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May Tax Evasion Help Control Public Debt?

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Abstract

Tolerating tax evasion may increase debt less than an equivalent tax cut. In our model, utility-maximizing entrepreneurs earn income from risky production technologies and risk-free bonds. The government uses income taxes and bonds to finance its expenses. Entrepreneurs can evade taxes at the risk of being audited and fined. Aggregate tax evasion and debt-to-GDP are positively related in equilibrium. Nevertheless, reducing effective tax rates by tolerating evasion may generate a lower debt-to-GDP ratio (but also lower growth) than equivalent debt-financed nominal tax cuts. Policies are equivalent with log utility. (JEL: D5 E6 H2)

Keywords: Dynamic tax evasion; general equilibrium; public debt.

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1 Introduction

Tax evasion is a pervasive global phenomenon which may harm welfare (Albarea et al., 2020; Davison, 2021). In fact, public budget shortfalls may induce the government to increase taxes or cut its expenditure (Pappa et al., 2015). If neither occurs, there will be an increase in public debt. Moreover, public debt may foster tax evasion, as government bonds can be used to "hedge" against uninsurable auditing risk (He et al., 2019; Brunnermeier et al., 2024).

This paper investigates the general equilibrium relationship between tax evasion and public debt dynamics. In particular, it provides a theoretical foundation for the counter-intuitive result that allowing tax evasion may generate lower public debt than equivalent nominal tax cuts but reduces growth more. For this purpose, we extend the model by Gersbach et al. (2023) by considering a Constant Relative Risk Aversion (CRRA) utility function and the possibility of evading taxes, as in Bernasconi et al. (2015).

In our model, utility-maximizing entrepreneurs allocate their net worth between risky capital production and risk-free government bonds and pay income taxes. They cannot diversify idiosyncratic productivity shocks due to market incompleteness but can evade taxes at the risk of being audited and fined. The government uses taxes and bonds to finance productivity-enhancing expenditures, as in Barro (1990).

We solve the model for its competitive equilibrium and find (analytically) a positive feedback loop between aggregate tax evasion and public debt-to-GDP levels. Next, we compare the model's steady state and transition dynamics under different fiscal policies using numerical simulations.

We show that reducing effective tax rates by allowing tax evasion is not the same as implementing equivalent nominal tax cuts and driving tax evasion to zero unless entrepreneurs are myopic (i.e., they have logarithmic preferences). The reason is that, in equilibrium, tax evasion increases the risk-free interest rate (debt financing costs) less than the nominal tax cut due to the higher demand for government bonds as a hedge against audit risk. On the other hand, tax evasion reduces aggregate savings (hence capital accumulation and GDP) by boosting individual income expectations (consumption).

Our model predicts that tax audit uncertainty generates sizeable macroeconomic outcomes by influencing agents' consumption-saving (or investment) decisions. To our knowledge, no empirical or theoretical research has yet explored this channel.

To relate our theory to the empirical literature, one can view tax evasion as a source of uncertainty in the effective tax rate, with changes occurring infrequently (audits occur in about 2.5% of cases, according to the U.S. Government Accountability Office) but having a significant impact (fines are about 175% of evaded/under-reported taxes according to Title 26 of the US Internal Revenue Code and the IRS).[1] This association is motivated by evidence of a strong relationship between tax avoidance (defined as the potential loss of tax savings when challenged by tax authorities) and tax uncertainty at the firm level (Dyreng et al., 2019; Guenther et al., 2019).[2]

From this perspective, we connect with the literature showing that tax uncertainty significantly and negatively affects a firm's investment decisions at both the individual (Hassett and Metcalf, 1999) and aggregate levels (Fernández-Villaverde et al., 2015).

[1]Information on the tax audit frequency can be found at *https://www.gao.gov/assets/gao-22-104960.pdf*

[2]A key distinction between tax uncertainty related to evasion and that which is unrelated to evasion is that, in the former case, the degree of exposure to uncertainty is endogenously determined by taxpayers.

Alternatively, we can think of tax evasion as investing in an asset with low-frequency and high-intensity risk. This feature is akin to the so-called "disaster" risk, which has been shown to have substantial macroeconomic effects. On this point, see He et al. (2023) and the literature referred therein.

Outline The paper is organized as follows. Section 2 presents the model and characterizes its competitive equilibrium. Section 3 discusses the main results. Section 4 concludes.

2 Model

Time $t \in [0, \infty)$ is continuous. A mass of competitive enterprises, indexed $i \in [0, 1]$, and the government populate the economy. One capital good (the numéraire) can be consumed or invested. One financial asset, risk-free debt, can be issued by the government.

Preferences and production technology Each enterprise is owned and controlled by one entrepreneur with a net worth $n_{t,i}$. Entrepreneurs have the following CRRA preferences on consumption $c_{t,i}$:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{c_{t,i}^{1-\gamma}}{1-\gamma} dt \right], \tag{1}$$

in which γ and ρ measure RRA and subjective discounting.

Entrepreneurs invest the amounts $b_{t,i}$ and $k_{t,i}$ in bonds and capital production respectively, such that $k_{t,i} + b_{t,i} = n_{t,i}$. Bonds return ($r_t$) is determined in equilibrium. Capital production $y_{t,i}$ has a stochastic dynamics

$$dy_{t,i} = AG_{t,i}^{\beta} k_{t,i}^{1-\beta} dt + k_{t,i} \sigma dW_{t,i},$$

in which public spending is $G_{t,i} = gk_{t,i}$, with $g > 0$, as in Barro (1990).[3] $W_{t,i}$ is a Brownian process scaled by a volatility parameter σ, β is the output elasticity to public spending, and A is the total factor productivity. Shocks are i.i.d. across enterprises, but entrepreneurs cannot diversify them due to incomplete financial markets, as in Gersbach et al. (2023).

The government collects taxes on expected capital income at a fixed rate $\tau \in [0, 1]$.[4] Entrepreneurs can conceal a share $e_{t,i}$ of production, exposing themselves to stochastic audit. Auditing events are mutually independent Poisson processes $(J_{t,i})$ with constant intensity λ, as in Bernasconi et al. (2015). Fines are a fixed share η of evaded income, as in Yitzhaki (1979). Accordingly, individual entrepreneurs' net worth evolves with dynamics

$$dn_{t,i} = \left[r_t n_{t,i} + k_{t,i}\left(Ag^\beta\left(1-\tau+e_{t,i}\tau\right) - r_t\right) - c_{t,i}\right]dt + k_{t,i}\left(\sigma dW_{t,i} - \eta e_{t,i} A g^\beta dJ_{t,i}\right). \quad (2)$$

Entrepreneurs' problem Formally, entrepreneurs choose $\{c_{t,i}, k_{t,i}, e_{t,i}\}$ to maximize (1) subject to (2). As shown in the appendix, entrepreneurs' optimal policy is independent of i. The consumption rate equals:

$$\frac{c^*_{t,i}}{n_{t,i}} = \left(\int_t^\infty e^{\frac{1-\gamma}{\gamma}\int_t^s a_u du} ds\right)^{-1} := h_t, \quad (3)$$

in which

$$a_t := r_t + \frac{1}{2}\frac{\left(\left(1-\tau\right)Ag^\beta - r_t\right)^2}{\gamma\sigma^2} - \frac{\rho + \lambda\left(1 - \left(\frac{\eta\lambda}{\tau}\right)^{\frac{1-\gamma}{\gamma}}\right)}{1-\gamma} - \frac{\tau}{\eta}\left(1 - \left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\gamma}}\right). \quad (4)$$

[3]We interpret public spending as pure public goods (e.g., broadband and mobility infrastructures), benefiting enterprises proportionally to their private activity.

[4]Taxing bonds would be immaterial because the risk-free interest rate would adjust in equilibrium.

Optimal capital and tax evasion shares are:

$$\frac{k^*_{t,i}}{n_{t,i}} = \frac{(1-\tau)\, Ag^{\beta} - r_t}{\gamma\sigma^2}, \tag{5}$$

$$e^*_{t,i} = \frac{\gamma\sigma^2}{Ag^{\beta}\eta} \frac{1-\left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\gamma}}}{\left[(1-\tau)\, Ag^{\beta} - r_t\right]}. \tag{6}$$

Eqs. (3)-(5) are similar to those obtained in a standard consumption-portfolio problem. Tax evasion depends on income and fiscal parameters, as in Bernasconi et al. (2015). Consumption depends on tax evasion via (4). All controls depend on public debt through r_t, which will be determined in equilibrium.

Note that entrepreneurs do not internalize the impact of their decisions on the dynamics of public debt. In other words, they are subject to "fiscal illusion".

Public debt dynamic The government provides public spending, collects taxes, audits evasion, and issues bonds. The overall stock of public debt B_t evolves with the following dynamics:

$$\frac{dB_t}{dt} = r_t B_t + \underbrace{gK_t - \tau \int_{[0,1]} (1-e_{t,i})\, Ag^{\beta} k_{t,i} \cdot di + \eta\lambda \int_{[0,1]} e_{t,i} Ag^{\beta} k_{t,i} \cdot di}_{\text{Primary deficit/surplus}}\,. \tag{7}$$

Eq. (7) is deterministic because the sum of infinitely many iid shocks coincides with its average. Therefore, $\int_{[0,1]} dW_{t,i} \cdot di = 0$ and $\int_{[0,1]} dJ_{t,i} \cdot di = \lambda dt$. Aggregation is straightforward because $k^*_{t,i}/n_{t,i}$ and $e^*_{t,i}$ are independent of i.

2.1 General equilibrium and aggregation

A competitive equilibrium is a set of macroeconomic aggregates (N_t, K_t) and price (r_t) such that entrepreneurs maximise (1), public debt evolves as in (7), and all markets (capital and bonds) clear.

By using (5), the market clearing condition for capital is

$$\underbrace{\int_{[0,1]} k^*_{t,i} \cdot di}_{:=K_t} = \frac{(1-\tau)Ag^\beta - r_t}{\gamma\sigma^2} \cdot \underbrace{\int_{[0,1]} n_{t,i} \cdot di}_{:=N_t}, \tag{8}$$

in which K_t is aggregate capital and N_t denotes aggregate net worth. Accordingly, the equilibrium risk-free rate equals

$$r_t = (1-\tau)Ag^\beta - \frac{K_t}{N_t}\gamma\sigma^2. \tag{9}$$

The market clearing condition for bonds is

$$\underbrace{\int_{[0,1]} b^*_{t,i} \cdot di}_{:=B_t} = \left(1 - \frac{(1-\tau)Ag^\beta - r_t}{\gamma\sigma^2}\right) \cdot \int_{[0,1]} n_{t,i} \cdot di. \tag{10}$$

Matching (8) and (10) yields the aggregate balance sheet condition $N_t = B_t + K_t$.

As for public debt, total tax revenues T_t are deterministic because capital depreciation shocks and auditing events are iid across enterprises. Aggregation is straightforward because entrepreneurs' optimal strategy is linear in net worth and independent of i (see (3)-(6)).

Therefore, aggregate tax revenues have the following law of motion:

$$dT_t = \tau \left(\frac{(1-\tau)Ag^{\beta} - r_t}{\gamma\sigma^2} \right) (1 - e_t^*) \, Ag^{\beta} \cdot \int_{[0,1]} n_{t,i} \cdot di \cdot dt + \\ + \eta e_t^* Ag^{\beta} \frac{(1-\tau)Ag^{\beta} - r_t}{\gamma\sigma^2} \underbrace{\int_{[0,1]} n_{t,i} \cdot dJ_{t,i} \cdot di}_{=N_t \lambda dt}, \quad (11)$$

in which λN_t denotes the mass of audited entrepreneurs.

Equilibrium characterization As shown in the appendix, equilibrium objects can be written as functions of h_t and the public debt-to-GDP ratio

$$x_t := \frac{B_t}{Y_t}, \quad (12)$$

whose motion obeys the following ODE:

$$\frac{d \ln x_t}{dt} = \left[g + Ag^{\beta} \left(x_t h_t - \tau \right) + \left(\frac{\tau}{\eta} - \lambda \right) \left(1 - \left(\frac{\eta\lambda}{\tau} \right)^{\frac{1}{\gamma}} \right) \right] \frac{1 + Ag^{\beta} x_t}{Ag^{\beta} x_t} - \frac{\gamma\sigma^2}{1 + Ag^{\beta} x_t}. \quad (13)$$

By using (12), one can express the ratio between aggregate tax evasion and output as an affine function of x_t:

$$\frac{\int_{[0,1]} e_{t,i}^* Ag^{\beta} k_{t,i} di}{Y_t} = \frac{1}{\eta} \left(\frac{1}{Ag^{\beta}} + x_t \right) \left(1 - \left(\frac{\eta\lambda}{\tau} \right)^{\frac{1}{\gamma}} \right). \quad (14)$$

At first glance, (14) suggests that "financing" tax evasion by using debt rather than raising taxes may generate a self-fuelling mechanism generating additional debt. However, this is not necessarily true because tax evasion affects debt-to-GDP indirectly through entrepreneurs' consumption decisions in equilibrium. As we cannot disentangle these forces in closed form, the next section explores them via simulation.

3 Policy analysis

In this section, we parametrize the model and investigate the feedback between tax evasion and debt-to-GDP via numerical simulations. More specifically, we compare the transition dynamics and the steady state of "equivalent" economies where the effective tax rate is reduced either by allowing tax evasion or by implementing a nominal tax cut without tax evasion. The starting point of all simulations is a "benchmark" economy where:

(1) The tax rate $\tau^* = g^{1-\beta}/A$ is such that $dB_0 = B_0 = x_0 = 0$.

(2) The public good supply $g^* = (A\beta)^{1/(1-\beta)}$ maximises entrepreneurs' value function ("welfare").

(3) Parameters λ and η satisfy $\lambda\eta = \tau^*$, so that optimal tax evasion in (14) is zero.

A detailed derivation of the benchmark economy appears in Appendix A.3.

The first simulation (Policy (A)) considers a reduction in the tax rate τ with zero evasion (i.e. $\lambda\eta = \tau < \tau^*$). The second simulation (Policy (B)) implements an equivalent decrease in audit frequency (i.e. $\lambda < \tau/\eta = \tau^*/\eta$). The value of λ is such that the primary deficit at time zero (dB_0) is the same in both policies.

Parametrization Our exercise uses the parameters gathered in Table 1. Auditing fines η, subjective discount rate ρ, and relative risk aversion γ are in line with Bernasconi et al.

Parameter	Meaning	Value	Source
A	Total factor productivity	0.825	Target
β	Public good elasticity	0.3	Kamps (2006)
λ	Auditing intensity	[0.166-$\eta\tau^*$]	Target
η	Auditing fine	1.3	Bernasconi et al. (2020)
ρ	Subjective discount rate	0.01	Standard
τ	Tax rate	[0.24-τ^*=0.3]	OECD data/target
γ	Relative risk aversion	1.2	Standard
g	Public spending to GDP	0.3 [0.2-0.65]	OECD data
σ	Volatility	0.5 [0.2-1.5]	Herskovic (2016)

Table 1: Baseline parameters

(2020). Consistent with Herskovic (2016), $\sigma = 0.5$ is in the range of reasonable idiosyncratic equity return volatility levels for small- and medium-sized enterprises. The TFP parameter A generates a steady-state risk-free rate of about 2% in the benchmark model.

According to Kamps (2006), output elasticity to public capital across OECD countries ranges between 0.2 and 0.65. Thus, we set $\beta = 0.3$. According to the OECD, total public expenditure to GDP ranges between 0.2 and 0.6.[5] For setting the value of g we use that

$$\frac{G_t}{Y_t} = \frac{g^{1-\beta}}{A}.$$

Choosing $G_t/Y_t = 0.5$ and using $A = 0.825$ and $\beta = 0.3$, we get $g \approx 0.3$.

The auditing intensity parameter ranges from 0.166 (i.e., the tax-evasion-equivalent level to $\tau = 0.24$), and the no-tax evasion level $\lambda^* = \eta\tau$. The tax rate τ ranges between 0.24, which is the average corporate tax rate worldwide in 2022, and the "optimal" level in the benchmark model $\tau^* = 0.3$ (see Appendix A.3 for details).[6]

[5] The data can be found at *https://data.oecd.org/gga/general-government-spending.html*.

[6] Global corporate tax data can be found at *https://stats.oecd.org/index.aspx?DataSetCode=TableII1*.

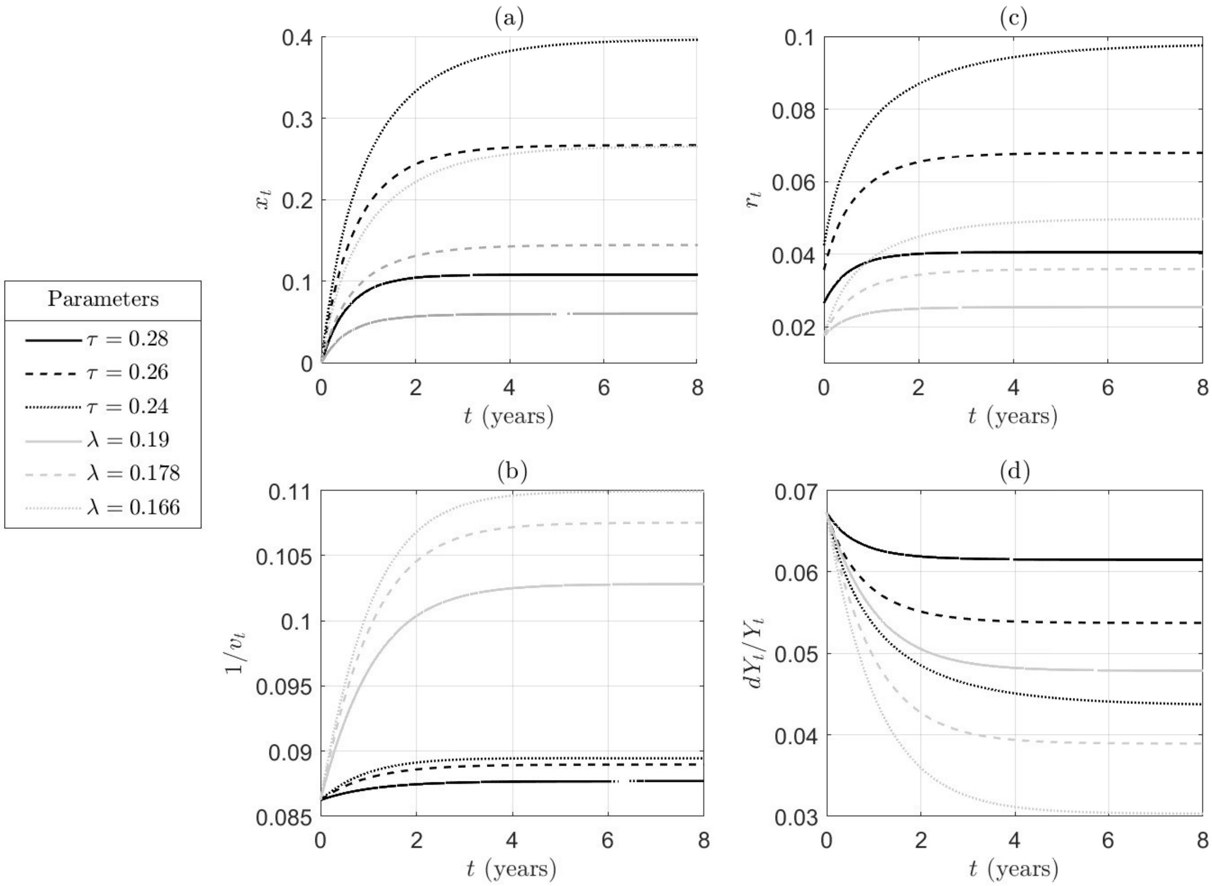

Figure 1: Transition towards the steady state in case of a debt-financed (black) and "evasion-equivalent" (grey) tax cuts for (a) the dynamics of debt-to-GDP, (b) the optimal consumption rate, (c) the risk-free rate, and (d) the GDP growth rate.

Results Figure 1 reports the outcomes of (A) and (B) in black and grey, respectively. Each panel shows the path of (a) debt-to-GDP, (b) optimal consumption rate, (c) risk-free rate, and (d) GDP growth rates. Different types of lines (solid, dashed, and dotted) correspond to varying levels of τ in (A) and the associated λ in (B).

The optimal initial consumption is the same between Policy (A) and (B), but their dynamics differ over time. Notably, long-run risk-free rates are lower when the (effective) tax cut is implemented by allowing tax evasion. This happens because of the higher volatility of disposable income due to evasion risk, fostering bond demand for hedging purposes. Lower risk-free rates explain lower debt-to-GDP, too.

Optimal consumption rates are higher with Policy (B) than with Policy (A). The reason is that entrepreneurs have higher risk-adjusted income expectations in (B). This is not true for log utility because the optimal consumption $c^*_{t,i}/n_{t,i} = \rho$ does not depend on public debt and fiscal parameters. For any other level of risk aversion, GDP growth is higher in (A) than in (B) because the production function is linear. Under both policies, the magnitude of these forces increases with the size of the effective tax cut, as one might expect.

To quantify the magnitude of the relationships among tax evasion, growth, and the risk-free rate under Policies (A) and (B), Table 2 reports their steady-state levels for different (but "equivalent") levels of τ and λ. According to these simulations, a six % point increase in aggregate tax evasion is associated with approximately a 1.2 % point rise in the risk-free interest rate and a 0.8 % point reduction in growth. Notably, the magnitude of the negative relationship between debt service costs (i.e., the risk-free rate) and economic growth is larger with than without tax evasion.

In summary, our results imply that allowing tax evasion to reduce effective tax rates can

	Policy (A)		
	E_{ss}	dY_{ss}/Y_{ss}	r_{ss}
$\tau = 0.28$	-	0.0615	0.0405
$\tau = 0.26$	-	0.0537	0.0680
$\tau = 0.24$	-	0.0434	0.0979
	Policy (B)		
	E_{ss}	dY_{ss}/Y_{ss}	r_{ss}
$\lambda = 0.194$	0.1160	0.0479	0.0254
$\lambda = 0.178$	0.1802	0.0389	0.0359
$\lambda = 0.166$	0.2447	0.0303	0.0497

Table 2: Steady-state levels of aggregate tax evasion (E_{ss}), growth (dY_{ss}/Y_{ss}), and the risk-free rate (r_{ss}) under Policies (A) and (B) for different levels of τ and λ.

generate a lower debt-to-GDP ratio than cutting nominal tax rates by an equivalent amount with no tax evasion. Importantly, this outcome is true only if consumption increments are not too large relative to the corresponding variations in the risk-free rate.[7]

4 Conclusion

Disentangling the forces connecting tax evasion and public debt is challenging because these two variables reinforce each other in equilibrium.

We have developed a general equilibrium dynamic framework to investigate these forces and compared two policy scenarios, starting from a benchmark steady state with no debt. The first policy implements a debt-financed nominal tax cut in the steady state but allows no tax evasion. The second one leaves the nominal tax rate unchanged, implementing an equivalent (effective) tax cut by enabling tax evasion. Counter-intuitively, the latter policy

[7] As shown in the appendix, the growth is decreasing in *both* x_t and h_t. Therefore, we cannot determine the total effect of lower debt-to-GDP and higher consumption rates ex-ante. We have checked that the result described above holds for a range of parameters.

produces lower debt-to-GDP ratios and growth rates than the former (the only exception arises when entrepreneurs have log preferences).

From a policy perspective, we find that more tax evasion relates positively to the level of public debt (and vice versa). However, we demonstrate that allowing tax evasion may increase debt-to-GDP less than equivalent tax cuts by reducing risk-free interest rates (i.e. the service cost of debt) in equilibrium.

Finally, even if tax evasion may reduce the equilibrium debt-to-GDP ratio, it may also worsen its sustainability. We leave the exploration of this phenomenon to future research.

A Appendix

A.1 Entrepreneurs' problem

By standard stochastic control arguments, entrepreneurs' value function H satisfies the following HJBE (omitting subscripts i and t):

$$(\rho+\lambda)H=\frac{\partial H}{\partial t}+\frac{\partial H}{\partial n}rn+\max_{c,\theta,e}\left\{\begin{array}{c}\frac{c^{1-\gamma}}{1-\gamma}+\frac{\partial H}{\partial n}n\left[\theta\left(\left(1-\tau+e\tau\right)Ag^{\beta}-r\right)-\frac{c}{n}\right]+\\ +\frac{1}{2}\frac{\partial^2 H}{\partial n^2}\theta^2n^2\sigma^2+\lambda H\left(n\left(1-e\eta Ag^{\beta}\theta\right)\right)\end{array}\right\}, \quad (15)$$

where $\theta=k/n$ and transversality condition $\lim_{t\to\infty}H_t e^{-(\rho+\lambda)t}=0$. The FOCs are:

$$c\colon\ c^{-\gamma}=\frac{\partial H}{\partial n},$$

$$\theta\colon\ \frac{\partial H}{\partial n}n\left[\left(1-\tau+e\tau\right)Ag^{\beta}-r\right]+\frac{\partial^2 H}{\partial n^2}\theta n^2\sigma^2=e\eta Ag^{\beta}\lambda\frac{\partial H\left(n\left(1-e\eta Ag^{\beta}\theta\right)\right)}{\partial\left(n\left(1-e\eta Ag^{\beta}\theta\right)\right)},$$

$$e\colon \frac{\partial H}{\partial n}\tau = \eta\lambda \frac{\partial H\left(n\left(1 - e\eta A g^{\beta}\theta\right)\right)}{\partial\left(n\left(1 - e\eta R\theta\right)\right)}.$$

To solve the problem, one can guess and verify that

$$H(t,n) = v(t)^{\gamma}\frac{n^{1-\gamma}}{1-\gamma}, \tag{16}$$

where $v(t) = v_t$ is an unknown function of time. By substituting the guess in the FOCs and rearranging, we obtain $\{c^*, k^*, e^*\}$ as they appear in the main text.

To obtain v_t, we substitute the guess function and the optimal policy in the HJBE and rearrange the result to obtain the following ODE:

$$\frac{dv_t}{dt} = v_t a_t \frac{\gamma - 1}{\gamma} - 1,$$

with boundary condition $\lim_{t\to\infty} v_t < \infty$. If $a_t > 0$ and $\gamma > 1$, this equation has a unique solution

$$v_t = \int_t^{\infty} e^{-\frac{\gamma-1}{\gamma}\int_t^s a_u du} ds.$$

Eq. (4) follows suit by setting $h_t \coloneqq 1/v_t$.

A.2 State variable

In the main text, we characterise the competitive equilibrium using the (endogenous) state variable $x_t := B_t/Y_t$. By integrating Eq. (2) over $i \in [0,1]$ and using that $G_{t,i} = gk_{t,i}$, it is straightforward to verify that $Y_t = Ag^{\beta}K_t$. Imposing that $N_t = K_t + B_t$, we can express the equilibrium risk-free rate in (9) as a function of x_t:

$$r(x_t) = Ag^{\beta}(1-\tau) - \frac{\gamma\sigma^2}{1+Ag^{\beta}x_t}. \tag{17}$$

By substituting (17) in (6), we can also express the aggregate tax evasion as a function of x_t (as in the main text).

We now substitute (3)-(6) and (11) in (7) to obtain the dynamics of debt:

$$\frac{dB_t}{dt} = r_tB_t + gK_t - N_tAg^{\beta}\frac{(1-\tau)Ag^{\beta}-r_t}{\gamma\sigma^2}\tau + N_t\left(\frac{\tau}{\eta}-\lambda\right)\left[1-\left(\frac{\tau}{\eta\lambda}\right)^{-\frac{1}{\gamma}}\right], \tag{18}$$

which is deterministic since the dynamics of an infinite number of i.i.d. agents coincides with the expected value of the single agent's dynamics.

The dynamics of entrepreneurs' aggregate net worth N_t is obtained by substituting (3)-(6) in (2), rearranging, and integrating over i:

$$\begin{aligned} dN_t = {} & r_t\underbrace{\int_{[0,1]} n_{t,i}di}_{=N_t}\cdot dt + \frac{\left(Ag^{\beta}(1-\tau+e_t^*\tau)-r_t\right)^2}{\gamma\sigma^2}N_tdt - \frac{N_t}{v_t}dt + \\ & + \frac{(1-\tau)Ag^{\beta}-r_t}{\gamma\sigma^2}\int_{[0,1]} n_{t,i}\sigma dZ_{t,i}\cdot di - Ag^{\beta}e_t^*\eta\frac{(1-\tau)Ag^{\beta}-r_t}{\gamma\sigma^2}N_t\lambda dt. \end{aligned} \tag{19}$$

Next, we differentiate x_t to get

$$\frac{dx_t}{x_t} = \frac{d\left(\frac{B_t}{Y_t}\right)}{\left(\frac{B_t}{Y_t}\right)} = \frac{dB_t}{B_t} - \frac{dY_t}{Y_t}. \tag{20}$$

The former term on the right-hand side of the equation comes from rearranging (18) using the definition of x_t and the market clearing condition $N_t = K_t + B_t$. The latter comes from matching $dK_t = dN_t - dB_t$ with (18) and (19), which yields

$$\frac{dK_t}{K_t} = \frac{dY_t}{Y_t} = Ag^{\beta} - g - \left(1 + Ag^{\beta}x\right) h^{-\frac{1}{\gamma}}. \tag{21}$$

A.3 Benchmark economy

This appendix solves the benchmark model and analytically derives the optimal tax rate and public expenditure τ^* and g^*. We start from $\lambda\eta = \tau$, which implies $e^*_{t,i} = E_t = 0$. Since public debt is zero, $K_t = N_t$ and the risk-free rate is constant $r = (1-\tau)Ag^{\beta} - \gamma\sigma^2$. Substituting these results in (15) yields the following linear ODE:

$$\frac{\partial v}{\partial t} = 1 + \frac{v}{\gamma} \underbrace{\left(\rho + \frac{\gamma\,(1-\gamma)}{2}\sigma^2 - (1-\gamma)\,(1-\tau)\,Ag^{\beta}\right)}_{=\tilde{\rho}(g)}$$

which, under the parametric assumption that $\tilde{\rho} > 0$, has solution

$$v = \left(-\frac{\gamma}{\tilde{\rho}(g)}\right)^{\gamma}.$$

The "optimal" tax rate τ^* is set so that $B_0 = 0$ and remains zero; that is,

$$B_0 = \frac{dB_{t=0}}{dt} = gK_0 - \tau^* Ag^{\beta} K_0 = 0,$$

and thus

$$\tau^* = \frac{g^{1-\beta}}{A}.$$

Substituting τ^* in the above, the level of public spending g^* that maximises welfare satisfies

$$\frac{\partial v}{\partial g} = -\left(1-\gamma\right)\frac{v(g^*)}{\tilde{\rho}(g^*)}\left(A\beta\left(g^*\right)^{\beta-1}-1\right) = 0.$$

Therefore, $g^* = \left(A\beta\right)^{1/(1-\beta)}$ and $\tau^* = \beta$.

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