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Endorsements and Referrals: Product Recommendations in Bilateral Trade

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Endorsements and Referrals: Product Recommendations in Bilateral Trade*

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Abstract

This paper examines how a monopoly seller strategically employs pricing strategies and incentive mechanisms to influence consumer learning in the presence of a third-party information provider. Without direct payments, the seller influences consumer learning indirectly through distinct pricing strategies, which either deter or induce information acquisition. With direct payments, the seller can influence recommendations directly. "Endorsements", which tie payments to recommendations, remove informativeness and unambiguously harm the buyer. In contrast, "referrals", which tie payments to sales, can enhance consumer surplus and can even lead to Pareto improvements.

1 Introduction

Product recommendations are critical drivers of consumer decision-making across a wide range of purchases, from high-value items such as real estate and automobiles to everyday goods like electronics and cosmetics. Consumers increasingly turn to various sources for advice, including independent experts, customer reviews on e-commerce platforms, evaluations on dedicated review websites, and endorsements or recommendations on social media. These information providers/experts, whether genuinely independent or merely perceived as such, generate revenue through channels like direct payments, subscriptions, advertisements,

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and in-kind benefits provided by sellers in exchange for referrals or endorsements. This paper investigates how the presence of such an expert affects a seller's pricing strategies and broader market outcomes. While expert advice can significantly benefit consumers by reducing uncertainty about product value, regulators and consumer advocates emphasize potential downsides, such as biased recommendations that distort markets and harm consumer welfare. Understanding these trade-offs is essential for evaluating both market efficiency and consumer surplus in environments influenced by expert-driven recommendations.

We propose a tractable framework to study such settings. In our model, a seller offers a product of uncertain value to a potential buyer. Before making a purchase decision, the buyer may consult an expert for information about the product's value, incurring an idiosyncratic, private cost to acquire this advice. In turn, the expert benefits from being consulted by the buyer, for example through advertising, but also receives compensation from the seller. The seller seeks to influence the expert's recommendations using different incentive mechanisms. We distinguish between *soft incentives*, such as favors or rewards of negligible cost that subtly influence the expert's recommendations, and *hard incentives*, which involve explicit monetary payments or contracts that directly shape the expert's behavior. To model the expert's decision-making in this environment, we assume they can commit to a recommendation policy. This setup captures the core trade-off the expert faces: maintaining credibility versus biasing recommendations to align with the seller's incentives. Using this framework, we analyze the seller's optimal pricing strategy in the presence of an expert, evaluating its implications for market efficiency and consumer welfare.

In the case of soft incentives, our analysis identifies three distinct pricing regimes based on the buyer's cost of acquiring information. When information acquisition costs are high, the seller adopts *deterrence pricing*, offering substantial discounts to dissuade the buyer from consulting the expert. Deterrence pricing is adversarial towards the expert, deliberately minimizing the buyer's reliance on expert advice. While the buyer receives a positive surplus under deterrence pricing, trade may be inefficient, as the uninformed buyer risks purchasing products that do not suit their needs. In contrast, when information acquisition costs are low, the seller employs *inducement pricing*. In this regime, the seller sets a high price to encourage some buyer types to seek expert advice, ensuring they purchase the product only when it is of high value. Unlike deterrence pricing, which discourages the buyer's reliance on expert advice, inducement pricing leverages the expert's ability to reveal the product's true value. This approach enables informed purchases, ensuring that only high-value products are sold. As a result, the seller can charge higher prices and capture a greater share of the surplus.

Deterrence and inducement pricing embody distinct equilibrium dynamics between the seller and the expert. Under deterrence pricing, the seller and the expert are in an adversarial relationship: the seller strategically sets a price that discourages the buyer from seeking the expert's advice, effectively sidelining the expert from the transaction. The discount offered by the seller to deter information acquisition ultimately benefits the buyer. In contrast, under inducement pricing, the relationship between the seller and the expert is synergistic: The seller's high price creates demand for expert advice, while the expert's recommendations allow the seller to set a high price. However, this synergistic relationship comes at the buyer's expense. Because the seller sets the price knowing that the buyer will purchase only after learning that the product is of high value through the expert, the seller can condition the price on this knowledge. By accurately anticipating the buyer's willingness to pay, the seller extracts all the surplus from the buyer, leaving them with zero net benefit.

Building on this baseline model, we then turn to examining the effects of hard incentives by considering two common incentive mechanisms: endorsement contracts and referral schemes. Endorsement contracts involve the seller making direct monetary payments to the expert in exchange for adopting specific recommendations, typically biased toward encouraging purchases. Such contracts are widely observed in practice, particularly in the form of celebrity endorsements and product placements. In our model, endorsement contracts are represented as mechanisms in which the seller sets the product price and designs a payment structure contingent on the expert's chosen recommendation policy. We show that, in equilibrium, the seller offers endorsement contracts that incentivize the expert to always recommend a purchase, regardless of the product's true value to the buyer-hence the term endorsement contracts.

However, this renders the expert's recommendation meaningless, as buyers, recognizing the lack of informativeness, stop relying on the expert's recommendation. Thus, endorsement contracts take an adversarial role in our setup, similar to deterrence pricing, as they are used by the seller to prevent the buyer from ever becoming informed. Endorsement contracts provide an alternative mechanism to deterrence pricing and are more profitable for the seller whenever it is cheaper to pay the expert than to deter the buyer through price discounts. We show that endorsement contracts typically harm consumer surplus and market efficiency, as they suppress the flow of valuable information. However, in some cases, endorsement contracts can trigger the seller to move away from a pricing regime that excludes some buyer types from trade, resulting in an increase in total trade surplus.

Referral schemes, on the other hand, involve the seller compensating the expert with payments contingent on successful purchases that result from the expert's recommendations. Unlike endorsement contracts, referral schemes do not require the seller to monitor the expert's recommendation policy; instead, payments are tied to observable sales outcomes. In our model, referral schemes are characterized by a mechanism in which the seller sets both the price and the referral payment to align the expert's incentives with their own. However, this introduces a *capture-conversion trade-off*. The expert must balance capturing buyer attention with converting it into purchases: more informative recommendations increase the ability to capture buyer attention but reduce conversion rates, while biased recommendations increase conversion rates but lower capture rates. This trade-off limits the effectiveness of referral schemes compared to endorsements, forcing the seller to set prices below the prior mean to sustain the expert's biased recommendations. The effect of referral schemes on consumer surplus and efficiency depends on the corresponding baseline equilibrium. Compared to deterrence pricing, the buyer is worse off, and total trade surplus remains unchanged. Compared to inducement pricing, some buyer types benefit from referral schemes due to savings on information acquisition costs and the lower price. Referral schemes may also induce the seller to shift away from a pricing regime that excludes certain buyer types, potentially resulting in a Pareto improvement in some cases.

Related Literature This paper contributes to the literature on pricing and value uncertainty. Beginning with the seminal work of Akerlof (1970), a large body of literature has examined value uncertainty, with a particular emphasis on the information asymmetry between firms and consumers. Salop and Stiglitz (1977) demonstrate that price dispersion can function as a signal of value disparities among products. Three papers are particularly relevant. Bester and Ritzberger (2001) allow consumers to pay for a perfectly revealing signal subsequent to observing the price, while Martin (2017) and Boyacı and Akçay (2018) analyze the same strategic pricing game under the assumption of rational inattention. All three papers conclude that prices are never fully revealing. Our paper diverges from these works in two aspects. First, we abstract from information asymmetry, meaning that the posted price does not convey any information about product value. This approach enables us to focus on the interplay between monopoly pricing and consumer information acquisition, centering our discussion around the role of prior knowledge and information acquisition cost. Second, we examine the environment in which information is provided by a third party, who is able to strategically design informative signals that are available to the customer.[1]

This paper is also related to the literature on third-party information provision. Lizzeri (1999) considers a third party information provider who offers certification to sellers for a fee. In the optimal certification scheme, all sellers seek certification, despite the certification

[1] In the baseline model, the payoff of the third party depends on how likely consumers acquire information, which implies that the optimal signal is always perfectly revealing. In the extension, we study the impact of side payment from the seller to the third party on information provision and information acquisition, as well as the pricing strategy.

itself being uninformative. Stahl and Strausz (2017) compare the effects of seller-induced and buyer-induced certification in a market in which sellers know the quality. They show that seller-induced certifications are socially preferable, but again, the certifier extracts the entire surplus. Indeed, Pollrich and Strausz (2023) show that this full surplus extraction argument applies whenever the certification intermediary has all the bargaining power, regardless of the fee structure employed. Zapechelnyuk (2020) studies the optimal value certification in a moral hazard setting. A key difference between this literature and our paper is that the expert is in a much weaker bargaining position in our model. In our baseline model, the expert cannot set the fee for information provision and generates profits solely from buyers paying attention. Furthermore, there is neither adverse selection nor moral hazard in our model.

The literature on Bayesian persuasion is also concerned with information design in bilateral trade.[2] The literature considers the information design problem faced by the monopoly seller as well as that by the buyer. The optimal policy for the seller is to provide no information and charge at the buyer's expected value. When the buyer is able to inspect the good at a cost, the optimal policy is partial disclosure (Anderson and Renault, 2006). Closer to our paper are Roesler and Szentes (2017) and Evans and Park (2024), because they concern the information design problem for the buyer. Roesler and Szentes (2017) study the buyer-optimal signal structure and show that the optimal signal structure generates efficient trade. Evans and Park (2024) study design and pricing of information by a monopoly information provider. They characterize the profit-maximizing information structure and show that it has a simple threshold structure.

These two studies differ from our paper in two dimensions: First, in both papers, the signal structure is chosen before the monopoly seller sets the price, while the seller moves first in our paper, which allows us to focus on how monopoly pricing affects third-party information provision.[3] Second, the information designer is a benevolent regulator in Roesler and Szentes (2017), while the information provider in Evans and Park (2024) is profit-maximizing with the ability to set fees. In our setting, the information provider is profit-maximizing but does not have the fee-setting capacity. Peitz and Sobolev (2025) consider a problem similar to

[2]Before Bayesian persuasion, early contributions include Admati and Pfleiderer (1986) and Lewis and Sappington (1994). Admati and Pfleiderer (1986) find that a monopoly seller of financial information may find it optimal to sell noisy information, and Lewis and Sappington (1994) show that the monopoly seller provides either no information or full information about the product value.

[3]Evans and Park (2024) also consider the case where the seller moves first. In their setting, given any price, the information provider also offers a single-threshold signal structure and charges a fee equal to the buyer's surplus. Therefore, the monopoly seller sets the monopoly price in the first place. Due to the flexibility in pricing, the information provider is able to subtract the entirety of the surplus for any given price, which is not true in our setting.

Evans and Park (2024), in which an intermediary commits to a recommendation policy and a profit-sharing rule before the seller sets the price, while the seller can sell both through the intermediary and a direct channel.

This paper contributes to the growing literature on market competition with intermediaries. Intermediaries often have an incentive to bias their recommendations in favor of firms that pay them most money (Armstrong and Zhou, 2011; Inderst and Ottaviani, 2012; Teh and Wright, 2022). Our model features a single firm, and the expert faces a trade-off between attracting buyers to pay attention to the recommendations and converting that attention into actual sales. We examine how the expert biases the recommendations under various incentive schemes.[4] Intermediaries play a crucial role as middlemen, steering consumers toward particular products (Armstrong and Zhou, 2011; Inderst and Ottaviani, 2012; Hagiu and Jullien, 2011; Teh and Wright, 2022). In our model, however, buyers can access the product independently of the expert, while the seller can bypass the expert by strategically adjusting the price. It introduces additional strategic considerations in pricing and the design of incentive payments. Moreover, intermediaries' recommendations can influence market prices (Hagiu and Jullien, 2011; Janssen and Williams, 2024). In our model, both the price and the incentive payment are determined before the expert makes recommendations, allowing us to focus on how these variables affect the subsequent recommendation strategy.[5]

Additionally, Fainmesser and Galeotti (2021) consider the market of intermediaries, where the intermediaries who provide more informative recommendations are matched with more followers, which is dictated by an exogenous functional form. In contrast, we explicitly model the buyer's learning decision. The buyer incurs a learning cost to access the expert's recommendation and decides whether to acquire this information. Pei and Mayzlin (2022) model how affiliation affects reviewers providing reviews to consumers who might be unaware of the product. The affiliation contract they consider is similar to the endorsement contract considered in section 4.1, and the seller dictates the information content of an affiliated review in their model as well, but in a different fashion.

2 Model

Consider a market with a single seller (they), a buyer (he) and an information intermediary we call an "expert" (she). The seller owns a product that has an idiosyncratic value $\theta \in$

[4] Inderst and Ottaviani (2009) also considers a setting with one firm, and they study how commission fees affect agent's marketing effort.

[5] Janssen and Williams (2024) use their setting to model macro influencers, and our setting could be interpreted as a model about micro influencers. Even for macro influencers, many of the products they review and recommend are from big brands and the prices will not change according to the recommendation.

$\{\theta_L, \theta_H\}$ to the buyer. The value of the product is determined by a random draw of Nature, where $\pi \in (0,1)$ is the probability that the value is high ($\theta = \theta_H$). Let $\tilde{\theta} = \pi\theta_H + (1-\pi)\theta_L$ denote the prior mean of θ, and denote the variance of the distribution by $\sigma = \pi(1-\pi)$. Further, let $\Delta = \theta_H - \theta_L$ represent the value difference for the buyer.

Before making a purchase decision, the buyer decides whether to acquire information from the expert, who observes the buyer's true value θ and provides a binary purchase recommendation.[6] We assume that prior to observing the buyer's value, the expert commits to a *recommendation policy*, which specifies the distribution over purchase recommendations.[7] A recommendation policy for the expert is characterized by a pair of conditional probabilities $q = (q_H, q_L) \in [0,1]^2$, where q_j denotes the probability that the expert recommends a purchase when the buyer's value is θ_j for $j \in \{H, L\}$. There are two types of buyers: a zero-cost type whose information acquisition cost is zero,[8] and a high-cost type whose information acquisition cost is $k > 0$. The probability that the buyer is a zero-cost type is denoted by $\lambda \in (0,1)$.

The timing of this game is as follows. First, the seller sets the price $p \geq 0$. After observing this price, the expert selects a recommendation policy, $q \in [0,1]^2$. The buyer then observes both p and q and makes an attention decision, $a \in \{0,1\}$, determining whether to seek a recommendation from the expert. Next, Nature determines the value of the product, $\theta \in \{\theta_H, \theta_L\}$. If the buyer has chosen to acquire information, he receives a purchase recommendation with probability q_θ. Finally, the buyer makes a purchase decision, $b \in \{0,1\}$, and the resulting payoffs are realized.

The payoffs are as follows: The seller's payoff is $b(p-c)$, where $c > 0$ is the unit cost of production. The payoff for the high-cost buyer is $b(\theta - p) - ak$, and the payoff for the zero-cost buyer is $b(\theta - p)$. The expert's payoff is av, where $v > 0$ is the benefit the expert receives from the buyer's attention (for example, through advertising revenue on the platform).

Throughout, we make the following assumptions:

Assumption 1 (High-cost buyer acquires information)**.** $k < \sigma\Delta$.

This assumption ensures that there exists a price range and some recommendation policy

[6] It is easy to see that, by a revelation-principle type argument, it is without loss to focus on binary recommendations. For any more complex communication strategy the expert might use that involves larger message spaces, there exists an equivalent direct mechanism where the expert communicates a binary recommendation in a way that preserves the buyer's incentives and outcomes.

[7] The commitment assumption is crucial for our model and results, and we view it as a reduced form representation of the commitment that can be generated through repeated interaction in a full-fledged dynamic model in which the choices of the expert and the seller would be disciplined by continuation play.

[8] Focusing on these two discrete buyer types allows us to capture the core equilibrium mechanics in the simplest and most transparent way. Extending the model to include a broader range of buyer types with varying information acquisition costs would add considerable complexity while offering limited additional insights.

such that the high-cost type finds it worthwhile to pay attention. We impose this assumption to rule out trivial cases in which the high-cost type never acquires information in any equilibrium.

Assumption 2 (Trade is ex-ante efficient)**.** $\tilde{\theta} > c$.

This assumption ensures that trade takes place even in the absence of the expert. We impose this assumption to focus on the more interesting scenario in which the seller can strategically set a price to exclude the expert from the transaction.

Assumption 3 (Trade is inefficient in low state)**.** $\theta_L < c < \theta_H$.

This assumption ensures that expert recommendations can improve trade efficiency. We adopt it to focus on scenarios in which the presence of the expert has more interesting welfare implications.

Assumption 4 (Share of zero-cost type is small)**.** $\lambda \leq \frac{\tilde{\theta}-c}{\pi(\theta_H - c)} =: \bar{\lambda}$.

Our last assumption limits the share of zero-cost type in the buyer population, ensuring that the high-cost type has a meaningful impact on the seller's pricing strategy. Specifically, if this assumption is not satisfied, the seller always prefers selling exclusively to the zero-cost type at $p = \theta_H$ to selling to both types at $p = \tilde{\theta}$. Therefore, we impose this assumption to focus on the scenarios in which the presence of the high-cost type is relevant.

We consider perfect Bayesian equilibrium, where we assume that, regardless of the buyer's prior belief about q, his posterior belief about θ is consistent with the information generated by the chosen recommendation policy.

3 Soft Incentives

Soft incentives are small, non-monetary rewards that a seller offers to an expert at negligible cost. Common examples include exclusive access, VIP experiences, or product samples, all designed to encourage the expert to recommend the seller's product without compromising the expert's reputation. In our model, we capture the impact of these incentives by focusing on seller-optimal equilibria. Because experts are indifferent among recommendation policies that generate the same level of buyer engagement, even a modest perk can prompt them to favor the policy that maximizes the seller's probability of making a sale. As a result, seller-optimal equilibria naturally reflect the influence of soft incentives and provide a convenient benchmark for comparing the effects of incentive payments in the following section.

To derive these equilibria, we begin by examining the buyer's decision to acquire the expert's recommendation. We assume that a buyer with zero cost of acquiring information always seeks the recommendation. We shall see that this is indeed without loss of generality. By contrast, a high-cost buyer consults the expert only if the expected gain exceeds the cost of the recommendation. When the seller's price p exceeds $\tilde{\theta}$, the buyer would otherwise forgo the purchase, so it becomes optimal to acquire the recommendation if

$$\pi\, q_H(\theta_H - p) \;+\; (1-\pi)\, q_L(\theta_L - p) \;-\; k \;\;\geq\;\; 0. \tag{1}$$

In contrast, if $p \leq \tilde{\theta}$, the buyer would purchase the good based on prior beliefs, and thus pays for the recommendation only if

$$\pi\, q_H(\theta_H - p) \;+\; (1-\pi)\, q_L(\theta_L - p) \;-\; k \;\;\geq\;\; \tilde{\theta} - p. \tag{2}$$

We refer to these two inequalities as the attention constraints, as they indicate when the high-cost buyer finds it worthwhile to consult the expert. We also assume that once the buyer obtains a recommendation, it is optimal to follow it. Ignoring the expert's advice would yield no benefit and thus would not justify the acquisition cost in the first place.

When the recommendation is fully informative (i.e., $q = (1, 0)$), the high-cost buyer acquires information at price p only if

$$\underline{p}_k := \theta_L + \frac{k}{1-\pi} \;\;\leq\;\; p \;\;\leq\;\; \theta_H - \frac{k}{\pi} := \bar{p}_k.$$

We call $\underline{p}_k$ and $\bar{p}_k$ the attention thresholds, since they mark the lowest and highest prices at which the buyer still finds it profitable to learn more about the product. Below $\underline{p}_k$, the good is too cheap to warrant further investigation; above $\bar{p}_k$, it is too expensive for additional information to matter. Thus, these thresholds delineate the precise range of prices over which the buyer consults the expert, forming a key component of our seller-optimal equilibria analysis.

We now take a step back to focus on the expert's choice of recommendation policy. Under the assumption that the seller uses soft incentives to nudge the recommendations in their favor, we assume that the expert chooses a recommendation policy that maximizes the probability of recommending a purchase while keeping the level of information acquisition by the buyer constant. Formally, we define a recommendation policy as *k-maximal* at price p if it maximizes the probability of purchase for the k-type buyer subject to inducing the k-type buyer to acquire information, if possible, at price p. Given price p, the k-maximal recommendation policy is the solution to the maximization problem

$$\max_{(q_H,q_L)\in[0,1]^2} \pi q_H + (1-\pi)q_L \tag{3}$$

subject to the attention constraint (1) if $p \geq \tilde{\theta}$ and (2) if $p \leq \tilde{\theta}$.

Because it benefits both the seller and the buyer to always recommend a purchase when $\theta = \theta_H$, any k-maximal recommendation policy must set $q_H = 1$ for all prices. Consequently, a k-maximal recommendation policy can be written as

$$q^k(p) = \big(1,\ q_L^k(p)\big),$$

where $q_L^k(p)$ is the probability of recommending a purchase when $\theta = \theta_L$. Since the maximization problem (3) is increasing in q_L, the relevant attention constraint binds at the optimum. Letting $q_H = 1$, we can then solve for q_L in a k-maximal recommendation policy explicitly:

$$q_L^k(p) = \begin{cases} 1 - \frac{k}{(1-\pi)(p-\theta_L)} & \text{if } \theta_L < p \leq \tilde{\theta}, \\ \frac{\pi(\theta_H - p) - k}{(1-\pi)(p-\theta_L)} & \text{if } \tilde{\theta} < p \leq \theta_H. \end{cases} \tag{4}$$

A 0-maximal recommendation policy, denoted $q^0(p)$, is defined analogously. In this case, we have

$$q_L^0(p) = \begin{cases} 1 & \text{if } \theta_L < p \leq \tilde{\theta}, \\ \frac{\pi(\theta_H - p)}{(1-\pi)(p-\theta_L)} & \text{if } \tilde{\theta} < p \leq \theta_H. \end{cases} \tag{5}$$

Note that a 0-maximal recommendation policy always recommends a purchase for prices below the prior mean $\tilde{\theta}$, making it completely uninformative. However, it becomes increasingly more informative as the price rises above $\tilde{\theta}$.

Now, consider the seller's pricing problem. In a seller-optimal equilibrium, the expert uses a recommendation policy that maximizes the probability of sale, subject to keeping constant the probability of information acquisition by the buyer. This means that for prices between $\underline{p}_k$ and $\bar{p}_k$, the expert uses a k-maximal recommendation policy. For prices below $\underline{p}_k$ and above $\bar{p}_k$, the high-cost buyer doesn't acquire information regardless of the recommendation policy, and thus, the expert uses a 0-maximal recommendation.

The seller's profit equals the per-unit profit $p - c$ times the overall probability of purchase. To see how this probability varies with price p, recall that $\underline{p}_k$ and $\bar{p}_k$ are the attention thresholds for the high-cost buyer. When $p < \underline{p}_k$, both buyer types purchase the product in both states (because the high-cost type does not find it worthwhile to acquire information, and the zero-cost type does so but always hears a purchase recommendation). Hence, the total probability of purchase is 1. When $p > \bar{p}_k$, the high-cost type never purchases the product, whereas the zero-cost type acquires the expert's recommendation and buys only if advised to do so. In that case, the probability of purchase is $\lambda\big(\pi + (1-\pi)\, q_L^0(p)\big)$. If instead $\underline{p}_k < p < \bar{p}_k$, then both types acquire the recommendation, and the probability of purchase is $\pi + (1-\pi)\, q_L^k(p)$. Substituting the corresponding expressions for q_L^k and q_L^0 into each region and simplifying yields the following piecewise profit function:

$$\Pi(p) \;=\; \begin{cases} p - c, & \text{if } p \in [\theta_L,\ \underline{p}_k], \\ \left(1 \;-\; \dfrac{k}{p-\theta_L}\right)(p-c), & \text{if } p \in (\underline{p}_k,\ \tilde{\theta}], \\ \dfrac{\pi\,\Delta \;-\; k}{p-\theta_L}\,(p-c), & \text{if } p \in (\tilde{\theta},\ \bar{p}_k], \\ \dfrac{\lambda\,\pi\,\Delta}{p-\theta_L}\,(p-c), & \text{if } p \in (\bar{p}_k,\ \theta_H]. \end{cases}$$

Because $\Pi(p)$ is piecewise linearly increasing in p but exhibits downward jumps at $p = \underline{p}_k$ and $p = \bar{p}_k$ (and a kink at $p = \tilde{\theta}$), any global maximum must occur at one of these three points: $\underline{p}_k$, $\bar{p}_k$, or θ_H. Comparing the seller's profit across these prices identifies the optimal choice.

To interpret the threshold prices in an equilibrium where the expert's recommendation policy aligns with the seller's interests, observe that at $p = \underline{p}_k$, no additional information is provided to the high-cost type; both types simply buy in both states, yielding

$$\Pi(\underline{p}_k) = \underline{p}_k \;-\; c = \theta_L \;+\; \frac{k}{1-\pi} \;-\; c.$$

Accordingly, we call $\underline{p}_k$ a *k-deterrence price*, since it effectively "deters" the high-cost buyer from acquiring information. By contrast, at $p = \bar{p}_k$, the expert provides full information and both buyer types purchase only in the high state, giving

$$\Pi(\bar{p}_k) = \frac{\pi\,\Delta - k}{\bar{p}_k - \theta_L}\,\big(\bar{p}_k - c\big) = \pi\,(\theta_H - c) \;-\; k.$$

Here, $\bar{p}_k$ is a *k-inducement price* because it "induces" the high-cost buyer to learn and then buy only when $\theta = \theta_H$. Finally, at $p = \theta_H$, the expert also provides full information, but only the zero-cost type purchases in the high state, yielding

$$\Pi(\theta_H) = \lambda\,\pi\,(\theta_H - c).$$

This price is referred to as a 0-*inducement price* because the high-cost type is never induced to learn at $p = \theta_H$. By Assumption (4), there exists k such that $\Pi(\underline{p}_k) > \Pi(\theta_H)$.

To identify the seller's optimal price, we compare the profit $\Pi(p)$ at the three candidate prices $\underline{p}_k$, $\bar{p}_k$, and θ_H. Straightforward algebra shows that $\Pi(\underline{p}_k) > \Pi(\bar{p}_k)$ if and only if

$$k \;>\; \left(\tfrac{1-\pi}{2-\pi}\right)\big[\pi(\theta_H - c) \;-\; (\theta_L - c)\big] \;=:\; k^*. \tag{6}$$

Hence, a deterrence price $\underline{p}_k$ is strictly more profitable than an inducement price $\bar{p}_k$ when $k > k^*$. Next, comparing the seller's profit at $\bar{p}_k$ and θ_H reveals that $\Pi(\bar{p}_k) > \Pi(\theta_H)$ if and only if

$$k \;<\; \pi\,(1-\lambda)\,(\theta_H - c) \;=:\; \underline{k}. \tag{7}$$

Thus, the inducement price $\bar{p}_k$ yields a higher profit than simply setting $p = \theta_H$ if $k < \underline{k}$. Finally, comparing $\underline{p}_k$ and θ_H shows that $\Pi(\underline{p}_k) > \Pi(\theta_H)$ if and only if

$$k \;>\; (1-\pi)\big[\lambda\,\pi\,(\theta_H - c) \;-\; (\theta_L - c)\big] \;=:\; \bar{k}. \tag{8}$$

In other words, the deterrence price $\underline{p}_k$ outperforms the high price θ_H whenever $k > \bar{k}$. Figure 1 illustrates $\Pi(p)$ at each of these three prices and shows how the upper envelope of these segments determines the seller's optimal choice.

In summary, for high values of k we have $\Pi(\underline{p}_k) > \max\{\Pi(\bar{p}_k), \Pi(\theta_H)\}$ which means that k-deterrence pricing outperforms the other two pricing strategies. This is because the price discount required to deter information acquisition is decreasing in k, and the k-deterrence price $\underline{p}_k$ is increasing in k. For low values of k, we have $\Pi(\bar{p}_k) > \max\{\Pi(\underline{p}_k), \Pi(\theta_H)\}$ which means that k-inducement pricing is the optimal pricing strategy for the seller. This is because the price premium the seller can charge while still inducing both buyer types to acquire information is increasing in k. On the other hand, the 0-inducement price θ_H is independent of information acquisition cost k. As a result, the corresponding profit $\Pi(\theta_H)$ is constant. This makes 0-inducement pricing more favorable for intermediate levels of k.

The following proposition provides a formal characterization of the seller-optimal equilibrium.

Proposition 1. *There is a seller-optimal equilibrium, with the following properties.*

(*i*) *k-Deterrence pricing: If $k \geq \max\{k^*, \bar{k}\}$, the seller sets price $p = \underline{p}_k < \tilde{\theta}$, the expert always recommends a purchase, only the zero-cost type seeks a recommendation and both types buy in both states.*

(*ii*) *0-Inducement pricing: If $\underline{k} < k < k^*$ or $k^* < k < \bar{k}$, the seller sets price $p = \theta_H$, the*

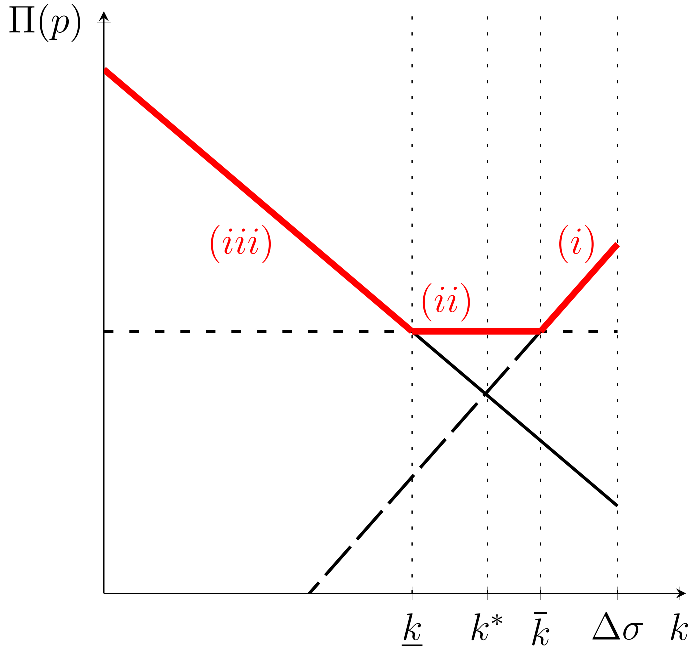

Figure 1: $\Pi(p)$ as a function of information acquisition cost k: (i) $\Pi(\overline{p}_k)$ (dashed), (ii) $\Pi(\theta_H)$ (dotted), and (iii) $\Pi(\underline{p}_k)$ (solid). The bold red line represents the upper-envelope of $\Pi(\overline{p}_k)$, $\Pi(\theta_H)$, and $\Pi(\underline{p}_k)$.

expert's recommendation is fully revealing, only the zero-cost type seeks a recommendation and buys only when the state is high, while the high-cost type never buys.

(iii) *k-Inducement pricing: If $k \leq \min\{k^*, \underline{k}\}$, the seller sets price $p = \bar{p}_k > \tilde{\theta}$, the expert's recommendation is fully revealing, both types seek a recommendation and buy only when the state is high.*

Moreover, this equilibrium is unique, except for knife-edge cases.

A detailed proof appears in the appendix. Proposition 1 describes on-path equilibrium behaviors as a function of the high-cost buyer's information acquisition cost k. Consistent with intuition, the buyer acquires less information on average as k increases.

The share of zero-cost buyers affects both the overall demand for information and the seller's optimal pricing strategy. In particular, both $\max\{k^*, \bar{k}\}$ and $\min\{k^*, \underline{k}\}$ depend on λ, and as λ increases, the seller is more likely to adopt 0-inducement pricing. This reflects the fact that a larger fraction of buyers can access expert recommendations without incurring a cost, making it more profitable for the seller to set a high price and extract surplus from this segment rather than encourage broader information acquisition. Consequently, even though more consumers can theoretically learn at lower cost, the seller's equilibrium strategy (raising prices to exploit that segment) may negate potential gains in sales or efficiency. Moreover, if

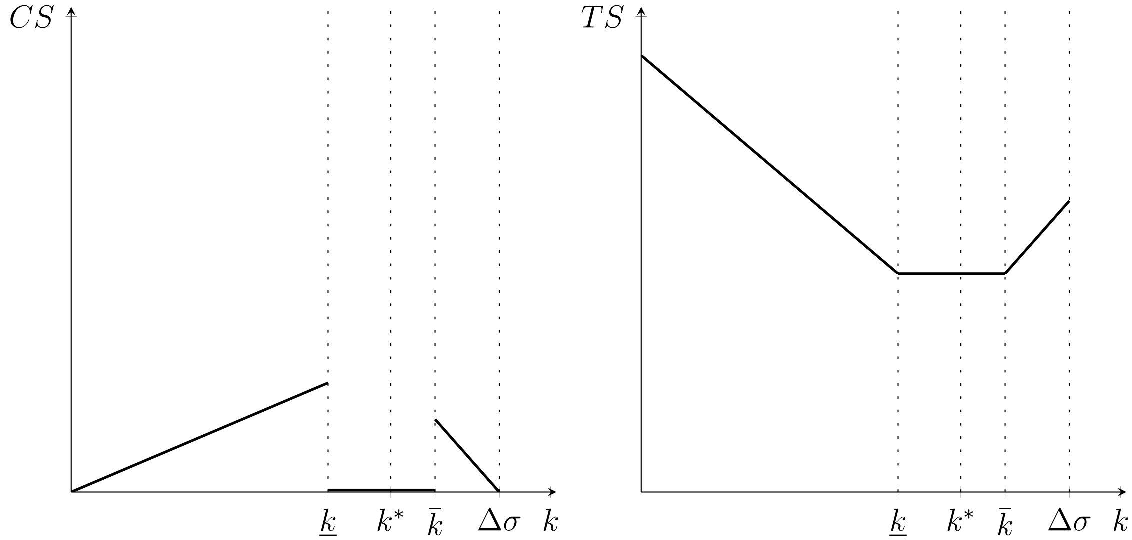

Figure 2: Consumer surplus (CS) and trade surplus (TS) as a function of information acquisition cost k.

θ_H increases relative to $\tilde{\theta}$, the threshold $\max\{k^*, \bar{k}\}$ also rises, making k-deterrence pricing less likely. This in turn can lead to a switch from deterrence to inducement, characterized by higher prices, fewer sales, and better-informed buyers. See Figure 2.

The welfare properties of the equilibrium, in terms of consumer surplus and trade surplus, depend crucially on the pricing regime. Here, we mean by trade surplus the total gains from trade net of any information-acquisition costs. We deliberately exclude the expert's payoff in order to isolate the impact of a third-party information provider on market efficiency. In a k-deterrence equilibrium, neither type of buyer acquires information, so that trade surplus remains constant; however, consumer surplus falls as k increases because the seller extracts more surplus through higher prices. By contrast, under 0-inducement pricing only the zero-cost type acquires information and buys in the high state, leaving both consumer and trade surplus unchanged as k varies. Finally, with k-inducement pricing the seller sets a price that induces even the high-cost buyer to acquire information, so that as k grows, consumer surplus increases while trade surplus declines, reflecting the additional friction introduced by higher information costs. We now formalize these results in the following proposition.

Proposition 2. *In a seller-optimal equilibrium,*

(*i*) *k-Deterrence pricing: Consumer surplus is decreasing in k, and total surplus is independent of k.*

(*ii*) 0-*Inducement pricing: Both consumer surplus and total surplus are independent of k.*

(*iii*) *k-Inducement pricing: Consumer surplus is increasing in k, and total surplus is decreasing in k.*

Proposition 2 highlights a non-monotonic relationship between the consumer's surplus and the information-acquisition cost k. In particular, when k lies above $\bar{k}$ (but remains below $\sigma\Delta$), both buyer types enjoy a positive surplus because the seller must offer a sufficiently low price to deter the high-cost type from acquiring information. As k decreases, this surplus initially grows: it becomes cheaper for the seller to discourage learning, and the price therefore falls. However, once k moves below a critical level, it becomes prohibitively expensive for the seller to keep discounting in order to deter the high-cost type from seeking information. At that juncture, the seller either chooses a high price and sells only to the zero-cost buyer or switches to an inducement price that allows both types to learn but raises the price considerably. In doing so, the seller captures more surplus by exploiting the high-cost type's newly affordable access to expert information.

Proposition 2 also shows that the seller's profit as a function of k is single-dipped. The key driver for this is the seller's focus on extracting surplus. Note that under k-inducement pricing, an increase in k forces the seller to share more surplus with the high-cost buyer, since the buyer's willingness to pay depends on obtaining valuable information. Conversely, under k-deterrence pricing, a higher k reduces the incentive to learn, allowing the seller to retain a larger share of the surplus by setting a higher price that both buyer types accept without further information. When the seller sets a 0-inducement price, total surplus is minimal because only the zero-cost type purchases in the high state—yet the seller fully captures whatever surplus remains.

Total surplus similarly displays a single-dipped shape. Under k-inducement pricing, trade is efficient in the high state, but the rising cost of information reduces overall welfare as k increases. In contrast, when the seller uses deterrence or 0-inducement pricing, no information costs are actually incurred, so total surplus stays constant—but fewer transactions occur in the high state or only the zero-cost buyer is served. In this way, the distribution of surplus between the buyer and the seller varies with k, while overall market efficiency changes most sharply when the buyer's incentive to learn is on the margin.

4 Hard incentives

We now explore scenarios where the seller offers non-negligible payments to the expert to sway the expert's recommendations in ways that may hurt the expert's reputation[9] and

[9] We use "reputation" here to refer to the anticipated informativeness of the expert's recommendation. This interpretation naturally fits our environment where the expert's perceived credibility shapes information

potentially reduce the buyer's incentive to acquire information. We consider two prominent payment structures: *endorsements* and *referrals*.

In an endorsement setup, the seller observes the expert's choice of recommendation policy and compensates the expert for any revenue lost when buyers acquire less information. By contrast, referrals tie payments directly to realized sales: the seller pays the expert for purchases made by buyers who follow her recommendations. Unlike endorsements, this approach does not require observing the expert's underlying policy, since rewards depend only on sales outcomes. However, referral schemes introduce a misalignment of incentives: the expert aims to maximize the purchases made through her own recommendations (requiring both buyer attention and successful conversions), whereas the seller seeks to maximize total sales—regardless of whether they stem from the expert's recommendations or not.

4.1 Endorsements

In online marketing, an endorsement typically involves influencers publicly expressing support or approval for a product or service to build trust and influence purchasing decisions among their audience. In the context of this paper, we interpret endorsements as a strategic tool used by the seller to influence the expert's recommendation behavior through direct monetary payments. For this strategy to be effective, the seller must observe the expert's recommendation policy. By offering an endorsement fee, the seller can incentivize the expert to bias her recommendations, favoring a recommendation policy that increases the likelihood of recommending a purchase. As a result, endorsements become a strategic tool for the seller to align the expert's recommendation with the seller's objectives, albeit at the risk of compromising the expert's integrity.

We formalize the concept of endorsements through a contracting problem between the seller and the expert. The seller chooses both the product price and a contract that specifies a payment to the expert, contingent on the recommendation policy adopted. In our model, the seller offers an endorsement payment that compensates the expert if she adopts a specific recommendation policy. This endorsement payment depends on the difference in outcomes between using the seller's preferred recommendation policy and other alternatives. The expert accepts the contract if the payoff from adopting the seller's preferred recommendation policy, along with the endorsement payment, exceeds what she would receive without the contract.

It is easy to see in our context and interpretation, an endorsement mechanism is the most general form of incentive mechanism for the seller. It can mimic or improve upon

acquisition incentives, but it is conceptually different form the standard use of the term in economic theory, where it refers to the public belief about the persistent type a long-lived player in a repeated game.

any other incentive schemes the seller could use, including the referral schemes discussed in the next section, incentive schemes based on realized recommendations, as in Condorelli et al. (2018), schemes that condition on whether the buyer seeks advice as in Fainmesser and Galeotti (2021), or any combination of these. From the seller's perspective, what matters is the expected transfer made to the expert for adopting a specific recommendation policy. While these alternative schemes condition on the recommendation policy indirectly through its effect on recommendations and the buyer's decisions, endorsement schemes allow the seller to directly condition on the distribution of recommendations. This direct conditioning offers the seller greater flexibility in aligning the expert's recommendation with the profit-maximizing objectives.

The following result characterizes the optimal endorsement contract.

Proposition 3. *There are thresholds k^E and v^E, such that if and only if $k^E < k < \Delta\sigma$ and $v < v^E$, in the seller-optimal equilibrium, the seller sets price $p^E = \tilde{\theta}$ and pays the expert the amount $E^* = (1-\lambda)v$ for an endorsement; the expert always recommends a purchase, only the zero-cost type seeks a recommendation and both types buy in both states. Otherwise, the seller-optimal equilibrium is as characterized in Proposition 1. Moreover, we have $k^E < \underline{k}$, where $\underline{k}$ is the threshold defined in* (7)*.*

In an equilibrium with endorsements, the price is equal to the prior mean $\tilde{\theta}$. The seller compensates the expert with an amount equal to the expected revenue loss caused by reduced buyer attention from the high-cost type, which results from the biased recommendations. Endorsements essentially function as an alternative to deterrence pricing. Instead of offering a price discount to discourage information acquisition as deterrence pricing, the seller shares surplus with the expert to incentivize uninformative recommendations which recommend a purchase with probability one. Endorsements are optimal if inducing uninformative recommendations is less costly than offering price discounts to the buyer. In particular, endorsements outperform deterrence pricing when the expert's gain from buyer attention v is sufficiently low.

Because endorsements are more attractive for the seller than deterrence pricing when the expert's gain from buyer attention is low, we would expect in practice that experts who earn little from providing honest recommendations are more likely to accept paid endorsements. This helps explain criticisms that reviewers prioritize financial incentives over impartial advice, especially when alternative revenue sources are limited.[10] Moreover, even with regulations requiring disclosure of endorsements,[11] sellers can still offer payments high

[10] Platforms like Patreon argue that diversifying income streams reduces reliance on brand partnerships and paid endorsements.

[11] See the Federal Trade Commission's Endorsement Guides.

enough to bias recommendations, ultimately shaping the information consumers receive.

Corollary 1. *Under endorsement schemes, both buyer types are weakly worse off, while the expert is strictly better off compared to the corresponding baseline equilibrium under soft incentives.*

In terms of welfare, endorsements are unsurprisingly bad for the buyer. Under any endorsement scheme, the buyer receives zero surplus, while the expert secures a guaranteed payoff of v. Furthermore, it is clear that the use of endorsements improves the seller's profit compared to the baseline case with soft incentives, as the seller always has the option to forego payments to the expert and rely on soft incentives instead. The inequality $k^E < \underline{k}$, as identified in Proposition 2, implies that, in an equilibrium with endorsements, the corresponding baseline equilibrium may be any of the three types of equilibrium described in Proposition 1. The effect of endorsements on trade surplus thus depends on the equilibrium that would have prevailed in the absence of endorsements.

The effect of endorsements on trade efficiency depends on the corresponding baseline equilibrium with soft incentives. Compared to an equilibrium with k-deterrence pricing, the trade outcome under endorsements remains unchanged. However, the seller redistributes surplus by raising the price and compensating the expert for biased recommendations, effectively shifting surplus from the buyer to the expert. Compared to an equilibrium with k-inducement pricing, overall trade surplus is lower under endorsements. This is because, with endorsements, both buyer types purchase without learning the product value, resulting in less efficient trade outcomes. Compared to an equilibrium with 0-inducement pricing, overall trade surplus is higher under endorsements. Endorsements generate additional surplus by influencing the expert's recommendations, which is captured by the seller and partially shared with the expert through endorsement payments, while still leaving the buyer with zero surplus.

4.2 Referrals

Referral schemes, which are commonly observed in practice through affiliate marketing, reward purchase recommendations with incentive payments that are directly linked to either customer referrals or completed sales. The affiliate marketing industry is substantial, with its market size projected to reach 15.7 billion USD by 2024.[12] In affiliate marketing, the process typically involves an individual recommending a product through a unique link or

[12] See Influencer Marketing Hub's report, `https://influencermarketinghub.com/ebooks/Affiliate_Marketing_Benchmark-Report-2023.pdf`.

discount code, allowing the seller to track sales attributable to that individual's recommendation. When a buyer uses the link or code to make a purchase, the person who made the recommendation earns a commission, which can be a percentage of the sale price or a fixed fee. In our model, we formalize referrals as a contracting problem, where the seller determines both the product price and a referral payment. The referral payment represents the amount the seller pays the expert whenever the expert's recommendation successfully leads to a sale.

Formally, we consider a variant of the seller's price-setting problem, in which the seller determines both a price p and a referral payment R. The referral payment R represents the nominal payment given to the expert when a sale is realized through the expert's recommendation. When the seller offers a referral scheme with payment R, the seller's payoff is $b(p-c) - abR$, and the expert's payoff is $av + abR$. Note that the distinction between referral payments based on conversion (paying only when a sale occurs) and leads (paying when the buyer shows interest, regardless of purchase) is immaterial in our model, as there is no new information revealed after the buyer receives a recommendation from the expert, and thus every referral results in a sale. Moreover, since the seller choosing both the price and the referral payment, the analysis does not depend on whether the payment is a percentage of the transaction value or a fixed amount. In other words, our specific choice of payment structure-whether a revenue-based commission or a flat fee-is without loss of generality.

For the the seller, the primary advantage of a referral scheme over an endorsement contract is the reduced need for monitoring the expert's recommendation policy. This makes them simpler to implement in practice. However, this simplicity comes at the cost of reduced effectiveness in shaping the expert's recommendations compared to endorsement contracts. The key limitation of referral schemes lies in the inherent tension between capturing the buyer's attention and converting that attention into sales. Informative recommendations help engage the buyer by building trust and credibility, yet they may reduce the likelihood that a buyer makes a purchase, as the buyer becomes more well-informed. On the other hand, recommendations which biased toward purchase will increase conversion rates, but are less informative, and thus they reduce the likelihood of attracting the buyer's attention. Thus, while referral schemes can incentivize the expert to align with the seller's interests to some extent, they do not provide the same level of influence over the expert's recommendation policy as endorsement contracts.

Because of the inherent capture–conversion trade-off, a seller-optimal equilibrium with referral payments exists only if the seller sets a sufficiently low price. To see this, consider the case where the seller sets a price above the buyer's prior mean $\tilde{\theta}$. At such a high price, the buyer will purchase only after receiving a favorable recommendation from the expert. In this scenario, the seller benefits from the expert providing sufficiently informative recommendations, as greater informativeness increases the buyer's willingness to pay. This dynamic mirrors inducement pricing, where the seller relies on the informativeness of the expert's recommendations to sustain a higher price while ensuring that sales occur.

However, even when the seller sets a price below the buyer's prior mean, referral payments may still fail to incentivize the expert to bias her recommendations, because the expert's strategic options depend on how the price influences buyer behavior. Specifically, when the price falls between $\underline{p}_k$ and $\tilde{\theta}$, the expert can either focus only on the zero-cost type, so that conversion within that group improves, or target both buyer types, so that the audience expands but the overall conversion rate declines. Since the relative appeal of these two strategies changes with the price, the expert finds it less attractive to focus only on the zero-cost type when the price is too high, because attracting both buyer types while maintaining a higher overall conversion rate becomes easier. As a result, if there is no strong incentive to exclude high-cost buyers, then referral payments do not effectively steer the expert toward a biased (0-maximal) recommendation policy.

The following lemma formalizes this reasoning and specifies the maximum price at which an equilibrium with referrals can be sustained.

Lemma 1. *There is no seller-optimal equilibrium with referrals, in which the seller sets a price $p \geq \min\{\tilde{\theta}, \hat{p}_k\}$ where*

$$\hat{p}_k = \theta_L + \frac{k}{1-\lambda},$$

Moreover, the expert always recommends a purchase in any seller-optimal equilibrium with referrals.

Here, $\hat{p}_k$ represents the highest price at which the expert finds it optimal to target a narrower audience (zero-cost type only) with a higher conversion rate, rather than a broader audience (both types) with a lower conversion rate. When the price exceeds $\hat{p}_k$, the expert adopts the same recommendation policy as if only soft incentives had been offered. A testable implication of this result is that, in real-world applications, referral schemes should be more prevalent for low-cost products and less common in premium markets.

An immediate consequence of Lemma 1 is that referral payments become ineffective whenever $\lambda < \pi$. In this case, we have $\hat{p}_k < \underline{p}_k$. Here, $\hat{p}_k$ represents the maximum price at which referral payments can successfully incentivize biased recommendations, while $\underline{p}_k$ denotes the lowest price at which high-cost buyers would have any incentive to acquire information. Since enabling referral-based incentives would require setting a price below $\hat{p}_k$, doing so would be unprofitable for the seller. Instead, the seller can achieve higher profits by choosing the k-deterrence price $\underline{p}_k$, where the high-cost type naturally avoids seeking recommendations. Therefore, we have

Corollary 2. *When $\lambda \leq \pi$, there is no seller-optimal equilibrium in which the seller pays for referrals.*

Having established that referral payments are ineffective when $\lambda \leq \pi$, we now turn to the case where $\lambda > \pi$ and focus on characterizing the seller-optimal equilibrium in this case. The following proposition provides the main characterization.

Proposition 4. *Suppose $\lambda > \pi$. There exist thresholds $\hat{k}, k^R$ and v^R such that for $k^R < k < \Delta\sigma$ and $v < v^R$, in the seller-optimal equilibrium,*

(i) If $k < \hat{k}$, the seller sets price $p^R = \hat{p}_k - \sqrt{\frac{vk\lambda}{1-\lambda}} < \tilde{\theta}$ and referral payment $R^* = \sqrt{\frac{kv}{\lambda(1-\lambda)}} - v$.

(ii) If $k \geq \hat{k}$, the seller sets price $p^R = \tilde{\theta}$ and referral payment $R^* = \frac{(1-\lambda)\pi\Delta v}{k-(1-\lambda)\pi\Delta}$.

In both cases, the expert always recommends a purchase, only the zero-cost type seeks a recommendation and both types buy in both states. Moreover, $k^R < \min\{k^*, \underline{k}\}$ where k^* and $\underline{k}$ are the thresholds defined in (6) *and* (7)*, respectively.*

This characterization reveals two distinct types of referral-based equilibria, depending on the high-cost buyer's information acquisition cost k. In either case, the seller's referral payment is set to the minimum amount needed to induce the expert to employ a 0-maximal recommendation policy instead of a k-maximal one. Although the seller seeks to raise the price while keeping referral costs as low as possible, these goals conflict because the minimum referral payment rises with the price. When the seller increases the price, it becomes more risky for the buyer buyer to purchase the good which increases the buyer's incentive to acquire additional information from the expert. The buyer's increased demand for information strengthens the expert's incentive to focus on capturing the high-cost type rather than simply converting the zero-cost type, which the seller must counter by raising the referral fee.

When k is strictly below the threshold $\hat{k}$, the buyer's strong demand for information makes it optimal for the seller to set a price below the threshold below the prior mean, where the price discount serves to regulate the cost of referral payments. As k increases, the equilibrium referral payment increases as well. This is because with larger k, it becomes less costly for the seller to raise the price, who then raises the price and thus must increase the referral payment to make it incentive-compatible.

When k lies above the threshold $\hat{k}$, the seller optimally sets the price to the prior mean $\tilde{\theta}$. The seller cannot raise the price above this threshold, as the high-cost type would no longer be willing to purchase without informative recommendations. In this equilibrium regime, a further increase in the information acquisition cost k, no longer has any price effect, as the price remains fixed at $\tilde{\theta}$, but a negative effect on referral payments. This occurs because an increase in the information acquisition cost of the high-cost type makes it more difficult for the expert to capture that type's attention, thereby reducing the referral payment required to sustain the expert's incentives.

Similar to the case of endorsements, the effect of referrals on trade efficiency depends on the corresponding baseline equilibrium with soft incentives. Compared to an equilibrium with k-deterrence pricing, the trade outcome under referrals remains unchanged. However, the seller redistributes surplus from the buyer to the expert by raising the price and compensating the expert through referral fees for purchase made by the zero-cost type. When $p^R = \tilde{\theta}$, the referral payment to the expert exceeds what they would earn under an endorsement contract.

Compared to an equilibrium with k-inducement pricing, overall trade surplus is lower under referrals, though the impact on consumer surplus is mixed. The high-cost type benefits from the lower price and avoids incurring information acquisition costs, leading to an increase in consumer surplus. The zero-cost type may also benefit from the lower price but could experience reduced expected utility if the recommendation becomes less informative.

Compared to an equilibrium with 0-inducement pricing, overall trade surplus is higher under referrals. The price adjustment leads to increased sales while maintaining the same level of information acquisition, ensuring market participation from both buyer types without introducing inefficiencies. Additionally, both buyer types can experience increased surplus, particularly if the price p^R is set below the prior mean, allowing them to achieve positive surplus compared to the zero surplus in the baseline case.

Overall, referral payments can reshape the allocation of surplus and trade efficiency compared to the equilibrium under soft incentives. In some cases, referral schemes increase the payoffs of the seller and the expert at the expense of buyer surplus and market efficiency, while in other cases, they can improve outcomes for all participants, and even result in a Pareto improvement.

5 Conclusion

This paper examines the role of third-party experts in influencing consumer purchasing decisions and how sellers strategically use different pricing and incentive mechanisms to shape market outcomes. Our analysis reveals that expert recommendations introduce new strategic considerations for sellers when buyers can acquire information at a cost. We show that the presence of an independent expert leads to three distinct pricing regimes, characterized by whether the seller seeks to deter, induce, or ignore information acquisition by the high-cost buyer. We further examine the effects of endorsement contracts and referral schemes on market outcomes. Under endorsement contracts, where the expert is compensated based on their choice of recommendation policy, consumers are unambiguously worse off. In contrast, referral schemes, where the expert is rewarded only when a sale is realized, introduce a *capture-conversion trade-off* for the expert between attracting attention and converting that attention into sales. This dynamic can, under certain conditions, lead to a Pareto improvement for all market participants.

A key assumption in this paper is that the expert is able to commit to a recommendation policy that is observable to the buyer. This assumption is necessary to ensure that the expert has an incentive to adhere to her recommendation policy in equilibrium, thereby avoiding cheap-talk babbling. We interpret this assumption as a reduced-form representation of a reputational mechanism that would arise in a long-term relationship between the buyer and the expert, where the buyer learns about the informational content of the expert's recommendations over time. This allows the expert to build a reputation for honesty that she would endanger by biasing her recommendations. This view is supported by Best and Quigley (2022), which shows that when the buyer observes a summary statistic of the expert's past truthfulness, any communication equilibrium the sender prefers to one-shot cheap talk can be supported.

There are several extensions to this model that we have not discussed. For example, it would be natural to allow for multiple buyer types or introduce multiple experts. In both cases, the key results would still hold. The introduction of multiple experts would create competitive pressure, potentially mitigating collusion between experts and the seller that results in biased recommendations. If consumers know which expert offers the most accurate recommendation, they will gravitate toward that expert, creating a Bertrand-type competition among experts. Such competition would likely lead to full information revelation in equilibrium. Moreover, competition among experts makes it more expensive for the seller to bias recommendations through endorsement payments. To prevent buyers from seeking more accurate information from a less-biased expert, the seller would need to offer equal payments to all experts, thereby increasing the cost of incentivizing biased recommendations.

Future research could also extend our model to consider competition among sellers or to allow for a richer structure of buyer types in terms of information acquisition costs. Additionally, exploring the dynamics of reputation and repeated interactions between buyers and experts could provide further insights into how trust and credibility affect market outcomes in such settings.

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A Proofs

Proof of Proposition 1. We begin by describing the buyer's and the expert's strategies that, for any given price p, form part of a seller-optimal equilibrium, and then derive the seller's payoff function as a function of price, given the expert's and buyer's strategies.

Expert In a seller-optimal equilibrium, the expert uses the sales-maximizing recommendation policy that targets the buyer type with the highest cost who would acquire information given the price. Specifically, if $p \in [\underline{p}_k, \bar{p}_k]$, then the expert offers the k-maximal recommendation policy, and otherwise, she offers the 0-maximal recommendation policy.

Buyer The zero-cost type acquires information for all $p \in (\theta_L, \theta_H]$. When $p \in [\underline{p}_k, \bar{p}_k]$, the expert offers the k-maximal recommendation policy, and the zero-cost type strictly prefers to acquire information. When $p \in (\bar{p}_k, \theta_H]$, the expert offers the 0-maximal recommendation policy, and the zero-cost type strictly prefers to acquire information. When $p \in (\theta_L, \underline{p}_k)$, the expert offer the 0-maximal recommendation policy, which is uninformative, and the zero-cost type is indifferent between acquiring information and not. If the zero-cost type does not acquire information, then the expert can always offer some more informative recommendation policy such that the zero-cost type acquires information, and doing so is a strictly more profitable for the expert. Similarly, the high-cost type acquires information for all $p \in (\underline{p}_k, \bar{p}_k]$. When $p \in (\underline{p}_k, \bar{p}_k)$, the expert offers the k-maximal recommendation policy and the high-cost type strictly prefers to acquire information. When $p = \bar{p}_k$, the high-cost type is indifferent between acquiring information and not. If the high-cost type does not acquire information, then he does not purchase the product. It is strictly more profitable for the seller to slightly reduce the price.

Seller If $p \leq \theta_L$, both types buy immediately without information acquisition, and the optimal price on this interval is $p = \theta_L$. If $p > \theta_H$, neither type buys or acquires information and the seller gets a payoff of 0. Therefore, we focus on $p \in [\theta_L, \theta_H]$. If $p \in [\theta_L, \underline{p}_k]$, regardless of information acquisition, both types always buy in both states. If $p \in (\underline{p}_k, \bar{p}_k]$, both types acquire information. The expert offers the k-maximal recommendation policy which recommends purchase with probability $\beta^k(p) =: \pi + (1-\pi)q_L^k(p)$. If $p \in (\bar{p}_k, \theta_H]$, only the zero-cost type acquires information. The expert offers the 0-maximal recommendation policy which recommends purchase with probability $\beta^0(p) =: \pi + (1-\pi)q_L^0(p)$. The seller's payoff is thus

$$\Pi(p) = \begin{cases} p-c & \text{if } p \in [\theta_L, \underline{p}_k], \\ \beta^k(p)(p-c) & \text{if } p \in (\underline{p}_k, \bar{p}_k], \\ \lambda\beta^0(p)(p-c) & \text{if } p \in (\bar{p}_k, \theta_H]. \end{cases}$$

It is easy to show that the function $\Pi(p)$ is piecewise linearly increasing with downward jumps. Its local maxima are thus located at $\underline{p}_k$, $\bar{p}_k$ and θ_H. A global maxima of $\Pi(p)$ must be one of these three.

We now show that given the strategies for the expert and the buyer above, the seller's optimal pricing strategy is as described in the proposition.

(i) We show that there exist thresholds k^* and $\bar{k}$ such that if $k > \max\{k^*, \underline{k}\}$, then $V(p)$ has a global maximum at $p = \underline{p}_k$, that is, $\Pi(\underline{p}_k) \geq \Pi(\bar{p}_k)$ and $\Pi(\underline{p}_k) \geq \Pi(\theta_H)$. For the first inequality, note that $\beta^k(\bar{p}_k) = \pi$, and thus

$$\begin{aligned} \Pi(\underline{p}_k) \geq \Pi(\bar{p}_k) &\Leftrightarrow \underline{p}_k - c \geq \beta^k(\bar{p}_k)(\bar{p}_k - c) \\ &\Leftrightarrow \theta_L + \frac{k}{1-\pi} - c \geq \pi\left(\theta_H - \frac{k}{\pi} - c\right) \\ &\Leftrightarrow k \geq \left(\frac{1-\pi}{2-\pi}\right)(\pi(\theta_H - c) - (\theta_L - c)) =: k^*. \end{aligned}$$

For the second inequality,

$$\begin{aligned} \Pi(\underline{p}_k) \geq \Pi(\theta_H) &\Leftrightarrow \underline{p}_k - c \geq \lambda\beta^0(\theta_H)(\theta_H - c) \\ &\Leftrightarrow \theta_L + \frac{k}{1-\pi} - c \geq \lambda\pi\left(\theta_H - c\right) \\ &\Leftrightarrow k \geq (1-\pi)\left(\lambda\pi\left(\theta_H - c\right) - (\theta_L - c)\right) =: \bar{k}. \end{aligned}$$

Therefore, when $k \geq \max\{k^*, \bar{k}\}$, $\underline{p}_k$ is a global maximum of $\Pi(p)$.

(ii) We show that there exist a third threshold $\underline{k}$, such that if $k \leq \min\{k^*, \underline{k}\}$, then $\Pi(p)$ has a global maximum at $p = \bar{p}_k$, that is, $\Pi(\bar{p}_k) \geq \Pi(\underline{p}_k)$ and $\Pi(\bar{p}_k) \geq \Pi(\theta_H)$. For the first inequality, note we have already showed that $k \leq k^*$ implies $\Pi(\bar{p}_k) \geq \Pi(\underline{p}_k)$. For the second inequality,

$$\begin{aligned}\Pi(\bar{p}_k) \geq \Pi(\theta_H) &\Leftrightarrow \pi(\bar{p}_k - c) \geq \lambda\beta^0(\theta_H)(\theta_H - c)\\ &\Leftrightarrow \pi\left(\theta_H - \frac{k}{\pi} - c\right) \geq \lambda\pi\left(\theta_H - c\right)\\ &\Leftrightarrow k \leq \pi(1-\lambda)(\theta_H - c) =: \underline{k}.\end{aligned}$$

Therefore, when $k \leq \min\{k^*, \underline{k}\}$, $\bar{p}_k$ is a global maximum of $\Pi(p)$.

(iii) Follows immediately by inverting the inequalities in Parts (i) and (ii).

□

Proof of Proposition 3. By definition, for any price $p < \underline{p}_k$ or $p > \bar{p}_k$, the high-cost type is unwilling to acquire information even under full revelation by the expert, and therefore paying for endorsement can never be optimal for the seller at these prices. For $p \in [\underline{p}_k, \bar{p}_k]$, the expert receives a payoff of v when using the k-maximal recommendation policy and λv if the 0-maximal recommendation recommendation policy. Without any endorsement contract, the expert offers the k-maximal recommendation policy. Therefore, it is never optimal for the seller to pay the expert anything to induce the k-maximal recommendation policy for $p \in [\underline{p}_k, \bar{p}_k]$. On the other hand, it is optimal for the seller to pay the expert the difference $(1-\lambda)v$, if the seller's gain from inducing the expert to offer the 0-maximal recommendation policy instead of the k-maximal one exceeds that value.

Denote the seller's payoff when the seller-optimal recommendation is offered by $\bar{\Pi}(p)$. We have

$$\bar{\Pi}(p) = \begin{cases} p - c & \text{if } p \in [\theta_L, \tilde{\theta}],\\ \max\{\beta^k(p)(p-c), \lambda\beta^0(p)(p-c)\} & \text{if } p \in (\tilde{\theta}, \bar{p}_k],\\ \lambda\beta^0(p)(p-c) & \text{if } p \in (\bar{p}_k, \theta_H].\end{cases}$$

We now argue that paying for endorsement is never optimal for $p > \tilde{\theta}$.

(i) Consider $p \in (\bar{p}_k, \theta_H]$. Since $p > \bar{p}_k$, the expert offers the 0-maximal recommendation policy even without endorsement payment. Additionally, $\bar{\Pi}(p)$ is strictly increasing in p and is maximized at $p = \theta_H$, and no endorsement is offered.

(ii) Consider $p \in (\tilde{\theta}, \bar{p}_k]$. If $\beta^k(p)(p-c) \geq \lambda\beta^0(p)(p-c)$, then no endorsement payment is needed. If $\beta^k(p)(p-c) < \lambda\beta^0(p)(p-c)$, then $\bar{\Pi}(p) = \lambda\beta^0(p)(p-c)$, which can be achieved by inducing the 0-maximal recommendation policy. However, $\lambda\beta^0(p)(p-c)$

is strictly increasing in p and maximized at $p = \theta_H$ for $p > \tilde{\theta}$, where no endorsement payment is needed.

We now consider $p \in (\underline{p}_k, \tilde{\theta}]$. $\bar{\Pi}(p)$ is strictly increasing in p and is maximized at $p = \tilde{\theta}$. Therefore, when the seller pays for endorsement, the optimal price is $p = \tilde{\theta}$ and the corresponding endorsement payment is $E^* = (1-\lambda)v$. The seller's associated payoff is $\Pi^E = \tilde{\theta} - c - (1-\lambda)v$.

Next, we check conditions in which this endorsement contract is optimal.

(i) Π^E exceeds $\Pi(\underline{p}_k)$, which corresponds to k-deterrence pricing, if

$$\begin{aligned}\Pi^E \geq \Pi(\underline{p}_k) &\Leftrightarrow \tilde{\theta} - (1-\lambda)v - c \geq \theta_L + \frac{k}{1-\pi} - c \\ &\Leftrightarrow k \leq \sigma\Delta - (1-\pi)(1-\lambda)v =: v_1.\end{aligned}$$

Since $k < \Delta\sigma$ by Assumption 1, it follows that for $v \leq v_1$, the inequality holds.

(ii) Π^E exceeds $\Pi(\bar{p}_k)$, which corresponds to k-inducement pricing, if

$$\begin{aligned}\Pi^E \geq \Pi(\bar{p}_k) &\Leftrightarrow \tilde{\theta} - (1-\lambda)v - c \geq \pi\left(\theta_H - \frac{k}{\pi} - c\right) \\ &\Leftrightarrow k \geq (1-\pi)(c-\theta_L) + (1-\lambda)v =: k^E.\end{aligned}$$

(iii) Π^E exceeds $\Pi(\theta_H)$, which corresponds to 0-inducement pricing, if

$$\begin{aligned}\Pi^E \geq \Pi(\theta_H) &\Leftrightarrow \tilde{\theta} - (1-\lambda)v - c \geq \lambda\pi(\theta_H - c) \\ &\Leftrightarrow v \leq \frac{1}{1-\lambda}(\tilde{\theta} - c - \lambda\pi(\theta_H - c)) =: v_2.\end{aligned}$$

Therefore, for $v \leq \min\{v_1, v_2\} =: v^E$ and $k \geq k^E$, the endorsement contract is indeed optimal.

Finally, we verify that $k^E < \underline{k}$. Note that

$$\begin{aligned}k^E &= (1-\pi)(c-\theta_L) + (1-\lambda)v \\ &< (1-\pi)(c-\theta_L) + v_2 \\ &= (1-\pi)(c-\theta_L) + (\tilde{\theta} - c - \lambda\pi(\theta_H - c)) \\ &= \pi(1-\lambda)(\theta_H - c) = \underline{k}.\end{aligned}$$

□

Proof of Lemma 1. Similar to endorsement contracts, there is no need to pay for referrals for $p < \underline{p}_k$ and $p > \bar{p}_k$.

First we define $V^0(p) =: \lambda\left(v + \beta^0(p)R\right)$ and $V^k(p) =: v + \beta^k(p)R$, which represents expert's payoff if always offering the 0-maximal and k-maximal recommendation policies, respectively.

We now find conditions on p under which paying for referrals is never optimal.

(*i*) Consider $p \in [\underline{p}_k, \tilde{\theta}]$. If the expert offers the 0-maximal recommendation policy, her corresponding payoff is

$$V^0(p) = \lambda(v + R).$$

If the expert offers the k-maximal recommendation policy, her corresponding payoff is

$$V^k(p) = v + \left(1 - \frac{k}{p - \theta_L}\right) R.$$

The expert prefers the 0-maximal recommendation policy over the k-maximal recommendation policy if $V^0(p) \geq V^k(p)$. Simple algebra shows that this is equivalent to

$$p \leq \theta_L + \frac{Rk}{(1-\lambda)(R+v)}.$$

It follows that for an equilibrium with referrals to exist for any $v > 0$, we need

$$p \leq \theta_L + \frac{Rk}{(1-\lambda)(R+v)} < \theta_L + \frac{k}{(1-\lambda)} =: \hat{p}_k.$$

(*ii*) Consider $p \in (\tilde{\theta}, \bar{p}_k]$. If the expert offers the 0-maximal recommendation policy, her corresponding payoff is

$$V^0(p) = \lambda\left(v + \frac{\pi\Delta}{p - \theta_L} R\right).$$

If the expert offers the k-maximal recommendation policy, her corresponding payoff is

$$V^k(p) = v + \frac{\pi\Delta - k}{p - \theta_L} R.$$

It is easy to see that when k is sufficiently small relative to λ, then the expert always prefers the k-maximal recommendation policy over the 0-maximal recommendation policy, so paying for referral is never optimal. One sufficient condition for $V^0(p) < V^k(p)$ is

$$k < \pi\Delta(1-\lambda).$$

Therefore, we consider $k \geq \pi\Delta(1-\lambda)$ hereafter. Then we can further show that the seller's payoff from offering any referral payment is strictly increasing in p on $(\tilde{\theta}, \bar{p}_k]$. This means that for $p > \tilde{\theta}$ the optimal price is $p = \theta_H$, and at this price, it can never be optimal for the seller to pay for referrals, hence $R = 0$.

By contradiction, assume $p > \tilde{\theta}$ and $R > 0$. The expert is indifferent between the 0-maximal and k-maximal recommendation policies if

$$V^0(p) = V^k(p) \Leftrightarrow \lambda \left(v + \frac{\pi\Delta}{p - \theta_L} R \right) = v + \frac{\pi\Delta - k}{p - \theta_L} R.$$

Solving for R yields the referral payment that renders the expert indifferent

$$R^*(p) = \frac{(1-\lambda)(p-\theta_L)v}{k - (1-\lambda)\pi\Delta},$$

which is non-negative, for $k \geq \pi\Delta(1-\lambda)$. The seller's payoff when offering referral payment $R^*(p)$ that induces 0-maximal recommendations is then given by

$$\Pi^{R^*}(p) = \begin{cases} \lambda \frac{\pi\Delta}{p-\theta_L}(p-c) - \frac{\lambda(1-\lambda)\pi\Delta}{k-(1-\lambda)\pi\Delta} v & \text{if } p \in (\tilde{\theta}, \overline{p}_k] \\ \lambda \frac{\pi\Delta}{p-\theta_L}(p-c) & \text{if } p \in (\overline{p}_k, \theta_H], \end{cases}$$

and the derivative with respect to p is

$$\frac{\partial \Pi^{R^*}}{\partial p} = \frac{\pi\Delta\lambda(c-\theta_L)}{(p-\theta_L)^2} > 0,$$

since $p > \tilde{\theta} > c > \theta_L$ by assumption. So that $\Pi^{R^*}(p)$ is strictly increasing in p and therefore is maximized at $p = \theta_H$. However, the expert offers the k-maximal recommendation policy even without any payment. This means that, for $p > \tilde{\theta}$, the seller offers no referral payments to the expert.

□

Proof of Corollary 2. Note that by Lemma 1, a seller-optimal equilibrium with referrals exists only when $p < \min\{\tilde{\theta}, \hat{p}_k\}$. But, when $\pi \geq \lambda$, then

$$\underline{p}_k = \theta_L + \frac{k}{1-\pi} \geq \theta_L + \frac{k}{1-\lambda} = \hat{p}_k.$$

This means that the maximum price at which a seller-optimal equilibrium with referrals exists is lower than the k-deterrence price, which implies that the seller would rather use

k-deterrence pricing than offering referral payments. $\square$

Proof of Proposition 4. First, we show that the seller's problem

$$\max_{p \in [\theta_L, \tilde{\theta}]} \Pi^{R^*}(p)$$

has a unique solution. At any price $p \in [\theta_L, \tilde{\theta}]$, the expert is indifferent between the 0-maximal and k-maximal recommendations if

$$V^0(p) = V^k(p) \Leftrightarrow \lambda(v + R) = v + \left(1 - \frac{k}{p - \theta_L}\right) R.$$

Solving for R yields the optimal referral payment for the seller for a given price p, we get

$$R^*(p) = \frac{v(1-\lambda)}{(\frac{k}{p-\theta_L} - (1-\lambda))} = \frac{p - \theta_L}{\theta_L + \frac{k}{1-\lambda} - p} v = \frac{p - \theta_L}{\hat{p}_k - p} v.$$

Second, we determine the profit-maximizing price p^* on the interval $[\theta_L, \min\{\tilde{\theta}, \hat{p}_k\}]$. The seller's profit function is

$$\Pi^{R^*}(p) = \begin{cases} p - c & \text{if } p \in [\theta_L, \underline{p}_k), \\ (p - c) - \lambda R^*(p) & \text{if } p \in [\underline{p}_k, \min\{\tilde{\theta}, \hat{p}_k\}]. \end{cases}$$

The derivative of the seller's profit function on this interval is

$$\frac{\partial \Pi^{R^*}}{\partial p} = \begin{cases} 1 & \text{if } p \in [\theta_L, \underline{p}_k), \\ 1 - \frac{kv\lambda}{(1-\lambda)(\hat{p}_k - p)^2} & \text{if } p \in [\underline{p}_k, \min\{\tilde{\theta}, \hat{p}_k\}]. \end{cases}.$$

Note that the derivative with respect to p is positive for $p = \underline{p}_k$ for v small enough, but becomes negative as $p \to \hat{p}_k$. Moreover, it is strictly decreasing in p in between, since

$$\frac{\partial^2 \Pi^{R^*}}{\partial p^2} = -\frac{2\lambda k v}{(1-\lambda)(\hat{p}_k - p)^3} < 0.$$

This implies that the seller's profit function Π^{R^*} has a unique maximum p^R in $[\theta_L, \min\{\tilde{\theta}, \hat{p}_k\}]$. The first-order condition yields the solution

$$p^R = \max\left\{\hat{p}_k - \sqrt{\frac{vk\lambda}{1-\lambda}}, \underline{p}_k\right\}.$$

Note that $p^R < \hat{p}_k$ for $v > 0$. Moreover, since $\lambda > \pi$, we have $p^R > \underline{p}_k$ if

$$k > \frac{(1-\pi)^2(1-\lambda)\lambda v}{(\lambda-\pi)^2} =: k_0.$$

and we have $p^R < \tilde{\theta}$ if

$$k < \frac{1}{2}(1-\lambda)\left(v\lambda + 2\Delta\pi + \sqrt{v\lambda\,(v\lambda + 4\Delta\pi)}\right) =: \hat{k}.$$

Note that when $v = 0$, $k_0 = 0$ and $\hat{k} = (1-\lambda)\pi\Delta < \Delta\sigma$. We choose v_0^R such that for all $v < v_0^R$, we have $k_0 < \hat{k}$.

When $k \geq \hat{k}$, $p^R \geq \tilde{\theta}$ and this implies that $\Pi^{R^*}(p)$ has a boundary solution at $p = \tilde{\theta}$. To find conditions under which p^R is a global maximum, we show that $\tilde{\theta}$ is a global maximum when $v = 0$. Then, using the fact that $\Pi^{R^*}(\tilde{\theta})$ is continuous in v with $\lim_{v\to 0}\Pi^{R^*}(\tilde{\theta}) = \tilde{\theta} - c$, we conclude that there must exist a threshold v_1^R such that for all $v < v_1^R$, $\tilde{\theta}$ is a global maximum. Note that when $v = 0$, we have $\Pi^{R^*}(\tilde{\theta}) > \Pi(\underline{p}_k)$ by construction. It remains to show that $\Pi^{R^*}(\tilde{\theta}) \geq \Pi(\underline{p}_k)$ and $\Pi^{R^*}(\tilde{\theta}) \geq \Pi(\theta_H)$.

(i) For the first inequality

$$\begin{aligned}
\Pi^{R^*}(\tilde{\theta}) \geq \Pi(\bar{p}_k) &\Leftrightarrow \tilde{\theta} - c \geq \beta^k(\bar{p}_k)(\bar{p}_k - c) \\
&\Leftrightarrow \pi\theta_H + (1-\pi)\theta_L - c \geq \pi\left(\theta_H - \frac{k}{\pi} - c\right) \\
&\Leftrightarrow k \geq c - (1-\pi)\theta_L =: k_1.
\end{aligned}$$

(ii) For the second inequality

$$\begin{aligned}
\Pi^{R^*}(\tilde{\theta}) \geq \Pi(\theta_H) &\Leftrightarrow \tilde{\theta} - c \geq \lambda\beta^0(\theta_H)(\theta_H - c) \\
&\Leftrightarrow \pi\theta_H + (1-\pi)\theta_L - c \geq \lambda\pi\,(\theta_H - c) \\
&\Leftrightarrow \lambda \leq 1 - \frac{1-\pi}{\pi}\frac{c-\theta_L}{\theta_H - c} = \bar{\lambda},
\end{aligned}$$

which holds by Assumption 4.

When $k_0 < k < \hat{k}$, $p^R < \tilde{\theta}$ and this implies that $\Pi^{R^*}(p)$ has an interior solution at $p = p^R$. Then $\Pi^{R^*}(p)$ has a local maximum at $p^R \in (\underline{p}_k, \tilde{\theta})$. To find conditions under which p^R is a global maximum, we show that $\hat{p}_k$ is a global maximum when $v = 0$. Then, using the fact that p^R is continuous in v with $\lim_{v\to 0} p^R = \hat{p}_K$, and that $\Pi^{R^*}(p)$ is continuous in p, we conclude that there must exist a threshold v_2^R such that for all $v < v_2^R$, p^R is a global maximum. Note that since $\lambda > \pi$, we have $\Pi^{R^*}(\hat{p}_k) \geq \Pi(\underline{p}_k)$ by construction. It remains to show that $\Pi^{R^*}(\hat{p}_k) \geq \Pi(\theta_H)$ and $\Pi^{R^*}(\hat{p}_k) \geq \Pi(\theta_H)$.

(i) For the first inequality

$$\begin{aligned}\Pi^{R^*}(\hat{p}_k) \geq \Pi(\bar{p}_k) &\Leftrightarrow \hat{p}_k - c > \beta^k(\bar{p}_k)(\bar{p}_k - c)\\ &\Leftrightarrow \theta_L + \frac{k}{1-\lambda} - c > \pi\left(\theta_H - \frac{k}{\pi} - c\right)\\ &\Leftrightarrow k \geq \left(\frac{1-\lambda}{2-\lambda}\right)(\pi(\theta_H - c) - (\theta_L - c)) =: k_2.\end{aligned}$$

(ii) For the second inequality

$$\begin{aligned}\Pi^{R^*}(\hat{p}_k) \geq \Pi(\theta_H) &\Leftrightarrow \hat{p}_k - c \geq \lambda\beta^0(\theta_H)(\theta_H - c)\\ &\Leftrightarrow \theta_L + \frac{k}{1-\lambda} - c \geq \lambda\pi\,(\theta_H - c)\\ &\Leftrightarrow k \geq (1-\lambda)\left[\lambda\pi\,(\theta_H - c) - (\theta_L - c)\right] =: k_3.\end{aligned}$$

We define $v^R =: \max\{v_0^R, v_1^R, v_2^R\}$ and $k^R =: \max\{k_0, k_1, k_2, k_3\}$. Then for $k \geq k^R$, $\hat{p}_k$ is indeed an optimal price for the seller, and thus, by extension, p^R is optimal for v sufficiently small. Note in particular that for $\lambda > \pi$ we have $k_1 < k^*$ and $k_2 < \bar{k}$. Since $k_0 \to 0$ for $v \to 0$, thus this implies that $k^R < \max\{k^*, \bar{k}\}$ for v small enough. $\square$