Discussion Paper Series – CRC TR 224

Discussion Paper No. 477
Project C 03

Bank Resolution, Deposit Insurance, and Fragility

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December 2023

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

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Bank Resolution, Deposit Insurance, and Fragility*

Alkis Georgiadis-Harris[†] Maxi Guennewig[‡]

September 7, 2023

Abstract

Since the Great Financial Crisis, the share of deposits—both insured and uninsured—in bank liabilities has increased substantially. In this paper, we document this fact for the largest US banks. We show that it can be theoretically explained by the introduction of resolution powers, i.e. the ability to impose losses on bank shareholders and creditors. In such a world, banks issue deposits in order to channel resources towards uninsured depositors, imposing losses on insured depositors and forcing the government to conduct bailouts. Our model suggests that resolution and deposit insurance must be complemented by equity or long-term debt requirements.

*We would like to thank Hendrik Hakenes, Martin Hellwig, Yuliyan Mitkov, Cyril Monnet, Farzad Saidi, Andreas Schaab, André Stenzel, Elu von Thadden, Ansgar Walther, Olivier Wang, Jing Zeng and seminar participants at the LSE (Economics, Finance) and the University of Bonn for helpful comments and conversations. Maxi Guennewig gratefully acknowledges support from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC TR 224 (Project C03).

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1 Introduction

Why do banks issue deposits? Considerable research has argued that deposits fulfill a vital role in the economy: they provide liquidity insurance (Diamond and Dybvig, 1983), help overcome information frictions in financial markets (Gorton and Pennacchi, 1990), and ensure market discipline (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). Deposits further allow banks to hedge their interest rate risk (Drechsler et al., 2021). Given their many functions, it is then unsurprising that banks heavily rely on deposit financing. In Figure 1, we see that total deposits amounted to around 70% of total liabilities for the 30 largest banks in the US before the Great Financial Crisis (GFC).[1] Since the GFC, we can see that deposit financing has increased significantly, accounting for almost 90% of bank liabilities in Q4 2022.

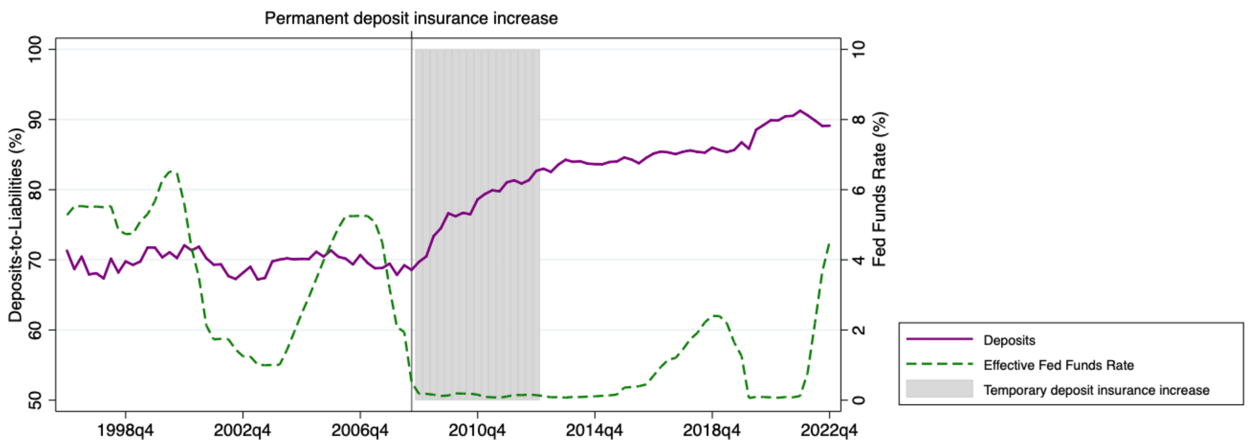

Figure 1: Deposits to liabilities for the 30 largest banks. Source: FDIC.

One possible explanation for this increase are changes in the level of deposit insurance which reduce the relative cost of deposit financing.[2] Figure 2 shows that the increase in deposits is driven by domestic *insured and uninsured* deposits which have both increased substantially since the pre-crisis period. Temporary changes to the level of deposit insurance can only explain the relationship

[1]We identify the 30 largest banks by asset size in Q4 2022. We choose Q4 2022 as the cut-off point for our sample given runs on the failure of Silicon Valley Bank in March 2023. See Appendix A for a list of banks in our sample.

[2]A temporary increase in the deposit insurance limit from USD 100k to USD 250k passed in October 2008 was made permanent in 2010. Furthermore, the Federal Deposit Insurance Corporation (FDIC) provided unlimited deposit insurance coverage to certain transaction accounts for institutions that chose to participate from October 2008 to December 2010. Under the Dodd-Frank Act, this program was expanded to all institutions for 2011 and 2012. For a more detailed description, see the FDIC report *Options for deposit insurance reform* (May 1, 2023).

between insured and uninsured deposits from 2008 to 2012. Permanent changes can perhaps explain the change in insured deposits but fail to explain the increase in uninsured deposits. Furthermore, one may be tempted to explain the increase in deposits by the low interest rates of the post-GFC period. However, the interest rate in the economy, here captured by the fed funds rate, seems to have little explanatory power for the share of uninsured deposits in bank liabilities.

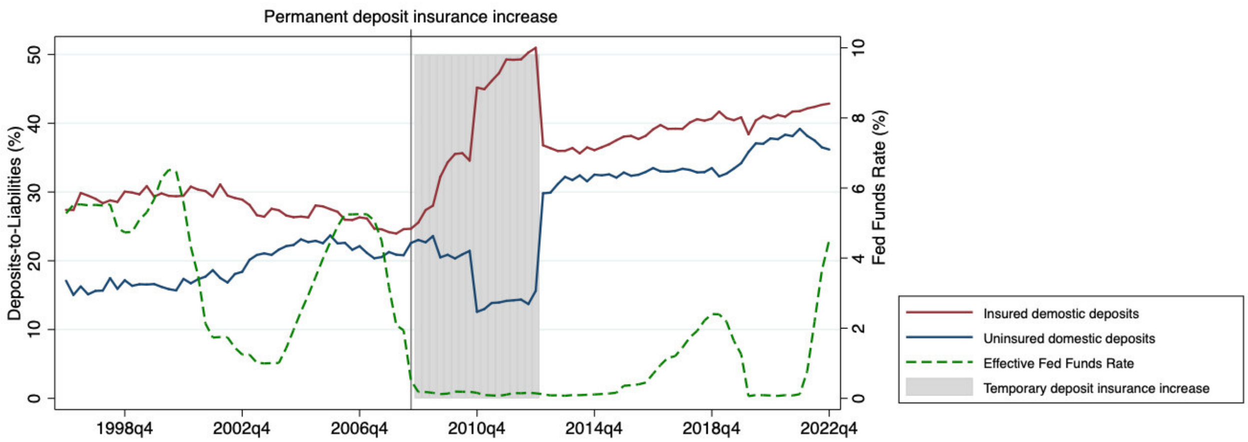

Figure 2: Domestic uninsured and insured deposits to liabilities for the 30 largest US banks. Source: FDIC.

So why do banks issue more deposits? And perhaps more importantly, why do banks issue *more uninsured deposits*? In this paper we offer an explanation, based on the shift in the policy environment in the aftermath of the GFC. Before 2008, governments could only rescue failing banks considered 'too big to fail' by injecting costly public funds, i.e. using bailouts. Nowadays, in order to avoid the fiscal and incentive costs of bailouts, governments can put failing banks into *resolution*. During a bank resolution, equity and debt write-downs as well as debt-to-equity conversions, often referred to as 'bail-ins', aim to recapitalize banks without the need for public funds. At the same time, governments continue to provide partial deposit insurance with the goal of curbing financial fragility.

We develop a model that highlights a delicate interaction between bank resolution and deposit insurance that operates through the banks' financing choices. If the government provides little deposit insurance, then banks finance using equity and long-term debt. Resolution effectively prevents bank failure in times of crisis without the need for bailouts, and the economy achieves efficiency. However, if the government provides a sufficiently large amount of deposit insurance, banks finance exclusively using insured and uninsured deposits. Then, upon the arrival of unfavourable news about the value of bank assets, uninsured depositors withdraw in anticipation of intervention. This imposes losses on insured depositors. The fragility resulting from the anticipation of bail-ins thus stands in the way of resolution, and the government is forced to conduct bailouts after all.

The aforementioned mechanism provides a rationale for why banks issue increasing amounts of uninsured deposits: front running a government with resolution powers. Resolution thus cannot replace bailouts but instead contributes to financial fragility. If so, then neither policy achieves its desired goal: resolution does not successfully recapitalize banks, and deposit insurance spurs rather than curbs fragility.

We obtain these insights in a model of a bank which seeks financing for a risky asset. The government allows the bank to partially finance using insured deposits. This deposit insurance is free.[3] The remaining financing need is covered by a combination of equity, long-term debt, and uninsured deposits, all issued to a market of fragmented investors. The key feature of uninsured deposits is that investors can demand to be repaid in full at any instant, forcing (partial) liquidation of the asset. The return of the asset is determined partly by an unknown state, which captures anything that exogenously affects the value of a bank's asset, i.e. both idiosyncratic or economy-wide shocks. Additionally, the asset's per-unit returns can be enhanced by costly and non-contractible investment. This investment occurs after the quality of the asset is revealed. It captures productive activities of banks such as monitoring borrowers and conducting due diligence, which affect returns positively, as well as the prevention of harmful activities such as risk shifting and diversion of funds.

The key friction in our model is the lack of commitment by all parties involved. First, the bank cannot commit to its ex-post investment strategy which gives rise to moral hazard. In particular, investment is inefficient if there is a debt overhang, generating a scope for government intervention. Second, the government is unable to commit to its crisis policy and seeks to recapitalize banks at minimal costs to the taxpayer, given the tools at hand. Thus, a government with resolution capabilities cannot commit not to impose losses on bank creditors once a bank is taken into resolution. Similarly, a government without resolution capabilities cannot commit not to bail out bank creditors. Essentially, we model banks as too big to fail, motivated by the many historical examples of government interventions, e.g. during the GFC or the 2023 banking crises of Silicon Valley Bank in the US and Credit Suisse in Switzerland.

[3]This assumption simplifies the exposition but is not necessary for our results. We require that the insurance premium is actuarially unfair, i.e. too cheap. See Section 5 for a discussion.

We derive our main result for a government with resolution capabilities. We find that banks prefer a *fragile* financing structure if the level of deposit insurance is sufficiently *high*. That is, banks issue as many insured deposits as allowed by the regulator and issue uninsured deposits otherwise. The resulting fragility channels resources towards investors and inflicts losses on insured depositors in times of crisis, forcing the government to conduct bailouts. Banks therefore trade off the liquidation loss, i.e. inefficient investment, against the bailout transfer that can be generated from the government. Whenever the transfer is larger than the liquidation loss, banks optimally choose a fragile financing structure which is illustrated in Panel A of Figure 3.

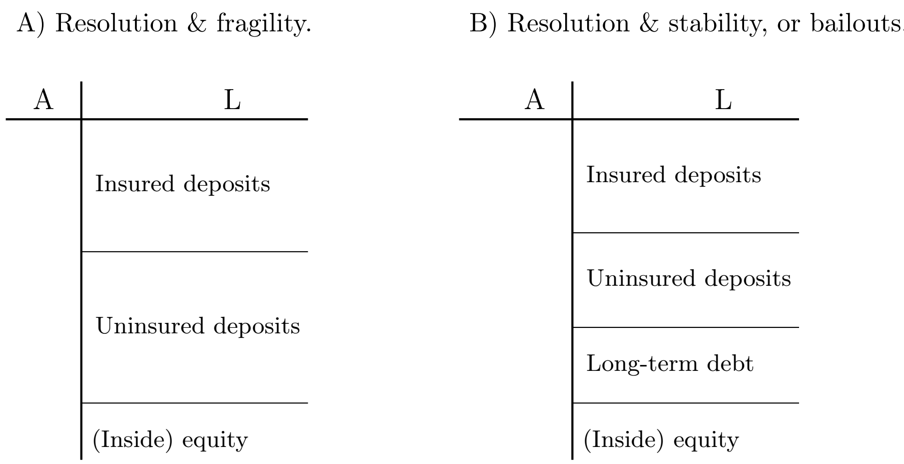

Figure 3: Balance sheets for the different types of intervention.

Whenever the level of deposit insurance and thus the corresponding transfer is small, banks choose a stable financing structure, issuing long-term debt rather than uninsured deposits. Issuing many uninsured deposits opens up the door for coordination among investors to run on the bank which the bank cannot always prevent. In particular, if all investors demand to be repaid immediately upon the arrival of unfavourable news, the bank has insufficient resources to do so. This justifies a run in the first place. With few uninsured deposits, banks prevent liquidation and resolution ensures efficient investment. Welfare is maximized. Panel B of Figure 3 illustrates the bank's balance sheets for a stable financing structure.

We contrast our main results to a scenario in which the government both lacks resolution capabilities and commitment not to to bailout banks at times of crises. In this scenario, banks again limit their exposure to uninsured deposits and fully lever up. Panel B of Figure 3 again illustrates the corresponding bank balance sheet. Issuing few uninsured deposits is privately optimal for the bank since it prevents asset liquidations and thus increases the scope for government bailouts. Issuing long-term debt rather than equity is optimal because it maximizes government transfers and thus minimizes the cost of financing. The market outcome therefore features stability but inefficiently high levels of debt.

Our model highlights a non-monotone welfare effect of deposit insurance in the presence of resolution, illustrated in Figure 4. Welfare is maximized if the level of deposit insurance is low. If the level of deposit insurance increases above a threshold, then the bank chooses a fragile financing structure. This fragility is privately optimal but socially costly for two reasons: liquidations are inefficient and transfers are socially costly. For high levels of deposit insurance, welfare is increasing in the level of deposit insurance. Replacing an additional unit of uninsured deposits with insured deposits comes at no further fiscal cost: one unit of the asset which otherwise would have been liquidated can now be used to fully cover the additional unit of insured deposits. With lower asset liquidations, deposit insurance then facilitates higher investment, leading to an increase in welfare.

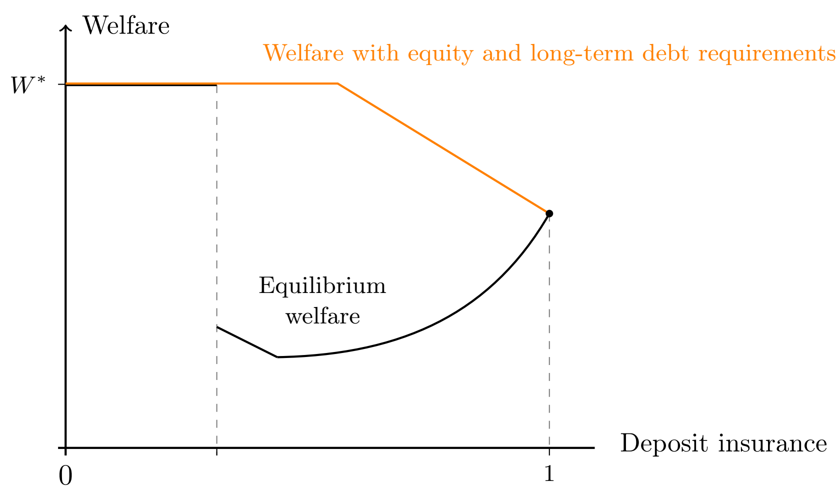

Figure 4: Welfare.

Our model has important policy implications. The model's results rely on cheap deposit insurance and therefore highlight the importance of computing fair deposit insurance premia correctly. Moreover, we stress that fair pricing requires taking the whole liability structure into consideration:

even though the asset is sufficient to fully cover all insured deposits for a stable financing structure, uninsured deposits force liquidation, inflicting losses on the deposit insurance facility for a fragile financing structure. In this context, complementing deposit insurance with long-term debt and equity requirements prevents fragility directly even if the insurance premium is unfair. Figure 4 illustrates that such a policy strictly increases welfare for a given level of deposit insurance whenever the equilibrium outcome features fragility. The model thus justifies the minimum maturity of bail-in debt of existing bank capital regulation.[4]

Literature Review.—Our paper is closely related to Brunnermeier and Oehmke (2013), in which financial institutions cannot commit their aggregate debt maturity structure and issue short maturity debt to dilute existing creditors. In our model, banks issue short-term debt—even if they can commit their debt maturity structure—in order to dilute the claims of insured depositors, and thus of the government. We describe newly introduced resolution capabilities as a rationale for the increase in uninsured deposit financing by banks.[5]

Highlighting this increase as an unintended consequence of resolution is also our contribution to the literature on bail-ins and fragility. Walther and White (2020) find that the regulator may conduct too small bail-ins when they are read as bad signals over bank fundamentals, triggering runs *after* intervention. We allow investors to act before the regulator. We find fragility *in anticipation* of intervention. Schilling (2023) features a non-strategic bank with a fixed balance sheet and a regulator that can commit its ex-post policy. The paper discusses the optimal timing of intervention if the resolution authority learns from the number of withdrawals; with earlier intervention, depositors withdraw in more states of the world. In Keister and Mitkov (2023), banks do not impose bail-ins if they anticipate bailouts. Informed investors may run on the bank if they expect some bail-ins to take place, forcing the government to conduct even larger bailouts. Furthermore, the fragility that we point out may impair the mechanism described by Philippon and Wang (2023). They show that a regulator can use the distribution of bailout funds to implement the ex-post efficient

[4]See, for example, the one year minimum maturity requirement for unsecured debt to contribute to "minimum requirement for own funds and eligible liabilities" (MREL) under the Bank Recovery and Resolution Directive (BRRD) of the European Union, or to the "total loss absorbing capacity" (TLAC) as defined by the Financial Stability Board (FSB).

[5]See also Flannery (1986), Diamond (1991) and Stein (2005) who find that debt maturity may be excessively short due to asymmetric information on borrower quality. Similarly, Segura and Suarez (2017) find that banks' maturity transformation may be excessive if they fail to internalize a pecuniary externality. See also He and Xiong (2012), Diamond and He (2014), and He and Milbradt (2016).

allocation—importantly, allowing the worst banks to fail in the process and thus correcting the ex-ante incentives usually associated with bailouts. However, this may not be feasible if the government supplies partial deposit insurance and banks issue ample amounts of uninsured deposits.[6]

Turning to the literature on deposit insurance and fragility, Diamond and Dybvig (1983) show that banks provide liquidity insurance by issuing demand deposits and investing into illiquid assets. This liquidity mismatch creates fragility which can be overcome using deposit insurance.[7] Demirgüç-Kunt and Detragiache (2002) provide empirical evidence that deposit insurance has adverse effects on bank system stability, and these effects are larger when the coverage is extensive and supplied by the government. Other studies have shown that deposit insurance may lead to excessive risk taking (Pennacchi, 1987a; Keeley, 1990; Matutes and Vives, 2000; Gropp and Vesala, 2004; Ioannidou and Penas, 2010). Recent work by Dávila and Goldstein (2022) characterizes optimal deposit insurance in a structural, heterogeneous depositor version of Diamond and Dybvig (1983), taking the deposit contract as given. The authors find that fairly-priced deposit insurance is neither necessary nor sufficient for optimal regulation since banks can adjust their asset allocations.[8] Our findings highlight adjustments on the liability side which lead to fragility if a government with resolution capabilities provides partial deposit insurance.

These findings resonate with Ahnert et al. (2019), where banks issue unsecured, demandable debt and encumber assets for secured long-term borrowing. If some of the demandable debt is insured by the government, banks optimally encumber more assets to issue long-term debt, and the uninsured holders of demandable debt run in more states of the world. The authors take

[6]Martynova et al. (2022) analyse the strategic interaction between banks and the resolution authority without commitment to resolve undercapitalized banks. Banks take inefficiently low private initiative to recapitalize, and more banks require resolution as a result. Clayton and Schaab (2020) discuss bail-ins and bailouts in the presence of a bank monitoring problem. Banks issue too little (long-term) bail-in debt because they do not internalize a pecuniary externality. In Shapiro and Skeie (2015), the regulator trades off injecting capital, revealing its cost of recapitalisation and potentially induces higher risk-taking in the process, against letting a bank fail, leading to the possibility of bank runs. In Pandolfi (2021), bail-ins are ex-post efficient but lead to a breakdown of financing markets ex-ante. A combination of bail-ins and bailouts are the optimal policy. See Bolton and Oehmke (2019) and Clayton and Schaab (2022) for bail-ins in multinational banks. Farhi and Tirole (2020) discuss bail-ins of shadow bank debt holders. See Bernard et al. (2022) for an analysis of bail-ins in a model of financial contagion. Mendicino et al. (2017) and Tanaka and Vourdas (2018) are concerned with optimal capital regulation and conduct numerical exercises for bail-ins and equity requirements. Dewatripont and Tirole (2018) discuss liquidity support in a bail-in environment.

[7]See further canonical papers by Gorton and Pennacchi (1990) and Goldstein and Pauzner (2005).

[8]See also Kareken and Wallace (1978), Acharya and Dreyfus (1989), Dreyfus et al. (1994), Matutes and Vives (1996), Hazlett (1997), Freixas and Gabillon (1999), Cooper and Ross (2002), Duffie et al. (2003), Pennacchi (2006), Acharya et al. (2009). See Iyer and Puri (2012) for a study of a bank run under deposit insurance using minute-by-minute deposit withdrawal data. See De Roux and Limodio (2021) for a recent study on the effect of deposit insurance increases on the size of insured bank deposits.

demandable debt as given; we highlight resolution as a rationale for this type of debt and offer an explanation for the observed increase in uninsured deposits. Finally, our results stand in contrast with the results of Allen et al. (2018) who augment the Goldstein and Pauzner (2005) model with both fundamental and panic-driven runs to include different forms of government guarantees. They find that a deposit insurance payment up to a threshold—as in our model and observed in reality—prevents fundamental runs and reduces the likelihood of panic runs.

Outline of paper. We introduce the model in Section 2. Section 3 presents our main results. Section 4 highlights welfare implications and policy options. Section 5 discusses the main results. Section 6 presents a benchmark for a government without commitment not to conduct bailouts.

2 Environment

Our model has three kinds of agents: a Banker, a Regulator, and a competitive market of Investors. The Banker has access to an asset of uncertain quality but is in need of initial financing. Investors have cash and compete to purchase securities from the Banker. The model evolves over three time periods. The Banker seeks financing in period 1. At the beginning of period 2 the quality of their asset is publicly revealed. In light of this information, and in anticipation of a return-enhancing investment opportunity in period 3, the Banker attempts to renegotiate with Investors. The game ends in period 3 after the investment takes places, all uncertainty is resolved, and funds are distributed among Banker and Investors. To this game, we add a Regulator who cannot commit not to intervene at the end of period 2, after observing the renegotiation outcome. Figure 5 presents a timeline of events.

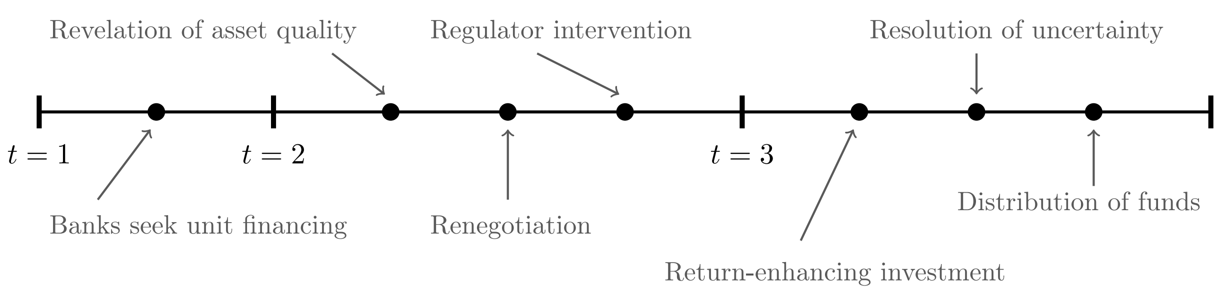

Figure 5: Timeline of events.

2.1 Banker, Asset & Investment

In period 1, the Banker has no funds but can access a unit of a fully divisible asset which generates returns in period 3. To initiate the project a unit initial investment is required. Each unit of the asset generates *base* returns $X^\theta \in \mathbb{R}^+$, where $\theta \in \{L, H\}$ is an ex-ante unknown state of the world, with $\mathbb{P}(\theta = H) = p$. The state θ is publicly revealed in the beginning of period 2, which triggers potential renegotiation (see Section 2.3). At that point in the game, any amount of the asset can be liquidated prematurely at a liquidation price X^θ.

In the spirit of Walther and White (2020), per-unit returns can be enhanced via *non-contractible investment* in period 3. Importantly, such an opportunity implies that the balance-sheet with which the bank enters period 3 matters, as it affects investment incentives and ultimately gives scope for a Regulator to intervene. More specifically, in the beginning of period 3 the bank has access to a set of state-independent[9] investment opportunities $\big\{\big(G(\cdot|\ e), h(e)\big) \in \Delta(\mathbb{R}^+) \times \mathbb{R}^+ : e \in [0, \bar{e}]\big\}$, where $G(\cdot|\ e)$ are full-support CDFs with $G(q|\ e) > G(q|\ \hat{e})$ for all realizations $q \in \mathbb{R}^+$, whenever $\hat{e} > e$.[10] Without loss of generality we set $G(0|\ 0) = 1$.

Investment enhances per unit base returns additively. That is, if the bank has $\chi \in [0, 1]$ units of the asset and the realization of the technology is $q \in \mathbb{R}^+$, the final returns are given by $\chi \cdot (X^\theta + q)$. The function $h(e)$ is the associated cost for $G(\cdot|\ e)$, which we assume to be convex in $e \in [0, \bar{e}]$. Importantly, this cost is paid by the *residual claimants* (inside and outside equity). That is, $h(e)$ is shared among holders of bank equity according to their respective shares.[11,12] Moreover, for a choice $e \in [0, \bar{e}]$, and for $\chi \in [0, 1]$ units of the asset, we denote the expected continuation return net of effort costs by $y(e, \chi)$, given by

$$y(e, \chi) \;=\; \chi \int_0^\infty q \, dG(q|e) - h(e) \tag{1}$$

For technical convenience, we assume that $y(e, 1)$ is twice continuously differentiable, and strictly

[9]This assumption is immaterial, as only investment opportunities in the low state matter for the results.

[10]That is, the CDFs are ranked according to first-order stochastic dominance on the 'effort' variable $e \in [0, \bar{e}]$.

[11]The non-contractable investment and the corresponding cost—borne by inside and outside equity—capture the agency problem between bank owners and bank managers in a reduced form. One could consider a model in which bank owners pay out working capital as bonuses to bank managers or as dividends among themselves. Bonuses induce managers to monitor borrowers, increasing the return on assets.

[12]This is a critical assumption differentiating our framework from that in Diamond and Rajan(2000, 2001, 2005, and 2012), where 'investment costs' are not transferable between the Banker and other contracting parties.

concave in e, which implies the same properties for $y(e,\chi)$ for all $\chi \in [0,1]$. We denote $y^* = \max_{e\in[0,\bar{e}]} y(e,1)$ which is the efficient continuation return, and by e^* the unique efficient investment.

Since investment is non-contractible, choices must be privately optimal given the liability structure. Equity is protected by limited liability, so investment is characterized by the solution to

$$\mathcal{E}(\chi, B|\,\theta) = \max_{e} \int_0^{+\infty} \max\Big\{\chi\cdot(X^\theta + q) - B,\ 0\Big\} dG(q|\,e) - h(e) \tag{E}$$

where B is the level of debt outstanding. We denote the resulting effort by $e(\chi, B|\,\theta)$.

Whenever there is strictly positive probability that $\chi\cdot(X^\theta+q) - B \le 0$, some of the returns to investment accrue to outside creditors and not the residual claimants who incur the investment costs. In these cases investment is distorted downwards. This is the classic debt-overhang problem. From the full-support assumption we can see that investment is inefficiently low whenever $X^\theta < B$; that is, when base returns cannot fully cover the outstanding debt.[13]

Finally, we make the following assumption regarding the asset returns.

Assumption 1. $\mathbb{E}[X] > 1 > X^L + y^*$ *and* $X^L \geq \int_0^\infty q\, dG(q|\bar{e})$

The first inequality implies that base returns are sufficient to make the asset ex-ante profitable. The second inequality implies that low quality assets are not profitable even with efficient investment. The third inequality says that the asset base returns exceed the asset enhancing investment returns even at the highest level of effort. Jointly these assumptions capture the idea that the high state corresponds to 'normal times' while the low state is a sufficiently bad crisis. Moreover, that investment is small relative to the outstanding asset. One interpretation is that X^θ is the accumulation of past investments. Note that Assumption 1 implies that $X^L > y^*$.

2.2 Investors & Contracts

In period 1, there is a market with an infinite measure of infinitesimally small potential Investors who are risk-neutral, are endowed with a unit of money, and do not discount the future. The Banker needs to cover the unit initial investment by borrowing from a total measure 1 of Investors. This assumption is warranted by the observed dispersion in banks' depositor bases. As an example, the

[13] Indeed, we have $0 < B - X^\theta \le \frac{B}{\chi} - X^\theta$, so there is an interval of positive values where $\chi\cdot(X^\theta+q) - B < 0$. Since all distributions have full support this interval has strictly positive probability.

two largest external depositors of Silicon Valley Bank were Circle Internet Financial and Sequioa Capital with deposits as a share of total assets of 1.53% and 0.46%, respectively.[14] Note that our model shares the assumption of fragmented investors, giving rise to coordination and commitment problems, with Brunnermeier and Oehmke (2013) and Admati et al. (2018).

The Banker chooses a portfolio of securities $c \in \mathcal{C}$, which is then priced competitively by the market. We call such $c \in \mathcal{C}$ a 'contract.' The contract space $\mathcal{C}$ comprises of mixtures of four kinds of securities: (i) equity ($\boldsymbol{E}$), (ii) long-term debt ($\boldsymbol{D}$), (iii) uninsured deposits ($\boldsymbol{d}$), and (iv) insured deposits ($\boldsymbol{\delta}$).[15] A contract $c \in \mathcal{C}$ can be identified with the shares of the unit initial investment raised by each of these securities:

$$\mathcal{C} = \left\{ (\xi_{\boldsymbol{E}}, \xi_{\boldsymbol{D}}, \xi_{\boldsymbol{d}}, \xi_{\boldsymbol{\delta}}) \in [0,1]^4 : \xi_{\boldsymbol{E}} + \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{\delta}} = 1 \right\}$$

For instance, a contract $c = (0.5, 0, 0, 0.5)$ raises half of the initial investment using equity and half using insured deposits. We now describe the properties of these component securities in more detail.

Equity.— An equity contract is parameterized by a share $\gamma \in [0,1]$ of the final equity value in Equation (E) that each equity Investor receives. Importantly, this implies that 'outside' equity participates in investment costs, as was explained in the previous section.

Long-term Debt.— A long-term debt contract is parameterized by a promised repayment $D \in \mathbb{R}^+$. It matures in period 3 after all uncertainty is resolved and has the right to seize *final* output after all more senior claims have been settled.

Uninsured deposits.— An uninsured deposit contract is parameterized by a promised repayment $d \in \mathbb{R}^+$. It matures in period 2. Uninsured deposits are a 'hard claim' on the asset in that holders can force *early* liquidation to obtain the promised repayment. Without loss of generality, we model uninsured deposits as senior to long-term debt.[16]

Insured Deposits.— We assume that the Regulator commits to *fully insure* some depositors *without cost* to the Banker. Insured deposits enjoy seniority over all other claims. The existence of such insurance is the identifying feature that makes our financing environment an abstraction of bank financing. Given this insurance, insured depositors demand a gross interest rate of one. We

[14] Source: Bloomberg News, June 23, 2023. The FDIC mistakenly released an unredacted list of Silicon Valley Bank deposits in response to a News Freedom of Information Act request.

[15] See Section 5 for a discussion of state-contingent contracts.

[16] This assumption is without loss given the threat of liquidation at the interim which renders uninsured deposits senior to long-term debt even if it is contractually junior.

assume that the Regulator limits the amount of insured deposit financing: $\xi_{\boldsymbol{\delta}} \leq \hat{\xi}_{\boldsymbol{\delta}}$. Consequently, the Banker needs to raise at least $1 - \hat{\xi}_{\boldsymbol{\delta}}$ from other types of Investors.

The assumption on free deposit insurance warrants discussion. Previous studies have shown that deposit insurance is not fairly priced (Merton, 1977; Pennacchi, 1987b; Allen and Saunders, 1993). Other studies have shown that fair pricing may not be feasible (Chan et al., 1992), not desirable (Freixas and Rochet, 1998), or not necessary and sufficient for efficiency (Dávila and Goldstein, 2022). Furthermore, as will become clearer in the analysis below, a zero deposit insurance premium is the fair premium for the *efficient* contract. One part of our contribution is to highlight negative consequences from deposit insurance that is 'too cheap' for the *equilibrium* contract.

We denote the set of Investors by $\mathbb{I}$, and the subset of Investors holding debt claims by $\mathbb{D}$. For each $i \in \mathbb{I}$, we write c_i for the security Investor i holds initially. For example, $c_i = \boldsymbol{D}$ means that Investor i is a long-term debt Investor. The securities different Investors hold determine a *seniority structure*, which we model as a total order, $\preceq$, on the set of Investors $\mathbb{I}$. We say that i is (weakly) more senior to j, whenever $i \preceq j$. For instance, if $c_i = \boldsymbol{\delta}$, $c_j = \boldsymbol{d}$ and $c_k = \boldsymbol{D}$, then $i \prec j \prec k$. Initially, Investors who hold the same security rank pari passu, i.e. $c_i = c_j$ implies $i \sim j$.

2.3 Renegotiation

A key friction in our model is the inability of parties to commit not to renegotiate the existing contracts. In particular, after the state $\theta \in \{L, H\}$ is revealed in period 2, the Banker renegotiates (on behalf of all equity) with debt Investors by making *simultaneous* take-it-or-leave-it offers of new securities to each type of Investor.[17] If an individual Investor rejects the new offer, they keep their existing security. If they accept, they forfeit the old security, and hold the new one.

In particular, the Banker offers each Investor a new security by choosing a profile from

$$\mathcal{S} = \left\{ \left(\alpha_i, B_i\right)_{i \in \mathbb{D}} : \ \alpha_i \geq 0, \int \alpha_i \mathrm{d}i \leq 1, \ \ B_i \geq 0 \right\}$$

That is, the Banker offers each Investor a share of equity, α_i, and a promised debt payment, B_i. The seniority structure, $\preceq$, is inherited from the initial contract and is not subject to renegotiation. As a consequence, if $i \prec j$, then Investor i can become (weakly) junior to Investor j only by accepting

[17] Simultaneity is essentially expressing the idea that Investors do not observe the Banker's offers to other Investors; nor do individual Investors communicate with each other.

an equity contract during renegotiation.

Each choice of security $s \in \mathcal{S}$ in state $\theta \in \{L, H\}$ by the Banker, induces an extensive-form game $\mathcal{G}_s^\theta(c)$ among Investors, the Regulator and the Banker. In $\mathcal{G}_s^\theta(c)$ Investors simultaneously accept or reject the Banker's offer $s \in \mathcal{S}$. Then, the Regulator observes Investors' choices and chooses a policy. Finally, the Banker chooses the continuation investment $e \in [0, \bar{e}]$. The precise definition of the game $\mathcal{G}_s^\theta(c)$ is notation-heavy and relegated to Appendix B. In what follows, we describe the Regulator's policy choices.

2.4 Regulator & Policies

As mentioned above, we are interested in studying the consequences of a Regulator who cannot commit their ex-post policy. The Regulator observes the outcome of the renegotiation game and chooses a policy action to maximize ex-post welfare.[18]

The Regulator has the following tools at hand. First, the Regulator can inject public funds which are used to reduce the outstanding debt. We call such injections *bailouts*. The use of public funds incurs a cost $\kappa > 1$ per unit. However, when conducting bailouts, we assume that the Regulator acts as if the use of public funds is not costly. This assumption has two consequences. First, it makes the lack of commitment problem more pronounced. Second, conditional on bailing out, the Regulator fully removes any debt overhang, which simplifies the exposition. This modelling assumption captures the idea that the Regulator does not fully internalize the full cost of bailouts at the time of intervention. Furthermore, bailouts frequently occur in reality, often with delayed transfers via loss guarantees.[19]

Second, we consider the case where the Regulator has coercive powers to alter existing debt and equity arrangements without having to respect voluntary participation constraints. In particular, the Regulator can mandate write downs of either debt or equity, or conduct debt-to-equity swaps. We interpret such intervention as *bail-ins*.

A policy for the Regulator is a choice of security $\hat{s} = (\hat{\alpha}_i, \hat{B}_i)_{i \in \mathbb{D}} \in \mathcal{S}$, together with a bailout transfer, $b \in \mathbb{R}^+$, which is incorporated in the total asset value. Such a choice alters the liabilities by changing promises to $(\hat{B}_i)_{i \in \mathbb{D}}$, and redistributes equity across (inside and outside) equity holders

[18] This is in contrast to Walther and White (2020), where regulators move *before* the rest of the market.

[19] E.g. the Swiss government provided loss guarantees of USD 9bn to UBS during the takeover of failing Credit Suisse in June 2023. See Acharya et al. (2021) for empirical evidence.

and debt Investors through the new equity shares $(\hat{\alpha}_i)_{i\in\mathbb{D}}$. While allowing for coercive powers, we assume that the Regulator does not treat some asset class preferentially:

Assumption 2. *Given a security profile s, security $\hat{s}$ satisfies the following restrictions:*

1. *Adherence to the face value of debt securities:*

$$\hat{B}_i + \hat{\alpha}_i \cdot \mathcal{E}(\chi, \hat{B} - b|\, \theta) \leq B_i \quad \textit{for all} \quad i \in \mathbb{D}, \quad \textit{where} \;\; \hat{B} = \int_{\mathbb{D}} \hat{B}_i di$$

2. *Adherence to the seniority structure:*

$$\hat{B}_j = \hat{\alpha}_j = 0 \quad \textit{if} \quad \hat{B}_i + \hat{\alpha}_i \cdot \mathcal{E}(\chi, \hat{B} - b|\, \theta) < B_i \quad \textit{for all} \quad i, j \in \mathbb{D} \quad \textit{s.t.} \quad i \prec j$$

To give an example, Assumption 2 implies that the Regulator cannot fully write down uninsured deposits while converting more junior long-term debt claims into equity. Similarly, the Regulator cannot fully write down all equity and long-term debt claims and convert all uninsured deposits into equity, if the post-resolution equity value exceeds the face value of uninsured deposits. Assumption 2 captures the idea that a government that does not satisfy these restrictions is subject to be challenged in court.

We now explain how different policy regimes can be thought of as constraints in the policy choices of the Regulator. We first consider the least constrained regime in which the Regulator can conduct *both* bailouts and bail-ins, which we consider in our main analysis (Section 3). Formally, this environment is characterized by the fact that the policy choice $(\hat{s}, b) \in \mathcal{S} \times \mathbb{R}^+$ by the Regulator does not have to respect voluntary participation constraints of Investors and the Banker. Effectively, the Regulator can *coerce* parties into 'accepting' $\hat{s}$.

Second, we consider a regime where only bailouts are available (Section 6). Formally, this environment is characterized by the constraint that the Regulator must respect voluntary participation. It is without loss of generality to assume that in this case $\hat{s} = s$. Indeed, if the Banker anticipates $\hat{s} \neq s$, which satisfies voluntary participation, then he can just offer s, which induces the same strategy profile for Investors.[20]

[20]If the Regulator could commit not to intervene, this can be viewed as a constraint that $\hat{s} = s$ and $b = 0$.

2.5 Ex-Ante Solution Concept

When the Banker posts an initial contract in period 1, the market 'prices' this contract by determining the parameters (γ, D, d, δ) of each component security defined in Section 2.2. Due to the externalities among Investors, the market must take into account the continuation strategy profile.

Whenever a contract $c \in \mathcal{C}$ achieves financing, it induces a symmetric information extensive-form game between the Banker, the Investors and the Regulator. We denote by $\Sigma(c)$ the set of subgame-perfect equilibria of this game. Formally, we derive a system of *pricing equations* for each $c \in \mathcal{C}$ and $\sigma \in \Sigma(c)$, whose solution—when it exists—determines (γ, D, d, δ). Whenever a solution does not exist, we say that $c \in \mathcal{C}$ fails to achieve financing under $\sigma \in \Sigma(c)$.

Because of the potential multiplicity of continuation equilibria following a contract posted by the Banker, we introduce the following notion of robustness. We denote by $V(c, \sigma)$ the Banker's ex-ante payoff from a profile of strategies σ, after posting contract $c \in \mathcal{C}$.

Definition 1. *A contract $c^* \in \mathcal{C}$ is* robustly *optimal if*

$$\inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) \geq \sup_{\sigma \in \Sigma(c)} V(c, \sigma) \quad \textit{for all } c \in \mathcal{C}$$

Essentially, a robustly optimal contract is optimal in a strong sense: even in the worst-case scenario for the continuation equilibrium, it delivers at least as high a payoff as the best-case scenario for any other contract. In what follows, we will assume that the Banker chooses robustly optimal contracts ex-ante.

2.6 Welfare

Welfare is given by the sum of Banker and Investor payoffs, minus the cost of bailouts:

$$W \;=\; \mathbb{E}\big[X + Y(c, \sigma|\theta) - 1 - (\kappa - 1) \cdot b(c, \sigma|\theta)\big]$$

where $b(c, \sigma|\theta)$ are government bailouts and $Y(c, \sigma|\theta)$ are the investment returns which depend on the chosen contract c, the equilibrium $\sigma \in \Sigma(c)$, and the state $\theta \in \{L, H\}$.

Welfare is maximized for a contract $c \in \mathcal{C}$ if investment is efficient without the use of government funds for all equilibria $\sigma \in \Sigma(c)$ and in all states $\theta \in \{L, H\}$:

$$W^* = \mathbb{E}[X] + y^* - 1$$

It is straightforward to show that such a contract exists. Consider a pure equity contract with $\xi_{\boldsymbol{E}} = 1$. Without any debt outstanding, assets are never liquidated, no renegotiations with any debt investors take place, and there is no debt overhang and hence no bailouts. Investment is efficient in all equilibria. An equity contract also achieves financing since $\mathbb{E}[X] + y^* > 1$.

In a benchmark model without a Regulator—or equivalently, in a model with a Regulator that can commit not to bailout and that does not supply deposit insurance—the Banker would issue a contract that ensures efficient investment in all states and in all equilibria, given its market power vis-à-vis Investors. This highlights that there is no efficiency role for debt, neither insured nor uninsured, neither demandable nor non-demandable. While this makes our model a stark abstraction of reality, it does not conflate arguments and allows us to focus on a particular channel: the effect of government intervention policies on the liability structure of banks, particularly on the level of uninsured deposits issued by banks.

We are now ready to commence the main analysis of our paper.

3 Equilibrium with resolution

Suppose the Regulator has coercive powers to alter contracts as outlined in Section 2.4.

During the financing stage, the Banker posts a contract $c = (\xi_{\boldsymbol{\delta}}, \xi_{\boldsymbol{d}}, \xi_{\boldsymbol{D}}, \xi_{\boldsymbol{E}}) \in \mathcal{C}$ to the market of competitive Investors. The prices of debt and equity, (d, D, γ), are not only a direct function of c, but also of the strategies of debt holders during renegotiation, of the Regulator during intervention, and of the ultimate shareholders of the bank during investment, all induced by c. In other words, there are many moving parts that affect the Banker's equilibrium payoff of each contract.

However, both Banker and Investors perfectly anticipate that the government faces a commitment problem at time 2. In particular, once a bank is taken into resolution, the Regulator will fully recapitalize the banks at minimal cost. First, she uses coercive powers, converting all uninsured deposits and long-term debt into equity. Importantly, she cannot commit not to bail in uninsured depositors. If bail-ins are insufficient to fully remove the debt overhang, the Regulator uses bailouts.

The Regulator's commitment problem allows us to simplify the expression of the Banker's payoff for each contract $c \in \mathcal{C}$ significantly (Lemma 1 below). To state the lemma, we introduce some more notation. For each profile σ, we denote by $\lambda_\sigma \in [0,1]$ the fraction of uninsured depositors rejecting the offer and thus withdrawing rather than rolling over. $\chi_\sigma \in [0,1]$ denotes the *share* of the asset still remaining in the bank after potential liquidation for profile σ.[21]

Since the Regulator fully removes the debt overhang using bail-ins and bailouts, bailouts in state L—for a given contract c and in an equilibrium $\sigma \in \Sigma(c)$ with a share of assets χ_σ remaining in the bank—are therefore given by

$$b(c,\sigma|L) \;=\; \max\{\xi_{\boldsymbol{\delta}} - \chi_\sigma \cdot X^L, 0\}$$

which is piece-wise linear and weakly decreasing in χ_σ. Once the debt overhang is alleviated, investment returns $Y(c,\sigma|L)$ in state L, in any equilibrium $\sigma \in \Sigma(c)$, are given by

$$Y(c,\sigma|L) \;=\; \max_e \;\chi_\sigma \cdot \int_0^\infty q\, dG(q|e) - h(e)$$

As a maximum of increasing linear functions in χ_σ, $Y(c,\sigma|L)$ is a strictly increasing, strictly convex function in χ_σ.

It turns out that these two items, bailouts and investment in the low state, are sufficient to describe the equilibrium payoff of any financing contract:

Lemma 1. *Consider contracts $c, c' \in \mathcal{C}$ and continuation equilibria $\sigma \in \Sigma(c), \sigma' \in \Sigma(c')$. Then $V(c,\sigma) > V(c',\sigma')$ if and only if $\psi(c,\sigma) > \psi(c',\sigma')$, where*

$$\psi(c,\sigma) \;=\; Y(c,\sigma|L) + b(c,\sigma|L)$$

Proof. See Appendix C.1.

Intuitively, the asset returns in the good state are sufficiently high such that there is never a debt overhang. In other words, the face value of debt outstanding for all conceivable contracts $c \in \mathcal{C}$ is always below X^H. Hence, the Banker optimally prevents liquidations, never receives any bailout funds, and achieves efficient investment. Given perfect competition in financing markets,

[21] For the formal definitions, see Appendix B.

differences in equilibrium payoffs across contracts are then determined by the differences in resources generated in the low state. That is, a contract that maximizes the Banker's payoff maximizes the sum of investment returns and the government transfer in the low state. Importantly, the function $\psi(c, \sigma)$ is strictly convex, as illustrated by Figure 6.

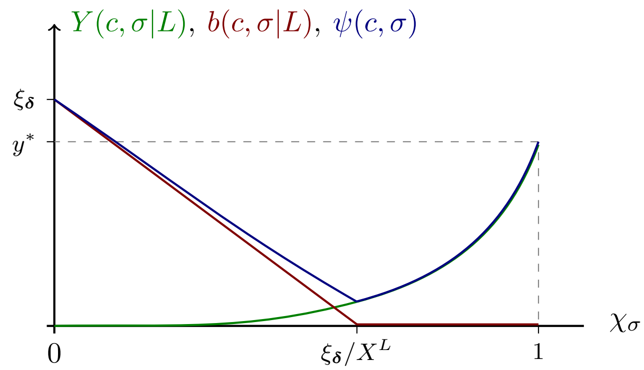

Figure 6: The strict convexity of $\psi(c, \sigma)$.

We are now ready to state the main result of our paper.

Proposition 1. *Let $\mathcal{C}^*$ be the set of (robustly) optimal contracts. Then, $\mathcal{C}^*$ is non-empty, and any $c = (\xi_{\boldsymbol{E}}, \xi_{\boldsymbol{D}}, \xi_{\boldsymbol{d}}, \xi_{\boldsymbol{\delta}}) \in \mathcal{C}^*$ satisfies the following:*

If $\hat{\xi}_{\boldsymbol{\delta}} > y^*$, then $c = \left(0, 0, 1 - \hat{\xi}_{\boldsymbol{\delta}}, \hat{\xi}_{\boldsymbol{\delta}}\right)$, and we have:

1. *Fragility: All uninsured depositors demand immediate repayment, $\lambda_\sigma = 1$ for all $\sigma \in \Sigma(c)$.*

2. *Maximal leverage: $\xi_{\boldsymbol{E}} = 0$.*

3. *Inefficiency in all $\sigma \in \Sigma(c)$.*

If $\hat{\xi}_{\boldsymbol{\delta}} \leq y^*$, there exists $\bar{B}_{\boldsymbol{d}}(c) \in \left[0, X^L + y^* - \xi_{\boldsymbol{\delta}}\right]$ such that $\xi_{\boldsymbol{d}} \leq \bar{B}_{\boldsymbol{d}}(c)$. Moreover, we have:

1. *Stability: No uninsured depositor demands immediate repayment, $\lambda_\sigma = 0$ for all $\sigma \in \Sigma(c)$.*

2. *Irrelevant leverage: For any $\xi \in [0, 1]$, there exists $c' = (\xi_{\boldsymbol{E}}, \xi_{\boldsymbol{D}}, \xi_{\boldsymbol{d}}, \xi_{\boldsymbol{\delta}}) \in \mathcal{C}^*$, with $\xi_{\boldsymbol{E}} = \xi$.*

3. *Efficiency in all $\sigma \in \Sigma(c)$.*

Proof. See Appendix C.2.

Proposition 1 states that—if the government supplies a sufficiently high level of deposit insurance—it is optimal to issue insured deposits up the regulatory limit and finance the remainder using uninsured deposits. Upon the arrival of bad news about asset returns, all uninsured depositors withdraw in anticipation of government bail-ins, forcing liquidation of the asset. This constitutes one source of inefficiency. Furthermore, in order to ensure full repayment of all insured deposits, the government is forced to transfer resources into the bank, which constitutes another source of inefficiency. Thus, the private financing arrangement is not socially optimal.

Whenever the government supplies little deposit insurance, it is optimal to finance using limited amounts of uninsured deposits and primarily issue insured deposits, long-term debt and equity. Since neither long-term debt nor equity Investors have the right to force liquidation upon the arrival of bad news, all of the asset is held within the bank until period 3. Bail-ins then ensure efficient investment without the use of public funds. The private financing arrangement is also socially optimal.

The provision of cheap deposit insurance presents a trade-off to the Banker: issuing many uninsured deposits leads to inefficient liquidation but forces the government to transfer resources into the bank, which reduces the cost of financing. Banks prefer to issue uninsured deposits if this transfer is sufficiently big.

We illustrate this result by considering a limited contract space in which the Banker can issue insured deposits up to the government-mandated limit but otherwise can only choose between long-term debt and uninsured deposits: $\xi_{\boldsymbol{\delta}} \in [0, \hat{\xi}_{\boldsymbol{\delta}}]$, $\xi_{\boldsymbol{d}} \in \{0, 1 - \xi_{\boldsymbol{\delta}}\}$, and $\xi_{\boldsymbol{D}} = 1 - \xi_{\boldsymbol{\delta}} - \xi_{\boldsymbol{d}}$.

Consider first a long-term debt contract c_D with $\xi_{\boldsymbol{d}} = 0$, inducing continuation equilibrium σ_D. Without any uninsured deposits outstanding, no assets are liquidated ($\chi_{\sigma_D} = 1$). During renegotiations, the Banker cannot do better than offering the same long-term debt contract again, given the Regulator's resolution strategy; Investors accept. The Regulator then bails in all long-term debt to remove the debt overhang, if feasible; any remaining debt overhang stemming from insured deposits requires bailouts: $b(c_D, \sigma_D|L) = \max\{\xi_{\boldsymbol{\delta}} - X^L, 0\}$. Investment is efficient. It follows that

$$\psi(c_D, \sigma_D) \;=\; y^* + \max\{\xi_{\boldsymbol{\delta}} - X^L, 0\} \tag{2}$$

The first panel of Figure 7 depicts Equation (2) as a function of $\xi_{\boldsymbol{\delta}}$. The Banker's payoff is a weakly increasing function of $\xi_{\boldsymbol{\delta}}$. It is strictly increasing whenever $\xi_{\boldsymbol{\delta}} > X^L$, and bailouts are required to remove the debt overhang; we call this region in $\xi_{\boldsymbol{\delta}}$ the *bail-in infeasibility* region.

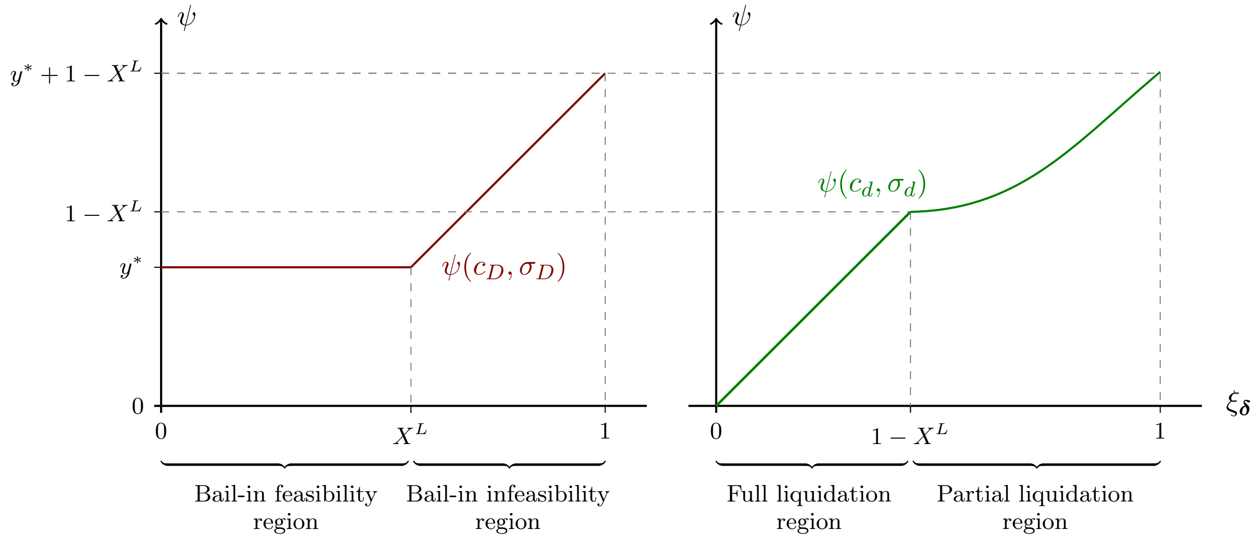

Figure 7: Resources with long-term debt ($\xi_{\boldsymbol{d}} = 0$, LHS) or uninsured deposits ($\xi_{\boldsymbol{d}} = 1 - \xi_{\boldsymbol{\delta}}$, RHS).

Consider next an uninsured deposit contract c_d with $\xi_{\boldsymbol{d}} = 1 - \xi_{\boldsymbol{\delta}}$, inducing continuation equilibrium σ_d. The liquidation value of the asset in the low state is given by X^L. Full liquidation occurs whenever $\xi_{\boldsymbol{d}} \geq X^L$ and $\lambda_{\sigma_d} = 1$, resulting in $\chi_{\sigma_d} = 0$. Intuitively, the Banker cannot repay all uninsured deposits even if all uninsured depositors roll over. All uninsured depositors withdraw. As a consequence, investment is zero, and the Regulator is forced to fully repay all insured deposits:

$$\psi(c_d, \sigma_d) \;=\; \xi_{\boldsymbol{\delta}} \tag{3}$$

Suppose $\xi_{\boldsymbol{d}} < X^L$ and thus $\chi_{\sigma_d} > 0$ for all equilibria. Since all uninsured deposits are safe even in the low state, their total face value is given by $\xi_{\boldsymbol{d}} = 1 - \xi_{\boldsymbol{\delta}}$. The bank generates the following resources as a function of χ_{σ_d} at the interim:

$$\psi(c_d, \sigma_d) \;=\; Y(c_d, \sigma_d | L) + \max\{\xi_{\boldsymbol{\delta}} - \chi_{\sigma_d} X^L, 0\}$$

Given the strict convexity of $\psi(c_d, \sigma_d)$, consider the two extremes where all uninsured depositors roll over, $\lambda_{\sigma_d} = 0$ and thus $\chi_{\sigma_d} = 1$, and where all uninsured depositors withdraw, $\lambda_{\sigma_d} = 1$ and thus $\chi_{\sigma_d} = 1 - \frac{1-\xi_{\boldsymbol{\delta}}}{X^L}$. If $\chi_{\sigma_d} = 1$, then the uninsured deposit contract delivers the same payoff as the long-term debt contract. If $\chi_{\sigma_d} = 1 - \frac{1-\xi_{\boldsymbol{\delta}}}{X^L}$, the uninsured deposit contract delivers

$$\psi(c_d, \sigma_d) \;=\; Y\left(c_d, \sigma_d | L\right) + (1 - X^L) \tag{4}$$

The second panel of Figure 7 depicts Equations (3) and (4) as a function of $\xi_{\boldsymbol{\delta}}$.

A robustly optimal contract maximizes the resources available in the low state (in all continuation equilibria), subject to the limit on insured deposits, $\xi_{\boldsymbol{\delta}} \leq \hat{\xi}_{\boldsymbol{\delta}}$. We say that a *fragility* region exists whenever there is a region of insured deposits $\xi_{\boldsymbol{\delta}}$ for which the equilibrium payoff from financing using uninsured deposits—featuring liquidation at the interim—is strictly larger than the equilibrium payoff from financing using long-term debt. Figure 8 combines both payoff functions and helps illustrate the result of Proposition 1. It also clearly shows that the existence of a fragility region, with its lower bound given by y^*.

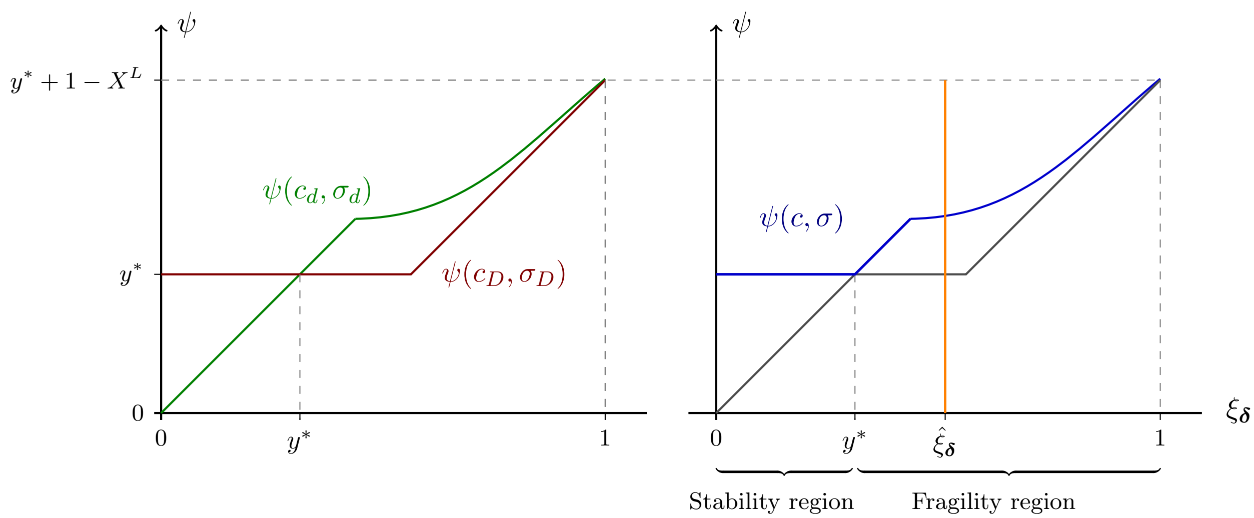

Figure 8: Characterising the fragility region and the bank's equilibrium resources.

In the fragility region, the Banker prefers to maximize the Regulator's deposit insurance transfer and forego investment returns correspondingly. If insured deposits are low, $\xi_{\boldsymbol{\delta}} < y^*$, the Regulator's ex-post transfer in the low state is small relative to the investment loss of having to liquidate all assets. We refer to this region as the *stability region.*

The second panel of Figure 8 contains the representation of the Banker's equilibrium investment and bailout resources $\psi(c, \sigma)$ in the low state as a function of $\xi_{\boldsymbol{\delta}}$, given by the upper envelope of

$\psi(c_D, \sigma_D)$ and $\psi(c_d, \sigma_d)$. Optimally, the Banker issues as many insured deposits as possible. If the constraint on insured deposits is binding in the fragility region, as illustrated here, then the Banker finances the remaining share of the asset using uninsured deposits.

The results generalize to mixed securities: by the convexity of $\psi(c, \sigma)$ in χ_σ, the relevant contracts to compare are the contracts with a) the highest level of liquidation, generating the same payoff as c_d, and b) with the lowest level of liquidation, generating the same payoff as c_D.

Having described the equilibrium contract, the next section derives the equilibrium welfare and discussed policies to address the ineffficencies.

4 Welfare & Policy

Equation (5) and Figure 9 depict the social welfare generated by the equilibrium financing contract as a function of the regulatory limit of insured deposits in the economy:

$$W(\hat{\xi}_{\boldsymbol{\delta}}) \;=\; W^* \;-\; (1-p)\cdot \begin{cases} 0 & \text{if } \hat{\xi}_{\boldsymbol{\delta}} \leq y^* \\ \left[y^* + (\kappa-1)\hat{\xi}_{\boldsymbol{\delta}}\right] & \text{if } \hat{\xi}_{\boldsymbol{\delta}} \in \left(y^*, 1-X^L\right] \\ \left[y^* - Y\left(c_d, \sigma_d | L\right) + (\kappa-1)\left[1-X^L\right]\right] & \text{otherwise} \end{cases} \tag{5}$$

Welfare is maximized in the stability region in which neither liquidations occur, nor bailouts are necessary in order to achieve efficient investment. Once the level of insured deposits crosses the threshold, $\hat{\xi}_{\boldsymbol{\delta}} > y^*$, the fragility induced by the uninsured deposit contracts reduces welfare both by forcing deposit insurance payments and by foregoing all continuation investment returns. Welfare is decreasing in $\hat{\xi}_{\boldsymbol{\delta}}$ as the required insured deposit insurance payments increase.

For high levels of deposit insurance, $\hat{\xi}_{\boldsymbol{\delta}} > 1 - X^L$, welfare is increasing in $\hat{\xi}_{\boldsymbol{\delta}}$. Replacing an additional unit of uninsured deposits with insured deposits comes at no further fiscal cost: one unit of the asset which otherwise would have been liquidated can now be used to fully cover the additional unit of insured deposits. With lower asset liquidations, deposit insurance then facilitates higher investment, leading to an increase in welfare.

Long-term debt and equity requirements.—Our model emphasizes a social cost as an unintended consequence of increasing the level deposit insurance or introducing a resolution regime, operating

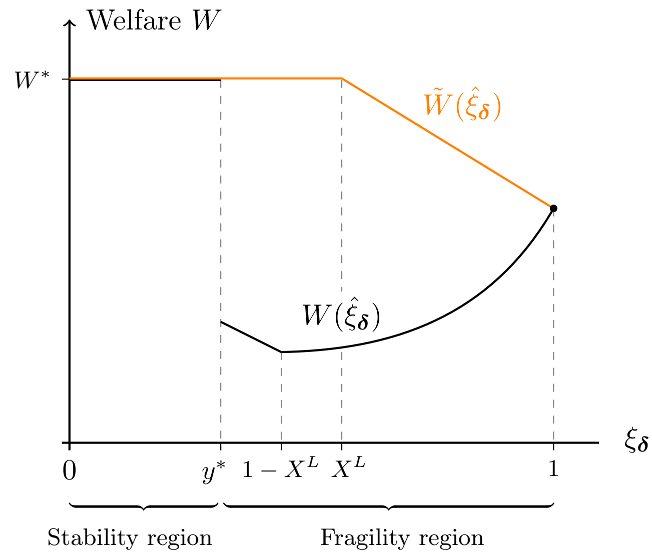

Figure 9: Welfare.

through the financing contract choice by the Banker. The Regulator can address the resulting fragility by requiring that all uninsured liabilities are issued in the form of long-term debt or equity. Welfare is then given by

$$\tilde{W}(\hat{\xi}_{\boldsymbol{\delta}}) \;=\; W^* \;-\; (1-p)\cdot \begin{cases} 0 & \text{if } \hat{\xi}_{\boldsymbol{\delta}} \leq X^L \\ (\kappa - 1)\left(\hat{\xi}_{\boldsymbol{\delta}} - X^L\right) & \text{otherwise} \end{cases} \tag{6}$$

Figure 9 illustrates that such a policy strictly increases welfare for all $\hat{\xi}_{\boldsymbol{\delta}} < 1$. We thus highlight the complementarities in deposit insurance and minimum maturity capital requirements when governments intend to resolve failing banks.

This is in line with existing regulation. According to the definition of the Financial Stability Board (FSB), debt securities contribute to a bank's 'total loss absorbing capacity' (TLAC) only with a maturity of one year.[22]

Fair deposit insurance premia.—First, an actuarially fair deposit insurance premium forces the Banker to internalize the fiscal cost of its liability choice. The trade-off between generating fiscal

[22]See also the minimum maturity requirement for unsecured debt to contribute to 'minimum requirement for own funds and eligible liabilities' (MREL) under the Bank Recovery and Resolution Directive (BRRD) of the European Union.

transfers from the Regulator and ensuring efficient ex-post investment tips in favour of the latter. Second, the Regulator can tighten the limit on insured deposits, and demand that $\hat{\xi}_{\boldsymbol{\delta}} \leq y^*$. This again lowers the fiscal transfer available to the Banker up to a point where the Banker prefers to issue long-term debt, inducing efficient ex-post investment.

In reality, it may be hard for the Regulator to compute the appropriate deposit insurance premium, and to determine at which level of $\hat{\xi}_{\boldsymbol{\delta}}$ the limit on insured deposits becomes binding. Furthermore, our result highlights that simply measuring the overall asset risk against the amount of deposit insurance is insufficient if uninsured depositors force liquidations and thus inflict losses on the more senior insured deposits. In other words, in order to compute the fair insurance premium, one has to consider the whole liability structure.

5 Discussion

The role of commitment. In our model lack of commitment to future actions is pervasive: the Banker cannot commit not to renegotiate with investors, nor commit to any investment policy; uninsured depositors cannot commit to roll-over; and the government cannot commit to any resolution policy. We believe this to be the most realistic framework for our application.

Nevertheless, it turns out that the Banker's inability to commit is irrelevant for our conclusions. This is because the Banker can fine-tune the ex-ante liability structure, perfectly anticipating everybody's future strategies, including the fragility induced by depositors as well as the government's recapitalization policy.

The Regulator's inability to commit is the critical aspect of our environment. The inability not to bail out if bail-ins are unsuccessful ensures that the bank receives a transfer which induces the Banker to opt for a fragile financing structure ex-ante. Furthermore, the inability to commit not to bail-in is essential: once the initial balance sheet is determined it is (interim) optimal for the Regulator to promise not to bail-in if the financing structure is fragile. This can be seen in Figure 9: conditional on the bank issuing uninsured deposits, it is optimal to extend deposit insurance to all uninsured depositors and achieve welfare $\tilde{W}(1) > W^*(\hat{\xi}_{\boldsymbol{\delta}})$ for all $\hat{\xi}_{\boldsymbol{\delta}} < 1$.

Disciplining effect of demandable debt.—The previous section has demonstrated that the Banker issues contracts that induce fragility whenever the limit on cheap deposit insurance is sufficiently high ($y^* < \hat{\xi}_{\boldsymbol{\delta}}$). Uninsured deposits become the means of extracting resources from the Regulator providing deposit insurance, at the social cost of inefficient investment. We interpret this efficiency loss as *ill-discipline*. This result stands in contrast with the findings in Diamond and Rajan (2000, 2001, 2005, and 2012). In their seminal papers, demandable debt holders can threaten to force liquidation of the asset. This disciplines a Banker who would like to extract private rents due to abilities inalienable to *inside* equity. Deposit insurance removes this threat of liquidation and consequently uninsured deposits and long-term debt become alike in terms of discipline.[23]

Our findings align with those of Diamond and Rajan in the sense that deposit insurance leads to ill-discipline. However, uninsured deposits are the means of extracting payments from the government. That is, they facilitate this breakdown of discipline.

An important distinction between our respective frameworks is that Diamond and Rajan (2000, 2001, 2005, and 2012) rule out resolution as a measure to achieve efficiency from the outset. The reason is that key productive resources are inalienable to *initial inside equity* holders, and therefore efficiency dictates that they remain part of the bank at all times. Alternatively, we assume that assets can be managed efficiently as long as they remain within the *institution*, and not necessarily in the hands of the initial inside equity holders. We believe this to be the more realistic assumption, especially in the context of large institutions like banks considered 'too big to fail'.

State-contingent contracts.—We could allow the Banker to issue state-contingent contracts. Our main result is that the Banker optimally issues insured deposits, which are not state-contingent and enforce liquidation upon the arrival of bad news on asset quality. The Banker could have alternatively issued an equity contract or a long-term debt contract which is renegotiated in the low state. Equity is naturally state-contingent, and resolution renders long-term debt contracts de facto state-contingent. However, it is not optimal to issue these type of state-contingent contracts.[24]

[23]In Repullo et al. (2013), banks can affect their asset returns by exerting unobservable effort. They find that short-term debt has a disciplining effect only for low profitability projects. Similarly, Huberman and Repullo (2015) find a disciplining effect of demandable debt if a bank can shift risks ex-ante. Our model suggests the exact opposite effect: demandable debt runs prevent efficient renegotiation of debt, inducing default of low profitability banks. Eisenbach (2017) finds that rollover risk disciplines financial intermediaries but only if asset returns are not correlated across states.

[24]If the government only has access to bailouts, it would again not be optimal to issue state-contingent debt contracts, since such contracts would generate strictly lower government transfers during crises. See Section 6 below.

6 Bailouts

To contrast the findings of Section 3, and argue that the introduction of resolution powers may have contributed to the rise in bank financing using uninsured deposits, suppose that bailouts are the only tool available to the Regulator at the interim. This arguably describes the recapitalization policy environment before and during the GFC.

Consider three special sets of contracts.

Definition 2 (**Liquidation-proof contracts**). *The set of liquidation-proof contracts, $\mathcal{P}$, is defined as*

$$\mathcal{P} := \left\{ c \in \mathcal{C} \mid \xi_{\boldsymbol{d}} \leq X^L \right\}$$

The significance of liquidation-proof contracts is captured by the following Lemma.

Lemma 2. *Suppose contract $c \in \mathcal{C}$ achieves financing. The following are equivalent:*

i. $c \in \mathcal{P}$.

ii. No liquidation occurs under any $\sigma \in \Sigma(c)$.

Proof. See Appendix C.3. □

Since the government fully removes any debt overhang using bailouts, any investment returns accrue to the Banker. The Banker thus prefers to avoid liquidation in order to maximize their investment returns. When $c \in \mathcal{P}$, the amount of uninsured deposits outstanding is limited such that—even if all uninsured depositors but one withdraw—the asset is not fully liquidated. Since the asset is not fully liquidated, the Banker can tempt uninsured depositors not to withdraw by marginally raising their claim since full repayment is guaranteed. Consequently, liquidation is avoided in all equilibria.

When $c \notin \mathcal{P}$, there is an equilibrium where uninsured depositors withdraw, forcing full liquidation. The reason is that Investors can always mis-coordinate on forcing full liquidation of the asset. Consider the decision problem of an individual uninsured depositor if all other uninsured depositors withdraw. Accepting any security offered by the Banker yields a zero payoff which justifies the choice to withdraw for each uninsured depositor.

Definition 3 (**Maximal-leverage contracts).** *The set of maximal-leverage contracts, $\mathcal{M}$, is defined as*

$$\mathcal{M} := \left\{ c \in \mathcal{C} \mid \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{\delta}} = 1 \right\}$$

The defining feature of all contracts $c \in \mathcal{M}$ is that the Banker obtains unit financing by only issuing debt claims, with the breakdown among insured and uninsured deposits as well as long-term debt unspecified. Any contract $c \in \mathcal{C}$ with $\xi_E > 0$ is not contained in $\mathcal{M}$.

Definition 4 (**Limited-leverage contracts).** *The set of limited-leverage contracts, $\mathcal{L}$, is defined as*

$$\mathcal{L} := \left\{ c \in \mathcal{C} \mid \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{\delta}} \leq X^L \right\}$$

The defining feature of all contracts $c \in \mathcal{L}$ is that the Banker obtains financing using equity and only partially by issuing debt claims. Any contract $c \in \mathcal{C}$ with $\xi_E < 1 - X^L$ is not contained in $\mathcal{L}$.

We are now ready to state and prove the main results of this section.

Proposition 2. *A contract c achieves efficiency in all $\sigma \in \Sigma(c)$ if and only if $c \in \mathcal{P} \cap \mathcal{L}$.*

Proposition 3. *Let $\mathcal{C}_2^*$ be the set of (robustly) optimal contracts. Then $\mathcal{C}_2^*$ is non-empty and every $c \in \mathcal{C}_2^*$ satisfies:*

1. *Stability: $c \in \mathcal{P}$.*
2. *Maximal leverage: $c \in \mathcal{M}$.*
3. *Inefficiency in all $\sigma \in \Sigma(c)$.*

Appendix C.4 proves Propositions 2 and 3 jointly. Intuitively, since the Regulator cannot commit her ex-post policy and only has access to bailouts, the debt overhang is fully removed using costly public funds upon intervention. Anticipating such transfers, the Banker optimally induces rollover by all uninsured depositors—which is feasible in all continuation equilibria only if the level of uninsured deposits is limited. Furthermore, the Banker does not offer any security to long-term debt holders which they would accept other than their initial debt contract; offering some equity compensation reduces not only government bailouts but also the Banker's equity share. The overall leverage of the bank thus remains unchanged. In this sense, private renegotiations fail to achieve efficient investment, and only government bailouts do. Since public funds are socially costly, the privately optimal financing arrangement is inefficient.

Maximising leverage at the contracting stage increases the size of bailouts in the low state, and therefore the share of the initial unit financing that is repaid by the government rather than the Banker. This in turn reduces the cost of financing and increases Banker payoffs in the high state. A maximal-leverage liquidation-proof contract achieves the highest feasible Banker payoff as it maximizes government transfers and achieves efficient investment. Since bailouts are increasing in leverage and socially costly, the optimal contract is inefficient in all continuation equilibria.

If $c \notin \mathcal{P}$, the Banker cannot prevent full liquidation in all equilibria in the low state. In these equilibria, the Banker not only loses the efficient investment in the low state, the cost of debt also increases in the high state given the government's response. Optimality then boils down to maximising leverage and preventing liquidation in all continuation equilibria.

In sum, we conclude that when the Regulator cannot commit her policy and can only conduct bailouts, banks are inefficiently leveraged but issue limited amounts of uninsured deposits.

7 Conclusion

This paper uncovers a delicate interaction between bank resolution policies and deposit insurance. We provide a rationale for why banks issue large amounts of uninsured deposit: front running the government which intends to impose losses on bank creditors. We highlight financial fragility as an unintended consequence of bail-ins, particularly when combined with high levels of deposit insurance.

The most recent episode of bank failures has reignited a policy debate, with some arguing in favour of greatly increasing the insurance limits on short-term bank debt. Our model suggests that the social cost of fragility—which may be induced by such policies—can be averted with other prudential policies in form of equity and long-term debt requirements.

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A Banks in sample

The following banks are contained in our sample:

UBS Bank USA, Discover Bank, Regions Bank, Northern Trust, American Express, HSBC Bank USA, BMO Harris Bank, Ally Bank, Huntington Bank, Keybank, Manufacturers and Traders Trust, Morgan Stanley Bank, Fifth Third Bank, Silicon Valley Bank, Morgan Stanley Private Bank, First Republic Bank, Citizens Bank, State Street Bank, The Bank of New York Mellon, Charles Schwab Bank, TD Bank, Capital One, Goldman Sachs, Truist Bank, PNC Bank, US Bank, Wells Fargo, Citibank, Bank of America, JP Morgan Chase.

B The Game

Strategies

Here we give all formal details of the game $\mathcal{G}_s^\theta(c)$. Each Investor chooses whether to accept (1) or reject (0) the offer $s \in \mathcal{S}$. Equity and insured depositors do not take any meaningful action by assumption and hence we focus on all other demandable and long-term debt Investors. In what follows, we suppress dependence on (θ, c, s) for clarity, whenever there is no risk of confusion. We denote pure strategies by $\sigma_i \in \{0, 1\}$, and the profile of strategies as $\sigma_I := (\sigma_i)_{i \in \mathbb{I}}$. For each profile σ_I, we denote by $\lambda_{\sigma_I} \in [0, 1]$ the fraction of uninsured depositors rejecting the offer and thus withdrawing rather than rolling over. Formally,

$$\lambda_{\sigma_I} = \int \mathbb{1}_{c_i = \boldsymbol{d}} \cdot (1 - \sigma_i) \, \mathrm{d}i$$

Given $\lambda_{\sigma_I} \in [0,1]$, we denote by $\chi_{\sigma_I} \in [0,1]$ the *share* of the asset still remaining in the bank after potential liquidation.[25] Formally,

$$\chi_{\sigma_I} \;=\; \max\left\{1 - \frac{\lambda_{\sigma_I}\xi_{\boldsymbol{d}} \cdot d}{X^{\theta}}, 0\right\}$$

The Regulator observes the profile σ_I as well as the state and chooses a policy. Consequently, the Regulator's strategy is a mapping $\sigma_R : \{0,1\}^I \mapsto \mathcal{S} \times \mathbb{R}^+$, where we write

$$\sigma_R(\sigma_I) = \left\{ \left(\hat{\alpha}_i(\sigma_I), \hat{B}_i(\sigma_I)\right)_{i \in \mathbb{I}},\ b(\sigma_I) \right\}$$

for the pair of security and bailout amount as a function of the security offered during renegotiation, and the acceptance/rejection decisions of each Investor.[26] The Regulator's strategy must satisfy the constraints laid out in Section 2.4. Since the Regulator maximizes ex-post welfare, his strategy depends only on χ_{σ_I}, and with slight abuse of notation we can write $\sigma_R(\sigma_I) = \sigma_R(\chi_{\sigma_I})$.

Finally, the Banker's investment strategy in state θ, given the strategy profile (σ_I, σ_R), is given by $\sigma_e : \{0,1\}^I \times \mathcal{S} \times \mathbb{R}^+ \mapsto \mathbb{R}^+$.

Payoffs

To define the payoffs to Investors we need some notation for the profile of debt promises out of *future* output, as a function of the strategy profile. To this end, define for each $i \in \mathbb{I}$,

$$\tilde{B}_i(\sigma_I, \sigma_R) = \sigma_i \cdot \hat{B}_i(1, \sigma_{-i}) + (1 - \sigma_i) \cdot \left[\mathbb{1}_{c_i = \mathbf{D}} \cdot \hat{B}_i(0, \sigma_{-i})\right]$$

$\tilde{B}_i$ is the promise Investor i ends up with depending on the strategy profile. If i accepts ($\sigma_i = 1$) security $s \in \mathcal{S}$, then his promise is given by $\hat{B}_i(1, \sigma_{-i})$—the promise the Regulator implements if i accepts and all other Investors play according to σ_{-i}. If he rejects ($\sigma_i = 0$), then his promise is as follows: if he is a long-term debt Investor it is $\hat{B}_i(0, \sigma_{-i})$—the promise the Regulator implements if i rejects and all other Investors play according to σ_{-i}. For equity Investors and uninsured depositors, the promised repayment out of future output is formally zero.[27]

[25] When we wish to emphasize the dependence on the state, we will write $\lambda^{\theta}_{\sigma_I}$, and $\chi^{\theta}_{\sigma_I}$ respectively.

[26] Again, when we wish to emphasize the dependence on the state θ and the security s, we will write $\sigma^{\theta}_R(\sigma_I \mid s)$

[27] Uninsured deposits are a promised repayment out of *current* assets in place; if they reject the offer they have no claim on future output.

We can now define the payoffs to Investor i, from accepting and rejecting security $s \in \mathcal{S}$ under profile σ_{-i}. We should stress that these payoffs are strictly speaking the per-unit returns on investment, and each individual Investor invests an infinitesimal unit. First, for each final output realization $z \in \mathbb{R}^+$, and investment $e \in [0, \bar{e}]$, the payoffs to i from accepting are given by

$$\begin{aligned} u_i\big(z|s, \sigma_{-i}, \sigma_R, e\big) = \hat{\alpha}_i(1, \sigma_{-i}) \cdot \left[\max\left\{z - \int \tilde{B}_j(1, \sigma_{-i}, \sigma_R)\,\mathrm{d}j,\ 0\right\} - h(e)\right] \\ + \min\left\{\frac{\tilde{B}_i(1, \sigma_{-i}, \sigma_R)}{\int_{[j\sim i]} \tilde{B}_j(1, \sigma_{-i}, \sigma_R)\mathrm{d}j} \cdot \left[\max\left\{z - \int_{[j\prec i]} \tilde{B}_j(1, \sigma_{-i}, \sigma_R)\,\mathrm{d}j,\ 0\right\}\right],\ \tilde{B}_i(1, \sigma_{-i}, \sigma_R)\right\} \end{aligned}$$

The first term captures the payoff to $i \in \mathbb{I}$ stemming from equity ownership: it is a fraction $\hat{\alpha}_i(1, \sigma_{-i})$ of the residual value after debt repayments, *net* of investment costs, corresponding to the security implemented by the Regulator after profile $(1, \sigma_{-i})$. The second term captures i's payoff from holding a debt claim $\tilde{B}_i(1, \sigma_{-i}, \sigma_R)$ under the seniority structure: if there are enough resources to pay all Investors with same seniority as i, after all strictly more senior Investors $j \prec i$ are paid, then all $j \sim i$ receive their promises. Otherwise, the set $[j \sim i]$ seizes output and distributes it according to the share corresponding to their promise. All claims are protected by limited liability.

The corresponding payoffs from rejecting the offer depend on i's initial security holding, c_i, under contract c. If i is a long-term debt Investor, $c_i = \mathbf{D}$, then for each realization of final output, $z \in \mathbb{R}^+$ the payoff to i from rejecting is given by

$$r_i^D\big(z|\ s, \sigma_{-i}\big) = \min\left\{\frac{\tilde{B}_i(0, \sigma_{-i}, \sigma_R)}{\int_{[j\sim i]} \tilde{B}_j(0, \sigma_{-i}, \sigma_R)\mathrm{d}j} \cdot \left[\max\left\{z - \int_{[j\prec i]} \tilde{B}_j(0, \sigma_{-i}, \sigma_R)\,\mathrm{d}j,\ 0\right\}\right],\ \tilde{B}_i(0, \sigma_{-i}, \sigma_R)\right\}$$

If i is a uninsured depositor then his payoff from rejection is the expected payoff from demanding early repayment which is independent of final output, depending only on the current value of assets $x \in \mathbb{R}^+$. That is, if $c_i = \mathbf{d}$,

$$r_i^d(x \mid s, \sigma_{-i}) = \begin{cases} d & \text{if } \lambda_{\sigma_I} \cdot \xi_{\boldsymbol{d}} d \leq x \\ \frac{x}{\lambda_{\sigma_I} \cdot \xi_{\boldsymbol{d}}} & \text{otherwise} \end{cases}$$

Note that λ_{σ_I} is also the fraction of uninsured depositors among $[j \neq i]$ rejecting the offer, since each individual Investor i has no aggregate impact.

At the time of acceptance/rejection decisions by Investors there is still residual uncertainty regarding final output. Ultimately, we want to determine Investors' expected payoffs at the time of decision, which amounts to determining the distribution of output. Recall that rejecting uninsured depositors force early liquidation which reduces the quantity of the asset to $\chi_{\sigma_I} \in [0,1]$. Furthermore, the Regulator injects an amount $b(\sigma_I) \in \mathbb{R}^+$. Finally, the distribution of per-unit returns depends on investment $e \in [0, \bar{e}]$. Consequently, output in state θ is given by $\chi_{\sigma_I} \cdot (X^\theta + q) + b(\sigma_I)$, where $q \sim G(\cdot| \, e)$.

We can now define the expected payoff to each Investor i from a profile $(\sigma_I, \sigma_R, \sigma_e)$ in the game $\mathcal{G}_s^\theta(c)$, depending on the class of debt they initially hold. If $c_i = \boldsymbol{D}$,

$$
\begin{aligned}
U_i(\sigma_i, \sigma_{-i}, \sigma_R, \sigma_e \mid \theta, s) = \sigma_i \cdot \mathbb{E}_e\Big[u_i\Big(\chi_{\sigma_I} \cdot (X^\theta + q) + b(\sigma_I) \mid s, \sigma_{-i}, \sigma_R, e\Big)\Big] + \\
+ (1 - \sigma_i) \cdot \mathbb{E}_e\Big[r_i^D\Big(\chi_{\sigma_I} \cdot (X^\theta + q) + b(\sigma_I) \mid s, \sigma_{-i}\Big)\Big]
\end{aligned}
$$

If $c_i = \boldsymbol{d}$,

$$
U_i(\sigma_i, \sigma_{-i}, \sigma_R, \sigma_e \mid \theta, s) = \sigma_i \cdot \mathbb{E}_e\Big[u_i\Big(\chi_{\sigma_I} \cdot (X^\theta + q) + b(\sigma_I) \mid s, \sigma_{-i}, \sigma_R, e\Big)\Big] + (1 - \sigma_i) \cdot r_i^d\Big(X^\theta \mid s, \sigma_{-i}\Big)
$$

Equilibria in $\mathcal{G}_s^\theta(c)$

We now describe subgame perfect equilibria in the continuation game after contract $c \in \mathcal{C}$ is posted.

Starting backwards with the Banker, σ_e is optimal whenever it maximizes the equity value in Equation E; that is,

$$
\sigma_e(\sigma_I, \sigma_R \mid \theta, s) = e(\chi_\sigma, \hat{B}(\sigma_I) \mid \theta) \tag{7}
$$

where $\chi_\sigma \in [0,1]$ is the amount of the asset remaining in the bank, and $\hat{B}(\sigma_I) \in \mathbb{R}^+$ is the total debt promises, after renegotiation.

Next, we consider the Regulator who maximizes ex-post welfare. Specifically, σ_R is optimal whenever it maximizes net continuation investment gains,

$$
\mathbb{E}\Big[\chi_\sigma \cdot q \Big| \; \sigma_e(\sigma_I, \sigma_R | \; \theta, s)\Big] - h\Big(\sigma_e(\sigma_I, \sigma_R | \; \theta, s)\Big) \geq \mathbb{E}\Big[\chi_\sigma \cdot q \Big| \; \sigma_e(\sigma_I, \widetilde{\sigma}_R | \; \theta, s)\Big] - h\Big(\sigma_e(\sigma_I, \widetilde{\sigma}_R | \; \theta, s)\Big) \tag{8}
$$

for all $\widetilde{\sigma}_R$. Finally, we consider Investors.

A profile $(\sigma_I, \sigma_R, \sigma_e)$ is an equilibrium of $\mathcal{G}_s^\theta(c)$, if

$$U_i(\sigma_i, \sigma_{-i}, \sigma_R, \sigma_e \mid \theta, s) \geq U_i(\widetilde{\sigma}_i, \sigma_{-i}, \sigma_R, \sigma_e \mid \theta, s), \text{ for all } i, \text{ and } \widetilde{\sigma}_i,$$

σ_R satisfies Equation (8), and σ_e satisfies Equation (7).

We denote the set of equilibria of $\mathcal{G}_s^\theta(c)$ by $\Sigma(c \mid \theta, s)$.

Continuation equilibria $\Sigma(c)$

Lastly, we consider the Banker's optimal choice of security $s \in \mathcal{S}$ after the state θ is realized, given that contract $c \in \mathcal{C}$ is in place. The Banker's pure strategies in the renegotiation game are mappings $\sigma_B : \{L, H\} \times \mathbb{R}^3 \mapsto \mathcal{S}$, assigning a security in each state for prices $(D, d, \gamma) \in \mathbb{R}^3$, of contract $c \in \mathcal{C}$.

Let $\sigma_{-B} = (\sigma_I, \sigma_R, \sigma_e) \in \times_{\theta \in \{L,H\}} \Big(\times_{s \in \mathcal{S}} \Sigma(c \mid \theta, s) \Big)$ be a profile of equilibrium strategies in $\mathcal{G}_s^\theta(c)$, for each possible $\theta \in \{L, H\}$ and each possible choice $s \in \mathcal{S}$.

The payoffs to the Banker are given by:

$$U_B\big(s, \sigma_{-B} \mid \theta\big) = \mathbb{E}\left[\chi_\sigma \cdot \Big(X^\theta + q\Big) \Big| \; \sigma_e(\sigma_I, \sigma_R \mid \theta, s)\right] - \int U_i(\sigma_{-B} \mid \theta, s)\, \mathrm{d}i - h\Big(\sigma_e(\sigma_I, \sigma_R \mid \theta, s)\Big)$$

A strategy σ_B is a best-response to σ_{-B} if and only if

$$U_B\big(\sigma_B, \sigma_{-B} \mid \theta\big) \geq U_B\big(s, \sigma_{-B} \mid \theta\big) \quad \text{for all } s \in \mathcal{S}, \text{ and all } \theta \in \{L, H\} \tag{9}$$

Finally, we denote by $\Sigma(c)$ the set of continuation equilibria following the initial offer of contract $c \in \mathcal{C}$.

$$\Sigma(c) = \Big\{(\sigma_B, \sigma_I, \sigma_R, \sigma_e) : \sigma_B \text{ satisfies (9) and } (\sigma_I, \sigma_R, \sigma_e) \in \times_{\theta \in \{L,H\}} \Big(\times_{s \in \mathcal{S}} \Sigma(c \mid \theta, s)\Big)\Big\}$$

C Proofs

C.1 Proof of Lemma 1

Proof. Let $\mathcal{E}(c,\sigma|\theta)$ denote the equity value of a Bank in state θ for contract c and continuation equilibrium $\sigma \in \Sigma(c)$. Furthermore, let $U_d(c,\sigma|\theta)$ and $U_D(c,\sigma|\theta)$ denote the equilibrium payoffs to each uninsured deposit and long-term debt Investor in state θ. The equity value is then given by

$$\mathcal{E}(c,\sigma|\theta) \;=\; \chi_\sigma^\theta X^\theta + Y(c,\sigma|\theta) - \xi_{\boldsymbol{\delta}} - (1-\lambda_\sigma^\theta)U_d(c,\sigma|\theta)\xi_{\boldsymbol{d}} - U_D(c,\sigma|\theta)\xi_{\boldsymbol{D}} + b(c,\sigma|\theta)$$

Intuitively, the value of equity is given by the total amount of resources within the bank, minus the equilibrium payoffs to insured depositors, uninsured deposits, and long-term debt Investors.

We first characterize the price of uninsured deposits, to pin down $U_d(c,\sigma|H)$. In particular, the payoff from rejecting any security during renegotiations for any $c_i = \boldsymbol{d}$, is

$$r_i(c,\sigma|\theta) \;=\; \begin{cases} d & \text{if } \lambda_\sigma^\theta \cdot \xi_{\boldsymbol{d}} d \le X^\theta \\ \frac{X^\theta}{\lambda_\sigma^\theta \cdot \xi_{\boldsymbol{d}}} & \text{otherwise} \end{cases}$$

Note that $U_d(c,\sigma|\theta) \ge r_i(c,\sigma|\theta)$: the equilibrium payoff is bounded from below by the rejection payoff. If the Banker successfully induces rollover, he optimally sets $U_d(c,\sigma|\theta) = r_i(c,\sigma|\theta)$.

First, if the investor gets fully 'repaid' in both states, then $d = 1$:

$$1 \;=\; p \cdot U_d(c,\sigma|H) + (1-p)\cdot U_d(c,\sigma|L) \;= p\cdot d + (1-p)\cdot d \;=\; d$$

Second, if the Investor gets the promised payoff in the low state, then he must be getting the promised payoff in the high state. Indeed, if not, we must have

$$1 \;=\; p \cdot U_d(c,\sigma|H) + (1-p)\cdot U_d(c,\sigma|L) \;\ge\; p \cdot \frac{X^H}{\lambda_\sigma^H \xi_{\boldsymbol{d}}} + (1-p)\cdot d \;\ge\; p\cdot X^H + (1-p)\cdot d$$

by the pricing equation. Since $X^H > 1$, this necessitates $d < 1$. But then

$$\lambda_\sigma^H \xi_{\boldsymbol{d}} \cdot d \;<\; \xi_{\boldsymbol{d}} \;<\; X^H$$

which implies $U_d(c,\sigma|H) = d$, a contradiction. Furthermore, failure of full repayment cannot occur in both states since then the pricing equation yields

$$1 \;\geq\; p \cdot \frac{X^H}{\lambda_\sigma^H \xi_{\boldsymbol{d}}} + (1-p) \cdot \frac{X^L}{\lambda_\sigma^L \xi_{\boldsymbol{d}}} \;\geq\; p \cdot X^H + (1-p) \cdot X^L$$

a contradiction. Therefore, failure of full repayment can only occur in the low state. The lower bound for the repayment to each $c_i = \boldsymbol{d}$ in the low state is given by $U_d(c,\sigma|\theta) \geq X^L$. The pricing equation then implies an upper bound to d:

$$1 \;\geq\; p \cdot d + (1-p) \cdot X^L$$

Since $\mathbb{E}[X] > 1$, it must be that $d < X^H$, and hence $U_d(c,\sigma|H) = d$. The value of equity in the high state is then given by

$$\mathcal{E}(c,\sigma|H) \;=\; X^H + Y(c,\sigma|H) - \xi_{\boldsymbol{\delta}} - d\xi_{\boldsymbol{d}} - U_D(c,\sigma|H)\xi_{\boldsymbol{D}} + \max\left\{\xi_{\boldsymbol{\delta}} - \chi_\sigma^H X^H, 0\right\}$$

We now show that $b(c,\sigma|H) = 0$. This follows from

$$\xi_{\boldsymbol{\delta}} - \chi_\sigma^H X^H \;\leq\; \xi_{\boldsymbol{\delta}} + d\xi_{\boldsymbol{d}} - X^H \;\leq\; d - X^H \;<\; 0$$

where the first inequality follows from the fact that $\chi_\sigma^H X^H \geq X^H - d\xi_{\boldsymbol{d}}$. The second inequality uses $\xi_{\boldsymbol{\delta}} + \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{E}} = 1$ and $d \geq 1$. The final inequality uses $d < X^H$.

We proceed to show that the bank equity value is increasing in χ_σ^H, implying $Y(c,\sigma,\chi_\sigma|H) = y^*$ in all continuation equilibria $\sigma \in \Sigma(c)$ and for all contracts $c \in \mathcal{C}$. For this, we need to show that

$$X^H - \xi_{\boldsymbol{\delta}} - d\xi_{\boldsymbol{d}} - U_D(c,\sigma|H)\xi_{\boldsymbol{D}} \;\geq\; 0$$

Note that $U_D(c,\sigma|H) = D$: the payoff to each long-term debt investor, if such debt has been issued, is given by the face value of its long-term debt claim. Indeed, if not, then no Investor purchases long-term debt. We consider two cases:

1. Suppose $\xi_{\boldsymbol{d}} > X^L$. The pricing equations for long-term debt and uninsured deposits imply

that

$$1 \;\geq\; p \cdot d + (1-p) \cdot \frac{X^L}{\xi_{\boldsymbol{d}}}$$

and

$$1 \;=\; p \cdot D + (1-p) \cdot U_D(c, \sigma|L), \;\; \text{where } U_D(c,\sigma|L) \geq 0 \qquad \Leftrightarrow \qquad 1 \;\geq\; p \cdot D$$

Using these inequalities, it follows that

$$\xi_{\boldsymbol{\delta}} + d\xi_{\boldsymbol{d}} + D\xi_{\boldsymbol{D}} \;\leq\; d\xi_{\boldsymbol{d}} + D(1 - \xi_{\boldsymbol{d}}) \;\leq\; \frac{1-(1-p)\cdot X^L}{p} \;<\; X^H$$

where the last inequality follows from $\mathbb{E}[X] > 1$.

2. Suppose $\xi_{\boldsymbol{d}} \leq X^L$. The pricing equations for long-term debt and uninsured deposits imply that $d = 1$ as well as

$$1 \;\geq\; p \cdot D + (1-p)\frac{X^L - \xi_{\boldsymbol{\delta}} - \xi_{\boldsymbol{d}}}{\xi_{\boldsymbol{D}}}$$

since

$$U_D(c,\sigma|L) \;\geq\; \frac{X^L - \xi_{\boldsymbol{\delta}} - \xi_{\boldsymbol{d}}}{\xi_{\boldsymbol{D}}}$$

Using these inequalities in exactly the same manner as above gives the desired result.

From here it follows that the bank equity value is increasing in $\chi^H_{\boldsymbol{\sigma}}$. Then, optimally, the Banker induces rollover by all uninsured depositors, offering a new security with face value $B_i = d + \varepsilon$ with $\varepsilon \searrow 0$ which is accepted, and hence $Y(c,\sigma|H) = y^*$. The Banker's equilibrium payoff is then given by

$$\begin{aligned}
V(c,\sigma) \;&=\; (1-\gamma)\cdot[p \cdot \mathcal{E}(c,\sigma|H) + (1-p)\cdot \mathcal{E}(c,\sigma|L)] \\
&=\; p \cdot \mathcal{E}(c,\sigma|H) + (1-p)\cdot \mathcal{E}(c,\sigma|L) - \xi_{\boldsymbol{E}} \\
&=\; \mathbb{E}[X] + p\cdot y^* + (1-p)\cdot \Big(Y(c,\sigma|L) + b(c,\sigma|L)\Big) \\
&\qquad\qquad - \xi_{\boldsymbol{\delta}} - \Big[p \cdot d + (1-p)\cdot U_d(c,\sigma|L)\Big]\xi_{\boldsymbol{d}} - \Big[p\cdot D + (1-p)\cdot U_D(c,\sigma|L)\Big]\xi_{\boldsymbol{D}} - \xi_{\boldsymbol{E}} \\
&=\; \mathbb{E}[X] + p \cdot y^* + (1-p)\cdot \psi(c,\sigma) - 1
\end{aligned}$$

where the first equality follows from the equity pricing equation:

$$\gamma \cdot \mathbb{E}\left[\mathcal{E}(c,\sigma|\theta)\right] \;=\; \xi_{\boldsymbol{E}}$$

and the last equality from $\xi_{\boldsymbol{\delta}} + \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{E}} \;=\; 1$. Clearly $V(c,\sigma) > V(c',\sigma')$ if and only if $\psi(c,\sigma) > \psi(c',\sigma')$ for some contracts $c, c' \in \mathcal{C}$ and equilibria $\sigma \in \Sigma(c)$, $\sigma' \in \Sigma(c')$. $\square$

C.2 Proof of Proposition 1

The proof of our main result follows from five lemmas. We first show that for any $\xi_{\boldsymbol{\delta}} \in [0,1]$, contract $c = (0, 0, 1-\xi_{\boldsymbol{\delta}}, \xi_{\boldsymbol{\delta}})$ uniquely achieves some level of liquidation $\chi_\sigma < 1$ in all $\sigma \in \Sigma(c)$, and thus also a unique payoff.

Second, consider some contract $c' = (\xi'_{\boldsymbol{E}}, \xi'_{\boldsymbol{D}}, \xi'_{\boldsymbol{d}}, \xi'_{\boldsymbol{\delta}}) \in \mathcal{C}$. We show that, if $\xi'_{\boldsymbol{d}} > X^L + y^* - \xi'_{\boldsymbol{\delta}}$, liquidation cannot be prevented in any equilibrium $\sigma' \in \Sigma(c')$. Thus, to have $\lambda_{\sigma'} = 0$ in all equilibria $\sigma' \in \Sigma(c')$, there must exist some $\bar{B}_{\boldsymbol{d}}(c') \in [0, X^L + y^* - \xi'_{\boldsymbol{\delta}}]$ such that $\xi'_{\boldsymbol{d}} \leq \bar{B}_{\boldsymbol{d}}(c')$.

Third, we characterize conditions for which $\psi(c,\sigma) > \psi(c',\sigma')$ for contract $c = (0, 0, 1-\xi_{\boldsymbol{\delta}}, \xi_{\boldsymbol{\delta}})$ and all $\sigma \in \Sigma(c)$, and for all contracts $c' \in \mathcal{C}$ with $\lambda_{\sigma'} = 0$ for all $\sigma' \in \Sigma(c')$.

Fourth, we show that it is never profitable to increase the level of liquidation by decreasing the level of insured deposits, $\xi_{\boldsymbol{\delta}}$.

Finally, we show that for a given level of $\xi_{\boldsymbol{\delta}}$, no other contract $\hat{c} \in \mathcal{C}$ achieves strictly higher payoffs than both contracts c and c' in any equilibrium $\hat{\sigma} \in \Sigma(\hat{c})$.

Lemma 3. *Contract $c = (0, 0, 1-\xi_{\boldsymbol{\delta}}, \xi_{\boldsymbol{\delta}})$ achieves a unique level of liquidation, $\chi_\sigma < 1$, if $\theta = L$, and thus also a unique payoff.*

Proof. We distinguish two regions in $\xi_{\boldsymbol{\delta}}$: a) $1 - \xi_{\boldsymbol{\delta}} \geq X^L$, and b) $1 - \xi_{\boldsymbol{\delta}} < X^L$.

a) We show that $\chi_\sigma = 0$. Since $\xi_{\boldsymbol{d}} \geq X^L$, $\chi_\sigma > 0$ requires $\lambda_\sigma < 1$. This in turn requires that

$$\chi_\sigma X^L + Y(c,\sigma|L) - \xi_{\boldsymbol{\delta}} + b(c,\sigma|L) \;\geq\; (1-\lambda_\sigma)\xi_{\boldsymbol{d}} \cdot U_d(c,\sigma|L)$$

The LHS captures the bank's equity value after a write-down of all equity and long-term debt, the RHS captures the total value of the uninsured depositors who could withdraw but decide to roll over. If the inequality is not satisfied, at least one more uninsured depositor prefers to withdraw even if all remaining uninsured depositors hold all the bank equity after a bail-in. Since uninsured depositors roll over, it must be that $U_d(c,\sigma|H) = U_d(c,\sigma|L) = d = 1$. Since

$$(1-\chi_\sigma)X^L = (1-\lambda_\sigma)d\xi_{\boldsymbol{d}}$$

we can rearrange the inequality to read

$$X^L + Y(c,\sigma|L) - \xi_{\boldsymbol{\delta}} + \max\{\xi_{\boldsymbol{\delta}} - \chi_\sigma X^L, 0\} \;\geq\; \xi_d$$

With bailouts, it must be that

$$(1-\chi_\sigma)X^L + Y(c,\sigma|L) \;\geq\; \xi_{\boldsymbol{d}}$$

where the LHS is a strictly convex function, maximized at $\chi_\sigma = 0$ since $X^L > y^*$. Since $\xi_{\boldsymbol{d}} \;\geq\; X^L$, we have

$$\xi_{\boldsymbol{d}} \;\geq\; X^L \;\geq\; (1-\chi_\sigma)X^L + Y(c,\sigma|L) \;>\; \xi_{\boldsymbol{d}}$$

for any $\chi_\sigma > 0$ for $\xi_{\boldsymbol{d}} \geq X^L$, a contradiction. Thus, we obtain $\chi_\sigma = 0$. Without bailouts, it must be that

$$1 \;>\; X^L + Y(c,\sigma|L) \;\geq\; \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{\delta}} \;= 1$$

which is a contradiction. It then follows that $\chi_\sigma = 0$ for all equilibria.

b) We show that $\lambda_\sigma = 1$ and thus $\chi_\sigma = 1 - \frac{1-\xi_{\boldsymbol{\delta}}}{X^L}$ for all $\sigma \in \Sigma(c)$. First, note that by $\xi_{\boldsymbol{d}} < X^L$ uninsured deposits are safe in both states and hence $U_d(c,\sigma|H) = U_d(c,\sigma|L) = d = 1$. The value of equity is given by

$$\begin{aligned}
\mathcal{E}(c,\sigma|L) \;&=\; \max\{\chi_\sigma X^L + Y(c,\sigma|L) - \xi_{\boldsymbol{\delta}} - (1-\lambda_\sigma)\xi_{\boldsymbol{d}} + \max\{\xi_{\boldsymbol{\delta}} - \chi_\sigma X^L, 0\},\; 0\} \\
&=\; \max\left\{X^L + Y(c,\sigma|L) - \xi_{\boldsymbol{\delta}} - \xi_{\boldsymbol{d}} + \max\{\xi_{\boldsymbol{\delta}} + \lambda_\sigma\xi_{\boldsymbol{d}} - X^L, 0\},\; 0\right\}
\end{aligned}$$

If $\xi_{\boldsymbol{\delta}} \geq X^L$, then

$$\mathcal{E}(c, \sigma | L) \;=\; Y(c, \sigma | L) - (1 - \lambda_\sigma)\xi_{\boldsymbol{d}} \tag{10}$$

where $\chi_\sigma = 1 - \frac{\lambda_\sigma \xi_{\boldsymbol{d}}}{X^L}$. Note that $\mathcal{E}(c, \sigma|L) > 0$ for $\lambda_\sigma = 1$ since $\xi_{\boldsymbol{d}} < X^L$.

If $\xi_{\boldsymbol{\delta}} < X^L$, equity value is only positive for a sufficiently large level λ_σ to induce bailouts, for which it is also given by Equation (10).

Note that the expression in Equation (10) is a strictly convex function in λ_σ which has its maximum point at either $\lambda_\sigma = 0$ or $\lambda_\sigma = 1$. The maximum point is at $\lambda_\sigma = 1$ if

$$y^* - (1 - \xi_{\boldsymbol{\delta}}) \;<\; \max_e \; \left[\left(1 - \frac{(1 - \xi_{\boldsymbol{\delta}})}{X^L}\right) \cdot \int_0^\infty q \, dG(q|e) - h(e)\right] \;\equiv\; f\big(\xi_{\boldsymbol{\delta}}, e(\chi_\sigma, 0)\big) \tag{11}$$

where $e(\chi_\sigma, 0)$ denotes the maximizing effort level given the level of liquidation on zero debt outstanding. Both LHS and RHS are given by y^* for $\xi_{\boldsymbol{\delta}} = 1$. Also note that $f\big(\xi_{\boldsymbol{\delta}}, e(\chi_\sigma, 0)\big)$ is a strictly increasing, strictly convex function and differentiable in $\xi_{\boldsymbol{\delta}}$ by Danskin's theorem, since there is a unique maximizer for any $\xi_{\boldsymbol{\delta}}$. Furthermore, we obtain by differentiating an upper bound on its derivative with respect to $\xi_{\boldsymbol{\delta}}$:

$$\frac{\partial f\big(\xi_{\boldsymbol{\delta}}, e(\chi_\sigma, 0)\big)}{\partial \xi_{\boldsymbol{\delta}}} \;=\; \frac{1}{X^L} \cdot \int_0^\infty q \, dG\big(q | e(\chi_\sigma, 0)\big) \;\leq\; \frac{1}{X^L} \cdot \int_0^\infty q \, dG\big(q | \bar{e}\big) \;<\; 1$$

where the first inequality follows from first order stochastic dominance, and the second inequality follows from Assumption 1. Reducing $\xi_{\boldsymbol{\delta}}$ reduces the LHS of Equation (11) by one, the RHS by less than one. Hence the inequality of Equation (11) is satisfied for all $1 - \xi_{\boldsymbol{\delta}} < X^L$ and $\xi_{\boldsymbol{\delta}} < 1$. It follows that $\lambda_\sigma = 1$ in all $\sigma \in \Sigma(c)$.

Combining the above, the resources from issuing a contract c for a given level of $\xi_{\boldsymbol{\delta}}$ and all equilibria $\sigma \in \Sigma(c)$ in the low state are given by

$$\psi(c, \sigma) \;=\; \begin{cases} \xi_{\boldsymbol{\delta}} & \text{if } \xi_{\boldsymbol{\delta}} \leq 1 - X^L \\ Y(c, \sigma | L) + 1 - X^L & \text{if } \xi_{\boldsymbol{\delta}} \in \big(1 - X^L, 1\big] \end{cases} \tag{12}$$

$\square$

Lemma 4. *Consider a contract $c' \in \mathcal{C}$. If $\xi_{\boldsymbol{d}} > X^L + y^* - \xi_{\boldsymbol{\delta}}$, then $\lambda_{\sigma'} > 0$ in all equilibria $\sigma' \in \Sigma(c')$.*

Proof. Suppose $\xi_{\boldsymbol{d}} > X^L + y^* - \xi_{\boldsymbol{\delta}}$. If all uninsured depositors roll over, $\lambda_{\sigma'} = 0$, then each Investor receives

$$U_d(c, \sigma|L) \;=\; \frac{X^L + y^* - \xi_{\boldsymbol{\delta}}}{\xi_{\boldsymbol{d}}} \;<\; 1$$

Rejecting, each investor receives

$$r_i(c, \sigma|L) \;=\; d$$

From the proof of Lemma 1, we know that $d \geq 1$. Uninsured depositors thus withdraw, and $\lambda_{\sigma'} > 0$. $\square$

Lemma 5. *Let $c^* = (0, 0, 1 - \hat{\xi}_{\boldsymbol{\delta}}, \hat{\xi}_{\boldsymbol{\delta}})$. Let c' be a contract such that $\lambda_{\sigma'} = 0$ in every equilibrium $\sigma' \in \Sigma(c')$.*

1. *If $\hat{\xi}_{\boldsymbol{\delta}} \in (y^*, 1)$, then $\psi(c^*, \sigma^*) > \psi(c', \sigma')$ for all $\sigma^* \in \Sigma(c^*)$.*
2. *If $\hat{\xi}_{\boldsymbol{\delta}} \leq y^*$, then $\psi(c^*, \sigma^*) \leq \psi(c', \sigma')$ for all $\sigma^* \in \Sigma(c^*)$, with equality if and only if $\hat{\xi}_{\boldsymbol{\delta}} \in \{y^*, 1\}$.*

Proof. To show the first part, note that $\psi(c^*, \sigma^*)$ is strictly increasing in $\xi_{\boldsymbol{\delta}}$ by Equation (12). Furthermore, if $\lambda_{\sigma'} = 0$ and $\xi_{\boldsymbol{\delta}} \leq X^L$, then $\psi(c', \sigma') = y^*$. If $\lambda_{\sigma'} = 0$ and $\xi_{\boldsymbol{\delta}} > X^L$, then $\psi(c', \sigma') = y^* + \xi_{\boldsymbol{\delta}} - X^L$. The result therefore immediately follows for $\hat{\xi}_{\boldsymbol{\delta}} \in (y^*, X^L]$. If $\hat{\xi}_{\boldsymbol{\delta}} \in (X^L, 1)$, then the result follows the proof to Lemma 3, which shows that the Banker's equity value of Equation (10) is maximized at $\lambda_\sigma = 1$ rather than $\lambda_\sigma = 0$.

The second part follows immediately by observing that $\psi(c^*, \sigma^*) = \hat{\xi}_{\boldsymbol{\delta}} \leq y^* = \psi(c', \sigma')$ if $\hat{\xi}_{\boldsymbol{\delta}} \leq y^*$, as well as $\psi(c^*, \sigma^*) = y^* + (1 - X^L) = \psi(c', \sigma')$. $\square$

Lemma 6. *It is never profitable to increase the level of liquidation by decreasing $\xi_{\boldsymbol{\delta}}$.*

Proof. The resources generated by contract c and thus the Banker's payoff are strictly increasing in $\xi_{\boldsymbol{\delta}}$. It follows that $\psi(c, \sigma) > \psi(c'', \sigma'')$ for any contract $c'' \in \mathcal{C}$ featuring $\xi''_{\boldsymbol{\delta}} < \xi_{\boldsymbol{\delta}}$, $\xi''_{\boldsymbol{d}} = 1 - \xi''_{\boldsymbol{\delta}}$ and $\lambda_{\sigma''} = 1$. It also follows that if $\psi(c', \sigma') > \psi(c, \sigma)$, then $\psi(c', \sigma') > \psi(c'', \sigma'')$. $\square$

Lemma 7. *For a given level of $\xi_{\boldsymbol{\delta}}$, no contract $c'' \in \mathcal{C}$, in any equilibrium $\sigma'' \in \Sigma(c'')$, achieves strictly higher payoffs than both contract $c = (0, 0, 1 - \xi_{\boldsymbol{\delta}}, \xi_{\boldsymbol{\delta}})$ and all contracts $c' \in \mathcal{C}$ with $\lambda_{\sigma'} = 0$ in all equilibria $\sigma' \in \Sigma(c')$.*

Proof. The resources generated by a contract $c'' \in \mathcal{C}$ in an equilibrium $\sigma'' \in \Sigma(c'')$ are given by

$$\psi(c'', \sigma'') \;=\; Y\left(c'', \sigma''|L\right) \;+\; \max\left\{\xi_{\boldsymbol{\delta}} - \chi_{\sigma''} X^L, 0\right\}$$

By the strict convexity of $\psi(c'', \sigma'')$ in $\chi_{\sigma''}$, it is maximized at either the highest or the lowest level of $\chi_{\sigma''}$ which can be implemented given $\xi_{\boldsymbol{\delta}}$. The result follows immediately from realizing that contract c achieves the highest level of liquidation given $\xi_{\boldsymbol{\delta}}$ in all equilibria $\sigma \in \Sigma(c)$, and any contract c' s.t. $\lambda_{\sigma'} = 0$ in all equilibria $\sigma' \in \Sigma(c')$ achieves the lowest level of liquidation given $\xi_{\boldsymbol{\delta}}$. □

From here we conclude on the robust optimality of contracts $c = (0, 0, 1 - \hat{\xi}_{\boldsymbol{\delta}}, \hat{\xi}_{\boldsymbol{\delta}})$ and $\lambda_\sigma = 1$ for all $\sigma \in \Sigma(c)$ if $\hat{\xi}_{\boldsymbol{\delta}} > y^*$, and otherwise on the robust optimality of contracts c' that feature $\lambda_{\sigma'} = 0$ for all $\sigma' \in \Sigma(c')$. □

C.3 Proof of Lemma 2

Proof. (i) ⇒ (ii): We show that it cannot be optimal for the Banker to allow for some liquidation to occur. Notice that the Banker can always offer $s = \left(\alpha_i, B_i\right)_{i \in \mathbb{D}} \in \mathcal{S}$ with $(\alpha_i, B_i) = (0, D)$ for all $c_i = \boldsymbol{D}$; and $(\alpha_i, B_i) = (0, 1 + \varepsilon)$ for all $c_i = \boldsymbol{d}$, with $\varepsilon \searrow 0$.

First, we show that s is accepted by all Investors in every equilibrium. Long-term debt Investors are indifferent by construction. Consider an uninsured depositor. Rejecting $\hat{s}$ delivers $r_i(\sigma_{-i}) = d = 1$ in every equilibrium (see Proof to Lemma 1). Consequently, if the Banker can 'repay' in full in period 3, Investors get $1 + \varepsilon \geq r_i(\sigma_{-i})$. Since $d\xi_{\boldsymbol{d}} \leq X^L$, and given the seniority of uninsured deposits, the Banker can promise full repayment, and thus s is accepted by all Investors.

With $\chi_\sigma = 1$, the Regulator optimally chooses

$$b(c, \sigma|L) \;=\; \max\left\{\int_{i \in \mathbb{D}} B_i \, \mathrm{d}i - X^L, 0\right\}$$

which alleviates debt overhang and induces efficient investment. Consequently, offering s yields a payoff of $U_B(s, \sigma_I, \sigma_R, \sigma_e|L) = y^*$ to the Banker. By a similar computation, any security s, which leads to some liquidation, $\chi_\sigma > 0$, generates payoff $U_B(s, \sigma_I, \sigma_R, \sigma_e|L) = Y(c, \sigma|L) < y^*$ Consequently, the Banker prevents liquidation in equilibrium.

(ii) $\Rightarrow$ (i): We show that if $c \notin \mathcal{P}$ there exists an equilibrium σ in which $\lambda_\sigma = 1$ and $\chi_\sigma = 0$. From the proof to Lemma 1, we know that $d \geq 1$. Then $d\xi_{\boldsymbol{d}} > X^L$. Suppose $\theta = L$ and $\lambda_\sigma = 1$. The payoff from withdrawing to each uninsured depositor is given by

$$r_i(c, \sigma|L) \ = \ \frac{X^L}{\xi_{\boldsymbol{d}}}$$

The payoff for each individual uninsured depositor who accepts a is zero. Hence $\lambda_\sigma = 1$ is indeed an equilibrium outcome for $c \notin \mathcal{P}$. $\square$

C.4 Proof of Propositions 2 and 3

Proof. To show optimality (Proposition 3), we need to show that

$$\inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) \geq \sup_{\sigma \in \Sigma(c)} V(c, \sigma) \quad \text{for all } c^* \in \mathcal{P} \cap \mathcal{M} \text{ and } c \notin \mathcal{P} \cap \mathcal{M}$$

To this end, we first compute $V(c^*, \sigma^*)$ for all $c^* \in \mathcal{P} \cap \mathcal{M}$, and show that it is unique. We then compute the lowest and the highest equilibrium payoffs $V(c, \sigma)$ for all contracts $c \in \mathcal{P} \cap \mathcal{M}^\complement$ and all contracts $c \in \mathcal{P}^\complement$.

Consider contract $c^* \in \mathcal{P} \cap \mathcal{M}$. From Lemma 2, we know that $\chi_\sigma = 1$ in all equilibria $\sigma^* \in \Sigma(c^*)$ and $d = 1$. As above, the Banker can always offer $s = \big(\alpha_i, B_i\big)_{i \in \mathbb{D}} \in \mathcal{S}$ with $(\alpha_i, B_i) = (0, D)$ for all $c_i = \boldsymbol{D}$, which long-term debt holders accept by construction. The Regulator's optimal bailouts are given by

$$b(c, \sigma|L) \ = \ \max\left\{\int_{i \in \mathbb{D}} \hat{B}_i \,\mathrm{d}i - X^L, 0\right\}$$

where $\hat{B}_i = B_i$ for all $i \in \mathbb{D}$. Since the government removes all debt overhang, it follows that $D = 1$, and thus $\int_{i \in \mathbb{D}} \hat{B}_i \,\mathrm{d}i = \xi_{\boldsymbol{\delta}} + d\xi_{\boldsymbol{d}} + D\xi_{\boldsymbol{D}} = 1$. Efficient investment ensues in both states. The payoffs are given by

$$V(c^*, \sigma) = p \cdot [X^H + y^* - 1] + (1 - p) \cdot y^* \quad \text{ for all } \sigma \in \Sigma(c^*),\, c^* \in \mathcal{P} \cap \mathcal{M}$$

Note that offering security $s' \in \mathcal{S}$ with $\alpha_i > 0$ for any $i \in \mathbb{D}$ cannot be optimal since it reduces $b(c, \sigma|L)$ directly but cannot induce higher levels of investment.

Next, consider any contract $c \in \mathcal{P} \cap \mathcal{M}^{\complement}$. Following the steps from above, we know that $\chi_\sigma = 1$ and $Y(c, \sigma|L) = y^*$ in all states as well as $d = D = 1$ which implies $\int_{i \in \mathbb{D}} \hat{B}_i \, \mathrm{d}i = \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{\delta}} = 1 - \xi_{\boldsymbol{E}} < 1$. Bailouts are given by

$$b(c, \sigma|L) = \max\left\{1 - \xi_{\boldsymbol{E}} - X^L, 0\right\}$$

and are thus directly reduced by $\xi_{\boldsymbol{E}}$ in the low state. The pricing Equation for equity is given by

$$1 = \gamma \cdot \left\{p \cdot [X^H + y^* - (1 - \xi_{\boldsymbol{E}})] + (1-p) \cdot y^*\right\}$$

and the Banker's payoffs are given by

$$\begin{aligned} V(c, \sigma) &= (1 - \gamma \xi_{\boldsymbol{E}}) \cdot \left\{p \cdot [X^H + y^* - (1 - \xi_{\boldsymbol{E}})] + (1-p) \cdot y^*\right\} \\ &= p \cdot [X^H + y^* - 1] + (1-p) \cdot (y^* - \xi_{\boldsymbol{E}}) \end{aligned}$$

Clearly, setting $\xi_{\boldsymbol{E}} = 0$ maximizes payoffs, and thus

$$\inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) > \sup_{\sigma \in \Sigma(c)} V(c, \sigma) \quad \text{for all } c^* \in \mathcal{P} \cap \mathcal{M} \text{ and } c \in \mathcal{P} \cap \mathcal{M}^{\complement}$$

Next, consider $c \in \mathcal{P}^{\complement}$. Suppose there is no liquidation, $\lambda_\sigma = 0$, which is consistent with equilibrium. To see this, note that the Banker optimally offers $s = \left(\alpha_i, B_i\right)_{i \in \mathbb{D}} \in \mathcal{S}$ with $(\alpha_i, B_i) = (0, d + \varepsilon)$, with $\varepsilon \searrow 0$, for all $c_i = \boldsymbol{d}$ which uninsured depositors accept and $(\alpha_i B_i) = (0, D)$ for all $c_i = \boldsymbol{D}$ which long-term debt holders accept by construction. Note that $\hat{B}_i = B_i$ for all $i \in \mathbb{D}$, and thus

$$b(c, \sigma, \chi_\sigma|\theta) \; = \; \max\left\{\int_{i \in \mathbb{D}} \hat{B}_i - X^\theta, 0\right\}$$

which implies $d = D = 1$. Following exactly the same steps as for contract $c \in \mathcal{P} \cap \mathcal{M}^{\complement}$, the Banker payoffs are again given by

$$V(c,\sigma) = p \cdot [X^H + y^* - 1] + (1-p) \cdot (y^* - \xi_{\boldsymbol{E}})$$

which is again maximized at $\xi_{\boldsymbol{E}} = 0$. Thus,

$$\inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) \;\geq\; \sup_{\sigma \in \Sigma(c)} V(c,\sigma) \quad \text{for all } c^* \in \mathcal{P} \cap \mathcal{M} \text{ and } c \in \mathcal{P}^{\complement}$$

It remains to be shown that

$$\inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) \;>\; \inf_{\sigma \in \Sigma(c)} V(c,\sigma) \quad \text{for all } c^* \in \mathcal{P} \cap \mathcal{M} \text{ and } c \in \mathcal{P}^{\complement}$$

Consider some contract $c \in \mathcal{P}^{\complement}$. By Lemma 2, if $c \in \mathcal{P}^{\complement}$, there exists an equilibrium in which full liquidation occurs, and $Y(c,\sigma|L) = 0$. With full liquidation, there is partial default on uninsured deposits and full default on long-term debt. By the pricing equations, we then know that $D > 1$ and $d > 1$. By the proof of Lemma 1, we know that $\xi_{\boldsymbol{\delta}} + d\xi_{\boldsymbol{d}} + D\xi_{\boldsymbol{D}} < X^H$ and hence $\chi_\sigma = 1$, $b(c,\sigma|H) = 0$ and $Y(c,\sigma|H) = y^*$ for all contracts $c \in \mathcal{P}^{\complement}$. Then

$$V(c,\sigma) \;=\; (1 - \gamma\xi_{\boldsymbol{E}}) \cdot \left\{ p \cdot [X^H + y^* - \xi_{\boldsymbol{\delta}} - d\xi_{\boldsymbol{d}} - D\xi_{\boldsymbol{D}}] + (1-p) \cdot 0 \right\}$$

The pricing equation for equity is given by

$$1 \;=\; \gamma \cdot p \cdot [X^H + y^* - \xi_{\boldsymbol{\delta}} - d\xi_{\boldsymbol{d}} - D\xi_{\boldsymbol{D}}]$$

and hence

$$\begin{aligned} V(c,\sigma) \;&=\; p \cdot [X^H + y^* - \xi_{\boldsymbol{\delta}} - d\xi_{\boldsymbol{d}} - D\xi_{\boldsymbol{D}}] - \xi_{\boldsymbol{E}} \\ &\leq\; p \cdot [X^H + y^* - (1 - \xi_{\boldsymbol{E}})] - \xi_{\boldsymbol{E}} \\ &\leq\; p \cdot [X^H + y^* - 1] \\ &<\; \inf_{\sigma^* \in \Sigma(c^*)} V(c^*, \sigma^*) \end{aligned}$$

for all $c^* \in \mathcal{P} \cap \mathcal{M}$, which completes the proof of optimality.

To show the claim of Proposition 2, consider contract $c \in \mathcal{L}$. From above we know that the Banker optimally offers $s = \big(\alpha_i, B_i\big)_{i \in \mathbb{D}} \in \mathcal{S}$ with $(\alpha_i, B_i) = (0, d + \varepsilon)$, with $\varepsilon \searrow 0$, for all $c_i = \boldsymbol{d}$ which uninsured depositors accept, and $(\alpha_i B_i) = (0, D)$ for all $c_i = \boldsymbol{D}$ which long-term debt holders accept by construction. Note that $\hat{B}_i = B_i$ for all $i \in \mathbb{D}$, and thus

$$b(c, \sigma|\theta) \;=\; \max\left\{\int_{i \in \mathbb{D}} \hat{B}_i - X^\theta, 0\right\}$$

which implies $d = D = 1$. Since $\int_{i \in \mathbb{D}} \hat{B}_i = \xi_{\boldsymbol{d}} + \xi_{\boldsymbol{D}} + \xi_{\boldsymbol{\delta}} \leq X^L < X^H$, it follows $b(c, \sigma|\theta) = 0$ and $Y(c, \sigma|\theta) = y^*$ for all states θ and equilibria $\sigma \in \Sigma(c)$.

Consider next $c \in \mathcal{L}^\complement \cap \mathcal{P}^\complement$. Lemma 2 has shown that there exists an equilibrium $\sigma \in \Sigma(c)$ in which $\chi_\sigma = 0$, which is inefficient. Consider $c \in \mathcal{L}^\complement \cap \mathcal{P}$. Since $b(c, \sigma|\theta) = \max\left\{\int_{i \in \mathbb{D}} \hat{B}_i - X^\theta, 0\right\}$, following the same steps as above yields that $b(c, \sigma|\theta) > 0$ if $c \in \mathcal{L}^\complement$ in at least the low state $\theta = L$, and the claim for inefficiency follows.

$\square$