Discussion Paper Series – CRC TR 224

Discussion Paper No. 433
Project B 04

Two-Dimensional Information Acquisition In Social Learning

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June 2023

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Two-Dimensional Information Acquisition in Social Learning*

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June 5, 2023

Abstract

We analyze a social learning model where the agents' utility depends on a common component and an idiosyncratic component. Each agent splits a learning budget between the two components. We show that information about the common component is fully aggregated if and only if agents do not have to sacrifice learning about their idiosyncratic component in order to learn marginally about the common component. If agents vary in how much they value their idiosyncratic component, then the order of agents can strictly impact how much information is aggregated.

JEL classification: D82, D83

Keywords: Information Acquisition, Social Learning, Information Aggregation

1 Introduction

Individuals often learn from the actions of other individuals in a society about some common unknown state of the world. Classic examples of this social learning are how much to invest in financial assets or the choice of insurance coverage (Banerjee, 1992; Bikchandani et al., 1992). Yet the preferences of individuals are typically not perfectly homogeneous. In the above examples, individuals might have some idiosyncratic taste or match realization of which they are not perfectly informed. For example, the optimal choice of health insurance coverage

*This is a revision of the paper "Learning what unites or divides us: information acquisition in social learning." We are grateful to S. Nageeb Ali, Stephan Lauermann, Ignacio Monzón, Mallesh Pai, seminar audiences at Aalto University, University of Bonn, Mannheim University, Stanford GSB, and Texas A&M, four anonymous referees and the editor Xavier Vives for helpful comments and suggestions. Financial support by the German Research Foundation (DFG) through CRC TR 224 (project B04) is gratefully acknowledged.

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might depend on a policyholder's predisposition to some illnesses that could require further testing; the optimal investment might depend on the composition of the portfolio that the investor already owns.

We introduce idiosyncratic uncertainty into a tractable and continuous model of information acquisition in social learning from Burguet and Vives (2000). In our setup, individuals face a common and idiosyncratic uncertainty, and split their attention between the two dimensions of this uncertainty. Will individuals successfully aggregate their private information so that in the long run society learns the common state of the world, or will the heterogeneity among agents bring the learning process to a halt? That is, will learning be *complete*? We show that even if all individuals have a fixed learning budget *for free*, learning is incomplete under fairly general conditions.

In our model, an infinite ordered sequence of agents faces a prediction problem: guessing a weighted sum of a common component and their idiosyncratic component. The common component is a Gaussian random variable and identical for every agent. Each agent's idiosyncratic component is an independently and identically distributed Gaussian random variable. Neither the common component nor any idiosyncratic one is known. First, an agent observes the predictions of all previous agents. Then, she obtains two additional private signals - one about the common component and another about her idiosyncratic component. The agent chooses the precision of each signal. The agent is endowed with some fixed learning budget and faces a trade-off: if she increases the precision of the common component signal, then she decreases the precision of the idiosyncratic component signal, and vice versa.

The information contained in the prediction about the idiosyncratic component leads to an inference problem: it impedes future agents' learning from previous actions. Another impediment for learning is that agents do not internalize the benefit of their learning about the common component for all future agents, leading to a free-rider problem.

Our main result is that learning is complete if and only if the learning technology satisfies a particular condition on the marginal rate of transformation. Specifically, the condition states that starting from an uninformative signal about the common component, learning marginally more about it does not decrease the precision of the idiosyncratic signal. If the condition is not satisfied, then agents eventually invest their entire learning budget into the idiosyncratic component. This is because the more information society has aggregated about the common component, the lower the marginal benefit of learning further about it. Eventually, this marginal benefit approaches zero, while the marginal cost from decreasing the idiosyncratic component precision remains constant (and strictly positive). Thus, there exists an upper bound on the public precision of the common component, and learning is incomplete. On the other hand, if the condition is fulfilled, then — in a sufficiently small neighborhood around zero — the marginal gain from learning about the common component outweighs the marginal loss from learning less about the idiosyncratic component. In that case, agents spend at least part of their learning budget on the common component irrespective of how much information about the common component has been already aggregated.

This reasoning relates to a result in Burguet and Vives (2000). They study a framework in which a continuum of agents faces the same one-dimensional uncertain state of the world in each period. Agents choose how much costly information to acquire in order to increase the precision of their signal. Burguet and Vives (2000) show that learning is complete if and only if the marginal cost of information at zero is zero. If this condition is fulfilled, learning continues ad infinitum and is bounded away from zero for a finite precision, a result similar to ours.

Ali (2018) studies information aggregation for more general action and signal spaces. Learning is complete in a responsive[1] decision problem, if arbitrarily cheap signals are available. Due to the structure of the sets of signals, arbitrarily uninformative signals do not exist. Thus, if learning goes on forever, then it is complete. In contrast, in both Burguet and Vives (2000) and our paper, an infinite sequence of agents acquiring information about the common component is not sufficient for complete learning.

A condition similar to the one in Ali (2018) is also relevant in Mueller-Frank and Pai (2016). In their model, agents face a common uncertainty about the payoff of finitely many actions. Agents can sequentially discover the utility of each action for a fixed cost. Complete learning occurs if and only if the distribution of search costs includes zero in its support.

We provide a broader perspective for why learning might be incomplete: we believe that it is reasonable to assume that there is more than one uncertain variable in an economy about which agents can learn.[2] In our setup, there are no fixed monetary costs associated with learning, in contrast to the setups in Burguet and Vives (2000), Mueller-Frank and Pai (2016), and Ali (2018). Instead, agents evaluate the nonmonetary opportunity costs of giving up learning about other components. These marginal nonmonetary opportunity costs are *endogenously* determined by how much has already been learned from previous agents. Each agent solves an information choice problem: how much information to acquire about which

[1] In a responsive decision problem, every change in the belief of an agent about the state changes her action, as is the case in Burguet and Vives (2000) and in our model.

[2] Known idiosyncratic types and no information acquisition have been considered, e.g., in Smith and Sørensen (2000) and Goeree et al. (2006). In Hendricks et al. (2012), agents decide whether to learn the sum of their idiosyncratic and common components perfectly or not at all. For information choice in other economic settings, see Deimen and Szalay (2019), Liang and Mu (2020), and Bobkova (2022).

component. Therefore, the necessary and sufficient conditions for complete learning in our model do not refer to a fixed marginal cost but rather to the marginal rate of transformation between two sources of information.

2 Model

There is an infinite ordered sequence of agents, indexed by $i = 1, 2, \ldots$. Each agent i chooses an irreversible action $a_i \in \mathbb{R}$. The payoff of each agent i from an action a_i depends on two unknown states of the world: a common component $\Theta \sim \mathcal{N}(0, \frac{1}{P})$ with finite $P > 0$, and a private component $T_i \sim \mathcal{N}(0, \frac{1}{Q})$ with finite $Q > 0$. All private components are i.i.d. draws and independent from the common component. Given agent i's action a_i, a realization θ of the common component, and a realization t_i of her idiosyncratic component, her utility is

$$u_i(a_i, \theta, t_i) = -(a_i - (1 - \mu)\theta - \mu t_i)^2 \tag{1}$$

where $\mu \in (0, 1)$. Agent i's objective is to minimize the distance between her action and a weighted sum of the common and idiosyncratic component.

Agent i observes all actions $a_1, \ldots, a_{i-1}$ of her predecessors. In addition, each agent i observes two independent private signals, $s_i^\Theta \sim \mathcal{N}\left(0, \frac{1}{\rho_i}\right)$, and $s_i^T \sim \mathcal{N}\left(0, \frac{1}{\tau_i}\right)$. The information choice of agent i consists of choosing the precision of the common component signal, ρ_i, and the private component signal, τ_i, along a *precision function* f that shows all feasible precision combinations (ρ_i, τ_i).

Assumption 1. *The precision function $f : \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable, $f'(\rho_i) < 0$ for $\rho_i > 0$ and $f''(\rho_i) \leq 0$. There exist finite $\overline{\rho}$ and $\overline{\tau}$ such that $0 = f(\overline{\rho})$ and $\overline{\tau} := f(0)$.*

The precision of the idiosyncratic component τ_i is determined by the choice of ρ_i. The derivative of the precision function, f', corresponds to the marginal rate of transformation between the two variables ρ_i and τ_i. As a microfoundation for the precision function, agents could have a budget B and face an increasing cost function $c(\rho_i, \tau_i)$ for their learning choice. Since it is always strictly optimal to spend the entire budget, we can restrict an agent's decision to choosing the precision of the common component such that $B = c(\rho_i, f(\rho_i))$.[3]

The timing is as follows: at time i, agent i observes the history of all previous actions $\mathcal{A}_i := \{a_1, \ldots, a_{i-1}\}$. She then chooses ρ_i, her information choice, which determines $\tau_i = f(\rho_i)$.

[3]See also Section 5 for a discussion of an endogenous learning budget.

Then, she observes her signal realizations s_i^Θ and s_i^T and chooses her irreversible action a_i. Finally, her utility is realized, unobserved by all future agents.

3 Main analysis

Given an information choice ρ_i and signals s_i^Θ and s_i^T, agent i's optimal action a_i^* is a weighted average of a two-dimensional belief about the common and idiosyncratic component, namely

$$a_i^* = E[(1-\mu)\Theta + \mu T_i | s_i^T, s_i^\Theta, \rho_i; \mathcal{A}_i].$$

As we will see, for every possible history $\mathcal{A}_i$, agent i faces a Gaussian distribution of the common component, $\Theta|\mathcal{A}_i$, with some precision P_i. We refer to P_i as the *public precision.*

Without an idiosyncratic component ($\mu = 0$) there is a bijection between signals and actions, and an agent can infer her predecessors' signals from their actions. In contrast, in the presence of an idiosyncratic component an inference problem arises: a high action can be the result of either a high private component signal s_i^T or a high common component signal s_i^Θ.

Lemma 1 (Precision Updating)**.** *For every $\mathcal{A}_i$, the random variable $\Theta|\mathcal{A}_i$ follows a Gaussian distribution. Let agent $i-1$ acquire a signal about the common component with precision ρ_{i-1}. Then, $P_i = P_{i-1} + \psi(\rho_{i-1}, P_{i-1})$ where $\psi(\rho_{i-1}, P_{i-1}) \coloneqq \rho_{i-1}\left[1 + \frac{\mu^2}{(1-\mu)^2}\frac{(\rho_{i-1}+P_{i-1})^2}{f(\rho_{i-1})+Q}\frac{f(\rho_{i-1})}{Q\rho_{i-1}}\right]^{-1}$.*

Agent i faces the precision $P_{i-1} + \psi(\rho_{i-1}, P_{i-1})$ which depends on the learning choice ρ_{i-1} of her predecessor but not on her action. The term $\psi(\rho_{i-1}, P_{i-1})$ is lower than ρ_{i-1} and reflects the loss in inference since only past actions and not past signals are observable.

After agent i chooses ρ_i and learns s_i^Θ and s_i^T, her updated common-component precision is $P_i + \rho_i$ and her private-component precision is $Q + f(\rho_i)$. With a quadratic payoff in (1), an agent's expected loss when choosing ρ_i is the variance of $(1-\mu)\Theta + \mu T_i$, given P_i and Q. Hence, the optimal information choice of agent i solves

$$\rho_i^* \in \arg\min_{\rho_i \in [0,\overline{\rho}]} \frac{(1-\mu)^2}{P_i + \rho_i} + \frac{\mu^2}{Q + f(\rho_i)}. \tag{2}$$

The agent faces a trade-off: decreasing the loss from the variance of the common component (by increasing ρ_i) increases the loss from the variance of the private component. We categorize the optimal learning choices into three regimes. If an agent learns exclusively about either the common component or the idiosyncratic component, then we say that she is in the *common regime* or the *idiosyncratic regime*, respectively. If the agent learns about both components ($\rho_i^* \in (0, \overline{\rho})$), then the agent is in the *splitting regime.*

Lemma 2 (Three Learning Regimes)**.** *There exists a unique ρ_i^* that solves (2). The optimal learning choice ρ_i^* is nonincreasing in P_i and nondecreasing in Q. Agent i is*

1. *in the common regime if $P_i \leq \alpha$,*
2. *in the splitting regime if $P_i \in (\alpha, \beta)$,*
3. *in the idiosyncratic regime if $P_i \geq \beta$,*

where $\alpha := \frac{1-\mu}{\mu}\frac{Q}{\sqrt{-f'(\overline{\rho})}} - \overline{\rho}$ and $\beta := \frac{1-\mu}{\mu}\frac{Q+f(0)}{\sqrt{-f'(0)}}$.[4]

The optimal information choice of every agent depends only on P_i and Q, not on the action history $\mathcal{A}_i$. Hence, the learning process $\{P_i\}_{i=1}^{\infty}$ follows a deterministic path. The higher the initial precision of a component (P_i or Q), the lower the marginal benefit from learning more about it. In the optimal learning choice, the agent chooses either a corner solution (in the common or idiosyncratic regime) or an interior optimum (in the splitting regime). For some precision functions, the solution is always interior. If $f'(0) = 0$, then β is unbounded and the idiosyncratic regime does not arise: the agent does not have to give up any precision about the idiosyncratic component when learning marginally about the common component. Similarly, if $f'(\overline{\rho}) = -\infty$, then the common regime never arises.

How much information about the common component does the public history accumulate? We say that learning is *complete* if and only if the limiting public precision, denoted by $P_\infty := \lim_{i\to\infty} P_i$, equals $+\infty$. Learning is *incomplete* otherwise. Our main theorem identifies β or P as the only possible values for the limiting public precision. Hence, for learning to be complete, we require $\beta = +\infty$, which occurs if and only if $f'(0) = 0$ by Lemma 2.

Theorem 1. *The limiting public precision is $P_\infty = \max\{P, \beta\}$. Learning is complete if and only if $f'(0) = 0$.*

By Lemma 2, if the initial precision of the common component is sufficiently high ($P \geq \beta$), it deters even the first agent from learning anything about Θ. Hence, agents never escape the idiosyncratic regime, and the public precision remains at the initial precision P. If the initial precision of the common component is sufficiently low ($P < \beta$), then the first agents are either in the common or the splitting regime. No agent would ever acquire so much precision about the common component as to raise her precision above β.[5] Furthermore, the limiting public precision cannot stop short of β: the learning choice of an agent converges to zero only as the public precision approaches β. As long as the public precision is strictly below β, agents keep

[4] If $f'(0) = 0$, we define β by $\beta := \lim_{\rho\to 0} \frac{1-\mu}{\mu}\frac{Q+\overline{\tau}}{\sqrt{-f'(\rho)}}$. If $\lim_{\rho\to\overline{\rho}} f'(\overline{\rho}) = \infty$, we set $\alpha = 0$.

[5] See Lemma 3 and Proposition 3 in the appendix for more details about the learning regime dynamics.

spending a constant amount of their learning budget on the common component, and learning continues.

Observable Signals. A first intuition might be that incomplete learning is due to signals being unobservable. Yet this is not the case: incomplete learning is caused by the binary uncertainty about two components and the free-rider problem, not by the inference lost through observational noise. Observing the signals $\{s_i^\Theta, s_i^T\}$ of all previous agents leads to a faster convergence of P_i to P_∞ since no precision is lost through noisy observations. However, learning is bounded by the same precision $\max\{P, \beta\}$ as in the case of unobservable signals. See Proposition 4 in the appendix for further details. A related observation has been made in Burguet and Vives (2000) and in Corollary 1 in Ali (2016), where it is not the loss of inference from signals to actions that drives the incomplete learning result, but the decreased incentives to buy costly information as public precision increases.

4 Heterogeneous agents

Next, we explore the effect of heterogeneous agents on information aggregation. Instead of a common weight μ as in (1), each agent i has a weight $\mu_i \in \mathcal{M}$, where $\mathcal{M} \subseteq [0,1]$ is the set of weights on the idiosyncratic component. The sequence of agents' weights $\{\mu_i\}_{i\geq 1}$, the *ordering*, is publicly known. In light of the paragraph on observable signals above and for simplicity of notation, we assume that all signals are observable. When is learning complete in a heterogeneous society?

Proposition 1. *Let $f'(0) \neq 0$. Then, learning is complete if and only if there exists a subsequence of agents $\{\mu_{i_k}\}_{k\geq 1}$ with $\mu_{i_k} \to 0$ as $k \to \infty$. Let $f'(0) = 0$. If $\mu_i \not\to 1$ as $i \to \infty$, learning is complete. If $\mu_i \to 1$ as $i \to \infty$, learning can be either complete or incomplete.*

One key qualitative takeaway is this: if for every agent i, $\mu_i \in [\underline{\mu}, \overline{\mu}]$ for some $0 < \underline{\mu} \leq \overline{\mu} < 1$, then our main result still holds, learning is complete if and only if $f'(0) = 0$. Only in the pathological case in which a subsequence of agents cares so much about the common component relative to the private one that they learn about the common component for any public belief, learning is complete even if $f'(0) \neq 0$. On the other hand, if agents eventually care (and thus, learn) only about the idiosyncratic component, then there is no guarantee that learning is complete even if $f'(0) = 0$.

Along the learning path, however, the ordering $\{\mu_i\}_{i\geq 1}$ has a quantitative impact: it influences the speed of learning and possibly even the limiting precision P_∞. The following proposition shows when agents with a lower μ_i should move later to facilitate learning.

Proposition 2. *Let there be two consecutive agents with $\mu_j < \mu_{j+1}$ in some sequence $\{\mu_i\}_{i\geq 1}$. Fix $\overline{\rho}$ and $\overline{\tau}$. Then, there exists $\epsilon > 0$ such that if $f'(0) - f'(\overline{\rho}) < \epsilon$, then swapping the positions j and $j+1$ yields (1) a weakly higher public precision P_i for every agent $i \geq j+2$, and (2) a weakly higher limiting precision P_∞.*

A sufficient condition for such a swap to increase the public precision is if the expression $\rho(\mu_i, P_i') - \rho(\mu_i, P_i)$ is nonincreasing in μ_i for all splitting-regime precisions $P_i' \geq P_i$. Intuitively, this condition says that agents who care more about the common component react less sensitively to an increase in the public precision (and thus, to moving later in the sequence when more is known) than agents who care less about the common component. If the slope of the precision function is sufficiently constant, the opportunity costs of learning more about the common component (i.e., the loss in idiosyncratic precision) are approximately similar at every learning choice, but the marginal gain is higher for an agent who cares relatively more about the common component.

The order of agents can strictly increase the limiting precision. Consider the linear case $f(\rho) = 1 - \rho$, $P_1 = Q = 1$, and a sequence $\{\mu_i\}_{i\geq 1}$ where $\mu_i = 0.6$ for $i \neq 1$, and $\mu_1 = 0.2$. Let $\rho(\mu_i, P_i)$ be the information choice of agent i with weight μ_i and public precision P_i.[6] If the agent with $\mu_1 = 0.2$ moves first, then she learns exclusively about the common component, and $P_2 = 2$. This is above the idiosyncratic-regime threshold of an agent with $\mu_i = 0.6$, so learning stops after the first agent and $P_\infty = 2$. However, swapping the positions of agents 1 and 2 strictly increases the limiting precision: at $P_1 = 1$, agent 2 is in the splitting regime, and chooses $\rho(0.6, 1) > 0$. After agent 2 moves, agent 1 is still in the common regime at $P_2 = 1 + \rho(0.6, 1)$, and thus, $P_\infty = 2 + \rho(0.6, 1) > 2$. Proposition 2 provides a recipe for how to increase the limiting public precision even further: the later in the sequence agent 1 moves, the strictly higher the limiting public precision P_∞ will be.[7]

If the slope of the precision function is not sufficiently constant, it may be optimal if the agent with the lower weight on the idiosyncratic component moves first. Consider two agents with $\mu_1 = 0.6$ and $\mu_2 = 0.4$ and a precision function $f(\rho) = 0.5$ if $\rho \in [0, 0.5]$ and $f(\rho) = 1 - \rho$ if $\rho \in (0.5, 1]$.[8] Agent 1's opportunity costs from increasing ρ beyond 0.5 are so high that it is never optimal to learn in the steep interval of the precision function for any $P_i \geq 1$; for this reason, she chooses $\rho(0.6, P_i) = 0.5$. Due to agent 2's higher weight on the common component,

[6] This is pinned down by Lemma 2, after substituting μ_i for μ everywhere.

[7] An optimal ordering does not exist because all agents cannot be sorted in decreasing order of μ_i.

[8] Although this example does not satisfy Assumption 1 (i.e., f decreasing and differentiable everywhere), it is simple to follow. One could easily approximate the precision function in this example via a strictly decreasing concave function that is differentiable everywhere.

and therefore, her higher marginal benefit from learning more about it, agent 2's optimal learning choice lies in the steep interval of the precision function: she chooses $\rho(0.4, 1) = 0.8$ when moving first, and $\rho(0.4, 1.5) = 0.6$ when moving second. In contrast to the previous example, now the agent with the lower μ_i reacts more to a change in public precision; hence, the public precision increases for all subsequent agents if agent 2 moves before agent 1.

5 Extensions

Two prediction problems. Instead of predicting a weighted average of two components, consider an alternative scenario in which each agent faces two separate prediction problems: (i) predict a variable Θ with action a_i^Θ, and (ii) predict a variable T_i (an independent investment problem) with action a_i^T, with the following utility function:

$$u_i(a_i^\Theta, a_i^T, \theta, t_i) = -(a_i^\Theta - \theta)^2 - (a_i^T - t_i)^2.$$

The expected utility reduces to $-\frac{1}{P_i+\rho_i} - \frac{1}{Q+f(\rho_i)}$; this is the expression in the two-dimensional prediction problem in (2) for $\mu = \frac{1}{2}$ multiplied by a constant. All our previous results apply, and learning is complete if and only if $f'(0) = 0$. If agents are uncertain about more than one prediction problem, then complete learning might also be difficult to achieve.

Beyond a concave precision function. The concavity of the precision function guarantees the existence of a unique optimal learning choice. However, even with a nonconcave learning technology (with f satisfying Assumption 1, but imposing no assumption on f''), our main result still holds. Specifically, if $f'(0) > 0$, then there exists some finite, sufficiently high public precision P, for which agents are in the idiosyncratic regime because their payoff is strictly decreasing in ρ on the whole domain; thus, the public precision can never increase beyond P, and learning is incomplete. On the other hand, if $f'(0) = 0$, then learning cannot stop at any finite P_∞. At any finite precision, all agents learn at least the same strictly positive amount about the common component, implying that learning cannot stop there. If the learning technology is convex (e.g., if it is easier to focus exclusively on one component than to learn about two components at the same time), we necessarily have $f'(0) > 0$, and learning is always incomplete.

Endogenous learning budget. So far, all agents have been choosing how much to learn along some common exogenous precision function. This can arise, for example, when agents have a fixed learning budget and decide how to split it between the two components. However, in many scenarios, agents might endogenously determine how much they spend overall on learning. Such an endogenous learning budget might depend on the public precision that agents are facing, and hence, might differ among agents.

To make this point concrete, consider the following setup:[9] Let the agents' utility function be $V(x_i) + u(a_i, \theta, t_i)$, where $x_i \in \mathbb{R}^+$ is consumption, $V'(x_i) > 0$, and $V''(x_i) \leq 0$, and u is as defined in (1). Suppose that agents face a budget constraint $B = x_i + c(\rho_i, \tau_i)$, where $c : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ determines the costs of acquiring the signals and is increasing in both arguments, strictly convex, and satisfies $c(0, 0) = 0$.

Let $x^*(P_i)$ be the optimal consumption choice of agent i given some public precision P_i, leaving agent i an endogenous learning budget of $B - x^*(P_i)$. Given $x^*(P_i)$, the optimal information choice $(\rho_i^*(P_i), \tau_i^*(P_i))$ coincides with the optimal point on the isocost curve, or precision function, which is given by $c(\rho_i, \tau_i) = B - x^*(P_i)$. Define $f_{P_i}(\rho)$ to be the implicit function of τ as a function of ρ, such that $c(\rho_i, f_{P_i}(\rho_i)) = B - x^*(P_i)$. If this implicit function satisfies Assumption 1 for every P_i, then we get the same sufficient condition for complete learning as before, pointwise for every P_i: learning is complete if $f'_{P_i}(0) = 0$ for all P_i and $\frac{\partial c(0,0)}{\partial \rho} = 0$.[10] Yet, these conditions are not necessary for complete learning. The condition $f'_{P_i}(0) = 0$ ought to hold only along the sequence of P_i's which are realized. Interestingly, reaching a single isocost curve in which an agent is in the idiosyncratic regime halts learning forever, even if $f'_{P_i}(0) = 0$ for any sufficiently high public precision P_i.

6 Discussion

This paper has developed a new model for information choice in observational learning. Our central interest is in investigating how agents resolve the trade-off between learning about a common component that affects all of their payoffs and learning about an idiosyncratic component that affects only one's own payoff. If others devote their resources to learning about the common component, later agents have a natural inclination to free-ride on that choice and to devote their resources to learning about their own idiosyncratic component. This incentive potentially impedes complete learning of the common component. In our study of this trade-off, we provide a necessary and sufficient condition for complete learning. As we show, learning is complete if agents do not have to sacrifice learning about their idiosyncratic component in order to learn marginally about the common component. We believe that for a

[9]We are very grateful to a referee for suggesting this interpretation.

[10]The latter condition guarantees that consumption does not amount to the entire budget B. By the implicit function theorem, learning is complete if $\frac{\partial c(0,\tau)}{\partial \rho} = 0$ for every $\tau > 0$. This is a generalization of a sufficiency result in Burguet and Vives (2000): they show that learning is complete if and only if the marginal cost of learning about the common (and only) component is zero at zero, $c'(0) = 0$.

variety of prediction problems — such as financial markets and health insurance choices — this condition might not be fulfilled. In such cases, a one-dimensional model incorrectly yields complete learning in these markets while a model with multidimensional uncertainty provides better predictions.

Appendix

Proof of Lemma 1. Using the standard Gaussian updating rule, the optimal action of agent $i-1$ who faces a public precision P_{i-1} and an expected common component γ_{i-1} is

$$a^*_{i-1} = (1-\mu)\frac{\rho_{i-1}}{\rho_{i-1}+P_{i-1}}\ s^{\Theta}_{i-1} + (1-\mu)\frac{P_{i-1}}{\rho_{i-1}+P_{i-1}}\gamma_{i-1} + \mu\frac{f(\rho_{i-1})}{f(\rho_{i-1})+Q}s^T_{i-1}.$$

From the perspective of agent i who does not learn s^{Θ}_{i-1} and s^T_{i-1} of her predecessor, observing a^*_{i-1} is equivalent to observing a variable $b_{i-1} := \left(a^*_{i-1} - (1-\mu)\frac{P_{i-1}}{\rho_{i-1}+P_{i-1}}\gamma_{i-1}\right)\frac{\rho_{i-1}+P_{i-1}}{\rho_{i-1}}\frac{1}{(1-\mu)}$. The conditional distribution of the random variable b_{i-1} that agent i learns via learning a^*_{i-1} (when anticipating ρ_{i-1} correctly) is

$$b_{i-1}|\theta \sim \mathcal{N}\left(\theta, \frac{1}{\rho_{i-1}} + \frac{\mu^2}{(1-\mu)^2}\frac{(\rho_{i-1}+P_{i-1})^2}{\rho^2_{i-1}}\frac{f(\rho_{i-1})}{(f(\rho_{i-1})+Q)}\frac{1}{Q}\right).$$

Applying the standard Gaussian updating rule and simplifying to compute $\theta|b_{i-1}$ for agent i, we get the expressions for $\psi(\rho_{i-1}, P_{i-1})$ and P_i as stated in the lemma. □

Proof of Lemma 2. The derivative of the function in (2) is

$$-\frac{(1-\mu)^2}{(P_i+\rho_i)^2} - \frac{\mu^2}{(Q+f(\rho_i))^2}f'(\rho_i), \tag{3}$$

which is increasing in P_i and nondecreasing in Q. If ρ_i is interior, then the first order condition is

$$\rho_i = \frac{Q+f(\rho_i)}{\sqrt{-f'(\rho_i)}}\frac{1-\mu}{\mu} - P_i. \tag{4}$$

Uniqueness. Any interior solution is *unique*: only one ρ_i satisfies (4) since $\frac{Q+f(\rho_i)}{\sqrt{-f'(\rho_i)}}$ is nonincreasing in ρ_i. If no interior solution exists, there is exactly one corner solution.

Idiosyncratic and common regimes. Let the derivative at $\rho_i = 0$ be positive. Then (as the second order derivative is nonnegative because $f''(\rho_i) \leq 0$), the derivative is positive for all $\rho_i \in [0, \overline{\rho}]$, and $\rho^*_i = 0$ is the unique solution. Thus, the following establishes when $\rho^*_i = 0$:

$$-\frac{(1-\mu)^2}{(P_i)^2} - \frac{\mu^2}{(Q+\overline{\tau})^2}f'(0) \geq 0.$$

The inequality is satisfied whenever $P_i \geq \beta$ where β is defined in the proposition.

Similarly, if the derivative at $\rho_i = \overline{\rho}$ is nonpositive, then it is negative at every feasible $\rho_i \in [0, \overline{\rho})$ and $\rho_i = \overline{\rho}$ is optimal. A sufficient and necessary condition for the first derivative being nonpositive at $\overline{\rho}$ is if $P_i \leq \alpha$ where α is given in the proposition.

In the remaining interval for $P_i \in (\alpha, \beta)$, an interior solution exists, since we are minimizing a continuous objective function over a closed feasible interval for ρ_i and the derivative changes its sign exactly once.

Comparative statics. The common regime threshold α is increasing in Q and independent of P_i. The idiosyncratic regime threshold β is increasing in Q and independent of P_i. In the splitting regime, ρ^* is nonincreasing in P_i and nondecreasing in Q (this follows from applying the implicit function theorem to (4)). □

Lemma 3. *Let P', P'' such that $P' < P'' \leq \beta$. Then, $P' + \rho^*(P') < P'' + \rho^*(P'')$.*

Proof. To simplify notation, let $\rho' := \rho^*(P')$ and $\rho'' := \rho^*(P'')$. First, let $P'' \leq \alpha$. Then, $\rho' = \rho'' = \overline{\rho}$ and thus, $P' + \rho' < P'' + \rho''$. Next, let $P'' > \alpha$. By Lemma 2, ρ'' is in the splitting regime. By Lemma 2, $\rho' \geq \rho''$, and ρ' is in the common or splitting regime. Then, ρ' and ρ'' satisfy the first order conditions

$$-\frac{(1-\mu)^2}{(P'+\rho')^2} - \frac{\mu^2}{(Q+f(\rho'))^2} f'(\rho') \leq 0,$$
$$-\frac{(1-\mu)^2}{(P''+\rho'')^2} - \frac{\mu^2}{(Q+f(\rho''))^2} f'(\rho'') = 0.$$

Furthermore, $\frac{f'(\rho)}{(Q+f(\rho))}$ strictly decreases in $\rho > 0$ because $f''(\rho) \leq 0$ by Assumption 1. Then, adding up the first order conditions yields

$$(1-\mu)^2 \left(\frac{1}{(P''+\rho'')^2} - \frac{1}{(P'+\rho')^2} \right) \leq \mu^2 \left(\frac{f'(\rho')}{(Q+f(\rho'))^2} - \frac{f'(\rho'')}{(Q+f(\rho''))^2} \right) < 0.$$

This can only be satisfied if $P' + \rho' < P'' + \rho''$. □

Proof of Theorem 1. We prove a more general result, connecting initial precision P to the long run learning choices.

Proposition 3.

1. *The learning process leaves the idiosyncratic regime in finite time: if $P \leq \alpha$, then there exists $N \in \mathbb{N}$ such that $\rho_N^* \in (0, \overline{\rho})$, and $P_\infty > \alpha$.*
2. *The learning process remains in the idiosyncratic regime forever: if $P \geq \beta$, $\rho_i^* = 0$ for all $i \geq 1$, and $P_\infty = P$.*
3. *The learning process remains in the splitting regime forever: if $P \in (\alpha, \beta)$, then $\rho_i^* \to 0$ and $\rho_i^* \in (0, \overline{\rho})$ for all $i \geq 1$, and $P_\infty = \beta$.*

Proof. Proof of point 1. Since $P \leq \alpha$, $\rho_1^*(P) = \overline{\rho}$. By Lemma 1, there is no inference problem, $P_2 = P_1 + \psi(P_1, \overline{\rho}) = P + \overline{\rho}$. Similarly, if $P_2 \leq \alpha$, by the same logic as for the first agent,

$\rho_2^*(P_2) = \overline{\rho}$ and $P_3 = P + 2\overline{\rho}$. After a finite number of $x := \lceil \frac{\alpha - P}{\overline{\rho}} \rceil$ agents, public precision equals $P_{x+1} = P + \lceil \frac{\alpha-P}{\overline{\rho}} \rceil \overline{\rho} > \alpha$. The idiosyncratic regime stops with agent $x+1$, and $P_\infty \geq P_{x+1} > \alpha$.

Proof of point 2. Let $P \geq \beta$. Then, by Lemma 2, $\rho_1^* = 0$. Since a_1^* contains no information about θ, $P_2 = P$. Thus, $\rho_2^* = \rho_i^* = 0$ for all i, and $P_\infty = P$.

Proof of point 3. Let $P \in (\alpha, \beta)$. By contradiction, let $\rho_i^* \not\to 0$. Since ρ_i^* is nondecreasing in i (Lemma 2), this implies that there exists $\delta > 0$ such that $\rho_i^* \geq \delta$ for all i, and $\rho_i^* \to \delta$.

First, let $P_\infty < \infty$. The increase in public precision in each period i is bounded away from zero: since $P_i \leq P_\infty$, it holds that $\psi(\rho_i^*, P_i) \geq \psi(\delta, P_i) \geq \psi(\delta, P_\infty) > 0$ (see the formula for ψ in Lemma 1). But then, $P_\infty = P + \sum_i^\infty \psi(\rho_i^*, P_i) \geq P + \sum_i^\infty \psi(\delta, P_\infty) = \infty$.

Second, let $P_\infty = \infty$. From the first-order condition of the minimization problem in (2), there exists some finite P' such that $\rho_i^*(P') = \delta$. But then, the public precision is bounded above, $P_i \leq P'$, to satisfy $\rho_i^* \geq \delta$ for every agent i. This contradicts $P_\infty = \infty$.

Hence, $\rho_i^* \to 0$. By Lemma 3, for any $P_i < \beta =: P''$ (using $\rho_i^*(\beta) = 0$),

$$P_{i+1} = P_i + \psi(\rho_i^*, P_i) \leq P_i + \rho_i^*(P_i) < \beta.$$

Thus, if $P \in (\alpha, \beta)$ then also $P_i \in (\alpha, \beta)$ for every i, and by Lemma 2, $\rho_i^* \in (0, \overline{\rho})$. This also establishes that $P_\infty \leq \beta$ for $P \in (\alpha, \beta)$.

By contradiction, let $P_\infty < \beta$. Then, for all $P' \in (P_\infty, \beta)$, we have $\rho_i^*(P')$=0 (this follows from $\rho_i^* \to 0$ since $P_i \to P_\infty$, and ρ_i^* nonincreasing in P_i.) But this contradicts that $\rho_i^* \in (0,1)$ interior for any $P' \in (\alpha, \beta)$ (Lemma 2). Hence, $P_\infty = \beta$. □

By Lemma 3, if $P \leq \alpha$ then $P_i < \beta$ for all i: learning transitions from the common to the splitting regime. This and Proposition 3 establishes that $P_\infty = \max\{P, \beta\}$. Hence, learning is complete if and only if $\beta = \infty$, which occurs if and only if $f'(0) = 0$ (Lemma 2). □

Proposition 4 (Observable Signals)**.** *For every $i \geq 1$, let the signals $\{s_i^\Theta, s_i^T\}$ be perfectly observable for all future agents $j > i$. Then,*

1. *learning is complete if and only if $f'(0) = 0$, and $P_\infty = \max\{P, \beta\}$,*
2. *for every agent, P_i is weakly higher with observable than unobservable signals.*

Proof. The proof of $P_\infty = \max\{P, \beta\}$ is the same as in Proposition 3, with substituting ρ_i^* for $\psi(\rho_i^*, P_i)$ since there is no inference problem with observable signals.

Let P_i^o be the public precision with observable signals, and P_i^u with unobservable signals. If $P \geq \beta$, then $P_\infty^o = P_\infty^u = P$. Next, let $P < \beta$. By Lemma 3, if $P_i^u \leq P_i^o$ then

$$P_{i+1}^u = P_i^u + \psi(\rho_i^*(P_i^u), P_i^u) \leq P_i^u + \rho_i^*(P_i^u) \leq P_i^o + \rho_i^*(P_i^o) = P_{i+1}^o.$$

If $P_i^u \leq P_i^o$ for any agent i, then the inequality $P_j^u \leq P_j^o$ also holds for all future agents $j > i$. Finally, since $P = P_1^o = P_1^u$, $P_i^o \geq P_i^u$ for all i. □

Proof of Proposition 1. First we show that if $f'(0) \neq 0$, then learning is complete if and only if there exists a subsequence of agents $\{\mu_{i_k}\}_{k \geq 1}$ with $\mu_{i_k} \to 0$ as $k \to \infty$. *Proof of* *sufficiency.* Assume there exists a subsequence $\{\mu_{i_k}\}_{k\geq 1}$ with $\mu_{i_k} \to 0$ as $k \to \infty$, and learning is not complete, $P_\infty < \infty$. Pick any $\hat{\mu} > 0$ such that

$$P_\infty < \frac{1-\hat{\mu}}{\hat{\mu}} \frac{Q + f(0)}{\sqrt{-f'(0)}}.$$

There exists a $\hat{k} \in \mathbb{N}$ such that for all $k \geq \hat{k}$, $\mu_{i_k} \leq \hat{\mu}$. But then, by Lemma 2, an infinite number of agents after $i_{\hat{k}}$ choose $\rho^*(\mu_k, P_k) \geq \rho^*(\hat{\mu}, P_\infty) > 0$, which contradicts $P_\infty < \infty$.

Proof of necessity. Assume that there does not exist a subsequence of μ_i's converging to zero and learning is complete. Then, there exists some $\hat{k} \in \mathbb{N}$ such that for all $j \in \mathbb{N}$ and $j > \hat{k}$, $\mu_j \geq \mu_{\hat{k}} > 0$. Let $P' = \frac{1-\mu_{\hat{k}}}{\mu_{\hat{k}}} \frac{Q+f(0)}{\sqrt{-f'(0)}}$ which is finite since $f'(0) \neq 0$. By Lemma 2, no agent $j \geq \hat{k}$ learns about the common component if $P_j > P'$. This contradicts $P_\infty = \infty$.

Now let $f'(0) = 0$. We show that if $\mu_i \not\to 1$ as $i \to \infty$, learning is complete. Let $\mu_i \not\to 1$. Then, there exists $\hat{\mu}$ such that for an infinite sequence of agents, $\mu_i \leq \hat{\mu} < 1$. Since $f'(0) = 0$, by Theorem 1, learning would be complete if an infinite sequence of agents had $\mu_i = \hat{\mu}$. Since ρ_i^* is nonincreasing in μ_i, learning is also complete if these agents learn even more about the common component.

Finally, we show that if $\mu_i \to 1$ as $i \to \infty$, learning can be complete or incomplete. We prove this by providing two examples for complete and incomplete learning. Note that $\rho_i^* \to 0$ in both cases. If learning is incomplete, we cannot have $\rho_i^* \not\to 0$ (otherwise, this contradicts learning being incomplete). If learning is complete, the marginal gain from any $\rho_i^* > 0$ goes to zero as $P_i \to P_\infty$.

For both examples, let $f(\rho_i) = 1 - \rho_i^5$. If $\mu_i = 1$ for every $i \geq 1$ (and thus, $\mu_i \to 1$ as $i \to \infty$) then learning is incomplete. Next, we provide an example of a sequence of μ_i's converging to 1 such that learning is complete. For this, we prove the following claim.

Claim 1. *For any given P_i there exists $\hat{\mu}_i$ such that $\hat{\mu}_i = 1 - \rho_i^*$.*

Proof. Consider the two functions

$$\begin{aligned} &\mu_i : [0,1] \to [0,1], \;\; \mu_i(\rho_i^*) = 1 - \rho_i^* \quad \text{ and} \\ &\rho_i^* : [0,1] \to [0,1], \;\; \rho_i^*(\mu_i) = \rho_i^*(\mu_i, P_i). \end{aligned}$$

We have to show that there exists a tuple $(\hat{\mu}_i, \hat{\rho}_i)$ such that $\hat{\rho}_i = \rho_i^*(\hat{\mu}_i)$ and $\hat{\mu}_i = \mu_i(\hat{\rho}_i)$. Equivalently, it must hold that $\hat{\mu}_i = \rho_i^*(\mu_i(\hat{\rho}_i))$ and $\hat{\mu}_i = \mu_i(\rho_i^*(\hat{\mu}_i))$. The functions $\rho_i^* \circ \mu_i$ and $\mu_i \circ \rho_i^*$ are continuous functions from a closed interval into itself. Thus, the existence of $\hat{\mu}_i$ and $\hat{\rho}_i$ is ensured by Brouwer's fixed point theorem. □

Let the sequence of μ_i's be as in the claim. Since $\rho_i^* \to 0$, it holds that $\mu_i \to 1$. By Lemma 2, for an infinite sequence of agents who are all in the splitting regime (as $f'(0) = 0$, there is no idiosyncratic regime), the optimal learning choice ρ_i^* satisfies

$$\rho_i^* = \frac{1-\mu_i}{\mu_i} \frac{Q + 1 - {\rho_i^*}^5}{\sqrt{5}{\rho_i^*}^2} - P_i = \frac{\rho_i^*}{1-\rho_i^*} \frac{Q + 1 - {\rho_i^*}^5}{\sqrt{5}{\rho_i^*}^2} - P_i = \frac{1}{1-\rho_i^*} \frac{Q + 1 - {\rho_i^*}^5}{\sqrt{5}\rho_i^*} - P_i. \tag{5}$$

By contradiction, let learning be incomplete, $P_\infty < \infty$. Then, $P_i \leq P_\infty < \infty$, and $\lim_{\rho_i^* \to 0} \frac{1}{1-\rho_i^*} \frac{Q+1-{\rho_i^*}^5}{\sqrt{5}\rho_i^*} \to \infty$. But this means that Equation 5 cannot be satisfied for ρ_i^* sufficiently small, yielding a contradiction. □

Proof of Proposition 2. Define a linear precision function for the given boundary values, $f^L(\rho) \coloneqq \overline{\tau} - \frac{\overline{\tau}}{\overline{\rho}}\rho$. Let $\rho^L(\mu, P)$ be the optimal learning choice of an agent with weight μ and public precision P with precision function $f^L(.)$, and $\alpha^L(\mu)$ and $\beta^L(\mu)$ be the common and idiosyncratic thresholds, respectively. The following auxiliary lemma will prove useful for the remainder of the proof:

Lemma 4. *Let $\mu_{j+1} > \mu_j$ and $\beta^L(\mu_{j+1}) > P_j > \alpha^L(\mu_{j+1})$. Then,*

$$\rho^L(\mu_{j+1}, P_j) - \rho^L(\mu_{j+1}, P_j + \rho^L(\mu_{j+1}, P_j)) > \rho^L(\mu_j, P_j) - \rho^L(\mu_j, P_j + \rho^L(\mu_{j+1}, P_j)). \quad (6)$$

Proof. First, note that the left-hand side of (6) is strictly positive because $P_j \in (\alpha(\mu_{j+1}), \beta(\mu_{j+1}))$. Also, note that $P_j + \rho^L(\mu_{j+1}, P_j + \rho^L(\mu_{j+1}, P_j)) < \beta^L(\mu_{j+1})$, since an agent never learns so much to jump from the splitting to the idiosyncratic regime.

If agent j is in the common regime at both precisions P_j and $P_j + \rho^L(\mu_{j+1}, P_j)$ (i.e., $\alpha^L(\mu_j) \geq P_j + \rho^L(\mu_{j+1}, P_j)$), then the right-hand side of the inequality is zero and (6) holds.

If agent j is in the splitting regime at both precisions P_j and $P_j + \rho^L(\mu_{j+1}, P_j)$ (i.e., $\alpha^L(\mu_j) < P_j$), then the strict inequality is also satisfied: plugging in $f(\rho) = \overline{\tau} - \frac{\overline{\tau}}{\overline{\rho}}\rho$ into the first-order condition (4) and analyzing the sign of the crossderivative $\frac{\partial^2 \rho^L(\mu,P)}{\partial\mu\partial P} < 0$ establishes the strict inequality in (6).

Finally, let agent j be in the common regime at precision P_j, and in the splitting regime at $P_j + \rho^L(\mu_{j+1}, P_j)$. Let $\tilde{\rho}(\mu_j, P_j)$ be the interior solution of the first-order condition as in 4. Then, note that $\rho^L(\mu_j, P_j) = \max\{\overline{\rho}, \tilde{\rho}(\mu_j, P_j)\} \leq \tilde{\rho}(\mu_j, P_j)$. Using $\frac{\partial^2 \tilde{\rho}^L(\mu,P)}{\partial\mu\partial P} < 0$ again shows that (6) holds. As $\beta^L(\mu_j) > \beta^L(\mu_{j+1}) \geq P_j$, this covers all possible cases, and hence, establishes the lemma. □

To prove the proposition, it is sufficient to prove that the public precision P_{j+2} after both agent j and $j+1$ move is higher if agent $j+1$ moves first. From this, statements (1) and (2) in the proposition follow using Lemma 3 since each agent $i \geq j+2$ faces a higher public precision.

For the given $\overline{\rho}$ and $\overline{\tau}$, consider any sequence of precision functions $\{f_{\epsilon_k}\}_{k\geq 1}$ that has $\epsilon_k \to 0$ and satisfies the following for every k: Assumption 1, $f_{\epsilon_k}(0) = \overline{\tau}$, $f_{\epsilon_k}(\overline{\rho}) = 0$ and $f'_{\epsilon_k}(0) - f'_{\epsilon_k}(\overline{\rho}) < \epsilon_k$. Note that $\lim_{k\to\infty} f_{\epsilon_k} = f^L$ pointwise, where $f^L(.)$ is the linear precision function defined above. Then, the optimal learning choice also converges to the linear solution, $\lim_{k\to\infty} \rho_{\epsilon_k}(\mu_i, P_i) = \rho^L(\mu_i, P_i)$. Similarly, the idiosyncratic and common thresholds also converge to the linear solution, $\alpha_{\epsilon_k}(\mu) \to \alpha^L(\mu)$ and $\beta_{\epsilon_k}(\mu) \to \beta^L(\mu)$.

If $P_j \geq \beta_{\epsilon_k}(\mu_{j+1})$, then agent $j+1$ is in the idiosyncratic regime irrespective of moving before or after agent j, and the proposition holds trivially. If $P_j \leq \alpha_{\epsilon_k}(\mu_{j+1})$, then both agent j and agent $j+1$ are in the common regime if moving first, and public precision after the first mover is $P_j + \overline{\rho}$. Then, public precision P_{j+2} is higher when agent j moves second because $\rho(\mu_j, P_j + \overline{\rho}) \geq \rho(\mu_{j+1}, P_j + \overline{\rho})$.

Finally, consider the remaining case for P_j such that $\alpha(\mu_{j+1}) < P_j < \beta(\mu_{j+1})$. We want to show that for k sufficiently high, public precision P_{j+2} after both agents j and $j+1$ move is higher when agent $j+1$ moves before agent j:

$$\underbrace{P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j) + \rho_{\epsilon_k}(\mu_j, P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j))}_{j+1 \text{ moves first}} \geq \underbrace{P_j + \rho_{\epsilon_k}(\mu_j, P_j) + \rho_{\epsilon_k}(\mu_{j+1}, P_j + \rho_{\epsilon_k}(\mu_j, P_j)).}_{j \text{ moves first}}$$

Note that $\rho_{\epsilon_k}(\mu_{j+1}, P_j + \rho_{\epsilon_k}(\mu_j, P_j)) \leq \rho_{\epsilon_k}(\mu_{j+1}, P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j))$ because ρ_{ϵ_k} is nonincreasing in μ and nonincreasing in P_j. Therefore,

$$\begin{aligned}
&\lim_{k\to\infty} \rho_{\epsilon_k}(\mu_{j+1}, P_j) - \rho_{\epsilon_k}(\mu_{j+1}, P_j + \rho_{\epsilon_k}(\mu_j, P_j)) - \rho_{\epsilon_k}(\mu_j, P_j) + \rho_{\epsilon_k}(\mu_j, P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j)) \\
\geq &\lim_{k\to\infty} \rho_{\epsilon_k}(\mu_{j+1}, P_j) - \rho_{\epsilon_k}(\mu_{j+1}, P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j)) - \rho_{\epsilon_k}(\mu_j, P_j) + \rho_{\epsilon_k}(\mu_j, P_j + \rho_{\epsilon_k}(\mu_{j+1}, P_j)) \\
= &\rho^L(\mu_{j+1}, P_j) - \rho^L(\mu_{j+1}, P_j + \rho^L(\mu_{j+1}, P_j)) - \rho^L(\mu_j, P_j) + \rho^L(\mu_j, P_j + \rho^L(\mu_{j+1}, P_j)) > 0,
\end{aligned}$$

where the last strict inequality follows via Lemma 4. Hence, for k sufficiently high, P_{j+2} is weakly higher if agent $j+1$ moves first, establishing the result. □

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