When and How Does Household Heterogeneity Matter for Aggregate Fluctuations?

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Abstract

I establish the existence of a distributionally neutral benchmark for aggregate shock transmission in incomplete-market heterogeneous-agent (HA) economies, where all agents are equally exposed to the shock. In this benchmark, aggregates satisfy the equilibrium conditions of a fictitious representative-agent (RA) economy. Leveraging this result, I develop a tractable framework to identify and quantify redistribution mechanisms that drive the divergence between HA and RA outcomes. The framework (i) uncovers the mapping from deep structural parameters to redistribution and (quantitatively) to general-equilibrium dynamics; (ii) clarifies the roles of fiscal policy and investment; (iii) provides rescalable sufficient statistics portable across shock types; and (iv) identifies new redistribution channels in two-asset HANK and overlapping generation models.

Keywords: Heterogeneous households; Monetary Policy; Fiscal Policy; Incomplete markets; Inequality; Business cycles. *JEL classification*: D31, E21, E43, E52, E62

1 Introduction

Bewley-Aiyagari-Huggett incomplete-market heterogeneous-agent models (henceforth, HA models) have become prevalent in macroeconomics. By introducing nominal rigidities, heterogeneous-agent New Keynesian (HANK) models can help understand how household heterogeneity affects economic fluctuations. Despite substantial progress, two foundational gaps remain. First, relative to the HA model, what does the representative-agent (RA) model miss in modeling aggregate dynamics? What is the nature of the differences in dynamics between HA and RA models? Answers to these questions decide the boundary of the RA paradigm and the additional benefits of incorporating complex household heterogeneity. Second, while tractable HA models are widely used to study the interaction between inequality, redistribution, and aggregate dynamics,[1] the literature lacks a well-defined distributionally neutral benchmark for a general HA economy. Consequently, the mapping from economic shocks to redistribution remains opaque. How is redistribution precisely correlated with households' marginal propensity to consume (MPCs)? Which source of redistribution is more important, and which plays a minor role in amplifying/stabilizing aggregate fluctuations? How do the answers depend on the economy's structure? These questions are challenging because quantitative HA models feature intractable dynamics that are costly to solve and even harder to interpret.

This paper makes two contributions. First, I broaden the RA paradigm by embedding it as a special case of a full HA economy. I prove the existence of a distributionally-neutral benchmark, in which agents are perfectly insured against aggregate shocks, but remain exposed to idiosyncratic income shocks. Once all agents are equally exposed to the aggregate shock, a full HA economy aggregates even though micro heterogeneity and precautionary saving motives persist in the background. Thus, any difference in dynamics between HA and RA can be attributed to redistribution induced by the aggregate shock. Second, leveraging the benchmark, I develop a tractable framework to identify and quantify redistribution mechanisms that drive the deviation from RA.

For measuring redistribution, a complete-market allocation is too stark a reference because it eliminates all movements of individual consumption shares, not just those induced by the aggregate shock. In the benchmark here, individual i's consumption satisfies $c_{it} = s_{it}C_t$ where shares s_{it} evolve only in response to idiosyncratic shocks but are invariant to aggregate consumption C_t. Absent the benchmark, aggregate shocks would additionally generate "cyclical heterogeneity" in consumption shares through redistribution, thereby altering precautionary saving motives. I establish the bench-

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[1]See Werning (2015), Auclert (2019), Acharya and Dogra (2020), Ravn and Sterk (2021), Bilbiie (2020), Bilbiie, Känzig and Surico (2022), Bilbiie (2024), and Debortoli and Galí (2024).

mark in a perfect-foresight environment by showing that, for any aggregate shock, there exist lump-sum transfers that exactly offset the endogenous redistribution the shock would otherwise induce, and the benchmark allocation consists of the equilibrium. These transfers are purely redistributive and sum to zero cross-sectionally. A useful analogue is that, in a stochastic environment, this is the allocation that a restricted set of Arrow–Debreu claims spanning only aggregate states would implement; with perfect foresight, these claims collapse to the transfers. I prove such a benchmark exists for: (i) a two-agent model with permanent-income households and hand-to-mouth households, (ii) the standard incomplete-market model, and (iii) standard models that incorporate illiquid assets or ex-ante heterogeneity in discount factors to reproduce the large MPCs observed in data.

The first contribution advances the aggregation literature. For incomplete-market models, Krusell and Smith (1998) established the "quasi-aggregation" result: the HA model's responses to TFP shocks closely track those in the RA benchmark. More recently, Werning (2015) studies special cases in which an HA economy behaves "as if" it were an RA economy.[2] Berger, Bocola and Dovis (2019) represents the HA economy as a complete-market RA economy with wedges on intertemporal discounting and labor supply conditions, and Acharya and Dogra (2020) derives aggregation under CARA preferences. The equilibrium allocation here coincides with Krueger and Lustig (2010) and Werning (2015) for the cases they analyze. This paper overcomes the limitations of those special cases and establishes exact aggregation for a broad class of quantitatively relevant HA models. A key feature is that the definition of the fictitious RA economy is invariant to realized shocks. This contrasts with Berger, Bocola and Dovis (2019), where the wedges depend on endogenous consumption shares and relative wages. Here, the parameters of the fictitious RA economy are pinned down once the steady state is solved (assuming the HA economy starts from its invariant steady-state distribution). This independence is crucial: a fully specified benchmark is necessary to identify deviations from it. Identification fails if the benchmark itself shifts with realized redistribution.

This paper's second contribution is a tractable framework to identify and quantify the redistribution mechanisms that drive deviations from the RA benchmark. The approach proceeds in two steps. In the first step, I make counterfactual transfers to households and consider the model's response to the aggregate shock with redistribution muted, which I refer to as RA effects. In the second step, I utilize these transfers to back out the underlying redistribution shock and consider its impacts on the HA economy, referred to as redistribution effects. To first order, the RA and redistribution

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[2]Werning (2015) shows that the HA economy can be aggregated as an RA economy under (i) zero liquidity and acyclical income risks, or (ii) proportional liquidity and log utility on consumption. The proportional-liquidity case is also studied in Krueger and Lustig (2010), with a greater emphasis on aggregate uncertainty and the risk premium.

effects sum to the total response of the HA model. Essentially, the transfers are introduced to render implicit redistribution explicit, thereby making tractable analysis possible. I further analytically decompose the redistribution shock to trace its origins and assess the impacts of each origin, following Auclert (2019). The key innovation is that the redistribution shock here is a structural object, rather than the endogenous perturbations to households' optimization problem as in Auclert (2019). In general equilibrium (GE), those perturbations are themselves the outcome of redistribution.

There are various new findings when I apply the framework to study monetary policy in a workhorse HANK model, as well as when applying it to the literature. First, the decomposition into granular channels directly uncovers the mapping from deep structural parameters to redistribution (analytically) and GE dynamics (quantitatively). This approach differs from functional-VAR studies such as Chang, Chen and Schorfheide (2024), which estimate the VAR parameters that match the observed time-series evidence but remain silent on how heterogeneity shapes the parameters. Counterfactuals that switch off a channel are obtained immediately, with no need to resolve the model. McKay, Nakamura and Steinsson (2016) proposes that incomplete-market models resolve the forward guidance puzzle through precautionary saving motives. Farhi and Werning (2019), Acharya and Dogra (2020), and Bilbiie (2024) argue that the dampening or amplification of the power of forward guidance depends critically on the cyclicality of income risk and liquidity. My decomposition clarifies the discussion by showing that the negative redistribution effects in McKay, Nakamura and Steinsson (2016) stem from model assumptions on profit distribution and taxation. Both assumptions implicitly redistribute from low-income to high-income households, dampening consumption responses. Quantitatively, redistribution due to profit distribution accounts for -80% of total consumption responses, while the redistribution due to taxation contributes -44%. Consider a counterfactual where the tax-induced redistribution is muted by assuming uniform tax exposure, the consumption response in GE simply increases by 44% with all the other channels intact. This clean inference is challenging when redistribution forces are entangled.[3]

Second, the framework is modular and extends to richer environments. In the HANK model, I add two common features: cyclical government bond supply and investment. This framework provides new insights into their roles in the HA economy. Due to the failure of Ricardian equivalence, time-varying bond supply has real effects. I extend Aiyagari and McGrattan (1998) to transition dynamics and show that, under

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[3]Given the revealed mapping from model specifications to structural redistribution, another approach is to implement robustness checks accordingly, for example, by assuming uniform tax exposure and resolving the model. Therefore, one can also interpret the contribution as this framework characterizes the needed model specification to neutralize the interested channel. Note that even with the revealed mapping, the decomposition has advantages. Fiscal policy and profit distribution affect both the steady state (MPCs distribution) and the transition dynamics (correlation between MPCs and redistribution), and it is cumbersome to isolate them by re-specifying and resolving the model.

uniform taxation, bond supply shocks are allocation-equivalent to shocks to the borrowing constraint. The supply of government debt affects how households borrow from and lend to each other, and I attribute it to the liquidity channel of bond supply. An increase in bond supply has the same effect on the economy as a shock that relaxes households' borrowing constraints, while a decrease in bond supply tightens the constraints, as in Guerrieri and Lorenzoni (2017). The literature lacks a clear evaluation of the investment's impact on HA dynamics, where this framework is especially useful. I show that investment induces a redistribution between equity (capital) holders and workers. Aggregate income comprises dividends and labor income, and investment responses affect their respective shares, which further decide households' income elasticities based on their income portfolios. The investment-induced redistribution displays an endogenous phase reversal. During capital accumulation, workers experience stronger income growth, as capital income is retained for investment. When the economy de-invests, equity holders receive dividend payments and their income grows more. In my HANK model, this channel accounts for more than one-fourth of the total redistribution effects of monetary policy.

Third, the framework yields a set of portable sufficient statistics that apply to any policy shock via simple rescaling, enabling streamlined policy evaluation. Consider an unexpected interest-rate cut. The two-step approach implies that to predict its effects, the policymaker needs only: (i) the RA economy's response to the interest rate cut; and (ii) the HA economy's response to the triggered redistribution shock. The RA effects are well established in the literature. The redistribution effects generally require solving a full HA model. Following Auclert (2019) and Patterson (2023), I derive estimable model moments of partial-equilibrium consumption response for each redistribution channel. The advancement here is that the scaling factors of these moments come from RA aggregates, rather than HA, which are unknown ex ante and vary across policies in an intractable manner. For a given policy, the RA effects pin down the magnitudes of all redistribution channels. Treating channels as independent blocks, I rescale and sum their contributions to obtain total redistribution effects. These estimates can then be reused for any other policy through rescaling alone.

Finally, the framework uncovers new, testable mechanisms of redistribution. Since redistribution effects exhaust the HA–RA gap, any omitted channel would show up as a residual and thus be detectable. Asset-illiquidity is important to match empirical wealth distribution and aggregate MPCs (Kaplan and Violante 2014, Kaplan, Moll and Violante 2018, Bayer et al. 2019, and Luetticke 2021). The framework reveals that its implications for transition dynamics have so far been neglected. The illiquid-asset return affects how much assets agents accumulate. When the return fluctuates, households are differentially affected depending on portfolios and rebalance opportunities. For the two-asset HANK model with typical calibrations, this is the most important

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redistribution channel of monetary policy. To my knowledge, there has been no prior discussion of this channel. When applied to the overlapping generation setting, the framework uncovers a subtle yet important redistribution mechanism between agents with positive and negative saving flows. Passive redistribution arises as consumption, income, and asset flows re-match after an aggregate shock. It is worth noting that this channel has negligible effects when agents are infinitely-lived. The life-cycle pattern of consumption-income flows endogenously generates this redistribution, and it is not clear how to manifest it without this framework.[4]

The unified framework allows me to connect the literature. One example is the fiscal policy. The literature finds that fiscal policy response is crucial for determining aggregate shock propagation. As discussed above, bond supply affects aggregate demand as it changes households' borrowing capacities. Guerrieri and Lorenzoni (2017) analyzes the tightened borrowing constraint shock, a negative demand shock forcing constrained households to deleverage. When studying monetary policy, Kaplan, Moll and Violante (2018) lets public debt absorb the majority of the fiscal imbalance in the short run and finds that the economy's responses are much smaller. The borrowing constraint shock implied by the decreased bond supply is precisely the deleveraging shock in Guerrieri and Lorenzoni (2017). The same argument applies to the analysis of fiscal multipliers. Auclert, Rognlie and Straub (2024) and Hagedorn, Manovskii and Mitman (2019) find that the deficit-financed fiscal multiplier is larger than the tax-financed multiplier. This result is due to the relaxed borrowing condition induced by the increasing government debt. Bayer, Born and Luetticke (2023) investigates fiscal policy in a HANK model with portfolio choices, showing that the increasing debt stimulates consumption while avoiding investment crowding out. Wolf (2021), Wolf (2023), and Angeletos, Lian and Wolf (2023) study deficit-financed lump-sum fiscal transfers as a stimulating policy tool, the effects of which are equivalent to relaxing borrowing constraints and thus weakening the precautionary saving motives.

I calibrate the HANK model to the US economy and consider its response to an expansionary monetary policy shock. The framework reveals that redistribution effects amplify the responses of output and consumption while dampening those of

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[4]This channel is closely related to the "unhedged interest rate exposure (URE)" channel in Auclert (2019). Besides the different aggregates used in perturbations discussed earlier, measures of this channel somewhat differ across these two papers. I measure this channel by the net exposure of goods flow $(y - c)$ and assets flow $(a' - a)$. Auclert (2019) instead implicitly includes goods flow exposure within the aggregate income channel and considers only the asset flow exposure in the "URE" channel. I account for goods flow exposure because a common one-percent income growth is not distributionally neutral: for agents with $y > c$, it is more than enough to raise consumption by one percent, while for agents with $y < c$, it is less. Especially, goods flow and asset flow exposures tend to offset each other endogenously when agents hold a large share of long-maturity assets, so net effects in this case tend to be minimal. In addition, I exclude zero-maturity bonds when measuring asset flows, and the related flow exposure is assigned to a dedicated "interest rate exposure" channel, but this is more of an inessential accounting distinction. In comparison, the sum of the URE channel and goods flow exposure in Auclert (2019) is equivalent to the sum of interest rate exposure and saving flow exposure in this paper.

Table 1: The redistribution following an expansionary monetary policy shock

	Interest rate exposure	Income portfolio exposure	Liquidity (bond)	Tax exposure
Low MPC	Creditors	Equity holders	Unconstrained	High labor income
	↓	S.R. ↓↑ L.R.	S.R. ↓↑ L.R.	↓
High MPC	Debtors	Workers	Constrained	Low labor income

Notes: S.R. refers to short run and L.R. refers to long run.

investment and the real interest rate, relative to the RANK benchmark. On impact, consumption rises by 0.65 percent. RANK effects account for 62% of the increase, followed by interest rate exposure (the redistribution between creditors and debtors) 16%, income exposure 14%, and liquidity channel of bond supply 8%. The income exposure channel's 14% contribution can be further broken down: income portfolio exposure (redistribution between equity holders and workers) accounts for 10.3%; tax exposure (redistribution among taxpayers with different tax elasticities) contributes 2.6%; and labor income exposure (redistribution among workers with different labor income elasticities) adds 1.1%. Table 1 summarizes the redistribution.

Related Literature. This paper contributes to the analytical HANK literature that makes simplifying assumptions to study how heterogeneity changes aggregate outcomes. The literature offers several different yet related approaches to this problem.

Auclert (2019) decomposes the aggregate consumption response to a transitory monetary policy change into substitution and income effects, and further breaks down the income effects into an "aggregate income channel" and three redistribution channels: the "earnings heterogeneity channel," the "Fisher channel," and the "interest rate exposure channel." I follow the spirit of Auclert (2019) in tracing the origins of redistribution and evaluating the partial-equilibrium effects, with a key distinction: I use aggregates from the RA model to define the redistribution, while Auclert (2019) employs the HA model. Leveraging aggregates from the RA model offers two key advantages. First, RA models are widely used by researchers and practitioners, and their dynamics are well understood. Second, they provide clearer counterfactuals. Consider, for example, the role of income exposures in consumption responses. Focusing only on the "earnings heterogeneity channel" of Auclert (2019) is insufficient. Changes in the parameter governing income exposures necessitate adjustments in the equilibrium interest rate, which then influences consumption through substitution effects and the "interest rate exposure channel". In equilibrium, income exposures affect the economy through all endogenous variables and channels defined in Auclert (2019). This paper's decomposition ensures that income exposures affect the economy exclusively through income-related redistribution channels, while the interest rate exposure channel operates independently of income-driven redistribution.

Closely related, Kaplan, Moll and Violante (2018) decomposes the aggregate consumption response to a monetary policy shock into direct and indirect effects. Direct effects affect consumption through interest rates, and indirect effects affect consumption through labor income. They emphasized two findings: (i) the indirect effects can be substantial, in contrast to RANK economies; (ii) the overall consumption response can be larger or smaller than in RANK, depending on various factors that are neutral in RANK. However, the relative size of direct and indirect effects is not one-to-one related to the overall consumption response of HANK compared to RANK.[5] Building on Kaplan, Moll and Violante (2018)'s second finding, this paper aims to explain how the factors they discuss impact the overall consumption response.

Werning (2015) discusses situations where an incomplete-market economy can be aggregated and behave "as if" it were an RA economy. Krueger and Lustig (2010) studies related allocations with a focus on aggregate risk premium. In more complex scenarios where their results do not hold, I introduce counterfactual transfers to maintain the aggregation. Berger, Bocola and Dovis (2019) uses an RA economy with wedges on the discount factor and the labor supply condition to study the implications of imperfect risk sharing in HA. Since these wedges are endogenous objects, their applicability in welfare or policy analyses is limited. This paper can explain these wedges by analyzing how micro-level redistribution affects consumption shares. In addition, the RA economy in Berger, Bocola and Dovis (2019) assumes complete financial markets. RA effects here preserve idiosyncratic risks and isolate the effects of the shock-induced redistribution. Hagedorn et al. (2019) also uses transfers to construct the "as if" RA economy of Werning (2015) and assess the imbalance between aggregate demand and supply off the equilibrium path in explaining the forward guidance puzzle. This paper has a different objective: I exploit the property of the first-order solution to decompose general equilibrium responses. For this aim, I prove the existence of such exogenous transfers and show how to construct them. Bilbiie (2020) analyzes the TANK model's amplification/dampening mechanism. This paper shares the emphasis on unequal exposures of agents. Bilbiie, Känzig and Surico (2022) explores the role of investment in amplification in a tractable TANK model. I study investment in a full HA model. Debortoli and Galí (2024) approximates the aggregate dynamics of HANK models with carefully specified and calibrated TANK models. Since the approximation quality depends on the context, the analytical characterization of structural redistribution shock here provides additional insights into their approach.

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[5]See the discussion in sections IV.B and IV.D of Kaplan, Moll and Violante (2018). In the Appendix, I apply their decomposition to a two-agent model and show that the relative size of direct-indirect effects is a function of the measure of hand-to-mouth households and the amplification/dampening parameter. Conditional on the measure of hand-to-mouth households, we can infer the amplification/dampening parameter. In a model with richer heterogeneity, the relative size of direct-indirect effects will be a function of various structural parameters, which also affect the overall responses, though not in a tractable manner.

This paper also adds to the quantitative HANK literature, which integrates nominal rigidities into incomplete-market models to study various macroeconomic questions.[6] My quantitative model re-evaluates monetary policy transmission through the lens of structural redistribution. This paper also reveals key redistribution channels driving model dynamics in the existing literature.

The rest of the paper is organized as follows. Section 2 illustrates the aggregation and decomposition framework in a general HA economy. Section 3 proves the existence of a no-distribution benchmark for a canonical HANK model and characterizes the redistribution channels. Section 4 extends the model with cyclical bond supply and investment. Section 5 derives estimable moments for redistribution effects in partial equilibrium. Section 6 quantitatively decomposes the model's responses to a monetary policy shock. Section 7 applies the framework to the literature. Section 8 concludes. The Appendix discusses several alternative models: a tractable TANK model, a canonical model without investment, and a model with illiquid assets.

2 Counterfactual Transfers and Exact Aggregation

Consider a heterogeneous-agent economy. The specifics of heterogeneity will be detailed in future sections. For the current discourse, A reduced form is employed to outline the idea. Time is discrete and extends indefinitely $t = 0, 1, \cdots$. There is no aggregate risk, and the perfect-foresight economy starts from its deterministic steady state. At time $t = 0$, there is a one-time unexpected aggregate shock (MIT shock) $\epsilon = (\epsilon_0, \epsilon_1, \cdots)'$ realized. In the infinite horizon, the economy is back to its initial state. I study the transition path following the aggregate shock. I first construct a counterfactual without redistribution, then propose a decomposition of the shock and the impulse response function of outcome variables.
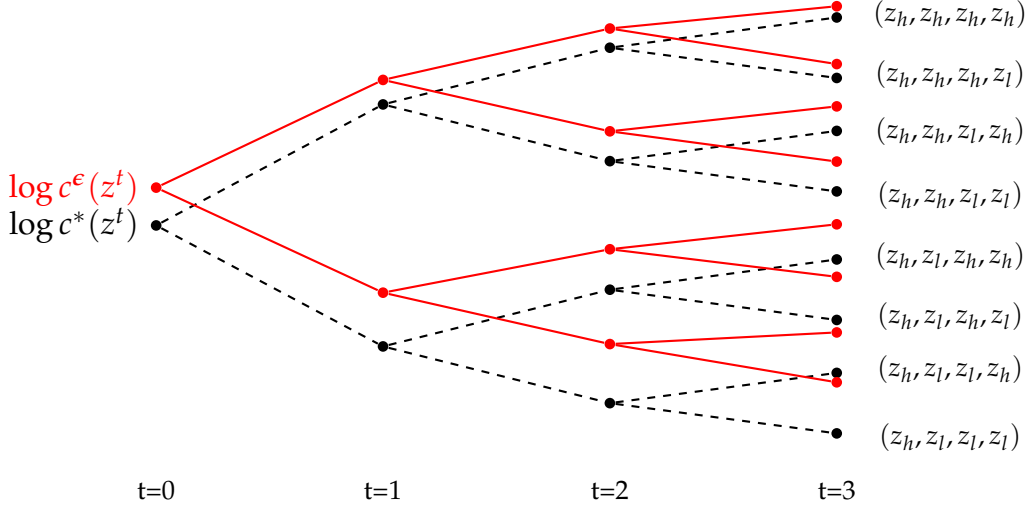
An aggregate variable Y's value in the steady state is denoted as Y^*, which is constant across time. Following the shock ϵ, Y's value at time t along the transition path is denoted as Y_t^ϵ, and the entire time path is denoted as $\mathbf{Y}^\epsilon = (Y_0^\epsilon, Y_1^\epsilon, \cdots)'$. Then we can define Y's impulse responses to the aggregate shock as

$$\tilde{\mathbf{Y}}^\epsilon \equiv \mathbf{Y}^\epsilon - Y^* \cdot \mathbf{1}, \tag{1}$$

where $\mathbf{1}$ is the identity vector with all elements equal to one. For an individual variable

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[6]These include fiscal transfers (Oh and Reis 2012), automatic fiscal stabilizers (McKay and Reis 2016), monetary policy transmission (Gornemann, Kuester and Nakajima 2016; McKay, Nakamura and Steinsson 2016; Kaplan, Moll and Violante 2018; Luetticke 2021; Auclert, Rognlie and Straub 2020), endogenous income risk (Ravn and Sterk 2017), de-leveraging (Guerrieri and Lorenzoni 2017), fiscal multipliers (Auclert, Rognlie and Straub 2024; Hagedorn, Manovskii and Mitman 2019), inequality and income risk shocks (Auclert and Rognlie 2018; Bayer et al. 2019), and business cycles (Bayer, Born and Luetticke 2024; Berger, Bocola and Dovis 2019; Bilbiie, Primiceri and Tambalotti 2023).

Figure 1: Individual Consumption Unequally Affected by the Aggregate Shock

$$\log c^{\epsilon}(z^t)$$
$$\log c^*(z^t)$$

(z_h, z_h, z_h, z_h)

(z_h, z_h, z_h, z_l)

(z_h, z_h, z_l, z_h)

(z_h, z_h, z_l, z_l)

(z_h, z_l, z_h, z_h)

(z_h, z_l, z_h, z_l)

(z_h, z_l, z_l, z_h)

(z_h, z_l, z_l, z_l)

t=0   t=1   t=2   t=3

y_i with individual index i, let y_{it}^* denote its value at time t in the steady state, and $\mathbf{y}_i^* = (y_{i0}^*, y_{i1}^*, \cdots)'$ denote the entire time path. Along the transition path, the path of variable y_i is denoted as $\mathbf{y}_i^{\epsilon} = (y_{i0}^{\epsilon}, y_{i1}^{\epsilon}, \cdots)'$. The impulse responses of the individual variable y_i are defined as

$$\tilde{\mathbf{y}}_i^{\epsilon} \equiv \mathbf{y}_i^{\epsilon} - \mathbf{y}_i^*. \tag{2}$$

A typical feature of the heterogeneous-agent model is that agents are unequally exposed to the shock. I use the standard incomplete-market model to illustrate this feature. Use z_t to denote the individual productivity level at time t, and $z^t = (z_0, z_1, \cdots, z_t)$ be a history of idiosyncratic states up to period t. Assume there are two idiosyncratic states z_h and z_l, representing high and low productivity, respectively. In Figure 1, the black dashed line shows the individual consumption path in the steady state. The x-axis is the time horizon, and the y-axis (not shown) is the level of (log) consumption. Due to imperfect insurance, household consumption is a function of its productivity history. The red line shows the individual consumption path after an expansionary shock. Overall, the consumption profile shifts up. However, some paths are more exposed to the shock, and some paths are less: at time $t = 3$, the bottom dot (z_h, z_l, z_l, z_l) shifts up more than the top dot (z_h, z_h, z_h, z_h). To remove redistribution induced by the aggregate shock, I introduce counterfactual transfers to individuals and construct a hypothetical scenario where all agents are equally exposed to the shock. Let I denote the set of individuals in the economy. I define a transfer scheme $\boldsymbol{\omega} = \{\boldsymbol{\omega}_i\}_{i \in I}$, where $\boldsymbol{\omega}_i = (\omega_{i0}, \omega_{i1}, \cdots)'$ and ω_{it} represents the transfer received by individual i at time t. Consider the joint shock $(\epsilon, \boldsymbol{\omega})$ comprising both the aggregate shock and the transfer scheme. By properly designing the transfers, all agents have the same equilibrium

Figure 2: Individual Consumption Equally Affected by the Aggregate Shock
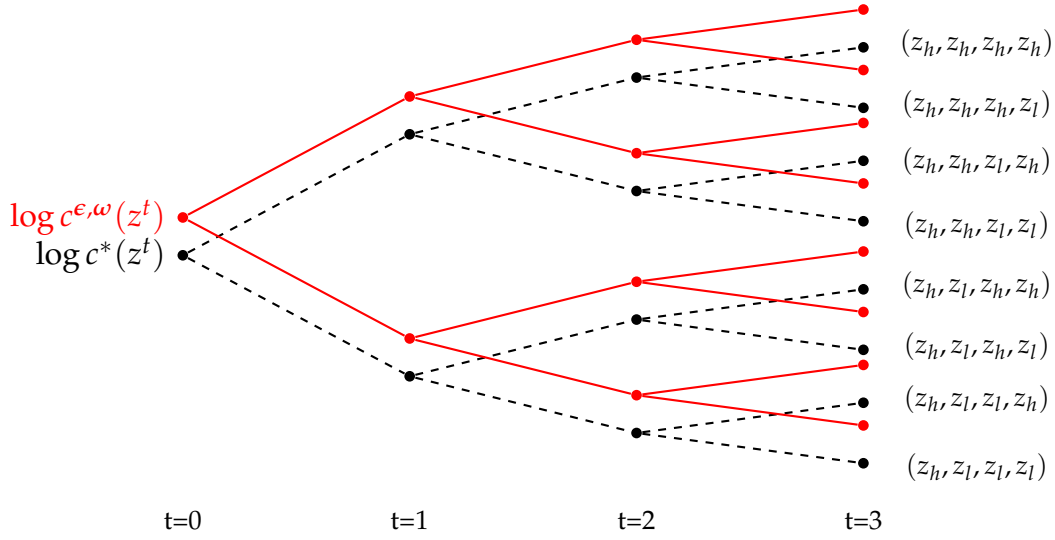
consumption response (in percentage) to the joint shock

$$\tilde{\mathbf{c}}_i^{\epsilon,\omega}/\mathbf{c}_i^* = \tilde{\mathbf{C}}^{\epsilon,\omega}/C^*, \forall i. \tag{3}$$

Figure 2 illustrates the consumption allocation. This paper shows that such transfers exist in various HA models. With transfers, the dynamics of aggregates can be characterized by the equilibrium conditions of a fictitious RA model. This counterfactual provides the benchmark for further analysis of redistribution.

With the transfer scheme ω, we can decompose the aggregate shock as

$$(\epsilon, \mathbf{0}) = \underbrace{(\epsilon, \omega)}_{\text{"pure" aggregate shock}} + \underbrace{(\mathbf{0}, -\omega)}_{\text{redistribution shock}}.$$

The aggregate shock ϵ is a sum of two components: a "pure" aggregate shock (ϵ, ω), which is redistribution-neutral; and a redistribution shock $-\omega$, which is defined as the negative of the transfer scheme. To first order, the impulse response function of an aggregate outcome variable admits an additive decomposition:

$$\tilde{Y}^{\epsilon,\mathbf{0}} = \underbrace{\tilde{Y}^{\epsilon,\omega}}_{\text{RANK effects}} + \underbrace{\tilde{Y}^{\mathbf{0},-\omega}}_{\text{redistribution effects}}.$$

Y's response to the sum of two shocks equals the sum of its response to each shock. For individual variables, we can define the decomposition in a similar manner.

3   A Canonical HANK Model

This section considers a canonical HANK model in the style of McKay, Nakamura and Steinsson (2016). I describe the model in Section 3.1. Section 3.2 shows that with counterfactual transfers, aggregate dynamics of the model are "as if" those of a text-book RANK model (Galí, 2015). In Section 3.3, the redistribution shock is decomposed into three channels: interest rate, income, and tax exposures. Section 3.4 writes the household's problem in recursive form and discusses the computation.

3.1   Model Description

Time is discrete and infinite. The economy is populated by households, firms, and fiscal and monetary policy authorities. Households face idiosyncratic uncertainty on incomes and have access to one-period riskless government bonds, subject to exogenous borrowing constraints. There is price stickiness in the firm's price setting. The government collects taxes to pay interest on the debt. Monetary policy follows the Taylor rule.

Households.   There is a unit continuum of households that face idiosyncratic productivity shocks $z_t \in Z_t$. Let $z^t = (z_0, z_1, \cdots, z_t)$ be a history of idiosyncratic states up to period t. For ease of notation, the initial state z_0 also indexes the initial bond holdings. At $t = 0$, the economy inherits an initial distribution over idiosyncratic states and bonds $\Phi_0(z_0)$. The stochastic process then induces a distribution $\Phi(z^t)$ over histories $z^t \in Z^t$. Households are infinitely lived and have preferences over consumption $\{c(z^t)\}$ and labor supply $\{n(z^t)\}$ flows given by the utility function

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c(z^t), n(z^t)) \right], \tag{4}$$

where β is the subjective discount factor. Households derive utility from consumption and disutility from working. The period utility function is given by power utilities

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \varphi \frac{n^{1+\nu}}{1+\nu}. \tag{5}$$

Households face budget constraints for all $t = 0, 1, \cdots$ and histories $z^t \in Z^t$

$$c(z^t) + b(z^t) = R_t b(z^{t-1}) + W_t z_t n(z^t) + \pi(z_t) - \tau(z^t). \tag{6}$$

Households face labor income risks so that if they work $n(z^t)$, they supply efficient labor $z_t n(z^t)$ to firms and receive labor income $W_t z_t n(z^t)$, where W_t is the real wage.

The idiosyncratic productivity z_t evolves according to the AR(1) process $\log z_t = \rho_e \log z_{it-1} + e_{it}$ with innovations $e_{it} \sim \mathcal{N}(-\sigma_e^2(1-\rho_e^2)^{-1}/2, \sigma_e^2)$ so that $\int z_t d\Phi_t(z^t) = 1$. Households also receive profits $\pi(z_t)$ from intermediate firms and pay taxes $\tau(z^t)$ to the government. The financial markets are incomplete. Households have access to a risk-free government bond with the gross real interest rate R_{t+1} between periods t and $t+1$. Households' bond holdings are subject to an exogenous borrowing constraint

$$b(z^t) \geq \phi. \tag{7}$$

I assume that ϕ is strictly higher than the natural borrowing limit.

Firms. A competitive final-good firm produces a final good from intermediate goods, indexed by j, according to the production function $Y_t = (\int y_{j,t}^{1/\mu} dj)^\mu$. The intermediate goods are produced by monopolistically competitive firms using labor as input with linear technology $y_{j,t} = Al_{j,t}$, where $l_{j,t}$ denotes the labor hired by firm j in period t. Due to symmetry, aggregate labor demand $L_t = l_{j,t}$. Each intermediate firm sets its price to maximize profits, subject to quadratic adjustment costs as in Rotemberg (1982)

$$\Theta_t(p_{j,t}, p_{j,t-1}) = \frac{\mu}{\mu-1}\frac{1}{2\kappa}[\log(p_{j,t}/p_{j,t-1})]^2 Y_t. \tag{8}$$

The Phillips curve can be derived as

$$\log(1 + \pi_t^p) = \kappa\left(\frac{W_t}{A} - \frac{1}{\mu}\right) + \frac{1}{R_{t+1}}\frac{Y_{t+1}}{Y_t}\log(1 + \pi_{t+1}^p), \tag{9}$$

where π_t^p is inflation. The price adjustment creates real costs Θ_t, and profits equal the output net of labor expenditure and price adjustment costs $\Pi_t = Y_t - W_t L_t - \Theta_t$.

Fiscal Policy. The government collects taxes T_t to pay interest on debt B^* to households. The government budget constraint is

$$B^* + T_t = R_t B^*, \tag{10}$$

and $T_t = r_t B^*$ is the aggregate tax. The government maintains a constant level of debt B^* and adjusts taxes to balance its budget. In the next section, I will allow the government to adjust the level of outstanding debt.

Monetary policy. The monetary authority sets the nominal interest rates on government bonds i_t according to the Taylor rule $i_t = r^* + \phi_\pi \pi_t^p + \epsilon_t$. The ex-post real interest rates satisfy the Fisher equation $R_t = 1 + r_t = (1 + i_{t-1})/(1 + \pi_t^p)$.

Equilibrium Definition. Given a monetary policy shock $\epsilon = (\epsilon_0, \epsilon_1, \cdots)'$, an equilibrium consists of the path of aggregates $\{Y_t, i_t, \pi_t^p, W_t\}$, profit distribution and tax payment rule, firm choices $\{l_{j,t}, p_{j,t}\}$, and household choices $\{c(z^t), n(z^t), b(z^t)\}$ such that:

(i) individual optimization: given initial bond holdings and the path of aggregates $\{R_t, W_t, \Pi_t, T_t\}$, households choose $\{c(z^t), n(z^t), b(z^t)\}$ to maximize their utility function subject to the budget constraints and borrowing constraints; given $\{Y_t, W_t\}$, firms choose $\{l_{j,t}, p_{j,t}\}$ to maximize profits, subject to price adjustment costs and aggregate demand;

(ii) the Phillips curve holds; the nominal interest rate follows the Taylor rule;

(iii) aggregation and market-clearing: for $t = 0, 1, \cdots$, the goods, labor, and bond markets clear:

$$C_t + \Theta_t = Y_t, \text{ where } C_t = \int c(z^t) d\Phi_t(z^t); \tag{11}$$

$$N_t = L_t, \text{ where } N_t = \int z_t n(z^t) d\Phi_t(z^t); \tag{12}$$

$$B_t^d = B^*, \text{ where } B_t^d = \int b(z^t) d\Phi_t(z^t). \tag{13}$$

In the economy's deterministic steady state, aggregate quantities and prices are constant, and inflation is zero. A variable Y's steady state value is denoted as Y^*, its level deviation from Y^* is denoted as $\tilde{Y}$, and percentage deviation is denoted as $\hat{Y}$.

3.2 RANK Effects

Assume the economy starts from the deterministic steady state. At time 0, there is an unexpected monetary policy shock ϵ realized and the shock evolves according to $\epsilon_t = \rho \epsilon_{t-1}$ where $\rho \in (0,1)$ is its persistence. In the infinite horizon, the economy returns to its initial state. Following the shock, I construct a transfer scheme $\omega = \{\omega(z^t), \forall z^t \in Z^t\}_{t=0}^{\infty}$ to redistribute resources among households, such that the household with the productivity path z^t receive a lump-sum transfer $\omega(z^t)$ at time t. The household's budget constraints then read

$$c(z^t) + b(z^t) = R_t b(z^{t-1}) + W_t z_t n(z^t) + \pi(z_t) - \tau(z^t) + \omega(z^t). \tag{14}$$

Proposition 1 establishes that we can find transfers such that all agents exhibit homogeneous consumption and labor supply responses to the joint shock (ϵ, ω), and the economy aggregates.

Proposition 1. *Given a monetary policy shock ϵ, there exist transfer schemes ω such that, under the joint shock (ϵ, ω):*

(i) The aggregates satisfy the equilibrium conditions of a fictitious RANK model, including the aggregate Euler equation

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{ra} R_{t+1}^{\epsilon,\omega}(C_{t+1}^{\epsilon,\omega})^{-\sigma}, \text{ where } \beta^{ra} \equiv 1/R^*; \tag{15}$$

the aggregate labor supply condition

$$W_t^{\epsilon,\omega}(C_t^{\epsilon,\omega})^{-\sigma} = \varphi^{ra}(N_t^{\epsilon,\omega})^{\nu}, \text{ where } \varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}; \tag{16}$$

the Phillips curve; the government budget constraint; the Taylor rule, and the market-clearing conditions.

(ii) The individual consumption and labor supply satisfy:

$$\frac{c^{\epsilon,\omega}(z^t)}{c^*(z^t)} = \frac{C_t^{\epsilon,\omega}}{C^*}, \quad \frac{n^{\epsilon,\omega}(z^t)}{n^*(z^t)} = \frac{N_t^{\epsilon,\omega}}{N^*}. \tag{17}$$

(iii) The transfers sum to zero cross-sectionally: $\int \omega(z^t) \, d\Phi_t(z^t) = 0$.

Proof. See Appendix.

The fictitious representative agent's subjective discount factor is defined as the steady-state real discount rate, $\beta^{ra} \equiv 1/R^*$. Proposition 1 aligns with the "as if" result in Werning (2015): although heterogeneity influences the level of the real interest rate (given the path of aggregate consumption), it does not necessarily alter the elasticity of aggregate consumption to the change in the real interest rate. To account for level effects, the fictitious representative agent's discount factor β^{ra} differs from the true agent's discount factor β. This reasoning similarly applies to labor supply, where the labor-supply disutility parameter is defined as $\varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}$.

To understand this result, it is useful to consider how the aggregate shock influence precautionary saving motives. Note that the economy remains an incomplete market economy, and the consumption (and labor supply) share of agents is not constant. Agents have precautionary saving motives due to idiosyncratic income risks. The counterfactual transfers, however, prevent the aggregate shock from inducing "cyclical heterogeneity" in individual shares. Conditional on the individual path z^t, the consumption (and labor supply) share remains constant before and after the shock. The precautionary saving motives are not affected by the aggregate shock.

Counterfactual transfers ensure that scaled individual choices satisfy budget constraints. Henceforth, the equilibrium in Proposition 1 is referred to as the "RANK" equilibrium. All variables in the equilibrium are denoted with the superscript "ra".

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We back out transfers from the household budget constraint. The elements of the budget constraint are obtained as follows. Solving the fictitious RANK model, we obtain the path of aggregates. The path of aggregate consumption and labor supply $\{C_t^{ra}, N_t^{ra}\}$, together with individual consumption and labor supply in the steady state $\{c^*(z^t), n^*(z^t)\}$, determine the individual's consumption and labor supply $\{c^{ra}(z^t), n^{ra}(z^t)\}$ according to equation (17). The aggregate firm profits Π_t^{ra} and the profits distribution rule determine the individual profits income $\pi^{ra}(z_t)$, and similar for individual taxes $\tau^{ra}(z^t)$. To recover the transfer term $\omega(z^t)$, we also need the bond demand function $b^{ra}(z^t)$. In the proof of Proposition 1, I impose the bond demand function $b^{ra}(z^t) = b^*(z^t)$, which is effectively a normalization. Without additional restrictions on the timing of transfers, the transfers can only be pinned down by first pinning down the bond demand function. As shown below, the bond demand function and the corresponding transfer scheme are indeterminate. The intuition is similar to the Ricardian equivalence of an RA model: the timing of taxes does not matter for the equilibrium. Here, for some households and to some extent, the income loss at time t can be compensated by future or past income, as they can utilize financial markets to transfer income across time.

Proposition 2. *Given individual consumption $\{c^{ra}(z^t)\}$ and the path of real interest rates $\{R_{t+1}^{ra}\}$, bond demand in the "RANK" equilibrium $\{b^{ra}(z^t)\}$ is characterized by:*

(i) The borrowing constraint and complementary slackness condition: $b^{ra}(z^t) \geq \phi, =$ if $u'(c^{ra}(z^t)) > \beta R_{t+1}^{ra} E[u'(c^{ra})(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi) = 0$;

(iii) Bond market clearing: $\int b^{ra}(z^t)d\Phi_t(z^t) = B^*$.

Proof. See Appendix.

Unconstrained households have indeterminate bond demand. The timing of transfers does not map one-to-one to their consumption and labor supply decisions. However, constrained households have a fixed bond demand at the borrowing constraint ϕ. The transversality condition is a necessary condition of optimality, and bond market clearing is one of the related equilibrium conditions. With $b^{ra}(z^t)$, the corresponding transfer $\omega(z^t)$ is recovered from the household budget constraint

$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra}b^{ra}(z^{t-1}) - W_t^{ra}z_t n^{ra}(z^t) - \pi^{ra}(z_t) + \tau^{ra}(z^t). \quad (18)$$

In practice, it is natural to choose $b^{ra}(z^t)$ as a function of $b^*(z^t)$ (see Section 3.4). Since $b^*(z^t)$ satisfies the stationary-equilibrium counterparts of the conditions in Proposition 2, it is feasible to verify that the chosen bond demand satisfies those conditions.

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3.3 Redistribution Channels in the Canonical Model

The Appendix shows that the redistribution shock can be decomposed as follows

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\tau^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}}$$

$$+ \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))}_{\text{saving flow exposure}} + \underbrace{(b^*(z^t) - b^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - b^{ra}(z^{t-1}))}_{\text{undetermined bond demand}}.$$

where $y(z^t) \equiv W_t z_t n(z^t) + \pi(z_t)$ is income. I define three sources of redistribution: income, interest rate, and tax exposure channels. The income exposure channel is

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t), \tag{19}$$

which captures the redistribution among households with different income elasticities to aggregate income ($\hat{y}^{ra}(z^t) \neq \hat{Y}_t^{ra}$). The interest exposure channel is

$$(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*), \tag{20}$$

which focuses on the redistribution between creditors and debtors. After consolidating the government budget constraint into the household budget constraint, we can see that the net bond position $b^*(z^{t-1}) - B^*$, rather than the gross position $b^*(z^{t-1})$, determines the exposure to the interest rate shock. The tax exposure channel is

$$(\tau^*(z^t) - r^*B^*) - (\tau^{ra}(z^t) - r_t^{ra}B^*), \tag{21}$$

which reflects different exposures to the change in taxes. In the case of uniform tax exposure $\tau^*(z^t) - r^*B^* = \tau^{ra}(z^t) - r_t^{ra}B^*$, this channel is muted because all households benefit equally from the tax reduction. For more general taxation schemes, households may benefit or lose from the tax response.

There are also two residual terms. Define $y(z^t) - c(z^t)$ as saving flows. Scaling of saving flows leads to passive redistribution. For models with infinitely-lived agents and typical calibrations, its effects are negligible.[7] In overlapping generation models, this channel can have non-trivial effects because there is a systemic correlation between saving flows and MPCs across the life cycle. The last term is due to the undetermined bond demand. Imposing $b^{ra}(z^t) = b^*(z^t)$, this term is zero.[8]

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[7]Here, I keep goods flow exposure $C_t^{ra}(y^*(z^t) - c^*(z^t))$, while assigning asset flow exposure related to the zero-maturity bond to a dedicated "interest rate exposure" channel, as this terminology is commonly used for that redistribution in the literature. In Section 4.2, I include exposure associated with long-maturity assets in measuring the total saving flow exposure. This channel's effects are minimal because (i) the saving flow is small relative to consumption/income flows; (ii) the MPCs' heterogeneity between net savers ($y^*(z^t) > c^*(z^t)$) and net borrowers ($y^*(z^t) < c^*(z^t)$) is small.

[8]We can also choose other bond demand functions that satisfy Proposition 2. Theoretically, a bond

3.4 Households' Problem in Recursive Form

To compute the redistribution effects, I write the household's problem in recursive form. First, we impose a bond demand function for the "RANK" equilibrium, which needs to satisfy Proposition 2 and is not unique:

$$b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi). \tag{22}$$

When government debt is constant, $B_t^{ra} = B^*$ and $b^{ra}(z^t) = b^*(z^t)$. When government debt is cyclical (see Section 4.1), the function $g_t(\cdot)$ shrinks or stretches the stationary-equilibrium bond demand function, keeping its lower bound at the borrowing constraint. The Appendix shows that $b^{ra}(z^t)$ satisfies Proposition 2.

Let $c^*(z, b^*)$, $b'^*(z, b^*)$, and $n^*(z, b^*)$ denote the household's consumption, bond demand, and labor supply policy functions in the steady state, where b^* is the household's wealth in the steady state. Note that from the path of aggregates and the household's states in the steady state, the transfers are fully pinned down:

$$\omega_t(z, b^*) = \frac{C_t^{ra}}{C^*}c^*(z, b^*) + g_t(b'^*(z, b^*)) - R_t^{ra}g_{t-1}(b^*) - W_t^{ra}z\frac{N_t^{ra}}{N^*}n^*(z, b^*) - \pi_t^{ra}(z) + \tau_t^{ra}(z).$$

It is unnecessary to track the entire individual history z^t is to determine the transfers. In the following, I use the steady state bond holdings b^* as an exogenous state variable to summarize the relevant part of an individual's history for determining the transfers. With the state-dependent transfers $\omega_t(z, b^*)$, the household's problem in recursive form is:

$$V_t^{ra}(z, b^*, b) = \max_{\{c,n,b'\}} u(c, n) + E[V_{t+1}^{ra}(z', b'^*, b')|z, b^*], \tag{23}$$

$$s.t. \quad c + b' = R_t b + W_t z n + \pi_t(z) - \tau_t(z) + \omega_t(z, b^*),$$

$$b' \geq \phi.$$

The law of motion for the exogenous state b^* is the bond demand policy function in the steady state $b'^* = b'^*(z, b^*)$. Along the equilibrium path, the household's policy functions satisfy, for $b = g_{t-1}(b^*)$,

$$c_t^{ra}(z, b^*, b)/c^*(z, b^*) = C_t^{ra}/C^*, \tag{24}$$

$$n_t^{ra}(z, b^*, b)/n^*(z, b^*) = N_t^{ra}/N^*, \tag{25}$$

$$b_t'^{ra}(z, b^*, b) = g_t(b'^*(z, b^*)). \tag{26}$$

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demand function different from $b^*(z^t)$ has real effects, as the equivalence between different transfer schemes is evaluated along the interest rate path $\{R_t^{ra}\}$, while the redistribution shock is evaluated at the model's steady state with interest rate path $\{R^*\}$. To the first order, we can ignore the difference.

However, there is still a high computational cost due to the additional state variable b^*. For two-asset models, this will make the computation infeasible. In Section B.1 of the Appendix, I further simplify the method and make transfers directly based on the household equilibrium state (z, b) and get practically the same results.

4 Time-varying Bond Supply and Investment

This section explores two extensions: cyclical bond supply and investment. These features are common in the literature and play an essential role in analyzing business cycles. The decomposition framework effectively sheds light on their roles in redistribution and HANK models. Section 4.1 discusses how the path of public debt influences household borrowing and lending. Section 4.2 shows that investment responses lead to a redistribution between equity holders and workers.

4.1 Time-varying Bond Supply

Assume the government can also adjust public debt to balance its budget, and the budget constraint is $B_t + T_t = R_t B_{t-1}$. The fiscal policy induces a time-varying bond supply such that $B_t > \phi$ and $\lim_{t \to \infty} B_t = B^*$. Due to the failure of Ricardian equivalence, changing the bond supply has real effects: the timing of taxes directly affects the consumption of non-Ricardian households. Let $\bar{b}(z^t)$, $\bar{\tau}(z^t)$, and $\bar{T}_t$ denote the bond demand, individual tax payment, and aggregate tax, respectively, **assuming** the bond supply is constant. When the bond supply is cyclical, the actual bond demand $b(z^t)$, individual taxes $\tau(z^t)$, and aggregate tax T_t deviate from these counterparts. In this case, the redistribution shock $-\omega(z^t)$ is decomposed as follows :

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}}$$

$$+ \underbrace{(\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity (bond)}} \quad (27)$$

$$+ \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))}_{\text{saving flow exposure}} + \underbrace{(b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}))}_{\text{undermined bond demand}}.$$

The income, interest rate, and tax exposure channels are defined as before and are independent of the path of public debt. The last term "undetermined bond demand" is zero after imposing the bond demand function $\bar{b}^{ra}(z^t) = b^*(z^t)$.

The newly defined liquidity channel (of bond supply) may seem obscure at first. To understand it better, consider the partial equilibrium when households have not adjusted their bond demand so $\bar{b}^{ra}(z^t) = b^{ra}(z^t)$. The liquidity channel (27) simplifies to $\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)$, capturing the change in the timing of taxes. When the government

shifts the timing of taxes through deficit financing, it transfers households' income across time. In the partial equilibrium, the consumption of unconstrained households hardly responds if the net present value of their income does not change. However, the consumption of constrained households responds one-to-one to the change in their disposable income. In a GE that fixes output, the interest rate will adjust to clear the markets, and unconstrained households absorb the change in government debt by reallocating their consumption over time.

To link the above mechanism more closely with the concept of "liquidity", I show that in the case of uniform taxation, the liquidity channel can be proxied by counterfactual shocks to the borrowing constraint ϕ. To eliminate the real effects on consumption when the government changes the timing of taxes, one approach is to introduce counterfactual transfers, as shown above. Another approach is to introduce counterfactual shocks to borrowing constraints, which force households to absorb the tax changes through bond holdings rather than consumption.

Proposition 3. *(a) When government debt B_t deviates from its long run-level B^*, if (i) agents are uniformly exposed to the tax change such that $\tau(z^t) - \bar{\tau}(z^t) = T_t - \bar{T}_t, \forall z^t$; and (ii) the borrowing constraint evolves with debt level according to $\phi_t = \phi + B_t - B^*$, then bond demand in the "RANK" equilibrium is characterized by:*

(i) The borrowing constraint and complementary slackness condition: $b^{ra}(z^t) \geq \phi_t^{ra}, =$ if $u'(c^{ra}(z^t)) > \beta R_{t+1}^{ra} E[u'(c^{ra}(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t \to \infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi_t^{ra}) = 0$;

(iii) Bond market clearing: $\int b^{ra}(z^t) d\Phi_t(z^t) = B_t^{ra}$.

(b) For any bond demand function $\bar{b}^{ra}(z^t)$ satisfying the conditions in Proposition 2, the shifted bond demand function $b^{ra}(z^t) \equiv \bar{b}^{ra}(z^t) + B_t^{ra} - B^*$ satisfies the above conditions. The counterfactual transfers $\omega(z^t)$ are invariant to the path of government debt.

Proof. See Appendix.

The argument is similar to those in Aiyagari (1994), Aiyagari and McGrattan (1998), and Bhandari et al. (2017). Proposition 3 extends their results to transition paths. Suppose government debt increases by ΔB. For the same consumption choice $c^{ra}(z^t)$, the household now holds ΔB additional units of bonds into the next period, shifting up the wealth distribution across all individual states. To satisfy the complementary slackness condition of constrained households, the borrowing constraint is also increased by the same amount ΔB. Proposition 3 implies that in the case of uniform taxation, we can use counterfactual shocks to the borrowing constraint to proxy the liquidity channel. In this case, equation (27) equals zero, and the liquidity channel are reflected in the economy's response to the borrowing constraint shock $-\Delta \phi \equiv -\{B_t^{ra} - B^*\}_{t=0}^{\infty}$.

For non-uniform taxation, we can use path-dependent counterfactual borrowing constraint shocks to proxy the liquidity channel. To illustrate, consider the bond demand function $b^{ra}(z^t)$ given by

$$\bar{b}^{ra}(z^t) - b^{ra}(z^t) = R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) - (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)). \tag{28}$$

When government taxes deviates from the constant-debt path $\bar{\tau}^{ra}(z^t) \neq \tau^{ra}(z^t)$, households absorb the change of taxes through bond holdings $b^{ra}(z^t)$. To ensure this is an equilibrium demand function, we can construct the path-dependent borrowing constraints $\phi^{ra}(z^t)$ such that[9]

$$b^{ra}(z^t) \geq \phi^{ra}(z^t), = \text{ if } u'(c^{ra}(z^t)) > \beta R_{t+1}^{ra} E[u'(c^{ra}(z^{t+1}))|z^t]. \tag{29}$$

The effects of altering the timing of taxes generally depend on the taxation scheme. Households may experience gains or losses in real terms based on the tax payment history, rather than merely reallocating income across time. Consider temporary tax cuts financed by future tax increases, and taxes are proportional to productivity. This policy impacts the expected net present value of household income. Households with low productivity, who expect to revert to higher productivity, would expect to lose from the timing shift. Conversely, high-productivity households would expect to benefit. This "real" redistribution dampens consumption responses. Combining these real redistributive effects with the "pure" liquidity effects results in an understatement of the "pure" liquidity effects. In the current framework, I simply attribute all effects resulting from the cyclical public debt to the liquidity channel.

4.2 Investment

This section incorporates investment into the model, following the approach of Auclert, Rognlie and Straub (2024). Capital (equity) is considered liquid and serves as a perfect substitute for bonds. In Section F, I extend the model with illiquid assets.

4.2.1 Model Description

Households. Households can also trade in firm shares with price P_t, which provides a dividend stream D_t each period. The household's budget constraint is

$$c(z^t) + b(z^t) + P_t v(z^t) = R_t b(z^{t-1}) + (P_t + D_t)v(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z^t).$$

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[9]The transversality condition is $\lim_{t\to\infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi^{ra}(z^t)) = 0$. The bond demand function given by equation (28) may become unbounded for some paths z^t depending on tax payment histories, even if B_t is bounded. The transversality condition incorporates the exogenous borrowing constraints leading to explosive individual bond holdings.

The non-arbitrage condition requires that $R_t = (P_t + D_t)/P_{t-1}$ from $t = 1$. Define wealth $a(z^t) \equiv b(z^t) + P_t v(z^t)$, from $t = 1$ the budget constraints can be written as

$$c(z^t) + a(z^t) = R_t a(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z^t). \tag{30}$$

At $t = 0$, the return on bonds and equity can be different. Bond return is subject to unexpected inflation, and equity return is subject to unexpected capital gains:

$$c(z^0) + a(z^0) = R_0 b_{-1} + (P_0 + D_0) v_{-1} + z_0 W_0 n(z^0) + \pi(z_0) - \tau(z^0). \tag{31}$$

Households are subject to the non-borrowing constraints $a(z^t) \geq 0$.

Firms. The intermediate goods firms have a Cobb-Douglas production function $y_{j,t} = A k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}$. The marginal cost also includes rents on capital. The Phillips Curve is

$$\log(1 + \pi_t^p) = \kappa \left(mc_t - \frac{1}{\mu} \right) + \frac{1}{R_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}^p). \tag{32}$$

where $mc_t = (r_t^K/\alpha)^{\alpha} (W_t/(1-\alpha))^{1-\alpha}/A$. Firms own capital K_{t-1} and choose investment I_t to obtain the capital of the next period $K_t = (1-\delta)K_{t-1} + I_t$, subject to quadratic capital adjustment cost. Dividends equal capital products plus post-tax monopolistic profits net of investment, capital adjustment cost, and price adjustment cost,

$$D_t = r_t^K K_{t-1} + \alpha \Pi_t - I_t - \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta^K \right)^2 - \Theta_t. \tag{33}$$

Firms choose investment to maximize $P_t + D_t$. Tobin's Q and capital evolve according to the standard Q-theory of investment:

$$\frac{I_t}{K_{t-1}} - \delta^K = \frac{1}{\Psi}(Q_t - 1), \tag{34}$$

$$R_{t+1} Q_t = r_{t+1}^K - \frac{I_{t+1}}{K_t} - \frac{\Psi}{2} \left(\frac{I_{t+1}}{K_t} - \delta^K \right)^2 + \frac{K_{t+1}}{K_t} Q_{t+1}. \tag{35}$$

The monopolistic profits Π_t are taxed, so firms receive an α fraction of the monopolistic profits. The remaining $1 - \alpha$ fraction is paid to households as a lump-sum transfer in proportion to household productivity. This profit taxation scheme fully neutralizes the impact of countercyclical markups and generates reasonable asset price responses.

Equilibrium Definition. In the equilibrium, households and firms optimize, the government budget constraint holds, nominal interest rates evolve according to the Taylor

rule, and markets clear:

$$\int a(z^t)d\Phi_t(z^t) = B_t + P_t, \tag{36}$$

$$\int z_t n(z^t)d\Phi_t(z^t) = L_t, \tag{37}$$

$$C_t + I_t + \frac{\Psi}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 + \Theta_t = Y_t^{GDP}. \tag{38}$$

4.2.2 Redistribution Channels with Investment

Imposing the bond demand function $\bar{b}^{ra}(z^t) = b^*(z^t)$. The Appendix shows that the redistribution shock can be decomposed as

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}}$$

$$+ \underbrace{(b^*(z^t) - b^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity (bond)}}$$

$$+ \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) - \hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))}_{\text{saving flow exposure}}$$

$$+ \underbrace{P_t^{ra}(v^*(z^t) - v^{ra}(z^t)) - P_t^{ra}(v^*(z^{t-1}) - v^{ra}(z^{t-1}))}_{\text{undetermined equity demand}}, \tag{39}$$

where $y(z^t) \equiv z_t W_t n(z^t) + \pi(z_t) + Dv(z^{t-1})$ is household's income, including labor income $y^L(z^t) \equiv z_t W_t n(z^t) + \pi(z_t)$[10] and dividend income $Dv(z^{t-1})$. On the aggregate level, aggregate income equals consumption $Y_t = W_t N_t + (1-\alpha)\Pi_t + D_t = C_t$.

When evaluating saving flow exposure with long-maturity assets, we need to consider asset flow exposure due to endogenous asset-price fluctuations $\hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$. The change in asset price affects traders rather than holders, consistent with the argument made in Fagereng et al. (2022).[11] The last residual term is due to the undetermined equity demand. After imposing $v^{ra}(z^t) = v^*(z^t)$, this term is zero.

To focus on investment, temporarily assume that household labor income is given by $y^L(z^t) = z_t Y_t^L$, implying that all households have the same labor income elasticities to aggregate labor income Y_t^L. The income exposure then simplifies to

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) = \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*\left(v^*(z^{t-1}) - z_t\right)}_{\text{income portfolio exposure}}, \tag{40}$$

---

[10]Income from labor in the broad sense, including profit income interpreted as the bonus.

[11]The effects of the saving flow exposure are minimal because first, monetary policy shock affects asset price and consumption in the same direction, implying that goods flow exposure $\hat{C}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$ and asset flow exposure $\hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$ counteract each other; second, the MPCs heterogeneity between net savers ($y^*(z^t) > c^*(z^t)$) and net borrowers ($y^*(z^t) < c^*(z^t)$) is small.

which reflects the redistribution between equity holders ($v^*(z^{t-1}) > z_t$) and workers ($v^*(z^{t-1}) < z_t$) as the share of dividends in aggregate income fluctuates ($\hat{D}_t^{ra} \neq \hat{Y}_t^{ra}$).

If dividends are more responsive than labor income, equity holders gain and workers lose. Conversely, the redistribution favors workers over equity holders. Equation (33) implies that dividends and investment responses are negatively correlated. Omitting capital and price adjustment costs and noting that $r_t^K K_{t-1} + \alpha \Pi_t = \alpha Y_t^{GDP}$, we have $D_t = \alpha Y_t^{GDP} - I_t = \alpha(C_t + I_t) - I_t = \alpha Y_t - (1-\alpha)I_t$. Dividends are less responsive than aggregate income $\hat{D}_t^{ra} < \hat{Y}_t^{ra}$ if and only if the investment is more responsive than aggregate consumption $\hat{I}_t^{ra} > \hat{C}_t^{ra}$.[12] For typical calibrations, investment is more responsive than consumption in the short run and less responsive in the long run, implying a redistribution from equity holders to workers in the short run and the reverse in the long run. When equity holders accumulate capital for future consumption, workers consume the additional income generated from producing capital. In the future, however, workers have to cut their consumption when the economy de-invests and consumes the accumulated capital. The redistribution allows workers to move their future consumption to the present, which has a similar flavor to the liquidity channel of bond supply. From this perspective, the income portfolio exposure can also be interpreted as the liquidity channel of productive assets.

5 Estimable Moments for Partial Equilibrium Responses

The framework implies that if the policymaker lowers the nominal interest rate and knows the RA model's response, then she only needs to know the HA model's response to the redistribution shock to get full responses. This generally require numerically solving a full HANK model. Below, I show we can gain insights from simple partial equilibrium analysis, following Auclert (2019). To simplify the analysis, I truncate the shock and only consider the redistribution at time $t = 0$. To the first order, the aggregate consumption response in partial equilibrium is

$$\partial C_0 = \int MPC_{i0} \cdot (-\omega_{i0})di = cov_I(MPC_{i0}, -\omega_{i0}). \tag{41}$$

The equation follows from the re-distributive nature of the transfers: $\int -\omega(z^t)d\Phi_t(z^t) = 0$. The consumption response in partial equilibrium is the cross-sectional covariance between households' MPCs and their redistribution shock. $cov_I(MPC_{i0}, -\omega_{i0}) > 0$ in the case of amplification; and $cov_I(MPC_{i0}, -\omega_{i0}) < 0$ in the case of dampening. Since each redistribution channel sums to zero cross-sectionally, the above argument also applies to the evaluation of each channel.

---

[12]From $D_t + (1-\alpha)I_t = \alpha Y_t$ we have $\hat{D}_t D^* + \hat{I}_t(1-\alpha)I^* = \hat{Y}_t(D^* + (1-\alpha)I^*)$ and then $(\hat{D}_t - \hat{Y}_t)D^* + (\hat{I}_t - \hat{Y}_t)(1-\alpha)I^* = 0$. Given $D^* > 0$ and $I^* > 0$ we know $\hat{D}_t < \hat{Y}_t$ if and only if $\hat{I}_t > \hat{Y}_t$.

Before deriving estimable moments at the channel level, I specify the functional form of household labor income and tax payment. I also specify the aggregate labor supply condition and fiscal policy to close the model.

5.1 The Full Model

Household Labor Income. Assume that households supply the same amount of labor and that the distribution of profits is proportional to productivity. I introduce the "incidence function", following Guvenen et al. (2017), Werning (2015), Auclert and Rognlie (2018), Alves et al. (2020), etc., to capture households' different labor income elasticities to aggregate labor income fluctuations. The specific function form is the same as Alves et al. (2020). Household gross labor income is given by

$$y^{GL}(z^t) = \frac{z_t(Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}}{E_I[z_t(Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}]}Y_t^{GL}, \tag{42}$$

where $Y_t^{GL} = W_t N_t + (1-\alpha)\Pi_t$ is aggregate gross labor income. In the steady state, household gross labor income is simply $y^{GL,*}(z^t) = z_t Y^{GL,*}$. Off the steady state, imposing the normalization $E_I[z_t\gamma(z_t)] = 1$, then $\gamma(z_t)$ is the elasticity of the type z_t gross labor income $y^{GL}(z^t)$ to aggregate gross labor income Y_t^{GL} evaluated at $Y^{GL,*}$ (see Alves et al. 2020). The household's budget constraint is

$$c(z^t) + b(z^t) + P_t v(z^t) = R_t b(z^{t-1}) + (P_t + D_t)v(z^{t-1}) + y^{GL}(z^t) - \tau(z^t). \tag{43}$$

Labor Supply. The modeling of the labor market is non-standard, borrowed from Alves et al. (2020) to simplify the labor-supply analysis. Households supply the same amount of labor $n(z^t) = N_t$ to firms, and the aggregate labor supply follows

$$W_t = W^*\left(\frac{N_t}{N^*}\right)^{\epsilon_w}. \tag{44}$$

In the wage schedule, if $\epsilon_w = 0$, wages are perfectly rigid, and employment is determined by only labor demand. If $\epsilon_w > 0$, there is pressure on wages whenever employment is different from its steady-state level.

Fiscal Policy. The taxes households pay to the government are

$$\tau(z^t) = \Gamma y^{GL}(z^t) + T_t^{uniform}, \tag{45}$$

where Γ is a constant tax rate on gross labor income, and $T_t^{uniform}$ is a uniform tax. The aggregate tax income of the government is then $T_t = \Gamma Y_t^{GL} + T_t^{uniform}$. I assume non-

standard fiscal policy responses to capture the relaxed borrowing conditions following an expansionary shock. The path of government debt evolves according to:

$$B_t - B^* = \rho_B(B_{t-1} - B^*) + \epsilon_t^B. \tag{46}$$

Following the monetary policy shock ϵ_t, there is also a shock to the level of government debt $\epsilon_t^B = \phi^B \epsilon_t$. When $\phi^B < 0$, the bond supply is procyclical (conditional on the monetary policy shock), and when $\phi^B > 0$, the bond supply is countercyclical. The uniform taxes $T_t^{uniform}$ are adjusted such that the government budget constraint holds:

$$B_t + \Gamma Y_t^{GL} + T_t^{uniform} = R_t B_{t-1} + G, \tag{47}$$

where G is the constant government spending.

5.2 Parameterize and Redistribution Channels

Given the full model specification, we can parameterize redistribution channels and simplify the algebras. It is then straightforward to see the mapping from structural parameters to redistribution. Proofs of this section can be found in the Appendix.

In the previous analysis, government spending is not considered, and aggregate taxes (under constant debt) cover interest expenses. With government spending G, taxes need to satisfy $\int \bar{\tau}(z^t) d\Phi_t(z^t) = G + r_t B^*$, resulting in aggregate income exceeding consumption. When defining redistribution channels, one restriction is that each channel sums to zero cross-sectionally, so I define net income below; I also incorporate taxes into income to streamline the decomposition:[13]

$$
\begin{aligned}
y(z^t) &= D_t v(z^{t-1}) + y^{GL}(z^t) - (\bar{\tau}(z^t) - r_t B^*) \\
&= \underbrace{D_t v(z^{t-1})}_{\text{dividend income}} + \underbrace{(y^{GL}(z^t) - \tau^G(z^t))}_{\text{net labor income } y^L(z^t)} - \underbrace{(\bar{\tau}(z^t) - \tau^G(z^t) - r_t B^*)}_{\text{net taxes } \tau^n(z^t)},
\end{aligned} \tag{48}
$$

where $\tau^G(z^t) = \frac{z_t(Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}}{E_I[z_t(Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}]}G$ represents the portion of individual taxes allocated to financing government spending.[14] Hereafter, labor income refers to net labor income $y^L(z^t) \equiv y^{GL}(z^t) - \tau^G(z^t)$ and $Y_t^L \equiv Y_t^{GL} - G$. The newly defined net taxes $\tau^n(z^t)$ exclude $\tau^G(z^t)$ and $r_t B^*$ from gross taxes $\bar{\tau}(z^t)$ and sum to zero cross-sectionally.

After imposing the bond demand function $\bar{b}^{ra}(z^t) = b^*(z^t)$ and the equity demand

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[13]Excluding taxes from income makes the exposition of the liquidity channel in Section 4.1 easier.

[14]The functional form of $\tau^G(z^t)$ minimally impacts the decomposition results as long as G remains constant. Changes in individual labor income and taxes are attributed to variations in aggregate labor income and interest payments, rather than government spending.

function $v^{ra}(z^t) = v^*(z^t)$, the sources of redistribution are

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} \tag{49}$$

$$+ \underbrace{(b^*(z^t) - b^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity (bond)}}$$

$$+ \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) - \hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))}_{\text{saving flow exposure}}.$$

The income channel can be further decomposed based on its composition:

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) = \underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*\left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}}\right)}_{\text{income portfolio exposure}}$$

$$+ \underbrace{\tau^{n,*}(z^t) - \tau^{n,ra}(z^t)}_{\text{tax exposure}} + \hat{Y}_t^{ra}\tau^{n,*}(z^t). \tag{50}$$

The first part, labor income exposure, is about the redistribution within the category of labor income: households may have different labor income elasticities to aggregate labor income. Given the labor income incidence function (42), it simplifies to

$$(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t) = (\gamma(z_t)\hat{Y}_t^{L,ra} - \hat{Y}_t^{L,ra})z_t Y^{L,*} = (\gamma(z_t) - 1)z_t\tilde{Y}_t^{L,ra}. \tag{51}$$

If $\gamma(z_t) > 1$, then the type z_t household's labor income is more elastic to aggregate labor income, making the labor income exposure term positive. The labor income elasticity $\gamma(z_t)$ is the target for calibration. The second part, income portfolio exposure, discussed earlier in Section 4.2, captures the redistribution between equity holders (households with $v^*(z^{t-1}) > y^{L,*}(z^t)/Y^{L,*}$) and workers (households with $v^*(z^{t-1}) < y^{L,*}(z^t)/Y^{L,*}$). The labor income incidence function (42) implies that $y^{L,*}(z^t) = z_t Y^{L,*}$; therefore, we have

$$(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*\left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}}\right) = (\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*(v^*(z^{t-1}) - z_t), \tag{52}$$

the same as in Section 4.2. The third part is the tax exposure, now consolidated into the income channel. Given the taxing scheme (45) and government budget constraint, this channel simplifies to

$$\Gamma\tilde{Y}_t^{L,ra}(1 - \gamma(z_t)z_t). \tag{53}$$

The last part of the income exposure $\hat{Y}_t^{ra}\tau^{n,*}(z^t)$ is the scaling of net taxes in the steady

Table 2: Consumption response to the redistribution shock in partial equilibrium

Redistribution channel		Consumption response	Value (% of C^*)
Interest rate exposure		$\tilde{R}^{ra} \cdot cov_I \left(MPC_i, b_i - B \right)$	0.066
Income exposure	labor	$\tilde{Y}^{L,ra} \cdot cov_I \left(MPC_i, (\gamma(z_i) - 1)z_i \right)$	0.002
	portfolio	$(\hat{D}^{ra} - \hat{Y}^{ra})D \cdot cov_I \left(MPC_i, v_i - z_i \right)$	0.051
	Tax	$\Gamma \tilde{Y}^{L,ra} \cdot cov_I \left(MPC_i, 1 - \gamma(z_i)z_i \right)$	0.009
Liquidity		$\tilde{B}^{ra} / (B - \phi) \cdot cov_I \left(MPC_i, B - b_i' \right)$	0.037

Notes: Partial-equilibrium consumption response to a transitory redistribution shock. MPC_i is the marginal propensity of consumption of individual i. $b_i, v_i, z_i, \gamma(z_i)$ denote individual i's bond position, dividend income share, labor income share, and labor income elasticities, respectively.

state. When the definition of income does not include taxes as in the last section, this term is absorbed into the channel "saving flow exposure (bond)". I also treat this term as a residual since it has negligible quantitative effects.

The liquidity channel of bond supply can also be simplified to

$$\frac{B_t^{ra} - B^*}{B^* - \phi}(B^* - b^*(z^t)) - \frac{R_t^{ra}(B_{t-1}^{ra} - B^*)}{B^* - \phi}(B^* - b^*(z^{t-1})). \tag{54}$$

5.3 Consumption Response in Partial Equilibrium

The partial equilibrium consumption responses to the redistribution shock are summarized in Table 2. I omit the time script and instead denote b_i as individual i's initial bond position and b_i' as its bond position at the beginning of $t = 1$. Similarly, v_i is the initial equity share and v_i' is the share at the beginning of $t = 1$.

The covariance terms in Table 2 can be estimated as in Auclert (2019) and Patterson (2023). The key advancement here is that, when applying these estimates, the aggregate quantities or prices are derived from the RANK model instead of the actual HANK economy used implicitly in Auclert (2019) and Patterson (2023). For example, the effects of the interest rate exposure channel are the covariance $cov_I \left(MPC_i, b_i - B \right)$ times the (level) response of the interest rate in the "RANK" economy, $\tilde{R}^{ra}$, rather than that in HANK, which is ex ante unknown. To predict the policy effects, the policy-maker needs only the responses of the "RANK" economy, which are ex ante known, and the covariance terms in Table 2. Each channel can be treated as an independent block, and the total effects are simply their sum. Once estimated, the statistics in Table 2 can be used to evaluate any policy shock by simply adjusting their scales that derive from the "RANK" economy.

In the third column, I use the calibrated model in Section 6 to compute those en-

dogenous moments and combine them with the response of the RANK model at time $t = 0$ (see Figure 3) to get the consumption response. Both the sign and relative magnitude of the responses align with the GE responses reported in Figure 5 of Section 6. In what follows, I briefly discuss the standard model's prediction of these moments and the implications for amplification/dampening of consumption responses.

Interest rate exposure. The incomplete-market model predicts that

$$cov_I \left(MPC_i, b_i - B \right) < 0. \tag{55}$$

Creditors ($b_i > B$) have lower MPCs than debtors ($b_i < B$), which implies a negative correlation between MPC and the exposure to interest rate changes. An interest rate cut taxes creditors and subsidizes debtors, amplifying consumption responses.

Labor income exposure. Households' labor income elasticities are exogenous to the model and need to be calibrated. Guvenen et al. (2017) estimate "worker betas" (i.e., systematic risk exposure) with respect to GDP for US workers. Since aggregate gross labor income is a constant share of output in the model, the elasticity of individual gross labor income to GDP equals its elasticity to aggregate gross labor income. The estimation shows a U-shaped elasticity, i.e., exposure is high at both the bottom and the top of the distribution ($\gamma(z_i) > 1$ for both low and high z_i).[15] As will be shown in the next section, using estimates from Guvenen et al. (2017) implies a slightly positive covariance:

$$cov_I \left(MPC_i, (\gamma(z_i) - 1)z_i \right) > 0. \tag{56}$$

Income portfolio exposure. Since asset-rich households receive a larger share of aggregate dividend income relative to their share of aggregate labor income ($v_i > z_i$), while the reverse holds for asset-poor households ($v_i < z_i$), we have

$$cov_I \left(MPC_i, v_i - z_i \right) < 0. \tag{57}$$

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[15]Guvenen et al. (2017)'s estimates capture the income dynamics of workers at both the lowest and highest ends of the income distribution. Extensive empirical evidence suggests that individuals with lower incomes are generally more exposed to economic fluctuations. Patterson et al. (2019) documents a positive covariance between workers' MPCs and their earnings elasticities to GDP in the US. Broer, Kramer and Mitman (2020) uses German data and finds that workers at the bottom of the income distribution are more exposed to aggregate earnings risk in general, and to monetary policy shocks in particular. Amberg et al. (2022) documents a similar pattern in Swedish administrative individual data: there is a higher sensitivity of labor income to monetary shocks at the bottom than elsewhere in the income distribution. Coibion et al. (2017) finds that contractionary monetary policy increases inequality in labor earnings. For Denmark, Andersen et al. (2022) finds that gains created by softer monetary policy through the labor channel are concentrated among relatively low-income workers.

As discussed earlier, in the short run $\hat{D}^{ra} < \hat{Y}^{ra}$. Impact investment responses redistribute from equity holders to workers and amplify consumption responses.

Tax Exposure. Households with low labor income ($\gamma(z_i)z_i < 1$) have higher MPCs than households with high labor income ($\gamma(z_i)z_i > 1$)

$$cov_I \left(MPC_i, 1 - \gamma(z_i)z_i \right) > 0. \tag{58}$$

When the aggregate tax on labor income $\Gamma Y^{L,ra}$ increases, the tax burden rises less for low-income workers and more for high-income workers because the former pay a smaller share of the aggregate tax. When the uniform tax is reduced to balance the government budget, however, all households benefit equally. Overall, the tax exposure channel benefits workers with low labor income and penalizes those with high labor income, with a positive effect on aggregate consumption.

Liquidity. Households with higher savings have smaller MPCs than those with lower savings:

$$cov_I \left(MPC_i, B - b_i' \right) > 0. \tag{59}$$

In the fiscal policy calibration, the bond supply is procyclical $\tilde{B}^{ra} > 0$. When the government cuts taxes through deficit finance, it transfers income from the future to the present. In GE, unconstrained households absorb debt more than the tax reduction, their "effective" income that can be used for present consumption decreases. And it is the opposite for constrained households. The bond demand function (22) implies that the net impact of debt issuance on one's "effective" income is inversely related to its distance from being constrained. Hence, we get the covariance term above. The liquidity channel eases borrowing conditions and amplifies consumption responses.

6 Quantitative Analysis

In this section, I implement the decomposition framework quantitatively. I calibrate the model and consider the model's response to an expansionary monetary policy shock. I use the sequence-space approach developed in Auclert et al. (2021) and Boppart, Krusell and Mitman (2018) to solve the model. To compute redistribution effects, I first solve for the steady-state asset policy function $a'^*(z, a^*)$. Then I add a^* as another exogenous individual state, derive the law of motion for the pair of exogenous individual states (z, a^*) from the policy function $a'^*(z, a^*)$, and input the redistribution shock as a function of the exogenous state (z, a^*).

6.1 Calibration

Table 4 summarizes the parameter values and calibration targets. I calibrate the model to the 2004 US economy, as in Kaplan, Moll and Violante (2018). The annual real interest rate is set at 5% in the steady state, equal to the average real return on equity and government bonds. The coefficient of risk aversion σ is set to 1. The value of total wealth relative to annual output is $(B + P)/Y^{GDP} = 3.21$, which is the sum of government debt to annual output $B/Y^{GDP} = 0.29$ and equity to annual output $P/Y^{GDP} = 2.92$. Following the categorization of Kaplan, Moll and Violante (2018), the value of equity to annual output P/Y^{GDP} is the **net** illiquid assets from the Flow of Funds (FoF) divided by annual GDP, and the value of government debt to annual output B/Y^{GDP} is the **gross** liquid assets from the Survey of Consumer Finances (SCF) divided by annual GDP.[16]

The capital share parameter in the production function α is set to 0.33. The capital depreciation rate is $\delta^K = 0.07$. The steady-state capital stock satisfies $rP = \alpha Y^{GDP} - \delta^K K$, which gives $K/Y^{GDP} = 2.63$. The capitalized markup on the annual output is then $P/Y^{GDP} - K/Y^{GDP} = 0.29$. The steady-state markup $1 - 1/\mu$ satisfies $\alpha(1 - 1/\mu)/r = 0.29$, giving $\mu = 1.05$. The capital share parameter and the markup together imply a capital share of 31% and a labor share of 64%. The slope of the Phillips curve is $\kappa = 0.1$ and the Taylor rule coefficient ϕ is set to 1.25, both standard values in the New Keynesian literature. The capital adjustment cost parameter Ψ is chosen so that the peak response of investment is about twice that of consumption in the HANK model, consistent with the empirical evidence in Christiano, Eichenbaum and Trabandt (2016). The wage elasticity is set to $\epsilon_w = 0.5$. The proportional tax rate on labor income (and profit income) is set to $\Gamma = 0.3$, and the value of the uniform tax to output is $T^{uniform}/Y^{GDP} = -0.06$. Government spending is then determined by the government budget constraint $G/Y^{GDP} = 0.13$.

Income process. The (log) income process is the quarterly process estimated in Kaplan and Violante (2022), which is the sum of two independent components. The first component is a typical AR(1) process with persistence 0.988 and variance of innovations 0.0108, and the second component is the IID with variance 0.2087 (see the second row of Table A.2 in Kaplan and Violante 2022). For the income incidence function, I include the estimates of Guvenen et al. (2017) and normalize to $E_I[z_t\gamma(z_t)] = 1$.

---

[16]The borrowing constraint is normalized to zero so I map bond to gross liquid assets in the data. From Section 4.1 we know that a model with borrowing constraint ϕ, unnormalized bond supply B^{un}, individual bond demand $b(z^t)$, and tax payment $\tau(z^t)$ is isomorphic to a model with borrowing constraint $\phi - \phi$, bond supply $B^{un} - \phi$, individual bond demand $b(z^t) - \phi$, and tax payment $\tau(z^t) - r\phi$. If $B^{un} = 0.26$ is mapped to net liquid assets, then ϕ is simply liquid loans (-0.03 in the data, most of which are consumer loans), and the normalized bond $B^{un} - \phi$ is gross liquid assets, which is 0.29.

Figure 3: RANK effects
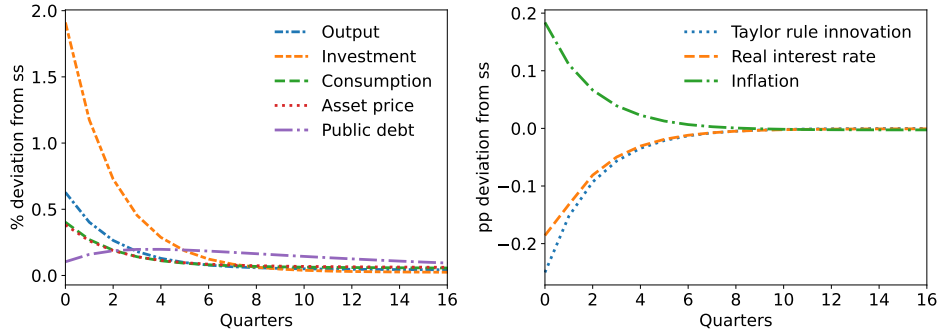
Notes: Impulse responses of the fictitious RANK model to a 25 bp monetary policy shock.

Aggregate MPC.   One-asset HANK models have difficulty matching aggregate MPCs and aggregate wealth simultaneously (Kaplan and Violante 2022). To overcome this issue, I introduce ex-ante heterogeneity in the discount factor as in Carroll et al. (2017). There are five equal-measure groups of households with ex-ante heterogeneous discount factors $\{\beta^m - 2\Delta, \beta^m - \Delta, \beta^m, \beta^m + \Delta, \beta^m + 2\Delta\}$ where β^m is the median household discount factor, and Δ is the dispersion parameter of the discount factor distribution. I calibrate Δ to hit the first (year 0) iMPC as in Auclert, Rognlie and Straub (2024). Figure 10 in the Appendix shows that the other iMPCs fit well with Fagereng, Holm and Natvik (2021)'s estimates.

Asset portfolio.   The household portfolio between bonds and equity is undetermined. I assume that households have the same portfolio between bonds and equity as the aggregate portfolio $b(z^t)/a(z^t) = B_t/A_t$.

Fiscal Policy.   The literature makes ad-hoc assumptions on fiscal policies. Section 4.1 implies that the level of public debt is related to the level of borrowing constraint, which is also clear from the normalization identity $B = B^{un} - \phi$. Based on this observation, I adopt a novel calibration strategy. I estimate the response of consumer loans ϕ to the shock and convert its response to public debt $B^{un} - \phi$ in the model. The estimated percent response is $\hat{\phi} = 1\%$ and level response is $\tilde{\phi} = -0.03 \cdot 1\%$. The parameter ϕ^B is chosen such that on impact the public debt rises in level by $0.03 \cdot 1\%$.

6.2   Decomposition of Aggregates

At time $t = 0$, there is an innovation in the Taylor rule of $\epsilon_0 = -0.25$ percent (-1 percent annually) with a quarterly persistence of 0.61. Figure 3 shows the responses of the RA model. Nominal interest rates fall, stimulating consumption and investment.

Figure 4: Decomposition of the HANK model's responses to a monetary policy shock
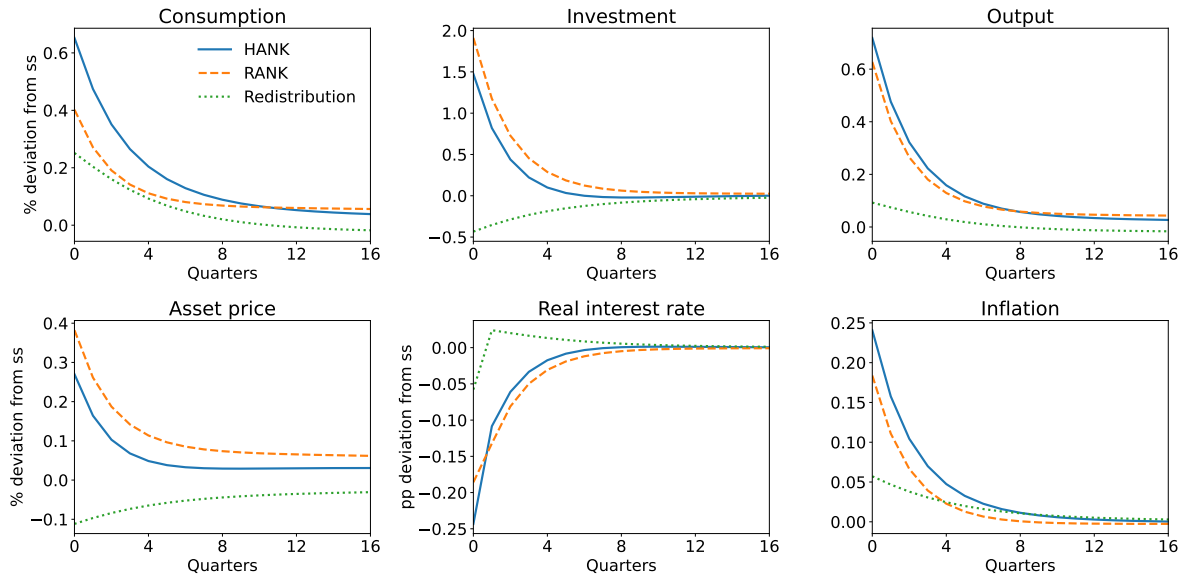
Notes: At time 0, the Taylor rule innovation $\epsilon_0 = 25$ basis points. RANK effects are the responses of the fictitious RANK model to the monetary policy shock, and redistribution effects are the HANK model's responses to the redistribution shock.

Given the sticky price, the increase in aggregate demand leads to an increase in output and inflation. The calibrated fiscal policy implies that public debt also increases.

The responses of aggregate variables are decomposed into RANK and redistribution effects in Figure 4. Using RANK effects as a benchmark, redistribution effects amplify the responses of output and consumption while dampening the responses of investment and real interest rates. Redistribution effects account for 38% of the consumption response and 14% of the output response on impact. Similar to its effects on output, redistribution also amplifies inflation responses. Since the redistribution shock raises real interest rates, it dampens the responses of asset prices.

To assess the impact of the redistribution channel, I input each channel into the model separately, and the decomposition of redistribution effects is shown in Figure 5. The effects of the terms "saving flow exposure" are close to zero and not shown. The interest rate exposure channel stands out as the largest amplifier of the consumption response, contributing to more than one-third of the total amplification. Consistent with the partial-equilibrium predictions of Section 5.3, the redistribution from creditors to debtors amplifies the consumption response. The income exposure channel is the second-largest amplifier. On impact, the three subchannels of income exposure collectively increase consumption by 0.09 percent. Most amplification effects within the income exposure channel are attributed to income portfolio exposure. From the RANK effects, we observe that investment is more responsive than consumption before quarter 8 and less responsive after quarter 8. This pattern implies a redistribution

Figure 5: Decomposition of redistribution effects
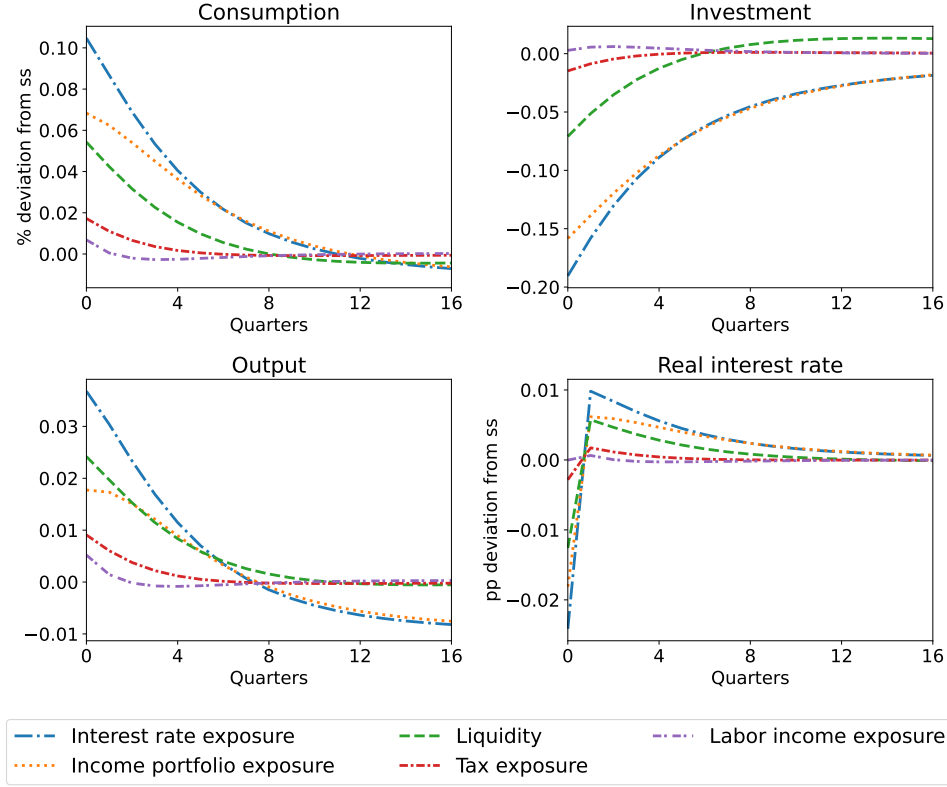
Notes: The redistribution shock's effects on consumption, investment, output, and real interest rates are decomposed into five channels. The redistribution shock is triggered by a monetary policy shock of 25 basis points. Section 5 gives the definitions of these redistribution channels.

from equity holders to workers before quarter 8 and the reverse after quarter 8, amplifying the consumption response. The liquidity channel is the third-largest amplifier. By increasing the supply of bonds, the government allows households to better insure themselves against income risks, leading to an increase in aggregate spending. Contrary to the commonly assumed stabilizing fiscal policy, the liquidity channel acts as an amplifier rather than a dampener. Low-labor-income households benefit from the overall tax reduction; the tax exposure channel slightly increases aggregate consumption. Estimates from Guvenen et al. (2017) suggest that both low- and high-labor-income households are more exposed to business cycle fluctuations. The net effect on consumption is positive but small.

In summary, all redistribution channels amplify consumption responses, with three playing the most critical roles: the interest rate exposure channel, the income portfolio exposure channel, and the liquidity channel. The labor income and tax exposure channels have a minor impact.

The decomposition for output is qualitatively similar to that for consumption, but with a smaller magnitude. This is because if one channel amplifies the consumption response, it dampens the investment response: households with a higher MPC have

Figure 6: Decomposition of individual consumption responses (impact)
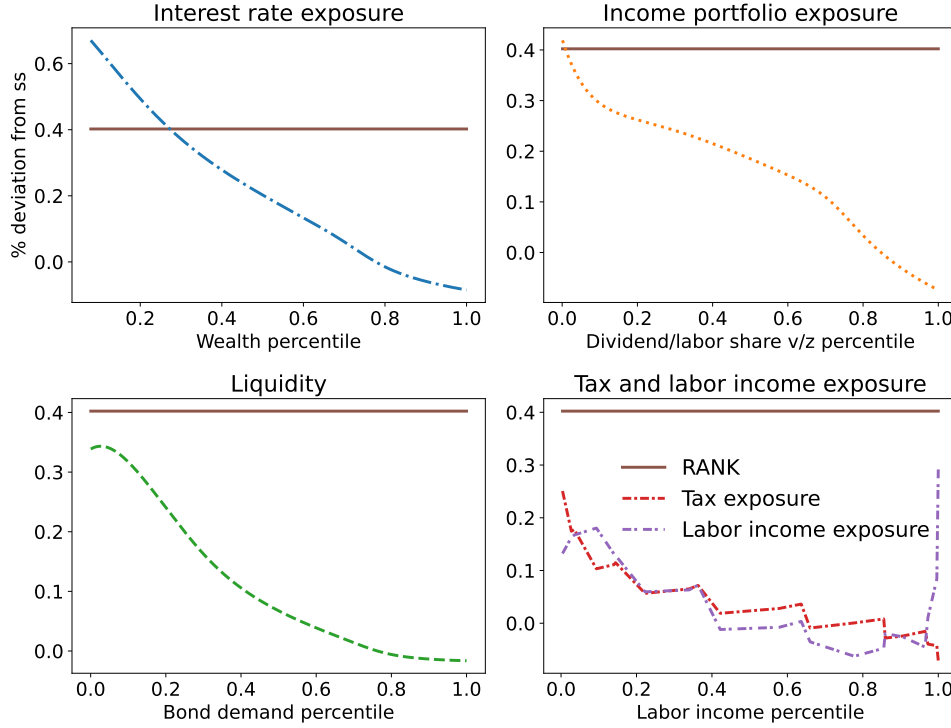
Notes: The redistribution effects on individual consumption (impact) are decomposed into five channels. I also show the RANK effects, which are homogeneous across individuals. Each channel is plotted against its redistribution dimension. Section 5 gives the definitions of these channels.

a lower marginal propensity to save. As a result, the net effects on output are smaller than on consumption. At the aggregate level, redistribution induces households to consume more and accumulate less. Due to the dampened investment responses, the long-run amplification of consumption and output responses can be reversed. Starting in quarter 8, the impact of the interest rate channel on output becomes negative as the capital stock declines, leading to a reduction in output.

6.3 Individual-level Decomposition

Figure 6 shows the decomposition of individual consumption responses (on impact).[17] The effects of each channel are plotted along its redistribution dimension. The effects of the interest rate exposure channel are shown across the wealth distribution because bond holdings determine a household's exposure to interest rate changes. The effects of the income portfolio exposure channel are shown across the distribution of the dividend income share relative to the labor income share (v/z). The ratio v/z

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[17]The effects of the interest rate exposure, income portfolio exposure, and liquidity channels are estimated using local linear regression on model-generated data with a Gaussian kernel and a bandwidth of 0.1. The effects of the tax and labor income exposure channels on consumption for a given level of productivity are the weighted consumption responses across the wealth (and discount factor) distribution.

Table 3: The contribution of redistribution (channels) to total consumption responses.

	Redistribution	Income Exposure			Interest Rate Exposure	Liquidity	
		Portfolio	Labor	Tax		Bond Supply	Illiquid Assets
Werning (2015)	0	0	0	0	0	0	N.A.
McKay, Nakamura and Steinsson (2016)	-99%	N.A.	-80%	-44%	25%	0	N.A.
Bilbiie (2020)	33%	N.A.	33%	0	0	0	N.A.
Auclert, Rognlie and Straub (2024)	143%	-7%	0	-4%	11%	160%	-17%
Wolf (2021); Wolf (2023); Angeletos, Lian and Wolf (2023)	100%	N.A.	0	0	0	100%	N.A.

Notes: In the quantitative models, the total consumption responses are calculated as the sum of consumption responses over the period from 0 to 300. The effects of "saving flow exposure" are omitted.

determines the income elasticity of a household when the share of dividends in aggregate income changes. The liquidity channel is represented by the bond demand percentile. The redistribution of the tax and labor income channels operates through the dimension of labor productivity.

The average consumption response of poor households is higher than that of rich households due to the interest rate cut. From the decomposition of aggregates, we know that redistributive effects account for 38% of the aggregate consumption response. At the individual level, however, redistributive effects can account for a much larger share of the consumption response. For households in the lowest wealth percentile, the impact of interest rate exposure on consumption is more than 150% of the RANK effects (0.6/0.4). For the richest households, the interest rate exposure channel has negative effects, dampening their consumption responses.

The income portfolio exposure channel allows households with a low dividend income share (v) but a high labor income share (z) to consume the additional income from producing capital. Conversely, households with a high dividend but a low labor income share reduce their immediate consumption and save the return on capital for future consumption. The liquidity channel eases households' borrowing constraints. Households far from borrowing constraints lend to those closer to them in a relatively homogeneous manner: the top 20% of the wealthy households in the bond demand b' distribution show similar consumption cuts. According to Guvenen et al. (2017)'s estimates, labor income elasticities exceed 1 ($\gamma(z_i) > 1$) at both the low and high ends of the labor income distribution, consistent with the consumption responses of workers. The consumption of the median household in the labor income distribution is negatively affected by the labor income channel. The tax exposure channel dampens the consumption of high-labor-income households while amplifying the consumption responses of low-labor-income households.

Figure 7: Output response decomposition: McKay, Nakamura and Steinsson (2016)
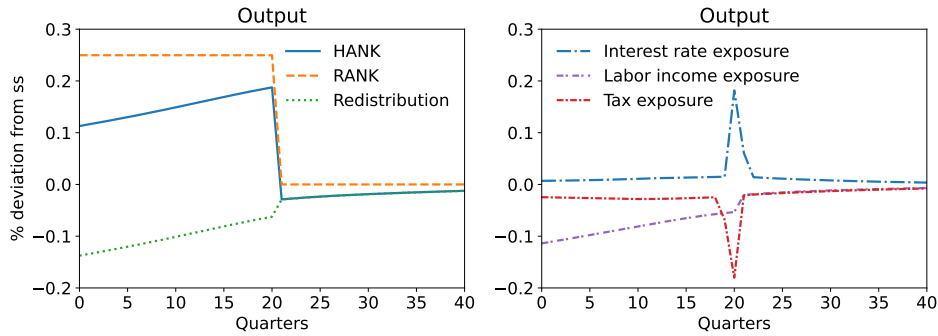
Notes: McKay, Nakamura and Steinsson (2016) consider the economy's response to a one-time 50-basis-point real rate shock in Quarter 20. The left panels decompose the HANK model's output response into RANK and redistribution effects. The right panel further decomposes the redistribution effects into the contribution of interest rate, (labor) income, and tax exposure channels.

7   Application to Literature

In this section, I apply the decomposition framework to the literature. Table 3 summarizes the contribution of redistribution (channels) to consumption responses in these models. The details of the decomposition can be found in the Appendix C.

Werning (2015) analyzes scenarios in which an incomplete market economy can be aggregated as an "as if" RA economy. With zero liquidity and acyclic income risk, the generalized Euler equation derived in Werning (2015) is consistent with that of a representative agent. In the Appendix, I show that the "as if" RA economy in Werning (2015) corresponds to the fictitious RA economy defined in Proposition 1. Substituting the assumptions of the "as if" economy (Section 3.2 and 4 in Werning 2015) into the definition of redistribution channels 49, we can verify that counterfactual transfers are zero and all redistribution channels are muted.

McKay, Nakamura and Steinsson (2016) studies the forward guidance puzzle in an incomplete market model. They consider a one-time 50 basis point real interest rate cut 20 quarters into the future, with real interest rates unchanged in all other quarters. In the RANK model, output immediately increases by 25 basis points and remains at that level for 20 quarters. In the HANK model, the initial increase in output is only about 10 basis points. Two model assumptions: (i) firm profits are distributed uniformly to households; (ii) only the highest-income households that pay taxes are the main drivers of the negative redistribution effects. The assumption that firm profits are equally distributed to households implies that countercyclical profits account for a larger share of total income for low-income households, resulting in lower income elasticities of low-income households. The second assumption, that only the highest-income households pay taxes, implies that only the highest-income households benefit from the tax cut of quarter 20. Both assumptions lead to a redistribution from low-

Figure 8: Consumption response decomposition: Auclert, Rognlie and Straub (2024)
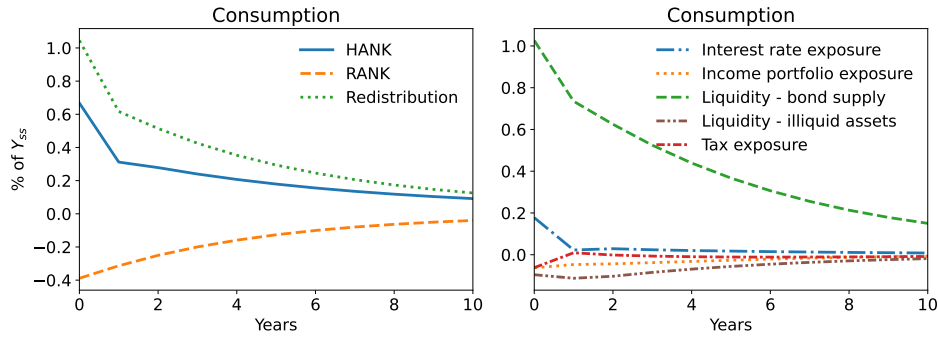
Notes: Auclert, Rognlie and Straub (2024) considers the economy's responses to a government spending shock (1% of the steady-state output) in a two-account quantitative model. The left panels decompose their model's consumption responses into RANK and redistribution effects. The right panel decomposes the redistribution effects into the contribution of redistribution channels.

income to high-income households. Figure 7 shows that the redistribution through the income channel dampens consumption responses from quarter 0 to quarter 19, while the redistribution due to taxation further dampens the response in quarter 20.

Bilbiie (2020) discusses the amplification mechanism in a TANK model.[18] In the TANK model, savers receive a smaller share of countercyclical firm profits compared to hand-to-mouth households, which leads to unequal income elasticities. The only active redistribution channel is the (labor) income exposure channel.[19]

Auclert, Rognlie and Straub (2024) studies fiscal multipliers in HANK models and finds that deficit-financed multipliers can be greater than one. Section 7 in Auclert, Rognlie and Straub (2024) considers a fully specified two-account quantitative model.[20] The decomposition of consumption responses is shown in Figure 8. The redistribution effects on consumption are positive, reversing the sign of the consumption response in RANK. When the government increases the supply of bonds, the borrowing conditions of households are eased, stimulating aggregate consumption. The liquidity channel of bond supply explains most of the redistribution effects.

Wolf (2021), Wolf (2023), and Angeletos, Lian and Wolf (2023) study the role of deficit-financed lump-sum fiscal transfers as a stimulating policy tool, which essen-

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[18]Section 5 of Bilbiie (2020) calibrates the TANK model to match the amplification magnitude of Kaplan, Moll and Violante (2018), in which the redistribution effects contribute to one-third of the total consumption responses.

[19]Note that labor income in Table 3 includes both earnings from labor supply and firm profits. According to this definition, all income in Bilbiie (2020) is labor income. The original definition in Bilbiie (2020) solely considers earnings derived from supplying labor $W_t n_t$ as labor income.

[20]Compared to the one-asset models, there is an additional channel to consider in two-asset models: the liquidity channel of illiquid assets (refer to section F.2 in the Appendix for the formal definition). The change in the return on illiquid assets impacts the illiquid assets that non-adjusters are compelled to accumulate. Roughly speaking, if the aggregate shock increases the return on illiquid assets, households are forced to accumulate more illiquid assets, dampening aggregate consumption.

tially reflects the liquidity channel defined here. Since Ricardian equivalence holds in the RANK model, all consumption responses in the HANK model are due to redistribution effects, and the only active channel is the liquidity channel of bond supply.

8   Conclusion

This paper proves the existence of a no-redistribution benchmark for incomplete-market HA models and develops a framework that decomposes the HA model's response to an aggregate shock into two components: the response of a fictitious RA model and the response of the HA model to the induced redistribution. The framework enhances the understanding of how HA models respond to aggregate shocks. I analytically characterize the redistribution channels and quantify their relative importances. This study offers a valuable tool for developing HA models where the strength of these channels is grounded in empirical evidence.

This paper also opens up avenues for future research. One key aspect not addressed here is the implications of aggregate uncertainty and endogenous portfolio choices for redistribution effects. In Section 6, ad-hoc assumptions are made regarding portfolios between bonds and equity. The decomposition result for one channel would depend on asset portfolios if bond or equity holdings enter its definition. In two-asset models, the realization of return influences households' accumulation of illiquid assets, which has substantial effects on consumption and output. Section F.3 and F.4 in the Appendix highlight how this mechanism depends on the composition of illiquid assets. Incorporating aggregate uncertainty would allow households to optimize their portfolios and better hedge against the associated risks. Explorations in this direction are made by Bhandari et al. (2023) and Auclert et al. (2024).

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Appendix

A Proofs

Proof of Proposition 1.   We need to show that the consumption and labor supply allocation in Proposition 1 satisfies the equilibrium conditions of the HANK model. The aggregation and the market-clearing conditions are easy to verify. In the following, I show that the individual allocation satisfies individual optimality conditions.

First, I impose the bond demand function $b^{ra}(z^t) = b^*(z^t)$ which satisfies the borrowing constraint $b^{ra}(z^t) \geq \phi$. I verify the F.O.C. with respect to bond demand

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t], = \text{if } b^{ra}(z^t) > \phi. \tag{60}$$

and the transversality condition

$$\lim_{t \to \infty} \beta^t E_0 (b^{ra}(z^t) - \phi) u'(c^{ra}(z^t)) = 0. \tag{61}$$

To see the F.O.C (60) holds, substituting the individual consumption allocation $c^{ra}(z^t) = c^*(z^t) \cdot C_t^{ra}/C^*$ into both sides of (60):

$$(c^{ra}(z^t))^{-\sigma} = (C_t^{ra}/C^*)^{-\sigma}(c^*(z^t))^{-\sigma}.$$
$$\beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t] = \beta R_{t+1}^{ra}(C_{t+1}^{ra}/C^*)^{-\sigma} E[(c^*(z^{t+1}))^{-\sigma}|z^t]$$
$$= \beta \frac{R_{t+1}^{ra}}{R^*} R^*(C_{t+1}^{ra}/C^*)^{-\sigma} E[(c^*(z^{t+1}))^{-\sigma}|z^t]$$
$$= \beta^{ra} R_{t+1}^{ra}(C_{t+1}^{ra}/C^*)^{-\sigma} \beta R^* E[(c^*(z^{t+1}))^{-\sigma}|z^t] \tag{62}$$
$$= (C_t^{ra}/C^*)^{-\sigma} \beta R^* E[(c^*(z^{t+1}))^{-\sigma}|z^t]. \tag{63}$$

Equations (62) and (63) hold because $\beta^{ra} \equiv 1/R^*$ and $(C_t^{ra})^{-\sigma} = \beta^{ra} R_{t+1}^{ra}(C_{t+1}^{ra})^{-\sigma}$. We know that in the steady state, the following F.O.C. holds

$$(c^*(z^t))^{-\sigma} \geq \beta R^* E[(c^*(z^{t+1})^{-\sigma}|z^t], = \text{if } b^*(z^t) > \phi. \tag{64}$$

Multiply both sides of (64) by $(C_t^{ra}/C^*)^{-\sigma}$, we have

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t].$$

In the case of $b^{ra}(z^t) > \phi$, it must be the case that $b^*(z^t) > \phi$ and the F.O.C in the steady state (64) holds with equality; therefore, (60) also holds with equality. We have a useful corollary.

Corollary. The Euler equation holds with equality in the "RANK" equilibrium if and only if it holds with equality in the steady state

$$u'(c^{ra}(z^t)) = \beta R_{t+1}^{ra} E[u'(c^{ra}(z^{t+1})|z^t] \iff u'(c^*(z^t)) = \beta R^* E[u'(c^*(z^{t+1})|z^t] \quad (65)$$

To see the transversality condition (61), substituting the individual bond demand and consumption allocation into (61)

$$\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) = \lim_{t\to\infty} (C_t^{ra}/C^*)^{-\sigma}\beta^t E_0(b^*(z^t) - \phi)(c^*(z^t))^{-\sigma}. \quad (66)$$

Since $\lim_{t\to\infty}(C_t^{ra}/C^*)^{-\sigma} = 1$ and $b^*(z^t)$ satisfy the transversality condition in the steady state $\lim_{t\to\infty} \beta^t E_0(b^*(z^t) - \phi)u'(c^*(z^t)) = 0$, we can see that the transversality condition holds.

Second, I verify the individual labor supply condition

$$W_t^{ra} z_t (c^{ra}(z^t))^{-\sigma} = \varphi(n^{ra}(z^t))^{\nu}. \quad (67)$$

Substituting the consumption and labor supply into each side of (67)

$$W_t^{ra} z_t (c^{ra}(z^t))^{-\sigma} = W_t^{ra} z_t (c^*(z^t))^{-\sigma}(C_t^{ra}/C^*)^{-\sigma}$$
$$= W^* z_t (c^*(z^t))^{-\sigma} W_t^{ra}/W^*(C_t^{ra}/C^*)^{-\sigma} \quad (68)$$
$$\varphi(n^{ra}(z^t))^{\nu} = \varphi(n^*(z^t))^{\nu}(N_t^{ra}/N^*)^{\nu}. \quad (69)$$

From the aggregate labor supply condition in the "RANK" equilibrium $W_t^{ra}(C_t^{ra})^{-\sigma} = \varphi^{ra}(N_t^{ra})^{\nu}$ where $\varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}$ we have

$$\frac{W_t^{ra}}{W^*}(\frac{C_t^{ra}}{C^*})^{-\sigma} = (\frac{N_t^{ra}}{N^*})^{\nu}. \quad (70)$$

Multiply the individual labor supply condition in the steady state

$$W^* z_t (c^*(z^t))^{-\sigma} = \varphi(n^*(z^t))^{\nu} \quad (71)$$

by (70) we verify the individual labor supply condition (67).

Finally, the transfer is recovered from the budget constraint :

$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - W_t^{ra} z_t n^{ra}(z^t) - \pi^{ra}(z_t) + \tau^{ra}(z^t). \quad (72)$$

Aggregating over transfers $\omega(z^t)$,

$$\int \omega(z^t) d\Phi_t(z^t) = C_t^{ra} + B^* - R_t^{ra} B^* - W_t^{ra} N_t^{ra} - \Pi_t^{ra} + T_t^{ra}. \quad (73)$$

The market clearing condition $C_t^{ra} = W_t^{ra} N_t^{ra} + \Pi_t^{ra}$ and the government's budget constraint $B^* + T_t^{ra} = R_t^{ra} B^*$ in the "RANK" equilibrium imply that $\int \omega(z^t) d\Phi_t(z^t) = 0$.

The quantitative model in Section 6 assumes permanent heterogeneity in discount factors β^i, and it is straightforward to verify the above proofs under this specification.

Proof of Proposition 2. By construction, the imposed bond demand satisfies the borrowing constraint and transversality condition. The F.O.C w.r.t bond demand is

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t], = \text{ if } b^{ra}(z^t) > \phi, \tag{74}$$

In the proof of Proposition 1 we show

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t]. \tag{75}$$

In the case of $b^{ra}(z^t) > \phi$, from term (i) of Proposition 2, it can only be the case that

$$(c^{ra}(z^t))^{-\sigma} = \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t]. \tag{76}$$

The complementary slackness condition is verified.

Sources of redistribution in the canonical model of Section 3.3. Rewrite the household's budget constraint in the steady state and the "RANK" equilibrium below,

$$0 = c^*(z^t) + b^*(z^t) - R^* b^*(z^{t-1}) - y^*(z^t) + \tau^*(z^t), \tag{77}$$
$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - y^{ra}(z^t) + \tau^{ra}(z^t). \tag{78}$$

Subtracting equation (78) from (77)

$$
\begin{aligned}
-\omega(z^t) =\, & c^*(z^t) - c^{ra}(z^t) + b^*(z^t) - b^{ra}(z^t) - [R^* b^*(z^{t-1}) - R_t^{ra} b^{ra}(z^{t-1})] \\
& - (y^*(z^t) - y^{ra}(z^t)) + (\tau^*(z^t) - \tau^{ra}(z^t)) \\
=\, & -\hat{C}_t^{ra} c^*(z^t) + (b^*(z^t) - \bar{b}^{ra}(z^t) + \bar{b}^{ra}(z^t) - b^{ra}(z^t)) \\
& - [R^* b^*(z^{t-1}) - R_t^{ra}(b^{ra}(z^{t-1}) - b^*(z^{t-1}) + b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}) + \bar{b}^{ra}(z^{t-1}))] \\
& + \hat{y}^{ra}(z^t) y^*(z^t) - \hat{Y}_t^{ra} y^*(z^t) + \hat{Y}_t^{ra} y^*(z^t) + (\tau^*(z^t) - \bar{\tau}^{ra}(z^t) + \bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) \\
=\, & (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) + \hat{Y}_t^{ra} y^*(z^t) - \hat{C}_t^{ra} c^*(z^t) + b^*(z^{t-1})(R_t^{ra} - R^*) - (\bar{\tau}^{ra}(z^t) - \tau^*(z^t)) \\
& + (\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) \\
& + (b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1})).
\end{aligned}
$$

(79)

(80)

(81)

Add equation $0 = -B^*(R_t^{ra} - R^*) - r^*B^* + r_t^{ra}B^*$ into $-\omega(z^t)$ we have

$$-\omega(z^t) = (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \tag{82}$$

$$+ (b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*) \tag{83}$$

$$+ (\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*) \tag{84}$$

$$+ (\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) \tag{85}$$

$$+ (b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1})). \tag{86}$$

In the case that government debt is constant $B_t = B^*$, we have $\bar{b}^{ra}(z^t) = b^{ra}(z^t)$ and $\bar{\tau}^{ra}(z^t) = \tau^{ra}(z^t)$, the term "liquidity channel" (85) is zero. In the case of $b^*(z^t) = \bar{b}^{ra}(z^t)$, the last term "undetermined bond demand" (86) is zero.

Proof of $b^{ra}(z^t)$ in Section 3.4 as an equilibirum bond demand function. In the case of constant bond supply $B_t = B^*$, the proof of Proposition 1 shows that $b^{ra}(z^t) = b^*(z^t)$ is an equilibrium bond demand function. Below, I verify that $b^{ra}(z^t)$ satisfies Proposition 2 if B_t is time-varying. Assume the fiscal rule induces a time-varying bond supply such that $B_t > \phi$ and $\lim_{t\to\infty} B_t = B^*$. First,

$$b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi) \geq \phi \tag{87}$$

satisfies the borrowing constraint. Next, from the proof of Proposition 1, we can see

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1})^{-\sigma}|z^t] \tag{88}$$

holds regardless of the choice of bond demand function. In the case of $b^{ra}(z^t) > \phi$, from $b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi)$ it follows $b^*(z^t) > \phi$. According to the Corollary in the proof of Proposition 1, households are unconstrained in the steady state as well as in the "RANK" equilibrium, thus the complementary slackness condition holds. The transversality condition also holds

$$\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) \tag{89}$$

$$= \lim_{t\to\infty} (C_t^{ra}/C^*)^{-\sigma}\beta^t E_0(\phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi) - \phi)(c^*(z^t))^{-\sigma} \tag{90}$$

$$= \lim_{t\to\infty} \beta^t E_0(b^*(z^t) - \phi)u'(c^*(z^t)) = 0. \tag{91}$$

The bond market clearing follows

$$\int b^{ra}(z^t)d\Phi_t(z^t) = \int [\phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi)]d\Phi_t(z^t) = \phi + \frac{\int b^*(z^t)d\Phi_t(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi) = B_t^{ra}.$$

Proof of Proposition 3.   It's easy to verify that $b^{ra}(z^t)$ satisfies the conditions in Proposition 3 if $\bar{b}^{ra}(z^t)$ satisfies the conditions in Proposition 2, since $b^{ra}(z^t)$, $\tau^{ra}(z^t)$ and ϕ_t^{ra} all shift by the same amount $B_t^{ra} - B^*$ from their constant-debt counterparts $\bar{b}^{ra}(z^t)$, $\bar{\tau}^{ra}(z^t)$ and ϕ.

To see that the transfers are invariant to the path of government debt,

$$b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) + \tau^{ra}(z^t) \tag{92}$$
$$= (\bar{b}^{ra}(z^t) + B_t^{ra} - B^*) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) + B_{t-1}^{ra} - B^*) + \bar{\tau}^{ra}(z^t) + T_t^{ra} - \bar{T}_t^{ra}$$
$$= \bar{b}^{ra}(z^t) - R_t^{ra}\bar{b}^{ra}(z^{t-1}) + \bar{\tau}^{ra}(z^t). \tag{93}$$
$$\text{Thus } \omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - y^{ra}(z^t) + \tau^{ra}(z^t) \tag{94}$$
$$= c^{ra}(z^t) + \bar{b}^{ra}(z^t) - R_t^{ra}\bar{b}^{ra}(z^{t-1}) - y^{ra}(z^t) + \bar{\tau}^{ra}(z^t). \tag{95}$$

Sources of redistribution with investment in Section 4.2.2.   For simplicity, first, assume that households only have access to equity. Define $y \equiv Dv_- + zWn + \pi$ as individual income. The budget constraints are

$$c^{ra}(z^t) + P_t^{ra} v^{ra}(z^t) = P_t^{ra} v^{ra}(z^{t-1}) + y^{ra}(z^t) + \omega(z^t). \tag{96}$$

Subtracting the budget constraint in the steady state from equation (96)

$$-\omega(z^t) = P_t^{ra}(v^*(z^{t-1}) + v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P^* v^*(z^{t-1}) + \hat{y}^{ra}(z^t)y^*(z^t)$$
$$\quad - (P_t^{ra}(v^*(z^t) + v^{ra}(z^t) - v^*(z^t)) - P^* v^*(z^t)) - \hat{C}_t^{ra} c^*(z^t) \tag{97}$$
$$= (P_t^{ra} - P^*)v^*(z^{t-1}) + (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) - (P_t^{ra} - P^*)v^*(z^t)$$
$$\quad + P_t^{ra}(v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P_t^{ra}(v^{ra}(z^t) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \tag{98}$$
$$= (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) + (P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))$$
$$\quad + P_t^{ra}(v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P_t^{ra}(v^{ra}(z^t) - v^*(z^t)). \tag{99}$$

Imposing $v^{ra}(z^t) = v^*(z^t)$, the last term "undetermined equity demand" (99) is zero. The saving flow exposure with equity is

$$(P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))$$

In the case that the budget constraint includes bonds, the above derivation also holds.

Decomposition of the income exposure channel in Section 5.2.   For simplicity, first, assume that there is no government spending and the income only includes dividends and labor income $y(z^t) = D_t v(z^{t-1}) + y^L(z^t)$. Aggregate income $Y = WN + (1 - \alpha)\Pi + D$ satisfies $C = Y$. Define $y^L \equiv zWn + \pi$ as individual labor income and

$Y^L \equiv WN + (1-\alpha)\Pi$ as aggregate labor income, then

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) \tag{100}$$

$$=y^{ra}(z^t) - y^*(z^t) - \hat{Y}_t^{ra}y^*(z^t) \tag{101}$$

$$=\hat{D}_t^{ra}D^*v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t)y^{L,*}(z^t) - \hat{Y}_t^{ra}y^*(z^t) \tag{102}$$

$$=\hat{D}_t^{ra}D^*v^*(z^{t-1}) + (\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t) + \hat{Y}_t^{L,ra}y^{L,*}(z^t) - \hat{Y}_t^{ra}y^*(z^t) \tag{103}$$

$$=(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*v^*(z^{t-1}) + (\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t) + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra})y^{L,*}(z^t) \tag{104}$$

$$=(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t) + (\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*v^*(z^{t-1}) + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra})y^{L,*}(z^t). \tag{105}$$

From $Y_t = Y_t^L + D_t$ we know $\hat{Y}_t Y^* = \hat{Y}_t^L Y^{L,*} + \hat{D}_t D^*$ and

$$(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^* + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra})Y^{L,*} = 0,$$

$$(\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra})y^{L,*}(z^t) = -(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*\frac{y^{L,*}(z^t)}{Y^{L,*}}. \tag{106}$$

Substituting equation (106) into equation (105)

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)$$

$$= \underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}})}_{\text{income portfolio exposure}}. \tag{107}$$

With net taxes in income $y(z^t) = D_t v(z^{t-1}) + y^L(z^t) - \tau^n(z^t)$,

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) \tag{108}$$

$$=y^{ra}(z^t) - y^*(z^t) - \hat{Y}_t^{ra}y^*(z^t) \tag{109}$$

$$=\hat{D}_t^{ra}D^*v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t)y^{L,*}(z^t) + \tau^{n,*} - \tau^{n,ra} - \hat{Y}_t^{ra}(D^*v^*(z^{t-1}) + y^{L,*}(z^t) - \tau^{n,*}) \tag{110}$$

$$=\hat{D}_t^{ra}D^*v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t)y^{L,*}(z^t) - \hat{Y}_t^{ra}(D^*v^*(z^{t-1}) + y^{L,*}(z^t)) + (1 + \hat{Y}_t^{ra})\tau^{n,*} - \tau^{n,ra} \tag{111}$$

The term $(1 + \hat{Y}_t^{ra})\tau^{n,*} - \tau^{n,ra}$ is the tax-related channels, and the remaining parts are labor and portfolio income exposure channels, which can be derived as in (105).

With positive government spending, redefine $y^L(z^t) \equiv y^{GL}(z^t) - \tau^G(z^t)$ and $Y_t^L \equiv Y_t^{GL} - G$ as equation (48), the derivations hold.

Simplify redistribution channles in Section 5.2. For the tax exposure channel, omit the difference between $\tau^{G,ra}(z^t)$ and $\tau^{G,*}(z^t)$ since government spending is constant,

$$\tau^{n,*}(z^t) - \tau^{n,ra}(z^t) = (\tau^*(z^t) - \tau^{G,*}(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - \tau^{G,ra}(z^t) - r_t^{ra}B^*)$$

$$= (\Gamma y^{GL,*}(z^t) + T^{uniform,*} - r^*B^*) - (\Gamma y^{GL,ra}(z^t) + \bar{T}_t^{uniform,ra} - r_t^{ra}B^*).$$

The budget constraint of the government implies $\bar{T}_t^{uniform} - r_t B^* = G - \Gamma Y_t^{GL}$, so

$$\Gamma y^{GL,*}(z^t) + T^{uniform,*} - r^* B^* - (\Gamma y^{GL,ra}(z^t) + \bar{T}_t^{uniform,ra} - r_t^{ra} B^*)$$
$$= \Gamma y^{GL,*}(z^t) - \Gamma y^{GL,ra}(z^t) - (\Gamma Y^{GL,*} - \Gamma Y_t^{GL,ra})$$
$$= -\Gamma \gamma(z_t) \hat{Y}_t^{GL,ra} y^{GL,*}(z^t) + \Gamma \hat{Y}_t^{GL,ra} Y^{GL,*}$$
$$= \Gamma \hat{Y}_t^{GL,ra} Y^{GL,*}(1 - \gamma(z_t) z_t)$$
$$= \Gamma (Y_t^{L,ra} - Y^{L,*})(1 - \gamma(z_t) z_t)$$

For the liquidity channel of bond supply, notice that the government adjusts uniform taxes to balance its budget, implying

$$\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t) = \bar{T}_t^{uniform,ra} - T_t^{uniform,ra} \text{ and,} \tag{112}$$
$$B_t^{ra} - B^* + T_t^{uniform,ra} - \bar{T}_t^{uniform,ra} = R_t^{ra}(B_{t-1}^{ra} - B^*). \tag{113}$$

B Quantitative Results

B.1 Computation

The method introduced in Section 3.4 has a high computational cost because the number of discretized states grows exponentially relative to the original problem. Section F introduces illiquid assets and models illiquidity a la Calvo, where households face the IID adjustment shock s_t that takes two values indicating whether households can adjust their illiquid assets or not. Consider a five-point Markov process and two asset grids with 50 points each for liquid and illiquid assets. The original problem with four state variables $(z, s, a^{liq}, a^{illiq})$ has $5 * 2 * 50 * 50 = 25,000$ individual states. With six state variables $(z, s, a^{liq,*}, a^{illiq,*}, a^{liq}, a^{illiq})$, there will be $5 * 2 * 50 * 50 * 50 * 50 = 62,500,000$ individual states. The computation is much more demanding than the original problem and will make the decomposition of two-asset models infeasible.

This section introduces a simplified computation method that yields results practically identical to those obtained by the method above. The idea is to make transfers directly based on the household's equilibrium states (z, b). Consider a simpler problem where we verify the "RANK" equilibrium numerically. Then we do not need the third state variable $b^*(z^t)$ because the relation between $b^{ra}(z^t)$ and $b^*(z^t)$ is known. We can build a mapping from the household's asset state in the "RANK" equilibrium $b^{ra}(z^t)$ to the household's asset state in the steady state $b^*(z^t)$ and make transfers based on $b^{ra}(z^t)$ (and z_t). Inverting the monotonic asset demand function imposed in the

"RANK" equilibrium $g_t(\cdot)$ to build this mapping:

$$b^*(z^t) = g_t^{-1}(b^{ra}(z^t)), \forall z^t. \tag{114}$$

Along the transition path of the "RANK" equilibrium, the household problem is

$$V_t^{ra}(z,b) = \max_{\{c,n,b'\}} u(c,n) + E[V_{t+1}^{ra}(z'',b')|z], \tag{115}$$

$$s.t. \quad c + b' = R_t b + W_t z n + \pi_t(z) - \tau_t(z) + \omega_t(z, g_{t-1}^{-1}(b)), \tag{116}$$

$$b' \geq \phi, \tag{117}$$

where $\omega_t(z, g_{t-1}^{-1}(b))$ is the transfers received by households of type (z, b^*). The pair of equilibrium asset state and the transfers received is a fixed point: Given the asset state b^{ra}, households receive $\omega_t(z, g_{t-1}^{-1}(b^{ra}))$; and given the transfers $\omega_t(z, g_{t-1}^{-1}(b^{ra}))$, the asset state of the household is b^{ra}. We can use the simplified method to verify the "RANK" equilibrium with only equilibrium states (z, b).

When computing redistribution effects, building the mapping from the equilibrium asset state $b(z^t)$ to $b^*(z^t)$ without knowing z^t is infeasible. However, as long as the deviation of $b(z^t)$ from $b^*(z^t)$ is small relative to the transfers received, we can approximate $b^*(z^t)$ with the equilibrium asset state $b(z^t)$ and directly make transfers based on $b(z^t)$:

$$V_t(z,b) = \max_{\{c,n,b'\}} u(c,n) + E[V_{t+1}^{ra}(z'',b')|z], \tag{118}$$

$$s.t. \quad c + b' = R_t b + W_t z n + \pi_t(z) - \tau_t(z) + \omega_t(z, b), \tag{119}$$

$$b' \geq \phi. \tag{120}$$

To estimate the impact of the approximation error on the decomposition results, consider the interest rate exposure channel as an example. The transfers households receive should be $(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)$, and the actual transfers households receive is $(b(z^{t-1}) - B^*)(R_t^{ra} - R^*)$. Omitting the term $B^*(R_t^{ra} - R^*)$, which is the same across all states, the relative approximation error is

$$\frac{b(z^{t-1})(R_t^{ra} - R^*) - b^*(z^{t-1})(R_t^{ra} - R^*)}{b^*(z^{t-1})(R_t^{ra} - R^*)} = \frac{b(z^{t-1}) - b^*(z^{t-1})}{b^*(z^{t-1})}, \tag{121}$$

which is the percentage response of the household's bond position $b(z^{t-1})$. For the interest rate exposure channel considered in Section 6, the (impact) individual bond demand responses across the wealth percentile are in the range of $-0.1\% - 2\%$. The bond position response induces a transfer error that is two orders of magnitude smaller than the transfer. So the effects of transfer approximation on decomposition results are minimal.

Table 4: Calibration of the HANK model in Section 6

Parameter	Description	Value	Target
r^*	Real interest rate (p.a.)	0.05	
β^m	Discount factor of median HH (p.a.)	0.851	Asset market clearing
Δ	Dispersion of discount factors (p.a.)	0.048	Aggregate MPC
σ	Risk aversion	1	
A	TFP	0.46	Unit quarterly output
α	Capital share	0.33	
Ψ	Capital adjustment cost	11.43	Christiano, Eichenbaum and Trabandt (2016)
δ^K	Depreciation of capital (p.a.)	0.07	Kaplan, Moll and Violante (2018)
K/Y^{GDP}	Capital to GDP (p.a.)	2.4	Internally calibrated
B/Y^{GDP}	Government debt to GDP (p.a.)	0.29	2004 SCF gross liquid assets
p/Y^{GDP}	Equity to GDP (p.a.)	2.92	2004 FoF net illiquid assets
$\mu - 1$	markup	0.046	Interally calibrated
κ	Slope of Phillips curve	0.1	Christiano, Eichenbaum and Rebelo (2011)
ϵ^w	Wage elasticity	0.5	Christiano, Eichenbaum and Trabandt (2016)
ϕ_π	Coefficient on inflation	1.25	
ρ_B	Debt Persistence	0.93	Auclert and Rognlie (2018)
ϕ^B	Magnitude of the shock to debt level	-0.43	IRFs of real loans to monetary policy shock
Γ	Labor income tax rate	0.3	
$T^{uniform}$	Uniform tax	-0.058	Kaplan, Moll and Violante (2018)
G^*	Government spending	0.13	Internally calibrated

For the one-asset model in Section 6, this simplified method yields results that are visually indistinguishable from those obtained using the method proposed in Section 3.4. Therefore, I use this approach for the two-asset models in Section F.

B.2 Calibration

Table 4 presents the values of parameters, and below are details on fiscal policy.

Calibration of fiscal policy. I show the estimated effects of the monetary policy shock on household loans. Nominal loans are estimated from Flow of Funds (FoF) data as the sum of consumer credit, depository institution loans, and other loans and advances in liabilities minus loans as assets and total other assets, then I deflate it by CPI and take the log of real loans. The responses are estimated by local projections with high-frequency monetary policy shocks identified in Gorodnichenko and Weber (2016):

$$Y_{t+h} = \beta_{h,0} + \beta_{h,1}t + \beta_{h,2}\epsilon_t + \beta_{h,3}\mathbf{X}_{t-1} + \nu_{t+h}, \quad h = 0,....16 \tag{122}$$

The aggregate real loans Y_t at the forecast horizon $h = 0,...,16$ is regressed on the current normalized monetary shock ϵ_t, a constant, a linear time trend, and lagged controls $\mathbf{X}_{t-1}$. To control for potential endogeneity in practice, the lagged controls are set as the federal funds rate i_{t-1}, the monetary shock ϵ_{t-1}, unemployment rate U_{t-1}, log of output Y_{t-1}, consumption C_{t-1}, investment I_{t-1}, TFP A_{t-1} and the consumer price index P_{t-1}. The monetary policy shock is normalized such that the nominal rate i_t^{ib} decreases by 25 basis points on impact. I use quarterly data from 1988Q4 to 2016Q2. The data on monetary policy shocks are from 1988Q4 to 2012Q2. The figure also shows

Figure 9: Responses of consumer loans and aggregates to a monetary policy shock

(a) Loans

(b) Output

(c) Consumption

(d) Investment

(e) Nominal Rate
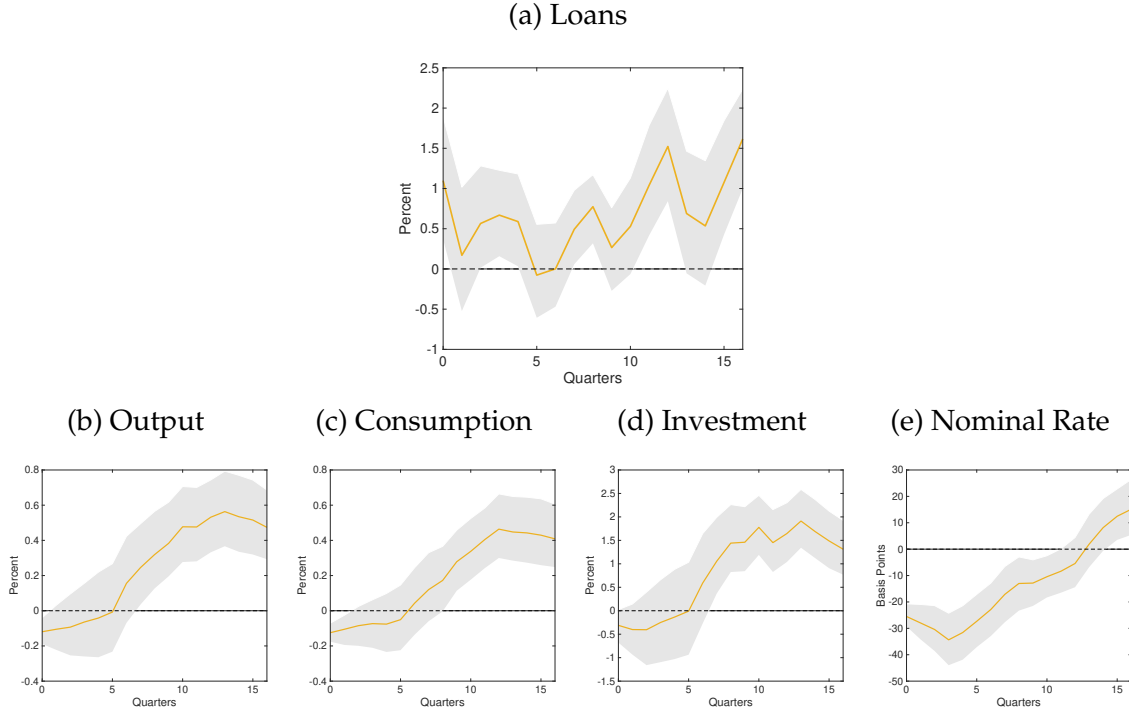
Notes: Estimated responses to a monetary policy shock. The monetary policy shock is normalized so that the reduction in the yield on the 3-month treasury bill is 25 basis points. I use quarterly data from 1988Q4 to 2016Q2 (the monetary policy shock data are from 1988Q4 to 2012Q2). The lagged controls are set as $\mathbf{X}_{t-1} = [i_{t-1}, \epsilon_{t-1}, U_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, A_{t-1}, P_{t-1}]$. The shaded area represents the bootstrapped 66% confidence limits.

.

the estimated responses of output Y_t, consumption C_t, investment I_t, nominal rate i_t^b (the return on the three-month treasury bill), and household loans in liquid assets L_t to the monetary shock with the same specification. All variables except the nominal rate are in real terms.

C   Application to literature

C.1   Werning (2015)

Werning (2015) analyzes scenarios in which an incomplete market economy can be aggregated as an "as if" representative agent economy. In the following, I first show that the "as if" representative agent corresponds to the fictitious representative agent defined in Proposition 1. Then, I show that the assumptions of the "as if" economy (Section 3.2 of Werning 2015) imply that counterfactual transfers are zero and all re-distribution channels are muted.

For simplicity, I omit the ex-ante heterogeneity and taste shocks in Werning (2015), as these do not affect the conclusion. The Euler equation in Werning (2015)'s Proposi-

Figure 10: iMPCs in the data and the model
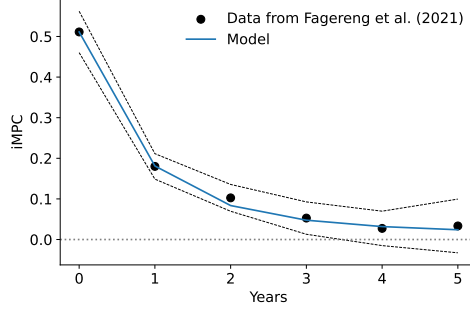
tion 2 also incorporates potentially time-varying idiosyncratic uncertainty (the stochastic process governing z_t). The time-varying idiosyncratic uncertainty implies a time-varying interest rate even in the steady state where aggregate consumption remains constant. In this case, the discount factor of the "as if" representative agent $\beta_t^{\text{as-if}}$ is time-varying. Normalizing aggregate consumption to 1, we can derive the steady state interest rate $\tilde{R}_t$ from equation 12 of Werning (2015). Equation 16 of Werning (2015) defines the discount factor of the "as if" representative agent $\beta_t^{\text{as-if}}$. We can find the following relation

$$\beta_t^{\text{as-if}} = 1/\tilde{R}_t. \tag{123}$$

This paper additionally assumes that idiosyncratic uncertainty is time-invariant and that the economy starts from its invariant distribution, leading to

$$\beta^{\text{as-if}} = 1/R^*, \tag{124}$$

which is the discount factor of the fictitious representative agent in Proposition 1.

To see that all the redistribution channels are muted in the "as if" economy, notice the following relations implied by Werning (2015)'s model (with notations of this paper):

$$y(z^t) = z_t Y_t, \tag{125}$$
$$b(z^t) = v(z^t) = 0 \text{ and } c(z^t) = y(z^t). \tag{126}$$

Equation (125) derives from the assumption of acyclical-income-risk, and equation (126) derives from the assumption of zero liquidity. Acyclical income risk implies that household income is proportional to aggregate income, and zero liquidity implies that household consumption is equal to their income.[21] Substituting the above relationships into the definition of redistribution channels (49), we see that all redis-

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[21]Households are assumed not to be able to borrow, and the equilibrium interest rate is low enough that no household has an incentive to save.

tribution channels are muted. Specifically, the income exposure channel is muted $(\hat{y}(z^t) - \hat{Y}_t)y^*(z^t) = 0$ since household income is proportional to aggregate income and all households have the same income elasticity $\hat{y}(z^t) = \hat{Y}_t$.

Section 4 of Werning (2015) extends the aggregation results to a positive, but acyclical, liquidity-to-income ratio with log utility of consumption. Households have access to the equity market, which pays dividends to households. The budget constraint is

$$c(z^t) + P_t v(z^t) = (P_t + D_t)v(z^{t-1}) + y^L(z^t). \tag{127}$$

The model assumptions in Section 4 of Werning (2015) imply:

$$y^L(z^t) = z_t Y_t^L,$$
$$\hat{D}_t^{ra} = \hat{Y}_t^{ra}, \tag{128}$$
$$\hat{C}_t^{ra} = \hat{P}_t^{ra}. \tag{129}$$

Household labor income $y^L(z^t)$ is proportional to aggregate labor income (and also to aggregate income) as before. Equation (128) follows from the assumption that dividends are proportional to aggregate income. Equation (129) is a result of log-utility on consumption: asset prices and consumption have the same responses. Substituting the above relations into the definition of redistribution channels (49), we find that both subchannels of income exposure are zero:

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) = \underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^* \left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}} \right)}_{\text{income portfolio exposure}} = 0.$$

About the channel "saving flow exposure", notice budget constraints in the steady state imply $y^*(z^t) - c^*(z^t) = P^*(v^*(z^t) - v^*(z^{t-1}))$ so this channel simplifies to

$$(P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \tag{130}$$
$$= -\hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1})) + \hat{C}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1})) \tag{131}$$
$$= (\hat{C}_t^{ra} - \hat{P}_t^{ra})P^*(v^*(z^t) - v^*(z^{t-1})), \tag{132}$$

which is zero since asset prices and consumption have the same response $\hat{C}_t^{ra} = \hat{P}_t^{ra}$.

In summary, Werning (2015) shows that if aggregate and individual consumption satisfy the following conditions

$$(C_t)^{-\sigma} = \beta^{ra} R_{t+1}(C_{t+1})^{-\sigma} \text{ and } c(z^t)/c^*(z^t) = C_t/C^* \tag{133}$$

then the allocation $\{c(z^t)\}$ satisfies individual optimality conditions. Werning (2015) provides examples where the scaled individual choices, together with the equilib-

rium prices, also satisfy budget constraints. For more general cases where budget constraints do not hold with scaled individual choices, this paper introduces counter-factual transfers to households.

C.2 McKay, Nakamura and Steinsson (2016)

McKay, Nakamura and Steinsson (2016) studies the forward guidance puzzle in an incomplete market model. They consider the response of the economy to a one-time 50 basis point real interest rate cut 20 quarters into the future, with real interest rates unchanged in all other quarters. The result of this experiment on their baseline model is shown in Figure 3 of the paper (reproduced below). In the RANK model, output immediately increases by 25 basis points and remains at that level for 20 quarters. In their HANK model, the initial increase in output is only about 10 basis points. I apply the decomposition to their model, and the results are shown in Figure 7. The negative redistribution effects are needed to solve the puzzle. Two model assumptions: (i) firm profits are distributed uniformly to households; (ii) only the highest-income house-holds pay taxes, and are the main drivers of the negative redistribution effects, as can be seen from the effects of income and tax exposure channels.

The assumption that firm profits are equally distributed to households implies that countercyclical profits Π account for a larger share of total income for low-income households, resulting in lower income elasticities for low-income households. Omitting differences in labor supply and assuming $n(z^t) = N$, individual income is $y = zWN + \Pi$. After some algebra, it can be shown that

$$(\hat{y}^{ra}(z) - \hat{Y}^{ra})y^*(z) = (\hat{Y}^{L,ra} - \hat{\Pi}^{ra})(\Pi^*/Y^* - \Pi^*/y^*(z)), \tag{134}$$

where $Y^L \equiv WN$. For low-income households with $z < 1$, profits Π are a larger share of aggregate income than average: $\Pi^*/Y^* < \Pi^*/y^*(z)$. After an expansionary shock, $\hat{Y}^{L,ra} > 0$ and $\hat{\Pi}^{ra} < 0$. Low-income households experience a smaller income increase $\hat{y}^{ra}(z) < \hat{Y}^{ra}$. The redistribution from low-income to high-income households dampens the output responses from quarter 0 onwards and accounts for most of the negative redistribution effects.

The second assumption, that only the highest-income households pay taxes, implies that only the highest-income households benefit from the tax cut in quarter 20 (taxes in other quarters do not change from steady-state levels because real interest rates only fall in quarter 20). For households with the highest skill level z^H, the tax exposure channel is

$$(\tau^*(z^H) - r^*B^*) - (\tau^{ra}(z^H) - r_t^{ra}B^*) = (r_t^{ra} - r^*)B^*(1 - 1/\lambda(z^H)) > 0 \tag{135}$$

A13

Figure 11: Decomposition of individual consumption responses to a government spending shock in the IKC environment of Auclert, Rognlie and Straub (2018)
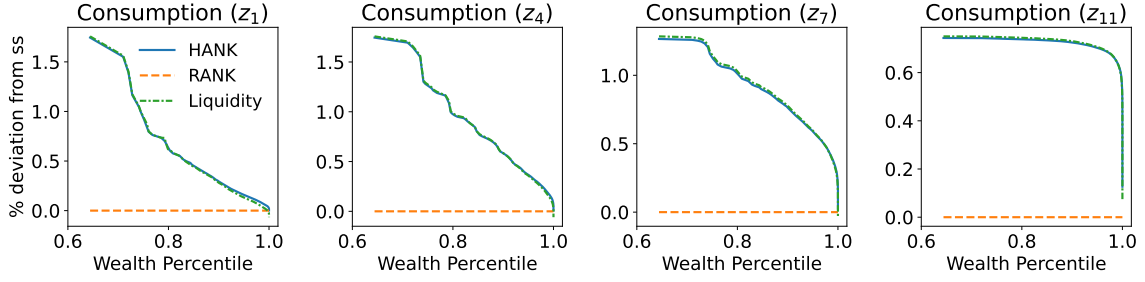

Figure 5a in Auclert, Rognlie and Straub (2024) shows the impact multiplier of a persistent government spending shock across different degrees of deficit finance in the "IKC" environment where the real interest rate is fixed. I consider their HA-one model with the deficit-finance parameter $\rho_B = 0.6$. The government spending shock is 1% of the steady-state output. The individual consumption responses are decomposed into RANK and redistribution effects.

where $\lambda(z^H)$ is the measure of households with the highest skill level z^H. For the remaining households that do not pay taxes, the tax exposure channel is $(r_t^{ra} - r^*)B^* < 0$. The redistribution from the remaining households to the highest-skilled households dampens the output response in quarter 20. It also counteracts the amplifying effects of the interest rate risk channel on output in quarter 20.

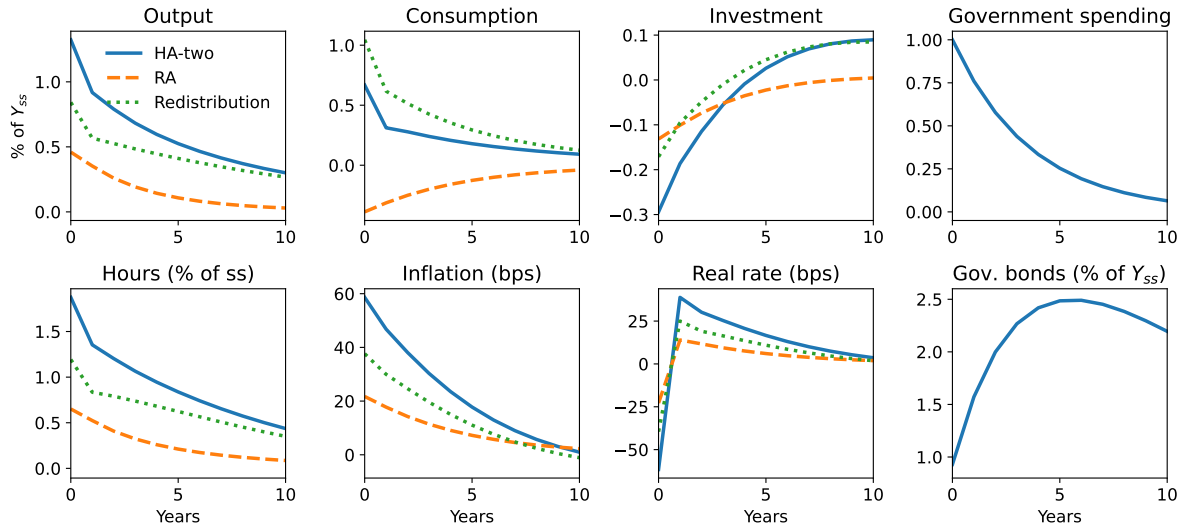## C.3   Auclert, Rognlie and Straub (2024)

Auclert, Rognlie and Straub (2024) examines fiscal multipliers and finds that deficit-financed multipliers can be greater than one. Figure 5a in Auclert, Rognlie and Straub (2024) shows the impact multiplier of a persistent government spending shock under different degrees of deficit financing in their "IKC" environment, where the real interest rate is fixed at the steady-state level and the government bond is the only asset with a positive supply. Figure 8 in Auclert, Rognlie and Straub (2024) relaxes the assumptions of the "IKC" environment and considers a fully specified two-account quantitative model. I discuss the decomposition of the one-asset model in its "IKC" setting and then move to their two-account HANK model.

C.3.1   The "IKC" Environment

The size of the shock dG_0 is 1% of steady-state output, and the persistence of the government spending shock is $\rho_G = 0.76$. After the shock $\{dG_t\}$, the government debt evolves as follows

$$dB_t = \rho_B(dB_{t-1} + dG_t) \tag{136}$$

Figure 12: Government spending shock in the quantitative environment of Auclert, Rognlie and Straub (2024)

Notes: Replication of Figure 8 in Auclert, Rognlie and Straub (2024). The HA-two model is a two-account heterogeneous-agent model.

where $dB_t \equiv B_t - B^*$ is the bond supply shock induced by the financing rule above. The government adjusts labor income taxes to satisfy its budget constraint. In the case of $\rho_B = 0$, government spending is fully financed by contemporaneous labor-income taxation. When $\rho_B > 0$, the government finances some of the spending through deficits and postpones raising taxes. I consider their HA-one model with the deficit financing parameter $\rho_B = 0.6$. From Figure 5a of the paper, we can see the impact output response of the HA-one model is 2.1%, implying an impact multiplier of $dY_0/dG_0 = 2.1$. In the RANK model, the impact multiplier is exactly one because the central bank fixes the real interest rate, and the government spending shock does not affect consumption. It is straightforward to see that the amplified multiplier in their HA-one model is due to the liquidity channel. When the government delays tax increases and funds spending through deficits, the supply of bonds increases, and more liquidity is injected into the economy. This allows constrained households to borrow from unconstrained households. This mechanism is illustrated in the household-level decomposition in Figure 11. It shows that poor households, who are more likely to be constrained, are the most responsive to the shock. The interest rate exposure channel is muted because the real interest rate is fixed at the steady-state level. The income exposure channel is muted because households pay taxes in proportion to their income.

When the expenditure shock is financed by contemporaneous taxation $dG_t = dT_t$, the increases in taxes and labor income counteract each other. After the shock, aggregate labor income increases by dG_t. For households with a productivity level z_t, pre-tax labor income increases by $z_t dG_t$, and taxes increase by the same amount $z_t dT_t$.

Figure 13: Decomposition of redistribution effects of a government spending shock
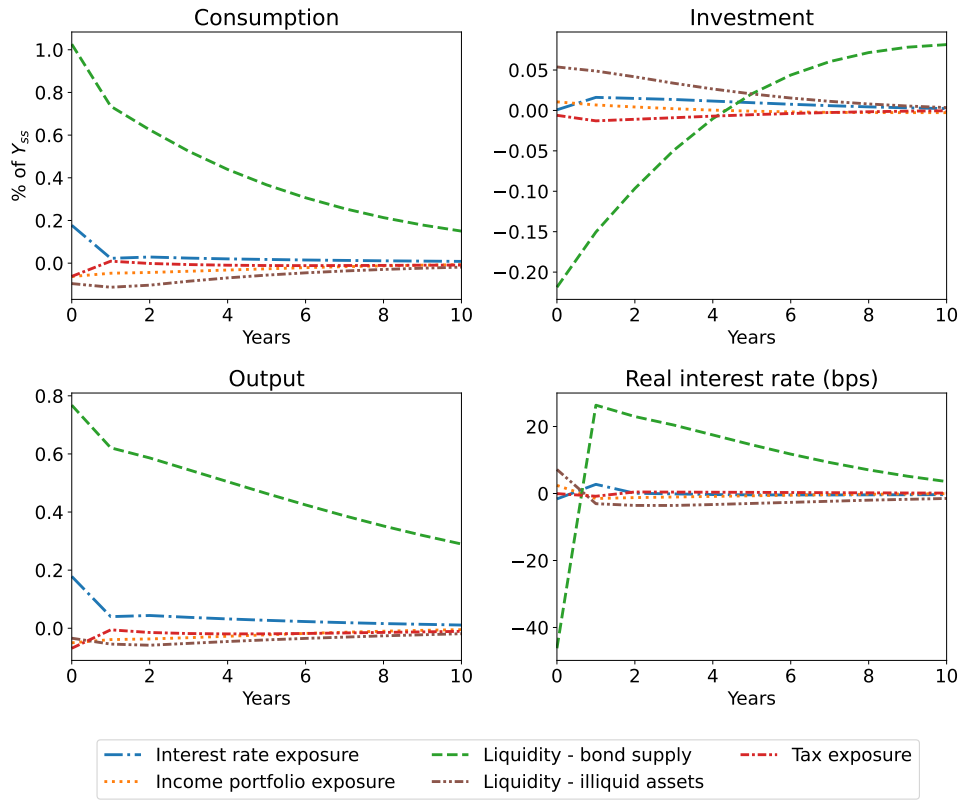
Notes: Decomposition of redistribution effects of the government spending shock on consumption, investment, output and real interest rate in Auclert, Rognlie and Straub (2024).

The after-tax labor income remains unchanged. The fiscal multiplier is exactly 1 under a balanced-budget fiscal policy and a fixed real interest rate. There is no redistribution, and household heterogeneity is irrelevant in determining the fiscal multiplier, which is consistent with Proposition 3 in Auclert, Rognlie and Straub (2024).

C.3.2 The Quantitative Environment

Figure 8 in Auclert, Rognlie and Straub (2024) shows the effect of the government spending shock in a two-account quantitative HANK model, which is replicated in Figure 12.[22] In the RANK (RA) model, consumption and investment are crowded out, limiting output expansion. The HA-two model is a two-account HANK model. In the HA-two model, consumption responds positively, offsetting the crowding out of the investment, and the fiscal multiplier is greater than one.

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[22]In the original model of Auclert, Rognlie and Straub (2024), aggregate taxes T_t distort labor supply by entering the wage Philipps curve, which implies that Ricardian equivalence does not hold even in the RANK model. To focus on the demand-side effects of time-varying bond supply, I instead assume that only taxes under the constant debt path $\bar{T}_t = G_t + r_t B^*$ enter the wage Philipps curve. If the government changes the timing of taxes, the wage Philipps curve is unaffected, ensuring Ricardian equivalence in the RANK model. Under this specification, the model responses are slightly different from Auclert, Rognlie and Straub (2024).

Figure 13 shows the decomposition of redistribution effects on consumption, investment, output, and the real interest rate. The redistributive effects on consumption are positive, reversing the sign of the consumption response in RANK. As in the "IKC" environment, when the government increases bond supply, the borrowing conditions of households are eased, stimulating aggregate consumption. The liquidity channel of bond supply explains most of the redistribution effects. Unexpected inflation in period 0 lowers the real interest rate on government bonds, which benefits debtors and hurts creditors through the interest rate exposure channel. This stimulates aggregate consumption. The stimulative effect of the unexpected inflation outweighs the dampening effect of the rising interest rate from period 1 onward, as the interest rate rises modestly in the RANK model. The income exposure channel dampens the consumption response due to heterogeneous income portfolio exposures.[23] Two mechanisms lead to a smaller income decrease for equity holders than for workers. First, the factor income of both capital and labor rises with output expansion, but only labor income is taxed to finance government spending. Second, capital owners can smooth consumption by reducing savings, as discussed in Section 4.2. The liquidity channel of illiquid assets dampens consumption responses. The rising real interest rate forces non-adjusters to accumulate more illiquid assets than in the steady state. Aggregate savings increase, and aggregate consumption falls.

D   Decomposing TANK

The decomposition can be analytically implemented in the Two-Agent New Keynesian (TANK) model. For comparison, the TANK model used here is kept identical to Bilbiie (2020).[24] I briefly describe the environment and characterize the equilibrium conditions. Details of the model can be found in Bilbiie (2020).

D.1   Model Description

There are two types of households with total unit mass. A fraction of λ households is hand-to-mouth H, who are excluded from financial markets and consume their current income. The budget constraint of H is given by

$$C_t^H = W_t N_t^H + D_t^H, \tag{137}$$

where W_t is real wage, N_t^H is H's labor supply, and D_t^H is the firm's profits received by H. The remaining fraction $1 - \lambda$ of households are savers S, trading one-period riskless

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[23]The labor income channel is muted because households have the same labor income elasticities.

[24]Bilbiie (2020) has aggregate uncertainty and log-linearizes the model. The solution is equivalent to the linearized perfect-foresight transition path (Boppart, Krusell and Mitman 2018).

real bonds. The budget constraint of S is given by

$$C_t^S + \frac{B_t}{R_t} = B_{t-1} + W_t N_t^S + D_t^S, \tag{138}$$

where N_t^S is S's labor supply and D_t^S is the firm's profits received by S. All households maximize their discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ subject to the sequence of their budget constraints. The utility function takes the form $U(C, N) = C^{1-1/\sigma}/(1-\sigma) - N^{1+\varphi}/(1+\varphi)$.

The supply side is standard. There is a continuum of firms, and each firm produces a differentiated good with linear technology $Y_t(i) = A_t N_t(i)$. In each period, firms have the possibility of θ to reset the price. The demand for each good is $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$ where $P_t = (\int_0^1 P_t(i)^{1-\epsilon} di)^{1/(1-\epsilon)}$ is the aggregate price index and Y_t is the aggregate output. The standard supply-side implies the canonical representation of the log-linearized Phillips Curve: $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$ where y_t is the log deviation of output from steady state.

The government implements standard NK optimal subsidy inducing marginal cost pricing financed by a lump-sum tax on the firms' profits. The profit function is $D_t(i) = (1+\tau)P_t(i)Y_t(i)/P_t - W_t N_t(i) - T_t^F$. With the optimal subsidy, $\tau = 1/(\epsilon - 1)$, firms' steady-state profits are zero. in the steady state, households have the same income and consumption. The central bank conducts monetary policy in the form of the Taylor rule: $i_t = r^* + \phi_\pi \pi_t + \epsilon_t$ where r^* is the steady state real interest rate, and ϵ_t is an exogenous monetary policy shock.

The key assumption in TANK is the distribution rule of the firm's profits. The government redistributes τ^D share of profits to H: $D_t^H = \tau^D D_t / \lambda$, and $1 - \tau^D$ share of profits to S: $D_t^S = (1 - \tau^D)D_t/(1-\lambda)$. When $\tau^D = \lambda$, H and S receive the same profits, and their income and consumption have the same responses in equilibrium. When $\tau^D \neq \lambda$, TANK deviates from this representative-agent benchmark.

Denote log deviations of variables from their steady-state values except for interest rates by small letters. After imposing the market-clearing condition, the aggregate Euler equation of TANK is derived as

$$c_t = E_t c_{t+1} - \delta^{-1} \sigma (r_t - r^*), \tag{139}$$

where $\delta^{-1} = (1-\lambda)/(1-\lambda\chi)$ and $\chi = 1 + \varphi(1 - \tau^D/\lambda)$. Though H have no access to financial markets and their consumption does not price the bond, one can infer the quantitative relation between their consumption and interest rates from the relation between H and S's equilibrium consumption.

From the aggregate Euler equation (139), we can see the amplifying/dampening mechanism in TANK. As already mentioned, if $\tau^D = \lambda$, it follows $\chi = 1$ and $\delta^{-1} = 1$.

The elasticity of contemporaneous aggregate consumption to interest rates is the same as RANK. In equilibrium, the income and consumption responses of H and S are the same. If $\tau^D < \lambda$, H receives a smaller amount of profits than S. With counter-cyclical profits, it implies that H's consumption responds more than S's consumption. As a weighted sum, aggregate consumption also responds more than S's consumption, and its elasticity to interest rates is larger than the consumption elasticity of S: $\delta^{-1}\sigma > \sigma$. For a given change in real interest rates, the aggregate consumption response in TANK is amplified relative to RANK.

With the full characterization of the equilibrium, I now consider the output response to an exogenous monetary policy shock. For simplicity, here I consider a monetary policy shock that lasts only one period: $E_t\epsilon_{t+1} = 0$. Given a monetary policy shock ϵ_t, the output response of TANK is

$$
y_t = -\frac{\delta^{-1}\sigma}{1 + \delta^{-1}\sigma\phi_\pi\kappa}\epsilon_t. \tag{140}
$$

In the case of amplifying, $\delta^{-1} > 1$, and the output response is larger (in abstract value) than that in RANK. In the case of dampening, $\delta^{-1} < 1$, the output is less responsive to monetary policy shocks relative to RANK.

D.2   Decomposition

I decompose the output response y_t into **RANK effects** y_t^{ra} and **redistribution effects** y_t^{re} such that $y_t = y_t^{ra} + y_t^{re}$. This decomposition is based on the observation that monetary policy shocks in TANK induce a redistribution between H and S due to their unequal exposures to the countercyclical profits, which affects their income elasticities to aggregate income. In a counterfactual scenario where this redistribution is eliminated, TANK behaves the same as RANK. To achieve this scenario, I construct lump-sum transfers to households. The difference between TANK and RANK is then attributed to the absence of these transfers.

Let ω_t^H and ω_t^S be the counterfactual transfers to H and S, respectively, that eliminate the redistribution effects of a monetary policy shock $\{\epsilon_t\}$. The counterfactual transfers are purely redistributive: $\lambda\omega_t^H + (1 - \lambda)\omega_t^S = 0$, where λ is the fraction of H in the population. The RANK effects of the shock on output y_t^{ra} are the response of output to the shock and the transfers $\{\epsilon_t, \omega_t^H, \omega_t^S\}$; and the redistribution effects of the shock on output y_t^{re} are the response of output to the redistribution shock $\{-\omega_t^H, -\omega_t^S\}$.

RANK effects.   The RANK effects on output y_t^{ra} are the output responses of a representative agent model:

$$y_t^{ra} = -\frac{\sigma}{1 + \sigma\phi_\pi\kappa}\epsilon_t. \tag{141}$$

In RANK effects, S and H have the same consumption responses, and it is easy to verify that the consumption of Savers $c_t^{S,ra}$ satisfies the Euler equation with interest rates $\{R_t^{ra}\}$. However, these consumption responses do not satisfy households' budget constraints. To satisfy the budget constraints, I construct lump-sum transfers $\{\omega_t^H, \omega_t^S\}$ to H and S. With lump-sum transfers, the budget constraints of households are

$$c_t^{H,ra} = w_t^{ra} + n_t^{H,ra} + \frac{\tau^D}{\lambda}d_t^{ra} + \omega_t^H,$$

$$c_t^{S,ra} = w_t^{ra} + n_t^{S,ra} + \frac{1 - \tau^D}{1 - \lambda}d_t^{ra} + \omega_t^S, \tag{142}$$

where ω_t^S and ω_t^H are the transfers (as a percentage of steady state output Y^*) to S and H, respectively. Assume that the optimal labor supply condition holds for both households, so $c_t^{S,ra} = c_t^{H,ra}$ implies $n_t^{S,ra} = n_t^{H,ra}$, the budget constraints require:

$$\omega_t^H = \left(1 - \frac{\tau^D}{\lambda}\right)d_t^{ra},$$

$$\omega_t^S = \left(1 - \frac{1 - \tau^D}{1 - \lambda}\right)d_t^{ra}. \tag{143}$$

With this transfer scheme, S and H have the same consumption response, and the aggregate Euler equation holds.

Redistribution effects.   Consider an exogenous transfer scheme such that $\lambda T_t^H + (1 - \lambda)T_t^S = 0$ where T_t^H and T_t^S are the transfers (as the percentage of steady-state output Y^*) to H and S, respectively. The proof below shows that the output response of TANK to such a transfer scheme is

$$y_t = -\frac{1}{\sigma\phi_\pi\kappa + \delta} \cdot \frac{1}{1 + (\sigma\varphi)^{-1}}T_t^S. \tag{144}$$

To obtain redistribution effects, I input the redistribution shock $\{-\omega_t^H, -\omega_t^S\}$ into the model. Letting $T_t^S = -\omega_t^S$ we have

$$y_t^{re} = \frac{1 - \delta}{\sigma\phi_\pi\kappa + \delta}y_t^{ra}. \tag{145}$$

Discussion. Expressing the output response (140) y_t in terms of RANK effects (141) y_t^{ra}:

$$y_t = \frac{1 + \sigma\phi_\pi\kappa}{\delta + \sigma\phi_\pi\kappa} y_t^{ra}. \tag{146}$$

In the case of amplification ($\tau^D < \lambda$, $\chi > 1$, and $\delta < 1$), the redistribution effects act in the same direction as RANK effects, and the total effects are greater than RANK effects (in absolute value). The endogenous redistribution through firms' profit distribution τ^D/λ in TANK amplifies the output response. To see this, consider an expansionary monetary policy shock $\epsilon_t < 0$, from equation (143) it follows $\omega_t^H < 0$ and $\omega_t^S > 0$. The redistribution shock $\{-\omega_t^H, -\omega_t^S\}$ subsidizes H by taxing S. In TANK, fiscal stimulus in the form of transfers from S to H is itself a policy instrument that stimulates the economy (see Bilbiie, Monacelli and Perotti 2013). In the case of dampening ($\tau^D > \lambda$, $\chi < 1$, and $\delta > 1$), the redistribution shock taxes H and subsidizes S, which will dampen the output's response relative to RANK effects.

Kaplan, Moll and Violante (2018) decomposes the response of output y_t into "direct effects" and "indirect effects", which are simply substitution and income effects in the TANK model. The substitution effects are the response of aggregate consumption, keeping the income of households unchanged. The income effects are the response of aggregate consumption, keeping the interest rates unchanged.[25] After some algebra, it can be shown that

$$c_t^{sub} = \beta(1 - \lambda\chi)y_t, \tag{147}$$
$$c_t^{inc} = [1 - \beta(1 - \lambda\chi)]y_t, \tag{148}$$

the sizes of substitution effect c_t^{sub} and income effect c_t^{inc} depend on H's measure λ and the amplifying/dampening parameter χ. One can easily see the difference between this paper's decomposition and the decomposition in Kaplan, Moll and Violante (2018) and also Auclert (2019) in the case of proportional distribution of firm profits ($\tau^D = \lambda$, $\chi = 1$, and $\delta = 1$). In this case, the economy's response is equivalent to RANK. But the size of substitution and income effects varies with H's measure λ. This is because the decomposition in Auclert (2019) and Kaplan, Moll and Violante (2018) captures both the heterogeneous MPCs across households (parameter λ) and the correlation between households' MPCs and income exposures (parameter χ). This paper's decomposition is designed to isolate the parameter χ. This paper's decompo-

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[25] Auclert (2019) further decomposes those effects into an aggregate and a redistribution component, respectively. For instance, the aggregate component of the income effects c_t^{inc} is the consumption response of an average household (whose MPC is the weighted average of S and H's MPCs) to the shock y_t, and the redistribution component of the income effects c_t^{inc} is the weighted sum of S's consumption response to $y_t^S - y_t$ and H's consumption response to $y_t^H - y_t$.

sition implies zero redistribution effects $y_t^{re} = 0$. All output response is due to RANK effects regardless of the mass of hand-to-mouth households because, in equilibrium, S and H are equally exposed to the shock.

Proof. The equilibrium of TANK can be characterized by the following equations

$$c_t = E_t c_{t+1} - \delta^{-1}\sigma(r_t - r^*), \tag{149}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa c_t, \tag{150}$$

$$i_t = r^* + \phi_\pi \pi_t + \epsilon_t. \tag{151}$$

For a transient shock $E_t \epsilon_{t+1} = 0$ and $E_t c_{t+1} = E_t \pi_{t+1} = 0$. The solution is simply

$$y_t = -\frac{\delta^{-1}\sigma}{1 + \delta^{-1}\sigma\phi_\pi\kappa}\epsilon_t. \tag{152}$$

The RANK effects are obtained by letting $\delta = 1$

$$y_t^{ra} = -\frac{\sigma}{1 + \sigma\phi_\pi\kappa}\epsilon_t. \tag{153}$$

Expressing y_t in terms of y_t^{ra},

$$y_t/y_t^{ra} = \frac{\delta^{-1}(1 + \sigma\phi_\pi\kappa)}{1 + \delta^{-1}\sigma\phi_\pi\kappa} = \frac{1 + \sigma\phi_\pi\kappa}{\delta + \sigma\phi_\pi\kappa}. \tag{154}$$

Consider a transfer scheme such that $\lambda T_t^H + (1 - \lambda)T_t^S = 0$ where T_t^H and T_t^S are the transfers (measured as the percentage of steady-state output Y^*) H and S receive, respectively. From the budget constraint of S, we can derive the relation between S's consumption c_t^S, output y_t, and T_t^S

$$c_t^S = w_t + n_t^S + \frac{1 - \tau^D}{1 - \lambda}d_t + T_t^S,$$

$$= (1 - \frac{1 - \tau^D}{1 - \lambda})w_t + \varphi^{-1}(w_t - \sigma^{-1}c_t^S) + T_t^S;$$

$$[1 + (\sigma\varphi)^{-1}]c_t^S = (1 - \frac{1 - \tau^D}{1 - \lambda} + \varphi^{-1})w_t + T_t^S,$$

$$c_t^S = \delta y_t + \frac{1}{1 + (\sigma\varphi)^{-1}}T_t^S. \tag{155}$$

Figure 14: Decomposition of a transient real-rate shock's effects
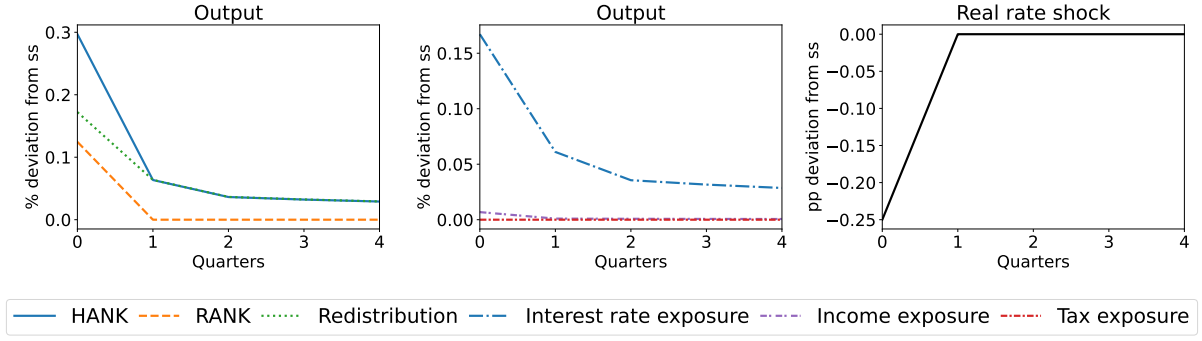
Notes: Decomposition of the output's response to a transient real-rate shock, $\tilde{r}_0 = -0.25\%$.

From S's Euler equation, Philips Curve, and Taylor rule it follows $c_t^S = -\sigma(r_t - r^*) = -\sigma\phi_\pi\kappa y_t$. Substituting into (155), the output response to the transfer scheme is

$$y_t = -\frac{1}{\sigma\phi_\pi\kappa + \delta}\frac{1}{1 + (\sigma\varphi)^{-1}}T_t^S. \tag{156}$$

To obtain redistribution effects, note that the transfer S receive is

$$T_t^S = -\omega_t^S = -(1 - \frac{1 - \tau^D}{1 - \lambda})d_t^{ra} = (1 - \frac{1 - \tau^D}{1 - \lambda})(\sigma^{-1} + \varphi)y_t^{ra}. \tag{157}$$

Substituting (157) into (156) it follows

$$y_t^{re} = -\frac{1}{\sigma\phi_\pi\kappa + \delta}\frac{1}{1 + (\sigma\varphi)^{-1}}(1 - \frac{1 - \tau^D}{1 - \lambda})(\sigma^{-1} + \varphi)y_t^{ra} = \frac{1 - \delta}{\delta + \sigma\phi_\pi\kappa}y_t^{ra}. \tag{158}$$

We can verify that $y_t = y_t^{ra} + y_t^{re}$.

E   Decomposition Without Investment

I implement the decomposition on the model presented in Section 3, where there is no productive capital and investment. To avoid making a distinction between ex-ante and ex-post interest rates, I modify the budget constraints of households:

$$c(z^t) + \frac{b(z^t)}{R_t} = b(z^{t-1}) + z_t W_t n(z^t) + \pi_t(z) - \tau_t(z). \tag{159}$$

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Figure 15: Transfers as a function of household characteristics
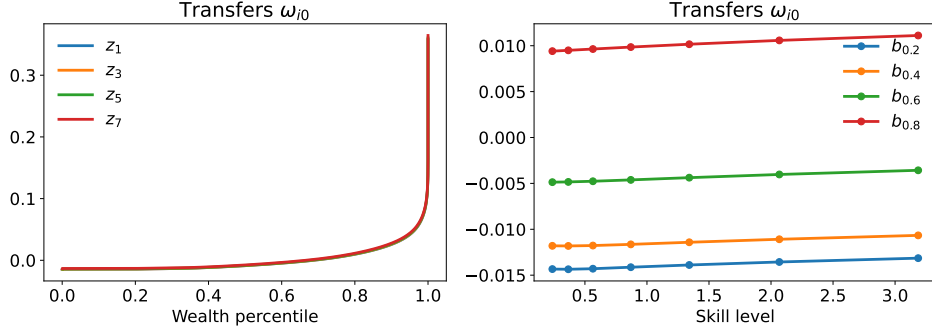
Notes: The left panel shows the transfers ω_{i0} as a function of wealth at four different productivity levels. The right panel shows ω_{i0} as a function of household productivity level at the wealth distribution's 20th, 40th, 60th, and 80th percentiles.

The channel level decomposition is, instead,

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^t) - B)(\frac{1}{R^*} - \frac{1}{R_t^{ra}})}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}}$$

$$+ \underbrace{\frac{\bar{b}^{ra}(z^t) - b^{ra}(z^t)}{R_t^{ra}} - (\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity}}$$

$$+ \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))}_{\text{saving flow exposure}} + \underbrace{\frac{b^*(z^t) - \bar{b}^{ra}(z^t)}{R_t^{ra}} - (b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}))}_{\text{undetermined bond demand}}, \qquad (160)$$

where $y \equiv zWn + \pi$ includes household labor and pofit income. To make the exercise more transparent, I assume that the central bank directly controls the real interest rate. At time $t = 0$ there is a quarterly real rate shock $\tilde{r}_0 = -0.25$ percent with the persistence of 0.61. By construction, the output response in the "RANK" equilibrium is given by the aggregate Euler equation:

$$(C_t^{ra})^{-\sigma} = \beta^{ra} R_t (C_{t+1}^{ra})^{-\sigma}. \qquad (161)$$

The redistribution effects are the economy's response to the redistribution shock, keeping the real interest rate at the steady state level. In the first two exercises, I assume a balanced budget fiscal policy. In the third exercise, I let the government adjust the outstanding debt.

E.1 Calibration

I consider a model with an annual real interest rate of 2% in the steady state. The coefficient of risk aversion σ is set to 2. The Frisch elasticity of labor supply is $1/\nu = 0.5$,

following Chetty (2012). For the idiosyncratic income process, I use $\rho_e = 0.966$ and $\sigma_e^2 = 0.017$, as in McKay, Nakamura and Steinsson (2016) and Guerrieri and Lorenzoni (2017). The supply of government bonds B is set to match the ratio of aggregate liquid assets to output $B/Y = 5.6$, as in McKay, Nakamura and Steinsson (2016). The borrowing constraint is zero $\phi = 0$. The discount factor $\beta = 0.98$ and disutility from labor $\varphi = 0.933$ are calibrated to deliver the values of annual real interest and unit quarterly output. On the supply side, the slope of the Phillips Curve is $\kappa = 0.1$ and the parameter of the markup of intermediate firms is $\mu = 1.2$. The Taylor rule coefficient ϕ is set to 1.25. In the baseline calibration, I assume that household tax payments are uniform. The firm profits are distributed to households proportional to their productivity $\pi(z_t) \sim z$, as in Kaplan, Moll and Violante (2018).

E.2   Purely Transient Shocks

To begin, consider a real rate shock that lasts only one period (the persistence $\rho = 0$), in the same spirit as the thought experiments in Auclert (2019). The result is shown in Figure 14. The real interest rates decrease and stimulate consumption. Given the sticky price, the rising aggregate demand leads to an increase in both output and inflation. Regarding decomposition, redistribution effects amplify the output response. Under the transient monetary policy shock, RANK effects last for only one period, the same as a representative-agent model. In contrast, the redistribution effects affect the economy for a long time, and all the economy's responses after time 0 are due to redistribution effects.

Figure 15 shows the transfers ω_{i0} as a function of the household's wealth and productivity. The left panel of Figure 15 shows the transfers ω_{i0} as a function of wealth at four different productivity levels. The right panel shows ω_{i0} as a function of household productivity level at the wealth distribution's 20th, 40th, 60th, and 80th percentiles. The transfers ω_{i0} increase with the household's wealth and (weakly) with productivity. To eliminate the exposure to the interest rate cut, creditors need positive transfers, and debtors need negative transfers. The transfers increase with productivity because profits are countercyclical. The income of the household is $y = zWn + z\Pi = z(WNn/N + \Pi)$. Due to labor supply heterogeneity, high-income households have a higher share of profit income, which is countercyclical. High-income households' income increases less and needs positive transfers. Overall, the redistribution shock $-\omega$ benefits high-MPC households by taxing low-MPC households: $cov_I(MPC_{i0}, -\omega_{i0}) > 0$. The redistribution effects stimulate aggregate consumption.

Figure 16: Decomposition of a persistent real-rate shock's effects
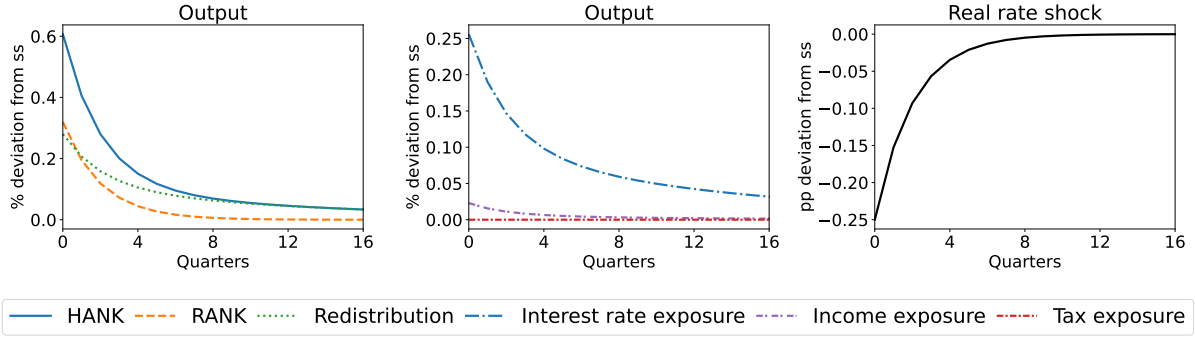
Notes: Decomposition of the output's response to a persistent real-rate shock. The redistribution effects on output are decomposed into three channels. The government keeps a constant debt and adjusts the uniform tax following the shock. Equation (160) gives the definitions of these redistribution channels. The government keeps a constant debt and adjusts the uniform tax to balance its budget.

E.3   Persistent Shocks

Consider the economy's response to a persistent real-rate shock. I apply the decomposition, and the result is shown in Figure 16. Output increases by 0.6% on impact. The decomposition result is qualitatively similar to the decomposition of the transient shock in Figure 14. Redistribution effects amplify the output's response to the real rate shock. On impact, RANK effects increase output by 0.31%, and redistribution effects increase output by 0.29%. The redistribution effects amplify the elasticity of output to real interest rates. Quantitatively, the interest exposure channel accounts for most of the redistribution effects. On impact, the interest exposure channel increases consumption by 0.25%. The interest rate cuts tax creditors and subsidizes debtors. Given that debtors have higher MPCs, the interest rate exposure channel stimulates aggregate consumption. The income exposure channel slightly contributes to the output amplification. Since I assume uniform taxation, all households benefit equally from the tax reduction, and the tax exposure channel is muted.

E.4   Including Liquidity Channel

Assuming the fiscal policy takes the following rule:

$$T_t = T^* + \rho_B * (B_{t-1} - B^*). \tag{162}$$

In the short run, the government uses debt to absorb most of the fiscal imbalance. In the long run, the government uses taxes to bring the debt back to its initial level. Similar fiscal policy specifications are assumed in Kaplan, Moll and Violante (2018), Alves et al. (2020), and Auclert, Rognlie and Straub (2024).

The decomposition result is shown in Figure 17. The redistribution effects are

Figure 17: Decomposition with liquidity channel
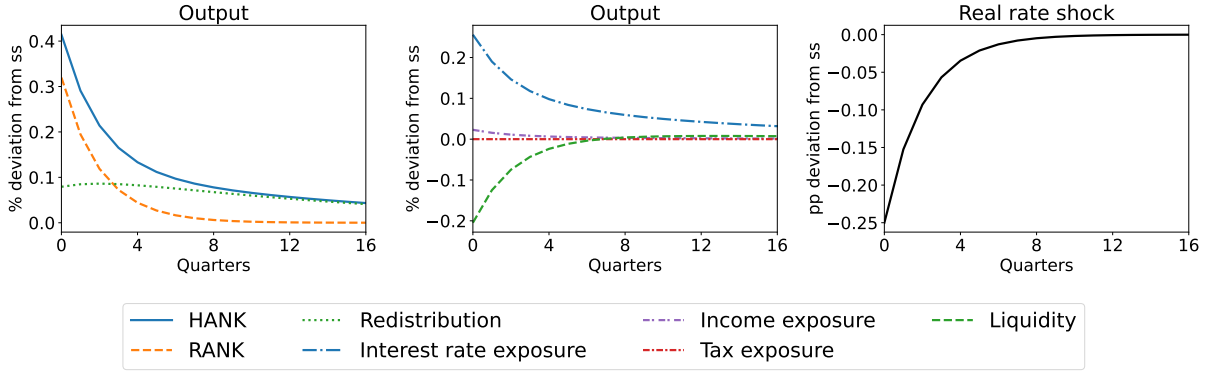
Notes: Decomposition of the output's response to a persistent real-rate shock, $\tilde{r}_0 = -0.25\%$, with fiscal policy $T_t = T^* + \rho_B * (B_{t-1} - B^*)$. The government uses debt to absorb most of the fiscal imbalance in the short run. In the long run, the government uses uniform taxes to bring the debt back to its initial level. Redistribution effects on output are decomposed into four channels.

smaller than Figure 16. On impact, redistribution effects increase output by less than 0.1%, rather than close to 0.3% under a balanced fiscal policy. The effects of interest exposure, income exposure, and tax exposure channels are invariant to the path of government debt. However, the liquidity channel decreases output by 0.2% on impact. The liquidity channel explains why the output response with the fiscal policy of (162) is smaller than a balanced budget. The fiscal rule (162) implies a countercyclical bond supply. As proved in Section 4.1, the liquidity channel can be proxied by a borrowing-constraint shock in the case of uniform taxation. Given the constant real interest rate, the output needs to decrease to clear the market. Figure 18 shows the decomposition of the households' impact consumption responses. The interest rate exposure channel increases the consumption of poor households and decreases the consumption of rich households. However, the liquidity channel forces the constrained households to hold the additional income from other channels. As a result, the redistribution effects on the consumption of poor households are smaller compared to a balanced fiscal policy.

F   Model with Illiquid Assets

In this section, I extend the model with illiquid assets. I model illiquidity a la Calvo as in Bayer et al. (2019) and Luetticke (2021). Section F.1 shows that aggregation holds with illiquid assets, and Section F.2 reveals that the presence of illiquid assets introduces a new channel that amplifies the effects of monetary policy shocks.

Figure 18: Household-level decomposition (on impact)
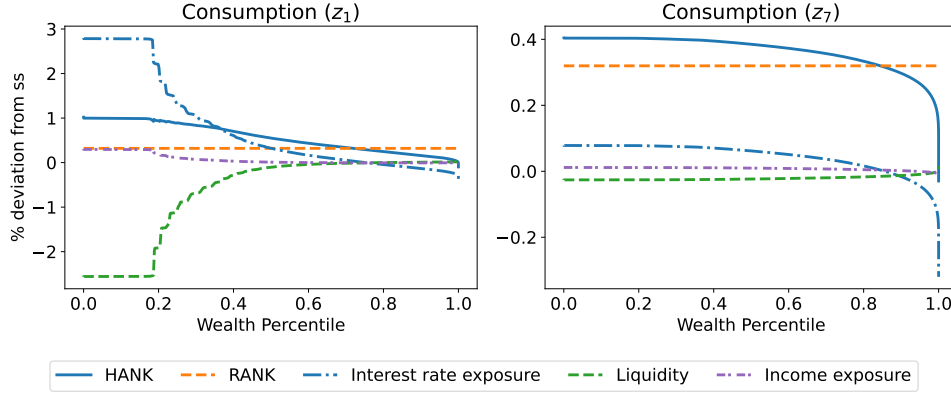
Notes: The redistribution shock's effects on individual consumption (impact) are decomposed into three channels. For comparison, I also show the RANK effects and the HANK model's responses. Equation (160) gives the definitions of these redistribution channels.

F.1 Model Description and RANK Effects

Households. Households have access to two assets: (i) liquid assets a^{liq} with gross real return R^{liq}; (ii) illiquid assets a^{illiq} with gross real return R^{illiq}. Households maximize subject to the following budget, adjustment, and borrowing constraints:

$$c(h^t) + a^{liq}(h^t) = R_t^{liq} a^{liq}(h^{t-1}) - d(h^t) + z_t W_t n(h^t) + \pi(h^t) - \tau(h^t), \tag{163}$$

$$a^{illiq}(h^t) = R_t^{illiq} a(h^{t-1}) + d(h^t), \tag{164}$$

$$a^{liq}(h^t) \geq 0, \quad a^{illiq}(h^t) \geq 0, \tag{165}$$

where $h^t \equiv ((b_{-1}, a_{-1}), (z_0, s_0), (z_1, s_1), \cdots, (z_t, s_t))$ is the individual's history of idiosyncratic shocks up to time t, including both productivity shock z and adjustment shock s. Households can only adjust their holdings on illiquid assets at period t when $s_t = 1$, which occurs with iid probability λ. So, in each period, a randomly selected λ fraction of households can adjust their holdings of illiquid assets. When $s_t = 0$, the illiquid assets accumulate in the background:

$$d(h^t) = 0, \text{ if } s_t = 0. \tag{166}$$

The liquid assets are invested in government bonds. The illiquid assets are invested in equity. Firms issue equity to households, the price of each share is P_t, and each share provides dividends D_t. The amount of equity held by households is given by $v(h^t) \equiv a^{illiq}(h^t)/P_t$. The return on illiquid assets satisfies $R_t^{illiq} = (P_t + D_t)/P_{t-1}$.

Firms. Firms own capital K_{t-1} and choose investment I_t to obtain the capital of the next period $K_t = (1 - \delta)K_{t-1} + I_t$, subject to quadratic capital adjustment cost. Div-

idends equal capital products plus post-tax monopolistic profits net of investment, capital adjustment cost, and price adjustment cost.

$$D_t = r_t^K K_{t-1} + \alpha \Pi_t - I_t - \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta^K \right)^2 - \Theta_t. \tag{167}$$

Firms choose investment to maximize $P_t + D_t$. Tobin's Q and capital evolve according to the standard Q-theory of investment:

$$\frac{I_t}{K_{t-1}} - \delta^K = \frac{1}{\Psi}(Q_t - 1), \tag{168}$$

$$R_{t+1}^{illiq} Q_t = r_{t+1}^K - \frac{I_{t+1}}{K_t} - \frac{\Psi}{2} \left(\frac{I_{t+1}}{K_t} - \delta^K \right)^2 + \frac{K_{t+1}}{K_t} Q_{t+1}. \tag{169}$$

Equilibrium. The other sectors of the economy are the same as in Section 5.1. In equilibrium, households and firms optimize, the government budget constraint holds, nominal interest rates evolve according to the Taylor rule, and markets clear:

$$\int a^{liq}(h^t) d\Phi_t(h^t) = B_t, \tag{170}$$

$$\int a^{illiq}(h^t) d\Phi_t(h^t) = P_t, \tag{171}$$

$$C_t + I_t + \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 + \Theta_t = Y_t^{GDP}. \tag{172}$$

Note that there are two asset-market clearing conditions, one for liquid assets and another for illiquid assets.

In the following, I show that proposition 1 holds with the presence of illiquid assets, and there is an aggregate Euler equation governing the return on illiquid assets given the path of aggregate consumption.

Proposition 4. *For a given monetary policy shock ϵ, there exist counterfactual transfers ω such that:*

(i) *The equilibrium of aggregates can be characterized with only aggregate conditions, including the aggregate Euler equation with respect to liquid assets*

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{liq,ra} R_{t+1}^{liq,(\epsilon,\omega)} \left(C_{t+1}^{\epsilon,\omega} \right)^{-\sigma}, \text{ where } \beta^{liq,ra} \equiv 1/R^{liq,*}; \tag{173}$$

the aggregate Euler equation with respect to illiquid assets

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{illiq,ra} R_{t+1}^{illiq,(\epsilon,\omega)} (C_{t+1}^{\epsilon,\omega})^{-\sigma}, \text{ where } \beta^{illiq,ra} \equiv 1/R^{illiq,*}; \tag{174}$$

the aggregate labor supply condition

$$W_t^{\epsilon,\omega}(C_t^{\epsilon,\omega})^{-\sigma} = \varphi^{ra}(N_t^{\epsilon,\omega})^{\nu}, \text{ where } \varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}; \qquad (175)$$

the Phillips curve; Q theory of investment; the government budget constraint; the Taylor rule; and market-clearing conditions.

(ii) *The individual consumption and labor supply satisfy:*

$$\frac{c^{\epsilon,\omega}(h^t)}{c^*(h^t)} = \frac{C_t^{ra}}{C^*}, \quad \frac{n^{\epsilon,\omega}(h^t)}{n^*(h^t)} = \frac{N_t^{ra}}{N^*}. \qquad (176)$$

(iii) *The transfers sum to zero crosssectionally $\int \omega(h^t)d\Phi_t(h^t) = 0$.*

Proof. The proof of the first-order condition (F.O.C) with respect to liquid assets is the same as the one-asset model. I prove the F.O.C. with respect to illiquid assets below. In the case of adjustment ($s_t = 1$), the F.O.C with respect to illiquid assets is:

$$(c(h^{t-1}, (z_t, 1)))^{-\sigma} \geq$$

$$\left\{ \beta \lambda R_{t+1}^{illiq} E_z[(c(h^t, (z_{t+1}, 1)))^{-\sigma} | h^t, s_{t+1} = 1] + \right.$$

$$\beta^2 \lambda (1 - \lambda) R_{t+1}^{illiq} R_{t+2}^{illiq} E_z[(c(h^t, (z_{t+1}, 0), (z_{t+2}, 1)))^{-\sigma} | h^t, s_{t+1} = 0, s_{t+2} = 1] +$$

$$\beta^3 \lambda (1 - \lambda)^2 R_{t+1}^{illiq} R_{t+2}^{illiq} R_{t+3}^{illiq} E_z[(c(h^t, (z_{t+1}, 0), (z_{t+2}, 0)), (z_{t+3}, 1)))^{-\sigma} | h^t, s_{t+1} = s_{t+2} = 0, s_{t+3} = 1] +$$

$$\left. \cdots \right\}, = \text{ if } a^{illiq}(h^t) > 0. \qquad (177)$$

Consider households save one additional unit of illiquid assets at time t; then, with probability λ, the (accumulated) one unit of illiquid assets can be used for consumption at time $t + 1$, generating expected marginal utility

$$R_{t+1}^{illiq} E_z[(c(h^t, (z_{t+1}, 1)))^{-\sigma} | h^t, s_{t+1} = 1] \qquad (178)$$

at time $t + 1$; with probability $\lambda(1 - \lambda)$, the (accumulated) one unit illiquid assets can be used for consumption at time $t + 2$, generating expected marginal utility

$$R_{t+1}^{illiq} R_{t+2}^{illiq} E_z[(c(h^t, (z_{t+1}, 0), (z_{t+2}, 1)))^{-\sigma} | h^t, s_{t+1} = 0, s_{t+2} = 1] \qquad (179)$$

at time $t + 2$; with probability $\lambda(1 - \lambda)^2$, the (accumulated) one unit illiquid assets can be used for consumption at time $t + 3$, generating expected marginal utility

$$R_{t+1}^{illiq} R_{t+2}^{illiq} R_{t+3}^{illiq} E_z[(c(h^t, (z_{t+1}, 0), (z_{t+2}, 0)), (z_{t+3}, 1)))^{-\sigma} | h^t, s_{t+1} = s_{t+2} = 0, s_{t+3} = 1]$$

at time $t + 3$, etc.. Then the marginal value of the one additional unit of illiquid assets is the expected value of the (discounted) utility flows.

In the steady state, the F.O.C. with respect to illiquid assets

$$(c^*(h^{t-1},(z_t,1)))^{-\sigma} \geq$$

$$\left\{ \beta\lambda R^{illiq,*} E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+ \right.$$

$$\beta^2\lambda(1-\lambda)(R^{illiq,*})^2 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+$$

$$\beta^3\lambda(1-\lambda)^2(R^{illiq,*})^3 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+$$

$$\left. \cdots \right\}, = \text{if } a^{illiq,*}(h^t) > 0. \tag{180}$$

Given (180) holds, we verify the consumption allocation $\{c^{ra}(h^t)\}$ satisfies the F.O.C (177) given the interest rate path $\{R_{t+1}^{illiq,ra}\}$. Substituting $\{c^{ra}(h^t)\}$ into the F.O.C (177). The left-hand side is

$$(c^{ra}(h^{t-1},(z_t,1)))^{-\sigma} = (C_t^{ra}/C^*)^{-\sigma}(c^*(h^{t-1},(z_t,1)))^{-\sigma}, \tag{181}$$

and the right-hand side follows

$$\beta\lambda R_{t+1}^{illiq,ra} E_z[(c^{ra}(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+$$

$$\beta^2\lambda(1-\lambda)R_{t+1}^{illiq,ra}R_{t+2}^{illiq,ra}E_z[(c^{ra}(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+$$

$$\beta^3\lambda(1-\lambda)^2 R_{t+1}^{illiq,ra}R_{t+2}^{illiq,ra}R_{t+3}^{illiq,ra}E_z[(c^{ra}(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+$$

$$\cdots \tag{182}$$

$$=\beta\lambda R_{t+1}^{illiq,ra}(\frac{C_{t+1}^{ra}}{C^*})^{-\sigma}E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+$$

$$\beta^2\lambda(1-\lambda)R_{t+1}^{illiq,ra}R_{t+2}^{illiq,ra}(\frac{C_{t+2}^{ra}}{C^*})^{-\sigma}E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+$$

$$\beta^3\lambda(1-\lambda)^2 R_{t+1}^{illiq,ra}R_{t+2}^{illiq,ra}R_{t+3}^{illiq,ra}(\frac{C_{t+3}^{ra}}{C^*})^{-\sigma}E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+$$

$$\cdots \tag{183}$$

$$=\beta\lambda \frac{R_{t+1}^{illiq,ra}}{R^{illiq,*}}(\frac{C_{t+1}^{ra}}{C^*})^{-\sigma}R^{illiq,*}E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+$$

$$\beta^2\lambda(1-\lambda)\frac{R_{t+1}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+2}^{illiq,ra}}{R^{illiq,*}}(\frac{C_{t+2}^{ra}}{C^*})^{-\sigma}(R^{illiq,*})^2 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+$$

$$\beta^3\lambda(1-\lambda)^2\frac{R_{t+3}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+2}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+3}^{illiq,ra}}{R^{illiq,*}}(\frac{C_{t+3}^{ra}}{C^*})^{-\sigma}(R^{illiq,*})^3 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+$$

$$\cdots . \tag{184}$$

Given $(C_t^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}(C_{t+1}^{ra})^{-\sigma}$, where $\beta^{illiq,ra} \equiv 1/R^{illiq,*}$, we have

$$\frac{R_{t+1}^{illiq,ra}}{R^{illiq,*}}(C_{t+1}^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}(C_{t+1}^{ra})^{-\sigma} = (C_t^{ra})^{-\sigma}, \tag{185}$$

$$\frac{R_{t+1}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+2}^{illiq,ra}}{R^{illiq,*}}(C_{t+2}^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}\beta^{illiq,ra}R_{t+2}^{illiq,ra}(C_{t+2}^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}(C_{t+1}^{ra})^{-\sigma} = (C_t^{ra})^{-\sigma}, \tag{186}$$

$$\frac{R_{t+1}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+2}^{illiq,ra}}{R^{illiq,*}}\frac{R_{t+2}^{illiq,ra}}{R^{illiq,*}}(C_{t+3}^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}\beta^{illiq,ra}R_{t+2}^{illiq,ra}\beta^{illiq,ra}R_{t+3}^{illiq,ra}(C_{t+3}^{ra})^{-\sigma}$$

$$= \beta^{illiq,ra}R_{t+1}^{illiq,ra}\beta^{illiq,ra}R_{t+2}^{illiq,ra}(C_{t+2}^{ra})^{-\sigma} = \beta^{illiq,ra}R_{t+1}^{illiq,ra}(C_{t+1}^{ra})^{-\sigma} = (C_t^{ra})^{-\sigma}, \tag{187}$$

$$\cdots .$$

So equation (184) simplifies to

$$
\begin{aligned}
(C_t^{ra}/C^*)^{-\sigma}\Big\{ &\beta\lambda R^{illiq,*} E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda)(R^{illiq,*})^2 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2(R^{illiq,*})^3 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+ \\
&\cdots \Big\},
\end{aligned}
\tag{188}
$$

which is the marginal value of illiquid assets in the steady state scaled by $(C_t^{ra}/C^*)^{-\sigma}$. Combined with (181), we can see that given the F.O.C in the steady state (180) holds, $\{c^{ra}(h^t)\}$ satisfies the F.O.C (177) with the interest rate path $\{R_{t+1}^{illiq,ra}\}$.

An important implication of Proposition 4 is that in the "RANK" equilibrium, the liquid and illiquid return satisfies $R_t^{liq} = \frac{R^{liq,*}}{R^{illiq,*}} R_t^{illiq}$ and the liquidity premium $R_t^{illiq} - R_t^{liq}$ is nearly acyclical in the "RANK" equilibrium. All the responses of the liquidity premium are due to redistribution effects.

F.2 Liquidity Channel of Illiquid Assets

Compared to the one-asset model, there is an additional channel to consider with illiquid assets: the liquidity channel of illiquid assets. The change in the return on illiquid assets R_t^{illiq} impacts the illiquid assets that non-adjusters are forced to accumulate. Roughly speaking, if the aggregate shock reduces the return on illiquid assets, non-adjusters accumulate fewer illiquid assets, and some of these assets become liquid. The eased constraints on accumulating illiquid assets stimulate aggregate consumption. To distinguish it from the liquidity channel of bond supply, this channel is defined as the liquidity channel of illiquid assets. Formally, when the return on illiquid assets changes, the illiquid assets non-adjusters accumulate can differ from the imposed asset demand: $R_t^{illiq,ra} a^{illiq,ra}(h^{t-1}) \neq a^{illiq,ra}(h^t)$. To achieve the "RANK" equilibrium, I introduce a counterfactual shock to non-adjusters' demand for illiquid assets $\Delta a^{illiq}(h^t) = a^{illiq,ra}(h^t) - R_t^{illiq,ra} a^{illiq,ra}(h^{t-1})$ such that, given illiquid asset holdings $a^{illiq,ra}(h^{t-1})$ and return $R_t^{illiq,ra}$, the illiquid-asset demand of non-adjusters satisfies the imposed asset demand $a^{illiq,ra}(h^t)$.

Consider two simple cases for illustration. In the first case, illiquid assets are invested in government bonds, and the government maintains a constant level of debt. Impose the asset demand function $a^{illiq,ra}(h^t) = b^*(h^t)$. After an expansionary shock, for non-adjusters, $R_t^{illiq,ra} a^{illiq,ra}(h^{t-1}) = R_t^{illiq,ra} b^*(h^{t-1}) < R^{illiq,*} b^*(h^{t-1}) = b^*(h^t) = a^{illiq,ra}(h^t)$. To satisfy the imposed illiquid-asset demand $a^{illiq,ra}(h^t) = b^*(h^t)$, the illiquid-asset demand shock is

$$
\Delta a^{illiq}(h^t) = b^*(h^t) - R_t^{illiq,ra} b^*(h^{t-1}) = (R^{illiq,*} - R_t^{illiq,ra}) b^*(h^{t-1}). \tag{189}
$$

Figure 19: Responses of aggregates in the two-asset HANK model
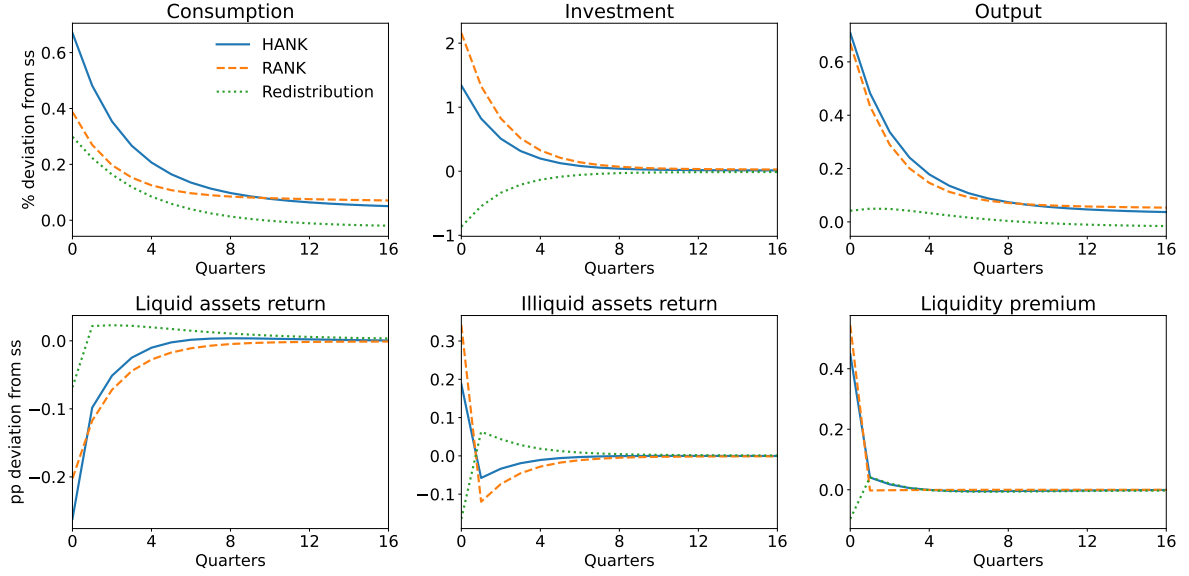
Notes: The responses of aggregate variables to a monetary policy shock in the two-asset HANK model. The liquid assets are invested in government bonds, and illiquid assets are invested in firm equity.

The decreasing return on illiquid assets implies that we need a positive shock $\Delta a^{illiq}(h^t) > 0$ to achieve the "RANK" equilibrium. In assessing the impact of the liquidity channel of illiquid assets, the negative of the counterfactual shock $\{-\Delta a^{illiq}(h^t)\}$ is input into the model, which is a shock that reduces non-adjusters' demand for illiquid assets. The falling interest rate relaxes the constraint on illiquid-asset accumulation. Some of the illiquid assets become liquid and aggregate consumption increases.

For the second case, consider that the illiquid assets are invested in firm equity and the imposed illiquid-asset demand is $a^{illiq,ra}(h^t) = P_t^{ra} v^*(h^t)$. For non-adjusters, the counterfactual shock to the illiquid-asset demand is

$$\Delta a^{illiq}(h^t) = P_t^{ra} v^*(h^t) - R_t^{illiq,ra} P_{t-1}^{ra} v^*(h^{t-1}) \tag{190}$$

$$= P_t^{ra} R^{illiq,*} v^*(h^{t-1}) - R_t^{illiq,ra} P_{t-1}^{ra} v^*(h^{t-1}) \tag{191}$$

$$= (P_t^{ra} R^{illiq,*} - P_{t-1}^{ra} R_t^{illiq,ra}) v^*(h^{t-1}). \tag{192}$$

If $P_t^{ra} R^{illiq,*} > P_{t-1}^{ra} R_t^{illiq,ra}$, we need a positive shock $\Delta a^{illiq}(h^t) > 0$ to achieve the "RANK" equilibrium, similar to the case that illiquid assets are invested in government bonds.

F.3 Monetary Policy in Two-asset HANK Model

This section decomposes the responses of a two-asset model to a monetary policy shock. The production side is calibrated as in the one-asset model in Section 6, except

Figure 20: Decomposition of redistribution effects in the two-asset HANK model
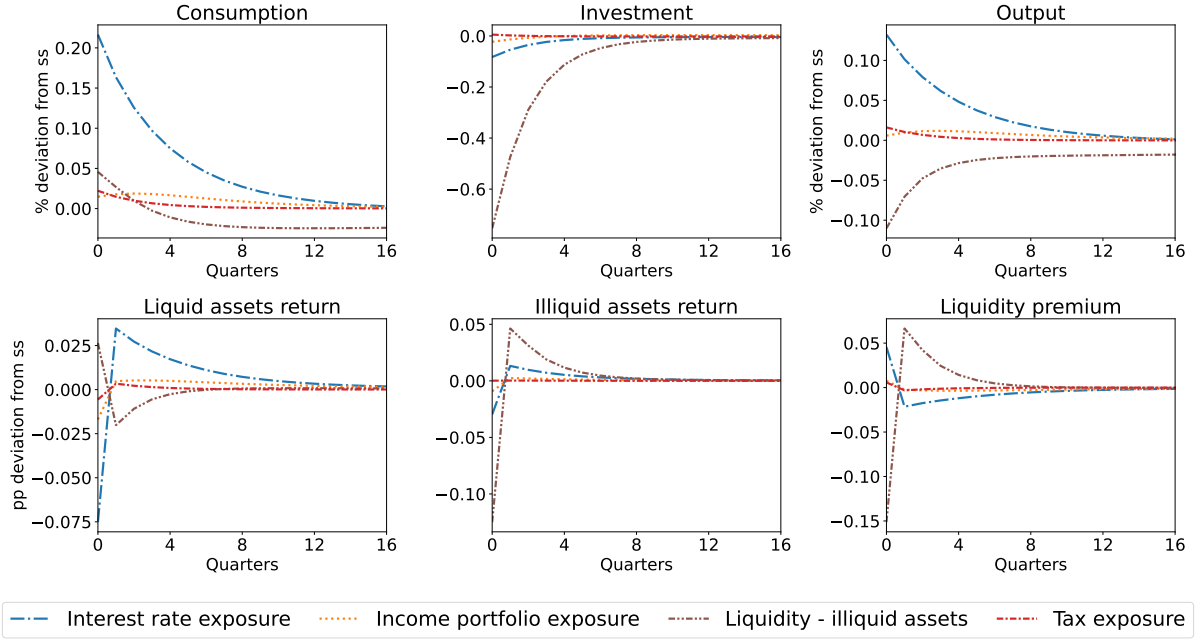
Notes: The decomposition of redistribution effects of a monetary policy shock in the two-asset HANK model. The liquid assets are invested in government bonds, and illiquid assets are invested in firm equity.

that the steady-state markup is set to zero. Set annual output to 1. The value of illiquid assets is set to $A = 2.375$ as in Kaplan and Violante (2022)[26], where they are calibrated to the 2019 U.S. economy. The annual real return on illiquid assets is $r^{illiq} = 0.06$ and on liquid assets is $r^{liq} = -0.02$. The borrowing constraint is normalized to zero. The value of liquid assets B, the adjustment probability λ, and the discount factor β are calibrated to match three targets: two market-clearing conditions for liquid and illiquid assets, and the year-0 iMPC estimated in Fagereng, Holm and Natvik (2021). The model performs well in hitting the three targets. The calibrated value of liquid assets is $B = 0.6$. Given the value of net liquid assets to annual output in the data is 0.375, the implied borrowing constraint (before normalization) in the model is $B^{un} - B = 0.375 - 0.6 = -0.225$, 1.3 times the quarterly average labor income. The calibrated adjustment probability is 0.06 and the discount factor is 0.982 (both quarterly).

For simplicity, I abstract from the labor income exposure channel and the liquidity channel of time-varying bond supply, as they depend on exogenous assumptions about labor income elasticities and fiscal policy responses. Individual labor income is given by $y^L(z^t) = z_t Y^L$. The value of government debt is assumed to be constant, and the government adjusts uniform taxes to balance its budget.

The responses of the aggregates are shown in Figure 19. The responses of the aggregates are close to the one-asset model in Section 6. However, the decomposition of the

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[26]The difference between net wealth and net liquid assets in their sample

Figure 21: Responses of aggregates in the two-account HANK model
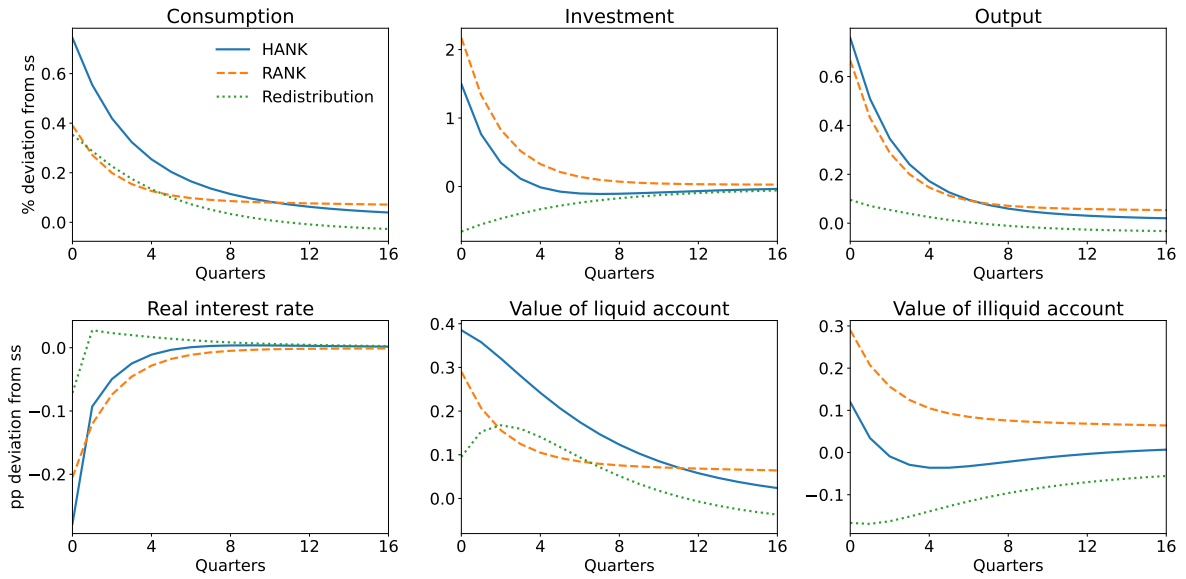
Notes: The responses of aggregate variables to a monetary policy shock in the two-account HANK model. Households can hold government bonds and firm equity in liquid and illiquid accounts as Auclert, Rognlie and Straub (2024).

redistribution effects shown in Figure 20 is quite different from the one-asset model. The effects of the interest rate exposure channel are twice as large as in the one-asset model. The impact of the income portfolio exposure channel is much smaller, being only about a quarter of its impact in the one-asset model. This is because the MPC from illiquid asset gains (and losses) is much smaller than that from liquid assets. The income portfolio exposure channel operates through redistribution between equity holders and workers, and in the model, equity is illiquid. The monetary policy shock pushes down the real return on illiquid assets, easing the constraints on holding illiquid assets. Its amplification effects on consumption responses are also more transitory than other channels, as it dampens investment responses and reduces the capital stock. The interest rate exposure channel has a large effect on consumption, but a much smaller effect on investment. Similarly, the liquidity channel of illiquid assets has a large effect on investment but a much smaller effect on consumption. Redistribution has an asymmetric impact on consumption/investment depending on whether it operates through liquid or illiquid assets.

F.4 "Two-account" HANK Model

The above decomposition assumes that liquid assets are invested in government bonds and illiquid assets are invested in firm equity, as in Kaplan, Moll and Violante (2018), Alves et al. (2020), Bayer et al. (2019), and Luetticke (2021). It implies that

households hold bonds $b(h^t) = a^{liq}(h^t)$ and the value of equity $P_t v(h^t) = a^{illiq}(h^t)$. In the next section, I consider another version of the two-asset model in which I follow Auclert, Rognlie and Straub (2024) and interpret $a^{liq}(h^t)$ and $a^{illiq}(h^t)$ as household savings in liquid and illiquid accounts, respectively. The savings in each account can be invested in government bonds and firm equity, and they are perfect substitutes (except at time 0, when the returns differ due to unexpected inflation and capital gains). The market-clearing condition for assets is instead

$$\int (a^{liq}(h^t) + a^{illiq}(h^t))d\Phi_t(h^t) = A_t = B_t + P_t, \qquad (193)$$

where A_t is the aggregate asset demand and $B_t + P_t$ is the aggregate asset supply. Liquid account returns are given by $R_t^{liq} = \frac{R^{liq,*}}{R^{illiq,*}} R_t^{illiq}$. The fraction of each account invested in government bonds and equity is assumed to be the same as the aggregate portfolio and homogeneous across agents. Thus, the individual holdings of bonds and equity are given by

$$b(h^t) = a^{liq}(h^t)\frac{B_t}{A_t} + a^{illiq}(h^t)\frac{B_t}{A_t}, \qquad (194)$$

$$P_t v(h^t) = a^{liq}(h^t)\frac{P_t}{A_t} + a^{illiq}(h^t)\frac{P_t}{A_t}. \qquad (195)$$

I calibrate the two-account model such that it has the same bond supply B, value of equity P, and adjustment probability λ as the baseline two-asset model in the previous section. The discount factor β is recalibrated to clear the asset market. The year-0 aggregate MPC is 46.3%, slightly lower than the baseline two-asset model's 49.8%. The value of the liquid account is 0.595, close to the value of liquid assets in the baseline two-asset model, which is 0.598.

Figure 21 shows the responses of aggregates under this two-account specification. Overall, the responses of aggregates are close to the baseline two-asset model. However, from Figure 22 we can see that the decomposition of redistribution effects is very different. The effect of interest rate exposure on consumption is only about half of its effect in the baseline two-asset model. This is because the MPC from gains (and losses) on illiquid assets is smaller than that from liquid assets. When bonds are held as illiquid assets, the interest rate change has a smaller effect on consumption. Instead, the interest rate change has a larger effect on investment than the baseline two-asset model because the marginal propensity of saving (MPS) from illiquid asset gains (and losses) is larger than the MPS from liquid assets. In this two-account specification, the liquidity channel of illiquid assets is the largest amplifier of consumption responses, rather than the interest rate exposure channel. From the responses of asset demand, we can also see that the relaxed constraints on accumulating illiquid assets increase

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Figure 22: Decomposition of redistribution effects in the two-account HANK model
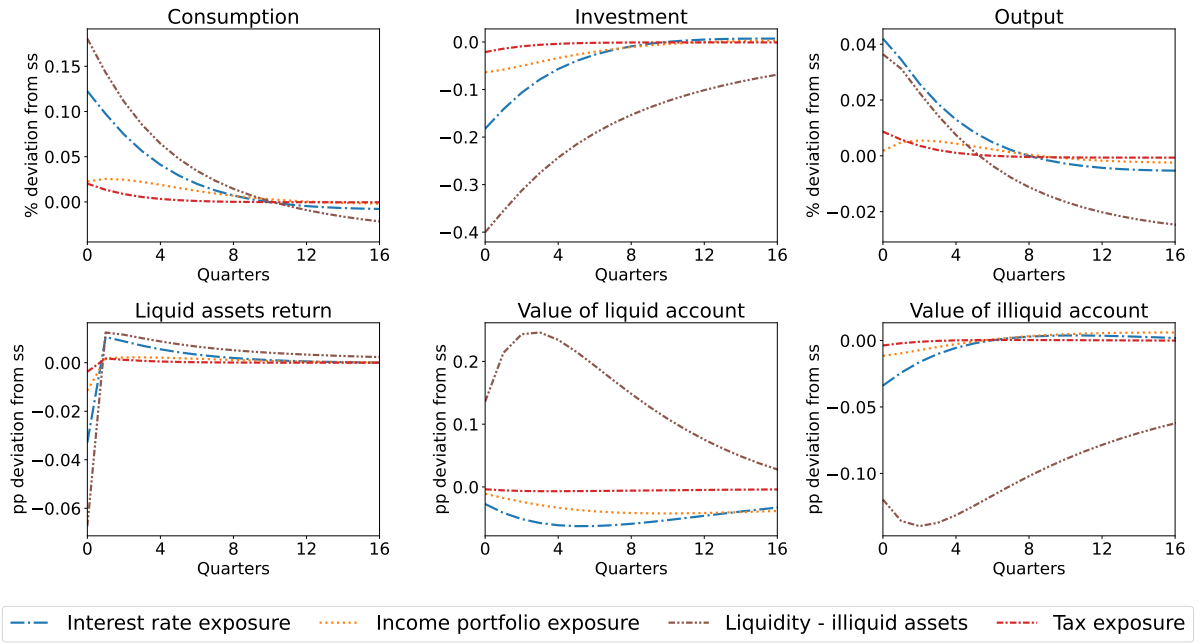
Notes: The decomposition of redistribution effects of a monetary policy shock in the two-account HANK model. Households can hold government bonds and firm equity in liquid and illiquid accounts as Auclert, Rognlie and Straub (2024).

households' demand for liquid assets and decrease the demand for illiquid assets.