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Endogenous Lemon Markets: Risky Choices and Adverse Selection

Avi Lichtig [1]
Ran Weksler [2]

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[1] University of Bonn, Email: alichtig@uni-bonn.de
[2] University of Haifa, Email: rweksler@econ.haifa.ac.il

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Avi Lichtig
University of Bonn

Ran Weksler
University of Haifa

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Abstract

The severity of adverse selection depends, to a great extent, on the underlying distribution of the asset. This distribution is commonly modeled as exogenous; however, in many real-world applications, it is determined endogenously. A natural question in this context is whether one can predict the severity of the adverse selection problem in such environments. In this paper, we study a bilateral trade model in which the distribution of the asset is affected by pre-trade unobservable actions of the seller. Analyzing general trade mechanisms, we show that the seller's actions are characterized by a risk-seeking disposition. In addition, we show that (location-independent) riskier underlying distributions of the asset induce lower social welfare. That is, "lemon markets" arise endogenously in these environments. (JEL: C72, D83, L15)

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E-mail: alichtig@uni-bonn.de (Lichtig); rweksler@econ.haifa.ac.il (Weksler)

1. Introduction

Akerlof's (1970) seminal paper has taught us that markets might fail due to asymmetric information. However, his model does not suggest that conditions particularly harmful to trade will predominate in markets. Akerlof (1970) assumes that the distribution of the asset is exogenous, and both informational conditions that lead to full trade, as well as those that lead to market unraveling, are consistent with his model. Yet, despite its lack of prediction power, adverse selection is commonly used as an explanation for various observed market malfunctions, e.g., the liquidity shocks in the financial markets during the financial crisis of 2007–2008.[1]

One divergence of some financial markets from Akerlof's (1970) model, as well as of other markets, is the endogeneity of the asset distribution. For example, investment banks issue loans and later sell them. Another example is an entrepreneur who makes private decisions (regarding hiring strategy, project choice, etc.) that affect the value of a traded startup. In this paper, we show that "lemon markets" are to be expected in such environments.

We study an adverse selection model in which, prior to the trade, the seller chooses the asset distribution without being observed. Next, the asset value is realized according to the relevant distribution and revealed to the seller privately. Finally, the seller decides whether to utilize the asset or sell it. To

1. See, for example, Philippon and Skreta (2012) and Tirole (2012).

keep the analysis tractable, our baseline model imposes the following three assumptions, which we will later relax. (1) The selling price is deterministic and is set by a competitive market. (2) The gains from trade are fixed; i.e., the utility of the buyers from consumption is equal to that of the seller plus a constant denoted by Δ. (3) The buyers are completely uninformed regarding the realized value of the asset.

The central takeaway of this paper is that the option value of the seller's equilibrium payoff leads her to choose risky distributions and that riskier distributions imply lower trade and social welfare in equilibrium. To capture the intuition behind this result, we present the following simple example that deals with alternatives that have the same expected value. Then, we discuss the case where the seller chooses between alternatives that have different expectations.

Example 1. Assume that the seller chooses between two alternatives, X_1 and X_2:

$$X_1 = \begin{cases} 1 & \text{w.p. } 1, \end{cases} \quad X_2 = \begin{cases} 0 & \text{w.p. } \frac{1}{2} \\ 2 & \text{w.p. } \frac{1}{2}. \end{cases}$$

Further assume that the seller's utility from consuming a good of type x is x, while the buyers' utility is $x + 1/2$. As in Akerlof (1970), our baseline model assumes that the seller is on the short side of the market and, in equilibrium, the market price is the expected utility of the buyers from consumption of the sold good.

The efficient choice of the seller – the alternative that maximizes social welfare – is X_1, which results in a competitive equilibrium with full trade. However, in the unique equilibrium of the extended trade game, the seller chooses X_2, and only "lemons" are traded. The seller pursues risky alternatives because every strategy she plays in the initial stage implies a competitive price $p \in [1/2, 3/2]$. That is, whatever their beliefs about the seller's choice are, the buyers are willing to pay at least 1/2 for the asset and, since a realization with a value of 2 is not traded, the price cannot exceed $1 + 1/2$. Therefore, in any candidate equilibrium that assigns positive probability to X_1, the seller's payoff when choosing X_1, $\max\{1, p\}$, is strictly below her expected payoff when choosing X_2, $(1/2) \cdot p + (1/2) \cdot 2$.

Example 1 demonstrates two general phenomena. First, the seller's equilibrium behavior is characterized by a risk-seeking disposition. The seller tends to choose risky alternatives due to the option value of her equilibrium payoff. We show that, given any two distributions with the same expectation, where one distribution is riskier than the other, there is no equilibrium in which the seller chooses the dominant distribution with positive probability. Since the seller's equilibrium payoff is convex in her type – i.e., her payoff is at least the market price, and she consumes realizations above this price – she chooses risky alternatives.

Second, as we can see in Example 1, the seller's equilibrium choice is the alternative that induces minimal trade and social welfare. We show that for every two distributions with the same expectation, one induces lower trade and social welfare than the other *for every* Δ (the gains from trade) if and only if it is location-independent riskier, as defined by Jewitt (1989).[2]

The above discussion is concerned with distributions that have the same expectation, though the intuition is not limited to this case. We study a problem with two distributions, ranked according to location-independent risk, where the dominant distribution also has a strictly higher expectation. In our model, the higher Δ is, the more likely the seller is to trade and, therefore, the more "tempted" she is to choose the risky distribution. We show that, for low values of Δ, the seller chooses the dominant distribution in the unique equilibrium of the game; for intermediate values of Δ, she mixes between the two distributions; and for high values of Δ, she chooses the dominated one. Notably, as Δ increases in the intermediate region, the seller's equilibrium strategy puts more and more weight on the risky distribution, such that the equilibrium price is constant. As a result, the equilibrium social welfare is also constant in this region; i.e., the seller's increasing "temptation" nullifies the potential increase (due to the rising gains from trade) in social welfare.

The driving force behind our results is the *unobservability* assumption. Because the potential buyers do not observe the seller's choices, she pursues

2. F_2 is said to be location-independent riskier than F_1, if, for every *quantile*, the area below the CDF of F_2, up to that quantile, is greater than the area below the CDF of F_1. See Section 3.1 for an extended discussion of this concept.

risky alternatives. Otherwise, if those choices were to affect the equilibrium price and trade (or, more generally, the trade mechanism), there would be a balancing force that would (at least partially) dissuade the seller from such decisions. Under the above assumption, we expect "lemon" conditions to materialize more frequently when the seller has more room to make decisions privately. Consider, for example, a startup vs. a public company, where the latter is more strictly obligated to disclose its actions than the former. Our results suggest that an entrepreneur, who invests in her startup, takes more risks and is more likely to bring about low trade and social welfare. The same can be said about traditional vs. innovative technologies. Even when the buyers observe the seller's decisions, if those are hard to interpret, e.g., if she is dealing with novel investment structures, the seller is more likely to pursue risky alternatives and thereby engender a "lemon market."

In the second part of the paper, we relax the three structural assumptions mentioned above, one at a time, and show that our results continue to hold. We begin with the analysis of a general trade mechanism in this environment. First, the mechanism, whose objective is to maximize the expected social welfare, recommends to the seller which distribution to choose. Subsequently, the seller makes her choice privately, observes the realization, and reports it to the mechanism. Finally, the mechanism determines the trade probability and a transfer from the buyer to the seller.[3] We require that the mechanism satisfy an obedience constraint, i.e., that the seller be willing to follow the mechanism's recommendation in the first stage. In addition, the mechanism needs to allow for the rational participation of both sides. That is, the informed seller prefers participation in the mechanism to self-consumption, and the buyer's expected utility is nonnegative. Lastly, the mechanism has to be incentive compatible. That is, the seller cannot profit from a non-truthful report of her type.

Going back to Example 1, note that no mechanism that induces trade with positive probability can recommend the choice of X_1. For each price that the buyer would agree to pay, assuming X_1 is the seller's choice, the seller would be better off deviating and choosing X_2. In general, for every type of seller, a mechanism defines a probability of trade and a transfer. That is, in any incentive-compatible mechanism the seller's utility is a maximum of linear functions and, as such, the seller's utility is a convex function of her type. Therefore, if the mechanism satisfies the obedience constraint, it cannot recommend a dominant distribution with positive probability.

We study the effect of the seller's risk-seeking disposition on social welfare in the optimal trade mechanism. In Example 1, the optimal trade mechanism for the risky alternative (X_2) performs strictly better than Akerlof's (1970)

3. Guerrieri and Shimer (2018) consider a multi-dimensional model, in which stochastic trade can be part of an *equilibrium*. Their equilibrium is a special case of our mechanism, where they also require an interim IR constraint for the partially informed buyers.

solution. In the optimal trade mechanism, the low type ($x = 0$) sells the asset for a price of 1/2 with probability 1, and the high type ($x = 2$) sells the asset for a price of 2 with probability 1/2. That is, the low type is indifferent between the two options on the menu (and prefers both to self-consumption), and the high type is indifferent between self-consumption and selling with probability 1/2 (and prefers both to selling with probability 1). However, even though the optimal mechanism induces higher trade and social welfare than the market solution, it cannot guarantee trade with probability 1 (as could have been achieved if the seller had chosen X_1).

The effect of the seller's risk-seeking disposition, observed in Example 1, is also a part of a general phenomenon. We show that the solution of the optimal mechanism is characterized by the maximum of an objective function subject to a "budget" constraint, which has two main implications. First, we show that under the assumptions considered in our baseline model – log-concavity of the asset distribution – the solution to the optimal mechanism is simply Akerlof's (1970) solution; i.e., more risk implies lower social welfare also in the optimal mechanism. Second, we show that the "budget" constraint is less restrictive when the distribution is less risky in the sense of location-independent risk. That is, given the optimal trade mechanism for some distribution, the designer can construct another incentive-compatible mechanism that, in any less risky distribution, achieves the same trade and social welfare. Therefore, we conclude that the optimal mechanism for the less risky distribution induces more trade and higher social welfare.

In the baseline model, we have assumed that the buyers' utility is $x + \Delta$, i.e., the realized value of the asset plus a constant gain from trade Δ. However, we show that our results hold for any concave valuation function of the buyers, for which the gains from trade are non-decreasing. The seller's risk-seeking disposition is not affected in any way by the buyers' valuations, as long as they are increasing. The seller's equilibrium utility is convex and therefore, she pursues risky alternatives. Regarding the implication of the seller's equilibrium behavior on social welfare, we show that for every two distributions with the same expectation, one induces lower trade and social welfare than the other for *any* such valuation, if and only if it is location-independent riskier.

Up until now, we have assumed that the buyers do not obtain any information after the seller's choice. However, in some environments, the buyers observe public signals before trade; e.g., a car buyer might deduce some information about the state of the asset by viewing it. To tackle this issue, we analyze a normal-distribution model in which, for every realization of the asset x, a public signal $x + \varepsilon$ is observed, where ε is a normally distributed noise. We show that, no matter what the variance of ε is, the results of our baseline model carry through. In this model, for some realizations of ε, a risky distribution results in lower profits for the seller. However, this does not dissuade the seller from pursuing it. Coupling every ε with $(-\varepsilon)$, we show that, when the market believes that the seller chooses some distribution, her expected profits increase if she deviates to a risky distribution. Therefore, the seller pursues the risky distribution, thereby causing trade to decrease.

Related Literature. Our main contribution is to the vast literature that analyzes the interaction between two of the most fundamental problems in (information) economics: agents with hidden information, i.e., adverse selection (Akerlof, 1970), and agents who take hidden actions, i.e., moral hazard (Arrow, 1963; Stiglitz, 1974). This literature establishes, in different setups, that adding moral hazard to the environment does not entail welfare losses compared to the case of pure adverse selection; see Guesnerie et al. (1989) for a survey. This prediction somewhat contrasts with our analysis that shows that adding hidden actions, in the manner we do, to Akerlof's (1970) model can dramatically worsen market performance.

A recent branch of this literature, starting with Gul (2001), considers costly unobserved pre-trade investment by the seller/buyer. Our paper is close to this literature in its economic motivation but diverges substantially in modeling and analysis. This literature assumes that the unobserved investment affects the values of both sides of the market in a deterministic way. Therefore, the adverse selection problem arises due to the nature of the equilibrium. In equilibrium, the side able to invest mixes between different levels of investment; thereby, introducing uncertainty and adverse selection into the market; for more recent contributions, see Hermalin and Katz (2009), Hermalin (2013), and Dilmé

(2019). Our setup is therefore fundamentally different in two ways. First, technically, the unobserved action in our model is costless, and its mapping to the value is stochastic. Second, since our model assumes a non-deterministic relation between the action and the value, we can characterize the relation between the riskiness of the seller's choices and the severity of the adverse selection problem.

Conceptually, the paper most closely related to our risk-seeking characterization, though it appears in a different branch of the information economics literature, is Ben-Porath et al. (2018). They study a generalized Dye (1985) disclosure game in which the asset is affected by the agent's preliminary unobserved choices, and they show that the agent is inclined to make risky choices. In the equilibrium of both, Akerlof's (1970) and Dye's (1985) models, the informed agent has an action that guarantees at least some utility, and she can choose a higher payoff when the realization is high. As a result, in both models, the endogenization of the asset distribution implies risk-seeking by the informed agent.[4] Our main contribution is concerned with the implications of the informed agent's equilibrium behavior on social welfare, a property absent from the discussion in the Dye (1985) model. In verifiable disclosure games, as long as the sender chooses between distributions with the same expectation,

4. For further discussion see Chen (2015), who studies risky behavior in Holmstrom's (1979) model, and Barron et al. (2017), who study risk-seeking arising from limited liability in a principal–agent context.

the social welfare in equilibrium is the same. However, in our model, the seller's tendency to choose risky distributions harms trade and social welfare even if the expectations of the distributions are equal. In addition, we also introduce novel characterizations of the informed player's tendency to pursue risk. First, as mentioned above, we discuss a choice between alternatives with different expectations and characterize the seller's behavior as a function of the gains from trade. Second, we extend the analysis to environments with public information, and we show that, in normal distributions, no matter how informative the public signal is, the seller pursues risky alternatives. Finally, in our paper, risk-seeking is not limited to equilibrium models. We show that, even in a mechanism design environment, the seller's payoff is necessarily convex, albeit in a more complicated way than in equilibrium and, therefore, the seller chooses risky alternatives.

Our novel characterization of the relation between risk and trade somewhat relates to the study of the role of the information structure in adverse selection environments. A riskier distribution of the asset value is, under some conditions, analogous to a better-informed seller. Levin (2001) and Kessler (2001) study welfare properties derived from different information structures and show that the relation between additional private information and trade is ambiguous. Our approach, however, points to a stark negative relation between risk and trade/social welfare. DeMarzo et al. (2019) study a Dye (1985) model in which the agent chooses between tests (information structures), and they show that, in equilibrium, the agent chooses the test that minimizes the no-disclosure price.[5] In addition, the literature that studies the role of the information structure in adverse selection environments contains other significant contributions that are less closely connected to our analysis. Kessler (1998) and Ravid et al. (2022) study information purchase by the buyer, and Athey and Levin (2018) study the value of information in general decision problems. Migrow and Severinov (2022) study the interaction between an investment decision and information purchase, and they show that, in equilibrium, the seller allocates too many resources to information purchase.

The rest of the paper is organized as follows. In Section 2 we introduce the baseline model and provide a preliminary analysis. In Section 3 we analyze the seller's equilibrium behavior and its implications on trade and social welfare. In Section 4 we extend our analysis to a mechanism design environment. In Section 5 we discuss general functional forms of the players' utilities. In Section 6 we consider the case in which the buyers observe an informative signal. Section 7 concludes.

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5. As was shown by Jung and Kwon (1988), the no-disclosure price is decreasing in the informativeness of the test.

2. Model and Preliminary Analysis

We begin our discussion with the presentation of the baseline model. For the sake of tractability, we make several simplifying assumptions. In Sections 4–6, we generalize the results.

The baseline model is a two-stage incomplete information game between a seller (she), Nature, and a competitive market of buyers. In the first stage of the game the seller chooses an alternative $X_i \in \{X_1, X_2, \dots X_n\}$ (e.g., a project), where $X_i \sim F_i$, $\mathcal{F} = \{F_1, \dots, F_n\}$. Following the *unobservable* choice of the seller, the asset value is realized according to the appropriate distribution and privately observed by the seller. Finally, the seller decides whether to utilize the asset or sell it to the market.[6]

We normalize the seller's utility from consumption to

$$v_s(x) = x,$$

and we assume that the buyers' utility from consumption is

$$v_b(x) = x + \Delta.$$

6. For a mechanism design approach, where a mechanism recommends an a alternative and then determines a stochastic trade and transfers, see Section 4.

That is, the buyers' utility is linear in the quality of the good, and the gains from trade are constant.[7]

Remark 1 (*Alternative Interpretation*). Another way to interpret the seller's utility from consumption is to view it as a production cost. That is, the seller chooses an alternative that determines the value of the object to the buyers, y, while the seller's production cost is $y - \Delta$. Since the conditions that define the equilibrium are analogous to the ones in our model, the analysis does not change.[8]

2.1. The Trade Stage

Next, we analyze the equilibrium of the F trade game, i.e., an Akerlof (1970) game in which the asset's distribution is F. To simplify the presentation of our results, we confine our analysis to continuous distributions over $\mathbb{R}_+$, in which there exists a unique interior equilibrium in the F trade game.

Assumption 1. The asset distribution F is continuous with a PDF f, where f is log-concave, and $f(x) > 0$ if and only if $x > 0$.

7. The model assumes that the gains from trade are deterministic, but the analysis extends to the stochastic case. As long as the gains from trade and the realization of the asset are not correlated, Δ can be interpreted as the expected gains from trade. The case in which they are correlated falls into the framework discussed in Section 5, where the general v_b can be interpreted as the expected value of the buyers from consumption.

8. We thank the editor for suggesting this interpretation of the model.

The log-concavity of f implies a unique solution to the equation[9]

$$\mathbb{E}_F\left[v_b\left(x\right)|x \leq p\right] = p. \tag{1}$$

That is, there exists a unique price, denoted by $\hat{p}_F$, that clears the market. Types below $\hat{p}_F$ sell the asset for a price $\hat{p}_F$, types above $\hat{p}_F$ consume the asset, and the price is the expected value of the buyers conditional on trade; i.e., the buyers are indifferent to purchasing and supply equals demand.[10]

Our assumption that F is unbounded from above limits the discussion to the adverse selection case; i.e., the price is in the interior, and some types of the seller consume. Note that we also assume that x is positive. In other words, x is a good, and consumption is always beneficial.[11]

In Section 4, we show that Assumption 1 also implies that the optimal trade mechanism is the Akerlof (1970) solution. In this sense, our focus on this solution is without loss of generality. When we study general trade

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9. The function $\beta_F\left(p\right) := \mathbb{E}_F\left[v_b\left(x\right)|x \leq p\right] - p$ is a decreasing function, with $\beta_F\left(0\right) > 0$ and $\lim_{p\to\infty}\beta_F\left(p\right) = -\infty$. That is, there exists a unique p such that $\beta_F\left(p\right) = 0$.

10. This is a reduced-form presentation of the play in the trade stage. For different structural assumptions on the trade game form, see Wilson (1980) and Mas-Colell et al. (1995).

11. We characterize the relation between risky choices of the seller and social welfare. In this context, it is worth noting that if x could be smaller than 0 – i.e., it could generate negative consumption utility – and we had assumed free disposal, risky distributions would not necessarily be socially undesirable.

mechanisms that allow for random trade and allow the transfer to depend on the seller's type, we lift the log-concavity assumption and characterize the optimal mechanism for any F.

In the following sections we compare the efficiency of different distributions in equilibrium, and we use the following two expressions:

$$PT(F) := F(\hat{p}_F), \tag{2}$$

which denotes the probability of trade in the unique equilibrium of the F trade game, and

$$SW(F) := \int_0^{\hat{p}_F} v_b(x)\, dF(x) + \int_{\hat{p}_F}^{\infty} v_s(x)\, dF(x), \tag{3}$$

which denotes social welfare in this equilibrium.

2.2. The Extended Trade Game

A strategy for the seller in the initial stage is $\alpha \in \Delta^n$, where α_i denotes the probability that the strategy assigns to distribution F_i.

Let $R_i(p) := \mathbb{E}_{F_i}[\max\{x, p\}]$ denote the seller's expected utility when choosing F_i given a market price p. A perfect Bayesian equilibrium (PBE) of the extended trade game is defined by a strategy $\hat{\alpha}$, a price $\hat{p}$, and a market belief $Q \in \Delta(\mathcal{F})$, that together satisfy the following conditions: the seller chooses optimally, i.e.,

$$\hat{\alpha}_i > 0 \Longrightarrow R_i(\hat{p}) \geq R_j(\hat{p}) \, \forall j \in \{1, 2, \dots n\}; \tag{4}$$

the market clears, i.e.,

$$\mathbb{E}_Q [v_b(x) | x \leq \hat{p}] = \hat{p}; \tag{5}$$

and the market belief is correct,[12] i.e.,

$$Q(x) = \Sigma_i \alpha_i F_i(x). \tag{6}$$

The extended trade game might admit multiple equilibria, especially so because we place minimal restrictions on the set $\mathcal{F}$. However, our characterization of the seller's choice is concerned with all equilibria of the game. In addition, we show that the extended trade game admits a simple equilibrium in which the seller's strategy assigns positive probability to, at most, two distributions.

Proposition 1. *There exists an equilibrium in the extended trade game that is either pure or involves mixing between exactly two distributions.*

12. Since a deviation of the seller is unobservable, off-path beliefs do not play a role in equilibria selection.

The proof of Proposition 1 is standard, and, as with all other proofs, is deferred to the Appendix.

3. Risky Choices and Trade

We now proceed to the analysis of the seller's choice in the extended trade game. The equilibrium of the stage game is characterized by an option value. The seller is paid at least $\hat{p}$, and she can obtain a higher payoff if the realized value is above $\hat{p}$. We show that this aspect of the seller's equilibrium payoff implies risk-seeking and low trade. First, we discuss the notion of risk we use in our analysis.

3.1. Location-independent Risk

In many environments, the assumption that second-order stochastic dominance implies monotone comparative statics, although natural, is incorrect. For example, one risk-averse agent may be willing to pay a certain amount of money for partial insurance, whereas a more risk-averse agent, as defined by Pratt (1964) and Arrow (1971), may not be willing to do so. Another example is in Rothschild and Stiglitz (1971), where an agent, who invests in two assets, one risky and one riskless, does not necessarily decrease the share of the risky asset in his portfolio when it becomes riskier in the sense of second-order stochastic dominance. See Chateauneuf et al. (2004) for more such examples.

The notion of risk we use, defined by Jewitt (1989), was shown to be not only sufficient to guarantee intuitive monotone comparative statics but also necessary. First, let us present a (strict) definition of this notion.

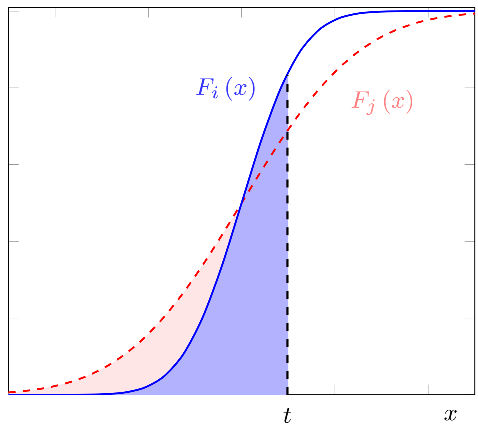
Definition 1 (*Strict Location-independent Risk*). We say that distribution F_i is strictly[13] location-independent less risky than F_j, denoted by $F_i \succ_{LIR} F_j$, if

$$\int_0^{F_i^{-1}(q)} F_i(x)\,dx < \int_0^{F_j^{-1}(q)} F_j(x)\,dx, \tag{7}$$

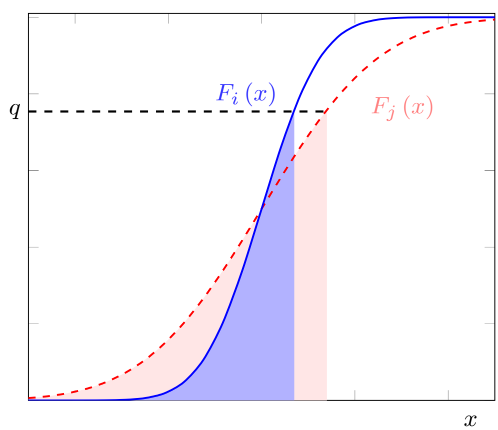
for every $q \in (0,1)$.

Location-independent risk is somewhat similar to the well-known definition of second-order stochastic dominance. The difference is that we compare the areas below the CDF with regard to quantiles instead of realizations (see Figure 1.) This condition holds, for example, in normal distributions; i.e., a larger variance of a normal distribution implies more location-independent risk. As can be seen from Figure 1, second-order stochastic dominance and location-independent risk do not imply each other. However, Landsberger and Meilijson (1994) show that if $\mathbb{E}_{F_i}[x] \geq \mathbb{E}_{F_j}[x]$ and $F_i \succeq_{LIR} F_j$ then F_i dominates F_j also in the sense of second-order stochastic dominance.

13. We use a strict version of location-independent risk to obtain stark results. We denote by $\succeq_{LIR}$ the non-strict version of this notion, i.e., with weak inequality.



(a) Second-order Stochastic Dominance: the area below the dominant distribution is smaller than the area below the dominated distribution, *for every* t.



(b) Location-independent Risk: the area below the dominant distribution is smaller than the area below the dominated distribution, *for every* q.

FIGURE 1. Second-order Stochastic Dominance and Location-independent Risk

Jewitt (1989) shows that *any* more risk-averse agent would agree to pay higher premium in order to face F_i and not F_j, if and only if $F_i \succeq_{LIR} F_j$. Vergnaud (1997) shows that $\succeq_{LIR}$ is also the weakest notion that guarantees the optimality of Arrow's (1971) deductible contracts. Chateauneuf et al. (2004) show that $\succeq_{LIR}$ is a necessary and sufficient condition under which a more informative distribution of a signal implies a longer search. We show that $\succ_{LIR}$ is a necessary and sufficient condition for a monotone relation between risk and social welfare in a bilateral trade environment.

3.2. Risky Choices

First, we show that, for any two distributions with the same expectation, ranked according to $\succ_{LIR}$, the seller strictly prefers the riskier one.

PROPOSITION 2. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. If $F_i \succ_{LIR} F_j$, then in any equilibrium of the extended trade game $\hat{\alpha}_i = 0$.*

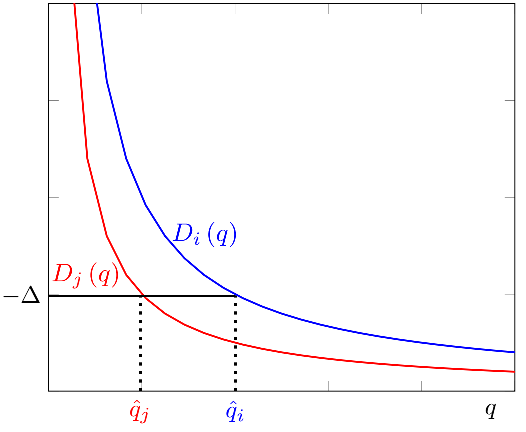
To see why Proposition 2 is true, note that the seller's utility in the equilibrium of any F trade game has an option value. The seller consumes the asset if its realized value is above the market price, and she sells it if its realized value is below the price. That is, the seller's equilibrium utility is a convex function of the asset realization. Therefore, given any candidate strategy of the seller in the first stage that assigns positive probability to the choice of F_i, the seller is strictly better off if she deviates and moves that probability mass to F_j. Such deviation is not observable by the buyers; i.e., the equilibrium price stays the same, and, since the seller's utility is convex, her expected profits increase. To prove the claim, we use a strict version of second-order stochastic dominance, implied by $F_i \succ_{LIR} F_j$ (when $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$.) Similar to Ben-Porath et al.'s (2018) proof regarding risky choices in Dye's (1985) model, we apply Rothschild and Stiglitz's (1970) representation of mean-preserving spreads, and show that the seller is strictly better off if, contrary to market beliefs, she deviates to a riskier alternative.

3.3. Increasing Adverse Selection

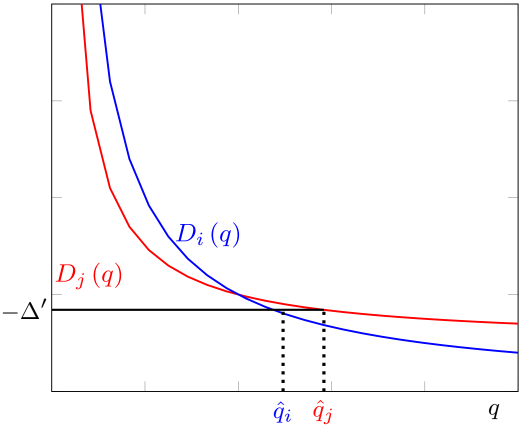
We now turn to analyze the implications of the seller's risk-seeking disposition on trade and social welfare. We start by characterizing the conditions under which a riskier distribution results in less trade and lower social welfare. Then, we discuss distributions with different expectations.

Proposition 3. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. $SW(F_i) > SW(F_j)$ for every $\Delta > 0$ if and only if $F_i \succ_{LIR} F_j$.*

In order to prove Proposition 3 we let $D_F(q)$ denote the following difference: $D_F(q) := \mathbb{E}_F\left[x|x \leq F^{-1}(q)\right] - F^{-1}(q)$. The equilibrium of the F trade game is obtained at a quantile $\hat{q}$ satisfying $D_F(\hat{q}) = -\Delta$. Since F is log-concave we know that $D_F(q)$ is a decreasing function, and we show that D_{F_i} and D_{F_j} do not cross if and only if $F_i \succ_{LIR} F_j$. Thus, if $F_i \succ_{LIR} F_j$ then, for every $\Delta > 0$, we have $\hat{q}_i > \hat{q}_j$; see Figure 2(a). If, however, $F_i \not\succ_{LIR} F_j$ then there exists $\Delta' > 0$ such that $\hat{q}_i \leq \hat{q}_j$; see Figure 2(b). Note that social welfare is equal to $\mathbb{E}_F[x] + PT(F) \cdot \Delta$. Therefore, a monotone relation between the probabilities of trade in the two distributions is equivalent to a monotone relation between the levels of social welfare.



(a) $F_i \succ_{LIR} F_j$



(b) $F_i \nsucc_{LIR} F_j$

FIGURE 2. Location-independent Risk and Social Welfare

Propositions 2 and 3 show that the seller's risk-seeking disposition can have dire implications on market performance. The seller chooses distributions on the "risk frontier" and engenders "lemon markets." Note that even though this result deals with distributions that have the same expectation, the economic intuition extends beyond this case. The seller's incentive to choose risky distributions persists even if they have a lower expectation. Therefore, the seller's equilibrium behavior, in addition to exacerbating adverse selection, may reduce the average quality.

The following lemma facilitates the characterization of the seller's choice and its implications when she faces two distributions with different expectations. Lemma 1 states that $\succ_{LIR}$ also implies a monotone relation between the equilibrium price of the different distributions.

Lemma 1. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] \geq \mathbb{E}_{F_j}[x]$. If $F_i \succ_{LIR} F_j$ then $\hat{p}_i > \hat{p}_j$.*

To see why Lemma 1 is true, consider first the case where $\mathcal{F} = \{F_i, F_j\}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$, and assume to the contrary $\hat{p}_j \geq \hat{p}_i$. By Proposition 2, the seller chooses F_j. Now, if $\hat{p}_j \geq \hat{p}_i$ the seller's expected profit when deviating to F_i, $R_i(\hat{p}_j)$, is greater than $R_i(\hat{p}_i)$. However, this cannot be true since $R_i(\hat{p}_i) = SW(F_i)$, and, by Proposition 3, we have $SW(F_i) > SW(F_j) = R_j(\hat{p}_j)$. That is, if $\hat{p}_j \geq \hat{p}_i$ we have $R_i(\hat{p}_j) > R_j(\hat{p}_j)$, and the seller would profit from deviating and choosing F_i, which is in contradiction to Proposition 2. Finally, if $\mathbb{E}_{F_i}[x]$ is strictly greater than $\mathbb{E}_{F_j}[x]$ and $F_i \succ_{LIR} F_j$, we can add a constant to X_j, $X'_j = X_j + c$, to make both expectations equal. Such an addition does not change the $\succ_{LIR}$ relation, and thus we know that $\hat{p}'_j < \hat{p}_i$. Taking into account the fact that $\hat{p}_j < \hat{p}'_j$ completes the proof of Lemma 1.

Lemma 1 allows us to characterize the seller's choice between two distributions with different expectations as a function of the gains from trade.

Proposition 4. *Let $\mathcal{F} = \{F_1, F_2\}$ where $\mathbb{E}_{F_1}[x] > \mathbb{E}_{F_2}[x]$ and $F_1 \succ_{LIR} F_2$. If there is a single crossing between F_1 and F_2, i.e., there exists $x^\star$ satisfying $F_1(x) \leq F_2(x)\ \forall x \leq x^\star$ and $F_1(x) > F_2(x)\ \forall x > x^\star$, then there exist Δ_1 and Δ_2, $0 \leq \Delta_1 < \Delta_2 < \infty$, that satisfy the following conditions:*

- *The seller's choice: in the unique equilibrium of the extended trade game,*
  - *for every $\Delta \leq \Delta_1$, the seller chooses F_1;*
  - *for every $\Delta \in (\Delta_1, \Delta_2)$, the seller mixes between F_1 and F_2;*
  - *for every $\Delta \geq \Delta_2$, the seller chooses F_2.*
- *Social welfare: let $p(\Delta)$ and $SW(\Delta)$ denote the equilibrium price and social welfare as a function of Δ, respectively. Then:*
  - *for every $\Delta \leq \Delta_1$ and $\Delta \geq \Delta_2$, $p(\Delta)$ and $SW(\Delta)$ are strictly increasing functions;*
  - *for every $\Delta \in (\Delta_1, \Delta_2)$, $p(\Delta)$ and $SW(\Delta)$ are constant.*

To prove Proposition 4, we show that if there is a single crossing of F_1 and F_2 then there is a single crossing of $R_1(p)$ and $R_2(p)$; i.e., there exists $p^\star$ such that for every $p < p^\star$ we have $R_1(p) > R_2(p)$, and for every $p > p^\star$ we have $R_1(p) < R_2(p)$. The intuition is as follows. If the price is low, the seller consumes the product quite often. Therefore, she prefers the distribution with the higher expectation. However, if the price is high, the seller rarely consumes it. Thus, the basic intuition behind Proposition 2 extends, and she prefers the risky distribution.[14] For intermediate values of Δ, there is no pure equilibrium. A choice of F_1 would induce $p > p^\star$ and thus would render the choice of F_1 suboptimal, while a choice of F_2 would induce $p < p^\star$ and would render the choice of F_2 suboptimal. Therefore, in the unique equilibrium, the seller mixes between F_1 and F_2 such that the equilibrium price of the trade game is exactly $p^\star$. Note that $p^\star$ does not depend on Δ; i.e., for every Δ in this region, the equilibrium price is the same. Now, for a fixed price, the seller's profit from choosing, say, F_1, does not depend on Δ. Therefore, for every Δ in this region, the social welfare is also the same. That is, for $\Delta \in (\Delta_1, \Delta_2)$, the higher Δ is, the more weight the seller's equilibrium strategy puts on F_2, such that the decrease in the expected value of the good counterbalance the increase in Δ. The seller's increasing "temptation" to take risks introduces more and more choice inefficiency and completely nullifies the potential increase in social welfare.

By Proposition 4, the seller's risk-seeking disposition incentivizes her not only to choose distributions with higher risk but also to choose distributions with lower expectation. However, the toll on social welfare is limited to the potential gains from trade, Δ. Whatever the equilibrium price is, the seller can guarantee a utility of $\max\limits_{F_i \in \mathcal{F}} \mathbb{E}_{F_i}[x]$ by choosing the distribution with the highest mean and never sell. That is, the social welfare in equilibrium is bounded from

14. Recall that the distributions are assumed to be unbounded from above (Assumption 1).

below by $\max\limits_{F_i \in \mathcal{F}} \mathbb{E}_{F_i}[x]$. In addition, if the seller were to choose optimally, the social welfare would be $\max\limits_{F_i \in \mathcal{F}} SW(F_i) \leq \max\limits_{F_i \in \mathcal{F}} \mathbb{E}_{F_i}[x + \Delta]$. That is, since the source of inefficiency is the option value of the seller, which arises from the trade opportunity, the harm to social welfare cannot exceed Δ. Despite the above upper bound, the losses in welfare can be quite substantial. For $\Delta > \Delta_2$, the losses are strictly larger than the difference between the expectations of F_1 and F_2, which can be arbitrarily large. If $F_1 \succ_{LIR} F_2$ and $\mathbb{E}_{F_1}[x] > \mathbb{E}_{F_2}[x]$ then $PT(F_1) > PT(F_2)$ for every Δ. That is, social welfare decreases not only because of the direct effect of the choice of the distribution with lower expectation, but also because there is less trade.

In the following sections, we study three extensions of the baseline model. First, we extend our analysis beyond market pricing to general trade mechanisms. Second, we analyze general functional forms of the players' utilities. Finally, we analyze the case where the buyers observe partial information about the asset value.

4. A Mechanism Design Approach

Up until now, we have focused our analysis on market mechanisms based on the classic Akerlof (1970) model, in which an equilibrium condition determines the price. However, our results extend far beyond such a model. We proceed now to study the problem in a mechanism design environment.

The central economic force that generates inefficiencies in Akerlof's (1970) model is the seller's ability to make her decision after she learns her type. Otherwise, if the seller could commit to selling the asset in all states, she would trade efficiently. Therefore, when studying our problem in a mechanism design environment, we preserve this property: after learning the realization of the asset, the seller can quit the mechanism and consume it. Next, we define an extended trade mechanism and characterize the feasible set.

4.1. An Extended Trade Mechanism

An extended (direct) trade mechanism Ψ is defined by a recommendation $Y \in \Delta(\mathcal{F})$, an allocation rule $A : \mathbb{R}_+ \to [0,1]$, and a transfer $T : \mathbb{R}_+ \to \mathbb{R}$. In the first stage of the game, the mechanism recommends to the seller which distribution to choose. The mechanism can recommend a mixture, and we denote by Y_i the probability that the recommendation assigns to the choice of F_i. Then, after the seller chooses a distribution privately and sees the realization x, she reports to the mechanism a number in $\mathbb{R}_+$. Finally, the mechanism determines the allocation A and the transfer T. Note that $A \in [0,1]$; i.e., the mechanism can randomize also in the trade stage.[15]

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15. In this model, the transfer T does not depend on the trade realization. Given a report by the seller, the buyer pays the same amount whether the mechanism has determined that they trade or that they do not. However, when implementing such a mechanism, one

Remark 2 (*Log-concavity*). In Section 2, where we were dealing with Akerlof's (1970) model, we assumed that all distributions in $\mathcal{F}$ are log-concave. The purpose of this assumption was mainly to derive a unique equilibrium of the stage game. In this section, where we consider a mechanism solution, we get uniqueness "for free." That is, when multiple outcomes are possible, the mechanism can choose the preferred one. Therefore, in this section, we no longer require the assumption that every $F \in \mathcal{F}$ is log-concave. We do provide one result for the log-concave case, but we assume only that every $F \in \mathcal{F}$ is unbounded from above, continuous, has a PDF, and its mean is finite.

We assume that the mechanism designer's (the principal's) goal is to maximize social welfare, which, in our model, is equivalent to the maximization of the probability of trade. First, let us define the set of possible mechanisms the principal can choose from. We distinguish between the extended trade mechanism $\Psi = (Y, A(\cdot), T(\cdot))$, and the trade mechanism $\Psi_F = (A_F(\cdot), T_F(\cdot))$. That is, the mechanism Ψ includes the choice stage, whereas the mechanism Ψ_F takes the distribution of the asset as given.

4.1.1. Feasibility.

can condition the payment on trade. Since we study a linear environment and only the expectations are relevant to the discussion, the analysis does not change.

Definition 2 (*F-Feasibility*). A mechanism $\Psi_F = (A(\cdot), T(\cdot))$ is F-feasible if the following conditions hold:

$$\forall x \in \mathbb{R}_+, (1 - A(x))\, x + T(x) \geq x, \tag{IRS}$$

$$\mathbb{E}_F \left[A(x)(x + \Delta) - T(x)\right] \geq 0, \tag{IRB}$$

$$\forall x, x' \in \mathbb{R}_+, (1 - A(x))\, x + T(x) \geq \left(1 - A\left(x'\right)\right) x + T\left(x'\right). \tag{ICS}$$

A trading mechanism Ψ_F is F-feasible if both players agree to participate. That is, the informed seller's expected utility from participation is above her utility from consumption for every x (IRS), and the buyer's expected utility is positive (IRB). In addition, the mechanism incentivizes the seller to report truthfully (ICS).

Definition 3 (*Feasibility*). Let M_i denote the expected utility of the seller when choosing distribution F_i, $M_i := \mathbb{E}_{F_i}\left[(1 - A(x))\, x + T(x)\right]$. A mechanism $\Psi = (Y, A(\cdot), T(\cdot))$ is feasible if it is obedient, i.e.,

$$Y_i > 0 \Longrightarrow M_i \geq M_j \text{ for every } j \in \{1, \ldots, n\}, \tag{OBS}$$

and $\Psi_G = (A(\cdot), T(\cdot))$ is G-feasible, where $G(x) = \sum_{i=1}^{n} Y_i F_i(x)$.

The extended trade mechanism Ψ is feasible if the seller is willing to follow the mechanism's recommendation Y, and the trade mechanism is G-feasible for the induced distribution.

Finally, we adapt our efficiency measurements to this general environment. We denote by

$$PT^{\Psi_F}(F) := \int_0^\infty A_F(x)\, dF(x) \tag{8}$$

the probability of trade in trading mechanism Ψ_F, and by

$$SW^{\Psi_F}(F) := \int_0^\infty \left[A_F(x)(x+\Delta) + (1 - A_F(x))\, x\right] dF(x) \tag{9}$$

its social welfare.

4.2. Risky Choices

First, we show that Proposition 2 extends to the mechanism design environment, and the seller pursues risky alternatives.

PROPOSITION 5. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$, and let $\Psi = (Y, A(\cdot), T(\cdot))$ be a feasible mechanism that induces trade with positive probability. If $F_i \succ_{LIR} F_j$ then $Y_i = 0$.*

In fact, any mechanism that satisfies ICS and OBS cannot assign a positive probability to a dominant distribution. To see why this is true, note that the function $A(x)z - T(x)$ is a linear function of z. That is, if Ψ satisfies ICS, the function $A(x)x - T(x)$ is a maximum of linear functions, and is therefore convex. Since the utility of the seller, when she reports truthfully in the second stage, is a convex function of her type, a mechanism that satisfies OBS cannot recommend to the seller the choice of F_i. Note that the proof of this result is somewhat more involved since we need to characterize A in order to show that the seller *strictly* prefers the dominated distribution.

By Proposition 5, the seller chooses distributions on the "risk frontier." In what follows, we discuss the implication of this property. That is, we characterize the optimal trade mechanism and analyze the effect of the seller's choice on trade and social welfare. To do so, we first show that if F is log-concave then the market solution, i.e., Akerlof's (1970) solution, is the optimal solution, and thus Proposition 3 trivially extends. We then show that even for a general F, where the characterization of the optimal mechanism is only partial, it can still be shown that the seller's risk-seeking disposition decreases social welfare.

4.3. An Optimal Trade Mechanism

Given a choice F of the seller in the first stage, the principal maximizes the probability of trade subject to F-feasibility. The following lemma characterizes the solution of the optimal mechanism.

LEMMA 2. *The optimal trade mechanism $\Psi_F^\star = \left(A_F^\star(\cdot), T_F^\star(\cdot)\right)$ is the solution to the following problem:*

$$\max_{A(\cdot)} \mathbb{E}_F\left[A(x)\right],$$

where $A(\cdot)$ is weakly decreasing and satisfies the following "budget" constraint:

$$\Delta \cdot \mathbb{E}_F\left[A(x)\right] \geq \int_0^\infty A(x)\,F(x)\,dx. \tag{BC}$$

We use the ICS constraint to show that $A_F^\star$ is non-increasing and to obtain $T_F^\star$ up to a constant, as in Myerson (1981). Then, we deduce that the IRS constraint holds for every x if and only if it holds for $x \to \infty$, and the optimality of $\Psi_F^\star$ gives us the minimal constant that satisfies ICS in the limit. By posting it in the IRB constraint, we obtain the "budget" constraint.

The first conclusion we derive from Lemma 2 is that if the distribution is log-concave then the optimal mechanism is implemented by the Akerlof (1970) solution considered above. That is, our assumptions on $\mathcal{F}$ in Section 2, which were needed to derive a unique equilibrium of the stage game, also imply that restricting attention to the Akerlof (1970) solution is without loss of generality.

Proposition 6. *Let $F \in \mathcal{F}$. If f is log-concave then the optimal mechanism $\Psi_F^\star = \left(A_F^\star(x), T_F^\star(x)\right)$ is defined by $x^\star$, where*

$$A(x) = \begin{cases} 1 & x \leq x^\star \\ 0 & x > x^\star \end{cases}$$

and

$$T(x) = \begin{cases} \mathbb{E}_F\left[x + \Delta | x \leq x^\star\right] & x \leq x^\star \\ 0 & x > x^\star \end{cases}.$$

Moreover, $x^\star$ is the unique solution to the equation $\mathbb{E}_F\left[x + \Delta | x \leq x^\star\right] = x^\star$. That is, the optimal mechanism induces the same allocation and transfers as in the Akerlof (1970) solution.

To prove Proposition 6, we rewrite the constraint (BC) as

$$\int_0^\infty A(x)\left(\Delta - \frac{F(x)}{f(x)}\right) dF(x) \geq 0. \tag{10}$$

The log-concavity of f implies that $\frac{F(x)}{f(x)}$ is a decreasing function. As a result, it is easier to sustain the "budget" constraint when low types trade and, in the optimal solution, low types trade with probability 1, as in Akerlof (1970). Note that Jehiel and Pauzner (2006) study partnership dissolution, and they prove such result, under similar assumptions on F, for the case of decreasing gains from trade.

4.4. Trade and Social Welfare

An immediate consequence of Propositions 5 and 6 is that under log-concavity of the distributions in $\mathcal{F}$, the seller's choices lead to low levels of social welfare also in the optimal mechanism. Since the optimal trading mechanism is the Akerlof (1970) solution, our Proposition 3 applies, and we know that $\succ_{LIR}$ is a necessary and sufficient condition for less trade. However, when we do not assume log-concavity of the asset distribution, it is not clear anymore that the optimal mechanism is deterministic. Nevertheless, even without a complete characterization of the optimal mechanism, we can show that the seller's risk-seeking disposition induces a decrease in trade and social welfare.

Proposition 7. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. If $F_i \succ_{LIR} F_j$ then $SW^{\Psi^\star_{F_i}}(F_i) > SW^{\Psi^\star_{F_j}}(F_j)$ for every $\Delta > 0$.*

Let $\Psi_{F_j} = \left(A_{F_j}(x), T_{F_j}(x)\right)$ be any F_j-feasible mechanism. We construct another mechanism Ψ_{F_i} for distribution F_i that "mimics" the allocation $A_{F_j}(x)$ *quantile by quantile*:

$$A_{F_i}(x) = A_{F_j}\left(F_j^{-1}(F_i(x))\right). \tag{11}$$

That is, for every quantile $q \in [0,1)$, allocation A_{F_i} provides the same probability of trade as allocation A_{F_j}, and therefore the overall probabilities of trade, $PT^{\psi_{F_i}}(F_i)$ and $PT^{\psi_{F_j}}(F_j)$ are equal. To prove Proposition 7, we show that if Ψ_{F_j} is F_j-feasible then Ψ_{F_i} is F_i-feasible. Therefore, we can conclude that the probability of trade induced by the optimal mechanism for distribution F_j, $\Psi^{\star}_{F_j}$, is lower than the probability of trade induced by the optimal mechanism for distribution F_i, $\Psi^{\star}_{F_i}$.

5. General Utility Functions

Thus far, we have assumed that the gains from trade are constant. We now discuss more general functional forms. In this section and the next, we focus again on the environment considered in Sections 2 and 3. That is, an equilibrium condition determines trade and Assumption 1 applies.

First, the assumption $v_s(x) = x$ can be viewed as a normalization; i.e., the random variable we are dealing with is the seller's utility from consumption. In this context, we can limit our discussion to the functional form of the buyers' valuation, v_b. We now extend our main results to a concave v_b with non-decreasing gains from trade $(v_b(x) - x)$. We assume weak concavity, and thus our environment includes also the Akerlof (1970) example, $v_b(x) = ax$, for some $a > 1$. Note however that Assumption 1 excludes $x = 0$ from the discussion; i.e., the buyers' valuation is always strictly above that of the seller. Therefore, unlike in Akerlof's (1970) model, the market never breaks down completely.

The seller's risk-seeking disposition stems from the convexity of her equilibrium utility. Therefore, Proposition 2 extends to any (increasing) v_b; i.e., in equilibrium, the seller chooses risky distributions. To prove that Proposition 3 also extends, we first prove the following lemma.

LEMMA 3. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$, and $F_i \succ_{LIR} F_j$. Assume that $v_b(x)$ is increasing and weakly concave, and that the gains from trade $(v_b(x) - x)$ are positive and non-decreasing. If $PT(F_i) > PT(F_j)$ then $SW(F_i) > SW(F_j)$.*

Lemma 3 shows that to compare the equilibrium social welfare of two trade games, one needs only to compare the probability of trade. Intuitively, when the gains from trade are increasing, social welfare is impacted more by the trading of high realizations of x than by low realizations. In addition, the equilibrium trade is defined by a cutoff. Therefore, a higher trade probability in the equilibrium of distribution F_i implies a more frequent trade of products that contribute more to social welfare. By Lemma 3, it can be shown that $\succ_{LIR}$ is a necessary and sufficient condition for a monotone relation between risk and social welfare.

PROPOSITION 8. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. $SW(F_i) > SW(F_j)$ for any increasing and weakly concave v_b, where the gains from trade $(v_b(x) - x)$ are non-decreasing, if and only if $F_i \succ_{LIR} F_j$.*

We show that, as in the case with constant gains from trade, $\succ_{LIR}$ is a necessary and sufficient condition for a monotone relation between risk and equilibrium trade. Applying Lemma 3, we deduce that $\succ_{LIR}$ is also a necessary and sufficient condition for a monotone relation between risk and social welfare.

Finally, note that our results do not hold for a non-concave v_b. The seller still pursues risky alternatives, but this is not necessarily an undesirable characteristic. If, for example, v_b is strictly convex, then the buyers' utility from consumption is higher when the distribution is more dispersed. Therefore, social welfare may increase in equilibrium due to the seller's risk-seeking disposition.

6. An Informative Signal

Our baseline model does not allow the buyers to use public information to resolve any uncertainty about the asset value. By contrast, one can easily extend Akerlof's (1970) model to allow the buyers to update their beliefs after viewing the asset: one simply needs to interpret the distribution of the asset as a description of the uncertainty unresolved by all public information. This reduction is not appropriate in our model since the joint distribution of the signal and the asset value typically depends on the seller's initial choice. Therefore, our option-value argument does not hold in the presence of public information, and the seller might be dissuaded from pursuing risky alternatives. To demonstrate the general applicability of our results to Akerlof (1970), we study a normal-distribution model in which an informative signal is publicly observed before the trade.[16]

6.1. Timing of the game

1. The seller chooses a distribution $F_i \in \mathcal{F}$, $X_i \sim N\left(\mu_i, \sigma_i^2\right)$.
2. The seller observes the realization x, sampled according to the distribution she has chosen.
3. The buyers (and the seller) observe a public signal $S = X + \mathcal{E}$. We assume that $\mathcal{E} \sim N\left(0, \sigma_{\mathcal{E}}^2\right)$ and that it is independent of X. We denote by s and ε the realizations of S and $\mathcal{E}$, respectively.
4. A market price $p(s)$ is determined such that $p(s)$ is equal to the expected value of the buyers conditional on the seller agreeing to sell the asset. That is,

$$p(s) = \mathbb{E}_\eta \left[x + \Delta | x \leq p(s)\right], \tag{12}$$

   where η denotes the beliefs of the buyers about the unconditional distribution of the asset.

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16. A normal distribution of x somewhat contradicts our view of x as a good since x might be less than 0; i.e., the seller's utility from consumption is negative. Nevertheless, we can choose μ and σ such that the probability that $x < 0$ is as small as we wish it to be. Therefore, to gain the tractability that accompanies normal distributions, we are making this assumption, while focusing on parameters under which negative values of x are a negligible case.

For each realization of the signal, the market updates its beliefs on the asset distribution, and a standard Akerlof (1970) game is played. Since the distributions of both the good and the noise are normal, the beliefs of the market are also normally distributed. Let $\eta_i|s$ denote the market's beliefs given a realization s of the signal, in the case where it believes that the seller has chosen distribution F_i. The market's beliefs $\eta_i|s$ are normally distributed, with mean $\tilde{\mu}_i(s) = \frac{\sigma_{\mathcal{E}}^2 \mu_i + \sigma_i^2 s}{\sigma_{\mathcal{E}}^2 + \sigma_i^2}$ and variance $\tilde{\sigma}_i^2 = \frac{\sigma_{\mathcal{E}}^2 \sigma_i^2}{\sigma_{\mathcal{E}}^2 + \sigma_i^2}$.

6.2. Analysis

Lemma 4 states that the trade volume in equilibrium does not depend on the realization of the signal.

LEMMA 4. *Let $F_i = N\left(\mu_i, \sigma_i^2\right)$ be the seller's equilibrium distribution choice. For every $s, s' \in \mathbb{R}$, $PT\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_i^2\right)\right) = PT\left(N\left(\tilde{\mu}_i(s'), \tilde{\sigma}_i^2\right)\right)$.*

The variance of the buyers' beliefs does not depend on the realization of the signal, and the gains from trade are assumed to be constant; i.e., the players participate in shifts of the same trade game. Thus, the equilibrium price is obtained at the same quantile for every s.

Next, we study the seller's incentive to pursue risk in this model. Assume that the market expects the seller to choose some distribution F_i. When she deviates to a more risky distribution, there are two effects. (a) The buyers, who expect a less dispersed distribution, underweight the signal they receive. (b)

Fixing the signal realization, the distribution of the asset is more dispersed than the buyers believe it to be. Relative to the less dispersed distribution, effect (a) hurts the seller when the signal realization is above the prior mean and favors her when below. However, the effect is not symmetric. If the signal realization is above the prior mean, the seller is relatively more likely to consume the good, while if it is below, she is more likely to sell it; i.e., she gains from the upside more than she looses from the downside. Finally, for effect (b), the option value argument, analyzed in the previous sections, is applicable. Therefore, the seller's expected profit is higher if she deviates and chooses the more dispersed distribution.

Proposition 9. *Let $F_i = N\left(\mu, \sigma_i^2\right)$, $F_j = N\left(\mu, \sigma_j^2\right)$. If $\sigma_j^2 > \sigma_i^2$, then in any equilibrium of the extended trade game with the signal S, $\alpha_i = 0$.*

Let $\pi\left(F_j, F_i\right)$ be a random variable denoting the seller's profits following a deviation to F_j when the market believes she chooses F_i. To prove Proposition 9, we assume to the contrary that the seller chooses F_i in equilibrium, and we show that, for every a, $\mathbb{E}_{F_j}\left[\pi\left(F_j, F_i\right) | \varepsilon \in \{-a, +a\}\right] > \mathbb{E}_{F_i}\left[\pi\left(F_i, F_i\right) | \varepsilon \in \{-a, +a\}\right]$. That is, if the market believes that the seller chooses F_i she can profit from deviating to F_j. The intuition for this inequality is demonstrated in Figures 3–5.

The difference between the market's beliefs and the actual distribution of x, following an unobservable deviation of the seller to F_j, is depicted in Figure 3.

Conditional on every realization of the signal, the variance of the asset value is higher than the buyers believe it to be. In addition, the relative informativeness of s is higher, and therefore the conditional expectation is weighted more toward the signal. Note that the price p is determined according to the (wrong) conjectured choice, F_i.

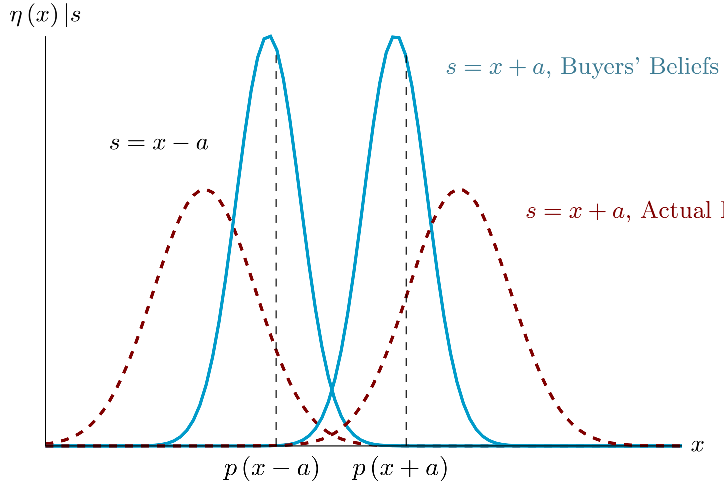

FIGURE 3. A Deviation to F_j

The blue (solid) lines represent the market's beliefs following both realizations of the signal s. The red (dashed) lines represent the true distributions of the asset when the seller has chosen the riskier distribution.

To show that the seller's profit is higher if she chooses F_j, we derive the difference between the distributions in two steps. First, conditional on each realization we add a noise term; see Figure 4. As we have already shown, due to the option value of the seller's equilibrium payoff, such an addition strictly benefits her.

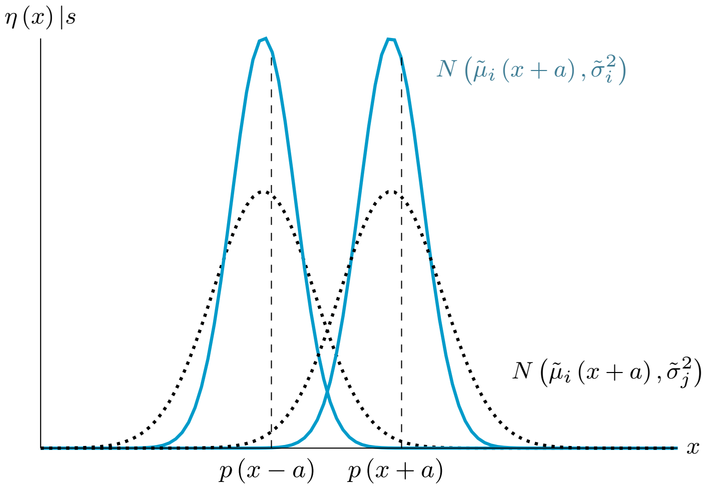

FIGURE 4. A Deviation to F_j: More Risk

The blue (solid) lines represent the market's beliefs following both realizations of the signal s. The black (dotted) lines represent two distributions of the asset with the same expectation as conjectured by the market but with the true variance induced by the riskier choice of the seller.

Second, the buyers' beliefs are obtained by a rightward shift of the good state distribution, which increases the seller's payoff, and a leftward shift of the bad state distribution, which generates losses for the seller. However, the profits are strictly greater than the losses. A rightward shift of a distribution F by a constant d generates additional profits greater than $(1 - PT(F))\,d$. Any realization consumed in the source distribution is consumed in the shifted distribution, and in each case, the seller's profits increase by d. By contrast, the losses caused by a leftward shift of a distribution F are less than $(1 - PT(F))\,d$. Any realization that is sold in the source distribution is sold in the shifted distribution too, and for any realization that is consumed in the source distribution, the seller loses at most d; see Figure 5.

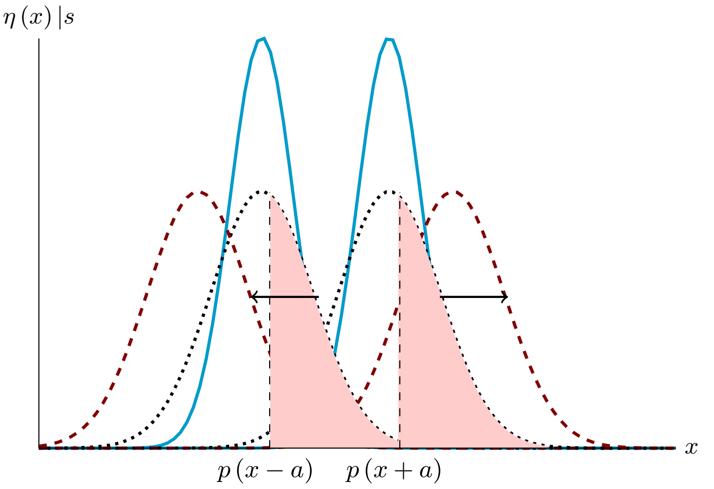

FIGURE 5. A Deviation to F_j – Total Shift of the Distributions

The blue (solid) lines represent the market beliefs. The red (dashed) lines represent the true distributions. The Black (dotted) lines represent distributions with the true variance but with the (wrong) conjectured expectations, and the shaded areas represent the probability of consumption by the seller that those imply, for each realization of the signal.

Finally, our social welfare characterization also follows and trade decreases due to the seller's risk-seeking disposition. Since both distributions are normal, $\sigma_j^2 > \sigma_i^2$ implies $F_i \succ_{LIR} F_j$, and, by Proposition 3, the chosen distribution induces lower social welfare. Note however that the implications of the seller's behavior on trade become less meaningful as the signal becomes more informative. In the limit where $\sigma_\varepsilon^2 \to 0$, all asymmetric information considerations vanish since the asset value is observable, i.e., $\lim_{\sigma_\varepsilon^2 \to 0} PT\left(N\left(\tilde{\mu}_i\left(s\right), \tilde{\sigma}_i^2\right)\right) = 1$.

7. Discussion and Conclusion

We conclude this paper by considering the implications of some changes to our assumptions and by discussing the significance of our results.

7.1. Partly Observable Distribution Choice

We have assumed that the seller's choice is unobservable. However, if it is observable, then the seller's equilibrium choice is efficient, as was noted in the Introduction. A natural question in this context is how robust is the risk-seeking disposition to a partly informative signal about the seller's initial choice. Next, we consider Example 1 again and analyze the effect of a symmetric signal on the seller's choice. That is, assume that the seller chooses between X_1 and X_2:

$$X_1 = \begin{cases} 1 & \text{w.p. } 1, \end{cases} \quad X_2 = \begin{cases} 2 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2}, \end{cases}$$

where the buyers' utility from consuming an asset of type x is $x + 1/2$. Let the buyers observe a signal $\gamma \in \{\gamma_1, \gamma_2\}$, and assume for simplicity $\Pr[\gamma = \gamma_i | X_i] = r > 1/2$ for $i \in \{1, 2\}$. First, note that for any $r \in (1/2, 1)$ there is no pure equilibrium in which the seller chooses X_1. A deviation from such a strategy is undetectable since the buyers expect γ_2 to be realized on the equilibrium path. As a result, the seller would always be "tempted" to deviate and choose X_2. Moreover, if $r \leq 3/4$ then in the unique equilibrium of the game the seller chooses X_2 with probability 1. To see why, note that the seller can be indifferent between X_1 and X_2 only if the market price given a realization of γ_2 is lower than the market price given γ_1. Now, in any mixed equilibrium, the lowest possible price given γ_2 is $1/2$ and the highest possible price given γ_1 is smaller than 1.5. Therefore, as long as $r \leq 3/4$, the seller would strictly prefer to choose X_2. For $r \in (3/4, 1)$, the game admits multiple equilibria and the seller might mix between both alternatives. However, a pure choice of X_2 (and a market price of $1/2$) is always an equilibrium of the game. Finally, as was noted above, if $r = 1$ a choice of X_1 (with a market price of 1.5 given γ_1 and $1/2$ given γ_2) is the unique equilibrium of the game.

7.2. Excessive Risk and Financial Crises

The failure of many financial markets to provide liquidity following the financial crisis of 2007–2008 is well documented; see Gorton (2009) and Chui et al. (2010). Stiglitz (2007), among others, argued that the introduction of mortgage-backed securities has brought about excessive risk. A standard argument, e.g., is that securitization lowers the banks' incentive to invest in reducing risk. Keys et al. (2010), for example, shows that securitization reduces the lenders' incentive for costly screening. Our model suggests a different rationale for the same phenomenon: the ability to trade in an adverse selection environment sets the stage for excessive risk-taking.[17] Since the lenders can offload low-quality loans on the market, they might choose risky alternatives even when safer ones (possibly with a higher expected return) are available at no extra cost.

Conclusion. In this paper, we have provided reasons to suspect that "lemon" markets are generated endogenously. The option-value structure of the seller's equilibrium payoff in a bilateral trade model implies that, as long as her pre-trade actions are unobservable, the seller pursues risky alternatives. As we have shown, under quite general assumptions, the seller's equilibrium behavior implies a substantial decrease in trade and social welfare.

17. We wish to extend our thanks to an anonymous referee for proposing this comparison.

Appendix: Proofs

A.1. Proof of Proposition 1

For every $F_i \in \mathcal{F}$, we denote by $p_i^\star$ the price in the unique equilibrium of the F trade game, i.e., $E\left[x + \Delta \mid x \leq p_i^\star\right] = p_i^\star$. Rearrange the set $\mathcal{F}$ such that if $i < j$ then $p_i^\star \leq p_j^\star$, and let $R_i\left(p\right) := F_i\left(p\right)p + \left(1 - F_i\left(p\right)\right)E_{F_i}\left[x \mid x > p\right]$. This function takes as an input a price p and returns the equilibrium utility of a seller whose asset is distributed according to F_i when the market price is p. It is easy to verify that for every $i \in \{1, 2, ..., n\}$ R_i is continuous. For every price $p \in [p_1, p_n]$, define the set $\max(p) := \{i \mid R_i\left(p\right) \geq R_j\left(p\right) \forall j \in \{1, 2, ..., n\}\}$. In addition, define $i(p) := \min\left[\max\left(p\right)\right]$. Lastly, define the function $H : [p_1, p_n] \longmapsto \{p_1, p_2, ..., p_n\}$ to be $H(p) := p_{i(p)}^\star$. Note that every discontinuity of H corresponds to an intersection between R_i functions at the frontier. Therefore, if there exists $i \in \{1, 2, ..., n\}$ such that $H(p_i^\star) = p_i^\star$ then $\alpha_i = 1$ is an equilibrium in the extended trade game. Otherwise, if for every $i \in \{1, 2, ..., n\}$ $H(p_i^\star) \neq p_i^\star$, we prove that there exists an equilibrium under which the seller mixes between exactly two distributions. First, note that if $H(p_i^\star) \neq p_i^\star$ for every i then it follows that $H(p_1^\star) > p_1^\star$ and $H(p_n^\star) < p_n^\star$. Under this condition we prove the following Lemma.

LEMMA A.1. *There exists a discontinuity point of H, $p \in (p_1^\star, p_n^\star)$, such that there exists a left neighborhood of p with $H(p') > p'$ for every p' in this* *neighborhood, and there exists a right neighborhood of p with $H(p') < p'$ for every p' in this neighborhood.*

Proof. Because we have that $H(p_1^\star) > p_1^\star$ and $H(p_n^\star) < p_n^\star$ and we assumed that there is no pure equilibrium it must be that the step function H and the identity function "cross" in the manner stated in Lemma A.1. This follows directly from the Intermediate Value Theorem when one of the functions is a step function and the other one is continuous. □

Denote the price from Lemma A.1 by $p^\star$. From the assumption that there is no pure equilibrium it must be the case that $p^\star \neq p_i^\star$ for every $i \in \{1, 2, ..., n\}$. We also know that there exists a left neighborhood of $p^\star$ in which $H(p') > p^*$ for every p' in this neighborhood, and that H is a step function so it is constant on a small enough neighborhood. There is an index $l \in \{1, 2, ..., n\}$ such that $H(p') = p_l^\star$ in this neighborhood. In the same manner there exists an index $k \in \{1, 2, ..., n\}$ such that $H(p') = p_k^\star$ at a small enough right neighborhood of $p^\star$. It is clear that $l > k$. We now argue that there is a mix between F_k and F_l which constitutes an equilibrium of the extended trade game.

Lemma A.2. *There exists an $\alpha \in (0, 1)$ such that the price in the trade game, in which the asset distribution is $\alpha F_l + (1 - \alpha) F_k$, is $p^\star$.*

Proof. We know $p_l^\star > p^\star > p_k^\star$, and it follows that $E_{F_l}[x \mid x \leq p^\star] > p^\star$ and $E_{F_k}[x \mid x \leq p^\star] < p^\star$. Therefore, there exists an $\alpha \in (0,1)$ such that $\alpha E_{F_l}[x \mid x \leq p^\star] + (1-\alpha) E_{F_k}[x \mid x \leq p^\star] = p^*$. $\square$

From Lemma A.1 we also have that $R_l(p^*) = R_k(p^*)$ and that $\{l,k\} \in \arg\max_i R_i(p^*)$. This ends the proof.

A.2. Proof of Proposition 2

First, we prove that $\succ_{LIR}$ implies a strict version of second-order stochastic dominance.

LEMMA A.3. *Let F, G be two distributions, $\mathbb{E}_F[x] \geq \mathbb{E}_G[x]$. If $F \succ_{LIR} G$ then, for every x,*

$$\int_0^x F(s)ds < \int_0^x G(s)ds.$$

Proof. First, by Landsberger and Meilijson (1994), if $\mathbb{E}_F[x] \geq \mathbb{E}_G[x]$ then $\succ_{LIR}$ implies second-order stochastic dominance. That is, for every $t \in \mathbb{R}^+$, we have

$$\int_0^t G(x)dx \geq \int_0^t F(x)dx. \tag{A.1}$$

Assume by the way of contradiction that there exists a $y \in \mathbb{R}^+$ such that

$$\int_0^y G(x)dx = \int_0^y F(s)dx. \tag{A.2}$$

We have that for every t in a neighborhood around y,

$$\int_0^t G(x)dx \geq \int_0^t F(x)dx. \tag{A.3}$$

It follows that the function $\mathbb{G}(t) := \int_0^t G(x)dx$ is tangent to the function $\mathbb{F}(t) := \int_0^t F(x)dx$ at y. Therefore, the derivative of $\mathbb{G}$ is equal to the derivative of $\mathbb{F}$ at y, i.e., $G(y) = F(y)$. This is a contradiction to $\succ_{LIR}$ because we have a quantile $q = G(y) = F(y)$ such that

$$\int_0^{G^{-1}(q)} G(x)dx = \int_0^{G^{-1}(q)} F(x)dx. \tag{A.4}$$

□

Now, after we have that $\succ_{LIR}$ implies strict second-order stochastic dominance, we show that the latter implies $\alpha_i = 0$. Assume by the way of contradiction $\alpha_i > 0$, and let H_α denote the cumulative distribution of x given the first-stage strategy α. That is, $H_\alpha(x) := \sum_i \alpha_i F_i(x)$.

The utility of the seller in equilibrium is

$$R_{H_\alpha}(p) = H_\alpha(p)\,p + \int_p^\infty x h_\alpha(x)\,dx, \tag{A.5}$$

where (integration by parts)

$$R_{H_\alpha}(p) = \int_0^p x h_\alpha(x)\,dx + \int_0^p H_\alpha(x)\,dx + \int_p^\infty x h_\alpha(x)\,dx. \tag{A.6}$$

Place

$$\mathbb{E}_{H_\alpha}[x] = \int_0^p x h_\alpha(x)\,dx + \int_p^\infty x h_\alpha(x)\,dx, \tag{A.7}$$

with (A.5) and (A.6) and obtain:

$$R_{H_\alpha}(p) = \mathbb{E}_{H_\alpha}[x] + \int_0^p H_\alpha(x)\,dx. \tag{A.8}$$

Now, consider the strategy α' obtained by moving all mass of play from F_i to F_j. That is, $\alpha'_i = 0$ and $\alpha'_j = \alpha_i + \alpha_j$. Since α is an equilibrium strategy we have

$$R_{H_\alpha}(p) \geq R_{H_{\alpha'}}(p). \tag{A.9}$$

That is,

$$\mathbb{E}_{H_\alpha}[x] + \int_0^p H_\alpha(x)\,dx \geq \mathbb{E}_{H_{\alpha'}}[x] + \int_0^p H_{\alpha'}(x)\,dx. \tag{A.10}$$

Thus, we have obtained a contradiction. We assumed $\mathbb{E}_{H_{\alpha'}}[x] = \mathbb{E}_{H_\alpha}[x]$ and, by strict second order stochastic dominance, we know $\int_0^p H_\alpha(x)\,dx < \int_0^p H_{\alpha'}(x)\,dx \;\forall p$.

A.3. Proof of Proposition 3

We prove the Proposition in two steps.

LEMMA A.4. *$PT(F_i) \geq PT(F_j)$ For every $\Delta > 0$ if and only if $E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q)$ for every $q \in (0, 1)$.*

Proof. Denote by $q_k(\Delta) := PT(F_k)(\Delta)$. For every log-concave CDF F_k and for every $q \in (0, 1)$, there exists a $\Delta > 0$ such that $q_k(\Delta) = q$. Assume that for every $q \in (0, 1)$ we have that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q). \tag{A.11}$$

This is true if and only if, for every $q \in (0, 1)$,

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - F_i^{-1}(q) \geq E_{F_j}[x \mid x \leq F_j^{-1}(q)] - F_j^{-1}(q). \tag{A.12}$$

In particular,

$$E_{F_i}[x \mid x \leq F_i^{-1}(q_j(\Delta))] - F_i^{-1}(q_j(\Delta)) \geq E_{F_j}[x \mid x \leq F_j^{-1}(q_j(\Delta))] - F_j^{-1}(q_j(\Delta)) = 0. \tag{A.13}$$

It follows that $q_i(\Delta) \geq q_j(\Delta)$. Now, assume that there exists $\hat{q} \in (0, 1)$ such that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(\hat{q})] - F_i^{-1}(\hat{q}) < E_{F_j}[x \mid x \leq F_j^{-1}(\hat{q})] - F_j^{-1}(\hat{q}). \tag{A.14}$$

Since there exists a $\hat{\Delta} > 0$ such that $q_j(\hat{\Delta}) = \hat{q}$, it follows that $q_i(\hat{\Delta}) < q_j(\hat{\Delta})$. $\square$

We now proceed to the second step.

Lemma A.5.

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q),$$

if and only if

$$\int_0^{F_i^{-1}(q)} F_i(x)dx \leq \int_0^{F_j^{-1}(q)} F_j(x)dx.$$

Proof. For every $H \in \{F_i, F_j\}$ and every $q \in [0,1]$:

$$E_H[x \mid x \leq H^{-1}(q)] = H^{-1}(q) - \frac{\int_0^{H^{-1}(q)} H(x)dx}{q}. \tag{A.15}$$

This allows us to rewrite the condition in Lemma A.5 as

$$F_i^{-1}(q) - \frac{\int_0^{F_i^{-1}(q)} F_i(x)dx}{q} - \left(F_j^{-1}(q) - \frac{\int_0^{F_j^{-1}(q)} F_j(x)dx}{q}\right) > F_i^{-1}(q) - F_j^{-1}(q), \tag{A.16}$$

Rearranging (A.16), we get:

$$\int_0^{F_i^{-1}(q)} F_i(x)dx \leq \int_0^{F_j^{-1}(q)} F_j(x)dx. \tag{A.17}$$

$\square$

A.4. Proof of Lemma 1

Consider first the case where $\mathcal{F} = \{F_i, F_j\}$, where $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. By Proposition 2, the seller chooses F_j, the equilibrium price is $\hat{p}_j$, and the seller's profit is $R_j(\hat{p}_j)$.

Now, assume to the contrary $\hat{p}_j \geq \hat{p}_i$. This implies:

$$R_i(\hat{p}_j) \geq R_i(\hat{p}_i). \tag{A.18}$$

By Proposition 3, since $F_i \succ_{LIR} F_j$ we have:

$$R_i(\hat{p}_i) = SW(F_i) > SW(F_j) = R_j(\hat{p}_j). \tag{A.19}$$

Put (A.18) with (A.19) and obtain $R_i(\hat{p}_j) > R_j(\hat{p}_j)$, which is in contradiction to Proposition 2. The seller would be better off if she deviates and chooses F_i.

Finally, assume $\mathbb{E}_{F_i}[x] > \mathbb{E}_{F_j}[x]$. There exists $c > 0$ such that $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x + c]$. Let $X_j' := X_j + c$; we know that $F_i \succ_{LIR} F_j'$, and thus $\hat{p}_j' < \hat{p}_i$. In addition, since the equilibrium price increases when we add a constant, we know $\hat{p}_j < \hat{p}_j'$. That is, $\hat{p}_j < \hat{p}_i$.

A.5. Proof of Proposition 4

Let $op_i(p) = R_i(p) - p$.

Lemma A.6. *If F_1 and F_2 single cross and $E_{F_1}[x] \geq E_{F_2}[x]$ then op_1 and op_2 single cross. That is, there exists $p^\star$ such that, for every $p \leq p^\star$ $op_1(p) \geq op_2(p)$, and for every $p \geq p^\star$ $op_1(p) \leq op_2(p)$.*

Proof. We start by computing the derivatives of $op_1(p)$ and $op_2(p)$:

$$\frac{d}{dp} op_i(p) = \frac{d}{dp} \int_p^\infty (x-p) f_i(x)\, dx$$

$$= \frac{d}{dp} \int_p^\infty x f_i(x)\, dx - \frac{d}{dp} p \int_p^\infty f_i(x)\, dx$$

$$= -p f_i(p) - \left(p \frac{d}{dp}(1 - F_i(p)) + (1 - F_i(p)) \right) = F_i(p) - 1.$$

It follows that:

$$\frac{d}{dp}(op_1 - op_2) = F_1(p) - F_2(p).$$

Since F_1 and F_2 single cross, the difference between $op_1(p)$ and $op_2(p)$ is decreasing until the crossing point of F_1 and F_2, and then it is increasing. We also know that:

$$\lim_{p \to 0} (op_1(p) - op_2(p)) = E_{F_1}[x] - E_{F_2}[x] \geq 0, \tag{A.20}$$

and

$$\lim_{p \to +\infty} \left(op_1(p) - op_2(p) \right) = 0. \tag{A.21}$$

We know that (A.20) holds since, in the limit in which the price goes to 0, the agent never sells under both distributions. It follows that the difference in payoffs must be the difference in the expectations of the distributions. The limit in (A.21) follows from the fact that when the price goes to plus infinity the agent will sell with probability one under both distributions, and the difference in payoffs is zero. Therefore, we know that if $E_{F_1}[x] = E_{F_2}[x]$ then $op_2(p) > op_1(p)$ for every p. This corresponds to Proposition 2. However, if $E_{F_1}[x] > E_{F_2}[x]$, then it must be that there exists a cutoff $p^\star$, such that $op_1(p) > op_2(p)$ for every $p < p^\star$, and $op_1(p) < op_2(p)$ for every $p > p^\star$. Note that we also know that $F_2(p^\star) > F_1(p^\star)$, i.e., $p^\star$ is smaller than the crossing point. $\square$

By Lemma A.6, there exists Δ_1, such that for every $\Delta \leq \Delta_1$ we have that $p_1(\Delta) < p^\star$ and $p_2(\Delta) < p^\star$. It follows that for every such Δ, in the unique equilibrium of the extended trade game, the seller chooses F_1. There also exists $\Delta_2 > \Delta_1$ such that for every $\Delta \geq \Delta_2$ we have that $p_1(\Delta) > p^\star$ and $p_2(\Delta) > p^\star$. It follows that for every such Δ, in the unique equilibrium of the extended trade game, the seller chooses F_2. It also follows that, for $\Delta \leq \Delta_1$ and $\Delta \geq \Delta_2$, the equilibrium social welfare is monotone in Δ. Social welfare $SW(F)$ is equal to $\mathbb{E}_F[x] + PT(F)\,\Delta$, and we know that the equilibrium price and the probability of trade are increasing in Δ. That is, as long as the the seller's choice does not change, social welfare is strictly increasing in the gains from trade.

Now, by Lemma 1, we know that, for every $\Delta \in (\Delta_1, \Delta_2)$, $p_1(\Delta) > p_2(\Delta)$. That is, if the seller chooses F_1 the equilibrium price $p_1(\Delta)$ is larger than $p^\star$; i.e., the seller would be better off deviating to F_2. If, however, the seller chooses F_2 the equilibrium price $p_2(\Delta)$ is smaller than $p^\star$; i.e., the seller would be better off deviating to F_1. That is, there is no pure equilibrium in this region. In addition, in a mixed equilibrium it must be that the seller is indifferent between F_1 and F_2, which can happen only if the equilibrium price is exactly $p^\star$. Let $\alpha \in [0,1]$ denote the probability that the seller's strategy assigns to the choice of F_1. Since the equilibrium price is a continuous function of α, for every $\Delta \in (\Delta_1, \Delta_2)$, there exists (a unique) $\alpha^\star$ such that the equilibrium price is exactly $p^\star$. Therefore, in the unique equilibrium of the extended trade game, the seller mixes between the two distributions and the equilibrium price is $p^\star$. Finally, since the equilibrium price is constant in this region so is social welfare. The social welfare in equilibrium is equal to the utility of the seller, which, as long as we fix the price, does not depend on Δ. That is, the utility of the seller when choosing, say, F_1 is $F_1(p^\star) + (1 - (F_1(p^\star)))\,\mathbb{E}_{F_1}[x|x > p^\star]$, which does not depend on Δ. Because for every $\Delta \in (\Delta_1, \Delta_2)$, the seller is indifferent between F_1 and F_2 and the equilibrium price is $p^\star$, we know that the equilibrium social welfare is constant.

A.6. Proof of Proposition 5

LEMMA A.7. *In any implementable trade mechanism, the allocation $A(\cdot)$ is non-increasing. Moreover, the allocation $A(\cdot)$ pins down the transfer T up to a constant:* $T(x) = xA(x) - \int_0^x A(s)ds + C$.

Proof. Given some $x, x' \in R^+$ we have from ICS that the following two inequalities must hold:

$$(1 - A(x))x + T(x) \geq (1 - A(x'))x + T(x'), \tag{A.22}$$

and

$$(1 - A(x))x' + T(x) \leq (1 - A(x'))x' + T(x'). \tag{A.23}$$

It follows that:

$$x'(A(x) - A(x')) \geq T(x) - T(x') \geq x(A(x) - A(x')). \tag{A.24}$$

That is,

$$(x' - x)(A(x) - A(x')) \geq 0. \tag{A.25}$$

Assume $x' > x$. In order to satisfy (A.25), it must hold that $A(x) \geq A(x')$; i.e., A is weakly decreasing.

We return to (A.24). Assume that $x' = x + \varepsilon$ for $\varepsilon > 0$. It follows that:

$$(x+\varepsilon)(A(x) - A(x+\varepsilon)) \geq T(x) - T(x+\varepsilon) \geq x(A(x) - A(x+\varepsilon)). \quad \text{(A.26)}$$

That is,

$$\varepsilon(A(x) - A(x+\varepsilon)) + x(A(x) - A(x+\varepsilon)) \geq T(x) - T(x+\varepsilon) \geq x(A(x) - A(x+\varepsilon)). \quad \text{(A.27)}$$

We can divide by ε and obtain:

$$(A(x) - A(x+\varepsilon)) + \frac{x(A(x) - A(x+\varepsilon))}{\varepsilon} \geq \frac{T(x) - T(x+\varepsilon)}{\varepsilon} \geq \frac{x(A(x) - A(x+\varepsilon))}{\varepsilon}. \quad \text{(A.28)}$$

For $x \in R^+$ in which $A(\cdot)$ is continuous we have that:

$$\lim_{\varepsilon \to 0} A(x) - A(x+\varepsilon) = 0. \quad \text{(A.29)}$$

Thus for $x \in R^+$ in which $A(\cdot)$ is differentiable we have that:

$$\lim_{\varepsilon \to 0} \frac{x(A(x) - A(x+\varepsilon))}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{T(x) - T(x+\varepsilon)}{\varepsilon}. \quad \text{(A.30)}$$

That is,

$$xA'(x) = T'(x). \quad \text{(A.31)}$$

It follows that $A(\cdot)$ pins down $T(\cdot)$ up to a constant, and that

$$T(x) = \int_0^x sA'(s)ds + C, \tag{A.32}$$

which we can rewrite as

$$T(x) = xA(x) - \int_0^x A(s)ds + C. \tag{A.33}$$

□

By Lemma A.7 we know that a non-increasing allocation function $A(\cdot)$ is implementable only if the payment scheme induces the following equilibrium expected utility as a function of the seller's type:

$$U_A(x) = x - \int_0^\infty A(s)ds + C, \tag{A.34}$$

for some constant C. We need to show that given two distributions F_i, F_j such that $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$ and $F_i \succ_{LIR} F_j$, the ex-ante expected utility of the seller U_A is strictly higher under F_j than under F_i. That is,

$$\int_0^\infty U_A(x)dF_j(x) > \int_0^\infty U_A(x)dF_i(x). \tag{A.35}$$

We have that for every $l \in \{i, j\}$ the following holds:

$$\int_0^\infty U_A(x)dF_l(x) = \mathbb{E}_{F_l}[x] + C - \int_0^\infty f_l(x)\left(\int_0^x A(s)ds\right)dx. \tag{A.36}$$

The first two expressions are equal under both distributions, thus we only need to show that

$$\int_0^\infty f_j(x)\left(\int_0^x A(s)ds\right)dx < \int_0^\infty f_i(x)\left(\int_0^x A(s)ds\right)dx. \tag{A.37}$$

Changing the integration order, we obtain the following: for every $l \in \{i, j\}$,

$$\int_0^\infty f_l(x)\left(\int_0^x A(s)ds\right)dx = \int_0^\infty A(s)\left(\int_s^\infty f_l(x)dx\right)ds. \tag{A.38}$$

Therefore, we have:

$$\int_0^\infty A(s)\left(\int_s^\infty f_l(x)dx\right)ds = \int_0^\infty A(s)(1-F_l(s))ds = \int_0^\infty A(s)ds - \int_0^\infty A(s)F_l(s)ds. \tag{A.39}$$

The expression $\int_0^\infty A(s)ds$ is independent of the distribution; i.e., we only need to show that

$$\int_0^\infty A(s)F_j(s)ds > \int_0^\infty A(s)F_i(s)ds. \tag{A.40}$$

We prove this by changing again the order in which we compute the integral. For every $l \in \{i, j\}$ we have:

$$\int_0^\infty A(s)F_l(s)ds = \int_0^\infty F_l(s)\left(\int_0^{A(s)} dy\right)ds = \int_0^1 \left(\int_0^{A^{-1}(y)} F_l(s)ds\right)dy, \tag{A.41}$$

where $A^{-1}(y) = \sup\{x \mid A(x) \geq y\}$, and if this set is empty it equals zero. Because $\succ_{LIR}$ implies strict SOSD we get that, for every y such that $A^{-1}(y) \neq$

$0, \infty$, the following holds.

$$\int_0^{A^{-1}(y)} F_j(s)ds > \int_0^{A^{-1}(y)} F_i(s)ds. \tag{A.42}$$

Note that it is infeasible that for every $y \in [0,1]$ it holds that $A^{-1}(y) = \infty$ because this means that mechanism implements full trade. In this case the transfer can be at most the ex-ante expectation plus Δ. This sum must be finite according to our assumptions, and thus it must be the case that sellers with types that are larger than this sum would choose not to sell, which is in contradiction to full trade. It follows that, as long as the allocation $A(\cdot)$ induces trade with strictly positive probability, Proposition 5 is true.

A.7. Proof of Lemma 2

By Lemma A.7, we have

$$T(x) = xA(x) - \int_0^x A(s)ds + C. \tag{A.43}$$

Plugging (A.43) into the IRS constraint we obtain:[18]

$$\forall x \in R^+, \quad (1 - A(x))x + xA(x) - \int_0^x A(s)ds + C \geq x. \tag{A.44}$$

18. As is well known, by the envelope theorem, in order for equation A.43 to hold, we do not need A to be continuous, we only need A to be integrable, which is true for every monotone function such as the allocation A.

That is,

$$\forall x \in R^{+}, \quad -\int_{0}^{x} A(s)ds + C \geq 0. \tag{A.45}$$

By (A.45), the IR constraint becomes harder to satisfy when x increases; i.e., all the IRS constraints are satisfied if and only if

$$-\int_{0}^{\infty} A(s)ds + C \geq 0. \tag{A.46}$$

It follows that the minimal constant C the enables all the IRS constraints to be satisfied is $C = \int_{0}^{\infty} A(s)ds$. Now, the smaller the constant C is the easier it is to satisfy the IRB constraint. That is, an allocation $A(\cdot)$ is implementable if and only if the IRB constraint is satisfied when $C = \int_{0}^{\infty} A(s)ds$. Plugging $C = \int_{0}^{\infty} A(s)ds$ into (A.43) we obtain:

$$T(x) = xA(x) - \int_{0}^{x} A(s)ds + \int_{0}^{\infty} A(s)ds = xA(x) + \int_{x}^{\infty} A(s)ds. \tag{A.47}$$

Plugging (A.47) into the IRB constraint we obtain:

$$\int_{0}^{\infty} A(x)(x+\Delta) - xA(x) - (\int_{x}^{\infty} A(s)ds)dF_i(x) \geq 0, \tag{A.48}$$

and we rewrite (A.48) as:

$$\Delta \int_{0}^{\infty} A(x)dF_i(x) - \int_{0}^{\infty} (\int_{x}^{\infty} A(s)ds)dF_i(x) \geq 0. \tag{A.49}$$

Finally, we can change the order of integration and obtain

$$\Delta \int_0^\infty A(x)dF_i(x) - \int_0^\infty A(x)F_i(x)dx \geq 0, \tag{A.50}$$

which we can write as:

$$\Delta \mathbb{E}_{F_i}[A(x)] \geq \int_0^\infty A(x)F_i(x)dx. \tag{A.51}$$

A.8. Proof of Proposition 6

We first show that if f_i is log-concave then the optimal mechanism is deterministic. That is, there exists a threshold x^* such that for every $x \leq x^*$ we have $A^*_{F_i}(x) = 1$ and for every $x > x^*$ we have $A^*_{F_i}(x) = 0$. To see why this is the case first recall that if f_i is log-concave then $\frac{F_i(x)}{f_i(x)}$ is weakly increasing. We can use this fact by rewriting (A.50) in the following way:

$$\int_0^\infty A(x)(\Delta - \frac{F_i(x)}{f_i(x)})dF_i(x) \geq 0. \tag{A.52}$$

In terms of this "budget" constraint, because $\frac{F_i(x)}{f_i(x)}$ is weakly increasing, trade is weakly cheaper for lower types. Therefore, an ex-ante probability of trade is feasible if and only if it is feasible with a deterministic mechanism. I.e., the set of optimal mechanisms contains a deterministic mechanism. Finally, among the deterministic mechanisms, the allocation that is induced by the Akerlof (1970) solution maximizes the probability of trade. First, the buyers' IR constraint is binding as they pay the expected value. Second, the IR constraint of a seller of type $x^\star$ is also binding, as she is indifferent, in equilibrium, between selling at the market price and self-consumption.

A.9. Proof of Proposition 7

We prove that any probability of trade (social welfare) that is feasible under F_j is strictly feasible under F_i. Then we show that we can make a small change to the allocation such that the probability of trade increases but the budget constraint still holds. Let $\Psi_{F_j} = (A_{F_j}(x), T_{F_j}(x))$ be some feasible mechanism under F_j. We construct a mechanism Ψ_{F_i} which is feasible under F_i and "mimics" the allocation $A_{F_j}(x)$ in the following sense:

$$A_{F_i}(x) = A_{F_j}(F_j^{-1}(F_i(x))). \tag{A.53}$$

That is, for every quantile $q \in [0,1]$ it holds that:

$$A_{F_i}(F_i^{-1}(q)) = A_{F_j}(F_j^{-1}(q)). \tag{A.54}$$

By this construction, it follows that:

$$\mathbb{E}_{F_i}[A_{F_i}(x)] = \mathbb{E}_{F_j}[A_{F_j}(x)]. \tag{A.55}$$

That is, we only need to show that the mechanism we have constructed, Ψ_{F_i}, is indeed feasible under F_i. First, we know that A_{F_j} is weakly decreasing because the mechanism Ψ_{F_j} is feasible and, by the proof of Lemma 2, this a necessary condition for a mechanism to be feasible. It follows from the definition of A_{F_i} that if A_{F_j} is weakly decreasing then A_{F_i} is also weakly decreasing. It is left to show that the second constraint (BC) from Lemma 2 holds strictly, i.e.,:

$$\Delta \mathbb{E}_{F_i}[A_{F_i}(x)] > \int_0^\infty A_{F_i}(x) F_i(x) dx. \tag{A.56}$$

Recall that by construction we have that:

$$\Delta \mathbb{E}_{F_i}[A_{F_i}(x)] = \Delta \mathbb{E}_{F_j}[A_{F_j}(x)]. \tag{A.57}$$

Therefore, we only need to show that:

$$\int_0^\infty A_{F_j}(x) F_j(x) dx > \int_0^\infty A_{F_i}(x) F_i(x) dx. \tag{A.58}$$

For every $r \in [0,1]$ and an allocation A_F, define $B_F(r) \subset R^+$ as follows. $B_F(r) := \{x \mid A_F(x) \geq r\}$. Notice that because the allocation of a feasible mechanism is weakly decreasing we have that for every $r \in [0,1]$ the set $B_F(r)$ is a segment $[0, k_F(r)]$ for some $k_F(r) \in [0,1]$. Now, we can rewrite (A.58) as:

$$\int_0^1 \left(\int_{B_{F_j}(r)} F_j(x) dx \right) dr > \int_0^1 \left(\int_{B_{F_i}(r)} F_i(x) dx \right) dr. \tag{A.59}$$

By the definition of the mimicking mechanism Ψ_{F_i}, we have that for every $r \in [0,1]$ it holds that $F_i(k_{F_i}(r)) = F_j(k_{F_j}(r))$. It follows that we can rewrite

(A.59) as:

$$\int_0^1 \left(\int_0^{k_{F_j}(r)} F_j(x)dx \right) dr > \int_0^1 \left(\int_0^{k_{F_i}(r)} F_i(x)dx \right) dr. \tag{A.60}$$

By the definition of $\succ_{LIR}$ we have that, for every $r \in [0,1]$,

$$\int_0^{k_{F_j}(r)} F_j(x)dx > \int_0^{k_{F_i}(r)} F_i(x)dx. \tag{A.61}$$

Therefore, we have that (A.60) holds and thus also (A.56). Finally, by equation (A.52), we can make a small change to the allocation such that the probability of trade increases and the budget constraint holds. This ends the proof.

A.10. Proof of Lemma 3

By Lemma A.3, we have that F_i dominates F_j in the sense of strict second-order stochastic dominance, denoted by $F_i \succ_{SOSD} F_j$. Now, for every $q \in (0,1)$, let

$$\widetilde{F}^q(x) := \begin{cases} \frac{F(x)}{q} & x \leq F^{-1}(q) \\ 1 & x > F^{-1}(q) \end{cases}.$$

Claim 1. *If $F_i \succ_{SOSD} F_j$, then $\widetilde{F}_i^q \succ_{SOSD} \widetilde{F_j}^q$ $\forall q \in (0,1)$.*

Proof. $F_i \succ_{SOSD} F_j$: $\int\limits_0^t F_j(x)\,dx > \int\limits_0^t F_i(x)\,dx$ $\forall t$, hence, $\int\limits_0^t \frac{F_j(x)}{q} dx > \int\limits_0^t \frac{F_i(x)}{q} dx$ $\forall t$. □

Having this result we turn to prove that if $F_i \succ_{SOSD} F_j$ and $F_i(\hat{p}_i) > F_j(\hat{p}_j)$ then $SW(F_i) > SW(F_j)$.

Let $\hat{q}_i = F_i(\hat{p}_i)$. If the seller chooses F_j in equilibrium the social welfare is

$$SW(F_j) := \int_0^{\hat{p}_j} v_b(x) f_j(x)\, dx + \int_{\hat{p}_j}^{\infty} x f_j(x)\, dx. \tag{A.62}$$

Equation (A.62) can be written as

$$\int_0^{\hat{p}_j} (v_b(x) - x) f_j(x)\, dx + \mathbb{E}_{F_j}[x]. \tag{A.63}$$

And, since $\hat{p}_j < F_j^{-1}(\hat{q}_i)$, we have that (A.63) is smaller than

$$\int_0^{F_j^{-1}(\hat{q}_i)} ((v_b(x) - x)) f_j(x)\, dx + \mathbb{E}_{F_j}[x]. \tag{A.64}$$

Now, let $u(x) := v_b(x) - x$. An agent whose preferences over lotteries are represented by the VNM utility function u is risk-averse since $u' = v_b' - 1 > 0$ and $u'' = v_b'' < 0$. Hence, we can write ($A.64$) as

$$\int_0^{F_j^{-1}(\hat{q}_i)} u(x) f_j(x)\, dx + \mathbb{E}_{F_j}[x] \tag{A.65}$$

$$< \int_0^{F_i^{-1}(\hat{q}_i)} u(x) f_i(x)\, dx + \mathbb{E}_{F_i}[x] = SW(F_i), \tag{A.66}$$

where ($A.65$) is smaller than ($A.66$) since $\widetilde{F}_{i_q} \succ_{SOSD} \widetilde{F_j}_q$ and $\mathbb{E}_{F_i}[x] = \mathbb{E}_{F_j}[x]$. Thus, the proof of Lemma 3 is concluded.

A.11. Proof of Proposition 8

LEMMA A.8. *Assume $v_b(\cdot)$ is an increasing and concave function of the asset value x, such that the gains from trade, $v_b(x) - v_s(x) = v_b(x) - x$, are non-decreasing in x. If $F_i \succ_{LIR} F_j$ and $\mathbb{E}_{F_j}[x] = \mathbb{E}_{F_i}[x]$ then $PT_{v_b}(F_i) \geq PT_{v_b}(F_j)$.*

Proof.

We prove the Lemma with the help of another lemma:

LEMMA A.9. *Assume $F_i \succ_{LIR} F_j$ and $\mathbb{E}_{F_j}[x] = \mathbb{E}_{F_i}[x]$. For every v_b that satisfies the conditions in Lemma A.8, if*

$$\forall q \in [0,1] E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q)$$

then

$$\forall q \in [0,1] E_{F_i}[v_b(x) \mid x \leq F_i^{-1}(q)] - E_{F_j}[v_b(x) \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q).$$

Proof. For every $q \in [0,1]$, the following equation holds:

$$E_{F_i}[v_b(x) \mid x \leq F_i^{-1}(q)] - E_{Fj}[v_b(x) \mid x \leq F_j^{-1}(q)] =$$

$$= E_{F_i}[x + (v_b(x) - x) \mid x \leq F_i^{-1}(q)] - E_{F_j}[x + (v_b(x) - x) \mid x \leq F_j^{-1}(q)] \tag{A.67}$$

Rearranging the RHS, we obtain:

$$E_{F_i}\left[x \mid x \leq F_i^{-1}(q)\right] - E_{F_j}\left[x \mid x \leq F_j^{-1}(q)\right]$$
$$+E_{F_i}\left[(v_b(x) - x) \mid x \leq F_i^{-1}(q)\right] - E_{F_j}\left[(v_b(x) - x) \mid x \leq F_j^{-1}(q)\right] \tag{A.68}$$

It follows from the antecedent of Lemma A.9 that we only need to prove that

$$E_{F_i}[(v_b(x) - x) \mid x \leq F_i^{-1}(q)] - E_{F_j}[(v_b(x) - x) \mid x \leq F_j^{-1}(q)] \geq 0. \tag{A.69}$$

We know that $F_i \succ_{LIR} F_j$ and that $\mathbb{E}_{F_j}[x] = \mathbb{E}_{F_i}[x]$. It follows that F_j is a mean-preserving spread of F_i. In addition, for every quantile $q \in [0,1]$, the distribution F_i truncated at $F_i^{-1}(q)$ dominates the distribution F_j truncated at $F_j^{-1}(q)$ in the sense of second-order stochastic dominance.[19] Thus, we can deduce that every risk-averse agent would prefer the distribution F_i truncated

19. The distribution F_k truncated at $F_k^{-1}(q)$ is the distribution with support $[0, F_k^{-1}(q)]$ and CDF $\frac{F_k(x)}{q}$.

at $F_i^{-1}(q)$ to the distribution F_j truncated at $F_j^{-1}(q)$. Specifically, an agent with the VNM utility function $v_b(x) - x$, who is risk-averse because $v_b(x) - x$ is increasing and concave, would prefer F_i truncated at $F_i^{-1}(q)$ to the distribution F_j truncated at $F_j^{-1}(q)$. That is, we have:

$$E_{F_i}[(v_b(x) - x) \mid x \leq F_i^{-1}(q)] \geq E_{F_j}[(v_b(x) - x) \mid x \leq f_j^{-1}(q)]. \tag{A.70}$$

This ends the proof of Lemma A.9. □

We already know that a sufficient condition for $PT_{v_b}(F_i) \geq PT_{v_b}(F_j)$ is that for every quantile $q \in [0, 1]$ the next inequality holds:

$$E_{F_i}[v_b(x) \mid x \leq F_i^{-1}(q)] - F_i^{-1}(q) \geq E_{F_j}[v_b(x) \mid x \leq F_j^{-1}(q)] - F_j^{-1}(q). \tag{A.71}$$

Rearranging (A.71) we obtain:

$$E_{F_i}[v_b(x) \mid x \leq F_i^{-1}(q)] - E_{F_j}[v_b(x) \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q). \tag{A.72}$$

By Lemma A.9, we have that if, for every $q \in [0, 1]$, it holds that

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q), \tag{A.73}$$

then (A.72) also holds for every $q \in [0, 1]$. It follows that if (A.73) holds for every $q \in [0, 1]$ then $PT_{v_b}(F_i) \geq PT_{v_b}(F_j)$, and we have already proved that (A.73) holds for every $q \in [0, 1]$ if and only if $F_i \succ_{LIR} F_j$.

□

A.12. Proof of Proposition 9

Let $\pi(F, F')$ denote the seller's utility in the case where the buyers believe that the asset is distributed according to F, while the true distribution of the asset is F'. We need to show that

$$\mathbb{E}\left[\pi\left(F_i, F_j\right)|\varepsilon \in \{-a, +a\}\right] > \mathbb{E}\left[\pi\left(F_i, F_i\right)|\varepsilon \in \{-a, +a\}\right]. \tag{A.74}$$

First, it is easy to see that $\pi\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_i^2\right), N\left(\tilde{\mu}_i(s), \tilde{\sigma}_j^2\right)\right) > \pi\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_i^2\right), N\left(\tilde{\mu}_i(s), \tilde{\sigma}_i^2\right)\right)$ If the true variance is higher than the buyers' belief, the option value of the seller's payoff implies that her profits are larger.

Second, we argue that

$$\mathbb{E}\left[\pi\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_j^2\right), N\left(\tilde{\mu}_j(s), \tilde{\sigma}_j^2\right)\right)|\varepsilon \in \{-a, a\}\right] > \tag{A.75}$$
$$> \mathbb{E}\left[\pi\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_j^2\right), N\left(\tilde{\mu}_i(s), \tilde{\sigma}_j^2\right)\right)|\varepsilon \in \{-a, a\}\right].$$

In the case where $\varepsilon = a$ we have

$$R_{N\left(\tilde{\mu}_j(s), \tilde{\sigma}_j^2\right)}\left(\hat{p}_i(s)\right) - R_{N\left(\tilde{\mu}_i(s), \tilde{\sigma}_j^2\right)}\left(\hat{p}_i(s)\right) > \left(\tilde{\mu}_j(s) - \tilde{\mu}_i(s)\right)\Delta, \tag{A.76}$$

while in the case where $\varepsilon = -a$ we have

$$R_{N\left(\tilde{\mu}_j(-s),\tilde{\sigma}_j^2\right)}\left(\hat{p}_i\left(-s\right)\right) - R_{N\left(\tilde{\mu}_i(-s),\tilde{\sigma}_j^2\right)}\left(\hat{p}_i\left(-s\right)\right) < -\left(\tilde{\mu}_j\left(-s\right) - \tilde{\mu}_i\left(-s\right)\right)\Delta. \tag{A.77}$$

Since both realization are equally likely the inequality is implied.

Finally, note that we showed that, for every pair of realizations of the noise term, the seller's expected profit is strictly higher. Thus, a deviation to F_j is necessarily profitable.

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