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Search-Cost-Based Market Segmentation Through Product Design

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Abstract

We study pricing and product design choices in a duopoly model of directed search. Product design determines both the expected valuation and the dispersion of consumers' match values. While greater dispersion increases the option value of search, it also increases expected search duration. Consumers with lower search costs therefore benefit more from products with high dispersion than consumers with higher search costs. When firms choose product designs with different levels of dispersion, consumers sort by search cost in their initial search decisions. Firms thereby gain prominence for different consumer segments, which softens price competition. We show that, in equilibrium, firms choose maximum differentiation.

Keywords: product design, market segmentation, directed search, consumer search, search cost heterogeneity

JEL classification: D43, D83, L15, M31

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1 Introduction

Firms use product design to shape not only how much consumers value their products on average, but also how variable these valuations are. They can make consumer valuations relatively predictable, or create greater variability, allowing for exceptionally good matches but potentially also exposing consumers to poorer ones. Such product-design choices arise in several settings:

First, consider *algorithmic product design.* An e-commerce platform decides which algorithm to implement. A more refined algorithm removes products that are less likely to be suitable, thereby reducing product variability and increasing the expected valuation of the products a consumer inspects. However, platforms implement different levels of precision. For example, in the market for handmade and vintage goods, the two platforms *Etsy* and *Amazon Handmade* provide different shopping experiences. The platform *Etsy* operates a discovery-driven model, targeting consumers who are willing to browse a wider set of products. According to Nick Daniel, Chief Product Officer at Etsy,

> *"[T]oo often, shopping online feels like deja vu. We've all been there: you're searching for a shirt or a lamp and suddenly you find yourself sorting through nearly identical items across the internet that blur into a sea of sameness (...) Etsy offers something fundamentally different. With over 100 million items (...), our inventory is as vast and varied as shoppers' unique interests and tastes"* [Daniel, 2025].

In contrast to Etsy's discovery-driven model, its main competitor, Amazon Handmade, primarily targets consumers who already have a specific product in mind [Darji, 2025].

Second, consider *product assortment design.* A retailer that chooses the degree of variability in its product design can either choose high variability and offer products that are a good fit for some consumers and a bad fit for others, or instead offer a more standardized product design that serves most consumers equally well. For example, Bershka offers a more variable fashion assortment. According to Bershka, *"Everyday wardrobe staples — like denim, tees, hoodies, and basics — get the Bershka treatment through bold cuts, edgy prints, and unexpected detailing"* [Bershka, n.d.]. In contrast, Uniqlo takes a more generic approach to product design through LifeWear, its core clothing concept: *"Our LifeWear clothes are simple. That is because we want to highlight the individuality of wearers, rather than highlight the clothing"* [Fast Retailing, 2024].

Third, consider a firm's *innovation choice*. Incremental innovation preserves familiar product characteristics and keeps consumer fit relatively predictable. Breakthrough innovation changes core characteristics more substantially, increasing the chance that a new product is an exceptional match for some consumers but a poor match for others. For example, in the market for household appliances, Miele and SharkNinja illustrate two distinct innovation strategies: Miele positions itself around reliability, promising *"no compromises when it comes to quality and durability"* [Miele, n.d.], while SharkNinja explicitly emphasizes disruption, describing itself as having *"a proven track record of bringing disruptive innovation to market"* [SharkNinja, 2025].

These applications share the same economic structure: competing firms choose different levels of variability in consumers' match values. A more curated design raises predictability and may increase the average match value by narrowing the distribution of match values. A more exploratory design preserves greater variability, allowing for very good matches but potentially also exposing consumers to poorer ones. We explain this product-design differentiation by focusing on consumers' search costs. Greater variability increases the option value of search because consumers can reject poor realizations and continue searching for a better match. This option value is larger for consumers with low search costs, for whom continued search is less costly, and smaller for consumers with high search costs. In light of this pattern, we show that competing firms strategically choose their product design by setting different levels of variability. These asymmetric designs segment consumers by search cost and soften price competition.

Preview of the Results. The analysis builds on a model of directed search, where consumers observe firms' product designs and aggregate prices before deciding where to search. Firms first choose product design and then compete in prices. We consider two settings. In the main model, firms offer multiple products. We then show that the same logic applies when each firm offers only one product.

We show that search-cost heterogeneity creates endogenous tastes for dispersion in match values. Greater dispersion increases the option value of search, but this option value is more valuable to consumers with low search costs than to consumers with high search costs. Consequently, when firms choose different levels of dispersion, consumers sort by search cost according to a cutoff rule: lower-search-cost consumers start at the firm offering the more dispersed product design, whereas higher-search-cost consumers start at the firm offering the less dispersed product design. In the multiproduct case, consumers who observe a poor match continue searching within the same firm; in the single-product case, continued search entails visiting the rival firm.

The sorting result has direct implications for pricing. Firms value prominence, that is, being visited first. When product designs are symmetric, firms undercut each other's prices in order to become the preferred first firm for all consumers. Asymmetric product designs weaken this undercutting incentive: because consumers sort by search cost, a small price reduction shifts the cutoff search cost rather than inducing all consumers to visit the firm first. This softens price competition. In equilibrium, this force leads firms to choose asymmetric product designs, both in the multiproduct and in the single-product model.

Finally, we characterize which firm benefits more from differentiation. When greater dispersion mainly creates more valuable high match-value realizations, the firm that chooses the more dispersed design earns higher profits. When lower dispersion reflects effective curation and therefore mainly removes poor match-value realizations, the firm that chooses the less dispersed design earns higher profits. These results have practical implications for how firms evaluate design choices in search markets.

Managerial implications. Firms should carefully balance variability and curation based not only on consumer preferences, but also on the competitive environment. In particular, adopting a product design or recommendation strategy that contrasts with those of its competitors can soften price competition and increase profitability, even when it does not maximize average consumer utility.

This strategic logic also affects how firms should interpret performance metrics. An important implication is that firms should not necessarily view lower conversion rates relative to competitors as evidence of an inferior recommendation system or shopping experience. Instead, differences in conversion rates can be indicative of successful search-cost-based market segmentation, where the firm with the lower conversion rate attracts consumers who value exploratory search environments more.

For such segmentation to arise, however, firms must clearly communicate their algorithm, innovation style, or assortment strategy, because consumers' search decisions depend on their expectations about the search environment. Brand positioning, interface design, and recommendation transparency therefore become complements to product design itself. They help consumers understand the degree of variability and curation offered by competing firms before deciding where to search, allowing market segmentation to arise and competition to be softened.

1.1 Plan of the paper

The following subsection provides an overview of the relevant literature. In Section 2, we present the model for multiproduct firms, which we solve by backward induction in the subsequent sections: Section 3 characterizes consumer search, Section 4 studies pricing, and Section 5 studies product design. In Section 6, we consider the case in which both firms offer one product. We conclude in Section 7. Most proofs are collected in the Appendix.

1.2 Related literature

This paper builds on the literature on consumer search for differentiated products and, in particular, on research concerning product design in search markets. Most of the work on product design in search markets relies on random search models.[1] Bar-Isaac et al. [2012]

[1]Models of consumer search can be categorized into three groups. In random search models [Wolinsky, 1986, Bakos, 1997, Anderson and Renault, 1999, Moraga-González et al., 2017], consumers inspect ex ante identical products in random order. In contrast, models of ordered search [Armstrong et al., 2009, Zhou, 2011] assume that consumers are obligated to follow an exogenously determined search order. Arbatskaya [2007] examines ordered price search with homogeneous products. Models of directed search [Choi et al., 2018, Haan et al., 2018] endogenize the search order. Consumers determine which product to inspect first based on observable characteristics such as price.

study the product design choices of single-product firms whose products differ in qualities. They model changes in product design as demand rotations à la Johnson and Myatt [2006] and show that low-quality firms choose the most niche design and high-quality firms choose the broadest design. Bar-Isaac et al. [2012] show that a decrease in (homogeneous) search costs leads to more polarizing product design choices in equilibrium. They argue that this finding constitutes a potential explanation for the empirical observation of the "long tail effect" [Anderson, 2004, Brynjolfsson et al., 2011], which describes the increasing importance of fringe products for sales in digital markets. Larson [2013] studies a similar model without heterogeneous qualities, employing a special form of demand rotations. Depending on the search cost, equilibrium design choices entail only the most niche design, only the broadest design or both extreme designs. Bar-Isaac et al. [2023] propose another framework for product design that allows for the optimality of intermediate design choices and characterize the optimal product design in search markets.[2]

We contribute to the literature by studying the product design choices of both multiproduct firms and single-product firms in a directed search model, highlighting a different role of product design as a tool to segment the market.[3] Under random search, firms set product design to optimally exploit a given set of consumers that inspect their products. Under directed search, the product design choices influence the share and composition of consumers that visit a firm. Our model studies the role of product design as an integral part of a firm's brand reputation that distinguishes it from competitors.

Some of the economic forces that arise in our model share similarities with models of vertical product differentiation, which were pioneered by Gabszewicz and Thisse [1979] and Shaked and Sutton [1982]. In these models, consumers exhibit heterogeneous tastes for

[2]Kuksov [2004] models product design as a binary choice. Different product designs introduce horizontal differentiation between otherwise homogeneous products. Johnen and Leung [2022] study product design in a search model with limited attention. In contrast to classical models of search, consumers face a tradeoff between breadth and depth of search.

[3]To the best of our knowledge, Song [2017] constitutes the only paper that studies directed search based on product design. However, the focus of that paper lies on which search orders can be sustained in equilibrium. Asymmetric choices of product design are given exogenously. Homogeneous search costs, single-product firms and the non-observability of prices constitute other important differences from our model.

quality. Similar to horizontal differentiation, vertical differentiation leads to dispersed utilities among consumers and enables firms to charge higher prices.[4] We show that this mechanism can arise endogenously in a consumer search framework. In the two polar cases discussed above, product design resembles quality, as all consumers prefer to visit the firm with the higher (lower) product variability when prices are equal. The desirability of a more dispersed design decreases with search cost. Consequently, search cost heterogeneity gives rise to heterogeneous tastes for product design.

Our model also relates to the literature on information design in search settings. Most related, Au and Whitmeyer [2023] study the optimal design of match value signals by single-product firms when consumers with homogeneous search costs observe these information policies and direct their search accordingly. Under fixed prices, Au and Whitmeyer [2023] show that firms face a tradeoff between attraction and persuasion. While firms are most likely to be visited by consumers when providing more informative signals ("attraction"), less-precise signals can increase the conversion rate from visit to purchase ("persuasion"). This resembles the trade-off in our model of single-product firms. While more dispersed match values help a firm to attract more consumers initially, consumers that visit the firm offering the higher product variability first are more likely to continue searching.

Lastly, our work relates to oligopoly models of sequential search involving multiproduct firms. While most of the literature on consumer search involves single-product firms, there are papers considering competition between firms selling multiple independent products [Zhou, 2014, Rhodes, 2015, Rhodes and Zhou, 2019] or multiple variants of the same product [Cachon et al., 2008, Zia and Kuksov, 2023]. Our work falls into the latter category and explicitly considers within-firm evaluation costs.[5]

[4] Cremer and Thisse [1991] even show that a large class of Hotelling-type models can be written as a special case of a vertical differentiation model. However, corresponding vertical differentiation models require marginal costs that are increasing in quality. A similar result is thus not obtained for the representation of vertical differentiation models as models of horizontal differentiation.

[5] Liu and Dukes [2013] consider within-firm evaluation costs in a model of simultaneous search.

2 Setting

There are two multiproduct firms, each offering a continuum of products, and a continuum of risk-neutral consumers of mass 1. Consumers have unit demand and the value of their outside option is normalized to zero. Each consumer j's tastes are described by a conditional utility function (net of any search costs) of the form:

$$u_{jki}(p_{ki}) = \epsilon_{jki} - p_{ki}$$

if buying product i from firm k at price p_{ki}. The stochastic match value between consumer j and firm k's product i is denoted by ϵ_{jki}. Match values and prices of specific products are ex ante unknown to consumers and can be discovered through sequential search. While consumers can decide whether to search an additional product of firm 1 or of firm 2 (directed search), they search through the firms' products at random.[6] Consumers are heterogeneous with respect to search costs. Consumer j's search cost per variant s_j is drawn independently from a differentiable cumulative distribution function (CDF) G with support $[\underline{s}, \bar{s}]$. We assume that the corresponding density g is strictly positive and that G and $1 - G$ are both log-concave.[7]

Both firms produce a continuum of products at zero marginal cost. They choose prices and product design for their products. Product design affects the distribution of match values. Specifically, we follow Larson [2013] and assume that match values take the following form:

$$\epsilon_{jki} = \epsilon_{\mu}(\alpha_k) + \alpha_k \cdot z_{jki}$$

Product design is represented by a firm-level parameter α_k, which is chosen by a firm for all of its products from an interval $[\underline{\alpha}, \bar{\alpha}]$, where $\bar{\alpha} > \underline{\alpha} > 0$. There is no cost associated with

[6]The cost of searching for an additional variant at any firm does not depend on whether a consumer has previously inspected products at that firm. While this assumption seems appropriate for online settings, consumers might incur an upfront travel cost when physically visiting a store. As it turns out to be optimal for consumers to inspect products at only one firm, such a cost would not affect our results.

[7]A function that maps a concave set into the positive real numbers is said to be log-concave if the composition of the natural logarithm and that function is concave. See Bagnoli and Bergstrom [2005] for a technical discussion of log-concavity.

product design. The product design parameter affects both the baseline utility $\epsilon_\mu(\alpha_k)$ and the idiosyncratic match value component $\alpha_k \cdot z_{jki}$.

The second term $\alpha_k \cdot z_{jki}$ captures idiosyncratic consumer preferences for specific products. The realization of z_{jki} is assumed to be drawn independently across consumers, firms and products from a CDF F, which is assumed to be continuously differentiable, to have a strictly positive density f and to represent a random variable of mean zero. Its support is $[\underline{z}, \bar{z}]$, where $\bar{z} > 0 > \underline{z}$.

A firm's product design choice affects the dispersion of the consumers' match values. The random component of the match value is scaled by α_k. A higher α_k therefore represents a higher dispersion of match values being offered. In the spirit of Anderson and Renault [1999],[8] the product design choices thus affect the degree of horizontal differentiation between products within a firm.[9]

The baseline utility for the good is $\epsilon_\mu(\alpha_k) > 0$ and assumed to be sufficiently high that no consumer would end up taking the outside option. We also assume that the derivative of $\epsilon_\mu(\alpha_k)$ with respect to α_k exists and that

$$0 \leq -\epsilon'_\mu(\alpha_k) \leq \bar{z}.$$

Hence, the baseline utility is weakly decreasing in α_k. This captures the idea that a more dispersed product assortment might lead to a decrease in the average quality. The assumption also implies that a lower degree of dispersion reduces the set of match values offered and does not result in firms supplying any match values that were unavailable under a more dispersed product design.

We assume that the product design choice and the average price of each firm are observable to consumers.[10] While consumers have to search for the individual characteristics and price of each product, they obtain information at the firm level. This captures the idea that firms

[8]Anderson and Renault [1999] use a scaling parameter that affects the match value distribution of all products. Here, this scaling parameter is firm-specific.

[9]For an in-depth motivation and discussion of this approach to model product design, see Larson [2013].

[10]In the literature on consumer search, it is commonly assumed that consumers do not observe any information on prices prior to search. Under passive beliefs of consumers and multiproduct firms offering a continuum of products, this would, however, place upward pressure on prices in the spirit of the Diamond Paradox.

build a brand reputation, which allows consumers to infer their product design and average price.[11]

The timing of the game is as follows:

1. Firms simultaneously and publicly choose product design $\alpha_k \in [\underline{\alpha}, \bar{\alpha}]$.

2. Firms simultaneously set (non-negative) prices p_{ki} for their products.

3. Consumers observe the average prices of both firms and engage in sequential search.

At the third stage of the game, consumers need to form beliefs about the price distributions set by firms given their average prices. We assume passive beliefs, which means that consumers do not revise their beliefs based on the prices they come across. Henceforth, 'equilibrium' refers to a perfect Bayesian equilibrium with passive beliefs.

In Section 4, we show that in equilibrium a firm sets the same price for each of its products at the second stage of the game. In the following section, we solve for the optimal consumer search strategy on-path when consumers believe in symmetric pricing, i.e., that each firm sets a degenerate price distribution. Consumer search under more general beliefs is discussed in the Appendix. Furthermore, we assume that even absent price dispersion the gains of search are high enough for each product design such that no consumer would always buy the first product she visits. This is formalized by the following assumption:[12]

$$\underline{\alpha} \cdot \int_{\underline{z}}^{\bar{z}} (z - \underline{z}) \, dF(z) \geq \bar{s}$$

3 Consumer search

Consider a consumer with search cost $s \in [\underline{s}, \bar{s}]$. In principle, she could search products of both firms in any order. However, it turns out that it is optimal to exclusively search for products from a single firm. This follows from the optimal search rule of Weitzman [1979], which prescribes that consumers should search through products in the order determined by their

[11] A similar assumption is made by Ke et al. [2023]. They assume that consumers observe only the average location of a firm's products on a Hotelling line.

[12] We will later assume that z_{jki} is uniformly distributed on $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Then, this assumption corresponds to $\bar{s} \leq \frac{1}{2} \cdot \underline{\alpha}$.

reservation prices. Since products from a given firm are ex ante identical, their reservation prices are identical as well. If one of the firms offers products with a higher reservation price, the consumer should therefore search through all of its products first. As the firm offers a continuum of products, the consumer never visits the other firm. The optimal rule of search can be characterized in two steps. First, the optimal within-firm search is characterized. Second, reservation prices are compared across firms.

Suppose that the consumer searches only among the products of firm k, which chooses design α_k and charges price p_k for all its products. The benefit of search consists of the chance of getting a higher draw from distribution F and thus a higher match value. As shown by Kohn and Shavell [1974], optimal search is determined by a threshold value. We define this threshold value relative to the match value distribution F and call it the reservation draw r_k.[13] It is determined by the following equation:

$$\alpha_k \cdot \int_{r_k}^{\bar{z}} (z - r_k)\, dF(z) = s \tag{1}$$

At the reservation draw, the benefits of an additional search through a potentially higher match value equal the search costs. Whenever the consumer obtains a higher realization from F than the reservation draw, she stops searching and buys. Otherwise, she continues to search.

The reservation draw depends on s and α_k. Using the implicit function theorem on (1), the following comparative statics can be obtained:

$$\frac{\partial r_k}{\partial \alpha_k} = \frac{s}{\alpha_k^2 \cdot (1 - F(r_k))} > 0 \qquad\qquad \frac{\partial r_k}{\partial s} = -\frac{1}{\alpha_k \cdot (1 - F(r_k))} < 0$$

The reservation draw is decreasing in s and increasing in α_k. Consumers are "pickier" when facing a lower search cost and when the firm offers a more dispersed product design. The latter implies that consumers, on average, search more products as α_k increases.

Given the optimal within-firm search rule, the expected consumer surplus CS_k of searching products from firm k can be calculated. Using (1), this corresponds to the reservation price

[13] The reservation price of a product is a strictly increasing linear transformation of its reservation draw.

of each of the firm's products:

$$CS_k = \underbrace{\epsilon_\mu(\alpha_k) + \alpha_k \cdot \left(r_k + \frac{\int_{r_k}^{\bar{z}} (z - r_k)\, dF(z)}{1 - F(r_k)} \right)}_{\text{expected match value}}$$

$$- \underbrace{\frac{s}{1 - F(r_k)}}_{\substack{\text{expected total}\\ \text{search cost}}} - p_k = \underbrace{\epsilon_\mu(\alpha_k) + \alpha_k \cdot r_k - p_k}_{\text{reservation price}}$$

The consumer ranks firms according to this measure and visits only the firm with the higher surplus. She keeps searching until she discovers a good enough match. A firm's demand is equal to the share of consumers that visit that firm.

The comparative statics of consumer surplus with respect to α_k are as follows:

$$\frac{\partial CS_k}{\partial \alpha_k} = \epsilon'_\mu(\alpha_k) + \frac{\int_{r_k}^{\bar{z}} z\, dF(z)}{1 - F(r_k)}$$

The derivative shows that a change in α_k has two opposing effects on consumer surplus. The baseline utility is weakly decreasing in α_k, capturing the idea that more dispersion might negatively affect the baseline utility. The second term is increasing in α_k: The benefit of search is driven by the chance of obtaining a high match value. Because an increase in α_k implies a higher degree of dispersion, the option value of finding a higher match value increases.[14]

While all consumers prefer more dispersion, holding the baseline utility fixed, the increment in expected utility depends on their search cost:

$$\frac{\partial^2 CS_k}{\partial \alpha_k \partial s} = -\frac{f(r_k) \cdot s}{\alpha_k^2 \cdot (1 - F(r_k))^3} < 0 \tag{2}$$

This result shows how different product designs can introduce product differentiation by inducing consumers with low search costs to value dispersed product designs more strongly than consumers with high search costs. The reason is intuitive due to the comparative statics

[14] Note that the second term corresponds to the derivative with respect to $\alpha_k \cdot r_k$:

$$\begin{aligned} \frac{\partial \alpha_k \cdot r_k}{\partial \alpha_k} &= r_k + \alpha_k \cdot \frac{\partial r_k}{\partial \alpha_k} r_k + \frac{s}{\alpha_k \cdot (1 - F(r_k))} \\ &= r_k + \frac{\int_{r_k}^{\bar{z}} (z - r_k)\, dF(z)}{1 - F(r_k)} = \frac{\int_{r_k}^{\bar{z}} z\, dF(z)}{1 - F(r_k)} > 0 \end{aligned}$$

The third equality follows from (1) and the inequality arises due to the fact that F has mean zero.

of the reservation draw with respect to product design. As α_k increases, consumers become "pickier" and search more products on average. An increased duration of search is less harmful for consumers with a lower search cost.

4 Pricing

We first show that, in equilibrium, firms set the same price for each of their products. Consumers can therefore infer the price of each product from the observable average prices.[15]

Lemma 1. *Any (pure-strategy) equilibrium involves symmetric pricing of both firms.*

Proof: See Appendix.

There is a simple intuition behind this result: Regardless of their beliefs, which are only affected by changes in the average price, consumers would be more likely to buy products with lower price realizations when a firm sets a non-degenerate price distribution. It would always be optimal for this firm to deviate to price distributions that eliminate some of the dispersion in prices while keeping the mean price constant and obtain a higher average purchase price.

The same reasoning establishes that symmetric pricing is optimal for any symmetric pricing equilibrium candidate. While the number of consumers who visit a firm (and eventually buy one of its products) solely depends on the observable average price, any pricing scheme involving price dispersion would lead to a lower average purchase price. In this spirit, we assume that consumers also believe in symmetric pricing when observing an off-path mean price.[16] It then suffices to check for deviations to non-degenerate price distributions, as these capture the most profitable deviation for any off-path mean price.

[15] The assumption that consumers only observe average prices remains crucial, even though there is no within-firm price dispersion in equilibrium. If consumers observed the entire price distribution, deviations to non-degenerate price distributions would affect the share of consumers visiting a given firm.

[16] Alternatively, the timing of the game could be modified. Firms could set prices in two stages: first, they set their average price. Second, they determine their price distribution conditional on its mean. In the spirit of Nocke and Rey [2024], each combination of average prices would then give rise to a proper subgame of incomplete information. Correct beliefs in symmetric pricing would be necessary for an equilibrium in each subgame.

In order to obtain tractable conditions for equilibrium existence and uniqueness, from now on, we assume that F follows a uniform distribution with support $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Also, we assume that $\epsilon_\mu(\alpha_k) = \epsilon - c \cdot \alpha_k$, where $c \in [0, 1/2)$. Using (1), explicit expressions for the reservation draw and the expected consumer surplus can be derived as functions of the search cost:

$$r_k(s) = \frac{1}{2} - \sqrt{2 \cdot \frac{s}{\alpha_k}} \qquad CS_k(s) = \epsilon_\mu - c\alpha_k + \frac{\alpha_k}{2} - \sqrt{2s\alpha_k} - p_k$$

We define the net consumer surplus (NCS) to capture the consumer surplus net of the price:

$$NCS_k(s) = \epsilon + \alpha_k \frac{1-2c}{2} - \sqrt{2s\alpha_k}$$

If firms set the same product design ($\alpha_1 = \alpha_2$), products are ex ante homogeneous. Firms engage in Bertrand competition and price at marginal cost. In the following, we solve for the pricing equilibrium with asymmetric product design. Without loss of generality, we assume that $\alpha_1 > \alpha_2$. The following lemma characterizing demand follows from the discussion in Section 3.

Lemma 2. *There exists a unique $\hat{s} \in [\underline{s}, \bar{s}]$ such that consumers visit*

- *firm 1 if $s < \hat{s}$ and*
- *firm 2 if $s > \hat{s}$.*

It holds that $\hat{s} = \bar{s}$ if $NCS_1(\bar{s}) - NCS_2(\bar{s}) \geq p_1 - p_2$ and $\hat{s} = \underline{s}$ if $NCS_1(\underline{s}) - NCS_2(\underline{s}) \leq p_1 - p_2$. Otherwise, $\hat{s} \in (\underline{s}, \bar{s})$ and solves $NCS_1(\hat{s}) - NCS_2(\hat{s}) = p_1 - p_2$.

Proof: Follows from the fact that $NCS_1(s) - NCS_2(s)$ is continuous and strictly decreasing in s, which is implied by (2).

The heterogeneous taste for product design leads to a selection of consumers according to their search costs. If $\hat{s}$ solves $NCS_1(\hat{s}) - NCS_2(\hat{s}) = p_1 - p_2$, there is a threshold search cost at which a consumer is exactly indifferent between visiting firm 1 and firm 2. Consumers with a lower (resp. higher) search cost have a stronger (resp. weaker) preference for a more dispersed design and buy at firm 1 (resp. 2).

Remark 1. *We define (α_1, α_2) to be vertically differentiated if at equal prices, all consumers visit the same firm. This holds true with firm 1 providing the high-quality product design if $NCS_1(\bar{s}) \geq NCS_2(\bar{s})$. Conversely, if $NCS_2(\underline{s}) \geq NCS_1(\underline{s})$, product designs are vertically differentiated and firm 2 provides the high-quality product design.*

In the first case, all consumers prefer to visit firm 1 at equal prices, because for every consumer the higher option value of search outweighs the lower baseline utility. Note that this always holds for $c = 0$, since a change in α only increases the dispersion while leaving the baseline utility unchanged. Conversely, in the second case, all consumers prefer to visit firm 2 at equal prices because for every consumer the higher baseline utility outweighs the lower option value of search. This always holds for $c \to 1/2$, as a decrease in α induces a first-order stochastic dominance shift in the distribution of match values, which is preferred by all consumers at equal prices. If neither condition is met, neither design is preferred by all consumers at equal prices. In this case, the interaction between average match value and option value generates differentiation with a horizontal component, because consumers with different search costs rank the two product designs differently.

Using Lemma 2, the profit functions of both firms can be derived:

$$\pi_1(p_1, p_2) = p_1 \cdot G\big(\hat{s}(p_1, p_2)\big) \qquad \pi_2(p_1, p_2) = p_2 \cdot \Big(1 - G\big(\hat{s}(p_1, p_2)\big)\Big)$$

The following Lemma shows that it is optimal for firms to set prices such that consumers are indeed indifferent at the threshold search cost:

Lemma 3. *Given the price of firm 2 (resp. 1), firm 1 (resp. 2) is weakly better off setting a price such that*

$$NCS_1(\hat{s}) - NCS_2(\hat{s}) = p_1 - p_2 \tag{3}$$

Proof: See Appendix.

Under this restriction, there is a one-to-one mapping between prices that could be optimal for a firm and the resulting threshold search costs given the price of the other firm. The pricing problem of each firm can be rewritten into an equivalent formulation, in which firms choose demand via determining the threshold search cost:[17]

$$1\colon \max_{\hat{s}\in[\underline{s},\bar{s}]} \big(p_2 + NCS_1(\hat{s}) - NCS_2(\hat{s})\big)\cdot G(\hat{s})$$

$$2\colon \max_{\hat{s}\in[\underline{s},\bar{s}]} \big(p_1 - NCS_1(\hat{s}) + NCS_2(\hat{s})\big)\cdot \big(1 - G(\hat{s})\big)$$

We first focus on interior equilibria with $\hat{s} \in (\underline{s}, \bar{s})$. The first-order conditions can be solved for the equilibrium prices depending on the implicitly defined threshold search cost $\hat{s}^*$:

$$p_1^* = (\sqrt{\alpha_1} - \sqrt{\alpha_2})\cdot\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{G(\hat{s}^*)}{g(\hat{s}^*)}$$

$$p_2^* = (\sqrt{\alpha_1} - \sqrt{\alpha_2})\cdot\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{1-G(\hat{s}^*)}{g(\hat{s}^*)}$$

Using these equilibrium prices, the necessary condition (3) takes the following form:

$$\underbrace{(\sqrt{\alpha_1} - \sqrt{\alpha_2})\cdot\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{2G(\hat{s}^*)-1}{g(\hat{s}^*)}}_{\Delta p} = \underbrace{(\alpha_1 - \alpha_2)\cdot\frac{1-2c}{2} - (\sqrt{\alpha_1} - \sqrt{\alpha_2})\cdot\sqrt{2\hat{s}^*}}_{NCS_1(\hat{s}^*)-NCS_2(\hat{s}^*)} \tag{4}$$

Condition (4) can be simplified to:

$$\frac{2G(\hat{s}^*)-1}{g(\hat{s}^*)} = (1-2c)\cdot(\sqrt{\alpha_1} + \sqrt{\alpha_2})\cdot\frac{\sqrt{\hat{s}^*}}{\sqrt{2}} - 2\hat{s}^* \tag{5}$$

The LHS of this equation is weakly increasing in s due to the log-concavity of G and $1 - G$. If the RHS is strictly decreasing in s for $s \geq \underline{s}$, there is at most one candidate for an interior equilibrium. The slope of the RHS equals $(\sqrt{\alpha_1} + \sqrt{\alpha_2})\cdot\frac{1-2c}{2^{1.5}\cdot\sqrt{s}} - 2$. A sufficient condition for the uniqueness of the interior equilibrium candidate in any subgame is given by $(1-2c)^2\cdot\bar{\alpha} < 8\cdot\underline{s}$.

With this condition, the existence of an interior equilibrium candidate is determined by two simple criteria. There is an interior equilibrium candidate if and only if (i) the RHS of

[17] Strictly speaking, a restriction of the support of firm 2's action set would be needed. Given a low p_1, setting low threshold search costs can involve setting a negative price.

equation (5) exceeds its LHS when both are evaluated at $\underline{s}$ and (ii) the LHS of equation (5) exceeds its RHS when both are evaluated at $\bar{s}$. This holds true under the following conditions:

$$(1-2c)\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right) > -\frac{\sqrt{2}}{g(\underline{s})\cdot\sqrt{\underline{s}}} + 2^{3/2}\cdot\sqrt{\underline{s}} \tag{6}$$

and

$$(1-2c)\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right) < \frac{\sqrt{2}}{g(\bar{s})\cdot\sqrt{\bar{s}}} + 2^{3/2}\times\sqrt{\bar{s}} \tag{7}$$

If either condition fails to hold, there is no equilibrium in which both firms earn strictly positive profits. Nevertheless, an equilibrium exists in which one firm earns zero profits. Whether this applies to the firm offering the more or less dispersed product design depends on which condition is violated. In the following, we refer to those equilibria as corner pricing equilibria.

If condition (6) is violated, the LHS in equation (5) always exceeds the RHS. In this case, an equilibrium requires that $p_1 = 0$. Otherwise, if $p_1 > 0$, either firm 2 could set a higher price and obtain the entire demand or firm 1 could set a lower price and make positive profits. Firm 2 solves the following optimization problem:

$$\max_{\hat{s}\in[\underline{s},\bar{s}]} \left(NCS_2(\hat{s}) - NCS_1(\hat{s})\right)\cdot(1-G(\hat{s})) \tag{8}$$

A corner pricing equilibrium obtains if the objective function is maximized at $\underline{s}$. A necessary condition is local optimality, i.e., the derivative of the objective function evaluated at $\underline{s}$ must be weakly negative. Notably, this condition is satisfied if and only if (6) is violated.

Conversely, if condition (7) is violated, the RHS in equation (5) always exceeds the LHS. In this case, an equilibrium requires that $p_2 = 0$. Firm 1 solves the following optimization problem:

$$\max_{\hat{s}\in[\underline{s},\bar{s}]} \left(NCS_1(\hat{s}) - NCS_2(\hat{s})\right)\cdot G(\hat{s}) \tag{9}$$

In this case, a corner pricing equilibrium is obtained if the objective function is maximized at $\bar{s}$. Again, local optimality is ensured as the derivative of the objective function evaluated at $\bar{s}$ is weakly positive if and only if (7) is violated.

Note that a necessary condition for a corner equilibrium is that the products are vertically differentiated. Only in this case can one firm serve the entire market while charging a higher price. Which of the two corner pricing equilibria might arise depends on the parameter c. If

c is sufficiently large, firm 2 may serve the entire market, as even the lowest search cost type $\underline{s}$ prefers firm 2's portfolio due to its higher baseline utility. Conversely, if c is sufficiently small, a corner equilibrium may arise in which even the highest search cost type $\bar{s}$ prefers firm 1's product design due to the option value of a more dispersed portfolio.

One might have anticipated that a corner equilibrium would occur if the product design choices are sufficiently differentiated. Instead, the term $\sqrt{\alpha_1} + \sqrt{\alpha_2}$ determines whether a corner equilibrium occurs. A necessary condition for firm 2 to serve the whole market is that $\sqrt{\alpha_1} + \sqrt{\alpha_2}$ is sufficiently small. In this case, visiting firm 2 yields a high net consumer surplus, and at the same time, firm 2 is not sufficiently differentiated to poach high-search-cost types through a low price. In the same vein, a necessary condition for firm 1 to serve the whole market is that $\sqrt{\alpha_1} + \sqrt{\alpha_2}$ is sufficiently large. In this case, visiting firm 1 yields a high net consumer surplus and firm 2 might not be sufficiently differentiated to poach high search types through a low price.

Under the formal specification, interior and corner equilibria are mutually exclusive and there is exactly one equilibrium candidate for each subgame. Using additional sufficient conditions, which are derived in the Appendix, the existence of a unique (pure-strategy) equilibrium can be proven. The following Proposition summarizes the results of the pricing stage when $\alpha_1 > \alpha_2$.

Proposition 1. *Suppose that* $(1-2c)\sqrt{\bar{\alpha}} < 2\sqrt{2\underline{s}} - \frac{1}{\sqrt{2\underline{s}}}\frac{1}{g(\underline{s})}$.

- *If (6) and (7) hold, there is an interior equilibrium with* $\hat{s}^* \in (\underline{s}, \bar{s})$*, where* $\hat{s}^*$ *is implicitly defined by (5). Equilibrium prices are characterized by*

$$p_1^* = \frac{\sqrt{\alpha_1} - \sqrt{\alpha_2}}{\sqrt{2\hat{s^*}}} \times \frac{G(\hat{s}^*)}{g(\hat{s}^*)}, \; p_2^* = \frac{\sqrt{\alpha_1} - \sqrt{\alpha_2}}{\sqrt{2\hat{s^*}}} \times \frac{1 - G(\hat{s}^*)}{g(\hat{s}^*)}.$$

- *Otherwise, there is a corner pricing equilibrium:*

  – *If (6) is violated,* $\hat{s}^* = \underline{s}$, $p_2^* = NCS_2(\underline{s}) - NCS_1(\underline{s})$, $p_1^* = 0$.

  – *If (7) is violated,* $\hat{s}^* = \bar{s}$, $p_1^* = NCS_1(\bar{s}) - NCS_2(\bar{s})$, $p_2^* = 0$.

Proof: See Appendix.

5 Product design

Section 4 determines equilibrium payoffs of the pricing subgame. Comparative statics of the equilibrium profits in an interior pricing equilibrium yield the following Lemma.[18]

Lemma 4. *While firm 1's interior pricing equilibrium profits are strictly increasing in α_1, firm 2's interior pricing equilibrium profits are strictly decreasing in α_2. Thus, any equilibrium that gives rise to an interior pricing equilibrium requires that one firm sets $\bar{\alpha}$ and the other firm sets $\underline{\alpha}$.*

Proof: See Appendix.

This result shows that asymmetric product design choices, which formally introduce differentiation, constitute a source of market power. Although a firm may face a competitive disadvantage when adopting a differentiated product design, it benefits from reduced competition, leading to higher equilibrium prices. It follows from Lemma 4 that there is a unique pair of equilibria under a restriction on c and the support of product designs $[\underline{\alpha}, \bar{\alpha}]$. First, condition (6) must hold when evaluated at $\bar{\alpha}$ and $\underline{\alpha}$:

$$(1-2c)\left(\sqrt{\bar{\alpha}}+\sqrt{\underline{\alpha}}\right) > -\frac{\sqrt{2}}{g(\underline{s})\cdot\sqrt{\underline{s}}} + 2^{3/2}\cdot\sqrt{\underline{s}}. \tag{10}$$

Then, there cannot exist an equilibrium that leads to a corner solution in which firm 1 earns zero profits. By setting $\alpha_1 = \bar{\alpha}$, firm 1 can always ensure that condition (10) holds and thus earn positive profits.

Second, also condition (7) must hold when evaluated at $\bar{\alpha}$ and $\underline{\alpha}$:

$$(1-2c)\left(\sqrt{\bar{\alpha}}+\sqrt{\underline{\alpha}}\right) < \frac{\sqrt{2}}{g(\bar{s})\cdot\sqrt{\bar{s}}} + 2^{3/2}\times\sqrt{\bar{s}} \tag{11}$$

Under this condition, there cannot exist an equilibrium that leads to a corner solution in which firm 2 earns zero profits. By setting $\alpha_2 = \underline{\alpha}$, firm 2 can always ensure that condition (11) holds and thus earn positive profits. Hence, if conditions (10) and (11) both hold, no firm can unilaterally enforce a corner pricing equilibrium. By Lemma 4, in equilibrium, one

[18] Throughout this section, we maintain the notation that $\alpha_1 > \alpha_2$. Firm 1 (resp. 2) generally refers to the firm with the more dispersed (resp. less dispersed) product design in equilibrium.

firm opts for the most dispersed design $\bar{\alpha}$ while the other firm selects the least dispersed product design $\underline{\alpha}$.

If either condition (10) or (11) is violated, maximal differentiation leads to a corner pricing equilibrium. In this case, any equilibrium requires that one firm adopts the product design that maximizes net consumer surplus for all search types (hereafter, the superior product design). If condition (10) is violated, this corresponds to $\underline{\alpha}$ and if condition (11) is violated, this corresponds to $\bar{\alpha}$. Otherwise, by Proposition 1, the firm serving the entire market could increase its profits by deviating to the superior product design. Lemma 5 summarizes this observation.

Lemma 5. *If (10) is violated, in any equilibrium, at least one firm sets the least dispersed product design $\underline{\alpha}$. If (11) is violated, then in any equilibrium, at least one firm sets the most dispersed product design $\bar{\alpha}$.*

Proof: See Appendix.

If condition (10) or (11) is violated, and the rival firm chooses the superior product design, any product design yields zero profits and is therefore a best response. However, for an equilibrium to exist, choosing the superior product design must also constitute a best response. We have shown that choosing the superior product design yields higher profits than any alternative when $\alpha_1 > \alpha_2$. However, a deviation to a product design that induces an interior equilibrium with $\alpha_2 > \alpha_1$ could, in principle, be profitable. The associated demand reduction may be offset by gains from increased differentiation.

We derive conditions under which such deviations are unprofitable. In particular, this holds when the rival's product design is either sufficiently high or sufficiently low.

To see this, suppose first that condition (11) is violated. In this case, $\alpha_1 = \bar{\alpha}$ must be optimal given α_2^*. According to Lemma 4, the most profitable deviation with $\alpha_1 < \alpha_2$ is to set $\alpha_1 = \underline{\alpha}$. If α_2^* exceeds a certain threshold, deviating to $\underline{\alpha}$ leads to a corner pricing equilibrium in which firm 1 earns zero profits. Conversely, if α_2^* is sufficiently small, deviating to the most dispersed product design results in a pricing outcome close to Bertrand competition. Consequently, there exists a continuum of equilibria, including the special cases of Bertrand competition ($\alpha_1^* = \bar{\alpha}, \alpha_2^* = \bar{\alpha}$) and maximum differentiation ($\alpha_1^* = \bar{\alpha}, \alpha_2^* = \underline{\alpha}$). The conditions under which $\alpha_2 = \underline{\alpha}$ is optimal when (10) is violated are analogous. A potential deviation from $\underline{\alpha}$ is to choose the most dispersed product design, $\bar{\alpha}$. As in the previous case, such a deviation is not profitable if firm 1's product design α_1^* is either sufficiently large or sufficiently small.

While consumer surplus is maximized when both firms set the superior product design and compete à la Bertrand, the most profitable corner pricing equilibrium is obtained under maximum differentiation. The following Proposition summarizes the equilibrium outcomes at the product design stage.

Proposition 2. *The equilibrium product design choices can be characterized as follows:*

- *If (10) and (11) hold: $\alpha_1^* = \bar{\alpha}$, $\alpha_2^* = \underline{\alpha}$. An interior pricing equilibrium is obtained.*
- *If (10) does not hold: $\alpha_2^* = \underline{\alpha}$, $\alpha_1^* \in [\underline{\alpha}, \bar{\alpha}]$ such that $\underline{\alpha} \in BR_2(\alpha_1^*)$. A corner pricing equilibrium is obtained.*
- *If (11) does not hold: $\alpha_1^* = \bar{\alpha}$, $\alpha_2^* \in [\underline{\alpha}, \bar{\alpha}]$ such that $\bar{\alpha} \in BR_1(\alpha_2^*)$. A corner pricing equilibrium is obtained.*

The best responses $\underline{\alpha} \in BR_2(\alpha_1^*)$ and $\bar{\alpha} \in BR_1(\alpha_2^*)$ are characterized in the Appendix.

Proof: See Appendix.

Having characterized the equilibrium for any parameter values c, $\underline{\alpha}$, and $\bar{\alpha}$, we compare the firms' equilibrium profits depending on c. To this end, define the threshold

$$\tilde{c} = \frac{1}{2} - \frac{\sqrt{2med(G)}}{\sqrt{\underline{\alpha}} + \sqrt{\bar{\alpha}}} \in (0, 1/2),$$

where $\tilde{c} > 0$ follows from the uniqueness condition $(1-2c)^2 \cdot \bar{\alpha} < 8\underline{s}$.

Corollary 1. *In equilibrium, firm 1 makes higher profits than firm 2 if $c < \tilde{c}$. Conversely, firm 2 makes higher profits than firm 1 if $c > \tilde{c}$.*

Proof: See Appendix.

Corollary 1 highlights the two channels that determine which firm is better off. The parameter c governs how strongly baseline utility declines as product dispersion increases. When c is low, most consumers prefer higher dispersion despite a modest reduction in baseline utility, so firm 1 is better off.

Conversely, when c is high, the opposite holds: a highly dispersed product design entails a substantial loss in baseline utility, leading more consumers to favor a firm offering higher baseline utility over greater dispersion. This includes the case of *algorithmic product design*, where reduced dispersion arises from the exclusion of poor match values.

At $c = \tilde{c}$, these effects exactly offset each other, and the median consumer is indifferent between the two firms at equal prices. Consequently, consumers split evenly between firms, and equilibrium prices and profits are symmetric.

6 Single-product model

In the main part of the paper, we have shown that asymmetric product design choices lead to market power. The underlying mechanism is that consumers differ in their search costs and, consequently, in how much they value a dispersed product assortment. Heterogeneity in product design therefore generates differentiation between firms and relaxes price competition.

In the baseline model, we assume that firms offer a continuum of products, implying that consumers never visit both firms. If firms instead offer a finite number of products, some consumers only observe bad matches and might decide to visit the second firm. In that case, even under symmetric product designs, firms can ensure positive profits because the firm setting the higher price still serves consumers who visit both firms. This raises the question of whether our results depend on the result that with a continuum of products, consumers never visit both firms.

In the following extension, we demonstrate that this is not the case. We consider the same setup but assume that each firm offers a single product. Product design refers to the choice of the ex ante distribution of match values. We show that when firms can choose product designs that affect both baseline utility and the dispersion of match values, they optimally differentiate, as in the multiproduct model. The intuition closely parallels that of the main model. Following Weitzman [1979], consumers optimally inspect the product with the higher reservation price first. Introducing variation in dispersion affects consumers' reservation prices asymmetrically: the reservation price increases more strongly for consumers with lower search costs. As a result, an asymmetric product design segments the market and softens price competition.

As in the multiproduct model, consumer utility is

$$\epsilon_{jk} = \epsilon - c \cdot \alpha_k + \alpha_k \cdot z_k,$$

where the random match value z_k is uniformly distributed on $[-1, 1]$. In contrast to the main model, each firm offers only one product.

The reservation draw for firm j when starting search at firm k can be shown to be

$$r_k(s) = \begin{cases} \frac{\alpha_j + p_k - p_j - c\cdot(\alpha_j - \alpha_k) - 2\sqrt{\alpha_j s}}{\alpha_k} & \text{if } s \leq \alpha_j \\ \frac{p_k - p_j - c\cdot(\alpha_j - \alpha_k) - s}{\alpha_k} & \text{else.} \end{cases}$$

For $z_k < r_k(s)$, a search type s optimally continues to search. Note that $r_k(s)$ is decreasing in s. Consumers become pickier when their search cost are low and inspect the second product with a higher probability. Define the reservation price as the utility from consuming the match value $r_k(s)$:

$$\tau_k(s) = \begin{cases} \alpha_j - c \cdot \alpha_j - p_j - 2\sqrt{\alpha_j s} & \text{if } s \leq \alpha_j \\ -c \cdot \alpha_j - p_j - s & \text{else.} \end{cases}$$

According to Weitzman [1979], consumers inspect the product with the higher reservation price first. Therefore, having pinned down the reservation prices allows us to characterize the search order. Note that for $s < \alpha_j$,

$$\frac{\partial \tau_k(s)}{\partial \alpha_j \partial s} = -\frac{1}{2\sqrt{\alpha_j s}} < 0. \tag{12}$$

This result mirrors equation (2) in the multiproduct case and indicates that search-cost-based market segmentation extends to the single-product case. The effect of a change in α_j through an increased option value is weaker for consumers with higher search costs.

For convenience, we introduce $\Delta p \equiv p_1 - p_2$ and $\Delta\alpha \equiv \alpha_1 - \alpha_2$. WLOG, we assume that $\Delta\alpha \geq 0$. Moreover, denote by $\Delta \equiv \Delta p + c \cdot \Delta\alpha$ the expected utility difference between the two products. The following Lemma characterizes the search order depending on the search cost.

Lemma 6. *For $\Delta \leq 0$, all consumers visit firm 1 first. For $\Delta > \Delta\alpha$, all consumers visit firm 2 first. For $\Delta \in (0, \Delta\alpha]$, there exists a search type $\hat{s} \in [0, \alpha_1)$ that is indifferent which product to inspect first. It is described by*

$$\hat{s} = \begin{cases} \left(\sqrt{\alpha_1} - \sqrt{\Delta}\right)^2 & \text{if } \Delta \in [0, \left(\sqrt{\alpha_1} - \sqrt{\alpha_2}\right)^2) \\ \frac{1}{4}\left(\frac{\Delta\alpha - \Delta}{\sqrt{\alpha_1} - \sqrt{\alpha_2}}\right)^2 & \text{if } \Delta \in [\left(\sqrt{\alpha_1} - \sqrt{\alpha_2}\right)^2, \Delta\alpha]. \end{cases}$$

In this region, consumers visit firm 1 *first if $s < \hat{s}$ and firm* 2 *first otherwise.*

Proof: See Appendix.

From equation (12), the difference in reservation prices between firm 1 and firm 2 is decreasing in s. Hence, there exists a unique cutoff such that consumers with search costs below the cutoff visit firm 1 first, while those above it visit firm 2.

In order to get a better intuition, consider $\Delta \in (0, \Delta\alpha]$, where an indifferent consumer exists. When deciding where to start, a consumer considers the expected utility from purchasing the first product and the expected surplus from continued search under the optimal search rule. The first component is independent of the search type and larger when starting at firm 2. In contrast, the second component is larger when starting with firm 1 and decreases with search costs. Inspecting the more dispersed product first resolves more uncertainty, thereby making it clearer in which states of the world continued search is worthwhile. For the indifferent consumer, both effects balance each other out.

Having solved the consumers' search behavior, we now turn to the firms' pricing and product design decisions. We begin by considering the case in which both firms choose the same product design α. In the symmetric subgame, no equilibrium in pure strategies exists. The reason is that each firm has an incentive to slightly undercut its rival's price in order to attract all consumers to visit its firm first. However, if $\underline{s} < \alpha$, competition for the first visit does not fully dissipate profits, as a strictly positive fraction of consumers proceeds to inspect the second product after observing a sufficiently low match value at the first firm. Consequently, firms retain some market power, and equilibrium profits remain strictly positive. In the following Lemma, we describe an upper bound for the firms' equilibrium profits if firms are symmetric.

Lemma 7. *Consider the case $\alpha_1 = \alpha_2 = \alpha$ and assume that s is distributed according to the CDF $G(s)$, where $\underline{s} < \alpha$. There is no pure-strategy Nash equilibrium. However, there exists a symmetric Nash equilibrium in mixed strategies with*

$$\pi_1 = \pi_2 \leq \frac{G(\alpha)}{2} \left(\alpha - E[s|s < \alpha]\right).$$

Proof: See Appendix.

We can show that in a symmetric mixed strategy NE, the highest price that is played must be below α. Given that a firm sets the highest price, demand only consists of consumers that have visited the rival firm's product before and is therefore less than 1/2.

When firms set different product designs, the discontinuity in the firms' profit functions at $\Delta = 0$ vanishes. In the next Lemma, we provide sufficient conditions for a unique Nash equilibrium in pure strategies to exist. In order to obtain a tractable solution, we assume that

$$s \sim \begin{cases} \bar{s} \text{ with probability } \beta \\ \underline{s} \text{ with probability } 1-\beta, \end{cases}$$

where $\underline{s} \in (1/4\alpha_2, \alpha_2]$ and $\bar{s} \geq \alpha_1$. In this case, we can state clear conditions that ensure existence and uniqueness of a Nash equilibrium in pure strategies.

Lemma 8. *Suppose that $\alpha_1 > 20\alpha_2$ and $c \in [0, \alpha_2/\Delta\alpha]$. There exists a unique pure strategy Nash equilibrium for*

$$\beta \leq \hat{\beta}(\kappa) = \frac{4\kappa^2 - 3\sqrt{9\kappa^2 + 2\kappa + 9} + \kappa + 9}{2(2\kappa^2 + \kappa - 1)},$$

where $\kappa = (\underline{s} - c \cdot \Delta\alpha)/\alpha_1$. The corresponding equilibrium profits are

$$\pi_1 = \frac{1-\beta}{18\alpha_1} \left(\frac{3-\beta}{1-\beta}\alpha_1 + \underline{s} - c \cdot \Delta\alpha\right)^2 \text{ and } \pi_2 = \frac{1-\beta}{18\alpha_1} \left(\frac{3+\beta}{1-\beta}\alpha_1 - \underline{s} + c \cdot \Delta\alpha\right)^2.$$

Proof: See Appendix.

Consumers with high search costs always purchase firm 2's product that provides a higher expected utility. In contrast, consumers with lower search costs first inspect the more dispersed product despite its lower expected utility, and visit the second firm with positive probability. Because firms are highly differentiated, neither finds it profitable to attract additional consumers to inspect its product first by lowering prices. As a result, the equilibrium corresponds to an equilibrium in which firms take the consumers' search order as given. This equilibrium only holds if the share of high-cost types is sufficiently small. Otherwise, the same undercutting incentive as in the symmetric case prevails and the pure strategy equilibrium breaks down.[19]

If the conditions for a pure strategy NE in the asymmetric subgame are met, firms differentiate in the product design stage.

Proposition 3. *Consider the product design stage, where firms can choose from $\alpha \in \{\underline{\alpha}, \bar{\alpha}\}$, where $\bar{\alpha} > 20\underline{\alpha}$. Moreover, assume that $c \in [0, \underline{\alpha}/(\bar{\alpha} - \underline{\alpha})]$. For $\beta \leq \hat{\beta}((\underline{s} - c \cdot (\bar{\alpha} - \underline{\alpha}))/\bar{\alpha})$, firms choose different product designs in equilibrium.*

Proof: See Appendix.

Both firms prefer the asymmetric equilibrium to the symmetric equilibrium that would result from adopting the rival's design. The reason is that both firms benefit from consumers visiting them first and, in contrast to the symmetric equilibrium, this surplus is not competed away.

Corollary 2 shows that, as in the main model, there exists a cutoff value c that determines which firm makes higher profits.

Corollary 2. *Under the parameter specification, firm 1 makes higher profits if $c < \underline{s}/\Delta\alpha$. Conversely, firm 2 makes higher profits if $c > \underline{s}/\Delta\alpha$.*

[19] In this case, a mixed-strategy equilibrium exists (see Dasgupta and Maskin [1986]). However, since the equilibrium is generally asymmetric, we cannot derive a sufficiently tight upper bound that would permit a meaningful comparison with profits in the symmetric subgame.

Proof: See Appendix.

The intuition closely resembles that of the main model. The parameter c governs how strongly baseline utility declines with α. For sufficiently high c, firm 1's advantage in serving low search-cost consumers first is outweighed by the disadvantage of offering a product with lower baseline valuation. For sufficiently low c, the opposite holds. At $c = \underline{s}/\Delta\alpha$, the two effects exactly offset each other, and both firms earn identical profits.

7 Conclusion

Our framework provides a rationale for several real-world observations. Retailers choose different degrees of variability in their product assortments, e-commerce platforms vary in the precision of their recommendation algorithms, and firms pursue innovation strategies ranging from incremental improvements to radical breakthroughs. We explain why firms systematically adopt such different design strategies: When consumers face heterogeneous search costs, their ability to find a suitable product depends on those costs. We show that this heterogeneity creates scope for differentiation and thereby reduces competition.

Our analysis gives rise to additional applications. For instance, choosing the degree of dispersion may be interpreted as a firm's information-provision decision. Suppose consumers receive only a noisy signal about their match with a given product. A retailer can influence the precision of this signal by providing information about product attributes. More precise information enables consumers to assess products more accurately, thereby increasing the dispersion in match values. Finally, we believe that heterogeneity in search costs may also be used to segment the market through other choice variables. Consider, for example, two retailers that choose the number of products they offer, while product variability is taken as given. Suppose that consumers visit only one store due to travel costs. In this setting, firms may differentiate in the size of their assortments: consumers with lower search costs value a larger number of products within a store relatively more, for the same reason as in the present model. For them, the option value of continued search is greater. Therefore, retailers could induce search-cost-based market segmentation by offering assortments of different sizes.

A Consumer search under general beliefs

Even when consumers do not believe in symmetric pricing, each firm's products are ex ante identical to them. As in Section 3, consumers only visit the firm whose products have a higher reservation price.

Consider a consumer with search cost $s \in [\underline{s}, \bar{s}]$, which believes that firm k sets prices according to some CDF $L_k(.)$ with support $[\underline{p}, \bar{p}]$, where $\underline{p} \geq 0$.[20] Firm k's reservation price y_k is determined by the following equation:

$$\int_{\underline{z}}^{\bar{z}} \int_{\underline{p}}^{\bar{p}} \mathbb{1}\{\epsilon_\mu(\alpha_k) + \alpha_k \cdot z - p \geq y_k\} \cdot (\epsilon_\mu(\alpha_k) + \alpha_k \cdot z - p - y_k) \, dL_k(p) \, dF(z) = s$$

Consumers only search for products of the firm with the higher y_k. They stop if and only if the net utility of a product (weakly) exceeds y_k. Two immediate observations are useful for the following analysis. First, the reservation price y_k is strictly decreasing in s. Second, lowering the price of a particular product strictly increases the probability that a consumer buys that product. This follows from the fact that f is strictly positive and implies that consumers on average buy at a strictly lower price than the average price of firm k unless $L_k(.)$ is degenerate.

Furthermore, a useful lemma can be established:

Lemma 9. *Fix any product design $\alpha_k \in [\underline{\alpha}, \bar{\alpha}]$ and mean price $p_\mu > 0$ for firm k. Then, a consumer's reservation price y_k is higher under any non-degenerate price distribution than under symmetric pricing.*

Proof: The distribution of net utilities[21] under any form of price dispersion is a mean-preserving spread of the distribution of net utilities under symmetric pricing. It thus suffices to show that a consumer's incremental benefit function[22] would be strictly lower for any outside option y when facing a distribution of net match values H_1 than when facing a distribution of net match values H_2 which constitutes a mean preserving spread of H_1. Suppose that random variable X is distributed according to H_1 and that random variable Z

[20] The same argument holds for a CDF with support $[\underline{p}, \infty)$.

[21] The net utility of a product corresponds to its match value minus its price.

[22] As in Dogan and Hu [2022], we define the incremental benefit function as each consumer's incremental gain from one more search with an outside option of y at hand.

is distributed according to H_2. Then, there is some random variable ϵ from a conditional probability distribution $K(.|x)$ such that $Z = X + \epsilon$. Define $f(v) = \mathbb{1}\{v \geq y\} \cdot (v - y)$ and note that $f(.)$ is a convex function. For any $y \in \mathbb{R}$, Jensen's inequality can be used to show that:

$$\int_{-\infty}^{\infty} f(z)dH_2(z) = \int_{-\infty}^{\infty} \mathbb{E}[f(x+\epsilon)|x]dH_1(x) \geq \int_{-\infty}^{\infty} f(\mathbb{E}[x+\epsilon|x])dH_1(x) = \int_{-\infty}^{\infty} f(x)dH_1(x)$$

B Proofs of the Main Results

Proof of Lemma 1

Suppose for a contradiction that there is an equilibrium candidate where at least one firm k sets different prices for its products according to a non-degenerate CDF L_k with mean $p_\mu > 0$ and support $[\underline{p}, \bar{p}]$, where $\underline{p} \geq 0$.

Suppose first that only one firm sets a non-degenerate price distribution and obtains no demand in equilibrium.[23] We index this firm with k and the other firm with $-k$. Then, there exists a search cost $s \in [\underline{s}, \bar{s}]$ such that a consumer with search cost s is indifferent between searching products of the two firms. Otherwise, firm $-k$ could profitably deviate by slightly increasing its price while still obtaining the entire demand.[24] Firm k could thus profitably deviate by setting an arbitrarily small, but strictly positive price p'_k, which is smaller than p_μ, for all of its products and capture a positive share of consumers.[25]

Suppose now that firm k sets a non-degenerate price distribution and obtains a positive share of consumers in equilibrium. Note from the discussion on consumer search under general beliefs that any consumer of firm k on average pays a price of less than p_μ and that a consumer's reservation price y_k is strictly decreasing in her search cost. Denote by $\underline{s}_k$ the

[23] This implies that the other firm employs symmetric pricing and obtains the entire demand.

[24] This hold true independent of consumers' off-path beliefs about price dispersion at firm $-k$. If consumers believed that firm $-k$ would set a non-degenerate price distribution when observing an off-path average price, this would not lead to a lower reservation price of firm $-k$ than under symmetric pricing due to Lemma 9.

[25] Consumers' reservation price is minimized when they believe in symmetric pricing at p'_k. The deviation price p'_k can however be chosen low enough such that consumers are better off – even under symmetric pricing – than under any non-degenerate price distribution $L_k(.)$ with mean $p_\mu > 0$.

infimum of the set of $s \in [\underline{s}, \bar{s}]$ such that consumers with search cost s buy at firm k in equilibrium. Denote by $\underline{y}_k$ the reservation price of firm k's products for a consumer with search cost $\underline{s}_k$. Define $\hat{p}$ as the lowest price at which a consumer with search cost $\underline{s}_k$ would buy according to her optimal rule of search when facing a product the highest possible match value of $\epsilon_\mu(\alpha_k) + \alpha_k \cdot \bar{\epsilon}$:

$$\hat{p} = \epsilon_\mu(\alpha_k) + \alpha_k \cdot \bar{\epsilon} - \underline{y}_k$$

Suppose first that $\hat{p} \geq p_\mu$. Then, firm k could profitably deviate to symmetric pricing at p_μ. As the average price remains constant, demand would not be affected. Suppose now that $\hat{p} < p_\mu$ and that there is no mass on prices strictly below $\hat{p}$, i.e., that $\lim_{p \to \hat{p}^-} L_k(p) = 0$. Then, the expected gains from an additional search when facing a product with a match value of $\epsilon_\mu(\alpha_k) + \alpha_k \cdot \bar{\epsilon}$ and a price of $\hat{p}$ equal zero. This implies that $\underline{y}_k < \epsilon_\mu(\alpha_k) + \alpha_k \cdot \bar{\epsilon} - \hat{p}$ which contradicts the definition of $\hat{p}$. Lastly, suppose that $\hat{p} < p_\mu$ and that there is a positive mass on prices strictly below $\hat{p}$, i.e., that $\lim_{p \to \hat{p}^-} L_k(p) > 0$. Then, define distribution $L'_k(.)$ as follows:

$$L'_k(p) = \begin{cases} 0, & \text{if } p \in [0, \hat{p}), \\ L_k(p), & \text{if } p \geq \hat{p}, \end{cases}$$

Under $L'_k(.)$, all mass below $\hat{p}$ is shifted to a mass point at $\hat{p}$. Denote by $p'_\mu > p_\mu$ the mean of that distribution. Define $\tilde{p}$ such that:

$$\int_{\tilde{p}}^{\infty} (p - \tilde{p}) dL_k(p) = p'_\mu - p_\mu$$

Clearly $\tilde{p} > p_\mu$, as $\hat{p} < p_\mu$. Define $L''_k(.)$ as follows:

$$L''_k(p) = \begin{cases} 0, & \text{if } p \in [0, \hat{p}), \\ L_k(p), & \text{if } p \in [\hat{p}, \tilde{p}), \\ 1, & \text{if } p \geq \tilde{p}, \end{cases}$$

Under $L''_k(.)$, the entire mass above $\tilde{p}$ is shifted to a mass point on $\tilde{p}$ in order to compensate for the increase in average price through $L'_k(.)$. It is straightforward to see that the mean of $L''_k(.)$ equals p_μ. Firm k could profitably deviate to setting prices according to $L''_k(.)$. As the average price remains constant, consumers would not adjust their search behavior. They would however buy at a higher average price under $L''_k(.)$. Switching from $L_k(.)$ to $L'_k(.)$ would increase the average purchase price conditional on consumer's equilibrium rule of search by more than $p'_\mu - p_\mu$.[26] This is due to the fact that consumers are more likely to purchase at lower prices. By the same reasoning, switching from $L'_k(.)$ to $L''_k(.)$ would decrease the average purchase price conditional on consumer's equilibrium rule of search by less than $p'_\mu - p_\mu$.

Proof of Lemma 3

Suppose that firm 2 sets price $p_2 \geq 0$. If firm 1 sets $p_1 \in [NCS_1(\bar{s}) - NCS_2(\bar{s}) + p_2, NCS_1(\underline{s}) - NCS_2(\underline{s}) + p_2]$, condition (3) holds. Suppose that $p_1 < NCS_1(\bar{s}) - NCS_2(\bar{s}) + p_2$. Then firm 1 could deviate to $p_1 = NCS_1(\bar{s}) - NCS_2(\bar{s}) + p_2$ and sell the same quantity at a higher price. Suppose that $p_1 > NCS_1(\underline{s}) - NCS_2(\underline{s}) + p_2$. Then firm 1 obtains zero profits, which is the lowest possible profit and is thus weakly better off to set any other price.[27] The symmetric argument applies to firm 2.

Proof of Proposition 1

In the main body of the paper, we have already shown that there is a unique equilibrium candidate for each subgame. We now derive a sufficient condition under which this candidate actually constitutes an equilibrium. For this purpose, we establish log-concavity of the profit functions.

Suppose first that (7) does not hold. The unique equilibrium candidate constitutes a corner-pricing equilibrium in which firm 1 serves all consumers. Clearly, it is optimal for firm 2 to set $p_2 = 0$. Any higher price would also generate zero demand and lead to zero profits. Given firm 2's price, firm 1 solves the optimization problem given by (9). Taking logs and

[26]There are three arguments to consider here. First, the probability to obtain a purchase price smaller or equal than $\hat{p}$ under $L_k(.)$ is higher than $L_k(\hat{p})$. Second, the expected purchase price conditional on obtaining a purchase price smaller or equal than $\hat{p}$ increases by more than $\frac{p'_\mu - p_\mu}{L_k(\hat{p})}$ when switching to $L'_k(.)$. Third, the probability to obtain a higher purchase price than $\hat{p}$ is higher under $L'_k(.)$.

[27]For firm 1, it is straightforward to show a strictly profitable deviation here. However, this argument would not apply to firm 2.

differentiating twice with respect to s yields:

$$\underbrace{\frac{1}{\left(\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{\sqrt{2}}\cdot\sqrt{s}-2s\right)^2}}_{>0}\cdot\left(\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{2^{1.5}\cdot\sqrt{s}}-2\right)+\underbrace{\frac{g'(s)\cdot G(s)-g(s)^2}{G(s)^2}}_{\leq 0 \text{ due to log-concavity of } G}$$

Thus, the profit function is strictly log-concave if $\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{2^{1.5}\cdot\sqrt{s}}-2<0$ for all $s\in[\underline{s},\bar{s}]$. This holds true for all subgames if $\bar{\alpha}(1-2c)^2<8\cdot\underline{s}$.

Suppose now that (7) holds. In this situation, two scenarios can arise. Either, (6) also holds and we are in an interior pricing equilibrium, or (6) does not hold and the unique equilibrium candidate constitutes a corner-pricing equilibrium in which firm 2 serves all consumers and firm 1 sets $p_1=0$. In the following, we cover both cases simultaneously.

Given p_2^*, firm 1 solves the following problem:

$$\max_{\hat{s}\in[\underline{s},\bar{s}]}\left(p_2^*+NCS_1(\hat{s})-NCS_2(\hat{s})\right)\cdot G(\hat{s})$$

Taking logs, differentiating twice and plugging in for p_2^* yields:

$$\begin{aligned}&\underbrace{\frac{1}{\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{1-G(\hat{s}^*)}{g(\hat{s}^*)}+\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{2}-\sqrt{2s}}\cdot\frac{1}{2s}}_{>0 \text{ for } p_1=p_2^*+NCS_1(s)-NCS_2(s)>0}\\&\cdot\left(\frac{1}{\sqrt{2s}}-\frac{1}{\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{1-G(\hat{s}^*)}{g(\hat{s}^*)}+\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{2}-\sqrt{2s}}\right)\\&+\underbrace{\frac{g'(s)G(s)-g(s)^2}{G(s)^2}}_{\leq 0 \text{ due to log-concavity of } G}.\end{aligned}$$

According to Lemma 3, firm 1's profit-maximizing price can be rewritten to $p_1=p_2^*+NCS_1(s)-NCS_2(s)$. Note that WLOG, we can restrict attention to positive prices (as negative prices are always weakly dominated by setting $p_1=0$). This is useful, because if $p_1=p_2^*+NCS_1(s)-NCS_2(s)$ is positive, we get the condition that the profit function is strictly log-concave if $\frac{1}{\sqrt{2s}}-\frac{1}{\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{1-G(\hat{s}^*)}{g(\hat{s}^*)}+\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)\cdot\frac{1-2c}{2}-\sqrt{2s}}<0$ for all $s\in[\underline{s},\bar{s}]$ and $\hat{s}^*\in[\underline{s},\bar{s}]$. This holds true for all subgames if $(1-2c)\sqrt{\bar{\alpha}}<2\sqrt{2\underline{s}}-\frac{1}{\sqrt{2\underline{s}}}\cdot\frac{1}{g(\underline{s})}$. Note that this condition implies that $(1-2c)^2\bar{\alpha}<8\cdot\underline{s}$.

For firm 2, consider the pricing representation of the profit maximization problem. Given p_1^*, firm 2 solves:

$$\max_{p_2\geq 0}p_2\cdot\left(1-G(\hat{s}(p_1^*,p_2))\right)$$

Lemma 3 shows that firm 2 is (weakly) better off setting a price such that $NCS_1(\hat{s}) - NCS_2(\hat{s}) = p_1 - p_2$. Using that restriction, we can explicitly solve this equation for $\hat{s}$: $\hat{s} = \frac{1}{2} \cdot \left((\sqrt{\alpha_1} + \sqrt{\alpha_2}) \cdot \frac{1-2c}{2} - \frac{p_1^* - p_2}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} \right)^2$. It turns out that $\hat{s}$ is strictly increasing and strictly convex in p_2:

$$\frac{\partial \hat{s}}{\partial p_2} = \left((\sqrt{\alpha_1} + \sqrt{\alpha_2}) \cdot \frac{1-2c}{2} - \frac{p_1^* - p_2}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} \right) \cdot \frac{1}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} > 0$$

$$\frac{\partial^2 \hat{s}}{\partial p_2^2} = \frac{1}{(\sqrt{\alpha_1} - \sqrt{\alpha_2})^2} > 0$$

The SOC of the logarithm of the profit function of firm 2 takes the following form:

$$-\frac{1}{p_2^2} - \frac{g(\hat{s})}{1 - G(\hat{s})} \cdot \frac{\partial^2 \hat{s}}{\partial p_2^2} - \underbrace{\frac{g'(\hat{s})\big(1 - G(\hat{s})\big) + g(\hat{s})^2}{\big(1 - G(\hat{s})\big)^2}}_{> 0 \text{ due to log-concavity of } 1-G} \cdot \left(\frac{\partial \hat{s}}{p_2} \right)^2 < 0$$

Thus, the profit function is strictly log-concave.

Proof of Lemma 4

The comparative statics of $\hat{s}^*$ with respect to α_1 and α_2 are useful for the proof. Applying the implicit function theorem to (5) yields:

$$\frac{\partial \hat{s}^*}{\partial \alpha_k} = \frac{\frac{1}{2^{1.5}} \cdot \frac{1}{\sqrt{\alpha_k}} \cdot \sqrt{\hat{s}^*}(1-2c)}{\frac{2 \cdot g(\hat{s}^*)^2 - (2 \cdot G(\hat{s}^*) - 1) \cdot g'(\hat{s}^*)}{g(\hat{s}^*)^2} + 2 - \frac{\sqrt{\alpha_1} + \sqrt{\alpha_2}}{2^{1.5} \cdot \sqrt{\hat{s}^*}}(1-2c)} > 0 \text{ for } k \in \{1, 2\}$$

where the denominator is greater than zero due to the restriction on $\bar{\alpha}$ and the log-concavity of G and $1 - G$. The equilibrium threshold search cost is increasing in α_1 and α_2.

It follows that the interior equilibrium profits of firm 2 ($\pi_2^i(\alpha_1, \alpha_2)$) are strictly decreasing in α_2 (for $\alpha_2 < \alpha_1$):

$$
\begin{aligned}
\frac{\partial \pi_2^i}{\partial \alpha_2} &= \frac{\partial \ (\sqrt{\alpha_1} - \sqrt{\alpha_2}) \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{\left(1-G(\hat{s}^*)\right)^2}{g(\hat{s}^*)}}{\partial \alpha_2} \\
&= -\frac{1}{2\sqrt{\alpha_2}} \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{\left(1-G(\hat{s}^*)\right)^2}{g(\hat{s}^*)} \\
&\quad - (\sqrt{\alpha_1} - \sqrt{\alpha_2}) \cdot \frac{1}{(2\hat{s}^*)^{1.5}} \cdot \frac{\partial \hat{s}^*}{\partial \alpha_2} \cdot \frac{\left(1-G(\hat{s}^*)\right)^2}{g(\hat{s}^*)} \\
&\quad - (\sqrt{\alpha_1} - \sqrt{\alpha_2}) \frac{1}{\sqrt{2\hat{s}^*}} \\
&\quad \cdot \underbrace{\frac{2 \cdot \left(1-G(\hat{s}^*)\right) \cdot g(\hat{s}^*)^2 + g'(\hat{s}^*) \cdot \left(1-G(\hat{s}^*)\right)^2}{g(\hat{s}^*)^2}}_{\geq 0 \text{ due to log-concavity of } 1-G} \cdot \frac{\partial \hat{s}^*}{\partial \alpha_2} \\
&< 0
\end{aligned}
$$

We now turn to the interior equilibrium profits of firm 1 ($\pi_1^i(\alpha_1, \alpha_2)$). Taking the first derivative of the logarithmized profits yields:

$$
\begin{aligned}
\frac{\partial \log \pi_1^i}{\partial \alpha_1} &= \frac{\partial \ \log(\sqrt{\alpha_1} - \sqrt{\alpha_2}) - \log(\sqrt{2\hat{s}^*}) + \log\left(\frac{G(\hat{s}^*)^2}{g(\hat{s}^*)}\right)}{\partial \alpha_1} \\
&= \frac{1}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\alpha_1}} \ - \ \frac{1}{2\hat{s}^*} \cdot \frac{\partial \hat{s}^*}{\partial \alpha_1} \ + \ \underbrace{\frac{2g(\hat{s}^*)^2 - g'(\hat{s}^*) \cdot G(\hat{s}^*)}{g(\hat{s}^*) \cdot G(\hat{s}^*)}}_{\geq 0 \text{ due to log-concavity of } G} \cdot \frac{\partial \hat{s}^*}{\partial \alpha_1}
\end{aligned}
$$

A sufficient condition for strictly increasing profits is given by:

$$
\frac{1}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\alpha_1}} > \frac{1}{2\hat{s}^*} \cdot \frac{\partial \hat{s}^*}{\partial \alpha_1} \tag{13}
$$

It follows from log-concavity of G and $1 - G$ that:

$$
\frac{\partial \hat{s}^*}{\partial \alpha_1} = \frac{\frac{1}{2^{1.5}} \cdot \frac{1}{\sqrt{\alpha_1}} \cdot \sqrt{\hat{s}^*}(1-2c)}{\frac{2 \cdot g(\hat{s}^*)^2 - (2 \cdot G(\hat{s}^*) - 1) \cdot g'(\hat{s}^*)}{g(\hat{s}^*)^2} + 2 - \frac{\sqrt{\alpha_1} + \sqrt{\alpha_2}}{2^{1.5} \cdot \sqrt{\hat{s}^*}}(1-2c)} \leq \frac{\frac{1}{2^{1.5}} \cdot \frac{1}{\sqrt{\alpha_1}} \cdot \sqrt{\hat{s}^*}}{\frac{2}{1-2c} - \frac{\sqrt{\alpha_1} + \sqrt{\alpha_2}}{2^{1.5} \cdot \sqrt{\hat{s}^*}}}
$$

A sufficient condition for (13) is thus:

$$
\frac{1}{\sqrt{\alpha_1} - \sqrt{\alpha_2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\alpha_1}} > \frac{1}{2\hat{s}^*} \cdot \frac{\frac{1}{2^{1.5}} \cdot \frac{1}{\sqrt{\alpha_1}} \cdot \sqrt{\hat{s}^*}}{\frac{2}{1-2} - \frac{\sqrt{\alpha_1} + \sqrt{\alpha_2}}{2^{1.5} \cdot \sqrt{\hat{s}^*}}}
$$

This can be simplified to:

$$
2\sqrt{2\hat{s}^*} > (1-2c)\sqrt{\alpha_1}.
$$

The condition is implied by $(1-2c)\sqrt{\bar{\alpha}} < 2\sqrt{2\underline{s}} - \frac{1}{\sqrt{2\underline{s}}} \cdot \frac{1}{g(\underline{s})}$

In any equilibrium that leads to an interior pricing equilibrium and does not satisfy the conditions of the Lemma, there would thus be a profitable deviation for one of the two firms.

Proof of Lemma 5

First consider that condition (10) is violated. Consider any equilibrium candidate (α_1, α_2) such that $\min(\alpha_1, \alpha_2) \neq \underline{\alpha}$. If $\alpha_1 = \alpha_2$, both firms make zero profits and could profitably deviate to $\underline{\alpha}$. From now on, suppose without loss of generality that $\alpha_1 > \alpha_2$. Consider (α_1, α_2) such that condition (6) or condition (7) are violated. Then, there is a corner pricing equilibrium and one firm makes zero profits. This firm could profitably deviate to $\underline{\alpha}$ and get the entire demand at a strictly positive price. Now consider (α_1, α_2) such that condition (6) and (7) hold, yielding an interior pricing equilibrium. In this case, there exists $\alpha' < \alpha_2$ such that (α_1, α') still satisfies condition (6). According to Lemma 4, firm 2 should profitably deviate to α'.

Second, consider that condition (11) is violated. Consider any equilibrium candidate (α_1, α_2) such that $\max(\alpha_1, \alpha_2) \neq \bar{\alpha}$. If $\alpha_1 = \alpha_2$, both firms make zero profits and could profitably deviate to $\bar{\alpha}$. From now on, suppose without loss of generality that $\alpha_1 > \alpha_2$. Consider (α_1, α_2) such that condition (6) or condition (7) are violated. Then, there is a corner pricing equilibrium and one firm makes zero profits. This firm could profitably deviate to $\bar{\alpha}$ and get the entire demand at a strictly positive price. Now consider (α_1, α_2) such that condition (6) and (7) hold, yielding an interior pricing equilibrium. In this case, there exists $\alpha' > \alpha_1$ such that (α', α_2) still satisfies condition (7). According to Lemma 4, firm 1 should profitably deviate to α'.

Proof of Proposition 2

Suppose that conditions (10) and (11) hold. Then, the equilibrium characterization follows directly from Lemma 4. Suppose now that condition (11) does not hold. By Lemma 5, firm 1 sets $\bar{\alpha}$ in any equilibrium. Any $\alpha_2 \in [\underline{\alpha}, \bar{\alpha}]$ yields zero profits and constitutes a best response to $\bar{\alpha}$. It thus remains to be shown that $\bar{\alpha}$ constitutes a best response to $\alpha_2 \in [\underline{\alpha}, \bar{\alpha}]$ under the conditions of Proposition 2.

Fix any $\alpha_2 \in [\underline{\alpha}, \bar{\alpha}]$. First, consider

$$\alpha_2 \geq \tilde{\alpha}_2 \equiv \max\Big\{\frac{1}{(1-2c)^2}\big(\frac{\sqrt{2}}{g(\bar{s})\sqrt{\bar{s}}} + 2\sqrt{2}\sqrt{\bar{s}} - \sqrt{\underline{\alpha}}\big)^2, \underline{\alpha}\Big\}$$

This implies that any $\alpha_1 \in [\underline{\alpha}, \bar{\alpha}]$ leads to a corner pricing equilibrium. Since $\alpha_1 \leq \alpha_2$ yields zero profits for firm 1 and firm 1's corner pricing equilibrium profits are strictly increasing in α_1 for $\alpha_1 > \alpha_2$, setting $\bar{\alpha}$ constitutes the unique best response for firm 1.

Consider now $\alpha_2 < \tilde{\alpha}$. Define $\hat{\alpha}_1$ such that, given α_2, condition (7) does not hold true when $\alpha_1 \geq \hat{\alpha}_1$, that is:

$$\hat{\alpha}_1 = \frac{1}{(1-2c)^2}\Big(\frac{\sqrt{2}}{g(\bar{s}) \cdot \sqrt{\bar{s}}} + 2 \cdot \sqrt{2} \cdot \sqrt{\bar{s}} - \sqrt{\alpha_2}\Big)^2$$

Suppose first that $\hat{\alpha}_1 > \alpha_2$. Clearly, firm 1's corner pricing equilibrium profits are strictly increasing in α_1 for $\alpha_1 \geq \hat{\alpha}_1$. Two possibilities to deviate remain to be checked: first, firm 1 could deviate to $\alpha_1' \in (\alpha_2, \hat{\alpha_1})$, which leads to an interior pricing equilibrium with a competitive advantage for firm 1. Such deviations are not profitable according to the following Lemma.

Lemma 10. *Fix $\alpha_2 \in [\underline{\alpha}, \bar{\alpha})$ and suppose that condition (11) does not hold. Then, firm 1 is weakly better off to choose $\bar{\alpha}$ than any $\alpha_1 \in [\alpha_2, \bar{\alpha}]$.*

Proof: By Lemma 4, firm 1's interior pricing equilibrium profits are strictly increasing in α_1 for $\alpha_1 \in (\alpha_2, \hat{\alpha}_1)$. Furthermore, firm 1's corner pricing equilibrium profits are strictly increasing in α_1 for $\alpha_1 \in [\hat{\alpha}_1, \bar{\alpha}]$. It thus remains to be shown that the corner pricing equilibrium profits at $\bar{\alpha}$ are higher than the limit of interior pricing equilibrium profits when $\alpha_1 \to \hat{\alpha}_1$. The corner pricing equilibrium profits at $\bar{\alpha}$ are equal to:

$$(\bar{\alpha} - \alpha_2) \cdot \frac{1-2c}{2} - (\sqrt{\bar{\alpha}} - \sqrt{\alpha_2}) \cdot \sqrt{2\bar{s}}$$

An upper bound for the interior equilibrium pricing profits can be derived as follows:[28]

$$\begin{aligned}
&(\sqrt{\alpha_1} - \sqrt{\alpha_2}) \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{G(\hat{s}^*)^2}{g(\hat{s}^*)} \\
&\xrightarrow{\alpha_1 \to \hat{\alpha}_1} (\sqrt{\hat{\alpha}_1} - \sqrt{\alpha_2}) \cdot \frac{1}{\sqrt{2\bar{s}}} \cdot \frac{1}{g(\bar{s})} \\
&\leq (\sqrt{\bar{\alpha}} - \sqrt{\alpha_2}) \cdot \frac{1}{\sqrt{2\bar{s}}} \cdot \frac{1}{g(\bar{s})}
\end{aligned}$$

[28] Again, $\hat{s}^*$ is determined by equation (5).

Corner pricing equilibrium profits lie above that upper bar if and only if the following condition is satisfied:

$$(1-2c)(\sqrt{\bar{\alpha}}+\sqrt{\alpha_2}) \geq 2\sqrt{2\bar{s}}+\frac{\sqrt{2}}{\sqrt{\bar{s}}\cdot g(\bar{s})}$$

This is implied by the fact that condition (11) does not hold, which completes the proof of the Lemma.

Second, firm 1 could deviate to $\alpha_1' \in [\underline{\alpha}, \alpha_2]$. By Lemma 4, the best deviation in this interval is to set $\underline{\alpha}$. This deviation is not profitable if and only if

$$(\bar{\alpha}-\alpha_2)\cdot\frac{1-2c}{2}-(\sqrt{\bar{\alpha}}-\sqrt{\alpha_2})\cdot\sqrt{2\bar{s}} \geq (\sqrt{\alpha_2}-\sqrt{\underline{\alpha}})\cdot\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{(1-G(\hat{s}^*))^2}{g(\hat{s}^*)}$$

where $\hat{s}^*$ is determined by equation (5) when evaluated at α_2 and $\underline{\alpha}$. Clearly, the right-hand side of this equation converges to zero as $\alpha_2 \to \underline{\alpha}$, while the left-hand side converges to a strictly positive value. Thus, there exists $\underset{\sim}{\alpha}_2 > \underline{\alpha}$ such that this condition is satisfied for $\alpha_2 \in [\underline{\alpha}, \underset{\sim}{\alpha}_2)$.

Suppose now that $\hat{\alpha}_1 \leq \alpha_2$. Then, deviations to $\alpha_1' \in [\hat{\alpha}_1, \alpha_2]$ yield zero profits and thus only deviations to $\alpha_1' \in [\underline{\alpha}, \hat{\alpha}_1)$ need to be considered. By the previous arguments, the same condition as under $\hat{\alpha}_1 > \alpha_2$ is obtained.

To summarize, if 11 does not hold: $\alpha_1^* = \bar{\alpha}$, $\alpha_2^* \in [\underline{\alpha}, \bar{\alpha}]$ such that $\bar{\alpha} \in BR_1(\alpha_2^*)$. A corner pricing equilibrium is obtained. $\bar{\alpha} \in BR_1(\alpha_2^*)$ if and only if:

- $\alpha_2^* \geq \tilde{\alpha}_2$ or
- $(\bar{\alpha}-\alpha_2^*)\cdot\frac{1-2c}{2}-\left(\sqrt{\bar{\alpha}}-\sqrt{\alpha_2^*}\right)\cdot\sqrt{2\bar{s}} \geq \left(\sqrt{\alpha_2^*}-\sqrt{\underline{\alpha}}\right)\cdot\frac{1}{\sqrt{2\hat{s}^*}}\cdot\frac{(1-G(\hat{s}^*))^2}{g(\hat{s}^*)}$, where $\hat{s}^*$ is determined by equation (5) when evaluated at α_2^* and $\underline{\alpha}$. There exists $\underset{\sim}{\alpha}_2 > \underline{\alpha}$ such that this condition is satisfied for $\alpha_2^* \in [\underline{\alpha}, \underset{\sim}{\alpha}_2)$.

Now suppose that (10) does not hold. By Lemma 5, firm 2 sets $\underline{\alpha}$ in any equilibrium. Any $\alpha_1 \in [\underline{\alpha}, \bar{\alpha}]$ yields zero profits and constitutes a best response to $\underline{\alpha}$. It thus remains to be shown that $\underline{\alpha}$ constitutes a best response to $\alpha_1 \in [\underline{\alpha}, \bar{\alpha}]$ under the conditions of Proposition 2.

Fix any $\alpha_1 \in [\underline{\alpha}, \bar{\alpha}]$. First, consider

$$\alpha_1 \leq \tilde{\alpha}_1 \equiv \min\Big\{\frac{1}{(1-2c)^2}\big(-\frac{\sqrt{2}}{g(\underline{s})\sqrt{\underline{s}}}+2\sqrt{2}\sqrt{\underline{s}}-\sqrt{\bar{\alpha}}\big)^2, \bar{\alpha}\Big\}.$$

This implies that any $\alpha_2 \in [\underline{\alpha}, \bar{\alpha}]$ leads to a corner pricing equilibrium. Since $\alpha_2 \geq \alpha_1$ yields zero profits for firm 2 and firm 2's corner pricing equilibrium profits are strictly decreasing in α_2 for $\alpha_2 < \alpha_1$, setting $\underline{\alpha}$ constitutes the unique best response for firm 2.

Consider now $\alpha_1 > \tilde{\alpha}_1$. Define $\hat{\alpha}_2$ such that, given α_1, condition (6) does not hold true when $\alpha_2 \geq \hat{\alpha}_2$, that is:

$$\hat{\alpha}_2 = \frac{1}{(1-2c)^2}\Big(-\frac{\sqrt{2}}{g(\underline{s}) \cdot \sqrt{\underline{s}}} + 2 \cdot \sqrt{2} \cdot \sqrt{\underline{s}} - \sqrt{\alpha_1}\Big)^2$$

Suppose first that $\hat{\alpha}_2 < \alpha_1$. Clearly, firm 2's corner pricing equilibrium profits are strictly decreasing in α_2 for $\alpha_1 \leq \hat{\alpha}_2$. Two possibilities to deviate remain to be checked: first, firm 1 could deviate to $\alpha_2' \in (\hat{\alpha}_2, \alpha_1)$. Such deviations are not profitable according to the following Lemma.

Lemma 11. *Fix $\alpha_1 \in [\underline{\alpha}, \bar{\alpha})$ and suppose that condition (10) does not hold. Then, firm 2 is weakly better off to choose $\underline{\alpha}$ than any $\alpha_2 \in (\underline{\alpha}, \alpha_1]$.*

Proof: By Lemma 4, firm 2's interior pricing equilibrium profits are strictly decreasing in α_2 for $\alpha_2 \in (\hat{\alpha}_2, \alpha_1)$. Furthermore, firm 2's corner pricing equilibrium profits are strictly decreasing in α_2 for $\alpha_2 \in [\underline{\alpha}, \hat{\alpha}_2]$. It thus remains to be shown that the corner pricing equilibrium profits at $\underline{\alpha}$ are higher than the limit of interior pricing equilibrium profits when $\alpha_2 \to \hat{\alpha}_2$. The corner pricing equilibrium profits at $\underline{\alpha}$ are equal to:

$$(\sqrt{\alpha_1} - \underline{\alpha}) \cdot \sqrt{2\underline{s}} - (\alpha_1 - \underline{\alpha}) \cdot \frac{1-2c}{2}$$

An upper bound for the interior equilibrium pricing profits can be derived as follows:[29]

$$\begin{aligned}
&(\sqrt{\alpha_1} - \sqrt{\alpha_2}) \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{(1-G(\hat{s}^*))^2}{g(\hat{s}^*)} \\
&\xrightarrow{\alpha_2 \to \hat{\alpha}_2} (\sqrt{\alpha_1} - \sqrt{\hat{\alpha}_2}) \cdot \frac{1}{\sqrt{2\underline{s}}} \cdot \frac{1}{g(\underline{s})} \\
&\leq (\sqrt{\alpha_1} - \sqrt{\underline{\alpha}}) \cdot \frac{1}{\sqrt{2\underline{s}}} \cdot \frac{1}{g(\underline{s})}
\end{aligned}$$

Corner pricing equilibrium profits lie above that upper bar if and only if the following condition is satisfied:

$$(1-2c)(\sqrt{\alpha_1} + \sqrt{\underline{\alpha}}) \leq 2\sqrt{2\bar{s}} - \frac{\sqrt{2}}{\sqrt{\underline{s}} \cdot g(\underline{s})}$$

[29] Again, $\hat{s}^*$ is determined by equation (5).

This is implied by the fact that condition (10) does not hold, which completes the proof of the Lemma.

Second, firm 2 could deviate to $\alpha_2' \in [\alpha_1, \bar{\alpha}]$. By Lemma 4, the best deviation in this interval is to set $\bar{\alpha}$. This deviation is not profitable if and only if

$$(\alpha_1 - \underline{\alpha}) \cdot \frac{1-2c}{2} - \left(\sqrt{\alpha_1} - \sqrt{\underline{\alpha}}\right) \cdot \sqrt{2\underline{s}} \geq \left(\sqrt{\bar{\alpha}} - \sqrt{\alpha_1}\right) \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{(G(\hat{s}^*))^2}{g(\hat{s}^*)}$$

where $\hat{s}^*$ is determined by equation (5) when evaluated at α_1 and $\bar{\alpha}$. Clearly, the right-hand side of this equation converges to zero as $\alpha_1 \to \bar{\alpha}$, while the left-hand side converges to a strictly positive value. Thus, there exists $\underset{\sim}{\alpha}_2 < \bar{\alpha}$ such that this condition is satisfied for $\alpha_1 \in (\underset{\sim}{\alpha}_1, \bar{\alpha}]$.

Suppose now that $\hat{\alpha}_2 \geq \alpha_1$. Then, deviations to $\alpha_2' \in [\alpha_1, \hat{\alpha}_2]$ yield zero profits and thus only deviations to $\alpha_2' \in (\hat{\alpha}_1, \bar{\alpha}]$ need to be considered. By the previous arguments, the same condition as under $\hat{\alpha}_1 > \alpha_2$ is obtained.

To summarize, if 10 does not hold: $\alpha_2^* = \underline{\alpha}$, $\alpha_1^* \in [\underline{\alpha}, \bar{\alpha}]$ such that $\underline{\alpha} \in BR_2(\alpha_1^*)$. A corner pricing equilibrium is obtained. $\underline{\alpha} \in BR_2(\alpha_1^*)$ if and only if:

- $\alpha_1^* \leq \tilde{\alpha}_1$ or
- $(\alpha_1^* - \underline{\alpha}) \cdot \frac{1-2c}{2} - \left(\sqrt{\alpha_1^*} - \sqrt{\underline{\alpha}}\right) \cdot \sqrt{2\underline{s}} \geq \left(\sqrt{\bar{\alpha}} - \sqrt{\alpha_1^*}\right) \cdot \frac{1}{\sqrt{2\hat{s}^*}} \cdot \frac{(G(\hat{s}^*))^2}{g(\hat{s}^*)}$, where $\hat{s}^*$ is determined by equation (5) when evaluated at α_1^* and $\bar{\alpha}$. There exists $\underset{\sim}{\alpha}_1 < \bar{\alpha}$ such that this condition is satisfied for $\alpha_1^* \in [\underset{\sim}{\alpha}, \bar{\alpha}]$.

Proof of Corollary 1

For $c = \tilde{c}$, condition (5) is solved at $\hat{s}^* = med(G)$ for $\alpha_1^* = \bar{\alpha}$ and $\alpha_2^* = \underline{\alpha}$. Hence, an interior pricing equilibrium is obtained, in which according to Proposition 1, both firms set the same price and earn the same profits. First consider $c > \tilde{c}$. This corresponds to a decrease in the LHS of condition (10) and (11) compared to $c = \tilde{c}$. Hence, either both conditions are satisfied, or condition (10) is violated. If condition (10 is violated, a corner pricing equilibrium is obtained in which firm 2 serves the whole market and thus earns higher profits. If both conditions are satisfied, it follows from condition (5), that $\hat{s}^* < med(G)$. According to Proposition 1, firm 2 earns higher profits. Second, consider $c < \tilde{c}$. This corresponds to an increase in the LHS of condition (10) and (11) compared to $c = \tilde{c}$. Hence, either both conditions are satisfied, or condition (11) is violated. If condition (11 is violated, a corner pricing equilibrium is obtained in which firm 1 serves the whole market and thus earns higher profits. If both conditions are satisfied, it follows from condition (5), that $\hat{s}^* > med(G)$. According to Proposition 1, firm 1 earns higher profits.

Single Product, consumer search

With 1 product per firm, after visiting one firm, consumers might decide to also visit the other firm. If a consumer has visited both firms, he prefers the product of firm j over the one of firm k if

$$z_j \geq \tilde{z}_j(z_k) = \frac{\alpha_k z_k + p_j - p_k + c \cdot (\alpha_j - \alpha_k)}{\alpha_j}.$$

The expected surplus from continuing search after having visited firm k is

$$ES(z_k) = \int_{\min\{\max\{\tilde{z}_j(z_k),-1\},1\}}^{1} (\alpha_j z_j - \alpha_k z_k + p_k - p_j - c \cdot (\alpha_j - \alpha_k))\, d\frac{z_j}{2}$$

which can be rewritten as

$$ES(z_k) = \begin{cases} -p_j - \alpha_k z_k + p_k - c \cdot (\alpha_j - \alpha_k) & \text{if } \tilde{z}_j(z_k) < -1 \\ \frac{1}{4\alpha_j}(\alpha_j + p_k - p_j - c \cdot (\alpha_j - \alpha_k) - \alpha_k z_k)^2 & \text{if } \tilde{z}_j(z_k) \in [-1, 1] \\ 0 & \text{if } \tilde{z}_j(z_k) > 1. \end{cases}$$

A search type s only continues the search if $ES(z_k) > s$. If $\tilde{z}_j(z_k) \in [-1, 1]$, a consumer with search cost s continues searching if

$$z_k \leq r_k(s) = \frac{\alpha_j + p_k - p_j - c \cdot (\alpha_j - \alpha_k) - 2\sqrt{\alpha_j s}}{\alpha_k}.$$

If $\tilde{z}_j(z_k) < -1$, a consumer with search cost s continues searching if

$$z_k < r_k(s) = \frac{p_k - p_j - c \cdot (\alpha_j - \alpha_k) - s}{\alpha_k}$$

Therefore, we get the rule that consumer type s continues to search if

$$z_k < r_k(s) = \begin{cases} \frac{\alpha_j + p_k - p_j - c \cdot (\alpha_j - \alpha_k) - 2\sqrt{\alpha_j s}}{\alpha_k} & \text{if } s \leq \alpha_j \\ \frac{p_k - p_j - c \cdot (\alpha_j - \alpha_k) - s}{\alpha_k} & \text{else.} \end{cases}$$

Proof of Lemma 6

According to Weitzman [1979], consumers inspect the product with the highest reservation price first, where the reservation price is defined as the utility from consuming the match value $r_k(s)$:

$$\tau_k(s) = \epsilon - c \cdot \alpha_k + \alpha_k \cdot r_k(s) - p_k = \begin{cases} \alpha_j - c \cdot \alpha_j - p_j - 2\sqrt{\alpha_j s} & \text{if } s < \alpha_j \\ -c \cdot \alpha_j - p_j - s & \text{else.} \end{cases}$$

In the following, we consider firm 1 and firm 2 and assume WLOG that $\alpha_1 - \alpha_2 \equiv \Delta\alpha \geq 0$. Denote the price difference by $p_1 - p_2 \equiv \Delta p$. Moreover, denote by $\Delta \equiv \Delta p + c \cdot \Delta\alpha$ the average utility difference between the two products.

We need to distinguish between $s < \alpha_2$, $s \in [\alpha_2, \alpha_1)$, and $s \geq \alpha_1$. This determines a consumer's choice depending on Δ: For $s < \alpha_2$, a consumer inspects product 1 first if

$$s \leq \hat{s} \equiv \frac{1}{4}\left(\frac{\Delta\alpha - \Delta}{(\sqrt{\alpha_1} - \sqrt{\alpha_2}}\right)^2.$$

It holds that $\hat{s} \in [0, \alpha_2)$ if $\Delta \in ((\sqrt{\alpha_1} - \sqrt{\alpha_2})^2, \Delta\alpha]$.

For $s \in [\alpha_2, \alpha_1)$, a consumer inspects product 1 first if

$$s \leq \hat{s} \equiv \left(\sqrt{\alpha_1} - \sqrt{\Delta}\right)^2.$$

It holds that $\hat{s} \in [\alpha_2, \alpha_1)$ if $\Delta \in [0, (\sqrt{\alpha_1} - \sqrt{\alpha_2})^2]$. For $s > \alpha_1$, a consumer inspects product 1 first and always buys it if $\Delta < 0$. Note that for $\Delta \in (0, \Delta\alpha]$, $\hat{s}$ is decreasing in Δ. Moreover, note that in all segments, the difference $\tau_1(s) - \tau_2(s)$ is strictly decreasing in Δ.

Hence, a consumer's search order is uniquely pinned down by $\hat{s}$. There exists a unique Δ, such that a given search type is indifferent. For higher Δ, the search type inspects product 2 first, whereas for lower Δ, the search type inspects product 1 first.

Demand

Define

$$\tilde{s}_1 = \begin{cases} \frac{1}{4\alpha_2}(\alpha_1 + \alpha_2 + \Delta)^2 & \text{if } \Delta < -\Delta\alpha \\ \alpha_1 + \Delta & \text{if } \Delta \geq -\Delta\alpha \end{cases}$$

and

$$\tilde{s}_2 = \begin{cases} \alpha_2 - \Delta & \text{if } \Delta < -\Delta\alpha \\ \frac{1}{4\alpha_1}(\alpha_1 + \alpha_2 - \Delta)^2 & \text{if } \Delta \geq -\Delta\alpha. \end{cases}$$

Consumers that visit firm j first never continue their search and thus certainly buy firm j's product if $s > \tilde{s}_j$.

For consumers that visit both firms with positive probability, the demand can be characterized by first deriving the demand absent any search cost and then subtracting consumers who purchase the rival firm's product despite its lower utility and adding consumers who purchase the own product despite its lower utility.

In the following, we restrict attention to firm 1's demand D_1. Firm 2's demand directly follows from D_1 because $D_1 + D_2 = 1$. Absent search cost, demand for firm 1 is equal to

$$\begin{cases} 0 & \text{if } \Delta > \alpha_1 + \alpha_2 \\ \frac{1}{8\alpha_1\alpha_2}\left(\alpha_1 + \alpha_2 - \Delta\right)^2 & \text{if } \Delta \in (\Delta\alpha, \alpha_1 + \alpha_2] \\ \frac{1}{2} - \frac{1}{2}\frac{\Delta}{\alpha_1} & \text{if } \Delta \in (-\Delta\alpha, \Delta\alpha] \\ 1 - \frac{1}{8}\left(\frac{\alpha_1+\alpha_2+\Delta}{\alpha_1\alpha_2}\right)^2 & \text{if } \Delta \in (-\alpha_1 - \alpha_2, -\Delta\alpha] \\ 1 & \text{if } \Delta \leq -\alpha_1 - \alpha_2 \end{cases}$$

For a given search type $s < \tilde{s}_1$ that visits firm 1 first, the probability of buying product 1 when it gives a lower utility is

$$\begin{cases} \frac{1}{2}\frac{s}{\alpha_1} & \text{if } \Delta \leq \Delta\alpha \\ \max\left\{0, \frac{1}{2\alpha_1}\left(s - \frac{1}{4\alpha_2}(\Delta\alpha - \Delta)^2\right)\right\} & \text{if } \Delta > \Delta\alpha \end{cases}$$

Similarly, a given search type $s < \tilde{s}_2$ that visits firm 2 first, the probability of buying product 2 when it gives a lower utility is

$$\begin{cases} \max\{0, \frac{1}{2\alpha_2}\left(s - \frac{1}{4\alpha_1}(\Delta\alpha - \Delta)^2\right)\} & \text{if } \Delta \leq \Delta\alpha \\ \frac{1}{2}\frac{s}{\alpha_2} & \text{if } \Delta > \Delta\alpha. \end{cases}$$

Bringing all terms together, substituting in the expressions for $\tilde{s}_1$ and $\tilde{s}_2$, and incorporating our results on which firm consumers visit first, demand for firm 1 is

$$
D_1(\Delta) = \begin{cases} 0 & \text{if } \Delta > \alpha_1 + \alpha_2 \\ \frac{1}{2\alpha_2}\left(G(\tilde{s}_2)\tilde{s}_2 - \int_{\underline{s}}^{\tilde{s}_2} s dG(s)\right) & \text{if } \Delta \in (\Delta\alpha, \alpha_1 + \alpha_2], \\ \frac{G(\tilde{s}_2)}{2\alpha_1}\left(\alpha_1 - \Delta + E[s|s \leq \tilde{s}_2]\right) & \\ \qquad -\frac{1}{2\alpha_1\alpha_2}\int_{\hat{s}}^{\tilde{s}_2}\left((\alpha_1+\alpha_2)s - \frac{1}{4}(\Delta\alpha - \Delta)^2\right) dG(s) & \text{if } \Delta \in ((\sqrt{\alpha_1} - \sqrt{\alpha_2})^2, \Delta\alpha] \\ \frac{G(\hat{s})}{2\alpha_1}(\alpha_1 - \Delta + E[s|s \leq \hat{s}) & \text{if } \Delta \in (0, (\sqrt{\alpha_1} - \sqrt{\alpha_2})^2], \\ 1 - G(\tilde{s}_1) + \frac{G(\tilde{s}_1)}{2\alpha_1}(\alpha_1 - \Delta + E[s|s \leq \tilde{s}_1) & \text{if } \Delta \in (-\Delta\alpha, 0] \\ 1 - \frac{1}{2\alpha_1}\left(G(\tilde{s}_1)\tilde{s}_1 - \int_{\underline{s}}^{\tilde{s}_1} s dG(s)\right) & \text{if } \Delta \in (-\alpha_1 - \alpha_2, -\Delta\alpha] \\ 1 & \Delta \leq -\alpha_1 - \alpha_2 \end{cases}
$$

Proof of Lemma 7

Suppose that firms set $\alpha_1 = \alpha_2 = \alpha$. In this case, demand simplifies to

$$
D_1(\Delta p) = \begin{cases} 0 & \text{if } \Delta p > 2\alpha \\ \frac{1}{2\alpha}\left(\tilde{s}_2 G(\tilde{s}_2) - \int_{\underline{s}}^{\tilde{s}_2} s dG(s)\right) & \text{if } \Delta p \in (0, 2\alpha] \\ 1 - \frac{1}{2\alpha}\left(\tilde{s}_1 G(\tilde{s}_1) - \int_{\underline{s}}^{\tilde{s}_1} s dG(s)\right) & \text{if } \Delta p \in (-2\alpha, 0] \\ 1 & \text{if } \Delta p \leq -2\alpha \end{cases}
$$

Taking the derivative wrt Δp for $\Delta p \neq 0$ gives

$$
D_1'(\Delta p) = \begin{cases} 0 & \text{if } \Delta p > 2\alpha \\ -\frac{1}{4\alpha^2}(2\alpha - \Delta p)G(\tilde{s}_2) = -\frac{1}{2\alpha}\sqrt{\tilde{s}_2/\alpha}\,G(\tilde{s}_2) & \text{if } \Delta p \in (0, 2\alpha] \\ -\frac{1}{4\alpha^2}(2\alpha + \Delta p)G(\tilde{s}_1) = -\frac{1}{2\alpha}\sqrt{\tilde{s}_1/\alpha}\,G(\tilde{s}_1) & \text{if } \Delta p \in (-2\alpha, 0) \\ 0 & \text{if } \Delta p \leq -2\alpha \end{cases}
$$

Next, we show that there cannot exist a pure-strategy Nash equilibrium. First, observe that there cannot exist a symmetric pure-strategy Nash equilibrium. For a symmetric pure-strategy Nash equilibrium with positive prices, both firms would have an incentive to slightly undercut. For a symmetric pure-strategy Nash equilibrium at $p = 0$, both firms would have an incentive to set a slightly higher price. As long as $\underline{s} < \alpha$, positive profits can be ensured.

Next, we show that there also cannot exist an asymmetric pure-strategy Nash equilibrium. To see this, consider the marginal return for firm 1 for $\Delta p < 0$:

$$D_1(\Delta p) + p_1 D_1'(\Delta p) = 1 - \frac{1}{2\alpha}\left(\tilde{s}_1 G(\tilde{s}_1) - \int_{\underline{s}}^{\tilde{s}_1} s dG(s) + p_1\sqrt{\tilde{s}_1/\alpha}\, G(\tilde{s}_1)\right)$$

Note that because $\Delta p < 0$, we get $\tilde{s}_1 < \alpha$. Hence, the term is larger than

$$1 - \frac{1}{2\alpha}\left(\alpha G(\alpha) + p_1 G(\alpha)\right)$$

and therefore positive for $p_1 < p_2 \leq \alpha$. Hence, in any PSNE, the firm setting the higher price must set $p > \alpha$, as otherwise, the rival firm marginally undercuts, which cannot be optimal.

Now consider firm 1's marginal return when $\Delta p > 0$:

$$D_1(\Delta p) + p_1 D_1'(\Delta p) = \frac{1}{2\alpha}\left(\tilde{s}_2 G(\tilde{s}_2) - \int_{\underline{s}}^{\tilde{s}_2} s dG(s) - p_1\sqrt{\tilde{s}_2/\alpha}\, G(\tilde{s}_2)\right)$$

Because $\Delta p > 0$, we have $\tilde{s}_2 < \alpha$. Note that for $p_1 > \alpha$, $\tilde{s}_2 < p_1\sqrt{\tilde{s}_2/\alpha}$ and hence, firm 1's marginal return is negative. This implies that the firm setting the higher price must set a price below α. But as shown before, this induces the rival firm to marginally undercut. Hence, no Nash equilibrium in pure strategies exists.

In the following, we show that our model satisfies the assumptions of Dasgupta and Maskin [1986] (Lemma 7), ensuring the existence of a symmetric Nash equilibrium in mixed strategies.

In the symmetric subgame with product design α, the gross utility from consuming a product is bounded above by $\epsilon - c\alpha + \alpha$. Hence, any price $p_i > \epsilon - c\alpha + \alpha$ yields zero demand and therefore zero profits, independently of the rival firm's price. Since a firm can guarantee strictly positive profits by setting a sufficiently small positive price, no equilibrium assigns positive probability to prices above $\epsilon - c\alpha + \alpha$. We can therefore restrict the strategy space of each firm to the compact interval

$$P_i = [0, \epsilon - c\alpha + \alpha].$$

An equilibrium of this restricted game is also an equilibrium of the original game with price space $\mathbb{R}_+$, because deviations to prices outside P_i yield zero profits and are therefore not profitable.

On the compact strategy space $P_1 \times P_2$, the payoff functions satisfy the conditions of Dasgupta and Maskin's existence theorem. In particular, payoff discontinuities arise only at price ties, and the sum of firms' payoffs is upper semicontinuous. Hence, a mixed-strategy Nash equilibrium exists. Since the restricted game is symmetric, there exists a symmetric mixed-strategy equilibrium.

Next, we derive an upper bound for the firms' equilibrium profits in the symmetric MSNE. To see this, we show that the marginal revenue for $\bar{p} > \alpha$ in the case of $\Delta p > 0$ is negative. This implies that the highest price played in a symmetric mixed strategy equilibrium $\bar{p}$ must be below α. To see this, first observe that the highest price $\bar{p}$ cannot be set with positive mass, as there would be a positive probability to get a discontinuously higher demand by slightly undercutting this price. Therefore, it follows that when firm 1 sets $p_1 = \bar{p}$, it holds that $\Delta p > 0$. In this case, the marginal return for $\bar{p} \geq \alpha$ is negative for the same reason as argued above:

$$D_1(\Delta p) + \bar{p} D_1'(\Delta p) = \frac{1}{2\alpha}\left(\tilde{s}_2 G(\tilde{s}_2) - \int_{\underline{s}}^{\tilde{s}_2} s dG(s) - \bar{p}\sqrt{\frac{\tilde{s}_2}{\alpha}} G(\tilde{s}_2)\right) < -\frac{1}{2\alpha}\int_{\underline{s}}^{\tilde{s}_2} s dG(s) < 0.$$

The inequality follows by substituting in α for $\bar{p}$ and observing that it holds that $\tilde{s}_2 \leq \alpha$. Therefore, the last term is larger than the first term and canceling both gives a negative upper bound. Moreover, for $p_2 \to \bar{p}$, we get an upper bound for the demand equal to $\frac{G(\alpha)}{2\alpha}(\alpha - E[s|s \leq \alpha])$. Hence, equilibrium profits cannot exceed

$$\frac{G(\alpha)}{2}(\alpha - E[s|s \leq \alpha]).$$

Proof of Lemma 8

Consider $\alpha_1 = \bar{\alpha}$ and $\alpha_2 = \underline{\alpha}$. In order to obtain a tractable solution, we assume that

$$s \sim \begin{cases} \bar{s} \text{ with probability } \beta \\ \underline{s} \text{ with probability } 1-\beta, \end{cases}$$

where $\underline{s} \in (1/4\underline{\alpha}, \underline{\alpha}]$ and $\bar{s} \geq \bar{\alpha}$. The assumption ensures that for $\Delta \in [-\Delta\alpha, \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})]$, search type $\underline{s}$ inspects product 1 first and continues to firm 2 with positive probability, whereas search type $\bar{s}$ always buys the first product she inspects. Moreover, we assume that $c < \frac{\underline{\alpha}}{\bar{\alpha} - \underline{\alpha}}$.

In this case, we can state clear conditions that ensure existence and uniqueness of a NE in pure strategies.

Demand is equal to

$$D_1(\Delta p) = \begin{cases} 0 & \text{if } \Delta \geq \bar{\alpha} + \underline{\alpha} - 2\sqrt{\bar{\alpha}\underline{s}} \\ \frac{1-\beta}{2\underline{\alpha}}(\tilde{s}_2 - \underline{s}) & \text{if } \bar{\alpha} + \underline{\alpha} - 2\sqrt{\bar{\alpha}\underline{s}} \geq \Delta > \Delta\alpha \\ \frac{1-\beta}{2\bar{\alpha}}(\bar{\alpha} - \Delta) - \frac{1-\beta}{2\alpha_2}(\underline{s} - \frac{1}{4\bar{\alpha}}(\Delta\alpha - \Delta)^2) & \text{if } \Delta\alpha \geq \Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}}) \\ \frac{1-\beta}{2\bar{\alpha}}\left(\bar{\alpha} - \Delta + \underline{s}\right) & \text{if } \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}}) \geq \Delta > 0 \\ \beta + \frac{1-\beta}{2\bar{\alpha}}\left(\bar{\alpha} - \Delta + \underline{s}\right) & \text{if } 0 \geq \Delta > -\Delta\alpha \\ 1 - \frac{1-\beta}{2\bar{\alpha}}(\tilde{s}_1 - \underline{s}) & \text{if } -\Delta\alpha \geq \Delta > -\bar{\alpha} - \underline{\alpha} + 2\sqrt{\bar{\alpha}\underline{s}} \\ 1 & \text{if } -\bar{\alpha} - \underline{\alpha} + 2\sqrt{\bar{\alpha}\underline{s}} > \Delta. \end{cases}$$

We first derive conditions for which there exists a unique NE candidate for $\Delta \in [-\Delta\alpha, \Delta\alpha - 2\sqrt{\bar{\alpha}\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})]$. Here, firms are facing linear demand except for $\Delta = 0$, where both demand functions exhibit a discontinuity. There are three possible best responses for both firms. Firms might optimally set the best response given that $\Delta > 0$ or $\Delta < 0$, or they might marginally undercut the rival firm. Note that when marginally undercutting the rival firm's price is a best response, this cannot constitute an equilibrium. Therefore, there are two equilibrium candidates left. We first consider the equilibrium candidate with $\Delta < 0$. In this case, all consumers inspect product 1 first. Clearly, firm 1's best response function is strictly larger than firm 2's best response function and thus, both functions do not intersect for $\Delta < 0$. Therefore, there cannot be an equilibrium in $\Delta \in [-\Delta\alpha, 0]$. Solving for the equilibrium prices in the equilibrium candidate with $\Delta > 0$ gives

$$p_1^* = \frac{1}{3}\left(\frac{3-\beta}{1-\beta}\bar{\alpha} + \underline{s} - c \cdot \Delta\alpha\right)$$

and

$$p_2^* = \frac{1}{3}\left(\frac{3+\beta}{1-\beta}\bar{\alpha} - \underline{s} + c \cdot \Delta\alpha\right).$$

The resulting price difference is

$$\Delta p = \frac{2}{3}\left(\underline{s} - c \cdot \Delta\alpha - \frac{\beta}{1-\beta}\bar{\alpha}\right).$$

In the following, we derive conditions that there is no profitable deviation which will give us that the price difference is larger than $-c \cdot \Delta\alpha$ and thus, $\Delta > 0$.

Corresponding equilibrium profits are

$$\pi_1 = \frac{1-\beta}{18\bar{\alpha}} \left(\frac{3-\beta}{1-\beta}\bar{\alpha} + \underline{s} - c \cdot \Delta\alpha \right)^2$$

and

$$\pi_2 = \frac{1-\beta}{18\bar{\alpha}} \left(\frac{3+\beta}{1-\beta}\bar{\alpha} - \underline{s} + c \cdot \Delta\alpha \right)^2$$

These prices constitute an equilibrium candidate if firm 1 does not have an incentive to deviate to $p_2 - c \cdot \Delta\alpha - \epsilon$ in order to attract the share of search types β that never search. This deviation is not profitable if

$$\frac{1-\beta}{18\bar{\alpha}} \left(\frac{3-\beta}{1-\beta}\bar{\alpha} + \underline{s} - c \cdot \Delta\alpha \right)^2 > \frac{1}{3} \left(\frac{3+\beta}{1-\beta}\bar{\alpha} - \underline{s} - 2c \cdot \Delta\alpha \right) \cdot \frac{1-\beta}{2\bar{\alpha}} \left(\frac{1+\beta}{1-\beta}\bar{\alpha} + \underline{s} \right),$$

where the LHS are firm 1's profits in the equilibrium candidate and the RHS are firm 1's profits when undercutting firm 2's price by $-c\cdot\Delta\alpha$ and therefore having all consumers visiting firm 1 first. Now consider the following upper bound for the RHS:

$$\frac{1-\beta}{6\bar{\alpha}} \left(\frac{3+\beta}{1-\beta}\bar{\alpha} - \underline{s} - 2c \cdot \Delta\alpha \right) \cdot \left(\frac{1+\beta}{1-\beta}\bar{\alpha} + \underline{s} \right) < \frac{1-\beta}{6\bar{\alpha}} \left(\frac{3+\beta}{1-\beta}\bar{\alpha} - \underline{s} + c \cdot \Delta\alpha \right) \cdot \left(\frac{1+\beta}{1-\beta}\bar{\alpha} + \underline{s} - c \cdot \Delta\alpha \right).$$

This holds true if

$$c \cdot \Delta\alpha < 4\underline{s} + \frac{2\beta}{1-\beta}\bar{\alpha}.$$

Hence, because our parameter assumptions imply that $4\underline{s} > c \cdot \Delta\alpha$, the inequality holds.

Making use of the inequality allows us to characterize the unprofitability of the deviation depending on $\kappa \equiv \frac{\underline{s} - c \cdot \Delta\alpha}{\bar{\alpha}}$. A sufficient condition for the inequality to hold is

$$\beta \leq \tilde{\beta}(\kappa) = \frac{4\kappa^2 - 3\sqrt{9\kappa^2 + 2\kappa + 9} + \kappa + 9}{2(2\kappa^2 + \kappa - 1)}.$$

It turns out that under the parameter assumptions, it holds that $|\kappa| \leq 1/20$ and therefore, the term is well-defined.

Next, we consider possible deviations and other equilibria for $\Delta < -\Delta\alpha$ and $\Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$.

The most relevant possible deviation that determines how dispersed the product designs must be, is the potential deviation from firm 2 to $\Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$ to attract low search type consumers to visit firm 2 first. To exclude this possibility, we derive a sufficient condition for which this is not profitable. The deviation is unprofitable if

$$\frac{1-\beta}{18\bar{\alpha}}\left(\frac{3+\beta}{1-\beta}\bar{\alpha}-\underline{s}+c\cdot\Delta\alpha\right)^2 > \frac{1}{3}\left(\frac{3-\beta}{1-\beta}\bar{\alpha}+\underline{s}-c\cdot\Delta\alpha\right)-\left(\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}}\right)^2+c\cdot\Delta\alpha$$

where we make use of the fact that $\Delta\alpha-2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}}) \geq (\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}})^2$. In order to derive an upper bound for $\underline{\alpha}$ depending on $\bar{\alpha}$ such that the equation holds, assume that $\underline{\alpha} \geq c\cdot\Delta\alpha$. In this case, a sufficient condition is $\bar{\alpha} > 20\underline{\alpha}$.

To see that there are no other deviations and that the equilibrium is unique, we first show that for any equilibrium, it must hold that $\Delta \in [-\Delta\alpha, \Delta\alpha]$. Then, we argue that p_2 never induces $\Delta \in [\Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}}), \Delta\alpha]$ for $p_1 < p_1^*$ and p_1 never induces $\Delta \in [\Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}}), \Delta\alpha]$ for $p_2 > p_2^*$. Hence, any equilibrium must be in the region $[-\Delta\alpha, \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}}-\sqrt{\underline{\alpha}}]$, in which demand is linear. However, for this region, we have shown that there is only one equilibrium which concludes the proof.

First, consider firm 1's best response for $\Delta > \Delta\alpha$. firm 1's objective function is

$$p_1\left(\frac{1-\beta}{2\bar{\alpha}\underline{\alpha}}(\bar{\alpha}+\underline{\alpha}-\Delta p-c\cdot\Delta\alpha)^2-\frac{1-\beta}{\underline{\alpha}}\underline{s}\right)$$

Note that omitting the negative constant in the demand function and solving for the optimal price yields an upper bound on the price that solves the original objective function. Hence, we obtain the upper bound $p_1(p_2) = 1/3(\bar{\alpha}+\underline{\alpha}+p_2-c\cdot\Delta\alpha)$. Therefore, we get that

$$\Delta = \frac{1}{3}(\bar{\alpha}+\underline{\alpha}-c\cdot\Delta\alpha)+c\cdot\Delta\alpha-\frac{2}{3}p_2 < \Delta\alpha$$

which holds if $p_2 > 2\underline{\alpha}-\bar{\alpha}+c\cdot\Delta\alpha$. However, this term is strictly negative for $\bar{\alpha} > 20\underline{\alpha}$ and given our assumption on c. Hence, firm 1 never induces $\Delta > \Delta\alpha$. In the same way, we show that for firm 2, inducing $\Delta < -\Delta\alpha$ is not optimal. Given $\Delta < -\Delta\alpha$, an upper bound for firm 2's best response function is $p_2(p_1) = 1/3(\bar{\alpha}+\underline{\alpha}+c\cdot\Delta\alpha+p_1)$. Therefore, we get the result that

$$\Delta = \frac{2}{3}p_1-\frac{1}{3}(\bar{\alpha}+\underline{\alpha}+c\cdot\Delta\alpha)+c\cdot\alpha > -\Delta\alpha$$

which holds if $p_1 > 2\underline{\alpha}-\bar{\alpha}-c\cdot\Delta\alpha$. The term is negative under our parameter specification. To sum up, we have shown that for $\bar{\alpha} > 20\underline{\alpha}$, it must be that $\Delta \in [-\Delta\alpha, \Delta\alpha]$.

Moreover, note that firm 1's demand for $\Delta \in [\Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}}), \Delta\alpha]$ is

$$\frac{1-\beta}{2\bar{\alpha}}(\bar{\alpha} - \Delta) - \frac{1-\beta}{2\alpha_2}(\underline{s} - \frac{1}{4\bar{\alpha}}(\Delta\alpha - \Delta)^2).$$

Note that this demand function is smaller and has a smaller derivative due to the second term than $\frac{1-\beta}{2\bar{\alpha}}(\bar{\alpha} - \Delta)$. Thus, we can again derive an upper bound for p_1, using the second linear demand. Hence, when we can show that with the upper bound, it never holds that $\Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$ we have a sufficient condition sufficient condition that firm 1 never induces such a price difference at the actual demand. In order to derive the optimal pricing given the upper bound, note that firm 1's best response given this upper bound is

$$p_1 = \frac{1}{2}(\bar{\alpha} + p_2 + \underline{s} - c \cdot \Delta\alpha)$$

Hence, $\Delta = \frac{1}{2}(\bar{\alpha} + p_2 + \underline{s} - c \cdot \Delta\alpha) - p_2 + c \cdot \Delta\alpha$. Because we have already shown that for this demand, $\Delta < \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$ when $p_2 = p_2^*$ and Δ is decreasing in p_2, we can conclude that it cannot be optimal to induce $\Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$ for $p_2 > p_2^*$.

Lastly, we show that firm 2 does not induce $\Delta > \Delta\alpha - 2\sqrt{\underline{s}}(\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})$ for $p_1 < p_1^*$. A sufficient condition for this to hold is

$$\frac{1-\beta}{8\bar{\alpha}}\left(\frac{1+\beta}{1-\beta}\bar{\alpha} + c \cdot \Delta\alpha - \underline{s} + p_1\right)^2 > p_1 + c \cdot \Delta\alpha - (\sqrt{\bar{\alpha}} - \sqrt{\underline{\alpha}})^2.$$

To see why this is not profitable, first observe that we have shown already that the condition holds for $p_1 = p_1^*$ if $\bar{\alpha} > 20\underline{\alpha}$. Moreover, note that taking the derivative wrt p_1 on the LHS and plugging in $p_1 = p_1^*$ gives

$$\frac{(1-\beta)}{4\bar{\alpha}}\left(\frac{1+\beta}{1-\beta}\bar{\alpha} + c \cdot \Delta\alpha - \underline{s} + p_1\right) < \frac{(1-\beta)}{6\bar{\alpha}}\left(\frac{3+2\beta}{1-\beta}\bar{\alpha} + 2(c \cdot \Delta\alpha - \underline{s})\right) = \frac{3+2\beta}{6} - \frac{1-\beta}{3}\kappa.$$

Note that it follows from our assumptions on κ and β that this term is smaller than 1. Hence, the LHS increases at a lower rate than the RHS which increases at rate 1 and thus, because the inequality holds for $p_1 = p_1^*$, we know that it also holds for $p_1 < p_1^*$.

Proof of Proposition 3

From now on, we assume that firms can choose $\alpha \in \{\underline{\alpha}, \bar{\alpha}\}$. It is left to be shown that both firms prefer the asymmetric subgame. Note that under the stated conditions, a unique pure-strategy Nash equilibrium arises with profits characterized in the previous Lemma. Hence firm 2 prefers to set $\underline{\alpha}$ in the asymmetric subgame because

$$\frac{1-\beta}{18\bar{\alpha}}\left(3\bar{\alpha}+4\frac{\beta}{1-\beta}\bar{\alpha}-\underline{s}+c\cdot\Delta\alpha\right)^2 > \frac{1-\beta}{2\bar{\alpha}}\left(\bar{\alpha}-\frac{1}{3}\underline{s}\right)^2 > \frac{1-\beta}{2}\left(\bar{\alpha}-\frac{2}{3}\underline{s}\right).$$

Clearly, the last term is strictly larger than the upper bound derived in the symmetric subgame, $\frac{1-\beta}{2}(\bar{\alpha}-\underline{s})$. Similarly, for firm 1,

$$\frac{1-\beta}{18\bar{\alpha}}\left(3\bar{\alpha}+2\frac{\beta}{1-\beta}\bar{\alpha}+\underline{s}-c\cdot\Delta\alpha\right)^2 > \frac{1-\beta}{2\bar{\alpha}}\left(\bar{\alpha}-\frac{1}{3}(c\cdot\Delta\alpha-\underline{s})\right)^2 > \frac{1-\beta}{2}\left(\bar{\alpha}-\frac{2}{3}(c\cdot\Delta\alpha-\underline{s})\right).$$

For $\bar{\alpha} > 20\underline{\alpha}$, the last term is clearly larger than $\frac{1-\beta}{2}(\underline{\alpha}-\underline{s})$ which corresponds to the upper bound in the symmetric game if firm 1 deviates to $\underline{\alpha}$.

Proof of Corollary 2

Note that π_1 and π_2 can be rewritten to

$$\pi_1 = \frac{(1-\beta)\bar{\alpha}}{18}\left(\frac{3-\beta}{1-\beta}+\kappa\right)^2 \text{ and } \pi_2 = \frac{(1-\beta)\bar{\alpha}}{18}\left(\frac{3+\beta}{1-\beta}-\kappa\right)^2.$$

Hence, $\pi_1 \geq \pi_2$ if and only if

$$\kappa \geq \frac{\beta}{1-\beta}$$

Under the constraint $\beta < \hat{\beta}(\kappa)$ and the admissible range of κ, the condition holds if and only if $\kappa \geq 0$ and hence, $\underline{s} \geq c\cdot\Delta\alpha$.

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