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When Does Household Heterogeneity Matter for Aggregate Fluctuations?

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Abstract

This paper decomposes a Heterogeneous-Agent New Keynesian (HANK) model's responses to aggregate shocks into Representative-Agent (RANK) and redistribution effects. RANK effects are obtained by introducing counterfactual transfers neutralizing redistribution, ensuring homogeneous agent responses. Redistribution effects stem from the HANK model's response to the derived redistribution shock, which breaks down into interest rate exposure, income exposure, and liquidity channels. Following a monetary policy shock, RANK effects explain 62% of the consumption response; the interest rate, income, and liquidity channels contribute 16%, 14%, and 8% respectively. The decomposition is also applied to literature to identify key redistribution channels driving model dynamics.

Keywords: Heterogeneous households; Monetary Policy; Fiscal Policy; HANK model; Inequality; Business cycles. *JEL classification*: D31, E21, E43, E52, E62

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1 Introduction

Heterogeneous-agent models have become increasingly popular in macroeconomics. By introducing nominal rigidities, heterogeneous-agent New Keynesian (HANK) models can help us understand how household heterogeneity affects aggregate demand and economic fluctuations. HANK models can amplify/dampen the general equilibrium effects of aggregate shocks, relative to representative-agent New Keynesian (RANK) models. Consider the response of consumption to an expansionary monetary policy shock. If households who benefit from the shock have higher marginal propensities to consume (MPCs) than those who lose, the consumption response will be amplified; otherwise, the response will be dampened.

There is an extensive literature on heterogeneity in MPCs. However, the redistribution mechanism remains less understood. Quantitative HANK models can incorporate multiple redistribution channels. It is unclear how model specifications and parameterizations affect the redistribution and how the redistribution is correlated with households' MPCs. Is the consumption response always amplified? Which redistribution channel is amplifying and which channel is dampening? In terms of amplification, which channel is more important, and which channel plays a minor role? Answering these questions is challenging because these models have intractable dynamics and usually involve complicated numerical methods for solving.

The contribution of this paper is to provide a structural analysis and quantitative assessment of the redistribution mechanism in HANK models. The structural analysis contributes to the analytical HANK literature that makes simplifying assumptions to study aggregate shock transmission.[1] Two contributions within this literature are particularly relevant to this paper's approach. First, Werning (2015) shows that when agents are equally exposed to the aggregate shock, the heterogeneous-agent economy acts "as-if" it were a representative-agent economy, providing a benchmark for analyzing amplification/dampening mechanisms. Second, Auclert (2019) emphasizes the importance of the covariance between households' MPCs and the sensitivity of their incomes in amplifying monetary policy shocks — an approach this paper builds upon. Auclert (2019) characterizes the redistribution channels by aggregating individual consumption responses to perturbations to individual optimization problems. Since the perturbations are themselves the outcome of redistribution in general equilibrium, the identified redistribution channels are not considered structural shocks, limiting their use for counterfactual analysis.

Building on the insights of Werning (2015) and Auclert (2019), I develop a novel two-step framework to understand the model's response to an aggregate shock. In

[1] See Werning (2015), Auclert (2019), Acharya and Dogra (2020), Ravn and Sterk (2021), Bilbiie (2020), Bilbiie, Känzig and Surico (2022), Bilbiie (2024), and Debortoli and Galí (2024).

the first step, I consider the response with all redistribution channels muted, such that the consumption (and labor supply) of agents is equally affected by the aggregate shock. Importantly, in this case, an aggregation result akin to Werning (2015) arises, and the responses of aggregates are equivalent to those of a (fictitious) representative-agent model. In the second step, I unmute those redistribution channels one by one, allowing for a granular analysis of their effects, as in Auclert (2019). This two-step approach enables a clear distinction between two key components: the impact of a ”pure” aggregate shock — characterized by homogeneous responses across agents — and the additional effects resulting from the redistribution triggered by the shock.

Formally, I introduce counterfactual lump-sum transfers that counteract the redistribution implied by the aggregate shock, ensuring that all agents have the same consumption (and labor supply) responses. These transfers are designed to be purely redistributive and sum to zero cross-sectionally. With the transfers in place, the equilibrium can be characterized by the aggregate conditions of a (fictitious) RANK model. Then I utilize these transfers to back out the underlying redistribution shock. A key methodological innovation is that the redistribution shock functions as a structural shock — marking a significant departure from Auclert (2019)'s approach. Consequently, the general equilibrium impulse responses in HANK can be decomposed into two components: the responses of the (fictitious) RANK model to the aggregate shock (RANK effects), and the HANK model's responses to the redistribution shock (redistribution effects). Essentially, the transfers are introduced to make the implicit redistribution explicit, facilitating both an analytical decomposition of redistribution channels and their quantitative assessment.

I prove the existence of such transfers in various heterogeneous-agent models. These models include a two-agent model with a fraction of permanent-income households and a fraction of hand-to-mouth households, the standard Bewley-Aiyagari-Huggett model of incomplete markets, and standard incomplete market models that incorporate illiquid assets or ex-ante heterogeneity in household discount factors.[2] In the standard incomplete-market model and its variants, transfers depend on the history of idiosyncratic shocks households experience.

Consider an unexpected interest-rate cut. The two-step approach implies that to predict the policy's effects, the policymaker needs to know (i) the RANK model's response to the interest rate cut; and (ii) the HANK model's response to the redistribution shock triggered by the interest-rate cut. The RANK model's response is well-established in the literature. The HANK model's response to the redistribution shock generally requires solving numerically a full heterogeneous-agent model. However, we can obtain insights from a simple partial equilibrium analysis. Since the transfers

[2] These features allow the standard incomplete-market model to reproduce the large aggregate MPC observed in data.

are purely redistributive, the first-order consumption response to a transitory redistribution shock in partial equilibrium is given by the covariance between household MPCs and the redistribution shock they receive. I derive the model moments of the partial-equilibrium consumption response that can be estimated from data, following Auclert (2019) and Patterson (2023). Those moments can help identify important redistribution channels from data and predict the total effects of an interest-rate cut.

I first characterize the redistribution channels in a canonical HANK model in the style of McKay, Nakamura and Steinsson (2016). With counterfactual transfers, the aggregates satisfy the equilibrium conditions of a textbook RANK model (Galí 2015). In the canonical model, I identify three channels of redistribution: interest rate exposure, income exposure, and tax exposure. The interest rate exposure channel captures the redistribution between creditors and debtors. The income exposure channel reflects the redistribution among households with different income elasticities to aggregate income. The tax exposure channel focuses on the redistribution among households that are unequally affected by the change in tax payments.

Then I consider two model extensions: time-varying bond supply and investment. If the bond supply varies cyclically, a liquidity channel is also present. Due to the failure of Ricardian equivalence, the time-varying bond supply has real effects on the economy, captured by this liquidity channel. The responses of public debt affect how households borrow from and lend to each other. I extend the result of Aiyagari and McGrattan (1998) to transition dynamics and show that, under uniform taxation, bond supply shocks are equivalent to borrowing constraint shocks. Specifically, an increase in bond supply has the same effect on the economy as a shock that relaxes households' borrowing constraints, while a decrease in bond supply tightens these constraints, as in Guerrieri and Lorenzoni (2017).

For the second extension, I add investment to the model. Firms own capital, make investment decisions, issue equity, and pay dividends to households. Investment induces a redistribution between equity holders and workers via the income channel. Aggregate income comprises dividends and labor income, and investment responses affect their shares within aggregate income. Fluctuations in these shares impact the income elasticities of equity holders and workers in opposite directions. During capital accumulation, workers experience a higher income increase than equity holders, as equity holders save the equity return for future consumption. When the economy de-invests, equity holders' income increases more as they receive dividend payments.

I calibrate the model to the US economy and consider the model's response to an expansionary monetary policy shock. The decomposition reveals that redistribution effects amplify the responses of output and consumption while dampening those of investment and the real interest rate. On impact, consumption rises by 0.65 percent. The decomposition shows that RANK effects account for 62% of the increase, followed

Table 1: The redistribution following an expansionary monetary policy shock

	Interest rate exposure	Income portfolio exposure	Liquidity	Tax exposure
Low MPC	Creditors	Equity holders	Unconstrained	High labor income
	↓	S.R. ↓↑ L.R.	S.R. ↓↑ L.R.	↓
High MPC	Debtors	Workers	Constrained	Low labor income

Notes: S.R. refers to short run and L.R. refers to long run.

by interest rate exposure 16%, income exposure 14%, and liquidity 8%. The income exposure channel's 14% contribution can be further broken down: income portfolio exposure (redistribution between equity holders and workers) accounts for 10.3% percent; tax exposure (redistribution among taxpayers with different tax elasticities) contributes 2.6% percent; and labor income exposure (redistribution among workers with different labor income elasticities) adds 1.1%. The majority of the income exposure effect is driven by income portfolio exposure resulting from investment responses. Table 1 summarizes the redistribution following the shock.

Finally, I apply the decomposition to the literature to identify key redistribution channels that drive model dynamics. McKay, Nakamura and Steinsson (2016) argue that incomplete-market models can resolve the forward guidance puzzle through precautionary saving motives. However, Farhi and Werning (2019), Acharya and Dogra (2020), and Bilbiie (2024) reveal that the dampening or amplification of the power of forward guidance depends critically on the cyclicality of income risk and liquidity. My decomposition clarifies how market incompleteness resolves the forward guidance puzzle by showing that the negative redistribution effects in McKay, Nakamura and Steinsson (2016) stem from delicate model assumptions about profit distribution and taxation. Both channels redistribute from low-income to high-income households, dampening aggregate consumption responses. Quantitatively, redistribution due to profit distribution accounts for -80% of total consumption responses, while the redistribution due to taxation contributes -44%. I further apply this decomposition to analyze Auclert, Rognlie and Straub (2018)'s findings on fiscal multipliers in a HANK model with illiquid assets, where deficit-financed multipliers exceed one. The decomposition reveals that the amplified consumption and output responses primarily result from increased bond supply following government spending shocks, which effectively relax household borrowing constraints through the liquidity channel.

Related Literature. This paper contributes to the analytical HANK literature that makes simplifying assumptions to study how heterogeneity changes aggregate outcomes. The literature offers several different yet related approaches to this problem.

Auclert (2019) decomposes the aggregate consumption response to a transitory monetary policy change into substitution and income effects. The paper further breaks down the income effects into an ”aggregate income channel” and three redistribution channels: the ”earnings heterogeneity channel,” the ”Fisher channel,” and the ”interest rate exposure channel.” I follow the approach of Auclert (2019) in evaluating the partial-equilibrium effects of redistribution, but with a key distinction: I use aggregate prices and quantities from the RANK model to construct the redistribution shock, while Auclert (2019) employs the HANK model.

Leveraging the prices and quantities from the RANK model offers two key advantages over using a HANK model. First, RANK models are appealing due to their tractability, and their dynamics are well understood by researchers and practitioners. Second, they provide clearer counterfactuals. Consider, for example, the role of unequal income exposures in consumption responses. Focusing only on the ”earnings heterogeneity channel” of Auclert (2019) is insufficient. Changes in the parameter governing income exposures necessitate adjustments in the equilibrium interest rate, which then influences consumption through substitution effects and the ”interest rate exposure channel”. In equilibrium, income exposures affect the economy through all endogenous variables and channels defined in Auclert (2019). This limitation, common in partial-equilibrium analysis, can be addressed by constructing redistribution shocks from prices and quantities of the RANK model. This paper's decomposition ensures that unequal income exposures affect the economy exclusively through income-related redistribution channels, while the interest rate exposure channel operates independently of income-driven redistribution.[3]

Closely related to the decomposition of Auclert (2019), Kaplan, Moll and Violante (2018) study monetary policy transmission in a HANK model with rich heterogeneity and decompose the aggregate consumption responses into direct and indirect effects. Direct effects are consumption responses to the change in interest rates and indirect effects affect consumption through labor income responses. They emphasized two findings, that: (i) the indirect effects can be substantial, in contrast to RANK economies; (ii) the overall consumption response can be larger or smaller than in RANK, depending on various factors that are neutral in RANK. However, the relative size of direct and indirect effects is not directly related to the overall consumption response of HANK compared to RANK.[4] Building on Kaplan, Moll and Violante (2018)'s second finding,

[3]In the special case where an endogenous variable becomes exogenous — such as when the central bank directly controls real interest rates — the definition of the corresponding channel in both approaches coincides. The ”interest rate exposure channel” would then take the same form because the equilibrium interest rates would be identical in both the HANK and RANK models.

[4]As discussed in sections IV.B and IV.D of Kaplan, Moll and Violante (2018), the direct elasticity of consumption to interest rates is stable even across specifications where the total elasticity differs greatly. In the Appendix, I apply their decomposition to a two-agent model and show that the relative size of direct-indirect effects is a function of the measure of hand-to-mouth households and the amplification/dampening parameter. Conditional on the measure of hand-to-mouth households, we can infer the amplification/dampening parameter. In a model with richer heterogeneity, the relative size of direct-indirect effects will be a function of various structural parameters, which also affect the overall

my decomposition aims to characterize the channels through which the factors they discuss impact the overall consumption response and to quantify their roles.

Berger, Bocola and Dovis (2019) use a representative-agent economy augmented with wedges on the discount factor and the labor supply condition to study the aggregate implications of imperfect risk sharing. These wedges are functions of households' consumption shares and relative wages, while abstracting from the microeconomic details. Since these wedges are endogenous objects, their applicability in welfare assessments or policy analyses is limited. This paper explains the fluctuations of these wedges in fully structural models by analyzing the micro-level redistribution that affects consumption shares. This approach directly maps model primitives to redistribution and then to consumption shares, quantifying how much each redistribution channel contributes to the wedge fluctuations.

Werning (2015) examines situations where an incomplete-market economy can be aggregated to behave "as if" it were a representative-agent economy, corresponding to the RANK effects defined in this paper. In more complex scenarios where the "as if" result does not hold, I introduce counterfactual transfers to maintain aggregation. Hagedorn et al. (2019) use counterfactual transfers to construct the "as if" RANK economy and assess the imbalance between aggregate demand and aggregate supply off-the-equilibrium path in explaining the forward guidance puzzle. This paper has a different objective: I use the transfers to decompose the effects of aggregate shocks in general equilibrium. For this aim, I prove the existence of such transfers and show how to construct them. Bilbiie (2020) analytically characterize a TANK model's amplification/dampening mechanism. This paper shares the emphasis on unequal exposures of agents. Bilbiie, Känzig and Surico (2022) explore the role of investment in amplification in a tractable TANK model. I extend their analysis to a standard incomplete-market model. Debortoli and Galí (2024) show that TANK models can approximate the aggregate dynamics of HANK models when carefully specified and calibrated to embed the redistribution in HANK. Since the quality of the approximation depends on the context, the precise characterization and quantitative relevance of the redistribution channels presented in this paper provide additional insights into their approach.

This paper also adds to the quantitative HANK literature, which integrates nominal rigidity into incomplete-market models to study various macroeconomic questions.[5] Instead of studying a specific question, this paper assesses the redistribution

responses, though not in a tractable manner.

[5]These include fiscal transfers (Oh and Reis 2012), automatic fiscal stabilizers (McKay and Reis 2016), monetary policy transmission (Gornemann, Kuester and Nakajima 2016; McKay, Nakamura and Steinsson 2016; Kaplan, Moll and Violante 2018; Luetticke 2021; Auclert, Rognlie and Straub 2020), endogenous income risk (Ravn and Sterk 2017), de-leveraging (Guerrieri and Lorenzoni 2017), fiscal multipliers (Auclert, Rognlie and Straub 2018; Hagedorn, Manovskii and Mitman 2019), inequality and income risk shocks (Auclert and Rognlie 2018; Bayer et al. 2019), and business cycles (Bayer, Born and Luetticke 2024; Berger, Bocola and Dovis 2019; Bilbiie, Primiceri and Tambalotti 2023).

mechanism in a relatively general environment.

The characterization of redistribution channels allows me to discuss the literature in a unified framework. One example is the role of fiscal policy in the HANK literature. Previous studies found that the fiscal policy response is crucial for determining the effects of aggregate shocks. As discussed in the liquidity channel, the time-varying bond supply affects aggregate demand as it changes households' borrowing conditions. Guerrieri and Lorenzoni (2017) analyze the tightened borrowing constraint shock, a negative demand shock forcing constrained households to cut spending. When discussing the role of fiscal policy, Kaplan, Moll and Violante (2018) let public debt absorb the majority of the fiscal imbalance in the short run and find that the economy's responses to the monetary policy shock are much smaller. The borrowing constraint shock implied by the decreased bond supply is the deleveraging shock in Guerrieri and Lorenzoni (2017). The same argument applies to the analysis of fiscal multipliers. Auclert, Rognlie and Straub (2018) and Hagedorn, Manovskii and Mitman (2019) find that the deficit-financed fiscal multiplier is larger than the tax-financed multiplier. This result is due to the relaxed borrowing condition induced by the increasing bond supply. Bayer, Born and Luetticke (2023) investigate fiscal policy in a HANK model with portfolio choices, showing that the increasing bond supply stimulates consumption while avoiding investment crowding out. Wolf (2021), Wolf (2023), and Angeletos, Lian and Wolf (2023) study the role of deficit-financed lump-sum fiscal transfers as a stimulating policy tool, the effects of which are equivalent to relaxing borrowing constraints. The increasing bond supply allows constrained households to borrow, weakening the precautionary saving motive and stimulating aggregate consumption.

The rest of the paper is organized as follows. Section 2 defines the decomposition in a general heterogeneous-agent economy. Section 3 shows the existence of transfers in a canonical HANK model and discusses the redistribution channels. Section 4 extends the model with time-varying bond supply and investment. Section 5 derives the estimable moments for redistribution effects in partial equilibrium. Section 6 quantitatively decomposes the model's responses to a monetary policy shock. Section 7 applies the framework to existing literature. Section 8 concludes. The Appendix discusses several alternative models: a tractable TANK model, a canonical HANK model without investment, and a model with illiquid assets.

2 Aggregate Shock Decomposition

Consider a heterogeneous-agent economy. The specifics of heterogeneity will become detailed in future sections. For the current discourse, A reduced form is employed to outline the decomposition. Time is discrete and extends indefinitely $t =$

$0, 1, \cdots$. There is no aggregate risk and the perfect-foresight economy starts from its stationary equilibrium. At time $t = 0$, there is a one-time unexpected aggregate shock (MIT shock) following a mean-reverting process $\boldsymbol{\epsilon} = (\epsilon_0, \epsilon_1, \cdots)'$. In the infinite horizon, the economy is back to its initial equilibrium. I study the transition path following the aggregate shock. I first define the impulse responses and then discuss the decomposition of the shock and the impulse responses.

An aggregate variable Y's value in the stationary equilibrium is denoted as Y^*, which is constant across time. Following the shock $\boldsymbol{\epsilon}$, Y's value at time t along the transition path is denoted as $Y_t^{\boldsymbol{\epsilon}}$, and the entire time path is denoted as $\mathbf{Y}^{\boldsymbol{\epsilon}} = (Y_0^{\boldsymbol{\epsilon}}, Y_1^{\boldsymbol{\epsilon}}, \cdots)'$. Then we can define Y's impulse responses to the aggregate shock $\boldsymbol{\epsilon}$ as

$$\tilde{\mathbf{Y}}^{\boldsymbol{\epsilon}} \equiv \mathbf{Y}^{\boldsymbol{\epsilon}} - Y^* \cdot \mathbf{1}, \tag{1}$$

where $\mathbf{1}$ is the identity vector with all elements equal to one. For an individual variable y_i with individual index i, let y_{it}^* denote its value at time t in the stationary equilibrium, and $\mathbf{y}_i^* = (y_{i0}^*, y_{i1}^*, \cdots)'$ denotes the entire time path. Along the transition path, the path of variable y_i is denoted as $\mathbf{y}_i^{\boldsymbol{\epsilon}} = (y_{i0}^{\boldsymbol{\epsilon}}, y_{i1}^{\boldsymbol{\epsilon}}, \cdots)'$. The impulse responses of the individual variable y_i are defined as

$$\tilde{\mathbf{y}}_i^{\boldsymbol{\epsilon}} \equiv \mathbf{y}_i^{\boldsymbol{\epsilon}} - \mathbf{y}_i^*. \tag{2}$$

In the standard Bewley-Aiyagari-Huggett model of incomplete markets, the individual outcome is a function of the path of the individual's idiosyncratic shocks. In this case, the individual impulse responses are defined conditional on the path of idiosyncratic shocks.

Consider the responses of consumption. To isolate the role of redistribution, I introduce counterfactual transfers to individuals to construct a hypothetical scenario in which all agents exhibit identical consumption responses (in percentage terms) to the aggregate shock. Denote the set of individuals in the economy as I and consider the transfer scheme: $\boldsymbol{\omega} = \{\boldsymbol{\omega}_i\}_{i \in I}$, where $\boldsymbol{\omega}_i = (\omega_{i0}, \omega_{i1}, \cdots)'$ and ω_{it} is the transfer received by individual i at time t. The aggregate shock $\boldsymbol{\epsilon}$ can be written as

$$(\boldsymbol{\epsilon}, \mathbf{0}) = (\boldsymbol{\epsilon}, \boldsymbol{\omega}) + (\mathbf{0}, -\boldsymbol{\omega}). \tag{3}$$

The aggregate shock $\boldsymbol{\epsilon}$ is decomposed as a sum of two shocks. The first shock includes the aggregate shock $\boldsymbol{\epsilon}$ and the transfer scheme $\boldsymbol{\omega}$; and the second shock is the redistribution shock, which is defined as the negative of the transfer scheme $-\boldsymbol{\omega}$. By properly constructing the transfers, the redistribution induced by the aggregate shock can be removed, and all agents have the same consumption responses to $(\boldsymbol{\epsilon}, \boldsymbol{\omega})$:

$$\tilde{\mathbf{c}}_i^{\epsilon,\omega} / \mathbf{c}_i^* = \tilde{\mathbf{C}}^{\epsilon,\omega} / C^*, \forall i \tag{4}$$

This paper shows that such transfers exist in various heterogeneous-agent models. For power utilities, the dynamics of aggregates in this case can be characterized by the equilibrium conditions of a (fictitious) representative agent model.

Following the decomposition of the aggregate shock, the first-order impulse responses of an outcome variable Y admit an additive decomposition:

$$\tilde{\mathbf{Y}}^{\epsilon,\mathbf{0}} = \tilde{\mathbf{Y}}^{\epsilon,\omega} + \tilde{\mathbf{Y}}^{\mathbf{0},-\omega}. \tag{5}$$

Y's responses to the sum of two shocks equal the sum of its responses to each shock. This first-order relation also holds for the impulse responses of individual variables:

$$\tilde{\mathbf{y}}_i^{\epsilon,\mathbf{0}} = \tilde{\mathbf{y}}_i^{\epsilon,\omega} + \tilde{\mathbf{y}}_i^{\mathbf{0},-\omega}. \tag{6}$$

With the transfer scheme $\boldsymbol{\omega}$, I define the decomposition.

Definition 1. *The RANK effects of the aggregate shock $\boldsymbol{\epsilon}$ on variable Y are variable Y's responses to the aggregate shock $\boldsymbol{\epsilon}$ and the transfer scheme $\boldsymbol{\omega}$:*

$$\tilde{\mathbf{Y}}^{ra} \equiv \tilde{\mathbf{Y}}^{\epsilon,\omega}. \tag{7}$$

The redistribution effects of the aggregate shock $\boldsymbol{\epsilon}$ on variable Y are Y's responses to the redistribution shock, which is defined as the negative of the transfer scheme $-\boldsymbol{\omega}$:

$$\tilde{\mathbf{Y}}^{re} \equiv \tilde{\mathbf{Y}}^{\mathbf{0},-\omega}. \tag{8}$$

For an individual variable y_i, we can define the decomposition similarly. The RANK effects provide the benchmark for further analysis of redistribution.

3 Decomposing a Canonical HANK Model

This section considers a one-asset HANK model and decomposes the economy's response to a monetary policy shock. I describe the model in Section 3.1. In Sections 3.2, I show that with counterfactual transfers, aggregate variables satisfy the equilibrium conditions of a textbook RANK model (Galí, 2015). In Section 3.3, the redistribution shock is decomposed into three redistribution channels: interest rate exposure, income exposure, and tax exposure. Section 3.4 writes the household's problem in recursive form and discusses the computation of redistribution effects.

3.1 Model Description

Time is discrete and infinite. The economy is populated by households, firms, a fiscal and monetary policy authorities. In this economy, households face idiosyncratic uncertainty on incomes and have access to one-period risk-less government bonds, subject to exogenous borrowing constraints. There is price stickiness in the firm's price setting. The government collects taxes from households to pay interest on the debt. Monetary policy follows the Taylor rule. I analyze the economy's response to an innovation to the Taylor rule.

Households. There is a unit continuum of households that face idiosyncratic productivity shocks $z_t \in Z_t$. Let $z^t = (z_0, z_1, \cdots, z_t)$ be a history of idiosyncratic states up to period t. For ease of notation, the initial state z_0 also indexes the initial bond holdings. At $t = 0$, the economy inherits an initial distribution over idiosyncratic states and bonds $\Phi_0(z_0)$. The stochastic process then induces a distribution $\Phi(z^t)$ over histories $z^t \in Z^t$. Households are infinitely lived and have preferences over consumption $c(z^t)$ and labor supply $n(z^t)$ given by the utility function

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c(z^t), n(z^t))\right], \tag{9}$$

where β is the subjective discount factor. I also assume that the period utility function is given by power utilities

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \varphi\frac{n^{1+\nu}}{1+\nu}. \tag{10}$$

Households derive utility from consumption and dis-utility from working. Households face budget constraints

$$c(z^t) + b(z^t) = R_t b(z^{t-1}) + W_t z_t n(z^t) + \pi(z_t) - \tau(z^t), \tag{11}$$

for all $t = 0, 1, \cdots$ and histories $z^t \in Z^t$. Households face labor income risks so that if they work $n(z^t)$, they supply efficient labor $z_t n(z^t)$ to firms and receive labor income $W_t z_t n(z^t)$, where W_t is the real wage. The idiosyncratic productivity z_t evolves according to the first-order auto-regressive process $\log z_t = \rho_e \log z_{it-1} + e_{it}$ with normal innovations $e_{it} \sim \mathcal{N}(-\sigma_e^2(1-\rho_e^2)^{-1}/2, \sigma_e^2)$ so that $\int z_t d\Phi_t(z^t) = 1$. Households also receive (type-speific) profits $\pi(z_t)$ from intermediate firms and pay taxes $\tau(z^t)$ to government. The financial markets are incomplete. Households have access to a risk-free government bond with the gross real interest rate R_{t+1} between periods t and $t+1$.

However, households' bond holdings are subject to the borrowing constraints

$$b(z^t) \geq \phi, \tag{12}$$

where ϕ is the exogenous borrowing limit and is strictly higher than the natural borrowing limit.

Firms. A competitive final-good firm produces a final good from intermediate goods, indexed by j, according to the production function $Y_t = (\int y_{j,t}^{1/\mu} dj)^{\mu}$. The intermediate goods are produced by monopolistic competitive firms using labor as the only input with linear technology $y_{j,t} = A l_{j,t}$, where $l_{j,t}$ denotes the labor hired by firm j in period t. Each intermediate firm sets its price to maximize profits subject to quadratic price adjustment costs as in Rotemberg (1982)

$$\Theta_t(p_{j,t}, p_{j,t-1}) = \frac{\mu}{\mu - 1} \frac{1}{2\kappa} [\log(p_{j,t}/p_{j,t-1})]^2 Y_t \tag{13}$$

where $\kappa > 0$. The corresponding Philips curve can be derived as

$$\log(1 + \pi_t^p) = \kappa \left(\frac{W_t}{A} - \frac{1}{\mu} \right) + \frac{1}{R_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}^p), \tag{14}$$

where π_t^p is the inflation. The price adjustment creates real costs Θ_t, and profits equal the output net of labor expenditure and price adjustment costs $\Pi_t = Y_t - W_t L_t - \Theta_t$.

Fiscal Policy. The government collects taxes from households to pay interest on the debt, giving the budget constraint

$$B^* + T_t = R_t B^*, \tag{15}$$

where $T_t = r_t B^*$ is the aggregate tax. In the canonical model, the government maintains a constant level of debt B^* and adjusts taxes to balance its budget when interest rates change. In the next section, I will allow the government to adjust the level of outstanding debt and document a liquidity channel of the time-varying bond supply.

Monetary policy. The monetary authority sets the nominal interest rates on government bonds i_t according to the Taylor rule $i_t = r^* + \phi_\pi \pi_t^p + \epsilon_t$. The ex-post real interest rates satisfy the Fisher equation $R_t \equiv 1 + r_t = (1 + i_{t-1})/(1 + \pi_t^p)$.

Equilibrium Definition. Given a monetary policy shock $\epsilon = (\epsilon_0, \epsilon_1, \cdots)'$, an equilibrium consists of the path for aggregates $\{C_t, Y_t, N_t, L_t, R_t, W_t, B_t^d, \pi_t^p, \Pi_t, T_t\}$, profit distribution and tax payment $\{\pi(z_t), \tau(z^t)\}$, and household choices $\{c(z^t), n(z^t), b(z^t)\}$ such that:

(i) given initial bond holdings, the path of aggregates, profits distribution, and tax payment rules, households choose $\{c(z^t), n(z^t), b(z^t)\}$ to maximize their utility function subject to the budget constraints and borrowing constraints; firms optimize; government budget constraint holds; nominal interest rates evolve according to the Taylor rule;

(ii) aggregation and market-clearing: for $t = 0, 1, \cdots$, the good, labor, and bond markets clear:

$$C_t + \Theta_t = Y_t, \textit{ where } C_t = \int c(z^t) d\Phi_t(z^t); \tag{16}$$

$$N_t = L_t, \textit{ where } N_t = \int z_t n(z^t) d\Phi_t(z^t); \tag{17}$$

$$B_t^d = B^*, \textit{ where } B_t^d = \int b(z^t) d\Phi_t(z^t). \tag{18}$$

In the economy's stationary equilibrium, aggregate quantities and prices are constant, and inflation is zero. An outcome variable Y's stationary equilibrium value is denoted as Y^*, and Y's deviation from its stationary equilibrium value is denoted as $\tilde{Y}$. The percentage deviation is denoted as $\hat{Y}$.

3.2 RANK Effects

Assume the economy starts from the stationary equilibrium and consider the economy's response to an expansionary monetary policy shock $\boldsymbol{\epsilon} = (\epsilon_0, \epsilon_1, \cdots)'$. The shock evolves according to $\epsilon_t = \rho \epsilon_{t-1}$ where $\rho \in (0,1)$ is its persistence. I decompose the impulse responses of outcome variables into RANK and redistribution effects. To do this, I construct a transfer scheme $\boldsymbol{\omega} = \{\omega(z^t), \forall z^t \in Z^t\}_{t=0}^{\infty}$ where $\omega(z^t)$ is the lump-sum transfer received by the household conditional on the productivity path z^t. The household's budget constraints with the counterfactual transfers then read

$$c(z^t) + b(z^t) = R_t b(z^{t-1}) + W_t z_t n(z^t) + \pi(z_t) - \tau(z^t) + \omega(z^t). \tag{19}$$

Proposition 1 shows that for a given monetary policy shock $\boldsymbol{\epsilon}$, there exist counterfactual transfers $\boldsymbol{\omega}$ such that the heterogeneous-agent model is "as if" a representative-agent model.

Proposition 1. *For a given monetary policy shock $\boldsymbol{\epsilon}$, there exist counterfactual transfers $\boldsymbol{\omega}$ such that:*

(i) The aggregates satisfy the equilibrium conditions of a (fictitious) RANK model, including the aggregate Euler equation

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{ra} R_{t+1}^{\epsilon,\omega} (C_{t+1}^{\epsilon,\omega})^{-\sigma}, \text{ where } \beta^{ra} \equiv 1/R^*; \tag{20}$$

the aggregate labor supply condition

$$W_t^{\epsilon,\omega} (C_t^{\epsilon,\omega})^{-\sigma} = \varphi^{ra} (N_t^{\epsilon,\omega})^{\nu}, \text{ where } \varphi^{ra} \equiv W^* (C^*)^{-\sigma} (N^*)^{-\nu}; \tag{21}$$

the Philips curve; government budget constraint; Taylor rule; and market-clearing conditions.

(ii) The individual consumption and labor supply satisfy:

$$\frac{c^{\epsilon,\omega}(z^t)}{c^*(z^t)} = \frac{C_t^{\epsilon,\omega}}{C^*}, \quad \frac{n^{\epsilon,\omega}(z^t)}{n^*(z^t)} = \frac{N_t^{\epsilon,\omega}}{N^*}. \tag{22}$$

(iii) The transfers sum to zero cross-sectionally: $\int \omega(z^t)\, d\Phi_t(z^t) = 0$.

Proof. *See Appendix.*

The fictitious representative agent's subjective discount factor is defined as the steady-state real discount rate, $\beta^{ra} \equiv 1/R^*$. Proposition 1 aligns with the "as if" result in Werning (2015): although heterogeneity influences the level of the real interest rate (given the path of aggregate consumption), it does not necessarily alter the elasticity of aggregate consumption to the change in the real interest rate. To account for level effects, the fictitious representative agent's discount factor β^{ra} differs from the true agent's discount factor β. This reasoning similarly applies to labor supply, where the labor-supply disutility parameter is defined as $\varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}$.

It is also useful to consider how aggregate shocks influence precautionary saving motives. Although the aggregates meet the equilibrium conditions of a complete market model, the economy remains an incomplete market economy. There is a distribution of agents who move across individual states z_t. The consumption (and labor supply) share of agents is not constant. Agents have precautionary saving motives due to idiosyncratic income risks. However, counterfactual transfers prevent the aggregate shock from inducing "cyclical heterogeneity", ensuring that, conditional on the individual path z^t, the consumption (and labor supply) share remains constant. The precautionary saving motives regarding consumption inequality across different individual paths are not affected by the aggregate shock.

Counterfactual transfers ensure that scaled individual choices satisfy budget constraints. With such transfers, we can define the decomposition as in Section 2. I use the terminology "RANK" equilibrium to refer to the equilibrium in Proposition 1. Henceforth, all variables in the "RANK" equilibrium are denoted with superscript "ra".

From the equilibrium conditions of the fictitious representative agent model, we can obtain the path of aggregates $\{R_t^{ra}, W_t^{ra}, C_t^{ra}, Y_t^{ra}, \pi_t^{p,ra}, \Pi_t^{ra}, T_t^{ra}\}$ in the "RANK" equilibrium. The elements of the household budget constraint are obtained as follows. The path of aggregate consumption and labor supply $\{C_t^{ra}, N_t^{ra}\}$ and individual consumption and labor supply in the stationary equilibrium $\{c^*(z^t), n^*(z^t)\}$ determine the individual's consumption and labor supply $\{c^{ra}(z^t), n^{ra}(z^t)\}$ according to equation 22. The aggregate profit Π_t^{ra} and the profit distribution rule determine the individual profits income $\pi^{ra}(z_t)$. The aggregate tax T_t^{ra} and tax payment rule determine individual taxes $\tau^{ra}(z^t)$. To recover the transfer term $\omega(z^t)$ from the household budget constraint, we also need the bond demand function $b^{ra}(z^t)$. In the proof of Proposition 1, I impose the bond demand function $b^{ra}(z^t) = b^*(z^t)$, which is effectively a normalization. Without additional restrictions on the timing of transfers, the transfers can only be pinned down by pinning down the bond demand function.

As shown below, the bond demand function $b^{ra}(z^t)$ and the corresponding transfer scheme $\boldsymbol{\omega}$ are indeterminate. The intuition is similar to the Ricardian equivalence of a representative agent model. Under Ricardian equivalence, the timing of taxes does not affect the equilibrium. In our case, the timing of transfers does not affect agents' consumption and labor supply decisions. The income loss at time t can be compensated by future or past income, and households with access to financial markets will use bonds to move income across time. However, the timing of transfers does affect the path of individual bond holdings.

Proposition 2. *Given individual consumption $\{c^{ra}(z^t)\}$ and the path of real interest rates $\{R_{t+1}^{ra}\}$, bond demand in the "RANK" equilibrium $\{b^{ra}(z^t)\}$ is characterized by:*

(i) The borrowing constraint and complementary slackness condition: $b^{ra}(z^t) \geq \phi, =$ if $u'(c^{ra}(z^t)) > \beta R_{t+1}^{ra} E[u'(c^{ra})(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi) = 0$;

(iii) Bond market clearing: $\int b^{ra}(z^t) d\Phi_t(z^t) = B^*$.

Proof. *See Appendix.*

Households with access to the financial markets have indeterminate bond demand. However, constrained households have a fixed bond demand at the borrowing limit ϕ. The transversality condition is a necessary condition of household optimization, and bond market clearing is one of the market clearing conditions. With $b^{ra}(z^t)$, the corresponding transfer $\omega(z^t)$ is recovered from the household budget constraint

$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - W_t^{ra} z_t n^{ra}(z^t) - \pi^{ra}(z_t) + \tau^{ra}(z^t). \tag{23}$$

In practice, it is natural to choose $b^{ra}(z^t)$ as a function of $b^*(z^t)$ (see Section 3.4). Since $b^*(z^t)$ satisfies the stationary-equilibrium counterparts of the conditions in Proposition 2, it is feasible to verify that the chosen bond demand $b^{ra}(z^t)$ satisfies those conditions.

3.3 Redistribution Channels

Define $y(z^t) \equiv W_t z_t n(z^t) + \pi(z_t)$ as income. The Appendix shows that the redistribution shock can be decomposed as follows

$$-\omega(z^t) = \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\tau^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}}$$
$$+ \underbrace{\hat{C}_t^{ra}(b^*(z^t) - R^*b^*(z^{t-1}) + \tau^*(z^t))}_{\text{saving flow exposure}} + \underbrace{(b^*(z^t) - b^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - b^{ra}(z^{t-1}))}_{\text{undetermined bond demand}}. \tag{24}$$

I define three sources of redistribution: income exposure, interest rate exposure, and tax exposure channels. The income exposure channel is defined as

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t), \tag{25}$$

which captures the redistribution among households with different income elasticities to aggregate income ($\hat{y}^{ra}(z^t) \neq \hat{Y}_t^{ra}$). The interest exposure channel is defined as

$$(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*), \tag{26}$$

which foucses the redistribution among creditors and debtors. After consolidating the government budget constraint into the household budget constraint, we can find that, the net bond position $b^*(z^{t-1}) - B^*$, rather than the gross position $b^*(z^{t-1})$, determines the bondholder's exposure to the interest rate shock. The tax exposure channel is defined as

$$(\tau^*(z^t) - r^*B^*) - (\tau^{ra}(z^t) - r_t^{ra}B^*), \tag{27}$$

which reflects different exposures to the change in taxes. In the case of uniform taxation $\tau(z^t) = T_t = r_t B^*$ we have $\tau^*(z^t) - r^*B^* = \tau^{ra}(z^t) - r_t^{ra}B^*$, and the tax exposure channel is muted because all households benefit equally from the tax reduction. For more general taxing schemes, households may benefit or lose from the tax change.

There are also two residual terms. Define $b(z^t) - R_t b(z^{t-1}) + \tau(z^t) = y(z^t) - c(z^t)$ as saving flows. In the stationary equilibrium, the individual saving flows are generally not zero, and the first residual term "saving flow exposure" is to compensate for the scaling of the saving flows. For the quantitative model with a reasonable calibration, its effects are negligible because (i) the saving flow is small relative to consumption and income; (ii) the MPCs' heterogeneity between households with positive and negative flows is small. The last term is due to the undetermined bond demand function. After imposing $b^{ra}(z^t) = b^*(z^t)$, this term is zero.[6]

3.4 Households' Problem in Recursive Form

To compute the redistribution effects with the method of policy function iteration, I write the household's problem in recursive form. First, we need to impose a bond demand function of the "RANK" equilibrium, which needs to satisfy Proposition 2 and is not unique. I use the following bond demand function

$$b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi). \tag{28}$$

When the government maintains a constant-debt path, $B_t^{ra} = B^*$ and $b^{ra}(z^t) = b^*(z^t)$. When government debt is time-varying (see Section 4.1), the function $g_t(\cdot)$ shrinks or stretches the stationary-equilibrium bond demand function, keeping the lower bound of bond demand at the borrowing limit. I show that $b^{ra}(z^t)$ satisfies Proposition 2 in the Appendix.

Let $c^*(z, b^*)$, $b'^*(z, b^*)$, and $n^*(z, b^*)$ denote the household's consumption, bond demand, and labor supply policy functions in the stationary equilibrium, where b^* is the household's wealth in the stationary equilibrium. Note that from the path of aggregates and the household's states in the stationary equilibrium, transfers are fully pinned down:

$$\omega_t(z, b^*) = \frac{C_t^{ra}}{C^*}c^*(z, b^*) + g_t(b'^*(z, b^*)) - R_t^{ra}g_{t-1}(b^*) - W_t^{ra}z\frac{N_t^{ra}}{N^*}n^*(z, b^*) - \pi_t^{ra}(z) + \tau_t^{ra}(z). \tag{29}$$

I use the household's wealth in stationary equilibrium b^* as an exogenous state variable to summarize an individual's history relevant for determining the transfers. The household's problem with state-dependent transfers $\omega_t(z, b^*)$ in recursive form is:

$$\begin{aligned} V_t^{ra}(z, b^*, b) = \max_{\{c,n,b'\}} \; & u(c, n) + E[V_{t+1}^{ra}(z', b'^*, b')|z, b^*], \\ s.t. \quad & c + b' = R_t b + W_t z n + \pi_t(z) - \tau_t(z) + \omega_t(z, b^*), \\ & b' \geq \phi. \end{aligned} \tag{30}$$

[6] Theoretically, a bond demand function different from $b^*(z^t)$ has real effects, as the equivalence between different transfer scheme is evaluated along the interest rate path $\{R_t^{ra}\}$, while the redistribution shock is input into the model's steady state with interest rates R^*.

The law of motion for the exogenous state b^* is the bond demand policy function in the stationary equilibrium $b'^* = \mathscr{b}'^*(z, b^*)$. Along the equilibrium path, the household's policy functions satisfy, for $b = g_{t-1}(b^*)$,

$$c_t^{ra}(z, b^*, b)/c^*(z, b^*) = C_t^{ra}/C^*, \tag{31}$$

$$n_t^{ra}(z, b^*, b)/n^*(z, b^*) = N_t^{ra}/N^*, \tag{32}$$

$$\mathscr{b}_t'^{ra}(z, b^*, b) = g_t(\mathscr{b}'^*(z, b^*)). \tag{33}$$

The method above demonstrates that tracking the entire individual history z^t is unnecessary to determine transfers. Household's bond demand in the stationary equilibrium is enough to pin down the transfers. However, there is still a high computational cost due to the additional state variable b^*. In Section B.1 of the Appendix, I simplify the method and make transfers based on the household equilibrium state (z, b) and yield results that are practically the same as those obtained using the method above.

4 Time-varying Bond Supply and Investment

The previous section establishes the decomposition framework within a simple heterogeneous-agent model. In this section, I explore two extensions: cyclical bond supply and investment. These features are common in the literature and play an essential role in analyzing business cycles. The decomposition effectively sheds light on their roles in redistribution and HANK models. Section 4.1 discusses how the path of public debt influences household borrowing and lending. Section 4.2 shows that investment responses lead to a redistribution between equity holders and workers.

4.1 Time-varying Bond Supply

In the baseline model, the government maintains a constant debt level. Previous studies in the quantitative HANK literature found that the fiscal policy response is crucial for the transmission of aggregate shocks.[7] Following an aggregate shock, the government can also adjust public debt to balance its budget. This section attributes the effects induced by time-varying bond supply to the liquidity channel.

Assume the fiscal policy induces a time-varying bond supply such that $B_t > \phi$ and $\lim_{t\to\infty} B_t = B^*$. The government budget constraint is then $B_t + T_t = R_t B_{t-1}$. Due to the failure of Ricardian equivalence, changing the bond supply has real effects: the timing of taxes directly affects the consumption of non-Ricardian households. Let $\bar{b}(z^t)$, $\bar{\tau}(z^t)$, and $\bar{T}_t$ denote the bond demand function, individual tax payment, and

[7]See Kaplan, Moll and Violante (2018), Alves et al. (2020), Auclert, Rognlie and Straub (2018), Hagedorn, Manovskii and Mitman (2019), Wolf (2021) and Wolf (2023).

aggregate tax, respectively, **assuming** the bond supply is constant. When the bond supply is cyclical, the actual bond demand $b(z^t)$, individual taxes $\tau(z^t)$, and aggregate tax T_t will deviate from these counterparts.

In this case, the redistribution shock $-\omega(z^t)$ can be decomposed as follows :

$$
\begin{aligned}
-\omega(z^t) = & \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}} \\
& + \underbrace{(\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity}} \\
& + \underbrace{\hat{C}_t^{ra}(b^*(z^t) - R^*b^*(z^{t-1}) + \tau^*(z^t))}_{\text{saving flow exposure}} + \underbrace{(b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}))}_{\text{undermined bond demand}}.
\end{aligned}
$$

The income, interest rate, and tax exposure channels are defined as before and are independent of the path of public debt. The liquidity channel is defined as

$$
(\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)). \tag{34}
$$

As in the previous section, after imposing the bond demand function $\bar{b}^{ra}(z^t) = b^*(z^t)$, the last term "undetermined bond demand" is zero.

The liquidity channel may seem obscure at first. To understand it better, consider the subgroup of households that remain constrained $\bar{b}^{ra}(z^t) = b^{ra}(z^t) = \phi$. With uniform taxation, $\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t) = \bar{T}_t^{ra} - T_t^{ra}$. For these households, equation (34) simplifies to $\bar{T}_t^{ra} - T_t^{ra}$. The liquidity channel captures the effects of altering the timing of taxes. When the government shifts the timing of taxes through deficit financing, it transfers income across time for households. In partial equilibrium, the consumption of unconstrained households hardly changes because the net present value of the tax change is zero. However, the consumption of constrained households responds one-to-one to the change in their disposable income. In general equilibrium, the interest rate will adjust to clear the bond markets, and unconstrained households absorb the change in government debt by reallocating their consumption over time.

To link the above mechanism more closely with the concept of "liquidity", I show that in the case of uniform taxation, the liquidity channel can be proxied by counterfactual shocks to the borrowing constraint ϕ. To eliminate the real effects on consumption when the government changes the timing of taxes, one approach is to introduce counterfactual transfers, as shown above. Another approach is to introduce counterfactual shocks to borrowing constraints, which force households to absorb the tax changes through bond holdings rather than consumption.

Proposition 3. *Off the constant-debt path, assume (i) uniform taxation such that $\tau(z^t) - \bar{\tau}(z^t) = T_t - \bar{T}_t$; (ii) borrowing constraint $\phi_t = \phi + B_t - B^*$. Given individual consumption $\{c^{ra}(z^t)\}$ and the path of real interest rates $\{R_{t+1}^{ra}\}$, the bond demand in the "RANK"* *equilibrium is characterized by the following conditions:*

(i) The borrowing constraint and complementary slackness condition: $b^{ra}(z^t) \geq \phi_t^{ra}, =$ *if* $u'(c^{ra}(z^t)) > \beta R_{t+1}^{ra} E[u'(c^{ra}(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi_t^{ra}) = 0$;

(iii) Bond market clearing: $\int b^{ra}(z^t) d\Phi_t(z^t) = B_t^{ra}$.

For any bond demand function $\bar{b}^{ra}(z^t)$ *satisfying the conditions in Proposition 2, the shifted bond demand function* $b^{ra}(z^t) \equiv \bar{b}^{ra}(z^t) + B_t^{ra} - B^*$ *satisfies the above conditions. The counterfactual transfers* $\omega(z^t)$ *are invariant to the path of public debt.*

Proof. *See Appendix.*

The argument is similar to those in Aiyagari (1994), Aiyagari and McGrattan (1998), and Bhandari et al. (2017). Proposition 3 extends their results to transition paths. Suppose government debt increases by ΔB. For the same consumption choice $c^{ra}(z^t)$, the household now holds ΔB additional units of bonds into the next period, shifting the wealth distribution for each household across all states. To satisfy the complementary slackness condition of constrained households, the borrowing limit is also increased by the same amount ΔB.

Proposition 3 implies that in the case of uniform taxation, we can use counterfactual shocks to the borrowing constraint to proxy the liquidity channel. In this case, the term (34) equals zero, and the effects of the liquidity channel are reflected in the economy's response to the borrowing constraint shock $-\Delta\boldsymbol{\phi} \equiv -\{B_t^{ra} - B^*\}_{t=0}^{\infty}$.

A common specification of fiscal policy in the quantitative HANK literature is to use government debt to offset fiscal imbalances in the short run and use taxes to restore the debt in the long run. This fiscal rule implies that, after a decrease in interest rates, government debt drops on impact and gradually returns to its steady-state level. When assessing the effects of the liquidity channel, $-\Delta\boldsymbol{\phi}$ is exactly the deleveraging shock in Guerrieri and Lorenzoni (2017). The binding borrowing constraint compels poor households to deleverage. The deleveraging shock lowers equilibrium real interest rates and dampens the consumption response.

In the context of non-uniform taxation, we can use path-dependent counterfactual borrowing constraint shocks to proxy the liquidity channel. To illustrate, consider the bond demand function $b^{ra}(z^t)$ given by

$$\bar{b}^{ra}(z^t) - b^{ra}(z^t) = R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) - (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)). \tag{35}$$

When government debt deviates from the constant-debt path $\bar{\tau}^{ra}(z^t) \neq \tau^{ra}(z^t)$, households absorb the change of tax payment through bond holdings $b^{ra}(z^t)$. To ensure this is an equilibrium demand function, we can construct the path-dependent borrowing constraints $\phi^{ra}(z^t)$ such that[8]

$$b^{ra}(z^t) \geq \phi^{ra}(z^t), = \text{ if } u'(c^{ra}(z^t)) > \beta R^{ra}_{t+1} E[u'(c^{ra}(z^{t+1}))|z^t]. \tag{36}$$

The effects of altering the timing of taxes depend on the taxation scheme. Households may experience gains or losses in real terms from changes in tax timing, depending on their tax payment histories. In the current decomposition framework, I attribute all effects — including those that are 'real' redistributive effects — resulting from the varying path of government debt to the liquidity channel.[9]

4.2 Investment

This section incorporates investment into the model, following the approach of Auclert, Rognlie and Straub (2018). Capital (equity) is considered liquid and serves as a perfect substitute for bonds. In Section F, I extend the model with illiquid assets.

4.2.1 Model Description

Households. Households can also trade in firm shares with price P_t, which provides a dividend stream D_t each period. The household's budget constraint is

$$c(z^t) + b(z^t) + P_t v(z^t) = R_t b(z^{t-1}) + (P_t + D_t) v(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z^t). \tag{37}$$

The non-arbitrage condition requires that $R_t = (P_t + D_t)/P_{t-1}$ from $t = 1$. Define total wealth $a(z^t) \equiv b(z^t) + P_t v(z^t)$, from $t = 1$ the budget constraints faced by households can be written as

$$c(z^t) + a(z^t) = R_t a(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z^t). \tag{38}$$

[8] The transversality condition is $\lim_{t\to\infty} \beta^t E_0 u'(c^{ra}(z^t))(b^{ra}(z^t) - \phi^{ra}(z^t)) = 0$. The bond demand function given by equation 35 may become unbounded for some paths z^t depending tax payment histories, even B_t is bouned. The transversality condition incorporates the exogenous borrowing constraints leading to explosive individual bond holdings.

[9] Consider the case of productivity-based taxation with temporary tax cuts financed by future tax increases. The 'pure' liquidity effects stimulate consumption as the disposable income of low-productivity households increases in the current period and decreases in the future, similar to uniform taxation. However, this change also impacts the net present value of household income. Households with low productivity, who expect to revert to higher productivity, may lose from the change in tax timing. Conversely, high-productivity households benefit from the timing shift. This 'real' redistribution dampens consumption responses. Combining these real redistributive effects with the 'pure' liquidity effects results in an understatement of the 'pure' liquidity effects.

At $t = 0$, the return on bonds and equity can be different. Bond return is subject to unexpected inflation, and equity return is subject to unexpected capital gains:

$$c(z^0) + a(z^0) = R_0 b_{-1} + (P_0 + D_0)v_{-1} + z_0 W_0 n(z^0) + \pi(z_0) - \tau(z^0). \tag{39}$$

Households are subject to the non-borrowing constraints $a(z^t) \geq 0$.

Firms. The intermediate goods firms have a Cobb Douglas production function $y_{j,t} = Ak^{\alpha}_{j,t-1} n^{1-\alpha}_{j,t}$. The marginal cost also includes rents on capital. The Philips Curve is

$$\log(1 + \pi^p_t) = \kappa \left(mc_t - \frac{1}{\mu} \right) + \frac{1}{R_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi^p_{t+1}). \tag{40}$$

where $mc_t = (r^K_t/\alpha)^{\alpha} (W_t/(1-\alpha))^{1-\alpha}/A$. Firms own capital K_{t-1} and choose investment I_t to obtain the capital of the next period $K_t = (1-\delta)K_{t-1} + I_t$, subject to quadratic capital adjustment cost. Dividends equal capital products plus post-tax monopolistic profits net of investment, capital adjustment cost, and price adjustment cost,

$$D_t = r^K_t K_{t-1} + \alpha \Pi_t - I_t - \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta^K \right)^2 - \Theta_t. \tag{41}$$

Firms choose investment to maximize $P_t + D_t$. Tobin's Q and capital evolve according to the standard Q-theory of investment:

$$\frac{I_t}{K_{t-1}} - \delta^K = \frac{1}{\Psi}(Q_t - 1), \tag{42}$$

$$R_{t+1} Q_t = r^K_{t+1} - \frac{I_{t+1}}{K_t} - \frac{\Psi}{2} \left(\frac{I_{t+1}}{K_t} - \delta^K \right)^2 + \frac{K_{t+1}}{K_t} Q_{t+1}. \tag{43}$$

The monopolistic profits Π_t are taxed, so firms receive an α fraction of the monopolistic profits. The remaining $1 - \alpha$ fraction is paid to households as a lump-sum transfer in proportion to household productivity. This profit taxation scheme fully neutralizes the impact of countercyclical markups and generates reasonable asset price responses.

Equilibrium Definition. In the equilibrium, households and firms optimize, nominal interest rates evolve according to the Taylor rule, and markets clear:

$$\int a(z^t) d\Phi_t(z^t) = B_t + P_t, \tag{44}$$

$$\int z_t n(z^t) d\Phi_t(z^t) = L_t, \tag{45}$$

$$C_t + I_t + \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 + \Theta_t = Y^{GDP}_t. \tag{46}$$

4.2.2 Redistribution Channels with Investment

The Appendix shows that the redistribution shock can be decomposed as

$$
\begin{aligned}
-\omega(z^t) = & \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}} \\
& + \underbrace{(\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity}} \\
& + \underbrace{(\hat{C}_t^{ra} - \hat{P}_t^{ra})P^*(v^*(z^t) - v^*(z^{t-1}))}_{\text{saving flow exposure (equity)}} + \underbrace{\hat{C}_t^{ra}(b^*(z^t) - R^*b^*(z^{t-1}) + \tau^*(z^t))}_{\text{saving flow exposure (bond)}} \\
& + \underbrace{(b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}))}_{\text{undetermined bond demand}} \\
& + \underbrace{P_t^{ra}(v^*(z^t) - v^{ra}(z^t)) - P_t^{ra}(v^*(z^{t-1}) - v^{ra}(z^{t-1}))}_{\text{undetermined equity demand}},
\end{aligned}
\tag{47}
$$

where $y(z^t) \equiv z_t W_t n(z^t) + \pi(z_t) + Dv(z^{t-1})$ is defined as the household's income, including labor income $y^L(z^t) \equiv z_t W_t n(z^t) + \pi(z_t)$[10] and dividend income $Dv(z^{t-1})$. On the aggregate level, aggregate income $Y_t = W_t N_t + (1-\alpha)\Pi_t + D_t$ equals aggregate consumption $Y_t = C_t$.

There is a new term "saving flow exposure (equity)"

$$
(\hat{C}_t^{ra} - \hat{P}_t^{ra})P^*(v^*(z^t) - v^*(z^{t-1})), \tag{48}
$$

similar to the term "saving flow exposure (bond)". The term $\hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$ implies that the change in asset price affects traders rather than holders, consistent with the argument made in Fagereng et al. (2022). When evaluating the net saving flow exposures we need to take into account the endogenous fluctuations in asset price.[11]

The last residual term is due to the undetermined equity demand. After imposing $v^{ra}(z^t) = v^*(z^t)$, this term is zero.

How does the investment response affect the household's income exposure? First, to simplify the analysis, assume that household labor income satisfies $y^L(z^t) = z_t Y_t^L$, which implies that all households have the same labor income elasticities to aggregate labor income Y_t^L (this assumption is relaxed later). Under this assumption, there is no redistribution due to unequal labor income exposures. The Appendix shows that the

[10] Income from labor in the broad sense, including profit income interpreted as the bonus.

[11] The quantitative effects of the "saving flow exposure (equity)" are negligible, similar to the "saving flow exposure (bond)". First, monetary policy shock belongs to the class of shocks affecting asset price and consumption in the same direction, implying that $\hat{C}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$ and $\hat{P}_t^{ra}P^*(v^*(z^t) - v^*(z^{t-1}))$ tend to counteract each other thus the saving flow exposure (equity) is small compared to other channels; second, the MPC difference between asset buyers ($v(z^t) > v(z^{t-1})$) and sellers ($v(z^t) < v(z^{t-1})$) is small.

income exposure channel simplifies to

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) = (\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^* \left(v^*(z^{t-1}) - z_t \right), \tag{49}$$

which reflects the redistribution between equity holders ($v^*(z^{t-1}) > z_t$) and workers ($v^*(z^{t-1}) < z_t$) as the share of dividends (and labor income) in aggregate income fluctuates ($\hat{D}_t^{ra} \neq \hat{Y}_t^{ra}$).

Household income elasticities depend on the household's income portfolio and the responses of each component of aggregate income. If dividends are more responsive than aggregate income ($\hat{D}_t^{ra} > \hat{Y}_t^{ra}$), the share of dividends in aggregate income increases while the share of labor income decreases. As a result, equity holders gain and workers lose. Conversely, if labor income is more responsive, the redistribution will favor workers over equity holders. This redistribution due to heterogeneous income portfolios among households is defined as the **income portfolio exposure** channel.

From equation 41 we can see dividends' responses are negatively correlated with investment responses. Omitting capital adjustment cost and price adjustment cost and noting that $r_t^K K_{t-1} + \alpha \Pi_t = \alpha Y_t^{GDP}$, we have $D_t = \alpha Y_t^{GDP} - I_t = \alpha(C_t + I_t) - I_t = \alpha Y_t - (1-\alpha) I_t$. Dividends are less responsive than aggregate income $\hat{D}_t^{ra} < \hat{Y}_t^{ra}$ if and only if the investment is more responsive than aggregate consumption $\hat{I}_t^{ra} > \hat{C}_t^{ra}$.[12] For typical calibrations, investment is more responsive than consumption in the short run and less responsive than consumption in the long run, implying that the redistribution is from equity holders to workers in the short run and the reverse in the long run.

When equity holders accumulate capital for future consumption, workers consume additional income generated from producing capital in the current period. In the future, however, workers will have to cut their consumption when the economy deinvests and consumes the accumulated capital. Essentially, the redistribution allows workers to move their future consumption to the present, which has a similar flavor to the liquidity channel of the time-varying supply of government bonds discussed in Section 4.1. From this perspective, the income portfolio exposure can also be interpreted as the liquidity channel of productive assets.

5 Estimable Moments for Partial Equilibrium Responses

The decomposition implies that if the policymaker lowers the nominal interest rate and knows the representative agent model's response, then she only needs to know the heterogeneous agent model's response to the redistribution shock $-\boldsymbol{\omega}$ to get the full responses. The responses to the redistribution shock $-\boldsymbol{\omega}$ generally require nu-

[12] From $D_t + (1-\alpha)I_t = \alpha Y_t$ we have $\hat{D}_t D^* + \hat{I}_t(1-\alpha)I^* = \hat{Y}_t(D^* + (1-\alpha)I^*)$ and then $(\hat{D}_t - \hat{Y}_t)D^* + (\hat{I}_t - \hat{Y}_t)(1-\alpha)I^* = 0$. Given $D^* > 0$ and $I^* > 0$ we know $\hat{D}_t < \hat{Y}_t$ if and only if $\hat{I}_t > \hat{Y}_t$.

merically solving a full HANK model. Below I show we can gain insights from simple partial equilibrium analysis, following the approach of Auclert (2019).

The redistribution shock is persistent if the monetary policy shock is persistent or if the model features investment. To simplify the analysis, I truncate the redistribution shock from time $t = 1$ and only consider the redistribution at time $t = 0$. To the first order, the aggregate consumption response in partial equilibrium is

$$\partial C_0 = \int MPC_{i0} \cdot (-\omega_{i0}) di = cov_I(MPC_{i0}, -\omega_{i0}). \tag{50}$$

The equation follows from the re-distributive nature of the transfers: $\int -\omega(z^t) d\Phi_t(z^t) = 0$. The consumption response in partial equilibrium is the cross-sectional covariance between households' MPCs and their redistribution shock. $cov_I(MPC_{i0}, -\omega_{i0}) > 0$ in the case of amplification; and $cov_I(MPC_{i0}, -\omega_{i0}) < 0$ in the case of dampening. Since each redistribution channel sums to zero cross-sectionally, the above argument also applies to the evaluation of each channel.

Before deriving estimable moments at the channel level, I specify the functional form of household labor income and tax payment. I also specify the aggregate labor supply condition and fiscal policy to close the model.

5.1 The Full Model

Household Labor Income. Assume that households supply the same amount of labor and that the distribution of the profits is proportional to productivity. I introduce the "incidence function", following Guvenen et al. (2017), Werning (2015), Auclert and Rognlie (2018), Alves et al. (2020), etc., to capture households' different labor income elasticities to aggregate labor income fluctuations. The specific function form is the same as Alves et al. (2020). Household gross labor income is given by

$$y^{GL}(z^t) = \frac{z_t (Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}}{E_I[z_t (Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}]} Y_t^{GL}, \tag{51}$$

where $Y_t^{GL} = W_t N_t + (1-\alpha)\Pi_t$ is aggregate gross labor income. In the stationary equilibrium, household gross labor income is simply $y^{GL,*}(z^t) = z_t Y^{GL,*}$. Off the stationary equilibrium, imposing the normalization $E_I[z_t \gamma(z_t)] = 1$, then $\gamma(z_t)$ is the elasticity of the type z_t gross labor income $y^{GL}(z^t)$ to aggregate gross labor income Y_t^{GL} evaluated at $Y^{GL,*}$ (see Alves et al. 2020).

The household's budget constraint is

$$c(z^t) + b(z^t) + P_t v(z^t) = R_t b(z^{t-1}) + (P_t + D_t) v(z^{t-1}) + y^{GL}(z^t) - \tau(z^t). \tag{52}$$

Labor Supply. The modeling of the labor market is non-standard, borrowed from Alves et al. (2020) to simplify the labor-supply analysis. Households supply the same amount of labor $n(z^t) = N_t$ to firms, and the aggregate labor supply follows the wage schedule,

$$W_t = W^* \left(\frac{N_t}{N^*} \right)^{\epsilon_w}. \tag{53}$$

If $\epsilon_w = 0$, wages are perfectly rigid, and employment is determined by only labor demand. If $\epsilon_w > 0$, there is pressure on wages whenever employment is different from its steady-state level.

Fiscal Policy. The taxes households pay to the government are

$$\tau(z^t) = \Gamma y^{GL}(z^t) + T_t^{uniform}, \tag{54}$$

where Γ is a common constant tax rate on gross labor income, and $T_t^{uniform}$ is a uniform tax. The aggregate tax income of the government is then $T_t = \Gamma Y_t^{GL} + T_t^{uniform}$.

I assume non-standard fiscal policy responses to capture the relaxed borrowing conditions following an expansionary shock. The path of government debt evolves according to:

$$B_t - B^* = \rho_B (B_{t-1} - B^*) + \epsilon_t^B. \tag{55}$$

Following the monetary policy shock ϵ_t, there is also a shock to the level of government debt $\epsilon_t^B = \phi^B \epsilon_t$. When $\phi^B < 0$, the bond supply is procyclical (conditional on the monetary policy shock), and when $\phi^B > 0$, the bond supply is countercyclical. The uniform taxes $T_t^{uniform}$ are adjusted such that the government budget constraint holds:

$$B_t + \Gamma Y_t^{GL} + T_t^{uniform} = R_t B_{t-1} + G, \tag{56}$$

where G is the constant government spending.

5.2 The Parameterizations of Redistribution Channels

In the previous analysis, government spending is not considered, and aggregate taxes cover interest expenses (in the case of constant bond supply) $\int \bar{\tau}(z^t) d\Phi_t(z^t) = r_t B^*$. After introducing government spending, taxes also cover government spending $\int \bar{\tau}(z^t) d\Phi_t(z^t) = G + r_t B^*$, resulting in aggregate income exceeding aggregate consumption. When defining redistribution channels, one restriction is that each channel sums to zero cross-sectionally, so I define net labor income below. I also incorporate taxes into income to streamline the decomposition.[13] Household income is defined as

$$\begin{aligned} y(z^t) =& D_t v(z^{t-1}) + y^{GL}(z^t) - (\bar{\tau}(z^t) - r_t B^*) \\ =& \underbrace{D_t v(z^{t-1})}_{\text{dividend income}} + \underbrace{(y^{GL}(z^t) - \tau^G(z^t))}_{\text{net labor income } y^L(z^t)} - \underbrace{(\bar{\tau}(z^t) - \tau^G(z^t) - r_t B^*)}_{\text{net taxes } \tau^n(z^t)}, \end{aligned} \tag{57}$$

where $\tau^G(z^t) = \frac{z_t (Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}}{E_I[z_t (Y_t^{GL}/Y^{GL,*})^{\gamma(z_t)}]} G$ represents the portion of individual taxes allocated to financing government spending.[14] Hereafter, labor income refers to net labor income $y^L(z^t) \equiv y^{GL}(z^t) - \tau^G(z^t)$ and $Y_t^L \equiv Y_t^{GL} - G$. The newly defined net taxes $\tau^n(z^t)$ exclude $\tau^G(z^t)$ and $r_t B^*$ from gross taxes $\bar{\tau}(z^t)$ and sum to zero cross-sectionally.

Impose the bond demand function $\bar{b}^{ra}(z^t) = b^*(z^t)$ and the equity demand function $v^{ra}(z^t) = v^*(z^t)$, the sources of redistribution are

$$\begin{aligned} -\omega(z^t) =& \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*)}_{\text{interest rate exposure}} \\ &+ \underbrace{(b^*(z^t) - b^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity}} \\ &+ \underbrace{(\hat{C}_t^{ra} - \hat{P}_t^{ra}) P^* (v^*(z^t) - v^*(z^{t-1}))}_{\text{saving flow exposure (equity)}} + \underbrace{\hat{C}_t^{ra}(b^*(z^t) - R^* b^*(z^{t-1}) + r^* B^*)}_{\text{saving flow exposure (bond)}}. \end{aligned} \tag{58}$$

The Appendix shows that the income exposure channel can be further decomposed based on the composition of income:

$$\begin{aligned} (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) =& \underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* \left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}} \right)}_{\text{income portfolio exposure}} \\ &+ \underbrace{\tau^{n,*}(z^t) - \tau^{n,ra}(z^t)}_{\text{tax exposure}} + \hat{Y}_t^{ra} \tau^{n,*}(z^t). \end{aligned} \tag{59}$$

The first part, labor income exposure, captures the redistribution within the category of labor income: households may have different labor income elasticities to aggregate labor income. Given the labor income incidence function (51), we have

$$(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t) = (\gamma(z_t) \hat{Y}_t^{L,ra} - \hat{Y}_t^{L,ra}) z_t Y^{L,*} = (\gamma(z_t) - 1) z_t \tilde{Y}_t^{L,ra}. \tag{60}$$

If $\gamma(z_t) > 1$, then type z_t household's labor income is more elastic to aggregate labor

[13] Excluding taxes from income makes the exposition of the liquidity channel in Section 4.1 easier.

[14] The functional form of $\tau^G(z^t)$ minimally impacts the decomposition results as long as G remains constant. Changes in individual labor income and taxes are attributed to variations in aggregate labor income and interest payments, rather than government spending.

income, making the labor income exposure term positive. The labor income elasticity $\gamma(z_t)$ is the target for calibration.

The second part, income portfolio exposure, captures the redistribution between equity holders (households with $v^*(z^{t-1}) > y^{L,*}(z^t)/Y^{L,*}$) and workers (households with $v^*(z^{t-1}) < y^{L,*}(z^t)/Y^{L,*}$), as discussed in Section 4.2. The labor income incidence function (51) implies that $y^{L,*}(z^t) = z_t Y^{L,*}$, therefore we have

$$(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^* \left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}} \right) = (\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*(v^*(z^{t-1}) - z_t). \quad (61)$$

The third part is the tax exposure, now consolidated into the income channel. Given the taxing scheme (54) and the government budget constraint, the tax exposure channel simplifies to[15]

$$\Gamma \tilde{Y}_t^{L,ra}(1 - \gamma(z_t)z_t). \quad (62)$$

The last part $\hat{Y}_t^{ra}\tau^{n,*}(z^t)$ is the scaling of net taxes in the stationary equilibrium. When the definition of income does not include taxes as in the last section, this term is absorbed into the channel ”saving flow exposure (bond)”. I also treat this term as a residual since it has negligible quantitative effects.

The liquidity channel can also be simplified. Substitute the bond demand function 28 into the definition of the liquidity channel and notice that the government adjusts uniform taxes to balance its budget[16], the liquidity channel simplifies to:

$$\frac{B_t^{ra} - B^*}{B^* - \phi}(B^* - b^*(z^t)) - \frac{R_t^{ra}(B_{t-1}^{ra} - B^*)}{B^* - \phi}(B^* - b^*(z^{t-1})). \quad (63)$$

5.3 Consumption Responses in Partial Equilibrium

The partial equilibrium consumption responses to the redistribution shock at the redistribution-channel level are summarized in Table 2. I omit the time script and instead denote b_i as the individual i's initial bond holding at time $t = 0$ and b_i' as his bond holding at the beginning of $t = 1$ in the steady state. Similarly, v_i is the initial equity and v_i' is the equity held at the beginning of $t = 1$.

[15]Omit the difference bwteen $\tau^{G,ra}(z^t)$ and $\tau^{G,*}(z^t)$ since government spending is constant. From $\tau^{n,*}(z^t) = \tau^*(z^t) - \tau^G(z^t) - r^*B^*$ and $\tau^{n,ra}(z^t) = \bar{\tau}^{ra}(z^t) - \tau^G(z^t) - r_t^{ra}B^*$ we have $\tau^{n,*}(z^t) - \tau^{n,ra}(z^t) = \tau^*(z^t) - r^*B^* - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*) = \Gamma y^{GL,*}(z^t) + T^{uniform,*} - r^*B^* - (\Gamma y^{GL,ra}(z^t) + \bar{T}_t^{uniform,ra} - r_t^{ra}B^*)$. From the budget constraint of government we know $\bar{T}_t^{uniform} - r_t B^* = G - \Gamma Y_t^{GL}$. Then $\Gamma y^{GL,*}(z^t) + T^{uniform,*} - r^*B^* - (\Gamma y^{GL,ra}(z^t) + \bar{T}_t^{uniform,ra} - r_t^{ra}B^*) = \Gamma y^{GL,*}(z^t) - \Gamma y^{GL,ra}(z^t) - (\Gamma Y^{GL,*} - \Gamma Y_t^{GL,ra}) = -\Gamma\gamma(z_t)\hat{Y}_t^{GL,ra}y^{GL,*}(z^t) + \Gamma\hat{Y}_t^{GL,ra}Y^{GL,*} = \Gamma\hat{Y}_t^{GL,ra}Y^{GL,*}(1 - \gamma(z_t)z_t) = \Gamma(Y_t^{L,ra} - Y^{L,*})(1 - \gamma(z_t)z_t)$.

[16]Government adjusting uniform taxes to balance its budget implies that $\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t) = \bar{T}_t^{uniform,ra} - T_t^{uniform,ra}$ and $B_t^{ra} - B^* + T_t^{uniform,ra} - \bar{T}_t^{uniform,ra} = R_t^{ra}(B_{t-1}^{ra} - B^*)$.

Table 2: Consumption response to the redistribution shock in partial equilibrium

Redistribution channel		**Consumption response**	**Value (% of C^*)**
Interest rate exposure		$\tilde{R}^{ra} \cdot cov_I\left(MPC_i, b_i - B\right)$	0.066
Income exposure	labor	$\tilde{Y}^{L,ra} \cdot cov_I\left(MPC_i, (\gamma(z_i) - 1)z_i\right)$	0.002
	portfolio	$(\hat{D}^{ra} - \hat{Y}^{ra})D \cdot cov_I\left(MPC_i, v_i - z_i\right)$	0.051
	Tax	$\Gamma\tilde{Y}^{L,ra} \cdot cov_I\left(MPC_i, 1 - \gamma(z_i)z_i\right)$	0.009
Liquidity		$\tilde{B}^{ra}/(B - \phi) \cdot cov_I\left(MPC_i, B - b_i'\right)$	0.037

Notes: Partial-equilibrium consumption response to a transitory redistribution shock. MPC_i is the marginal propensity of consumption of individual i. b_i, v_i, z_i, $\gamma(z_i)$ denote individual i's bond position, dividend income share, labor income share, and labor income elasticities, respectively.

The covariance terms in Table 2 can be estimated as in Auclert (2019) and Patterson (2023). The key advancement here is that, when applying these estimates, the changes in aggregate quantities or prices are derived from the RANK model. For example, the effects of the interest rate exposure channel on consumption are the covariance $cov_I\left(MPC_i, b_i - B\right)$ times the change in the real interest rate in the counterfactual "RANK" equilibrium, $\tilde{R}^{ra}$, rather than the interest rate change in the actual HANK economy, which is implicitly used in Auclert (2019) and Patterson (2023). The responses of the actual HANK economy can only be observed ex post or by numerically solving the HANK model. To predict the effects of aggregate shocks, the policymaker needs only to know the responses of aggregates in the "RANK" equilibrium and the covariance terms in Table 2.

In the third column, I use the calibrated model in Section 6 to compute those endogenous moments in the stationary equilibrium and combine them with the response of the RANK model at time $t = 0$ (see Figure 1) to get the partial equilibrium consumption responses. Both the direction and relative size of the responses match closely with the general equilibrium responses presented in Figure 3 of Section 6.

In what follows, I briefly discuss the standard model's prediction of these moments and the effects of redistribution channels in partial equilibrium following an expansionary monetary policy shock.

Interest rate exposure. The incomplete-market model predicts that

$$cov_I\left(MPC_i, b_i - B\right) < 0. \tag{64}$$

Creditors ($b_i > B$) have lower MPCs than debtors ($b_i < B$), which implies a negative correlation between MPC and the exposure to interest rate changes. An interest rate cut $\tilde{R}^{ra} < 0$ taxes creditors and subsidizes debtors, amplifying consumption responses.

Labor income exposure. I use estimates from Guvenen et al. (2017) to calibrate the cyclical labor income elasticities.[17] Guvenen et al. (2017) estimate "worker betas" (i.e.. systematic risk exposure) with respect to GDP using data from the US Social Security Administration's Master Earnings File and find a U-shaped elasticity, i.e., exposure is high at both the bottom and the top of the distribution ($\gamma(z_i) > 1$ for both low and high z_i). As will be shown in the next section, using estimates from Guvenen et al. (2017) implies that the net effects are positive:

$$cov_I\left(MPC_i, (\gamma(z_i) - 1)z_i\right) > 0. \tag{65}$$

Income portfolio exposure. Since rich households on average receive relatively more dividend income ($v_i > z_i$) and poor households receive relatively more labor income ($v_i < z_i$), we have

$$cov_I\left(MPC_i, v_i - z_i\right) < 0. \tag{66}$$

As discussed in the last section, in the short run $\hat{D}^{ra} < \hat{Y}^{ra}$. Investment responses induce a redistribution from equity holders to workers in the short run and amplify consumption responses.

Tax Exposure. Households with low labor income ($\gamma(z_i)z_i < 1$) have higher MPCs than households with high labor income ($\gamma(z_i)z_i > 1$)

$$cov_I\left(MPC_i, 1 - \gamma(z_i)z_i\right) > 0. \tag{67}$$

When the aggregate tax on labor income $\Gamma Y^{L,ra}$ increases, the tax burden rises less for low-income workers and more for high-income workers because the former pay a smaller share of the aggregate tax. However, when the uniform tax is reduced to balance the government budget, all households benefit equally. Overall, the tax expo-

[17] Guvenen et al. (2017)'s estimates have the advantage of capturing the income dynamics of workers at both the lowest and highest ends of the income distribution. Extensive empirical evidence suggests that individuals with lower incomes are generally more exposed to economic fluctuations. Patterson et al. (2019) documents a positive covariance between workers' MPCs and their earnings elasticities to GDP in the US. Broer, Kramer and Mitman (2020) uses German data and finds that workers at the bottom of the income distribution are more exposed to aggregate earnings risk in general, and to monetary policy shocks in particular. Amberg et al. (2022) documents a similar pattern in Swedish administrative individual data: there is a higher sensitivity of labor income to monetary shocks at the bottom than elsewhere in the income distribution. Coibion et al. (2017) finds that contractionary monetary policy systematically increases inequality in labor earnings. For Denmark, Andersen et al. (2022) find that gains created by softer monetary policy through the labor channel are concentrated among relatively low-income workers.

sure channel benefits workers with low labor income and penalizes workers with high labor income, with a positive effect on aggregate consumption.

Liquidity. Households with higher savings have smaller MPCs than those with lower savings:

$$cov_I \left(MPC_i, B - b_i' \right) > 0. \tag{68}$$

When the government shifts the timing of taxes through deficit financing, it transfers income across time. For constrained households, changes in disposable income have a one-to-one effect on consumption. However, unconstrained households absorb changes in government debt in general equilibrium, resulting in a negative impact of debt issuance on their consumption. The bond demand function (28) implies that the "effective" income change available for consumption is inversely correlated with the distance of being constrained. Hence, we get the covariance term above. In the fiscal policy calibration, $\phi^B < 0$, the bond supply is procyclical $\tilde{B}^{ra} > 0$. The liquidity channel eases household borrowing conditions and amplifies consumption responses.

6 Quantitative Analysis

In this section, I implement the decomposition quantitatively. I calibrate the model and then consider the model's response to a one-time unexpected monetary policy shock. At time $t = 0$, there is an innovation in the Taylor rule of $\epsilon_0 = -0.25$ percent (-1 percent annually) with a quarterly persistence of 0.61. I use the sequence-space approach developed in Auclert et al. (2021) and Boppart, Krusell and Mitman (2018) to solve the model. To implement the decomposition, I first solve the stationary equilibrium of the model without transfers and obtain the law of motion of steady-state states (z, b^*). Then I add b^* as an exogenous state variable and input the redistribution shock as a function of household steady-state states (z, b^*) into the model.

6.1 Calibration

Table 4 summarizes the parameter values and calibration targets. I calibrate the model to the 2004 US economy, as in Kaplan, Moll and Violante (2018). The annual real interest rate is set at 5% in the stationary equilibrium, equal to the average real return on equity and government bonds. The coefficient of risk aversion σ is set to 1. The value of total wealth relative to annual output is $(B + P)/Y^{GDP} = 3.21$, which is the sum of government debt to annual output $B/Y^{GDP} = 0.29$ and equity to annual output $P/Y^{GDP} = 2.92$. Following the categorization of Kaplan, Moll and Violante (2018), the value of equity to annual output P/Y^{GDP} is the **net** illiquid assets from the Flow of Funds (FoF) divided by annual GDP, and the value of government debt to annual output B/Y^{GDP} is the **gross** liquid assets from the Survey of Consumer Finances (SCF) divided by annual GDP.[18]

The capital share parameter in the production function α is set to 0.33. The capital depreciation rate is $\delta^K = 0.07$. The steady-state capital stock satisfies $rP = \alpha Y^{GDP} - \delta^K K$, which gives $K/Y^{GDP} = 2.63$. The capitalized markup on the annual output is then $P/Y^{GDP} - K/Y^{GDP} = 0.29$. The steady-state markup $1 - 1/\mu$ satisfies $\alpha(1 - 1/\mu)/r = 0.29$, giving $\mu = 1.05$. The capital share parameter and the markup together imply a capital share of 31% and a labor share of 64%. The slope of the Phillips curve is $\kappa = 0.1$ and the Taylor rule coefficient ϕ is set to 1.25, both standard values in the New Keynesian literature. The capital adjustment cost parameter Ψ is chosen so that the peak response of investment is about twice that of consumption in the HANK model, consistent with the empirical evidence in Christiano, Eichenbaum and Trabandt (2016). The wage elasticity is set to $\epsilon_w = 0.5$. The proportional tax rate on labor income (and profit income) is set to $\Gamma = 0.3$ and the value of the uniform tax to output is $T^{uniform}/Y^{GDP} = -0.06$. Government spending is then determined by the government budget constraint $G/Y^{GDP} = 0.13$.

Income process. The (log) income process is the quarterly process estimated in Kaplan and Violante (2022), which is the sum of two independent components. The first component is a typical AR(1) process with persistence 0.988 and variance of innovations 0.0108, and the second component is the IID with variance 0.2087 (see the second row of Table A.2 in Kaplan and Violante 2022).

Aggregate MPC. One-asset HANK models have difficulty matching aggregate MPC and aggregate wealth simultaneously (Kaplan and Violante 2022). To match aggregate MPCs and wealth, I introduce ex-ante heterogeneity in the discount factor as in Carroll et al. (2017) and Kaplan and Violante (2022). There are five equal-measure groups of households with ex-ante heterogeneous discount factors $\{\beta^m - 2\Delta, \beta^m - \Delta, \beta^m, \beta^m + \Delta, \beta^m + 2\Delta\}$ where β^m is the median household discount factor, and Δ is the dispersion parameter of the discount factor distribution. Auclert, Rognlie and Straub (2018)

[18] I normalize the borrowing limit to zero so I use gross liquid assets as the measure of bond in the model. Consider the steady state of the model. From the discussion of the liquidity channel of bond supply, we know that a model with borrowing limit ϕ, bond supply B, individual bond demand $b(z^t)$, and tax payment $\tau(z^t)$ is isomorphic to a model with borrowing limit $\phi + \Delta B$, bond supply $B + \Delta B$, individual bond demand $b(z^t) + \Delta B$, and tax payment $\tau(z^t) + r\Delta B$. Normalizing the borrowing limit to zero $\phi' = \phi + \Delta B = 0$, the normalized bond supply $B' = B + \Delta B$ is the sum of the unnormalized bond supply B (calibrated to net liquid assets $B^{net} = 0.26$) and the negative of the borrowing limit $-\phi$ (calibrated to liquid loans $B^{loan} = 0.03$ which are mostly consumer loans). In the data, the sum of net liquid assets and liquid loans is the gross liquid assets.

show that the interaction between intertemporal marginal propensities (iMPC) and fiscal deficits is a sufficient statistic for the model's response to government spending shocks. I calibrate Δ to hit the first (year 0) iMPC from the data. Figure 9 in the Appendix shows that the other iMPCs fit well with Fagereng, Holm and Natvik (2021)'s estimates.

Income incidence function. I include the estimates of Guvenen et al. (2017) and normalize to $E_I[z_t\gamma(z_t)] = 1$. Since aggregate labor income is a constant share of output in the model, the elasticity of individual labor income to GDP equals its elasticity to aggregate labor income.

Asset portfolio. The household portfolio between bonds and equity is undetermined. I assume that households have the same portfolio between bonds and equity as the aggregate portfolio $b(z^t)/a(z^t) = B_t/A_t$.

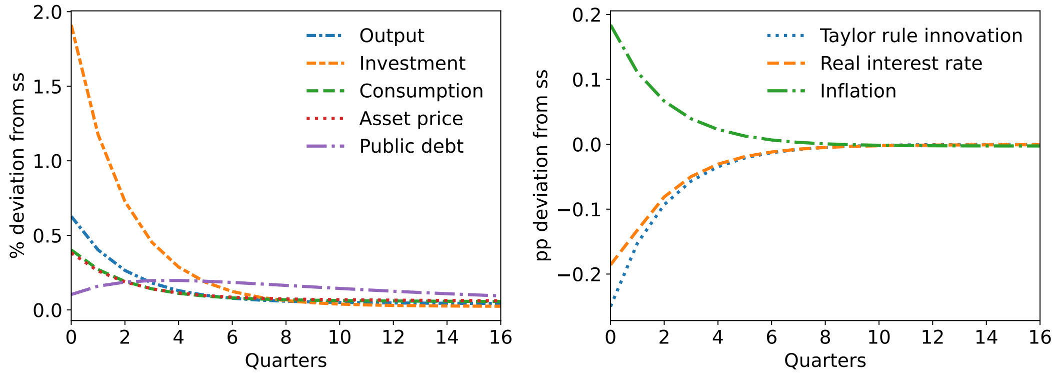
Fiscal Policy. The current literature makes ad-hoc assumptions about fiscal policy responses. The liquidity channel discussed in Section 4.1 implies that the path of public debt B is related to the path of the borrowing condition. Based on this observation, I adopt a novel calibration strategy for fiscal policy. To capture the effects of the expansionary shock on borrowing conditions, I estimate the effects of monetary policy shocks on borrowing constraints, which are mapped to liquid loans in the data. Then I convert the path of borrowing constraints to the path of public debt in the model. I specify the path of B_t such that $B_t = B^{net} + B_t^{loan}$. The estimated response is $\hat{B}_t^{loan} = 1\%$, implying the public debt B_t increases by $\hat{B}_t^{loan} \cdot B^{loan,*}/B^* = 1\% \cdot 0.03/0.29 = 0.1\%$. I calibrate the parameter ϕ^B so that the impact increase in public debt is 0.1%. This approach is equivalent to keeping public debt constant and decreasing the level of the borrowing limit ϕ by $1\% \cdot 0.03$.

6.2 Decomposition of Aggregates

Figure 1 shows the responses of the fictitious representative agent model. In response to an expansionary monetary policy shock, nominal interest rates fall, stimulating consumption and investment. Given the sticky price, the increase in aggregate demand leads to an increase in output and inflation. The calibrated fiscal policy implies that public debt also increases.

The responses of aggregate variables are decomposed into RANK and redistribution effects in Figure 2. Using RANK effects as a benchmark, redistribution effects amplify the responses of output and consumption while dampening the responses of

Figure 1: RANK effects



Notes: Impulse responses of the fictitious RANK model to a 25 bp monetary policy shock.

investment and real interest rates. Redistribution effects account for 38% of the consumption response and 14% of the output response on impact. Similar to its effects on output, redistribution also amplifies inflation responses. Since the redistribution shock raises real interest rates, it dampens the responses of asset prices.

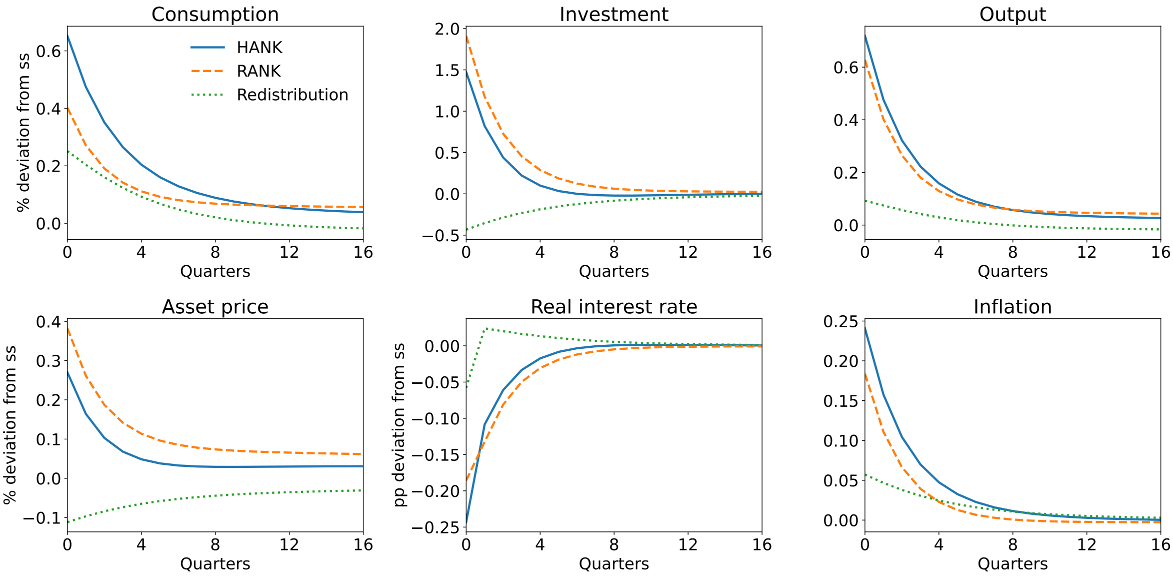
To assess the impact of the redistribution channel, I input each channel into the model separately, and the decomposition of redistribution effects is shown in Figure 3. The effects of the terms "saving flow exposure" are close to zero and not shown. The interest rate exposure channel stands out as the largest amplifier of the consumption response, contributing to more than one-third of the total amplification. Consistent with the partial-equilibrium predictions of Section 5.3, the redistribution from creditors to debtors amplifies the consumption response.

The income exposure channel is the second-largest amplifier. On impact, the three subchannels of income exposure collectively increase consumption by 0.09 percent. Most amplification effects within the income exposure channel are attributed to income portfolio exposure. From the RANK effects, we observe that investment is more responsive than consumption before quarter 8 and less responsive after quarter 8. This pattern implies a redistribution from equity holders to workers before quarter 8 and the reverse after quarter 8, amplifying the consumption response.

The liquidity channel is the third-largest amplifier. By increasing the supply of bonds, the government allows households to insure themselves against income risks better, leading to an increase in aggregate spending. Contrary to the commonly assumed stabilizing fiscal policy, the liquidity channel acts as an amplifier rather than a dampener.

Low-labor-income households benefit from the overall tax reduction the tax exposure channel slightly increases aggregate consumption. With respect to heterogeneous labor income elasticities, estimates from Guvenen et al. (2017) suggest that both low

Figure 2: Decomposition of the HANK model's responses to a monetary policy shock



Notes: Decomposition of aggregate variables' responses to a monetary policy shock, $\epsilon_0 = 25$ basis points. The RANK effects are these variables' responses to the monetary policy shock in the fictitious RANK model, and the redistribution effects are these variables' responses to the redistribution shock in the HANK model.

and high-labor-income households are more exposed to business cycle fluctuations. The net effect on consumption is positive but small.

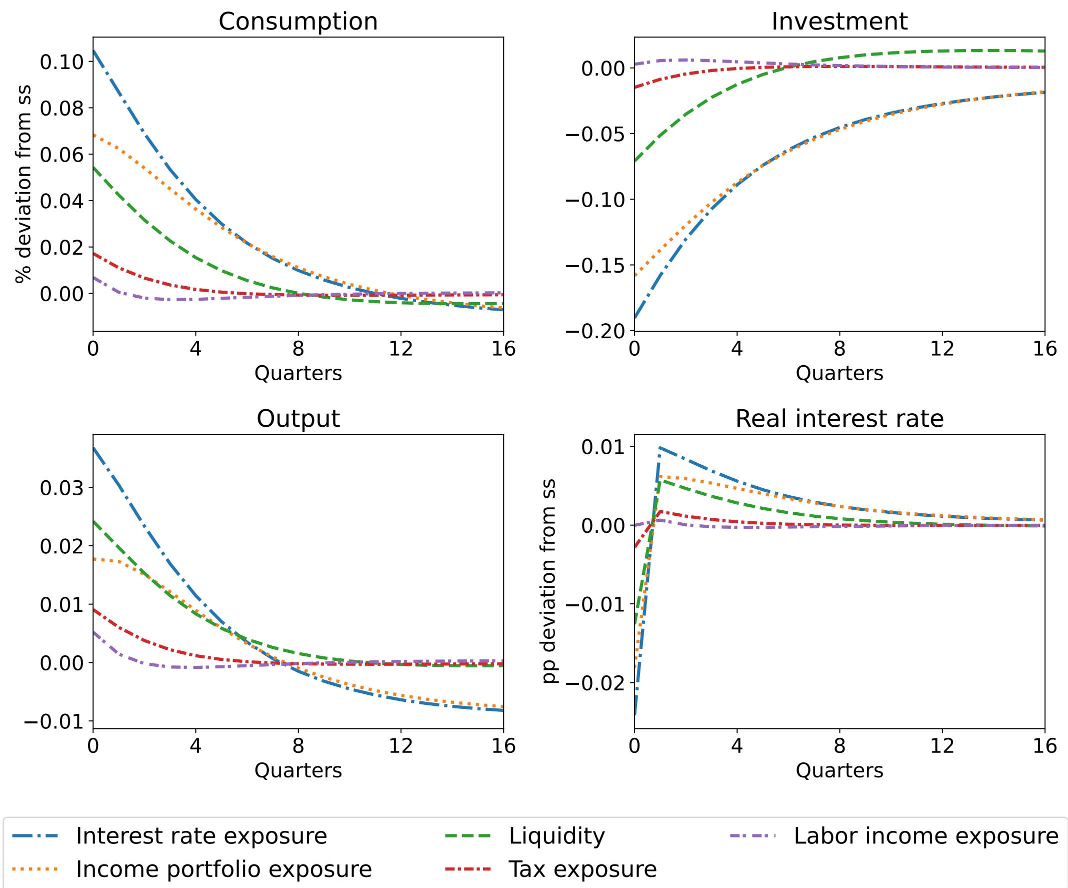
Qualitatively, the decomposition for output is similar to that for consumption, but with a smaller magnitude. This is because if one channel amplifies the consumption response, it dampens the investment response: households with a higher MPC have a lower marginal propensity to save. As a result, the net effects on output are smaller than on consumption. At the aggregate level, redistribution induces households to consume more and accumulate less. Due to the dampened investment responses, the long-run amplification of consumption and output responses can be reversed. Starting in quarter 8, the impact of the interest rate channel on output becomes negative as the capital stock declines, leading to a reduction in output.

6.3 Individual-level Decomposition

Figure 4 shows the decomposition of individual consumption responses (on impact).[19] The effects of each channel are plotted along its redistribution dimension. The effects of the interest rate exposure channel are shown across the wealth distribution

[19]The effects of the interest rate exposure, income portfolio exposure, and liquidity channels are estimated using local linear regression on model-generated data with a Gaussian kernel and a bandwidth of 0.1. The effects of the tax and labor income exposure channels on consumption for a given level of productivity are the weighted consumption responses across the wealth (and discount factor) distribution.

Figure 3: Decomposition of redistribution effects

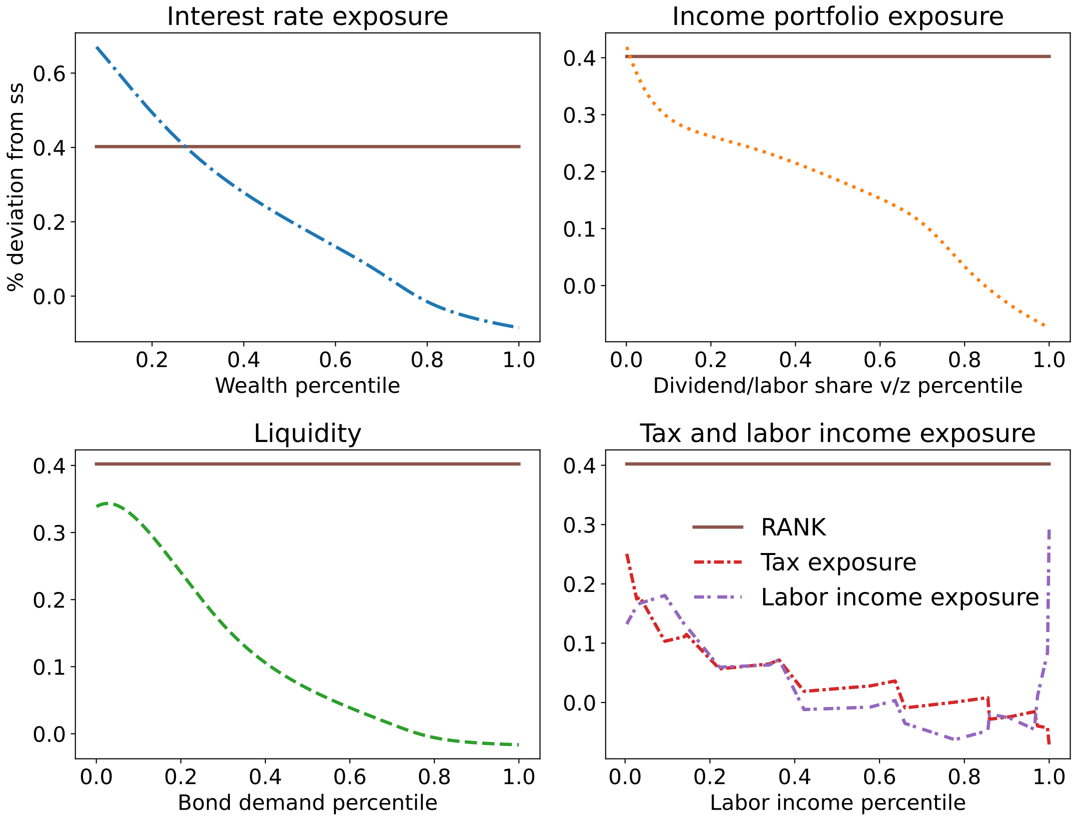


Notes: The redistribution shock's effects on consumption, investment, output, and real interest rates are decomposed into five channels. The redistribution shock is triggered by a monetary policy shock of 25 basis points. Section 5 gives the definitions of these redistribution channels.

because bond holdings determine a household's exposure to interest rate changes. The effects of the income portfolio exposure channel are shown across the distribution of the dividend income share relative to the labor income share (v/z). The ratio v/z determines the income elasticity of a household when the share of dividends in aggregate income changes. The liquidity channel is represented by the bond demand percentile. The redistribution of the tax and labor income channels operates through the dimension of labor productivity.

The average consumption response of poor households is higher than that of rich households due to the interest rate cut. From the decomposition of aggregates, we know that redistributive effects account for 38% of the aggregate consumption response. At the individual level, however, redistributive effects can account for a much larger share of the consumption response. For households in the lowest wealth percentile, the impact of interest rate exposure on consumption is more than 150% of the RANK effects (0.6/0.4). For the richest households, the interest rate exposure channel has negative effects, dampening their consumption responses.

Figure 4: Decomposition of individual consumption responses (impact)



Notes: The redistribution shock's effects on individual consumption (impact) are decomposed into five channels. For comparison, I also show the RANK effects, which are homogeneous across individuals. The effects of each channel are shown across its redistribution dimension. Section 5 gives the definitions of the redistribution channels.

The income portfolio exposure channel allows households with a low dividend income share (v) but a high labor income share (z) to consume the additional income from producing capital. Conversely, households with a high dividend but a low labor income share reduce their immediate consumption and save the return on capital for future consumption.

The liquidity channel eases households' borrowing constraints. Households far from borrowing constraints lend to those closer to them in a relatively homogeneous manner: the top 20% of the wealthy households in the bond demand b' distribution show similar consumption cuts.

According to Guvenen et al. (2017)'s estimates, labor income elasticities exceed 1 ($\gamma(z_i) > 1$) at both the low and high ends of the labor income distribution, consistent with the consumption responses of workers. The consumption of the median household in the labor income distribution is negatively affected by the labor income channel. The tax exposure channel dampens the consumption of high-labor-income households while amplifying the consumption responses of low-labor-income households.

Table 3: The contribution of redistribution (channels) to total consumption responses.

	Redistribution	Income Exposure			Interest Rate Exposure	Liquidity	
		Portfolio	Labor	Tax		Bond Supply	Illiquid Assets
Werning (2015)	0	0	0	0	0	0	N.A.
McKay, Nakamura and Steinsson (2016)	-99%	N.A.	-80%	-44%	25%	0	N.A.
Bilbiie (2020)	33%	N.A.	33%	0	0	0	N.A.
Auclert, Rognlie and Straub (2018)	143%	-7%	0	-4%	11%	160%	-17%
Wolf (2021); Wolf (2023); Angeletos, Lian and Wolf (2023)	100%	N.A.	0	0	0	100%	N.A.

Notes: In the quantitative models, the total consumption responses are calculated as the sum of consumption responses over the period from 0 to 300. The effects of "saving flow exposure" are omitted.

7 Application to Literature

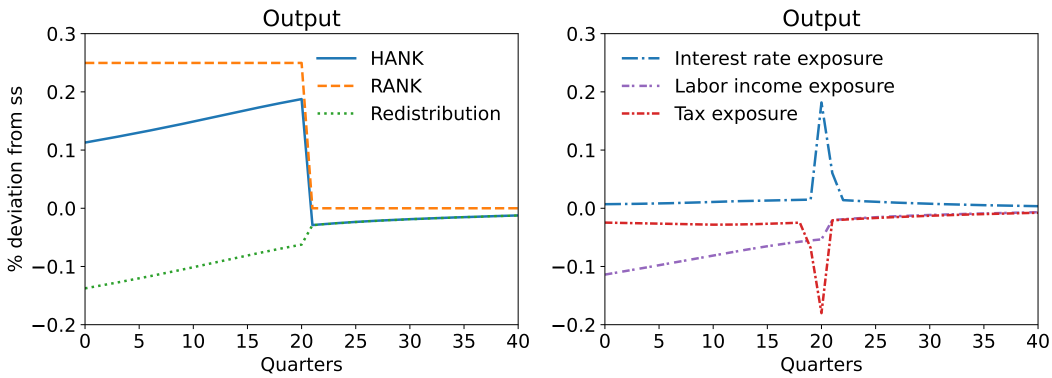
In this section, I apply the decomposition to several incomplete-market models in the literature. Table 3 summarizes the contribution of redistribution (channels) to consumption responses in these models. The details of the decomposition can be found in the Appendix C.

Werning (2015) analyzes scenarios in which an incomplete market economy can be aggregated as an "as if" representative agent economy. With zero liquidity and acyclic income risk, the generalized Euler equation derived in Werning (2015) is consistent with that of a representative agent. In the Appendix, I show that the "as if" representative agent in Werning (2015) corresponds to the fictitious representative agent defined in Proposition 1. Substituting the assumptions of the "as if" economy (Section 3.2 and 4 in Werning 2015) into the definition of redistribution channels 58, we can verify that counterfactual transfers are zero and all redistribution channels are muted.

McKay, Nakamura and Steinsson (2016) study the forward guidance puzzle in an incomplete market model. They consider the response of the economy to a one-time 50 basis point real interest rate cut 20 quarters into the future, with real interest rates unchanged in all other quarters. In the RANK model, output immediately increases by 25 basis points and remains at that level for 20 quarters. In the HANK model, the initial increase in output is only about 10 basis points.

Two model assumptions: (i) firm profits are distributed uniformly to households; (ii) only the highest-income households pay taxes, are the main drivers of the negative redistribution effects, as can be seen from the effects of income and tax exposure channels. The assumption that firm profits are equally distributed to households implies that countercyclical profits account for a larger share of total income for low-income households, resulting in lower income elasticities for low-income households.

Figure 5: Decomposition of output (consumption) responses in McKay, Nakamura and Steinsson (2016)



Notes: McKay, Nakamura and Steinsson (2016) consider the economy's response to a one-time 50-basis-point real rate shock in Quarter 20. The left panels decompose their HANK model's output response into RANK and redistribution effects. The right panel further decomposes the redistribution effects into the contribution of interest rate, (labor) income, and tax exposure channels.

The second assumption, that only the highest-income households pay taxes, implies that only the highest-income households benefit from the tax cut in quarter 20 (taxes in other quarters do not change from steady-state levels because real interest rates only fall in quarter 20). Both assumptions lead to a redistribution from low-income to high-income households. The redistribution through the income channel dampens consumption responses from quarter 0 to quarter 19, while the redistribution due to taxation further dampens the response in quarter 20.

Bilbiie (2020) discuss the amplification mechanism in a TANK model.[20] In the TANK model, savers receive a smaller share of countercyclical firm profits compared to hand-to-mouth households, which leads to unequal income elasticities. The only active redistribution channel is the (labor) income exposure channel.[21]

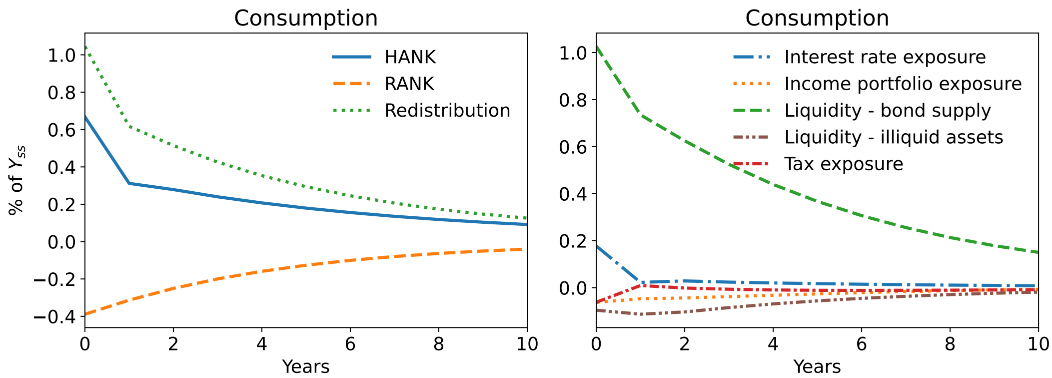
Auclert, Rognlie and Straub (2018) studies fiscal multipliers in HANK models and finds that deficit-financed multipliers can be greater than one. Section 7 in Auclert, Rognlie and Straub (2018) considers a fully specified two-account quantitative model.[22] The decomposition of consumption responses is shown in Figure 6. The redistribution effects on consumption are positive, reversing the sign of the consumption response

[20]Section 5 of Bilbiie (2020) calibrates the TANK model to match the amplification magnitude of Kaplan, Moll and Violante (2018), in which the consumption response is 50% higher than RANK. The redistribution effects contribute to one-third of the total consumption responses.

[21]Note that labor income in Table 3 includes both earnings from labor supply and firm profits. According to the this definition, all income in Bilbiie (2020) is labor income. The original definition in Bilbiie (2020) solely considers earnings derived from supplying labor $W_t n_t$ as labor income.

[22]Compared to the one-asset models, there is an additional channel to consider in two-asset models: the liquidity channel of illiquid assets (refer to section F.2 in the Appendix for the formal definition). The change in the return on illiquid assets impacts the illiquid assets that non-adjusters are compelled to accumulate. Roughly speaking, if the aggregate shock increases the return on illiquid assets, households are forced to accumulate more illiquid assets, dampening aggregate consumption.

Figure 6: Decomposition of consumption responses in Auclert, Rognlie and Straub (2018)



Notes: Auclert, Rognlie and Straub (2018) considers the economy's responses to a government spending shock in a two-account quantitative model. The government spending shock is 1% of the steady-state output. The left panels decompose their model's consumption responses into RANK and redistribution effects. The right panel further decomposes the redistribution effects into the contribution of redistribution channels.

in RANK. When the government increases the supply of bonds, the borrowing conditions of households are eased, stimulating aggregate consumption. The liquidity channel of bond supply explains most of the redistribution effects.

The liquidity channel of bond supply can be of interest independent of aggregate shocks. Wolf (2021), Wolf (2023), and Angeletos, Lian and Wolf (2023) study the role of deficit-financed lump-sum fiscal transfers as a stimulating policy tool, which essentially reflects the liquidity channel defined here. Since Ricardian equivalence holds in the RANK model, all consumption responses in the HANK model are due to redistribution effects, and the only active channel is the liquidity channel of bond supply.

8 Conclusion

This paper decomposes the heterogeneous-agent New Keynesian (HANK) model's response to an aggregate shock into two components: the response of a fictitious representative agent and the response of the HANK model to the redistribution shock induced by the policy. By further breaking down the latter, I provide an analytical characterization of the redistribution channels within the HANK model and quantitatively assess how each channel contributes to the model's divergence from a representative-agent New Keynesian (RANK) model.

The quantitative analysis reveals that redistribution effects significantly amplify the responses of output and consumption to a monetary policy shock while dampening the response of investment and real interest rates. On impact, redistribution effects account for 38% of the consumption response and 14% of the output response. All redistribution channels contribute to this amplification, with three playing the most critical roles: the interest rate exposure channel (redistributing between creditors and debtors), the income portfolio exposure channel (redistributing between equity holders and workers), and the liquidity channel (redistributing between constrained and unconstrained households). In contrast, the labor income and tax exposure channels have a more minor impact.

This decomposition framework enhances understanding of how heterogeneous-agent models respond to aggregate shocks. I apply it to the existing literature to measure the strength of redistribution channels. This study is one of the first to quantitatively assess the relevance of different redistribution channels, offering a valuable tool for developing HANK models where the strength of these channels is grounded in empirical evidence.

This paper also opens up avenues for future research. One key aspect not addressed here is the role of aggregate uncertainty and endogenous portfolio choices, which have important implications for redistribution effects. In Section 6, ad-hoc assumptions are made regarding portfolios between bonds and equity. The decomposition results would be influenced by asset portfolios if bond holdings $b(z^t)$ or equity $v(z^t)$ enter one of the channels. In two-asset models, the realization of return influences households' accumulation of illiquid assets, which has substantial effects on ex-post consumption and output. Section F.3 and F.4 in the Appendix highlight how this mechanism depends on the portfolio composition of illiquid assets. Incorporating aggregate uncertainty and portfolio choices would allow households to optimize their illiquid-asset holdings and portfolio allocation, better hedging against risks associated with these channels. Explorations in this direction are made by Bhandari et al. (2023) and Auclert et al. (2024).

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Appendix

A Proofs

Proof of Proposition 1. We need to show that the consumption and labor supply allocation in Proposition 1 satisfies the equilibrium conditions of the HANK model. The aggregation and the market-clearing conditions are easy to verify. In the following, I show that the individual allocation satisfies individual optimality conditions.

First, I impose the bond demand function $b^{ra}(z^t) = b^*(z^t)$ which satisfies the borrowing constraint $b^{ra}(z^t) \geq \phi$. I verify the F.O.C with respect to bond demand

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R^{ra}_{t+1} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t], = \text{if } b^{ra}(z^t) > \phi. \tag{69}$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) = 0. \tag{70}$$

To see the F.O.C (69) holds, substituting the individual consumption allocation $c^{ra}(z^t) = c^*(z^t) \cdot C^{ra}_t / C^*$ into each side of (69):

$$\begin{aligned}
(c^{ra}(z^t))^{-\sigma} &= (C^{ra}_t/C^*)^{-\sigma}(c^*(z^t))^{-\sigma}. \\
\beta R^{ra}_{t+1} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t] &= \beta R^{ra}_{t+1}(C^{ra}_{t+1}/C^*)^{-\sigma} E[(c^*(z^{t+1}))^{-\sigma}|z^t] \\
&= \beta \frac{R^{ra}_{t+1}}{R^*} R^*(C^{ra}_{t+1}/C^*)^{-\sigma} E[(c^*(z^{t+1}))^{-\sigma}|z^t] \\
&= \beta^{ra} R^{ra}_{t+1}(C^{ra}_{t+1}/C^*)^{-\sigma} \beta R^* E[(c^*(z^{t+1}))^{-\sigma}|z^t] \qquad (71) \\
&= (C^{ra}_t/C^*)^{-\sigma} \beta R^* E[(c^*(z^{t+1}))^{-\sigma}|z^t]. \qquad (72)
\end{aligned}$$

Equations (71) and (72) hold because $\beta^{ra} \equiv 1/R^*$ and $(C^{ra}_t)^{-\sigma} = \beta^{ra} R^{ra}_{t+1}(C^{ra}_{t+1})^{-\sigma}$. We know that in the stationary equilibrium, the following F.O.C holds

$$(c^*(z^t))^{-\sigma} \geq \beta R^* E[(c^*(z^{t+1})^{-\sigma}|z^t], = \text{if } b^*(z^t) > \phi. \tag{73}$$

Multiply both sides of (73) by $(C^{ra}_t/C^*)^{-\sigma}$, we have

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R^{ra}_{t+1} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t].$$

In the case of $b^{ra}(z^t) > \phi$, it must be the case that $b^*(z^t) > \phi$ and the F.O.C in the stationary equilibrium (73) holds with equality, therefore (69) also holds with equality. We have a useful corollary.

Corollary. The Euler equation holds with equality in the "RANK" equilibrium

$$u'(c^{ra}(z^t)) = \beta R^{ra}_{t+1} E[u'(c^{ra}(z^{t+1})|z^t]$$

if and only if it holds with equality in the stationary equilibrium

$$u'(c^*(z^t)) = \beta R^* E[u'(c^*(z^{t+1})|z^t].$$

To show the transversality condition (70), substituting the individual bond demand and consumption allocation into (70)

$$\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) = \lim_{t\to\infty} (C^{ra}_t/C^*)^{-\sigma} \beta^t E_0(b^*(z^t) - \phi)(c^*(z^t))^{-\sigma} \quad (74)$$

Since $\lim_{t\to\infty}(C^{ra}_t/C^*)^{-\sigma} = 1$ and $b^*(z^t)$ satisfy the transversality condition in the stationary equilibrium

$$\lim_{t\to\infty} \beta^t E_0(b^*(z^t) - \phi)u'(c^*(z^t)) = 0, \quad (75)$$

we can see that $\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) = 0$.

Second, I verify the individual labor supply condition in the "RANK" equilibrium

$$W^{ra}_t z_t (c^{ra}(z^t))^{-\sigma} = \varphi(n^{ra}(z^t))^\nu. \quad (76)$$

To see it, substituting the consumption and labor supply into each side of (76)

$$\begin{aligned} W^{ra}_t z_t (c^{ra}(z^t))^{-\sigma} &= W^{ra}_t z_t (c^*(z^t))^{-\sigma}(C^{ra}_t/C^*)^{-\sigma} \\ &= W^* z_t (c^*(z^t))^{-\sigma} W^{ra}_t/W^*(C^{ra}_t/C^*)^{-\sigma} \quad (77) \\ \varphi(n^{ra}(z^t))^\nu &= \varphi(n^*(z^t))^\nu (N^{ra}_t/N^*)^\nu. \quad (78) \end{aligned}$$

From the aggregate labor supply condition in the "RANK" equilibrium $W^{ra}_t(C^{ra}_t)^{-\sigma} = \varphi^{ra}(N^{ra}_t)^\nu$ where $\varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}$ we have

$$\frac{W^{ra}_t}{W^*}\left(\frac{C^{ra}_t}{C^*}\right)^{-\sigma} = \left(\frac{N^{ra}_t}{N^*}\right)^\nu. \quad (79)$$

Multiply the individual labor supply condition in the stationary equilibrium

$$W^* z_t (c^*(z^t))^{-\sigma} = \varphi(n^*(z^t))^\nu$$

by (79) we verify the individual labor supply condition (76).

Finally, the transfer is recovered from the budget constraint:

$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - W_t^{ra} z_t n^{ra}(z^t) - \pi^{ra}(z_t) + \tau^{ra}(z^t). \tag{80}$$

Aggregating over transfers $\omega(z^t)$,

$$\begin{aligned}\int \omega(z^t) d\Phi_t(z^t) &= \int [c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra} b^{ra}(z^{t-1}) - W_t^{ra} z_t n^{ra}(z^t) - \pi^{ra}(z_t) + \tau^{ra}(z^t)] d\Phi_t(z^t) \\ &= C_t^{ra} + B^* - R_t^{ra} B^* - W_t^{ra} N_t^{ra} - \Pi_t^{ra} + T_t^{ra}. \end{aligned} \tag{81}$$

The market clearing condition $C_t^{ra} = W_t^{ra} N_t^{ra} + \Pi_t^{ra}$ and the government's budget constraint $B^* + T_t^{ra} = R_t^{ra} B^*$ in the "RANK" equilibrium implies that $\int \omega(z^t) d\Phi_t(z^t) = 0$.

The quantitative model in Section 6 assumes permanent heterogeneity in discount factors β^i, and it is straightforward to verify the above proofs under this specification.

Proof of Proposition 2. By construction the imposed bond demand function satifies borrowing constraint and transversality condition. The F.O.C w.r.t bond demand reads

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma} | z^t], = \text{if } b^{ra}(z^t) > \phi, \tag{82}$$

In the proof of Proposition 1 we already show

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma} | z^t]. \tag{83}$$

In the case of $b^{ra}(z^t) > \phi$, from term (i) of Proposition 2 it can only be the case that

$$(c^{ra}(z^t))^{-\sigma} = \beta R_{t+1}^{ra} E[(c^{ra}(z^{t+1}))^{-\sigma} | z^t].$$

Proof of $b^{ra}(z^t)$ in Section 3.4 as an equilibirum bond demand function. In the case of constant bond supply $B_t = B^*$, the proof of Proposition 1 shows that $b^{ra}(z^t) = b^*(z^t)$ is an equilibrium bond demand function. Below I verify $b^{ra}(z^t)$ satisfies the conditions imposed on bond demand in Proposition 2 if B_t is time-varying. Assume the fiscal rule induces a time-varying bond supply such that $B_t > \phi$ and $\lim_{t \to \infty} B_t = B^*$.

First,

$$b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi) \geq \phi$$

satisfies the borrowing constraint. The bond market clearing follows

$$\int b^{ra}(z^t) d\Phi_t(z^t) = \int [\phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi)] d\Phi_t(z^t) = \phi + \frac{\int b^*(z^t) d\Phi_t(z^t) - \phi}{B^* - \phi}(B_t^{ra} - \phi) = B_t^{ra}. \tag{84}$$

From the proof of Proposition 1, we can see that

$$(c^{ra}(z^t))^{-\sigma} \geq \beta R^{ra}_{t+1} E[(c^{ra}(z^{t+1})^{-\sigma}|z^t]$$

holds regardless of the choices of bond demand function. In the case of $b^{ra}(z^t) > \phi$, from $b^{ra}(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t)-\phi}{B^*-\phi}(B^{ra}_t - \phi)$ we can see $b^*(z^t) > \phi$. From the Corollary in the proof of Proposition 1 we know that households are unconstrained in the stationary equilibrium as well as in the "RANK" equilibrium

$$(c^{ra}(z^t))^{-\sigma} = \beta R^{ra}_{t+1} E[(c^{ra}(z^{t+1}))^{-\sigma}|z^t]. \tag{85}$$

To see the transversality condition holds

$$\lim_{t\to\infty} \beta^t E_0(b^{ra}(z^t) - \phi)u'(c^{ra}(z^t)) \tag{86}$$
$$= \lim_{t\to\infty} (C^{ra}_t/C^*)^{-\sigma}\beta^t E_0(\phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B^{ra}_t - \phi) - \phi)(c^*(z^t))^{-\sigma} \tag{87}$$
$$= \lim_{t\to\infty} \beta^t E_0(b^*(z^t) - \phi)u'(c^*(z^t)) = 0. \tag{88}$$

Sources of redistribution in the canonical model. Rewrite the household's budget constraint in the stationary and the "RANK" equilibrium below,

$$0 = c^*(z^t) + b^*(z^t) - R^*b^*(z^{t-1}) - y^*(z^t) + \tau^*(z^t), \tag{89}$$
$$\omega(z^t) = c^{ra}(z^t) + b^{ra}(z^t) - R^{ra}_t b^{ra}(z^{t-1}) - y^{ra}(z^t) + \tau^{ra}(z^t). \tag{90}$$

Subtracting equation (90) from (89)

$$\begin{aligned}
-\omega(z^t) =& c^*(z^t) - c^{ra}(z^t) + b^*(z^t) - b^{ra}(z^t) - [R^*b^*(z^{t-1}) - R^{ra}_t b^{ra}(z^{t-1})] \\
&- (y^*(z^t) - y^{ra}(z^t)) + (\tau^*(z^t) - \tau^{ra}(z^t)) \qquad (91)\\
=& - \hat{C}^{ra}_t c^*(z^t) + (b^*(z^t) - \bar{b}^{ra}(z^t) + \bar{b}^{ra}(z^t) - b^{ra}(z^t)) \\
&- [R^*b^*(z^{t-1}) - R^{ra}_t(b^{ra}(z^{t-1}) - b^*(z^{t-1}) + b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}) + \bar{b}^{ra}(z^{t-1}))] \\
&+ \hat{y}^{ra}(z^t)y^*(z^t) - \hat{Y}^{ra}_t y^*(z^t) + \hat{Y}^{ra}_t y^*(z^t) + (\tau^*(z^t) - \bar{\tau}^{ra}(z^t) + \bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) \qquad (92)\\
=& (\hat{y}^{ra}(z^t) - \hat{Y}^{ra}_t)y^*(z^t) + \hat{Y}^{ra}_t y^*(z^t) - \hat{C}^{ra}_t c^*(z^t) + b^*(z^{t-1})(R^{ra}_t - R^*) - (\bar{\tau}^{ra}(z^t) - \tau^*(z^t)) \\
&+ (\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R^{ra}_t(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) \\
&+ (b^*(z^t) - \bar{b}^{ra}(z^t)) - R^{ra}_t(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1})). \qquad (93)
\end{aligned}$$

Add equation $0 = -B^*(R_t^{ra} - R^*) - r^*B^* + r_t^{ra}B^*$ into $-\omega(z^t)$ we have

$$
\begin{aligned}
-\omega(z^t) =& (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) && (94)\\
&+ (b^*(z^{t-1}) - B^*)(R_t^{ra} - R^*) && (95)\\
&+ (\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*) && (96)\\
&+ (\bar{b}^{ra}(z^t) - b^{ra}(z^t)) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t)) && (97)\\
&+ (b^*(z^t) - \bar{b}^{ra}(z^t)) - R_t^{ra}(b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1})). && (98)
\end{aligned}
$$

In the case that government debt is constant $B_t = B^*$, we have $\bar{b}^{ra}(z^t) = b^{ra}(z^t)$ and $\bar{\tau}^{ra}(z^t) = \tau^{ra}(z^t)$, the term "liquidity channel" (97) is zero. In the case of $b^*(z^t) = \bar{b}^{ra}(z^t)$, the last term "undetermined bond demand" (98) is zero.

Proof of Proposition 3. It's easy to verify that $b^{ra}(z^t)$ satisfies the conditions in Propositition 3 if $\bar{b}^{ra}(z^t)$ satisfies the conditions in Propositition 2, since $b^{ra}(z^t)$, $\tau^{ra}(z^t)$ and ϕ_t^{ra} all shift by the same amount $B_t^{ra} - B^*$ from their constant-debt counterparts $\bar{b}^{ra}(z^t)$, $\bar{\tau}^{ra}(z^t)$ and ϕ.

To see that the transfers are invariant to the path of government debt,

$$
\begin{aligned}
& b^{ra}(z^t) - R_t^{ra}b^{ra}(z^{t-1}) + \tau^{ra}(z^t) && (99)\\
=& (\bar{b}^{ra}(z^t) + B_t^{ra} - B^*) - R_t^{ra}(\bar{b}^{ra}(z^{t-1}) + B_{t-1}^{ra} - B^*) + \bar{\tau}^{ra}(z^t) + T_t^{ra} - \bar{T}_t^{ra} && (100)\\
=& \bar{b}^{ra}(z^t) - R_t^{ra}\bar{b}^{ra}(z^{t-1}) + \bar{\tau}^{ra}(z^t). && (101)
\end{aligned}
$$

So

$$
\begin{aligned}
\omega(z^t) &= c^{ra}(z^t) + b^{ra}(z^t) - R_t^{ra}b^{ra}(z^{t-1}) - y^{ra}(z^t) + \tau^{ra}(z^t) && (102)\\
&= c^{ra}(z^t) + \bar{b}^{ra}(z^t) - R_t^{ra}\bar{b}^{ra}(z^{t-1}) - y^{ra}(z^t) + \bar{\tau}^{ra}(z^t). && (103)
\end{aligned}
$$

Sources of redistribution with outside assets. For simplicity, first, assume that households only have access to equity. The budget constraints of households are

$$c(z^t) + P_t v(z^t) = (P_t + D_t)v(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) + \omega(z^t). \tag{104}$$

Define $y \equiv zWn + \pi + Dv_-$ as the individual income, including labor income $zWn + \pi$ and dividend income Dv_-. The redistribution shock is

$$-\omega(z^t) = P_t^{ra}v^{ra}(z^{t-1}) + y^{ra}(z^t) - P_t^{ra}v^{ra}(z^t) - c^{ra}(z^t). \tag{105}$$

In the stationary equilibrium, the transfers are zero. Subtracting the budget constraint in the stationary equilibrium from equation (105)

$$
\begin{aligned}
-\omega(z^t) =& P_t^{ra}(v^*(z^{t-1}) + v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P^* v^*(z^{t-1}) + \hat{y}^{ra}(z^t) y^*(z^t) \\
& - (P_t^{ra}(v^*(z^t) + v^{ra}(z^t) - v^*(z^t)) - P^* v^*(z^t)) - \hat{C}_t^{ra} c^*(z^t) \qquad (106) \\
=& (P_t^{ra} - P^*) v^*(z^{t-1}) + (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) - (P_t^{ra} - P^*) v^*(z^t) \\
& + P_t^{ra}(v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P_t^{ra}(v^{ra}(z^t) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \\
& \qquad (107) \\
=& (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) + (P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \\
& + P_t^{ra}(v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P_t^{ra}(v^{ra}(z^t) - v^*(z^t)) \qquad (108)
\end{aligned}
$$

From the budget constraint in the stationary equilibrium $y^*(z^t) - c^*(z^t) = P^*(v^*(z^t) - v^*(z^{t-1}))$

$$
\begin{aligned}
& (P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \qquad (109) \\
=& -\hat{P}_t^{ra} P^*(v^*(z^t) - v^*(z^{t-1})) + \hat{C}_t^{ra} P^*(v^*(z^t) - v^*(z^{t-1})) \qquad (110) \\
=& (\hat{C}_t^{ra} - \hat{P}_t^{ra}) P^*(v^*(z^t) - v^*(z^{t-1})) \qquad (111)
\end{aligned}
$$

so

$$
\begin{aligned}
-\omega(z^t) =& (\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) + (\hat{C}_t^{ra} - \hat{P}_t^{ra}) P^*(v^*(z^t) - v^*(z^{t-1})) \\
& + P_t^{ra}(v^{ra}(z^{t-1}) - v^*(z^{t-1})) - P_t^{ra}(v^{ra}(z^t) - v^*(z^t)) \qquad (112)
\end{aligned}
$$

In the case of $v^{ra}(z^t) = v^*(z^t)$, the last term ”undetermined equity demand” (112) is zero.

When the budget constraint includes bond as in the main text

$$
y^*(z^t) - c^*(z^t) = P^*(v^*(z^t) - v^*(z^{t-1})) + b^*(z^t) - R^* b(z^{t-1}) + \tau^*(z^t),
$$

and instead

$$
\begin{aligned}
& (P_t^{ra} - P^*)(v^*(z^{t-1}) - v^*(z^t)) + \hat{C}_t^{ra}(y^*(z^t) - c^*(z^t)) \qquad (113) \\
=& (\hat{C}_t^{ra} - \hat{P}_t^{ra}) P^*(v^*(z^t) - v^*(z^{t-1})) + \hat{C}_t^{ra}(b^*(z^t) - R^* b(z^{t-1}) + \tau^*(z^t)). \qquad (114)
\end{aligned}
$$

which are the channels ”saving flow exposure (equity)” and ”saving flow exposure (bond)”.

Decomposition of income exposure channel. For simplicity, first, assume that there is no government spending and the income only includes dividend and labor income.

$$y(z^t) = D_t v(z^{t-1}) + y^L(z^t) \tag{115}$$

The aggregate income is $Y = WN + (1-\alpha)\Pi + D$ satisfying $C = Y$. Define $y^L \equiv zWn + \pi$ as the individual labor income and $Y^L \equiv WN + (1-\alpha)\Pi$ as the aggregate labor income then

$$
\begin{aligned}
&(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) && (116)\\
=&y^{ra}(z^t) - y^*(z^t) - \hat{Y}_t^{ra} y^*(z^t) && (117)\\
=&\hat{D}_t^{ra} D^* v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t) y^{L,*}(z^t) - \hat{Y}_t^{ra} y^*(z^t) && (118)\\
=&\hat{D}_t^{ra} D^* v^*(z^{t-1}) + (\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t) + \hat{Y}_t^{L,ra} y^{L,*}(z^t) - \hat{Y}_t^{ra} y^*(z^t) && (119)\\
=&(\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* v^*(z^{t-1}) + (\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t) + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra}) y^{L,*}(z^t) && (120)\\
=&(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t) + (\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* v^*(z^{t-1}) + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra}) y^{L,*}(z^t). && (121)
\end{aligned}
$$

From $Y_t = Y_t^L + D_t$ we know $\hat{Y}_t Y^* = \hat{Y}_t^L Y^{L,*} + \hat{D}_t D^*$ and

$$
\begin{aligned}
(\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* + (\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra}) Y^{L,*} = 0,&\\
(\hat{Y}_t^{L,ra} - \hat{Y}_t^{ra}) y^{L,*}(z^t) = -(\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* \frac{y^{L,*}(z^t)}{Y^{L,*}}.& \qquad (122)
\end{aligned}
$$

Substituting equation (122) into (121)

$$
\begin{aligned}
&(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t)\\
=&\underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra}) y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra}) D^* \left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}}\right)}_{\text{income portfolio exposure}}. \qquad (123)
\end{aligned}
$$

With net taxes in income $y(z^t) = D_t v(z^{t-1}) + y^L(z^t) - \tau^n(z^t)$,

$$
\begin{aligned}
&(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra}) y^*(z^t) && (124)\\
=&y^{ra}(z^t) - y^*(z^t) - \hat{Y}_t^{ra} y^*(z^t) && (125)\\
=&\hat{D}_t^{ra} D^* v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t) y^{L,*}(z^t) + \tau^{n,*} - \tau^{n,ra} - \hat{Y}_t^{ra}(D^* v^*(z^{t-1}) + y^{L,*}(z^t) - \tau^{n,*}) && (126)\\
=&\hat{D}_t^{ra} D^* v^*(z^{t-1}) + \hat{y}^{L,ra}(z^t) y^{L,*}(z^t) - \hat{Y}_t^{ra}(D^* v^*(z^{t-1}) + y^{L,*}(z^t)) + (1 + \hat{Y}_t^{ra})\tau^{n,*} - \tau^{n,ra} && (127)
\end{aligned}
$$

The term $(1 + \hat{Y}_t^{ra})\tau^{n,*} - \tau^{n,ra}$ is the tax-related channels and the remaining parts are labor and portfolio income exposure channels, which can be derived as in (121).

With positive government spending, redefine $y^L(z^t) \equiv y^{GL}(z^t) - \tau^G(z^t)$ and $Y_t^L \equiv Y_t^{GL} - G$ as equation 57, the derivations hold.

B Quantitative Results

B.1 Computation

The method introduced in Section 3.4 has a high computational cost because the number of discretized states grows exponentially with respect to the original problem. Consider discretizing the productivity process using a 7-point Markov process and setting the number of asset grid points to 500. In the original problem with two state variables (z, b), there are $7 * 500 = 3500$ individual states. When computing the redistribution effects with three state variables (z, b^{ss}, b), the number of individual states is $7 * 500 * 500 = 1,750,000$. The computation of redistribution effects is much more demanding than the original problem and will make the decomposition of a two-asset model infeasible.[23]

This section introduces a simplified computation method that yields results practically identical to those obtained by the method above. The idea is to make transfers directly based on the household's equilibrium states (z, b). Consider a simpler problem where we verify the "RANK" equilibrium numerically. Then we do not need the third state variable $b^*(z^t)$ to know the transfers received by households because the relation between $b^{ra}(z^t)$ and $b^*(z^t)$ is known. We can build a mapping from the household's asset state in the "RANK" equilibrium $b^{ra}(z^t)$ to the household's asset state in the stationary equilibrium $b^*(z^t)$ and make transfers based on $b^{ra}(z^t)$ (and z_t). Inverting the monotonic asset demand function imposed in the "RANK" equilibrium $g_t(\cdot)$ to build this mapping:

$$b^*(z^t) = g_t^{-1}(b^{ra}(z^t)), \forall z^t. \tag{128}$$

Along the transition path of the "RANK" equilibrium, the household problem is

$$V_t^{ra}(z, b) = \max_{\{c,n,b'\}} u(c, n) + E[V_{t+1}^{ra}(z'', b')|z], \tag{129}$$

$$s.t. \quad c + b' = R_t b + W_t zn + \pi_t(z) - \tau_t(z) + \omega_t(z, g_{t-1}^{-1}(b)), \tag{130}$$

$$b' \geq \phi, \tag{131}$$

where $\omega_t(z, g_{t-1}^{-1}(b))$ is the transfers received by households of type (z, b^*). The pair of equilibrium asset state and the transfers received is a fixed point: Given the asset state b^{ra}, households receive $\omega_t(z, g_{t-1}^{-1}(b^{ra}))$; and given the transfers $\omega_t(z, g_{t-1}^{-1}(b^{ra}))$,

[23] Section F models illiquidity a la Calvo, where households face the IID adjustment shock s_t. When $s_t = 0$, illiquid assets accumulate, and when $s_t = 1$, households can adjust their illiquid assets. Consider a five-point Markov process and two asset grids with 50 points each for liquid and illiquid assets. The original problem with four state variables $(z, s, a^{liq}, a^{illiq})$ has $5 * 2 * 50 * 50 = 25,000$ individual states. With six state variables $(z, s, a^{liq,*}, a^{illiq,*}, a^{liq}, a^{illiq})$, there will be $5 * 2 * 50 * 50 * 50 * 50 = 62,500,000$ individual states.

the asset state of the household is b^{ra}. We can use the simplified method to verify the "RANK" equilibrium with only equilibrium states (z, b).

When computing redistribution effects, building the mapping from the equilibrium asset state $b(z^t)$ to $b^*(z^t)$ without knowing z^t is infeasible. However, as long as the deviation of $b(z^t)$ from $b^*(z^t)$ is small relative to the transfers received, we can approximate $b^*(z^t)$ with the equilibrium asset state $b(z^t)$ and directly make transfers based on $b(z^t)$:

$$V_t(z, b) = \max_{\{c,n,b'\}} u(c, n) + E[V^{ra}_{t+1}(z'', b')|z], \tag{132}$$

$$s.t. \quad c + b' = R_t b + W_t zn + \pi_t(z) - \tau_t(z) + \omega_t(z, b), \tag{133}$$

$$b' \geq \phi. \tag{134}$$

Consider the interest rate exposure channel as an example, the transfers households receive should be $(b^*(z^{t-1}) - B^*)(R^{ra}_t - R^*)$, and the actual transfers households receive is $(b(z^{t-1}) - B^*)(R^{ra}_t - R^*)$. Omitting the term $B^*(R^{ra}_t - R^*)$ which is the same across all states, the relative approximation error is

$$\frac{b(z^{t-1})(R^{ra}_t - R^*) - b^*(z^{t-1})(R^{ra}_t - R^*)}{b^*(z^{t-1})(R^{ra}_t - R^*)} = \frac{b(z^{t-1}) - b^*(z^{t-1})}{b^*(z^{t-1})}, \tag{135}$$

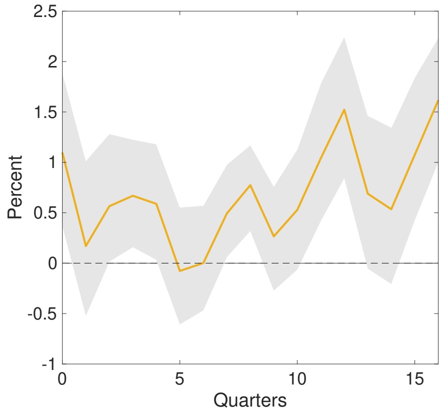
which is the percentage deviation of the household's bond position $b(z^{t-1})$ after the channel shock. For the interest-rate-exposure-channel shock considered in Section 6, the (impact) individual bond demand responses across the wealth percentile are in the range of $-0.1\% - 2\%$.

To estimate the impact of the approximation error on the decomposition results, we assume a simple linear relationship between the transfers received by poor households and their effect on contemporaneous aggregate consumption.[24] From Section 6 we can see that the interest rate exposure channel's effects on aggregate consumption at $t = 1$ is around 0.1%. If poor households increase their bond demand by 2% at time 0, we can adjust the transfers received by poor households at $t = 1$ by 2%. This adjustment implies that aggregate consumption at $t = 1$ would instead increase by $0.1\% * 98\%$. Consequently, the effect of the approximation error on aggregate consumption is $0.1\% * 2\%$, which is two orders of magnitude smaller than the already minimal aggregate consumption response without the approximation.

For the one-asset model in Section 6, this simplified method yields results that are quantitatively indistinguishable from those obtained using the method proposed in Section 3.4. Therefore, I use this approach for the decomposition of two-asset models in Section F.

[24] This linear assumption is not based on microeconomic foundations and is used solely for illustration purposes to gauge the magnitude of the approximation error's effects.

Figure 7: Households loans' responses to a monetary policy shock



Notes: Estimated responses of real (liquid) loans to a monetary policy shock. The monetary policy shock is normalized so that the reduction in the yield on the 3-month treasury bill is 25 basis points. I use quarterly data from 1988Q4 to 2016Q2 (the monetary policy shock data are from 1988Q4 to 2012Q2). The lagged controls are set as $\mathbf{X}_{t-.1} = [i_{t-1}, \epsilon_{t-1}, U_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, A_{t-1}, P_{t-1}]$. The shaded area represents the bootstrapped 66% confidence limits.

.

B.2 Calibration

Calibration of fiscal policy. I show the estimated effects of the monetary policy shock on household loans in Figure 7. Nominal loans are estimated from Flow of Funds (FoF) data as the sum of consumer credit, depository institution loans, and other loans and advances in liabilities minus loans as assets and total other assets, then I deflate it by CPI and take the log of real loans. The responses are estimated by local projections with high-frequency monetary policy shocks identified in Gorodnichenko and Weber (2016):

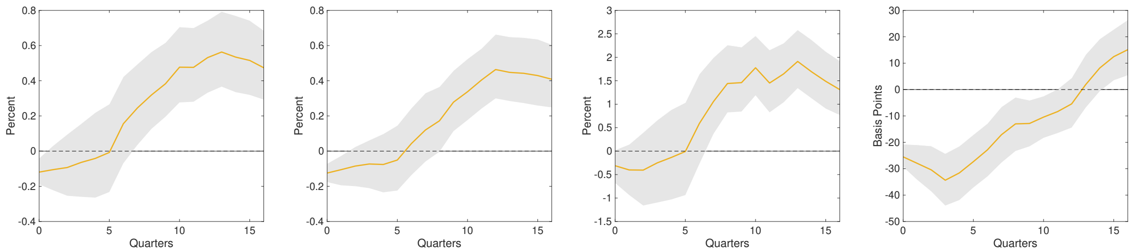
$$\mathrm{Y}_{t+h} = \beta_{h,0} + \beta_{h,1}t + \beta_{h,2}\epsilon_t + \beta_{h,3}\mathbf{X}_{t-1} + \nu_{t+h}, \quad h = 0, ...16 \tag{136}$$

The aggregate real loans Y_t at the forecast horizon $h = 0, ..., 16$ is regressed on the current normalized monetary shock ϵ_t, a constant, a linear time trend, and lagged controls $\mathbf{X}_{t-1}$. To control for potential endogeneity in practice, the lagged controls are set as the federal funds rate i_{t-1}, the monetary shock ϵ_{t-1}, unemployment rate U_{t-1}, log of output Y_{t-1}, consumption C_{t-1}, investment I_{t-1}, TFP A_{t-1} and the consumer price index P_{t-1}. The monetary policy shock is normalized such that the nominal rate i_t^b decreases by 25 basis points on impact. I use quarterly data from 1988Q4 to 2016Q2. The data on monetary policy shocks are from 1988Q4 to 2012Q2.

Figure 8 also shows the estimated responses of output Y_t, consumption C_t, investment I_t, nominal rate i_t^b (the return on the three-month treasury bill), and household loans in liquid assets L_t to the monetary shock with the same specification. All variables except the nominal rate are in real terms.

Figure 8: Aggrregate responses to a monetary shock

(a) Output (b) Consumption (c) Investment (d) Nominal Rate



Notes: Estimated response of output, consumption, investment, and nominal rates to a monetary policy shock. Monetary policy shock is normalized such that the impact decrease of the return on the 3-month treasury bill return is 25 basis points. I estimate the responses by local projections with high-frequency identified monetary policy shocks in Gorodnichenko and Weber (2016) on quarterly data from 1988Q4 to 2016Q2 (the data on monetary policy shocks is from 1988Q4 to 2012Q2). The lagged controls are set as $\mathbf{X}_{t-1} = [i_{t-1}, \epsilon_{t-1}, U_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, A_{t-1}, P_{t-1}]$. The shadow area represents the bootstrapped 66% confidence bounds.

Figure 9: iMPCs in the data and the model

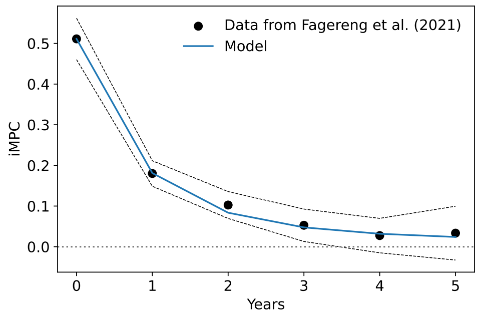


Table 4 lists the value of parameters.

C Application to literature

C.1 Werning (2015)

Werning (2015) analyzes scenarios in which an incomplete market economy can be aggregated as an ”as if” representative agent economy. With zero liquidity and acyclic income risk, the generalized Euler equation derived in Werning (2015) is consistent with that of a representative agent. In the following, I first show that the ”as if” representative agent corresponds to the fictitious representative agent defined in Proposition 1. Then, I show that the assumptions of the ”as if” economy (Section 3.2 of Werning 2015) imply that counterfactual transfers are zero and all redistribution channels are muted.

For simplicity, I omit the ex-ante heterogeneity and taste shocks in Werning (2015), as these do not affect the conclusion. The Euler equation in Werning (2015)’s Proposi-

Table 4: Calibration of the HANK model in Section 6

Parameter	Description	Value	Target
r^*	Real interest rate (p.a.)	0.05	
β^m	Discount factor of median HH (p.a.)	0.851	Asset market clearing
Δ	Dispersion of discount factors (p.a.)	0.048	Aggregate MPC
σ	Risk aversion	1	
A	TFP	0.46	Unit quarterly output
α	Capital share	0.33	
Ψ	Capital adjustment cost	11.43	Christiano, Eichenbaum and Trabandt (2016)
δ^K	Depreciation of capital (p.a.)	0.07	Kaplan, Moll and Violante (2018)
K/Y^{GDP}	Capital to GDP (p.a.)	2.4	Internally calibrated
B/Y^{GDP}	Government debt to GDP (p.a.)	0.29	2004 SCF gross liquid assets
p/Y^{GDP}	Equity to GDP (p.a.)	2.92	2004 FoF net illiquid assets
$\mu - 1$	markup	0.046	Interally calibrated
κ	Slope of Phillips curve	0.1	Christiano, Eichenbaum and Rebelo (2011)
ϵ^w	Wage elasticity	0.5	Christiano, Eichenbaum and Trabandt (2016)
ϕ_π	Coefficient on inflation	1.25	
ρ_B	Debt Persistence	0.93	Auclert and Rognlie (2018)
ϕ^B	Magnitude of the shock to debt level	-0.43	IRFs of real loans to monetary policy shock
Γ	Labor income tax rate	0.3	
$T^{uniform}$	Uniform tax	-0.058	Kaplan, Moll and Violante (2018)
G^*	Government spending	0.13	Internally calibrated

tion 2 also incorporates potentially time-varying idiosyncratic uncertainty (the stochastic process governing z_t). The time-varying idiosyncratic uncertainty implies a time-varying interest rate even in the stationary equilibrium where aggregate consumption remains constant. In this case, the discount factor of the "as if" representative agent $\beta_t^{\text{as-if}}$ is time-varying. Normalizing aggregate consumption to 1, we can derive the stationary equilibrium interest rate $\tilde{R}_t$ from equation 12 of Werning (2015). Equation 16 of Werning (2015) defines the discount factor of the "as if" representative agent $\beta_t^{\text{as-if}}$. We can find the following relation

$$\beta_t^{\text{as-if}} = 1/\tilde{R}_t. \tag{137}$$

This paper additionally assumes that idiosyncratic uncertainty is time-invariant and that the economy starts from its invariant distribution, leading to

$$\beta^{\text{as-if}} = 1/R^*, \tag{138}$$

which is also the discount factor of the fictitious representative agent defined in Proposition 1.

To see that all the redistribution channels are muted in the "as if" economy, notice the following relations implied by Werning (2015)'s model (with notations of this paper):

$$y(z^t) = z_t Y_t, \tag{139}$$

$$b(z^t) = v(z^t) = 0 \text{ and } c(z^t) = y(z^t). \tag{140}$$

Equation (139) derives from the assumption of acyclical-income-risk, and equation

(140) derives from the assumption of zero liquidity. Acyclical income risk implies that household income is proportional to aggregate income, and zero liquidity implies that household consumption is equal to their income.[25] Substituting the above relationships into the definition of redistribution channels (58), we see that all redistribution channels are muted. Specifically, the income exposure channel is

$$(\hat{y}(z^t) - \hat{Y}_t)y^*(z^t) = 0 \tag{141}$$

since household income is proportional to aggregate income and all households have the same income elasticity $\hat{y}(z^t) = \hat{Y}_t$.

Section 4 of Werning (2015) extends the aggregation results to a positive, but acyclical, liquidity-to-income ratio with log utility of consumption. Households have access to the equity market, which pays dividends to households. The budget constraint is

$$c(z^t) + P_t v(z^t) = (P_t + D_t)v(z^{t-1}) + y^L(z^t). \tag{142}$$

The model assumptions in Section 4 of Werning (2015) imply the following relationships:

$$y^L(z^t) = z_t Y_t^L,$$

$$\hat{D}_t^{ra} = \hat{Y}_t^{ra}, \tag{143}$$

$$\hat{C}_t^{ra} = \hat{P}_t^{ra}. \tag{144}$$

Household labor income $y^L(z^t)$ is proportional to aggregate labor income (and also to aggregate income) as before. The equation (143) follows from the assumption that dividends are proportional to aggregate income. The equation (144) is a result of log-utility on consumption: asset prices and consumption have the same responses.

Substituting the above relations into the definition of redistribution channels (58), we find that both subchannels of income exposure are zero:

$$(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t) = \underbrace{(\hat{y}^{L,ra}(z^t) - \hat{Y}_t^{L,ra})y^{L,*}(z^t)}_{\text{labor income exposure}} + \underbrace{(\hat{D}_t^{ra} - \hat{Y}_t^{ra})D^*\left(v^*(z^{t-1}) - \frac{y^{L,*}(z^t)}{Y^{L,*}}\right)}_{\text{income portfolio exposure}} = 0. \tag{145}$$

In addition, the term "saving flow exposure (equity)" is also zero

$$(\hat{C}_t^{ra} - \hat{P}_t^{ra})P^*(v^*(z^t) - v^*(z^{t-1})) = 0 \tag{146}$$

[25] Households are assumed not to be able to borrow, and the equilibrium interest rate is low enough that no household has an incentive to save.

because asset prices and consumption have the same responses $\hat{C}_t^{ra} = \hat{P}_t^{ra}$. The counterfactual transfers and redistributive effects are zero.

In summary, Werning (2015) shows that if aggregate and individual consumption satisfy the following conditions

$$(C_t)^{-\sigma} = \beta^{ra} R_{t+1} (C_{t+1})^{-\sigma}, \tag{147}$$

$$c(z^t)/c^*(z^t) = C_t/C^*, \tag{148}$$

then the allocation $\{c(z^t)\}$ satisfies individual optimality conditions. Werning (2015) provides examples where the scaled individual choices, together with the equilibrium prices, also satisfy budget constraints. For more general cases where budget constraints do not hold with scaled individual choices, this paper introduces counterfactual transfers to households.

C.2 McKay, Nakamura and Steinsson (2016)

McKay, Nakamura and Steinsson (2016) study the forward guidance puzzle in an incomplete market model. They consider the response of the economy to a one-time 50 basis point real interest rate cut 20 quarters into the future, with real interest rates unchanged in all other quarters. The result of this experiment on their baseline model is shown in Figure 3 of the paper (reproduced below). In the RANK model, output immediately increases by 25 basis points and remains at that level for 20 quarters. In their HANK model, the initial increase in output is only about 10 basis points.

I apply the decomposition to their model, and the results are shown in Figure 5. The negative redistribution effects solved the forward guidance puzzle. Two model assumptions: (i) firm profits are distributed uniformly to households; (ii) only the highest-income households pay taxes, are the main drivers of the negative redistribution effects, as can be seen from the effects of income and tax exposure channels.

The assumption that firm profits are equally distributed to households implies that countercyclical profits Π account for a larger share of total income for low-income households, resulting in lower income elasticities for low-income households. Omitting differences in labor supply and assuming $n(z^t) = N$, individual income is $y = zWN + \Pi$. After some algebra, it can be shown that

$$(\hat{y}^{ra}(z) - \hat{Y}^{ra}) y^*(z) = (\hat{Y}^{L,ra} - \hat{\Pi}^{ra})(\Pi^*/Y^* - \Pi^*/y^*(z)), \tag{149}$$

where $Y^L \equiv WN$. For low-income households with $z < 1$, profits Π are a larger share of aggregate income than average: $\Pi^*/Y^* < \Pi^*/y^*(z)$. After an expansionary shock, $\hat{Y}^{L,ra} > 0$ and $\hat{\Pi}^{ra} < 0$. Low-income households experience a smaller income increase $\hat{y}^{ra}(z) < \hat{Y}^{ra}$. The redistribution from low-income to high-income households

Figure 10: Decomposition of output's responses to a government spending shock

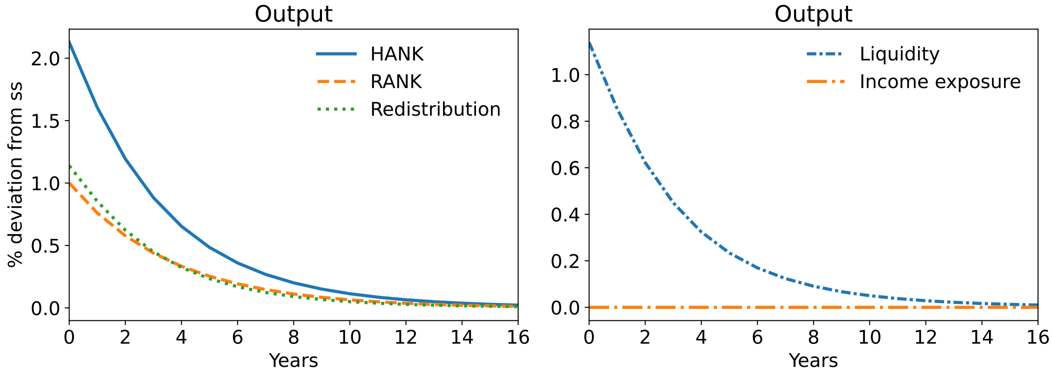


Figure 5a in Auclert, Rognlie and Straub (2018) shows the impact multiplier of a persistent government spending shock under different degrees of deficit finance in an "IKC" environment, where the real interest rate is fixed. I consider their HA-one model with the deficit finance parameter $\rho_B = 0.6$. The government spending shock is 1% of steady-state output. The left panels decompose the output response of their HANK model into RANK and redistribution effects. The right panel decomposes the redistribution effects into the contribution of the liquidity and income exposure channels.

dampens the output responses from quarter 0 onwards and accounts for most of the negative redistribution effects.

The second assumption, that only the highest-income households pay taxes, implies that only the highest-income households benefit from the tax cut in quarter 20 (taxes in other quarters do not change from steady-state levels because real interest rates only fall in quarter 20). For households with the highest skill level z^H, the tax exposure channel is

$$(\tau^*(z^H) - r^*B^*) - (\tau^{ra}(z^H) - r_t^{ra}B^*) = (r_t^{ra} - r^*)B^*(1 - 1/\lambda(z^H)) > 0 \tag{150}$$

where $\lambda(z^H)$ is the measure of households with the highest skill level z^H. For the remaining households that do not pay taxes, the tax exposure channel is $(r_t^{ra} - r^*)B^* < 0$. The redistribution from the remaining households to the highest-skilled households dampens the output response in quarter 20. It also counteracts the amplifying effects of the interest rate risk channel on output in quarter 20.

C.3 Auclert, Rognlie and Straub (2018)

Auclert, Rognlie and Straub (2018) examines fiscal multipliers and finds that deficit-financed multipliers can be greater than one. Figure 5a in Auclert, Rognlie and Straub (2018) shows the impact multiplier of a persistent government spending shock under different degrees of deficit financing in their "IKC" environment, where the real interest rate is fixed at the steady-state level and the government bond is the only asset with a positive supply. Figure 8 in Auclert, Rognlie and Straub (2018) relaxes the

Figure 11: Decomposition of individual consumption responses to a government spending shock keeping real interest rate fixed

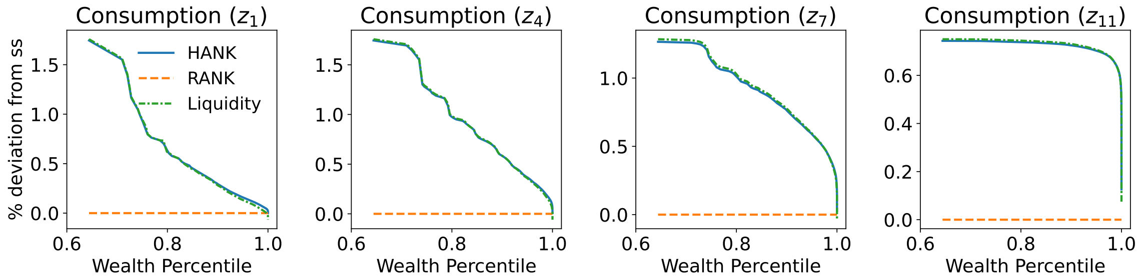


Figure 5a in Auclert, Rognlie and Straub (2018) shows the impact multiplier of a persistent government spending shock across different degrees of deficit finance in the "IKC" environment where the real interest rate is fixed. I consider their HA-one model with the deficit-finance parameter $\rho_B = 0.6$. The government spending shock is 1% of the steady-state output. The individual consumption responses are decomposed into RANK and redistribution effects.

assumptions of the "IKC" environment and considers a fully specified two-account quantitative model with a more realistic supply side. I discuss the decomposition of the one-asset model in its "IKC" setting and then move to the two-account HANK model.

C.3.1 The "IKC" Environment

The size of the shock dG_0 is 1% of steady-state output, and the persistence of the government spending shock is $\rho_G = 0.76$. After the shock $\{dG_t\}$, the government debt evolves as follows

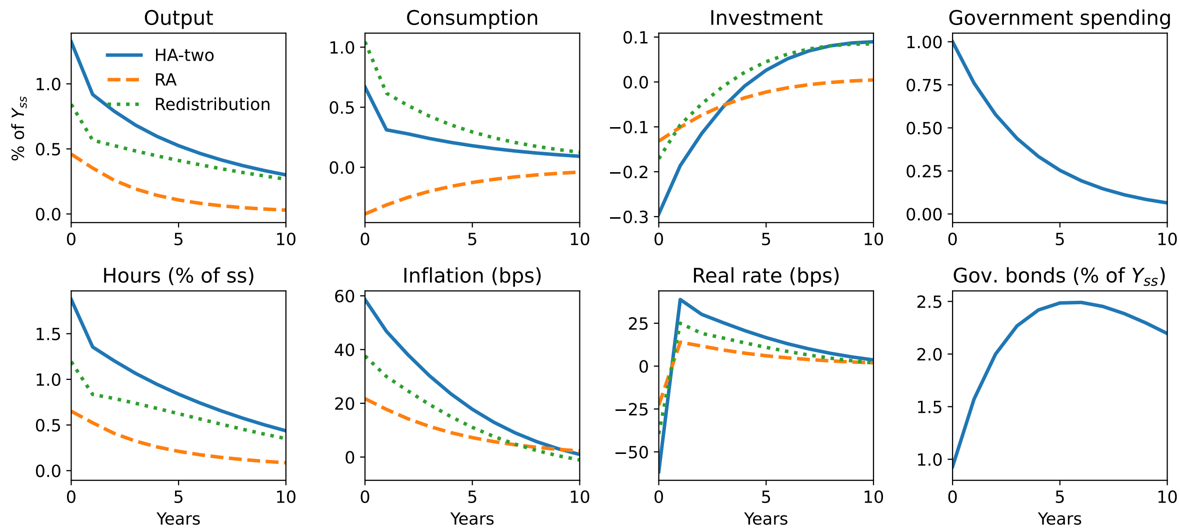
$$dB_t = \rho_B(dB_{t-1} + dG_t) \tag{151}$$

where $dB_t \equiv B_t - B^*$ is the bond supply shock induced by the financing rule above. The government adjusts labor income taxes to satisfy its budget constraint. In the case of $\rho_B = 0$, government spending is fully financed by contemporaneous labor-income taxation. When $\rho_B > 0$, the government finances some of the spending through deficits and postpones raising taxes. I consider their HA-one model with the deficit financing parameter $\rho_B = 0.6$. The decomposition result is shown in Figure 10. The impact output response of the HA-one model is 2.1%, implying an impact multiplier of $dY_0/dG_0 = 2.1$. In the RANK model, the impact multiplier is exactly one because the central bank fixes the real interest rate, and the government spending shock does not affect consumption.

From the decomposition[26], we can see that the amplified multiplier in the HA-one model is attributed to the liquidity channel. When the government delays tax

[26] Income is defined as in equation 57 and the redistribution channels are defined as in equation 58.

Figure 12: Government spending shock in the quantitative environment of Auclert, Rognlie and Straub (2018)



Notes: Replication of Figure 8 in Auclert, Rognlie and Straub (2018). The HA-two model is a two-account heterogeneous-agent model.

increases and funds spending through deficits, the supply of bonds increases and more liquidity is injected into the economy. This allows constrained households to borrow from unconstrained households. This mechanism is illustrated in the household-level decomposition in Figure 11. It shows that poor households, who are more likely to be constrained, are the most responsive to the shock.

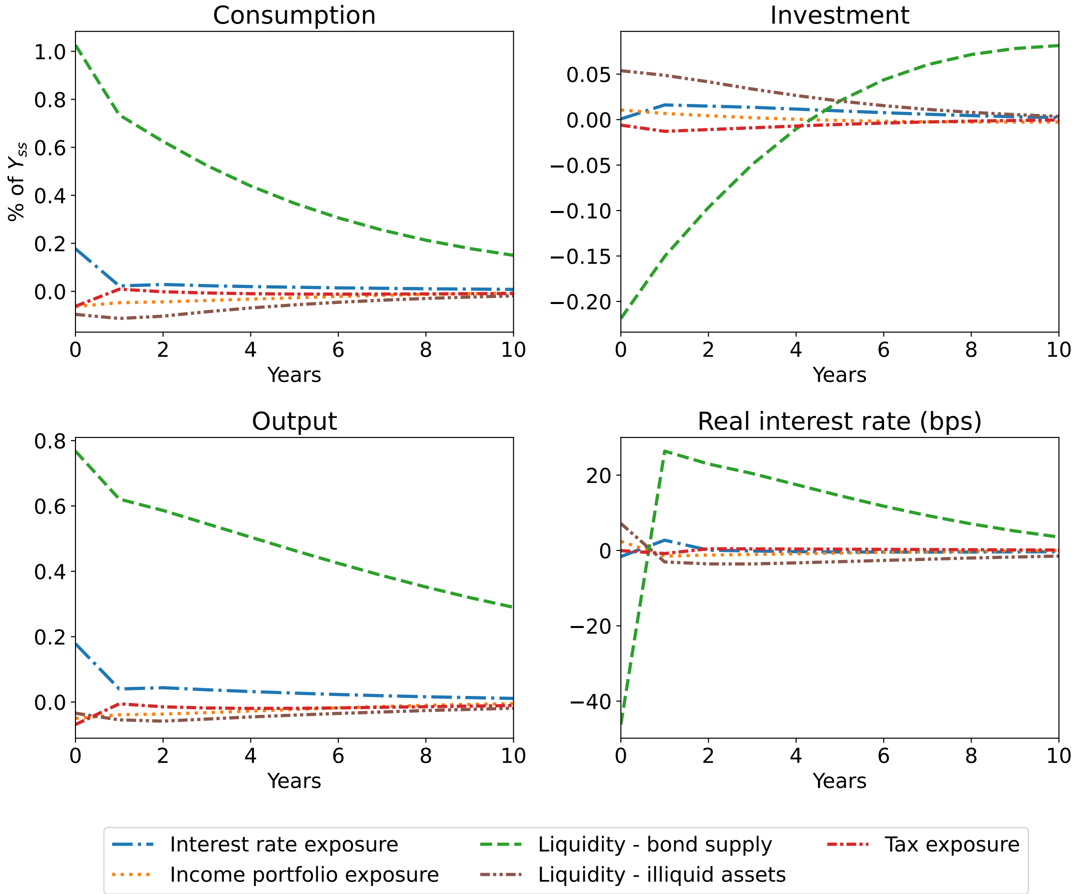
The interest rate exposure channel is muted because the real interest rate is fixed at the steady-state level. The income exposure channel is muted because households pay taxes in proportion to their income. When the expenditure shock is financed by contemporaneous taxation $dG_t = dT_t$, the increases in taxes and labor income counteract each other. After the shock, aggregate labor income increases by dG_t. For households with a productivity level z_t, pre-tax labor income increases by $z_t dG_t$, and taxes increase by the same amount $z_t dT_t$. The after-tax labor income remains unchanged.

The fiscal multiplier is exactly 1 under a balanced-budget fiscal policy and fixed real interest rate. There is no redistribution and household heterogeneity is irrelevant in determining the fiscal multiplier, which is consistent with Proposition 3 in Auclert, Rognlie and Straub (2018).

C.3.2 The Quantitative Environment

Figure 8 in Auclert, Rognlie and Straub (2018) shows the effect of the government spending shock in a two-account quantitative HANK model, which is replicated in

Figure 13: Decomposition of redistribution effects of a government spending shock



Notes: Decomposition of redistribution effects of the government spending shock on consumption, investment, output and real interest rate in Auclert, Rognlie and Straub (2018).

Figure 12.[27] In the RANK (RA) model, consumption and investment are crowded out, limiting output expansion. The HA-two model is a two-account HANK model. In the HA-two model, consumption responds positively, offsetting the crowding out of the investment, and the fiscal multiplier is greater than one.

Figure 13 shows the decomposition of redistribution effects on consumption, investment, output, and the real interest rate. The redistributive effects on consumption are positive, reversing the sign of the consumption response in RANK. As in the "IKC" environment, when the government increases bond supply, the borrowing conditions of households are eased, stimulating aggregate consumption. The liquidity channel of bond supply explains most of the redistribution effects.

[27] In the original model of Auclert, Rognlie and Straub (2018), aggregate taxes T_t distort labor supply by entering the wage Philipps curve, which implies that Ricardian equivalence does not hold even in the RANK model. To focus on the demand-side effects of time-varying bond supply, I instead assume that only taxes under the constant debt path $\bar{T}_t = G_t + r_t B^*$ enter the wage Philipps curve. If the government changes the timing of taxes, the wage Philipps curve is unaffected, ensuring Ricardian equivalence in the RANK model. Under this specification, the model responses are slightly different from Auclert, Rognlie and Straub (2018).

Unexpected inflation in period 0 lowers the real interest rate on government bonds, which benefits debtors and hurts creditors through the interest rate exposure channel. This stimulates aggregate consumption. The stimulative effect of the unexpected inflation outweighs the dampening effect of the rising interest rate from period 1 onward, as the interest rate rises modestly in the RANK model.

The income exposure channel dampens the consumption response due to heterogeneous income portfolio exposures.[28] Two mechanisms lead to a smaller income decrease for equity holders than for workers. First, the factor income of both capital and labor rises with output expansion, but only labor income is taxed to finance government spending. Second, capital owners can smooth consumption by reducing savings, as discussed in Section 4.2.

The liquidity channel of illiquid assets dampens consumption responses. The rising real interest rate forces non-adjusters to accumulate more illiquid assets than in the stationary equilibrium. Aggregate savings increase and aggregate consumption falls.

D Decomposing TANK

The decomposition can be analytically implemented in the Two-Agent New Keynesian (TANK) model. For comparison, the TANK model used here is kept identical to Bilbiie (2020).[29] I briefly describe the environment and characterize the equilibrium conditions. Details of the model can be found in Bilbiie (2020).

D.1 Model Description

There are two types of households with total unit mass. A fraction of λ households is hand-to-mouth H, who are excluded from financial markets and consume their current income. The budget constraint of H is given by

$$C_t^H = W_t N_t^H + D_t^H, \tag{152}$$

where W_t is real wage, N_t^H is H's labor supply, and D_t^H is the firm's profits received by H. The remaining fraction $1 - \lambda$ of households are savers S, trading one-period riskless real bonds. The budget constraint of S is given by

$$C_t^S + \frac{B_t}{R_t} = B_{t-1} + W_t N_t^S + D_t^S, \tag{153}$$

[28]The labor income channel is muted because households have the same labor income elasticities.

[29]Bilbiie (2020) has aggregate uncertainty and log-linearize the model. The solution is equivalent to the linearized perfect-foresight transition path (Boppart, Krusell and Mitman 2018).

where N_t^S is S's labor supply and D_t^S is the firm's profits received by S. All households maximize their discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ subject to the sequence of their budget constraints. The utility function takes the form $U(C, N) = C^{1-1/\sigma}/(1-\sigma) - N^{1+\varphi}/(1+\varphi)$.

The supply side is standard. There is a continuum of firms, and each firm produces a differentiated good with linear technology $Y_t(i) = A_t N_t(i)$. In each period, firms have the possibility of θ to reset the price. The demand for each good is $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$ where $P_t = (\int_0^1 P_t(i)^{1-\epsilon} di)^{1/(1-\epsilon)}$ is the aggregate price index and Y_t is the aggregate output. The standard supply-side implies the canonical representation of the log linearized Philips Curve: $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$ where y_t is the log deviation of output from steady state.

The government implements standard NK optimal subsidy inducing marginal cost pricing financed by a lump-sum tax on the firms' profits. The profit function is $D_t(i) = (1+\tau) P_t(i) Y_t(i)/P_t - W_t N_t(i) - T_t^F$. With the optimal subsidy, $\tau = 1/(\epsilon - 1)$, firms' steady-state profits are zero. in the stationary equilibrium, households have the same income and consumption. The central bank conducts monetary policy in the form of the Taylor rule: $i_t = r^* + \phi_\pi \pi_t + \epsilon_t$ where r^* is the steady state real interest rate, and ϵ_t is an exogenous monetary policy shock.

The key assumption in TANK is the distribution rule of the firm's profits. The government redistributes τ^D share of profits to H: $D_t^H = \tau^D D_t/\lambda$, and $1 - \tau^D$ share of profits to S: $D_t^S = (1-\tau^D) D_t/(1-\lambda)$. When $\tau^D = \lambda$, H and S receive the same profits, and their income and consumption have the same responses in equilibrium. When $\tau^D \neq \lambda$, TANK deviates from this representative-agent benchmark.

Denote log deviations of variables from their steady-state values except for interest rates by small letters. After imposing the market clearing condition, the aggregate Euler equation of TANK is derived as

$$c_t = E_t c_{t+1} - \delta^{-1} \sigma (r_t - r^*), \tag{154}$$

where $\delta^{-1} = (1-\lambda)/(1-\lambda\chi)$ and $\chi = 1 + \varphi(1 - \tau^D/\lambda)$. Though H have no access to financial markets and their consumption does not price the bond, one can infer the quantitative relation between their consumption and interest rates from the relation between H and S's equilibrium consumption.

From the aggregate Euler equation (154), we can see the amplifying/dampening mechanism in TANK. As already mentioned, if $\tau^D = \lambda$, it follows $\chi = 1$ and $\delta^{-1} = 1$. The elasticity of contemporaneous aggregate consumption to interest rates is the same as RANK. In equilibrium, the income and consumption responses of H and S are the same. If $\tau^D < \lambda$, H receives a smaller amount of profits than S. With counter-cyclical profits, it implies that H's consumption responds more than S's consumption. As a weighted sum, aggregate consumption also responds more than S's consumption, and its elasticity to interest rates is larger than the consumption elasticity of S: $\delta^{-1}\sigma > \sigma$. For a given change in real interest rates, the aggregate consumption response in TANK is amplified relative to RANK.

With the full characterization of the equilibrium, I now consider the output response to an exogenous monetary policy shock. For simplicity, here I consider a monetary policy shock that lasts only one period: $E_t \epsilon_{t+1} = 0$. Given a monetary policy shock ϵ_t, the output response of TANK is

$$y_t = -\frac{\delta^{-1}\sigma}{1+\delta^{-1}\sigma\phi_\pi\kappa}\epsilon_t. \tag{155}$$

In the case of amplifying, $\delta^{-1} > 1$, and the output response is larger (in abstract value) than that in RANK. In the case of dampening, $\delta^{-1} < 1$, the output is less responsive to monetary policy shocks relative to RANK.

D.2 Decomposition

I decompose the output response y_t into **RANK effects** y_t^{ra} and **redistribution effects** y_t^{re} such that $y_t = y_t^{ra} + y_t^{re}$. This decomposition is based on the observation that monetary policy shocks in TANK induce a redistribution between H and S due to their unequal exposures to the countercyclical profits, which affects their income elasticities to aggregate income. In a counterfactual scenario where this redistribution is eliminated, TANK behaves the same as RANK. To achieve this scenario, I construct lump-sum transfers to households. The difference between TANK and RANK is then attributed to the absence of these transfers.

Let ω_t^H and ω_t^S be the counterfactual transfers to H and S, respectively, that eliminate the redistribution effects of a monetary policy shock $\{\epsilon_t\}$. The counterfactual transfers are purely redistributive: $\lambda\omega_t^H + (1-\lambda)\omega_t^S = 0$, where λ is the fraction of H in the population. The RANK effects of the shock on output y_t^{ra} are the response of output to the shock and the transfers $\{\epsilon_t, \omega_t^H, \omega_t^S\}$; and the redistribution effects of the shock on output y_t^{re} are the response of output to the redistribution shock $\{-\omega_t^H, -\omega_t^S\}$.

RANK effects. The RANK effects on output y_t^{ra} are the output responses of a representative agent model:

$$y_t^{ra} = -\frac{\sigma}{1+\sigma\phi_\pi\kappa}\epsilon_t. \tag{156}$$

In RANK effects, S and H have the same consumption responses and it is easy to verify the consumption of Savers $c_t^{S,ra}$ satisfies the Euler equation with interest rates $\{R_t^{ra}\}$. However, these consumption responses do not satisfy households' budget constraints. To satisfy the budget constraints, I construct lump-sum transfers $\{\omega_t^H, \omega_t^S\}$ to H and S. With lump-sum transfers, the budget constraints of households are

$$\begin{aligned} c_t^{H,ra} &= w_t^{ra} + n_t^{H,ra} + \frac{\tau^D}{\lambda} d_t^{ra} + \omega_t^H, \\ c_t^{S,ra} &= w_t^{ra} + n_t^{S,ra} + \frac{1-\tau^D}{1-\lambda} d_t^{ra} + \omega_t^S, \end{aligned} \tag{157}$$

where ω_t^S and ω_t^H are the transfers (as a percentage of steady state output Y^*) to S and H, respectively. Assuming that both households satisfy their optimal labor supply condition in equilibrium, so $c_t^{S,ra} = c_t^{H,ra}$ implies $n_t^{S,ra} = n_t^{H,ra}$, the budget constraints require:

$$\begin{aligned} \omega_t^H &= \left(1 - \frac{\tau^D}{\lambda}\right) d_t^{ra}, \\ \omega_t^S &= \left(1 - \frac{1-\tau^D}{1-\lambda}\right) d_t^{ra}. \end{aligned} \tag{158}$$

With this transfer scheme, S and H have the same consumption response, and the aggregate Euler equation holds.

Redistribution effects. Consider an exogenous transfer scheme such that $\lambda T_t^H + (1-\lambda) T_t^S = 0$ where T_t^H and T_t^S are the transfers (as the percentage of steady-state output Y^*) to H and S, respectively. The proof below shows that the output response of TANK to such a transfer scheme is

$$y_t = -\frac{1}{\sigma\phi_\pi\kappa + \delta} \cdot \frac{1}{1 + (\sigma\varphi)^{-1}} T_t^S. \tag{159}$$

To obtain redistribution effects, I input the redistribution shock $\{-\omega_t^H, -\omega_t^S\}$ into the model. Letting $T_t^S = -\omega_t^S$ we have

$$y_t^{re} = \frac{1-\delta}{\sigma\phi_\pi\kappa + \delta} y_t^{ra}. \tag{160}$$

Discussion. Expressing the output response (155) y_t in terms of RANK effects (156) y_t^{ra}:

$$y_t = \frac{1 + \sigma\phi_\pi\kappa}{\delta + \sigma\phi_\pi\kappa} y_t^{ra}. \tag{161}$$

In the case of amplification ($\tau^D < \lambda$, $\chi > 1$ and $\delta < 1$), the redistribution effects act in the same direction as RANK effects, and the total effects are greater than RANK effects (in absolute value). The endogenous redistribution through firms' profit distribution τ^D/λ in TANK amplifies the output response. To see this, consider an expansionary monetary policy shock $\epsilon_t < 0$, from (158) it follows $\omega_t^H < 0$ and $\omega_t^S > 0$. The redistribution shock $\{-\omega_t^H, -\omega_t^S\}$ subsidizes H by taxing S. In TANK, fiscal stimulus in the form of transfers from S to H is itself a policy instrument that stimulates the economy (see Bilbiie, Monacelli and Perotti 2013). In the case of dampening ($\tau^D > \lambda$, $\chi < 1$ and $\delta > 1$), the redistribution shock tax H and subsidize S, which will dampen the output's response relative to RANK effects.

Another way to decompose the response of output y_t is to decompose it into substitution and income effects, as in Auclert (2019), which are also closely related to the "direct effects" and "indirect effects" of Kaplan, Moll and Violante (2018). The substitution effects are the response of aggregate consumption keeping the income of households unchanged. When interest rates fall, households save less for the future and consume more today due to intertemporal substitution. The income effects are the response of aggregate consumption keeping the interest rates unchanged.[30] After some algebra, it can be shown that

$$c_t^{sub} = \beta(1 - \lambda\chi)y_t, \tag{162}$$

$$c_t^{inc} = [1 - \beta(1 - \lambda\chi)]y_t, \tag{163}$$

the sizes of substitution effect c_t^{sub} and income effect c_t^{inc} depend on H's measure λ and the amplifying/dampening parameter χ. One can easily see the difference between this paper's decomposition and the decomposition in Kaplan, Moll and Violante (2018) and Auclert (2019) in the case of proportional distribution of firm profits ($\tau^D = \lambda$, $\chi = 1$ and $\delta = 1$). In this case, the economy's response is equivalent to RANK. This paper's decomposition implies zero redistribution effects $y_t^{re} = 0$. All output response is due to RANK effects regardless of the mass of hand-to-mouth households because, in equilibrium, S and H are equally exposed to the aggregate shock. But the size of substitution and income effects simply varies with H's measure λ. This is because the decomposition in Auclert (2019) and Kaplan, Moll and Violante (2018) captures both the heterogeneous MPCs across households (parameter λ) and the correlation between households' MPCs and income exposures (parameter χ). This paper's decomposition is designed to isolate the parameter χ.

[30] Auclert (2019) further decompose those effects into an aggregate and a redistribution component, respectively. For instance, the aggregate component of the income effects c_t^{inc} is the consumption response of an average household (whose MPC is the weighted average of S and H's MPCs) to the shock y_t, and the redistribution component of the income effects c_t^{inc} is the weighted sum of S's consumption response to $y_t^S - y_t$ and H's consumption response to $y_t^H - y_t$.

Proof. The equilibrium of TANK can be characterized by the following equations

$$c_t = E_t c_{t+1} - \delta^{-1}\sigma(r_t - r^*), \tag{164}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa c_t, \tag{165}$$

$$i_t = r^* + \phi_\pi \pi_t + \epsilon_t. \tag{166}$$

For a transient shock $E_t\epsilon_{t+1} = 0$ and $E_t c_{t+1} = E_t\pi_{t+1} = 0$. The solution is simply

$$y_t = -\frac{\delta^{-1}\sigma}{1+\delta^{-1}\sigma\phi_\pi\kappa}\epsilon_t. \tag{167}$$

The RANK effects are obtained by letting $\delta = 1$

$$y_t^{ra} = -\frac{\sigma}{1+\sigma\phi_\pi\kappa}\epsilon_t. \tag{168}$$

Expressing y_t in terms of y_t^{ra},

$$y_t/y_t^{ra} = \frac{\delta^{-1}(1+\sigma\phi_\pi\kappa)}{1+\delta^{-1}\sigma\phi_\pi\kappa} = \frac{1+\sigma\phi_\pi\kappa}{\delta+\sigma\phi_\pi\kappa}. \tag{169}$$

Consider a transfer scheme such that $\lambda T_t^H + (1-\lambda)T_t^S = 0$ where T_t^H and T_t^S are the transfers (measured as the percentage of steady-state output Y^*) H and S receive, respectively. From the budget constraint of S, we can derive the relation between S's consumption c_t^S, output y_t, and T_t^S

$$\begin{aligned} c_t^S &= w_t + n_t^S + \frac{1-\tau^D}{1-\lambda}d_t + T_t^S, \\ &= (1 - \frac{1-\tau^D}{1-\lambda})w_t + \varphi^{-1}(w_t - \sigma^{-1}c_t^S) + T_t^S; \\ [1+(\sigma\varphi)^{-1}]c_t^S &= (1 - \frac{1-\tau^D}{1-\lambda} + \varphi^{-1})w_t + T_t^S, \\ c_t^S &= \delta y_t + \frac{1}{1+(\sigma\varphi)^{-1}}T_t^S. \end{aligned} \tag{170}$$

From S's Euler equation, Philips Curve, and Taylor rule it follows $c_t^S = -\sigma(r_t - r^*) = -\sigma\phi_\pi\kappa y_t$. Substituting into (170), the output response to the transfer scheme is

$$y_t = -\frac{1}{\sigma\phi_\pi\kappa+\delta}\frac{1}{1+(\sigma\varphi)^{-1}}T_t^S. \tag{171}$$

To obtain redistribution effects, note that the transfer S receive is

$$T_t^S = -\omega_t^S = -(1 - \frac{1-\tau^D}{1-\lambda})d_t^{ra} = (1 - \frac{1-\tau^D}{1-\lambda})(\sigma^{-1}+\varphi)y_t^{ra}. \tag{172}$$

Substituting (172) into (171) it follows

$$y_t^{re} = -\frac{1}{\sigma\phi_\pi\kappa + \delta}\frac{1}{1+(\sigma\varphi)^{-1}}(1 - \frac{1-\tau^D}{1-\lambda})(\sigma^{-1}+\varphi)y_t^{ra} = \frac{1-\delta}{\delta+\sigma\phi_\pi\kappa}y_t^{ra}. \tag{173}$$

We can verify that $y_t = y_t^{ra} + y_t^{re}$.

E Decomposition Without Investment

I implement the decomposition on the model presented in Section 3, where there is no productive capital and investment. To avoid making a distinction between ex-ante and ex-post interest rates, I modify the budget constraints of households:

$$c(z^t) + \frac{b(z^t)}{R_t} = b(z^{t-1}) + z_t W_t n(z^t) + \pi_t(z) - \tau_t(z). \tag{174}$$

The channel level decomposition is, instead,

$$\begin{aligned} -\omega(z^t) = & \underbrace{(\hat{y}^{ra}(z^t) - \hat{Y}_t^{ra})y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^t) - B)(\frac{1}{R^*} - \frac{1}{R_t^{ra}})}_{\text{interest rate exposure}} + \underbrace{(\tau^*(z^t) - r^*B^*) - (\bar{\tau}^{ra}(z^t) - r_t^{ra}B^*)}_{\text{tax exposure}} \\ & + \underbrace{\frac{\bar{b}^{ra}(z^t) - b^{ra}(z^t)}{R_t^{ra}} - (\bar{b}^{ra}(z^{t-1}) - b^{ra}(z^{t-1})) + (\bar{\tau}^{ra}(z^t) - \tau^{ra}(z^t))}_{\text{liquidity}} \\ & + \underbrace{\hat{C}_t^{ra}(y^*(z^t) - c^*(z^t))}_{\text{saving flow exposure}} + \underbrace{\frac{b^*(z^t) - \bar{b}^{ra}(z^t)}{R_t^{ra}} - (b^*(z^{t-1}) - \bar{b}^{ra}(z^{t-1}))}_{\text{undetermined bond demand}}, \end{aligned} \tag{175}$$

where $y \equiv zWn + \pi$ is household income, including labor income zWn and profit income π.

To make the exercise more transparent, I assume that the central bank directly controls the real interest rate. At time $t = 0$ there is a quarterly real rate shock $\tilde{r}_0 = -0.25$ percent with the persistence of 0.61. By construction, the output response in the "RANK" equilibrium is given by the aggregate Euler equation:

$$(C_t^{ra})^{-\sigma} = \beta^{ra} R_t (C_{t+1}^{ra})^{-\sigma}. \tag{176}$$

The redistribution effects are the economy's response to the redistribution shock keeping the real interest rate at the steady state level.

In the first two exercises, I assume a balanced budget fiscal policy. In the third exercise, I let the government adjust the outstanding debt.

Table 5: Calibrated Parameter Values

Parameter	Description	Value	Target
β	Discount factor (p.q.)	0.98	2 percent annual interest rate
σ	Risk aversion	2	
$1/\nu$	Frisch elasticity	1/2	Chetty (2012)
φ	Disutility of labor	0.933	Output
ρ_e	Autocorrelation of earnings	0.966	McKay, Nakamura and Steinsson (2016)
σ_e^2	Innovation variance	0.017	McKay, Nakamura and Steinsson (2016)
B	Supply of assets (p.q.)	5.6	Aggregate liquid assets
μ	Markup of intermediate firms	1.2	Christiano, Eichenbaum and Rebelo (2011)
κ	Slope of Phillips curve	0.1	Christiano, Eichenbaum and Rebelo (2011)
ϕ	Coefficient on inflation	1.25	
$\pi(z)$	Profits distribution		Proportional to productivity
$\tau(z)$	Tax payment		Uniform across households
ρ_B	Debt reverting rate	0.1	

E.1 Calibration

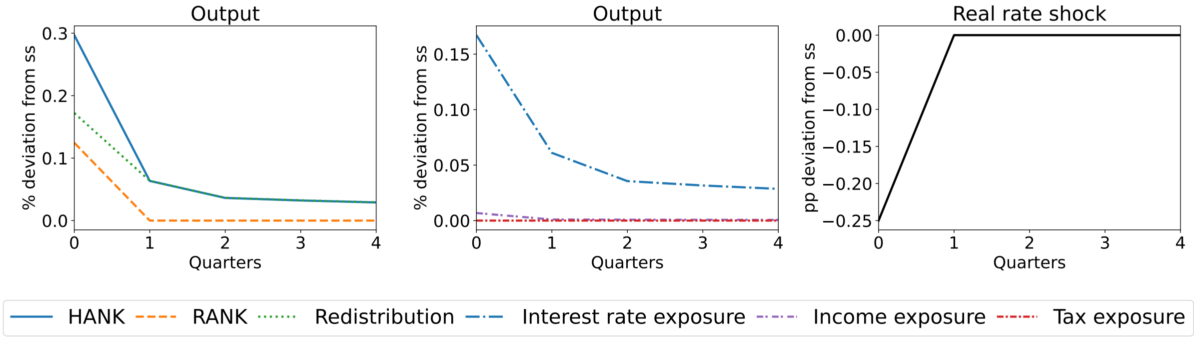
I consider a model with an annual real interest rate of 2% in the stationary equilibrium. The coefficient of risk aversion σ is set to 2. The Frisch elasticity of labor supply is $1/\nu = 0.5$, following Chetty (2012). For the idiosyncratic income process, I use $\rho_e = 0.966$ and $\sigma_e^2 = 0.017$, as in McKay, Nakamura and Steinsson (2016) and Guerrieri and Lorenzoni (2017). The supply of government bonds B is set to match the ratio of aggregate liquid assets to output $B/Y = 5.6$, as in McKay, Nakamura and Steinsson (2016). The borrowing constraint is zero $\phi = 0$. The discount factor $\beta = 0.98$ and disutility from labor $\varphi = 0.933$ are calibrated to deliver the values of annual real interest and unit quarterly output. On the supply side, the slope of the Phillips Curve is $\kappa = 0.1$ and the parameter of the markup of intermediate firms is $\mu = 1.2$. The Taylor rule coefficient ϕ is set to 1.25. In the baseline calibration, I assume that household tax payments are uniform. The firm profits are distributed to households proportional to their productivity $\pi(z_t) \sim z$, as in Kaplan, Moll and Violante (2018). Table 5 summarizes the parameter values.

E.2 Purely Transient Shocks

To begin, consider a real rate shock that lasts only one period (the persistence $\rho = 0$), in the same spirit as the thought experiments in Auclert (2019). The result is shown in Figure 14. The real interest rates decrease and stimulate consumption. Given the sticky price, the rising aggregate demand leads to an increase in both output and inflation. Regarding decomposition, redistribution effects amplify the output response. Under the transient monetary policy shock, RANK effects last for only one period, the same as a representative-agent model. In contrast, the redistribution effects affect the economy for a long time, and all the economy's responses after time 0 are due to redistribution effects.

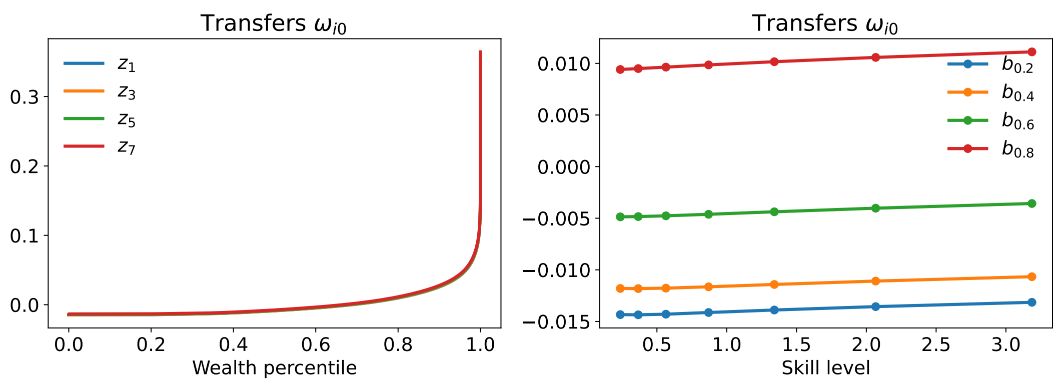
Figure 15 shows the transfers ω_{i0} as a function of the household's wealth and productivity. The left panel of Figure 15 shows the transfers ω_{i0} as a function of wealth at

Figure 14: Decomposition of a transient real-rate shock's effects



Notes: Decomposition of the output's response to a transient real-rate shock, $\tilde{r}_0 = -0.25\%$.

Figure 15: Transfers as a function of household characteristics

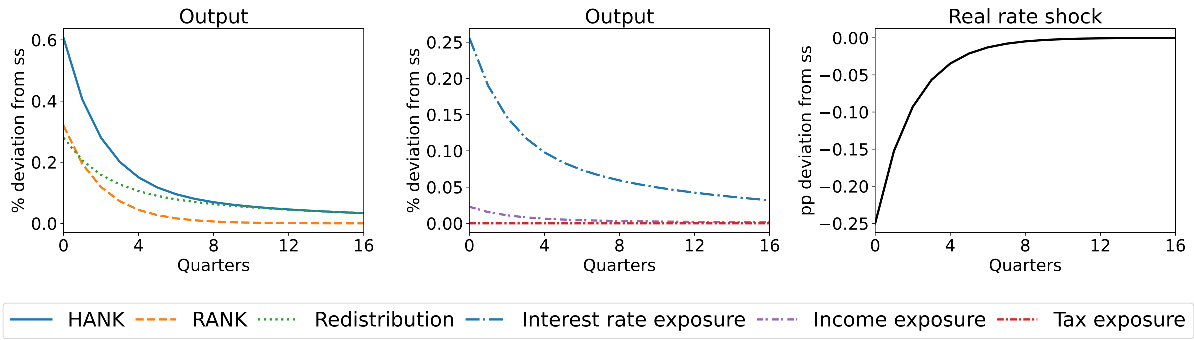


Notes: The left panel shows the transfers ω_{i0} as a function of wealth at four different productivity levels. The right panel shows ω_{i0} as a function of household productivity level at the wealth distribution's 20th, 40th, 60th, and 80th percentiles.

four different productivity levels. The right panel shows ω_{i0} as a function of household productivity level at the wealth distribution's 20th, 40th, 60th, and 80th percentiles. The transfers ω_{i0} increase with the household's wealth and (weakly) with productivity. Transfers increase with wealth because to eliminate the exposure to the interest rate cut, creditors need positive transfers, and debtors need negative transfers. The transfers increase with productivity because profits are countercyclical. The income of the household is $y = zWn + z\Pi = z(WNn/N + \Pi)$. Due to labor supply heterogeneity, high-income households have a higher share of profit income, which is countercyclical. High-income households' income increases less and needs positive transfers.

Overall, the redistribution shock $-\boldsymbol{\omega}$ benefits high-MPC households by taxing low-MPC households: $cov_I(MPC_{i0}, -\omega_{i0}) > 0$. The redistribution effects stimulate aggregate consumption.

Figure 16: Decomposition of a persistent real-rate shock's effects



Notes: Decomposition of the output's response to a persistent real-rate shock, $\tilde{r}_0 = -0.25\%$. The redistribution shock's effects on output are decomposed into three channels. The government keeps a constant debt and adjusts the uniform tax following the shock. Equation (175) gives the definitions of these redistribution channels. The government keeps a constant debt and adjusts the uniform tax to balance its budget.

E.3 Persistent Shocks

Consider the economy's response to a persistent real-rate shock. I apply the decomposition, and the result is shown in Figure 16. Output increases by 0.6% on impact. The decomposition result is qualitatively similar to the decomposition of the transient shock in Figure 14. Redistribution effects amplify the output's response to the real rate shock. On impact, RANK effects increase output by 0.31%, and redistribution effects increase output by 0.29%. The redistribution effects amplify the elasticity of output to real interest rates.

Quantitatively, the interest exposure channel accounts for most of the redistribution effects. On impact, the interest exposure channel increases consumption by 0.25%. The interest rate cuts tax creditors and subsidizes debtors. Given that debtors have higher MPCs, the interest rate exposure channel stimulates aggregate consumption. The income exposure channel slightly contributes to the output amplification. Since I assume uniform taxation, all households benefit equally from the tax reduction, and the tax exposure channel is muted.

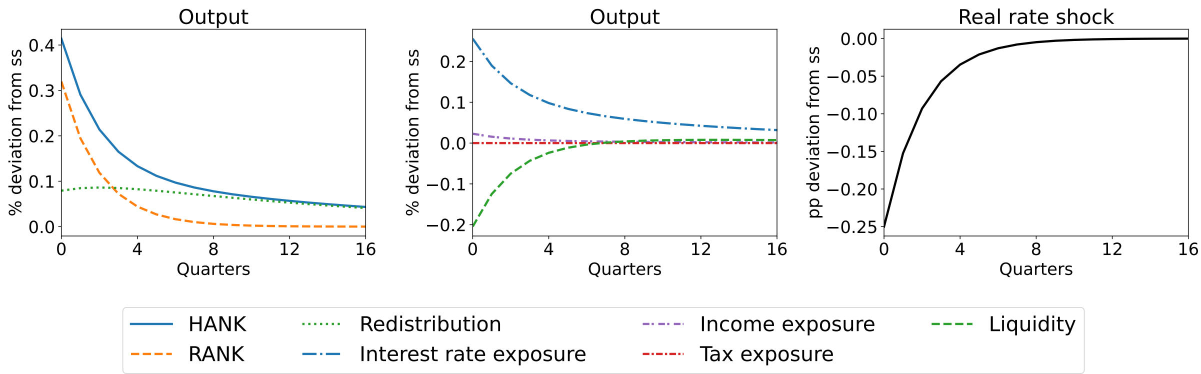
E.4 Including Liquidity Channel

Assuming the fiscal policy takes the following rule:

$$T_t = T^* + \rho_B * (B_{t-1} - B^*). \tag{177}$$

In the short run, the government uses debt to absorb most of the fiscal imbalance. In the long run, the government uses taxes to bring the debt back to its initial level. Similar fiscal policy specifications are assumed in Kaplan, Moll and Violante (2018),

Figure 17: Decomposition with liquidity channel



Notes: Decomposition of the output's response to a persistent real-rate shock, $\tilde{r}_0 = -0.25\%$, with fiscal policy $T_t = T^* + \rho_B * (B_{t-1} - B^*)$. The government uses debt to absorb most of the fiscal imbalance in the short run. In the long run, the government uses uniform taxes to bring the debt back to its initial level. Redistribution effects on output are decomposed into four channels.

Alves et al. (2020), and Auclert, Rognlie and Straub (2018).

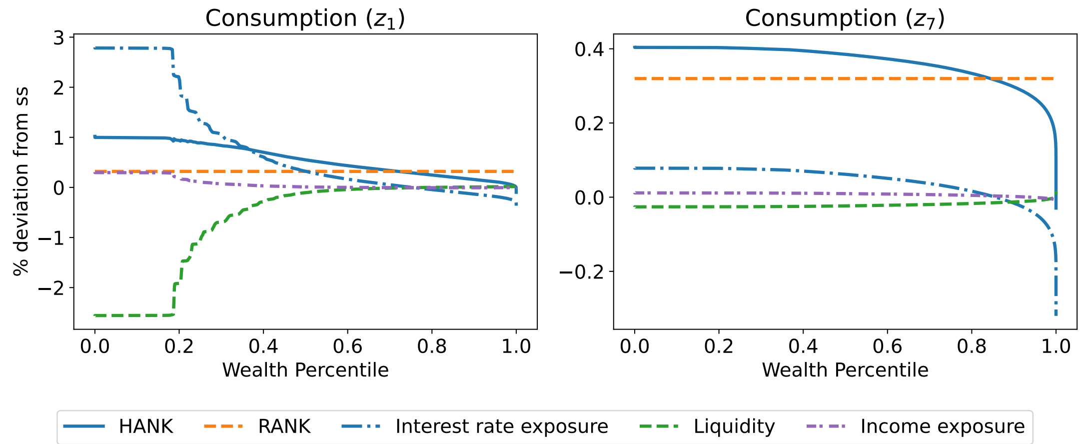
The decomposition result is shown in Figure 17. The redistribution effects are smaller than Figure 16. On impact, redistribution effects increase output by less than 0.1%, rather than close to 0.3% under a balanced fiscal policy. The effects of interest exposure, income exposure, and tax exposure channels are invariant to the path of government debt. However, the liquidity channel decreases output by 0.2% on impact. The liquidity channel explains why the output response with the fiscal policy of (177) is smaller than a balanced budget.

The fiscal rule (177) implies a countercyclical bond supply. As proved in Section 4.1, the liquidity channel can be proxied by a borrowing-constraint shock in the case of uniform taxation. Given the constant real interest rate, the output needs to decrease to clear the market. Figure 18 shows the decomposition of the households' impact consumption responses. The interest rate exposure channel increases the consumption of poor households and decreases the consumption of rich households. However, the liquidity channel forces the constrained households to hold the additional income from other channels. As a result, the redistribution effects on the consumption of poor households are smaller compared to a balanced fiscal policy.

F Model with Illiquid Assets

In this section, I extend the model with illiquid assets as in Kaplan, Moll and Violante (2018), Bayer et al. (2019), Luetticke (2021), Auclert, Rognlie and Straub (2018), Auclert, Rognlie and Straub (2020) and Kaplan and Violante (2022). Section F.1 shows that Proposition 1 holds with illiquid assets when modeling illiquidity a la Calvo as in Bayer et al. (2019) and Luetticke (2021). Section F.2 reveals that the presence of illiquid

Figure 18: Household-level decomposition (on impact)



Notes: The redistribution shock's effects on individual consumption (impact) are decomposed into three channels. For comparison, I also show the RANK effects and the HANK model's responses. Equation (175) gives the definitions of these redistribution channels.

assets introduces a new redistribution channel that amplifies the effects of monetary policy shocks.

F.1 Model Description and RANK Effects

Households. Households have access to two assets: (i) liquid assets a^{liq} with gross real return R^{liq}; (ii) illiquid assets a^{illiq} with gross real return R^{illiq}. Households maximize subject to the following budget, adjustment, and borrowing constraints:

$$c(h^t) + a^{liq}(h^t) = R_t^{liq} a^{liq}(h^{t-1}) - d(h^t) + z_t W_t n(h^t) + \pi(h^t) - \tau(h^t), \tag{178}$$

$$a^{illiq}(h^t) = R_t^{illiq} a(h^{t-1}) + d(h^t), \tag{179}$$

$$a^{liq}(h^t) \geq 0, \quad a^{illiq}(h^t) \geq 0, \tag{180}$$

where $h^t \equiv ((b_{-1}, a_{-1}), (z_0, s_0), (z_1, s_1), \cdots, (z_t, s_t))$ is the individual's history of idiosyncratic shocks up to time t, including both productivity shock z and adjustment shock s. Households can only adjust their holdings on illiquid assets at period t when $s_t = 1$, which occurs with iid probability λ. So, in each period, a randomly selected λ fraction of households can adjust their holdings of illiquid assets. When $s_t = 0$, the illiquid assets accumulate in the background:

$$d(h^t) = 0, \text{ if } s_t = 0. \tag{181}$$

The liquid assets are invested in government bonds. The illiquid assets are invested in equity. Firms issue equity to households, the price of each share is P_t, and each share provides dividends D_t. The amount of equity held by households is given by

$v(h^t) \equiv a^{illiq}(h^t)/P_t$. The return on illiquid assets satisfies $R_t^{illiq} = (P_t + D_t)/P_{t-1}$.

Firms. Firms own capital K_{t-1} and choose investment I_t to obtain the capital of the next period $K_t = (1-\delta)K_{t-1} + I_t$, subject to quadratic capital adjustment cost. Dividends equal capital products plus post-tax monopolistic profits net of investment, capital adjustment cost, and price adjustment cost,

$$D_t = r_t^K K_{t-1} + \alpha \Pi_t - I_t - \frac{\Psi}{2}(\frac{I_t}{K_{t-1}} - \delta^K)^2 - \Theta_t. \tag{182}$$

Firms choose investment to maximize $P_t + D_t$. Tobin's Q and capital evolve according to the standard Q-theory of investment:

$$\frac{I_t}{K_{t-1}} - \delta^K = \frac{1}{\Psi}(Q_t - 1), \tag{183}$$

$$R_{t+1}^{illiq} Q_t = r_{t+1}^K - \frac{I_{t+1}}{K_t} - \frac{\Psi}{2}(\frac{I_{t+1}}{K_t} - \delta^K)^2 + \frac{K_{t+1}}{K_t} Q_{t+1}. \tag{184}$$

Equilibrium. The other sectors of the economy are the same as in Section 5.1. In the equilibrium, households and firms optimize, government budget constraint holds, nominal interest rates evolve according to the Taylor rule, and markets clear:

$$\int a^{liq}(h^t) d\Phi_t(h^t) = B_t, \tag{185}$$

$$\int a^{illiq}(h^t) d\Phi_t(h^t) = P_t, \tag{186}$$

$$C_t + I_t + \frac{\Psi}{2}(\frac{I_t}{K_{t-1}} - \delta)^2 + \Theta_t = Y_t^{GDP}. \tag{187}$$

In the following, I show that proposition 1 holds with the presence of illiquid assets, and there is an aggregate Euler equation governing the return on illiquid assets given the path of aggregate consumption.

Proposition 4. *For a given monetary policy shock ϵ, there exist counterfactual transfers ω such that:*

(i) The equilibrium of aggregates can be characterized with only aggregate conditions, including the aggregate Euler equation with respect to liquid assets

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{liq,ra} R_{t+1}^{liq,(\epsilon,\omega)} \left(C_{t+1}^{\epsilon,\omega}\right)^{-\sigma}, \text{ where } \beta^{liq,ra} \equiv 1/R^{liq,*}; \tag{188}$$

the aggregate Euler equation with respect to illiquid assets

$$(C_t^{\epsilon,\omega})^{-\sigma} = \beta^{illiq,ra} R_{t+1}^{illiq,(\epsilon,\omega)} (C_{t+1}^{\epsilon,\omega})^{-\sigma}, \text{ where } \beta^{illiq,ra} \equiv 1/R^{illiq,*}; \tag{189}$$

the aggregate labor supply condition

$$W_t^{\epsilon,\omega}(C_t^{\epsilon,\omega})^{-\sigma} = \varphi^{ra}(N_t^{\epsilon,\omega})^{\nu}, \textit{ where } \varphi^{ra} \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}; \tag{190}$$

the Philips curve; Q theory of investment; government budget constraint; Taylor rule; and market clearing conditions.

(ii) The individual consumption and labor supply satisfy:

$$\frac{c^{\epsilon,\omega}(h^t)}{c^*(h^t)} = \frac{C_t^{ra}}{C^*}, \quad \frac{n^{\epsilon,\omega}(h^t)}{n^*(h^t)} = \frac{N_t^{ra}}{N^*}. \tag{191}$$

(iii) The transfers sum to zero crosssectionally $\int \omega(h^t)d\Phi_t(h^t) = 0$.

Proof. The proof of the first-order condition (F.O.C) with respect to liquid assets is the same as the one-asset model. I prove the F.O.C with respect to illiquid assets below. In the case of adjustment ($s_t = 1$), the F.O.C with respect to illiquid assets is:

$$\begin{aligned}
&(c(h^{t-1},(z_t,1)))^{-\sigma} \geq \\
&\Big\{\beta\lambda R_{t+1}^{illiq} E_z[(c(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda)R_{t+1}^{illiq}R_{t+2}^{illiq}E_z[(c(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2R_{t+1}^{illiq}R_{t+2}^{illiq}R_{t+3}^{illiq}E_z[(c(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+ \\
&\cdots\Big\}, = \text{if } a^{illiq}(h^t) > 0.
\end{aligned} \tag{192}$$

Consider households save one additional unit of illiquid assets at time t; then, with probability λ, the (accumulated) one unit of illiquid assets can be used for consumption at time $t+1$, generating expected marginal utility

$$R_{t+1}^{illiq}E_z[(c(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]$$

at time $t+1$; with probability $\lambda(1-\lambda)$, the (accumulated) one unit illiquid assets can be used for consumption at time $t+2$, generating expected marginal utility

$$R_{t+1}^{illiq}R_{t+2}^{illiq}E_z[(c(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]$$

at time $t+2$; with probability $\lambda(1-\lambda)^2$, the (accumulated) one unit illiquid assets can be used for consumption at time $t+3$, generating expected marginal utility

$$R_{t+1}^{illiq}R_{t+2}^{illiq}R_{t+3}^{illiq}E_z[(c(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]$$

at time $t+3$, etc.. Then the marginal value of the one additional unit of illiquid assets is the expected value of the (discounted) utility flows.

In the stationary equilibrium, the F.O.C with respect to illiquid assets

$$
\begin{aligned}
&(c^*(h^{t-1},(z_t,1)))^{-\sigma} \geq \\
&\Big\{\beta\lambda R^{illiq,*} E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t, s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda)(R^{illiq,*})^2 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t, s_{t+1}=0, s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2(R^{illiq,*})^3 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t, s_{t+1}=s_{t+2}=0, s_{t+3}=1]+ \\
&\cdots\Big\}, = \text{ if } a^{illiq,*}(h^t) > 0.
\end{aligned} \tag{193}
$$

Given (193) holds, we verify the consumption allocation $\{c^{ra}(h^t)\}$ satisfies the F.O.C (192) given the interest rate path $\{R^{illiq,ra}_{t+1}\}$. Substituting $\{c^{ra}(h^t)\}$ into the F.O.C (192). First,

$$
(c^{ra}(h^{t-1},(z_t,1)))^{-\sigma} = (C^{ra}_t/C^*)^{-\sigma}(c^*(h^{t-1},(z_t,1)))^{-\sigma}, \tag{194}
$$

and

$$
\begin{aligned}
&\beta\lambda R^{illiq,ra}_{t+1} E_z[(c^{ra}(h^t,(z_{t+1},1)))^{-\sigma}|h^t, s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda) R^{illiq,ra}_{t+1} R^{illiq,ra}_{t+2} E_z[(c^{ra}(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t, s_{t+1}=0, s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2 R^{illiq,ra}_{t+1} R^{illiq,ra}_{t+2} R^{illiq,ra}_{t+3} E_z[(c^{ra}(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t, s_{t+1}=s_{t+2}=0, s_{t+3}=1]+ \\
&\cdots
\end{aligned} \tag{195}
$$

$$
\begin{aligned}
=&\beta\lambda R^{illiq,ra}_{t+1}(\frac{C^{ra}_{t+1}}{C^*})^{-\sigma} E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t, s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda) R^{illiq,ra}_{t+1} R^{illiq,ra}_{t+2}(\frac{C^{ra}_{t+2}}{C^*})^{-\sigma} E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t, s_{t+1}=0, s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2 R^{illiq,ra}_{t+1} R^{illiq,ra}_{t+2} R^{illiq,ra}_{t+3}(\frac{C^{ra}_{t+3}}{C^*})^{-\sigma} E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t, s_{t+1}=s_{t+2}=0, s_{t+3}=1]+ \\
&\cdots
\end{aligned} \tag{196}
$$

$$
\begin{aligned}
=&\beta\lambda \frac{R^{illiq,ra}_{t+1}}{R^{illiq,*}}(\frac{C^{ra}_{t+1}}{C^*})^{-\sigma} R^{illiq,*} E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t, s_{t+1}=1]+ \\
&\beta^2\lambda(1-\lambda)\frac{R^{illiq,ra}_{t+1}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+2}}{R^{illiq,*}}(\frac{C^{ra}_{t+2}}{C^*})^{-\sigma}(R^{illiq,*})^2 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t, s_{t+1}=0, s_{t+2}=1]+ \\
&\beta^3\lambda(1-\lambda)^2\frac{R^{illiq,ra}_{t+3}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+2}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+3}}{R^{illiq,*}}(\frac{C^{ra}_{t+3}}{C^*})^{-\sigma}(R^{illiq,*})^3 E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t, s_{t+1}=s_{t+2}=0, s_{t+3}=1]+ \\
&\cdots.
\end{aligned} \tag{197}
$$

Given $(C^{ra}_t)^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}(C^{ra}_{t+1})^{-\sigma}$, where $\beta^{illiq,ra} \equiv 1/R^{illiq,*}$, we have

$$
\frac{R^{illiq,ra}_{t+1}}{R^{illiq,*}}(C^{ra}_{t+1})^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}(C^{ra}_{t+1})^{-\sigma} = (C^{ra}_t)^{-\sigma}, \tag{198}
$$

$$
\frac{R^{illiq,ra}_{t+1}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+2}}{R^{illiq,*}}(C^{ra}_{t+2})^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}\beta^{illiq,ra} R^{illiq,ra}_{t+2}(C^{ra}_{t+2})^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}(C^{ra}_{t+1})^{-\sigma} = (C^{ra}_t)^{-\sigma}, \tag{199}
$$

$$
\begin{aligned}
&\frac{R^{illiq,ra}_{t+1}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+2}}{R^{illiq,*}}\frac{R^{illiq,ra}_{t+2}}{R^{illiq,*}}(C^{ra}_{t+3})^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}\beta^{illiq,ra} R^{illiq,ra}_{t+2}\beta^{illiq,ra} R^{illiq,ra}_{t+3}(C^{ra}_{t+3})^{-\sigma} \\
&= \beta^{illiq,ra} R^{illiq,ra}_{t+1}\beta^{illiq,ra} R^{illiq,ra}_{t+2}(C^{ra}_{t+2})^{-\sigma} = \beta^{illiq,ra} R^{illiq,ra}_{t+1}(C^{ra}_{t+1})^{-\sigma} = (C^{ra}_t)^{-\sigma},
\end{aligned} \tag{200}
$$

$\cdots.$

So equation (197) simplifies to

$$
\begin{aligned}
&(C_t^{ra}/C^*)^{-\sigma}\Big\{\beta\lambda R^{illiq,*}E_z[(c^*(h^t,(z_{t+1},1)))^{-\sigma}|h^t,s_{t+1}=1]+\\
&\beta^2\lambda(1-\lambda)(R^{illiq,*})^2E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},1)))^{-\sigma}|h^t,s_{t+1}=0,s_{t+2}=1]+\\
&\beta^3\lambda(1-\lambda)^2(R^{illiq,*})^3E_z[(c^*(h^t,(z_{t+1},0),(z_{t+2},0)),(z_{t+3},1)))^{-\sigma}|h^t,s_{t+1}=s_{t+2}=0,s_{t+3}=1]+\\
&\cdots\Big\},
\end{aligned}
\tag{201}
$$

which is the marginal value of illiquid assets in the stationary equilibrium scaled by $(C_t^{ra}/C^*)^{-\sigma}$. Combined with (194), we can see that given the F.O.C in the stationary equilibrium (193) holds, $\{c^{ra}(h^t)\}$ satisfies the F.O.C (192) with the interest rate path $\{R_{t+1}^{illiq,ra}\}$.

An important implication of Proposition 4 is that in the "RANK" equilibrium, the liquid and illiquid return satisfies $R_t^{liq} = \frac{R^{liq,*}}{R^{illiq,*}}R_t^{illiq}$ and the liquidity premium $R_t^{illiq} - R_t^{liq}$ is nearly acyclical in the "RANK" equilibrium. All the responses of the liquidity premium are due to redistribution effects.

F.2 Liquidity Channel of Illiquid Assets

We can construct counterfactual transfers and define redistribution channels as in the one-asset model. Compared to the one-asset model, there is an additional channel to consider: the liquidity channel of illiquid assets. The change in the return on illiquid assets R_t^{illiq} impacts the illiquid assets that non-adjusters are forced to accumulate. Roughly speaking, if the aggregate shock reduces the return on illiquid assets, non-adjusters accumulate fewer illiquid assets for the future, and some of these assets become liquid for consumption. The eased constraints on accumulating illiquid assets stimulate aggregate consumption. To distinguish it from the liquidity channel of time-varying bond supply, this channel is defined as the **liquidity channel of illiquid assets**.

Formally, when the return on illiquid assets changes, the illiquid assets non-adjusters accumulate can differ from the imposed asset demand: $R_t^{illiq,ra}a^{illiq,ra}(h^{t-1}) \neq a^{illiq,ra}(h^t)$. To achieve the "RANK" equilibrium, I introduce a counterfactual shock to non-adjusters' demand for illiquid assets $\Delta a^{illiq}(h^t) = a^{illiq,ra}(h^t) - R_t^{illiq,ra}a^{illiq,ra}(h^{t-1})$ such that, given illiquid asset holdings $a^{illiq,ra}(h^{t-1})$ and return $R_t^{illiq,ra}$, the illiquid-asset demand of non-adjusters satisfies the imposed asset demand $a^{illiq,ra}(h^t)$.

Consider two simple cases for illustration. In the first case, illiquid assets are invested in government bonds and the government maintains a constant level of debt. Impose the asset demand function $a^{illiq,ra}(h^t) = b^*(h^t)$. After an expansionary shock, for non-adjusters, $R_t^{illiq,ra}a^{illiq,ra}(h^{t-1}) = R_t^{illiq,ra}b^*(h^{t-1}) < R^{illiq,*}b^*(h^{t-1}) = b^*(h^t) = a^{illiq,ra}(h^t)$. To satisfy the imposed illiquid-asset demand $a^{illiq,ra}(h^t) = b^*(h^t)$, the illiquid-asset demand shock is

$$\Delta a^{illiq}(h^t) = b^*(h^t) - R_t^{illiq,ra} b^*(h^{t-1}) = (R^{illiq,*} - R_t^{illiq,ra}) b^*(h^{t-1}). \tag{202}$$

The decreasing return on illiquid assets implies that we need a positive shock $\Delta a^{illiq}(h^t) > 0$ to achieve the "RANK" equilibrium. In assessing the impact of the liquidity channel of illiquid assets, the negative of the counterfactual shock $\{-\Delta a^{illiq}(h^t)\}$ is input into the model, which is a shock that reduces non-adjusters' demand for illiquid assets. The falling interest rate relaxes the constraint on illiquid-asset accumulation. Some of the illiquid assets become liquid and aggregate consumption increases.

For the second case, consider that the illiquid assets are invested in firm equity and the imposed illiquid-asset demand is $a^{illiq,ra}(h^t) = P_t^{ra} v^*(h^t)$. For non-adjusters, the counterfactual shock to the illiquid-asset demand is

$$\Delta a^{illiq}(h^t) = P_t^{ra} v^*(h^t) - R_t^{illiq,ra} P_{t-1}^{ra} v^*(h^{t-1}) \tag{203}$$

$$= P_t^{ra} R^{illiq,*} v^*(h^{t-1}) - R_t^{illiq,ra} P_{t-1}^{ra} v^*(h^{t-1}) \tag{204}$$

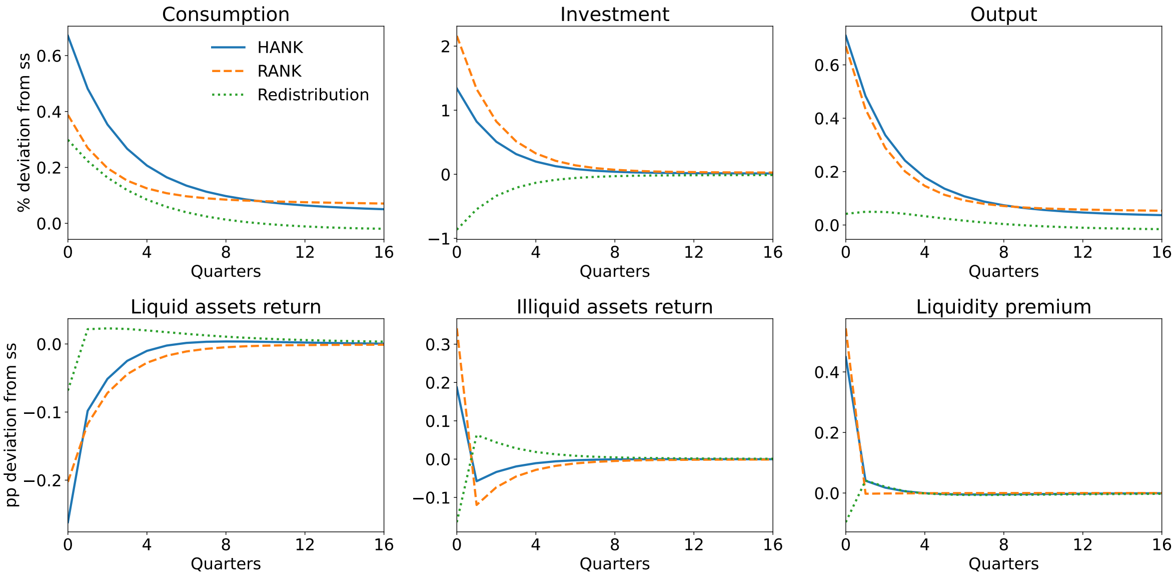
$$= (P_t^{ra} R^{illiq,*} - P_{t-1}^{ra} R_t^{illiq,ra}) v^*(h^{t-1}). \tag{205}$$

If $P_t^{ra} R^{illiq,*} > P_{t-1}^{ra} R_t^{illiq,ra}$, we need a positive shock $\Delta a^{illiq}(h^t) > 0$ to achieve the "RANK" equilibrium, similar to the case that illiquid assets are invested in government bonds.

F.3 Monetary Policy in Two-asset HANK Model

This section decomposes the responses of a two-asset model to a monetary policy shock. The production side is calibrated as in the one-asset model in Section 6, except that the steady-state markup is set to zero. The value of illiquid assets (equity) relative to annual output is set to $A/Y^{GDP} = 2.375$, which is the residual between net wealth and net liquid assets in Kaplan and Violante (2022), where they are calibrated to the 2019 U.S. economy. The annual real return on illiquid assets is $r^{illiq} = 0.06$ and on liquid assets is $r^{liq} = -0.02$. The value of gross liquid assets relative to annual output B/Y^{GDP}, the adjustment probability λ, and the discount factor β are calibrated to match three targets: two market clearing conditions for liquid and illiquid assets, and the year-0 iMPC estimated in Fagereng, Holm and Natvik (2021). The model performs well in hitting the three targets. The calibrated value of gross liquid assets for annual output is $B/Y^{GDP} = 0.6$. Given the value of net liquid assets from the data (SCF 2019) $B^{net}/Y^{GDP} = 0.375$, the implied borrowing limit for annual output (before normalization) is $B^{net}/Y^{GDP} - B/Y^{GDP} = -0.225$, 1.3 times quarterly average labor income. The calibrated adjustment probability is 0.06 and the discount factor is 0.982

Figure 19: Responses of aggregates in the two-asset HANK model



Notes: The responses of aggregate variables a monetary policy shock in the two-asset HANK model. The liquid assets are invested in government bonds and illiquid assets are invested in firm equity.

(both quarterly).

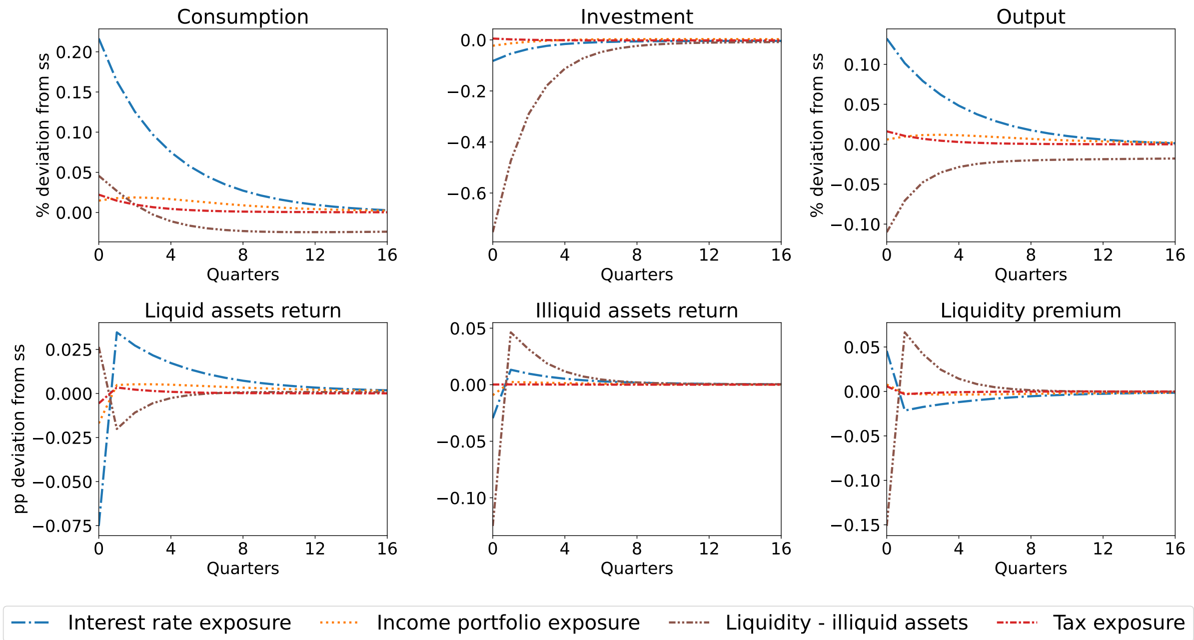
For simplicity, I abstract from the labor income exposure channel and the liquidity channel of time-varying bond supply, as they depend on exogenous assumptions about labor income elasticities and fiscal policy responses. Individual labor income is given by $y^L(z^t) = z_t Y^L$. The value of government debt is assumed to be constant, and the government adjusts uniform taxes to balance its budget.

The responses of the aggregates are shown in Figure 19. The responses of the aggregates are close to the one-asset model in Section 6. However, the decomposition of the redistribution effects shown in Figure 20 is quite different from the one-asset model. The effects of the interest rate exposure channel are twice as large as in the one-asset model. The impact of the income portfolio exposure channel is much smaller, being only about a quarter of its impact in the one-asset model. This is because the MPC from illiquid asset gains (and losses) is much smaller than that from liquid assets. The income portfolio exposure channel operates through redistribution between equity holders and workers, and in the model equity is illiquid.

The monetary policy shock pushes down the real return on illiquid assets, easing the constraints on holding illiquid assets. Its amplification effects on consumption responses are also more transitory than other channels, as it dampens investment responses and reduces the capital stock.

The interest rate exposure channel has a large effect on consumption, but a much smaller effect on investment. Similarly, the liquidity channel of illiquid assets has a large effect on investment but a much smaller effect on consumption. Redistribution

Figure 20: Decomposition of redistribution effects in the two-asset HANK model



Notes: The decomposition of redistribution effects of a monetary policy shock in the two-asset HANK model. The liquid assets are invested in government bonds and illiquid assets are invested in firm equity.

has an asymmetric impact on consumption/investment depending on whether it operates through liquid or illiquid assets.

F.4 "Two-account" HANK Model

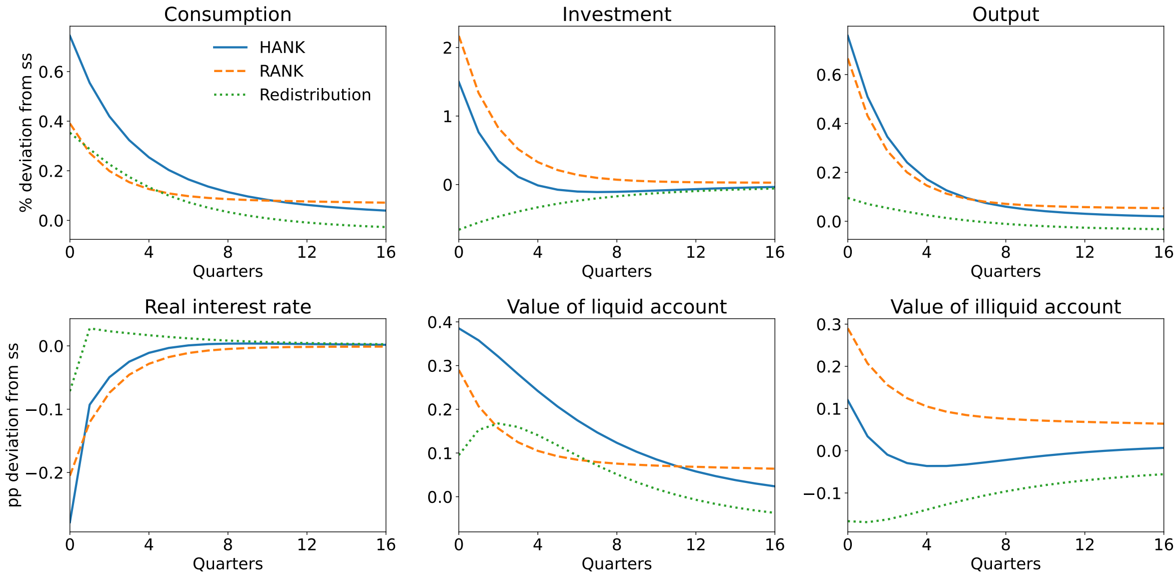
The above decomposition assumes that liquid assets are invested in government bonds and illiquid assets are invested in firm equity, as in Kaplan, Moll and Violante (2018), Alves et al. (2020), Bayer et al. (2019) and Luetticke (2021). It implies that households hold bonds $b(h^t) = a^{liq}(h^t)$ and the value of equity $P_t v(h^t) = a^{illiq}(h^t)$. In the next, I consider another version of the two-asset model in which I follow Auclert, Rognlie and Straub (2018) and interpret $a^{liq}(h^t)$ and $a^{illiq}(h^t)$ as household savings in liquid and illiquid accounts, respectively. The savings in each account can be invested in government bonds and firm equity and they are perfect substitutes (except at time 0, when the returns differ due to unexpected inflation and capital gains).

The market clearing conditions for assets are instead

$$\int (a^{liq}(h^t) + a^{illiq}(h^t))d\Phi_t(h^t) = A_t = B_t + P_t, \tag{206}$$

where A_t is the aggregate asset demand and $B_t + P_t$ is the aggregate asset supply. Liquid account returns are given by $R_t^{liq} = \frac{R^{liq,*}}{R^{illiq,*}} R_t^{illiq}$. I calibrate the two-account model such that it has the same bond supply B, value of equity P, and adjustment

Figure 21: Responses of aggregates in the two-account HANK model



Notes: The responses of aggregate variables to a monetary policy shock in the two-account HANK model. Households can hold government bonds and firm equity in liquid and illiquid accounts as Auclert, Rognlie and Straub (2018).

probability λ as the baseline two-asset model in the previous section.[31]

The fraction of each account invested in government bonds and equity is assumed to be the same as the aggregate portfolio and homogeneous across agents. Given this assumption, the individual holdings of bonds and equity are given by

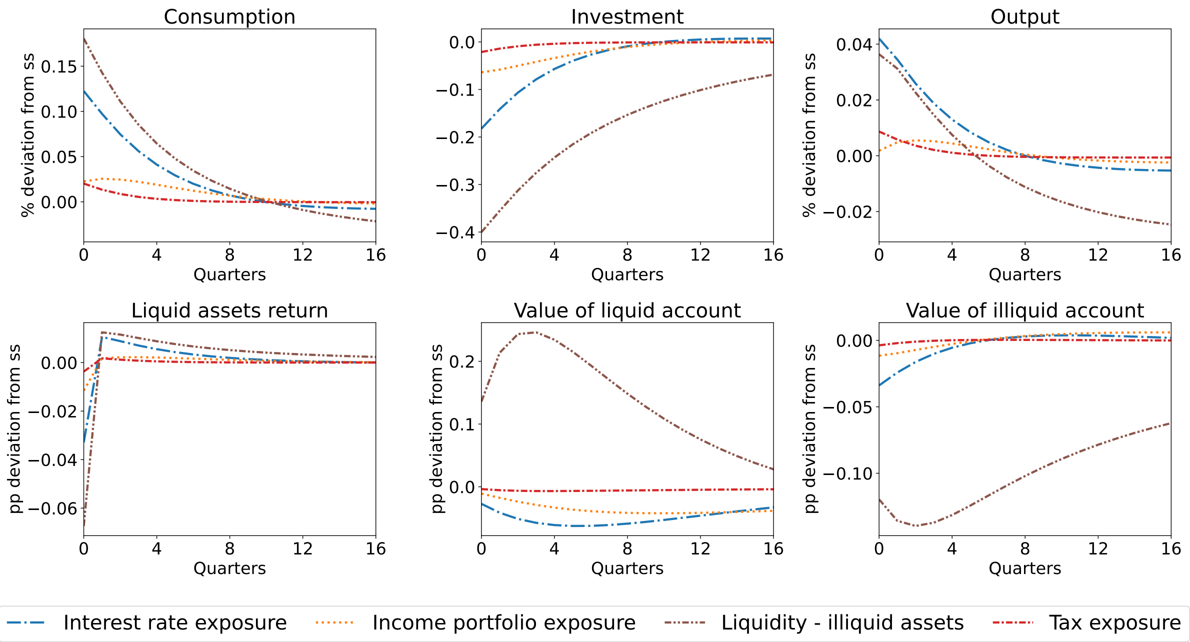
$$b(h^t) = a^{liq}(h^t)\frac{B_t}{A_t} + a^{illiq}(h^t)\frac{B_t}{A_t}, \tag{207}$$

$$P_t v(h^t) = a^{liq}(h^t)\frac{P_t}{A_t} + a^{illiq}(h^t)\frac{P_t}{A_t}. \tag{208}$$

Figure 21 shows the responses of aggregates under this two-account specification. Overall, the responses of aggregates are close to the baseline two-asset model. However, from Figure 22 we can see that the decomposition of redistribution effects is very different. The effect of interest rate exposure on consumption is only about half of its effect in the baseline two-asset model. This is because the MPC from gains (and losses) on illiquid assets is smaller than that from liquid assets. When bonds are held as illiquid assets, the interest rate change has a smaller effect on consumption. Instead, the interest rate change has a larger effect on investment than the baseline two-asset model because the marginal propensity of saving (MPS) from illiquid asset gains (and

[31]The discount factor β is recalibrated to clear the asset market. The year-0 aggregate MPC is 46.3%, slightly lower than the baseline two-asset model's 49.8%. The value of the liquid account (to annual output) in the two-account model is 0.595, close to the value of liquid assets (to annual output) of the baseline two-asset model, which is 0.598.

Figure 22: Decomposition of redistribution effects in the two-account HANK model



Notes: The decomposition of redistribution effects of a monetary policy shock in the two-account HANK model. Households can hold government bonds and firm equity in liquid and illiquid accounts as Auclert, Rognlie and Straub (2018).

losses) is larger than the MPS from liquid assets.

In this alternative two-account specification, the liquidity channel of illiquid assets is the largest amplifier of consumption responses, rather than the interest rate exposure channel.

From the responses of asset demand, we can also see that the relaxed constraints on accumulating illiquid assets increase households' demand for liquid assets (account) and decrease households' demand for illiquid assets (account).