemma I.1. *At labor cost $C_l > 0$, the largest attainable nest output is $X_l^W = C_l \mathcal{T}_l^{1/(\sigma-1)}$. At total labor cost $C > 0$, the largest attainable industry output is*

$$X^A = C \left(\sum_l \mathcal{T}_l^\beta \right)^{1/E}. \quad (\text{I.18})$$

The efficient fraction of labor assigned to nest l is ν_l from (I.16). Both resource-efficient allocations are unique.

Proof. Within nest l , maximize $\sum_i a_i^{1/\sigma} x_i^\alpha$ subject to $\sum_i c_i x_i = C_l$. Strict concavity and infinite marginal products at zero imply a unique, interior solution, satisfying $x_i = (C_l/\mathcal{T}_l) a_i c_i^{-\sigma}$. Substitution gives the claimed X_l^W . Since X is increasing in every X_l , solving these problems independently gives $\mathbf{x}^W$. After this step, maximizing X over nest labor allocations is equivalent to maximizing

$$\sum_l \left(\mathcal{T}_l^\beta \right)^{1/\varepsilon} C_l^\delta \quad \text{subject to} \quad \sum_l C_l = C.$$

Again the objective is strictly concave, and the unique solution is $C_l = C \mathcal{T}_l^\beta / \sum_m \mathcal{T}_m^\beta = C \nu_l$. Substituting this solution proves (I.18). $\square$

For use in the exact formulas, define

$$g_l = \frac{R_l}{\alpha} A_l^{1/(\sigma-1)}, \quad D = \sum_l s_l g_l^\delta, \quad \lambda_l = \frac{s_l R_l}{R}. \quad (\text{I.19})$$

Here g_l is the factor by which nest output rises when its internal allocation is made efficient at its observed labor use; λ_l is its observed share of industry labor.

Proposition I.2. *The three components admit the closed-form expressions*

$$M^W = -\varepsilon \log D, \quad (\text{I.20})$$

$$M^A = \varepsilon \log \sum_l \nu_l^{1/\varepsilon} \lambda_l^\delta = \varepsilon \log D - \log B - E \log(R/\alpha), \quad (\text{I.21})$$

$$M^L = E [\log(R/\alpha) + \alpha - R]. \quad (\text{I.22})$$

Both M^W and M^A are nonpositive, and $M^L < 0$. Moreover:

1. $M^W = 0$ if and only if firms have identical conditional shares within every nest, equivalently identical types within every nest.
2. $M^A = 0$ if and only if $\lambda_l = \nu_l$ for every nest, equivalently R_l/A_l^β is constant across nests.

3. $M^W + M^A = 0$ if and only if relative markups are identical across all products, equivalently u_f is identical across all firms.

The fraction of benchmark industry employment lost to oligopolistic conduct is $1 - R/\alpha$.

Proof. Conditional demand at the oligopoly allocation gives

$$X_l^1 = Es_l/P_l = Es_l H_l^{1/(\sigma-1)}.$$

Hence

$$\frac{X_l^W}{X_l^1} = R_l(\mathcal{T}_l/H_l)^{1/(\sigma-1)} = g_l.$$

Moreover, $(X_l^1)^\delta/(X^1)^\delta = s_l$. Consequently, $X^W/X^1 = D^{1/\delta}$. Labor use is unchanged, so $M^W = -E \log(X^W/X^1) = -\varepsilon \log D$.

At the same total labor cost, Lemma I.1 gives

$$\frac{X^W}{X^A} = \left(\sum_l \nu_l^{1/\varepsilon} \lambda_l^\delta \right)^{1/\delta}.$$

This proves the first expression for M^A . Alternatively, $X^1 = EH(1)^{1/E}$ and (I.18) imply

$$M^W + M^A = -E \log(X^A/X^1) = -\log B - E \log(R/\alpha), \quad (\text{I.23})$$

which proves the second expression. Finally, $\mathbf{x}^A$ and $\mathbf{x}^0$ are both efficient at their respective labor levels, so their outputs are in the ratio R/α . Their labor costs differ by $E(R - \alpha)$, proving (I.22) and the decomposition.

By strict convexity of $r \mapsto r^{-(\sigma-1)}$,

$$A_l = \alpha^{\sigma-1} \sum_f \tilde{s}^f r_f^{-(\sigma-1)} \geq (\alpha/R_l)^{\sigma-1}.$$

Thus $g_l \geq 1$, with equality precisely when the r_f in that nest coincide. Since r_f strictly decreases in $\tilde{s}^f$ at fixed s_l , and all weights are positive, this proves the sign and equality condition for M^W . Equation (I.8) makes equal conditional shares equivalent to equal types within the nest.

For the across-nest component, strict concavity of z^δ gives

$$\sum_l \nu_l^{1/\varepsilon} \lambda_l^\delta = \sum_l \nu_l (\lambda_l/\nu_l)^\delta \leq \left(\sum_l \lambda_l \right)^\delta = 1,$$

with equality precisely when $\lambda_l/\nu_l = 1$ for all l . Using the definitions of λ_l and ν_l gives the stated equivalent condition. If both internal components vanish, then $r_f = r_l$ within nest l and $A_l = (\alpha/r_l)^{\sigma-1}$. The across-nest equality condition becomes r_l^ε/α^E constant across nests, so all r_l coincide. The converse is immediate.

Lastly, put $z = R/\alpha \in (0, 1)$. Since $\log z < z - 1$,

$$M^L = E[\log z + \alpha(1 - z)] < -E(1 - \alpha)(1 - z) < 0.$$

This also proves $\Delta AS < 0$ and completes the proof. $\square$

I.5 Approximations around Monopolistic Competition Conduct

To interpolate between monopolistic competition and Nash pricing, multiply firms' perceived price derivatives of the common demand factor $H_l^{\beta-1}/H$ by $\theta \in [0, 1]$, retaining the direct derivative of $p_i^{-\sigma}$. Primitives are fixed as θ varies. The resulting markup relation is

$$\mu^f(\theta) = \frac{1}{1 - \alpha\theta u_f(\theta)}, \quad r_f(\theta) = \alpha \frac{1 - \theta u_f(\theta)}{1 - \alpha\theta u_f(\theta)}, \quad u_f(\theta) = [1 - \beta + \beta s_l(\theta)] \tilde{s}^f(\theta). \quad (\text{I.24})$$

The cases $\theta = 0$ and $\theta = 1$ give monopolistic competition and Nash pricing, respectively. For intermediate conduct, the solution is defined by these perceived first-order conditions and the unchanged demand relations. For the local approximations below, these equations define a smooth solution near $\theta = 0$. Indeed, in log inverse markups their Jacobian at zero is the identity: the share-dependent terms in (I.24) are multiplied by θ . The implicit function theorem therefore applies. The formulas that follow are accounting identities at any solution of the conduct equations.

Exact formulas for arbitrary conduct. Define $R_l(\theta)$, $A_l(\theta)$, $R(\theta)$, $B(\theta)$, $g_l(\theta)$, $D(\theta)$, $\lambda_l(\theta)$, and $\nu_l(\theta)$ using (I.13), (I.16), and (I.19), with all shares and inverse markups evaluated at conduct θ . Then (I.14) and (I.20)–(I.22) hold unchanged for every θ . In these identities, $\nu_l(\theta) = \mathcal{T}_l^\beta / \sum_m \mathcal{T}_m^\beta$ is actually independent of conduct. At $\theta = 0$, $R_l = R = \alpha$, $A_l = B = g_l = D = 1$, and all distortions vanish. For $\theta > 0$, the signs and equality conditions in Proposition I.2 continue to hold.

Define the conditional concentration indices and associated moments

$$\begin{aligned} h_l &= \sum_{f \in \mathcal{F}_l} (\tilde{s}^f)^2, & k_l &= \sum_{f \in \mathcal{F}_l} (\tilde{s}^f)^3, & m_l &= \gamma_l h_l, & v_l &= \gamma_l^2 k_l, \\ \bar{m} &= \sum_l s_l m_l, & \bar{v} &= \sum_l s_l v_l, & m_2 &= \sum_l s_l m_l^2, \\ V_W &= \bar{v} - m_2 = \sum_l s_l \gamma_l^2 (k_l - h_l^2), & V_A &= m_2 - \bar{m}^2 = \sum_l s_l (m_l - \bar{m})^2. \end{aligned} \quad (\text{I.25})$$

All quantities on the right-hand sides below, including nest weights, are evaluated at the equilibrium shares under conduct θ .

Proposition I.3. *As $\theta \downarrow 0$, the welfare distortions satisfy*

$$\Delta \text{CS}(\theta) = -\frac{E}{\sigma} \bar{m} \theta - \frac{E}{2\sigma} [(1+2\alpha)\bar{v} + (\beta-1)\alpha m_2 - \beta\alpha \bar{m}^2] \theta^2 + O(\theta^3), \quad (\text{I.26})$$

$$\Delta \Pi(\theta) = \frac{E\alpha}{\sigma} \bar{m} \theta + \frac{E\alpha^2}{\sigma} \bar{v} \theta^2 + O(\theta^3), \quad (\text{I.27})$$

$$\Delta \text{AS}(\theta) = -\frac{E}{\sigma^2} \bar{m} \theta - \frac{E}{2\sigma} [(1+2\alpha-2\alpha^2)\bar{v} + (\beta-1)\alpha m_2 - \beta\alpha \bar{m}^2] \theta^2 + O(\theta^3). \quad (\text{I.28})$$

The three resource components satisfy

$$M^W(\theta) = -\frac{E}{2\sigma} V_W \theta^2 + O(\theta^3), \quad (\text{I.29})$$

$$M^A(\theta) = -\frac{E\varepsilon}{2\sigma^2} V_A \theta^2 + O(\theta^3), \quad (\text{I.30})$$

$$M^L(\theta) = -\frac{E}{\sigma^2} \bar{m} \theta - \frac{E}{\sigma^2} \left(\alpha \bar{v} + \frac{1}{2} \bar{m}^2 \right) \theta^2 + O(\theta^3). \quad (\text{I.31})$$

Proof. For $0 \leq u \leq 1$, uniformly as $\theta \rightarrow 0$,

$$\begin{aligned} \frac{r}{\alpha} &= 1 - \frac{u}{\sigma} \theta - \frac{\alpha u^2}{\sigma} \theta^2 + O(\theta^3), \\ \left(\frac{\alpha}{r} \right)^{\sigma-1} &= 1 + \alpha u \theta + \frac{\alpha(1+2\alpha)}{2} u^2 \theta^2 + O(\theta^3). \end{aligned}$$

Weighting by $\tilde{s}^f$, and using $u_f = \gamma_l \tilde{s}^f$, gives

$$\frac{R_l}{\alpha} = 1 - \frac{m_l}{\sigma} \theta - \frac{\alpha v_l}{\sigma} \theta^2 + O(\theta^3), \quad A_l = 1 + \alpha m_l \theta + \frac{\alpha(1+2\alpha)}{2} v_l \theta^2 + O(\theta^3). \quad (\text{I.32})$$

Consequently,

$$B = 1 + \beta\alpha \bar{m} \theta + \frac{\beta\alpha}{2} [(1+2\alpha)\bar{v} + (\beta-1)\alpha m_2] \theta^2 + O(\theta^3).$$

Expand $-\log B$ and use $\beta\alpha = E/\sigma$ to obtain (I.26). Averaging the expansion of R_l yields (I.27); adding gives (I.28).

For the within-nest component, the linear terms in $\log g_l = \log(R_l/\alpha) + (\sigma-1)\log A_l$ cancel, leaving

$$\log g_l = \frac{v_l - m_l^2}{2\sigma} \theta^2 + O(\theta^3).$$

Thus $\log D = \delta V_W \theta^2 / (2\sigma) + O(\theta^3)$, which proves (I.29). For the across-nest component,

$$\lambda_l = s_l \left[1 - \frac{m_l - \bar{m}}{\sigma} \theta + O(\theta^2) \right], \quad \nu_l = s_l [1 + \beta\alpha(m_l - \bar{m})\theta + O(\theta^2)].$$

Hence, with $q_l = \lambda_l/\nu_l - 1$,

$$q_l = -\frac{\varepsilon}{\sigma}(m_l - \bar{m})\theta + O(\theta^2), \quad \sum_l \nu_l q_l = 0.$$

Expand the exact expression for M^A :

$$\begin{aligned} M^A &= \varepsilon \log \sum_l \nu_l (1 + q_l)^\delta \\ &= \frac{\varepsilon \delta (\delta - 1)}{2} \sum_l \nu_l q_l^2 + O(\theta^3) = -\frac{E\varepsilon}{2\sigma^2} V_A \theta^2 + O(\theta^3). \end{aligned}$$

Replacing ν_l by s_l in the quadratic coefficient changes only the remainder. Finally,

$$R/\alpha = 1 - \bar{m}\theta/\sigma - \alpha \bar{v}\theta^2/\sigma + O(\theta^3).$$

Expanding $E[\log(R/\alpha) + \alpha - R]$ proves (I.31). All remainders used above are uniform over shares near the strictly positive benchmark shares. Thus the formulas hold along the endogenous equilibrium path; no assumption that shares remain fixed is being made. $\square$

Within-nest misallocation is therefore of second order and is governed by conditional cubic Hannah–Kay indices minus squared conditional Herfindahl indices. Across-nest misallocation is also of second order, and is governed by the expenditure-weighted variance of $m_l = (1 - \beta + \beta s_l)h_l$. The first-order aggregate-surplus distortion comes entirely from the leisure/inside-goods component. Its concentration statistic can be written as

$$\bar{m} = (1 - \beta) \sum_l s_l h_l + \beta \sum_f (s^f)^2. \quad (\text{I.33})$$

Thus both conditional concentration and industry-wide concentration matter when nests are economically distinct.