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Harvesting Ratings

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Abstract

Ratings play a crucial role in online marketplaces, shaping consumer decisions and firm strategies. We investigate how firms strategically use pricing to influence ratings, and how this undermines ratings as signals of product quality. We develop a two-period model of price competition between an established firm and a potentially high- or low-quality entrant, capturing the challenge high-quality newcomers face in building reputation. Consumers rate based on value-for-money, but cannot distinguish whether positive ratings result from genuine quality or discounted prices. Low-quality entrants take advantage of this and may offer low prices to harvest good ratings in the future, or mimic high prices to signal high quality. We show that ratings harvesting inflates positive ratings, reducing their informativeness. This exacerbates the cold-start problem and discourages high-quality entry. Our results mirror empirical patterns and generate implications for how rating design affects market outcomes: reducing effort costs to rate induces more but less-informative ratings, and discourages entry. Thus, actions by major marketplaces to encourage ratings could backfire and induce less precise ratings that discourage entry. To mitigate these effects, policymakers can consider balancing rating effort-costs to preserve informativeness, discouraging excessive discounts for new sellers, and incorporating price-paid into rating displays. While the effects of individual entrants' harvesting may appear temporary, harvesting hinders high-quality entrants from building reputation, discouraging entry and causing lasting distortions.

JEL: D21, D83, L10

Keywords: Ratings, digital economy, reputation, cold-start problem, biased ratings

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1 Introduction

Ratings are central to decision-making on online marketplaces. Consumers rely on ratings when choosing products, hiring freelancers, or booking accommodation because ratings are meant to summarize past consumers' experiences and signal product quality. But ratings may also reflect prices: consumers tend to rate products more favorably when they perceive better value-for-money. This creates scope for firms to strategically manipulate ratings through pricing. By temporarily lowering prices, even low-quality firms may obtain favorable ratings and subsequently benefit from an inflated reputation.

This paper studies how firms strategically use prices to shape ratings, and how this affects the informativeness of rating systems, market entry, and competition. We develop a model in which firms can engage in "ratings harvesting": pricing aggressively to generate favorable ratings that do not necessarily reflect product quality. We show that ratings harvesting inflates reputations, reduces the ability of ratings to distinguish high- from low-quality firms, and worsens the cold-start problem faced by newcomers.

Empirical evidence suggests a two-way relationship between ratings and prices. Higher-rated firms charge higher prices, consistent with ratings conveying information about quality (Cabral & Hortaçsu, 2010; Jin & Kato, 2006; Reimers & Waldfogel, 2021). At the same time, lower prices improve ratings (Carnehl et al., 2025; Li & Hitt, 2010; Luca & Reshef, 2021), indicating that consumers evaluate products relative to value-for-money. If ratings depend partly on prices, firms may strategically manipulate ratings through pricing decisions.

To capture how prices affect ratings, we assume consumers leave positive (negative) ratings when realized value-for-money is sufficiently above (below) their outside option. This formulation is consistent with evidence that prices shape evaluations and with mechanisms such as reciprocity (Fradkin et al., 2021; Rabin, 1993), and that consumers are more inclined to rate when they experience more extreme outcomes.

We analyze a two-period model of entry and price competition between an incumbent and a newcomer with privately known quality. Consumers observe current prices and past ratings, but cannot infer whether favorable ratings reflect high quality or low prices. After consumption, consumers learn realized quality and may leave a rating. Importantly, leaving a rating is costly: consumers incur an effort cost, so only some consumers find it worthwhile to rate. As a result, ratings can be informative, but they are endogenous to both realized value-for-money and the decision to incur rating effort. This friction plays a central role in shaping how prices translate into observed ratings.

The model captures online marketplaces such as Amazon, Airbnb, and freelance platforms, where entry and reputation formation are central market features. Entry is quantitatively important in these settings: platforms regularly experience substantial turnover, with frequent entry of new sellers and providers alongside exit of existing ones (Dendorfer & Seibel, 2024; Farronato & Fradkin, 2022). At the same time, entrants face a well-documented cold-start problem, as they must build reputation from scratch in environments where ratings are both highly influential and potentially noisy in early stages (Dendorfer & Seibel, 2024; Hui et al., 2024; Pallais, 2014). Our framework formalizes how strategic pricing distorts this early reputation formation and thereby affects entry incentives.

Our key mechanism is a trade-off faced by low-quality newcomers. They can either: (i) engage in "ratings harvesting" by setting low prices to induce favorable ratings, or (ii) engage in "price mimicking" by charging prices similar to high-quality newcomers.

Ratings harvesting improves future ratings through better value-for-money, but also reduces the informativeness of the rating system because favorable ratings increasingly reflect low prices rather than high quality. By contrast, price mimicking generates worse consumer experiences and less favorable ratings, allowing consumers to better distinguish quality over time.

Our key trade-off generates a central equilibrium implication: ratings harvesting reduces the informativeness of ratings, and this distortion in turn shapes entry and competition.

When low-quality firms engage in more ratings harvesting, favorable ratings increasingly reflect low prices rather than high underlying quality. As a result, ratings become less reliable signals of quality differences between newcomers. This reduces the informativeness of the rating system in equilibrium.

A major implication is that ratings harvesting worsens the *cold-start problem*—the difficulty newcomers face in building reputation. We show that entry occurs only when ratings remain sufficiently informative. Harvesting erodes the informational value of ratings so much that consumers become reluctant to buy from newcomers, even when they receive a good rating. This discourages entry, including by high-quality firms that cannot credibly distinguish themselves from low-quality firms. Thus, ratings harvesting simultaneously inflates reputations and discourages entry.

These results have important implications for platform design. Because ratings harvesting reduces the informativeness of ratings, platform policies that unintentionally encourage harvesting may worsen the cold-start problem and distort entry.

We then study how platform design affects incentives for ratings harvesting. Many platforms actively try to increase the number of ratings by reducing the effort required to leave feedback. For example, Amazon replaced its earlier written-review requirement with a one-click rating system, arguing that more ratings would "more accurately [...] reflect the experience of all purchasers."[1] We show that this reasoning overlooks firms' strategic responses. When ratings become easier to leave, low-quality firms benefit more from ratings harvesting because favorable experiences induced by temporary discounts are more likely to translate into positive ratings. As a result, lowering rating effort costs can increase the quantity of ratings while reducing their informativeness.

[1] Quote from Rey (2020), vox.com. Prior to 2020, Amazon required at least 20 written words per review; see (Amazon Customer, 2012; crebel, 2017).

Evidence supports that this mechanism is relevant. Cabral and Li (2015) show that paying eBay buyers $1 to leave a rating—compensating them for rating effort—lowers negative ratings by 22%.

Taken together with broader evidence that platforms have increasingly facilitated the rating process over time, these results suggest that platforms may be engaged in a race toward less-informative ratings that discourage entry. Hence, platforms seeking to preserve the quality of their rating systems and encourage entry may need to re-balance efforts that indiscriminately encourage rating.

Increasing rating effort is not the only way platforms can discourage harvesting. Incorporating paid prices into rating systems can mitigate ratings harvesting by helping consumers distinguish whether favorable ratings stem from high quality or low prices. By making the source of positive ratings more transparent, such systems improve rating informativeness without discouraging ratings for high-quality firms.

Finally, we study implications for competition and welfare. More informative ratings encourage entry and intensify competition by helping high-quality newcomers build reputation. At the same time, conditional on entry, informative ratings soften price competition by increasing product differentiation. When designing rating systems, platforms therefore face a trade-off between encouraging entry and relaxing competition.

A key implication of our results is that ratings harvesting can generate persistent and economically meaningful distortions. By impeding early reputation-building, it creates a reputational bottleneck that affects entry and market outcomes even if ratings eventually become informative. Given high entry and exit rates on major platforms, these early distortions can have substantial and persistent effects.

Section 2 connects our results to the literature. We introduce the basic model in Section 3, and discuss the equilibrium in Section 4. Section 5 shows how various features in the rating system influence how well ratings reflect quality. We then discuss implications on surplus in Section 6. We discuss extensions and robustness in Section 7. In particular, we extend our results to a three-period model to illustrate how results extend to longer time horizons. We show that harvesting induces low-quality newcomers to stay longer in the market, reinforcing the cold-start problem. In Section 8, we discuss the implications for rating management. Section 9 concludes. All proofs and extensions are in the Online Appendix.[2]

2 Related Literature

Our key novelty, which we have not seen elsewhere, is that we endogenize if and how consumers rate based on value-for-money to study how firms price to free-ride on the reputation of others. Based on this mechanism, we derive novel predictions for how informative ratings are, entry and

[2] The Online Appendix is available at https://robinng.com/research/Harvesting_Ratings_Online_Appendix_B.pdf.

the cold-start problem, competition and surplus allocation, and the design of rating systems.

We connect to the wider theoretical literature on **trust and information transmission in the digital economy**. Platforms may recommend products (Benkert & Schmutzler, 2024; Hagiu & Jullien, 2011; Ng, 2025; Peitz & Sobolev, 2022), shroud additional fees and features of third-party sellers (Johnen & Somogyi, 2024), and marketplaces may have fake reviews (He et al., 2022). We contribute by studying information transmission via ratings, and how firms can use prices to affect their own ratings.

A growing literature studies the **cold-start problem** (Bergemann & Välimäki, 1997, 2000; Che & Hörner, 2018; Kremer et al., 2014; Vellodi, 2018). In existing models, newcomers and their consumers have symmetric information. So newcomers may offer discounts to encourage experimentation, but they cannot distort the type of signal that is generated. Our key contribution here is that newcomers have private information about quality, which seems a reasonable feature in many markets. This induces rating harvesting and its various novel implications, i.e. that harvesting makes ratings less precise, discourages entry also of high-quality newcomers, and makes it harder to build a reputation.

We contribute to the **theoretical literature on reputation** (Bar-Isaac & Tadelis, 2008; Cabral, 2000; Holmström, 1999; Hörner, 2002; Jullien & Park, 2014; Kovbasyuk & Spagnolo, 2024; Martin & Shelegia, 2021; Tadelis, 1999, etc.), and **word-of-mouth** (Chakraborty, Deb, et al., 2023). In existing work usually (i) buyers do not endogenously choose if and how to rate, and (ii) ratings mostly reflect quality, and prices do not affect how consumers rate. While some papers relax some of these assumptions (e.g. Chakraborty, Deb, et al. (2023) and Martin and Shelegia (2021) relax (i), Carnehl et al. (2024) and Sobolev et al. (2021) relax (ii)), no article seems to feature both that buyers choose strategically if and how to rate, and sellers price to free-ride on the ratings of others. So our results on rating harvesting and its various implications are new.

Our model features an extensive and an intensive margin for ratings. How consumers rate (intensive margin) depends on whether their value-for-money is positive or negative—i.e. whether low-quality firms mimic prices or harvest ratings—and if they rate (extensive margin) on whether it is sufficiently extreme relative to their effort cost of rating. Many existing articles focus on either of the two. E.g. Aleksenko and Kohlhepp (2025), Hui et al. (2024) and Sobolev et al. (2021) focus on the extensive margin. As in Hui et al. (2024), our extensive margin results from continuously distributed effort-costs to leave a rating. However, since we have endogenous prices, we also have an intensive margin. Martin and Shelegia (2021) focus on the intensive margin. Since we feature both, we can make novel predictions about how lowering the rating effort leads to more, but also less informative ratings.

Recent models incorporate different drivers for ratings. According to the surprise hypothesis, the difference between expected and actual quality drives ratings (Martin & Shelegia, 2021). In Hui et al. (2024), users rate more if they learn more from the experience. Our model follows other recent articles (Carnehl et al., 2024) and focuses on value-for-money. But since the purchase decision depends on expected quality, our equilibrium captures aspects of the other two hypotheses: in equilibrium, high-quality products induce a better-than-expected experience and the highest value-for-money, so they get the best ratings more often. Conversely a low-quality product induces a (weakly) worse-than-expected experience and lower value-for-money, and therefore worse ratings.

Maybe the first theoretical article on how **value-for-money** affects ratings is Carnehl et al. (2024). They focus on prices in long-run equilibria where ratings transmit precise information about quality. Instead, we focus on the shorter-run challenge of newcomers to establish reputation after entry, so both approaches are highly complementary. In particular, we study how firms can harvest ratings to free-ride on the reputation of other sellers, biasing ratings also on the path of play. Also in Sobolev et al. (2021) ratings may be a noisy signal of quality on the path of play. But their mechanism is very different: they start with the premise that more sales can lead to more or less precise ratings, e.g. because the additional raters might know the products better or worse than existing raters. Aleksenko and Kohlhepp (2025) study when a high-quality monopolist underprices to build reputation, so that ratings are always good news. In contrast to these papers, we study how firms price their products to free-ride on the reputation of others, and we provide novel insights about how the design of rating environments leads to more informative ratings.

Some researchers argue that consumers should get **paid to rate**. One argument is that sellers should be allowed to pay for feedback: because high-quality firms are more inclined to pay for feedback, feedback is a credible signal for quality (Halliday & Lafky, 2019; Kihlstrom & Riordan, 1984; Milgrom & Roberts, 1986; Nelson, 1974). Others argue that feedback is like a public good that is underprovided (Avery et al., 1999; Bolton et al., 2004; Chen et al., 2010). In contrast, we show that encouraging feedback via ratings encourages low-quality firms to harvest ratings, leading possibly to more ratings, but also less-informative ratings, and discouraging entry. This result is in line with evidence by Cabral and Li (2015) which we discuss above.

We provide a theoretical explanation for why identical products may get **different ratings across platforms** (Chevalier & Mayzlin, 2006). Some evidence suggests that this is due to user self-selection onto marketplaces (Granados et al., 2012; Raval, 2020). We provide a complementary explanation and show that differences in features of the rating system can lead to different ratings for identical products. Our results align with experimental evidence on how the design of rating systems can influence ratings (Lafky & Ng, 2024; Schneider et al., 2021).

We contribute to the literature on **consumer information about differentiated products**. Prior work shows that firms benefit from well-informed consumers, as this strengthens product differentiation and relaxes competition (Anderson & Renault, 2006; Armstrong & Zhou, 2022; Hefti et al., 2022; Johnen & Leung, 2025). By contrast, we show that with entry and rating harvesting, firms may prefer less informative ratings than consumers to deter entry.

3 Basic Model

We set up a two-period model of incomplete information where a newcomer competes with an established firm over a consumer in each period.

Firms. The established firm A has quality $q^A > 0$, which is common knowledge. The newcomer B has type $B \in \{h, l\}$ with qualities $q^h > q^A > q^l$, where $\Pr(B = h) = \gamma \in (0, 1)$ and $\Pr(B = l) = 1-\gamma$. This distribution is common knowledge. The newcomer privately observes its realized quality, which remains constant across periods. To simplify illustration and focus on the striking case in which the low-quality newcomer may sell despite producing no value, we assume $q^l = 0$.

After firms learn their quality and before setting prices, the newcomer chooses whether to enter. To model entry parsimoniously, we assume the newcomer enters if and only if it would attract strictly positive demand.

In each period $t \in \{1, 2\}$, firm $j \in \{A, B\}$ sets price p_t^j to maximize lifetime profit $\sum_{t=1}^{2} p_t^j \cdot d_t^j$, where $d_t^j \in \{0, 1\}$ denotes demand in period t. We assume zero marginal cost for both firms regardless of quality.[3] After a sale in period 1, the seller may receive a consumer rating R_1. This rating becomes common knowledge in all subsequent periods.

Consumers. Each consumer participates in exactly one period, and a new consumer arrives in each period. In period t, consumers observe current prices, past ratings, and q^A, but not the newcomer's quality nor the price paid by previous buyers. They choose whether to buy; if they purchase, they observe realized quality and then decide whether to leave a rating.

Consumers choose if and how to rate, i.e. $R_t \in \{1, 0, -1\}$. The informational content of a rating is determined in equilibrium, but we say a rating is positive if $R_t = 1$, negative if $R_t = -1$, and when $R_t = 0$ consumers choose not to rate. Without loss of generality, a newcomer starts with no prior rating. For brevity, we sometimes drop the subscript and write R for R_1.

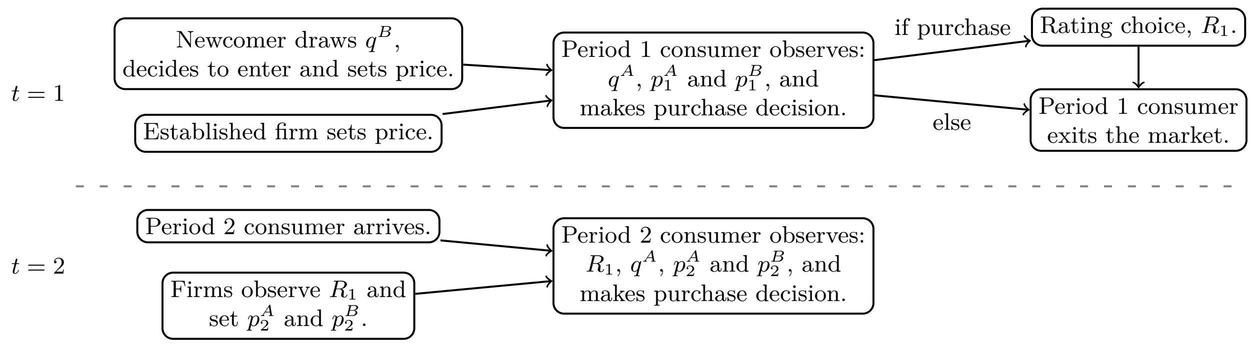
Throughout, we focus on ratings for the newcomer. Because the established firm's quality is common knowledge, ratings do not affect beliefs about it. This captures evidence that an additional positive rating boosts sales for newcomers with few ratings but has little effect on established firms' sales (Dendorfer & Seibel, 2024; Hollenbeck, 2018; Hui et al., 2024; Livingston, 2005; Luca & Zervas, 2016; Resnick et al., 2006).

We distinguish consumption utility from rating utility for two reasons. First, this captures evidence that consumers do not incorporate the intention to rate into their purchase decision.[4] Second, it simplifies the presentation of results. Consumption utility from buying from firm j in period $t \in \{1, 2\}$ is $u_t = q^j - p_t^j$, and we normalize the utility of the outside option to zero.

[3] We assume zero marginal cost to focus on information transmission via ratings. With sufficiently different marginal costs, cost-based signaling may arise as in Bagwell and Riordan (1991).

[4] Cabral and Li (2015) find that incentivizing consumers to rate does not change their willingness to pay.

Figure 1: Timing of the game.



Rating utility captures consumers' incentives to rate. In period t,

$$v_t = \begin{cases} (q^j - p_t^j) - e & \text{if } R_t = 1, \\ -(q^j - p_t^j) - e & \text{if } R_t = -1, \\ 0 & \text{if } R_t = 0, \end{cases}$$

where $e \sim F[0, \bar{e}]$ is the time and effort cost of rating. We assume F is uniform, i.e. $e \sim U[0, \bar{e}]$, and that $\bar{e}$ is large enough that some consumers do not rate. Hence, consumers leave a positive (negative) rating when value-for-money is sufficiently far above (below) their outside option: $R_t = 1$ if $q^j - p_t^j \geq e$, $R_t = -1$ if $-(q^j - p_t^j) \geq e$, and $R_t = 0$ otherwise.

We provide two foundations for this mechanism. First, the rating utility is a parsimonious reduced form of intrinsic reciprocity (Dufwenberg & Kirchsteiger, 2004; Rabin, 1993): consumers reciprocate sufficiently high (low) value-for-money with a positive (negative) rating. Second, ratings can reflect self-expression: consumers rate when they feel good or bad about a purchase, and firms can induce these feelings through the value-for-money they offer.

Figure 1 summarizes the timing of the game.

Equilibrium and Restrictions. To simplify exposition, we restrict attention to $\gamma q^h < q^A$, so newcomers do not sell absent a rating system.[5]

We study perfect Bayesian equilibria and impose the following restrictions. We impose selection assumptions to rule out equilibria sustained by pessimistic off-path beliefs that deter entry.

Selection Assumption 1. [Entry] *We select equilibria without entry if and only if there exists no equilibrium where at least one type of newcomer enters.*

This rules out no-entry equilibria sustained by pessimistic off-path beliefs, e.g. that any entrant has quality q^l, and therefore would not attract demand. Because our focus is entry and competition, we

[5] Without this assumption, newcomers may always enter in equilibrium.

select a no-entry equilibrium only when no equilibrium with entry exists. Relaxing this assumption can only reduce entry, so any conclusions that high-quality newcomers do no enter enough are strengthened if the assumption is dropped.

The next selection assumption rules out equilibria where firms are not competing in the spirit of Bertrand competition with vertical differentiation.

Selection Assumption 2. [Competition after Entry]

(a) If entry occurs, consumers are indifferent between the newcomer and the established firm, and they purchase from the firm that earns strictly larger marginal profits from that sale.

(b) If entry occurs, offers with zero demand are optimal also if a negligible share of indifferent consumers would purchase them.

This assumption selects equilibria with effective post-entry competition. Part (a) ensures firms can undercut rival's prices when profitable. In particular, it excludes equilibria in which pessimistic off-path beliefs prevent newcomers from profitably undercutting the incumbent.[6] Similarly, it excludes equilibria in which consumers strictly prefer the newcomer, but the newcomer does not raise its price because pessimistic off-path beliefs would destroy demand at higher prices.[7] Part (b) rules out non-credible offers by firms with zero demand, that is, offers that would strictly reduce profits if even a negligible mass of indifferent consumers accepted them.

The next restriction also ensures effective Bertrand competition with vertical differentiation. We assume the p.d.f. of the rating-effort distribution F is sufficiently flat.[8] This ensures firms that sell choose the highest price that still wins demand. Otherwise the period 1 consumer could strictly prefer the newcomer at the equilibrium price, meaning that firms would not be competing.

Discussion of Modeling Assumptions. Our model applies to platforms such as Airbnb, Amazon, Taobao, eBay, Yelp, and Google Reviews, where consumers heavily rely on ratings to form expectations about product quality.

First, our rating utility captures growing evidence that ratings are driven by value-for-money rather than quality alone. The effect can be substantial: in digital camera markets, a 1% price increase reduces ratings by 0.36 stars (on a 5-star scale) and by 0.71 stars (on a 10-star scale) (Li & Hitt, 2010). On Airbnb, higher prices reduce ratings (Gutt & Kundisch, 2016; Neumann et al., 2018);

[6] In such equilibria, consumers may buy from both the incumbent and the newcomer with positive probability, even though only the newcomer earns profits. In competitive markets à la Bertrand, this cannot occur, as the profitable firm would undercut its rival. Here, however, such outcomes can be sustained by beliefs that any lower off-path price is set by newcomer L.

[7] Since in such equilibria, consumers strictly prefer the newcomer over the incumbent, they are also not in the spirit of Bertrand competition with vertical differentiation, where such price increases would occur.

[8] A sufficient condition is $f(x) = \frac{1}{\overline{e}} < \frac{1}{\pi_2(R=1)}$ for all x, where $\pi_2(R = 1)$ denotes period 2 profit conditional on a positive rating. This condition holds if $\overline{e}$ is sufficiently large. The weaker conditions used in the proofs are (1), (2), and (3).

on Yelp, a 1% price increase lowers average ratings by 3–5% (Luca & Reshef, 2021); and in hotels, a 1% price increase reduces ratings by roughly one star (on a 10-star scale) (Abrate et al., 2021). Because these studies control for product quality, they suggest that value-for-money—not quality alone—is a key driver of consumer ratings.

Second, ratings are coarse: since consumers rarely observe the exact price paid by the rater, consumers typically cannot infer whether a high rating reflects high quality or a low past price.[9]

Third, our two-period entry model captures, in reduced form, why ratings are especially important—and arguably less informative—for newcomers. Reimers and Waldfogel (2021) find that book ratings affect consumer surplus about ten times more than New York Times reviews, largely because many genres and titles have few reviews, so even a small number of ratings can influence demand. Ratings are therefore central to early reputation building. Consistent with this, the marginal effect of a positive rating on sales is large for the first 20–30 reviews but diminishes thereafter (Dendorfer & Seibel, 2024; Hui et al., 2024). Even if ratings eventually reveal quality, this creates an early-stage asymmetry: entrants with few or no reviews face an uphill battle against established sellers. Our model targets this critical phase in which newcomers struggle to build a reputation.

Finally, online job platforms such as Freelancer.com and Upwork provide another natural application for our framework. Workers compete directly for jobs, public reputation systems shape hiring and visibility, and entrants face a cold-start problem relative to established workers.[10] A growing literature shows that reputation affects hiring and pricing in online freelance markets (Lin et al., 2018; Moreno & Terwiesch, 2014; Yoganarasimhan, 2013), and that early public feedback improves workers' later employment outcomes (Pallais, 2014).

4 Equilibrium

We now characterize equilibrium and summarize the main results in three steps. First, we introduce notation and impose parameter restrictions to focus on the equilibrium of interest. Second, we characterize equilibrium conditional on entry by both newcomer types. Third, we present the main results, including the newcomer's entry decision.

We now introduce notation. The key driver of our mechanism is the low-quality newcomer's period 1 pricing: in equilibrium it chooses one of two prices. Let δ^* denote the equilibrium probability that a low-quality newcomer mimics the high price $\overline{p}$ charged by high-quality newcomers. With probability $1 - \delta^*$ it sets a lower "harvesting" price $\underline{p}$ to increase value-for-money and hence the likelihood of a positive rating.

[9] Raters may comment on price (e.g., "good product for that price"), but rarely report the exact amount paid. Consumers may consult external databases for historical prices, yet it is difficult to map those to the specific prices past raters paid.

[10] For current platform documentation, see Freelancer.com's pages on ratings and Upwork's documentation on Job Success Scores; these sources illustrate how reputation metrics are displayed and used on major platforms (Freelancer.com, 2026; Upwork, 2026).

Equilibrium is unique up to off-path beliefs. To simplify the exposition, we impose two conditions on parameters to focus on the equilibrium of interest. Both require that a positive rating is sufficiently valuable, which holds for large enough q^h. Online Appendix A characterizes equilibrium when they do not hold.

First, we focus on parameter regions where the low-quality newcomer *mixes*, i.e. $\delta^* \in (0, 1)$. Otherwise it mimics $\overline{p}$ with probability one. But when reputation is sufficiently valuable ("large" q^h), the low type also harvests ratings, yielding the mixed-strategy equilibrium.[11]

Second, we focus on the "*silence is bad news*" region: empirically, newcomers with no reviews struggle to attract demand (Bolton et al., 2013; Cabral & Hortaçsu, 2010; Dellarocas & Wood, 2008; Dendorfer & Seibel, 2024; Hollenbeck, 2018; Hui et al., 2024; Luca & Zervas, 2016; Nosko & Tadelis, 2015; Resnick et al., 2006; Tadelis, 2016). Accordingly, we impose a sufficient condition under which a newcomer who receives no rating in period 1 does not attract demand in period 2.[12] This condition holds for sufficiently large q^h: high-quality entrants receive positive ratings so often that "no rating" carries too little reputation to induce a sale.

We now characterize equilibrium. First, we show how ratings shape beliefs and period 2 competition: conditional on entry, a positive rating creates a reputation premium that lets the newcomer win the period 2 market, whereas a non-positive rating does not. Second, we characterize period 1 pricing.

Period 2: Reputation premium. Suppose both types of newcomer enter. The following lemma characterizes period 2 competition.

Lemma 1. *There exists an equilibrium that is unique up to off-path beliefs. Suppose both types of newcomer enter. Then in period 2:*

1. ***Ratings build reputation:*** $E[q_2^B \mid R = 1] > E[q_2^B \mid R = 0] > E[q_2^B \mid R = -1]$.
2. ***Ratings are valuable:*** $p_2^B = E[q_2^B \mid R = 1] - q^A > 0$. *Firm A sells in period 2 if and only if* $R \in \{-1, 0\}$.[13]

Good ratings help newcomers build reputation and earn higher profits in period 2, because high-quality entrants deliver better value-for-money and therefore receive good ratings more often. The intuition has two steps. First, Bertrand-type competition implies that in period 1, customers of the newcomer must ex-ante expect the same utility from the newcomer as from the incumbent who prices at marginal cost, i.e. a utility of q^A. Second, in period 1 the low-quality newcomer mimics the high price $\overline{p}$, so both types of newcomer charge $\overline{p}$ with strictly positive probability. But since $q^h > q^l$, high-quality newcomers deliver higher ex-post utility and thus obtain better ratings

[11] The precise condition is Condition (6) in Online Appendix A, Proposition 4.

[12] The precise condition is Condition (5) in Online Appendix A, Proposition 4. If the condition does not hold, a qualitatively similar equilibrium exists but the newcomer may also sell after $R = 0$; we treat this case in Online Appendix A.

[13] The exact expressions for these expectations and period 2 prices can be found in Online Appendix A Lemma 11.

$(q^h - \overline{p} > q^A > q^l - \overline{p})$. Consumers therefore infer that positive ratings signal quality, enabling positively rated newcomers to earn higher profits in period 2. Moreover, under our "silence is bad news" condition, a positive rating is required for the newcomer to sell in period 2.

Period 1: Price mimicking vs rating harvesting. Lemma 1 implies that the newcomer's continuation value hinges on obtaining a positive rating, so period 1 pricing trades off current profits against the probability of generating $R = 1$. The next lemma characterizes period 1 pricing.

Lemma 2. *Suppose both types of newcomer enter. There exists a unique $\delta^* \in (0, 1)$, such that in period 1:*

1. ***Firm** A sets $p_1^A = 0$ and gets no demand. **Firm** h sets $\overline{p} = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A$ and gets $R \in \{0, 1\}$.*
2. ***Firm** l randomizes over prices:*
   (a) *It charges $\overline{p} > 0$ with probability δ^* and receives $R \in \{-1, 0\}$.*
   (b) *It charges $\underline{p} \equiv -q^A < 0$ with probability $1 - \delta^*$ and receives $R \in \{0, 1\}$.*

If a newcomer enters, it must sell in period 1; otherwise it cannot obtain a positive rating and, by Lemma 1, will not sell in period 2 either. Hence, conditional on entry, Bertrand competition drives the incumbent to price at cost and make no sale in period 1.

Entrants choose between two pricing strategies. The high-quality entrant always charges the high price $\overline{p}$, i.e., the highest price at which consumers weakly prefer the entrant to the incumbent. If types were observable, this price would equal the entrant's quality advantage $q^h - q^A$. However, since the low type can mimic by charging the same price, consumers anticipate pooling with positive probability, which lowers $\overline{p}$. The low price $\underline{p}$ instead makes even a low-quality entrant attractive when consumers correctly expect quality q^l: it incurs a loss in period 1 but raises the chance of a positive rating and thus increases period 2 profits.

This sets up the key trade-off faced by low-quality newcomers in period 1:

Price Mimicking: Charging the high price $\overline{p}$ to imitate high-quality firms. However, since this yields lower value-for-money, the firm obtains worse ratings and thus earns lower profits in the next period (Lemma 2, Point 2a).

Rating Harvesting: Charging the low price $\underline{p} = -q^A < q^l = 0$ to induce a positive rating. A good rating allows the firm to free-ride on the reputation of high-quality entrants and charge a higher price in period 2 (Lemma 2, Point 2b).

The probability that a low-quality firm chooses price mimicking over rating harvesting—denoted δ^*—captures how firms resolve this trade-off in equilibrium. Why do low-quality firms mix prices? Intuitively, they harvest ratings to free-ride on the reputation of high-quality entrants. For this to be profitable, reputation must be sufficiently valuable, which is the case we focus on in the main text. However, as more low-quality firms engage in rating harvesting, the equilibrium beliefs associated with a positive rating deteriorate. This weakens the incentive to harvest, until firms are indifferent between harvesting and mimicking—hence the emergence of a mixed strategy.

Remark 1. The fact that low-quality newcomers may charge a negative price $\underline{p}$ is only an artifact of normalizing marginal cost and q^l to zero. In general, the results do not predict negative prices, but negative margins to build reputation.

Remark 2. For tractability, we assume that consumers have homogeneous preferences, which leads to fierce competition. With (vertically or horizontally) differentiated preferences, competition would be less intense in period 1. This could also induce sales of incumbents in period 1 despite entry.

4.1 Pricing, Informative Ratings, and Entry

Our equilibrium illustrates the two-way relationship of prices and ratings highlighted in the Introduction: lower prices allow firms to build a reputation, enabling them to charge higher prices in the future. In equilibrium, all firms that receive a good rating raise their prices in period 2.

We begin with baseline comparative statics of δ^*. First, $\frac{\partial \delta^*}{\partial q^h} < 0$: higher quality leads to more harvesting. Intuitively, an increase in q^h raises both the value of price mimicking and of harvesting, but the latter effect is stronger. Higher quality increases $\overline{p}$ and the value of a good rating directly; but it also induces high-quality entrants to offer greater value for money $(q^h - \overline{p})$, further increasing the returns to a good rating. As a result, harvesting becomes relatively more attractive, and low-quality entrants engage in it more. Second, the effect of γ on δ^* is ambiguous. While a higher γ increases the likelihood of a high-quality entrant, it also reduces the value for money they offer $(q^h - \overline{p})$, making the net effect on harvesting unclear.

In the remainder of the paper, we often study how outcomes vary with δ^*. While δ^* is endogenously determined in equilibrium, we analyze its variation directly as a shortcut. The reason is that primitives such as q^h and γ affect outcomes of interest not only through δ^*, but, as the above comparative statics show, also directly (i.e., consumer expectations or the probability to get a rating), so their comparative statics do not isolate the effect of informativeness on these outcomes of interest. In principle, one could introduce additional primitives that shift first-period profits—and hence δ^*—without directly affecting other objects of interest. For brevity, we instead vary δ^* directly.[14]

Pricing and Informativeness of Ratings. The equilibrium links firms' pricing strategies with the informativeness of ratings. Specifically, for larger δ^*, low-quality firms more frequently mimic high-quality newcomers' pricing, making them less likely to receive positive ratings. After consumers adjust their expectations, ratings become more indicative of quality. Thus, changes in the firms' pricing strategy δ^* affect how much information ratings convey. This yields a key insight: rating harvesting reduces the informativeness of ratings. The proposition formalizes this:

[14] Examples include: (i) an exogenous probability that dissatisfied consumers receive a refund, (ii) capital costs for charging negative prices, or (iii) a discount factor governing the relative weight of the two periods.

Proposition 1. *Suppose both types of newcomer enter. If* δ^* *increases, then* $E[q_2^B \mid R = 1]$ *increases strictly.*

This result implies that rating harvesting leads to rating inflation—the phenomenon where most ratings cluster at the top of the scale, e.g. 5 out of 5 stars (Filippas & Horton, 2022; Filippas et al., 2022; Nosko & Tadelis, 2015; Zervas et al., 2021). A central concern with rating inflation is that it weakens ratings' ability to distinguish quality. Our model reinforces this concern: rating harvesting increases the number of positive ratings, thereby diluting their informational content.[15]

Entry and the Cold-start Problem. Entry and exit are major features of online platforms. On Airbnb, for instance, Dendorfer and Seibel (2024) report monthly entry and exit rates of hosts of 3–4%. Farronato and Fradkin (2022) find substantial supply elasticities in this market, suggesting entry responds to changing market conditions. Amazon also experiences significant seller turnover.[16]

To understand implications of rating harvesting on entry, we now characterize the entry decision of newcomers.

Proposition 2. *There exists a unique* $\underline{\delta} \in (0, 1)$ *such that both types of newcomers enter if and only if* $\delta^* > \underline{\delta}$*. Otherwise, neither enters.*

Proposition 2 provides new insight into how ratings influence entry decisions. Rating harvesting discourages entry: newcomers enter if and only if there is not too much harvesting, i.e., if $\delta^* > \underline{\delta}$. In other words, newcomers enter if and only if positive ratings are sufficiently informative to induce future sales. Clearly, if they enter, they must get positive ratings sometimes. Otherwise, if they do not sell with a positive rating in period 2—i.e. the best reputation they could have—they will also not sell with any lower reputation. But then they never sell and do not enter. In turn, if newcomers get positive ratings, they must enter: In the above mixed-strategy equilibrium this is immediate, as newcomers sell at weakly positive prices with strictly positive probability in each period. Thus, informative ratings foster entry and help high-quality newcomers gain traction.

Empirical evidence supports this mechanism. Luca (2016) and Hollenbeck (2018) show that Yelp ratings spur entry by small, independent restaurants, intensifying competition for large chains. Similarly, Leyden (2025) shows that when Apple stopped resetting average App Store ratings after product updates, developers released more upgrades—suggesting that more informative ratings encouraged participation.

Our analysis reveals that it is difficult to deter entry only of low-quality sellers. That is, there is no equilibrium in which only high-quality firms enter. If that were the case, consumers would expect high quality from all newcomers, encouraging low-quality firms to enter and exploit those

[15] Note that our result also holds when we allow for a wider parameter range such that we can have pure strategy equilibria with $\delta^* = 1$. Then $\delta^* = 1$ induces the most informative ratings.

[16] www.marketplacepulse.com, accessed June 5, 2025, reports 4 million new sellers on Amazon from 2020 to 2024.

expectations.[17] Free-riding is thus a robust feature of equilibrium.

These results are closely related to the cold-start problem on product platforms (Dendorfer & Seibel, 2024; Li et al., 2020) and online job platforms (Pallais, 2014), in which newcomers struggle to build reputation—even when they offer higher quality than incumbents.

A key takeaway is that rating harvesting exacerbates the cold-start problem in two ways. First, when $\delta^* < \underline{\delta}$, a good rating is no longer a strong-enough signal of quality to induce sales in period 2. Consumers prefer to buy from incumbents, and high-quality newcomers are discouraged from entering. Second, even when entry occurs, increased rating harvesting lowers the value of a good rating, reducing period 2 profits for high-quality firms (by Proposition 1).

Platforms are acutely aware of the cold-start problem and often encourage sellers to offer steep discounts to build a reputation. Airbnb, for example, recommends that new hosts offer a 20% discount to their first guests (Dendorfer & Seibel, 2024). Amazon permits sellers to offer discounted products in exchange for ratings and reviews.[18] Our model suggests that low-quality entrants are especially likely to pursue this strategy—further undermining the informativeness of ratings.

Instead, our findings suggest that platforms should discourage rating harvesting if they aim to promote entry. While this will not fully prevent low-quality entry, it ensures that such firms are sorted out more quickly. In the next section, we discuss how platform design can reduce rating harvesting and improve the informativeness of ratings.

5 Designing Ratings Environments

In this section, we examine how the rating environment influences the informativeness of ratings. We identify two strategies to discourage rating harvesting: (i) linking ratings to the prices raters paid, and (ii) increasing the cost of leaving a rating.

Link Ratings with Paid Prices. While platforms typically provide easy access to past ratings, they do not connect these ratings to the prices that raters actually paid.[19] We show that this is a key driver of rating harvesting: consumers cannot tell whether a rating reflects high product quality or simply a low purchase price. If consumers in period 2 knew what raters paid, they could distinguish genuine high-quality firms from those using steep discounts to harvest ratings. This would eliminate the incentive to harvest ratings. In equilibrium, low-quality firms would then mimic high-quality pricing with probability 1, and ratings would become more informative.

[17] Indeed, even in the pure-strategy equilibrium, firm h enters if and only if firm l enters with strictly positive probability.

[18] See https://sell.amazon.com/tools/vine, accessed June 5, 2025.

[19] Amazon does not reveal historical prices in product listings, nor do they disclose the price a reviewer or rater paid. Third-party sites like https://camelcamelcamel.com/ and https://keepa.com/ track price histories, but they do not link prices to individual ratings, and thus cannot reveal whether price influenced a given rating.

Facilitating Ratings. Platforms can encourage ratings by adjusting the effort required to leave them. Verification steps, multi-dimensional evaluations, rewards, or rebates and reminders all affect the cognitive and time costs associated with rating. These design choices directly influence the "cost of rating", which we model via the upper bound $\overline{e}$ of the effort-cost distribution. Changes in $\overline{e}$ induce first-order stochastic dominant shifts in the cost distribution.

At first glance, making ratings easier seems desirable. Holding firm behavior fixed, lower rating effort costs yield more ratings. However, this intuition is misleading, as it ignores how firms adjust their pricing in response.

In our setting, lower rating costs induce more ratings but reduce their informativeness. When $\overline{e}$ falls, the rating utility increases, leading to more positive ratings. Anticipating this, low-quality firms find it more attractive to harvest ratings, increasing the likelihood of rating harvesting in equilibrium. As a result, positive ratings become more likely to stem from low-quality firms, making ratings less informative about quality. The proposition formalizes this relationship:

Proposition 3. *If $\delta^* > \underline{\delta}$, then $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.*

The key takeaway is that—conditional on entry—lowering $\overline{e}$ increases the number of ratings but reduces their informativeness.

Empirical evidence supports this mechanism. Cabral and Li (2015) use shipping speed as a proxy for quality and show that offering rebates for ratings reduces negative reviews especially for low-quality products. Since rebates lower the cost of leaving a rating, their findings are consistent with our model: reducing rating costs can make ratings less informative.

We use this result to explore how rating-effort costs affect entry.

Corollary 1. *There exists a constant $\alpha > 0$ such that newcomers enter if and only if $\overline{e} \geq \alpha$.*

Encouraging consumers to rate, via lower rating effort-costs, discourages entry. Encouraging consumers to rate encourages rating harvesting (Proposition 3), which makes ratings less informative (Proposition 1). But if ratings are less informative, newcomers with positive ratings may no longer sell, so they will no longer enter (by Proposition 2).

These insights cast a new light on platform efforts to encourage ratings. Many platforms have worked to encourage ratings over time. Yelp and Google offer perks such as invitations to exclusive events or discounts to active raters.[20] Google also prompts users to leave quick feedback, allowing for one-tap reviews. Amazon similarly simplified its rating system: prior to 2020, users had to write a review alongside their rating; now, one-click ratings are allowed. They justified the shift by claiming that it would increase the accuracy of ratings through higher volume.[21]

[20] See the Yelp Elite Squad and Google Local Guides programs, as described by Yelp (Yelp, 2022) and Donaker et al. (2019).

[21] See Forbes (Masters, 2021), and TechCrunch reporting in (Perez, 2019).

However, our results suggest that these changes may have unintended consequences. First, while these measures increase rating volume, they also encourage rating harvesting, leading to a greater number of ratings that are less informative. Second, less-informative ratings make it harder for newcomers to sell after a positive rating, discouraging entry. Ultimately, both effects weaken the ability of high-quality newcomers to build a reputation and exacerbate the cold-start problem.

Rather than simply increasing the quantity of ratings, platforms may want to consider how to preserve or enhance their informational value. One way to do this is by increasing the cost of leaving a rating (raising $\overline{e}$), which discourages rating harvesting. Although this would reduce the overall number of ratings, it would increase their informativeness and encourage entry.

Comparing Policies. Raising the cost of leaving a rating induces fewer ratings, but also discourages rating harvesting. This may not discourage entry of low-quality sellers, but it helps to weed them out more quickly. However, the effect on high-quality sellers is ambiguous: more-informative ratings increase their profits after a good rating, but larger rating costs lower the probability they get one (we show this formally in the next section). Instead, linking ratings to the prices raters paid does not directly harm high-quality newcomers and might therefore fight the cold-start problem more effectively.

6 Surplus Analysis

We now study how rating harvesting shapes surplus. We proceed in two steps. First, we vary rating informativeness (the intensive margin). Second, we vary rating-effort costs, which changes both informativeness and the likelihood of leaving a review (the extensive margin).

More Informative Ratings. Recall that a higher δ^* means less rating harvesting and thus more informative ratings. Varying δ^* therefore captures changes along the *intensive margin* of the rating system.

The comparative statics reflect two forces. Without entry ($\delta^* \leq \underline{\delta}$), the incumbent is a monopolist. With entry ($\delta^* > \underline{\delta}$), competition increases consumer surplus relative to monopoly. However, conditional on entry, more informative ratings better differentiate products: positively rated entrants are more clearly identified as high quality and can charge more. In turn, entrants with non-positive ratings are perceived as worse competitors, allowing the incumbent who faces such an entrant to raise prices. This relaxes competition and reduces consumer surplus.

Let π^B, π^h, and π^l denote expected total profits of the average newcomer, the high-quality newcomer, and the low-quality newcomer, respectively. π^A denotes the incumbent's profits, and CS the expected consumer surplus, which we equate with consumption utility.[22]

[22]Results are qualitatively robust if rating utility is included, provided it gets a lower weight than consumption utility such that consumer surplus increases in $q - p$.

Corollary 2. *If $\delta^* \leq \underline{\delta}$, then $\pi^A = 2q^A$, $\pi^B = 0$, and $CS = 0$. If $\delta^* > \underline{\delta}$, then $\pi^A < q^A$, $\pi^B > 0$, and $CS > 0$; conditional on entry, $\frac{\partial \pi^A}{\partial \delta^*} > 0$, $\frac{\partial \pi^B}{\partial \delta^*} > 0$, and $\frac{\partial CS}{\partial \delta^*} < 0$.*

Corollary 2 summarizes the trade-off: sufficiently informative ratings induce entry and competition, but *overly* informative ratings (high δ^*) soften post-entry competition by increasing effective differentiation. Thus, consumers prefer somewhat, but never fully informative ratings.

In line with this mechanism, evidence suggests that less-precise quality signals intensify competition. Gandhi et al. (2024) show that when firms are exposed to more fake reviews of their rivals—which makes them less informative—firms that do not purchase fake reviews lower their prices.

Rating Costs. We study how increasing the rating-effort cost $\overline{e}$ affects equilibrium outcomes. A higher $\overline{e}$ increases δ^* and thus makes ratings more informative (intensive margin), but it also reduces the frequency of reviews (extensive margin). Because these forces work in opposite directions, the effect of $\overline{e}$ on incumbents, high-quality entrants, average entrant profits, and consumer surplus is generally ambiguous. The exception is low-quality entrants, who are hurt by both fewer reviews and greater informativeness.

Corollary 3. *Suppose $\delta^* > \underline{\delta}$. Then $\frac{\partial \pi^l}{\partial \overline{e}} < 0$, and an increase in $\overline{e}$ can increase or decrease π^B, π^h, π^A, and CS.*

The same trade-off shapes consumer surplus: more reviews improve matches, while more informative reviews soften post-entry competition. Despite these ambiguous effects, consumer-optimal rating costs must be high enough to induce entry.

Corollary 4. *Consumer-optimal rating effort satisfies $\overline{e}^{CS} \geq \alpha$.*

The corollary suggests that a marketplace that wants to attract consumers should ensure that ratings encourage entry. More broadly, by encouraging or discouraging ratings, marketplaces affect both the information content of ratings, but also how surplus is split between buyers, incumbents, and entrants.

7 Extensions and Robustness

Negative ratings. If our "silence is bad news" condition is violated, low-quality newcomers also sell in period 2 after no rating. First, newcomers always enter: low-quality newcomers sell in period 2 with strictly positive probability, making entry profitable. Second, also here, lower rating-effort costs $\overline{e}$ induce more rating harvesting. Details are in the Online Appendix B.

More generally, our results extend to rating systems with even more messages like 5-star ratings. Intuitively, in our framework, the value of ratings is determined endogenously in equilibrium. Thus, also with more complex rating systems, there exist equilibria where one rating has the same informational content as our good rating, other ratings have the same informational content as our bad rating, and the others are uninformative or not used in equilibrium. This equilibrium is plausible because it reflects the common finding that ratings are strongly bimodal and raters leave either five stars or one star (Dellarocas & Wood, 2008; Filippas & Horton, 2022; Filippas et al., 2022; Hu et al., 2009; Nosko & Tadelis, 2015).

Longer Horizon Model. Our model focuses on the short-run challenge of newcomers to establish a reputation, and its effects on entry. In this extension we indicate how our results extend to longer post-entry time horizons by studying a three-period model. In this model, newcomers who enter in period 1 can also choose to exit in period 2. First, we establish equilibria that are similar to our main model. In particular, there are equilibria where newcomers enter (and do not exit in period 2) if and only if low-quality newcomers do not harvest too much. In equilibrium, low-quality firms play mixed strategies as in our main model's period 1 in every non-terminal period. Thus, harvesting is not just driven by endgame effects: low prices in period 1 pay off already in period 2, since low-quality newcomers with a good rating charge a large price with positive probability. Second, low-quality firms who enter get a positive rating in every non-terminal period with strictly positive probability. Intuitively, if they only received a good rating in period 1, but not in period 2, then the value of reputation would skyrocket in period 3; but that induces strong incentives to also receive a good rating in period 2. Third, a lower rating effort $\bar{e}$ encourages rating harvesting in periods 1 and 2. Thus, low-quality firms that harvest more ratings may also stay longer in the market. This suggests another dimension through which rating harvesting reinforces the cold-start problem: low-quality firms that harvest ratings stay longer, making it harder for high-quality newcomers to establish a reputation. Details are in the Online Appendix B.

8 Implications for Platform Management

Our results carry important implications for the design and management of platform rating systems, summarized as follows:

First, as discussed, many major platforms have made concerted efforts to increase consumer participation in ratings. However, we show that overly incentivizing ratings can be counterproductive. When rating becomes too easy, low-quality firms are more likely to harvest ratings, reducing the informativeness of ratings. Eventually, this also discourages entry and reinforces dominant positions of established firms. Both effects exacerbate the cold-start problem. Thus, encouraging entry and maintaining an informative rating system may require not encouraging ratings too much.

Second, the ideal solution to eliminate the cold-start problem is preventing entry of low-quality sellers. Our results highlight the difficulty of doing so in practice. Free-riding is a robust feature of equilibrium: if only high-quality firms enter, reputation skyrockets, which encourages free-riding. Similarly, encouraging newcomers to offer steep discounts in order to build reputation can inadvertently promote rating harvesting. A more effective approach is to discourage harvesting—even if this does not prevent entry of low-quality entrants, it helps weed them out quicker. This can be done by increasing the relative profitability of price mimicking—for example, by making ratings more costly, by tying ratings to the price paid, or by implementing penalties that target deteriorating ratings.

Third, while platforms typically provide easy access to past ratings, they rarely link them to the prices paid by raters. This disconnect enables rating harvesting: consumers cannot distinguish whether a high rating reflects genuine product quality or simply a low price. Platforms seeking to discourage harvesting could design rating systems that better account for the price raters paid. One current practice that may be effective in doing so is asking consumers to specifically rate 'value-for-money' in addition to an overall rating.[23] Alternatively, a more direct intervention by platforms would be to assign lower weights to ratings from buyers who paid lower prices. Such policies could make ratings more reflective of true quality, which could help high-quality sellers build reputation and alleviate the cold-start problem. A key advantage is that it discourages harvesting without directly discouraging ratings for high-quality newcomers. However, these adjustments must be implemented carefully, as they may also reduce sellers' incentives to lower prices. For example, one may apply such adjustments only to sellers with few ratings, where the risk of harvesting is most acute.

Fourth, even though more informative ratings stimulate entry, they differentiate products and relax competition. Conditional on entry, more informative ratings enhance surplus extraction particularly for established firms, because consumers' outside option—purchasing from a newcomer with non-positive ratings—becomes less attractive.

Fifth, our results offer a broader insight for two-sided platforms: rating systems can be used to shift surplus between buyers and sellers. In our model, consumers may prefer more informative ratings than established sellers, primarily because these ratings facilitate entry and intensify competition. As such, platforms can influence the distribution of surplus—and ultimately platform participation—by shaping the informativeness of their rating systems.

9 Conclusion

We study how firms use prices to influence their own ratings. We highlight a qualitatively novel trade-off between rating harvesting and price mimicking, which connects closely to empirical evidence on the dynamic interplay between prices and ratings. We identify the drivers of rating harvesting and show that it can lead to less-informative ratings. We also examine implications for entry and the cold-start problem, as well as for buyer and seller surplus.

In practice, consumers may also read product reviews to form expectations about quality. In principle, such reviews could help consumers disentangle the effects of quality and price on observed ratings. However, we argue that reviews are unlikely to fully resolve this ambiguity. First, even

[23] Using experiments and data from Yelp and Airbnb, Chen et al. (2018), Gutt and Kundisch (2016), and Schneider et al. (2021) show multidimensional ratings affect overall rating. Thus, separate value-for-money ratings could induce raters to focus more on quality in their overall rating.

diligent consumers read only a small, selective sample of reviews. Second, even when reviews mention value-for-money or price-quality-ratio, they rarely state the exact price paid—making it impossible to assess whether the value was good relative to that price. Third, empirical evidence supports the disproportionate influence of ratings relative to reviews: Liu and Reimers (2025) estimate that, on Airbnb, ratings alone increase consumer surplus four times as much as reviews alone.

Our model focuses on consumers who rate based on the value-for-money they receive. Consumers may also rate for other reasons—for example, to help others by signaling product quality or out of an intrinsic motivation to report the truth. Importantly, such motivations would lead consumers to rate based on quality alone, especially as prices fluctuate over time. Still, as long as a subset of consumers rates based on value-for-money, we would expect price dynamics consistent with rating harvesting. Our results are therefore robust to a range of consumer motivations, provided value-for-money-sensitive raters remain active.

In practice, another factor that undermines the informativeness of ratings is the presence of fake reviews (He et al., 2022). If low-quality firms are more likely to acquire fake reviews, this also reduces the ability of review systems to signal true quality. In contrast, in our setting, firms use pricing strategies—rather than overt manipulation—to boost their ratings. This distinction is crucial: while fake reviews distort consumer beliefs about reviews, rating harvesting directly affects prices of raters, which has a range of implications we study above.

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Online Appendix for ‘Harvesting Ratings’

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Appendix A Proofs

A.1 Primitives

Towards proving our main Proposition 4, we first show a series of primitives that hold beyond our baseline model: in particular, they hold (i) for any finite $T \geq 2$ periods, (ii) for any q^l such that $q^l < q^A$, (iii) if firm B sells following a positive or no rating. We use them to prove our main proposition and extensions.

In the proofs, we use $\mathbb{H}_t$ to denote histories until t.

Lemma 3. *If the high-quality firm faces no demand in a given period t, the low-quality firm faces no demand in period t.*

Proof of Lemma 3.
Suppose towards a contradiction that in some period t, h is inactive and l is active. Then for any equilibrium price where l sells in t, we have beliefs $E[q_t^B|p] = q^l$. Because $q^l < q^A$, l only sells in t if it charges a price below costs, and since $q^l - q^A < q^l$, it gets no rating with probability strictly less than one in any $t < T$. This is clearly suboptimal in $t = T$, so we must have $t < T$. Next, we show that l earns weakly negative profits for subsequent periods if it gets a positive or a negative rating in period $t < T$. To see this, note that if l sells in t and receives a positive or negative rating, it is identified as a low-quality firm in period $t + 1$ and any subsequent period. Thus, since $q^l < q^A$, following a history with $R_t \in \{1, -1\}$, firm l has zero demand and is inactive after any such histories. But if l earns non-positive profits after $R_t \in \{1, -1\}$, and since it sells below cost in t, it must earn strictly positive profits after $R_t = 0$. But then l has a profitable deviation to not selling in t to get $R_t = 0$ with probability one, contradicting that l is active in period t. □

Lemma 4. *If the low-quality firm sells in period $t + 1$, it must sell in period t.*

Proof of Lemma 4.
Towards a contradiction, suppose l sells in $t + 1$ but not in t.

First consider the case where h is also inactive in period t. But then beliefs are the same in $t+1$ as they were in t, contradicting that l sells in $t+1$.

Next, consider the case where h sells in period t, setting some price p_t with strictly positive probability. Since h sells, we must have $p_t \geq 0$. Because the high-quality firm sells, it obtains $R_t = 1$ with some probability. Since only h gets $R_t = 1$, this rating perfectly identifies h for subsequent periods. If the low-quality firm does not sell in period t, it receives $R_t = 0$ with probability one. Thus, observing $R_t = 0$, and since l has that rating with probability one and h only with probability strictly less than one, beliefs after $R_t = 0$ in $t+1$ must be strictly lower than beliefs in period t. Hence, prices in $t+1$ following $R_t = 0$ must be strictly lower than prices the high-quality firm sets in period t.

We now distinguish two cases. First, if p_t is above q^l then l has a profitable deviation to set p_t in period t and sell, obtaining $R_t \in \{0, -1\}$. Recalling that p_t must be strictly greater than any price following no rating in period $t+1$ if it did not sell, it must be a strictly profitable deviation for the low-quality firm to set p_t, since it sells at a larger price and, since it did not yet get $R_t = -1$, has a larger demand than in $t+1$, contradicting that l does not sell in t.

Second, if p_t is below q^l, then l has a profitable deviation by starting to set p_t in period t and sell. This deviation induces $R_t \in \{1, 0\}$. Again recalling that p_t must be strictly greater than any price following no rating in period $t+1$ if it did not sell, it must be a strictly profitable deviation for the low-quality firm to set p_t. Additionally, since $R_t = 1$ perfectly identifies a high-quality firm, l also earns strictly larger profits after such histories. This contradicts that l does not sell in period t.

We conclude that if the low-quality firm sells in period $t+1$ it must sell in period t. □

Corollary 5. *If the low-quality firm has positive demand in period t, then the high-quality firm must have positive demand in all periods up to and including period t.*

Proof of Corollary 5.
Suppose the low-quality firm receives demand in period t. From Lemma 4 it also faces demand in period $t-1$. Then from Lemma 3 the high-quality firm faces demand in every period for which the low-quality does. Hence, it must be that the high-quality firm faces demand in both period t and $t-1$. Induction then implies the claim. □

Corollary 6. *If the high-quality firm sells in period t, it only obtains a good rating or no rating with strictly positive probability.*

Proof of Corollary 6.
Note that ratings only follow from a sale, and sales require winning the competition against A which has quality $q^A > 0$. This implies that in any period t where a rating could occur for h, we have $E[q_t^B|\mathbb{H}_t] \geq p_t$. Further, observe that expected quality of firm B is a convex combination of q^h and q^l, which is weakly less than q^h. Therefore, due to competition with A, it must be that $q^h > p_t$

and the high-quality firm obtains a good rating or no rating with strictly positive probability, and cannot obtain a bad rating. □

Corollary 7. *In any history $\mathbb{H}_t$ where firm B obtains at least one bad rating, consumer beliefs are the lowest possible, $E[q_t^B|\mathbb{H}_t] = q^l$.*[1]

Proof of Corollary 7.
By Corollary 6, h cannot receive bad ratings. Hence, for any equilibrium history with a bad rating the firm is perfectly identified as l. □

Lemma 5. *Firm A does not employ a mixed strategy in any period t.*

Proof of Lemma 5.
If firm A has zero demand after some histories, then by Selection Assumption 2, firm A plays $p_t = 0$ with probability 1 following those histories. To see this, note that by Selection Assumption 2, some consumers are indifferent, so offers with zero demand must be optimal. Prices below cost would be suboptimal if they would induce sales; prices above costs would be suboptimal if some indifferent consumers purchased, since consumers are indifferent between both firms, a marginally lower price would induce a discrete jump in demand. Thus, if A has zero demand, it charges marginal cost.

Next, consider histories after which firm A sells with strictly positive probability.

Note first that firm A cannot charge multiple mass points with strictly positive probability. If it sells, it must earn strictly positive profits at some of these mass points. Applying standard Bertrand arguments, either firm B earns strictly positive profits, then either firm can profitably deviate by shifting probability mass for some of its mass points downwards. Or firm B earns zero profits and by Selection Assumption 2, consumers must be indifferent between both firms. Then by Selection Assumptions 2, such offers must be best responses to A's offers, which is why B sets price at marginal cost with probability one (by the same argument used in the first paragraph of this proof). But then firm A cannot be indifferent between multiple mass points and shifts probability mass away from the least profitable ones. We conclude that A does not have multiple mass points.

We show next that both firm A and firm B cannot mix over intervals. Towards a contradiction, suppose firm A mixes over an interval of prices. Since firm A mixes over an interval, firm B must also mix over an interval; otherwise firm A would sell with probability zero (which we ruled out above), or with probability one (in which case mixing over an interval is clearly suboptimal).

Now take one such price $p^A > 0$. Note that since A must earn weakly positive profits in t, and therefore does not charge prices below cost, such a $p^A > 0$ must exist whenever A mixes over an interval. By our Selection Assumption 2, consumers are indifferent between both firms, implying that all prices of B, p^B, must be such that consumers are indifferent between p_A and p^B. Thus, all

[1]For the two-period model, possible equilibrium histories with negative ratings are $\mathbb{H}_t \in \{\{-1\}\}$. For the three period model, they are $\mathbb{H}_t \in \{\{-1\}, \{1, -1\}, \{0, -1\}, \{-1, 1\}, \{-1, 0\}, \{-1, -1\}\}$.

prices of B must induce the same expected utility with firm B as p^A does from firm A. But then deviating to a marginally smaller price induces a discrete jump in demand for firm A while only marginally reducing margins. Thus, such a deviation must be strictly profitable, contradicting that A mixes over an interval. This concludes the proof. □

Lemma 6. *When firm B enters and is high-quality, it sets a unique price, $\overline{p}_t$ conditional on the history if in period t:*

- *Good ratings are not beneficial (i.e. conditional on the same history, $R_t = 1$ leads to lower future profits than $R_t = 0$); or*
- *Good ratings are beneficial and* (1) *holds.*

The final period is a special case of obtaining good ratings being not beneficial.

Proof of Lemma 6.
We show that a high-quality firm B sets a unique price in each period conditional on the history.

First, suppose firm h earns zero profits in period t. By our Selection Assumption 2, consumers must be indifferent between h and A and by the same selection assumption, offers with zero demand of h must be optimal even if some indifferent consumers purchase, implying that h charges a price at marginal cost (by the same argument we used in the proof of Lemma 5). Thus, if h earns zero profits, it sets a unique price.

Next, suppose h earns strictly positive profits. Then h must have strictly positive demand for all its prices. Additionally, by Corollary 6, h gets good and no ratings with strictly positive probability. By Selection Assumption 2, consumers are ex-ante indifferent between h and A, which is why firm A must earn zero profits in period t; otherwise A could strictly increase profits by marginally reducing its price. Thus, since A earns zero profits from selling in t and h earns strictly positive profits, our Selection Assumption 2 implies that h sells with probability one for all prices it charges in period t.

We now distinguish two cases, whether good ratings are beneficial and raise continuation profits, or whether they are not beneficial.

Suppose first that good ratings are not beneficial such that obtaining a good rating in period t does not improve the expected continuation payoff. In other words, the sum of expected future profits conditional on obtaining a good rating in period t is weakly less than the sum of expected future profits conditional on obtaining no rating in period t. Then, since all prices in t induce the same demand in t, firm h has a profitable deviation and moves all probability mass in t to its highest price in t from the candidate equilibrium. This strictly raises its current profits and its future profits, contradicting that h plays a mixed strategy. Therefore, when good ratings are not beneficial firm h sets a unique price in period t.

Note the special case of the last period. Since ratings from the last period do not influence any future decision, following a history of ratings, if the firm sells in the last period, it sets the highest possible price which induces demand.

Suppose next that, following a history $\mathbb{H}_t$, good ratings are beneficial such that obtaining a good rating in period t improves the expected continuation payoff. In other words, the sum of expected future profits conditional on obtaining a good rating in period t is strictly larger than the sum of expected future profits conditional on obtaining no rating in period t. Here, we have to consider that although firm B would receive the same demand at any price over which it mixes in period t, shifting probability from a lower price to a higher price reduces the probability of receiving a good rating, reducing continuation profits. Therefore, the distribution of effort to leave ratings has to be sufficiently flat such that the probability of receiving a good rating does not decrease by too much if the firm shifts probability mass to the higher price. To derive such a condition, note the total profit of the firm is $p + F(q^h - p)\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) + (1 - F(q^h - p))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, where $\pi_{t+1}(\{\mathbb{H}_t, R_t\})$ is the expected continuation profit from obtaining R_t. Note $\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) > \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$ for ratings to be beneficial. Then a marginal price increase raises profits if for all $p \in (-q^h, q^h)$, we have

$$1 - f(q^h - p)(\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow$$
$$f(q^h - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})} \tag{1}$$

Therefore if (1) holds for all prices $p \in (-q^h, q^h)$, firm h earns strictly larger profits from its larger candidate equilibrium prices, contradicting that h plays a mixed strategy. The same condition also implies that if h sets a unique price that satisfies Selection Assumption 2, lowering the price cannot increase profits.

Therefore, we conclude that a high-quality firm B sets a unique price $\overline{p}_t$ if in period t good ratings are not beneficial or (1) holds on the support of f. □

Lemma 7. *Suppose firm l enters.*

Firm l charges the following prices when ratings are beneficial. If $\overline{p}_t > q^l$, and (2) *and* (3) *hold in period t:*

- *Firm l charges only $\overline{p}_t$ or $\underline{p}_t$ with positive probability in period t.*
- *Firm l charges $\overline{p}_t$ with probability $\delta_t \in [0, 1]$, obtaining a bad rating with probability $F(\overline{p}_t - q^l)$ and no rating otherwise.*
- *And $\underline{p}_t = q^l - q^A \leq q^l$ with probability $1 - \delta_t$, obtaining a good rating with probability $F(q^l - \underline{p}_t)$ and no rating otherwise.*

If $\overline{p}_t \leq q^l$ and (1) holds, $\delta_t = 1$ and firm l obtains either a good rating with probability $F(q^l - \overline{p}_t)$ or no rating otherwise.

If good ratings are not beneficial, then $\delta_t = 1$.

Proof of Lemma 7.
Selection Assumption 2 implies that consumers are, in expectation, indifferent between firm A and B. Hence, the highest price firm l can set in each period is one that leads to this indifference. This price is the unique price, $\overline{p}_t$, from Lemma 6 that also h sets. We start by assuming good ratings are beneficial in period t, showing the low-quality firm B mixes between this price and a unique lower price in period t. This proof follows in two parts. First, by considering $\overline{p}_t > q^l$. Then we consider $\overline{p}_t \leq q^l$.

Consider first the scenario where $\overline{p}_t > q^l$ and good ratings are beneficial in period t. When the low-quality firm sets $\overline{p}_t$, because $\overline{p}_t > q^l$ the firm receives a bad rating with probability $F(\overline{p}_t - q^l)$ and no rating with probability $1 - F(\overline{p}_t - q^l)$. If the firm deviates to a price above $\overline{p}_t$ it never makes a sale and gets no rating with probability 1. If the firm deviates to a price between q^l and $\overline{p}_t$ it sells at most with probability one and gets a bad rating with a lower probability. Hence, by deviating to a lower price it may be possible to increase the continuation payoff. Such a deviation is not profitable if f is sufficiently flat such that the higher probability of receiving the continuation payoff is dominated by the lower profits the firm earns in period t. In other words, for prices above q^l at which firm l sells, the derivative of the total expected profit, $p + F(p - q^l)\pi_{t+1}(\{\mathbb{H}_t, R_t = -1\}) + (1 - F(p - q^l))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, from playing the price p must be positive for all $p - q^l$ on the support of f. This holds if for all such prices, we have

$$1 + f(p - q^l)(\pi_{t+1}(\{\mathbb{H}_t, R_t = -1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow f(p - q^l) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})}. \tag{2}$$

Where the equivalence follows since by Corollary 7, negative ratings perfectly identify l and induce zero continuation profits. If (2) holds for all prices above $p - q^l$ on the support of f, the low-quality firm sets $\overline{p}_t$ in period t if it sets a price above q^l.

Next consider when the low-quality firm B may receive a good rating with some positive probability in period t. For this to occur the firm has to set prices weakly below q^l. Suppose the low-quality firm B sets more than one such price. This implies that it makes a positive profit at all such prices. However, if f is sufficiently flat such that an increase in price only changes the probability of receiving a rating by a small amount, it must be that raising prices in period t is beneficial as long as firm l continues to sell. In other words, the firm l gathers all its probability mass for prices below q^l at a mass point. This is true if the derivative of the total expected profit of setting a $p \leq q^l$, $p + F(q^l - p)\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) + (1 - F(q^l - p))\pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})$, is positive for all $q^l - p$

on the support of f:

$$1 - f(q^l - p)(\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})) > 0 \Leftrightarrow$$
$$f(q^l - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\}) - \pi_{t+1}(\{\mathbb{H}_t, R_t = 0\})}. \tag{3}$$

If (3) holds for all $q^l - p$ on the support of f in period t then the low-quality firm sets a single price $\underline{p}_t$ below q^l in period t.

The low-quality mixes over $\overline{p}_t$ and $\underline{p}_t$ in period t such that it is indifferent between the two expected continuation payoffs. This concludes that if $\overline{p}_t > q^l$ and good ratings are beneficial in period t, a low-quality firm B mixes between $\overline{p}_t$ and some unique $\underline{p}_t$ below q^l in period t.

Next consider the scenario where $\overline{p}_t \leq q^l$ and good ratings are beneficial. The same argument in the previous paragraph implies that l sets a unique price. In this scenario a low-quality firm B would receive a good rating with some strictly positive probability. When $\overline{p}_t \leq q^l$, then at any price above $\overline{p}_t$ consumers must have beliefs such that deviating to such prices induces no sales. This rules out any upward deviation by firm B. Moreover, no firm receives bad ratings. However, there are potential downward deviations from $\overline{p}_t$, since firm l can set lower prices and receive more good ratings. Such a deviation would not be profitable if f is sufficiently flat such that (3) holds.

Next consider the scenario where ratings are not beneficial. Then to prevent receiving good ratings the low-quality firm trivially plays the highest price at which it is able to sell in period t, playing $\overline{p}$ with probability 1.

Finally, because the high-quality firm B never sets $\underline{p}_t$, then it must be that following any price $\underline{p}_t$ the low-quality firm is perfectly identified. To make a sale, the firm has to set a price which provides at least as much utility as firm B, that is to say $\underline{p}_t \leq q^l - q^A$, and because the firm prefers to set the highest possible price following (3), $\underline{p}_t = q^l - q^A$. This concludes the proof. □

We refer to equations (1), (2) and (3) as sufficiently flat conditions for f.

Lemma 8. *If firm B enters, it plays a unique price in period T, which depends on its rating history.*

Proof of Lemma 8.
Given the consumer's information set in the final period, they condition their beliefs only on historical ratings and current prices. Note first that since there is no future period, future ratings do not affect profits. Selection Assumption 2 implies that consumers are ex-ante indifferent between firms A and B. Additionally, one of the firms must earn zero profits. Otherwise, if both firms earn strictly positive profits, firm A can marginally decrease its price to increase demand by a discrete amount, strictly increasing profits. By Selection Assumption 2, the firm earning zero profits charges a price at costs and earns zero profits (using the same argument as in Lemma 5). Thus, if firm B does not sell, it sets a unique price at marginal cost. If firm B sells, by Selection Assumption

2, consumers must be indifferent between both firms, and by the above result firm A earns zero profits, so firm A will set the largest price at which it can sell. This pins down the price of firm B uniquely. We conclude that firm B sets a unique price in period T, conditional on its rating history. $\square$

Corollary 8. *Suppose h and l enter. In period t, firm A sets $p_t^A = max\{q^A - E[q_t^B|\mathbb{H}_t, p_t], 0\}$. Firm B sets either $\overline{p}_{\mathbb{H}_t} = max\{E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A, 0\}$ and $\underline{p}_{\mathbb{H}_t} = q^l - q^A$.*

Proof of Corollary 8.
First recall from Lemma 7 that $\underline{p}_{\mathbb{H}_t} = q^l - q^A$, for any history $\mathbb{H}_t$. Next, consider firm B playing $\overline{p}_{\mathbb{H}_t}$ in each period t. Selection Assumption 2 implies that consumers are ex-ante indifferent between firms A and B. Additionally, one of the firms must earn zero profits. Otherwise, if both firms earn strictly positive profits, firm A can marginally decrease its price to increase demand by a discrete amount, strictly increasing profits. By Selection Assumption 2, the firm earning zero profits charges a price at costs and earns zero profits (using the same argument as in Lemma 5). Thus, if $E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] \geq q^A$, firm A charges a price at cost and earns zero profits in that period and firm B charges $\overline{p}_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A$. If, instead, $E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] < q^A$, then firm B sets the price $\overline{p}_{\mathbb{H}_t} = 0$ and firm A sets the price $q^A - E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}]$. This concludes the proof. $\square$

Lemma 9. *Suppose h and l enter. If (1), (2) and (3) hold, first period beliefs are without loss of generality*

$$E[q_1^A|p] = q^A \quad \forall p, \qquad\qquad E[q_1^B|p] = \begin{cases} \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*} & \forall p \;\; if\, \overline{p}_1 \leq q^l \\ \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*} & \forall p > q^l \;\; if\, \overline{p}_1 > q^l \\ q^l & \forall p \leq q^l \;\; if\, \overline{p}_1 > q^l. \end{cases}$$

Proof of Lemma 9.
Recall that (1), (2) and (3) are conditions such that the high-quality firm B sets a unique price $\overline{p}_t$ and the low-quality firm B sets either $\overline{p}_t$ or $\underline{p}_t$. The rest of the proof focuses on period 1.

Since firm A's quality is common knowledge, its quality is known to be q^A regardless of price.

Consider the case where $\overline{p}_1 \leq q^l$. Then firms charge $\overline{p}_1$ with probability one. It is straightforward to check that for the equilibrium price $\overline{p}_1$, the above expectations apply Bayes rule. Additionally, $E[q_1^B|\overline{p}_1]$ is such that consumers believe that firm B provides just as much utility as firm A. At $\underline{p}_1$, consumers would buy from a low-quality firm B. This holds, since $E[q_1^B|\underline{p}_1]$ in expectation provides consumers at least as much utility as firm A. Since $q^l < \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*}$, consumers buy at both prices if $E[q_1^B|p] = \frac{\gamma q^h + (1-\gamma)\delta^* q^l}{\gamma + (1-\gamma)\delta^*}$ $\forall p$. For all other off-equilibrium prices, the above beliefs are consistent with our selection assumptions and the necessary equilibrium conditions we derived so far, since deviations to prices above the equilibrium price induce zero demand. Thus, these off-equilibrium beliefs are without loss of generality.

Next, consider the case where $\overline{p}_1 > q^l$. Recall from Lemma 7 that $\underline{p}_1 < q^l$. Applying Bayes rule shows that the beliefs are correct for equilibrium prices $\overline{p}_1$ and $\underline{p}_1$. For all off-equilibrium prices, the above beliefs are consistent with our selection assumptions and the necessary equilibrium conditions we derived so far, since deviations to prices in $(\underline{p}_1, q^l)$ and to prices above $\overline{p}_1$ strictly reduce demand. Thus, these off-equilibrium beliefs are without loss of generality. □

A.2 Proof of Lemmas 1 and 2

We show a more general result than what we discuss in the main text. Proposition 4 fully characterizes equilibrium. In particular, we also allow for the case where the pricing strategy of low-quality newcomers is in pure strategies. But to simplify exposition, we focus on the pure-strategy pricing equilibria where the low-quality newcomer enters either with probability one or zero. This is the case if (4) holds. Intuitively, in the pure-strategy equilibrium, low-quality newcomers earn negative profits in period 1 and positive profits in period 2. The condition ensures that overall profits are weakly positive. This is the case if q^h is sufficiently large such that reputation is sufficiently valuable. Without this assumption, low-quality newcomers may enter with strictly positive probability less than one, but our results remain qualitatively unaffected.

As discussed in the main text, we also assume that silence is bad news such that a newcomer who receives no rating in period 1 does not attract demand in period 2. This is Condition (5), where δ^* is pinned down in equilibrium by parameters. As $q^h - \overline{p}$ approaches $\overline{e}$, the right-hand-side goes to infinity, and the left-hand-side is bounded away from infinity. If q^h is sufficiently large, $q^h - \overline{p}$ increases and the condition holds. Thus, also this condition holds for sufficiently large q^h. Below, we also discuss the case if the condition does not hold. Then, a qualitatively similar equilibrium exists but the newcomer may also sell after $R = 0$.

Proposition 4. *Suppose the following conditions hold:*

$$\gamma q^h - q^A + F(q^A - \gamma q^h)\left[\frac{\gamma F(q^A + (1-\gamma)q^h)}{\gamma F(q^A + (1-\gamma)q^h) + (1-\gamma)F(q^A - \gamma q^h)} q^h - q^A\right] \geq 0 \tag{4}$$

and

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)\left[\delta^*(1 - F(|\overline{p}|)) + (1-\delta^*)(1 - F(q^A))\right]}{\gamma(1 - F(q^h - \overline{p}))}. \tag{5}$$

Then an equilibrium exists that is unique up to off-path beliefs. There exists a unique $\underline{\delta} \in (0, 1)$ such that both types of newcomers enter if and only if $\delta^* > \underline{\delta}$; otherwise, neither enters. If newcomers enter:

1. ***Ratings build reputation:*** $E[q_2^B \mid R = 1] > E[q_2^B \mid R = 0] > E[q_2^B \mid R = -1]$.
2. ***Ratings are valuable:*** $p_2^B = E[q_2^B \mid R = 1] - q^A > 0$. *Firm A sells in period 2 if and only if $R \in \{-1, 0\}$.*

Furthermore, in period 1:

3. ***Firm*** *A sets $p_1^A = 0$ and gets no demand.*
4. ***Firm*** *h charges $\overline{p} = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A$ with probability 1 and receives $R \in \{0, 1\}$.*
5. ***Firm*** *l randomizes over prices in period 1 if*

$$\frac{\gamma(q^h - q^A)}{(1-\gamma)q^A} > \underline{\delta} \quad and \quad F(q^A) > \frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2}. \tag{6}$$

In that case:

(a) *It charges $\overline{p} > 0$ with probability δ^* and receives $R \in \{-1, 0\}$, where $\delta^* \in \left(\underline{\delta}, \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}\right)$.*

(b) *It charges $\underline{p} \equiv -q^A < 0$ with probability $1 - \delta^*$ and receives $R \in \{0, 1\}$.*

Otherwise, firm l sets $\delta^* = 1$ such that $\overline{p} \leq 0$ and receives $R \in \{0, 1\}$.

Lemma 1 follows directly from statement 1 and 2 of Proposition 4.

In Lemma 2, the pricing strategy of firm A and firm h follows directly from statements 3 and 4 of Proposition 4; and the pricing strategy of firm l follows directly from the mixed strategy, where $\delta^* \in (\underline{\delta}, \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A})$, in statements 5(a) and 5(b) in Proposition 4.

We prove Proposition 4 in a series of Lemmas and Corollaries.

To apply the previous Lemmas and Corollaries, it is useful to make the following observations: (i) In a two-period model $\mathbb{H} = R$, i.e. all relevant histories are period 1 ratings of the newcomer. (ii) We apply that $q^l = 0$ to save on notation. (iii) In a model where firm B sells only following a good rating, this occurs when $E[q_2^B | R = 0] \leq q^A$ which implies $\pi_2(R = 0) = 0$. Then (2) always holds. And (3) evaluates to $f(q^l - p) < \frac{1}{\pi_{t+1}(\{\mathbb{H}_t, R_t = 1\})}$. Towards proving the proposition, we establish further results in the following Lemmas.

Lemma 10. *Firm B's decision to sell in period 1 is independent of its quality realization. Thus, in period 1, a high-quality firm B sells if and only if a low-quality firm B also sells. If firm B sells in the second period, it must sell in the first period. Thus, whenever newcomers enter, they sell in period* 1.

Proof of Lemma 10.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now consider firm B selling only when it is high-quality.

Suppose instead firm B is only selling when it is high-quality. This means $E[q^B | p] = q^h \; \forall p$. The high-quality firm B receives a good rating with some positive probability at all prices at which it sells. This allows it to to charge $q^h - q^A > 0$ in both periods. But then l has a profitable deviation to enter the market with strictly positive profits in period 1. Hence it cannot be that only the high-quality firm sells in the market in period 1.

We show next that if firm B sells in period 2, it also sells in period 1. Suppose instead a high-quality firm B sells only in period 2 and not period 1. Then also firm l cannot sell in period 1. But then period 2 is the same as period 1 but without continuation profits, so since both firms did not sell in period 1, they will not sell in period 2, a contradiction. Similarly, if l sold only in period 2, then either h sold in period 1, in which case consumer beliefs about newcomers would be q^h and also l would deviate and mimic h in period 1, or h did not sell in period 1, in which case both firms B did not sell in period 1 and the above result implies that h also does not sell in period 2, contradicting that l sells in period 2. We conclude that if firm B sells in period 2, it also sells in period 1.

Therefore, we can conclude that if firm B enters, it sells in the first period and its entry decision is independent of its quality realization. □

This implies that we cannot have efficient entry - that is there is no situation where all high-quality firm Bs enter the market and low-quality firm Bs do not.

Lemma 11. *If* (1), (3) *hold, and* $\overline{p} > 0$*, second period beliefs over firm B's quality are*

$$E[q_2^B|R] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} & \text{if } R=1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]} & \text{if } R=0 \\ 0 & \text{if } R=-1 \end{cases}$$

where δ^* *is the equilibrium probability with which a low-quality firm B plays* $\overline{p}$ *in period 1.*

If (1) *and* (3) *hold, and instead* $\overline{p} \leq 0$*, second period beliefs are*

$$E[q_2^B|R] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)F(-\overline{p})} & \text{if } R=1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)(1-F(-\overline{p}))} & \text{if } R=0 \\ 0 & \text{if } R=-1. \end{cases}$$

In either case, ratings are informative in equilibrium as $E[q_2^B|R=1] > E[q_2^B|R=0] > E[q_2^B|R=-1]$*.*

These are the beliefs after both newcomer types enter. After other histories, consumers believe the quality of newcomers is q^l *in both periods.*

Proof of Lemma 11.
We first describe how consumer beliefs are constructed when $\overline{p} > 0$, then we show a good rating is beneficial. Then describe beliefs when $\overline{p} \leq 0$.

It is immediate to see consumer beliefs result from applying Bayes rule to the pricing strategies from Lemmas 6 and 7. A high-quality firm always gets good ratings with some positive probability, this probability depends on the price it sets. Conversely, a low-quality firm B obtains a good rating with some positive probability only if it sets a negative price (which occurs with probability $(1-\delta^*)$ or if $\overline{p} \leq 0$). Otherwise, it obtains no rating.

To see that good ratings induce higher beliefs than no rating, we apply $\underline{p} = -q^A$ to the above expectations and get,

$$\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} > \frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)(\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))}$$
$$\Leftrightarrow F(q^h - \overline{p}) - F(q^A) > (F(q^h - \overline{p})F(\overline{p}) - F(q^A))\delta^*,$$

We now argue that this always holds. Lemma 7 tells us $\underline{p}$ is the maximum price that induces demand for l if consumers know the firm's type. This price provides exactly q^A utility to consumers. Hence the firm receives a good rating with probability $F(q^A)$. Moreover, to obtain any demand, a firm h must provide at least expected utility q^A, which is why the ex-post utility satisfies $q^h - \overline{p} \geq q^A$ and therefore $F(q^h - \overline{p}) \geq F(q^A)$. Hence, we know that $F(q^h - \overline{p}) \geq F(q^A)$. This implies $F(q^h - \overline{p}) - F(q^A) > (F(q^h - \overline{p})F(\overline{p}) - F(q^A))\delta^*$ because $\delta^* \in (0, 1]$ and $F(\overline{p}) < 1$. It is straightforward to show that no rating induces higher expectations than a bad rating.

When $\overline{p} \leq 0$, firm B sets a single price in period 1. Hence, there is no mixed strategy involved, and both high- and low-quality firm B would receive good ratings with some positive probability. Note that because $q^h > 0$, obtaining a good rating must be beneficial in equilibrium. It is straightforward to show that no rating induces higher expectations than a bad rating.

Finally, these beliefs apply if both newcomers enter, i.e. if newcomers sell in period 1. By Lemma 10, all other histories are off the path of play, and we set beliefs to q^l. □

Lemma 12. *Suppose h and l enter. Both firms A and B receive some positive demand if and only if firm A sells in period 2, and firm B sells in period 1 with probability* 1. *Both firms sell in period 2 after some ratings if* (7) *holds and* $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$.

Proof of Lemma 12.
Recall that by Lemma 10 the low-quality firm B sells in period 1 if and only if the high-quality firm B also sells in period 1. Furthermore, by Lemma 10 if firm B sells in period 2, it must sell in period 1. This implies the high-quality firm B must sell in period 1 for firm B to sell at all. If the high-quality firm B sells in period 1, the low-quality firm B must also sell in period 1. Hence, firm B must sell with probability 1 in period 1. This means the only possibility for firm A to sell is in period 2.

Thus, we need to check (i) under which conditions firm A sells in period 2; and (ii) under which conditions a high-quality firm B sells in period 2. (i) holds if firm A sells if it faces a rival without rating, i.e. that $q^A > E[q_2^B|R=0] > E[q_2^B|R=-1]$. Using the expression of $E[q_2^B|R=0]$ and rearranging, this condition becomes

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A)))}{\gamma(1-F(q^h-\overline{p}))} \text{ when } \overline{p} > 0 \text{ and}$$

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(1-F(-\overline{p}))}{\gamma(1-F(q^h-\overline{p}))} \text{ when } \overline{p} \leq 0.$$

Observing that when $\overline{p} \leq 0$, $\delta^* = 1$ we can equivalently combine and more generally state that $q^A > E[q_2^B|R=0]$ whenever

$$\frac{q^h - q^A}{q^A} < \frac{(1-\gamma)(\delta^*(1-F(|\overline{p}|)) + (1-\delta^*)(1-F(q^A)))}{\gamma(1-F(q^h-\overline{p}))}. \tag{7}$$

Finally, (ii) requires that a firm with a good rating $R = 1$ in period 2 sells when competing against firm A and earns a positive profit, i.e. if $E[q_2^B|R=1] > q^A$. Using the expression for the conditional expectation and rearranging leads to $\delta^* > 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{q^A(1-\gamma)F(q^A)}$. Moreover, because δ^* is a probability, $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ for firm B to sell in period 2 after a good rating. □

Corollary 9. *Good ratings are instrumental (i.e. affect beliefs and outcomes on the path of play) if and only if a high-quality firm B enters, sells in period 2 and firm B sells in period 1 with probability 1. A high-quality firm B sells after a good rating if* $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$.

Proof of Corollary 9.
If a high-quality firm B sells in period 2, we know from Lemma 12 that both high- and low-quality firm B must have sold in period 1. Then we also know from Lemma 11 that good ratings cause consumers to positively update beliefs.

In turn, if good ratings are instrumental, a high-quality firm B must sell after a good rating, which by Lemma 10 implies it sold in 1.

Thus, good ratings are instrumental if and only if a high-quality firm B enters and sells in period 2, which is equivalent to $E[q_2^B|R=1] > q^A$, from Lemma 12, $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$. □

Corollary 10. *Suppose h and l enter and $\overline{p} \leq 0$. Then $\overline{p} = \gamma q^h - q^A < 0$ and $p_1^A = 0$, and (4) implies that firms h and l attract demand and earn positive total profits at this price and therefore enter.*

Proof of Corollary 10.
This is immediate because Lemmas 6 and 7 show that if firm B sells and $\overline{p} \leq 0$, it always sets $\overline{p}$. In other words, $\delta^* = 1$ and consumer beliefs in period 1 about firm B are γq^h (Lemma 9). Therefore the consumer only buys from firm B if it offers as much surplus as firm A does, $\overline{p} = \gamma q^h - q^A$.

Because firms compete in a Bertrand fashion, the firm not selling charges price at marginal cost and the firm selling in period 1 charges a price equal to the expected difference in quality.

Finally, for firm l to want to sell it must face a positive total profit (across both periods), which is the case if (4) holds. Since h has a higher probability to get a positive rating and therefore earns strictly higher profits, the condition implies that h and l enter for the above prices. $\square$

Corollary 11. *If $\overline{p} > 0$ and $\frac{q^h - q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$, we have $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$, $\underline{p} = -q^A$, $p_1^A = 0$ and $\delta^* \in (\max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$. Both newcomer types attract demand and enter. If $\frac{q^h-q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$ is violated, either we must have $\overline{p} \leq 0$, or $\delta^* > \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ is violated and newcomers do not enter.*

Proof of Corollary 11.
Note that if $\overline{p} > 0$ it must be that firm B prefers to sell in period 1 with probability one. This occurs if $\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A > 0$.

Rearranging leads to

$$\delta^* < \frac{\gamma(q^h - q^A)}{(1-\gamma)q^A}.$$

Therefore, combined with Corollary 9, we know $\delta^* \in (\max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$, and this is possible only when $\frac{q^h-q^A}{q^h} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$.

In this case, firms h and l sell at $\overline{p} > 0$ in period 1 and therefore attract demand and enter.

If $\overline{p} \leq 0$, Corollary 10 implies that B still sells. If $\overline{p} > 0$ holds, but $\frac{q^h-q^A}{q^A} > \frac{F(q^A)(1-\gamma)}{\gamma(F(q^A)+F(q^h-\overline{p}))}$ is violated, either $\delta^* < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$, and therefore we must have $\overline{p} \leq 0$, or δ^* is too small for newcomers to sell after a good rating so that there is no entry. $\square$

Corollary 12. *Suppose h and l enter. In period 2, firm A sets $\pi_2^A = p_2^A = max\{q^A - E[q_2^B|R], 0\}$ and firm B sets $\pi_2^B(R) = p_2^B = max\{E[q_2^B|R] - q^A, 0\}$.*

Proof of Corollary 12.
This follows from Lemma 8. And profits in period 2 are equivalent to the price set in period 2. $\square$

Next, we prove the existence and uniqueness of a mixed strategy.

Given (7) and supposing h and l enter, we can characterize an equilibrium δ^*, which satisfies the following: A low-quality firm B must be indifferent between setting $\overline{p}$ in period 1 and getting no or negative rating, obtaining a profit of $\overline{p} + 0$, and setting $\underline{p}$ in period 1 and getting a good rating with some positive probability, obtaining a profit of $\underline{p} + F(q^A)(E[q_2^B|R = 1] - q^A)$.

Lemma 13. *Suppose h and l enter, (4), (7), and (1) and (3) hold. If $\delta^* \geq \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$, newcomers have strictly positive demand and there exists a unique δ^* such that:*

- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} < F(q^A)$ *and* $\overline{p} > 0$, $\delta^* \in (\frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$.
- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} < F(q^A)$ *and* $\overline{p} \leq 0$, $\delta^* = 1$,
- *If* $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} \geq F(q^A)$, $\delta^* = 1$.

Proof of Lemma 13.
We first characterize the profits that a low-quality firm B would receive if it plays $\overline{p}$ in period 1, then its profits when playing $\underline{p}$ in period 1. We show the former is decreasing in δ^* while the latter is increasing. Then, we characterize when $\delta^* \in (0, 1)$.

Total profits of the low-quality firm B setting $\overline{p}$ is $\overline{p}$. We know from above that a mixed-strategy equilibrium requires $\overline{p} > 0$. Therefore, the low-quality firm B earns $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$. It is immediate to see that this is decreasing in δ^*.

Total profits of l setting $\underline{p}$ is $\underline{p}$ plus a probability of obtaining a good rating and selling in period 2. Therefore, l earns $-q^A + F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} - q^A)$. This profit is increasing in δ^*. To see this, an increase in δ^* places less emphasis on the firm being low-quality following a good rating. Additionally, an increase in δ^* has an indirect effect of decreasing $\overline{p}$, which increases $F(q^h - \overline{p})$, placing a higher emphasis on the firm being high-quality following a good rating.

In mixed-strategy equilibria, low-quality firms must be indifferent between both strategies, which requires

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} = F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A). \tag{8}$$

Note that $\delta^* \geq \max\{0, 1 - \frac{(q^h-q^A)\gamma F(q^h-\overline{p})}{(1-\gamma)F(q^A)q^A}\}$ ensures that newcomers with a good rating sell in period 2. Then, since for all $\overline{p} > 0$, firms earn attract strictly positive demand and earn positive profits.

To see when the solution is interior to $\delta^* \in (0, 1)$, consider $\delta^* = 0$. Then, evaluating equation (8), we get $\frac{\gamma}{\gamma+(1-\gamma)\delta^*}q^h = q^h$ and $F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} - q^A) = F(q^A)(\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)F(q^A)} - q^A)$. It is then immediate to see

$$q^h > F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)F(q^A)} - q^A).$$

This always holds since the term in brackets on the right-hand-side is strictly lower that q^h, and multiplied by a term that is strictly less than one.

Next, note that by Corollary 11, δ^* is bound above, i.e. $\delta^* < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$. Note that $\frac{\gamma(q^h-q^A)}{(1-\gamma)q^A} \leq 1$ if $\gamma q^h \leq q^A$, which we assume throughout. In particular, when $\delta^* = \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$ this implies $\overline{p} = 0 \Leftrightarrow \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} = q^A$, which can be rearranged to $(1-\gamma)(1-\delta^*) = \frac{q^A-\gamma q^h}{q^A}$. Hence, evaluating (8) at this upper bound, a mixed-strategy equilibrium requires that

$$q^A < F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + \frac{q^A - \gamma q^h}{q^A} F(q^A)} - q^A).$$

Recall that F is uniform such that $F(x) = x/\overline{e}$. Then we can replace F to obtain

$$q^A < F(q^A)(\frac{\gamma q^h q^h}{q^A} - q^A).$$

Then recall that the terms in the bracket are the second period profits of firm B conditional on a good rating which must be positive when firm B enters. Therefore,

$$\frac{(q^A)^2}{\gamma(q^h)^2 - (q^A)^2} < F(q^A)$$

must hold in an interior solution. Otherwise, if this condition is violated, l prefers to set $\delta^* = 1$.

We now show that when (7) holds, $\overline{p} > 0$ and $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} < F(q^A)$ holds such that there is an interior solution, we must have $\delta^* > \frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}$. Evaluating (8) at $\delta^* = \frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}$, we show that the left-hand side is greater than the right-hand side:

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}} > F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1 - \frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)})F(q^A)} - q^A)$$

$$\gamma q^h(F(q^A) + F(q^h - \overline{p}))(1 - F(q^A)) > -q^A F(q^A)(\gamma F(q^h - \overline{p}) + F(q^A))$$

which is always true. Recall that the left-hand side is decreasing in δ^* and the right-hand side increasing. Therefore, it must be that $\delta^* > \frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}$.

Next, we show that the interval where $\delta^* \in (\frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)}, \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A})$ can exist. This is the case if $\frac{F(q^A)}{F(q^h-\overline{p})+F(q^A)} < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$. Since F is uniform, we can rewrite this as $\frac{q^A}{q^h-\overline{p}+q^A} < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$.

To see existence, note, $\frac{q^A}{q^h+q^A} < \frac{\gamma(q^h-q^A)}{(1-\gamma)q^A}$ holds if $(q^A)^2 < \gamma(q^h)^2$, and therefore the range can exist if q^h is sufficiently large.

Next, if (7) holds but $\overline{p} \leq 0$, from Lemma 7 we know firm B plays $\overline{p}$ regardless of its type. In other words, the low-quality firm B never mixes and $\delta^* = 1$.

Next, if $\frac{(q^A)^2}{\gamma(q^h)^2-(q^A)^2} \geq F(q^A)$, then low-quality firm strictly prefers to mimic prices and we get $\delta^* = 1$.

Finally, we have already shown above that in the mixed-strategy equilibrium, newcomers sell and therefore enter. If $\delta^* = 1$, (4) ensures that newcomers attract strictly positive demand and enter. Thus, newcomers enter with probability one. This concludes the proof. □

We can now use the above results to prove each statement in Proposition 4 .

Proof of Proposition 4.
Suppose (4), (7), and (1) and (3) hold. It remains to show that there exists a unique $\underline{\delta} \in (0,1)$ such that newcomers enter if and only if $\delta^* > \underline{\delta}$. We know already from Lemma 10 that either both types of newcomers enter or none. By our Selection Assumption 1, newcomers enter whenever an equilibrium exists where they enter. By Lemma 13, they enter if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$, i.e. if newcomers with a good rating sell in period 2. Clearly, if they do not sell after a good rating, they never sell. Thus, they enter if and only if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$.

To see that $\underline{\delta} > 0$, note that for $\delta^* = 0$, $1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A} > 0$ if and only if $q^A > \gamma q^h$, which holds by assumption. Thus, the condition is violated for $\delta^* = 0$, implying no entry and $\underline{\delta} > 0$. Next, we know from (4) that even l earns weakly positive profits and attracts demand for $\delta^* = 1$. Since h must earn strictly larger profits than l, both types enter, implying that $\underline{\delta} < 1$. Finally, note that the left-hand-side of $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$ strictly increases in δ^*, while the right-hand-side decreases in δ^*, implying that $\underline{\delta}$ is unique.

Next, note that Lemma 13 shows that conditional on entry, strategies are unique up to off-path beliefs. Additionally, Selection Assumption 1 implies that the same holds conditional on no entry.

We can now prove each statement about the newcomer entry in turn:

- Statement 1 follows directly from Lemma 11.
- Statement 2 follows directly from Corollary 9.
- Statement 3 follows directly from Corollaries 10 and 11.
- Statement 4 follows directly from Corollaries 8 and 6 and Lemma 9.
- The conditions in statement 5 follow from Lemma 13 together with our result that newcomers firms enter if and only if $\delta^* > \underline{\delta}$.
- The prices in statement 5 follow from Corollary 11 and the ratings of the low-quality firm from Lemma 7.
- The equilibrium level of δ^* and its support comes from Lemma 13 and our result that newcomers firms enter if and only if $\delta^* > \underline{\delta}$.

This concludes the proof. □

A.3 Remaining proofs for the main text.

Proof of Proposition 1.
We define the informativeness of ratings as a good rating being able to identify high-quality firms. This way, a good rating becomes more informative if $E[q_2^B|R=1]$ increases in δ^* when $\delta^* > \underline{\delta}$. To

see this is true, first consider the case where $\overline{p} > 0$ then

$$E[q_2^B|R=1] = \frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)}$$

then note an increase in δ^* decreases $\overline{p}$ which means $\gamma F(q^h - \overline{p})$ increases. Hence, the expectation increases in δ^* and good ratings become more informative.

Suppose instead $\overline{p} \leq 0$, i.e. that δ^* is sufficiently large, and consider the more general beliefs where the low-quality firm may choose to mix between $\overline{p}$ and $\underline{p}$, then we have

$$E[q_2^B|R=1] = \frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)\left[\delta^* F(|\overline{p}|) + (1-\delta^*)F(q^A)\right]}.$$

Observe that $F(q^A) > F(|\overline{p}|)$ because $-(\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A) < q^A \Leftrightarrow 0 < \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*}$ which is always true. Further, because $\overline{p}$ decreases in δ^*, then it must be that any increase in δ^* reduces $\delta^* F(|\overline{p}|) + (1-\delta^*)F(q^A)$ and increases $F(q^h - \overline{p})$. Therefore, $E[q_2^B|R=1]$ is increasing in δ^*. Overall, good ratings become more informative in δ^*. □

Proof of Proposition 3.
To show this, recall that the indifference condition for the low-quality firm must hold in a mixed-strategy equilibrium, define

$$G = \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - F(q^A)(\frac{\gamma F(q^h - \overline{p})q^h}{\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A).$$

Then applying the uniform distribution, G becomes

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - \frac{q^A}{\overline{e}}\left[\frac{\gamma(q^h - \overline{p})q^h}{\gamma(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)q^A} - q^A\right].$$

We know from the proof of Lemma 13 that G is strictly decreasing in δ^*. Next, we consider the derivative of G with respect to $\overline{e}$, which is clearly strictly increasing. Then, using the implicit-function Theorem, it follows that $\frac{\partial \delta^*}{\partial \overline{e}} = -\frac{\frac{\partial G}{\partial \overline{e}}}{\frac{\partial G}{\partial \delta^*}} > 0$. This concludes the proof. □

Proof of Corollary 1.
First, we show that $\underline{\delta}$ is independent of $\overline{e}$. We know from the proof of Proposition 4 that (i) $\underline{\delta} \in (0,1)$, and (ii) that newcomers enter if and only if $\delta^* > \max\{0, 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}\}$. Thus, $\underline{\delta}$ is implicitly defined by $\underline{\delta} = 1 - \frac{(q^h - q^A)\gamma F(q^h - \overline{p})}{(1-\gamma)F(q^A)q^A}$, where also $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\underline{\delta}} - q^A$. Using that on its support, $F(x) = \frac{x}{\overline{e}}$, it follows that $\underline{\delta}$ is independent of $\overline{e}$.

Second, we know from Proposition 3 that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$. Together with our results of Proposition 4 that newcomers enter if and only if $\delta^* > \underline{\delta}$, and since $\underline{\delta}$ is independent of $\overline{e}$, this implies that newcomers enter if and only if $\overline{e}$ is above some constant (which has to be strictly positive since we focus on $\overline{e}$ where newcomers get no rating with strictly positive probability). The result follows. $\square$

Proof of Corollary 2.
We start by showing the results on profits.

First suppose $\delta^* \leq \underline{\delta}$. From Proposition 4 we know firm B is inactive and equivalently makes the profit $\pi^B = 0$. Hence, firm A charges monopoly prices, allowing it to extract the full surplus from consumers, q^A, in each period, making a total profit of $\pi^A = 2q^A$.

Now suppose $\delta^* > \underline{\delta}$. Firm B sells and its expected profit is

$$\gamma\left[\overline{p} + F(q^h - \overline{p})\pi_2(R=1)\right] + (1-\gamma)\left[\delta^*\overline{p} - (1-\delta^*)q^A + (1-\delta^*)F(q^A)\pi_2(R=1)\right],$$

where π_2 represents firm B's period 2 profit following the rating $R = 1$. Then substituting $\overline{p}$ and $\pi_2(R=1)$,

$$\gamma q^h(1 + F(q^h - \overline{p})) - q^A\left[1 + \gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)\right].$$

Note that $\overline{p}$ is decreasing in δ^*, which means $F(q^h - \overline{p})$ is increasing in δ^*. Then increases in δ^* increases $\gamma F(q^h - \overline{p})(q^h - q^A)$ and decreases $(1-\gamma)(1-\delta^*)F(q^A)$. Therefore, for increases in δ^*, the expected profits of firm B is increasing.

Firm A's expected profits are

$$\left[\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)\left[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))\right]\right]\left[q^A - E[q_2^B|R=0]\right] +$$
$$(1-\gamma)\delta^*F(\overline{p})\left[q^A - E[q_2^B|R=-1]\right],$$

and substituting the expectations from Lemma 11, we get

$$\gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)\left[1 - F(q^A)(1-\delta^*)\right]q^A.$$

Observing that an increase in δ^* decreases $\overline{p}$, it must be that $\gamma(1 - F(q^h - \overline{p}))(q^A - q^h)$ becomes larger (since $q^A - q^h < 0$), and also $(1 - F(q^A)(1-\delta^*))$ becomes larger. Therefore the profits of firm A increase in δ^*.

Note that firm A's profits are strictly below monopoly level when firm B sells.

$$2q^A > \gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)\left[1 - F(q^A)(1-\delta^*)\right]q^A \Leftrightarrow$$
$$q^A(1 + \gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)) > -\gamma(1 - F(q^h - \overline{p}))q^h$$

which is always true.

Therefore the profits of firm A are first flat, then discontinuously decreases in δ^* as firm B begins to sell, and then increasing but remains strictly lower than when it was a monopolist. This concludes the proof.

We now show the results on consumer surplus.

First suppose $\delta^* > \underline{\delta}$. To derive consumer surplus, note that total surplus is

$$\gamma q^h + \gamma \left[F(q^h - \overline{p})q^h + (1 - F(q^h - \overline{p}))q^A\right] + (1-\gamma)\left[\delta^* q^A + (1-\delta^*)(1 - F(q^A))q^A\right].$$

Then consumer surplus is given by total surplus minus the expected profits of A and B. Rearranging leads to the consumer surplus

$$q^A + \left[\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)\right] q^A + \gamma(1 - F(q^h - \overline{p}))q^h.$$

We can further simplify this to

$$q^A + \gamma q^h + \gamma F(q^h - \overline{p})(q^A - q^h) + (1-\gamma)(1-\delta^*)F(q^A)q^A.$$

Note that an increase in δ^* leads to a decrease in $\overline{p}$. Thus, since $(q^A - q^h) < 0$, it is immediate to see that consumers are worse off when δ^* increases.

Suppose now that $\delta^* \leq \underline{\delta}$. Then firm B does not enter and firm A is a monopolist. This way firm A is able to extract all surplus and consumer surplus is zero. □

Proof of Corollary 3.

Suppose $\delta^* > \underline{\delta}$ and consider an increase in $\overline{e}$ such that there is a first order stochastic dominant shift in the uniform rating cost distribution.

We first show the effect of a change in $\overline{e}$ on π^B, recall from the previous corollary that the profit function is $\gamma q^h(1 + \frac{q^h - \overline{p}}{\overline{e}}) - q^A$. Then its derivative w.r.t. $\overline{e}$ is $-\gamma q^h \left[\frac{q^h - \overline{p}}{\overline{e}^2} + \frac{\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}}}{\overline{e}}\right]$. Since the first term in squared brackets is positive and the second one negative, the effect on π^B is ambiguous.

We next look at the effect on π^l. Recall that the low-quality firm B is indifferent between the payoff from setting $\overline{p}$ and $\underline{p}$. Hence, it suffices to show that the profits $\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A$ is decreasing in $\overline{e}$. Notice that the derivative is $-\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*)^2}\frac{\partial \delta^*}{\partial \overline{e}} < 0$.

We now look at the effect on π^h. Observe that the profits of the high-quality firm B is

$$\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} - q^A + \frac{q^h - \overline{p}}{\overline{e}}\left[\frac{\gamma(q^h - \overline{p})q^h}{\gamma(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)(q^A)} - q^A\right].$$

We can see that the term in squared brackets increases in $\overline{e}$ since it increases δ^*. But a larger $\overline{e}$ also decreases the probability that h gets a rating $\frac{q^h - \overline{p}}{\overline{e}}$. Thus, the overall effect is ambiguous.

We now argue that a change in $\overline{e}$ has an ambiguous effect on consumer surplus. To see this, observe that consumer surplus, using the term from the proof of Corollary 2, is

$$q^A + \gamma q^h + \gamma F(q^h - \overline{p})(q^A - q^h) + (1-\gamma)(1-\delta^*)F(q^A)q^A.$$

Then recall from Proposition 3 that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$ and from above note $\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}} < 0$. Then the first and second terms are constant in $\overline{e}$. In the third term, $q^h - \overline{p}$ increase in $\overline{e}$, but $F(q^h - \overline{p})$ decreases in $\overline{e}$, so the effect is ambiguous. In the final term, an increase in $\overline{e}$ decreases $1-\delta^*$ and $F(q^A)$. Therefore, any increase in $\overline{e}$ can have an ambiguous effect on consumer surplus.

Finally, we show $\overline{e}$ has an ambiguous effect on π^A. To see this, consider firm A's profit

$$\gamma(1 - F(q^h - \overline{p}))(q^A - q^h) + (1-\gamma)[\delta^* + (1-\delta^*)(1 - F(q^A))]q^A$$

Again apply that $\frac{\partial \delta^*}{\partial \overline{e}} > 0$ and $\frac{\partial \overline{p}}{\partial \delta^*}\frac{\partial \delta^*}{\partial \overline{e}} < 0$. Then we know the first term increases in $q^h - \overline{p}$, but $\overline{e}$ has a direct negative effect in F, so the overall effect is ambiguous. The second term becomes more positive as δ^* increases, but also via the direct effect of $\overline{e}$ on $F(q^A)$. Hence, the overall effect is ambiguous. □

Proof of Corollary 4.
By Corollary 1, newcomers enter if and only if $\overline{e}$ is sufficiently large. Additionally, by Corollary 2 consumer surplus is zero without entry and strictly positive with entry, the result follows. □

Appendix B Extensions

B.1 Selling after no rating

In the base model we focus on the scenario where firm B does not sell after receiving no rating. We establish two results. First, we show that a qualitatively identical equilibrium arises. Second, we show that our comparative statics of $\frac{\partial \delta^*}{\partial \overline{e}}$ are also robust, that is $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Note that if firm B does not sell only following a negative rating, this means $E[q_2^B|R=-1] = 0 \leq q^A < E[q_2^B|R=0]$.

Proposition 5. *Suppose the following condition holds:*

$$\frac{q^h - q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1-F(|\overline{p}|)) + (1-\delta^*)(1-F(q^A))]}{\gamma(1-F(q^h-\overline{p}))}. \tag{9}$$

Then in equilibrium both newcomers enter:

1. ***Ratings build reputation:*** $E[q_2^B \mid R=1] > E[q_2^B \mid R=0] > E[q_2^B \mid R=-1]$.
2. ***Ratings are valuable:*** $p_2^B(R=1) = E[q_2^B \mid R=1] - q^A > 0$ *and* $p_2^B(R=0) = E[q_2^B \mid R=0] - q^A > 0$*. If* $R=-1$*, firm* A *sells in period 2.*

Furthermore, in period 1:

3. ***Firm*** *A sets $p_1^A = 0$ and faces no demand.*
4. ***Firm*** *h charges $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A$ with probability 1 and receives $R \in \{0, 1\}$.*
5. ***Firm*** *l randomizes over prices in period 1 if $\frac{\gamma(q^h-2q^A)}{2(1-\gamma)q^A} > 0$ and (11) hold. In that case:*
   (a) It charges $\overline{p} > 0$ with probability δ^* and receives $R \in \{-1, 0\}$, where $\delta^* \in \left(\underline{\delta}, \frac{\gamma(q^h-2q^A)}{2(1-\gamma)q^A}\right)$.
   (b) It charges $\underline{p} \equiv -q^A < 0$ with probability $1 - \delta^*$ and receives $R \in \{0, 1\}$.

   Otherwise, firm l sets $\delta^* = 1$ such that $\overline{p} \leq 0$ and receives $R \in \{0, 1\}$.

Finally, if (12) holds, this equilibrium is unique up to off-path beliefs.

(12) is a technical condition. It ensures that if the mixed-strategy equilibrium is played, it is unique. Otherwise, there might be multiple $\delta^* \in (0, 1)$ that induce a mixed-strategy equilibrium.

Recall that our results from Appendix A.1 still apply here, i.e. Lemmas 3 to 9 and Corollary 5 to 8 hold also for the two-period model in which the firm B sells after receiving no rating in period 1. We proceed by establishing Lemmas and Corollaries as we did towards the proof of Proposition 4.

Lemma 14. *Suppose both types of newcomer enter. If (1), (2) and (3) hold, and $\overline{p} > 0$, second period beliefs over firm B's quality are*

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)(1-\delta^*)F(q^A)} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]} & \text{if } R = 0 \\ 0 & \text{if } R = -1 \end{cases}$$

where δ^* is the equilibrium probability with which a low-quality firm B plays $\overline{p}$ in period 1.

If (1), (2) and (3) hold, and instead $\overline{p} \leq 0$, second period beliefs are

$$E[q_2^B] = \begin{cases} \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p})+(1-\gamma)F(|\overline{p}|)} & \text{if } R = 1 \\ \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)(1-F(|\overline{p}|))} & \text{if } R = 0 \\ 0 & \text{if } R = -1. \end{cases}$$

In either case, ratings are beneficial in equilibrium as $E[q_2^B|R = 1] > E[q_2^B|R = 0] > E[q_2^B|R = -1]$.

If not both types of newcomers enter, consumers believe the quality of newcomers is q^l.

Proof of Lemma 14.
Suppose that both newcomer types enter.

When $\overline{p} > 0$, the argument for the case with $R = 1$ follows directly from Lemma 11. When $\overline{p} \leq 0$, the proofs for $R = 1$ and $R = 0$ follow directly from Lemma 11.

What remains to show is consumer beliefs when $\overline{p} > 0$ for $R = 0$ and $R = -1$, and when $\overline{p} \leq 0$ for $R = -1$.

Suppose first $\overline{p} > 0$. Then when $R = 0$, Corollary 6 and 7 show that both high and low-quality firms can get no rating. This occurs if prices are set sufficiently low but with some probability $1-F(q^B-\overline{p})$ consumers face a high cost of rating and do not want to rate or in the case of the low-quality firm setting a sufficiently high price with some probability $1-F(\overline{p})$ consumers face a high cost of ratings and do not want to rate. Therefore, $E[q_2^B|R=0] = \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]}$.

Whenever $R = -1$, Corollary 7 shows only low-quality firms may receive bad ratings. In other words, high-quality firms receive bad ratings with probability 0, and $R = -1$ clearly identifies low-quality firms. Therefore, for any p, $E[q_2^B|R=-1] = 0$.

Finally, to show that ratings are beneficial, we have to show $E[q_2^B|R = 1] > E[q_2^B|R = 0] > E[q_2^B|R=-1]$. The second inequality is immediately satisfied. To see the first inequality, consider first $\overline{p} > 0$:

$$\begin{aligned}
&\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} \\
> &\frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))]} \\
&\delta^* F(q^h-\overline{p})(1-F(\overline{p})) + (1-\delta^*)(F(q^h-\overline{p}) - F(q^A)) > 0.
\end{aligned}$$

This inequality always holds because $F(\cdot) \in [0,1]$, $\delta^* \in [0,1]$, $q^h - \overline{p} > q^A$ imply that $F(q^h-\overline{p}) > F(q^A)$. Consider next $\overline{p} \leq 0$:

$$\begin{aligned}
\frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)F(|\overline{p}|)} &> \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)(1-F(|\overline{p}|))} \\
(1-F(|\overline{p}|))F(q^h-\overline{p})q^h &> (1-F(q^h-\overline{p}))F(|\overline{p}|)q^h \\
\frac{F(q^h-\overline{p})}{F(|\overline{p}|)} &> \frac{1-F(q^h-\overline{p})}{1-F(|\overline{p}|)}.
\end{aligned}$$

This inequality always holds as $q^h > 0 \to 1 > F(q^h-\overline{p}) > F(|\overline{p}|) > 0$. Hence, $\frac{F(q^h-\overline{p})}{F(|\overline{p}|)} > 1 > \frac{1-F(q^h-\overline{p})}{1-F(|\overline{p}|)}$.

Finally, for the off-equilibrium case after not both types of newcomers enter, consumers believe the quality of newcomers is q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in period 1. □

Lemma 15. *Firm B's decision to sell in period 1 is independent of its quality realization. Thus, in period 1, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in the second period, it must also sell in the first period.*

Proof of Lemma 15.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now consider firm B selling only when it is high-quality.

Suppose instead firm B only sells when it is high-quality. This means $E[q^B|p] = q^h \ \forall p$. The high-quality firm B receives a positive rating with some positive probability at all prices at which it sells. This allows it to get a continuation profit of $q^h - q^A > 0$ in period 2. But then there is a profitable deviation from a low-quality firm B to enter the market with positive prices in period 1. Hence it cannot be that only the high-quality firm sells in the market in period 1.

Suppose instead a high-quality firm B chooses to enter the market only in period 2 and not period 1. Then it faces the same payoffs as entering in the first period, while forgoing to continuation profits it may get from entering the market in period 1. Thus, h has a profitable deviation to enter in period 1.

Therefore, we can conclude that if firm B enters, it enters the market in the first period and its entry decision is independent of its quality realization. □

Lemma 16. *Suppose h and l enter. Both firms receive some positive demand only if firm A sells in period 2, and firm B sells in period 1 with probability* 1*. Firm A sells in period 2 if and only if* $\overline{p} > 0$ *(such that consumers give negative ratings) and firm B sells in period 2 if* (9) *holds.*

Proof of Lemma 16.
The proof takes two parts. First, proving that if firm B enters, it sells in period 1 with probability 1 and firm A only sells in period 2. Second, we derive the conditions for firm A to sell in period 2 and firm B with no rating to sell in period 2.

To show the first statement, recall that by Lemma 15 the low-quality firm B sells in period 1 if and only if the high-quality firm B sells in period 1. Furthermore, by Lemma 15 if firm B sells in period 2, it must also sell in period 1 if it enters. This implies the high-quality firm B must sell in period 1 if it enters. If the high-quality firm B sells in period 1, the low-quality firm B must also sell in period 1. Hence, firm B must sell with probability 1 in period 1 if it enters. This means the only possibility for firm A to sell is in period 2.

Thus, we need to check (i) firm A sells in period 2, (ii) firm B sells in period 2 if it has no rating. To show (i) is immediate as $q^A > 0$ implies that firm A sells if and only if firm B gets a negative rating. Note firm B only gets negative ratings if $\overline{p} > 0$.

To check for (ii) to hold,

$$\frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p}))+(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]} > q^A$$
$$\Leftrightarrow \frac{q^h-q^A}{q^A} > \frac{(1-\gamma)[\delta^*(1-F(\overline{p}))+(1-\delta^*)(1-F(q^A))]}{\gamma(1-F(q^h-\overline{p}))} \tag{9}$$

□

Corollary 13. *Suppose h and l enter. Ratings are instrumental (i.e. affect beliefs and outcomes) if and only if a high-quality firm B sells in period 2 and firm B sells in period 1 with probability 1. A high-quality firm B sells in period 2 if (9) holds.*

Proof of Corollary 13.
If a high-quality firm B sells in period 2, we know from Lemma 16 that both high- and low-quality firm B must have sold in period 1. Then we also know from Lemma 14 that ratings change consumer beliefs.

In turn, if ratings are instrumental, a high-quality firm B must sell in period 1, which requires that it sells in period 2.

Thus, ratings are instrumental if and only if a high-quality firm B sells in period 2, which holds if $E[q_2^B|R=1] > q^A$, i.e. if (9) is satisfied. □

Lemma 17. *Suppose (4) and (9) hold, as well as (1), (2) and (3). Then*

1. *If $\overline{p} > 0$ and (11) hold, there exists a mixed strategies where $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$.*
2. *If $\overline{p} > 0$ and (11) hold, (12) is a sufficient condition for a unique mixed strategy exists where $\delta^* \in (0, \frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$.*
3. *If $\overline{p} > 0$ holds and (11) is violated, then there exists a unique $\delta^* = 1$.*
4. *If $\overline{p} \leq 0$, then newcomers sell in period 1 and $\delta^* = 1$.*

Proof of Lemma 17.
We first characterize the total profits that a low-quality firm B would receive if it plays $\overline{p}$ in period 1. Then its total profits when playing $\underline{p}$ in period 1. We then find the conditions for an interior solution $\delta^* \in (0,1)$ exists, and when such a solution is unique.

We now characterize the profits the low-quality firm B would receive if it plays $\overline{p}$. When the firm does so, it receives a profit of $\overline{p}$ in the first period, and with probability $F(\overline{p})$ a bad rating and with complementary probability $(1-F(\overline{p}))$ no rating. This leaves the firm with a continuation profit of 0 with probability $F(\overline{p})$ and $\pi_2(R=0)$ with probability $(1-F(\overline{p}))$. Note that the firm is able to sell at a strictly positive price with strictly positive probability and therefore enters.

We next characterize the profits the low-quality firm B would receive if it plays $\underline{p}$. When the firm does so, it receives a profit of $\underline{p}$ in the first period, and with probability $F(-\underline{p}) = F(q^A)$ it receives a good rating and with complementary probability $(1-F(q^A))$ it receives no rating. Hence leaving the firm with a continuation profit of $\pi_2(R=1)$ with probability $F(q^A)$ and $\pi_2(R=0)$ with probability $(1-F(q^A))$.

Recall from Lemma 7 that $\delta^* = 1$ if $\overline{p} \leq 0$. Additionally, note that by (4), l gets strictly positive demand if it only sells after a positive rating, so this condition implies it also gets strictly positive demand if it sells after a positive and no rating. This proves statement 4. Hence, for the remainder of the proof we focus on the situation where $\overline{p} > 0$.

Note the following 3 conditions must be satisfied for an interior solution $\delta^* \in (0,1)$ to exist: (i) At any $\delta^* \in (0,1)$, the low-quality firm B has to be indifferent between setting $\overline{p}$ obtaining either no or bad ratings, and setting $\underline{p}$ obtaining either good or no ratings. (ii) At $\delta^* = 0$, the benefit from setting $\overline{p}$ must be strictly larger than the benefit of setting $\underline{p}$. (iii) At $\delta^* = 1$, the benefit of setting $\underline{p}$ must be strictly larger than the benefit of setting $\overline{p}$.

Thus, for an interior solution, (i) implies the following equation must hold:

$$\begin{aligned}
&\overline{p} + (1-F(\overline{p}))\pi_2(R=0) = \underline{p} + F(q^A)\pi_2(R=1) + (1-F(q^A))\pi_2(R=0) \\
&\Leftrightarrow \overline{p} - F(\overline{p})\pi_2(R=0) = \underline{p} + F(q^A)\pi_2(R=1) - F(q^A)\pi_2(R=0) \\
&\Leftrightarrow \frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*} + (F(q^A) - F(\overline{p}))\pi_2(R=0) - F(q^A)\pi_2(R=1) = 0,
\end{aligned} \tag{10}$$

defining the LHS of (10) as K. Where

$$\begin{aligned}
\pi_2(R=0) &= \frac{\gamma(1-F(q^h-\overline{p}))q^h}{\gamma(1-F(q^h-\overline{p})) + (1-\gamma)[\delta^*(1-F(\overline{p})) + (1-\delta^*)(1-F(q^A))]} - q^A, \\
\pi_2(R=1) &= \frac{\gamma F(q^h-\overline{p})q^h}{\gamma F(q^h-\overline{p}) + (1-\gamma)(1-\delta^*)F(q^A)} - q^A.
\end{aligned}$$

Then (ii) means $K > 0$ at $\delta^* = 0$ is required for an interior solution. Note that at $\delta^* = 0$, $\overline{p} = \frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*} - q^A = q^h - q^A$. Then, evaluated at $\delta^* = 0$

$$\begin{aligned}
K =& q^h - F(q^A)(\frac{\gamma F(q^A)q^h}{\gamma F(q^A) + (1-\gamma)F(q^A)} - q^A) \\
&+ (F(q^A) - F(q^h - q^A))(\frac{\gamma(1-F(q^A))q^h}{\gamma(1-F(q^A)) + (1-\gamma)(1-F(q^A))} - q^A) \\
=& q^h - F(q^A)(\gamma q^h - q^A) + (F(q^A) - F(q^h-q^A))(\gamma q^h - q^A) \\
=& q^h - F(q^h - q^A)(\gamma q^h - q^A),
\end{aligned}$$

because $F(\cdot) \in (0,1)$ and $q^h > \gamma q^h > 0$, when evaluated at $\delta^* = 0$, $K > 0$.

Then (iii) means $K < 0$ at $\delta^* = 1$ is required for an interior solution. Note that at $\delta^* = 1$, $\overline{p} = \gamma q^h - q^A$. Then, evaluated at $\delta^* = 1$

$$K = \gamma q^h + (F(q^A) - F(\overline{p}))(\frac{\gamma(1 - F(q^h - \overline{p}))q^h}{\gamma(1 - F(q^h - \overline{p})) + (1 - \gamma)(1 - F(\overline{p}))} - q^A) < 0, \tag{11}$$

is required for an interior solution. This implies $F(\overline{p}) > F(q^A)$ is a necessary condition for an interior solution. Note that since $\overline{p}$decreases in δ^*, (11) implies that $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0, 1]$.

Hence two conditions are required for an interior solution: $\overline{p} > 0$ and (11)$\Rightarrow$ $F(\overline{p}) > F(q^A)$.

To show that the interior solution is unique, it suffices to show that K is decreasing in δ^*.

$$\begin{aligned}
\frac{\partial K}{\partial \delta^*} &= \frac{\partial \overline{p}}{\partial \delta^*} - F(q^A)\frac{\partial \pi_2(R=1)}{\partial \delta^*} - f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*}(F(\overline{p}) - F(q^A)), \\
\frac{\partial \overline{p}}{\partial \delta^*} &= -\frac{\gamma(1-\gamma)q^h}{(\gamma + (1-\gamma)\delta^*)^2} < 0, \\
\frac{\partial \pi_2(R=1)}{\partial \delta^*} &= \frac{F(q^A)\gamma(1-\gamma)q^h)(F(q^h - \overline{p}) - (1-\delta^*)f(q^h - \overline{p})\frac{\partial \overline{p}}{\partial \delta^*})}{(\gamma F(q^h - \overline{p}) + (1-\gamma)(1-\delta^*)F(q^A))^2} > 0, \\
\frac{\partial \pi_2(R=0)}{\partial \delta^*} &= \frac{\gamma(1-\gamma)q^h\frac{\partial \overline{p}}{\partial \delta^*}(\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A)))f(q^h - \overline{p})}{(\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))])^2} \\
&+ \frac{\gamma(1-\gamma)q^h(\delta^* f(\overline{p})\frac{\partial \overline{p}}{\partial \delta^*} + F(\overline{p}) - F(q^A))(1 - F(q^h - \overline{p}))}{(\gamma(1 - F(q^h - \overline{p})) + (1-\gamma)[\delta^*(1 - F(\overline{p})) + (1-\delta^*)(1 - F(q^A))])^2}.
\end{aligned}$$

Recall that (11) is required for an interior solution, which implies $F(\overline{p}) > F(q^A)$ for all $\delta^* \in (0, 1]$. Then $\frac{\partial \pi_2(R=0)}{\partial \delta^*} > 0$ is a sufficient but not necessary condition for $\frac{\partial K}{\partial \delta^*} < 0$. Rewriting this sufficient condition,

$$\begin{aligned}
&\frac{\partial \pi_2(R=0)}{\partial \delta^*} \geq 0 \Leftrightarrow \\
&F(\overline{p}) - F(q^A) \geq \frac{-\frac{\partial \overline{p}}{\partial \delta^*}((1 - F(q^A))f(q^h - \overline{p}) + (1 - F(q^h - \overline{p}))\delta^* f(\overline{p}))}{1 - F(q^h - \overline{p}) - f(q^h - \overline{p})\frac{\partial \overline{p}}{\partial \delta^*}\delta^*} > 0.
\end{aligned} \tag{12}$$

Finally, because (11) implies $F(\overline{p}) > F(q^A)$, then it must be that $\overline{p} > q^A$, which means $\frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)} > \delta^*$.

With these conditions, we may now show the statements in the Lemma.

Statement 1. If $\overline{p} > 0$ and (11) holds, then there exists solutions to (10) such that all solutions are interior, $\delta^* \in (0, \frac{\gamma(q^h - 2q^A)}{2q^A(1-\gamma)})$.

Statement 2. A sufficient condition for these interior solutions to be unique is (12).

Statement 3. If $\overline{p} > 0$ and (11) is violated, then there exists a unique $\delta^* = 1$.

Statement 4. If $\overline{p} \leq 0$ there exists a unique $\delta^* = 1$.

□

Proof of Proposition 5.

Suppose (4) and (9) hold, as well as (1), (2) and (3).

Note first that by Lemma 17, (4) and (9) imply that newcomers always have strictly positive demand. Together with our Selection Assumption 1, this implies newcomers enter with probability one.

Statement 1 follows directly from Lemma 14.

Statement 2 follows directly from Corollary 8.

Statement 3 follows directly from Corollary 13 and Lemma 5.

Statement 4 follows directly from Lemma 9, Corollary 6 and 8

The conditions in statement 5 follow from Lemma 17.

The prices in statement 5 follow from the proof of Lemma 17 and the ratings of the low-quality firm from Lemma 7.

The equilibrium level of δ^* and its support follow from Lemma 17.

□

B.1.1 Comparative statics

Again, we focus on mixed-strategy equilibria. We also impose (12) to ensure it is unique. If the mixed-strategy equilibrium is not unique, then we cannot ensure that changes in parameters induce a jump to another mixed-strategy equilibrium.

Corollary 14. $\frac{\partial \delta^*}{\partial \overline{e}} > 0$.

Proof of Corollary 14.
We apply the uniform distribution to (10), which leads to

$$\overline{p} + q^A + \frac{q^A - \overline{p}}{\overline{e}} \pi_2(R = 0) - \frac{q^A}{\overline{e}} \pi_2(R = 1) = 0.$$

Now, we calculate the total derivative with respect to $\overline{e}$ as

$$-\frac{\gamma(1-\gamma) q^h \frac{\partial \delta^*}{\partial \overline{e}}}{(\gamma + (1-\gamma)\delta^*)^2} + \frac{q^A}{\overline{e}^2} \left[\pi_2(R=1) - \pi_2(R=0)\right] + \frac{q^A}{\overline{e}} \left[\frac{\partial \pi_2(R=0)}{\partial \delta^*} - \frac{\partial \pi_2(R=1)}{\partial \delta^*}\right] \frac{\partial \delta^*}{\partial \overline{e}} - \left[\frac{\partial \overline{p}}{\partial \delta^*} \frac{\partial \delta^*}{\partial \overline{e}} - \frac{\overline{p}}{\overline{e}^2}\right] \pi_2(R=0) - \frac{\partial \pi_2(R=0)}{\partial \delta^*} \frac{\partial \delta^*}{\partial \overline{e}} \frac{\overline{p}}{\overline{e}} = 0.$$

Rearranging leads to

$$\frac{\partial\delta^*}{\partial\bar{e}}\left[\frac{\partial\overline{p}}{\partial\delta^*}\left[1-\frac{\pi_2(R=0)}{\bar{e}}\right]+\frac{\partial\pi_2(R=0)}{\partial\delta^*}\frac{q^A-\overline{p}}{\bar{e}}-\frac{\partial\pi_2(R=1)}{\partial\delta^*}\frac{q^A}{\bar{e}}\right]=\frac{q^A-\overline{p}}{\bar{e}^2}\pi_2(R=0)-\frac{q^A}{\bar{e}^2}\pi_2(R=1).$$

Recall from the proof of Lemma 17 that $\frac{\partial\overline{p}}{\partial\delta^*}=-\frac{\gamma(1-\gamma)q^h}{(\gamma+(1-\gamma)\delta^*)^2}<0$, $\frac{\partial\pi_2(R=1)}{\partial\delta^*}>0$, and by (12) also $\frac{\partial\pi_2(R=0)}{\partial\delta^*}>0$.

Observe that the right side of this equation is negative. Applying (2), $1-\frac{\partial\pi_2(R=0)}{\partial\bar{e}}>0$ and because $\delta^*\in(0,\frac{\gamma(q^h-2q^A)}{2q^A(1-\gamma)})$, by the proof of Lemma 17, in mixed-strategy equilibria we must have $q^A-\overline{p}<0$.

Therefore $\frac{\partial\overline{p}}{\partial\delta^*}\left[1-\frac{\pi_2(R=0)}{\bar{e}}\right]<0$, $\frac{\partial\pi_2(R=0)}{\partial\delta^*}\frac{q^A-\overline{p}}{\bar{e}}<0$ and $\frac{\partial\pi_2(R=1)}{\partial\delta^*}\frac{q^A}{\bar{e}}>0$, which implies the term in the large squared brackets is negative. Hence, $\frac{\partial\delta^*}{\partial\bar{e}}>0$. □

B.2 Three-period model

We extend our main model and allow for three periods. To start, we explain how we change the main model.

We introduce notation to keep track of histories, denoting $\mathbb{H}_t$ a history of the game until period t, where a history is characterized by the historic ratings until and not including t. To simplify notation, we may suppress the subscript t. We may also use histories as subscripts to clarify arguments, i.e. $\overline{p}_{\{0,1\}}$ is the large newcomer price in period 3 for a history with $R_1=0$ and $R_2=1$, or $\overline{p}_{\{0\}}$ is the large newcomer price in period 2 for a history with $R_1=0$. Throughout, we denote $\emptyset$ as the history in period 1 when the game begins.

Entry. The newcomer B chooses to enter/exit in each period 1 and 2 and they do so if and only if their subsequent demand is non-zero. Due to this assumption, this extension also captures the possibility of exit better than our baseline model.

Low-quality newcomers enter with probability 1 or 0. We also have to adjust Condition (4) to this setting and replace it with Conditions (17) and (18). The first one ensures that low-quality newcomers enter with probability 1 or 0 in period 1, and the second one ensures this for period 2.

Silence is bad news. We extend Condition (5) to this setting in the following way. We focus on equilibria where firm B sells in period 2 and 3 if and only if it has a history of exclusively good ratings. Again, this is in line with evidence we discuss in the main text that silence is bad news. In other words, firm B only sells if $\mathbb{H}_t\in\{\emptyset,\{1\},\{1,1\}\}$. A sufficient condition is that $\max\{E[q_3^B|\{1,0\}],E[q_2^B|\{0\}]\}<q^A$. I.e. if the newcomer does not sell after a history of one good and no rating, and not after a history of no rating, they also do not sell after other histories that involve no rating or a negative rating, since expectations must be lower in such histories. Intuitively, this is satisfied if q^h is sufficiently large, since then the high-quality firm is unlikely to receive no

rating, lowering expectations for every history with no rating.

Other conditions and restrictions. The conditions that the PDF of F is sufficiently flat are the same as in the proof of Proposition 4. Also the equilibrium-selection assumptions are the same as in the main text. We also continue to assume $\gamma q^h < q^A$.

Proposition 6. *If f is sufficiently flat (i.e.* (1), (2) *and* (3) *hold),* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$*, and* (17) *and* (18) *hold. There exists an equilibrium that is unique up to off-path beliefs. In this equilibrium, there exist unique values $\underline{\delta}_\emptyset \in (0,1)$ and $\underline{\delta}_{\{1\}} \in (0,1)$ such that the following holds. Both types of newcomer B enter in period 1 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$; otherwise none enters. Both types of newcomer B enter in period 2 if and only if $\delta^*_\emptyset \geq \underline{\delta}_\emptyset$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ hold; otherwise none enters in period 2.*

Furthermore, if the newcomer enters in period t:

1. ***Ratings build reputation:*** *For $t > 1$, $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] \geq E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] \geq E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$, where the inequalities are strict if $\mathbb{H}_{t-1}$ only includes positive ratings.*
2. ***Ratings are valuable:*** *$p_{\mathbb{H}_3} = E[q_3^B|\mathbb{H}_3] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings, and for $t < 3$, we have $p_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, p_{\mathbb{H}_t}] - q^A > 0$ if $\mathbb{H}_t$ only contains positive ratings. Otherwise, $p_{\mathbb{H}_t} = 0$ and firm A sells in period t.*

Furthermore, in period $t < 3$ for any history $\mathbb{H}_t = \{\emptyset, \{1\}\}$:

3. ***Firm** A sets $p_t^A = 0$ and faces no demand.*
4. ***Firm** h charges $\overline{p}_{\mathbb{H}_t} = E[q_t^B|\mathbb{H}_t, \overline{p}_{\mathbb{H}_t}] - q^A$ with probability 1 and receives $R_t \in \{0,1\}$.*
5. ***Firm** l either charges a mixed-strategy equilibrium such that:*
   (a) *It charges $\overline{p}_{\mathbb{H}_t} > 0$ with probability $\delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{-1, 0\}$, where $\delta^*_{\mathbb{H}_t} \in (\underline{\delta}_t, 1]$.*
   (b) *It charges $\underline{p}_{\mathbb{H}_t} = -q^A < 0$ with probability $1 - \delta^*_{\mathbb{H}_t}$ and receives $R_t \in \{0,1\}$.*

   Otherwise, if this mixed-strategy equilibrium does not exist, firm l sets $\delta^*_{\mathbb{H}_t} = 1$ such that $\overline{p}_{\mathbb{H}_t} < 0$ and receives either $R_t = 1$ or $R_t = 0$.

Towards proving this proposition, we will show a range of lemmas and corollaries that follow along the lines of the proof of Proposition 4. Recall that Lemmas 3 to 9 and Corollary 5 to 8 hold also for the three period model.

Lemma 18. *Given* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ *such that firm B sells if and only if $\mathbb{H} = \{\emptyset, \{1\}, \{1,1\}\}$:*

The period 2 beliefs if both types of newcomers enter in period 1 and 2 are

$$E[q_2^B|\mathbb{H}_1,p_2] =$$

$$\begin{cases} \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}(1-F(|\overline{p}_\emptyset|))} & \text{for } \mathbb{H}_1=\{0\},\ \forall p_2=\overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset\leq 0 \\ \frac{\gamma(1-F(q^h-\overline{p}_\emptyset))q^h}{\gamma(1-F(q^h-\overline{p}_\emptyset))+(1-\gamma)\delta^*_{\{0\}}\left[\delta^*_\emptyset(1-F(\overline{p}_\emptyset))+(1-\delta^*_\emptyset)(1-F(q^A))\right]} & \text{for } \mathbb{H}_1=\{0\},\ \forall p_2=\overline{p}_{\{0\}} \text{ if } \overline{p}_\emptyset> 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)\delta^*_{\{1\}}F(|\overline{p}_\emptyset|)} & \text{for } \mathbb{H}_1=\{1\},\ \forall p_2=\overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset\leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)q^h}{\gamma F(q^h-\overline{p}_\emptyset)+(1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}}F(q^A)} & \text{for } \mathbb{H}_1=\{1\},\ \forall p_2=\overline{p}_{\{1\}} \text{ if } \overline{p}_\emptyset> 0 \\ 0 & \text{otherwise.} \end{cases}$$

And the period 3 beliefs, if both types of newcomers entered in period 1 and 2, are

$$E[q_3^B|\mathbb{H}_2,p_3] =$$

$$\begin{cases} \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)(1-F(|\overline{p}_{\{1\}}|))} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset,\overline{p}_{\{1\}}\leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-F(|\overline{p}_{\{1\}}|))} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset>0,\ \overline{p}_{\{1\}}\leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)F(|\overline{p}_\emptyset|)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset\leq 0,\ \overline{p}_{\{1\}}> 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))q^h}{\gamma F(q^h-\overline{p}_\emptyset)(1-F(q^h-\overline{p}_{\{1\}}))+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)\left[\delta^*_{\{1\}}(1-F(\overline{p}_{\{1\}}))+(1-\delta^*_{\{1\}})(1-F(q^A))\right]} \\ \qquad \text{for } \mathbb{H}_2=\{1,0\},\ \forall p_3 \text{ if } \overline{p}_\emptyset,\overline{p}_{\{1\}}> 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset,\overline{p}_{\{1\}}\leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset>0,\ \overline{p}_{\{1\}}\leq 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset\leq 0,\ \overline{p}_{\{1\}}> 0 \\ \frac{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})q^h}{\gamma F(q^h-\overline{p}_\emptyset)F(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)} \\ \qquad \text{for } \mathbb{H}_2=\{1,1\},\ \forall p_3 \text{ if } \overline{p}_\emptyset,\overline{p}_{\{1\}}> 0 \\ 0 \qquad \text{for } \mathbb{H}_2=\{1,-1\},\ \forall p_3 \\ E[q_2^B|\mathbb{H}_1,p_2] \qquad \text{otherwise.} \end{cases}$$

where $\delta^*_\emptyset$ *is the mixed strategy in the first period,* $\delta^*_{\mathbb{H}_t}$ *is the mixed strategy in the period* t *following* *a history $\mathbb{H}_t$. And $\overline{p}_{\mathbb{H}_t}$ and $\underline{p}_{\mathbb{H}_t}$ are the prices following a history $\mathbb{H}_t$.*

After histories where not both types of newcomers enter, beliefs are $q^l = 0$.

Proof of Lemma 18.
We now explain how we apply Bayes Rule to construct the above beliefs.

Recall from Corollary 8 that $\underline{p}_{\mathbb{H}_t} = -q^A$.

Corollary 7 implies that for any history which includes -1, consumer beliefs are q^l. Such histories are $\mathbb{H} = \{\{-1\}, \{1, -1\}, \{0, -1\}, \{-1, 1\}, \{-1, 0\}, \{-1, -1\}\}$.

We focus on the histories where firm B does not sell following no rating. In other words, following no rating, any subsequent beliefs must be the same as the previous period. Hence, following $\mathbb{H}_1 = 0$, it must be that beliefs are unchanged (for $\mathbb{H}_2 = \{\{0, 1\}, \{0, 0\}, \{0, -1\}\}$).

In any equilibrium where $\overline{p}_{\mathbb{H}_t} \leq 0$, Lemma 7 shows that $\delta^*_{\mathbb{H}_t} = 1$.

We now construct beliefs for $\mathbb{H}_1 = \{\{0\}, \{1\}\}$. Suppose $\overline{p}_\emptyset \leq 0$, then $\delta^*_\emptyset = 1$ such that low-quality firms may only get no rating or a good rating. If the low-quality firm gets $R_1 = 1$, it does so with probability $F(|\overline{p}_\emptyset|)$. High-quality firms get a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firms only play $\overline{p}_{\{1\}}$ with some probability $\delta^*_{\{1\}}$, otherwise they play $\underline{p}_{\{1\}}$.

If, instead, the low-quality firm gets $R_1 = 0$, it does so with complementary probability $1 - F(|\overline{p}_\emptyset|)$. Likewise, the high-quality firm gets $R_1 = 0$ with complementary probability $1 - F(q^h - \overline{p}_\emptyset)$. Following $R_1 = 0$, low-quality firms only play $\overline{p}_{\{0\}}$ with some probability $\delta^*_{\{0\}}$, or they play $\underline{p}_{\{0\}}$ with probability $1 - \delta^*_{\{0\}}$.

Suppose instead that $\overline{p}_\emptyset > 0$, then $\delta^*_\emptyset \in (0, 1)$. From Lemmas 6 and 7, high-quality firm B plays $\overline{p}_\emptyset$ with probability 1, obtaining a good rating with probability $F(q^h - \overline{p}_\emptyset)$. Low-quality firm B mixes between $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$ with probabilities $\delta^*_\emptyset$ and $1 - \delta^*_\emptyset$, respectively. The low-quality firm may only obtain $R_1 = 1$ with probability $F(q^A)$ if it plays $\underline{p}_\emptyset$, which it does with probability $1 - \delta^*_\emptyset$.

Consider instead when firms get the rating $R_1 = 0$. The high-quality firm gets this rating with probability $(1 - F(q^h - \overline{p}_\emptyset))$, and l with probability $\left[\delta^*_\emptyset(1 - F(\overline{p}_\emptyset)) + (1 - \delta^*_\emptyset)(1 - F(q^A))\right]$.

This characterizes all period 2 beliefs.

We now characterize beliefs in period 3 following $\mathbb{H}_2 = \{1, 1\}$. If prices are $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$. Firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})$, and l if it charges a low price in period 1 an draws sufficiently low rating effort, i.e. $(1 - \delta^*_\emptyset)F(q^A)F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in each period and obtains a sequence of good ratings with probability $F(|\overline{p}_\emptyset|) \cdot F(|\overline{p}_{\{1\}}|)$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\overline{p}_\emptyset$ in the first period, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$. Then the high-quality firm obtains a sequence of good ratings with probability $F(q^h - \overline{p}_\emptyset) \cdot F(q^h - \overline{p}_{\{1\}})$, and the low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains a good rating with probability $F(q^A)$.

Next, we characterize beliefs in the in period 3 following $\mathbb{H} = \{1, 0\}$. If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} \leq 0$, firm h gets these ratings with probability $F(q^h - \overline{p}_\emptyset)(1 - F(q^h - \overline{p}_{\{1\}}))$. Firm l sets a low price in period 1, leading to $(1 - \delta^*_\emptyset)F(q^A)(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset, \overline{p}_{\{1\}} \leq 0$, then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_{\mathbb{H}_t}$ with probability 1 in both periods, obtaining a good rating in period 1 with probability $F(|\overline{p}_\emptyset|)$ and no rating in period 2 with probability $(1 - F(|\overline{p}_{\{1\}}|))$.

If $\overline{p}_\emptyset \leq 0$ and $\overline{p}_{\{1\}} > 0$ such that $\delta^*_\emptyset = 1$. Then the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\overline{p}_\emptyset$ in the first period with probability 1, obtaining $R_1 = 1$ with probability $F(|\overline{p}_\emptyset|)$. In the second period, the low-quality firm sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A)$. Additionally, it sets the price $\overline{p}_{\{1\}}$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If $\overline{p}_\emptyset > 0$ and $\overline{p}_{\{1\}} > 0$, the high-quality firm obtains a good rating in the first period with probability $F(q^h - \overline{p}_\emptyset)$ and no rating in the second period with probability $(1 - F(q^h - \overline{p}_{\{1\}}))$. The low-quality firm sets the price $\underline{p}_\emptyset$ with probability $(1 - \delta^*_\emptyset)$ in the first period, obtaining $R_1 = 1$ with probability $F(q^A)$ and in the second period sets $\underline{p}_{\{1\}}$ with probability $(1 - \delta^*_{\{1\}})$ and obtains no rating with probability $(1 - F(q^A))$. Additionally, it sets the price $\overline{p}_{\{1\}} > 0$ with probability $\delta^*_{\{1\}}$ and obtains no rating with probability $(1 - F(\overline{p}_{\{1\}}))$.

If not both newcomers enter either in period 1 or 2, these histories are off-equilibrium and we set beliefs to q^l. By Corollary 5, histories where only the low-quality newcomer enters are off-path. Similarly, if only the high-quality firm enters, beliefs about newcomers are q^h and the newcomer sells at $q^h - q^A$ in each subsequent period. But then l has an incentive to deviate and enter and charge $q^h - q^A$ in the period where they enter, a contradiction. We conclude that histories after which not both types enter are off-equilibrium.

This characterizes consumer beliefs in every period following every history $\mathbb{H}$ given firm B sells only after a history where both types of newcomers enter and sell only after positive ratings.

We now find the condition where firm B does not sell in any history following at least a single instance of no rating or a bad rating. To do so, we find the most optimistic history following at least a single instance of no rating or a bad rating. Then we find the condition for which that belief is worse than q^A—such that consumers prefer to purchase from firm A instead of firm B following this history.

To find the most optimistic history, note that any such history cannot include $R_t = -1$. Following any history with $R_t = -1$, beliefs are $q^l = 0$ which are the most pessimistic beliefs possible. Next, note that the histories $\mathbb{H}_t = \{\{0,0\},\{0,1\}\}$ are off-path as firm B does not sell following $\mathbb{H}_t = \{0\}$. We fix these beliefs equal $q^l = 0$. Moreover, since there are no sales on the path-of-play following $\{0\}$, this implies that $\overline{p}_{\{0\}} = \underline{p}_{\{0\}} = 0$, which implies $\delta^*_{\{0\}} = 1$. Given these statements, we know the most optimistic beliefs following a history which includes no rating or a bad rating must be either $\{0\}$ or $\{1,0\}$. Thus, $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that if both newcomers enter, they sell only after histories with no ratings if $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ holds. □

Lemma 19. *Suppose* $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ *holds and* f *is sufficiently flat such that* (1)*,* (2) *and* (3) *hold. Then for all* $\mathbb{H}_t$ *on the path of play, there exists a unique* $\delta^*_{\mathbb{H}_t}$ *such that in period 3, firms charge* $p_3 = \max\{E[q_3^B|\mathbb{H}_t] - q^A, 0\}$*. Additionally, for all* $\mathbb{H}_t$ *on the path of play.*

1. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, then* $\delta^*_\emptyset \in (0,1)$*.*
2. *If* $\overline{p}_\emptyset > 0$*,* (14)*,* $\overline{p}_{\{1\}} > 0$ *and* (13) *hold, then* $\delta^*_{\{1\}} \in (0,1)$*.*
3. *If* $\overline{p}_\emptyset > 0$ *and* (14) *hold, and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$*, then* $\delta^*_{\{1\}} = 1$*.*
4. *If* $\overline{p}_\emptyset \leq 0$*, then* $\delta^*_\emptyset = 1$*, and the low-quality firm* B *obtains good ratings with some positive probability.*
5. *If* $\overline{p}_\emptyset \leq 0$*, then* $\overline{p}_{\{1\}} > 0$ *and* (13) *holds, then* $\delta^*_{\{1\}} \in (0,1)$*.*
6. *If* $\overline{p}_\emptyset \leq 0$*, and either* (13) *is violated or* $\overline{p}_{\{1\}} \leq 0$*, then* $\delta^*_{\{1\}} = 1$*.*

For all other histories, newcomers charge a price at marginal cost.

There exist parameters where entry does or does not occur. Indeed, if $q^A \to q^h$*, newcomers will not sell even after a positive rating, implying that newcomers never enter. However, if* q^h *is sufficiently large, reputation becomes increasingly valuable so that newcomers sell after a good rating and enter.*

Proof of Lemma 19.
We proceed as follows. We start by looking at the low-quality firm B's strategy in the third period, then in the second period, and finally in the first period.

In the third period, our Selection Assumption 2 implies that a low-quality firm B sets the highest possible positive price at which it would sell. This is because the third period is the terminal period and there is no continuation reputation effect that the firm needs to consider. Hence, to maximize profits it sets the highest price at which it sells with probability 1, given this price is positive.

In the second period, by Lemma 18, $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that a low-quality firm B only sells if it has a history of $\mathbb{H}_t = \{1\}$. By Lemma 7, when the low-quality firm B sells, it mixes between $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. If the low-quality firm sets the price $\overline{p}_{\{1\}} = E[q_2^B|\{1\}, \overline{p}_{\{1\}}] - q^A > 0$, it makes the expected profits of $\overline{p}_{\{1\}} + 0$. The firm makes no continuation profit since we focus on the scenario where the firm B sells if it only has a history of good ratings. If the low-quality firm B sets $\underline{p}_{\{1\}} = -q^A$, it makes the expected profits of $\underline{p}_{\{1\}} + F(q^A)(E[q_2^B|\{1,1\}] - q^A)$ as it only sells if it obtains a good rating in period 3 with probability $F(q^A)$. Hence, $\delta^*_{\{1\}}$ makes the firm indifferent between these two choices. To find the unique $\delta^*_{\{1\}}$, we first show that the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$ and the continuation profits after setting $\underline{p}_{\{1\}}$ is strictly increasing in $\delta^*_{\{1\}}$. Then we show that when $\delta^*_{\{1\}} = 0$, the continuation profits after setting $\overline{p}_{\{1\}}$ is strictly greater than setting $\underline{p}_{\{1\}}$, which implies that $\delta^*_{\{1\}} > 0$. Finally, we fix $\delta^*_{\{1\}} = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_{\{1\}} = 1$.

To see that the continuation profits after $\overline{p}_{\{1\}}$ is strictly decreasing in $\delta^*_{\{1\}}$, note the derivative of the continuation profits is

$$-\frac{\gamma F(q^h - \overline{p}_\emptyset)(1-\gamma)(1-\delta^*_\emptyset)F(q^A)q^h}{(\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)\delta^*_{\{1\}})^2} < 0.$$

To see that the continuation profits after $\underline{p}_{\{1\}}$ are strictly increasing in $\delta^*_{\{1\}}$, note first that $\underline{p}_{\{1\}} = -q^A$, and second, for the case $\overline{p}_\emptyset > 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)(1-\delta^*_{\{1\}})F(q^A)}.$$

Since by the previous argument, $\overline{p}_{\{1\}}$ decreases in $\delta^*_{\{1\}}$, this expression strictly increases in $\delta^*_{\{1\}}$. Similarly, for the case $\overline{p}_\emptyset \leq 0$, the expectation term in period 3 is

$$\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)(1-\delta^*_{\{1\}})F(q^A)},$$

which is strictly increasing for the same reason.

Then let $\delta^*_{\{1\}} = 0$, the continuation profits after $\overline{p}_{\{1\}}$ becomes $q^h - q^A$ and, for $\overline{p}_\emptyset > 0$, the continuation profits after $\underline{p}_{\{1\}}$ becomes $-q^A + F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]$. This is strictly smaller than $q^h - q^A$, since

$$\begin{aligned} q^h - q^A &> \frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A \\ &> F(q^A)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)F(q^A)} - q^A\right]. \end{aligned}$$

The argument for $\overline{p}_\emptyset \leq 0$ follows directly from the same steps.

Finally, let $\delta^*_{\{1\}} = 1$. Then the continuation profits after $\overline{p}_{\{1\}}$ becomes $\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} - q^A$ and the continuation profits after $\underline{p}_{\{1\}}$, for $\overline{p}_\emptyset > 0$, become $-q^A + F(q^A)\left[q^h - q^A\right]$. Then if the continuation profits after $\overline{p}_{\{1\}}$ is still larger than the continuation profits after $\underline{p}_{\{1\}}$, we have $\delta^*_{\{1\}} = 1$. The interior solution exists if and only if

$$\begin{aligned}
\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} - q^A &< -q^A + F(q^A)\left[q^h - q^A\right] \\
\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} &< +F(q^A)(q^h - q^A) \\
\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)} &< F(q^A)(q^h - q^A). \qquad (13)
\end{aligned}$$

Following the same argument for $\overline{p}_\emptyset \leq 0$ shows that the same condition applies to this case.

We now turn our attention to period 1.

In the first period the low-quality firm B sets $\overline{p}_\emptyset$ with strictly positive probability and obtains a continuation profit of $\frac{\gamma q^h}{\gamma + (1-\gamma)\delta^*_\emptyset} - q^A$. Otherwise, it may set $\underline{p}_\emptyset$ and make a continuation profit of

$$-q^A + F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)(1-\delta^*_\emptyset)F(q^A)\delta^*_{\{1\}}} - q^A\right).$$

Note that firm l only sets $\underline{p}_\emptyset$ with strictly positive probability if $\overline{p}_\emptyset > 0$.

Applying the same procedure, we show that the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$ and the continuation profits after $\underline{p}_\emptyset$ is strictly increasing, implying that any $\delta^*_\emptyset$ is unique. Then allowing $\delta^*_\emptyset = 0$ we show that the continuation profits after setting $\overline{p}_\emptyset$ is strictly greater than setting $\underline{p}_\emptyset$, which tells us $\delta^*_\emptyset > 0$. We then let $\delta^*_\emptyset = 1$ and find the conditions for which an interior solution exists, failing which $\delta^*_\emptyset = 1$.

To see the continuation profits after $\overline{p}_\emptyset$ is strictly decreasing in $\delta^*_\emptyset$, note the derivative of the continuation profits is

$$-\frac{\gamma(1-\gamma)q^h}{(\gamma + (1-\gamma)\delta^*_\emptyset)^2} < 0.$$

To see the continuation profits after $\underline{p}_\emptyset$ is strictly increasing in $\delta^*_\emptyset$, note that by the previous argument, $\overline{p}_\emptyset$ decreases in $\delta^*_\emptyset$, implying that these continuation profits strictly increase in $\delta^*_\emptyset$.

Then for $\delta^*_\emptyset = 0$, the continuation profit from $\overline{p}_\emptyset$ becomes $q^h - q^A$, and the continuation profit from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)\left(\frac{\gamma F(q^h - \overline{p}_\emptyset) q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A\right)$. The continuation profits after a high price are strictly larger, since

$$q^h - q^A > \frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A$$
$$> F(q^A)(\frac{\gamma F(q^h - \overline{p}_\emptyset)q^h}{\gamma F(q^h - \overline{p}_\emptyset) + (1-\gamma)F(q^A)\delta^*_{\{1\}}} - q^A).$$

Finally, for $\delta^*_\emptyset = 1$, the continuation profits after $\overline{p}_\emptyset$ is γq^h, and the continuation profits from $\underline{p}_\emptyset$ becomes $-q^A + F(q^A)(q^h - q^A)$. Then if the continuation profits after $\overline{p}_\emptyset$ is still larger than the continuation profits after $\underline{p}_\emptyset$, we have $\delta^*_\emptyset = 1$. The interior solution exists if and only if

$$\gamma q^h - q^A < -q^A + F(q^A)(q^h - q^A)$$
$$\gamma q^h < F(q^A)(q^h - q^A). \tag{14}$$

We now address each of the statements in Lemma 19 in turn.

Statement 1. If $\overline{p}_\emptyset > 0$ and (14) hold, then there exists a unique solution which is interior in period 1, $\delta^*_\emptyset \in (0,1)$.

Statement 2. If, in addition to Statement 1, $\overline{p}_{\{1\}} > 0$ and (13) hold, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 3. If, in addition to Statement 1 we have $\overline{p}_{\{1\}} \leq 0$, then Lemma 7 implies $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 1, (13) is violated, then $\delta^*_{\{1\}} = 1$.

Statement 4. If $\overline{p}_\emptyset \leq 0$, Lemma 7 implies $\delta^*_\emptyset = 1$. Additionally, because $\overline{p}_\emptyset \leq 0$, the low-quality firm B obtains good ratings with some positive probability.

Statement 5. If, in addition to Statement 4, $\overline{p}_{\{1\}} > 0$ and (13) holds, then there exists a unique solution which is interior in period 2, $\delta^*_{\{1\}} \in (0,1)$.

Statement 6. If, in addition to Statement 4, $\overline{p}_{\{1\}} \leq 0$ then by Lemma 7, we have $\delta^*_{\{1\}} = 1$. And if, in addition to Statement 4, (13) is violated, then $\delta^*_{\{1\}} = 1$.

These statements describe $\delta^*_{\mathbb{H}_t}$ after histories that occur in equilibrium with strictly positive probability and shows they are unique up to off-path-beliefs.

All other histories occur with probability zero on the path of play, so we set newcomer prices to marginal cost.

This concludes the proof. □

Lemma 20. *After any history $\mathbb{H}_t$, a high-quality firm B sells if and only if a low-quality firm B sells. If firm B sells in any period $t+1$, it also sells in period t.*

Suppose (17) *and* (18) *hold. Then there exist unique values* $\underline{\delta}_{\emptyset} \in (0,1)$ *and* $\underline{\delta}_{\{1\}} \in (0,1)$ *such that the following holds. Both types of newcomer* B *enter in period 1 if and only if* $\delta^*_{\emptyset} \geq \underline{\delta}_{\emptyset}$*; otherwise none enters. Both types of newcomer* B *enter in period 2 if and only if* $\delta^*_{\emptyset} \geq \underline{\delta}_{\emptyset}$ *and* $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$ *hold; otherwise none enters in period 2.*

Proof of Lemma 20.
First, we know from Corollary 5 that if low-quality firm B sells, a high-quality firm B must also sell.

We now show that if h enters, then also l enters. Towards a contradiction, suppose only firm h enters. Then $E[q^B_t|p_t] = q^h \forall p_t$. For any rating, newcomers are identified as h, which allows the firm to always get the profit of $q^h - q^A$ in each period. But then l has a profitable deviation to enter the market with positive prices $q^h - q^A$ in period t, contradicting that l does not enter. We conclude that after any history, h enters if and only if l enters.

Suppose instead the high-quality firm B chooses to enter the market in some period t but not the period $t-1$. Then it faces the same payoffs as entering in $t-1$ as it is unable to build its reputation. Thus if the high-quality firm chooses to enter the market in a period t rather than $t-1$, it forgoes the additional continuation profit it could make from a good reputation. Therefore, it must be that if the high-quality firm chooses to sell at all, it must sell in period 1.

Since, if the high-quality firm B sells at all it sells in period 1 and the low-quality firm B chooses to sell if the high-quality firm is selling, then it must be that if firm B enters the market it enters in the first period and this decision is independent of its quality realization.

We now check for the conditions which ensure firm B sells after a history of good ratings. In other words, firm B has to be able to sell following a good rating, $E[q^B_t|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Suppose to the contrary that $E[q^B_t|\mathbb{H}_t, p_t] < q^A$ for some $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$. Then it must be that firm B does not sell in period t. This means there is no incentive for the low-quality firm B to harvest ratings and set the low price in period $t-1$. Hence $\delta^*_{\mathbb{H}_t} = 1$ and firm B becomes inactive in period t. Therefore, firm B sells after a good rating if $E[q^B_t|\mathbb{H}_t, p_t] \geq q^A$ for $\mathbb{H}_t \in \{\{1\}, \{1,1\}\}$.

We now derive more precise conditions for selling, conditional on entry, for these separate histories.

We start with $\mathbb{H}_t = \{\{1\}\}$ Rearranging $E[q^B_t|\{1\}, p_t] > q^A$, implies that

$$\begin{gathered} E[q^B_2|\{1\}, p_2] > q^A \Leftrightarrow \\ \gamma F(q^h - \overline{p}_{\emptyset})(q^h - q^A) > (1-\gamma)(1-\delta^*_{\emptyset})\delta^*_{\{1\}} F(q^A) q^A \end{gathered} \tag{15}$$

where the left-hand side is strictly increasing in $\delta^*_{\emptyset}$ and, since by Lemma 19 $\delta^*_{\{1\}} > 0$, the right-hand side strictly decreasing in $\delta^*_{\emptyset}$. Therefore, if $\delta^*_{\emptyset}$ is sufficiently large, firm B sells after $\mathbb{H}_t = \{\{1\}\}$.

Next, consider $\mathbb{H}_t = \{1, 1\}$. Firm B sells if

$$E[q_3^B|\{1,1\}, p_3] > q^A \Leftrightarrow$$
$$\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})(q^h - q^A) > (1-\gamma)(1-\delta_\emptyset^*)F(q^A)^2(1-\delta_{\{1\}}^*)q^A \tag{16}$$

where the left-hand side is strictly increasing in $\delta_{\{1\}}^*$ and the right-hand side strictly decreasing if $\delta_\emptyset^* < 1$. If $\delta_\emptyset^* = 1$, the right had side is zero, and the condition always holds. Therefore, conditional on entry, firm B sells in period 3 if $\delta_{\{1\}}^*$ is sufficiently large.

We now characterize entry decisions in period 1 and 2. We start with period 1. Note that if $\overline{p}_\emptyset > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta_\emptyset^* > 0$, implying they earn strictly positive profits. If $\overline{p}_\emptyset \leq 0$, we know from Lemma 7 that $\delta_\emptyset^* = 1$ such that $\overline{p}_\emptyset < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$\gamma q^h - q^A + F(q^A - \gamma q^h)\Bigg[\frac{\gamma F(q^A + (1-\gamma)q^h)q^h}{\gamma F(q^A + (1-\gamma)q^h) + (1-\gamma)F(q^A - \gamma q^h)} - q^A +$$
$$F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right]\Bigg] > 0, \tag{17}$$

This is the profits when both $\overline{p}_\emptyset < 0$ and $\overline{p}_{\{1\}} < 0$. Clearly, since h must earn weakly larger profits, this implies that also h sells with strictly positive probability. We conclude that if (15) holds, both types of firm B have strictly positive demand in period 1 and therefore enter. Since they clearly do not enter if this condition is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_\emptyset \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_\emptyset \in (0,1)$ such that all types of newcomers enter in period 1 if and only if $\delta_\emptyset^* \geq \underline{\delta}_\emptyset$.

We continue with period 2. Note that if $\overline{p}_{\{1\}} > 0$, then newcomers charge strictly positive prices in period 1 with probability $\delta_{\{1\}}^* > 0$, implying they earn strictly positive profits. If $\overline{p}_{\{1\}} \leq 0$, we know from Lemma 7 that $\delta_{\{1\}}^* = 1$ such that $\overline{p}_{\{1\}} < 0$. Then the following condition implies that firm l sells with strictly positive probability and earns strictly positive profits:

$$E[q_2^B|\{1\}, p_2] - q^A + F(|\overline{p}_{\{1\}}|)\left[\frac{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}})q^h}{\gamma F(q^h - \overline{p}_\emptyset)F(q^h - \overline{p}_{\{1\}}) + (1-\gamma)F(|\overline{p}_\emptyset|)F(|\overline{p}_{\{1\}}|)} - q^A\right] > 0, \tag{18}$$

Firm B can only sell if it obtained a positive rating in period 1. Then firm l sells despite a negative price in period 2 if this condition holds. Thus, since firm h earns weakly larger profits, also firm h sells.

We conclude that if (17) and (18) hold, both types of newcomers enter in period 2 if and only if (15) and (16) hold. Since they clearly do not enter in period 2 if (16) is violated, and since by our above arguments, it holds with equality for a unique $\underline{\delta}_{\{1\}} \in (0,1)$, we conclude that there exists a unique $\underline{\delta}_{\{1\}} \in (0,1)$ such that all types of newcomers enter in period 2 if and only if $\delta^*_{\emptyset} \geq \underline{\delta}_{\emptyset}$ and $\delta^*_{\{1\}} \geq \underline{\delta}_{\{1\}}$.

Finally, by our Selection Assumption 1, the equilibria with entry are played whenever they exist. This concludes the proof. □

Proof of Proposition 6.

First, note that the claims on entry follow directly from Lemma 20.

Statement 1: Suppose $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$. Then Lemma 18 shows that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}] = q^l = 0$ and $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] \in (0, q^A)$. Next, from Lemma 20 we know that $E[q_t^B|\mathbb{H}_t, p_t] > q^A$ for $\mathbb{H}_t = \{\{1\},\{1,1\}\}$ whenever firm B enters. Therefore, we know that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > q^A > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}]$. This shows that $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] > E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$ if the history $\mathbb{H}_{t-1}$ includes only positive ratings. Now recall that if the history includes negative or no rating, then there is no sales and no updating. Hence, obtaining a rating in the period t does not change consumer expectations and $E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 1\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = 0\}] = E[q_t^B|\{\mathbb{H}_{t-1}, R_t = -1\}]$.

Statement 2: $\max\{E[q_3^B|\{1,0\}], E[q_2^B|\{0\}]\} < q^A$ implies that firm B does not sell if it receives either no or negative ratings. By Lemma 20, newcomers enter after a history of positive ratings. Then Lemma 19 characterized the prices in the statement.

Statement 3: Follows directly from Lemmas 19 and 20. Lemma 20 characterized which histories induce positive demand, and Lemma 19 characterizes the prices.

Statement 4: Comes directly from Corollary 6.

Statement 5: The strategies are characterized in Lemma 19. The price levels set by the firm B is follows from Corollary 8.

□

B.3 Comparative statics

As before, we focus on mixed-strategy equilibria for our comparative statics. Our arguments in Lemma 19 imply that such equilibria indeed exist for some parameters, i.e. if $\frac{F(q^A)(q^h - q^A)}{\gamma q^h}$ is sufficiently large.

Corollary 15. *Consider an increase in $\bar{e}$. In each period, the low-quality firms harvest ratings more, $\frac{\partial \delta^*_{\emptyset}}{\partial \bar{e}} > 0$ and $\frac{\partial \delta^*_{\{1\}}}{\partial \bar{e}} > 0$.*

Proof of Corollary 15.
In period 2, in equilibrium, the low-quality firm B is indifferent between the profit it obtains from

setting $\overline{p}_{\{1\}}$ and $\underline{p}_{\{1\}}$. This means

$$\frac{\gamma(q^h-\overline{p}_\emptyset)q^h}{\gamma(q^h-\overline{p}_\emptyset)+(1-\gamma)(1-\delta^*_\emptyset)\delta^*_{\{1\}}q^A}=\frac{q^A}{\overline{e}}(\frac{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})q^h}{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2}-q^A).$$

Then for a given $\delta^*_\emptyset$, it is immediate that setting a higher $\delta^*_{\{1\}}$ decreases the left side of this equation and changes in $\overline{e}$ only affects this equation through $\delta^*_{\{1\}}$.

On the right side, from Lemma 19 we know that the payoff $\frac{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})q^h}{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2}-q^A$ is increasing in $\delta^*_{\{1\}}$. Further, fixing $\delta^*_{\{1\}}$, increases in $\overline{e}$ leads to a decrease in the probability of obtaining the period 3 payoff $\frac{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})q^h}{\gamma(q^h-\overline{p}_\emptyset)(q^h-\overline{p}_{\{1\}})+(1-\gamma)(1-\delta^*_\emptyset)(1-\delta^*_{\{1\}})(q^A)^2}-q^A$.

Taken together, this means that following an increase in $\overline{e}$, the direct effect is the right side of the equation decreases because the probability of obtaining a period 3 payoff decreases. Hence, to compensate for this decrease and restore indifference, an increase in $\delta^*_{\{1\}}$ is required as this simultaneously decreases the left side of the equation and increases the period 3 payoff in the right side of the equation. Therefore, it follows that $\frac{\partial\delta^*_{\{1\}}}{\partial\overline{e}}>0$.

We now turn our attention to the first period. In equilibrium, the low-quality firm is indifferent between the profit it obtains from setting $\overline{p}_\emptyset$ and $\underline{p}_\emptyset$. This means

$$\frac{\gamma q^h}{\gamma+(1-\gamma)\delta^*_\emptyset}=\frac{q^A}{\overline{e}}(\frac{\gamma(q^h-\overline{p}_\emptyset)q^h}{\gamma(q^h-\overline{p}_\emptyset)+(1-\gamma)(1-\delta^*_\emptyset)(q^A)\delta^*_{\{1\}}}-q^A).$$

Observe here that the left side of the equation is decreasing in $\delta^*_\emptyset$ and that changes in $\overline{e}$ affects the left side of the equation only through $\delta^*_\emptyset$. On the right side of the equation, increases in $\overline{e}$ have a direct effect of reducing the probability of obtaining a continuation payoff. Note that the size of the continuation payoff is increasing in $\delta^*_\emptyset$.

Hence, considering an increase in $\overline{e}$, the direct effect leads to a decrease to the right side of the equation. To restore indifference, an increase in $\delta^*_\emptyset$ decreases the left side of the equation and simultaneously increases the probability of the continuation payoff following a good rating (the terms in brackets on the right side of the equation). Therefore, to restore equilibrium, it must be that $\frac{\partial\delta^*_\emptyset}{\partial\overline{e}}>0$. □