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Information Design with Costly State Verification

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Information Design with Costly State Verification*

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Abstract

We study a persuasion problem when the receiver has the ability to probabilistically verify the state at a cost. The sender wants to convince the receiver to accept a project but the receiver is only willing to accept the project when the quality is above a threshold. The optimal disclosure policy balances between influencing the receiver's decisions to accept and to verify the quality. The optimal disclosure is deterministic and involves at most three messages, each consisting of an action recommendation and a verification recommendation. In the optimal disclosure, the action recommendation has a cutoff structure while the verification recommendation has a negative assortative structure. Specifically, the optimal disclosure recommends acceptance when the quality is above a threshold. When the quality is below this threshold, rejection without verification is recommended. Above this threshold, verification is not recommended when the quality lies in the middle range of the interval. The optimal disclosure reveals more information compared to the case where verification is exogenous.

Keywords: Bayesian persuasion, Information design, Costly information acquisition, Costly state verification, Product recommendation

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1 Introduction

The literature on Bayesian persuasion has made significant progress in understanding strategic communication when the sender has commitment power. However there is a limited understanding of the optimal disclosure policy when the receiver has the option to acquire additional information. This is because the inclusion of endogenous information acquisition introduces a second dimension into the action space. In this paper, we study how the sender strikes a balance between influencing the receiver's action choice and the information acquisition choice and characterize the optimal disclosure policy within our framework.

To motivate the receiver's option to acquire additional information, consider a regulatory authority conducting stress tests on financial institutions and publicly disclosing the results. As discussed in Leitner (2005) and Goldstein and Leitner (2018), such disclosure aims to prevent disruptions in financial activities. However, market participants have the capacity to generate their own information by launching independent investigations and gaining private insights. As another example, an online platform provides product information to persuade customers to make purchases, while customers also have the option to seek product information and read reviews on other websites. In a third scenario, an applicant submits information to a certifying body for certification, and the certifying body can independently gather firsthand information about the applicant through tests and interviews, as explored in Bizzotto et al. (2020). In these three examples and many other applications of Bayesian persuasion, the key feature of our model that the receiver can acquire additional information besides the sender's disclosure, is a natural assumption. This assumption is particularly relevant in environments where the party that designs the disclosure has no control over alternative sources of information. For instance, while the platform can provide information through product reviews and recommendations, it is difficult to forbid customers from learning about the product outside the platform. In this paper, we study how to design a disclosure policy when the receiver can acquire additional information about the asset, product or applicant.

Environment. Consider an environment with a sender and a receiver. The primary objective of the sender is to persuade the receiver to accept a particular project. The receiver's payoff of acceptance is equal to the project's quality net of the cost of implementing the project and the payoff of rejection is normalized to zero. Consequently, the receiver accepts the project when the quality is high enough, but the sender simply wants to maximize the chance of acceptance.

The sender designs and commits to a disclosure policy about the quality. Following the sender's initial disclosure, the receiver can choose whether to acquire information about the quality at a cost or to make the decision to accept or reject the project immediately. We assume that if the receiver decides to acquire information, the quality is revealed with some positive probability, and otherwise the receiver remains uninformed about the quality except what the sender has disclosed. We refer to this information technology as a *state-verification* technology. After observing the result of the verification, the receiver decides whether to accept the project.

In our model, the receiver is endogenously privately informed. Disclosure affects the receiver through three distinct channels. First, disclosure affects how an uninformed receiver updates his belief about the quality. Second, disclosure affects the receiver's incentive to become privately informed through the verification decision. Third, disclosure affects how an endogenously informed receiver updates his belief. Our assumption of a state-verification technology eliminates the need to consider the last channel. In case of a successful verification, the quality is fully revealed, rendering the sender's disclosure irrelevant. Because of our assumption of state-independent success probability, the receiver gains no information about the quality from a failed verification attempt. In this case, disclosure influences the belief updating in the same manner as how it affects the updating of an uninformed receiver. Thus, the focus of this paper is on the first and second channels, exploring how the interplay between these two aspects influences the design of the optimal disclosure policy.

Main result. We show that the optimal disclosure policy is deterministic, and it is without loss to assume that it sends a maximum of three distinct messages, each consists of an action recommendation and a verification recommendation. The optimal disclosure policy combines a cutoff structure for action recommendation and a negative assortative structure for verification recommendation. On the one hand, there is a quality cutoff for action recommendation, and acceptance is only recommended above the cutoff. On the other hand, verification is not recommended for any quality below the cutoff. Above the cutoff, lowest and highest qualities are pooled together and verification is recommended. See Figure 1.

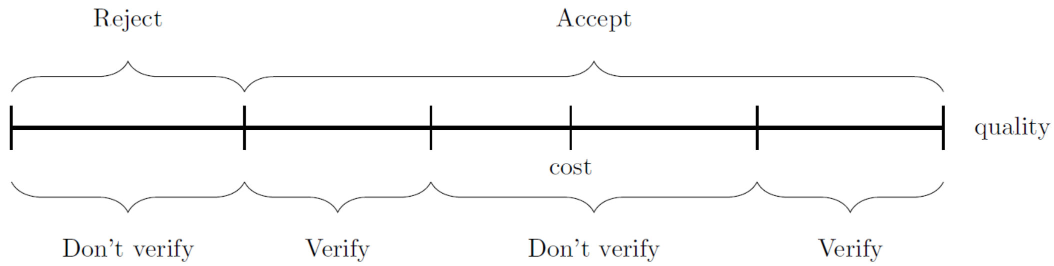


Figure 1. The optimal disclosure policy

Since the optimal disclosure policy may not make use of all three messages, the optimal disclosure policy can take one of four different forms. First, *no disclosure* can be optimal. In this case, no verification occurs and the project is accepted for sure. Second, a *cutoff* rule can be optimal. In this case, no verification occurs, and the project is accepted if and only if the quality is above a threshold. Third, a *negative assortative* rule can be optimal. In this case, acceptance is always recommended but verification is only recommended for the highest and lowest qualities.[1] Finally, a *three-message* rule, which makes use of all three messages, can be optimal. We further explore the conditions for the optimality of each of these rules, when the quality is uniformly distributed.

The implementation of the optimal disclosure policy could be straightforward. To implement a three-message rule, the sender can announce a quality threshold. Only projects with quality above the threshold are recommended, and supplemental information is provided for projects with intermediate quality levels above the threshold. The receiver rejects projects that are not recommended, and accepts projects without verification when additional information is provided. For projects that receive recommendation but no additional information is provided, the receiver verifies the quality and accepts the project only if the project quality is revealed to be above the cost.

Related literature. This paper contributes to the literature on Bayesian persuasion initiated by Rayo and Segal (2010) and Kamenica and Gentzkow (2011). A large body of the existing literature focuses on the linear case where the receiver's action depends only on the posterior

[1] Our terminology here loosely follows Kolotilin et al. (2023), which refers to the situation where the states are pooled in a negatively assortative manner.

mean (Gentzkow and Kamenica, 2016; Dworczak and Martini, 2019; Kleiner et al., 2021; Arieli et al., 2023).[2] In our model, the receiver's verification decision depends not only on the posterior mean but also on the entire posterior distribution.

Most of the existing literature also assumes that the sender is the exclusive source of information. Notable exceptions include Au (2015), Kolotilin et al. (2017), and Guo and Shmaya (2019), all of which consider the persuasion problem when the receiver has private information. A recent work by Kolotilin et al. (2023) considers the persuasion problem with non-linear preferences, which accommodates both the environments explored in Kolotilin et al. (2017), in which private information is independent of the quality, and Guo and Shmaya (2019), in which private information depends on the quality. In contrast, this paper studies an environment where the receiver's private information is endogenously determined. Consequently, actions are multidimensional in our model, while both the action and the state are unidimensional in Kolotilin et al. (2023).[3]

This paper considers a persuasion problem with endogenous information acquisition, and is closest to Matyskova and Montes (2023) and Bizzotto et al. (2020). Matyskova and Montes (2023) consider an environment with uniformly posterior separable information cost, a condition not applicable in this paper. They demonstrate that, under their assumptions, the persuasion problem with endogenous information acquisition can be solved as a conventional persuasion problem, albeit under a receiver-never-learn constraint. In contrast, this paper explores an environment where the information cost is *not* uniformly posterior separable and, as a result, one cannot assume that the receiver would not learn under the optimal disclosure policy.[4] Bizzotto et al. (2020) also consider information acquisition with information cost that is not uniformly posterior separable so that the receiver-never-learn result does not hold. They consider a persuasion problem with a binary state, while this paper considers a continuous state space. The assumption of a sufficiently rich state space allows more general optimal disclosure policies to arise. With a binary state space, all the optimal disclosure policies in their model has intrinsically a cutoff

[2]The terminology here follows Kolotilin et al. (2023). This specification is called linear case because it is without loss to assume the receiver's action is equal to posterior mean.

[3]In Appendix B, we reinterpret our two-dimensional action space as a one-dimensional action space, and show that it is impossible to order the actions in our model so that the assumptions in Kolotilin et al. (2023) are satisfied. Therefore, the results in Kolotilin et al. (2023) do not apply in our setting.

[4]Matyskova and Montes (2023) also show the receiver-never-learn result fails when their assumptions are relaxed. See their Examples C.1 and C.2.

structure.[5] Additionally, the information technology considered in Bizzotto et al. (2020) is also different from ours. They assume that the receiver can run a binary test on the qualities, which means that the receiver always becomes privately informed once he chooses to acquire information. In our model, the receiver is not privately informed with positive probability even verification is conducted.

In terms of our assumption concerning the receiver's information technology, this paper is related to a recent literature on cheap talk with detectable deception. Dziuda and Salas (2018), Balbuzanov (2019), and Ederer and Min (2022) have explored lie detection within this context. Sadakane and Tam (2022) and Zhao (2018) consider state verification rather than lie detection, and Zhao (2018) also endogenizes state verification. Levkun (2021) has examined fact checking provided by a third party, which is equivalent to state verification in the binary-state setting he considers. When the sender lacks commitment power, the truthful revelation of the lowest states in any equilibrium becomes unattainable, while optimal disclosure policies frequently involve the revelation of the lowest states.

Regarding the structure of our optimal disclosure policy, our findings exhibit certain similarities with the negative assortative information structure observed in Goldstein and Leitner (2018), Guo and Shmaya (2019), and Kolotilin et al. (2023). Goldstein and Leitner (2018) study the design of optimal stress tests, showcasing instances of non-monotonic scoring rules. Guo and Shmaya (2019) consider a persuasion problem with a privately informed receiver and show that the optimal disclosure policy has a nested-interval structure. Kolotilin et al. (2023) provide conditions for the optimality of negative assortative patterns of information disclosure and show that their model accommodates both Goldstein and Leitner (2018) and Guo and Shmaya (2019). Our characterization of the optimal disclosure policy does not follow from their results. It is also worth highlighting that although our optimal disclosure policy shares similarities with those in Goldstein and Leitner (2018) and Guo and Shmaya (2019), the underlying reasons behind the structure are fundamentally distinct. Goldstein and Leitner (2018) obtain the non-monotone structure because the gain-to-cost ratio, which is crucial to their analysis of the optimal stress test, is non-monotone in types. Depends on the shape of the gain-to-cost ratio, the optimal

[5] From a technical viewpoint, the binary state assumption also simplifies the analysis so that both the receiver's action and information acquisition decision depend only on the posterior mean, which is not true in our case.

stress test can take different forms. Guo and Shmaya (2019) establish the nest-interval structure because the acceptance set of a lower type is always a subset of that of a higher type. In this paper, the negative assortative structure emerges because it is optimal to disclose the quality levels that are closer to the project cost, in order to reduce the benefit of state verification.

2 Model

We consider a persuasion game, where a sender (she) selects an information structure to reveal information to a receiver (he). The receiver, upon receiving the disclosed information, decides whether to acquire additional information and then takes an action.

Disclosure. Let θ represent the state of nature. We assume that θ is drawn from a cumulative distribution function (CDF) F over $[0,1]$, which admits the strictly positive probability density function (PDF) f. The sender chooses an information structure. After the state of nature is drawn, the receiver observes a message generated by the chosen information structure. An information structure is a combination $(M, G(.))$ of a message set M and a function $G : [0,1] \to \Delta(M)$ such that if the state is θ, then a message $m \in M$ is drawn according to distribution $G(\theta)$ and observed by the receiver. For *no disclosure*, $G(\theta) = G(\theta')$ for all $\theta, \theta' \in [0,1]$. For *full disclosure* policy, the supports of $G(\theta)$ are disjoint across θ, so that the state is fully revealed. A *deterministic* policy has a degenerated $G(\theta)$ for each $\theta \in [0,1]$, and can be summarized by a function $\mathbf{m} : \theta \to M$.

State verification. The receiver can verify the state θ at a cost $c > 0$. Denote the state-verification effort by $e \in \{0,1\}$. The state verification generates a signal $s \in [0,1] \cup \{\phi\}$. The expertise level $q \in (0,1]$ determines the probability that the state is revealed. With probability q, the verification is successful and $s = \theta$; with probability $1-q$, the verification is unsuccessful and $s = \phi$. The expertise q is independent of the state, so there is no belief updating when the verification is unsuccessful. We refer to the pair (q, c) as the state-verification technology, and define $C = c/q$ as the quality-adjusted verification cost. We assume that the receiver would not verify whenever he is indifferent.

Payoffs. The receiver chooses a binary action $a \in \{0,1\}$, and we refer to $a = 1$ and $a = 0$ as *Accept* and *Reject*, respectively. The sender's utility v is independent of the state, and $v(a) = a$. The receiver's utility u depends on both the receiver's action a and the state θ, and $u(a,\theta) = a(\theta - R) - ec$, where $R \in (0,1)$ is the project cost and c is the state-verification cost. Therefore, the sender's objective is to maximize the acceptance probability, and the receiver chooses *Accept* only when the expected value of θ is above R. We further assume that the receiver accepts whenever he is indifferent.

Timeline. The game proceeds as follows. First, the sender chooses an information structure (M, G). Second, nature draws θ according to F, and a message $m \in M$ is sent to the receiver according to $G(\theta)$. Third, the receiver observes the message m, and decides whether to verify the state. Fourth, the receiver observes the signal s, and decides whether to accept or reject. The payoffs are realized.

2.1 An example

Consider an E-commerce platform that receives commission fees according to the sales volume, so its payoff is independent of the product quality. The customer's payoff from purchasing the product depends on the product quality θ, which is uniformly distributed on $[0,1]$. The product price p is 0.6, so the customer's payoff is $\theta - 0.6$ if he purchases the product, and 0 otherwise. The platform's revenue is 1 if the customer purchases the product, and 0 otherwise.

The platform designs a recommendation rule and can commit to it. If the customer has no private information, the optimal recommendation rule takes the form of a cutoff rule. Specifically, a product is recommended if its quality is above 0.2. The average quality of the recommended products is 0.6. The customer is indifferent between making a purchase and not, and therefore follows the recommendation. The average quality of the products that do not receive a recommendation is 0.1, and the customer therefore does not make a purchase if the product is not recommended. Moreover, the platform gets a payoff of 0.8 and the customer gets zero payoff.

Now suppose the customer is able to verify the quality at a cost of 0.01. When the customer verifies the quality, he can learn the exact quality with probability 0.9. The optimal recommendation rule is no longer a simple cutoff rule, and is depicted in Figure 2.

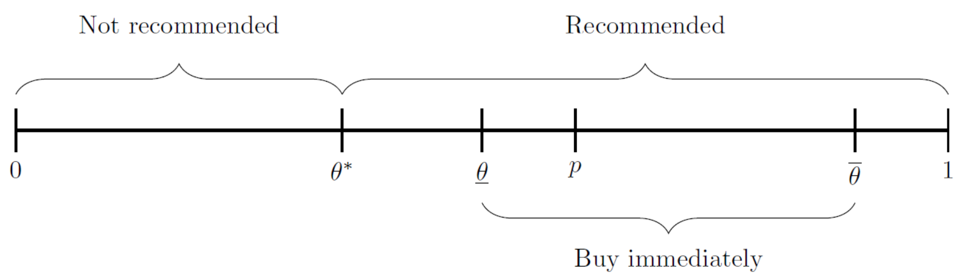


Figure 2. The optimal recommendation rule

In this numerical example, $\overline{\theta} = 0.92$, $\underline{\theta} = 0.50$ and $\theta^* = 0.34$. According to the optimal recommendation rule, a product is recommended if its quality is above 0.34. Furthermore, the platform recommends the customer to buy the product immediately, if its quality is between 0.50 and 0.92. The customer follows the recommendation, and only purchases a recommended product. When a product receives the *Buy immediately* recommendation, the customer makes a purchase without verification. Otherwise, the customer purchases the recommended product only after going through the verification process. The customer buys the recommended product unless the quality is found to be below 0.6.

The average quality of the recommended products is 0.67. The platform gets a payoff of 0.516 and the customer gets a payoff of 0.0755. This example shows that the average quality of recommended products are higher when the customer has access to external information beyond what is provided by the platform, and the customer gets a higher payoff.

3 Analysis

In this section, we first discuss the benchmark case with exogenous state verification and show that exogenous state verification has no effect on the optimal disclosure policy. We then study the optimal disclosure policy when state verification is endogenous.

3.1 Benchmark: exogenous state verification

When state verification is exogenous, the receiver is perfectly informed with probability q, and remains uninformed with probability $1 - q$.[6] When the receiver is perfectly informed, no infor-

[6] When $q = 1$, the receiver is always perfectly informed. As a result, no information provided by the sender can alter the receiver's action and any disclosure policy is optimal. Therefore, we focus on the case with $q \in (0, 1)$.

mation provided by the sender can influence the receiver's action. Consequently, the design of the disclosure policy aims only to affect the action of an uninformed receiver, same as the case where there is no privately informed receiver.

When there is no private information, the persuasion problem is a simple one: the sender, who wants to maximize the acceptance probability, chooses an information structure and sends a message to the receiver accordingly. The receiver accepts if and only if the state is on average higher than the project cost given the message received.

The optimal disclosure policy is a cutoff rule. There is a cutoff $\theta^e \in [0, R]$ such that the sender reveals whether θ is above the cutoff θ^e, and the recommendation is *Accept* when the state is above the cutoff and *Reject* otherwise. Denote the recommended action for the uninformed receiver by $a_\phi(m)$. That is, $\mathbf{m}(\theta) = m'$ for all $\theta < \theta^e$, and $\mathbf{m}(\theta) = m''$ for all $\theta \geq \theta^e$; $a_\phi(m') = 0$ and $a_\phi(m'') = 1$. The uninformed receiver adopts the recommended action, and the perfectly informed receiver chooses according to the state revealed. When $\mathbb{E}(\theta) \geq R$, $\theta^e = 0$. Otherwise, θ^e is uniquely defined by

$$\int_{\theta^e}^{1} (\theta - R) f(\theta) \, d\theta = 0.$$

Figure 3 illustrates the optimal disclosure policy and its action recommendation.

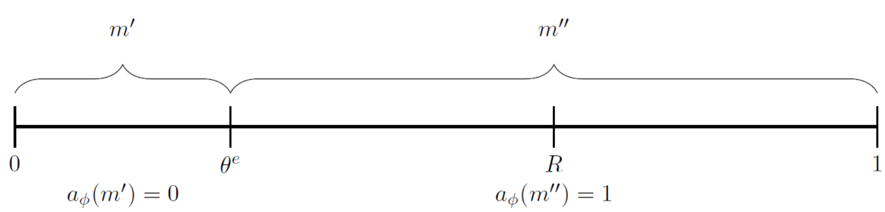


Figure 3. Optimal disclosure policy when state verification is exogenous

The optimal disclosure policy in the benchmark case only aims to influence an uninformed receiver's belief updating. It does not distort disclosure to influence an informed receiver's belief updating, because successful verification perfectly reveals the state and a receiver's updated belief would then be independent of the disclosure policy.[7] Nor does it distort disclosure to influence whether a receiver becomes informed, because verification is exogenous. In our main model where the verification is endogenous, an informed receiver's belief updating remains unaffected

[7] However, the optimal disclosure policy could influence an informed receiver's belief updating under alternative assumptions for the receiver's private information (Guo and Shmaya, 2019).

by disclosure, but the optimal disclosure policy strikes a balance between shaping an uninformed receiver's belief updating and influencing whether a receiver becomes informed.

3.2 Preliminary analysis

We now move to our main model. We focus on the case where $q < 1$. All our results apply when $q = 1$, and the assumption that $q = 1$ allows us to further pin down the optimal disclosure policy and perform comparative statics analysis analytically, which is discussed in Section 3.6.

The space of all information structures is the space of all functions that map the state space $[0, 1]$ to the space of all distributions over the message space M. Our first result shows that we can narrow down the search of the optimal information structure to information structures that involve only three distinct messages.

Denote the receiver's verification decision given message m by $e(m)$. If $e(m) = 0$, i.e., the receiver decides not to verify the state, the receiver's action depends only on message m, which we denote by $a_{NV}(m)$. If $e(m) = 1$, i.e., the receiver decides to verify the state, the receiver's action depends also on signal s. We denote it by $a_s(m)$, where $s \in [0, 1] \cup \{\phi\}$.

Two distinct characteristics of our state-verification technology allow us to significantly simplify the characterization of the receiver's action. First, upon successful verification, the receiver receives perfect information about the state. Therefore, any information provided by the sender becomes irrelevant and it must be the case that $a_s(m) = 1$ if $s \geq R$ and $a_s(m) = 0$ if $s < R$. Consequently, we do not need to track the recommended action for all $s \in [0, 1]$. Second, in the case of unsuccessful verification, the receiver does not receive additional information about the state, as the probability of successful verification is independent of the state. This implies that $a_{NV}(m) = a_\phi(m)$. Therefore, the receiver's recommended action is completely characterized by a single function $a_\phi(m)$, which we refer to as the *(default) action recommendation* of message m. In a similar fashion, we refer to $e(m)$ as the *verification recommendation* of message m. A message thus consists of an action recommendation and a verification recommendation.

Despite the potential complexity of the optimal disclosure policy, given that the receiver's action and verification decisions are both binary, any disclosure policy is outcome-equivalent to one characterized by four distinct types of messages: 1) *Verify then Accept*, $e(m) = 1$, $a_\phi(m) =$

1; 2) *Accept without verification*, $e(m) = 0$, $a_\phi(m) = 1$; 3) *Verify then Reject*, $e(m) = 1$, $a_\phi(m) = 0$; 4) *Reject without verification*, $e(m) = 0$, $a_\phi(m) = 0$. For instance, when the message is *Verify then Accept*, the receiver first verifies the state as recommended. When the verification is successful, the receiver takes the action according to the revealed state. Only when the verification is unsuccessful, would the receiver accept as recommended. Such a message can also be interpreted as *Accept unless finding bad news*.[8]

Now we show that the message *Verify then Reject* is not a part of any optimal disclosure policy. In the main text, we provide heuristic derivations of all the results, focusing on deterministic policies. The formal proofs for the general case can be found in Appendix A.

Lemma 1 *In the optimal disclosure policy, Verify then Reject is never recommended, i.e., if* $a_\phi(m) = 0$ *and* $e(m) = 1$, *then* $\Pr(m) = 0$.

Suppose there exists a message $m \in M$ in the optimal disclosure policy such that $a_\phi(m) = 0$ and $e(m) = 1$ but $\Pr(m) > 0$. Given the verification recommendation $e(m) = 1$, there must exist some $\theta > R$ that this message is sent. Otherwise, the receiver will not follow the recommendation to verify. Given the action recommendation $a_\phi(m) = 0$, for all $\theta < R$ that this message is sent, the acceptance probability is zero; for all $\theta > R$ that this message is sent, the acceptance probability is $q < 1$. Now, consider an alternative disclosure policy, which differs from the original disclosure policy only in that all θ's that the message *Verify then Reject* is sent are fully revealed. Full disclosure of all $\theta < R$ still results in certain rejection. However, full disclosure of all $\theta > R$ results in certain acceptance, which is a strict improvement. This means that $\Pr(m) = 0$.

Denote the three messages in the optimal disclosure policy by m_ϕ, m_0, and m_1, respectively, which correspond to 1) *Reject without verification*: $e(m_\phi) = 0$ and $a_\phi(m_\phi) = 0$; 2) *Accept without verification*: $e(m_0) = 0$ and $a_\phi(m_0) = 1$; and 3) *Verify then Accept*: $e(m_1) = 1$ and $a_\phi(m_1) = 1$.

For m_0 and m_1, the recommended action is *Accept*. It implies that both $\mathbb{E}(\theta|m_0)$ and $\mathbb{E}(\theta|m_1)$ should be above R. Moreover, no verification is recommended for m_0. It implies that the cost of verification must be higher than the benefit. Therefore, by individual rationality and incentive

[8] Similarly, the other three messages can be interpreted as *Immediate accept*, *Reject unless finding good news*, and *Immediate reject*.

compatibility, we have

$$\begin{aligned}\mathbb{E}(\theta|m_0) &\geq R, &\text{(A0)}\\ q\Pr(\theta<R|m_0)(R-\mathbb{E}(\theta|\theta<R,m_0)) &\leq c, &\text{(V0)}\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}(\theta|m_1) &\geq R, &\text{(A1)}\\ q\Pr(\theta<R|m_1)(R-\mathbb{E}(\theta|\theta<R,m_1)) &> c. &\text{(V1)}\end{aligned}$$

The left-hand sides of $(V0)$ and $(V1)$ are the benefits of verification. With probability q, verification is successful. Since the recommended action is *Accept*, verification only changes the receiver's action for $\theta < R$ and the resulting change in payoff is $R - \theta$.

The presence of the constraints $(V0)$ and $(V1)$ indicates that the receiver's verification decision depends on the entire posterior distribution, rather than the posterior mean only. As noted in the literature review, this aspect separates our model from a large body of the persuasion literature where the receiver's action depends only on the posterior mean.

Our next lemma shows that the optimal disclosure policy has a cutoff structure for action recommendation.

Lemma 2 (Cutoff for action recommendation) *There is some $\theta^* \in [0, R)$ such that in the optimal disclosure policy, m_ϕ is sent if and only if $\theta < \theta^*$.*

We prove the lemma formally in Appendix A using calculus of variations. To provide some intuition, suppose that, for some θ's above R, the message is m_ϕ (*Reject without verification*). Now, consider an alternative disclosure policy, which only differs in fully revealing all θ's that m_ϕ is sent in the original disclosure policy. Full disclosure of $\theta < R$ leads to certain rejection, which is the same as under the original one. On the other hand, full disclosure of $\theta > R$ leads to certain acceptance, and it is a strict improvement. Consequently, for all $\theta > R$, m_ϕ is never sent in the optimal disclosure policy.

Now suppose that the message is m_ϕ for some $\theta' < R$, and the message is m_0 (*Accept without verification*) for some $\theta'' < \theta'$. For illustration simplicity, assume $F(\theta) = \theta$. Consider an alternative disclosure policy, which only differs in switching messages for θ' and θ''. Under the alternative disclosure policy, $\mathbb{E}(\theta|m_0)$ and $\mathbb{E}(\theta|\theta < R, m_0)$ are higher, while $\Pr(\theta < R|m_0)$ remains the same. Such a change relaxes both $(A0)$ and $(V0)$, which enables the alternative disclosure policy to send m_0 for some other θ's. It leads to a strictly higher acceptance probability for $\theta < R$, and no change in the acceptance probability for $\theta > R$. The same logic applies to m_1 (*Verify then Accept*). Figure 4 illustrates the cutoff structure for action recommendation.

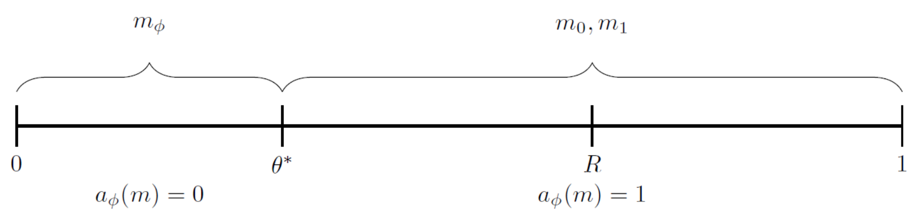


Figure 4. Cutoff structure for action recommendation

Next, we develop a notion of informativeness that will be useful in our setting. This is because there may be multiple disclosure policies that are outcome-equivalent and, as a result, give the receiver the same payoff but differ in the Blackwell ordering.

Note that our optimal disclosure policy with three messages pools all the states below θ^* together and discourages the receiver from both acceptance and verification in these states. This disclosure policy is outcome-equivalent to one that truthful reveals all $\theta < \theta^*$ while keeping the messages for all $\theta \geq \theta^*$ the same. Our notion of informativeness requires that when we compare the informativeness of two disclosure policies, we compare the rules that are outcome-equivalent to them, in which all $\theta < \theta^*$ are truthfully revealed rather than pooled together. Formally,

Definition 1 *Given a message set $M = \{m_\phi, m_0, m_1\}$ and two disclosure policies G and G' that send m_ϕ below cutoffs θ^* and $(\theta^*)'$, respectively. We say that G is more informative than G' if and only if the disclosure policy that is outcome-equivalent to G and truthful reveals all $\theta < \theta^*$ is more informative in the sense of Blackwell than the disclosure policy that is outcome-equivalent to G' and truthful reveals all $\theta < (\theta^*)'$.*

The receiver is indifferent to whether all $\theta < \theta^*$ is fully revealed or the same message is sent for all $\theta < \theta^*$. However, when G is more informative to than G' according to Definition 1, the receiver gets a higher payoff under G. Therefore, the redefined informativeness is more indicative of the disclosure policy's value to the receiver, given the structure of our optimal disclosure policy.

With Definition 1, we are ready to compare the informativeness of the optimal disclosure policies under endogenous and exogenous verification. Given $(A0)$ and $(A1)$, we have $\mathbb{E}(\theta|m_0) \geq R$ and $\mathbb{E}(\theta|m_1) \geq R$. Therefore, $\theta^* \geq \theta^e$, which implies that the optimal disclosure policy reveals more $\theta < R$ when verification is endogenous. Moreover, the optimal disclosure policy sends only one message for $\theta > \theta^e$ when verification is exogenous and sends potentially two messages for $\theta > \theta^*$ when verification is endogenous. Therefore, we have

Corollary 1 *More information is disclosed when state verification is endogenous than when it is exogenous.*

3.3 Sender's problem

Lemma 1 reduces the number of messages sent in an optimal disclosure policy to three, and Lemma 2 establishes the cutoff structure for action recommendation. From these two lemmas, we have

Proposition 1 (Sender's problem) *The sender's problem can be formulated as*

$$\max_{\substack{x_0(\theta)\in[0,1]\\ \theta^*\in[0,R]}} \int_{\theta^*}^{R} \left[x_0(\theta) + (1 - x_0(\theta))(1-q)\right] f(\theta)\, d\theta + \int_{R}^{1} f(\theta)\, d\theta \tag{S}$$

s.t.

$$\int_{\theta^*}^{1} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0, \tag{A0}$$

$$\int_{\theta^*}^{1} (\theta - R)(1 - x_0(\theta))\, f(\theta)\, d\theta \geq 0, \tag{A1}$$

$$c\int_{\theta^*}^{1} x_0(\theta)\, f(\theta)\, d\theta + q\int_{\theta^*}^{R} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0. \tag{V}$$

Constraints $(A0)$ and $(A1)$ follow from the fact that the recommended action for both m_0 and m_1 is *Accept*. These two constraints require that, given both messages, the state on average is above R. Constraint (V) follows from the fact that the verification recommendation for m_0 is *No verification*. This constraint requires that, given this message, the verification benefit must be smaller than c. Otherwise, the receiver would not follow the recommendation and verify the state instead.

Constraints $(A0)$ and $(A1)$ can be interpreted as *persuasion constraints*. The sender's objective is to convince the receiver to accept. Therefore, the states that receive recommendation for acceptance must be above R on average. Essentially, the sender pools states above R and below R, maintaining the average above R. Moreover, constraints $(A0)$ and $(A1)$ can also be interpreted as *credibility constraints*, similar to constraint (6) in Goldstein and Leitner (2018), where states above R provide credibility and states below R require credibility. Each state above R produces credibility resource in the amount $\theta - R$, and each state below R requires credibility resource in the amount $R - \theta$.

Constraint (V) can be interpreted as a *verification constraint*, which is required to dissuade the receiver from verification after receiving message m_0. The cost of verification is always c, while the benefit depends on how often the state is revealed to be below R, and how much the receiver can benefit from choosing *Reject* instead of *Accept* as recommended. Each state above R imposes verification cost in the amount c, and each state below R yields verification benefit in the amount $q(R - \theta) - c$.

It is worth noting that constraint $(V1)$, which requires that the receiver finds it optimal to verify upon receiving m_1, is ignored in our formulation of the sender's problem. This is because, if $(V1)$ is violated in the optimal disclosure policy under our formulation, the receiver would *not* verify the state given m_1. This means that we can simply replace m_1 with m_0 and the new disclosure policy is also feasible under our formulation. Thus, there is no loss to ignore $(V1)$.

Note that, from (V), for $\theta \in (R - c/q, R)$, the net verification benefit is negative. This means that the sender can always pool states slightly below R together with states higher than R such that the state is higher than R on average and send m_0 to the receiver. As a result, we conclude that

Lemma 3 *Full disclosure is never optimal. Moreover, in the optimal disclosure policy,* $\Pr(m_0) > 0$.

We close this section with a lemma that allows us to simplify the search for the optimal disclosure policy, which will be useful in later sections.

Lemma 4 *For any optimal disclosure policy, if* $\Pr(m_1) > 0$*, then* (V) *and* $(A1)$ *are binding.*

To understand Lemma 4, note that, by $(A0)$ and $(A1)$, the states that m_0 is sent and the states that m_1 is sent are both larger than R on average. Thus, if (V) was slack, it would be possible to move some mass from m_1 to m_0 so that all the constraints remain satisfied and the resulting acceptance probability is higher. As a result, m_1 will not be sent as long as (V) is slack. On the other hand, if $(A1)$ was slack, it would be possible to replace m_1 with m_0 for some states above R. This would result in a slack (V) after the change, which contradicts the optimality of the disclosure policy.

3.4 Optimal disclosure policy

In this section, we characterize the optimal disclosure policy in Theorem 1 and discuss its properties.

Theorem 1 *Any optimal disclosure policy is outcome-equivalent to a disclosure policy with at most three messages, which is characterized by three cutoffs* θ^*, $\underline{\theta}$, *and* $\overline{\theta}$ *such that* $0 \leq \theta^* \leq \underline{\theta} < \overline{\theta} \leq 1$ *and satisfies*

1. $$\mathbf{m}(\theta) = \begin{cases} m_\phi & \text{if } \theta < \theta^*, \\ m_0 & \text{if } \underline{\theta} \leq \theta \leq \overline{\theta}, \\ m_1 & o.w. \end{cases}$$

2. $a_\phi(m_\phi) = 0$, $a_\phi(m_0) = a_\phi(m_1) = 1$, $e(m_\phi) = e(m_0) = 0$, *and* $e(m_1) = 1$.

The optimal disclosure policy is deterministic. In terms of action recommendation, it adopts a cutoff structure, with the cutoff θ^*. Specifically, for $\theta < \theta^*$, the recommended action is *Reject*; for $\theta \geq \theta^*$, the recommended action is *Accept*. The receiver follows the recommended action whenever he is not privately informed. In terms of verification recommendation, the optimal disclosure policy has a negative assortative structure, given the same action recommendation. For $\theta < \theta^*$, *No verification* is always recommended. This verification recommendation, together with the action recommendation, is equivalent to full disclosure for $\theta < \theta^*$. For $\theta \geq \theta^*$, *No verification* is recommended for $\theta \in [\underline{\theta}, \overline{\theta}]$. A typical optimal disclosure policy with three messages is illustrated in the following figure.

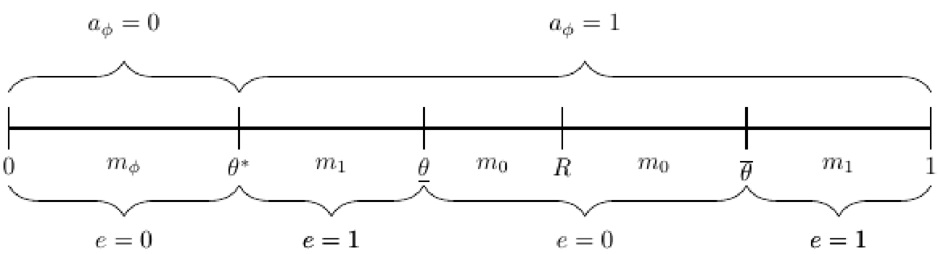


Figure 5. The optimal disclosure policy

Proposition 1 formulates the sender's choice of cutoff θ^* and choice between message m_0 and m_1 as an infinite-dimensional maximization problem, which we use to prove Theorem 1 in Appendix A. The proof shows that, for any cutoff θ^*, it is optimal to send m_0 on an interval nested in the interval $[\theta^*, 1]$ that contains R. In the following, we provide a heuristic proof of Theorem 1 for such a negative assortative structure, focusing on deterministic disclosure policies.

By Lemma 4, we can show that, for any disclosure policy that does not have a negative assortative structure for verification recommendation, there is an alternative disclosure policy that leads to a strictly higher acceptance probability or a weakly higher acceptance probability with a slack (V), which, by Lemma 4, would imply that the original disclosure policy is suboptimal.

Consider states above R. Each $\theta > R$ contributes to the pool of credibility resource in the amount $\theta - R$, while all $\theta > R$ induce the same amount of verification cost. Importantly, a lower value of θ induces the same amount of verification cost, yet generates fewer credibility resource. Consequently, when generating the same amount of credibility resource, opting for message m_0 for lower values of θ, instead of higher values of θ, induces higher verification cost. This, in turn, relaxes the constraint (V). The acceptance probability is one for all $\theta > R$, which implies that such a change does not reduce the overall acceptance probability.

Consider states below R. Each $\theta < R$ requires credibility resource in the amount $R - \theta$, while yielding a verification benefit in the amount $q(R - \theta) - c$. Importantly, a higher value of θ corresponds to a reduced requirement for credibility resource and lower verification benefit. Consequently, given the same requirement of credibility resource, opting for message m_0 for higher values of θ, instead of lower values of θ, leads to more frequent acceptance, while inducing lower verification benefit. This, in turn, relaxes the constraint (V) and raises the overall acceptance probability at the same time.

Hence, it is evident that for any disclosure policy lacking a negative assortative structure for verification recommendation for states above the cutoff θ^*, one can always construct an alternative disclosure policy, which achieves either the same acceptance probability while allowing for a slack (V) or a strictly higher overall acceptance probability.

3.5 Properties of optimal disclosure policy

Based on Theorem 1, we can further classify the optimal disclosure policy into four types.

Proposition 2 *Without loss of generality, the optimal disclosure policy is one of the following rules:*

1. *No disclosure, and the message sent is Accept without verification;*
2. *Cutoff rule, and the messages sent are Accept without verification above a cutoff and Reject without verification otherwise;*
3. *Negative assortative rule, and the messages sent are Accept without verification on an interval around R and Verify then Accept otherwise;*
4. *Three-message rule, the messages sent are Reject without verification below a cutoff, Accept without verification on an interval around R, and Verify then Accept otherwise.*

Figure 6 illustrates all four rules.

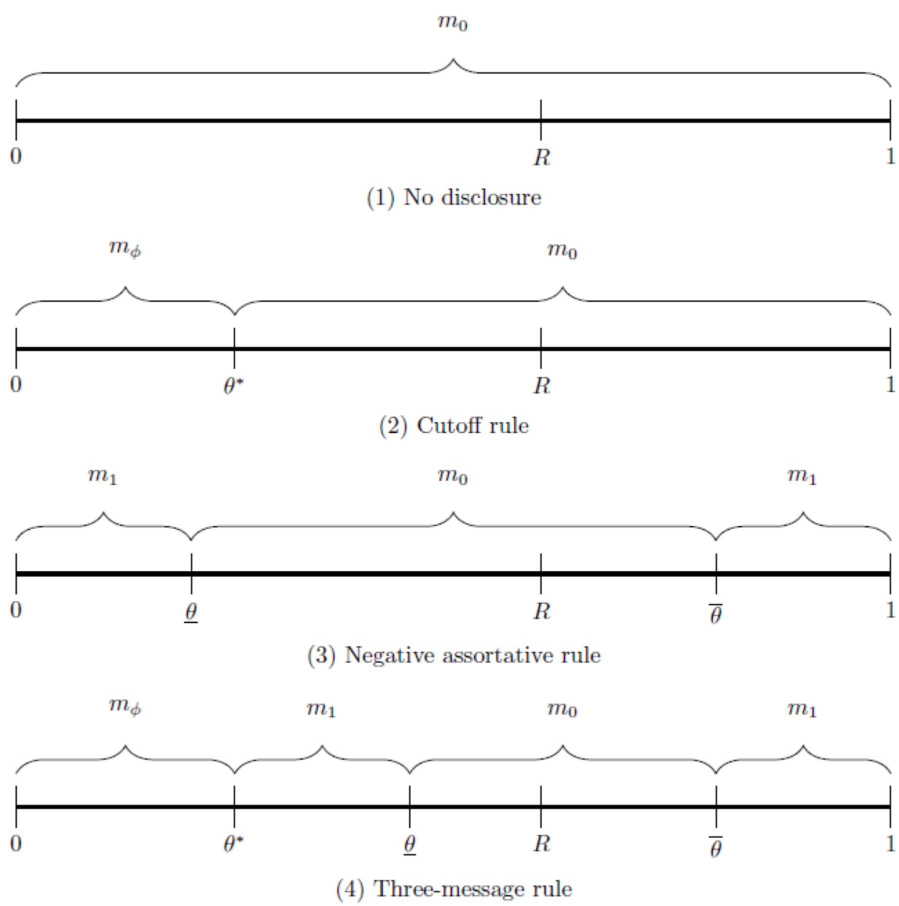


Figure 6. Classification of optimal disclosure policies

No disclosure can be optimal. When the optimal disclosure policy discloses no information, by Lemma 3, the message sent is always *Accept without verification*, and the receiver always accepts. To see that, suppose the message sent is *Reject without verification* instead. In this case, the sender's payoff is zero, while full disclosure induces certain acceptance for all states above R and generates strictly higher payoff for the sender. Similarly, if the message sent is *Verify then Accept*, then the sender can send m_0 for states in a small neighborhood around R and generate a strictly higher payoff.

No disclosure corresponds to a disclosure policy that only recommends *Accept without verification* (m_0). This gives the sender the highest possible payoff. Using constraints ($A0$) and (V), we can obtain the necessary and sufficient condition for the optimality of no disclosure.

Proposition 3 *No disclosure is optimal if and only if*

$$\mathbb{E}(\theta) \geq R \text{ and } q \int_0^R (R-\theta) f(\theta) d\theta \leq c.$$

When the project quality improves in the first-order stochastic dominance (FOSD) sense, it becomes more likely for no disclosure to be optimal. If F' FOSD F, the expected state $\mathbb{E}(\theta)$ is higher under F', and the verification benefit $\int_0^R (R-\theta) f'(\theta) d\theta$ is lower as well. In other words, when the quality is improving, the receiver is more inclined to accept the project without verification because he anticipates a higher expected quality and verification becomes less profitable.

On the other hand, when the project cost gets higher, it becomes less likely for no disclosure to be optimal. Under no disclosure, a higher R requires a higher expected state $\mathbb{E}(\theta)$, and simultaneously induces higher verification benefit. In other words, when the project cost gets higher, it is more difficult for the sender to convince the receiver to accept the project without verification because such a recommendation requires a higher expected quality. Additionally, verification becomes more profitable, because the project quality is now more likely to be below its cost, while the benefit is also higher whenever low qualities are discovered.

A negative assortative rule corresponds to a disclosure policy that does not recommend *Reject without verification* (m_ϕ). The sender recommends *Accept without verification* (m_0) on an interval around R, and recommends *Verify then Accept* (m_1) for both the lowest states and highest states. By ($A0$) and ($A1$), this means that the unconditional expected state must be higher than R, i.e., $\mathbb{E}(\theta) \geq R$. However, the optimal disclosure policy sometimes would recommend *Reject without verification* (m_ϕ) even when $\mathbb{E}(\theta) \geq R$. The sender is facing a trade-off between recommending *Verify then Accept* (m_1), which leads to acceptance with probability $1-q$, and recommending *Accept without verification* (m_0), which leads to certain acceptance. This trade-off is crucial in determining the optimal disclosure policy. For states below R, sending m_1 and m_0 requires the same amount of credibility resource, however sending m_0 raises the verification benefit, which makes it harder to satisfy (V). When the verification cost is high, ($A0$) binds while (V) is slack. Such a trade-off leads to sending m_ϕ for some of the lowest states in order to send m_0 for more states below R, despite that $\mathbb{E}(\theta) \geq R$.

When the prior belief is less optimistic, the sender is forced to reveal some of the lowest states to convince the receiver. Therefore, we have

Corollary 2 *If $\mathbb{E}(\theta) < R$, the optimal disclosure policy is either a cutoff rule or a three-message rule.*

A cutoff rule corresponds to a disclosure policy that does not recommend *Verify then Accept* (m_1). It is outcome-equivalent to an upper censorship that reveals all states below a certain cutoff and pools all states above that cutoff. In this case, verification is never recommended. On the other hand, a three-message rule makes all three recommendations with positive probability, which has been discussed in detail following Theorem 1.

We can also examine the consequences of varying the state-verification technology (c, q) and the project cost R on the sender's payoff.

Corollary 3 *The sender's payoff under the optimal disclosure policy is increasing in c and decreasing in q and R.*

Corollary 3 follows directly from Proposition 1 and the envelope theorem. The intuition is straightforward. First, an increase in verification cost c relaxes (V). In other words, when verification becomes more costly, the receiver has less incentive to verify the state. Therefore, the sender can recommend *Accept without verification* (m_0) more often, leading to a higher acceptance probability. Second, an increase in the project cost R makes it more likely that the state is below R and tightens all three constraints $(A0)$, $(A1)$ and (V). To be more specific, when the project cost gets higher, it is more difficult to persuade the receiver to accept and verification also becomes more profitable. Third, an increase in q tightens (V) and decreases the objective function (S). For the receiver, verification becomes more profitable. For the sender, the benefits from recommending *Verify then Accept* (m_1) diminishes. Both factors contribute to a reduction in the sender's payoff.

However, the envelope theorem does not provide us with comparative statics on the information revealed or the receiver's payoff. To gain further insights, we examine a special case with $q = 1$ in Section 3.6.

It is also worth highlighting three nice features of our optimal disclosure policy. First, the acceptance probability is increasing in the state. In the context of the platform interpretation, this implies that products with higher quality are sold more frequently, and the products with quality above the price are always sold. Suppose, instead of independently testing the products, the platform relies on the seller to provide information, and the seller can potentially underreport by providing insufficient information. If the probability of a sale is not increasing in quality, then some sellers may have an incentive to manipulate the quality report downward, posing potential problems for the platform. A similar concern is raised in Goldstein and Leitner (2018), where the optimal scoring rule is non-monotonic, resulting in some banks with lower asset values receiving higher scores. Such a concern does not arise under our optimal disclosure policy.

Second, the policy has some form of monotonicity of the acceptance probability and the posterior mean in the message sent. The sender induces certain acceptance by sending m_0 and certain rejection by sending m_ϕ, and the acceptance probability is 1 for $\theta \geq R$ and $1-q$ for $\theta < R$ if the sender sends m_1.[9] Thus, in terms of the acceptance probability, we can rank the three recommendations as $m_0 \succeq m_1 \succeq m_\phi$. In terms of posterior mean, there is a similar ranking: $E(\theta|m_0) \geq E(\theta|m_1) \geq E(\theta|m_\phi)$. The states that m_ϕ is sent are always lower than R on average, otherwise the sender can send m_1 instead. In contrast, by Lemma 4, the expected value of the states that m_1 is sent is exactly equal to R. Finally, the states that m_0 is sent are always higher than R on average. This is due to $(A0)$. When $(A0)$ is slack, the expected value of the states that m_0 is sent is strictly higher than R.

Third, the implementation of the optimal disclosure policy can be straightforward. To implement a three-message rule, the sender can 1) recommend the project when the quality is above a certain cutoff, and 2) provide additional information for some quality levels around R. As a response, the receiver immediately rejects any project that is not recommended. If a project is recommended, the receiver accepts the project right away whenever additional information is provided. Otherwise, verification is initiated.

[9] This also implies that there is no point in sending m_1 when $q = 1$. See detailed discussion of the case where $q = 1$ in Section 3.6.

3.6 Special case: $q = 1$

Given the characterization of the optimal disclosure policy, deriving comparative statics on the information revealed or the receiver's payoff is not straightforward. This challenge arises because the disclosure policy is not characterized by a single cutoff, as an upper censorship (Kolotilin et. al., 2023), or as in an environment with a binary state space (Bizzotto et. al. 2020). In this section, we consider the special case where $q = 1$ and examine the effect of varying the other parameters within the model.

When $q = 1$, verification always results in state discovery. Therefore, whenever the receiver decides to verify, he always learns the state. This implies that recommending verification for any state below R does not bring any benefit to the sender. For states above R, recommending verification takes away credibility resource and reduces verification cost of sending m_0. As a result, when $q = 1$, the optimal disclosure policy never sends m_1.

Proposition 4 *Suppose $q = 1$. The optimal disclosure policy is either no disclosure or a cutoff rule.*

Thus, the optimal disclosure policy can be characterized by a single cutoff θ^*, and the sender's problem is reduced to

$$\min_{\theta^* \in [0,R]} \theta^*$$

s.t.

$$\int_{\theta^*}^{1} (\theta - R) f(\theta) \, d\theta \geq 0, \tag{A}$$

$$c \int_{\theta^*}^{1} f(\theta) \, d\theta + \int_{\theta^*}^{R} (\theta - R) f(\theta) \, d\theta \geq 0. \tag{V}$$

The first constraint is the persuasion constraint, and the second constraint is the verification constraint. When the optimal θ^* is equal to zero, the optimal disclosure policy is no disclosure. Otherwise, it follows a cutoff rule. Recall that we use an outcome-equivalent policy that perfectly reveals states below θ^* when comparing the amount of information disclosed across disclosure policies. As a result, the amount of information revealed increases with θ^*.[10] Our next result follows directly from the envelope theorem.

Proposition 5 *Suppose $q = 1$. Increasing c or decreasing R reduces the amount of information disclosed.*

An increase in the verification cost c loosens the verification constraint, while an increase in the project cost R tightens both constraints. The intuition is straightforward: as verification becomes more expensive, the sender can disclose less information without encouraging verification; as the project cost increases, it requires more credibility resource to recommend *Accept*, and it also raises verification benefit. The sender must disclose more information as a result.

The monotone comparative statics come from the assumption of certain state discovery, which implies that the optimal disclosure policy never recommends verification, which corresponds to the aggressive disclosure in Bizzotto et. al. (2020). Therefore, the non-monotonicity due to shifting between aggressive disclosure and conservative disclosure, which is observed in their model, does not occur in our setting. Instead, we have

Proposition 6 *Suppose $q = 1$. Increasing c reduces the receiver's payoff.*

Since the optimal disclosure policy never recommends verification, a change in verification cost c affects the receiver's payoff exclusively through its effect on disclosure. This implies that an increase in c unambiguously results in a decrease in the receiver's payoff, due to an increase in θ^*.

The effect of a change in the project cost R is less straightforward. An increase in the project cost R leads to more information disclosure, but it also results in a lower payoff conditional on acceptance for all θ. The overall effect of such a change is ambiguous. The following example demonstrates that, when θ follows a uniform distribution, increasing R reduces the receiver's payoff.

Example 1 *Suppose $F(\theta) = \theta$ and $q = 1$. When $\sqrt{2c} < R < 1 - 4c$, the verification constraint is the binding constraint, and $\theta^* = R - c - \sqrt{2c(1-R) + c^2}$. When $R > \max\left\{1 - 4c, \frac{1}{2}\right\}$, the*

[10] As noted previously, this approach can be justified by considering value of information to the receiver, as demonstrated in Proposition 6.

persuasion constraint is the binding constraint, and $\theta^* = 2R - 1$. *When* $R < \min\left\{\sqrt{2c}, \frac{1}{2}\right\}$, *neither of the constraints is binding,* $\theta^* = 0$. *In all three cases, we can show that the receiver's payoff decreases with* R.

4 Optimal disclosure policy under uniform distribution

In this section, we consider a special case with the uniform distribution, i.e., for all $\theta \in [0,1]$, $F(\theta) = \theta$. Our objective is to demonstrate how our characterization result can be applied to derive the optimal disclosure policy in this specific case.

Given the uniform distribution, by Proposition 3, we have

Corollary 4 *Suppose* $F(\theta) = \theta$. *No disclosure is optimal if and only if* $R \leq \frac{1}{2}$ *and* $R \leq \sqrt{2C}$.

Next, we provide the necessary and sufficient condition for the optimality of recommending verification, i.e., m_1 is sent with positive probability.

Proposition 7 *Suppose* $F(\theta) = \theta$. *When* $q \leq \frac{2}{3}$, *the optimal disclosure policy recommends verification with positive probability if and only if*

$$\sqrt{2C} < R < 1 - 4C.$$

When $\frac{2}{3} < q < 1$, *the optimal disclosure policy recommends verification with positive probability if and only if*

$$\sqrt{2C} < R < 1 - Q(q)\,C,$$

where

$$Q(q) := \frac{4q}{3 - 2q - q^2 - (1-q)^{\frac{3}{2}}\sqrt{9-q}}.$$

When $q = 1$, *the optimal disclosure policy never recommends verification.*

To understand Proposition 7, note that we can identify the necessary and sufficient condition for the optimality of recommending verification by establishing the necessary and sufficient condition for the optimality of a cutoff rule. This is because Corollary 4 has already established the condition for the optimality of no disclosure. By Proposition 2, if we can further establish the condition for the optimality of a cutoff rule, we are left with rules that recommend verification, a negative assortative rule and a three-message rule. We illustrate how to find the condition for $q \leq \frac{2}{3}$, and leave the rest of the proof to Appendix A.

By Theorem 1, any optimal disclosure policy is characterized by three cutoffs, θ^*, $\underline{\theta}$, and $\overline{\theta}$. A cutoff rule corresponds to the case where $\underline{\theta} = \theta^* > 0$ and $\overline{\theta} = 1$. By Corollary 4, no disclosure is never optimal when $R > \frac{1}{2}$. It implies that the optimal disclosure policy is either a cutoff rule or a rule that recommends verification with positive probability.

Given any $\overline{\theta} \in [R, 1]$, if (V) is binding, $\underline{\theta}$ is uniquely pinned down by

$$C\left(\overline{\theta} - \underline{\theta}\right) + \int_{\underline{\theta}}^{R} (\theta - R)\, d\theta = 0.$$

Denote its solution by $\underline{\theta}_V\left(\overline{\theta}\right)$. On the other hand, if $(A0)$ is binding, $\underline{\theta}$ is uniquely pinned down by

$$\int_{\underline{\theta}}^{\overline{\theta}} (\theta - R)\, d\theta = 0.$$

Denote its solution by $\underline{\theta}_{A0}\left(\overline{\theta}\right)$. $(A0)$ is equivalent to $\underline{\theta} \geq \underline{\theta}_{A0}\left(\overline{\theta}\right)$ while (V) is equivalent to $\underline{\theta} \geq \underline{\theta}_V\left(\overline{\theta}\right)$.

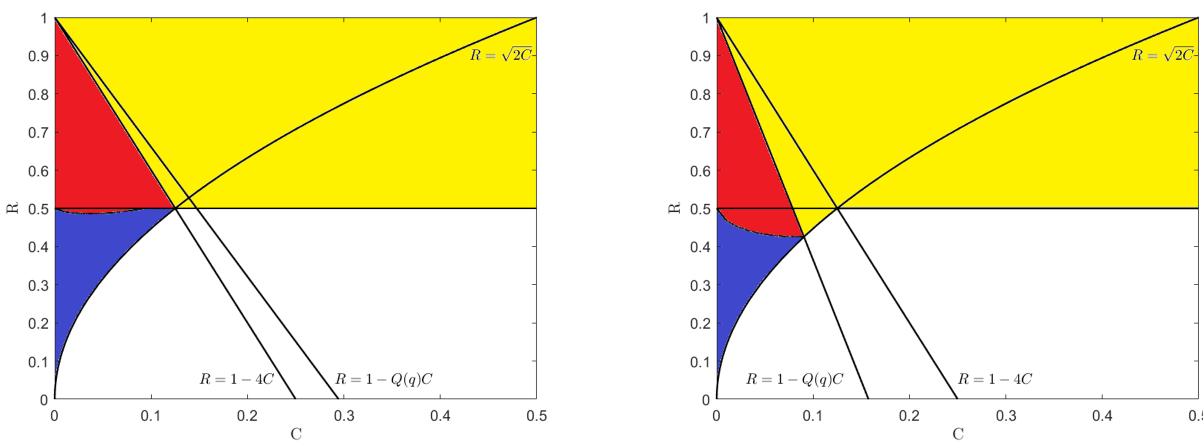
We can further show that $\underline{\theta}_V\left(\overline{\theta}\right) - \underline{\theta}_{A0}\left(\overline{\theta}\right)$ is strictly increasing in $\overline{\theta}$, and $\underline{\theta}_V\left(\overline{\theta}\right) - \underline{\theta}_{A0}\left(\overline{\theta}\right) = 0$ when $\overline{\theta} = R + 4C$. This implies that, when $R + 4C > 1$, $\underline{\theta}_V\left(\overline{\theta}\right) < \underline{\theta}_{A0}\left(\overline{\theta}\right)$ for all $\overline{\theta} \in [R, 1]$. Thus, (V) cannot be binding if $R + 4C > 1$. By Lemma 4, this means that the optimal disclosure policy does not recommend verification at all. As a result, we get the following sufficient condition for the optimality of a cutoff rule:

$$R > \frac{1}{2} \text{ and } R \geq 1 - 4C.$$

When $q \leq \frac{2}{3}$, it can be shown that this condition is also necessary. Together with Corollary 4, this implies that when $q \leq \frac{2}{3}$, the optimal disclosure policy recommends verification with positive probability if and only if $\sqrt{2C} < R < 1 - 4C$.

Corollary 4 and Proposition 7 are illustrated in Figures 7(a) and 7(b). In both figures, no disclosure is optimal in the white region, and a cutoff rule is optimal in the yellow region.

Both rules recommend no verification. A three-message rule is optimal in the red region and a negative assortative rule is optimal in the blue region. Both rules recommend verification with positive probability. Figure 7(a) depicts the case where $q < \frac{2}{3}$, when the verification technology is relatively imprecise. In this case, the line $R = 1 - Q\,(q)\,C$ always lies above the line $R = 1 - 4C$. Consequently, the combined area of the blue and red regions remains constant, which corresponds to the first part of Proposition 7. When $q = \frac{2}{3}$, the two lines coincide. Figure 7(b) depicts the case where $q > \frac{2}{3}$, when the verification technology is relatively precise. In this case, the line $R = 1 - Q\,(q)\,C$ always lies below the line $R = 1 - 4C$ and determines the boundary between the yellow region and the combined blue and red regions, which corresponds to the second part of Proposition 7. The boundary between the blue and red regions does not admit a closed-form expression and is determined numerically.



White: no disclosure; Yellow: cutoff rule; Blue: negative assortative rule; Red: three-message rule

Figure 7(a). Optimal disclosure policy ($q = 0.6$)   Figure 7(b). Optimal disclosure policy ($q = 0.8$)

Comparing Figures 7(a) and 7(b), we can see that the combined area for negative assortative and three-message rules (blue and red regions) shrinks as q increases. This is because when q gets higher, the benefit of sending m_1 gets lower. Consequently, the sender sends m_1 less often.

When $R > \frac{1}{2}$, by Corollary 2, the optimal disclosure policy is either a cutoff rule or a three-message rule. In this case, as illustrated in the Figures 7(a) and 7(b), the optimal disclosure policy is a cutoff rule when C is high, and a three-message rule when C is low. Intuitively, when

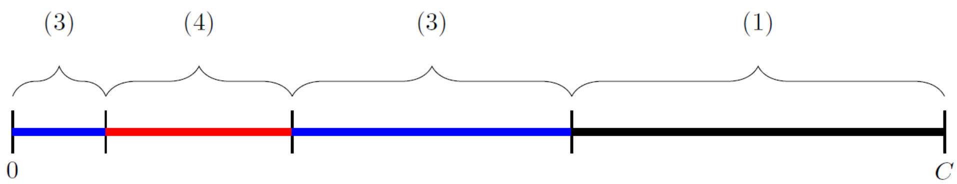
C gets lower, it is more difficult to convince the receiver not to verify. The sender sends m_1 more often. To summarize, we have

Corollary 5 *Suppose* $F(\theta) = \theta$ *and* $R > \frac{1}{2}$*. A cutoff rule is optimal if and only if* $C \geq \min\{\frac{1-R}{4}, \frac{1-R}{Q(q)}\}$*, and a three-message rule if and only if* $C < \min\{\frac{1-R}{4}, \frac{1-R}{Q(q)}\}$*.*

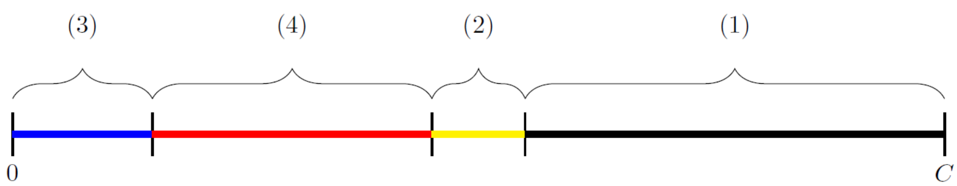
When $R < \frac{1}{2}$, the sender faces a more intricate trade-off. Once C gets below $\frac{R^2}{2}$, no disclosure can no longer induce immediate acceptance. The sender faces three options: a cutoff rule, which does not send m_1, a negative assortative rule, which does not send m_ϕ, and a three-message rule, which sends both m_ϕ and m_1. Sending m_1 leads to potential rejection but imposes no addition constraint on the persuasion problem aside from the requirement of credibility resource. Sending m_0 ensures certain acceptance but is very costly, requiring credibility resource and inducing verification benefit. A slack ($A0$) means that sending m_0 requires extra credibility resource, relative to sending m_1. In order to send m_0 for more states below R, the sender needs to opt for m_ϕ over m_1 due to the extra requirement of credibility resource. Thus, when C decreases, verification becomes less costly, but the optimal disclosure policy does not necessarily recommend verification more often.

For instance, consider the case when q is smaller than $\frac{2}{3}$ and R is smaller than but close to $\frac{1}{2}$, as depicted in Figure 8(a). The optimal disclosure policy is to have no disclosure when $C \geq \frac{R^2}{2}$. When $C < \frac{R^2}{2}$, the optimal disclosure policy transitions from a negative assortative rule for high values of C, to a three-message rule for intermediate values of C, and back to a negative assortative rule for low values of C. The rationale behind this shift is as follows: as C drops below $\frac{R^2}{2}$, the sender must disclose some information to the receiver, otherwise the receiver would verify the state. The sender starts replacing m_0 with m_1 to reduce verification benefit, resulting in a negative assortative rule. As C decreases further, sending m_0 becomes more costly in terms of the extra requirement of credibility resource. To met the requirement for credibility resource, the sender further reveals the lowest states, sending m_ϕ alongside m_1, resulting in a three-message rule. When C gets close to zero, sending m_0 becomes prohibitively costly, and as a result, the sender rarely sends m_0, freeing up most credibility resource. This, in turn, eliminates the need to send m_ϕ. This reasoning leads to the non-monotone behavior of the optimal disclosure policy regarding the revelation of the lowest states (sending m_ϕ) as C changes.

Furthermore, for some R, all four rules can occur as C varies. For instance, consider again the case when R is smaller than but close to $\frac{1}{2}$. However, assume instead $q > \frac{2}{3}$, as depicted in Figure 8(b). The optimal disclosure policy is to have no disclosure for all $C \geq \frac{R^2}{2}$. When $C < \frac{R^2}{2}$, the optimal disclosure policy transitions from a cutoff rule for high values of C, to a three-message rule for intermediate values of C, and to a negative assortative rule for low values of C.



(a) $R = 0.49, q = 0.6$



(b) $R = 0.45, q = 0.8$

(1) No disclosure, (2) cutoff rule, (3) negative assortative rule, (4) three-message rule

Figure 8. The effect of changing C on the recevier's payoff

5 Discussions

5.1 Multiple rounds of state verification

One limitation of our information acquisition technology is that the receiver is restricted to verifying the state only once. This is, however, without loss of generality. Consider a scenario where the receiver can choose to verify the state for a maximum of k rounds, where $k \in \{1, 2, ...\}$. Each round of verification incurs a cost of c. We have

Lemma 5 *Suppose the receiver can verify the state for a maximum of k rounds, where $k \in \{1, 2, ...\}$. The receiver either does not verify the state or verify as many rounds as possible until* *the state is discovered. When k goes to infinity, the receiver either does not learn the state or learns the state for sure in the limit.*

To gain some intuition for the lemma, consider the scenario, in which the receiver has the ability to verify for unlimited rounds. After an unsuccessful round of verification, the receiver's belief remains unchanged. This implies that the receiver faces the same decision problem as at the beginning of the previous round. Consequently, if a receiver has incentive to verify for one round, then he has incentive to verify for any number of rounds as long as the state is not revealed. Conversely, if a receiver lacks incentive to verify for one round, then he lacks incentive to verify at all. Given Lemma 5, we immediately have

Proposition 8 *Given $k \in \{1, 2, ...\}$, the sender's problem can be formulated as*

$$\max_{\substack{x_0(\theta)\in[0,1] \\ \theta^*\in[0,R]}} \int_{\theta^*}^{R} \left[x_0(\theta) + (1 - x_0(\theta))(1-q)^k\right] f(\theta)\, d\theta + \int_R^1 f(\theta)\, d\theta$$

s.t.

$$\int_{\theta^*}^{1} (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0, \qquad (A0)$$

$$\int_{\theta^*}^{1} (\theta - R)(1 - x_0(\theta))\, f(\theta)\, d\theta \geq 0, \qquad (A1)$$

$$C\int_R^1 x_0(\theta) f(\theta)\, d\theta + \int_{\theta^*}^{R} (\theta - R + C)\, x_0(\theta)\, f(\theta)\, d\theta \geq 0. \qquad (V)$$

Thus, if we consider the transformation $q' = 1 - (1-q)^k$ and $C' = C$, the problem reduces to the one in our baseline model and Theorem 1 applies. This implies that when the receiver can choose unlimited rounds of verification, the optimal disclosure policy is the same as when $q = 1$, and Proposition 4 applies.

5.2 Role of commitment power

Communication with detectable deception has been studied in the cheap-talking setting (Dziuda and Salas, 2018; Balbuzanov, 2019; Levkun, 2021; Sadakane and Tam, 2022; Zhao, 2018). In cheap talk, detectable deception often leads to equilibrium communication strategies with a structure that pools the highest types with the lowest types. In a cheap talk game, low types can always mimic high types. Therefore, there is no way to truthfully reveal the lowest types as in a persuasion problem. Such feature can be observed in settings with both exogenous and endogenous deception detection. With exogenous lie detection, for instance, Dziuda and Salas (2018) show that the moderate types and the highest types tell the truth, and the lowest types pretend to be the highest types. With endogenous state verification, for instance, Zhao (2018) shows that the sender-preferred equilibrium also pools the lowest and the highest types.

Furthermore, Zhao (2018) shows that the receiver never accepts without successful verification in a cheap-talk setting, otherwise every type would lie. In equilibrium, the lowest types never benefit from lying. Only two types of messages are sent in equilibrium, *Verify then Reject* and *Reject without verification*. The fewer low types are lying, the higher the verification effort, because the purpose of verification is to find out the high types. In contrast, the sender is always able to reveal the lowest types in information design. In the optimal disclosure policy, the lowest types are revealed to persuade the sender to accept without successful verification. The optimal disclosure policy never recommends *Verify then Reject*, and moderate types are pooled and no verification is recommended. The better the pooling moderate types, the lower the incentive for verification, because the purpose of verification is to find out the low types.

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Appendix A

Proof of Lemma 1. The proof here is identical to the one in the main text except that the disclosure policy is not necessarily deterministic and is included here for completeness. Suppose $q < 1$ and there exists a message $m \in M$ in the optimal disclosure policy such that $a_\phi(m) = 0$ and $e(m) = 1$ but $\Pr(m) > 0$. There must be a positive measure of $\theta \geq R$ such that $\Pr(m|\theta) > 0$. Otherwise, state verification always leads to rejection and it would be suboptimal to verify the state. Consider another information structure that, for all $\theta \in [0, 1]$, given which m is sent with positive probability, truthfully reveals the state with the same probability instead. This results in a strict improvement, which contradicts the optimality of the original disclosure policy. ■

Proof of Lemma 2. Our information design problem can be formulated as

$$\max_{x_0(\theta), x_1(\theta) \in [0,1]} \int_0^R [x_0(\theta) + x_1(\theta)(1-q)] f(\theta) d\theta + \int_R^1 (x_0(\theta) + x_1(\theta)) f(\theta) d\theta$$

s.t.

$$\begin{aligned} \int_0^1 (\theta - R) x_0(\theta) f(\theta) d\theta &\geq 0, && \text{(A0)} \\ \int_0^1 (\theta - R) x_1(\theta) f(\theta) d\theta &\geq 0, && \text{(A1)} \\ C \int_R^1 x_0(\theta) f(\theta) d\theta + \int_0^R (\theta - R + C) x_0(\theta) f(\theta) d\theta &\geq 0, && \text{(V)} \\ \forall \theta \in [0,1], \quad 1 - x_0(\theta) - x_1(\theta) &\geq 0, && \text{(TP)} \end{aligned}$$

where $x_0(\theta)$ and $x_1(\theta)$ are probabilities that m_0 and m_1 are sent, respectively, and the last inequality follows from the law of total probability, i.e., the probabilities that m_ϕ, m_0 and m_1 are sent must sum up to 1. Let λ_0, λ_1, μ and $\eta(\theta)$ be the Lagrange multipliers corresponding to the constraints $(A0)$, $(A1)$, (V) and (TP), respectively, and $x_0^*(\theta)$ and $x_1^*(\theta)$ be the maximizers. The Euler–Lagrange equations are given by

$$\forall\theta > R, \quad \begin{array}{l} \lambda_0(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) > 0 \Rightarrow x_0^*(\theta) = 1, \\ \lambda_0(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) < 0 \Rightarrow x_0^*(\theta) = 0, \end{array} \tag{1}$$

$$\forall\theta > R, \quad \begin{array}{l} \lambda_1(\theta - R) + 1 - \hat{\eta}(\theta) > 0 \Rightarrow x_1^*(\theta) = 1, \\ \lambda_1(\theta - R) + 1 - \hat{\eta}(\theta) < 0 \Rightarrow x_1^*(\theta) = 0, \end{array} \tag{2}$$

$$\forall\theta < R, \quad \begin{array}{l} (\lambda_0 + \mu)(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) > 0 \Rightarrow x_0^*(\theta) = 1, \\ (\lambda_0 + \mu)(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) < 0 \Rightarrow x_0^*(\theta) = 0. \end{array} \tag{3}$$

$$\forall\theta < R, \quad \begin{array}{l} \lambda_1(\theta - R) + 1 - q - \hat{\eta}(\theta) > 0 \Rightarrow x_1^*(\theta) = 1, \\ \lambda_1(\theta - R) + 1 - q - \hat{\eta}(\theta) < 0 \Rightarrow x_1^*(\theta) = 0, \end{array} \tag{4}$$

where $\hat{\eta}(\theta) := \eta(\theta)/f(\theta)$. For $\theta > R$, define the marginal benefits of sending messages m_0 and m_1, $\overline{B}_0(\theta)$ and $\overline{B}_1(\theta)$, by

$$\begin{aligned} \overline{B}_0(\theta) &:= \lambda_0(\theta - R) + 1 + \mu C, \\ \overline{B}_1(\theta) &:= \lambda_1(\theta - R) + 1. \end{aligned}$$

For all $\theta > R$, $\overline{B}_0(\theta), \overline{B}_1(\theta) > 0$. This means that we cannot have $x_0^*(\theta) + x_1^*(\theta) < 1$. This is because, in that case, (1) and (2) imply $\hat{\eta}(\theta) \geq \max\left\{\overline{B}_0(\theta), \overline{B}_1(\theta)\right\} > 0$, which in turn implies that (TP) is binding. Thus, m_ϕ is not sent for any $\theta > R$.

Similarly, for $\theta < R$, define the marginal benefits of sending messages m_0 and m_1, $\underline{B}_0(\theta)$ and $\underline{B}_1(\theta)$, by

$$\begin{aligned} \underline{B}_0(\theta) &:= (\lambda_0 + \mu)(\theta - R) + 1 + \mu C, \\ \underline{B}_1(\theta) &:= \lambda_1(\theta - R) + 1 - q. \end{aligned}$$

Suppose $\lambda_0 = \mu = 0$, then $\underline{B}_0(\theta) > 0$. This means that m_ϕ is not sent for any $\theta < R$. Otherwise, $\hat{\eta}(\theta) \geq \underline{B}_0(\theta) > 0$, which in turn implies that (TP) is binding. Similarly, if $\lambda_1 = 0$, then $\underline{B}_1(\theta) > 0$, which implies that m_ϕ is not sent for any $\theta < R$.

Finally, suppose λ_1 and at least one of λ_0 and μ are strictly larger than 0. Define

$$\theta^* := \max\left\{\min\left\{R - \frac{1-q}{\lambda_1}, R - \frac{1+\mu C}{\lambda_0 + \mu}\right\}, 0\right\}.$$

For all $\theta < \theta^*$, we must have $x_0^*(\theta) = x_1^*(\theta) = 0$ and m_ϕ is sent with probability 1. This is because, by construction, $\theta^* > 0$ implies that for all $\theta < \theta^*$, $\underline{B}_0(\theta), \underline{B}_1(\theta) < 0$. Since $\hat{\eta}(\theta) \geq 0$, (3) and (4) imply that $x_0^*(\theta) = x_1^*(\theta) = 0$. For all $\theta \in (\theta^*, R)$, at least one of $\underline{B}_0(\theta)$ and $\underline{B}_1(\theta)$ is strictly larger than 0. Suppose $\underline{B}_0(\theta) > 0$. We must have $x_0^*(\theta) = 1$ or $\hat{\eta}(\theta) \geq \underline{B}_0(\theta) > 0$. In both cases, (TP) is binding. Similarly, (TP) must be binding when $\underline{B}_1(\theta) > 0$. Thus, for all $\theta \in (\theta^*, R)$, m_ϕ is not sent. ■

Proof of Corollary 1. In the main text. ■

Proof of Proposition 1. Follows immediately from Lemma 2. ■

Proof of Lemma 3. Follows immediately from Theorem 1. ■

Proof of Lemma 4. From the proof of Theorem 1, the only case in which m_1 is sent with positive probability in the optimal information structure is when $\lambda_0 < \lambda_1$ and $\mu > 0$. Since $\mu > 0$, (V) is binding. Since $\lambda_1 > 0$, $(A1)$ is binding. ■

Proof of Theorem 1. Let λ_0, λ_1 and μ be the Lagrange multipliers corresponding to the constraints $(A0)$, $(A1)$, and (V), respectively, in the sender's problem formulated in Proposition 1 and let $x_0^*(\theta)$ be the maximizer. The Euler–Lagrange equations are given by

$$\forall\theta > R, \quad \begin{array}{l} (\lambda_0 - \lambda_1)(\theta - R) + \mu C > 0 \Rightarrow x_0^*(\theta) = 1, \\ (\lambda_0 - \lambda_1)(\theta - R) + \mu C < 0 \Rightarrow x_0^*(\theta) = 0, \end{array} \tag{5}$$

$$\forall\theta < R, \quad \begin{array}{l} (\lambda_0 - \lambda_1 + \mu)(\theta - R) + q + \mu C > 0 \Rightarrow x_0^*(\theta) = 1, \\ (\lambda_0 - \lambda_1 + \mu)(\theta - R) + q + \mu C < 0 \Rightarrow x_0^*(\theta) = 0. \end{array} \tag{6}$$

Moreover, the optimality of θ^* implies

$$\begin{array}{ll} \begin{array}{c} [x_0^*(\theta)+(1-x_0^*(\theta))(1-q)]+[\lambda_0 x_0^*(\theta)+\lambda_1(1-x_0^*(\theta))](\theta^*-R) \\ +\mu(\theta^*-R+C)x_0^*(\theta)=0 \end{array} & \text{if } \theta^*>0, \\ \begin{array}{c} [x_0^*(\theta)+(1-x_0^*(\theta))(1-q)]+[\lambda_0 x_0^*(\theta)+\lambda_1(1-x_0^*(\theta))](\theta^*-R) \\ +\mu(\theta^*-R+C)x_0^*(\theta)\geq 0 \end{array} & \text{if } \theta^*=0. \end{array} \tag{7}$$

Suppose first $\lambda_0 > \lambda_1$. (5) implies that $x_0^*(\theta) = 1$ for all $\theta > R$. $(A1)$ then implies that $x_0^*(\theta) = 1$ for all $\theta > \theta^*$. Similarly, if $\lambda_0 = \lambda_1$ and $\mu > 0$, then $x_0^*(\theta) = 1$ for all $\theta > R$. As a result, $x_0^*(\theta) = 1$ for all $\theta > \theta^*$.

If $\lambda_0 \leq \lambda_1$ and $\mu = 0$, then $x_0^*(\theta) = 1$ for all $\theta \in (\theta^*, R)$. Since m_1 is not sent below R, if m_1 is sent for a positive measure of θ above R, then $(V1)$ is violated and the receiver would not verify the state. This means that $x_0^*(\theta) = 1$ for all $\theta > R$ as well.

Next, suppose $\lambda_0 < \lambda_1$ and $\mu > 0$. Consider

$$\overline{\theta} := R + \frac{\mu C}{\lambda_1 - \lambda_0}.$$

If $\overline{\theta} \geq 1$, then m_1 is not sent with positive probability above R. $(A1)$ then implies m_1 is not sent with positive probability. If $\overline{\theta} < 1$, then $x_0^*(\theta) = 1$ for all $\theta \in \left(R, \overline{\theta}\right)$ and $x_0^*(\theta) = 0$ for all $\theta \in (\overline{\theta}, 1]$. Since $\lambda_1 > 0$, $(A1)$ implies $x_0^*(\theta) = 0$ for some $\theta \in (\theta^*, R)$. Since $q > 0$, (6) implies that $x_0^*(\theta) = 1$ for θ smaller than but close enough to R. Moreover, by (6), in order to have $x_0^*(\theta) = 0$ for some $\theta < R$, we must have $\lambda_0 - \lambda_1 + \mu > 0$. Define

$$\underline{\theta} := R - \frac{q + \mu C}{\lambda_0 - \lambda_1 + \mu}.$$

In this case, $x_0^*(\theta) = 1$ for all $\theta \in (\underline{\theta}, R)$ and $x_0^*(\theta) = 0$ for all $\theta \in (\theta^*, \underline{\theta})$.

In all cases, $\mathbf{m}(\theta)$ satisfies the descriptions in Theorem 1. ■

Proof of Proposition 2. Follows immediately from Theorem 1. ■

Proof of Proposition 3. Follows immediately from $(A0)$, (V) and the fact that the sender achieves her highest possible payoff when only m_0 is sent. ■

Proof of Corollary 2. Follows immediately from $(A0)$, $(A1)$ and Proposition 2. ■

Proof of Corollary 3. Let λ_0, λ_1 and μ be the Lagrange multipliers corresponding to the constraints $(A0)$, $(A1)$, and (V), respectively, in the sender's problem formulated in Proposition 1. The Lagrangian is given by

$$\begin{aligned}
\mathcal{L}(x_0, \theta^*) &= \int_{\theta^*}^{R} [x_0(\theta) + (1 - x_0(\theta))(1-q)] f(\theta)\, d\theta + \int_{R}^{1} f(\theta)\, d\theta \\
&\quad + \lambda_0 \int_{\theta^*}^{1} (\theta - R) x_0(\theta) f(\theta)\, d\theta + \lambda_1 \int_{\theta^*}^{1} (\theta - R)(1 - x_0(\theta)) f(\theta)\, d\theta \\
&\quad + \mu \left(C \int_{\theta^*}^{1} x_0(\theta) f(\theta)\, d\theta + \int_{\theta^*}^{R} (\theta - R) x_0(\theta) f(\theta)\, d\theta \right).
\end{aligned}$$

We have

$$\begin{aligned}
\frac{\partial \mathcal{L}(x_0, \theta^*)}{\partial c} &= \frac{\mu}{q} \int_{\theta^*}^{1} x_0(\theta) f(\theta)\, d\theta > 0, \\
\frac{\partial \mathcal{L}(x_0, \theta^*)}{\partial R} &= -(1 - x_0(R)) q f(R) - \lambda_0 \int_{\theta^*}^{1} x_0(\theta) f(\theta)\, d\theta \\
&\quad - \lambda_1 \int_{\theta^*}^{1} (1 - x_0(\theta)) f(\theta)\, d\theta - \mu \int_{\theta^*}^{R} x_0(\theta) f(\theta)\, d\theta \\
&< 0, \\
\frac{\partial \mathcal{L}(x_0, \theta^*)}{\partial q} &= - \int_{\theta^*}^{R} (1 - x_0(\theta)) f(\theta)\, d\theta - \mu c q^{-2} \int_{R}^{1} x_0(\theta) f(\theta)\, d\theta < 0.
\end{aligned}$$

The result then follows from the envelope theorem. ■

Proof of Proposition 4. When $q = 1$, verification never fails. As a result, the messages *Verify then Accept* and *Verify then Reject* can be considered as the same message, which we denote by m_1, and constraint $(A1)$ can be dropped. Thus, our information design problem can be formulated as

$$\max_{x_0(\theta), x_1(\theta) \in [0,1]} \int_{0}^{R} x_0(\theta) f(\theta)\, d\theta + \int_{R}^{1} (x_0(\theta) + x_1(\theta)) f(\theta)\, d\theta$$

s.t.

$$\begin{aligned} \int_0^1 (\theta - R)\, x_0(\theta)\, f(\theta)\, d\theta &\geq 0, && \text{(A0)} \\ C\int_R^1 x_0(\theta)\, f(\theta)\, d\theta + \int_0^R (\theta - R + C)\, x_0(\theta)\, f(\theta)\, d\theta &\geq 0, && \text{(V)} \\ \forall \theta \in [0,1]\,,\;\; 1 - x_0(\theta) - x_1(\theta) &\geq 0, && \text{(TP)} \end{aligned}$$

where $x_0(\theta)$ and $x_1(\theta)$ are probabilities that m_0 and m_1 are sent, respectively, and the last inequality follows from the law of total probability, i.e., the probabilities that m_ϕ, m_0 and m_1 are sent must sum up to 1. Let λ_0, μ, and $\eta(\theta)$ be the Lagrange multipliers corresponding to the constraints $(A0)$, (V), and (TP), respectively, and $x_0^*(\theta)$ and $x_1^*(\theta)$ be the maximizers. The Euler–Lagrange equations are given by

$$\forall \theta > R, \quad \begin{array}{l} \lambda_0(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) > 0 \Rightarrow x_0^*(\theta) = 1, \\ \lambda_0(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) < 0 \Rightarrow x_0^*(\theta) = 0, \end{array}$$

$$\forall \theta > R, \quad \begin{array}{l} 1 - \hat{\eta}(\theta) > 0 \Rightarrow x_1^*(\theta) = 1, \\ 1 - \hat{\eta}(\theta) < 0 \Rightarrow x_1^*(\theta) = 0, \end{array}$$

$$\forall \theta < R, \quad \begin{array}{l} (\lambda_0 + \mu)(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) > 0 \Rightarrow x_0^*(\theta) = 1, \\ (\lambda_0 + \mu)(\theta - R) + 1 + \mu C - \hat{\eta}(\theta) < 0 \Rightarrow x_0^*(\theta) = 0. \end{array}$$

$$\forall \theta < R, \quad \begin{array}{l} -\hat{\eta}(\theta) > 0 \Rightarrow x_1^*(\theta) = 1, \\ -\hat{\eta}(\theta) < 0 \Rightarrow x_1^*(\theta) = 0, \end{array}$$

where $\hat{\eta}(\theta) := \eta(\theta)/f(\theta)$. For $\theta > R$, define the marginal benefits of sending messages m_0 and m_1, $\overline{B}_0(\theta)$ and $\overline{B}_1(\theta)$, by

$$\begin{aligned} \overline{B}_0(\theta) &: = \lambda_0(\theta - R) + 1 + \mu C, \\ \overline{B}_1(\theta) &: = 1. \end{aligned}$$

Similarly, for $\theta < R$, define the marginal benefits of sending messages m_0 and m_1, $\underline{B}_0(\theta)$ and

$\underline{B}_1(\theta)$, by

$$\begin{aligned}\underline{B}_0(\theta) &:= (\lambda_0+\mu)(\theta-R)+1+\mu C,\\ \underline{B}_1(\theta) &:= 0.\end{aligned}$$

Suppose $\lambda_0 = \mu = 0$, then $\underline{B}_0(\theta) = 1 > 0 = \underline{B}_1(\theta)$. m_1 is not sent for $\theta < R$. If m_1 is sent for a positive measure of θ above R, then the receiver would not verify the state. This means that m_1 is not sent for all $\theta \in [0,1]$. Moreover, since $\overline{B}_0(\theta) > 0$ for all $\theta > R$ and $\underline{B}_0(\theta) > 0$ for all $\theta < R$, we have m_0 is sent with probability 1 for all $\theta \in [0,1]$.

If at least one of λ_0 and μ is nonzero, then for all $\theta > R$, $\overline{B}_0(\theta) > \overline{B}_1(\theta)$, which implies that m_1 is not sent for all $\theta > R$. If m_1 is sent for a positive measure of θ below R, then the receiver would not verify the state. This means that m_1 is not sent for all $\theta \in [0,1]$. Define

$$\theta^* := \max\left\{R - \frac{1+\mu C}{\lambda_0+\mu}, 0\right\}.$$

Thus, m_0 is sent with probability 1 for all $\theta \geq \theta^*$ and m_ϕ is sent with probability 1 for all $\theta < \theta^*$. The optimal disclosure policy must be no disclosure or a cutoff rule. ■

Proof of Proposition 5. Let λ and μ be the Lagrange multipliers corresponding to the constraints (A) and (V), respectively, in the sender's problem when $q = 1$ in the main text. The Lagrangian is given by

$$\mathcal{L}(\theta^*) = -\theta^* + \lambda\int_{\theta^*}^1 (\theta-R)\,x_0(\theta)\,f(\theta)\,d\theta + \mu\left(c\int_{\theta^*}^1 x_0(\theta)\,f(\theta)\,d\theta + \int_{\theta^*}^R (\theta-R)\,x_0(\theta)\,f(\theta)\,d\theta\right).$$

We have

$$\begin{aligned}\frac{\partial\mathcal{L}(\theta^*)}{\partial c} &= \mu\int_{\theta^*}^1 x_0(\theta)\,f(\theta)\,d\theta > 0,\\ \frac{\partial\mathcal{L}(\theta^*)}{\partial R} &= -\lambda\int_{\theta^*}^1 x_0(\theta)\,f(\theta)\,d\theta - \mu\int_{\theta^*}^R x_0(\theta)\,f(\theta)\,d\theta < 0.\end{aligned}$$

The result then follows from the envelope theorem. ■

Proof of Proposition 6. Since verification does not occur under the optimal disclosure policy, the receiver's expected payoff W_R is given by

$$W_R = \left(\frac{1+\theta^*}{2} - R\right)(1-\theta^*).$$

Moreover,

$$\frac{\partial W_R}{\partial \theta^*} = R - \theta^* > 0.$$

By Proposition 5, θ^* decreases with c. As a result, the receiver's expected payoff also decreases with c. ■

Proof of Example 1. Suppose $F(\theta) = \theta$ and $q = 1$. By Proposition 4, the sender will never send m_1. Thus, the sender's problem becomes

$$\min_{\theta^* \in [0,R]} \theta^*$$

s.t.

$$\frac{1+\theta^*}{2} \geq R, \tag{A}$$

$$c(1-\theta^*) - \frac{(R-\theta^*)^2}{2} \geq 0. \tag{V}$$

By Corollary 4, the optimal disclosure policy is no disclosure if $R \leq \min\left\{\sqrt{2c}, \frac{1}{2}\right\}$. In this case, the receiver's expected payoff W_R is $\frac{1}{2} - R$, which is strictly decreasing in R.

If $R > \min\left\{\sqrt{2c}, \frac{1}{2}\right\}$, the optimal disclosure policy is a cutoff rule, i.e., $\theta^* > 0$. Moreover, from (8) and (9) in the proof of Proposition 7, we find that when $c > \frac{1}{4}(1-R)$, (A) is the binding constraint, and $\theta^* = 2R - 1$; when $c < \frac{1}{4}(1-R)$, (V) is the binding constraint, and $\theta^* = R - c - \sqrt{2c(1-R) + c^2}$. When (A) is the binding constraint, the receiver's payoff W_R is equal to 0. Therefore, we focus on the case where (V) is the binding constraint.

The receiver's payoff W_R in this case is

$$\begin{aligned} W_R &= \int_{\theta^*}^{1} (\theta - R)\, d\theta = \frac{1}{2}\left(1 + \theta^* - 2R\right)\left(1 - \theta^*\right) \\ &= \frac{1}{2}\left(1 - R - c - \sqrt{2c\left(1-R\right) + c^2}\right)\left(1 - R + c + \sqrt{2c\left(1-R\right)+c^2}\right). \end{aligned}$$

We have

$$\frac{\partial W_R}{\partial R} = R + c - 1 + \frac{c^2}{\sqrt{2c\left(1-R\right)+c^2}}.$$

Moreover,

$$\frac{\partial^2 W_R}{\partial c \partial R} = 1 + \frac{3c^2\left(1-R\right)+c^3}{\left(2c\left(1-R\right)+c^2\right)^{\frac{3}{2}}} > 0.$$

Thus, $\frac{\partial W_R}{\partial R}$ is maximized at $c = \frac{1}{4}\left(1-R\right)$.

$$\frac{\partial W_R}{\partial R}\Big|_{c=\frac{1}{4}(1-R)} = -\frac{2}{3}\left(1-R\right) < 0.$$

Thus, W_R must decrease with R. ■

Proof of Corollary 4. By Proposition 3, no disclosure is optimal if and only if $\mathbb{E}\left(\theta\right) = \frac{1}{2} \geq R$ and $\int_0^R \left(R-\theta\right) d\theta = \frac{R^2}{2} \leq C$. ■

Proof of Proposition 7. Consider $\sqrt{2C} < R < 1 - 4C$. By Corollary 4, the optimal disclosure policy is not no disclosure. This means that one of $(A0)$ and (V) must bind. Otherwise, the sender can send more m_0 and increase his payoff.

To determine which constraint is binding, note that given $\overline{\theta} \in [R, 1]$, if (V) is binding, $\underline{\theta}$ is uniquely pinned down by (V) and has the expression

$$\underline{\theta}_V\left(\overline{\theta}\right) := R - C - \sqrt{2C\left(\overline{\theta} - R\right) + C^2}. \tag{8}$$

Similarly, given $\overline{\theta} \in [R, 1]$, if $(A0)$ is binding, $\underline{\theta}$ is uniquely pinned down by $(A0)$ and has the expression

$$\underline{\theta}_{A0}\left(\overline{\theta}\right) := 2R - \overline{\theta}. \tag{9}$$

Note that

$$\frac{\partial}{\partial\theta}\left(\underline{\theta}_V(\theta)-\underline{\theta}_{A0}(\theta)\right)=1-\frac{C}{\sqrt{2C(\theta-R)+C^2}}>0$$

for all $\theta>R$. Moreover, $\underline{\theta}_V(\theta)=\underline{\theta}_{A0}(\theta)$ if $\theta=R+4C$. As a result, $(A0)$ is binding if $\overline{\theta}<R+4C$ and (V) is binding if $\overline{\theta}>R+4C$.

We are now ready to show that (V) must be binding. Suppose, by way of contradiction, that it is not. By Lemma 4, we must have $\overline{\theta}=1$ under the optimal disclosure policy. But since $1>R+4C$ by assumption, we have $\underline{\theta}_V(1)\geq\underline{\theta}_{A0}(1)$ and (V) is binding. Therefore, under the optimal information structure, (V) must be binding and $\overline{\theta}\in[R+4C,1]$.

In the remaining of the proof, we check that under the conditions given in Proposition 7, $\overline{\theta}\neq 1$ in the optimal disclosure policy. This means that m_1 is sent with positive probability. To do so, we first write down the sender's problem as an optimization problem over $\overline{\theta}$ and show the first order condition at $\overline{\theta}=1$ is not satisfied under the stated conditions. Then, we show the first order condition is necessary and sufficient for $\overline{\theta}=1$ to be the maximizer.

Define

$$\theta^*_{A1}\left(\overline{\theta},\underline{\theta}\right):=R-\sqrt{(R-\underline{\theta})^2+(1-R)^2-\left(\overline{\theta}-R\right)^2}.$$

If $\theta^*=\theta^*_{A1}\left(\overline{\theta},\underline{\theta}\right)$, then θ^* is the solution to

$$\left(1-\overline{\theta}\right)\left(\frac{1+\overline{\theta}}{2}-R\right)=(\underline{\theta}-\theta)\left(R-\frac{\underline{\theta}+\theta}{2}\right),$$

which means that $(A1)$ binds. Thus, given $\underline{\theta}$ and $\overline{\theta}$, $\theta^*_{A1}\left(\overline{\theta},\underline{\theta}\right)$ identifies θ^* under the assumption that $(A1)$ binds.

Suppose that (V) is binding and $\theta^*=0$, the sender's objective function becomes

$$\Pr(a=1)=1-\underline{\theta}_V\left(\overline{\theta}\right)+(1-q)\,\underline{\theta}_V\left(\overline{\theta}\right)=1-q\left(R-C-\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\right),$$

which is strictly increasing in $\overline{\theta}$.

Next, suppose $(A1)$ and (V) are binding and $\theta^* > 0$, the sender's objective function becomes

$$\begin{aligned}
& \Pr(a=1) \\
= \; & \Pr(\theta \geq R) + \Pr(\theta < R)\left[\Pr(m = m_0|\theta < R) + (1-q)\Pr(m = m_1|\theta < R)\right] \\
= \; & 1 - R + R - \underline{\theta}_V\left(\overline{\theta}\right) + (1-q)\left(\underline{\theta}_V\left(\overline{\theta}\right) - \theta^*_{A1}\left(\overline{\theta}, \underline{\theta}_V\left(\overline{\theta}\right)\right)\right) \\
= \; & 1 - R + \left(C + \sqrt{2C\left(\overline{\theta} - R\right) + C^2}\right) \\
& + (1-q)\left(R - C - \sqrt{2C\left(\overline{\theta} - R\right) + C^2} - \left(R - \sqrt{(R - \underline{\theta})^2 + (1-R)^2 - \left(\overline{\theta} - R\right)^2}\right)\right) \\
= \; & 1 - R + q\left(C + \sqrt{2C\left(\overline{\theta} - R\right) + C^2}\right) \\
& + (1-q)\left(\sqrt{\left(C + \sqrt{2C\left(\overline{\theta} - R\right) + C^2}\right)^2 + (1-R)^2 - \left(\overline{\theta} - R\right)^2}\right).
\end{aligned}$$

Differentiating the objective function yields

$$\frac{d\Pr(a=1)}{d\overline{\theta}} = \frac{qC + (1-q)\frac{\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\left(C+R-\overline{\theta}\right)+C^2}{\sqrt{\left(C+\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\right)^2+(1-R)^2-\left(\overline{\theta}-R\right)^2}}}{\sqrt{2C\left(\overline{\theta} - R\right) + C^2}}.$$

Suppose $q = 1$. Then, $\frac{d\Pr(a=1)}{d\overline{\theta}} > 0$, which implies that $\overline{\theta} = 1$ is optimal. The optimal disclosure policy would never recommend state verification.

Consider next $q \in (0,1)$. Let $A := \frac{1}{1-q}$. We have

$$\begin{aligned}
& \tfrac{d\Pr(a=1)}{d\overline{\theta}}|_{\overline{\theta}=1} < 0 \\
\Leftrightarrow \; & (A-1)C\left(C + \sqrt{2C(1-R) + C^2}\right) + \sqrt{2C(1-R) + C^2}\,(C + R - 1) + C^2 < 0 \\
\Leftrightarrow \; & AC^2 - \sqrt{2C(1-R) + C^2}\,(1 - R - AC) < 0
\end{aligned}$$

If $R + AC \geq 1$, then $\frac{d\Pr(a=1)}{d\overline{\theta}}|_{\overline{\theta}=1} > 0$. Thus, to identify the condition for $\frac{d\Pr(a=1)}{d\overline{\theta}}|_{\overline{\theta}=1} < 0$, we only need to focus on the case when $R + AC < 1$. Thus,

$$\begin{aligned}
&\tfrac{d\Pr(a=1)}{d\overline{\theta}}|_{\overline{\theta}=1} < 0 \\
\Leftrightarrow\quad & A^2C^4 - \left(2C(1-R) + C^2\right)(1-R-AC)^2 < 0 \\
\Leftrightarrow\quad & -C(1-R)\left(2A(A-1)C^2 - (1-R)(4A-1)C + 2R^2 - 4R + 2\right) < 0 \\
\Leftrightarrow\quad & 2A(A-1)C^2 - (1-R)(4A-1)C + 2R^2 - 4R + 2 > 0
\end{aligned}$$

Let

$$H(R) := 2A(A-1)C^2 - (1-R)(4A-1)C + 2R^2 - 4R + 2.$$

The function $H(.)$ is quadratic and $H(\infty) = H(-\infty) = \infty$. Moreover,

$$H(1-AC) = -AC^2 < 0$$

and for $R \in (0, 1-AC)$ and $q \in (0,1)$,

$$H'(R) = (4A-1)C + 4R - 4 < (4A-1)C - 4AC = -C < 0.$$

Thus, there is at most one root of $H(.)$ on $[0, 1-AC]$ and it is given by

$$R^*(C) = 1 - \frac{4A(A-1)}{4A - 1 - \sqrt{8A+1}}C.$$

Thus, $\frac{d\Pr(a=1)}{d\overline{\theta}}|_{\overline{\theta}=1} < 0$ if and only if $R < R^*(C)$.

Finally, we show that the first order condition is sufficient. Let

$$\Omega\left(\overline{\theta}\right) := \frac{\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\left(C+R-\overline{\theta}\right)+C^2}{\sqrt{\left(C+\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\right)^2 + (1-R)^2 - \left(\overline{\theta}-R\right)^2}}.$$

Note that $\frac{d\Pr(a=1)}{d\overline{\theta}} \lesseqgtr 0$ if and only if $\Omega\left(\overline{\theta}\right) \lesseqgtr -\frac{qC}{1-q}$. The first order condition is sufficient if for all $\overline{\theta} \in [R+4C, 1]$, $\Omega'\left(\overline{\theta}\right) < 0$.

$$
\begin{aligned}
& \Omega'\left(\overline{\theta}\right) \\
= & -\frac{C}{\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\left(\left(C+\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\right)^2+(1-R)^2-\left(\overline{\theta}-R\right)^2\right)^{\frac{3}{2}}} \\
& \times\{2C^3+\left(\overline{\theta}-R\right)\left(6C^2-3CR+3C\overline{\theta}+2R^2+2R\overline{\theta}-6R-\overline{\theta}^2+3\right) \\
& +2C\sqrt{2C\left(\overline{\theta}-R\right)+C^2}\left(C+2\left(\overline{\theta}-R\right)\right)\}.
\end{aligned}
$$

which is negative if

$$\Phi\left(\overline{\theta}, C\right) = 6C^2 - 3CR + 3C\overline{\theta} + 2R^2 + 2R\overline{\theta} - 6R - \overline{\theta}^2 + 3 > 0.$$

We have

$$
\begin{aligned}
\frac{d\Phi\left(\overline{\theta}, C\right)}{d\overline{\theta}} &= 3C + 2R - 2\overline{\theta} < 3C + 2R - 2(R+4C) = -5C < 0, \\
\frac{d\Phi\left(\overline{\theta}, C\right)}{dC} &= 12C + 3\left(\overline{\theta} - R\right) > 0.
\end{aligned}
$$

Thus, $\Phi\left(\overline{\theta}, C\right)$ is minimized at $\left(\overline{\theta}, C\right) = (1, 0)$. Since

$$\Phi(1,0) = 2(1-R)^2 > 0,$$

for all $\overline{\theta} \in [R+4C, 1]$, $\Phi\left(\overline{\theta}, C\right) > 0$. ■

Proof of Corollary 5. Since $R > \frac{1}{2}$, $(A0)$ and $(A1)$ imply that the optimal disclosure policy cannot be no disclosure or a negative assortative rule. The result then follows immediately from Proposition 7. ■

Proof of Lemma 5. Suppose the receiver is given k rounds to verify the state. Let $\overline{V}(m)$ be the expected payoff of choosing the action knowing the state θ given message m, $V_\phi(m)$ be the expected payoff of choosing the action without verification, and $V_t(m)$ be the receiver's expected payoff at the beginning of the t-th round. Denote the verification decision for the t-th round by

e_t.

Suppose by way of contradiction that the receiver only verifies until $\bar{t} < k$. Then $e_{\bar{t}} = 1$ and $e_{\bar{t}+1} = 0$. For round $\bar{t}$, $e_{\bar{t}} = 1$ implies that

$$q\overline{V}(m) + (1-q)V_{\bar{t}+1}(m) - c > V_{\bar{t}+1}(m) \Leftrightarrow q\left(\overline{V}(m) - V_{\bar{t}+1}(m)\right) > c \Leftrightarrow q\left(\overline{V}(m) - V_{\phi}(m)\right) > c;$$

For round $T+1$, $e_{\bar{t}+1} = 0$ implies that

$$q\overline{V}(m) + (1-q)V_{\phi}(m) - c \leq V_{\phi}(m) \Leftrightarrow q\left(\overline{V}(m) - V_{\phi}(m)\right) \leq c,$$

which is a contradiction. As a result, if the receiver decides to verify the state, he must continue until it is successful or round k is reached. ■

Proof of Proposition 8. Follows immediately from Proposition 1 and Lemma 5. ■

Appendix B

In Kolotilin et al. (2023), the authors discuss the difficulty to extend their analysis of persuasion with non-linear preferences to allow multidimensional actions and suggest that it is unclear what the appropriate generalization is in this situation. In our model, actions are two-dimensional. We show here that it is impossible to transform the receiver's actions into a unidimensional model so that the assumptions in Kolotilin et al. (2023) are satisfied. As a result, our model is not a special case of their model and our results cannot be obtained by directly applying their results.

Since the receiver always chooses optimally when verification is successful, we can define the sender's utility $V(b,\theta)$ and the receiver's utility $U(b,\theta)$ according to the corresponding entry in the following table.

		$\theta \geq R$		$\theta < R$	
		V	U	V	U
b_1	$a_\phi = 1$ and $e = 0$	1	$\theta - R$	1	$\theta - R$
b_2	$a_\phi = 1$ and $e = 1$	1	$\theta - R - c$	$1 - q$	$(1-q)(\theta - R) - c$
b_3	$a_\phi = 0$ and $e = 1$	q	$q(\theta - R) - c$	0	$-c$
b_4	$a_\phi = 0$ and $e = 0$	0	0	0	0

Since the receiver's action choice b is discrete, assumptions in Kolotilin et al. (2023), which presume differentiability, are naturally not satisfied. We check instead the discrete version of their assumptions. Assumption 2 in Kolotilin et al. (2023) requires that the receiver's expected utility is single-peaked in action given any posterior belief. Assumption 4 requires that the receiver prefers higher actions at higher states and the sender prefers always higher actions.

There is a natural order of b such that the sender always prefers higher actions, i.e., $b_1 \succ b_2 \succ b_3 \succ b_4$. However, according to this ordering, $U(b, \theta)$ is not single-peaked in b, which implies that the receiver's expected utility is not single-peaked in action for some posterior belief. For example, for $\theta = R$, $U(b_1, \theta) = U(b_4, \theta) = 0$, and $U(b_2, \theta) = U(b_3, \theta) = -c$. This suggests that the common theoretical structure considered by Kolotilin et al. (2023), which applies to several other applications (Zhang and Zhou, 2016; Guo and Shmaya, 2019; Goldstein and Leitner, 2018) does not apply to our setting.