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# Aggregate and Distributional Effects of School Closure Mitigation Policies: Public versus Private Education

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## Abstract

We use a human capital formation model to compare extending school time to private education subsidies in mitigating the adverse effects of school closures. The impact on inequality and mobility depends crucially on the substitutability between private and public inputs.

**Keywords:** School Closures, Inequality, Intergenerational Mobility, Parental Investments, Substitutability

**JEL codes:** E24, I24, J24

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# 1 Introduction

During the COVID-19 pandemic, many governments unprecedentedly closed schools for extensive periods. This not only incurs considerable learning losses among affected children in the short run (Blanden et al., 2023; Werner and Woessmann, 2023), but may also entail significant adverse long-run consequences in terms of future income and welfare losses (Agostinelli et al., 2022; Fuchs-Schündeln et al., 2022; Jang and Yum, forthcoming). Moreover, as the learning losses and parental behavioral responses are heterogeneous across the income distribution, the school closures may lead to higher inequality and impair intergenerational mobility (Jang and Yum, forthcoming).

Against this backdrop, various policy interventions have been discussed to counteract the detrimental consequences of school closures (Zviedrite et al., 2021). To highlight the potential long-run implications of such mitigation policies, we present a model, which is simple yet considers the sophisticated nature of how private and public education investments interact as inputs into the production of human capital. We explore two mitigation policies: extending public schooling time, such as during the summer, and implementing means-tested subsidies for private education. Our results suggest that the elasticity of substitution between private and public inputs plays an important role in shaping the effects of these policies. In particular, if private and public inputs are highly substitutable, both mitigation policies can bring down inequality and improve intergenerational mobility. If the two inputs are complementary, however, extra public schooling can aggravate inequality and harm mobility, which is in contrast to government subsidies to private education.

This is informative, as even though much of the existing macro literature assumes that the degree of substitutability between public and private is very large, empirical evidence on their elasticity is far less clear. For example, Gelber and Isen (2013) find evidence that larger public investments *crowd in* parental investments into their children, which is incompatible with a model in which public and private investments are substitutes (e.g., Becker and Tomes, 1979). Generally, the degree to which private investments can replace public schooling in the production of child human capital likely depends, among other things, on the period length, the age and education stage of the child, and the presence and quality of private education markets.

The contribution of our paper is thus to offer a novel policy insight as to the importance of substitutability between public and private education for the long-term effects of mitigation policies on inequality and mobility. We thereby complement the analysis in Fuchs-Schündeln et al. (2023), who consider a school-time extension mitigation policy using a rich quantitative model, but do not consider mobility

consequences and the role of substitutability between public and private education.

## 2 Model

The model consists of three periods, indexed by  $t = 0, 1, 2$ . A household consists of an adult parent and a child and draws a time-constant endowment  $m \in \{m_l, m_h\}$  with an equal probability, such that we have low- and high-income households. We abstract from savings.

At the beginning of  $t = 0$ , the child draws a learning ability  $\phi \in \{\phi_l, \phi_h\}$ , which is correlated with  $m$ . Specifically, for  $k = l, h$ , households holding  $m_k$  draw  $\phi_k$  with a probability of  $p_\phi$ . The learning ability affects the production of a child's human capital over time,  $h_t$ .<sup>1</sup> The initial human capital level  $h_0$  is set to one. Human capital then evolves as a function of past human capital, learning ability, and private parental-, as well as public schooling inputs. We think of the initial period  $t = 0$  as capturing the early education stage of a child (e.g., pre- and primary school), and of period  $t = 1$  as the second education stage of a child (e.g., secondary school). The final period  $t = 2$  then captures the adult period of the child generation, where the final human capital level  $h_2$  realizes.

In  $t = 0$ , a household with endowment  $m$  and child learning ability  $\phi$  then solves:

$$\begin{aligned}
 V(\phi, m, t = 0) &= \max_{c_t, e_t \geq 0} \{ \log c_t + V(h_{t+1}, \phi, m, t = 1) \} \\
 \text{subject to} & \\
 c_t + e_t &= m \\
 h_{t+1} &= \phi \left\{ (e_t/\bar{e})^\psi + (\varsigma g)^\psi \right\}^{\frac{\alpha}{\psi}} h_t^{1-\alpha},
 \end{aligned} \tag{1}$$

where  $c_t$  denotes consumption and  $e_t$  denotes all private investments into the production of human capital, divided by its mean.<sup>2</sup> Private investments and time-invariant public investments, which we denote by  $g$ , are aggregated using a CES aggregator. The elasticity of substitution between the two investments is shaped by  $\psi \leq 1$ . The parameter  $\varsigma < 1$  captures the productivity loss of public schooling due to school closures and is used to simulate (unexpected) school closure shocks later (Fuchs-Schündeln et al., 2022; Jang and Yum, forthcoming). As is common, the production

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<sup>1</sup>The correlation of the learning ability of children and their parents' endowments allows us to parsimoniously capture various sources of intergenerational persistence not due to endogenous investments.

<sup>2</sup>Private investments may also include the time parents spend with their children, which we can think of as incurring an opportunity cost measured by foregone wages.

of human capital is then of the Cobb-Douglas form, where total investments and past human capital are the input factors with unit elasticity of substitution and the factor shares are given by  $\alpha$ .<sup>3</sup> Finally, the learning ability  $\phi$  plays a role of total factor productivity. We abstract from future discounting.

The decision problem in  $t = 1$  is similar,

$$\begin{aligned}
V(h_t, \phi, m, t = 1) &= \max_{c_t, e_t \geq 0} \{\log c_t + \eta \log h_{t+1}\} \\
\text{subject to} & \\
c_t + (1 - s(m))e_t &= m \\
h_{t+1} &= \phi \left\{ (e_t/\bar{e})^\psi + (\gamma g)^\psi \right\}^{\frac{\alpha}{\psi}} h_t^{1-\alpha},
\end{aligned} \tag{2}$$

where we assume a warm-glow altruism motive for parents governed by  $\eta > 0$ . Moreover, we introduce two policy tools that have been discussed as measures to counteract the learning losses induced by school closures: (i) prolonged school periods that make up for (some) of the lost time in public schools as governed by  $\gamma > 1$  (Fuchs-Schündeln et al., 2023); and (ii) means-tested subsidies to private education (Yum, 2023), given by  $s(m_l) = s \geq 0$  and  $s(m_h) = 0$ .

## 3 Results

### 3.1 Calibration of Two Baseline Model Economies

In light of the unclear evidence on the elasticity of substitution between private and public investments in the human capital formation of children in the literature, we calibrate two versions of the baseline model: In Model 1, we set  $\psi = 0.6$ , such that the substitution elasticity is 2.5, so that private and public investments are gross substitutes, albeit imperfect ones.<sup>4</sup> In Model 2, we set  $\psi = -1$  (so that the elasticity is 0.5), implying that both inputs are gross complements, reflecting pertinent findings in the micro literature (Gelber and Isen, 2013).

In both versions of the baseline model, there are no school closures, such that  $\varsigma = 1$ , and no government interventions (i.e.,  $\gamma = 1$  and  $s(m) = 0$ ). Moreover, we set  $\alpha = 0.25$  throughout, but our qualitative results are robust to this parameter. We parameterize  $m_l = 1 - m_\delta$ ,  $m_h = 1 + m_\delta$ , and  $\phi_l = \phi_\mu(1 - \phi_\delta)$  and  $\phi_h = \phi_\mu(1 + \phi_\delta)$ . We then have five parameters,  $\{p_\phi, \eta, m_\delta, \phi_\delta, \phi_\mu\}$ , which we internally calibrate to

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<sup>3</sup>We have explored a specification that allows for strong dynamic complementarity, and our qualitative findings remain robust to this consideration.

<sup>4</sup>For example, Kotera and Seshadri (2017) estimate an elasticity of substitution of 2.43, and Arcalean and Schiopu (2010) one of 1.31 for primary and secondary education stages.

**Table 1:** Internally Calibrated Parameters

Parameter	Model 1	Model 2	Target Statistics	
	$\psi = 0.6$ (ES = 2.5)	$\psi = -1.0$ (ES = 0.5)	Description	Value
$p_\phi$	0.567	0.604	IGE	0.34
$\eta$	0.920	1.251	Avg $e/\text{income}$	0.10
$m_\delta$	0.800	0.800	Gini Adult	0.40
$\phi_\delta$	0.419	0.449	Gini Child	0.40
$\phi_\mu$	0.723	1.140	Avg $h_2$	1.00

ensure that the equilibrium distribution in both baseline model versions exactly matches the intergenerational elasticity, the ratio of average monetary investments to average income, and the Gini coefficient of incomes in the US, as summarized in Table 1. The last parameter,  $\phi_\mu$ , determines the scale of  $\phi$ , which is used to normalize the average  $h_2$  to one.

### 3.2 Aggregate and Distributional Effects of School Closures

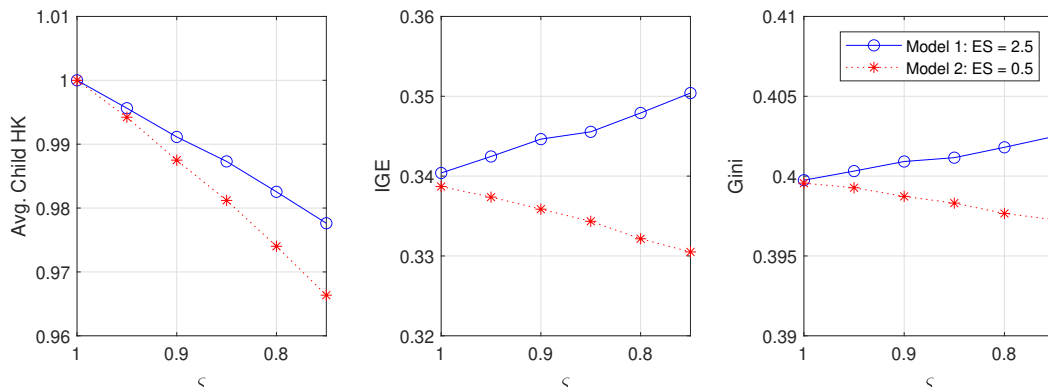
We first assess our simple model’s predictions regarding the aggregate and distributional effects of school closures in  $t = 0$  by varying the parameter  $\varsigma \leq 1$ , which lowers the public (schooling) input into the human capital formation in  $t = 0$ . We focus on three outcomes: (i) average final human capital of the child generation in  $t = 2$ ,  $h_2$ , which serves as our measure of long-run efficiency; (ii) the intergenerational elasticity, the slope coefficient from regressing  $\log(h_2)$  on  $\log(m)$ , which we take as our measure of intergenerational mobility; and (iii) the Gini coefficient of  $h_2$ , serving as our measure of long-term cross-sectional inequality for the child generation.

As shown in Figure 1, Model 1 (blue solid line) predicts that school closures lead to aggregate losses in terms of human capital, lower intergenerational mobility, and larger cross-sectional inequality. Thus, this simple model qualitatively replicates the findings of Jang and Yum (forthcoming). In contrast, Model 2 (red dotted line) predicts that, when the elasticity of substitution between private and public inputs in the production of human capital is very low, school closures can even lead to higher mobility and lower inequality, while average human capital still drops at a faster rate than in Model 1.<sup>5</sup>

Fundamentally, the different results arise because when private investments are good substitutes for lower public investments, richer parents can more easily offset the

<sup>5</sup>Jang and Yum (forthcoming) do not explore the case where public and private education investments are gross complements, as investigated in Model 2 in this study.

**Figure 1: Effects of School Closures without Mitigation Policies**



*Notes:* A longer duration of school closures is captured by a lower value of  $\zeta$ .

effects of lost schooling time by increasing their private investment than less wealthy parents. Thus, the human capital of children from different parental backgrounds diverges, which increases inequality and reinforces the correlation between child and parental economic outcomes in Model 1. If private and public inputs are more complementary in producing child human capital, however, this mechanism reverses. That is, the higher private inputs of richer parents become less effective when schools close as they are only productive when complemented with public inputs. Thus, differences between children from different parental backgrounds can even decrease, resulting in lower inequality and higher intergenerational mobility.

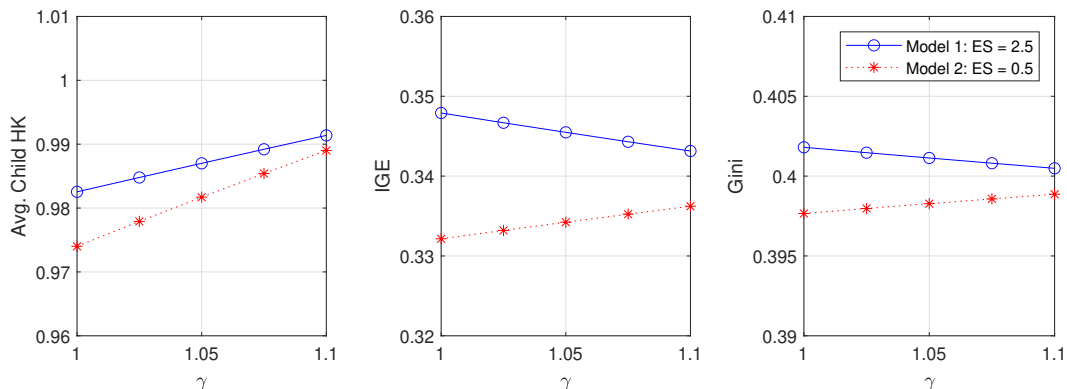
### 3.3 Mitigating the School Closure Effects

As our main exercises, we now explore the effects of two school closure mitigation policies: extra public schooling and means-tested private education subsidies. To that end, we consider different degrees of each mitigation policy in  $t = 1$  in both model versions, after all decisions in  $t = 0$  with school closures are made.<sup>6</sup> We focus on the case with  $\zeta = 0.8$ .

In the first exercise, we increase  $\gamma \geq 1$  in  $t = 1$ . Analogously to the school closures, this can be interpreted as prolonged schooling during the second education stage, for example by leaving schools open during the holidays, or by extending regular schooling days. As shown in the left panel of Figure 2, such policies indeed succeed in alleviating the human capital losses associated with school closures. This is true for both Model 1 and 2. However, while make-up schooling raises intergenerational

<sup>6</sup>Note that, under our logarithmic assumptions on the utility from  $h_2$  in (2), optimal private investments  $e_1$  are independent of  $h_1$ . For that reason, even if the policy interventions were anticipated, our results remain unchanged.

**Figure 2:** Mitigation Policy 1: Prolonged Schooling



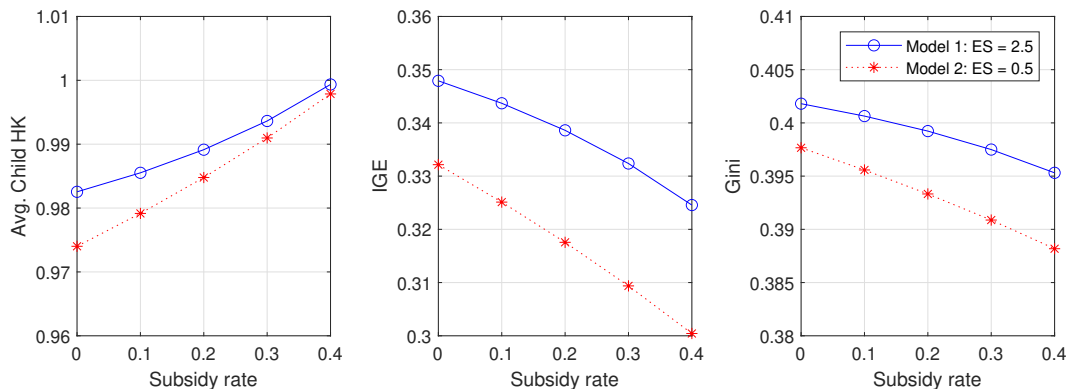
*Notes:* Prolonged schooling ( $\gamma$ ) is considered as a mitigation policy following school closures with  $\varsigma = 0.8$ .

mobility and lowers cross-sectional inequality in Model 1, it does the opposite in Model 2. This again reflects the argument that when private and public investments are substitutable, as in Model 1, children from poorer households benefit relatively more from make-up public schooling, as their parents could not compensate the learning losses resulting from closures through private inputs as effectively as richer parents. Thus, the differences in human capital between rich and poor children decrease, and the correlation between parental and child outcomes drops. In contrast, when the two inputs are complementary, children from richer households disproportionately gain from prolonged schooling as they also benefit from higher private inputs that make schooling productive. In Model 2, universal make-up schooling can therefore aggravate inequalities and hamper social mobility.

In the second exercise, we raise  $s \geq 0$  in  $t = 1$ , the subsidy rate for private education spending such as coupons for purchasing a tablet or online courses, given to parents of the low-endowment type. The effects are shown in Figure 3. In both Model 1 and Model 2, the policy successfully mitigates the average human capital losses resulting from school closures. At the same time, intergenerational mobility increases and cross-sectional inequality falls, regardless of the substitutability between public and private investments in human capital production. Thus, a means-tested private education subsidy can potentially prevent the exacerbation of inequality and adverse effects on mobility, in cases where the elasticity of substitution is especially low. Of course, in such a world, inequality and immobility after school closures would already be lower to begin with.



**Figure 3:** Mitigation Policy 2: Means-tested Private Education Subsidy



*Notes:* Education subsidies provided to low-income parents ( $s(m)$ ) are considered as a mitigation policy following school closures with  $\varsigma = 0.8$ .

## 4 Conclusion and Discussion

This paper demonstrates that the long-term consequences of school closure mitigation policies in terms of inequality and intergenerational mobility depend crucially on the elasticity of substitution between public and private investments in the human capital formation of children.<sup>7</sup> Our results illustrate that in a stark case when both inputs are complementary, untargeted mitigation policies such as universal schooling extensions may lead to the perhaps unintentional consequences of increasing inequality and lowering mobility. An important task for researchers and policymakers in the design of such policies is thus to consider how private, parental, and public schooling investments interact across different contexts, such as in the short- or long run, at different ages and educational stages of children, across different domains of skills like cognitive and non-cognitive, or in the presence of professional private education and tutoring markets.

Finally, despite our calibration, our analysis here serves the purpose of delivering these arguments mostly qualitatively. A serious quantitative evaluation of school closure mitigation policies would require a richer overlapping generations model that incorporates several further potentially important aspects. For example, a more accurate comparison of policies should take into account the financing costs of policies. In addition, a combination of the two policies we consider or a targeted prolongation of public schooling just for disadvantaged children is conceivable. These interesting and important investigations are left for future work.

<sup>7</sup>See [Glomm and Kaganovich \(2003\)](#) and [Aliprantis and Carroll \(2018\)](#) who make related points about the sensitivity of distributional or sorting outcomes to this elasticity in different contexts.

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