Discussion Paper Series – CRC TR 224

Discussion Paper No. 278
Project B 05

Partial Compatibility in Oligopoly

Federico Innocenti [1]
Domenico Menicucci [2]

March 2021

[1] University of Mannheim, Department of Economics, L7, 3-5, 68161 Mannheim, Germany;
email: finnocen@mail.unimannheim.de; orcid id 0000-0003-0650-7438
[2] Corresponding author: Università degli Studi di Firenze, Dipartimento di Scienze per l'Economia e l'Impresa, Via delle Pandette 9, I-50127 Firenze, Italy; phone: +39-055-2759666,
email: domenico.menicucci@unifi.it; orcid id 0000-0002-1891-2599

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Partial compatibility in oligopoly

Federico Innocenti* and Domenico Menicucci†

March 9, 2021

Abstract

This paper examines the issue of product compatibility in an oligopoly with three multi-product firms. Whereas most of the existing literature focuses on the extreme cases of full compatibility or full incompatibility, we look at asymmetric settings in which some firms make their products compatible with a standard technology and others do not. Our analysis reveals each firm's individual incentive to adopt the standard, and allows to study a two-stage game in which first each firm chooses its technological regime (compatibility or incompatibility), then price competition occurs given the regime each firm has selected at stage one. When firms are ex ante symmetric, we find that for each firm, compatibility weakly dominates incompatibility. In a setting in which a firm's products have higher quality than its rivals' products, individual incentives to make products incompatible emerge, first for the firm with higher quality products, then also for the other firms, as the quality difference increases. This paper sheds lights on markets in which some firms adopt the standard technology but other firms use proprietary systems.

Keywords: Compatibility, Spatial competition, Vertical differentiation, Asymmetric equilibrium, Competitive Bundling

JEL numbers: D43, L13.

Acknowledgments We thank one anonymous associate editor, two anonymous reviewers, Carl-Christian Groh, Daniil Larionov, Volker Nocke, Martin Peitz, Fabian Schmitz for helpful comments, and the participants to the Bonn Mannheim PhD Workshop 2019, where our work has been presented.

Funding: Funding by the German Research Foundation (DFG) through CRC TR 224 (Project B05) is gratefully acknowledged.

Declarations of interest: None.

*University of Mannheim, Department of Economics, L7, 3-5, 68161 Mannheim, Germany; email: finnocen@mail.uni-mannheim.de; orcid id 0000-0003-0650-7438

†Corresponding author: Università degli Studi di Firenze, Dipartimento di Scienze per l'Economia e l'Impresa, Via delle Pandette 9, I-50127 Firenze, Italy; phone: +39-055-2759666, email: domenico.menicucci@unifi.it; orcid id 0000-0002-1891-2599

1 Introduction

During our routine life we make extensive use of objects that are made of several complementary components, which generate utility only as a system. Examples include a printer and its cartridges; a coffee maker and its capsules; an operating system and other software; a smartphone and its battery charger. Consumers' behavior is affected by whether a system's components produced by different firms are compatible or not. Most of the times, these industries are characterized by a standard technology that each firm may decide to adopt. In alternative, a firm may develop and use a proprietary technology. Precisely, we consider perfectly complementary goods for which the producers can introduce technological barriers limiting their usage (note that our examples above meet these conditions). In practice, each firm chooses whether to make the components it offers compatible or not with the standard. If the components are compatible, then each consumer can "mix and match" them in his preferred way; otherwise a system works only if its components are produced by a same firm. Clearly, compatibility makes consumers better off, holding prices fixed, because it allows each consumer to buy his preferred variety of each component. We examine in this paper whether compatibility is profitable from each firm's perspective.

We study oligopoly competition when each firm either adopts the standard (i.e., chooses compatibility, denoted with C in the following) - this makes its components compatible with those of the competitors that adopt the standard - or a proprietary technology (i.e., chooses incompatibility, denoted with NC) which is used only by that firm. The existing literature has compared the extreme cases of competition under full compatibility (here it arises when each firm chooses C) with competition under full incompatibility (here it arises if each firm chooses NC), and has identified sufficient conditions for one regime to be more profitable than the other. We extend the analysis to intermediate cases with partial compatibility, that arise when some firms choose C but some do not.[1]

In our model there are three firms.[2] Each firm offers two components, A and B. Each consumer gets a positive utility only from consumption of both components, i.e. of a system. We model product differentiation by assuming that for each component consumers are independently and uniformly distributed over a Salop's circle (Salop, 1979). On each circle the firms are located equidistantly, in the same way on the two circles, and a firm's location on a circle represents the variety of the component (to which the circle refers) the firm is offering. In this context, we examine the following two-stage game:

- at stage one, each firm chooses its technological regime: either C or NC;
- at stage two, firms compete in prices given the technological regimes chosen at stage one.[3]

Our first result is that in a setting with ex ante symmetric firms, C weakly dominates NC for each firm. We know from Kim and Choi (2015) that under our assumptions, firms' collectively prefer full compatibility to full incompatibility. Unlike Kim and Choi (2015), we allow firms to choose different technological regimes, but prove that each firm prefers C to NC, independently of its rivals' choices; thus all firms choose C in each undominated equilibrium. In other words, even though each firm can make a distinct compatibility choice, full compatibility emerges, and as a dominant strategy equilibrium.

[1] To the best of our knowledge, only Chen (1997) addresses a related question, in the context of bundling. Later on in the introduction we describe the model in Chen (1997) and point out the differences with our paper.

[2] In a duopoly we cannot examine the effect of different compatibility choices because as soon as one firm chooses NC, in practice each consumer must buy the entire system either from one firm or from the other.

[3] Asymmetric regimes lead to asymmetric price competition games. This requires to apply a case-by-case approach that depends on the number of firms choosing NC and on the locations of these firms. Therefore, for reasons of tractability we restrict attention to a setting with three firms. In Innocenti and Menicucci (2021) we allow for four firms, which does not modify our results for the case of symmetric firms.

In order to develop an intuition for this result, consider a firm i that chooses NC and assume that all other firms choose C. This restricts consumers' choice set and forces some consumers to select a second best option because they cannot mix and match one component of firm i and one component of another firm. It turns out that this inflicts a profit loss to firm i. As a starting point, assume that the price of the system of firm i is the sum of the prices of firm i's individual components in the equilibrium with full compatibility. Then, consider the "mix and match consumers" of firm i that under full compatibility buy only one component of firm i. These consumers must now decide whether to buy firm i's entire system or not. Only a minority of them, those located not far away from firm i on both circles, do so. Thus, firm i loses most of its mix and match consumers. The reduced market share induces a price cut by firm i, and since prices are strategic complements, competition becomes more intense. At the equilibrium prices, firm i recovers its original market share but with a substantially reduced price that makes its profit smaller than under full compatibility.

This result suggests that even if firms can choose different technological regimes, ex ante symmetric firms all adopt the standard. Then, we introduce vertical differentiation assuming that one firm offers components with higher quality than its rivals' components (or, equivalently, one firm bears lower production costs than its rivals), and prove that this overturns the previous result.[4] The firm offering higher quality components has a higher market share. Therefore, we call it *large* firm, and call *small* the other firms. We let a positive parameter α represent the quality difference, and show that incompatibility is profitable for the large firm if α is above a threshold α'. Precisely, given a high α the market share effect of incompatibility is positive for the large firm: a majority of its mix and match consumers choose to buy the system of the large firm. Moreover, a high α also affects the intensity of price competition, making the demand of the large firm less elastic under incompatibility. This softens price competition and increases the profit of all firms with respect to full compatibility. Thus, partial compatibility may arise in equilibrium under vertical differentiation.

We also show that there exists another threshold α'' (larger than α') such that also each small firm chooses NC (given that the large firm chooses NC) when α is above α''. This occurs because, under full incompatibility, the large firm faces less competition as consumers cannot mix and match the components of the small firms. This increases the demand for the large firm, but also induces it to be less aggressive in pricing. This latter effect benefits the small firms and ultimately dominates the initial demand loss, increasing their profits. Therefore, full incompatibility emerges.

Our initial examples may provide real world cases of equilibrium with partial compatibility. Let us consider the smartphone industry. Currently, this oligopolistic industry is characterized by different technological regimes, meaning that some smartphones are incompatible with some battery chargers. In particular, Apple is using its proprietary technology, called lightning, whereas the other firms have a common standard called USB-C. Therefore, in order to charge an iPhone it is necessary to use a battery charger offered by Apple, whereas a Samsung phone (for instance) can be charged by any USB-C battery charger. In our setting, this is analogous to Apple choosing NC whereas the other firms choose C, and may suggest that Apple has a quality advantage over its competitors, perhaps due to a higher intrinsic value of its products compared to the competitors' products.

In the next subsection we briefly discuss some related literature. Then, Section 2 introduces the model. In Section 3 we deal with stage two of the game, whereas Section 4 is about stage one. In Section 5 we analyze the setting with vertical differentiation. Finally, Section 6 contains a few suggestions for future research. Since some proofs of our results are long, the appendix includes only a partial version of the proofs. The complete proofs can be found in Innocenti and Menicucci (2021).

[4] Hahn and Kim (2012) and Hurkens et al. (2019) examine a similar setting with two firms. However, as we remarked in footnote 2, when there are only two firms, in practice there cannot be asymmetric technological regimes.

1.1 Related literature

Our analysis also applies to the study of the incentives of multi-product firms to engage in bundling of products that have independent values (rather than being perfect complements). Precisely, the effect of incompatibility in our setting is equivalent to that of bundling when products have independent values: if a firm i uses its own technology, then each consumer either buys both components from firm i, or buys no component at all from firm i, just as if firm i were offering only the pure bundle of its products. Conversely, compatibility is equivalent to separate sales (no bundling) of firm i's products. Therefore, our paper is related to the literature on compatibility and also to the literature on competitive bundling.[5]

A seminal paper for both these literatures is Matutes and Regibeau (1988), which shows that in a two-dimensional Hotelling duopoly, competition under full incompatibility yields lower profits than competition under full compatibility.[6] However, more recent research shows that this result may not hold when more than two firms compete. In the random utility setting of Perloff and Salop (1985), Zhou (2017) shows that under suitable assumptions on the distribution of consumers' valuations, bundling essentially reduces the heterogeneity in consumer valuation. In particular, the density of the average per-product value has thinner tails compared to the density of the original single-product valuation. If the number of competing firms is sufficiently large, then thinner tails lead to higher profits for competition under bundling than under separate sales. Kim and Choi (2015) study system compatibility in a spatial model in which the market for each product is represented by a Salop's circle. They prove that with at least four firms, there exists a way to symmetrically locate the firms in the two circles (but not in the same way on the two circles) such that full incompatibility generates higher profits than full compatibility.

The above papers focus on the extreme cases of competition under full compatibility (separate sales) or under full incompatibility (bundling). By contrast, we examine competition when some firms choose compatibility and some do not, modelling product differentiation as in Kim and Choi (2015).[7] For ex ante symmetric firms, Kim and Choi (2015) show that if firms are located in the same way on the two circles, then full incompatibility reduces each firm's profit with respect to full compatibility. Thus, firms have no *collective* incentive to adopt proprietary technologies. We establish that also no single firm has an *individual* incentive to use a proprietary technology, regardless of the choices of the other firms. However, among vertically differentiated firms, a significant asymmetry generates incentives towards incompatibility first for the large firm, and then also for small firms.[8]

Our paper is also related to Chen (1997), in which two firms offer homogeneous products and each firm decides whether to offer only a single product, product A (for which consumers have homogeneous preferences), or a bundle of product A with another product, product B (for which consumers have

[5] We remark that the bundling interpretation makes the timing of the game less compelling. A sequential game is appropriate in a situation in which a firm's choice in stage one is irreversible, which is the case for a decision of compatibility/incompatibility. However, if a firm can costlessly and quickly switch from one regime to the other, then it may be appropriate to merge stage one and stage two, such that at a single stage each firm chooses bundling and the price for the bundle, or separate sales and the prices for the single products.

[6] See also Economides (1989) and Nalebuff (2000).

[7] An alternative approach is to use the random utility setting as in Zhou (2017). For the case of symmetric firms, that leads to the same result we obtain in Proposition 1. Moreover, the spatial distribution for consumers we consider allows to employ a graphical analysis to support the intuition about the firms' incentives towards C or NC (see our Subsections 4.2 and 5.5), because the consumers' space is in a one-to-one correspondence with the square $[0, 1) \times [0, 1)$. Such analysis is infeasible in a random utility model.

[8] There also exists a relationship with the literature on network goods. See for instance Crémer et al. (2000), who study a duopoly in which a firm has a larger installed base of customers than the other firm. A main result is that the large firm has a greater incentive to degrade connectivity (which is analogous to incompatibility in our setting) than the small firm, in order to have an advantage in the competition for unattached customers. However, our model is significantly different as there are no network goods; the quality advantage is independent of stage-one's choice; incentives to choose incompatibility emerge also for small firms when the large firm's quality advantage is sufficiently strong.

heterogeneous preferences). There is also a large number of perfectly competitive firms offering product B, hence no firm can make a profit by offering product B alone. After the firms have selected the products to offer, price competition takes place. Chen (1997) shows that in each equilibrium one firm offers the bundle and the other offers only product A; thus bundling emerges endogenously in equilibrium. This occurs as bundling differentiates the firms' products and softens price competition with respect to competition between homogeneous products.[9] Our paper is different because the firms offer differentiated products and there are no perfectly competitive firms which offer one of the products.

2 The setting

We consider competition among three symmetric firms, denoted firm 1, firm 2 and firm 3, each offering two different components, A and B. We let $N \equiv \{1, 2, 3\}$ denote the set of all firms. We use A_i (B_i) to denote component A (component B) offered by firm i, for each $i \in N$. The two components are perfectly complementary goods and there is no value from consumption of just one component. Each consumer has a unit demand for a system given by the union of the two components. In the following, with S_{ij}, or "system ij", we denote the system consisting of components A_i and B_j; notice that i may be equal to j.

The firms offer differentiated components and we represent this differentiation using a spatial model in which each firm is located on two Salop's circles (Salop, 1979). More precisely, like in Kim and Choi (2014, 2015) the market for each component is represented by a circle with unit length, in which a point is denoted "origin". Each point on the circle is identified by a number $x \in [0, 1)$ which represents the distance between the origin and that point, moving clockwise from the origin.

Fig. 1 A point's coordinate on the circle

Fig. 2 Distance between two points x and y on the circle

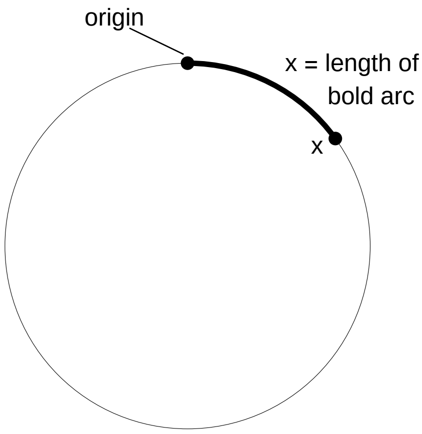

Figure 1

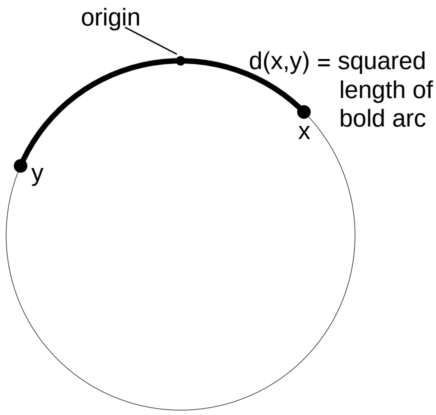

Figure 2

Each firm i is located at a point x_A^i on the circle for component A (circle A from now on) and at a point x_B^i on the circle for component B (circle B) such that $x_A^i = x_B^i$. On each circle, firms are equally-spaced (see for instance Figure 3 below). There is a unit mass of consumers and each consumer has a location x_A on circle A and a location x_B on circle B. The consumers' locations are independently and uniformly distributed on the two circles. A consumer's locations represent the consumer's ideal versions of the two components, and for a consumer with locations x_A, x_B, the utility from buying system S_{ij} is

$$V - d(x_A, x_A^i) - d(x_B, x_B^j) - \text{total payment to buy } S_{ij} \tag{1}$$

[9] A related mechanism applies also in Carbajo *et al.* (1990).

In the above expression, $V > 0$ represents the consumer's gross utility from consuming his preferred system. With $d(x, y)$ we denote the quadratic distance between two generic points x and y on the circle: $d(x, y) = d(y, x)$ and for any x, y such that $0 \leq x \leq y < 1$ (without loss of generality) we have

$$d(x,y) = \begin{cases} (y-x)^2 & \text{if } 0 \leq y - x < \frac{1}{2} \\ (1-y+x)^2 & \text{if } \frac{1}{2} \leq y - x < 1 \end{cases} \tag{2}$$

Hence, $d(x, y)$ is the quadratic length of the shortest path that connects x to y. This sometimes requires to move clockwise from x, sometimes counter-clockwise: see Figure 2. The term $d(x_A, x_A^i)$ in (1) is the distance between x_A and x_A^i on circle A. It represents the reduction in the consumer's utility from consuming a version of component A which differs from his ideal one. A similar interpretation applies to $d(x_B, x_B^j)$.[10]

We assume that there exists a unique standard technology that each firm can freely adopt - choosing *Compatibility* (C). The alternative for any firm is to adopt an exclusive proprietary technology - choosing *Incompatibility* (NC). When firm i chooses C, it sets a price for its component A_i and a price for its component B_i, but since $x_A^j = x_B^j$ for each $j \in N$ and we assume below that the marginal cost for each component is the same, we focus on the case in which firm i sets the same price p_i for both its components (about this, see footnote 15). If instead firm i chooses NC, then *de facto* it offers its components A_i and B_i jointly, and for this system, S_{ii}, what matters is just the sum of the components' prices, which we denote P_i. From a consumer's viewpoint, if firm i chooses C then the consumer can combine – mix and match – component A_i (B_i) with component B_j (A_j) as long as also firm j has chosen C. Conversely, if firm i chooses NC then a consumer either buys S_{ii}, or buys no component at all from firm i.

After the firms' choices of technological regimes, each consumer faces a set of available systems. For instance, if firms 1 and 2 have both chosen C but firm 3 has chosen NC, then $S_{11}, S_{12}, S_{21}, S_{22}, S_{33}$ are the available systems, whereas $S_{13}, S_{31}, S_{23}, S_{32}$ are not available. We assume that V in (1) is high enough to make each consumer buy a system in equilibrium. Hence, each consumer chooses the available system that yields the highest utility as evaluated in (1). That is equivalent to choosing the system with the lowest total cost. For a consumer located at (x_A, x_B), the total cost of S_{ij} is

$$d(x_A, x_A^i) + d(x_B, x_B^j) + \text{total payment to buy } S_{ij} \tag{3}$$

For each firm i, let c denote the marginal production cost for component A_i and for component B_i. Since marginal costs have an additive effect on prices, without loss of generality we simplify the notation by setting $c = 0$ and interpret prices as profit margins.[11]

The timing of the game we analyze is as follows:

- Stage one: Each firm simultaneously chooses C or NC.
- Stage two: Each firm simultaneously sets the prices of its single components or the price of its system.
- After stage two, consumers make their purchases as we have described above.

[10] We may multiply $d(x_A, x_A^i)$ and $d(x_B, x_B^j)$ by a positive number $t \neq 1$, representing the importance for a consumer of consuming a component different from his ideal one, but that would not change our results qualitatively.

[11] Precisely, consider full compatibility (to fix the ideas) and the market for component A. For each $i \in N$, let $m_i = p_i - c$ denote firm i's unit profit margin. For a consumer located at x_A in circle A, the cost of A_i can be written as $d(x_A, x_A^i) + m_i + c$. Then, c does not affect the comparisons among the costs of A_1, A_2, A_3, and since the market is fully covered, m_1, m_2, m_3 play the same roles as p_1, p_2, p_3 when $c = 0$. Thus, the demands for A_1, A_2, A_3 and the firms' profits are as when $c = 0$; hence also stage one is unchanged. The same logic applies if the two components have different marginal costs, equal across firms. The setting we examine in Section 5 is equivalent to one in which a firm has a cost advantage: see footnote 22.

We denote the whole game with Γ. We apply to Γ the notion of Subgame Perfect Nash Equilibrium (SPNE), which requires to determine a Nash Equilibrium (NE) for each subgame of Γ that may be entered at stage two. Next section is devoted to this analysis.

3 The second stage

In this section we examine stage two in Γ, in which firms compete in prices given the technological regimes determined at stage one. Precisely, we determine the equilibrium prices for each possible combination of regimes, that is for each subgame of Γ; notice that each subgame of Γ starts at stage two.

In order to distinguish different subgames, we let N' be a generic subset of N. Then we use $\gamma^{N'}$ to denote the subgame which is entered after at stage one each firm in N' has chosen NC and each firm in $N \backslash N'$ has chosen C. Hence, $\gamma^{\varnothing}$ is the subgame which is played after each firm has chosen C; $\gamma^{\{j\}}$ (from now on γ^j) is the subgame entered after only firm j has chosen NC; and γ^{123} is the subgame played after all firms have chosen NC. It is important to note that $\gamma^{12}, \gamma^{13}, \gamma^{23}$ are all equivalent to γ^{123}. Indeed, if two firms have chosen NC, then a consumer will buy the system of one of these firms, or the two components of the other firm; in each case the consumer buys both components from a same firm, as in γ^{123}. Hence, in subgames $\gamma^{12}, \gamma^{13}, \gamma^{23}, \gamma^{123}$ competition occurs among the systems S_{11}, S_{22}, S_{33}.

3.1 Competition under full compatibility: Subgame $\gamma^{\varnothing}$

Here we consider the subgame $\gamma^{\varnothing}$ that is entered if each firm chooses C at stage one. Given that each consumer's total cost is separable in the cost of the two components (see (3)), competition for the sale of component A is independent of competition for the sale of component B. Then, it is immediate to identify a symmetric NE for each single market: See Proposition 1 in Kim and Choi (2015).

Lemma 1 *In subgame $\gamma^{\varnothing}$ there exists a NE such that in the market for component A (B) the price of each component is $\frac{1}{9}$. For each firm the equilibrium profit in each market is $\frac{1}{27}$ and the total profit is $\frac{2}{27}$.*

3.2 Competition under full incompatibility: Subgame γ^{123}, or $\gamma^{12}, \gamma^{13}, \gamma^{23}$

Here we consider the case in which at least two firms have chosen NC at stage one. Then subgame γ^{123} (or an equivalent one) is entered, and for it Kim and Choi (2014) determine the following symmetric NE.

Lemma 2 (Kim and Choi (2014)) *In subgame γ^{123} there exists a NE such that the price for the system of each firm is $\frac{1}{6}$, and the equilibrium profit for each firm is $\frac{1}{18}$.*

From Lemmas 1 and 2 it is immediate to see that each firm's profit is greater in $\gamma^{\varnothing}$ than in γ^{123}. Therefore, all firms prefer that competition takes place among compatible components rather than among proprietary systems.

3.3 Asymmetric subgames: $\gamma^1, \gamma^2, \gamma^3$

In this subsection we examine the subgames $\gamma^1, \gamma^2, \gamma^3$, in which just one firm offers incompatible components. We call them *asymmetric* subgames because in these subgames firms are not in a symmetric situation. To fix the ideas, here we examine γ^2, the subgame played after only firm 2 has chosen NC; but the results we obtain apply also to γ^1 and to γ^3. In γ^2 it is computationally convenient (without loss of generality) to assume that in both circles firm 1 is located at $\frac{1}{6}$, firm 2 is located at $\frac{1}{2}$, firm 3 is located at $\frac{5}{6}$,[12] as described in Figure 3:

Fig. 3
Distribution of firms over the circle for each component :

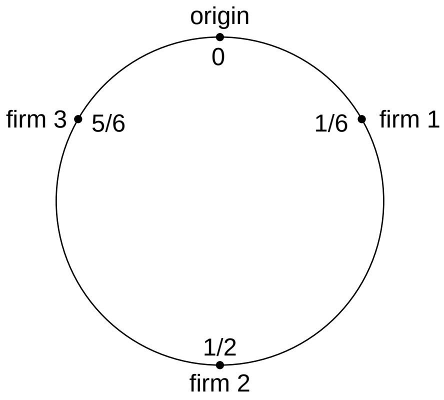

Figure 3

We denote with p_1 (p_3) the price firm 1 (firm 3) charges for each of its components, and with P_2 the price charged by firm 2 for its system. Since firms 1,3 are in a symmetric position, we focus on NE such that $p_1 = p_3$. In order to derive such NE, we now derive the demand functions for firms 2 and 3.

Demand function for firm 2 In γ^2, the available systems are $S_{11}, S_{22}, S_{13}, S_{31}, S_{33}$, and each consumer buys the system that is the cheapest for him, given p_1, P_2, p_3 and given his locations. Then, for a consumer located at (x_A, x_B), (3) reveals that the cost of S_{22} is

$$C_{22}(x_A, x_B) = (x_A - \frac{1}{2})^2 + (x_B - \frac{1}{2})^2 + P_2 \tag{4}$$

and the cost of S_{ij}, for $i = 1, 3$ and $j = 1, 3$, is

$$C_{ij}(x_A, x_B) = C_i(x_A) + C_j(x_B), \qquad \text{in which}$$

$$C_1(x) = \begin{cases} (x - \frac{1}{6})^2 + p_1 & \text{if } 0 \le x < \frac{2}{3} \\ (1 - x + \frac{1}{6})^2 + p_1 & \text{if } \frac{2}{3} \le x < 1 \end{cases} \quad \text{and} \quad C_3(x) = \begin{cases} (\frac{1}{6} + x)^2 + p_3 & \text{if } 0 \le x < \frac{1}{3} \\ (x - \frac{5}{6})^2 + p_3 & \text{if } \frac{1}{3} \le x < 1 \end{cases} \tag{5}$$

In order to derive the demand function for S_{22}, we exploit the fact that the set of consumers can be viewed as the square $[0, 1) \times [0, 1)$ in which locations are uniformly distributed. We need to identify the set of consumers for which S_{22} is the cheapest system, that is the set of solutions to the inequality $C_{22}(x_A, x_B) < \min\{C_{11}(x_A, x_B), C_{13}(x_A, x_B), C_{31}(x_A, x_B), C_{33}(x_A, x_B)\}$, and to evaluate the area of this set. Although this is conceptually straightforward, it requires some algebraic steps that we describe in the appendix. Here we only describe the result.

We let p denote the common equilibrium value of p_1 and p_3, and let R_{22} denote the subset of $[0, 1) \times [0, 1)$ in which S_{22} is the cheapest system. Then, we find that R_{22} depends on $P_2 - 2p$ as follows: R_{22} is the whole $[0, 1) \times [0, 1)$ if $P_2 - 2p < -\frac{4}{9}$, R_{22} is empty if $P_2 - 2p \ge \frac{2}{9}$. If $P_2 - 2p$ is between $-\frac{4}{9}$ and $\frac{2}{9}$, then R_{22} is a more complicated convex polygon which we describe by listing its vertices. In particular, given $\mathbf{x} = (x_A, x_B) \in [0, 1) \times [0, 1)$, we use $\bar{\mathbf{x}}$ to denote the point that is obtained by permuting the coordinates of $\mathbf{x}$, that is $\bar{\mathbf{x}} = (x_B, x_A)$. It turns out that R_{22} is the octagon in Figure 4 if $-\frac{4}{9} \le P_2 - 2p < -\frac{1}{9}$; R_{22}

[12]The reason is that $d(x, \frac{1}{2})$ is equal to $(x - \frac{1}{2})^2$ for each $x \in [0, 1)$, whereas if $y \neq \frac{1}{2}$ then $d(x, y)$ is a piecewise defined function of x as in (2). Thus, $x_A^2 = x_B^2 = \frac{1}{2}$ simplifies $d(x_A, x_A^2) + d(x_B, x_B^2)$.

is the square in Figure 5 if $-\frac{1}{9} \leq P_2 - 2p < \frac{2}{9}$:[13]

Fig. 4 The set R_{22} of consumers that buy the system of firm 2 in γ^2 when $-\frac{4}{9} \leq P_2 - 2p < -\frac{1}{9}$

Fig. 5 The set R_{22} of consumers that buy the system of firm 2 in γ^2 when $-\frac{1}{9} \leq P_2 - 2p < \frac{2}{9}$

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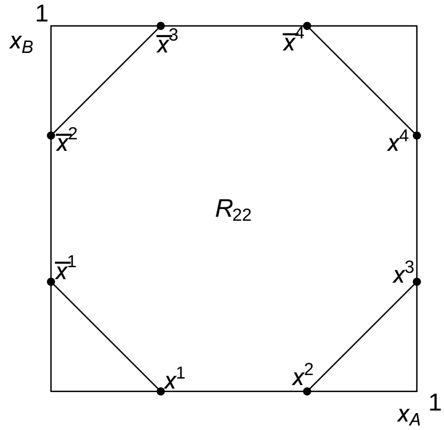

Figure 4

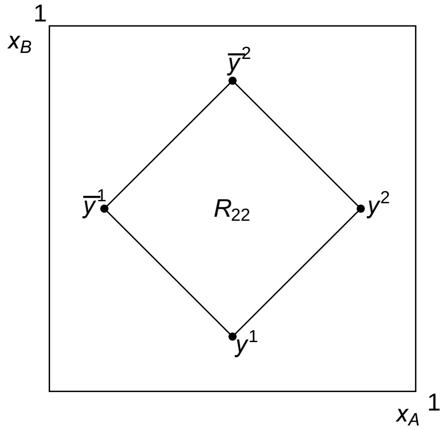

Figure 5

The demand for S_{22} is the area of R_{22}, hence

$$D_2(P_2) = \begin{cases} 1 & \text{if } P_2 < 2p - \frac{4}{9} \\ 1 - 2(\frac{2}{3} - 3p + \frac{3}{2}P_2)^2 & \text{if } 2p - \frac{4}{9} \leq P_2 < 2p - \frac{1}{9} \\ 2(\frac{1}{3} + 3p - \frac{3}{2}P_2)^2 & \text{if } 2p - \frac{1}{9} \leq P_2 < 2p + \frac{2}{9} \\ 0 & \text{if } 2p + \frac{2}{9} \leq P_2 \end{cases} \tag{6}$$

The profit of firm 2 is $P_2 D_2(P_2)$, and we denote with $br_2(p, \gamma^2)$ the profit maximizing value of P_2, that is the best reply of firm 2 given p:[14]

$$br_2(p, \gamma^2) = \begin{cases} \frac{2}{3}p + \frac{2}{27} & \text{if } p \leq \frac{5}{36} \\ \frac{4}{3}p - \frac{8}{27} + \frac{1}{27}\sqrt{324p^2 - 144p + 70} & \text{if } p > \frac{5}{36} \end{cases} \tag{7}$$

Demand function for firm 3 and the equilibrium prices Here we are interested in the demand function for the components of firm 3, which depends on p_1, P_2, p_3. We focus on the case of p_3 close to p_1, to obtain a first order condition for p_3 (recall that we are searching for a NE such that $p_1 = p_3$). This can be combined with the best reply function of firm 2 in (7) to identify prices p, P and a candidate NE for γ^2 such that $p_1 = p$, $P_2 = P$, $p_3 = p$. Lemma 3 below establishes that this is indeed a NE of γ^2.

In order to derive the demand for firm 3, we need to determine the subsets of $[0,1) \times [0,1)$ for which either S_{13} or S_{31} or S_{33} is the cheapest system. In the appendix we provide the details, which lead to

$$D_3(p_3) = 1 + \frac{9}{2}(p_1 - p_3) - \frac{9}{2}(\frac{2}{9} + p_1 + p_3 - P_2)^2 \tag{8}$$

Therefore, for p_3 close to p_1 the profit function of firm 3 is $p_3 D_3(p_3)$.[15] From this we derive a first order condition for p_3, which must hold at $p_3 = p_1$. Combining it with (7) yields the equilibrium prices.

[13] In Figure 4, $\mathbf{x}^1 = (\lambda, 0)$, $\mathbf{x}^2 = (1-\lambda, 0)$, $\mathbf{x}^3 = (1, \lambda)$, $\mathbf{x}^4 = (1, 1-\lambda)$, with $\lambda = \frac{2}{3} + \frac{3}{2}(P_2 - 2p)$. In Figure 5, $\mathbf{y}^1 = (\frac{1}{2}, \lambda - \frac{1}{2})$, $\mathbf{y}^2 = (\frac{3}{2} - \lambda, \frac{1}{2})$, with the same λ.

[14] The best reply of firm 2 in (7) is such that if $p \leq \frac{5}{36}$ (if $p > \frac{5}{36}$), then it is optimal for firm 2 to choose P_2 that makes R_{22} equal to a square as in Figure 5 (equal to an octagon as in Figure 4).

[15] As we explain in Section 2, when a firm chooses C we focus on the case in which the firm sets the same price for both its components. If firm 3 can set $p_{3A} \neq p_{3B}$, then for p_{3A} and p_{3B} close to p_1 we find that the demand for A_3 is $D_{3A}(p_{3A}, p_{3B}) = \frac{1}{2} + \frac{9}{4}p_1 - \frac{9}{4}p_{3A} - \frac{9}{4}\left(\frac{2}{9} + p_1 + \frac{1}{2}p_{3A} + \frac{1}{2}p_{3B} - P_2\right)^2$ and the demand for B_3 is $D_{3B}(p_{3A}, p_{3B}) = \frac{1}{2} + \frac{9}{4}p_1 - \frac{9}{4}p_{3B} - \frac{9}{4}\left(\frac{2}{9} + p_1 + \frac{1}{2}p_{3A} + \frac{1}{2}p_{3B} - P_2\right)^2$. Given $p_1 = p_1^*, P_2 = P_2^*$ in (9), we can prove that the profit of firm 3, $p_{3A}D_{3A}(p_{3A}, p_{3B}) + p_{3B}D_{3B}(p_{3A}, p_{3B})$, is maximized at $(p_{3A}, p_{3B}) = (p_3^*, p_3^*)$, consistently with Lemma 3.

Lemma 3 *Consider the subgame γ^2 which is entered if firm 2 chooses NC and firms 1, 3 choose C at stage one. In γ^2 there exists a NE such that*

$$p_1^* = p_3^* = \frac{3\sqrt{4681} - 137}{720} = 0.0948, \qquad P_2^* = \frac{\sqrt{4681} - 19}{360} = 0.1373 \tag{9}$$

The equilibrium profit of firm 2 is 0.0466*; the equilibrium profit of each other firm is* 0.0626.

Figure 6 represents the equilibrium distribution of consumers among the available systems in γ^2, in which R_{ij}, for $i = 1, 3$ and $j = 1, 3$, is the set of consumers that buy S_{ij}:[16]

Fig. 6
The distribution of consumers among S_{11}, S_{13}, S_{31}, S_{33}, S_{22} in the NE of γ^2 :

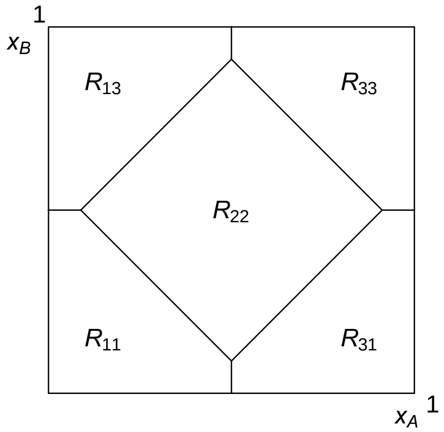

Figure 6

From Lemmas 1-3 we see that in γ^2 the profit of each firm is smaller than in $\gamma^{\varnothing}$. Moreover, firm 2 (firm 1, firm 3) has a lower (higher) profit in γ^2 than in γ^{123}. In Section 4 we provide an intuition for these results and we discuss their consequences on firms' choices at stage one.

4 The first stage

In this section we examine stage one, in which each firm chooses its technological regime, either C or NC. To this purpose, we study the stage one reduced game with simultaneous moves in which each firm's set of feasible actions is {C, NC} and given any action profile $(a_1, a_2, a_3) \in \{\text{C,NC}\}^3$, the firms' profits are given by the equilibrium profits in the subgame which is entered given (a_1, a_2, a_3). Note that (NC, NC, NC) is a NE of the reduced game. Indeed, if all firms different from firm i play NC, then firm i has no incentive to deviate by choosing C as the resulting subgame is equivalent to γ^{123}. Hence, there always exists a SPNE of Γ in which each firm offers its own proprietary system; we call it the *trivial* SPNE.

4.1 The stage one reduced game

Using Lemmas 1-3 we obtain the following stage one reduced game, in which firm 1 chooses a row, firm 2 chooses a column, firm 3 chooses a matrix:[17]

$a_3 = \text{C}$

	$a_2 = \text{C}$	$a_2 = \text{NC}$
$a_1 = \text{C}$	0.0741,0.0741,0.0741	0.0626,0.0466,0.0626
$a_1 = \text{NC}$	0.0466,0.0626,0.0626	0.0556,0.0556,0.0556

$a_3 = \text{NC}$

	$a_2 = \text{C}$	$a_2 = \text{NC}$
$a_1 = \text{C}$	0.0626,0.0626,0.0466	0.0556,0.0556,0.0556
$a_1 = \text{NC}$	0.0556,0.0556,0.0556	0.0556,0.0556,0.0556

[16] The vertices in Figure 6 are the vertices in Figure 5 (see footnote 13) with the equilibrium prices in Lemma 3.

[17] In each entry, the i^{th} number is the profit of firm i, for $i = 1, 2, 3$.

It is immediate to see that in this game, for each firm action C weakly dominates action NC because $0.0741 > 0.0466$ and $0.0626 > 0.0556$.

Proposition 1 *In the stage one reduced game for* Γ*, C weakly dominates NC for each firm and the unique non-trivial SPNE of* Γ *is such that each firm plays C at stage one.*[18]

Proposition 1 establishes that unless firms coordinate on the trivial SPNE, the only equilibrium outcome is that all firms choose C and competition occurs among fully compatible components, as full compatibility is the (unique) dominant strategy equilibrium for the stage one reduced game. We have remarked in Subsection 3.2 that all firms are better off in $\gamma^{\varnothing}$ than in γ^{123}, hence starting from (C, C, C) firms have no collective incentive to move to (NC, NC, NC). Proposition 1 establishes that no individual incentive for a firm to offer incompatible components exists either, regardless of the technological regimes adopted by the other firms, because C weakly dominates NC for each firm. In the rest of this section we explore in detail the causes of this result.

4.2 The unprofitability of incompatibility

In this subsection we explain why NC is weakly dominated by C for each firm. We rely on two notions described by Hurkens *et al.* (2019): the *demand size effect* and the *demand elasticity effect*.

Incompatibility is unprofitable when all other firms choose compatibility Without loss of generality, we focus on firm 2 and examine its incentive to choose NC given $a_1 = C$, $a_3 = C$. For the demand size effect we start from the NE in $\gamma^{\varnothing}$ (i.e., the NE under full compatibility), in which each components' price is $p^{\varnothing} = \frac{1}{9}$. Then, suppose firm 2 offers a proprietary system and sets its price P_2 equal to $2p^{\varnothing}$, the total equilibrium price of the individual components A_2, B_2 in $\gamma^{\varnothing}$; firms 1,3 still offer compatible components at the unit price $p^{\varnothing}$. The demand size effect inquires each firm's profit change due to the choice of NC by firm 2, with unchanged prices. From (6) we know that $P_2 = 2p^{\varnothing}$ and $p_1 = p_3 = p^{\varnothing}$ make the demand for the system of firm 2 equal to $\frac{2}{9}$, smaller than $\frac{1}{3}$, the demand for each component of firm 2 in $\gamma^{\varnothing}$. Therefore, firm 2 loses (firms 1,3 gain) market share and profit with respect to $\gamma^{\varnothing}$.

In order to see why, notice that NC by firm 2 makes unavailable the systems $S_{12}, S_{21}, S_{32}, S_{23}$. Hence, each consumer who buys one of these systems in $\gamma^{\varnothing}$ must change his purchase in γ^{2},[19] and firm 2's revenue comes only from the sale of S_{22}. Figure 7 represents the sets of the consumers that buy one or both components of firm 2 in $\gamma^{\varnothing}$, denoted with $R_{ij}^{\varnothing}$ for $ij = 12, 21, 23, 32, 22$. Figure 8 shows the set R_{22} of consumers that buy S_{22} in γ^{2} given $p_1 = p_3 = p^{\varnothing}$, $P_2 = 2p^{\varnothing}$. This set includes $R_{22}^{\varnothing}$ and a subset of $R_{ij}^{\varnothing}$ for $ij = 12, 21, 23, 32$; to fix the ideas, we focus on $R_{32}^{\varnothing}$ without loss of generality. For each consumer in $R_{32}^{\varnothing}$, incompatibility doubles firm 2's revenue from the consumer if the latter buys S_{22} (i.e., if the consumer is in R_{22}), but reduces the revenue to 0 if the consumer buys a different system. As Figure 8 suggests, the consumers in $R_{32}^{\varnothing}$ that belong to R_{22} are fewer than those that do not; thus, relative to the set $R_{32}^{\varnothing}$, NC makes firm 2 lose more consumers than those that eventually buy S_{22}. For instance, a consumer located at $\mathbf{x} = (0.8, 0.4) \in R_{32}^{\varnothing}$ buys S_{32} under $\gamma^{\varnothing}$. However S_{32} becomes unavailable after $a_2 = NC$, and S_{31} is more convenient for the consumer since it has the same monetary cost as S_{22}, but

[18] The complete SPNE strategies (which include each firm's behavior at stage two) are obtained from Lemmas 1-3.

[19] Conversely, there is no change in the purchase of any consumer that in $\gamma^{\varnothing}$ buys one of the other systems, as they remain available at the same price.

x is closer to S_{31} than to S_{22}. Hence, the demand size effect reduces firm 2's market share and profit.

Fig. 7 The sets of consumers that buy at least one component of firm 2 in the NE of $\gamma^{\varnothing}$

Fig. 8 The set of consumers that buy S_{22} in γ^2 given $p_1 = p_3 = p^{\varnothing}$, $P_2 = 2p^{\varnothing}$

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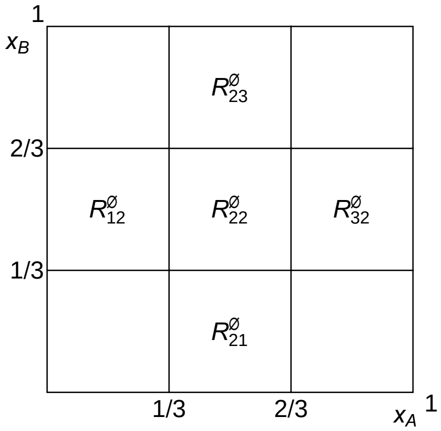

Figure 7

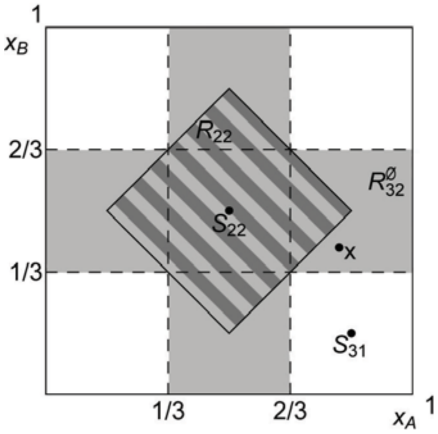

Figure 8

The above analysis neglects the demand elasticity effect, that is the firms' incentives to change prices given $a_2 = NC$. Precisely, from (7) we see that given $p_1 = p_3 = p^{\varnothing}$, the optimal price for firm 2 is $\frac{4}{27}$, smaller than $2p^{\varnothing}$; thus firm 2 wants to reduce P_2. This occurs because firm 2's lower demand reduces its loss from reducing the price to inframarginal consumers, but also because firm 2's demand in γ^2 reacts more to a price decrease than its demand in $\gamma^{\varnothing}$. Firms 1,3, if P_2 were fixed at $2p^{\varnothing}$, would increase slightly p_1, p_3 above $p^{\varnothing}$. However since prices are strategic complements, the decrease in P_2 induces firms 1,3 to reduce p_1, p_3 below $p^{\varnothing}$. This pushes firm 2 to further reduce P_2, and the NE is reached at the prices in Lemma 3.

Combining the two effects yields the equilibrium outcome under γ^2, in which firm 2's market share is 0.3392. Although this is greater than $\frac{1}{3}$, the price of S_{22} is low enough that firm 2 is worse off with respect to $\gamma^{\varnothing}$, and also with respect to γ^{123}. The stronger price competition hurts also firms 1,3 as they have about the same market share as in $\gamma^{\varnothing}$ but charge a price for each component lower than $p^{\varnothing}$.

Incompatibility is unprofitable when only another firm chooses it Now we suppose that $a_1 = C$, $a_2 = NC$ and illustrate why NC is unprofitable for firm 3. If firm 3 offers a proprietary system, then γ^{123} is entered. We examine the demand size effect given $P_1 = P_3 = 2p^*$ and $P_2 = P_2^*$ ($p^* = 0.0948$, $P_2^* = 0.1373$ as in the NE in γ^2: see (9)). Figure 9 describes how the set of consumers of firm 3 changes in moving from γ^2 to γ^{123} with unchanged prices. Precisely, let R^2_{ij} denote the set of consumers that buy S_{ij} in γ^2, for $ij = 13, 31$. The boundaries of these sets are represented by dashed segments (see also Figure 6). The solid segments are the boundaries of the set R_{33} of consumers that buy S_{33} in γ^{123}. Firm 3 keeps all the consumers that buy S_{33} in γ^2 (the set R^2_{33}) but, as Figure 9 suggests, loses most of the consumers in $R^2_{13} \cup R^2_{31}$ as they buy S_{22} or S_{11} rather than S_{33}.[20] For example, the consumer located at $\mathbf{x} = (0.85, 0.4)$ and the consumer located at $\mathbf{x}' = (0.85, 0.2)$ both buy S_{31} in γ^2, but in γ^{123} the first

[20] Precisely, the demand for S_{33} is 0.2779, whereas firm 3's market share in γ^2 is 0.3304 for both components.

consumer buys S_{22}, the second consumer buys S_{11}. Hence, the demand size effect is negative for firm 3.

Fig. 9
The set R_{33} of consumers that buy S_{33} in γ^{123}, given $P_1 = 2p^*$, $P_2 = P_2^*$, : $P_3 = 2p^*$ (p^*, P_2^* are the NE prices in γ^2: see Lemma 3)

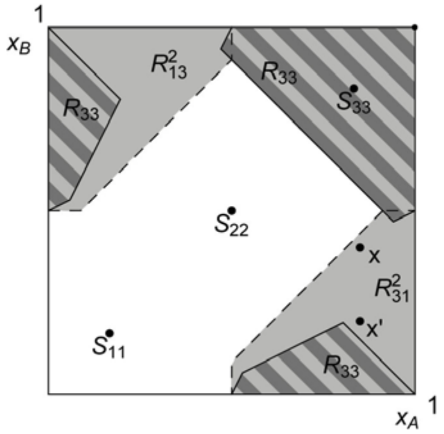

Figure 9

Also in this case there is a demand elasticity effect that modifies the firms' pricing incentives. In particular, at $P_2 = P_2^*$, $P_1 = P_3 = 2p^*$ firms 1,3 want to reduce P_1, P_3, whereas firm 2 wants to increase P_2.[21] Consistently with these incentives, the equilibrium price for each system in γ^{123} is $\frac{1}{6}$, such that $P_2^* < \frac{1}{6} < 2p^*$. At the equilibrium prices, the market share of firm 3 is slightly higher than in γ^2, $\frac{1}{3}$ instead of 0.3304, but the price $\frac{1}{6}$ of its system is smaller than $2p^*$; this makes its profit 0.0556, smaller than the profit 0.0626 under γ^2. Thus, it is unprofitable for firm 3 to choose NC when $a_1 = C$, $a_2 = NC$.

5 A setting with vertical differentiation

In this section, we examine a setting with vertical differentiation in which one firm offers components with higher quality with the respect to the other firms' components. We inquire whether this asymmetry generates incentives to choose incompatibility, or instead leads to results analogous to Proposition 1.[22]

We assume, without loss of generality, that it is firm 2 that offers higher quality components, and firms are located in both circles as described by Figure 3. The higher quality of A_2, B_2 is represented by a higher gross utility: a consumer receives gross utility $V + \alpha$, with $\alpha > 0$, from a system that includes one component made by firm 2, receives gross utility $V + 2\alpha$ from system S_{22}. We can equivalently interpret α as a cost reduction from a consumer's viewpoint, such that for each system the cost reduction is α times the number of components (in the system) supplied by firm 2. For instance, for a consumer located at (x_A, x_B), the cost of S_{22} is $d(x_A, \frac{1}{2}) + d(x_B, \frac{1}{2}) + P_2 - 2\alpha$. Since the quality difference leads to a higher market share for firm 2, sometimes we call it "large firm", and use "small firms" for firms $1, 3$.

As in the previous sections, we suppose that each firm incurs zero marginal production costs. However, since the large firm offers higher quality components, it may be plausible that firm 2's marginal cost is higher than that of the small firms, as in Bos and Marini (2019) and Bos, Marini and Saulle (2020), for instance. Indeed, this possibility is covered by our analysis:

[21] This occurs because in γ^{123}, firm 2 faces softer competition than in γ^2 as systems S_{13}, S_{31} are not available. In Subsection 5.5 we provide more details about this effect.

[22] We remark that the same results arise if all firms' components have the same quality but one firm is more efficient than the others in the sense that it has lower production costs, for instance because it employs more skilled workers, or can procure raw materials more cheaply then the other firms. The empirical literature identifies large differences in firms' productivity levels, and consequently in firms' costs, even within narrowly defined industries: see Syverson (2004).

Proposition 2 *The game in which the large firm has quality advantage $\alpha > 0$ and marginal cost $c \in (0, \alpha)$ is payoff-equivalent to the game in which the large firm's quality advantage is $\hat{\alpha} = \alpha - c$ and its marginal cost is zero.*

Therefore, the assumption of zero marginal cost for the large firm we make in the following is without loss of generality when α is interpreted as the large firm's quality advantage net the cost of producing such quality. That is, if higher quality can be achieved only via higher costs, then this is equivalent to a reduction in the quality advantage.[23] Therefore, our results below can be seen as determined by the relationship between the quality advantage and the associated cost.

In the rest of this section, we use Γ_α to denote the game which differs from Γ only because firm 2 offers higher quality components, and with $\gamma_\alpha^{N'}$ the stage two subgame of Γ_α that is entered if $N' \subseteq \{1, 2, 3\}$ is the set of the firms that at stage one choose NC. Since vertical differentiation introduces an ex ante asymmetry among firms which was absent in the previous sections, Lemmas 1 and 2 do not apply to $\gamma_\alpha^\varnothing$ and to γ_α^{123}, respectively, and we need to distinguish between γ_α^2 and γ_α^3 (γ_α^1 is equivalent to γ_α^3 up to a relabelling of firms) as it is relevant if the unique firm choosing NC is the large firm or a small firm. However, $\gamma_\alpha^{12}, \gamma_\alpha^{13}, \gamma_\alpha^{23}$ are each still equivalent to γ_α^{123}.

5.1 Full compatibility: Subgame $\gamma_\alpha^\varnothing$

Here we consider competition under full compatibility. Then, competition for component A is independent of competition for component B even under vertical differentiation. We focus on the market for component A, and for a consumer located at x_A the cost of component A_j, for $j = 1, 3$, is $C_j(x_A)$ in (5); the cost of component A_2 is $C_2(x_A) = (x_A - \frac{1}{2})^2 + p_2 - \alpha$.

Since firms 1,3 are in a symmetric position, we examine NE of $\gamma_\alpha^\varnothing$ such that firms 1,3 charge the same price. Given $p_1 = p_3 = p$, we derive the demand function for firm 2 by solving the inequality $C_2(x_A) < \min\{C_1(x_A), C_3(x_A)\}$ and obtain[24]

$$D_2(p_2) = \begin{cases} 1 & \text{if } p_2 < p + \alpha - \frac{2}{9} \\ 3p + 3\alpha - 3p_2 + \frac{1}{3} & \text{if } p + \alpha - \frac{2}{9} \leq p_2 < p + \alpha + \frac{1}{9} \\ 0 & \text{if } p + \alpha + \frac{1}{9} \leq p_2 \end{cases} \tag{10}$$

$$br_2(p, \gamma_\alpha^\varnothing) = \begin{cases} \frac{1}{18} + \frac{1}{2}p + \frac{1}{2}\alpha & \text{if } p + \alpha < \frac{5}{9} \\ p + \alpha - \frac{2}{9} & \text{if } p + \alpha \geq \frac{5}{9} \end{cases} \tag{11}$$

The best reply for firm 2 in (11) follows from (10). Now we derive the demand function for firm 3. Assume that firms 1,3 have both a positive market share in the NE, that is $p_1 = p_3 = p$ with $p < p_2 - \alpha + \frac{2}{9}$. Then, for p_3 close to p, $C_3(x_A) < \min\{C_1(x_A), C_2(x_A)\}$ reduces to $x_A \in (\frac{2}{3} + \frac{3}{2}(p_3 + \alpha - p_2), 1 - \frac{3}{2}(p_3 - p))$; hence

$$D_3(p_3) = \frac{1}{3} + \frac{3}{2}(p + p_2 - \alpha - 2p_3) \tag{12}$$

From (12) we derive a first order condition for p_3, which combined with (11) delivers the equilibrium prices when all firms have a positive market share. Next lemma also determines that firm 2 captures the whole market when $\alpha \geq \frac{5}{9}$.

[23] In order to see why, consider full compatibility (to fix the ideas) and the market for component A. Let $m_2 = p_2 - c$ denote the unit profit margin for firm 2. For a consumer located at x_A in circle A, the cost of A_2 is $d(x_A, x_A^2) + p_2 - \alpha$, or $d(x_A, x_A^2) + m_2 - (\alpha - c)$. Since the market is fully covered, the demands for A_1, A_2, A_3 are determined by the comparisons among the costs for these components . Thus, the demand functions (and profits) are the same as when firm 2's marginal cost is zero and its quality advantage is $\hat{\alpha} = \alpha - c$, with m_2 playing the same role as p_2.

[24] We have that $\min\{C_1(x_A), C_3(x_A)\}$ is equal to $C_1(x_A)$ if $x_A \in [0, \frac{1}{2})$, is equal to $C_3(x_A)$ if $x_A \in [\frac{1}{2}, 1)$. Hence, $C_2(x_A) < \min\{C_1(x_A), C_3(x_A)\}$ holds for each $x_A \in [0, 1)$ if $p_2 < p + \alpha - \frac{2}{9}$, holds for $x_A \in (\frac{1}{3} + \frac{3}{2}(p_2 - p - \alpha), \frac{2}{3} + \frac{3}{2}(p + \alpha - p_2))$ if $p + \alpha - \frac{2}{9} \leq p_2 < p + \alpha + \frac{1}{9}$, is violated for each $x_A \in [0, 1)$ if $p_2 \geq p + \alpha + \frac{1}{9}$.

Lemma 4 *In game Γ_α, consider the subgame $\gamma_\alpha^\varnothing$ which is entered if each firm chooses C at stage one. In this subgame, for each market there exists a NE such that $p_1 = p_3 = p^*$ and $p_2 = p_2^*$ with*

$$p^* = \frac{1}{9} - \frac{1}{5}\alpha, \qquad p_2^* = \frac{1}{9} + \frac{2}{5}\alpha \qquad \text{if } \alpha \in (0, \frac{5}{9}) \tag{13}$$

$$p^* = 0, \qquad p_2^* = \alpha - \frac{2}{9} \qquad \text{if } \alpha \geq \frac{5}{9} \tag{14}$$

In the following of this section we assume that $\alpha \in (0, \frac{5}{9})$, so that each firm has positive market share and profit in $\gamma_\alpha^\varnothing$.

5.2 Incompatibility by the large firm only: Subgame γ_α^2

Subgame γ_α^2 is entered if only the large firm chooses NC at stage one. Therefore, γ_α^2 is similar to γ^2 examined in Subsection 3.3, but the firm that has chosen a proprietary technology also offers higher quality components. As a consequence, $C_{22}(x_A, x_B)$ is not given by (4) but is $(x_A - \frac{1}{2})^2 + (x_B - \frac{1}{2})^2 + P_2 - 2\alpha$.

As in Subsection 3.3, we consider NE in which firm 1 and firm 3 charge a same price p for each single component they offer. Then, we notice that firm 2's per-component advantage α has the same effect as an increase in p by α. As a consequence, we can derive the demand function for S_{22} from (6), by replacing p with $p + \alpha$, and from (7) we obtain $br_2(p + \alpha, \gamma^2)$. Hence,

$$br_2(p, \gamma_\alpha^2) = \begin{cases} \frac{2}{3}p + \frac{2}{3}\alpha + \frac{2}{27} & \text{if } p + \alpha \leq \frac{5}{36} \\ \frac{4}{3}(p+\alpha) - \frac{8}{27} + \frac{1}{27}\sqrt{324(p+\alpha)^2 - 144(p+\alpha) + 70} & \text{if } p + \alpha > \frac{5}{36} \end{cases} \tag{15}$$

For firm 3 we can argue like in the appendix of Subsection 3.3 to derive the demand function when p_3 is close to p_1, but in fact it is simpler to recall, from the second paragraph in Section 5, that the quality difference is equivalent to a reduction in the monetary cost of S_{22} by 2α. Thus, we can obtain the demand function for firm 3 from (8), after replacing P_2 with $P_2 - 2\alpha$. This allows to derive a first order condition for p_3, which combined with (15) (when $p + \alpha \leq \frac{5}{36}$) identifies the equilibrium prices when α is close to zero: see p^*, P_2^* in (16) below in Lemma 5. In this case the set $[0,1) \times [0,1)$ of consumers is partitioned among the available systems as described by Figure 10, which is similar to Figure 6:[25]

Fig. 10 Equilibrium partition of consumers in γ_α^2 among $S_{11}, S_{22}, S_{13}, S_{31}, S_{33}$ when $\alpha \leq \frac{13}{180}$

:

Fig. 11 Equilibrium partition of consumers in γ_α^2 among $S_{11}, S_{22}, S_{13}, S_{31}, S_{33}$ when $\alpha \in (\frac{13}{180}, \frac{5}{9})$

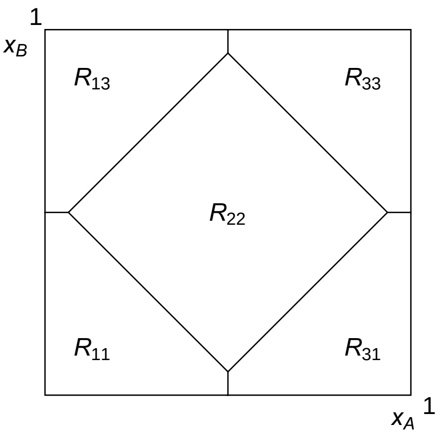

Figure 10

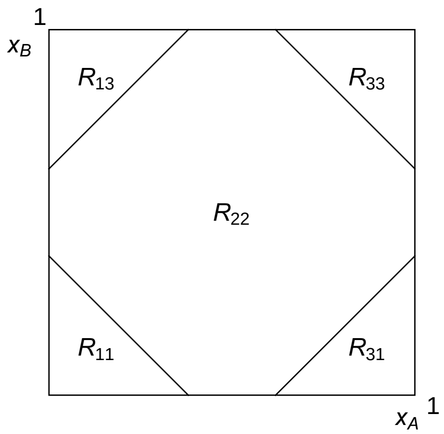

Figure 11

In order for the equilibrium prices to be given by (16), it is necessary that the set R_{22} is a square as in Figure 10. This occurs if $\alpha \leq \frac{13}{180}$, but R_{22} widens as α increases (as it is intuitive), for $\alpha = \frac{13}{180}$ the

[25] The vertices in Figure 10 are as in Figure 5 with $\lambda = \frac{2}{3} + \frac{3}{2}(P_2^* - 2\alpha - 2p^*)$ and p^*, P_2^* in (16): See footnote 13.

vertices of R_{22} hit the edges of the square $[0, 1) \times [0, 1)$, and for $\alpha > \frac{13}{180}$ the set R_{22} is an octagon like in Figure 11. Since R_{13}, R_{31}, R_{33} are triangles and not pentagons as in Figure 10, the expression of D_3 for p_3 slightly larger than p_1 is not anymore derived from (8): see the proof to Lemma 5 in the appendix. From it we derive a first order condition for p_3, which together with (15) (when $p + \alpha > \frac{5}{36}$) yields the equilibrium prices in (17) in Lemma 5, and the consumers partition as described by Figure 11.[26]

Lemma 5 *In game Γ_α, consider the subgame γ_α^2 which is entered if firm 2 chooses NC and firms 1,3 choose C at stage one. In γ_α^2 there exists a NE with $p_1 = p_3 = p^*$ and $P_2 = P_2^*$ such that*

$$\left.\begin{aligned} p^* &= \tfrac{1}{240}\sqrt{5184\alpha^2 + 10\,224\alpha + 4681} - \tfrac{7}{10}\alpha - \tfrac{137}{720} \\ P_2^* &= \tfrac{1}{360}\sqrt{5184\alpha^2 + 10\,224\alpha + 4681} + \tfrac{1}{5}\alpha - \tfrac{19}{360} \end{aligned}\right\} \quad \text{if } \alpha \leq \tfrac{13}{180} \tag{16}$$

$$\left.\begin{aligned} p^* &= \tfrac{1}{855}\sqrt{8100\alpha^2 - 3600\alpha + 2110} - \tfrac{2}{19}\alpha + \tfrac{4}{171} \\ P_2^* &= \tfrac{24}{19}\alpha + \tfrac{7}{855}\sqrt{8100\alpha^2 - 3600\alpha + 2110} - \tfrac{16}{57} \end{aligned}\right\} \quad \text{if } \tfrac{13}{180} < \alpha < \tfrac{5}{9} \tag{17}$$

5.3 Incompatibility by a small firm only: Subgame γ_α^3

Here we study the case in which only a small firm (either firm 1 or firm 3) chooses NC. Since the subgame γ_α^1 is equivalent to γ_α^3, up to a relabelling of firms, we examine γ_α^3. This subgame is more complicated than γ_α^2 as there is no symmetry between any two firms: firm 2 has a quality advantage over firms 1,3 and firm 3 offers a proprietary system whereas the others do not. In γ_α^3, a NE is a triplet (p_1^*, p_2^*, P_3^*).

One complication of the equilibrium analysis is that the expressions of the firms' demand functions, which lead to the equilibrium prices, change as α varies, like in the previous subsection. Precisely, if α is close to 0, then γ_α^3 is only slightly different from the subgame γ^2 examined in Subsection 3.3 (apart from the fact that in γ_α^3 it is firm 3 that has chosen NC rather than firm 2). This suggests that given p^* and P_2^* in Lemma 3, for α close to 0 the equilibrium prices p_1^*, p_2^* are close to p^*, and P_3^* is close to P_2^*. This is useful because it is cumbersome to derive the complete demand functions in γ_α^3, but is simpler to derive them for p_1, p_2 close to p^* and P_3 close to P_2^*. Since the expressions we obtain are complicated, we leave them to the proof of Lemma 6 in the appendix. From them we determine a NE of γ_α^3 for $\alpha \leq \frac{26}{77}$, and for this case the partition of consumers among the available systems is described by Figure 12.

Fig. 12 Equilibrium partition of consumers in γ_α^3 among S_{11}, $S_{22}, S_{33}, S_{12}, S_{21}$ when $\alpha \leq \frac{26}{77}$

Fig. 13 Equilibrium partition of consumers in γ_α^3 among S_{11}, $S_{22}, S_{33}, S_{12}, S_{21}$ when $\alpha \in (\frac{26}{77}, \frac{5}{9})$

:

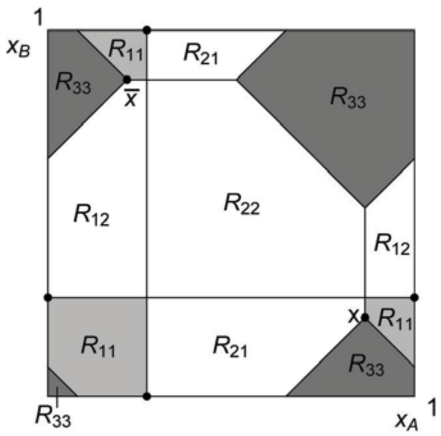

Figure 12

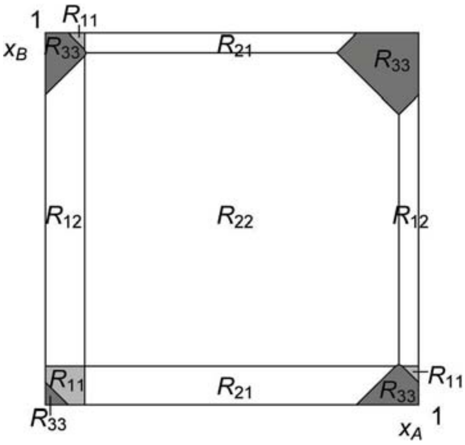

Figure 13

However, as α increases, the set R_{22} widens and for $\alpha > \frac{26}{77}$ it "absorbs" point $\mathbf{x}$ in Figure 12, which is the location of a consumer indifferent between S_{21}, S_{33}, S_{11}; a similar remark applies to point $\bar{\mathbf{x}}$,

[26] The vertices in Figure 11 are as in Figure 4 with $\lambda = \frac{2}{3} + \frac{3}{2}(P_2^* - 2\alpha - 2p^*)$ and p^*, P_2^* in (17). See footnote 13.

the location of a consumer indifferent between S_{12}, S_{33}, S_{11}. As a result, when $\alpha > \frac{26}{77}$ the consumers partition is as in Figure 13, and since the sets R_{11}, R_{33}, R_{21}, R_{12} have different shapes with respect to Figure 12, the demand functions have different expressions (see the proof to Lemma 6).[27]

Since the system of the first order conditions is highly non-linear, it cannot be solved in closed form and for subgame γ_α^3 (unlike for γ_α^2) we do not have a closed form expression for the equilibrium prices. For this reason we resort to a numerical approach, but the proof of Lemma 6 verifies that the solution we obtain numerically constitutes a NE of γ_α^3. Figure 14 plots the equilibrium prices as function of α.

Fig. 14
Equilibrium prices in γ_α^3
p_1^* (thin), p_2^* (dashed), P_3^* (thick) :

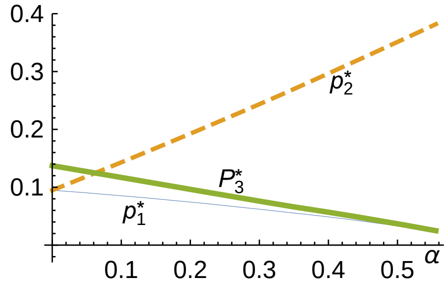

Figure 14

Lemma 6 *In game Γ_α, consider the subgame γ_α^3 which is entered if only firm 3 chooses NC at stage one. For each $\alpha \in (0, \frac{5}{9})$, there exists a NE of γ_α^3.*

5.4 Incompatibility by all firms: Subgame γ_α^{123}

Subgame γ_α^{123} is entered if each firm chooses NC; if only two firms choose NC, then a subgame equivalent to γ_α^{123} is entered. In this case, competition occurs under full incompatibility as in γ^{123} (see Subsection 3.2), but firm 2 offers higher quality components. Since firms 1,3 are in a symmetric position, we consider NE in which firms 1,3 charge the same price P for S_{11}, S_{33}.

The demand functions for the three systems have somewhat complicated expressions which are left to the proof of Lemma 7 in the appendix, and as in the Subsections 5.2, 5.3 the relevant expressions of the demand functions depend on α. Figure 15 represents the consumers' partition among S_{11}, S_{22}, S_{33} in equilibrium when α is close to zero. As α increases, the set R_{22} widens and for $\alpha = \frac{71}{630}$ four of its vertices, included the vertex $\mathbf{x}$, hit the edges of the square $[0, 1) \times [0, 1)$. As a consequence, when $\alpha > \frac{71}{630}$ there is no consumer with a strong preference for component A_2 that is indifferent among S_{11}, S_{22}, S_{33}: each consumer with a strong preference for A_2 buys S_{22}. This changes the shapes of the sets R_{11}, R_{22}, R_{33},

[27] For instance, in the southeast of $[0, 1) \times [0, 1)$, R_{11} is a triangle rather than a quadrilateral because there are consumers indifferent between S_{11} and S_{33} (or between S_{21} and S_{33}), but no consumer is indifferent among S_{11}, S_{21}, S_{33}.

and Figure 16 describes the resulting equilibrium partition of consumers when $\frac{71}{630} < \alpha < \frac{5}{9}$:

Fig. 15 Equilibrium partition of consumers in γ_α^{123} among S_{11}, S_{22}, S_{33} when $\alpha \leq \frac{71}{630}$

Fig. 16 Equilibrium partition of consumers in γ_α^{123} among S_{11}, S_{22}, S_{33} when $\alpha \in (\frac{71}{630}, \frac{5}{9})$

:

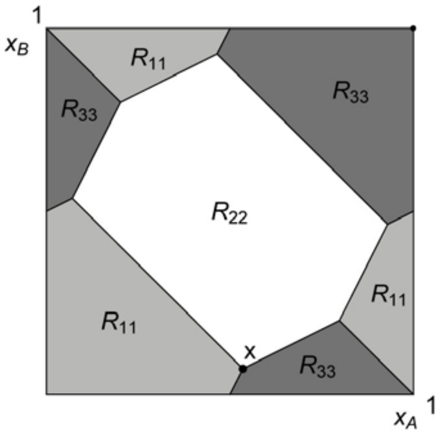

Figure 15

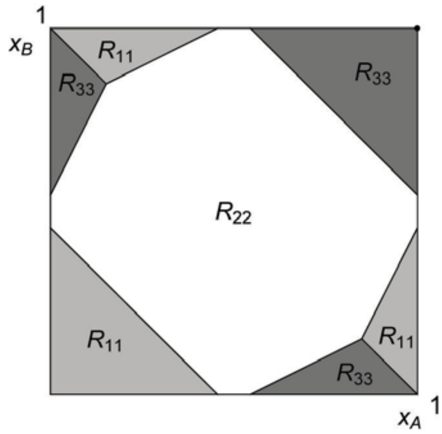

Figure 16

Lemma 7 *In game Γ_α, consider the subgame γ_α^{123} which is entered if all firms choose NC at stage one. For each $\alpha \in (0, \frac{5}{9})$, there exists a NE of γ_α^{123}.*

5.5 The first stage

Here we study the firms' choices in stage one of Γ_α: As in Section 4, we consider the stage one reduced game with simultaneous moves in which, for each $(a_1, a_2, a_3) \in \{\text{C,NC}\}^3$, the firms' profits given (a_1, a_2, a_3) are equal to the profits in the equilibrium of the subgame which is determined by (a_1, a_2, a_3): see Subsections 5.1-5.4. Precisely, we denote with $\Pi_i^\varnothing$ the profit of firm i in $\gamma_\alpha^\varnothing$ (i.e., $a_1 = a_2 = a_3 = C$), with Π_i^j the profit of firm i in γ_α^j (i.e., a_j =NC, $a_k = a_h = C$), and with Π_i^{123} the profit of firm i in γ_α^{123} (i.e., at least two firms have chosen NC). In the normal form below, firm 1 chooses a row, firm 2 chooses a column, firm 3 chooses a matrix.

$a_3 = \text{C}$

	$a_2 = \text{C}$	$a_2 = \text{NC}$
$a_1 = \text{C}$	$\Pi_1^\varnothing, \Pi_2^\varnothing, \Pi_3^\varnothing$	$\Pi_1^2, \Pi_2^2, \Pi_3^2$
$a_1 = \text{NC}$	$\Pi_1^1, \Pi_2^1, \Pi_3^1$	$\Pi_1^{123}, \Pi_2^{123}, \Pi_3^{123}$

$a_3 = \text{NC}$

	$a_2 = \text{C}$	$a_2 = \text{NC}$
$a_1 = \text{C}$	$\Pi_1^3, \Pi_2^3, \Pi_3^3$	$\Pi_1^{123}, \Pi_2^{123}, \Pi_3^{123}$
$a_1 = \text{NC}$	$\Pi_1^{123}, \Pi_2^{123}, \Pi_3^{123}$	$\Pi_1^{123}, \Pi_2^{123}, \Pi_3^{123}$

As we mentioned in Section 4, (NC,NC,NC) is a *trivial* NE for each α. Hence, in the following we examine the existence of other NE. To this purpose, it is useful to compare:

- Π_2^2 with $\Pi_2^\varnothing$ and Π_2^{123} with Π_2^3, in order to inquire the large firm's incentives to choose incompatibility, when no small firm does so or when only one small firm does so.
- Π_3^3 with $\Pi_3^\varnothing$, Π_3^{123} with Π_3^1, and Π_3^{123} with Π_3^2, in order to learn about the incentives of a small firm to choose incompatibility when: a) no other firm does that; b) only the other small firm does that; c) only the large firm does that.

We rely on our results from Subsections 5.1-5.4 and numeric analysis to perform the above comparisons. For instance, Figure 17 below plots Π_2^2 and $\Pi_2^\varnothing$ as a function of α; the appendix includes the plots of the profit functions involved in the other comparisons.

Fig. 17
Π_2^2 (solid) vs $\Pi_2^{\varnothing}$ (dashed) :

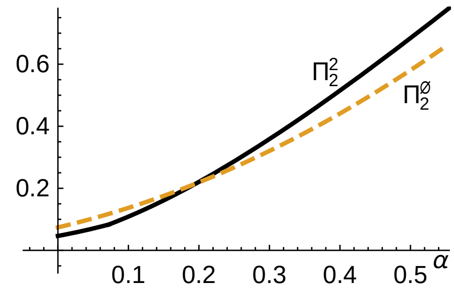

Figure 17

The profit comparisons for firm 2 reveal that

$$\Pi_2^2 < \Pi_2^{\varnothing} \quad \text{if} \quad \alpha \in (0, \alpha'), \qquad \Pi_2^2 > \Pi_2^{\varnothing} \quad \text{if} \quad \alpha \in (\alpha', \frac{5}{9}), \qquad \text{with } \alpha' = 0.1953 \tag{18}$$

From (18) we see that if the small firms adopt the standard technology, then the large firm has incentive to adopt it as well if its advantage is small (coherently with the results in Section 4 obtained for $\alpha = 0$); otherwise it wants to develop a proprietary system. Moreover,

$$\Pi_2^{123} < \Pi_2^3 \quad \text{if} \quad \alpha \in (0, 0.1709), \qquad \Pi_2^{123} > \Pi_2^3 \quad \text{if} \quad \alpha \in (0.1709, \frac{5}{9}) \tag{19}$$

Similarly, (19) reveals that if one small firm develops a proprietary technology, then the large firm still wants to adopt the standard if its advantage is small; otherwise it develops a proprietary system. However, note that NC by a small firm lowers the threshold for α above which the large firm chooses NC.

From (18)-(19) jointly we deduce that (i) if $\alpha < 0.1709$, then firm 2 plays C in any non-trivial NE; (ii) if $\alpha > \alpha'$, then NC is weakly dominant for firm 2; (iii) if α is between 0.1709 and α', then firm 2's best reply is C when $a_1 = a_3 = C$, and it is NC when $a_1 = NC$ or $a_3 = NC$.

The profit comparisons for firm 3 reveal that

$$\Pi_3^3 < \Pi_3^{\varnothing} \quad \text{if} \quad \alpha \in (0, 0.496), \qquad \Pi_3^3 > \Pi_3^{\varnothing} \quad \text{if} \quad \alpha \in (0.496, \frac{5}{9}) \tag{20}$$

$$\Pi_3^{123} < \Pi_3^1 \quad \text{for each} \quad \alpha \in (0, \frac{5}{9}) \tag{21}$$

$$\Pi_3^{123} < \Pi_3^2 \text{ if } \alpha \in (0, 0.051) \cup (0.1, \alpha''), \; \Pi_3^{123} > \Pi_3^2 \text{ if } \alpha \in (0.051, 0.1) \cup (\alpha'', \frac{5}{9}), \; \text{with } \alpha'' = 0.1981 \tag{22}$$

The most relevant takeaway from (20)-(22) is that firm 3 wants to choose NC if $a_1 = a_2 = C$ and $\alpha > 0.496$, or if $a_1 = C$, $a_2 = NC$ and $\alpha > \alpha''$.[28]

With these information we can identify the NE of the reduced game for each $\alpha \in (0, \frac{5}{9})$, distinguishing three intervals for α: $(0, \alpha')$, (α', α''), $(\alpha'', \frac{5}{9})$.

When α is in the interval $(0, \alpha')$, firm 2 wants to play NC only if at least one of the small firms plays NC and $\alpha > 0.1709$. However, for $\alpha < \alpha'$ firm 3 (firm 1) plays NC only if $a_1 = C$, $a_2 = NC$ and $\alpha \in (0.051, 0.1)$. Hence, (C,C,C) is the unique non-trivial NE. This extends Proposition 1, which covers the case of $\alpha = 0$.

We obtain different results when $\alpha \in (\alpha', \alpha'')$, because then firm 2 wants to choose NC even if no small firm does so. However, (22) reveals that firm 3 (firm 1) does not choose NC if only firm 2 does so. Hence (C,C,C) is not a NE, and (C,NC,C) is the unique non-trivial NE.

[28] If $a_1 = C$ and $a_2 = NC$, for firm 3 $a_3 = NC$ is optimal also when α is between 0.051 and 0.1. However, this does not affect the equilibrium behavior in the reduced game.

Finally, for $\alpha \in (\alpha'', \frac{5}{9})$ there is a change in the preference of firm 3 (firm 1): now firm 3 wants to choose NC if only firm 2 does so. Hence, in each NE at least two firms choose NC and each NE is equivalent to (NC,NC,NC). Next proposition summarizes these results.

Proposition 3 *In the stage one reduced game for Γ_α, (NC,NC,NC) is a NE for each α and*
(i) when $\alpha \in (0, \alpha')$, there exists a unique non-trivial NE, (C,C,C);
(ii) when $\alpha \in (\alpha', \alpha'')$, there exists a unique non-trivial NE, (C,NC,C);
(iii) when $\alpha \in (\alpha'', \frac{5}{9})$, each other NE is equivalent to (NC,NC,NC).

Unlike Proposition 1, Proposition 3 establishes that, in the setting with vertical differentiation we examine, firms may have individual incentives to offer proprietary systems. These incentives depend on the magnitude of α and on the other firms' technological regime choices. For $\alpha < \alpha'$ full compatibility remains the unique non-trivial equilibrium. By contrast, for $\alpha \in (\alpha', \alpha'')$ there exists an asymmetric equilibrium where only the large firm offers a proprietary system. Finally, for $\alpha > \alpha''$ full incompatibility is the unique equilibrium. The difference between Proposition 1 and Proposition 3 is determined by the inequalities $\Pi_2^2 > \Pi_2^{\varnothing}$ and $\Pi_3^{123} > \Pi_3^2$ in (18) and (22). In the rest of this section we explore why these inequalities hold for $\alpha > \alpha'$ and for $\alpha > \alpha''$, respectively, even though they are violated when $\alpha = 0$.

Incompatibility is profitable for the large firm when the small firms choose compatibility and $\alpha > \alpha'$. As we have explained in Section 4, NC reduces the profit of firm 2 (given $a_1 = a_3 = C$) when $\alpha = 0$ because of a negative demand size effect, and because the demand elasticity effect makes price competition fiercer. However, a different result emerges if $\alpha > 0$ is not small. Starting with the demand size effect, we consider γ_α^2 with $p_1 = p_3 = p^*$, $P_2 = 2p_2^*$ (p^*, p_2^* are the NE prices in $\gamma_\alpha^{\varnothing}$: see (13)). As in Section 4, we focus on the set $R_{32}^{\varnothing}$ of consumers that buy S_{32} in $\gamma_\alpha^{\varnothing}$, the rectangle $[\frac{2}{3} + \frac{3}{5}\alpha, 1) \times [\frac{1}{3} - \frac{3}{5}\alpha, \frac{2}{3} + \frac{3}{5}\alpha)$ shown in Figure 18 with three dashed edges.[29] Since S_{32} is unavailable in γ_α^2, each consumer in $R_{32}^{\varnothing}$ will buy either S_{22} or S_{33} or S_{31}. With respect to S_{32}, all these systems reduce the utility of such consumer. However, simple algebra shows that the utility decrease with S_{22} is decreasing in α, whereas the utility decrease with S_{33} or S_{31} is increasing in α, in such a way that more than half of the consumers in $R_{32}^{\varnothing}$ buy S_{22} if $\alpha > \frac{5}{36}$. Essentially, when $\alpha > \frac{5}{36}$ for a majority of consumers in $R_{32}^{\varnothing}$ it is not convenient to give up component B_2, even though that requires to buy A_2 which they like less than A_3. Figure 18 also represents the set R_{22} of consumers that buy S_{22} in γ_α^2 when $\alpha > \frac{5}{36}$, $p_1 = p_3 = p^*$, $P_2 = 2p_2^*$.

Fig. 18:
The set $R_{32}^{\varnothing}$ (with dashed edges) of the consumers that in $\gamma_\alpha^{\varnothing}$ buy S_{32}, and the set R_{22} (with solid edges) of consumers that buy S_{22} in γ_α^2 given $p_1 = p_3 = p^*$, $P_2 = 2p_2^*$ (p^*, p_2^* are NE prices in $\gamma_\alpha^{\varnothing}$: see (13))

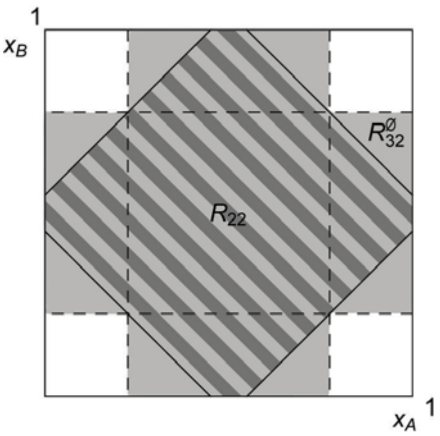

Figure 18

[29] Similar arguments apply to the sets $R_{12}^{\varnothing}, R_{21}^{\varnothing}, R_{23}^{\varnothing}$, and there is no change in the purchases of consumers that in $\gamma_\alpha^{\varnothing}$ buy no component of firm 2 or both components of firm 2.

Therefore, the demand size effect for firm 2 is negative if $\alpha < \frac{5}{36}$ but is positive if $\alpha > \frac{5}{36}$ (compare Figure 8 with Figure 18). We also remark that this effect is weak if α is close to $\frac{5}{9}$, as then the market share of firm 2 is already very large in $\gamma_\alpha^\varnothing$, hence the set $R_{32}^\varnothing$ is small.

Also the demand elasticity effect depends on α: given $p_1 = p_3 = p^*$, we find that at $P_2 = 2p_2^*$, D_2 is elastic if $\alpha < 0.3115$, but it is inelastic if $\alpha > 0.3115$. In Section 4.2 we have mentioned the first case when $\alpha = 0$; the second case is simple to see for $\alpha = \frac{5}{9}$. Precisely, when $\alpha = \frac{5}{9}$ the equilibrium prices for $\gamma_\alpha^\varnothing$ are $p_1^* = p_3^* = 0$, $p_2^* = \frac{1}{3}$ and all consumers buy A_2 and B_2. If in γ_α^2 firm 2 increases P_2 above $2p_2^* = \frac{2}{3}$ by $\Delta P_2 > 0$ close to zero, then the set of consumers that firm 2 loses is the union of four right triangles at the corners of $[0,1) \times [0,1)$, each with edges proportional to ΔP_2 (see Figure 4). This set has an area proportional to $(\Delta P_2)^2$, hence the increase in P_2 has a zero first order effect on the demand for S_{22} and this demand has zero elasticity at $P_2 = \frac{2}{3}$.

The incentive of firm 2 to reduce P_2 for $\alpha < 0.3115$ has the effect of inducing firms 1,3 to reduce their prices, as we remarked for the case of $\alpha = 0$. Moreover, except for values of α close to zero, firms 1,3 want to reduce p_1, p_3 below p^* even if $P_2 = 2p_2^*$. This harms firm 2, therefore $\alpha > \frac{5}{36}$ is not sufficient to make firm 2 prefer γ_α^2 to $\gamma_\alpha^\varnothing$. Indeed, (18) reveals that $\Pi_2^2 > \Pi_2^\varnothing$ holds if and only if $\alpha > \alpha'$.

Incompatibility is profitable for a small firm if the large firm chooses incompatibility and $\alpha > \alpha''$. In Section 4 we have explained why firm 3 prefers C to NC given $a_1 = C$, $a_2 = NC$ when $\alpha = 0$. Now we illustrate why the opposite holds, that is $\Pi_3^{123} > \Pi_3^2$, when $\alpha > \alpha''$.

We first notice that the demand size effect is negative for firm 3. Comparing the NE of γ_α^2 with the outcome in γ_α^{123} given $P_1 = P_3 = 2p^*$, $P_2 = P_2^*$ (p^*, P_2^* are the NE prices of γ_α^2: see (17)) reveals that in the latter case the market share and profit of firm 3 is reduced. In order to see why, notice that $\alpha'' > \frac{13}{180}$, therefore in γ_α^2 the equilibrium partition of consumers is described by Figure 11. Moving to γ_α^{123} with unchanged prices makes S_{13}, S_{31} unavailable, and firm 3's profit derives only from the sale of S_{33}. The consumers that buy S_{33} in the NE of γ_α^2 still buy S_{33} in γ_α^{123}. Hence, the demand size effect is determined by the purchases of the consumers that buy S_{31} in γ_α^2 (similar arguments apply to S_{13}); let R_{31}^2 denote this set, a triangle with vertices $\mathbf{x}^1$, $\mathbf{x}^3$, $(1,0)$ in Figure 19. In γ_α^{123}, suppose for one moment that S_{22} is not available. Then, the consumers in R_{31}^2 split equally between S_{11} and S_{33} since $P_1 = P_3$ and one half of them is closer to S_{11}, whereas the other half is closer to S_{33}. However, the presence of S_{22} is relevant since for the consumers in R_{31}^2, S_{11} and S_{33} are inferior to S_{31} but the consumers on the segment $\mathbf{x}^1, \mathbf{x}^3$ (the border between R_{31}^2 and R_{22}^2: see Figure 11) are indifferent between S_{22} and S_{31}. Hence, the consumers in R_{31}^2 close to this segment prefer S_{22} to both S_{11} and S_{33}: these are the consumers in the triangle $\mathbf{x}^1\mathbf{x}^2\mathbf{x}^3$. Therefore, the consumers in R_{31}^2 that buy S_{33} are less than one half of R_{31}^2: they are a half of the quadrilateral with vertices $\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3$, $(1,0)$ in Figure 19. Thus, moving from γ_α^2 to γ_α^{123} with unchanged prices worsens the situation of firm 3, but improves that of firm 2. The latter faces relaxed competition as S_{13}, S_{31} are not available anymore: Firm 2 wins over the consumers in the triangles $\mathbf{x}^1\mathbf{x}^2\mathbf{x}^3$ and $\bar{\mathbf{x}}^1\bar{\mathbf{x}}^2\bar{\mathbf{x}}^3$.

Fig. 19:
The set R^2_{31} (triangle $\mathbf{x}^1, \mathbf{x}^3, (1,0)$) of the consumers that in γ^2_α buy S_{31}, and the set (triangle $\mathbf{x}^1, \mathbf{x}^2, (1,0)$) of consumers in R^2_{31} that buy S_{33} in γ^{123}_α given $P_1 = P_3 = 2p^*$, $P_2 = P^*_2$ (p^*, P^*_2 are NE prices in γ^2_α: see (17)) :

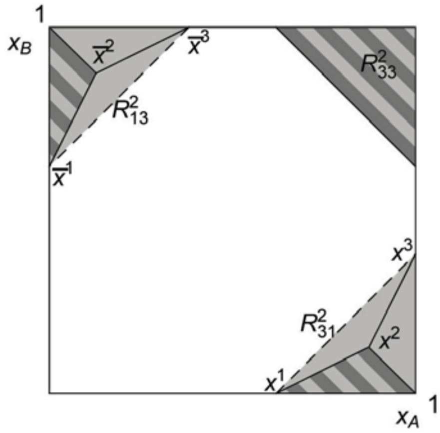

Figure 19

Nevertheless, also firms 1,3 prefer γ^{123}_α to γ^2_α for large α because firm 2 is less aggressive in γ^{123}_α than in γ^2_α. Precisely, suppose that $p_1 = p_3 = p$ in γ^2_α and $P_1 = P_3 = 2p$ in γ^{123}_α; thus S_{11} (S_{33}) has the same price in γ^2_α as in γ^{123}_α, but in γ^2_α also S_{13}, S_{31} are available. Comparing $br_2(p, \gamma^2_\alpha)$ in (15) with the best reply of firm 2 in γ^{123}_α, denoted $br_2(2p, \gamma^{123}_\alpha)$ [see (41) in the appendix], shows that $br_2(2p, \gamma^{123}_\alpha) > br_2(2p, \gamma^2_\alpha)$, thus firm 2 is less aggressive in γ^{123}_α than in γ^2_α. This occurs because a higher number of inframarginal consumers for firm 2 in γ^{123}_α than in γ^2_α (due to the positive demand size effect for firm 2) makes it more profitable to increase P_2, and also because a same increase in P_2 leads to a smaller loss of consumers in γ^{123}_α than in γ^2_α (this is a consequence of how the triangles $\mathbf{x}^1\mathbf{x}^2\mathbf{x}^3$ and $\bar{\mathbf{x}}^1\bar{\mathbf{x}}^2\bar{\mathbf{x}}^3$ depend on P_2). Although the demand size effect is negative for firm 3, when α is large that effect is weak as the market share of firm 3 is already small in γ^2_α. It follows that for firm 3 this effect is dominated by the demand elasticity effect, which induces less aggressive pricing by firm 2, and allows firm 3 (firm 1) to increase p_3 (p_1) and earn a higher profit than in γ^2_α.[30] We remark that, by reducing the own competitiveness through a reduction of the number of systems, firm 3 (firm 1) increases the own profit as the less competitive environment induces firm 2 to charge a higher price, which has a more important effect on the profit of firm 3.[31]

6 Discussion and conclusions

We have examined an oligopoly model in which multi-product firms compete for the sale of a system made of complementary components. Each firm can choose to offer components that are incompatible with those supplied by rivals. In particular, we studied a two-stage game where first firms simultaneously choose whether to adopt a standard technology or not; and then firms compete in prices given their compatibility choices. We show that, with ex-ante symmetric firms, for each firm compatibility weakly dominates incompatibility. It follows that full compatibility arises in equilibrium if firms avoid weakly dominated actions. This result provides a ground to the previous literature which focuses on the comparison between full compatibility and full incompatibility.

We then show that individual incentives to use proprietary technologies may exist when firms are ex-ante asymmetric because of vertical differentiation. A firm offering higher quality components has incentive to use a proprietary technology if its quality advantage is large. Therefore, partial compatibility

[30] This is similar to what happens in Hurkens et al. (2019), in a case with two firms for which footnote 2 applies.

[31] We point out that firms 1,3 prefer to play $\gamma^{\varnothing}_\alpha$ rather than γ^{123}_α for $\alpha < 0.429$. Conversely, firm 2 prefers γ^{123}_α to $\gamma^{\varnothing}_\alpha$ for each $\alpha > \alpha''$.

may arise in equilibrium when firms are vertically differentiated, and ruling it out a priori may entail some loss of generality. Furthermore, if the vertical differentiation is sufficiently large, then also the small firms want to develop proprietary technologies. In this case, full incompatibility is the unique equilibrium.

We have modelled asymmetry among firms in a specific way, but it would be interesting to find out whether other specifications lead to the result that asymmetric technological regimes can arise with ex ante asymmetric firms. For instance, one firm may have a quality advantage for only one component and a second firm an advantage for the other component. Alternatively, we may allow firms to be not equidistantly located in the two markets. Finally, a very interesting extension would be relaxing the assumption of a unique standard, in order to study whether clusters of firms may form around different standards (allowing for more than three firms).

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7 Appendix

7.1 Appendix to Subsection 3.3

7.1.1 Derivation of the demand function for firm 2

In order to derive the demand function for S_{22}, we employ two steps as follows.

Step 1 We pretend that consumers can buy only from firms 1 and 3, as if there were no firm 2, and derive the resulting distribution of consumers among $S_{11}, S_{13}, S_{31}, S_{33}$. Since $p_1 = p_3$ in equilibrium and firm 1 (3) is located at $\frac{1}{6}$ (at $\frac{5}{6}$), in each market a consumer buys from firm 1 (from firm 3) if the consumer is located between 0 and $\frac{1}{2}$ (between $\frac{1}{2}$ and 1). We let Q_{ij} denote the region of consumers that buy system S_{ij}, for $i = 1, 3$ and $j = 1, 3$, when there is no firm 2. Hence

$$Q_{11} = [0, \frac{1}{2}) \times [0, \frac{1}{2}), \quad Q_{13} = [0, \frac{1}{2}) \times [\frac{1}{2}, 1), \quad Q_{31} = [\frac{1}{2}, 1) \times [0, \frac{1}{2}), \quad Q_{33} = [\frac{1}{2}, 1) \times [\frac{1}{2}, 1) \qquad (23)$$

Fig. 20
Consumers' purchases in γ^2 when S_{22} is not available and $p_1 = p_3$:

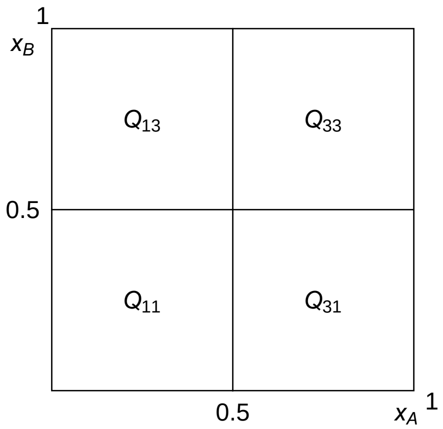

Figure 20

Step 2 For each region in (23) we identify the consumers that prefer S_{22} to the best alternative offered by firms 1, 3. Precisely, for $i = 1, 3$ and $j = 1, 3$ we solve $C_{22}(x_A, x_B) < C_{ij}(x_A, x_B)$ for (x_A, x_B) in Q_{ij} to determine the consumers that prefer S_{22} to S_{ij}. For instance, consider $i = 1$, $j = 3$ and let p be the common equilibrium value of p_1 and p_3. Then (4)-(5) yield $C_{13}(x_A, x_B) = (x_A - \frac{1}{6})^2 + (x_B - \frac{5}{6})^2 + 2p$ for $(x_A, x_B) \in Q_{13}$ and $C_{22}(x_A, x_B) < C_{13}(x_A, x_B)$ reduces to $x_B < \frac{1}{3} - \frac{3}{2}(P_2 - 2p) + x_A$. More generally,

$$\begin{array}{lll} \text{for } (x_A, x_B) \in Q_{11}, & C_{22}(x_A, x_B) < C_{11}(x_A, x_B) \text{ reduces to } & x_B > \frac{2}{3} + \frac{3}{2}(P_2 - 2p) - x_A \\ \text{for } (x_A, x_B) \in Q_{13}, & C_{22}(x_A, x_B) < C_{13}(x_A, x_B) \text{ is reduces to } & x_B < \frac{1}{3} - \frac{3}{2}(P_2 - 2p) + x_A \\ \text{for } (x_A, x_B) \in Q_{31}, & C_{22}(x_A, x_B) < C_{31}(x_A, x_B) \text{ reduces to } & x_B > -\frac{1}{3} + \frac{3}{2}(P_2 - 2p) + x_A \\ \text{for } (x_A, x_B) \in Q_{33}, & C_{22}(x_A, x_B) < C_{33}(x_A, x_B) \text{ reduces to } & x_B < \frac{4}{3} - \frac{3}{2}(P_2 - 2p) - x_A \end{array} \qquad (24)$$

The resulting subset of $[0, 1) \times [0, 1)$ depends on $P_2 - 2p$ as illustrated in the main text just before Figures 4 and 5.

7.1.2 Derivation of the demand function for firm 3

In order to derive the demand for firm 3 for p_3 close to p_1, we follow two steps as in Subsection 7.1.1.

Step 1 Given p_3 slightly larger than p_1, we examine the consumers' purchases when only $S_{11}, S_{13}, S_{31}, S_{33}$ are available, as if there were no firm 2. Since $p_3 > p_1$, solving $C_3(x) \le C_1(x)$ (see (5)) reveals that the consumers buying component A_3 (B_3) are those with x_A (x_B) in the interval $[y, z)$, with $y = \frac{1}{2}+\frac{3}{4}(p_3-p_1) > \frac{1}{2}$ and $z = 2-2y < 1$; conversely, the consumers with x_A (x_B) in $[0, y) \cup [z, 1)$ buy component A_1 (B_1). As a consequence, we define the sets Q_{11}, Q_{13}, Q_{31}, Q_{33} as follows (see Figure 21):

$$Q_{11} = ([0, y) \cup [z, 1)) \times ([0, y) \cup [z, 1)), \qquad Q_{13} = ([0, y) \cup [z, 1)) \times [y, z),$$
$$Q_{31} = [y, z) \times ([0, y) \cup [z, 1)), \qquad Q_{33} = [y, z) \times [y, z) \tag{25}$$

Step 2 Since we are interested in the demand for firm 3, we neglect Q_{11} but for the other regions in (25) we identify the consumers that prefer a system offered by firms 1 and 3 to S_{22}. Precisely, for $ij = 13, 31, 33$ we solve $C_{ij}(x_A, x_B) < C_{22}(x_A, x_B)$ for $(x_A, x_B) \in Q_{ij}$. For instance, (4)-(5) reveal that in Q_{33} the inequality $C_{33}(x_A, x_B) < C_{22}(x_A, x_B)$ reduces to $x_B > \frac{4}{3} + \frac{3}{2}(2p_3 - P_2) - x_A$. Therefore, the set of consumers that buy S_{33} is given by region R_{33} in Figure 22.[32] Arguing likewise for Q_{13} and Q_{31} shows that the set of consumers that buy just one component from firm 3 is $R_{13} \cup R_{31}$ in Figure 22.[33]

Fig. 21 The sets $Q_{11}, Q_{13}, Q_{31}, Q_{33}$ in (25)

Fig. 22 The sets R_{13}, R_{31}, R_{33}

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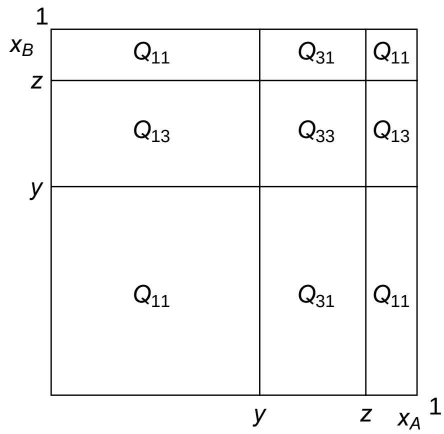

Figure 21

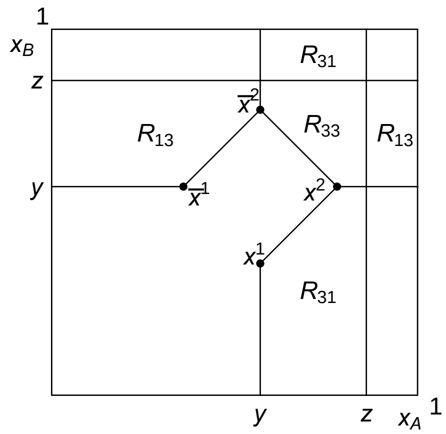

Figure 22

The demand for firm 3 given p_3 slightly larger than p_1 is equal to twice the area of R_{33} plus the area of $R_{13} \cup R_{31}$; this yields (8). In fact, from the proof of Lemma 3 we see that the demand for firm 3 has the expression in (8) also for p_3 slightly smaller than p_1.

7.1.3 Proof of Lemma 3

From (8) we derive the following first order condition for p_3, at $p_3 = p_1 = p$:

$$-\frac{9}{2}P_2^2 + 27P_2p + 2P_2 - 36p^2 - \frac{21}{2}p + \frac{7}{9} = 0 \tag{26}$$

Together with $P_2 = \frac{2}{3}p + \frac{2}{27}$ from (7) (given $p < \frac{5}{36}$), (26) identifies the prices in Lemma 3. In this proof we show that such prices constitute a NE for γ^2. We use p^* to denote the common value of p_1^* and p_3^*.

For firm 2, from (7) we know that $P_2^* = br_2(p^*, \gamma^2)$ is a best reply given that $p_1 = p_3 = p^*$.

In the rest of this proof we suppose that firm 1, firm 2 play $p_1 = p^*$, $P_2 = P_2^*$. We derive the complete demand function for firm 3 and prove that playing $p_3 = p^*$ is a best reply for firm 3.

[32] In Figure 22, $\mathbf{x}^1 = (y, 3y - \eta)$, $\mathbf{x}^2 = (\eta - y, y)$ with $\eta = \frac{4}{3} + \frac{3}{2}(2p_3 - P_2)$.

[33] Figure 22 is obtained assuming that $\frac{1}{2}P_2 + \frac{1}{18} > p_3$; this indeed holds in equilibrium by Lemma 3, otherwise the vertical coordinate of $\bar{\mathbf{x}}^2$ (the horizontal coordinate of $\mathbf{x}^2$) would be greater than z and R_{33} would be a triangle.

The demand function of firm 3 First we consider $p_3 < p^*$ and argue as in Steps 1 and 2 of Subsection 7.1.2, assuming initially that S_{22} is not available. The inequality $C_3(x) < C_1(x)$ holds for $x \in [0, y) \cup [z, 1)$, with $y = \frac{3}{2}(p^* - p_3)$, $z = \frac{1}{2} - \frac{1}{2}y$. Therefore the consumers partition among $S_{11}, S_{13}, S_{31}, S_{33}$ as follows:

$$\begin{cases} Q_{11} = [y, z) \times [y, z), & Q_{13}^S = [y, z) \times [0, y), & Q_{13}^N = [y, z) \times [z, 1), \\ Q_{31}^W = [0, y) \times [y, z), & Q_{31}^E = [z, 1) \times [y, z), & Q_{33}^{SW} = [0, y) \times [0, y), \\ Q_{33}^{SE} = [z, 1) \times [0, y), & Q_{33}^{NW} = [0, y) \times [z, 1), & Q_{33}^{NE} = [z, 1) \times [z, 1) \end{cases} \tag{27}$$

see Figure 23:

Fig. 23 The partition of $[0, 1) \times [0, 1)$ described in (27)

Fig. 24 The sets $R_{33}^{SW}, R_{33}^{SE}, R_{33}^{NW}, R_{33}^{NE}, R_{31}^W, R_{31}^E, R_{13}^S, R_{13}^N$ when $p_3 < p^*$

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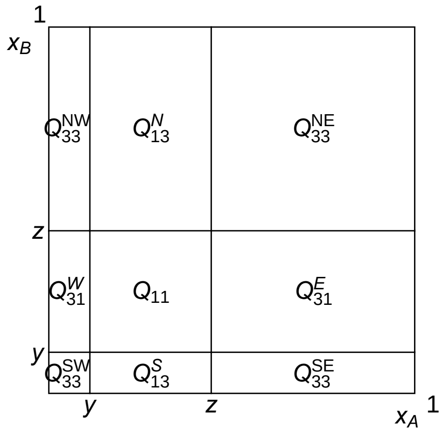

Figure 23

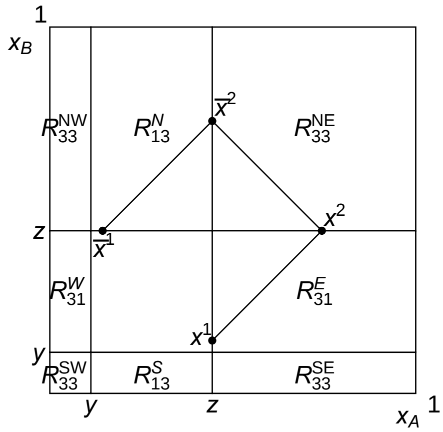

Figure 24

We neglect Q_{11} and solve $C_{22}(x_A, x_B) > C_{ij}(x_A, x_B)$ in Q_{ij} for $ij \neq 11$ to determine the set R_{ij} of consumers that prefers S_{ij} to S_{22}: see Figure 24. We use $\mathcal{A}(R_{ij})$ to denote the area of the set R_{ij}.

In Q_{33}^{SW}, $C_{22}(x_A, x_B) > C_{33}(x_A, x_B)$ reduces to $x_B < \frac{3}{4}P_2^* - \frac{3}{2}p_3 - x_A + \frac{1}{3}$, which is satisfied for each $(x_A, x_B) \in Q_{33}^{SW}$, for each $p_3 < p^*$. Hence $R_{33}^{SW} = Q_{33}^{SW}$ and $\mathcal{A}(R_{33}^{SW}) = y^2$.

In Q_{33}^{SE}, $C_{22}(x_A, x_B) > C_{33}(x_A, x_B)$ reduces to $x_B < \frac{3}{4}P_2^* - \frac{3}{2}p_3 + \frac{1}{2}x_A - \frac{1}{6}$, which holds for each $(x_A, x_B) \in Q_{33}^{SE}$, for each $p_3 < p^*$. Hence $R_{33}^{SE} = Q_{33}^{SE}$ and $\mathcal{A}(R_{31}^{SE}) = y(1 - z)$. Likewise, $R_{33}^{NW} = Q_{33}^{NW}$ and $\mathcal{A}(R_{33}^{NW}) = y(1 - z)$.

In Q_{33}^{NE}, $C_{22}(x_A, x_B) > C_{33}(x_A, x_B)$ is equivalent to $x_B > \frac{4}{3} + 3p_3 - \frac{3}{2}P_2^* - x_A$. Hence R_{33}^{NE} coincides with Q_{33}^{NE} except for a triangle in the left bottom of Q_{33}^{NE} with vertices (z, z), $\mathbf{x}^2 = (\frac{4}{3} + 3p_3 - \frac{3}{2}P_2^* - z, z)$, $\bar{\mathbf{x}}^2 = (z, \frac{4}{3} + 3p_3 - \frac{3}{2}P_2^* - z)$ (see Figure 24) and $\mathcal{A}(R_{33}) = (1 - z)^2 - \frac{1}{2}(\frac{4}{3} + 3p_3 - \frac{3}{2}P_2^* - 2z)^2$.

In Q_{31}^W, $C_{22}(x_A, x_B) > C_{31}(x_A, x_B)$ is equivalent to $x_B < \frac{3}{2}P_2^* - \frac{3}{2}p^* - \frac{3}{2}p_3 - 2x_A + \frac{2}{3}$, which holds for each $(x_A, x_B) \in Q_{31}^W$, for each $p_3 < p^*$. Hence $R_{31}^W = Q_{31}^W$ and $\mathcal{A}(R_{31}^W) = y(z - y)$. Likewise, $R_{13}^S = Q_{13}^S$ and $\mathcal{A}(R_{13}^S) = y(z - y)$.

In Q_{31}^E, $C_{22}(x_A, x_B) > C_{31}(x_A, x_B)$ reduces to $x_B < \frac{3}{2}P_2^* - \frac{1}{3} - \frac{3}{2}p^* - \frac{3}{2}p_3 + x_A$, which makes R_{31}^E equal to Q_{31}^E minus a triangle in the top left of Q_{31}^E with vertices (z, z), $\mathbf{x}^1 = (z, \frac{3}{2}P_2^* - \frac{1}{3} - \frac{3}{2}p^* - \frac{3}{2}p_3 + z)$, $\mathbf{x}^2$ (see Figure 24). Hence $\mathcal{A}(R_{31}^E) = (z - y)(1 - z) - \frac{1}{2}(\frac{1}{3} + \frac{3}{2}p^* + \frac{3}{2}p_3 - \frac{3}{2}P_2^*)^2$. Likewise, $\mathcal{A}(R_{13}^N) = (z - y)(1 - z) - \frac{1}{2}(\frac{1}{3} + \frac{3}{2}p^* + \frac{3}{2}p_3 - \frac{3}{2}P_2^*)^2$.

Hence the total demand of firm 3 when $p_3 < p^*$ is

$$\begin{aligned} & 2\mathcal{A}(R_{33}^{SW}) + 2\mathcal{A}(R_{33}^{SE}) + 2\mathcal{A}(R_{33}^{NW}) + 2\mathcal{A}(R_{33}^{NE}) + \mathcal{A}(R_{31}^W) + \mathcal{A}(R_{31}^E) + \mathcal{A}(R_{13}^S) + \mathcal{A}(R_{13}^N) \\ = \; & 1 + \frac{9}{2}p^* - \frac{9}{2}p_3 - \frac{9}{2}(\frac{2}{9} + p^* + p_3 - P_2^*)^2 \end{aligned}$$

In the Supplementary Material we derive the total demand for firm 3 when $p_3 > p^*$ and find that the complete demand function for firm 3 is

$$D_3(p_3)=\begin{cases} 1+\frac{9}{2}p^*-\frac{9}{2}p_3-\frac{9}{2}(\frac{2}{9}+p_3+p^*-P_2^*)^2 & \text{if } 0\le p_3\le \frac{1}{5}p^*+\frac{2}{5}P_2^*+\frac{2}{45} \\ \frac{981}{32}p_3^2-(\frac{153}{8}P_2^*+\frac{369}{16}p^*+\frac{77}{8})p_3+\frac{9}{8}(P_2^*)^2 \\ +\frac{117}{8}P_2^*p^*+\frac{13}{4}P_2^*-\frac{99}{32}(p^*)^2+\frac{25}{8}p^*+\frac{61}{72} & \text{if } \frac{1}{5}p^*+\frac{2}{5}P_2^*+\frac{2}{45}<p_3\le \frac{1}{2}p^*+\frac{1}{4}P_2^*+\frac{1}{9} \\ (\frac{1}{2}+\frac{9}{4}p^*-\frac{9}{4}p_3)(\frac{5}{4}-\frac{27}{8}p^*+\frac{9}{2}P_2^*-\frac{45}{8}p_3) & \text{if } \frac{1}{2}p^*+\frac{1}{4}P_2^*+\frac{1}{9}<p_3\le 2P_2^*-3p^*+\frac{2}{9} \\ \frac{27}{2}(\frac{2}{9}+\frac{1}{2}P_2^*-p_3)^2 & \text{if } 2P_2^*-3p^*+\frac{2}{9}<p_3\le \frac{1}{2}P_2^*+\frac{2}{9} \\ 0 & \text{if } p_3>\frac{1}{2}P_2^*+\frac{2}{9} \end{cases} \tag{28}$$

Since $\pi_3(p_3)=p_3D_3(p_3)$, from (28) we obtain $\pi_3'(p_3)<0$ for $p_3\in(0,p^*)$, $\pi_3'(p_3)>0$ for $p_3\in(p^*,\frac{1}{2}P_2^*+\frac{2}{9})$.

7.2 Proof of Lemma 5

In Subsection 5.2 we have established that given $p_1=p_3=p$, the best reply for firm 2 is given by $br_2(p,\gamma_\alpha^2)$ in (15). In the Supplementary Material we consider the point of view of firm 3 (a similar argument applies for firm 1). Given that firm 1, firm 2 play $p_1=p$, $P_2=P$, we derive the demand function of firm 3 when p_3 is slightly larger than p and obtain

$$D_3(p_3)=1+\frac{9}{2}(p_1-p_3)-\frac{9}{2}(\frac{2}{9}+2\alpha+p_1+p_3-P_2)^2 \tag{29}$$

if $p\le\frac{1}{2}P-\alpha+\frac{1}{18}$ (this is the demand function for firm 3 from (8), after replacing P_2 with $P_2-2\alpha$). If instead $p>\frac{1}{2}P-\alpha+\frac{1}{18}$, then

$$D_3(p_3)=\frac{63}{2}p_3^2+(45\alpha-18p_1-\frac{45}{2}P-10)p_3+\frac{9}{2}(P-2\alpha)^2+\frac{9}{2}Pp_1+4P+\frac{9}{2}p_1^2-9p_1\alpha+2p_1-8\alpha+\frac{8}{9} \tag{30}$$

From (29) and $\pi_3(p_3)=p_3D_3(p_3)$ it follows that

$$\pi_3'(p_3)=-\frac{27}{2}p_3^2+(18P-18p-36\alpha-13)p_3+\frac{9}{2}p+1-\frac{9}{2}(p+2\alpha-P+\frac{2}{9})^2 \tag{31}$$

and the first order condition for p_3, at $p_3=p$, is $-\frac{9}{2}P^2+27Pp+18P\alpha+2P-36p^2-54p\alpha-\frac{21}{2}p-18\alpha^2-4\alpha+\frac{7}{9}=0$. Jointly with $P=\frac{2}{3}p+\frac{2}{3}\alpha+\frac{2}{27}$ from (15), this yields p^*,P_2^* in (16), which satisfy the inequalities $p\le\frac{1}{2}P-\alpha+\frac{1}{18}$ and $p+\alpha\le\frac{5}{36}$ for each $\alpha\le\frac{13}{180}$, but violate them if $\alpha>\frac{13}{180}$.

From (30) we obtain

$$\pi_3'(p_3)=\frac{189}{2}p_3^2+(90\alpha-36p-45P-20)p_3+\frac{9}{2}(P-2\alpha)^2+\frac{9}{2}Pp+4P+\frac{9}{2}p^2-9p\alpha+2p-8\alpha+\frac{8}{9} \tag{32}$$

and the first order condition with respect to p_3, at $p_3=p$, is $\frac{9}{2}P^2-\frac{81}{2}Pp-18P\alpha+4P+63p^2+81p\alpha-18p+18\alpha^2-8\alpha+\frac{8}{9}=0$. Jointly with $P=\frac{1}{27}\sqrt{324(p+\alpha)^2-144(p+\alpha)+70}+\frac{4}{3}(p+\alpha)-\frac{8}{27}$ from (15), this yields p^*,P_2^* in (17), which satisfy $p>\frac{1}{2}P-\alpha+\frac{1}{18}$ and $p+\alpha>\frac{5}{36}$ for each $\alpha\in(\frac{13}{180},\frac{5}{9})$.

In the Supplementary Material we derive firm 3's complete demand function and show, also using the software Mathematica, that for each $\alpha\in(0,\frac{5}{9})$, $p_3=p^*$ is a best reply for firm 3.

7.3 Proof of Lemma 6

In the Supplementary Material we show that if

$$\begin{cases} \max\{p_2-\alpha,\frac{1}{2}P_3\}<p_1\le\frac{2}{5}P_3+\frac{1}{5}(p_2-\alpha)+\frac{2}{45}, & \alpha-2P_3+5p_1-\frac{2}{9}\le p_2<\alpha+p_1 \\ \frac{5}{2}p_1-\frac{1}{2}(p_2-\alpha)-\frac{1}{9}\le P_3<p_1+p_2-\alpha+\frac{2}{9} \end{cases} \tag{33}$$

then the demand functions are

$$\begin{cases} D_1(p_1) = & (9P_3 - 9p_2 + 9\alpha - \frac{13}{2})p_1 - \frac{9}{2}p_1^2 - \frac{9}{2}\alpha^2 - 9\alpha P_3 + \frac{7}{9} \\ & +9\alpha p_2 - \frac{5}{2}\alpha - \frac{9}{2}P_3^2 + 9P_3p_2 + 2P_3 - \frac{9}{2}p_2^2 + \frac{5}{2}p_2 \\ D_2(p_2) = & (9P_3 - 9p_1 + 9\alpha - \frac{13}{2})p_2 - \frac{9}{2}p_2^2 - \frac{9}{2}\alpha^2 - 9\alpha P_3 + \frac{7}{9} \\ & +9\alpha p_1 + \frac{13}{2}\alpha - \frac{9}{2}P_3^2 + 9P_3p_1 + 2P_3 - \frac{9}{2}p_1^2 + \frac{5}{2}p_1 \\ D_3(P_3) = \frac{9}{2}P_3^2 - (9p_1 + 9p_2 - 9\alpha + 2)\, P_3 + \frac{1}{18}\,(9p_1 + 9p_2 - 9\alpha + 2)^2 \end{cases} \tag{34}$$

and from them we derive the following first order conditions for p_1, p_2, P_3:[34]

$$\begin{cases} (18P_3 - 18(p_2 - \alpha) - 13)\, p_1 - \frac{27}{2}p_1^2 - \frac{9}{2}P_3^2 \\ +9P_3(p_2 - \alpha) + 2P_3 - \frac{9}{2}(p_2 - \alpha)^2 + \frac{5}{2}(p_2 - \alpha) + \frac{7}{9} = 0 \\ -\frac{27}{2}p_2^2 + (18\alpha + 18P_3 - 18p_1 - 13)\, p_2 - \frac{9}{2}\alpha^2 - 9\alpha P_3 \\ +9\alpha p_1 + \frac{13}{2}\alpha - \frac{9}{2}P_3^2 + 9P_3p_1 + 2P_3 - \frac{9}{2}p_1^2 + \frac{5}{2}p_1 + \frac{7}{9} = 0 \\ P_3 - (\frac{1}{3}p_1 - \frac{1}{3}\alpha + \frac{1}{3}p_2 + \frac{2}{27}) = 0 \end{cases} \tag{35}$$

By solving (35) numerically, we obtain a solution that satisfies (33) for $\alpha \in (0, \frac{26}{77}]$; the equilibrium partition of consumers among the available systems is described by Figure 12. However, for $\alpha > \frac{26}{77}$ solving (35) numerically yields a solution that violates $p_1 \leq \frac{2}{5}P_3 + \frac{1}{5}(p_2 - \alpha) + \frac{2}{45}$, therefore also $\alpha - 2P_3 + 5p_1 - \frac{2}{9} \leq p_2$ and $\frac{5}{2}p_1 - \frac{1}{2}(p_2 - \alpha) - \frac{1}{9} \leq P_3$ fail to hold.

In the Supplementary material we show that if

$$\begin{cases} \frac{2}{5}P_3 + \frac{1}{5}(p_2 - \alpha) + \frac{2}{45} < p_1 < \frac{1}{3}P_3 + \frac{1}{3}(p_2 - \alpha) + \frac{2}{27}, \qquad \alpha - P_3 + 3p_1 - \frac{2}{9} < p_2 < \alpha - 2P_3 + 5p_1 - \frac{2}{9} \\ \max\{4p_1 - 2(p_2 - \alpha) - \frac{4}{9}, \frac{1}{2}p_1 + \frac{3}{2}(p_2 - \alpha) - \frac{1}{9}\} < P_3 < \frac{5}{2}p_1 - \frac{1}{2}(p_2 - \alpha) - \frac{1}{9} \end{cases} \tag{36}$$

then the demand functions are

$$\begin{cases} D_1(p_1) = & \frac{981}{32}p_1^2 - \left(\frac{153}{8}P_3 + \frac{369}{16}(p_2 - \alpha) + \frac{77}{8}\right) p_1 - \frac{99}{32}\alpha^2 - \frac{117}{8}\alpha P_3 + \frac{61}{72} \\ & +\frac{99}{16}\alpha p_2 - \frac{25}{8}\alpha + \frac{9}{8}P_3^2 + \frac{117}{8}P_3p_2 + \frac{13}{4}P_3 - \frac{99}{32}p_2^2 + \frac{25}{8}p_2 \\ D_2(p_2) = & \left(\frac{153}{16}\alpha + \frac{63}{8}P_3 - \frac{99}{16}p_1 - \frac{53}{8}\right) p_2 - \frac{153}{32}p_2^2 - \frac{153}{32}\alpha^2 - \frac{63}{8}\alpha P_3 + \frac{55}{72} \\ & +\frac{99}{16}\alpha p_1 + \frac{53}{8}\alpha - \frac{45}{8}P_3^2 + \frac{117}{8}P_3p_1 + \frac{7}{4}P_3 - \frac{369}{32}p_1^2 + \frac{25}{8}p_1 \\ D_3(P_3) = & \frac{9}{4}P_3^2 + \left(\frac{9}{4}p_1 - \frac{45}{4}(p_2 - \alpha) - \frac{5}{2}\right) P_3 - \frac{153}{16}p_1^2 + \frac{63}{16}\alpha^2 - \frac{117}{8}\alpha p_1 + \frac{7}{36} \\ & -\frac{63}{8}\alpha p_2 - \frac{7}{4}\alpha - \frac{153}{16}p_1^2 + \frac{117}{8}p_1p_2 + \frac{13}{4}p_1 + \frac{63}{16}p_2^2 + \frac{7}{4}p_2 \end{cases} \tag{37}$$

and (37) yields the following first order conditions for p_1, p_2, P_3:

$$\begin{cases} \frac{2943}{32}p_1^2 - \left(\frac{153}{4}P_3 + \frac{369}{8}(p_2 - \alpha) + \frac{77}{4}\right) p_1 + \frac{9}{8}P_3^2 + \frac{13}{4}P_3 \\ +\frac{117}{8}P_3(p_2 - \alpha) - \frac{99}{32}(p_2 - \alpha)^2 + \frac{25}{8}(p_2 - \alpha) + \frac{61}{72} = 0 \\ -\frac{459}{32}p_2^2 + \left(\frac{153}{8}\alpha + \frac{63}{4}P_3 - \frac{99}{8}p_1 - \frac{53}{4}\right) p_2 - \frac{153}{32}\alpha^2 - \frac{63}{8}\alpha P_3 + \frac{99}{16}\alpha p_1 \\ +\frac{53}{8}\alpha - \frac{45}{8}P_3^2 + \frac{117}{8}P_3p_1 + \frac{7}{4}P_3 - \frac{369}{32}p_1^2 + \frac{25}{8}p_1 + \frac{55}{72} = 0 \\ \frac{27}{4}P_3^2 + \left(\frac{9}{2}p_1 - \frac{45}{2}(p_2 - \alpha) - 5\right) P_3 - \frac{153}{16}p_1^2 + \frac{13}{4}p_1 \\ +\frac{117}{8}p_1(p_2 - \alpha) + \frac{63}{16}(p_2 - \alpha)^2 + \frac{7}{4}(p_2 - \alpha) + \frac{7}{36} = 0 \end{cases} \tag{38}$$

[34]The first order condition for P_3 is written taking into account that the derivative with respect to P_3 of the profit function of firm 3, given D_3 in (34), factors into $\frac{27}{2}\left(P_3 - (\frac{1}{3}p_1 - \frac{1}{3}\alpha + \frac{1}{3}p_2 + \frac{2}{27})\right)\left(P_3 - (p_1 - \alpha + p_2 + \frac{2}{9})\right)$, and since $\frac{1}{3}p_1 - \frac{1}{3}\alpha + \frac{1}{3}p_2 + \frac{2}{27} < p_1 - \alpha + p_2 + \frac{2}{9}$ (because $p_1 + \alpha - \frac{2}{9} < p_2$ in equilibrium, otherwise firm 1 has zero demand), it follows that $P_3 = p_1 - \alpha + p_2 + \frac{2}{9}$ is a minimum point for the profit of firm 3, $P_3 = \frac{1}{3}p_1 - \frac{1}{3}\alpha + \frac{1}{3}p_2 + \frac{2}{27}$ is a maximum point.

By solving (38) numerically, we obtain a solution that satisfies (36) for each $\alpha \in (\frac{26}{77}, \frac{5}{9})$. In the Supplementary Material we show, also using the software Mathematica, that the solution we obtain is a NE for each $\alpha \in (0, \frac{5}{9})$.

7.4 Proof for Lemma 7

The demand for firm 2 is the area of the set of (x_A, x_B) which satisfy

$$C_{22}(x_A, x_B) < \min\{C_{11}(x_A, x_B), C_{33}(x_A, x_B)\} \tag{39}$$

We use P to denote the common value of P_1 and P_3 and define $\delta = \frac{1}{2}P_2 - \frac{1}{2}P - \alpha$. Then we notice that (39) holds for each (x_A, x_B) if $\delta < -\frac{2}{9}$; hence $D_2(P_2) = 1$ in this case. Conversely, if $\delta \geq \frac{1}{9}$ then (39) is violated for each (x_A, x_B) and $D_2(P_2) = 0$. In the intermediate case of $\delta \in [-\frac{2}{9}, -\frac{1}{18})$, the set of (x_A, x_B) such that (39) holds is the convex decagon in Figure 25,[35] with area $1 - 15(\frac{2}{9} + \delta)^2$. If $\delta \in [-\frac{1}{18}, \frac{1}{9})$, then the set of (x_A, x_B) which satisfy (39) is the hexagon in Figure 26,[36] with area $3(1 - 3\delta)(\frac{1}{9} - \delta)$.

Fig. 25 The set of consumers that buy S_{22} in γ_α^{123} given that $\delta = \frac{1}{2}P_2 - \frac{1}{2}P - \alpha$ is between $-\frac{2}{9}$ and $-\frac{1}{18}$

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Fig. 26 The set of consumers that buy S_{22} in γ_α^{123} given that $\delta = \frac{1}{2}P_2 - \frac{1}{2}P - \alpha$ is between $-\frac{1}{18}$ and $\frac{1}{9}$

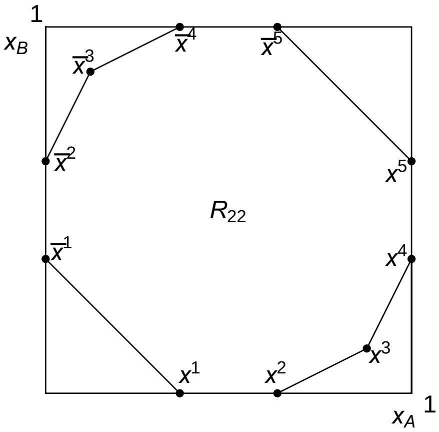

Figure 25

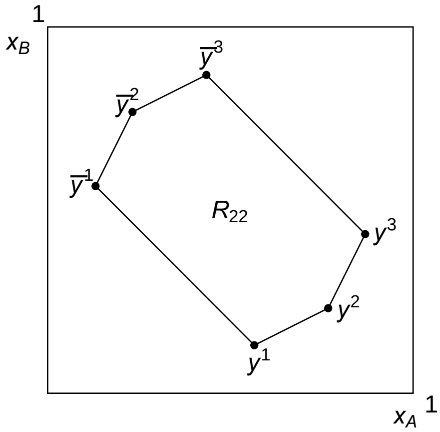

Figure 26

Using $\delta = \frac{1}{2}P_2 - \frac{1}{2}P - \alpha$, we write the demand for firm 2 as a function of P_2 as follows:

$$D_2(P_2) = \begin{cases} 1 & \text{if } P_2 < P + 2\alpha - \frac{4}{9} \\ 1 - 15(\frac{2}{9} + \frac{1}{2}P_2 - \frac{1}{2}P - \alpha)^2 & \text{if } P + 2\alpha - \frac{4}{9} \leq P_2 < P + 2\alpha - \frac{1}{9} \\ \frac{1}{12}(9P + 18\alpha - 9P_2 + 2)(3P + 6\alpha - 3P_2 + 2) & \text{if } P + 2\alpha - \frac{1}{9} \leq P_2 \leq P + 2\alpha + \frac{2}{9} \\ 0 & \text{if } P + 2\alpha + \frac{2}{9} \leq P_2 \end{cases} \tag{40}$$

hence P_2 that maximizes firm 2's profit is

$$br_2(P, \gamma_\alpha^{123}) = \begin{cases} \frac{2}{3}P + \frac{4}{3}\alpha + \frac{8}{27} - \frac{1}{27}\sqrt{81(P + 2\alpha)^2 + 72(P + 2\alpha) + 28} & \text{if } P + 2\alpha \leq \frac{31}{90} \\ \frac{2}{3}P + \frac{4}{3}\alpha - \frac{8}{27} + \frac{1}{27}\sqrt{81(P + 2\alpha)^2 - 72(P + 2\alpha) + \frac{404}{5}} & \text{if } P + 2\alpha > \frac{31}{90} \end{cases} \tag{41}$$

The demand for firm 3 is the area of the set of (x_A, x_B) which satisfy

$$C_{33}(x_A, x_B) < \min\{C_{11}(x_A, x_B), C_{22}(x_A, x_B)\} \tag{42}$$

and in order to solve this inequality we define $\mu = P_3 - P_1$, $\theta = P_3 - P_2 + 2\alpha$. We consider P_3 close to P_1, that is μ close to 0. First we examine the case of α close to zero, which implies that P_3 is close to P_2

[35] In Figure 25, $\mathbf{x}^1 = (\frac{2}{3} + 3\delta, 0)$, $\mathbf{x}^2 = (\frac{1}{3} - 3\delta, 0)$, $\mathbf{x}^3 = (\frac{7}{9} - \delta, \frac{2}{9} + \delta)$, $\mathbf{x}^4 = (1, \frac{2}{3} + 3\delta)$, $\mathbf{x}^5 = (1, \frac{1}{3} - 3\delta)$.

[36] In Figure 26, $\mathbf{y}^1 = (\frac{5}{9} + \delta, \frac{1}{9} + 2\delta)$, $\mathbf{y}^2 = (\frac{7}{9} - \delta, \frac{2}{9} + \delta)$, $\mathbf{y}^3 = (\frac{8}{9} - 2\delta, \frac{4}{9} - \delta)$.

in equilibrium, therefore also θ is close to zero. Then the set of (x_A, x_B) which satisfy (42) is the union of the three convex sets in Figure 27: the two quadrilaterals with vertices $\mathbf{x}^1, ..., \mathbf{x}^4$ and $\bar{\mathbf{x}}^1, ..., \bar{\mathbf{x}}^4$, and the hexagon with vertices $\mathbf{x}^5, \mathbf{x}^6, ..., \bar{\mathbf{x}}^5$:[37]

Fig. 27 The set of consumers that buy S_{33} in γ_α^{123} given that $\mu = P_3 - P_1$ is close to 0 and $\theta = P_3 - P_2 + 2\alpha$ is close to 0

Fig. 28 The set of consumers that buy S_{33} in γ_α^{123} given that $\mu = P_3 - P_1$ is close to 0 and $\theta = P_3 - P_2 + 2\alpha$ is greater than $\frac{1}{9}$

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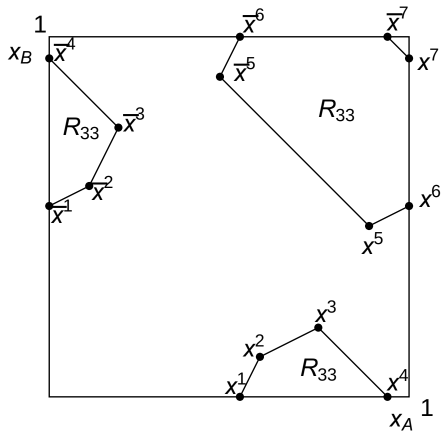

Figure 27

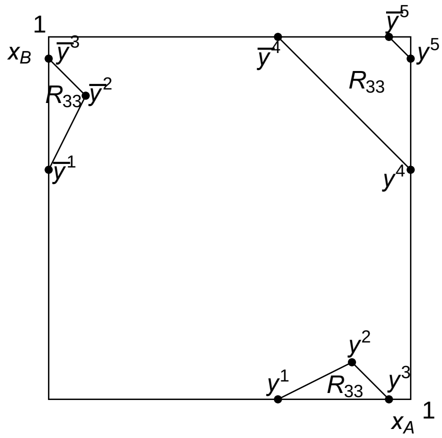

Figure 28

This is a disconnected set with area equal to

$$D_3(P_3) = \frac{9}{4}P_3^2 + \left(\frac{9}{2}\alpha - \frac{9}{4}P_1 - \frac{9}{4}P_2 - 2\right)P_3 - 9P_1\alpha + \frac{9}{2}P_1P_2 - \frac{9}{8}P_1^2 - \frac{9}{8}P_2^2 + P_1 - \frac{9}{2}\alpha^2 + \frac{9}{2}\alpha P_2 - 2\alpha + P_2 + \frac{1}{3} \quad (43)$$

Precisely, (43) applies as long as $\frac{1}{2}\mu + \frac{1}{9} \geq \theta$.[38] When instead μ is about 0 but $\theta > \frac{1}{9}$, then the set of (x_A, x_B) satisfying (42) is the union of the three convex sets in Figure 28: the two triangles with vertices $\mathbf{y}^1, \mathbf{y}^2, \mathbf{y}^3$ and $\bar{\mathbf{y}}^1, \bar{\mathbf{y}}^2, \bar{\mathbf{y}}^3$, and the quadrilateral with vertices $\mathbf{y}^4, \mathbf{y}^5, \bar{\mathbf{y}}^5, \bar{\mathbf{y}}^4$;[39] the area is given by (44)

$$D_3(P_3) = 3P_3^2 + \left(\frac{21}{2}\alpha - \frac{3}{4}P_1 - \frac{21}{4}P_2 - \frac{7}{3}\right)P_3 - \frac{3}{8}P_1^2 - 3P_1\alpha + \frac{3}{2}P_1P_2 + \frac{2}{3}P_1 + \frac{15}{2}\alpha^2 - \frac{15}{2}\alpha P_2 - \frac{10}{3}\alpha + \frac{15}{8}P_2^2 + \frac{5}{3}P_2 + \frac{10}{27} \quad (44)$$

7.4.1 The equilibrium prices

For α close to zero, from (43) we obtain $\Pi_3'(P_3) = \frac{27}{4}P_3^2 + \left(9\alpha - \frac{9}{2}P - \frac{9}{2}P_2 - 4\right)P_3 + \frac{9}{2}PP_2 - 9P\alpha - \frac{9}{8}P^2 + P - \frac{9}{2}\alpha^2 + \frac{9}{2}\alpha P_2 - 2\alpha - \frac{9}{8}P_2^2 + P_2 + \frac{1}{3}$ and the first order condition for P_3, at $P_3 = P$, is $\frac{9}{8}P^2 - 3P - \frac{9}{2}\alpha^2 + \frac{9}{2}\alpha P_2 - 2\alpha - \frac{9}{8}P_2^2 + P_2 + \frac{1}{3} = 0$. Combining this with (41) (for the case of $P + 2\alpha \leq \frac{31}{90}$) yields the following prices:

$$P^* = \frac{1}{13 - 54\alpha}\left(108\alpha^2 + 108(P_2^*)^2 - 270\alpha P_2^* - 23\alpha - \frac{727}{18} - 89\rho(\alpha) + \frac{89(18\alpha - 972\alpha^2 - 173)}{2916\rho(\alpha)}\right) \quad (45)$$

$$P_2^* = \alpha + \frac{11}{18} + \rho(\alpha) + \frac{972\alpha^2 - 18\alpha + 173}{2916\rho(\alpha)} \quad (46)$$

[37] In Figure 27, $\mathbf{x}^1 = (\frac{1}{2} + \frac{3}{4}\mu, 0)$, $\mathbf{x}^2 = (\frac{5}{9} + \mu - \frac{1}{2}\theta, \frac{1}{9} + \frac{1}{2}\mu - \theta)$, $\mathbf{x}^3 = (\frac{7}{9} + \frac{1}{2}\theta - \mu, \frac{2}{9} - \frac{1}{2}\mu - \frac{1}{2}\theta)$, $\mathbf{x}^4 = (1 - \frac{3}{2}\mu, 0)$ and $\mathbf{x}^5 = (\theta - \frac{1}{2}\mu + \frac{8}{9}, \frac{1}{2}\theta + \frac{1}{2}\mu + \frac{4}{9})$, $\mathbf{x}^6 = (1, \frac{3}{4}\mu + \frac{1}{2})$, $\mathbf{x}^7 = (1, 1 - \frac{3}{2}\mu)$.

[38] Otherwise the vertical coordinate of $\mathbf{x}^2$ (the horizontal coordinate of $\bar{\mathbf{x}}^2$) in Figure 27 is negative, and the horizontal coordinate of $\mathbf{x}^5$ (the vertical coordinate of $\bar{\mathbf{x}}^5$) is greater than 1.

[39] In Figure 28, $\mathbf{y}^1 = (\frac{3}{2}\theta + \frac{1}{3}, 0)$, $\mathbf{y}^2 = (\frac{7}{9} + \frac{1}{2}\theta - \mu, \frac{2}{9} - \frac{1}{2}\mu - \frac{1}{2}\theta)$, $\mathbf{y}^3 = (1 - \frac{3}{2}\mu, 0)$ and $\mathbf{y}^4 = (1, \frac{3}{2}\theta + \frac{1}{3})$, $\mathbf{y}^5 = (1 - \frac{3}{2}\mu, 1)$.

in which $\rho(\alpha) = \frac{1}{18}\sqrt[3]{162\alpha^2 - 453\alpha - \frac{76}{3} + (13-54\alpha)\sqrt{-432\alpha^4 - 184\alpha^3 - \frac{857}{3}\alpha^2 - \frac{506}{3}\alpha - \frac{27869}{729}}}$.[40] The prices in (45)-(46) satisfy $\frac{1}{2}\mu + \frac{1}{9} \geq \theta$ for $\alpha \in (0, \frac{71}{630}]$, but violate this inequality if $\alpha > \frac{71}{630}$.

For the case of $\alpha > \frac{71}{630}$, from (44) we obtain $\Pi_3'(P_3) = 9P_3^2 + \left(\frac{21}{2}\alpha - \frac{3}{4}P - \frac{21}{4}P_2 - \frac{7}{3}\right)2P_3 + 3\left(\frac{1}{2}P - \alpha + \frac{1}{2}P_2 + \frac{2}{9}\right)^2 + \frac{1}{2}\left(\frac{3}{2}P_2 - 3\alpha + \frac{2}{3}\right)^2 - \frac{9}{8}P^2$ and the first order condition with respect to P_3, at $P_3 = P$, is $\frac{57}{8}\left(P - (P_2 + \frac{4}{9} - 2\alpha)\right)\left(P - (\frac{5}{19}P_2 + \frac{20}{171} - \frac{10}{19}\alpha)\right) = 0$. Combining this with (41) (for the case of $P + 2\alpha > \frac{31}{90}$) yields

$$P^* = \frac{5}{117} - \frac{5}{26}\alpha + \frac{1}{1638}\sqrt{99\,225\alpha^2 - 44\,100\alpha + 29\,470} \quad (47)$$

$$P_2^* = \frac{33}{26}\alpha - \frac{11}{39} + \frac{19}{8190}\sqrt{99\,225\alpha^2 - 44\,100\alpha + 29\,470} \quad (48)$$

In the Supplementary Material we show, also using the software Mathematica, that (P^*, P_2^*) in (45)-(46) are a NE of γ_α^{123} for each $\alpha \in (0, \frac{71}{630}]$, and that (P^*, P_2^*) in (47)-(48) are a NE of γ_α^{123} for each $\alpha \in (\frac{71}{630}, \frac{5}{9})$.

7.5 Profit comparisons for the study of the stage one reduced game in Γ_α: (19)-(22)

Here we report the plots of the profit functions linked to (19)-(22).

Fig. 29 Π_2^{123}, solid curve, vs Π_2^3, dashed curve: see (19)

Fig. 30 Π_3^3, solid curve, vs $\Pi_3^\varnothing$, dashed curve: see (20)

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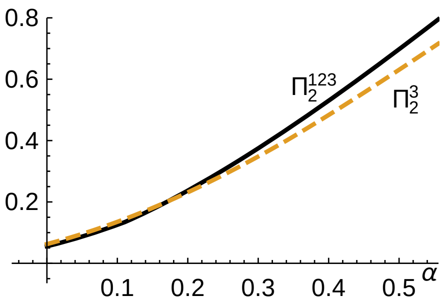

Figure 29

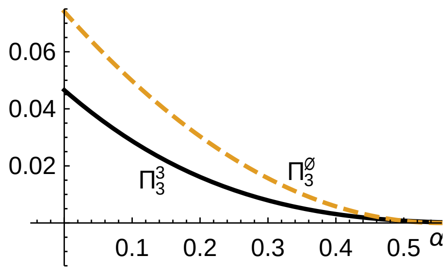

Figure 30

Fig. 31 Π_3^{123}, solid curve, vs Π_3^1, dashed curve: see (21)

Fig. 32 Π_3^{123}, solid curve, vs Π_3^2, dashed curve: see (22)

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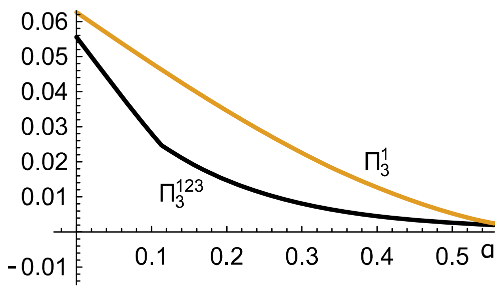

Figure 31

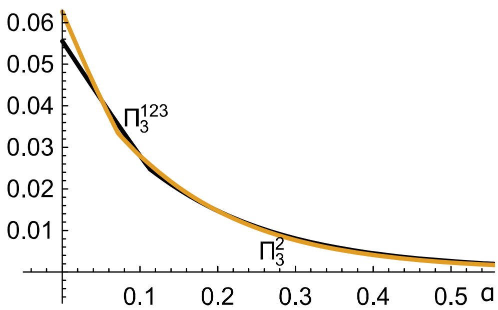

Figure 32

[40] Solving the system consisting of $\frac{9}{8}P^2 - 3P - \frac{9}{2}\alpha^2 + \frac{9}{2}\alpha P_2 - 2\alpha - \frac{9}{8}P_2^2 + P_2 + \frac{1}{3} = 0$ and (41) leads to a third degree equation in P, $-3P^3 + (\frac{53}{6} - 3\alpha)P^2 + (6\alpha^2 + \frac{59}{18}\alpha - \frac{226}{81})P - \frac{10}{9}\alpha^2 - \frac{40}{81}\alpha + \frac{227}{972} = 0$, for which no solution can be expressed in terms of real radicals. Indeed, $\rho(\alpha)$ is a complex number, although P^* and P_2^* in (45)-(46) are real numbers.