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Dynamic Tax Evasion and Growth With Heterogeneous Agents

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Dynamic Tax Evasion and Growth with Heterogeneous Agents*

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Abstract

We develop a tractable model of a production economy in which public capital improves aggregate productivity and the taxpayers have heterogeneous evasion opportunities. We show that, by issuing bonds, compliant taxpayers supply the evaders with an instrument to hedge against auditing risks, thereby expanding their evasion capacity. Moreover, we demonstrate that a higher share of tax evaders reduces the economy's total factor productivity but has a hump-shaped relationship with the growth rate of aggregate capital. (JEL: E20, G11, H26)

Keywords: Dynamic tax evasion; general equilibrium; growth; heterogeneous agents

1 Introduction

The idea that tax evasion generates capital misallocation and hinders economic growth appears in the works of Fullerton and Karayannis (1994) and Roubini and Sala-i-Martin (1995). More recently, and along the same lines, Ordonez (2014) and López (2017) develop models in which tax evasion redistributes resources towards firms that reduce their size and productivity with the aim to remain undetected. Close to these studies, Di Nola et al. (2021)

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show that tax evaders can be less productive than tax-compliant firms, precisely because of their fiscal advantage.

An important aspect which is not accounted for in this literature is that, due to institutional constraints (i.e., policies, laws, and regulations), several taxpayers have limited or even no evasion opportunities. Employees, for example, cannot avoid taxes because their compensation is taxed at the source. Conversely, self-employed workers are always able to under-report their earnings.

With this in mind, this note develops a tractable macroeconomic model in which production involves both private and (tax-financed) public capital. Furthermore, only a fraction of taxpayers can conceal their income. In this context, we demonstrate that, by selling bonds, tax-compliant households provide tax evaders with a hedging instrument against auditing risks, thereby fostering their incentives to evade. Then, we show that having a larger share of households who cannot evade taxes improves the total factor productivity but has a hump-shaped relationship with the growth rate of aggregate capital. Finally, we find that having heterogeneous evasion opportunities increases the tax rate that maximises public revenues compared to a representative-agent economy featuring only tax evaders.

Tax evasion choices are made in a general equilibrium dynamic economy. This way, we generalise previous works developed either in a static environment (e.g, Dessy and Pallage, 2003) or in a dynamic but partial equilibrium framework (Lin and Yang, 2001; Dzhumashev and Gahramanov, 2011; Bernasconi et al., 2015; Levaggi and Menoncin, 2013, 2016). Concerning the link between evasion and growth, we relate to the work of Chen (2003), which first studies tax evasion in a macro model with public capital.

2 Model

The continuous-time economy, with time $t \in [0, \infty)$, is populated by a continuum of households indexed as $i \in \mathbb{I}$, a representative firm, and a public sector. Households are born at time zero with initial net worth $n_{0,i}$ and either classify as tax compliant ($i = h$) or tax evaders ($i = e$). Their net worth is continuously (and frictionlessly) allocated between a bond $b_{t,i}$ and private capital $k_{t,i}$. The former asset yields the risk-free rate r_t. The latter can be rented to the representative firm at the competitive rate A_t. However, its total (log) returns are uncertain and fluctuate with constant volatility σ^2. Therefore, holding capital yields

$$dk_{i,t} = k_{i,t}(A_t dt + \sigma dZ_t), \tag{1}$$

in which Z_t denotes a Brownian Motion defined on the filtered probability space $(\Omega, \mathbb{P}, \mathcal{H})$.

Income from the bond is tax-free. Conversely, the public sector levies a proportional tax $\tau \in [0, 1]$ on capital revenues, which is used to finance the supply of public capital G_t. Taking G_t as given, the firm uses aggregate capital $K_t := \int_{\mathbb{I}} k_{t,i} di$ to produce output with the technology

$$Y_t = \alpha K_t^{\beta} G_t^{1-\beta}, \tag{2}$$

in which α and β are positive constants. To characterize the rental rate of capital, we conjecture that $G_t = g_t K_t$; a condition that will be verified in equilibrium.[1] This allows us to rewrite Eq. (2) as

$$Y_t = \alpha g_t^{1-\beta} K_t. \tag{3}$$

Equipped with Eq. (3), the firm's zero-profits condition implies $A_t = \alpha g_t^{1-\beta}$.

The difference between tax-compliant and tax-evading households is that the former (e.g., employees) are taxed at source and, thus, cannot conceal their income from the public agency. Accordingly, by imposing the budget constraint $n_{t,h} = b_{t,h} + k_{t,h}$, their net worth evolves with dynamics

$$dn_{t,h} = \underbrace{(n_{t,h} - k_{t,h})}_{=b_{t,h}} r_t dt + dk_{t,h} (1 - \tau) - c_{t,h} dt, \tag{4}$$

in which $c_{t,h}$ labels instantaneous consumption flows.

As in Levaggi and Menoncin (2016), households who self-report their income (e.g., self-employed) may conceal from the public agency a certain amount of capital $\tilde{k}_{t,e}$ with the aim of avoiding tax payments. By doing so, they face the possibility of being audited and fined. Auditing events are modelled as independent Poisson processes with constant intensity λ and denoted as $d\Pi_{t,e}$; as in Allingham and Sandmo (1972), evasion fines are a fixed share η of evaded income. Therefore, tax evaders' net worth satisfies the budget

[1] In this context, G can be interpreted as a "pure" public good (i.e., non-rivalrous and non-excludable), such as broadband and mobility infrastructures, which benefits individual firms proportionally to the volume of their activity.

constraint $n_{t,e} = b_{t,e} + k_{t,e} + \tilde{k}_{t,e}$ and evolves as

$$dn_{t,e} = \underbrace{\left(n_{t,e} - k_{t,e} - \tilde{k}_{t,e}\right)}_{=b_{t,e}} r_t dt + dk_{t,e}\left(1-\tau\right) - c_{t,e}dt + d\tilde{k}_{t,e} - \eta\tilde{k}_{t,e}d\Pi_{t,e}. \tag{5}$$

2.1 Households' problem

All households have log preferences and discount future utility at the constant rate ρ. Their optimization problem is

$$\max_{\{c_{t,i},k_{t,i},\tilde{k}_{t,i}\}} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\ln c_{t,i}dt\right], \text{ with } i \in \{h, e\}, \tag{6}$$

subject to either Eq. (4) or Eq. (5). As we show in Appendix A.1, solving this problem yields the following.

Proposition 1. ***(Optimal policies)*** *When $i = h$ (without evasion), the optimal policies solving the problem in Eq. (6) are*

$$c_{t,h}/n_{t,h} = \rho, \tag{7}$$

$$k_{t,i}/n_{t,i} \coloneqq \theta_{t,h} = \frac{A_t\left(1-\tau\right) - r_t}{\sigma^2\left(1-\tau\right)^2}; \tag{8}$$

When $i = e$ (with evasion), they are

$$c_{t,e}/n_{t,e} = \rho, \tag{9}$$

$$k_{t,e}/n_{t,e} \coloneqq \theta_{t,e} = \frac{A_t\left(1-\tau\right) - r_t}{\sigma^2\left(1-\tau\right)^2} - \frac{1}{\eta\left(1-\tau\right)} + \frac{\lambda}{r_t\tau}, \tag{10}$$

$$\tilde{k}_{t,e}/n_{t,e} \coloneqq \tilde{\theta}_{t,e} = \frac{1}{\eta}\left(1 - \frac{\lambda\eta}{r_t}\frac{1-\tau}{\tau}\right). \tag{11}$$

Consumption rates are constant and equal to the subjective discount rate ρ. Portfolio choices, instead, vary over time, depending on the household type. Tax-compliant households implement a mean-variance strategy. Conversely, as in Levaggi and Menoncin (2016) tax evaders trade off the risk of being audited and fined for higher expected (tax-free) returns on capital.[2] Note that the optimal tax evasion rate in $\tilde{\theta}_{t,e}$ must lie in the interval $[0, 1]$, which

[2]Bond holdings do not appear as controls in Eq. (6); they are identified residually as $b_{t,h}/n_{t,h} = 1 - \theta_{t,h}$ and $b_{t,e}/n_{t,e} = 1 - \theta_{t,e} - \tilde{\theta}_{t,e}$ by using households' budget constraints.

happens if and only if $\lambda\eta(1-\tau)/(1-\eta)\tau \geq r_t \geq \lambda\eta(1-\tau)\tau$. Since the risk-free rate is determined in equilibrium, we are not able to verify this condition ex-ante; we will thus do it ex-post after having solved the model numerically.

2.2 Equilibrium

Definition 1. ***(Competitive equilibrium)*** *A competitive equilibrium, denoted as Ω, is a map of histories of shocks $\{Z_t\}$ to macroeconomic aggregates such that households solve the problem in Eq. (6) and all markets clear.*

The first clearing condition requires bonds to be in zero net supply:

$$(1-\theta_{t,h})\, N_{t,h} + (1-\theta_{t,e}-\tilde{\theta}_{t,e})N_{t,e} = 0, \tag{12}$$

in which $N_{t,h} = \int_{\mathbb{I}} n_{t,h} dh$ $N_{t,e} = \int_{\mathbb{I}} n_{t,e} de$. The second is that total capital equals aggregate net worth:

$$K_t = N_{t,h} + N_{t,e}. \tag{13}$$

The third is that public goods equal total taxes plus (average) auditing revenues:

$$G_t = \tau A_t \left[N_{t,h}\theta_{t,h} + N_{t,e}\theta_{t,e}\right] + \lambda\eta\tilde{\theta}_{t,e}N_{e,t}. \tag{14}$$

To characterize the subject of Definition 1, we look for an equilibrium that is Markovian in the wealth share of tax-compliant households $\phi \coloneqq N_{t,h}/\left(N_{t,h}+N_{t,e}\right)$, which acts as a unique state variable. Then, we verify that the equilibrium exists and identify Ω as $\{r,g\} : \phi \to \mathbb{R}^2$ (details appear in Appendices A.2 and A.3). For the sake of clear notation, we henceforth drop all time subscripts t.

Proposition 2. ***(State dynamics)*** *The state variable ϕ has the law of motion*

$$\frac{d\ln\phi}{dt} = r + \theta_h\left[\left(A-\sigma_K\sigma\right)(1-\tau) - r\right] - \rho - \iota + \sigma_K^2, \tag{15}$$

in which

$$\sigma_K = \sigma\phi\theta_h\,(1-\tau) + \sigma\,(1-\phi)\left[\theta_e + \theta_h\,(1-\tau)\right],$$

$$\iota = A[(1-\tau)(\theta_h\phi + \theta_e(1-\phi)) + (1-\phi)\tilde{\theta}_e] - A(1-\phi)\tilde{\theta}_e\lambda\eta - \rho,$$

with $g = A\tau(\phi\theta_h + (1-\phi)\theta_e) + A\tilde{\theta}_e(1-\phi)\eta\lambda$.

The equilibrium does not depend on the wealth distribution *within* household types, but only on that *between* them. This is because both optimal policies and taxes are linear in individual net worths. As a result, the supply of public goods is also linear in capital, which verifies the conjecture in Eq. (3). Another remark concerns the choice of modelling auditing processes that are independent across tax evaders. Due to this assumption, the total amount of fines reducing tax evaders' net worth enters the market clearing condition as a deterministic rather than a "jump" process. The third implication of Proposition 2 is that, even though the economy features aggregate uncertainty, the state variable has a deterministic law of motion.

Lemma 1. ***(Steady state)*** *The state variable* ϕ *has a steady state* $\bar{\phi} \in [0,1]$, *which satisfies*

$$\frac{d\ln\phi}{dt} = 0 \Longleftrightarrow \bar{r} + \bar{\theta}_h[(\bar{A} - \bar{\sigma}_K\sigma)(1-\tau) - \bar{r}] = \rho + \bar{\iota} - \bar{\sigma}_K^2. \tag{16}$$

As we are not able to further characterize the steady-state equilibrium and its transition dynamics analytically, we now explore them numerically.

3 Numerical analysis and discussion

In line with Bernasconi et al. (2020), we calibrate the model's parameters as follows: $\alpha = 0.45$, $\beta = 0.9$, $\tau = 0.35$, $\lambda = 0.1$, $\rho = 0.02$, $\sigma = 0.3$, $\eta = 0.55$.

The blue solid lines in Figure 1 show households' portfolios and the risk-free rate as functions of ϕ; the red stars mark the steady state $\bar{\phi}$. What stands out is that tax-compliant households finance capital holdings by issuing bonds (Panels (a) and (b)). By doing so, they supply evaders with a hedging instrument against auditing risk. Accordingly, risk-free rates are lower than what they would be in a homogeneous-agent economy ($\bar{\phi} = 1$, Panel (c)). Additional net worth in the hand of tax-compliant households corresponds to a higher supply of hedging instruments, which allows for higher evasion rates (Panel (e)).

Figure 2 reports the macroeconomic aggregates. In line with the result of Lemma 1, the state variable drifts deterministically towards a steady-state level $\bar{\phi}$ (Panel (a)) (i.e., $\phi\mu^\phi$ is positive when ϕ is small, and vice versa). The supply of the public good g, which basically determines the total factor productivity A, is strictly increasing in ϕ because a lower share

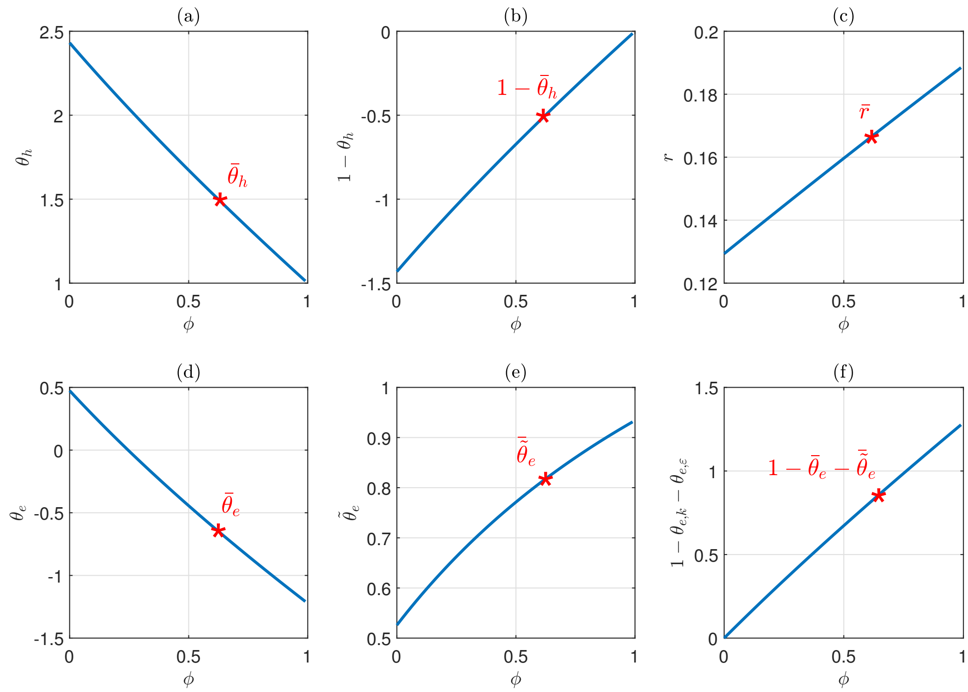

Figure 1: Capital allocations and the risk-free rate as functions of ϕ.

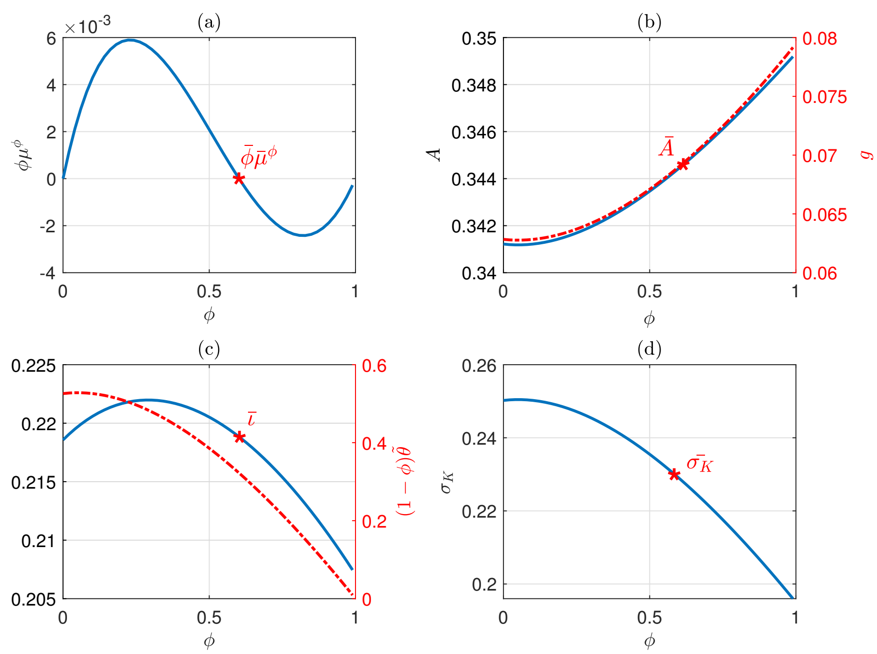

Figure 2: Macroeconomic aggregates as functions of ϕ.

of capital in the hands of tax evaders is associated with a wider tax base and a lower tax evasion in the aggregate (Panels (b) and (c)). The investment rate of capital ι is a hump-shaped function of ϕ (Panel (c)). This is because, when tax evaders are relatively few (ϕ is large), many resources are subtracted from private investments due to taxation. Conversely, when tax evaders are many (ϕ is small), a higher share of aggregate capital is concealed from taxes, thereby scaling down productivity (Panel (b)). The volatility of aggregate capital σ_K is overall decreasing in ϕ because, due to the presence of taxes, evaders' portfolios are more volatile than those of their tax-compliant peers.

To conclude the analysis, the solid blue lines in Figure 3 plot the Laffer curve in the heterogeneous-agents economy (Panel (a)) and the corresponding steady-state level $\bar{\phi}$ (Panel (b)) as functions of τ.[3] For comparison, the red dashed line depicts the same curve in a representative-agent economy in which all households are tax evaders. In both economies, the curve slopes upward when τ is low because more taxes improve aggregate productivity by fostering public spending (Panel (a)). After reaching its maximum, the curve slopes downward because increasing levels of tax evasion end up eroding the size of the tax base. Including tax-compliant households in the economy amplifies these trends, thereby increasing the tax rate level that maximizes public revenues. When τ is small, the curve is steeper than in the benchmark economy because tax-compliant households make the tax base less sensitive to variations in the fiscal policy. When τ is larger, the curve is steeper because tax evaders take over an increasingly higher share of aggregate capital (Panel (b)).

4 Conclusions

We develop a tractable model of a production economy where taxpayers have heterogeneous evasion opportunities. We solve the model for its competitive equilibrium and show that, by issuing bonds, tax-compliant households supply evaders with hedging instruments against auditing risks, thereby increasing their evasion capacity. Second, we find that aggregate productivity is decreasing in tax evaders' relative wealth share, but investments

[3]The Laffer curve can be (implicitly) derived by substituting households' optimal policies in the clearing condition for public goods and dividing by K, which yields

$$\alpha\bar{g}^{(1-\beta)} = (\tau\alpha)^{-1}[(\bar{g} - \lambda\eta\bar{\bar{\theta}}_e) + \bar{\phi}\lambda\eta\bar{\bar{\theta}}_e]/[(\bar{\theta}_h - \bar{\theta}_e)\bar{\phi} + \bar{\theta}_e].$$

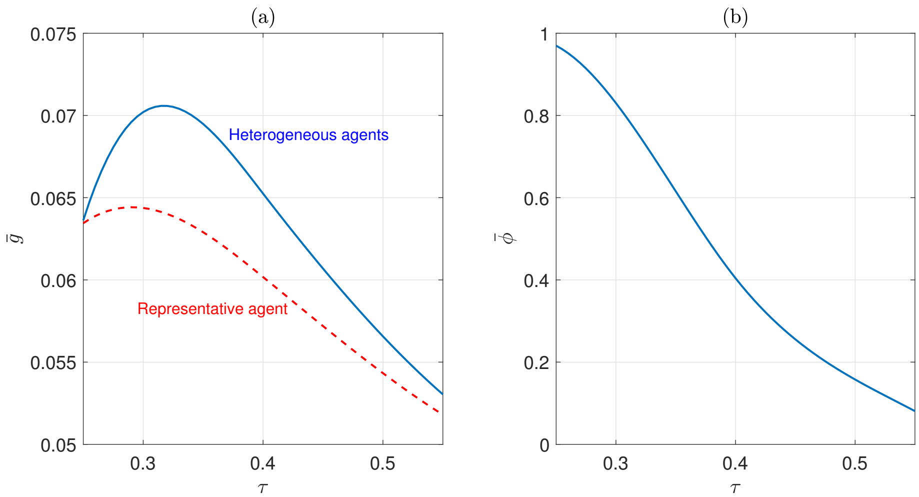

Figure 3: Laffer curve and the wealth distribution in the steady state.

are hump-shaped. When tax evaders are few (many), capital grows slowly due to high taxation (low total factor productivity). Third, we show that having heterogeneous evasion opportunities increases the tax rate which maximizes public revenues relative to the representative-agent economy featuring only tax evaders.

A Appendix

A.1 Proof of Proposition 1

Let $d\phi = \phi\left(\mu^{\phi}dt + \sigma^{\phi}\right)dZ$ be the dynamics of an arbitrary state variable. Then, the value function of tax evaders V solves the HJBE

$$(\rho+\lambda)\,V(n,\phi) = \max_{c,k,\tilde{k}} \left\{ \begin{array}{c} \log c + \frac{\partial V}{\partial n}\mu_n + \frac{1}{2}\frac{\partial^2 V}{\partial n^2}(\sigma_n)^2 + \frac{\partial V}{\partial \phi}\phi\mu^{\phi} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi^2}(\phi\sigma^{\phi})^2 + \\ +\frac{\partial^2 V}{\partial\phi\partial n}V\phi\sigma^{\phi}\sigma_n + \lambda V(n(1-\eta\tilde{k}/n),\phi) \end{array} \right\}, \tag{17}$$

in which μ_n and σ_n are the drift and diffusion terms in Eq. (5). By considering the guess function $V(n,\phi) = v(\phi) + \rho^{-1}\ln n$, the FOCs are

$$c: \;\; c = \rho n, \tag{18}$$

$$\tilde{k}: \;\; A - r = [k/n(1-\tau) + \tilde{k}/n]\sigma^2 + \eta\lambda(1-\eta\tilde{k}/n)^{-1}, \tag{19}$$

$$k: \; [A(1-\tau)-r]/\sigma^2(1-\tau) = k/n\,(1-\tau) + \tilde{k}/n. \tag{20}$$

Eqs. (18)-(20) can be rearranged to obtain those in Proposition 1. Optimal bond holdings satisfy $b = n - k - \tilde{k}$. By substituting these objects in Eq. (17) and rearranging, one obtains an ODE for the value of the unknown function v

$$\rho v = \Theta/\rho + \frac{\partial v}{\partial \phi}\phi\mu^{\phi} + \frac{1}{2}\frac{\partial^2 v}{\partial \phi^2}(\phi\sigma^{\phi})^2, \tag{21}$$

in which $\Theta = \frac{\log\rho - 1}{\rho} + r + \theta_e(A(1-\tau)-r) + \tilde{\theta}_e(A-r) - 0.5\sigma^2[\theta_e(1-\tau)+\tilde{\theta}_e]^2 + \lambda\log(1-\eta\tilde{\theta}_e)$. The problem of tax-compliant households can be solved by following the same steps while setting $\tilde{k} = \lambda = 0$.

A.2 Proof of Proposition 2

Considering the optimal strategies in Proposition 2, households' aggregate net worths are

$$dN_e = [r + \theta_e(A(1-\tau)-r) + \tilde{\theta}_e\,(A-r) - \rho - \tilde{\theta}_e\eta\lambda]dt + [\theta_e(1-\tau)+\tilde{\theta}_e]\sigma dZ, \tag{22}$$

$$dN_h = [r + \theta_h\,(A\,(1-\tau)-r) - \rho]\,dt + \theta_h\,(1-\tau)\,\sigma dZ. \tag{23}$$

By using that $N_h + N_e = K$ and applying Itô's lemma to the definition of ϕ, one gets

$$d\phi/\phi = dN_h/N_h - dK/K + dK^2/K^2 - dN_h dK/(N_h K), \tag{24}$$

$$dK/K = \phi dN_h/N_h + (1-\phi)dN_e/N_e. \tag{25}$$

By substituting Eqs. (22)-(23) in Eqs. (24)-(25) and using that $\phi\theta_h + (1-\phi)\,(\tilde{\theta}_e + \theta_e) = 1$, the diffusion term $\phi\sigma^{\phi}$ vanishes and the results of Proposition 2 follow suit.

A.3 Equilibrium characterization and numerical solution

To find $\Omega : \phi \to \mathbb{R}^2$, we match the results of Proposition 1 with Eqs. (12)-(14) to obtain

$$\begin{cases} g = \tau\alpha g^{1-\beta}[\frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2}]\phi + (1-\phi)[\tau\alpha g^{1-\beta}[\frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2} - \frac{1}{\eta(1-\tau)} + \frac{\lambda}{r\tau}] + \lambda - \frac{\lambda^2\eta}{r}\left(\frac{1-\tau}{\tau}\right)], \\ [1 - \frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2}]\phi + [1 - \frac{\lambda}{r\tau} - \frac{\alpha_\tau g^{1-\beta}-r}{\sigma_\tau^2} + \frac{1}{\eta}(\frac{\tau}{1-\tau})](1-\phi) = 0, \end{cases} \tag{26}$$

in which $\sigma_\tau \coloneqq (1-\tau)\sigma$ and $\alpha_\tau \coloneqq (1-\tau)\alpha$. We solve this system numerically by means of a non-linear solver to find the couple $\{r, g\}$ for each $\phi \in [0, 1]$

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