Discussion Paper Series – CRC TR 224

Discussion Paper No. 448
Project B 03, B 04, B 05

Ad Blocking, Whitelisting, and Advertiser Competition

Martin Peitz [1]
Anton Sobolev [2]
Paul Wegener [3]

August 2023

[1] University of Mannheim & MaCCI, Email: Martin.Peitz@gmail.com
[2] University of Mannheim & MaCCI, Email: anton.sobolev@uni-mannheim.de
[3] University of Mannheim & MaCCI, Email: pwegener@mail.uni-mannheim.de

Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

Ad Blocking, Whitelisting, and Advertiser Competition*

Martin Peitz† Anton Sobolev‡ Paul Wegener§

August 2, 2023

Abstract

Advertisers post ads on publishers' websites to attract the attention of consumers (who visit both available publishers). Since advertisers are competing in the product market, an advertiser may have an incentive to foreclose its competitor through excessive advertising. An ad blocker may be present and charge publishers for whitelisting. We fully characterize the equilibrium in which ad blocker, publishers, and advertisers make strategic pricing decisions. Under some conditions, the ad blocker sells whitelisting to one publisher and both publishers are strictly better off than without the ad blocker. Under other conditions, not only publishers but also advertisers or consumers are worse off.

Keywords: advertising, advertiser competition, ad blocker, whitelisting, imperfect competition

JEL-classification: L12, L13, L15, M37

*We are grateful to Özlem Bedre-Defolie, Bruno Carballa-Smichowski, Fabrizio Ciotti, Philipp Schmidt-Dengler, and various seminar audiences for useful comments. Martin Peitz and Anton Sobolev gratefully acknowledge support by the Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (projects B03, B04 and B05).

†Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; E-Mail: Martin.Peitz@gmail.com; also affiliated with CEPR and CESifo.

‡Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim, Germany; E-Mail: anton.sobolev@uni-mannheim.de.

§Department of Economics and MaCCI, University of Mannheim, 68131 Mannheim; E-Mail: pwegener@mail.uni-mannheim.de.

1 Introduction

Internet advertising is a main source of revenue for digital media. However, ad funding has come under the attack from ad blockers. According to surveys,[1] 34 % of internet users in the U.S. and 39 % in Germany state that they use an “ad blocker” on their computer desktop – third-party software that prevents advertisements from being displayed on websites.[2] The market structure for ad blockers is often monopolistic. For example, in Germany, Adblock Plus is the largest ad-blocking firm with a 95 market share in 2017 (OLG München, 2017, para. 20).

Ad-blocking firms earn money by allowing some select publishers to show ads, a practice called whitelisting.[3] To be part of the whitelist, large publishers (defined as those publishers that generate more than 10 million additional advertising impressions through whitelisting per month) have to pay 30 % of their additional revenue to the ad-blocker (Adblock Plus, n.d.).

In this paper, we model the strategic and welfare effects of whitelisting by a monopoly ad blocker, when users multi-home with publishers, advertisers may multi-home with publishers, and advertisers compete with each other in the product market. More precisely, we consider a parsimonious model with two publishers, two advertisers and a continuum of consumers.

[1] Numbers and sources for a large range of countries are reported by Statista, see https://www.statista.com/statistics/351862/adblocking-usage/

[2] On other devices, ad blocking tends to be less prominent: in 2020, only 17% of internet users in Germany stated in a survey that they use an ad blocker on their mobile device and 12% on their tablet. See https://www.statista.com/statistics/875612/ad-blocker-usage-in-germany-by-device/

[3] Not all types of ads are eligible. The two ad-block firms Adblock Plus and Adblock jointly run the Acceptable Ads Committee (AAC), a committee that determines criteria that define which ads are non-intrusive enough to be shown on whitelisted publishers’ websites (AAC, 2019, p. 27). The criteria refer to size and distinctiveness from the text (Adblock Plus, n.d.). While users can also change the settings on their ad blocker and see no ads at all, most do not, at least in Germany: 90 % of Adblock Plus users keep the default settings and see the filtered ads (Bundesgerichtshof, 2019, para. 3). Several ad blockers (including Adblock Plus) also offer a premium subscription model, but according to a survey from 2020, only 5 % of internet users in Germany stated that they subscribed to such a service whereas 93% stated that they did not; see https://de.statista.com/statistik/daten/studie/873815/umfrage/nutzung-von-kostenpflichtigen-werbeblockern-in-deutschland/.

In the presence of the ad blocker some consumers use the ad blocker, while others do not. We assume that consumers only consider buying a product that was advertised. Thus, it is essential for advertisers to reach consumers with their ads.

Advertisers engage in duopoly competition for consumers. There is one ad slot per publisher. Since consumers visit both publishers, an advertiser can foreclose the other advertiser by buying the ad slot for both publishers and thus achieve a monopoly position in the product market.

Depending on the intensity of competition between advertisers, one advertiser buys both ad slots or each advertiser buys one. An ad blocker will sell whitelisting to one or both publishers. When advertisers price discriminate between consumers who use the ad blocker and consumers who do not, some of the surplus that advertisers and publishers obtain without ad blocking is extracted by the ad blocker.

If advertisers can not price discriminate and thus resort to uniform pricing, the picture is richer. When publishers suffer due to ad blocking, the ad blocker's surplus is extracted only from publishers, from publishers and advertisers, or from publishers and consumers. However, it is also possible that publishers do better with the ad blocker in which case either advertisers or consumers are worse off.

Our model shows the importance of product market competition for the economic effects of an ad blocker. Our main insights hold when an ad blocker sells whitelisting to only one advertiser. They are robust to endogenous ad blocker installation where consumers experience advertising as a nuisance and, as a result, some consumers install an ad blocker only if the overall exposure to ads is reduced. Importantly, the ad blocker will only operate if only one of the publishers is whitelisted. Several extensions complement the main analysis.

Related literature. Previous work on the role of ad blockers has focused on the interaction between ad blocker and publishers abstracting from advertiser competition in the product market (Anderson and Gans, 2011; Despotakis, Ravi, and Srinivasan, 2021; Gritckevich, Katona, and Sarvary, 2022).[4]

[4] We do not address the interaction between ad targeting and ad blocking; see Johnson (2013) for an analysis. In a different vein, Chen and Liu (2022) focus on the signaling role of advertising following Nelson (1974) and Milgrom and Roberts (1986) and analyze how ad blocking affects the advertising cost.

We follow the advertising literature that views advertising as a way to increase the probability that consumers become aware of a product (because they did not know about it or because it was no longer part of their consideration set). According to this informative view of advertising, a consumer only considers buying a product if they have been exposed to an ad about this product (Butters, 1977; Grossman and Shapiro, 1984).[5]

One may suspect that ad blocking benefits consumers since consumers can reduce the intake of advertising, which reduces ad nuisance. However, Anderson and Gans (2011) and Gritckevich, Katona, and Sarvary (2022) show that ad-blocking can have a negative indirect effect on consumer welfare as it may lead to lower quality. In particular, Anderson and Gans (2011) show that publishers may increase ad volume in the presence of ad blockers. Since consumers with a high nuisance cost of advertising install the ad blocker, the presence of the ad blocker changes the composition of those consumers who still see all the ads and makes it more attractive for publishers to increase the ad volume. Our analysis has a different focus: by construction, ad volumes can not increase with the introduction of an ad blocker and quality is exogenous. We uncover a different mechanism by which consumers can suffer from the presence of an ad blocker. The ad blocker may limit the exposure of consumers to ads from different advertisers and thereby lead to a less competitive outcome in the product market. As a result, consumers have to pay higher prices in the product market and thereby suffer from the presence of the ad blocker (under uniform pricing in the product market this holds for consumers who installed the ad blocker and those who did not).

One may also suspect that ad blocking hurts publishers. In particular, one may think that an ad blocker extracts rents without providing additional benefits to publishers and, thus, ad blocking hurts publishers. However, as shown by Despotakis, Ravi, and Srinivasan (2021), competing publishers sometimes benefit from ad blocking in a setting with heterogeneous consumers as ad blocking enables them to discriminate between consumers with a different sensitivity to advertising; for a similar finding with a monopoly publisher, see Aseri et al. (2020). We also find that ad blocking can be beneficial for publishers; in contrast

[5]Starting with Grossman and Shapiro (1984), one stream of this literature considers advertiser competition with differentiated products restricting attention to symmetric settings that have symmetric equilibrium outcomes; see Soberman (2004), Christou and Vettas (2008), and Amaldoss and He (2010).

to Despotakis, Ravi, and Srinivasan (2021), our argument again hinges on the competitive effects in the product market: advertisers compete in prices for consumers and advertising affects the rent extraction possibility of publishers and ad blocker.

Several empirical papers speak to these theoretical findings. Since an ad blocker reduces the number of ads shown to consumers and consumers tend to see display advertising on the internet as a nuisance, one would expect that consumers with an ad blocker are less annoyed and spend more time on a publisher's website; this is indeed the finding of Yan, Miller, and Skiera (2022). In line with the theoretical prediction that ad blocking hurts publishers (Anderson and Gans, 2011; Gritckevich, Katona, and Sarvary, 2022), ad blocking in commercial television tends to decrease tv channel revenues (Wilbur, 2008) and ad blocking for display ads on publishers' websites reduces publishers' revenues (Shiller, Waldfogel, and Ryan, 2018). These lower publisher revenues may then feed into a decrease in the quality of publishers' websites and overturn the positive effect of ad blocking on usage once publishers have responded to the revenue reduction.[6] The finding by Shiller, Waldfogel, and Ryan (2018) that ad blocking reduces consumers' site visits of publishers' websites can be seen as supportive evidence. Todri (2022) provides evidence in line with ad blocking affecting product market competition, which is the novel mechanism in our model: they find that ad blocking significantly decreases spending for products that consumers had not been exposed to in the past and partially shifts it to products they are familiar with.

More broadly, our paper relates to the work on two-sided platforms that cater to two groups, sellers and buyers, and that manage competition between sellers (e.g., Nocke, Peitz, and Stahl, 2007; Hagiu, 2009; Belleflamme and Peitz, 2019; Karle, Peitz, and Reisinger, 2020; Teh, 2022) as in our paper the ad blocker affects competition between advertisers in the product market. Our paper also relates to work on media platforms since publishers do not make profits directly from consumers, but charge advertisers (Anderson and Coate, 2005). In these works, in contrast to this paper, network effects figure prominently. Advertiser competition not only features in the literature on the economics of advertising (Butters, 1977; Grossman and Shapiro, 1984), but it has also been introduced in models for competing media platforms (Dukes and Gal-Or, 2003). A key economic mechanism in our model is that an

[6] The wide adoption of ad blockers may also make it more attractive for publishers to raise paywalls.

advertiser may advertise with both publishers and thus foreclose its competitor in the product market, which is reminiscent of Prat and Valletti (2022) who analyzed the competitive effects of media mergers when consumers multi-home. However, this literature does not consider the role of ad blockers.

The paper proceeds as follows. In section 2 we present the base model in absence of the ad blocker and analyze how publishers, advertisers and consumers interact; here, we introduce two examples of product market competition that we use throughout for illustration (horizontal product differentiation à la Hotelling with linear and quadratic transport costs). In section 3 we introduce the model with the ad blocker and an exogenous fraction of consumers installing the ad blocker. This model is analyzed in Section 4 in two different product market settings: in the first, advertisers can price discriminate between consumers who installed the ad blocker and those who did not; in the second, advertisers must set uniform retail prices. In section 5 we endogenize the consumers' decision on whether or not to install the ad blocker. Section 6 concludes. All proofs are relegated to Appendix A. The full analysis of our two examples of product market interaction are contained in Appendices B.1 and B.2.

2 Preliminaries: Model and analysis without an ad blocker

In a given product category, each publisher can post at most one ad that can be seen by a unit mass of consumers.[7] A publisher bundles own content with advertising and makes money by selling consumer attention to advertisers. Advertisers can only make a sale if they attract attention through at least one of the publishers. More specifically, there are two advertisers competing in the same product category. There are two publishers that are both frequented by consumers – in other words, consumers are multi-homers. Thus, for an advertiser it is sufficient to show an ad via one publisher to get the consumer's attention. If one advertiser advertises on one of the publisher's website and the other advertiser on the other publisher's

[7]The limit to one ad slot per publisher can be motivated by consumers' limited attention for ads when visiting a publisher's website. If consumers dislike advertising and can pay attention to at most one ad on a website, a publisher benefits the most by posting only one ad.

website, consumers learn about both products and there is duopoly competition. In the equilibrium in the product market, the gross profit of each advertiser is denoted by π^d. If an advertiser does not advertise, its gross profit will be equal to zero. Thus, each advertiser is willing to pay up to π^d to show the ad, provided the other advertiser shows an ad with the other publisher.

We consider the timing: (1) Publishers simultaneously set a price for the ad, after which (2) advertisers sequentially decide whether to accept the offer,[8] Upon accepting, (3) advertisers simultaneously set product prices, and, finally, (4) consumers make purchase decisions.

Suppose that publishers have set the same fee f. If each of the advertisers agrees to pay to advertise with one publisher, the net profit of each advertiser is $\pi^d - f$. Instead, one advertiser could exclude the other advertiser by buying the ad slot on both websites. This would give a net profit of $\pi^m - 2f$, where π^m is the maximal gross profit when the advertiser is a monopolist in the product market. If $f > \pi^d$, it does not pay for advertiser B to buy the second slot. Thus, advertiser A will not buy the second slot at a fee above π^d. Hence, if $\pi^m > 2\pi^d$, publishers set $f = \pi^d$ and advertiser A buys both ad slots. Here, advertiser A obtains a net surplus of $\pi^m - 2\pi^d$.

By contrast, if $\pi^m < 2\pi^d$, publishers extract the full gross profit from advertisers by setting $f = \pi^d$. Each advertiser buys one slot and there will be duopoly competition between advertisers.

Duopoly industry profits are larger than monopoly profits if the advertisers' products are sufficiently differentiated. In this case both advertisers buy an ad slot. Otherwise, one advertiser advertises on both websites. We illustrate the relationship between monopoly profits and duopoly industry profits in two versions of the well-known Hotelling model of price competition with differentiated products; in Example 1 we assume linear transport costs and in Example 2 quadratic transport costs.

Example 1: Hotelling with linear transport costs. *We first compare monopoly to industry duopoly profits in the Hotelling model with linear transport cost. Consumers are uniformly distributed on the unit interval, are of mass 1, and demand one unit of one of*

[8] If a slot has been taken by the first advertiser, the second advertiser is excluded from the respective publisher. By assuming sequential acceptance decisions, we avoid mixed-strategy equilibria.

the products or do not buy in the market. Consumer x obtains net utility $v - p_i - t|x - l_i|$ from product i sold at price p_i at location l_i; the net utility of not buying is normalized to 0. Advertiser A sells a product located at 0 and advertiser B at 1 on the unit interval. Both have constant marginal costs of production c. Advertisers set retail prices and, after observing prices of advertised products, consumers make purchasing decisions.

If only advertiser A advertisers it makes monopoly profit

$$\pi^m = \begin{cases} \frac{(v-c)^2}{4t}, & \text{if } c < v \leq c + 2t, \\ v - t - c, & \text{if } v > c + 2t. \end{cases}$$

If consumers see ads from both advertisers, a symmetric duopoly prevails. For some parameters, there are multiple equilibria. Then, we select the equilibrium that maximizes industry profits (which features symmetric choices). The equilibrium duopoly profit is

$$\pi^d = \begin{cases} \frac{(v-c)^2}{4t}, & \text{if } \frac{v-c}{t} < 1, \\ \frac{1}{2}\left(v - c - \frac{t}{2}\right), & \text{if } \frac{v-c}{t} \in \left[1, \frac{3}{2}\right], \\ \frac{t}{2}, & \frac{v-c}{t} > \frac{3}{2}. \end{cases}$$

For the derivation, see Appendix B.1. We find that $\pi^m \geq 2\pi^d$ if and only if $\frac{v-c}{t} \geq 2$ (see Lemma 2 in Appendix B.1).

Example 2: Hotelling with quadratic transport costs. *We modify the previous example by assuming quadratic transport costs; that is, consumer x obtains net utility $v - p_i - t|x - l_i|^2$ from product i sold at price p_i at location l_i. The monopoly profit is*

$$\pi^m = \begin{cases} \frac{2}{3}\sqrt{\frac{(v-c)^3}{3t}}, & \text{if } \frac{v-c}{t} \leq 3, \\ v - t - c, & \text{if } \frac{v-c}{t} > 3. \end{cases}$$

As in the example with linear transport costs, for some parameters, there are multiple equilibria, and we select the equilibrium that maximizes industry profits (which features symmetric choices). The equilibrium duopoly profit is

$$\pi^d = \begin{cases} \frac{2}{3}\sqrt{\frac{(v-c)^3}{3t}}, & \text{if } \frac{v-c}{t} < \frac{3}{4}, \\ \frac{1}{2}\left(v - \frac{t}{4} - c\right), & \text{if } \frac{v-c}{t} \in \left[\frac{3}{4}, \frac{5}{4}\right], \\ \frac{t}{2}, & \text{if } \frac{v-c}{t} > \frac{5}{4}. \end{cases}$$

We find that $\pi^m \geq 2\pi^d$ *if and only if* $\frac{v-c}{t} \geq (\frac{27}{4})^{1/3}$ *(see Lemma 4 in Appendix B.2).*

Both examples feature discrete consumer choice with perfectly negatively correlated match values and full consumer participation. In a discrete choice with independent match values due to Perloff and Salop (1985) there would only be partial market coverage. Also in such an example, industry duopoly profits are larger than monopoly profits, if the degree of product differentiation is sufficiently large.[9]

In the absence of an ad blocker, we have the following result on pure strategy subgame-perfect Nash equilibria (the proof is relegated to Appendix A).

Proposition 1. *Consider an environment without an ad blocker. If* $\pi^m < 2\pi^d$*, in any equilibrium, both publishers set fees* $f_1 = f_2 = \pi^d$ *and each advertiser buys an ad slot. If* $\pi^m > 2\pi^d$*, in any equilibrium, both publishers set fees* $f_1 = f_2 = \pi^d$ *and advertiser* A *buys the ad slot on each publisher's website. In the borderline case, both publishers set fees* $f_1 = f_2 = \pi^d$ *and either advertiser* A *buys both slots or each advertiser buys one slot.*

If $\pi^m > 2\pi^d$, advertiser A buys both ad slots. We note that buying the second ad slot may look like a wasted expense since all consumers are reached in any case. However, advertiser A buys the second slot to foreclose advertiser B; as alluded to in the introduction, this logic is reminiscent of Prat and Valletti (2022).[10]

Table 1 reports the surplus for the different market participants depending on whether or not monopoly profits are larger than duopoly industry profits in the product market. In both cases, each publisher makes a profit of π^d. In the latter case, publishers fully extract the advertisers' gross profit; in the former, advertisers obtain a net surplus of $\pi^m - 2\pi^d$.

The intuition for the lack of full rent extraction by publishers in the former case is that both publishers provide access to the consumers' attention. If a publisher asked for a higher fee, advertiser A would stop buying the ad without endangering its monopoly position in the product market. Up until π^d the Bertrand undercutting logic applies to publishers because

[9]The comparison of monopoly and duopoly industry profits can be analyzed in other imperfect competition models where a parameter different than the degree of product differentiation differs across industries; see, for instance, the discussion by Karle, Peitz, and Reisinger (2020).

[10]In their setting, an incumbent firm can reach consumers in any case; it may then take the ad slot of each publisher to foreclose a potential competitor for whom advertising is necessary to reach consumers.

Table 1: *Net surplus without an ad blocker*

	$\pi^m \leq 2\pi^d$	$\pi^m > 2\pi^d$
Publisher surplus	$2\pi^d$	$2\pi^d$
Advertiser surplus	0	$\pi^m - 2\pi^d$
Consumer surplus	$CS(p^d, p^d)$	$CS(p^m, \infty)$

for any fees f_1, f_2 with $\max\{f_1, f_2\} > \pi^d$ advertiser A will drop the publisher with the higher fee, while, at equal fees above π^d it would randomize between the two; advertiser B will not buy a slot at those fees.

3 The model with an ad blocker

We now introduce an ad blocker that offers whitelisting to publishers and asks for a uniform fee to be whitelisted. A publisher who buys whitelisting makes sure that the ad on its website is shown to all consumers, including those who installed the ad blocker. By contrast, an ad from a publisher who does not pay for whitelisting will not be visible to those consumers.

For an ad to be visible to a consumer without an ad blocker, the corresponding advertiser must obtain an ad slot with at least one publisher. For an ad to be visible to a consumer with an ad blocker, the corresponding advertiser must obtain an ad slot with at least one publisher and at least one of those publishers must have bought whitelisting from the ad blocker.

The timing is as follows:

1. The ad blocker sets a uniform whitelisting fee.

2. Publishers simultaneously decide whether to accept the ad blocker's offer.

3. Publishers simultaneously set the advertising fee.

4. Advertisers arrive in sequential order and decide on which publishers to advertise.

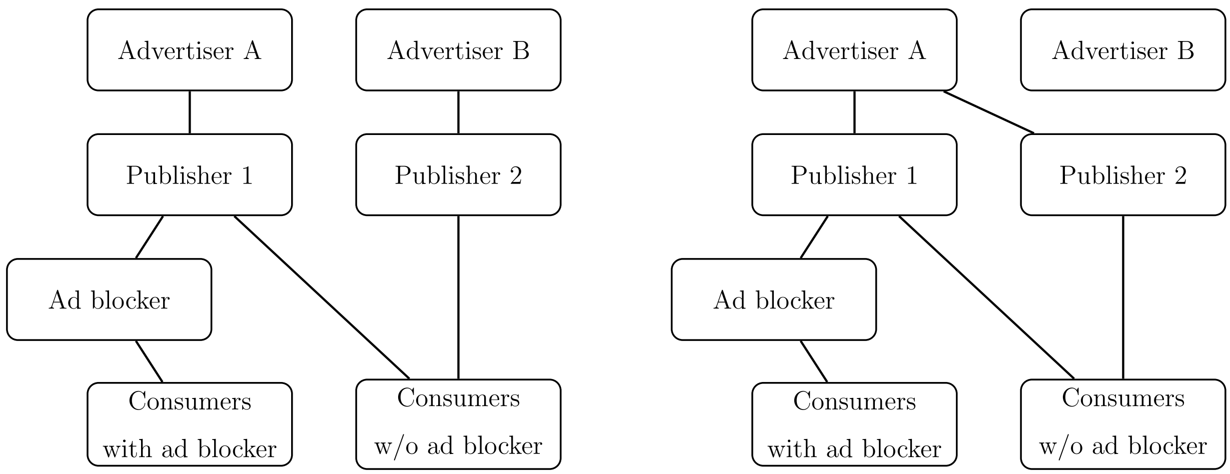

Figure 1: *Consumer choice sets when one publisher whitelists: in the left panel, advertisers A and B buy an ad slot each; in the right panel, advertiser A buys both ad slots*

5. Advertisers simultaneously set retail prices: advertisers price discriminate between consumers who use an ad blocker and those who do not (version a) or advertisers set the same retail price for all consumers (version b).

Figure 1 illustrates the consumer choice sets in two cases in which one publisher has bought whitelisting and the other has not. In the figure, a consumer can only buy those products for which there is a connecting line between the advertiser and consumer. In the figure on the left-hand side, each advertiser buys one ad slot; thus consumers without an ad blocker can choose between the two products, whereas consumers with the ad blocker can only buy the product from the advertiser who bought the ad slot on the whitelisted publisher. In the figure on the right-hand side, advertiser A buys both ad slots and thus no consumer can buy from advertiser B. In the following sections we establish conditions for such choice sets to emerge in equilibrium.

A fraction α of consumers use the ad blocker. We consider three different models of how consumers use the ad blocker. In the main model ("Fixed ad blocker installation"), we treat α as an exogenous parameter. Both, one, or none of the publishers buys whitelisting. If both publishers do, we are back to the outcome as in the previous section. If neither does, publishers make money only from consumers who are not using the ad blocker.

Ad blocker installation is endogenized in Section 5. In one version ("Upfront ad blocker installation"), a fraction α of consumers experience an ad nuisance and install the ad blocker if they expect fewer ads with an ad blocker. Here, consumers make the installation decision at stage 0. In the other version ("Committed ad blocker and subsequent ad blocker installation"), a fraction α of consumers experience an ad nuisance and install the ad blocker if the ad blocker has committed to reducing the amount of advertising.

4 Fixed ad blocker installation

4.1 Fixed ad blocker installation and retail price discrimination

A fraction α of consumers use the ad blocker. We treat α as an exogenous parameter; ad blocker installation becomes endogenous in Section 5. Both, one, or none of the publishers buy whitelisting. If both publishers do, we are back to the outcome as in the previous section. If neither does, publishers make money only from consumers who are not using the ad blocker.

The novel case arises if the ad of one advertiser (e.g., advertiser A) appears on the whitelisted website, whereas the ad of the other advertiser (advertiser B) is shown on the other website. In this case, advertiser A has exclusive access to the fraction α of consumers with an ad blocker.

If advertisers can price-discriminate between those consumers who use an ad blocker and those who do not, advertiser A makes per-consumer profit π^m from consumers with an ad blocker; both advertisers make per-consumer profit π^d from consumers without an ad blocker (by Proposition 1). Thus, advertiser A obtains profit $\alpha\pi^m + (1-\alpha)\pi^d$ and advertiser B obtains $(1-\alpha)\pi^d$. This implies that advertiser A is willing to pay the increment $\alpha\pi^m$ to advertise on website 1 instead of 2. In other words, publishers set fees $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and $f_2 = (1-\alpha)\pi^d$. Whitelisting is worth $\alpha\pi^m$, which can be extracted by the ad blocker. This is the fee set by the ad blocker in equilibrium if the ad blocker does not prefer to whitelist both advertisers in which case it can charge a whitelisting fee of $\alpha\pi^d$ and induce both publishers to accept the offer. The reason is that the publisher's gross profit would drop from π^d to $(1-\alpha)\pi^d$ if one of the publishers refused the offer. Hence, the ad blocker could make $2\alpha\pi^d$. The ad blocker prefers to admit only one publisher if $\alpha\pi^m > 2\alpha\pi^d$, which

Table 2: *Net surplus under price discrimination*

	$\pi^m \leq 2\pi^d$	$\pi^m > 2\pi^d$
Ad blocker surplus	$2\alpha\pi^d$	$\alpha\pi^m$
Publisher surplus	$2(1-\alpha)\pi^d$	$2(1-\alpha)\pi^d$
Advertiser surplus	0	$(1-\alpha)(\pi^m - 2\pi^d)$
Consumer surplus	$CS(p^d, p^d)$	$CS(p^m, \infty)$

is equivalent to monopoly profits being larger than industry duopoly profits. Otherwise, if $\pi^m < 2\pi^d$, for given α, the ad blocker would provide whitelisting to both publishers. Overall we see that the ad blocker can extract all profits made from consumers who have installed the ad blocker.

The following proposition summarizes this discussion.

Proposition 2. *Consider an environment with an ad blocker and price-discriminating advertisers. If $\pi^m < 2\pi^d$, then the ad blocker provides whitelisting to both publishers at a price $\alpha\pi^d$, both publishers buy whitelisting and set $f_1 = f_2 = \pi^d$, and each advertiser buys an ad slot.*
If $\pi^m \geq 2\pi^d$, then the ad blocker offers whitelisting at price $\alpha\pi^m$ and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. Advertiser A buys the ad slot on each publisher's website.

Table 2 reports the resulting net surplus for ad blocker, publishers, advertisers, and consumers. We recall that the condition $\pi^m > 2\pi^d$ holds if there is not too much product differentiation in the Hotelling example.

If $\pi^m < 2\pi^d$, there is no difference to the model without an ad blocker except that now each publisher pays $(1-\alpha)\pi^d$ to the ad blocker and, thus, parts of the publishers' rents are shifted to the ad blocker. Advertisers and consumers are not affected by the presence of the ad blocker, and total surplus is unchanged.

Consider now the opposite case $\pi^m \geq 2\pi^d$. With the ad blocker there is exclusive whitelisting and advertiser A obtains gross profits π^m and the other advertiser is not active. Again, consumers are not affected by the presence of the ad blocker, and total surplus is unchanged. Without the ad blocker, advertiser A obtains a net profit of $\pi^m - 2\pi^d$, while with the ad blocker it obtains $\pi^m - [\alpha\pi^m + 2(1-\alpha)\pi^d] = (1-\alpha)(\pi^m - 2\pi^d)$. Advertiser A is better off without the ad blocker if $\pi^m - 2\pi^d > (1-\alpha)(\pi^m - 2\pi^d)$, which always holds. Each publisher makes a net profit of $(1-\alpha)\pi^d$ with the ad blocker and thus is worse off than without the ad blocker. Overall, the ad blocker makes a profit at the expense of publishers *and* advertisers; total surplus and consumer surplus are not affected.

Corollary 1. *Consider an environment with price-discriminating advertisers. When an ad blocker enters, it extracts fraction α of the surplus from publishers and advertisers (in cases when the latter have any surplus at all).*

The result is in line with the complaints raised by publishers about the negative impact of ad blocking on their net revenues. However, when $\pi^m > 2\pi^d$, the ad blocker's profits materialize not only at the expense of publishers but also of advertiser A. As developed in Section 2, Examples 1 and 2 provide microfoundations for π^m and π^d and thereby the amount of rent shifting from publishers (and advertiser A when $\pi^m > 2\pi^d$) to the ad blocker. Consumers are unaffected when abstracting from the possible impact of ad load on consumer surplus.[11]

4.2 Fixed ad blocker installation and uniform retail prices

If advertisers cannot price-discriminate between consumers with and without an ad blocker and thus have to set uniform retail prices, a richer picture emerges. Under some conditions consumer surplus is no longer neutral to the introduction of an ad blocker: in some instances, consumers are better off and, under others, worse off with ad blocking. While under discriminatory retail pricing publishers are unambiguously worse off after the introduction of an ad blocker, there are now circumstances in which they are better off.

[11] With ad blocking, when $\pi^m > 2\pi^d$, the fraction α of consumers who use the ad blocker are exposed to less advertising since ads on the non-whitelisted publisher are blocked; we discuss nuisance costs of advertising in Section 5.

When only one publisher whitelists and both advertisers buy one ad slot each, product market competition is asymmetric: the advertiser with the whitelisted publisher enjoys a monopoly position regarding the consumers who installed the ad blocker, while there is duopoly competition for consumers without an ad blocker. Under uniform retail pricing these two market segments are interdependent and compared to the setting with price discrimination, the advertiser with the whitelisted publisher is a less aggressive competitor in the competitive consumer segment.[12]

For the analysis we need to introduce some extra notation. We denote profits as a function of prices by $\pi(p_A, p_B) = \pi_A(p_A, p_B) = \pi_B(p_B, p_A)$. If the competitor cannot reach consumers, an advertiser is in a monopoly position and makes a gross profit of $\pi(p, \infty)$ at price p and, at the profit-maximizing price p^m, we have $\pi(p^m, \infty) = \pi^m$. Under symmetric duopoly competition, provided that there is a unique price equilibrium or that we select the symmetric equilibrium among multiple equilibria, Nash duopoly prices are p_A^* and p_B^* with $p_A^* = p_B^* = p^d$ and yield gross profits of $\pi(p^d, p^d) = \pi^d$ for each advertiser.

When advertiser A is visible to all consumers and advertiser B only to consumers without an ad blocker, the gross profit of advertiser A is $\alpha\pi(p_A, \infty) + (1-\alpha)\pi(p_A, p_B)$ and advertiser B's gross profit is $(1-\alpha)\pi(p_B, p_A)$. We assume that there is a unique price equilibrium with prices p^w for the advertiser with the whitelisted publisher and p^{nw} for the other. Equilibrium gross profits of advertiser A are denoted by $\pi^w \equiv \alpha\pi(p^w, \infty) + (1-\alpha)\pi(p^w, p^{nw})$ and advertiser B's gross profit as $\pi^{nw} \equiv (1-\alpha)\pi(p^{nw}, p^w)$. Clearly, $\pi^d(p^{nw}, p^w) > \pi^d$ and $\pi^m > \pi(p^w, \infty)$ for $p^w \in (p^d, p^m)$ and $\alpha \in (0, 1)$.

We show the uniqueness of the asymmetric duopoly equilibrium in our two Hotelling examples (see Appendix B.1 and Appendix B.2).[13]

[12] Such asymmetric competition with uniform pricing has been looked at in the context of universal service obligations; see Anton, Vander Weide, and Vettas (2002) and Valletti, Hoernig, and Barros (2002); the latter considers a Hotelling duopoly similar to our example.

[13] More generally, we note that a sufficient condition to use first-order conditions to characterize the price equilibrium is the log-concavity of demand. The complication in our model is that the joint demand of the advertiser with the whitelisted publisher is the sum of its demand in the monopoly and the duopoly segment. Since log-concavity is not an additive property it is not sufficient to show that the demand in each segment is log-concave.

Proposition 3. *Consider an environment with an ad blocker and advertisers setting uniform prices.*

- *If $\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$, then the ad blocker provides whitelisting to both publishers at price $\pi^d - \pi^{nw}$, with both publishers buying whitelisting and setting $f_1 = f_2 = \pi^d$.*
- *If $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$, then the ad blocker whitelists a single publisher at price $\pi^w - (1-\alpha)\pi^d$. The whitelisted publisher sets its fee equal to π^w and the non-whitelisted publisher sets π^{nw}.*

If $\pi^m > f_1 + f_2$, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.

In the proof, which is relegated to Appendix A, we show that, in equilibrium, all ad slots will be filled and that advertiser A takes at least one of the two slots. This result can be seen as follows. Suppose that advertiser A did not buy any slots because it deemed them too expensive. Then advertiser B will buy only one ad slot, as it does not pay to reach consumers without an ad blocker through both publishers. It will buy ad slot 1 if $f_1 \leq \max\{\pi^m, f_2 + \alpha\pi^m\}$ and ad slot 2 if $f_2 \leq \max\{(1-\alpha)\pi^m, f_1 - \alpha\pi^m\}$, there would be asymmetric Bertrand competition between publishers for advertiser B resulting in $f_2 = 0$ and $f_1 = \alpha\pi^m$. Clearly, at those fees, advertiser A has a strict incentive to buy at least one ad slot. Therefore, for any fees such that advertiser A would not buy any ad slot, at least one of the publishers has an incentive to reduce its fee. Thus, in equilibrium, advertiser A buys at least one slot. Suppose that A takes slot 1 only. Advertiser B is willing to pay π^{nw} for slot 2 and publisher 2 has an incentive to sell the slot. Suppose that A takes slot 2 only. Advertiser A is willing to pay π^w for slot 1 and publisher 1 has an incentive to sell the slot. Hence, if advertiser A buys only one slot, advertiser B will buy the second. As a result, one of the following situations must arise in equilibrium: advertiser A buys both slots; advertiser A buys slot 1 and advertiser B slot 2; or advertiser A buys slot 2 and advertiser B slot 1.

If advertiser A buys both slots and both publishers whitelist, all consumers are reached through both publishers and the advertiser could still reach all consumers if it dropped one of the publishers. Similarly, if advertiser A buys both slots and one publisher whitelists, all consumers without the ad blocker are reached through both publishers and the advertiser

Table 3: *Net surplus under uniform pricing*

	$\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$		$\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$	
	$\pi^m \leq 2\pi^d$	$\pi^m > 2\pi^d$	$\pi^m \leq \pi^w + \pi^{nw}$	$\pi^m > \pi^w + \pi^{nw}$
Ad blocker surplus	$2(\pi^d - \pi^{nw})$	$2(\pi^d - \pi^{nw})$	$\pi^w - (1-\alpha)\pi^d$	$\pi^w - (1-\alpha)\pi^d$
Publisher surplus	$2\pi^{nw}$	$2\pi^{nw}$	$\pi^{nw} + (1-\alpha)\pi^d$	$\pi^{nw} + (1-\alpha)\pi^d$
Advertiser surplus	0	$\pi^m - 2\pi^d$	0	$\pi^m - (\pi^w + \pi^{nw})$
Consumer surplus	$CS(p^d, p^d)$	$CS(p^m, \infty)$	$\alpha CS(p^w, \infty)$ $+(1-\alpha)CS(p^w, p^{nw})$	$CS(p^m, \infty)$

could still reach these consumers if it dropped the non-whitelisted publishers. The only reason advertiser A may still want to buy both slots is to foreclose advertiser B.

Whether there is whitelisting of one or both publishers depends on how product market competition plays out among advertisers. If $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$, then, in equilibrium, there is whitelisting by one publisher. In such an equilibrium, publishers set fees equal to $f_1 = \pi^w$ and $f_2 = \pi^{nw}$, respectively. The product market outcome depends on whether or not inequality $\pi^m > f_1 + f_2$ is satisfied. If it is, advertiser A buys both slots and operates as a monopolist in the product market (and obtains a strictly positive net surplus). If it is not, there is some competition for users with the ad blocker and all consumers are better off than in the reverse case.

Table 3 reports equilibrium surplus for consumers, advertisers, publishers, and the ad blocker under all possible constellations. In the two columns on the left, both publishers buy whitelisting, while in the two columns on the right only one publisher does. In the first and third column, each advertiser buys one ad slot, while in the second and fourth column advertiser A buys both ad slots. Consumer surplus (per unit mass of consumers) is a function of the prices p_A, p_B set by the advertisers that reach consumers, denoted by $CS(p_A, p_B)$.

Example 1 continued. *In Appendix B.1, we fully characterize the set of pure strategy equilibria restricting attention to parameters* $(v-c)/t \in (3/2, 7/2)$. *When* $(v-c)/t > 3/2$, *we avoid the multiplicity of pure strategy equilibria that would arise outside this parameter* *region. With $(v-c)/t < 7/2$, we avoid possible mixed strategy equilibria. Therefore, for $(v-c)/t \in (3/2, 7/2)$, we can show that there is a unique pure strategy equilibrium for all $\alpha \in (0,1)$.*[14]

In Lemma 3 in Appendix B.1 we show that, for any given $\alpha \in (0,1)$, asymmetric duopoly industry profits satisfy $\pi^w + \pi^{nw} \in (\min\{2\pi^d, \pi^m\}, \max\{2\pi^d, \pi^m\})$. Thus, asymmetric duopoly industry profits lie between symmetric duopoly industry profits and monopoly profits. Then, the outcome cannot be in the third column of Table 3: if $2\pi^d < \pi^w + \pi^{nw} < \pi^m$, then $\pi^w + 2\pi^{nw} > 2\pi^d + \pi^{nw} > 2\pi^d + (1-\alpha)\pi^d = (3-\alpha)\pi^d$. We also show that inequality $\pi^m + 2\pi^{nw} \geq (3-\alpha)\pi^d$ is satisfied if and only if $(v-c)/t > 2$ (Proposition 8 in Appendix B.1), which is also the condition that $\pi^m > 2\pi^d$ (Lemma 2 in Appendix B.1). Thus, only the second and the fourth column in Table 3 apply and we have the following result: If $v \geq c+2t$, then the ad blocker whitelists a single publisher at price $\pi^m - (1-\alpha)\pi^d$; the whitelisted publisher sets its fee equal to π^w and the non-whitelisted publisher sets π^{nw}; and advertiser A buys the ad slot on each publisher's website. If $v < c+2t$, then the ad blocker whitelists both publishers at price $\pi^d - \pi^{nw}$; the publishers set fees π^d; and each advertiser buys one slot each.

To summarize, in Example 1, the results reported so far are qualitatively the same as in the setting with discriminatory pricing.

Example 2 continued. *With quadratic transport costs, given that one advertiser has a slot with the whitelisted publisher and the other with the non-whitelisted publisher, we analyze price equilibra in the parameter range $\frac{v-c}{t} \in \left(\frac{3}{2}, \frac{7}{2}\right)$ (see Appendix B.2).*

In Proposition 10 in Appendix B.2 we show that there exists a non-empty parameter region such that the inequality $\pi^w + 2\pi^{nw} \geq (3-\alpha)\pi^d$ holds and therefore the third column of Table 3 applies.

In Figure 2, we report how the parameters in the example map into the configurations in Table 3. The upper line gives the parameter values $(\alpha, (v-c)/t))$ such that $\pi^m = \pi^w + \pi^{nw}$ and the lower line gives the parameter values $(\alpha, (v-c)/t))$ such that $\pi^w + 2\pi^{nw} = (3-\alpha)\pi^d$. These lines delineate three parameter regions. In the bottom parameter region, both publishers

[14]Further below, we also consider a region with higher values of $(v-c)/t$ in which there also exists a unique pure strategy equilibrium but which is characterized differently.

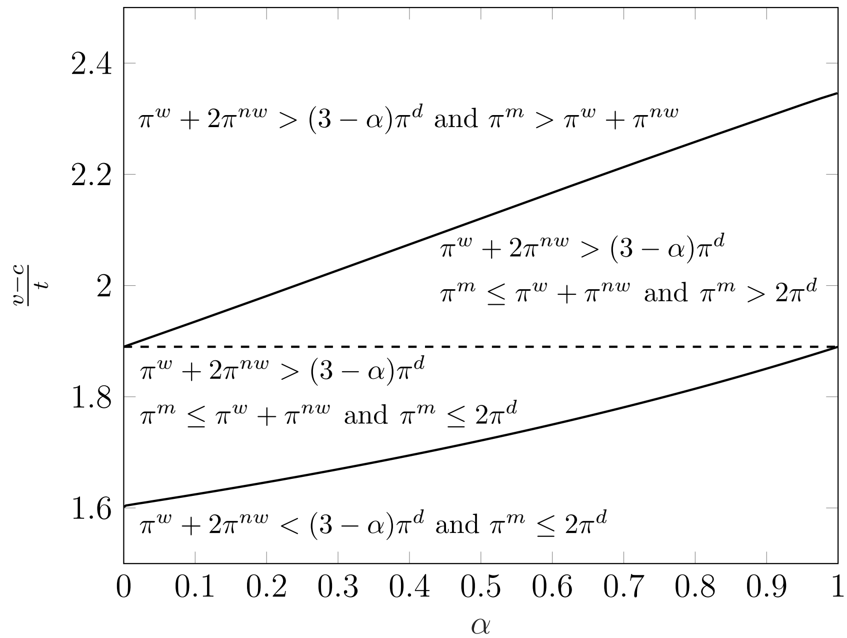

Figure 2: *Configurations of Table 3 in the Hotelling model with quadratic transport costs*

whitelist and each advertiser buys an ad slot – this corresponds to the first column of Table 3. In the top parameter region, one publisher whitelists and advertiser A buys both ad slots – this corresponds to the fourth column of Table 3. In the intermediate region, one publisher whitelists and sells its ad slot to advertiser A, while advertiser B buys the ad slot from the non-whitelisted publisher – this corresponds to the third column of Table 3. In this intermediate range, there is asymmetric competition in the product market along the equilibrium path.

While the conditions have been checked in particular functional form settings, the question may be what is the key difference between the two examples. When only advertiser A is visible to all consumers while advertiser B is visible to only a fraction of consumers, pricing will be asymmetric. In Example 2 this tends to further relax price competition between the two advertisers (in addition to the fact that an advertiser is active in a monopoly segment and may therefore set a higher price in any case) because demand reacts less sensitively to a price change starting from asymmetric prices than starting from symmetric prices. By contrast, the slope of the demand curve of advertiser A is constant in Example 1.

Exclusive whitelisting One may wonder if the ad blocker can do better if it were able to commit to exclusive whitelisting (i.e., a commitment to sign with at most one publisher does not help the ad blocker; if more than one asks for exclusive whitelisting, a random draw determines which publisher is selected). However, as we argue next, it is not in the interest of the ad blocker to do so. If one publisher is willing to accept the offer of exclusive whitelisting the other does as well. This implies that a deviation by a publisher not to accept will imply that the other one obtains exclusive whitelisting. The deviating publisher can then get $\pi^{nw} = (1-\alpha)\pi(p^{nw}, p^w)$, which is greater than $(1-\alpha)\pi^d$ for $p^w > p^d$. Thus, the ad blocker would be strictly worse off if it committed to exclusive whitelisting. We can also see this by taking a look at the payments received by the ad blocker. Under exclusive whitelisting, when both publishers ask for exclusive whitelisting, the ad blocker can extract $t^u \equiv \alpha\pi^w - \pi^{nw}$ because, when they do not accept to pay the ad blocker's fee, the other publisher will be whitelisted and, thus, the deviating publisher makes π^{nw}, which constitutes the publisher's outside option.[15]

Comparison to no ad blocking How does ad blocking affect total surplus and who pays for the ad blocker? In our two examples of product market competition, we compare how surpluses change with the introduction of an ad blocker.

The introduction of an ad blocker may reduce total surplus, increase it, or leave it unchanged. For the introduction of the ad blocker to reduce total surplus, it must hold that the ad blocker will induce an allocation in the product market such that consumers who installed the ad blocker will only consume advertiser A's product at price $p^w > p^d$ and consumers without the ad blocker face prices (p^{nw}, p^w) instead of (p^d, p^d) in the absence of an ad blocker. In other words, advertisers must be in asymmetric duopoly in the presence of the ad blocker, whereas they would be in symmetric duopoly in the absence of the ad blocker. For the introduction of an ad blocker to reduce total surplus former, advertisers must be in asymmetric duopoly in the presence of the ad blocker, whereas advertiser A would be monopolist in the product market in the absence of the ad blocker.

[15] In our model the ad blocker sets a price for whitelisting. Enriching the strategy of the ad blocker, one could allow the ad blocker to commit to exclusive whitelisting. However, the ad blocker is better off not committing to doing so.

The same holds for consumer surplus since, in our base model, consumers care only about the product market outcome. Thus, consumer surplus is unaffected by the presence of the ad blocker as long as product market competition does not change. Otherwise, consumers are better from the presence of the ad blocker if this leads to asymmetric duopoly instead of monopoly, while they are worse off if this leads to asymmetric duopoly instead of symmetric duopoly. While these are the possibilities in the general reduced-form setting, one may wonder which of the possible surplus effects arise in our examples.

Example 1 continued. *With linear transport costs, the first and the third column of Table 3 never apply. Thus, total surplus is not affected by the introduction of an ad blocker. Surplus changes for publishers, advertisers, and consumers are qualitatively the same as in the setting with discriminatory pricing. They depend on whether or not* $\pi^m > 2\pi^d$*. If* $\pi^m \leq 2\pi^d$*, in the presence of an ad blocker, both publishers buy whitelisting and the ad blocker extracts surplus from publishers only. If* $\pi^m > 2\pi^d$*, in the presence of an ad blocker, only one publisher buys whitelisting and, as a result, the ad blocker extracts some of the combined surplus of publishers and advertisers. Here, advertisers are necessarily worse off.*

Example 2 continued. *With quadratic transport costs, the surplus results are captured in Figure 3. As shown in Figure 2, for intermediate values of* $(v-c)/t$*, the parameter region can be divided into two subregions, one below and the other above the dashed line, which reports the values of* $(v-c)/t)$ *such that* $\pi^m = 2\pi^d$*. In the area above the dashed line, advertiser A would be a monopolist in the absence of the ad blocker, whereas, below the dashed line, advertisers would be in symmetric duopoly. Hence, in the upper subregion, advertiser A would buy both ad slots leading to the monopoly outcome in the product market, while below it both advertisers would buy one ad slot each, leading to the symmetric duopoly outcome. Introducing the ad blocker leads to more competition in the intermediate range above the dashed line and to less competition in the intermediate range below the dashed line. As a result, in the upper subregion, the introduction of the ad blocker leads to an increase in total surplus, while, in the lower subregion, this leads to a reduction (see Figure 3).*

Changes in total surplus and consumer surplus go hand in hand: consumers benefit from lower prices and more variety after the introduction of the ad blocker in the upper subregion, while the opposite is true in the lower subregion. Thus, consumers are better off with the *introduction of an ad blocker in the upper, but worse off in the lower subregion (see Figure 3).*

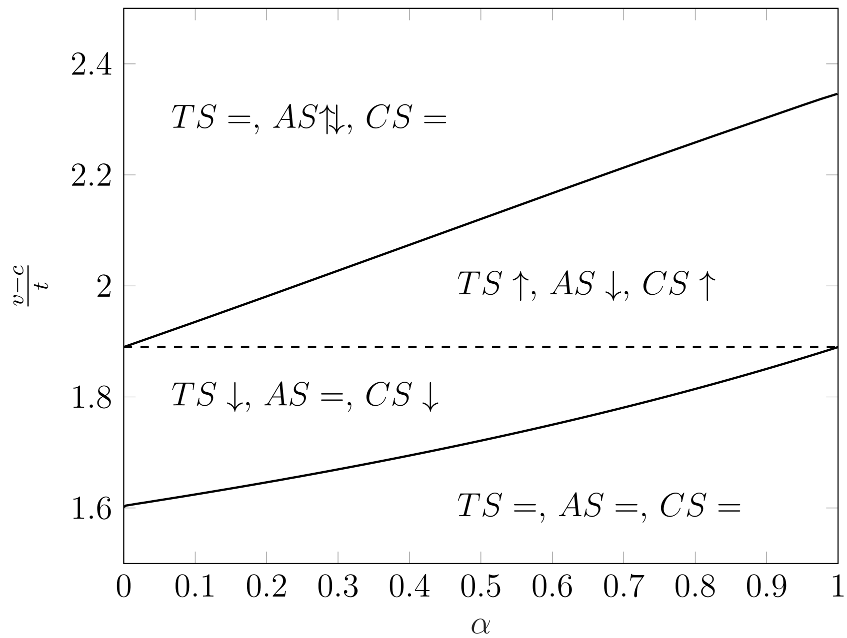

Figure 3: *Total surplus (TS), advertiser surplus (AS), and consumer surplus (CS) effects of the introduction of the ad blocker in the Hotelling model with quadratic transport costs*

One may also wonder whether the finding that publishers are necessarily worse off under ad blocking holds more generally. Under discriminatory pricing, the condition that publishers are better off under price discrimination is $2(1-\alpha)\pi^d > 2\pi^d$, which can never be satisfied. Under uniform pricing, the condition for publisher surplus to be higher with the ad blocker than without is that $\pi^{nw} + (1-\alpha)\pi^d > 2\pi^d$, which is equivalent to $\pi^{nw} > (1+\alpha)\pi^d$. The difference between uniform pricing and discriminatory pricing arises because the non-whitelisted publisher benefits from the other publisher's decision to whitelist, since its profit is $\pi^{nw} > (1-\alpha)\pi^d$. We show that the condition $\pi^{nw} > (1+\alpha)\pi^d$ can be satisfied and, thus, publishers may gain from the presence of the ad blocker.

Examples 1 and 2 continued. *With linear and quadratic transport costs, publisher surplus is higher in the presence of the ad blocker if $\alpha > \frac{3}{5}$ and $\frac{v-c}{t} \in \left(2 + \frac{1}{2}\frac{1+\alpha}{1-\alpha}, \frac{1+\alpha}{1-\alpha}\right)$; see Proposition 9 in Appendix B.1 and Proposition 12 in Appendix B.2.*

5 Ad blocker installation

If consumers install ad blockers to reduce the amount of advertising they are exposed to, the question arises as to which of our previous results are robust to endogenous ad blocker installation.

Suppose that a fraction α of consumers have "high" nuisance cost $\mu_h > 0$ per ad they are exposed to and the remaining $1 - \alpha$ fraction of consumers do not mind seeing ads (or have sufficiently "low" nuisance costs $\mu_l \geq 0$). We assume that the opportunity cost of installing the ad blocker, F_I is such that consumers with a high nuisance cost will install it if they reduce ad exposure by at least one ad (that is, $F_I < \mu_h$), while consumers with a low nuisance cost will not ($F_I > \mu_l$).

According to Propositions 2 and 3, the ad blocker provides whitelisting to one or to both publishers in equilibrium. If monopoly profits are larger than duopoly industry profits, advertiser A buys the ad slot from both publishers and thereby avoids retail price competition; under Assumption 1 this holds both under uniform and discriminatory pricing. In this case, the ad blocker offers the advantage that consumers are exposed to advertising via one publisher only. If both advertisers buy an ad slot and only one publisher whitelists, a trade-off arises for consumers, as the installation of the ad blocker implies that they are exposed to one advertiser only and thus forego the opportunity to buy from the other advertiser. Whether this has an impact on ad blocker installation depends on whether consumers rationally anticipate that their experience in the product market depends on their installation decision.

We consider two environments. First, we consider the consumers' ad blocker installation to be an inflexible decision and thus postulate that consumers move before the ad blocker sets its fee. Second, we consider the reverse situation in which consumers install the ad blocker after the ad blocker has committed to its price. As before, we distinguish the setting with retail price discrimination from the one with uniform retail prices.

5.1 Upfront ad blocker installation and retail price discrimination

Consider an extension in which consumers decide whether to install the ad blocker before the first period of the game.

With retail price discrimination, consumers correctly foresee that both publishers will be whitelisted if $2\pi^d > \pi^m$. This implies that ad blocking does not reduce ad exposure and, therefore, no consumer will install the ad blocker. Thus, the ad blocker can only be active if $2\pi^d \leq \pi^m$ in which case only one publisher will be whitelisted. Since advertiser A buys the ad slot from each publisher, the ad with the non-whitelisted publisher does not affect consumer choice in the product market and merely adds to the ad nuisance. For this reason, consumers with high nuisance cost have a strict incentive to install the ad blocker. Proposition 2 can thus be reformulated as follows:

Proposition 4. *Consider an environment with endogenous ad blocker installation and price-discriminating advertisers.*

- *If $\pi^m < 2\pi^d$, then none of the consumers installs the ad blocker; both publishers set $f_1 = f_2 = \pi^d$; and each advertiser buys an ad slot.*
- *If $\pi^m \geq 2\pi^d$, then the consumers with high nuisance costs install the ad blocker; the ad blocker offers whitelisting at price $\alpha\pi^m$; and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. Advertiser A buys the ad slot on each publisher's website.*

The proposition tells us that one should observe ad blockers in environments in which duopoly competition in the product market is intense.

5.2 Upfront ad blocker installation and uniform retail prices

We now turn to the case in which advertisers have to set uniform retail prices. Let us first assume that consumers have limited cognition when deciding whether to install the ad blocker in the sense that they do not internalize that this decision affects their experience in the product market. This means that the adoption decision is purely based on the comparison between the nuisance from advertising and the opportunity cost of ad blocker installation.

Proposition 5. *Consider an environment with endogenous ad blocker installation and advertisers setting uniform prices.*

- *If* $\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$, *then none of the consumers installs the ad blocker and both publishers set* $f_1 = f_2 = \pi^d$; *and each advertiser buys one slot.*

- *If* $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$, *then consumers with the high nuisance cost install the ad blocker and the ad blocker whitelists a single publisher at price* $\pi^w - (1-\alpha)\pi^d$. *The whitelisted publisher sets its fee equal to* π^w *and the non-whitelisted publisher sets* π^{nw}. *If* $\pi^m > 2\pi^d$, *advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

If consumers are fully rational, they take into account that ad blocker installation may lead to a worse experience in the product market since they can only buy advertiser A's product. When $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ and $\pi^m < 2\pi^d$, advertiser A forecloses advertiser B and thus the result of the proposition carries over. However, when $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ and $\pi^m > 2\pi^d$, ad blocker installation reduces a consumer's net surplus in the product market by $|CS(p^w, \infty) - CS(p^w, p^{nw})|$. Suppose that there are two groups of consumers, one with high nuisance costs of advertising and the other with low nuisance costs. If the former have sufficiently high nuisance cost they will continue to install the ad blocker despite the loss in consumer surplus in the product market, and our result continues to carry over.[16]

More generally, there may be consumers who are mostly concerned about ad nuisance (consumers with very high nuisance costs), while others find the product market experience relatively more important, and there is a continuum of types that will fall into two segments.

[16]For simplicity, suppose that the incremental ad nuisance from two instead of one ad is μ_h for the high nuisance cost type and μ_l for the low nuisance cost type. Suppose, furthermore, that the low nuisance type never installs the ad blocker. If all consumers expect that only the high type installs the ad blocker, then the condition to support the equilibrium is $\mu_h > CS(p^w, p^{nw}) - CS(p^w, \infty) > \mu_l$. Alternatively, consumers may expect that none of the other consumers installs the ad blocker. Not installing the ad blocker is preferred also by a consumer with high incremental nuisance costs if $CS(p^d, p^d) - CS(p^d) > \mu_h$. If $CS(p^w, p^{nw}) - CS(p^w, \infty) < \mu_h < CS(p^d, p^d) - CS(p^d)$ both outcomes can be supported in equilibrium. Moreover, for $\mu_h \in (\min\{CS(p^d, p^d) - CS(p^d), CS(p^w, p^{nw}) - CS(p^w, \infty)\}, \max\{CS(p^d, p^d) - CS(p^d), CS(p^w, p^{nw}) - CS(p^w, \infty)\})$, there is an equilibrium in which the high nuisance type mixes.

The former will install the ad blocker, while the latter will not. Thus, there is an endogenously determined fraction of consumers who will install the ad blocker; these are consumers with relatively high incremental nuisance costs. The take-away from this discussion is that our results carry over qualitatively when consumers are fully rational and anticipate the product market implications of their decision on whether or not to install the ad blocker.

5.3 Committed ad blocker and retail price discrimination

We now turn to an environment in which the ad blocker commits to its fee and consumers correctly infer the number of ads that they will be exposed to. This implies that the ad blocker will never profitably sell whitelisting to both publishers (with probability 1), as this would imply that no consumer installs the ad blocker. To make sure that the ad blocker does not prefer to set a whitelisting fee that induces an equilibrium in which the two publishers play a mixed strategy in their whitelisting decision, we assume that $\frac{F_I}{\mu_h}$ is sufficiently large in the case of $\pi^m < 2\pi^d$ (as spelled out in the proof of Proposition 6). Then, the ad blocker will set its fee such that only one publisher will buy whitelisting; hence, in contrast to our result under exogenous ad blocker installation, even if $\pi^m < 2\pi^d$, only one publisher will buy whitelisting. Whitelisting gives the advertiser on the website of the whitelisted publisher a monopoly position over high nuisance cost consumers as they are the ones who install the ad blocker. The corresponding increase in profits of $\alpha\pi^m$ can be charged as the increment in the advertising fee by the whitelisted publisher on top of the fee charged by the non-whitelisted publisher. Thus, whitelisting is worth $\alpha\pi^m$ to the publisher, which is extracted by the ad blocker. In our analysis, we set $\mu_l = 0$ for simplicity.

Proposition 6. *Consider an environment with endogenous ad blocker installation after the ad blocker has committed to its whitelisting fee and price-discriminating advertisers. The ad blocker offers whitelisting at price $\alpha\pi^m$ and one publisher accepts. The whitelisted publisher sets its fee equal to $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. If $\pi^m > 2\pi^d$, advertiser A buys the ad slot on each publisher's website and otherwise advertisers buy one slot each.*

5.4 Committed ad blocker and uniform retail prices

Consider the setting in which advertisers set uniform prices in the product market. Also in this setting and for the same reason as with discriminatory pricing, the ad blocker will never profitably sell whitelisting to both publishers with probability 1. As in the previous section, we restrict attention to the case in which the ad blocker does not prefer to set a whitelisting fee that induces an equilibrium in which the two publishers play a mixed strategy in their whitelisting decision. Assuming that $\frac{F_I}{\mu_h}$ is sufficiently large (as spelled out in the proof of Proposition 7; the exact condition is different to the one in the previous section), implies that the ad blocker will set its fee such that only one publisher will buy whitelisting. Whitelisting gives the advertiser on the website of the whitelisted publisher a monopoly position over consumers with an ad blocker.

If both advertisers buy one ad slot each, under uniform pricing the gross profit of the advertiser with the whitelisted publisher is π^w, and the gross profit of the other advertiser is π^{nw}, which the publishers can fully extract. If the whitelisted publisher deviated and did not buy whitelisting, its gross profit would be $(1-\alpha)\pi^d$. Hence, the ad blocker can extract $\pi^w - (1-\alpha)\pi^d$.

If advertiser A buys both ad slots, it makes a profit of π^m. If the fee charged by the non-whitelisted publisher is above π^{nw} it will not bother to buy slot 2 since advertiser B will not buy the slot at such a fee given that advertiser A bought slot 1. Correspondingly, if the fee charged by the whitelisted publisher is above π^w it will not buy slot 1 since advertiser B will not buy the slot at such a fee given that advertiser A bought slot 2. Thus, fees are $f_1 = \pi^w$ and $f_2 = \pi^{nw}$. As above, if the whitelisted publisher deviated and did not buy whitelisting, its gross profit would be $(1-\alpha)\pi^d$. Hence, the ad blocker can extract $\pi^w - (1-\alpha)\pi^d$. If advertiser A does not buy both ad slots, its profit is zero. If it buys both, its profit is $\pi^m - f_1 - f_2 = \pi^m - \pi^w + \pi^{nw}$. Thus, advertiser A prefers to buy both ad slots if and only if $\pi^m > \pi^w + \pi^{nw}$.

Proposition 7. *Consider an environment with endogenous ad blocker installation after the ad blocker committed to its whitelisting fee and advertisers set uniform prices. The ad blocker whitelists a single publisher at price $\pi^w - (1-\alpha)\pi^d$. The whitelisted publisher sets its fee* *equal to* π^w *and the non-whitelisted publisher sets* π^{nw}*. If* $\pi^m > \pi^w + \pi^{nw}$*, advertiser A buys the ad slot on each publisher's website and otherwise each advertiser buys one slot each.*

Comparing the surpluses under ad blocking versus no ad blocking, our insights boil down to the following result: the ad blocker extracts some surplus either from advertisers or consumers. Furthermore, publisher surplus can be higher or lower when the ad blocker is present. The condition for the former to hold (i.e., ad blocker and publisher interests are aligned) is $\pi^{nw} + (1-\alpha)\pi^d > 2\pi^d$.

6 Conclusion

While ad-blocking spares users from viewing annoying ads, it complicates publishers commercializing website traffic by showing ads to users. In Germany, the publishing company Axel Springer has tried to defend itself legally since 2014 without success. They accused Adblock Plus' business model of violating the right of freedom of the press. Their lawsuit was dismissed in 2019 by the Bundesgerichtshof, Germany's highest court of civil and criminal jurisdiction.

In this paper, we evaluated the equilibrium effects of ad blocking when an ad blocker can whitelist certain publishers and take a cut in the publishers' revenues from advertising. Our analysis applies to product markets operating as narrow oligopolies and sheds light on the endogenous prices in product and advertising markets. Our analysis is compatible with the view that publishers may be harmed by ad blocking. However, publishers may not be the only ones harmed by the ad blocker: there may also be harm to consumers or advertisers.

Our paper shows that there are no simple and unambiguous results on the surplus effects of the introduction of an ad blocker, when product market competition is taken seriously. Ad blocking is not simply a device to shift rents from publishers to the ad blocker.

What is more, as we show in the setting of uniform prices in the product market, publishers are not necessarily worse off: the presence of an ad blocker may relax price competition between advertisers under asymmetric competition. The ensuing higher advertiser revenues (off the equilibrium path) allow publishers to charge advertisers more. The ad blocker cannot fully extract these increased publisher revenues because the publisher that does not whitelist also makes higher revenues. As a result, publishers may actually be better off when the ad blocker is present.

Appendix

A Relegated proofs

Proof of Proposition 1. If advertiser A buys both ad slots its net profit will be $\pi^m - f_1 - f_2$ because it will operate as a monopolist. Instead, if advertiser A does not buy both slots, then it buys the slot at the lowest fee and advertiser B either buys the remaining slot or foregoes the possibility to advertise. Advertiser A makes profit $\pi^d - \min\{f_1, f_2\}$ and advertiser B makes $\max\{0, \pi^d - \max\{f_1, f_2\}\}$.

If $\pi^m < 2\pi^d$, both publishers set $f_i = \pi^d$ and each advertiser buys one slot. At a higher fee they would not be able to fill the ad slot, and they would obtain lower revenues if they were setting a lower fee. If, in equilibrium, at least one publisher set a fee strictly less than π^d, it would have an incentive to increase its fee. If, in equilibrium, at least one publisher set a fee strictly higher than π^d, the publisher with the (weakly) highest fee would not be able fill its ad slot with probability 1; it would make higher profit by undercutting the other publisher (and never charging above π^m) if that publisher's fee is strictly above π^d and by setting the fee equal to π^d otherwise.

If $\pi^m > 2\pi^d$, both publishers will also set $f_i = \pi^d$; in this case, advertiser A buys both slots. Publishers do not have an incentive to fill their slot at a lower fee. If one of the publishers were to increase its fee above π^d, advertiser A would decide not to buy this slot. At this fee, advertiser B prefers not to buy this slot since it can only make pi^d.

The equilibrium is unique, as we show next. If there were an equilibrium in which at least one publisher charged strictly more than π^d and publisher set different fees, advertiser A would not buy the more expensive, nor would advertiser B giving zero profit to the more expensive publisher; if publishers charged the same fee and this fee is larger than π^d each publisher increases its profit by undercutting (and never charging above π^m) because this implies that the ad slot will be filled with probability 1 instead of a probability in $(0, 1)$. If there were an equilibrium in which at least one publisher charged strictly less than π^d, the publisher with the weakly lower fee could increase its fee and continue to sell the ad slot. □

Proof of Propostion 2. In any equilibrium of the game with the ad blocker and price-discriminating advertisers at least one publisher buys whitelisting.

Consider the case in which both publishers buy whitelisting. Then, an ad slot of each publisher guarantees access to all consumers in the market. Consequently, if both publishers buy whitelisting the subgame that begins with publishers setting the advertising fees coincides with the game in which no ad blocker is present (barring the whitelisting fee that must be paid to the ad blocker). By proposition 1, publishers set fees $f_1 = f_2 = \pi^d$. If $\pi^m < 2\pi^d$, each advertiser buys an ad slot. Otherwise, advertiser A buys both. Each publisher's profit is π^d.

Next, we determine the highest fee the ad blocker can set to induce both publishers to buy whitelisting. Consider a publisher's deviation to opt out of buying whitelisting when the other publisher buys whitelisting on the equilibrium path. If one publisher is whitelisted it can charge $\alpha\pi^m$ for its ad slot on top of the fee it would set if it were not whitelisted. Thus, the whitelisted and the non-whitelisted publishers set $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and $f_2 = (1-\alpha)\pi^d$, respectively. As a result, if the competing publisher buys whitelisting the maximal whitelisting fee that a publisher is willing to pay for whitelisting is $\pi^d - (1-\alpha)\pi^d = \alpha\pi^d$. The profit of the ad blocker inducing both publishers to buy whitelisting is $2\alpha\pi^d$.

Now, consider the case in which only one publisher buys whitelisting in equilibrium. Then, the profit of the whitelisted publisher is $\alpha\pi^m + (1-\alpha)\pi^d$ and the profit of the non-whitelisted publisher is $(1-\alpha)\pi^d$. The willingness to pay for whitelisting if the other publisher opts out of whitelisting is $\alpha\pi^m + (1-\alpha)\pi^d - (1-\alpha)\pi^d = \alpha\pi^m$. Then, the maximal profit of the ad blocker inducing only one publisher to buy whitelisting is $\alpha\pi^m$.

In any equilibrium in which publishers randomize between buying and not buying whitelisting, the ad blocker profits are strictly lower than $\max\{\alpha\pi^m, 2\alpha\pi^d\}$. If $t \in [\alpha\pi^d, \alpha\pi^m]$, there is a symmetric mixed strategy equilibrium in which firms buy whitelisting with probability $\beta = \frac{\alpha\pi^m - t}{\alpha\pi^m - \alpha\pi^d}$. The ad blocker's profit is $\beta^2 2t + 2(1-\beta)\beta t = 2\beta t = \frac{2t(\alpha\pi^m - t)}{\alpha\pi^m - \alpha\pi^d}$. The fee that maximizes this profit is $\max\{\alpha\pi^m/2, \alpha\pi^d\}$. If $2\pi^d \geq \pi^m$, then the maximal profit is $2\alpha\pi^d$. Otherwise, if $2\pi^d < \pi^m$, the maximal profit in the mixed strategy equilibrium is $\alpha\pi^m \frac{\alpha\pi^m/2}{\alpha\pi^m - \alpha\pi^d} < \alpha\pi^m$. We conclude that the ad blocker chooses between setting $\alpha\pi^d$ and inducing both publishers to buy whitelisting and setting $\alpha\pi^m$ and inducing only one publisher to become whitelisted.

Hence, if $\pi^m \geq 2\pi^d$, then only one publisher becomes whitelisted. The whitelisted publisher sets $f_1 = \alpha\pi^m + (1-\alpha)\pi^d$ and the non-whitelisted publisher sets $f_2 = (1-\alpha)\pi^d$. Advertiser A buys both ad slots. By contrast, if $\pi^m < 2\pi^d$, then both publishers buy whitelisting at $\alpha\pi^d$. They set advertising fees $f_1 = f_2 = \pi^d$, and each advertiser buys an ad slot. □

Lemma 1. *Consider an environment with an ad blocker and advertisers setting uniform retail prices. Suppose that one publisher bought whitelisting. Then, in equilibrium, advertiser A buys at least one ad slot.*

Proof. Recall that $\pi^w = \alpha\pi(p^w, \infty) + (1-\alpha)\pi(p^w, p^{nw})$ and $\pi^{nw} = (1-\alpha)\pi(p^{nw}, p^w)$.

Suppose that publisher 1 and publisher 2 set f_1 and f_2 respectively. By contradiction, suppose that advertiser A does not buy any slot in equilibrium.

In such an equilibrium, we must have that advertiser A does not find it profitable to buy any slot. If advertiser A buys only slot 1, then its profit is equal to $\pi^w - f_1$ if advertiser B buys slot 2, and is equal to $\pi^m - f_1$ otherwise. This implies that $f_1 > \pi^w$ as otherwise advertiser A would find it profitable to buy only slot 1.

Now consider advertiser A buying only slot 2. Since $f_1 > \pi^w$ we have that advertiser B does not buy slot 1 and advertiser A makes monopoly profit from consumers who do not use ad blocker resulting in profits $(1-\alpha)\pi^m - f_2$. In equilibrium advertiser A does not find it profitable to buy only slot 2 implying that $f_2 > (1-\alpha)\pi^m$.

Note that $f_2 > (1-\alpha)\pi^m > \pi^{nw}$ as $\pi^m > \pi(p^{nw}, p^w)$. Thus, $f_2 > (1-\alpha)\pi^m$ implies that advertiser B would not buy slot 2 in case advertiser A decides to buy slot 1 only. This implies that advertiser A would be a monopoly if it decides to buy only slot 1. In the equilibrium, this deviation is unprofitable implying that $f_1 > \pi^m$.

We showed that $f_1 > \pi^m$ and $f_2 > (1-\alpha)\pi^m$ which implies that advertiser B does not buy any slot in the equilibrium either. This leads to non-positive profits for both publishers that cannot be in equilibrium, a contradiction.

□

Proof of Proposition 3. We have to distinguish between two possible pure-strategy equilibrium outcomes of the full game: either one publisher buys whitelisting or both publishers do so. It can not be an equilibrium that none buys whitelisting because the ad blocker would make zero profit, which is dominated by selling whitelisting at any positive price.

Consider the subgame in which both publishers bought whitelisting. Then Proposition 1 applies and each publisher sets $f_i = \pi^d$.

Consider now the subgame in which one publisher bought whitelisting (without loss of generality, publisher 1) and publishers have set fees f_1 and f_2. Recall that first advertiser A decides which ad slots to buy and then the remaining slots are offered to advertiser B. By Lemma 1 advertiser A buys at least one ad slot. Therefore, three cases remain to be considered.

First, suppose that advertiser A has bought both slots. It thus operates as a monopolist and makes profit $\pi^m - f_1 - f_2$.

Second, suppose that advertiser A has bought slot 2 only. Then advertiser B either buys slot 1 or foregoes the possibility to advertise. Advertiser A makes $\pi^{nw} - f_2$ if advertiser B buys the slot and $(1-\alpha)\pi^m - f_2$ otherwise. Advertiser B buys slot 1 if and only if $\pi^w - f_1 \geq 0$.

Third, suppose that advertiser A has bought slot 1 only. If advertiser B buys slot 2, advertiser A makes $\pi^w - f_1$ and otherwise $\pi^m - f_2$. Advertiser B buys slot 2 if and only if $\pi^{nw} - f_2 \geq 0$.

We show that, in equilibrium of this subgame, publishers set $f_1 = \pi^w$ and $f_2 = \pi^{nw}$ and both ad slots are taken by the advertisers. As shown above, if at fee $f_1 \leq \pi^w$ advertiser A does not buy slot 1 and buys slot 2 only, then advertiser B will buy slot 1. Thus, in equilibrium of the subgame starting with publishers simultaneously setting fees, f_1 can not be strictly lower than π^w. Correspondingly, f_2 can not be strictly lower than π^{nw}.

If exactly one publisher $i \in \{1, 2\}$ sets a higher fee (i.e. either $f_2 > \pi^{nw}$ or $f_1 > \pi^w$), advertiser B would not buy ad slot i. Would advertiser A have an incentive to buy ad slot i? First, if $f_2 > \pi^{nw}$, buying both slots gives $\pi^m - f_1 - f_2$, buying slot 1 only gives $\pi^m - f_1$, and buying slot 2 only gives $\pi^{nw} - f_2 < 0$. Thus, advertiser A buys slot 1 only and slot 2 remains idle. Second, if $f_1 > \pi^w$, buying both slots gives $\pi^m - f_1 - f_2$, buying slot 1 only gives $\pi^w - f_1 < 0$, and buying slot 2 only gives $(1-\alpha)\pi^m - f_2$. Since $\pi^w \geq \alpha\pi^m$, advertiser A will buy slot 2 only. Hence, slot i will remain idle and no single publisher has an incentive to set a higher fee.

If both publishers set higher fees with $f_1 \leq \pi^m$ and $f_2 \leq (1-\alpha)\pi^m$, advertiser A will select the ad slot that gives it the largest net surplus leading to asymmetric Bertrand competition between publishers. This implies that, in the equilibrium of the subgame in which one publisher is whitelisted, $f_1 = \pi^w$ and $f_2 = \pi^{nw}$.

Given publisher fees, we next characterize advertiser decisions given $f_1 = \pi^w$ and $f_2 = \pi^{nw}$. For advertiser A to buy both slots, it is necessary that at those fees $\pi^m - f_1 - f_2$ is non-negative. Thus, we must have $\pi^m \geq \pi^w + \pi^{nw}$. If $\pi^m < \pi^w + \pi^{nw}$, advertiser A will buy only one slot (it is indifferent as to which one). In this case, both advertisers are active and both make zero net surplus.

The next step is to analyze the whitelisting decisions of publishers for given t. When one publisher buys whitelisting, this publisher makes $\pi^w - t$, while it would make $(1-\alpha)\pi^d$ if it were to reject the whitelisting offer. Thus, for any $t \leq t^{ux} \equiv \pi^w - (1-\alpha)\pi^d$, each publisher is better off accepting the whitelisting offer given that the other publisher rejects it.

When both publishers buy whitelisting, each publisher makes $\pi^d - t$, while a publisher would make π^{nw} if it were to reject the whitelisting offer given the other publisher accepted the offer. Thus, for any $t \leq \pi^d - \pi^{nw}$, both publishers accept the whitelisting offer. First note that $\pi^w - (1-\alpha)\pi^d > \pi^d - \pi^{nw}$, which implies that for sufficiently high t only one publisher asks for whitelisting.

The last step is to determine the profit-maximizing whitelisting fee. If the ad blocker chooses t to induce a pure-strategy equilibrium among publishers it either sets $t = \pi^w - (1-\alpha)\pi^d$ and makes profit t or $t = \pi^d - \pi^{nw}$ and makes profits $2t$.

If the platform sets an intermediate whitelisting fee it induces a mixed-strategy equilibrium. In the unique symmetric mixed-strategy equilibrium, each publisher buys whitelisting with probability γ, where γ makes each publisher indifferent between buying and not buying whitelisting; that is, $\gamma\pi^d + (1-\gamma)\pi^w - t = \gamma\pi^{nw} + (1-\gamma)(1-\alpha)\pi^d$. This gives the explicit solution

$$\gamma = \frac{\pi^w - (1-\alpha)\pi^d - t}{\pi^w + \pi^{nw} - (2-\alpha)\pi^d}.$$

The ad blocker's expected profit is equal to $\gamma^2 2t + 2\gamma(1-\gamma)t = 2\gamma t$. Maximizing the ad blocker's profit for fees that give rise to a non-degenerate mixed-strategy equilibrium, we have to maximize $t(t^{ux} - t)$ with respect to t and obtain the profit-maximizing fee $\max\{t^{ux}/2, \pi^d - \pi^{nw}\}$. With a fee equal to $t^{ux}/2$, the ad blocker will make expected profit $2\gamma t^{ux}/2 < t^{ux}$. Hence, it is never optimal for the ad blocker to induce a mixed-strategy equilibrium.

It remains to compare the profits with $t = \pi^w - (1 - \alpha)\pi^d$ and $t = \pi^d - \pi^{nw}$. The ad blocker prefers the former if and only if $\pi^w + 2\pi^{nw} > (3 - \alpha)\pi^d$. □

Proof of Proposition 4. Suppose that a positive fraction of consumers install the ad blocker upfront. The subgame, which starts from the ad blocker setting the whitelisting fee, is characterized by Proposition 2.

First, if $\pi^m < 2\pi^d$, the ad blocker sells whitelisting to both publishers and all consumers become exposed to both ads irrespective of whether or not they initially installed the ad blocker. Since installing the ad blocker is costly ($F_I > 0$) and consumers can not reduce ad exposure with the ad blocker, no consumer installs the ad blocker in equilibrium, and the game is played according to Proposition 1.

Second, if $\pi^m \geq 2\pi^d$, then the ad blocker sells whitelisting to one publisher only. If consumers do not take into account that the ad blocker installation affects their surplus in the product market, then the high nuisance cost consumers install the ad blocker since $\mu_h > F_I$.[17] Thus, if $\pi^m \geq \pi^d$, α consumers install the ad blocker, and the game is played according to Proposition 2.

□

Proof of Proposition 5. Suppose that there is a positive fraction of consumers who installed the ad blocker. Then, Proposition 3 applies.

First, if $\pi^w + 2\pi^{nw} < (3 - \alpha)\pi^d$, the ad blocker sells whitelisting to both publishers and each consumer sees both ads independent of whether they installed the ad blocker. Thus, none of the consumers installs the ad blocker and Proposition 1 applies.

Second, if $\pi^w + 2\pi^{nw} > (3 - \alpha)\pi^d$, then only one publisher buys whitelisting. Since $\mu_h > F_I > \mu_l$ the consumers with high nuisance cost prefer to install the ad blocker.[18] This concludes the proof. □

[17]If consumers are fully rational, then the high nuisance cost consumers install the ad blocker if $\mu_h > F_I + (CS(p^d, p^d) - CS(p^m, \infty))$.

[18]If consumers are fully rational, then they install the ad blocker if $\mu_h > F_I + (CS(p^w, p^{nw}) - CS(p^w, \infty))$.

Proof of Proposition 6. We assume that F_I/μ_H is sufficiently large such that

$$\frac{F_I}{\mu_h} > \max\left\{2 - \frac{\pi^m}{\pi^d}, 0\right\}.$$

We first note that the ad blocker can make profits of $\alpha\pi^m$. If the ad blocker sets $t = \alpha\pi^m$, then only one publisher buys whitelisting and all high nuisance costs consumers prefer to install the ad blocker.

Next we explore the optimal whitelisting fee of the ad blocker. We start by showing that the ad blocker does not set a whitelisting fee $t \leq \alpha\pi^d$.

Suppose that $t \leq \alpha\pi^d$. By contradiction, suppose that all high nuisance cost consumers are strictly better off from installing the ad blocker. Then, by Proposition 2 both publishers buy whitelisting and consumers see both ads. This implies that consumers prefer not to install the ad blocker, a contradiction. Next, by contradiction, suppose that the high nuisance cost consumers are indifferent and a fraction $\alpha' < \alpha$ of consumers install the ad blocker. If $t \leq \alpha'\pi^d$, then both publishers buy whitelisting and the previous argument applies. If $t > \alpha'\pi^m$, then no publisher buys whitelisting and all high nuisance cost consumers are strictly better off installing the ad blocker. If $t \in (\alpha'\pi^d, \alpha'\pi^m]$ and only one publisher becomes whitelisted, then all high nuisance cost consumers are strictly better off installing the ad blocker, a contradiction. It remains to consider the publishers' mixed strategy. If publishers buy whitelisting with probability β, then the expected cost of ad nuisance of the high nuisance cost consumers if they install the ad blocker is $2\mu_h\beta^2 + 2\beta(1-\beta)\mu_h = 2\beta\mu_h$. In addition, they must bear the installation cost F_I. Since consumers are indifferent, we have that $\beta = 1 - \frac{F_I}{2\mu_h}$. The indifference condition of the publishers implies that $\beta = \frac{\alpha'\pi^m - t}{\alpha'\pi^m - \alpha'\pi^d}$. The resulting profit of the ad blocker is $2\beta t$ which is maximized at $\alpha' \max\{\pi^m/2, \pi^d\}$. If $\pi^m \geq 2\pi^d$, then the resulting profit $2\beta t \leq \frac{\alpha'\pi^m/2}{\alpha'\pi^m - \alpha'\pi^d}\alpha'\pi^m \leq \alpha'\pi^m < \alpha\pi^m$, where the last expression is the profit that the ad blocker can always obtain by setting $t = \alpha\pi^m$ and whitelisting only one publisher. Otherwise, if $\pi^m < 2\pi^d$, then $2\beta t = 2\left(1 - \frac{F_I}{2\mu_h}\right)t < \frac{\pi^m}{\pi^d}t \leq \alpha'\pi^m < \alpha\pi^m$. We conclude that $t \leq \alpha\pi^d$ will not be set by the ad blocker in equilibrium.

Next, suppose the ad blocker sets a whitelisting fee $t \in (\alpha\pi^d, \alpha\pi^m)$ and all high nuisance cost consumers are strictly better off from installing the ad blocker. In the pure strategy equilibrium in which only one publisher buys whitelisting, we have that all high nuisance cost consumers install the ad blocker. The resulting profit is t, which is less than what the ad blocker can make if it sets $t = \alpha\pi^m$. It remains to consider the mixed strategy equilibrium. Suppose that all high nuisance cost consumers install the ad blocker. Then, $\beta \leq 1 - \frac{F_I}{2\mu_h} < \min\left\{\frac{\pi^m}{2\pi^d}, 1\right\}$. The resulting profit of the ad blocker is $2\beta t$, where $\beta = \frac{\alpha\pi^m - t}{\alpha\pi^m - \alpha\pi^d}$. If $\pi^m \geq 2\pi^d$, then the profit is maximized at $\alpha\pi^m/2$ and is equal to $\beta\alpha\pi^m < \alpha\pi^m$. Otherwise, if $\pi^m < 2\pi^d$, then the profit $2\beta t < 2\frac{\pi^m}{2\pi^d}\alpha\pi^d = \alpha\pi^m$. The similar argument can be made if high nuisance cost consumers are indifferent and a fraction of $\alpha' < \alpha$ consumers install the ad blocker. This concludes the proof. □

Proof of Proposition 7. We assume that F_I/μ_H is sufficiently large such that

$$\frac{F_I}{\mu_h} > \max\left\{2 - \frac{\pi^w - (1-\alpha)\pi^d}{\pi^d - \pi^{nw}}, 0\right\}.$$

The ad blocker does not find it optimal to set $t > \pi^w - (1-\alpha)\pi^d$. Even if all high nuisance cost consumers install the ad blocker, Proposition 3 implies that no publisher buys whitelisting at such a high fee. Consider the case $t \leq \pi^w - (1-\alpha)\pi^d$. Suppose, by contradiction, that all consumers do not install the ad blocker. Then, publishers do not buy whitelisting. This, in turn, implies that high nuisance cost consumers can avoid two ads by installing the ad blocker. Thus, a positive fraction of high nuisance cost consumers install the ad blocker in equilibrium.

Since nuisance cost μ_h is the same for all high nuisance cost consumers, we have that either all high nuisance cost consumers are strictly better off from installing the ad blocker or they are indifferent and $\alpha' \in (0, \alpha]$ install the ad blocker. We analyze these two cases separately and show that in each case the ad blocker finds it optimal to set $t = \pi^w - (1-\alpha)\pi^d$.

First, suppose that $t \leq \pi^w - (1-\alpha)\pi^d$ and all high nuisance costs consumers strictly prefer to install the ad blocker. If $t = \pi^w - (1-\alpha)\pi^d$, then only one publisher buys whitelisting, and all high nuisance cost consumers install the ad blocker. The profit of the ad blocker is $\pi^w - (1-\alpha)\pi^d$. If $t \leq \pi^d - \pi^{nw}$, then by Proposition 3 both publishers buy whitelisting. In turn, all high nuisance cost consumers become exposed to two ads and refuse to install the ad blocker, a contradiction. If $t \in (\pi^d - \pi^{nw}, \pi^w - (1-\alpha)\pi^d)$, then there is an equilibrium in which only one publisher buys whitelisting that results in profits t for the ad blocker. Clearly, $t < \pi^w - (1-\alpha)\pi^d$ and the ad blocker can make strictly higher profits by setting fee $\pi^w - (1-\alpha)\pi^d$. It remains to consider the publishers' mixed strategy. If publishers buy whitelisting with probability γ, then the expected cost of ad nuisance of the high nuisance cost consumers if they install the ad blocker is $2\gamma\mu_h$. Since all consumers are strictly better off installing the ad blocker we have that $\gamma \leq 1 - \frac{F_I}{2\mu_h} < \min\left\{\frac{\pi^w-(1-\alpha)\pi^d}{2(\pi^d-\pi^{nw})}, 1\right\}$. The resulting profit of the ad blocker is $2\gamma t$, where $\gamma = \frac{\pi^w-(1-\alpha)\pi^d-t}{\pi^w+\pi^{nw}-(2-\alpha)\pi^d}$. If $\pi^w + 2\pi^{nw} \geq (3-\alpha)\pi^d$, then the profit of the ad blocker is maximized at $t = \frac{1}{2}(\pi^w - (1-\alpha)\pi^d)$ and is equal to $\gamma(\pi^w - (1-\alpha)\pi^d) < \pi^w - (1-\alpha)\pi^d$, where the last expression is the profit that the ad blocker can always obtain by setting $\pi^w - (1-\alpha)\pi^d$ and whitelisting only one publisher. If $\pi^w + 2\pi^{nw} < (3-\alpha)\pi^d$, then the profit of the ad blocker is $2\gamma t < 2\frac{\pi^w-(1-\alpha)\pi^d}{2(\pi^d-\pi^{nw})}(\pi^d - \pi^{nw}) = \pi^w - (1-\alpha)\pi^d$. We showed that if all high nuisance cost consumers install the ad blocker, then the ad blocker finds it optimal to set $t = \pi^w - (1-\alpha)\pi^d$ and whitelist only one publisher.

Second, suppose that the high nuisance costs consumers are indifferent and $\alpha' < \alpha$ consumers install the ad blocker in equilibrium. We note that π^w and π^{nw} correspond to profits of the whitelisted and the non-whitelisted publishers for α' respectively. If $t > \pi^w - (1-\alpha')\pi^d$, then no publisher buys whitelisting, and the high nuisance cost consumers are strictly better off from installing the ad blocker. If $t \leq \pi^d - \pi^{nw}$, then both publishers buy whitelisting and the high nuisance cost consumers will not install the ad blocker. If $t \in (\pi^d - \pi^{nw}, \pi^w - (1-\alpha')\pi^d)$, then if only one publisher buys whitelisting, then the high nuisance cost consumers are strictly better off from installing the ad blocker. It remains to consider the mixed strategy equilibrium. Suppose that publishers buy whitelisting with probability γ. Since the high nuisance cost consumers are indifferent, we have that $\gamma = 1 - \frac{F_I}{2\mu_h} < \min\left\{\frac{\pi^w-(1-\alpha')\pi^d}{2(\pi^d-\pi^{nw})}, 1\right\}$. The resulting profit of the ad blocker is $2\gamma t$, where $\gamma = \frac{\pi^w-(1-\alpha')\pi^d-t}{\pi^w+\pi^{nw}-(2-\alpha')\pi^d}$. Following the previous analysis for α we find that the profit of the ad blocker cannot be higher than $\pi^w - (1-\alpha')\pi^d$. This expression is higher for higher α' and is maximal when all the high nuisance cost consumers install the ad blocker.

We conclude that in the unique equilibrium, the ad blocker sets $t = \pi^w - (1-\alpha)\pi^d$, all the high nuisance cost consumers install the ad blocker, and only one publisher buys whitelisting.

□

B Full analysis of the two examples

In this section, we thoroughly analyze the monopoly, the equilibrium in symmetric duopoly, as well as the pure strategy equilibrium in the asymmetric Hotelling model in which only one firm has access to consumers who use the ad blocker – in an asymmetric duopoly, we restrict attention to the parameter range in which there exists a unique pure strategy equilibrium.[19] Recall that the consumer's gross valuation at the ideal location is v and the transport cost parameter is equal to t. In our first example, transport costs are linear, in our second they are quadratic. In the duopoly settings, we refer to firms 1 and 2, which, in the main text, are advertisers A and B respectively.

B.1 Full analysis of the Hotelling model with linear transport costs

We start our analysis with the monopoly problem.

Monopoly. Suppose that there is one firm located at 0. A consumer located at x buys a product at price p if $v - p - tx \geq 0$. The profit of this firm setting price p is

$$\pi(p) = (p - c) \min\left\{\frac{v - p}{t}, 1\right\}.$$

By solving for the profit-maximizing price we obtain that $p^m = (v + c)/2$ if $v \leq c + 2t$ and $p^m = v - t$ otherwise. The monopoly profit is

$$\pi^m = \begin{cases} \frac{(v-c)^2}{4t}, \text{ if } \frac{v-c}{t} < 2, \\ v - c - t, \text{ if } \frac{v-c}{t} \geq 2. \end{cases}$$

Symmetric competition. Consider a standard Hotelling duopoly model with linear transport cost and fully covered market. Suppose that firm 1 is located at 0 and firm 2 is located at 1.

We start by deriving the demand function of firm 1 setting price p_1. Suppose that firm 2 charges price p_2. A consumer located at x buys from firm 1 if and only if $v - p_1 - tx \geq$

[19] While a large IO literature has used the Hotelling model as a building block, we are not aware of such an analysis of the asymmetric model.

$v - p_2 - t(1-x)$. This implies that all consumers located closer to firm 1 than the marginal consumer with $\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$ choose between firm 1 and the outside option. If $v - p_1 - t\hat{x} > 0$ then $D_1 = \hat{x}$, otherwise if $p_1 \leq v$, then all consumers with $x < \frac{v-p_1}{t} < \hat{x}$ buy from firm 1 and $D_1 = \frac{v-p_1}{t}$. Thus, the demand function of firm 1 is given by

$$D_1(p_1, p_2) = \max\left\{0, \min\left\{\frac{1}{2} + \frac{p_2 - p_1}{2t}, \frac{v - p_1}{t}, 1\right\}\right\}.$$

We consider three possibilities: *i)* both firms act as local monopolists, *ii)* firms compete and the indifferent consumer located at $\hat{x}$ obtains positive surplus. *iii)* firms compete and the indifferent consumer obtains zero surplus.

First, suppose that both firms act as local monopolists and the demand of firm i in a small neighborhood of the price p^d is $D_i = \frac{v-p_i}{t}$. Then, the equilibrium price is $p^d = (c+v)/2$. The equilibrium demand is equal to $\frac{v-c}{2t}$. This constitutes an equilibrium if and only if $\frac{v-c}{t} < 1$.

Second, suppose that the market is full covered in equilibrium and the marginal consumer obtains a strictly positive surplus. The profit of firm i is $\pi_i = (p_i - c)\left(\frac{1}{2} + \frac{p^d - p_i}{2t}\right) = \frac{1}{2t}(p_i - c)\left(t + p^d - p_i\right)$. The first-order condition implies that $t + p^d - 2p_i + c = 0$ and the equilibrium price is $p^d = c + t$. The equilibrium demand is equal to $1/2$. The marginal consumer obtains strictly positive surplus if and only if $v - c - t - t/2 > 0$, which is equivalent to $\frac{v-c}{t} > \frac{3}{2}$.

Third, suppose that the marginal consumer located at $\hat{x}$ obtains zero surplus and the market is fully covered in equilibrium. This implies that the prices of firms 1 and 2 are

$$p_1 = v - t\hat{x} \quad \text{and} \quad p_2 = v - t + t\hat{x}.$$

The profit of firm i is $(p_i' - c)(v - p_i)/t$. Firm i does not find it optimal to deviate upwards to price $p_i' > p_i$ if $v + c - 2p_i' \leq 0$. This condition is satisfied if $v + c - 2p_i \leq 0$ or, equivalently, $p_i \leq \frac{v+c}{2}$ for $i \in \{1, 2\}$. Therefore, both firms cannot profitably deviate upwards if and only if

$$1 - \frac{1}{2}\frac{v-c}{t} \leq \hat{x} \leq \frac{1}{2}\frac{v-c}{t}.$$

This interval is non-empty if $\frac{v-c}{t} \geq 1$.

Next, consider a downward deviation to price $p_i' < p_i$. The resulting profit of firm i is $(p_i - c)(t + p_j - p_i)/2t$, where $j \neq i$. Firm i does not find it optimal to deviate downwards if $t + c + p_j - 2p_i' \geq 0$, $i \neq j$. This condition is satisfied if $t + c + p_j - 2p_i \geq 0$. By plugging in the equilibrium prices, we find that this condition is satisfied for

$$\frac{1}{3}\frac{v-c}{t} \leq \hat{x} \leq 1 - \frac{1}{3}\frac{v-c}{t}.$$

This interval is non-empty if $\frac{v-c}{t} \leq \frac{3}{2}$.

We conclude that for $\frac{v-c}{t} \in \left[1, \frac{3}{2}\right]$ there are multiple equilibria characterized by the location of the marginal consumer. In particular,

$$\hat{x} \in \begin{cases} \left[1 - \frac{1}{2}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left[1, \frac{6}{5}\right] \\ \left[\frac{1}{3}\frac{v-c}{t}, 1 - \frac{1}{3}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6}{5}, \frac{3}{2}\right]. \end{cases}$$

Equilibrium prices are given by $p_1 = v - t\hat{x}$ and $p_2 = v - t + t\hat{x}$. The corresponding profits are

$$\pi_1 = t\left(\frac{v-c}{t} - \hat{x}\right)\hat{x} \text{ and } \pi_2 = t\left(\frac{v-c}{t} - (1-\hat{x})\right)(1-\hat{x}).$$

We focus on the symmetric equilibrium in which $\hat{x} = \frac{1}{2}$, noting that this selects the equilibrium with maximal industry profits.

To sum up, we obtain that

$$p^d = \begin{cases} \frac{v+c}{2}, \text{ if } \frac{v-c}{t} < 1, \\ v - \frac{t}{2}, \text{ if } \frac{v-c}{t} \in \left[1, \frac{3}{2}\right], \\ c + t, \text{ if } \frac{v-c}{t} > \frac{3}{2}. \end{cases}$$

The equilibrium duopoly profit is

$$\pi^d = \begin{cases} \frac{(v-c)^2}{4t}, \text{ if } \frac{v-c}{t} < 1, \\ \frac{1}{2}\left(v - c - \frac{t}{2}\right), \text{ if } \frac{v-c}{t} \in \left[1, \frac{3}{2}\right], \\ \frac{t}{2}, \frac{v-c}{t} > \frac{3}{2}. \end{cases}$$

We are ready to state the lemma that compares the monopoly and the duopoly profits in the Hotelling model.

Lemma 2. *In the Hotelling model with linear transport cost* $\pi^m \geq 2\pi^d$ *if and only if* $\frac{v-c}{t} \geq 2$.

Proof. First, note that $\frac{v-c}{t} < 1$ we have that $\pi^m = \pi^d$ implying that the industry duopoly profit is higher than the monopoly profit – that is $\pi^m < 2\pi^d$.

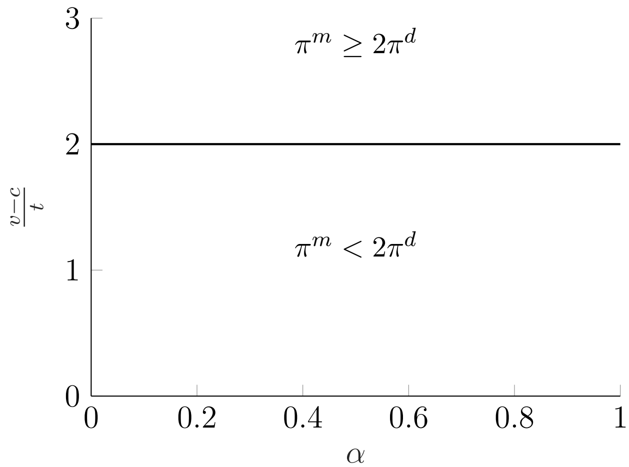

Figure 4: Parameter regions in the Hotelling model with linear transport costs.

Second, suppose that $\frac{v-c}{t} \in \left[1, \frac{3}{2}\right]$. Then, there is a multiplicity of equilibria under duopoly. The maximal industry profit is attained when firms coordinate on the symmetric equilibrium, i.e., $\hat{x} = \frac{1}{2}$. We show that the monopoly profit is higher than the duopoly profit in the symmetric equilibrium and therefore it is higher for all other possible equilibria. The difference in profits (divided by t) is

$$\frac{\pi^m - 2\pi^d}{t} = \frac{1}{4}\left(\frac{v-c}{t}\right)^2 - \frac{v-c}{t} + \frac{1}{2} = \left(1 - \frac{1}{2}\frac{v-c}{t}\right)^2 - \frac{1}{2}$$
$$< \left(1 - \frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4} < 0,$$

implying that $\pi^m < 2\pi^d$.

Third, if $\frac{v-c}{t} \in \left(\frac{3}{2}, 2\right]$, then $\pi^m - 2\pi^d = -\frac{t}{2} < 0$. It remains to explore the case of $\frac{v-c}{t} > 2$. In this case we always have that the monopoly profit, $v - t - c$, is greater than the total duopoly profit that is equal to t. This conludes the proof.

□

Asymmetric competition. Suppose that a fraction α of consumers buy either from firm 1 or take the outside option. This is the situation in which the fraction α of consumers is informed about firm 1 but not firm 2, whereas the remaining fraction is informed about both firms.[20] We characterize all pure strategy equilibria for $\frac{v-c}{t} > \frac{3}{2}$.

We suppose that firms play a pure-strategy equilibrium and afterwards verify the conditions under which this holds. Denote p^w and p^{nw} as the equilibrium prices of firm 1 and firm 2, respectively.

The demand function for product 1 when firm 1 sets price p_1 and firm 2 p^{nw} is given by

$$D_1(p_1, p^{nw}) = \alpha \max\left\{0, \min\left\{\frac{v-p_1}{t}, 1\right\}\right\} + (1-\alpha)\max\left\{0, \min\left\{\frac{1}{2} + \frac{p^{nw}-p_1}{2t}, \frac{v-p_1}{t}, 1\right\}\right\}.$$

The demand of firm 2 setting price p_2 playing against firm 1 setting price p^w is given by

$$D_2(p_2, p^w) = (1-\alpha)\max\left\{0, \min\left\{\frac{1}{2} + \frac{p^w-p_2}{2t}, \frac{v-p_2}{t}, 1\right\}\right\}.$$

Note that, in any equilibrium, firm 2 sells to a positive measure of consumers in the competitive segment. If this were not the case, then the effective price paid by a consumer located at 1, $p^w + t$, would have to be weakly lower than the lowest price firm 2 can charge, which is equal to c. Clearly, for any $t > 0$ there is no such a price p^w that would result in positive profits for firm 1. This implies that firm 2 always sells in the competitive segment.

Therefore, it is sufficient to consider four different possibilities: *i)* firm 1 does not sell in the competitive segment; *ii)* firm 1 sells in the competitive segment, it is fully covered, and the marginal consumer obtains a positive surplus; *iii)* firm 1 sells in the competitive segment, it is fully covered, and the marginal consumer obtains zero surplus and *iv)* firm 1 sells in the competitive segment and this segment is not fully covered.

***i)* Firm 1 sells in the competitive segment; the competitive segment is not fully covered.** In this case firms act as local monopolies. The profit of firm 1 setting price p_1 at which there are still some consumers in the competitive market who prefer to take the outside option equals $\pi_1(p_1, p^{nw}) = (p_1 - c)(v - p_1)/t$. The optimal price is $p^w = (v + c)/2$. The profit of firm 2 setting price p_2 is $(1-\alpha)(p_2 - c)(v - p_2)/t$ which is also maximized

[20]Several articles on informative advertising and differentiated products starting with Grossman and Shapiro (1984) and including Soberman (2004), Christou and Vettas (2008), and Amaldoss and He (2010) focused on symmetric settings. The asymmetric Hotelling model with a monopoly and a competitive segment has been analyzed by Valletti, Hoernig, and Barros (2002) under several parameter restrictions.

at price $p^{nw} = (v+c)/2$. The necessary condition for this to be in equilibrium is that the demand of firm 1 and the demand of firm 2 in the competing segment do not overlap – that is, $\frac{v-c}{2t} < 1 - \frac{v-c}{2t}$ implying $\frac{v-c}{t} < 1$. Note that no firm finds it optimal to deviate and lower its price as it would not do so even if there was no competitor present in the competing segments.

To sum up, if $\frac{v-c}{t} < 1$ there is an equilibrium in which firms act as local monopolists setting the monopoly prices

$$p^w = p^{nw} = \frac{v+c}{2}.$$

The respective profits are

$$\pi^w = \frac{(v-c)^2}{4t} \text{ and } \pi^{nw} = (1-\alpha)\frac{(v-c)^2}{4t}.$$

ii) **Firm 1 does not sell in the competitive segment.** If firm 1 does not sell in the competitive segment, then firm 2 must fully serve it in equilibrium. We consider the two, in principle, possible outcomes: either the monopoly segment of firm 1 is fully covered or it is not.

By contradiction, suppose the latter, namely that firm 1 does not serve all consumers in the monopoly segment. This can only occur if $\frac{v-c}{t} < 2$ as otherwise firm 1 would find it optimal to deviate and serve the entire monopoly segment. But if $\frac{v-c}{t} < 2$, then firm 2 serving all consumers in the competitive segment would find it profitable to lower its price, a contradiction. This implies that if firm 1 does not sell in the competitive segment, then it must serve all consumers in its monopoly segment.

Suppose now that firm 1 serves all consumers in its monopoly segment. If this scenario occurs in equilibrium, then the consumer located at 1 in the monopoly segment cannot enjoy a positive surplus as firm 1 could increase its price and make higher profits. This pins down the equilibrium price of firm 1, $p^w = v - t$. In order to solve for p^{nw} we note that the consumer located at zero has to be indifferent between firm 1 and firm 2 – that is, $v - t - p^{nw} = v - p^w$. By plugging in p^w and solving for p^{nw} we find that the possible equilibrium is represented by the following prices

$$p^w = v - t \text{ and } p^{nw} = v - 2t.$$

If this constitutes an equilibrium, then neither firm finds it profitable to deviate. We first establish the conditions under which firm 2 does not have incentives to deviate to a higher price. The profit of firm 2 deviating to $p_2 > p^{nw}$ is $\pi_2(p_2, p^{nw}) = \frac{1}{2t}(1-\alpha)(p_2-c)(t+p^w-p_2)$. The derivative of this profit function is

$$c + t + p^w - 2p_2 < c + t + p^w - 2p^{nw} = -v + c + 4t \leq 0,$$

if and only if $\frac{v-c}{t} \geq 4$. Under this condition the profit function decreases for all $p_2 > p^{nw}$ and firm 2 does not deviate to a higher price.

Next, we explore firm 1's incentive to deviate. Condition $\frac{v-c}{t} \geq 4$ implies that firm 1 does not deviate to higher prices (see the monopoly problem in the symmetric case). Thus, it remains to establish conditions under which firm 1 does not deviate to lower prices. If firm 1 sets a lower price, $p_1 < v - t$, then it would serve some consumers from the competitive markets resulting in total profits

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

The derivative of this profit (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned} c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p_1 &> c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2v + 2t \\ &= c + \frac{1+\alpha}{1-\alpha}t - v \geq 0, \end{aligned}$$

if and only if $\frac{v-c}{t} \leq \frac{1+\alpha}{1-\alpha}$. This condition ensures that firm 1 does not deviate to lower prices.

To sum up, we conclude that for $\alpha \geq \frac{3}{5}$ and $\frac{v-c}{t} \in \left[4, \frac{1+\alpha}{1-\alpha}\right]$ there exists an equilibrium in which firms set prices

$$p^w = v - t \quad \text{and} \quad p^{nw} = v - 2t,$$

all consumers in the monopoly segment buy from firm 1, all consumers in the competitive market buy from firm 2. The respective profits are given by

$$\pi^w = \alpha(v - c - t) \quad \text{and} \quad \pi^{nw} = (1-\alpha)(v - c - 2t).$$

***iii)* Firm 1 sells in the competitive, fully covered segment and the marginal consumer obtains a positive surplus.** Suppose that all $1-\alpha$ consumers in the competitive segment buy from either of the firms, the marginal consumer is in the interior (i.e. each firm has a positive market share) and enjoys a positive surplus. The profit of firm 1 setting price p_1 is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha \min\left\{\frac{v - p_1}{t}, 1\right\} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right).$$

We consider two cases taking into account whether the monopoly segment of firm 1 is fully covered or not.

Case 1.1: Monopoly segment of firm 1 is fully covered. The consumer located at 1 in the monopoly segment obtains a positive surplus. If this case can occur in equilibrium, then we have that $v - p^w - t > 0$. The profit of firm 1 setting a price p_1 at which all α consumers in the monopoly segment continue to buy and the marginal consumer in the competitive segment is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

The first-order condition at $p_1 = p^w$ is

$$\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p^w) - \frac{1-\alpha}{2t}(p^w - c) = 0,$$

implying that

$$p^w = \frac{1}{2}\left(c + p^{nw} + \frac{1+\alpha}{1-\alpha}t\right).$$

Since $v - p^w - t > 0$, the profit of firm 2 does not have kinks (if a consumer does not buy from firm 2 she always buys from firm 1 rather than taking the outside option). Therefore, the profit of firm 2 can be written as

$$\pi_2(p_2, p^{nw}) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p^w - p_2}{2t}\right).$$

Solving the first-order condition, we find that $p^{nw} = \frac{1}{2}(c + t + p^w)$. Plugging this back into the expression for p^w, we find that

$$p^w = c + \frac{3+\alpha}{3(1-\alpha)}t \quad \text{and} \quad p^{nw} = c + \frac{3-\alpha}{3(1-\alpha)}t.$$

The marginal consumer in the competitive segment is in the interior if $p^w - p^{nw} < t$. This is the case whenever $\alpha < \frac{3}{5}$.

The prices constitute an equilibrium if *i)* the monopoly segment of firm 1 is fully covered (this, in turn, would imply that the marginal consumer in the competitive market enjoys a positive surplus and firm 2 does not have incentives to deviate) and *ii)* firm 1 does not want to set a higher price such that some consumers from the monopoly segment do not buy.[21]

The first condition is satisfied if $v - p^w - t = v - c - \frac{2(3-\alpha)}{3(1-\alpha)}t > 0$, or equivalently, $\frac{v-c}{t} > \frac{2(3-\alpha)}{3(1-\alpha)}$. It remains to explore the second condition.

Consider firm 1 deviating to a price $p_1 \in (v-t, p^{nw}+t)$. Note that $p_1 < p^{nw}+t$ ensures that the marginal consumer in the competitive segment is in the interior. One can show that $v - t < p^{nw} + t$ if and only if $\frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$. By taking into account the first condition we obtain that $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$ which is non-empty for $\alpha < \frac{3}{5}$. We show that such a deviation is unprofitable. The profit of firm 1 is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p^{nw} - \frac{1+\alpha}{2t}p_1\right).$$

By taking the derivative of this profit function (multiplied by $2t$) and plugging in p^{nw} we find

$$\begin{aligned}
&2\alpha v + (1-\alpha)t + (1-\alpha)p^{nw} + (1+\alpha)c - 2(1+\alpha)p_1 \\
&\quad < 2\alpha v + (1-\alpha)t + (1-\alpha)p^{nw} + (1+\alpha)c - 2(1+\alpha)v + 2(1+\alpha)t \\
&\quad = (1-\alpha)t + 2c + \frac{3-\alpha}{3}t + 2(1+\alpha)t - 2v.
\end{aligned}$$

For all v satisfying the first condition (i.e. $v - c - \frac{2(3-\alpha)}{3(1-\alpha)}t > 0$) we have that the derivative can be evaluated from above by

$$\begin{aligned}
(1-\alpha)t + 2c + \frac{3-\alpha}{3}t + 2(1+\alpha)t - 2c - \frac{4(3-\alpha)}{3(1-\alpha)}t \\
= (3+\alpha)t + \frac{3-\alpha}{3}\left(1 - \frac{4}{1-\alpha}\right)t \\
= (3+\alpha)t - \frac{(3-\alpha)(3+\alpha)}{3(1-\alpha)}t = -\frac{2\alpha(3+\alpha)}{3(1-\alpha)}t < 0,
\end{aligned}$$

implying that the profit of firm 1 strictly decreases for all prices in $(v-t, p^{nw}+t)$. Thus, a deviation to such a p_1 is never optimal.

Next, consider a deviation of firm 1 to a price $p_1 > \max\{p^{nw}+t, v-t\}$. For these prices, the demand of firm 1 in the competitive segment drops down to zero and the profit is equal

[21] Obviously, firm 1 does not find it profitable to deviate to too low prices to serve the entire competitive segment.

to $\pi_1(p_1, p^{nw}) = \alpha(p_1 - c)(v - p_1)/t$. Note that $\frac{v-c}{t} > \frac{2(3-\alpha)}{3(1-\alpha)} = \frac{2}{3} + \frac{4}{3(1-\alpha)} > \frac{2}{3} + \frac{4}{3} = 2$. Thus, the analysis of the monopoly problem suggests that the profit of firm 1 decreases in p_1 and is maximal at $p_1 = \max\{p^{nw} + t, v - t\}$. First, suppose that $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$ (which holds true for $\alpha < \frac{3}{5}$) and, therefore, $v - t < p^{nw} + t$. In this case, we have shown that the equilibrium profit is higher than the profit at $p_1 = p^{nw} + t$. We conclude that a deviation to a price $p_1 > p^{nw} + t$ is unprofitable. Second, consider the case in which $\frac{v-c}{t} \geq \frac{9-7\alpha}{3(1-\alpha)}$ which ensures that $v - t \geq p^{nw} + t$. The maximal profit from such a deviation is equal to $\alpha(v - t - c)$. The equilibrium profit of firm 1 is weakly larger than the profit from this deviation if and only if $\frac{(3+\alpha)^2}{18(1-\alpha)}t \geq \alpha(v - t - c)$. By rearranging we obtain

$$\frac{v-c}{t} \leq 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}.$$

One can show that the function

$$g(\alpha) = 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)} - \frac{9-7\alpha}{3(1-\alpha)}$$

strictly decreases on $\alpha \in (0, 3/5)$ and is equal to 0 at $\alpha = \frac{3}{5}$. This implies that for all $\alpha < 3/5$ we have $\frac{9-7\alpha}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$ and a deviation to price $v - t$ is unprofitable.

To sum up, we conclude that if $\alpha < \frac{3}{5}$ and $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$, there is an equilibrium in which firms set prices

$$p^w = c + \frac{3+\alpha}{3(1-\alpha)}t \quad \text{and} \quad p^{nw} = c + \frac{3-\alpha}{3(1-\alpha)}t,$$

all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. Moreover, the marginal consumer in the competitive segment enjoys a positive surplus. The most remotely located consumer in the monopoly segment also enjoys a positive surplus. The respective profits are

$$\pi^w = \frac{(3+\alpha)^2}{18(1-\alpha)}t \quad \text{and} \quad \pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)}t.$$

Case 1.2: Monopoly segment of firm 1 is fully covered. The consumer located at 1 in the monopoly segment obtains zero surplus. Consider the possibility that a consumer located at 1 in the monopoly segment is indifferent between buying from firm 1 and taking the outside option. This implies that firm 1 sets a price $p^w = v - t$. From the analysis of the previous case, the best response of firm 2 is to set price $p^{nw} = \frac{1}{2}(c + t + p^w)$, implying that

$$p^w = v - t \ \text{ and } \ p^{nw} = \frac{v+c}{2}.$$

To ensure that firm 1 sells in the competitive segment, we must have that the location of the marginal consumer in the competitive segment is in the interior, $|p^w - p^{nw}| < t$, implying that $\frac{v-c}{t} < 4$. Note that under this condition firm 2 does not find it profitable to deviate.

It remains to check firm 1's incentives to deviate. If firm 1 sets a price $p_1 > p^w$ and the marginal consumer in the competitive segment is still in the interior, then its profit is given by

$$\begin{aligned} \pi_1(p_1, p^{nw}) &= (p_1 - c)\left(\alpha\frac{v - p_1}{t} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right) \\ &= (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p^{nw} - \frac{1+\alpha}{2t}p_1\right). \end{aligned}$$

The derivative of this profit (multiplied by $2t/(1+\alpha)$) is

$$\begin{aligned} &c + \frac{2\alpha v}{1+\alpha} + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p^{nw} - 2p_1 < c + \frac{2\alpha v}{1+\alpha} + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p^{nw} - 2p^w \\ &= -\frac{3+\alpha}{2(1-\alpha)}(v - c - 2t) \leq 0, \end{aligned}$$

if and only if $\frac{v-c}{t} \geq 2$. The analysis of the monopoly problem implies that under this condition firm 1 does find it profitable to deviate to an even higher price at which it would not sell in the competitive segment.

If firm 1 deviates to a price $p_1 < p^w$, then it does not increase sales in the monopoly segment and its profit function is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

The derivative of this profit function (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned} &c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p_1 > c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p^w \\ &= -\frac{3}{2}\left(v - c - \frac{2(3-\alpha)}{3(1-\alpha)}\right) \leq 0, \end{aligned}$$

if and only if $\frac{v-c}{t} \leq \frac{2(3-\alpha)}{3(1-\alpha)}$. This condition ensures that firm 1 does not deviate to lower prices.

To sum up, we conclude that, for $\alpha \leq \frac{3}{5}$ and $\frac{v-c}{t} \in \left[2, \frac{2(3-\alpha)}{3(1-\alpha)}\right]$ as well as for $\alpha > \frac{3}{5}$ and $\frac{v-c}{t} \in [2,4]$, there exists an equilibrium in which firms set prices

$$p^w = v - t \quad \text{and} \quad p^{nw} = \frac{v+c}{2},$$

all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. The marginal consumer in the competitive segment enjoys a positive surplus. The most remotely located consumer in the monopoly segment obtains zero surplus. The respective profits are

$$\pi^w = (v-c-t)\left(1-(1-\alpha)\frac{v-c}{4t}\right) \quad \text{and} \quad \pi^{nw} = (1-\alpha)\frac{(v-c)^2}{8t}.$$

Case 2: Monopoly segment of firm 1 is not fully covered. If this case, we must have that the most remotely located consumer in the monopoly segment does not buy from firm 1 – that is, $v - p^w - t < 0$. The profit of firm 1 setting price p_1 at which the monopoly segment of firm 1 is not fully covered and the marginal consumer in the competitive segment is in the interior is

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p^{nw} - \frac{1+\alpha}{2t}p_1\right).$$

By taking the first-order condition and solving for the profit-maximizing price of firm 1 we find that

$$p^w = \frac{1}{2}\left(c + \frac{1-\alpha}{1+\alpha}p^{nw} + \frac{2\alpha v + (1-\alpha)t}{1+\alpha}\right).$$

The problem of firm 2 is exactly the same as in the previous case implying that $p^{nw} = \frac{1}{2}(c + t + p^w)$. Solving this system of equations with respect to p^w and p^{nw}, we find that

$$p^w = \frac{3+\alpha}{3+5\alpha}c + \frac{3(1-\alpha)}{3+5\alpha}t + \frac{4\alpha}{3+5\alpha}v \quad \text{and} \quad p^{nw} = \frac{3(1+\alpha)}{3+5\alpha}c + \frac{3+\alpha}{3+5\alpha}t + \frac{2\alpha}{3+5\alpha}v.$$

We characterize conditions on the parameters such that at this price the monopoly segment is not fully covered, the marginal consumer's location in the competitive segment is in the interior.

The monopoly segment is not fully covered if and only if $v - p^w - t < 0$, which is equivalent to

$$\frac{3+\alpha}{3+5\alpha}v - \frac{3+\alpha}{3+5\alpha}c - \frac{2(3+\alpha)}{3+5\alpha}t < 0 \quad \Longleftrightarrow \quad \frac{v-c}{t} < 2.$$

Next, we check that the marginal consumer in the competitive segment is in the interior. This occurs if and only if $|p^w - p^{nw}| < t$. Since the monopoly segment is not fully covered we have that

$$p^{nw} - p^w = \frac{4\alpha}{3+5\alpha}t - \frac{2\alpha}{3+5\alpha}(v-c) = \frac{2\alpha}{3+5\alpha}(2t-(v-c)) > 0$$

and, moreover, $p^{nw} - p^w < \frac{4\alpha}{3+5\alpha}t < t$, implying that the marginal consumer is indeed in the interior.

The surplus of the marginal consumer is positive if

$$\begin{aligned} v - p^w - t\left(\frac{1}{2} + \frac{p^{nw}-p^w}{2t}\right) &= v - \frac{t}{2} - \frac{p^{nw}+p^w}{2} \\ &= \frac{3+2\alpha}{3+5\alpha}\left(v - c - \frac{9+3\alpha}{6+4\alpha}t\right) > 0 \iff \frac{v-c}{t} > \frac{9+3\alpha}{6+4\alpha}. \end{aligned}$$

Both conditions imply that if this equilibrium exists, then it must be that $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$.

To show that p^w and p^{nw} constitute an equilibrium, it remains to be shows that *i)* firm 1 does not have an incentive to set a lower price to fully serve either of the segments and *ii)* firm 2 does not have an incentive to increase its price such that some consumers from the competitive segment do not buy.

We start by exploring condition *i)* accounting for $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$. Suppose that firm 1 deviates from p^w and lowers its price to $p_1 \in (p^{nw} - t, v - t]$. In this case, it corners the monopoly segment but the competitive segment remains covered with both firms obtaining positive market shares. The profit from such a deviation is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

By taking the derivative (multiplied by $2t/(1-\alpha)$) we have that

$$\begin{aligned} c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p_1 &\geq c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2v + 2t \\ &= -\left(v - p^{nw} - \frac{1+\alpha}{1-\alpha}t\right) + (2t - (v-c)). \end{aligned}$$

Note that the second term is positive since $\frac{v-c}{t} < 2$. The first term $v - p^{nw} - \frac{1+\alpha}{1-\alpha}t$ is bounded from above by $v - p^w - t$, which is negative as the most remotely located consumer is not served. Thus, we showed that the profit function of firm 1 is increasing on $(p^{nw}, v - t]$.

Moreover, at price p^{nw}, firm 1 fully serves both segments and setting a price lower than it cannot be optimal. Therefore, we established that any deviation to a price lower than $v-t$ is unprofitable, which implies that firm 1 does not have an incentive to deviate from p^w.

Next, we show that *ii)* is satisfied and firm 2 does not find it profitable to set a price $p_2 > 2v - t - p^w$ implying that consumers with locations $\frac{v-p^w}{t} + \varepsilon$, when $\varepsilon > 0$ is small, do not buy from either firm – that is, $v - p_2 - t\left(1 - \frac{v-p^w}{t}\right) < 0$. In this case, the profit of firm 2 strictly decreases if $p_2 > \frac{v+c}{2}$ where the latter expression represents the price a local monopolist would set. Note that

$$\begin{aligned} p_2 - \frac{v+c}{2} &> v - t - p^w + \frac{v-c}{2} \\ &> \left(v - \frac{t}{2} - \frac{p^{nw}+p^w}{2}\right) + \frac{v-c-t}{2} > 0. \end{aligned}$$

The first term in brackets represents the surplus of the marginal consumer in the equilibrium and is always positive. The second term is also positive since $\frac{v-c}{t} > \frac{9+3\alpha}{6+4\alpha} > 1$. Hence, the profit function of firm 2 decreases for p_2 higher than $2v - t - p^w$ implying that firm 2's deviation to such a p_2 is unprofitable.

To sum up, we have established that for $\frac{v-c}{t} \in \left(\frac{9+3\alpha}{6+4\alpha}, 2\right)$ there is an equilibrium in which firms set prices

$$p^w = \frac{3+\alpha}{3+5\alpha}c + \frac{3(1-\alpha)}{3+5\alpha}t + \frac{4\alpha}{3+5\alpha}v \ \text{ and } \ p^{nw} = \frac{3(1+\alpha)}{3+5\alpha}c + \frac{3+\alpha}{3+5\alpha}t + \frac{2\alpha}{3+5\alpha}v,$$

the monopoly segment is not fully covered; the competitive segment is fully covered where the marginal consumer enjoys a positive surplus. The corresponding equilibrium profits are

$$\pi^w = \frac{1+\alpha}{2t}\left(\frac{4\alpha}{3+5\alpha}(v-c) + \frac{3(1-\alpha)}{3+5\alpha}t\right)^2 \text{ and } \pi^{nw} = \frac{1-\alpha}{2t}\left(\frac{2\alpha}{3+5\alpha}(v-c) + \frac{3+\alpha}{3+5\alpha}t\right)^2.$$

We have explored all the cases in which the marginal consumer in the competitive segment exists, is located in the interior, and enjoys a positive surplus. We come to the next possible equilibrium structure.

***iv)* Firm 1 sells in the competitive fully covered segment and the marginal consumer obtains zero surplus.** Note that if this type of equilibrium occurs, then firm 1 serves exactly the same fraction of consumers in both segment implying that the monopoly segment cannot be fully covered.

Define the location of marginal consumer in the competitive market as $\hat{x} \in (0,1)$. This consumer is indifferent between both firms and the outside option. This implies that

$$p^w = v - t\hat{x} \ \text{ and } \ p^{nw} = v - t + t\hat{x}.$$

We characterize all possible $\hat{x}$ that can constitute an equilibrium. First, firms do not have incentives to increase their prices if and only if $\max\left\{0, 1 - \frac{v-c}{2t}\right\} \leq \hat{x} \leq \min\left\{\frac{v-c}{2t}, 1\right\}$. This condition is satisfied for some $\hat{x} \in (0,1)$ if and only if $\frac{v-c}{t} \geq 1$. Second, consider a deviation of firm 2 to a price $p_2 < p^{nw}$. The profit of firm 2 is

$$\pi_2(p_2, p^w) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p^w - p_2}{2t}\right).$$

This profit function increases for prices $p_2 < p^w$ if and only if its derivative at p_2 is positive. The derivative of the profit function of firm 2 (multiplied by $2t$) is

$$\begin{aligned} c + t + p^w - 2p_2 &> c + t + p^w - 2p^{nw} \\ &= c + t + v - t\hat{x} - 2v + 2t - 2t\hat{x} \\ &= 3t\left(1 - \hat{x} - \frac{v-c}{t}\right) \geq 0, \end{aligned}$$

if and only if $\hat{x} \leq 1 - \frac{1}{3}\frac{v-c}{t}$. Next, we establish the conditions on $\hat{x}$ to ensure that firm 1 does not deviate to lower prices. The profit of firm 1 deviating to $p_1 < p^w$ is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{2\alpha v + (1-\alpha)t}{2t} + \frac{1-\alpha}{2t}p^{nw} - \frac{1+\alpha}{2t}p_1\right).$$

The derivative of this profit function (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned} & c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p^{nw} - 2p_1 > c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}p^{nw} - 2p^w \\ &= c + \frac{2\alpha}{1+\alpha}v + \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}v - \frac{1-\alpha}{1+\alpha}t + \frac{1-\alpha}{1+\alpha}t\hat{x} - 2v + 2t\hat{x} \\ &= \frac{3+\alpha}{1+\alpha}t\left(\hat{x} - \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}\right) \geq 0, \end{aligned}$$

if and only if $\hat{x} \geq \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}$.

Thus, $\hat{x} \in (0,1)$ can be supported in equilibrium if and only if

$$\begin{cases} \max\left\{0, 1 - \frac{v-c}{2t}\right\} \leq \hat{x} \leq \min\left\{\frac{v-c}{2t}, 1\right\}, \\ \hat{x} \leq 1 - \frac{1}{3}\frac{v-c}{t}, \\ \hat{x} \geq \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}. \end{cases}$$

Suppose that $\frac{v-c}{t} > 2$, then the first condition is always satisfied. Note that there exists $\hat{x}$ satisfying condition 2 and condition 3 if

$$\left(\frac{1+\alpha}{3+\alpha} + \frac{1}{3}\right)\frac{v-c}{t} = \frac{6+4\alpha}{9+3\alpha}\frac{v-c}{t} < 1.$$

It is straightforward to see that this condition is never satisfied for $\frac{v-c}{t} > 2$.

Next, suppose that $\frac{v-c}{t} \in [1,2]$. Then, condition 1 simplifies to $1 - \frac{v-c}{2t} \leq \hat{x} \leq \frac{v-c}{2t}$. We consider two cases of whether $\frac{v-c}{t} \in [1;6/5]$ or $\frac{v-c}{t} \in (6/5;2]$ separately. First, suppose that $\frac{v-c}{t} \in [1;6/5]$. Note that in this case $1 - \frac{1}{3}\frac{v-c}{t} \geq 1 - \frac{1}{3} \times \frac{6}{5} = \frac{1}{2}\frac{6}{5} \geq \frac{1}{2}\frac{v-c}{t}$. Moreover, since $\frac{1+\alpha}{3+\alpha} \leq \frac{1}{2}$ for all $\alpha \in [0,1]$ we have that there exists a non-empty interval of $\hat{x}$ satisfying all of the conditions. Note that $1 - \frac{1}{2}\frac{v-c}{t} \geq (<) \frac{1+\alpha}{3+\alpha}\frac{v-c}{t}$ if and only if $\frac{v-c}{t} \leq (>) \frac{6+2\alpha}{5+3\alpha}$. Therefore, we can conclude that $\hat{x}$ that satisfies all the conditions

$$\begin{cases} \hat{x} \in \left[1 - \frac{1}{2}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left[1, \frac{6+2\alpha}{5+3\alpha}\right] \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6+2\alpha}{5+3\alpha}, \frac{6}{5}\right]. \end{cases}$$

Next, consider the case in which $\frac{v-c}{t} \in (6/5;2]$. By following the above argumentation we can show that $\frac{1}{2}\frac{v-c}{t} > 1 - \frac{1}{3}\frac{v-c}{t}$. Moreover, $\frac{1+\alpha}{3+\alpha}\frac{v-c}{t} > \frac{1}{3} \times \frac{6}{5} = 1 - \frac{1}{2} \times \frac{6}{5} > 1 - \frac{1}{2}\frac{v-c}{t}$. This implies that $\hat{x}$ satisfying all the conditions belongs to $\left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, 1 - \frac{1}{3}\frac{v-c}{t}\right]$. This interval is non-empty if $\frac{v-c}{t} \leq \left(\frac{6}{5}, \frac{9+3\alpha}{6+4\alpha}\right]$.

To sum up, we conclude that for $\frac{v-c}{t} \in \left[1, \frac{9+3\alpha}{6+4\alpha}\right]$ there are multiple equilibria characterized by the location of the marginal consumer $\hat{x} \in (0,1)$. The possible equilibrium locations of the marginal consumer are summarized as follows

$$\begin{cases} \hat{x} \in \left[1 - \frac{1}{2}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left[1, \frac{6+2\alpha}{5+3\alpha}\right] \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, \frac{1}{2}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6+2\alpha}{5+3\alpha}, \frac{6}{5}\right], \\ \hat{x} \in \left[\frac{1+\alpha}{3+\alpha}\frac{v-c}{t}, 1 - \frac{1}{3}\frac{v-c}{t}\right], \text{ if } \frac{v-c}{t} \in \left(\frac{6}{5}, \frac{9+3\alpha}{6+4\alpha}\right]. \end{cases}$$

The equilibrium prices are given by

$$p^w = v - t\hat{x} \text{ and } p^{nw} = v - t + t\hat{x},$$

the monopoly segment is not fully covered, the competitive segment is fully covered and the marginal consumer obtains zero surplus. The corresponding profits are

$$\pi^w = t\left(\frac{v-c}{t} - \hat{x}\right)\hat{x} \text{ and } \pi^{nw} = (1-\alpha)t\left(\frac{v-c}{t} - (1-\hat{x})\right)(1-\hat{x}).$$

Note that $\hat{x}$ is always weakly lower than $\frac{1}{2}\frac{v-c}{t}$ implying that π^w increases in $\hat{x}$ and π^{nw} decreases in $\hat{x}$ for all $\hat{x}$ that might constitute an equilibrium.

The key results. We are ready to explore the condition of Proposition 3 in the Hotelling model with linear transport cost in the parameter range $\frac{v-c}{t} \in \left(\frac{3}{2}, \frac{7}{2}\right)$.

Lemma 3. *Consider the Hotelling model with linear transport costs and a positive fraction of consumers using the ad blocker for* $\frac{v-c}{t} \in \left(\frac{3}{2}, \frac{7}{2}\right)$. *Then, the following inequality holds*

$$\min\{\pi^m, 2\pi^d\} \leq \pi^d + \pi^{nw} \leq \max\{\pi^m, 2\pi^d\}.$$

Proof. We show that $2\pi^d \leq \pi^d + \pi^{nw} \leq \pi^m$ for $\frac{v-c}{t} \in \left[2, \frac{7}{2}\right)$ and $\pi^m \leq \pi^d + \pi^{nw} \leq 2\pi^d$ for $\frac{v-c}{t} \in \left(\frac{3}{2}, 2\right)$. We proceed by verifying these inequalities for all parameter regions with different profit characterizations separately.

First, we consider $\frac{v-c}{t} \in \left[2, \frac{7}{2}\right)$. In this interval, the monopoly and duopoly profits are given respectively by $\pi^m = v - c - t$ and $\pi^d = \frac{t}{2}$.

For $\alpha \leq 9/17$ and $\frac{v-c}{t} \in \left(\frac{2(3-\alpha)}{3(1-\alpha)}, \frac{7}{2}\right)$, the monopolistic market is fully covered and both firms serve some consumers in the competitive market. Resulting profits are $\pi^w = \frac{(3+\alpha)^2}{18(1-\alpha)}t$ and $\pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)}t$. The sum of profits is equal

$$\pi^w + \pi^{nw} = \frac{18 + 2\alpha^2}{18(1-\alpha)}.$$

Note that the sum of profits can be bounded from below by $2\pi^d$ and from above by π^m as

$$2\pi^d = t \leq \frac{18}{18(1-\alpha)}t \leq \pi^w + \pi^{nw} \leq \frac{18 + 6\alpha}{18(1-\alpha)}t = \left(\frac{2(3-\alpha)}{3(1-\alpha)} - 1\right)t < v - c - t = \pi^m.$$

If $\frac{v-c}{t} \in \left[2, \min\left\{\frac{2(3-\alpha),}{3(1-\alpha)}, \frac{7}{2}\right\}\right]$, firm 1 sets the monopoly price and serves all consumers in the monopoly segment but also a positive fraction in the competitive segment. The equilibrium profits are given by $\pi^w = (v-c-t)(1-(1-\alpha)\frac{v-c}{4t})$ and $\pi^{nw} = (1-\alpha)\frac{(v-c)^2}{8t}$. To show that the sum of profits is less than the monopoly profit, we note that

$$\pi^m - \pi^w - \pi^{nw} = \frac{1-\alpha}{4}(v-c)\left(\frac{v-c}{t} - 1 - \frac{v-c}{2t}\right) \geq 0,$$

as $\frac{v-c}{t} \geq 2$. Similarly, to show that the sum of profits is greater than the duopoly profits $2\pi^d$, we note that

$$\begin{aligned}\pi^w + \pi^{nw} - 2\pi^d &= \left(\frac{v-c}{t} - 2\right) t + \frac{1-\alpha}{8}(v-c)\left(2 - \frac{v-c}{t}\right) \\ &= \left(\frac{v-c}{t} - 2\right)\left(1 - \frac{1-\alpha}{8}\frac{v-c}{t}\right) t \geq 0.\end{aligned}$$

Second, we show that $\pi^m \leq \pi^w + \pi^{nw} \leq 2\pi^d$ for $\frac{v-c}{t} \in \left(\frac{3}{2}, 2\right)$. In this parameter range, some consumers in the monopoly segment do not buy, the competitive segment is fully covered where the marginal consumer enjoys a positive surplus. The firms' profits are given by $\pi^m = \frac{(v-c)^2}{4t}$, $\pi^d = \frac{t}{2}$, $\pi^w = \frac{1+\alpha}{2t}\left(\frac{4\alpha}{3+5\alpha}(v-c) + \frac{3(1-\alpha)}{3+5\alpha}t\right)^2$, and $\pi^{nw} = \frac{1-\alpha}{2t}\left(\frac{2\alpha}{3+5\alpha}(v-c) + \frac{3+\alpha)}{3+5\alpha}t\right)^2$.

We show that the sum of profits is greater than π^m:

$$\begin{aligned}\pi^w + \pi^{nw} &= \frac{1+\alpha}{2}\left(\frac{4\alpha}{3+5\alpha} + \frac{3(1-\alpha)}{3+5\alpha}\left(\frac{t}{v-c}\right)\right)^2 \frac{(v-c)^2}{t} \\ &+ \frac{1-\alpha}{2}\left(\frac{2\alpha}{3+5\alpha} + \frac{3+\alpha}{3+5\alpha}\left(\frac{t}{v-c}\right)\right)^2 \frac{(v-c)^2}{t} \\ &\geq \left(\frac{1+\alpha}{2}\left(\frac{4\alpha}{3+5\alpha} + \frac{3(1-\alpha)}{3+5\alpha}\frac{1}{2}\right)^2 + \frac{1-\alpha}{2}\left(\frac{2\alpha}{3+5\alpha} + \frac{3+\alpha}{3+5\alpha}\frac{1}{2}\right)^2\right)\frac{(v-c)^2}{t} \\ &= \frac{(v-c)^2}{4t} = \pi^m.\end{aligned}$$

Finally, we show that that the sum of profits is less than $2\pi^d$:

$$\begin{aligned}\pi^w + \pi^{nw} &= \frac{1+\alpha}{2}\left(\frac{4\alpha}{3+5\alpha}\frac{v-c}{t} + \frac{3(1-\alpha)}{3+5\alpha}\right)^2 t + \frac{1-\alpha}{2}\left(\frac{2\alpha}{3+5\alpha}\frac{v-c}{t} + \frac{3+\alpha}{3+5\alpha}\right)^2 t \\ &\leq \frac{1+\alpha}{2}\left(\frac{3+5\alpha}{3+5\alpha}\right)^2 t + \frac{1-\alpha}{2}\left(\frac{3+5\alpha}{3+5\alpha}\right)^2 t = t = 2\pi^d.\end{aligned}$$

This concludes the proof. □

Proposition 8. *Consider the Hotelling model with linear transport cost and a positive fraction of consumers using the ad blocker for* $\frac{v-c}{t} \in \left(\frac{3}{2}, \frac{7}{2}\right)$*. Then, the inequality* $\pi^m + 2\pi^{nw} \geq (3-\alpha)\pi^d$ *is satisfied if and only if* $v \geq c + 2t$*.*

Proof. We prove this lemma by considering all parameter regions corresponding to different equilibrium structures separately.

First, we show that this inequality holds for $\frac{v-c}{t} \in [2, 7/2)$. By Lemma 3, the industry profit of the asymmetric duopoly is greater than the industry profit of the symmetric duopoly

– that is, $\pi^w + \pi^{nw} \geq 2\pi^d$. Thus, if $\frac{v-c}{t} \in [2, 7/2)$, condition $\pi^{nw} \geq (1-\alpha)\pi^d$ is sufficient for inequality $\pi^m + 2\pi^{nw} \geq (3-\alpha)\pi^d$ to hold true. We show that $\pi^{nw} \geq (1-\alpha)\pi^d$ is satisfied for $\frac{v-c}{t} \in [2, 7/2)$. Recall that the symmetric duopoly profits is given by $\pi^d = \frac{t}{2}$.

Consider the case in which $\frac{v-c}{t} \in \left[2, \min\left\{\frac{2(3-\alpha)}{3(1-\alpha)}, \frac{7}{2}\right\}\right]$. In this case, all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. The respective profit of firm 2 is

$$\pi^{nw} = (1-\alpha)\frac{(v-c)^2}{8t} = (1-\alpha)\left(\frac{v-c}{2t}\right)^2 \frac{t}{2} \geq (1-\alpha)\frac{t}{2} = (1-\alpha)\pi^d.$$

It remains to consider the case in which $\alpha < \frac{9}{17}$ and $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < \frac{7}{2}$. In this case, all consumers in the monopoly segment buy from firm 1 and the market in the competitive segment is fully covered. The profit of firm 2 is

$$\pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)}t = \frac{\left(1-\frac{\alpha}{3}\right)^2}{2(1-\alpha)}t > \frac{(1-\alpha)^2}{2(1-\alpha)}t = (1-\alpha)\frac{t}{2} = (1-\alpha)\pi^d.$$

To conclude, we have established that for $\frac{v-c}{t} \in [2, 7/2)$ we have that $\pi^{nw} \geq (1-\alpha)\pi^d$ is satisfied and therefore, $\pi^m + 2\pi^{nw} \geq (3-\alpha)\pi^d$ holds true.

We turn to the case for which $\frac{v-c}{t} \in (3/2, 2)$. By Lemma 3, the industry profit of the asymmetric duopoly is strictly lower than $2\pi^d$. Thus, condition $\pi^{nw} \leq (1-\alpha)\pi^d$ implies that $\pi^m + 2\pi^{nw} < (3-\alpha)\pi^d$. We show that this condition is satisfied for $\frac{v-c}{t} \in (3/2, 2)$. Note that in this parameter region $\pi^d = \frac{t}{2}$. The profit of the non-whitelisted firm is lower than $(1-\alpha)\pi^d$ since

$$\begin{aligned}\pi^{nw} &= \frac{1-\alpha}{2}\left(\frac{2\alpha}{3+5\alpha}\frac{v-c}{t} + \frac{3+\alpha}{3+5\alpha}\right)^2 t < \frac{1-\alpha}{2}\left(\frac{2\alpha}{3+5\alpha}\times 2 + \frac{3+\alpha}{3+5\alpha}\right)^2 t \\ &= (1-\alpha)\frac{t}{2} = (1-\alpha)\pi^d.\end{aligned}$$

This concludes the proof. □

Next, we show that publisher surplus can be higher with the presence of the ad blocker.

Proposition 9. *Consider the Hotelling model with linear transport cost and a positive fraction of consumers using the ad blocker for* $\alpha > \frac{3}{5}$ *and* $\frac{v-c}{t} \in \left(2 + \frac{1}{2}\frac{1+\alpha}{1-\alpha}, \frac{1+\alpha}{1-\alpha}\right)$. *Then,* $\pi^{nw} > (1+\alpha)\pi^d$.

Proof. Note that for $\alpha > \frac{3}{5}$ we have that $2 + \frac{1}{2}\frac{1+\alpha}{1-\alpha} > 4$. Previously, we showed that for $\frac{v-c}{t} \in \left(4, \frac{1+\alpha}{1-\alpha}\right)$ firm 1 serves all consumers who use the ad blocker and firm 2 serves all consumers who do not use the ad blocker. The profit of firm 2 is given by $\pi^{nw} = (1-\alpha)(v-c-2t)$ and $\pi^{d} = \frac{t}{2}$. Therefore,

$$\pi^{nw} - (1+\alpha)\pi^{d} = (1-\alpha)t\left(\frac{v-c}{t} - 2 - \frac{1}{2}\frac{1+\alpha}{1-\alpha}\right) > 0.$$

This concludes the proof. □

B.2 Full analysis of the Hotelling model with quadratic transport costs

In this Appendix, we analyze the Hotelling setting with quadratic transport costs. The transport cost parameter is denoted by t as in the setting with linear transport costs, but now transport costs are t times the quadratic distance between consumer and a firm.

Monopoly. Suppose that there is one firm located at 0. A consumer located at x buys the product at price p if $v - p - tx^2 \geq 0$. The profit of this firm setting price p is

$$\pi(p) = (p - c) \min \left\{ \sqrt{\frac{v-p}{t}}, 1 \right\}.$$

Solving for the profit-maximizing price, we find that $p^m = \frac{2}{3}v + \frac{1}{3}c$ for $\frac{v-c}{t} \leq 3$ and $p^m = v - t$ for $\frac{v-c}{t} > 3$. The respective profits are

$$\pi^m = \begin{cases} \frac{2}{3}\sqrt{\frac{(v-c)^3}{3t}}, \text{ if } \frac{v-c}{t} \leq 3, \\ v - t - c, \text{ if } \frac{v-c}{t} > 3. \end{cases}$$

Symmetric competition. Consider the Hotelling duopoly with quadratic transport costs. Suppose that firm 1 is located at 0 and firm 2 is located at 1. The demand of firm 1 is given by

$$D_1(p_1, p_2) = \max \left\{ 0, \min \left\{ \frac{1}{2} + \frac{p_2 - p_1}{2t}, \sqrt{\frac{v - p_1}{t}}, 1 \right\} \right\}.$$

We consider three possibilities: *i)* both firms act as local monopolists, *ii)* firms compete and the indifferent consumer located at $\hat{x}$ obtains positive surplus. *iii)* firms compete and the indifferent consumer obtains zero surplus.

First, in a highly differentiated market, both firms act as monopolists. This holds if the consumer located at $x = 1/2$ does not buy, i.e., $v - (1/2)^2 t - 2/3v - 1/3c < 0$, which is the case for $\frac{v-c}{t} < 3/4$. The demand of firm i in a small neighborhood of the equilibrium prices is $D_i = \sqrt{\frac{v-p_i}{t}}$. The equilibrium price is $p^d = \frac{2}{3}v + \frac{1}{3}c$ and the equilibrium demand of each firm is $D_i(p^d, p^d) = \sqrt{\frac{v-c}{3t}}$.

Second, suppose that the market is fully covered and the marginal consumer obtains a strictly positive surplus. Then, firms compete and maximize $\pi_i = (p_i - c)(\frac{1}{2} + \frac{p_d - p_i}{2t})$. In equilibrium, both firms set $p^d = c + t$ and make profits of $\frac{t}{2}$. This constitutes an equilibrium if and only if the indifferent consumer at $x = 1/2$ obtains a positive surplus. This is the case if $v - (1/2)^2 t - (c + t) > 0$, i.e., $\frac{v-c}{t} > 5/4$.

Third, for $3/4 \leq \frac{v-c}{t} \leq 5/4$, there are multiple equilibria characterized by the location of the marginal consumer. In the symmetric equilibrium, the marginal consumer located at $x = 1/2$ is indifferent between either of the firms and the outside option. This implies that firms set $p = v - (1/4)t$.

To sum up, we obtain that

$$p^d = \begin{cases} \frac{2}{3}v + \frac{1}{3}c, \text{ if } \frac{v-c}{t} < \frac{3}{4}, \\ v - \frac{t}{4}, \text{ if } \frac{v-c}{t} \in \left[\frac{3}{4}, \frac{5}{4}\right], \\ c + t, \text{ if } \frac{v-c}{t} > \frac{5}{4}. \end{cases}$$

The equilibrium duopoly profit is

$$\pi^d = \begin{cases} \frac{2}{3}\sqrt{\frac{(v-c)^3}{3t}}, \text{ if } \frac{v-c}{t} < \frac{3}{4}, \\ \frac{1}{2}\left(v - \frac{t}{4} - c\right), \text{ if } \frac{v-c}{t} \in \left[\frac{3}{4}, \frac{5}{4}\right], \\ \frac{t}{2}, \text{ if } \frac{v-c}{t} > \frac{5}{4}. \end{cases}$$

In the following lemma, we compare the monopoly profit with the total duopoly profit.

Lemma 4. *In the Hotelling model with quadratic transport cost $\pi^m \geq 2\pi^d$ if and only if $\frac{v-c}{t} \geq (\frac{27}{4})^{1/3}$.*

Proof. We compare π^m and $2\pi^d$ in the four parameter regions one after the other. First, if $\frac{v-c}{t} < \frac{3}{4}$ we have that $\pi^m = \pi^d$ implying that $\pi^m < 2\pi^d$. Second, if $\frac{v-c}{t} \in \left[\frac{3}{4}, \frac{5}{4}\right]$ we have that the industry duopoly profit is given by $2\pi^d = v - \frac{t}{4} - c$. The difference in profits divided by t is

$$\begin{aligned} \frac{\pi^m - 2\pi^d}{t} &= \frac{2}{3}\frac{v-c}{t}\sqrt{\frac{(v-c)}{3t}} - \left(\frac{v-c}{t} - \frac{1}{4}\right) \\ &= \frac{v-c}{t}\left(\frac{2}{3}\sqrt{\frac{(v-c)}{3t}} - 1\right) + \frac{1}{4} < \frac{v-c}{t}\left(\frac{2}{3}\sqrt{\frac{1}{3} \times \frac{3}{4}} - 1\right) + \frac{1}{4} \\ &= -\frac{2}{3}\frac{v-c}{t} + \frac{1}{4} < -\frac{1}{2} + \frac{1}{4} < 0. \end{aligned}$$

Thus, we have that $2\pi^d > \pi^m$ for $\frac{v-c}{t} \in \left[\frac{3}{4}, \frac{5}{4}\right]$.

Third, consider the interval $\frac{v-c}{t} \in \left(\frac{5}{4}, 3\right]$. In this interval, the industry duopoly profit equals t. Thus, the difference in profits divided by t is given by

$$\frac{\pi^m - 2\pi^d}{t} = \sqrt{\frac{4}{27}\left(\frac{v-c}{t}\right)^3} - 1.$$

Note that $2\pi^d > \pi^m$ for $\frac{v-c}{t} \in \left(\frac{5}{4}, (\frac{27}{4})^{1/3}\right)$ and $\pi^m \geq 2\pi^d$ for $\frac{v-c}{t} \in \left[(\frac{27}{4})^{1/3}, 3\right]$.

Fourth, we consider the parameter region in which $\frac{v-c}{t} > 3$. In this case, the monopolist serves all consumers and makes profits $v - t - c$. Since $v > c + 3t$, these profits are higher than the industry duopoly profits, t. This concludes the proof.

□

Asymmetric competition. In the asymmetric case, a fraction α of consumers knows only about the existence of firm 1. We characterize pure strategy equilibria for $\frac{v-c}{t} \in \left(\frac{3}{2}, 4\right)$.

Denote p^w and p^{nw} as the equilibrium prices of firm 1 and firm 2 respectively. We start by deriving the demand function for firm 1 setting price p_1 when firm 2 price is p^{nw}; that is,

$$\begin{aligned} D_1(p_1, p^{nw}) &= \alpha \max\left\{0, \min\left\{\sqrt{\frac{v-p_1}{t}}, 1\right\}\right\} \\ &\quad + (1-\alpha)\max\left\{0, \min\left\{\frac{1}{2} + \frac{p^{nw}-p_1}{2t}, \sqrt{\frac{v-p_1}{t}}, 1\right\}\right\}. \end{aligned}$$

The demand of firm 2 setting price p_2 playing against firm 1 setting price p^w is given by

$$D_2(p_2, p^w) = (1-\alpha)\max\left\{0, \min\left\{\frac{1}{2} + \frac{p^w - p_2}{2t}, \sqrt{\frac{v-p_2}{t}}, 1\right\}\right\}.$$

It is straightforward to see that firm 2 serves a positive measure of consumers in the competitive segment. Therefore, it is sufficient to consider four different possibilities: *i)* firm 1 does not sell in the competitive segment; *ii)* firm 1 sells in the competitive segment, it is fully covered, and the marginal consumer obtains a positive surplus; *iii)* firm 1 sells in the competitive segment, it is fully covered, and the marginal consumer obtains zero surplus and *iv)* firm 1 sells in the competitive segment and this segment is not fully covered.

i) **Firm 1 sells in the competitive segment; the competitive segment is not fully covered.** In this case firms act as local monopolies. The profit of firm 1 setting price p_1 is $\pi_1(p_1, p^{nw}) = (p_1 - c)\sqrt{(v - p_1)/t}$. The optimal price is $p^w = \frac{2}{3}v + \frac{1}{2}c$. The profit of firm 2 setting price p_2 is $(1-\alpha)(p_2 - c)\sqrt{(v - p_2)/t}$. The optimal price of firm 2 is $p^{nw} = \frac{2}{3}v + \frac{1}{2}c$. Next, we check that the demand of firm 1 and the demand of firm 2 in the competing segment do not overlap – that is, $\sqrt{\frac{v-c}{3t}} < 1 - \sqrt{\frac{v-c}{3t}}$ implying $\frac{v-c}{t} < \frac{3}{4}$. Clearly, firms do not have profitable deviations.

To sum up, if $\frac{v-c}{t} < \frac{3}{4}$ there is an equilibrium in which firms act as local monopolists setting the monopoly prices

$$p^w = p^{nw} = \frac{2}{3}v + \frac{1}{3}c.$$

The respective profits are

$$\pi^w = \frac{2}{3}\sqrt{\frac{(v-c)^3}{3t}} \text{ and } \pi^{nw} = \frac{2}{3}(1-\alpha)\sqrt{\frac{(v-c)^3}{3t}}.$$

ii) **Firm 1 does not sell in the competitive segment.** Next, consider the case in which firm 1 does not sell in the competitive segment. Then firm 2 must fully serve it in equilibrium.

We show that the monopoly segment of firm 1 is fully covered. Suppose for a contradiction that that firm 1 does not serve all consumers in the monopoly segment. This implies that $\frac{v-c}{t} < 3$ as otherwise firm 1 would deviate to a lower price and serve the entire monopoly segment. But if $\frac{v-c}{t} < 3$, then firm 2 serving all consumers in the competitive segment would find it profitable to lower its price, a contradiction. This implies that if firm 1 does not sell in the competitive segment, then it must serve all consumers in its monopoly segment.

Next, we solve for the optimal prices. The fact that the demand of firm 1 consists of the entire monopoly segment, we have that $p^w - v - t$. The maximal price that firm 2 can set to serve all consumers from the competitive segment solves $v - t - p^{nw} = v - p^w$. By plugging in p^w and solving for p^{nw} we find that the possible equilibrium is represented by the following prices

$$p^w = v - t \text{ and } p^{nw} = v - 2t.$$

It remains to check that then neither firm finds it profitable to deviate. We first establish the conditions under which firm 2 does not have incentives to deviate to a higher price. The profit of firm 2 deviating to $p_2 > p^{nw}$ is $\pi_2(p_2, p^{nw}) = \frac{1}{2t}(1-\alpha)(p_2 - c)(t + p^w - p_2)$. The derivative of this profit function is

$$c + t + p^w - 2p_2 < c + t + p^w - 2p^{nw} = -v + c + 4t \leq 0,$$

if and only if $\frac{v-c}{t} \geq 4$. Under this condition, the profit function decreases for all $p_2 > p^{nw}$ and firm 2 does not deviate to a higher price.

Next, we explore firm 1's incentive to deviate. Condition $\frac{v-c}{t} \geq 4$ implies that firm 1 does not deviate to higher prices (see the monopoly problem in the symmetric case). Thus, it remains to establish conditions under which firm 1 does not deviate to lower prices. If firm 1 sets a lower price, $p_1 < v - t$, then it would serve some consumers from the competitive markets resulting in total profits

$$\begin{aligned}\pi_1(p_1, p^{nw}) &= (p_1 - c)\left(\alpha + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right) \\ &= (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).\end{aligned}$$

The derivative of this profit (multiplied by $2t/(1-\alpha)$) is

$$\begin{aligned}c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p_1 &> c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2v + 2t \\ &= c + \frac{1+\alpha}{1-\alpha}t - v \geq 0,\end{aligned}$$

if and only if $\frac{v-c}{t} \leq \frac{1+\alpha}{1-\alpha}$. This condition ensures that firm 1 does not deviate to lower prices.

To sum up, we conclude that for $\alpha \geq \frac{3}{5}$ and $\frac{v-c}{t} \in \left[4, \frac{1+\alpha}{1-\alpha}\right]$ there exists an equilibrium in which firms set prices

$$p^w = v - t \text{ and } p^{nw} = v - 2t,$$

all consumers in the monopoly segment buy from firm 1, all consumers in the competitive market buy from firm 2. The respective profits are given by

$$\pi^w = \alpha(v - c - t) \text{ and } \pi^{nw} = (1-\alpha)(v - c - 2t).$$

***iii)* Firm 1 sells in the competitive, fully covered segment and the marginal consumer obtains a positive surplus.** Next, we consider the case, in which firm 1 sells in the competitive segment, this segment is fully covered, and the marginal consumer obtains a positive surplus. The profit of firm 1 setting price p_1 is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha \min\left\{\sqrt{\frac{v-p_1}{t}}, 1\right\} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right).$$

We separately analyze the case in which the monopoly segment of firm 1 is fully covered and the one in which it is not.

Case1. The monopoly segment of firm 1 is fully covered. We start by analyzing the case in which all consumers from the monopoly segment are served by firm 1. We distinguish two subcases depending on whether or not the consumer located at $x = 1$ obtains a positive surplus.

Case 1.1: The consumer located at 1 in the monopoly segment obtains a positive surplus. If the consumer located at $x = 1$ obtains positive surplus we have that $v - p^w - t > 0$. Suppose that firm 1 sets a price p_1 and continues to serve all consumers from the monopoly segment, keeping the marginal consumer's location in the competitive segment in the interior. Then, firm 1's profit given $p_2 = p^{nw}$ is

$$\begin{aligned}\pi_1(p_1, p^{nw}) &= (p_1 - c)\left(\alpha + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right)\\ &= (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).\end{aligned}$$

Firm 2's profit given $p_1 = p^w$ is

$$\pi_2(p^w, p_2) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p^w - p_2}{2t}\right).$$

Note that these profit functions coincide with the profit functions (for this case) under linear transport costs. Thus, the equilibrium candidate is exactly the same; that is,

$$p^w = c + \frac{3+\alpha}{3(1-\alpha)}t \quad \text{and} \quad p^{nw} = c + \frac{3-\alpha}{3(1-\alpha)}t.$$

The marginal consumer's location has to be in the interior implying that $p^w - p^{nw} < t$ or equivalently, $\alpha < \frac{3}{5}$. The monopolistic market is fully covered if and only if $v - p^w - t = v - c - \frac{2(3-\alpha)}{3(1-\alpha)}t > 0$, or equivalently, $\frac{v-c}{t} > \frac{2(3-\alpha)}{3(1-\alpha)}$. It remains to consider firm 1 deviating to a price $p_1 > \max\{v - t, p^{nw} + t\}$.

First, suppose that $v - t < p^{nw} + t$. It can be shown that $v - t < p^{nw} + t$ if and only if $\frac{v-c}{t} < \frac{9-7\alpha}{3(1-\alpha)}$. Combining this with the condition that ensures the full coverage of the monopoly segment in equilibrium, we obtain that $\frac{v-c}{t}$ is in the interval $\left(\frac{2(3-\alpha)}{3(1-\alpha)}, \frac{9-7\alpha}{3(1-\alpha)}\right)$, which is non-empty for $\alpha < \frac{3}{5}$. Firm 1's profit for $p_1 \in (v - t, p^{nw} + t)$ is

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha\sqrt{\frac{v - p_1}{t}} + (1 - \alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right).$$

Taking the derivative of this profit function, we obtain that

$$\alpha\left(\sqrt{\frac{v - p_1}{t}} - \frac{1}{2t}\frac{p_1 - c}{\sqrt{\frac{v-p_1}{t}}}\right) + \frac{1-\alpha}{2t}(c + t + p^{nw} - 2p_1).$$

Since the profits from operating in the monopoly and in the competitive segment are both strictly concave in p_1, we have that the derivative of $\pi_1(p_1, p^{nw})$ strictly decreases and can be bounded from above by $\frac{d}{dp_1}\pi_1(v - t, p^{nw})$; that is,

$$\begin{aligned}
&\frac{\alpha}{2t}(c + 3t - v) + \frac{1-\alpha}{2t}(c + 3t + p^{nw} - 2v) \\
&= \frac{1}{2t}(c + 3t - v) - \frac{1-\alpha}{2t}(v - p^{nw}).
\end{aligned}$$

Multiplying by 2 and plugging in p^{nw} we have

$$\begin{aligned}
3 - \frac{v-c}{t} - (1-\alpha)\left(\frac{v-c}{t} - \frac{3-\alpha}{3(1-\alpha)}\right) &= \frac{12-\alpha}{3} - (2-\alpha)\frac{v-c}{t} \\
&< \frac{12-\alpha}{3} - (2-\alpha)\frac{2(3-\alpha)}{3(1-\alpha)}.
\end{aligned}$$

It is easy to check that $2(2-\alpha)(3-\alpha) > (12-\alpha)(1-\alpha)$ for any $\alpha \in (0, 1)$ implying that the derivative of firm 1 at price $v - t$ is negative. Therefore, a deviation to any price in $(v - t, p^{nw} + t)$ is not profitable.

Next, suppose that firm 1 deviates to $p_1 \geq p^{nw} + t$. Then, it does not sell to consumers in the competitive segment. The resulting profit is given by $\alpha(p_1 - c)\sqrt{(v - p_1)/t}$. If $\frac{v-c}{t} > 3$, then the monopoly profit decreases for all $p_1 > v - t$ which makes the deviation unprofitable. Suppose instead that $\frac{v-c}{t} \leq 3$. We show that $p^m < p^{nw} + t$. Note that $\frac{v-c}{t} \leq 3 < \frac{3-2\alpha}{1-\alpha}$. Then, multiplying by $\frac{2}{3}t$, we have that $\frac{2}{3}v - \frac{2}{3}c < \frac{3-\alpha}{3(1-\alpha)}t + t$. By adding c to both parts of this inequality we obtain that $p^m < p^{nw} + t$. Thus, the profit function strictly decreases at prices weakly higher than $p^{nw} + t$, and firm 1 does not find it profitable to deviate to $p_1 \geq p^{nw} + t$.

Second, suppose that $v - t \geq p^{nw} + t$. Then, $\frac{v-c}{t} \geq \frac{9-7\alpha}{3(1-\alpha)} > 3$. As follows from the analysis of the monopoly problem, firm 1's maximal profit from such a deviation is attained at $p_1 = v - t$ and is equal to $\alpha(v - c - t)$. The equilibrium profit of firm 1 is weakly larger than the profit from this deviation if and only if $\frac{(3+\alpha)^2}{18(1-\alpha)}t \geq \alpha(v - t - c)$. By rearranging, we obtain

$$1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)} \geq \frac{v-c}{t} \geq \frac{9-7\alpha}{3(1-\alpha)}.$$

One can show that the function

$$g(\alpha) \equiv 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)} - \frac{9-7\alpha}{3(1-\alpha)}$$

strictly decreases on $\alpha \in (0, 3/5)$ and is equal to 0 at $\alpha = \frac{3}{5}$. This implies that for all $\alpha < 3/5$ we have that for all $\frac{9-7\alpha}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$ and a deviation to price $v - t$ is unprofitable.

To sum up, we conclude that if $\alpha < \frac{3}{5}$ and $\frac{2(3-\alpha)}{3(1-\alpha)} < \frac{v-c}{t} < 1 + \frac{(3+\alpha)^2}{18\alpha(1-\alpha)}$, there is an equilibrium in which firms set prices

$$p^w = c + \frac{3+\alpha}{3(1-\alpha)}t \quad \text{and} \quad p^{nw} = c + \frac{3-\alpha}{3(1-\alpha)}t,$$

all consumers in the monopoly segment buy from firm 1, and the market in the competitive segment is fully covered. Moreover, the marginal consumer in the competitive segment enjoys a positive surplus, as does the consumer located at $x = 1$ in the monopoly segment. The respective profits are

$$\pi^w = \frac{(3+\alpha)^2}{18(1-\alpha)}t \quad \text{and} \quad \pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)}t.$$

Case 1.2: The consumer located at 1 in the monopoly segment obtains zero surplus. Suppose that all consumers from the monopoly segment buy from firm 1 and the consumer located at $x = 1$ obtains zero surplus. This pins down the price that firm 1 sets: $p^w = v - t$. From the analysis of the previous case we have that $p^{nw} = \frac{1}{2}(p^w + c + t) = \frac{v+c}{2}$.

To ensure that firm 1 sells in the competitive segment, we must have that the location of the marginal consumer in this segment is in the interior, $|p^w - p^{nw}| < t$, implying that $\frac{v-c}{t} < 4$. Note that, under this condition, firm 2 does not find it profitable to deviate.

We study the incentives of firm 1 to deviate. If firm 1 deviates to $p_1 < p^w$, then it makes profits

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

The analysis of this deviation coincides with the one in the case of linear transport costs, leading to the same condition. Thus, firm 1 does not have an incentive to deviate if and only if $\frac{v-c}{t} \leq \frac{2(3-\alpha)}{3(1-\alpha)}$.

Next, suppose that firm 1 deviates upwards to a price $p_1 > p^w$. Then, firm 1's profit is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha\sqrt{\frac{v-p_1}{t}} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw}-p_1}{2t}\right)\right).$$

Taking the derivative, we obtain

$$\alpha\left(\sqrt{\frac{v-p_1}{t}} - \frac{1}{2t}\frac{p_1-c}{\sqrt{\frac{v-p_1}{t}}}\right) + \frac{1-\alpha}{2t}(c+t+p^{nw}-2p_1).$$

Since the profit function is a concave in p_1, its derivative decreases and is larger at $p^w = v-t$ than at any other $p_1 > p^w$. Therefore, this derivative (multiplied by 2) can be bounded from above by

$$\begin{aligned} &3 - \frac{v-c}{t} - \frac{1-\alpha}{t}(v-p^{nw}) \\ &= 3 - \frac{v-c}{t} - \frac{1-\alpha}{2}\frac{v-c}{t} = 3 - \frac{3-\alpha}{2}\frac{v-c}{t} \leq 0 \end{aligned}$$

if and only if $\frac{v-c}{t} \geq \frac{6}{3-\alpha}$. By contrast, if $\frac{v-c}{t} < \frac{6}{3-\alpha}$, firm 1 has an incentive to slightly increase its price from $p^w = v-t$.

To sum up, we conclude that for $\alpha \leq \frac{3}{5}$ and $\frac{v-c}{t} \in \left[\frac{6}{3-\alpha}, \frac{2(3-\alpha)}{3(1-\alpha)}\right]$ as well as for $\alpha > \frac{3}{5}$ and $\frac{v-c}{t} \in \left[\frac{6}{3-\alpha}, 4\right]$ there exists an equilibrium in which firms set prices

$$p^w = v - t \quad \text{and} \quad p^{nw} = \frac{v+c}{2},$$

all consumers in the monopoly segment buy from firm 1; the market in the competitive segment is fully covered; the marginal consumer in the competitive segment enjoys a positive surplus; and the consumer located at $x = 1$ in the monopoly segment obtains zero surplus. The respective profits are

$$\pi^w = (v-c-t)\left(1 - (1-\alpha)\frac{v-c}{4t}\right) \quad \text{and} \quad \pi^{nw} = (1-\alpha)\frac{(v-c)^2}{8t}.$$

Case 2. The monopoly segment of firm 1 is not fully covered. In this case the consumer located at $x = 1$ in the monopoly segment does not buy from firm 1; that is,

$v - p^w - t < 0$. Since the marginal consumer in the competitive segment is in the interior and enjoys a positive surplus, we have that the problem of firm 2 is the same as in the previous case implying that $2p^{nw} = c + t + p^w$.

Firm 1's profit is

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha\sqrt{\frac{v - p_1}{t}} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right).$$

The first-order condition evaluated at $p_1 = p^w$ is

$$\frac{\alpha}{2t}\frac{2v + c - 3p^w}{\sqrt{\frac{v-p^w}{t}}} + \frac{1-\alpha}{2t}(c + t + p^{nw} - 2p^w) = 0.$$

By plugging in $p^{nw} = \frac{1}{2}(p^w + c + t)$ and multiplying it by $\frac{2}{3}t$ we obtain

$$\alpha\frac{\frac{2}{3}v + \frac{1}{3}c - p^w}{\sqrt{\frac{v-p^w}{t}}} - (1-\alpha)\frac{1}{2}(p^w - (c+t)) = 0.$$

Define $\tau \equiv \frac{v-c}{t}$ and $\tilde{p} \equiv \frac{p^w - c}{t}$. Since $0 < \frac{v-p^w}{t} < 1$ we have that $\tau - \tilde{p} \in (0,1)$ or equivalently $\tilde{p} \in (\tau - 1, \tau)$. The equation determining p^w can be rewritten as

$$\alpha\frac{\frac{2}{3}\frac{v-c}{t} - \frac{p^w-c}{t}}{\sqrt{\frac{v-c}{t} - \frac{p^w-c}{t}}} - \frac{1-\alpha}{2}\left(\frac{p^w - c}{t} - 1\right) = 0$$

$$\iff \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1-\alpha}{2}(\tilde{p} - 1)\sqrt{\tau - \tilde{p}} = 0.$$

Define a function

$$\phi(\tau, p, \alpha) \equiv \alpha\left(\frac{2}{3}\tau - p\right) - \frac{1-\alpha}{2}(p-1)\sqrt{\tau - p}.$$

Solution of $\phi(\tau, p, \alpha) = 0$. We show that there exists a unique $p = \tilde{p}$ that solves $\phi(\tau, p, \alpha) = 0$ for any positive τ. The partial derivative of $\phi(\tau, p, \alpha)$ with respect to p is given by

$$\frac{\partial\phi}{\partial p} = -\alpha - \frac{1-\alpha}{2}\left(\sqrt{\tau - p} - \frac{p-1}{2\sqrt{\tau - p}}\right) = -\alpha - \frac{1-\alpha}{2}\frac{2\tau - 3p + 1}{2\sqrt{\tau - p}}.$$

First, suppose that $\tau > \frac{3}{2}$. If a solution exists, then we must have that $\tilde{p} \in \left(1, \frac{2}{3}\tau\right)$. Note that for all prices in $\left(1, \frac{2}{3}\tau\right)$ we have that $\frac{2}{3}\tau - p + \frac{1}{3} > 0$ and therefore $\frac{\partial\phi}{\partial p} < 0$. Moreover, we have that $\phi(\tau, 1, \alpha) > 0$ and $\phi(\tau, 2/3\tau, \alpha) < 0$. Therefore, by the intermediate value theorem, there exists a unique $\tilde{p} \in (1, 2/3\tau)$ that solves $\phi(\tau, \tilde{p}, \alpha) = 0$. Second, suppose that $\tau < \frac{3}{2}$. Then, if a solution exists, then it must belong to $\left(\frac{2}{3}\tau, \min\{1, \tau\}\right)$. For any $p \in \left(\frac{2}{3}\tau, \min\{1, \tau\}\right)$

we have that $\frac{2}{3}\tau + \frac{1}{3} - p > 0$, and therefore $\frac{\partial \phi}{\partial p} < 0$. Moreover, $\phi(\tau, 2/3\tau, \alpha) > 0$ and $\phi(\tau, \min\{1,\tau\}, \alpha) < 0$. By the intermediate value theorem, we have that there exists a unique $\tilde{p} \in \left(\frac{2}{3}\tau, \min\{1,\tau\}\right)$ that solves $\phi(\tau, \tilde{p}, \alpha) = 0$.

Necessary conditions for the existence of equilibrium. We derive the conditions under which the monopoly segment is not fully covered. The monopoly segment is not fully covered if and only if $p^w > v - t \iff \tilde{p} > \tau - 1$. Since function $\phi(\tau, p, \alpha)$ strictly decreases in p, we have that $\tilde{p} > \tau - 1$ if and only if

$$\phi(\tau, \tau - 1, \alpha) > 0 \iff \alpha\left(1 - \frac{\tau}{3}\right) - \frac{1-\alpha}{2}(\tau - 2) \iff \tau < \frac{6}{3-\alpha}.$$

Next, we show that the marginal consumer in the competitive market is in the interior. We employ the restriction $\tau < \frac{6}{3-\alpha}$. The marginal consumer in the competitive market is in the interior if and only if $|p^{nw} - p^w| = \frac{1}{2}|c + t - p^w| \leq t$, which is equivalent to $|\tilde{p} - 1| < 2$. Note that if $\tau \geq 3/2$, then this condition is satisfied as $|\tilde{p} - 1| < \frac{2}{3}\tau - 1 < \frac{2}{3}\frac{6}{3-\alpha} - 1 < 1$. If $\tau < \frac{3}{2}$, then $\tilde{p} \in (0,1)$ and therefore $|\tilde{p} - 1| < 2$ is satisfied.

The surplus of the marginal consumer in the competitive segment is positive if and only if

$$v - p^w - t\left(\frac{1}{2} + \frac{p^{nw} - p^w}{2t}\right)^2 > 0 \iff \sqrt{\frac{v - p^w}{t}} > \frac{1}{2} + \frac{p^{nw} - p^w}{2t} \iff \sqrt{\tau - \tilde{p}} > \frac{3 - \tilde{p}}{4}.$$

Next, we show that there exists a function $\bar{\tau}(\alpha) \in \left(\bar{\tau}_1, \frac{5}{4}\right)$, where $\bar{\tau}_1$ solves $\sqrt{\tau/3} - (9 - 2\tau)/12 = 0$ (and approximately equals to 1.013), such that $\sqrt{\tau - \tilde{p}} > \frac{3-\tilde{p}}{4}$ for $\tau \in \left(\bar{\tau}(\alpha), \frac{6}{3-\alpha}\right)$ and $\sqrt{\tau - \tilde{p}} \leq \frac{3-\tilde{p}}{4}$ for $\tau \in \left[\frac{3}{4}, \bar{\tau}(\alpha)\right]$. Define a function

$$f(\tau, p) = \sqrt{\tau - p} - \frac{3 - p}{4}.$$

Note that when $\tau = \bar{\tau}_1 < \frac{3}{2}$, then the corresponding price $\tilde{p} > \frac{2}{3}\bar{\tau}_1$. Since $f(\bar{\tau}_1, p)$ strictly decreases in p, then $f(\bar{\tau}_1, \tilde{p}) < f(\bar{\tau}_1, 2/3\bar{\tau}_1) = \sqrt{\frac{\bar{\tau}_1}{3}} - \frac{9-2\bar{\tau}_1}{12} = 0$. When $\tau = \frac{5}{4} < \frac{3}{2}$ the corresponding price $\tilde{p} < \min\{1, \tau\} = 1$. Since $f(5/4, p)$ strictly decreases in p, then $f(5/4, \tilde{p}) > f(5/4, 1) = 0$.

The total derivative of $f(\tau, \tilde{p})$ with respect to τ is

$$\frac{df(\tau, \tilde{p})}{d\tau} = \frac{1}{2\sqrt{\tau - \tilde{p}}}\left(1 - \frac{d\tilde{p}}{d\tau}\right) + \frac{1}{4}\frac{d\tilde{p}}{d\tau}.$$

By applying the implicit function theorem to $\phi(\tau, \tilde{p}, \alpha)$, we obtain

$$\frac{2}{3}\alpha - \frac{1-\alpha}{2}\frac{\tilde{p}-1}{2\sqrt{\tau-\tilde{p}}} = \left(\alpha + \frac{1-\alpha}{2}\frac{2\tau-3\tilde{p}+1}{2\sqrt{\tau-\tilde{p}}}\right)\frac{d\tilde{p}}{d\tau}.$$

We explore $\frac{d\tilde{p}}{d\tau}$ for $\tau \in \left(\frac{3}{4}, \frac{3}{2}\right)$. Since $\tilde{p} < 1$, the term in the bracket on the right-hand side is positive. The term on the left-hand side is also positive. This implies that $\frac{d\tilde{p}}{d\tau} > 0$. Note that since $\tau - \tilde{p} > 0$, we have that

$$\frac{d\tilde{p}}{d\tau} < \left(\frac{2}{3}\alpha + \frac{1-\alpha}{2}\frac{1-\tilde{p}}{2\sqrt{\tau-\tilde{p}}}\right) \Big/ \left(\alpha + \frac{1-\alpha}{2}\frac{1-\tilde{p}}{2\sqrt{\tau-\tilde{p}}}\right) < 1.$$

Given that $\frac{d\tilde{p}}{d\tau} \in (0,1)$ we have that $\frac{df(\tau,\tilde{p})}{d\tau} > 0$ on $\tau \in \left(\frac{3}{4}, \frac{3}{2}\right)$. We established that function $f(\tau, \tilde{p})$ strictly increases on $\tau \in \left(\bar{\tau}_1, \frac{5}{4}\right)$, is negative at $\tau = \bar{\tau}_1$ and is positive at $\tau = \frac{5}{4}$. Thus, there exists a uniquely defined $\bar{\tau} = \bar{\tau}(\alpha) \in \left(\bar{\tau}_1, \frac{5}{4}\right)$ such that $\sqrt{\tau - \tilde{p}} > \frac{3-\tilde{p}}{4}$ if and only if $\tau > \bar{\tau}(\alpha)$.[22] We obtain that the surplus of the marginal consumer in the competitive segment is positive if and only if $\tau > \bar{\tau}(\alpha)$.

Necessary conditions for the existence of equilibrium are also sufficient. We show that the considered type of equilibrium exists for $\tau \in \left(\bar{\tau}(\alpha), \frac{6}{3-\alpha}\right)$. To show this, we check that neither of the firms has profitable deviations. In particular, firm 1 does not have an incentive to set a lower price to fully serve either of the segments, and firm 2 does not have an incentive to increase its price such that some consumers from the competitive segment do not buy at all.

We start by analyzing firm 1's deviation incentives. Suppose that firm 1 deviates to a price $p_1 \leq v-t$ and serves all consumers in the monopoly segment. First, consider a deviation to a price in $[p^{nw}-t, v-t]$. In this case, firm 1 serves all consumers in the monopoly segment and the marginal consumer in the competitive segment is still in the interior. The profit from such a deviation is

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\frac{1+\alpha}{2} + \frac{1-\alpha}{2t}(p^{nw} - p_1)\right).$$

Taking the derivative (multiplied by $2t/(1-\alpha)$) and using the fact that $p^{nw} > \frac{1}{2}(c+t+v-t) =$

[22] Note that when α tends to 1 we have that $\tilde{p}$ goes to $2/3\tau$ and therefore $\bar{\tau}(\alpha)$ converges to the solution of $\sqrt{\tau/3} - (9-2\tau)/12 = 0$, which is equal to $\bar{\tau}_1 \approx 1.013$. When $\alpha \to 0$ we have that $\tilde{p}$ tends to $\min\{\tau, 1\}$ and therefore $\bar{\tau}$ converges to $\frac{5}{4}$.

$\frac{v+c}{2}$ (since $p^w > v - t$), we have that

$$c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2p_1 \geq c + \frac{1+\alpha}{1-\alpha}t + p^{nw} - 2v + 2t$$
$$\geq c + \frac{1+\alpha}{1-\alpha}t + \frac{v+c}{2} - 2v + 2t = \frac{3}{2}\left(\frac{2(3-\alpha)}{3(1-\alpha)} - \frac{v-c}{t}\right)t > 0,$$

where the last inequality is due to $\frac{v-c}{t} < \frac{6}{3-\alpha} < \frac{2(3-\alpha)}{3(1-\alpha)}$ for any $\alpha \in (0,1)$. Therefore, firm 1 does not have incentives to lower its price to $p^1 <\leq v - t$. Second, if firm 1 deviates to a price $p_1 < p^{nw} - t$, it does not serve more consumers, which cannot be profitable.

Next, consider firm 2 setting $p_2 > v - t\left(1 - \sqrt{\frac{v-p^w}{t}}\right)^2$. At this price, consumers located at $\frac{v-p^w}{t} + \varepsilon$, when $\varepsilon > 0$ is small, do not buy from either firm - that is, $v - p_2 - t\left(1 - \sqrt{\frac{v-p^w}{t}}\right)^2 < 0$. Firm 2 acts as a local monopolist. Recall that since $\frac{v-c}{t} < \frac{6}{3-\alpha} < 3$, the monopolist would set $p^m = \frac{2}{3}v + \frac{1}{3}c$. Note that

$$\frac{p_2 - \frac{2}{3}v - \frac{1}{3}c}{t} > \frac{v - t\left(1 - \sqrt{\frac{v-p^w}{t}}\right)^2 - \frac{2}{3}v - \frac{1}{3}c}{t}$$
$$= \frac{1}{3}\tau - (1 - \sqrt{\tau - \tilde{p}})^2$$
$$= 2\sqrt{\tau - \tilde{p}} - \frac{2\tau}{3} + \tilde{p} - 1.$$

The derivative of $2\sqrt{\tau - \tilde{p}} - \frac{2\tau}{3} + \tilde{p} - 1$ with respect to $\tilde{p}$ is $1 - \frac{1}{\sqrt{\tau - \tilde{p}}} < 0$, since $\tau - \tilde{p} < 1$. Thus, this function strictly decreases in $\tilde{p}$. First, suppose that $\tau \in \left[\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Then, $\tilde{p} > 1$ and we can evaluate $2\sqrt{\tau - \tilde{p}} - \frac{2\tau}{3} + \tilde{p} - 1$ from below by $2\sqrt{\tau - 1} - \frac{2\tau}{3}$. The latter expression strictly increases in τ for $\tau < \frac{6}{3-\alpha}$ and is positive, since $2\sqrt{\tau - 1} - \frac{2\tau}{3} > 2\sqrt{\frac{3}{2} - 1} - 1 > 0$. Second, suppose that $\tau \in \left(\bar{\tau}(\alpha), \frac{3}{2}\right)$. Then, $\tilde{p} > \frac{2\tau}{3}$ and we can evaluate $2\sqrt{\tau - \tilde{p}} - \frac{2\tau}{3} + \tilde{p} - 1$ from below by $2\sqrt{\frac{\tau}{3}} - 1 > 0$ since $\tau > \bar{\tau}(\alpha) > \frac{3}{4}$.

Hence, we showed that $p_2 > p^m$. This implies that the profit function of firm 2 is decreasing for all $p_2 > v - t\left(1 - \sqrt{\frac{v-p^w}{t}}\right)^2$, implying that such a deviation cannot be profitable.

To sum up, we established that for $\frac{v-c}{t} \in \left(\bar{\tau}(\alpha), \frac{6}{3-\alpha}\right)$, where $\bar{\tau}(\alpha) \in \left(\bar{\tau}_1, \frac{5}{4}\right)$ and $\bar{\tau}_1$ solves $\sqrt{\bar{\tau}_1/3} - (9 - 2\bar{\tau})/12 = 0$, there is an equilibrium in which firms set prices

$$p^w \text{ that solves } \phi\left(\frac{v-c}{t}, \frac{p^w - c}{t}, \alpha\right) = 0 \quad \text{and} \quad p^{nw} = \frac{1}{2}(c + t + p^w),$$

the monopoly segment is not fully covered, the competitive segment is fully covered, and the marginal consumer in this segment enjoys a positive surplus. The corresponding equilibrium profits are

$$\pi^w = (p^w - c)\left(\alpha\sqrt{\frac{v - p^w}{t}} + \frac{1-\alpha}{4t}(3t + c - p^w)\right) \quad \text{and} \quad \pi^{nw} = (1-\alpha)\frac{(p^w - c + t)^2}{8t}.$$

iv) **Firm 1 sells in the competitive fully covered segment and the marginal consumer obtains zero surplus.** In this type of equilibrium, the indifferent consumer between firm 1 and firm 2 obtains a zero surplus. Therefore, firm 1 serves the same fraction of consumers in both segments and the monopoly segment is not fully covered.

We define the marginal consumer in the competitive market as $\hat{x} \in (0,1)$. Since the marginal consumer obtains a zero surplus, the equilibrium prices are determined by

$$p^w = v - t\hat{x}^2 \quad \text{and} \quad p^{nw} = v - t(1-\hat{x})^2.$$

Define $\hat{p} = \hat{p}(\hat{x}) \equiv \frac{p^w - c}{t}$ and $\tau \equiv \frac{v-c}{t}$. Then, we have that $\hat{x} = \sqrt{\tau - \hat{p}}$.

We characterize all possible $\hat{x}$ that can constitute an equilibrium. First, we characterize the conditions under which firms do not have incentives to increase prices. If $\frac{v-c}{t} \geq 3$, then the monopolist would serve all consumers, and therefore, firms do not have incentives to raise their prices. Otherwise, if $\frac{v-c}{t} \leq 3$, then the monopoly price is given by $p^m = \frac{2}{3}v + \frac{1}{3}c$. The firms do not have incentives to set higher prices if and only if $p^w \geq p^m$ and $p^{nw} \geq p^m$. By plugging in the respective expression for prices we obtain $1 - \sqrt{\frac{v-c}{3t}} \leq \hat{x} \leq \sqrt{\frac{v-c}{3t}}$, or equivalently, $1 - \sqrt{\frac{\tau}{3}} \leq \hat{x} \leq \sqrt{\frac{\tau}{3}}$. The set of $\hat{x}$ that satisfy this condition is not empty if and only if $\tau \geq \frac{3}{4}$.

Next, consider a deviation of firm 2 to $p_2 < p^{nw}$. The resulting duopoly profit of firm 2 is

$$\pi_2(p_2, p^w) = (1-\alpha)(p_2 - c)\left(\frac{1}{2} + \frac{p^w - p_2}{2t}\right).$$

The derivative of the profit function with respect to p_2 (multiplied by 2) is

$$\begin{aligned} \frac{1}{t}(c + t + p^w - 2p_2) > \frac{1}{t}(c + t + p^w - 2p^{nw}) &= -1 - \frac{p^w - c}{t} + 4\left(\frac{1}{2} + \frac{p^w - p^{nw}}{2t}\right) \\ &= 4(1-\hat{x}) - 1 - \hat{p} \geq 0, \end{aligned}$$

if and only if $\hat{x} \leq \frac{3-\hat{p}}{4}$. This implies that $\hat{p} < 3$. It is straightforward to show that the set of $\hat{x}$ satisfying this inequality is not empty if and only if $\tau < 3$. If $\tau \geq 3$, then for any $\hat{p} < 3$

we have that $\hat{x} = \sqrt{\tau - \hat{p}} > \sqrt{3 - \hat{p}} > \sqrt{3 - \hat{p}} \times \frac{\sqrt{3-\hat{p}}}{4} = \frac{3-\hat{p}}{4}$ and firm 2 finds it optimal to deviate to a price that is slightly lower than p^{nw}. This implies that the considered type of equilibrium can occur only for $\tau < 3$. In what follows, we restrict attention to the parameter region for which $\tau \in \left[\frac{3}{4}, 3\right)$.

We continue by establishing the condition under which firm 1 does not find it profitable to deviate to lower prices. Firm 1's profit from deviating downwards $p_1 < p^w$ such that firm 2 remains active is given by

$$\pi_1(p_1, p^{nw}) = (p_1 - c)\left(\alpha\sqrt{\frac{v - p_1}{t}} + (1-\alpha)\left(\frac{1}{2} + \frac{p^{nw} - p_1}{2t}\right)\right).$$

The first-order condition of the profit with respect to p_1 and multiplied by $2t$ is

$$\begin{aligned}
&2\alpha\left(\sqrt{\frac{v - p_1}{t}} - \frac{1}{2t}\frac{p_1 - c}{\sqrt{\frac{v-p_1}{t}}}\right) + \frac{1-\alpha}{t}\left(c + t + p^{nw} - 2p_1\right) \\
&\geq \frac{\alpha}{t}\frac{2v + c - 3p^w}{\sqrt{\frac{v-p^w}{t}}} + \frac{1-\alpha}{t}(c + t + p^{nw} - 2p^w) \\
&= 3\alpha\frac{\frac{2}{3}\frac{v-c}{t} - \frac{p^w-c}{t}}{\sqrt{\frac{v-p^w}{t}}} + (1-\alpha)\left(2\left(\frac{1}{2} + \frac{p^{nw} - p^w}{2t}\right) - \frac{p^w - c}{t}\right) \\
&= 3\alpha\frac{\frac{2}{3}\tau - \hat{p}}{\sqrt{\tau - \hat{p}}} - (1-\alpha)(\hat{p} - 2\hat{x}).
\end{aligned}$$

By multiplying the last expression by $\frac{1}{3}\sqrt{\tau - \hat{p}}$, we obtain that firm 1 does not find it profitable to deviate to a lower price if and only if

$$\alpha\left(\frac{2}{3}\tau - \hat{p}\right) - (1-\alpha)\frac{\hat{p} - 2\hat{x}}{3}\sqrt{\tau - \hat{p}} \geq 0.$$

Thus, $\hat{x} = \sqrt{\tau - \hat{p}} \in (0,1)$ can be supported in equilibrium if and only if $\tau \in \left[\frac{3}{4}, 3\right)$ and

$$\begin{cases}
1 - \sqrt{\frac{\tau}{3}} \leq \hat{x} \leq \sqrt{\frac{\tau}{3}}, \\
\hat{x} \leq \frac{3-\hat{p}}{4}, \\
\alpha\left(\frac{2}{3}\tau - \hat{p}\right) - (1-\alpha)\frac{\hat{p}-2\hat{x}}{3}\sqrt{\tau - \hat{p}} \geq 0.
\end{cases}$$

In the analysis of the case in which the monopoly segment is not fully covered, the competitive segment is fully covered and the marginal consumer obtains a positive surplus, we defined $\tau = \bar{\tau}(\alpha)$ that solves $f(\tau, \tilde{p}) = \sqrt{\tau - \tilde{p}} - \frac{3-\tilde{p}}{4}$, where $\tilde{p}$ solves $\phi(\tau, \tilde{p}, \alpha) = \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) -$

$\frac{1-\alpha}{2}(\tilde{p}-1)\sqrt{\tau-\tilde{p}} = 0$. Recall that the function $\bar{\tau}(\alpha)$ belongs to $\left(\bar{\tau}_1, \frac{5}{4}\right)$, where τ_1 solves $\sqrt{\tau_1/3} - (9-2\tau_1)/12 = 0$. We show that the set of $\hat{x}$ satisfying all three conditions is not empty if and only if $\tau \in \left[\frac{3}{4}, \bar{\tau}(\alpha)\right]$.

First, we show that this is a necessary condition. Suppose there is an $\hat{x}$ satisfying all the inequalities. Then, the first inequality $\hat{x} = \sqrt{\tau - \hat{p}} \leq \sqrt{\frac{\tau}{3}}$ implies that $\hat{p} \geq \frac{2}{3}\tau$. For any $\hat{p} \geq \frac{2}{3}\tau$ the third inequality is satisfied if $\hat{p} \leq 2\hat{x}$. Then, by the second inequality we have that $\hat{p} \leq 2\hat{x} \leq \frac{3-\hat{p}}{2} \iff \hat{p} \leq 1$. Thus, $\hat{p} \in \left[\frac{2}{3}\tau, \min\{1,\tau\}\right]$ and this interval is non-empty for $\tau \leq \frac{3}{2}$.

By the second inequality we have that $0 \geq \frac{\hat{p}-2\hat{x}}{3} \geq \frac{\hat{p}-(3-\hat{p})/2}{3} = \frac{\hat{p}-1}{2}$. Thus, we obtain

$$\begin{aligned}\phi(\tau,\hat{p},\alpha) &= \alpha\left(\frac{2}{3}\tau - \hat{p}\right) - \frac{1-\alpha}{2}(\hat{p}-1)\sqrt{\tau-\hat{p}} \\ &\geq \alpha\left(\frac{2}{3}\tau - \hat{p}\right) - (1-\alpha)\frac{\hat{p}-2\hat{x}}{3}\sqrt{\tau-\hat{p}} \geq 0 = \phi(\tau,\tilde{p},\alpha),\end{aligned}$$

which implies that $\hat{p} \leq \tilde{p}$, where $\tilde{p}$ is determined by $\phi(\tau,\tilde{p},\alpha) = 0$ (recall that function ϕ strictly decreases in p). Since $f(\tau,p) = \sqrt{\tau-p} - \frac{3-p}{4}$ strictly decreases in p for $\tau \leq \frac{3}{2}$, we have that the second inequality implies that $0 \geq f(\tau,\hat{p}) \geq f(\tau,\tilde{p})$. Recall that $f(\tau,\tilde{p}) \leq 0$ if and only if $\tau \leq \bar{\tau}(\alpha)$. We proved that if $\hat{x}$ satisfies all three inequalities, then $\tau \in \left[\frac{3}{4}, \bar{\tau}(\alpha)\right]$.

Second, we show that for any $\tau \in \left[\frac{3}{4}, \bar{\tau}(\alpha)\right]$ there is an $\hat{x}$ satisfying all three inequalities. We analyze the cases in which $\tau \in \left[\frac{3}{4}, \bar{\tau}_1\right]$ and $\tau \in (\bar{\tau}_1, \bar{\tau}(\alpha)]$ separately. Suppose that $\tau \in \left[\frac{3}{4}, \bar{\tau}_1\right]$ and consider $\hat{x} = \sqrt{\frac{\tau}{3}} < 1$. The first inequality is obviously satisfied. The corresponding price is equal to $\hat{p} = \frac{2}{3}\tau = 2\hat{x}^2$. Since $2\hat{x} - \hat{p} = 2\hat{x}\,(1-\hat{x}) > 0$ we have that the third inequality is satisfied as well. Since $\tau \leq \bar{\tau}_1$, the second inequality is also satisfied as

$$\hat{x} - \frac{3-\hat{p}}{4} = \sqrt{\frac{\tau}{3}} - \frac{3-\frac{2}{3}\tau}{4} \leq \sqrt{\frac{\bar{\tau}_1}{3}} - \frac{9-2\bar{\tau}_1}{12} = 0.$$

It remains to analyze the case in which $\tau \in (\bar{\tau}_1, \bar{\tau}(\alpha)]$. Consider $\hat{x} = \sqrt{\tau - \hat{p}}$, where $\hat{p}$ solves $f(\tau,\hat{p}) = 0$. Note that since $f(\tau,\tilde{p}) \leq 0$,

$$f(\tau, 2\tau/3) = \sqrt{\frac{\tau}{3}} - \frac{3-\frac{2}{3}\tau}{4} > \sqrt{\frac{\bar{\tau}_1}{3}} - \frac{9-2\bar{\tau}_1}{12} = 0,$$

and the fact that function $f(\tau,p)$ strictly decreases in p, we have that $\hat{p} \in \left(\frac{2}{3}\tau, \tilde{p}\right]$. The second condition is satisfied by the definition of $\hat{p}$ that solves $\hat{x} - \frac{3-\hat{p}}{4} = f(\tau,\hat{p}) = 0$. The third condition is also satisfied as

$$\alpha\left(\frac{2}{3}\tau-\hat{p}\right)-(1-\alpha)\frac{\hat{p}-2\hat{x}}{3}\sqrt{\tau-\hat{p}}=\alpha\left(\frac{2}{3}\tau-\hat{p}\right)-\frac{1-\alpha}{2}(\hat{p}-1)\sqrt{\tau-\hat{p}}$$
$$=\phi(\tau,\hat{p},\alpha)\geq\phi(\tau,\tilde{p},\alpha)=0,$$

where we used $\hat{p}\leq\tilde{p}$ and the fact that $\phi(\tau,p,\alpha)$ strictly decreases in p. It remains to check the first condition. By rearranging $f(\tau,\hat{p})$ and by using $\hat{p}\leq\tilde{p}<1$, we evaluate τ as follows, $\tau=\hat{p}+\frac{1}{16}(3-\hat{p})^2=\frac{1}{16}(9+10\hat{p}+\hat{p}^2)>\frac{3}{16}(1+\hat{p})^2$. Consequently, the first condition is satisfied as

$$1-\sqrt{\frac{\tau}{3}}<1-\sqrt{\frac{1}{3}\times\frac{3}{16}(1+\hat{p})^2}=\frac{3-\hat{p}}{4}=\hat{x}<\sqrt{\frac{\tau}{3}},$$

where the last inequality stems from $\hat{p}>\frac{2}{3}\tau$.

Therefore, we showed that the set of $\hat{x}$ satisfying all three conditions is not empty if and only if $\tau\in\left[\frac{3}{4},\bar{\tau}(\alpha)\right]$.

To sum up, we conclude that for $\frac{v-c}{t}\in\left[\frac{3}{4},\bar{\tau}(\alpha)\right]$, where $\bar{\tau}(\alpha)\in\left(\bar{\tau}_1,\frac{5}{4}\right)$ and $\bar{\tau}_1$ solves $\sqrt{\bar{\tau}_1/3}-(9-2\bar{\tau})/12=0$, there are multiple equilibria characterized by the location of the marginal consumer. The monopoly segment is not fully covered, the competitive segment is fully covered, and the marginal consumer obtains a zero surplus.

The key results. We are now in the position to evaluate the conditions of Proposition 3 in the Hotelling model with quadratic transport costs in the parameter range $\frac{v-c}{t}\in\left(\frac{3}{2},\frac{7}{2}\right)$. Note that the critical value $\left(\frac{27}{4}\right)^{1/3}$ is approximately 1.89.

First, in Proposition 10, we show that there exists a function $\hat{\tau}(\alpha)\in\left(\frac{3}{2},\left(\frac{27}{4}\right)^{1/3}\right)$ such that $\pi^w+2\pi^{nw}\geq(3-\alpha)\pi^d$ is satisfied if and only if $\frac{v-c}{t}>\hat{\tau}(\alpha)$. Second, in Proposition 11 we show that there exists a function $\tilde{\tau}(\alpha)\in\left(\left(\frac{27}{4}\right)^{1/3},\frac{6}{3-\alpha}\right)$ such that $\pi^w+\pi^{nw}\geq\pi^m$ if and only if $\frac{v-c}{t}>\tilde{\tau}(\alpha)$. Since $\hat{\tau}(\alpha)<\tilde{\tau}(\alpha)$ for any $\alpha\in(0,1)$, we have that the region in which $\pi^w+2\pi^{nw}\geq(3-\alpha)\pi^d$ and $\pi^m\leq\pi^w+\pi^{nw}$ is non-empty (see Figure 2).

Proposition 10. *Consider the Hotelling model with quadratic transport costs and a positive fraction α of consumers using the ad blocker in the parameter range $\frac{v-c}{t}\in\left(\frac{3}{2},\frac{7}{2}\right)$. Then, the inequality $\pi^w+2\pi^{nw}\geq(3-\alpha)\pi^d$ is satisfied for $\frac{v-c}{t}>\hat{\tau}(\alpha)$, where $\hat{\tau}(\alpha)$ together with*

$\tilde{p} \in \left(1, \frac{2}{3}\tau\right)$ *solves the following system of equations*

$$\begin{cases} \alpha\tilde{p}\sqrt{\tau - \tilde{p}} + \frac{1-\alpha}{4}(5\tilde{p} + 1) - \frac{3-\alpha}{2} = 0, \\ \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1}{2}(1-\alpha)(\tilde{p} - 1)\sqrt{\tau - \tilde{p}} = 0, \end{cases}$$

For all $\alpha \in (0,1)$*, the function* $\hat{\tau}(\alpha)$ *satisfies* $\hat{\tau}(\alpha) \in \left(\frac{3}{2}, \left(\frac{27}{4}\right)^{1/3}\right)$ *and* $\lim_{\alpha \to 1} \hat{\tau}(\alpha) = \left(\frac{27}{4}\right)^{1/3}$.

Proof. We define

$$\tau \equiv \frac{v-c}{t}.$$

First, suppose that $\tau \in \left[\frac{6}{3-\alpha}, \frac{7}{2}\right)$. We show that in this parameter region $\pi^m > 2\pi^d$ and $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$. Since $\frac{6}{3-\alpha} > 2 > \left(\frac{27}{4}\right)^{1/3}$ we have that $\pi^m > 2\pi^d$.

Suppose that $\tau \in \left[\frac{6}{3-\alpha}, \min\left\{\frac{7}{2}, \frac{2(3-\alpha)}{3(1-\alpha)}\right\}\right)$. The equilibrium profits of the firms are given by

$$\pi^w = (\tau - 1)\left(1 - (1-\alpha)\frac{\tau}{4}\right)t \quad \text{and} \quad \pi^{nw} = (1-\alpha)\frac{\tau^2}{8}t.$$

Thus, we obtain that

$$\begin{aligned} \pi^w + 2\pi^{nw} - (3-\alpha)\pi^d &= (\tau - 1)\left(1 - (1-\alpha)\frac{\tau}{4}\right)t + (1-\alpha)\frac{\tau^2}{4}t - \frac{3-\alpha}{2}t \\ &= \left(1 + \frac{1-\alpha}{4}\right)(\tau - 2)t > 0. \end{aligned}$$

Otherwise, if $\alpha \leq \frac{9}{17}$ and $\tau \in \left[\frac{2(3-\alpha)}{3(1-\alpha)}, \frac{7}{2}\right)$, then equilibrium profits of the firms are

$$\pi^w = \frac{(3+\alpha)^2}{18(1-\alpha)}t \quad \text{and} \quad \pi^{nw} = \frac{(3-\alpha)^2}{18(1-\alpha)}t.$$

Therefore,

$$\begin{aligned} \pi^w + 2\pi^{nw} - (3-\alpha)\pi^d &= \frac{(3+\alpha)^2}{18(1-\alpha)}t + \frac{(3-\alpha)^2}{9(1-\alpha)}t - \frac{3-\alpha}{2}t \\ &= \frac{\alpha(5-\alpha)}{3(1-\alpha)}t > 0. \end{aligned}$$

This implies that $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ for $\tau \in \left[\frac{6}{3-\alpha}, \frac{7}{2}\right)$.

Second, suppose that $\tau \in \left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Then, the price of firm 1 that has monopoly access to consumers using the ad blocker solves

$$\alpha\frac{\frac{2}{3}v + \frac{1}{3}c - p^w}{\sqrt{\frac{v-p^w}{t}}} - (1-\alpha)\frac{1}{2}(p^w - (c+t)) = 0.$$

Define $\tilde{p} = \frac{p^w - c}{t} \in \left(1, \frac{2}{3}\tau\right)$. Then, the equation determining p^w can be rewritten as

$$\alpha \frac{\frac{2}{3}\frac{v-c}{t} - \frac{p^w-c}{t}}{\sqrt{\frac{v-c}{t} - \frac{p^w-c}{t}}} - \frac{1-\alpha}{2}\left(\frac{p^w-c}{t} - 1\right) = 0$$
$$\iff \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1-\alpha}{2}(\tilde{p}-1)\sqrt{\tau - \tilde{p}} = 0.$$

Note that $\tilde{p}$ that solves this equation strictly increases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$, as we show next. By applying the implicit function theorem to the previous equation, we obtain

$$\frac{2}{3}\alpha - \frac{1-\alpha}{2}\frac{\tilde{p}-1}{2\sqrt{\tau-\tilde{p}}} = \left(\alpha + \frac{1-\alpha}{2}\frac{2\tau - 3\tilde{p} + 1}{2\sqrt{\tau - \tilde{p}}}\right)\frac{d\tilde{p}}{d\tau}.$$

Since $\tilde{p} < \frac{2}{3}\tau$, the term in the bracket on the right-hand side is positive. The term on the left-hand side is

$$\frac{2}{3}\alpha - \frac{1-\alpha}{2}\frac{\tilde{p}-1}{2\sqrt{\tau-\tilde{p}}} = \frac{2}{3}\alpha - \alpha\frac{\frac{2}{3}\tau - \tilde{p}}{2(\tau-\tilde{p})} = \frac{\alpha}{3}\frac{2\tau - \tilde{p}}{2(\tau - \tilde{p})} > 0.$$

This implies that $\frac{d\tilde{p}}{d\tau} > 0$.

We are now ready to determine the sign of $\pi^w + 2\pi^{nw} - (3-\alpha)\pi^d$. In the parameter range under consideration, the firms' equilibrium profits can be rewritten as

$$\pi^w = \tilde{p}\left(\alpha\sqrt{\tau - \tilde{p}} + \frac{1-\alpha}{4}(3-\tilde{p})\right)t,$$
$$\pi^{nw} = \frac{1-\alpha}{8}(\tilde{p}+1)^2 t.$$

We define the function

$$\begin{aligned} g(\tau, \tilde{p}, \alpha) &= (\pi^w + 2\pi^{nw} - (3-\alpha)\pi^d)/t \\ &= \tilde{p}\left(\alpha\sqrt{\tau-\tilde{p}} + \frac{1-\alpha}{4}(3-\tilde{p})\right) + \frac{1-\alpha}{4}(\tilde{p}+1)^2 - \frac{3-\alpha}{2} \\ &= \alpha\tilde{p}\sqrt{\tau-\tilde{p}} + \frac{1-\alpha}{4}(5\tilde{p}+1) - \frac{3-\alpha}{2}. \end{aligned}$$

Clearly, $\pi^w + 2\pi^{nw} > (3-\alpha)\pi^d$ if and only if $g(\tau, \tilde{p}, \alpha) > 0$.

We show that there exists a continuous function $\hat{\tau}(\alpha)$ that solves $g(\hat{\tau}(\alpha), \tilde{p}, \alpha) = 0$. First, note that $\lim_{\tau \downarrow \frac{3}{2}} \tilde{p} = 1$. Thus, $g(\tau, \tilde{p}, \alpha)$ tends to

$$g(3/2, 1, \alpha) = \alpha\sqrt{1/2} + \frac{3(1-\alpha)}{2} - \frac{3-\alpha}{2} = \alpha(\sqrt{1/2} - 1) < 0.$$

Second, we show that $g(\tau, \tilde{p}, \alpha) > 0$ at $\bar{\tau} = \left(\frac{27}{4}\right)^{1/3}$. Note that

$$\begin{aligned} g(\bar{\tau}, \tilde{p}, \alpha) &= \alpha\tilde{p}\sqrt{\bar{\tau} - \tilde{p}} + \frac{1-\alpha}{4}(5\tilde{p}+1) - \frac{3-\alpha}{2} \\ &= \alpha\tilde{p}\sqrt{\bar{\tau} - \tilde{p}} + \frac{5(1-\alpha)}{4}(\tilde{p}-1) - \alpha. \end{aligned}$$

By multiplying this by $\sqrt{\bar{\tau} - \tilde{p}}$ we have that

$$\begin{aligned} g(\bar{\tau}, \tilde{p}, \alpha)\sqrt{\bar{\tau} - \tilde{p}} &= \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + \frac{5(1-\alpha)}{4}(\tilde{p}-1)\sqrt{\bar{\tau} - \tilde{p}} - \alpha\sqrt{\bar{\tau} - \tilde{p}} \\ &= \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + \frac{5\alpha}{2}\left(\frac{2}{3}\bar{\tau} - \tilde{p}\right) - \alpha\sqrt{\bar{\tau} - \tilde{p}}. \end{aligned}$$

This expression strictly decreases in $\tilde{p}$ on $[1, 2\bar{\tau}/3]$. The first term strictly decreases in $\tilde{p}$ as $\bar{\tau} < 2$ and $\tilde{p} > 1$. The sign of the derivative of the sum of the second and the third terms is determined by $-5 + \frac{1}{\sqrt{\bar{\tau}-\tilde{p}}} < -5 + \frac{1}{\sqrt{\bar{\tau}-\frac{2}{3}\bar{\tau}}} = -5 + \frac{2}{3}\bar{\tau} < 0$. Thus,

$$\begin{aligned} g(\bar{\tau}, \tilde{p}, \alpha)\sqrt{\bar{\tau} - \tilde{p}} &> g(\bar{\tau}, 2\bar{\tau}/3, \alpha)\sqrt{\frac{1}{3}\bar{\tau}} \\ &= \alpha\sqrt{\frac{1}{3}\bar{\tau}}\left(\frac{2}{3}\bar{\tau}\sqrt{\frac{1}{3}\bar{\tau}} - 1\right) = 0, \end{aligned}$$

implying that $g(\tau, \tilde{p}, \alpha) > 0$ at $\tau = \left(\frac{27}{4}\right)^{1/3}$. We also note that $\left(\frac{27}{4}\right)^{1/3} < \frac{6}{3-\alpha}$ for all $\alpha \in (0, 1)$.

Third, we show that $g(\tilde{p}, \tau, \alpha)$ strictly increases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Taking the total derivative of g with respect to τ, we obtain

$$\begin{aligned} \frac{dg}{d\tau} &= \left(\alpha\left(\sqrt{\tau - \tilde{p}} - \frac{\tilde{p}}{2\sqrt{\tau - \tilde{p}}}\right) + \frac{5}{4}(1-\alpha)\right)\frac{d\tilde{p}}{d\tau} + \frac{\alpha\tilde{p}}{2\sqrt{\tau - \tilde{p}}} \\ &= \left(\frac{3}{2}\alpha\frac{\frac{2}{3}\tau - \tilde{p}}{\sqrt{\tau - \tilde{p}}} + \frac{5}{4}(1-\alpha)\right)\frac{d\tilde{p}}{d\tau} + \frac{\alpha\tilde{p}}{2\sqrt{\tau - \tilde{p}}} \\ &= \left(\frac{3}{4}(1-\alpha)(\tilde{p}-1) + \frac{5}{4}(1-\alpha)\right)\frac{d\tilde{p}}{d\tau} + \frac{\alpha\tilde{p}}{2\sqrt{\tau - \tilde{p}}} \\ &= \frac{1-\alpha}{4}(3\tilde{p}+2)\frac{d\tilde{p}}{d\tau} + \frac{\alpha\tilde{p}}{2\sqrt{\tau - \tilde{p}}} > 0, \end{aligned}$$

since $\frac{d\tilde{p}}{d\tau} > 0$. Thus, we have shown that, for all $\alpha \in (0, 1)$, the function $g(\tau, \tilde{p}, \alpha)$ is negative at $\tau \approx \frac{3}{2}$, positive at $\tau = \left(\frac{27}{4}\right)^{1/3}$, and strictly increasing on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Therefore, there exists a uniquely defined function $\hat{\tau}(\cdot)$ with values $\hat{\tau}(\alpha)$ that solves $g(\hat{\tau}(\alpha), \tilde{p}, \alpha) = 0$ with $\hat{\tau}(\alpha) \in \left(\frac{3}{2}, \left(\frac{27}{4}\right)^{1/3}\right)$.

Finally, we look at the limiting property of $\hat{\tau}(\alpha)$ when α goes to 1. Note that when α tends to 1 we have that $\tilde{p} \to \frac{2}{3}\tau$. By taking the limit of $g(\tau, \tilde{p}, \alpha)$ when α goes to 1 and $\tau = \hat{\tau}(\alpha)$ we obtain

$$\lim_{\alpha \to 1} g(\hat{\tau}(\alpha), \tilde{p}, \alpha) = \lim_{\alpha \to 1} g(\hat{\tau}(\alpha), 2/3\hat{\tau}(\alpha), \alpha) = \lim_{\alpha \to 1} \frac{2}{3}\hat{\tau}(\alpha)\sqrt{\frac{1}{3}\hat{\tau}(\alpha)} - 1 = 0.$$

This implies that $\lim_{\alpha \uparrow 1} \hat{\tau}(\alpha) = \left(\frac{27}{4}\right)^{1/3}$. □

Proposition 11. *Consider the Hotelling model with quadratic transport costs and a positive fraction α of consumers using the ad blocker in the parameter range $\frac{v-c}{t} \in \left(\frac{3}{2}, \frac{7}{2}\right)$. Then, the inequality $\pi^w + \pi^{nw} \geq \pi^m$ is satisfied for $\frac{v-c}{t} > \tilde{\tau}(\alpha)$, where $\tilde{\tau}(\alpha)$ together with $\tilde{p} \in \left(1, \frac{2}{3}\tau\right)$ solves the following system of equations*

$$\begin{cases} \alpha\tilde{p}\sqrt{\tau - \tilde{p}} + \frac{1-\alpha}{8}(-\tilde{p}^2 + 8\tilde{p} + 1) - \frac{2}{3}\sqrt{\frac{\tau^3}{3}} = 0, \\ \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1}{2}(1-\alpha)(\tilde{p} - 1)\sqrt{\tau - \tilde{p}} = 0, \end{cases}$$

For all $\alpha \in (0,1)$, the function $\tilde{\tau}(\alpha)$ satisfies $\tilde{\tau}(\alpha) \in \left(\left(\frac{27}{4}\right)^{1/3}, \frac{6}{3-\alpha}\right)$ and $\lim_{\alpha \to 0} \hat{\tau}(\alpha) = \left(\frac{27}{4}\right)^{1/3}$.

Proof. First, suppose that $\tau \equiv \frac{v-c}{t} \in \left(\frac{6}{3-\alpha}, \frac{7}{2}\right)$. In this case, the monopoly profit is given by $(\tau - 1)t$ for $\tau > 3$ and $\frac{2}{3}t\sqrt{\tau^3/3}$ for $\tau \leq 3$. This implies that in the considered region we have that $\pi^m \geq (\tau - 1)t$.

The equilibrium profits of the firms are given by

$$\pi^w = (\tau - 1)\left(1 - (1-\alpha)\frac{\tau}{4}\right)t \quad \text{and} \quad \pi^{nw} = (1-\alpha)\frac{\tau^2}{8}t.$$

Therefore,

$$\begin{aligned} \pi^w + \pi^{nw} - \pi^m &\leq \pi^w + \pi^{nw} - (\tau - 1)t \\ &= -\frac{1-\alpha}{4}\tau(\tau - 1 - \tau/2)t < 0, \end{aligned}$$

since $\tau > 6/(3-\alpha) > 2$. We conclude that $\pi^m > \pi^w + \pi^{nw}$ for $\tau \in \left(\frac{6}{3-\alpha}, \frac{7}{2}\right)$.

Second, suppose that $\tau \in \left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Then, $\tilde{p} \equiv \frac{p^w - c}{t} \in \left(1, \frac{2}{3}\tau\right)$ solves

$$\phi(\tau, \tilde{p}, \alpha) \equiv \alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1-\alpha}{2}(\tilde{p} - 1)\sqrt{\tau - \tilde{p}} = 0, \quad (1)$$

and $\tilde{p}$ strictly increases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. Note that $\tilde{p}$ is a function of τ and α; i.e., $\tilde{p} = \tilde{p}(\tau, \alpha)$. In the parameter range under consideration, the firms' equilibrium profits can be rewritten as

$$\pi^w = \tilde{p}\left(\alpha\sqrt{\tau - \tilde{p}} + \frac{1-\alpha}{4}(3 - \tilde{p})\right)t,$$
$$\pi^{nw} = \frac{1-\alpha}{8}(\tilde{p}+1)^2 t.$$

We define the function

$$\begin{aligned} h(\tau, p, \alpha) &= (\pi^w + \pi^{nw} - \pi^m)/t \\ &= p\left(\alpha\sqrt{\tau - p} + \frac{1-\alpha}{4}(3-p)\right) + \frac{1-\alpha}{8}(p+1)^2 - \frac{2}{3}\sqrt{\frac{\tau^3}{3}} \\ &= \alpha p\sqrt{\tau - p} + \frac{1-\alpha}{8}(-p^2 + 8p + 1) - \frac{2}{3}\sqrt{\frac{\tau^3}{3}}, \end{aligned}$$

where $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$ and $p \in (1, 2/3\tau)$. Clearly, $\pi^w + \pi^{nw} > \pi^m$ if and only if $h(\tau, \tilde{p}, \alpha) > 0$, where $\tilde{p}$ solves (1).

Next, we show that there exists a continuous function $\tilde{\tau}(\alpha) \in \left(\left(\frac{27}{4}\right)^{1/3}, \frac{6}{3-\alpha}\right)$ that solves $h(\tilde{\tau}(\alpha), \tilde{p}, \alpha) = 0$. Moreover, in the parameter region under consideration, we show that $h(\tau, \tilde{p}, \alpha) > 0$ if and only if $\tau > \tilde{\tau}(\alpha)$. To establish this result, we prove that $h(\tau, \tilde{p}, \alpha)$ strictly decreases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$; $h(\bar{\tau}, \tilde{p}, \alpha) > 0$, where $\bar{\tau} = \left(\frac{27}{4}\right)^{1/3}$; and $h(6/(3-\alpha), \tilde{p}, \alpha) < 0$.

Function $h(\tau, \tilde{p}, \alpha)$ is positive at $\tau = \left(\frac{27}{4}\right)^{1/3}$. First, we show that $h(\bar{\tau}, \tilde{p}, \alpha) > 0$. By multiplying $h(\bar{\tau}, \tilde{p}, \alpha)$ by $\sqrt{\bar{\tau} - \tilde{p}}$ and using the fact that $\frac{2}{3}\sqrt{\frac{\bar{\tau}^3}{3}} = 1$, we obtain that

$$\begin{aligned} h(\bar{\tau}, \tilde{p}, \alpha)\sqrt{\bar{\tau} - \tilde{p}} &= \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + (1-\alpha)\frac{-\tilde{p}^2 + 8\tilde{p} + 1}{8}\sqrt{\bar{\tau} - \tilde{p}} - \sqrt{\bar{\tau} - \tilde{p}} \\ &= \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + (1-\alpha)\frac{(\tilde{p}-1)(7-\tilde{p})}{8}\sqrt{\bar{\tau} - \tilde{p}} - \alpha\sqrt{\bar{\tau} - \tilde{p}} \\ &= \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + \alpha\left(\frac{2}{3}\bar{\tau} - \tilde{p}\right)\frac{7-\tilde{p}}{4} - \alpha\sqrt{\bar{\tau} - \tilde{p}} \\ &> \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + \frac{17\alpha}{12}\left(\frac{2}{3}\bar{\tau} - \tilde{p}\right) - \alpha\sqrt{\bar{\tau} - \tilde{p}}, \end{aligned}$$

where the last inequality stems from $\frac{7-\tilde{p}}{4} > \frac{7-2\bar{\tau}/3}{4} > \frac{7-4/3}{4} = \frac{17}{12}$. Note that the function $p(\bar{\tau} - p)$ decreases in p for $p > 1 > \bar{\tau}/2$. The derivative of $\frac{17\alpha}{12}\left(\frac{2}{3}\bar{\tau} - p\right) - \alpha\sqrt{\bar{\tau} - p}$ with respect to p is

$$-\frac{17\alpha}{12} + \frac{\alpha}{2\sqrt{\bar{\tau} - p}} < -\frac{17\alpha}{12} + \frac{\alpha}{2\sqrt{\bar{\tau} - \frac{2}{3}\bar{\tau}}} = \frac{\alpha}{12}(-17 + 4\bar{\tau}) < 0.$$

This implies that the function

$$\alpha p(\bar{\tau} - p) + \frac{17\alpha}{12}\left(\frac{2}{3}\bar{\tau} - p\right) - \alpha\sqrt{\bar{\tau} - p}$$

strictly decreases in p. Therefore, since $\tilde{p} < \bar{\tau}$ we have that

$$\begin{aligned} h(\bar{\tau}, \tilde{p}, \alpha)\sqrt{\bar{\tau} - \tilde{p}} &> \alpha\tilde{p}(\bar{\tau} - \tilde{p}) + \frac{17\alpha}{12}\left(\frac{2}{3}\bar{\tau} - \tilde{p}\right) - \alpha\sqrt{\bar{\tau} - \tilde{p}} \\ &> \frac{2}{3}\alpha\bar{\tau}(\bar{\tau} - \frac{2}{3}\bar{\tau}) + \frac{17\alpha}{12}\left(\frac{2}{3}\bar{\tau} - \frac{2}{3}\bar{\tau}\right) - \alpha\sqrt{\bar{\tau} - \frac{2}{3}\bar{\tau}} \\ &= \alpha\sqrt{\frac{1}{3}\bar{\tau}}\left(\frac{2}{3}\bar{\tau}\sqrt{\frac{1}{3}\bar{\tau}} - 1\right) = 0. \end{aligned}$$

Hence, we establish that $h(\bar{\tau}, \tilde{p}, \alpha) > 0$.

Function $h(\tau, \tilde{p}, \alpha)$ is negative at $\tau = \frac{6}{3-\alpha}$. Second, we show that $h(6/(3-\alpha), \tilde{p}, \alpha) < 0$. The function $h(\tau, \tilde{p}, \alpha)$ can be rewritten as

$$\begin{aligned} h(\tau, \tilde{p}, \alpha) &= \alpha\tilde{p}\sqrt{\tau - \tilde{p}} + \frac{1-\alpha}{8}(-\tilde{p}^2 + 8\tilde{p} + 1) - \frac{2}{3}\sqrt{\frac{\tau^3}{3}} \\ &= \alpha\left(\tilde{p}\sqrt{\tau - \tilde{p}} - \frac{2}{3}\sqrt{\frac{\tau^3}{3}}\right) + (1-\alpha)\left(\frac{(\tilde{p}-1)(7-\tilde{p})}{8} + 1 - \frac{2}{3}\sqrt{\frac{\tau^3}{3}}\right). \end{aligned}$$

The first term is negative since $p\sqrt{\tau - p}$ strictly increases in p on $\left(1, \frac{2}{3}\tau\right)$ and therefore $\tilde{p}\sqrt{\tau - \tilde{p}}$ is strictly lower than $\frac{2}{3}\tau\sqrt{\tau - \frac{2}{3}\tau} = \frac{2}{3}\sqrt{\frac{\tau^3}{3}}$. We show that the second term is negative as well.

In what follows, it is useful to state an auxiliary result and show that $\tilde{p} < 1 + \alpha$ for $\tau = \frac{6}{3-\alpha}$ and any $\alpha \in (0, 1)$. Note that $\phi(\tau, p, \alpha)$ strictly decreases in p on $\left(1, \frac{2}{3}\tau\right)$ and, by the definition of $\tilde{p}$, we have that $\phi(\tau, \tilde{p}, \alpha) = 0$. We show that function $\phi(\tau, p, \alpha)$ is negative at $\tau = \frac{6}{3-\alpha}$ and $p = 1 + \alpha$. By plugging $\tau = \frac{6}{3-\alpha}$ and $p = 1 + \alpha$ into $\phi(\tau, p, \alpha)$, we find that

$$\begin{aligned} \phi(6/(3-\alpha), 1+\alpha, \alpha) &= \alpha\left(\frac{4}{3-\alpha} - 1 - \alpha\right) - \frac{\alpha(1-\alpha)}{2}\sqrt{\frac{6}{3-\alpha} - 1 - \alpha} \\ &= \frac{\alpha(1-\alpha)^2}{3-\alpha}\left(1 - \sqrt{\frac{(3-\alpha)(2 + (1-\alpha)^2)}{4(1-\alpha)^2}}\right) < 0, \end{aligned}$$

since

$$\begin{aligned} &2(3-\alpha) + (3-\alpha)(1-\alpha)^2 - 4(1-\alpha)^2 \\ &> 2(1-\alpha) - (1+\alpha)(1-\alpha)^2 \\ &= (1-\alpha)(1+\alpha^2) > 0. \end{aligned}$$

Therefore, using that ϕ is strictly decreases in p and $\phi(6/(3-\alpha), 1+\alpha, \alpha) < 0$, we obtain that $\tilde{p} < 1+\alpha$ for $\tau = \frac{6}{3-\alpha}$.

We are ready to show that the second term of $h(6/(3-\alpha), \tilde{p}, \alpha)$ is negative for all $\alpha \in (0,1)$. Note that $(\tilde{p}-1)(7-\tilde{p})$ strictly increases in p on $(1,2)$. Thus, since $\tilde{p} < 1+\alpha < 2$ we have that

$$\frac{(\tilde{p}-1)(7-\tilde{p})}{8} + 1 - 2\left(\frac{2}{3-\alpha}\right)^{\frac{3}{2}} < \frac{\alpha(6-\alpha)}{8} + 1 - 2\left(\frac{2}{3-\alpha}\right)^{\frac{3}{2}} \equiv \psi(\alpha).$$

Function $\psi(\alpha)$ is strictly concave on $(0,1)$ as $\psi'' = -\frac{1}{4} - \frac{30}{\sqrt{2}}(3-\alpha)^{-\frac{7}{2}} < 0$ and attains its maximum at α^* that solves $\psi'(\alpha^*) = 0$ which is equivalent to

$$\frac{1}{6}(3-\alpha^*)^2 - 2\left(\frac{2}{3-\alpha^*}\right)^{\frac{3}{2}} = 0.$$

Note that $\psi'(0.3) \approx -0.02 < 0$, which implies that $\alpha^* < 0.3$. Function ψ evaluated at α satisfies

$$\begin{aligned}\psi(\alpha) \le \psi(\alpha^*) &= \frac{\alpha^*(6-\alpha^*)}{8} + 1 - \frac{1}{6}(3-\alpha^*)^2 \\ &= \frac{1}{24}(-7\alpha^{*2} + 42\alpha^* - 12) \\ &< \frac{1}{24}(-7 \times 0.3^2 + 42 \times 0.3 - 12) \approx -0.01 < 0,\end{aligned}$$

where the strict inequality comes from the fact that $-7\alpha^2 + 42\alpha - 12$ strictly increases in α on $[0,1]$ and $\alpha^* < 0.3$. This implies that the second term of $h(6/(3-\alpha), \tilde{p}, \alpha)$ is negative and therefore $h(6/(3-\alpha), \tilde{p}, \alpha) < 0$.

Function $h(\tau, \tilde{p}, \alpha)$ strictly decreases in τ. It remains to show that function

$$h(\tau, \tilde{p}, \alpha) = \alpha \underbrace{\left(\tilde{p}\sqrt{\tau-\tilde{p}} - 2\left(\frac{\tau}{3}\right)^{\frac{3}{2}}\right)}_{\equiv h_1} + (1-\alpha)\underbrace{\left(\frac{(\tilde{p}-1)(7-\tilde{p})}{8} + 1 - 2\left(\frac{\tau}{3}\right)^{\frac{3}{2}}\right)}_{\equiv h_2},$$

strictly decreases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$. We show that the first and the second terms are decreasing functions in τ separately.

In what follows, it is useful to establish that $\frac{d\tilde{p}}{d\tau} < \frac{2}{3}\frac{\tau-\tilde{p}/2}{2(\tau-\tilde{p})}$. By the implicit function theorem, we have that

$$\begin{aligned}\frac{d\tilde{p}}{d\tau} &= \left(\frac{2}{3}\alpha - \frac{1-\alpha}{2}\frac{\tilde{p}-1}{2\sqrt{\tau-\tilde{p}}}\right) \Big/ \left(\alpha + \frac{1-\alpha}{2}\frac{2\tau-3\tilde{p}+1}{2\sqrt{\tau-\tilde{p}}}\right) \\ &< \frac{2}{3} - \frac{1-\alpha}{2\alpha}\frac{\tilde{p}-1}{2\sqrt{\tau-\tilde{p}}} = \frac{2}{3} - \frac{\frac{2}{3}\tau-\tilde{p}}{2(\tau-\tilde{p})} = \frac{2}{3}\frac{\tau-\tilde{p}/2}{2(\tau-\tilde{p})}.\end{aligned}$$

First, we show that the first term of h that we denote by h_1 decreases in τ. The derivative of h_1 (divided by α) with respect to τ is

$$\frac{dh_1}{d\tau} = \frac{2\tau - 3\tilde{p}}{2\sqrt{\tau - \tilde{p}}}\frac{d\tilde{p}}{d\tau} + \frac{\tilde{p}}{2\sqrt{\tau - \tilde{p}}} - \sqrt{\frac{\tau}{3}}$$

By multiplying $\frac{dh_1}{d\tau}$ by $\sqrt{\tau - \tilde{p}}$ and using the fact that $\frac{d\tilde{p}}{d\tau} < \frac{2}{3}\frac{\tau - \tilde{p}/2}{2(\tau - \tilde{p})}$ and $\frac{\tilde{p}}{2} < \frac{2}{3}\tau \times \frac{1}{2} < 1$, we obtain that

$$\begin{aligned}\frac{dh_1}{d\tau}\sqrt{\tau - \tilde{p}} &< \left(\frac{2}{3}\tau - \tilde{p}\right)\frac{\tau - \tilde{p}/2}{2(\tau - \tilde{p})} + \frac{\tilde{p}}{2} - \sqrt{\frac{\tau}{3}(\tau - \tilde{p})} \\ &< \left(\frac{2}{3}\tau - \tilde{p}\right)\frac{\tau - \tilde{p}/2}{2(\tau - \tilde{p})} + 1 - \sqrt{\frac{\tau}{3}(\tau - \tilde{p})}.\end{aligned}$$

Define the function

$$\zeta(\tau, p) \equiv \left(\frac{2}{3}\tau - p\right)\frac{\tau - p/2}{2(\tau - p)} + 1 - \sqrt{\frac{\tau}{3}(\tau - p)},$$

where $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$ and $p \in (1, 2/3\tau)$. Note that we established that $\frac{dh_1}{d\tau}\sqrt{\tau - \tilde{p}} < \zeta(\tau, \tilde{p})$. We show that $\zeta(\tau, p)$ strictly decreases in τ for all $p \in (1, 2/3\tau)$. The derivative of $\zeta(\tau, p)$ with respect to τ is given by

$$\begin{aligned}\frac{d\zeta(\tau, p)}{d\tau} &= \frac{\frac{2}{3}(\tau - p/2)}{2(\tau - p)} - \left(\frac{2}{3}\tau - p\right)\frac{p}{4(\tau - p)^2} - \frac{\frac{2}{3}(\tau - p/2)}{2\sqrt{\frac{\tau}{3}(\tau - p)}} \\ &= \frac{\frac{2}{3}(\tau - p/2)}{2\sqrt{\tau - p}}\left(\frac{1}{\sqrt{\tau - p}} - \frac{1}{\sqrt{\frac{\tau}{3}}}\right) - \left(\frac{2}{3}\tau - p\right)\frac{p}{4(\tau - p)^2} < 0,\end{aligned}$$

where $p < \frac{2}{3}\tau$ implies that $\sqrt{\tau - p} > \sqrt{\tau/3}$ and that the first term is negative. Thus, we showed that $\zeta(\tau, p)$ strictly decreases in τ for all $p \in (1, 2/3\tau)$. Since $\tau > \frac{3}{2}p$ we have that, for $p = \tilde{p}$,

$$\frac{dh_1}{d\tau}\sqrt{\tau - \tilde{p}} < \zeta(\tau, \tilde{p}) < \zeta\left(\frac{3}{2}\tilde{p}, \tilde{p}\right) = \frac{\tilde{p}}{2} - \frac{\tilde{p}}{2} = 0.$$

Thus, $dh_1/d\tau$ is negative and we obtain that $h_1(\tau, \tilde{p}, \alpha)$ strictly decreases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$.

Second, we explore the second term of $h(\tau, \tilde{p}, \alpha)$. The derivative of h_2 (divided by $1 - \alpha$) with respect to τ is

$$\frac{dh_2}{d\tau} = \left(1 - \frac{\tilde{p}}{4}\right)\frac{d\tilde{p}}{d\tau} - \sqrt{\frac{\tau}{3}} < \frac{2}{3}\left(1 - \frac{\tilde{p}}{4}\right) - \sqrt{\frac{3}{2} \times \frac{1}{3}} < \frac{2}{3} - \sqrt{\frac{1}{2}} < 0.$$

The first inequality holds because $\frac{d\tilde{p}}{d\tau} < \frac{2}{3}$ and $\tilde{p} < \frac{2}{3}\tau < 2$ implying that $1 - \frac{\tilde{p}}{4} > 0$. For the second inequality, we use $1 - \frac{\tilde{p}}{4} < 1$ and $\tau > \frac{3}{2}$. We conclude that $h_2(\tau, \tilde{p}, \alpha)$ strictly decreases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$.

To sum up, we have shown that function $h(\tau, \tilde{p}, \alpha)$, where $\tilde{p}$ solves $\alpha\left(\frac{2}{3}\tau - \tilde{p}\right) - \frac{1-\alpha}{2}(\tilde{p} - 1)\sqrt{\tau - \tilde{p}} = 0$, strictly decreases in τ on $\left(\frac{3}{2}, \frac{6}{3-\alpha}\right)$; is positive at $\tau = \left(\frac{27}{4}\right)^{1/3}$ and is negative at $\tau = \frac{6}{3-\alpha}$. Thus, there exists a uniquely defined $\tilde{\tau}(\alpha)$ that belongs to $\left(\left(\frac{27}{4}\right)^{1/3}, \frac{6}{3-\alpha}\right)$ and solves $\pi^w + \pi^{nw} - \pi^m = 0$.

It remains to be shown that $\tilde{\tau}(\alpha)$ tends to $\bar{\tau} = \left(\frac{27}{4}\right)^{1/3}$ when α goes to 0. Note that $\tilde{p}$ tends to 1 when $\alpha \to 0$. By taking the limit of $h(\tau, \tilde{p}, \alpha)$ when α goes to 0 and $\tau = \tilde{\tau}(\alpha)$, we obtain

$$\lim_{\alpha \to 0} h(\tilde{\tau}(\alpha), \tilde{p}, \alpha) = \lim_{\alpha \to 0} \left(1 - 2\left(\frac{\tilde{\tau}(\alpha)}{3}\right)^{\frac{3}{2}}\right) = 0.$$

This implies that $\tilde{\tau}(\alpha)$ tends to $\left(\frac{27}{4}\right)^{1/3}$ when $\alpha \to 0$. □

Next, we show that publisher surplus can be higher with the presence of the ad blocker.

Proposition 12. *Consider the Hotelling model with quadratic transport cost and a positive fraction of consumers using the ad blocker for* $\alpha > \frac{3}{5}$ *and* $\frac{v-c}{t} \in \left(2 + \frac{1}{2}\frac{1+\alpha}{1-\alpha}, \frac{1+\alpha}{1-\alpha}\right)$*. Then,* $\pi^{nw} > (1+\alpha)\pi^d$ *.*

Proof. Note that for $\alpha > \frac{3}{5}$ we have that $2 + \frac{1}{2}\frac{1+\alpha}{1-\alpha} > 4$. Recall that for $\frac{v-c}{t} \in \left(4, \frac{1+\alpha}{1-\alpha}\right)$ the profits under symmetric and asymmetric competition coincide with the model with linear transport cost. Therefore, Proposition 9 can be applied. This concludes the proof. □

References

AAC (2019): *Bylaws 2019*.

Amaldoss, W. and C. He (2010): "Product Variety, Informative Advertising, and Price Competition," *Journal of Marketing Research*, 47, 146–156.

Anderson, S. P. and S. Coate (2005): "Market provision of broadcasting: A welfare analysis," *Review of Economic Studies*, 72, 947–972.

Anderson, S. P. and J. S. Gans (2011): "Platform siphoning: Ad-avoidance and media content," *American Economic Journal: Microeconomics*, 3, 1–34.

Anton, J. J., J. H. Vander Weide, and N. Vettas (2002): "Entry auctions and strategic behavior under cross-market price constraints," *International Journal of Industrial Organization*, 20, 611–629.

Aseri, M., M. Dawande, G. Janakiraman, and V. S. Mookerjee (2020): "Ad-blockers: A blessing or a curse?" *Information Systems Research*, 31, 627–646.

Belleflamme, P. and M. Peitz (2019): "Managing competition on a two-sided platform," *Journal of Economics & Management Strategy*, 28, 5–22.

Butters, G. R. (1977): "Equilibrium distributions of sales and advertising prices," *Review of Economic Studies*, 44, 465–491.

Chen, Y. and Q. Liu (2022): "Signaling through advertising when an ad can be blocked," *Marketing Science*, 41, 166–187.

Christou, C. and N. Vettas (2008): "On informative advertising and product differentiation," *International Journal of Industrial Organization*, 26, 92–112.

Despotakis, S., R. Ravi, and K. Srinivasan (2021): "The beneficial effects of ad blockers," *Management Science*, 67, 2096–2125.

Dukes, A. and E. Gal-Or (2003): "Negotiations and exclusivity contracts for advertising," *Marketing Science*, 22, 222–245.

Gritckevich, A., Z. Katona, and M. Sarvary (2022): "Ad blocking," *Management Science*, 68, 4703–4724.

Grossman, G. M. and C. Shapiro (1984): "Informative advertising with differentiated products," *Review of Economic Studies*, 51, 63–81.

Hagiu, A. (2009): "Two-sided platforms: Product variety and pricing structures," *Journal of Economics & Management Strategy*, 18, 1011–1043.

Johnson, J. P. (2013): "Targeted advertising and advertising avoidance," *Rand Journal of Economics*, 44, 128–144.

Karle, H., M. Peitz, and M. Reisinger (2020): "Segmentation versus agglomeration: Competition between platforms with competitive sellers," *Journal of Political Economy*, 128, 2329–2374.

Milgrom, P. and J. Roberts (1986): "Price and advertising signals of product quality," *Journal of Political Economy*, 94, 796–821.

Nelson, P. (1974): "Advertising as information," *Journal of Political Economy*, 82, 729–754.

Nocke, V., M. Peitz, and K. Stahl (2007): "Platform ownership," *Journal of the European Economic Association*, 5, 1130–1160.

Perloff, J. M. and S. C. Salop (1985): "Equilibrium with product differentiation," *Review of Economic Studies*, 52, 107–120.

Prat, A. and T. Valletti (2022): "Attention oligopoly," *American Economic Journal: Microeconomics*, 14, 530–57.

Shiller, B., J. Waldfogel, and J. Ryan (2018): "The effect of ad blocking on website traffic and quality," *Rand Journal of Economics*, 49, 43–63.

Soberman, D. A. (2004): "Research Note: Additional Learning and Implications on the Role of Informative Advertising," *Management Science*, 50, 1744–1750.

Teh, T.-H. (2022): "Platform governance," *American Economic Journal: Microeconomics*, 14, 213–254.

Todri, V. (2022): "Frontiers: The impact of ad-blockers on online consumer behavior," *Marketing Science*, 41, 7–18.

Valletti, T. M., S. Hoernig, and P. P. Barros (2002): "Universal service and entry: The role of uniform pricing and coverage constraints," *Journal of Regulatory Economics*, 21, 169–190.

Wilbur, K. C. (2008): "A two-sided, empirical model of television advertising and viewing markets," *Marketing Science*, 27, 356–378.

Yan, S., K. M. Miller, and B. Skiera (2022): "How Does the adoption of ad blockers affect news consumption?" *Journal of Marketing Research*, 59, 1002–1018.