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The Hockey Stick Phillips Curve and the Zero Lower Bound

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Abstract

The recently observed disconnect between inflation and economic activity can be explained by the interplay between the zero lower bound (ZLB) and the costs of external financing. In normal times, credit spreads and the nominal interest rate balance out; factor costs dominate firms' marginal costs. When nominal rates are constrained, larger spreads can more than offset the effect of lower factor costs and induce only moderate inflation responses. The Phillips curve is hence flat at the ZLB, but features a positive slope in normal times and thus a hockey stick shape. Via this mechanism, forward guidance may induce deflationary effects.

Keywords: Phillips Curve, Financial Frictions, Zero Lower Bound, Disinflation, Forward Guidance
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1 Introduction

What is the relationship between inflation and economic activity? Given the fundamental role of these two concepts, it is troubling that this question is currently puzzling the profession. After the Global Financial Crisis of 2007/2008 and the associated financial turmoil, inflation seemed disconnected from economic activity, leading to puzzles of both "missing disinflation" and "missing inflation" (Ball and Mazumder, 2011; Coibion and Gorodnichenko, 2015; Lindé and Trabandt, 2019). These observations sparked considerable interest in analyzing the determinants of the resulting seemingly flat Phillips curve.

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While the explanations put forward are numerous and manifold, we found one key contributing factor yet to be missing: the zero lower bound (ZLB) on nominal interest rates, which was reached by several central banks around the globe in recent years, coincidental with the observed inflation puzzles.[1]

In this paper, we show how the interplay of the ZLB and financial frictions reshapes the relationship between inflation and economic activity. From recent research, it is understood that financial distortions can be crucial for firms' price setting behavior and, thereby, for inflation dynamics (e.g. Gilchrist et al., 2017). We argue that during normal times, firms' marginal costs are dominated by the – procyclical – costs of employing real production factors, which hence determine their price setting. In the presence of financial frictions, marginal costs further contain the costs of external financing. These consist of the real safe interest rate and, caused by financial frictions, a countercyclical credit spread. While the effects of these two roughly balance in normal times, larger credit spreads can more than offset the effect of lower production factor costs if the nominal rate is constrained by ZLB. As a result, financial shocks at the ZLB induce only moderate deflation responses, and may in extreme cases even be inflationary. Taking the ZLB into account, the resulting *observational Phillips curve*[2] is thus shaped like a hockey stick: it features the usual positive slope in normal times, while – for large negative output gaps – being flat at the ZLB.

We show these results using an analytically tractable New Keynesian DSGE model featuring an explicit role for external financing. In the model, financial frictions result from the combination of a working capital channel in the spirit of Ravenna and Walsh (2006) and a costly state verification problem à la Townsend (1979) and Bernanke et al. (1999). Workers need to be paid before production, generating external financing needs for the entrepreneurs operating the firms. The costs of external finance thus consist of the real interest rate plus a risk premium, which depends positively on entrepreneurs' (countercyclical) leverage. We focus on the effects of financial shocks in the form of risk premium shocks in the spirit of Smets and Wouters (2007). These shocks are known to have a large explanatory power for the joint movement of consumption and investment following 2007/2008 recession (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., 2020).

Our first contribution is to show theoretically that, ceteris paribus, the expectation of a longer ZLB period can be associated with a weaker deflationary response, or even an increase in inflation. We provide closed-form solutions for the macroeconomic dynamics following financial shocks, both for normal times and for the case of a binding ZLB. After deriving the necessary conditions under which this scenario is possible, we argue that they are fairly regular: this case may occur for large financial shocks whenever the elasticity of the credit spread with respect to entrepreneur leverage is sufficiently large. The closed-form solutions furthermore highlight that *Neo-Fisherian* effects – in the form of an overall increase of inflation following contractionary financial shocks – are possible,

[1] Throughout this paper, we refer to the concept of a lower bound on nominal interest rates at zero. Our results equally hold when allowing for an effective lower bound (ELB) above zero or in negative territory.

[2] This refers to the realized values for inflation and output gap, i.e. the *observed* or *empirical* Phillips curve. As discussed below, this is not equivalent to the New Keynesian Phillips curve describing firms' price setting behavior.

and may in particular occur if the ZLB is expected to bind for an extended period of time.

We then investigate the implications of our mechanism numerically. This demonstrates that the simulated observational Phillips curve, i.e. the realized values of inflation and output gap that would be observed in general equilibrium, features a striking hockey stick shape. For normal times with positive or mildly negative output gaps, it exhibits a conventional positive slope in output gap - inflation space. In contrast, the slope is considerably flat for significantly negative output gaps when the ZLB is binding.

As our second contribution, we discuss the associated implications for monetary policy. We argue that the hockey stick Phillips curve does not constitute a threat to policymakers per se: at the ZLB, (further) contractionary financial shocks do not lead to an additional substantial decline of inflation. However, designing appropriate monetary policy at the ZLB in times of financial frictions is challenging. In particular, monetary policy shocks generate macroeconomic dynamics that are highly similar to financial shocks. As a consequence, forward guidance shocks with relatively low persistence can even be deflationary: their short-term effect of further decreasing expected refinancing costs may dominate their long-term effect of increasing the price level by stimulating consumption. Hence, this also provides an explanation for the forward guidance puzzle (Carlstrom et al., 2015; Del Negro et al., 2015a; Kiley, 2016) and suggests that any forward guidance measures must be undertaken with vigor.

The issue of missing (dis-)inflation in the recent years was first brought up by Ball and Mazumder (2011) and subsequently confirmed for many advanced economies by Friedrich (2016). As the authors argue, inflation did not fall as much as expected given the depth of the recession caused by the Global Financial Crisis. In subsequent years, however, inflation was lower than expected given the economic recovery. A manifold of explanations was put forward, encompassing anchored expectations (Ball and Mazumder, 2018; Coibion and Gorodnichenko, 2015), various measures of economic slack (Gordon, 2013; Watson, 2014; Krueger et al., 2014; Faccini and Melosi, 2019), supply shocks and wage rigidities (Daly and Hobijn, 2014; Lindé and Trabandt, 2019), optimal monetary policy, potentially in combination with financial frictions (Lieberknecht, 2019; McLeay and Tenreyro, 2020) or global factors (Bobeica and Jarociński, 2019; Forbes, 2019). Compared to this literature, our paper provides a complementary explanation for inflation dynamics that also matches the particular timing of the observed missing (dis-)inflation: the ZLB affects the cyclicality of marginal costs via the costs of external financing, thereby leading to a disconnect between economic activity and inflation.

A related strand of the literature investigates these recent inflation dynamics through the lens of New Keynesian DSGE models, notably Christiano et al. (2015), Del Negro et al. (2015b) and Gilchrist et al. (2017). In line with our paper, these contributions show that adding financial frictions to DSGE models helps to explain the missing disinflation puzzle in the US in the aftermath of the Global Financial Crisis. Closely related to our work, Gilchrist et al. (2017) explain inflation dynamics via financial distortions, i.e. larger credit spreads in recessions. While our paper shares this argument, we provide additional insights that a binding ZLB strongly amplifies the effects of financial frictions, such that credit spreads may even dominate inflation dynamics. This is in line with insights by Bianchi and Melosi (2017) and Boehl and Strobel (2020), who find that accounting for the ZLB substantially improves the empirical fit of estimated DSGE models.

Our hockey-stick Phillips curve is also well-supported by recent empirical work on the effects of financial shocks. In particular, expansionary financial shocks can be disinflationary if supply-side effects dominate demand effects. Consistent with prevailing supply effects of financial shocks, Barth III and Ramey (2001); Chowdhury et al. (2006); Tillmann (2008) and Abbate et al. (2016) find evidence in favor of such a cost channel. Similarly, Gaiotti and Secchi (2006) find this cost channel to be proportional to working capital, using Italian firm-level data. Relatedly, Acharya et al. (2020) find that cheap credit to impaired firms has a disinflationary effect by creating excess production capacity. Conversely, these contributions provides an empirical foundation for our channel, suggesting that contractionary financial shocks can indeed also be inflationary.

Lastly, our paper is related to the literature on Neo-Fisherianism, which argues that causality between the policy rate and inflation may be positive, or at least ambiguous. As such, Gabaix (2016) deems Neo-Fisherianism uncontroversial in the long run. Modern proponents challenging his rather conventional view are Cochrane (2011, 2016, 2017) – "when is the long run?" – and García-Schmidt and Woodford (2019). The latter find that in a perfect-foresight world, credible (long-run) changes in long-run targets can have immediate effects. In contrast, we share the view of Gerke and Hauzenberger (2017) that the above effect rather is an artifact of equilibrium selection instead of a "classic" macroeconomic effect.[3] Correspondingly, we show that Neo-Fisherian effects can exist even when expectations are anchored to the long-term interest rate, i.e. when assuming conventional terminal conditions.

The rest of the paper is structured as follows. Section 2 outlines the New Keynesian DSGE model with financial frictions and discusses the components of marginal costs in this framework. In Section 3, we derive closed-form solutions for macroeconomic dynamics following financial shocks. Section 4 complements by showing numerical solutions and analyzing the resulting observational Phillips curve. In Section 5, we investigate the implications for monetary policy at the ZLB. Section 6 concludes.

2 Model

Our analysis is based on a tractable New Keynesian DSGE model featuring an explicit role for external financing via financial frictions. The model setup is based on Boehl (2020) and Lieberknecht (2019), to which we refer for further details. We assume that production is subject to a working capital channel as in Ravenna and Walsh (2006). A distinct role for equity finance is motivated via a costly state verification problem in the spirit of Townsend (1979) and Bernanke et al. (1999). Entrepreneurs operating wholesale firms borrow money from the financial intermediary to finance production, and their shares are traded at the financial markets exchange. Their (homogeneous) good is sold to a monopolistic retail sector where diversification takes place. The resulting final goods are sold to a representative household, who consumes and supplies labor in a perfectly competitive labor market. A monetary authority sets the nominal interest rate, which is subject to a lower bound.

[3] Although it is clear that a New-Keynesian model with the ZLB produces a multiplicity of equilibrium paths (Benhabib et al., 2001), it is unclear how many of these paths can be stable equilibria.

2.1 *Households*

Households maximize the expected present value of lifetime utility by deciding over consumption of a composite good C_t and hours devoted to the labor market H_t. For each supplied unit of labor, they receive the real wage W_t. Households can deposit monetary savings D_t at the financial intermediary, for which they receive the gross nominal interest rate R_t in the next period. The final consumption good is composed of differentiated retail products and is sold in a market with monopolistic competition. The composite good and its respective aggregate price index are given by standard CES aggregators.

The household's optimization problem is completely standard and optimization yields the usual inter-temporal Euler equation and an intra-temporal labor supply equation

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}, \tag{2}$$

where Π_t is gross inflation. In the spirit of Smets and Wouters (2007), we understand U_t as a premium on the risk-free interest rate that reflects the state of the financial system. This type of shock features a hight explanatory power regarding the post-2000 macroeconomic dynamics across all standard shocks, and can explain a large share of the joint dynamics of consumption, investment and inflation following the 2007/2008 financial crisis (Gust et al., 2017; Kulish et al., 2017; Boehl and Strobel, 2020; Boehl et al., 2020). We label U_t as the *financial shock* in the following. The parameters σ, η and β are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of labor supply and the discount rate, respectively.

2.2 *Wholesale and Retail Firms*

The wholesale sector consists of a continuum of firms indexed by j. Each firm is operated by a risk-neutral entrepreneur. Labor is the only production factor, and the CRS production function for the homogeneous good is given by

$$Y_{j,t} = \omega_{j,t} H_{j,t}, \tag{3}$$

where $Y_{j,t}$ is output produced by firm j and $\omega_{j,t}$ is a firm-specific idiosyncratic productivity shock. Similar to Ravenna and Walsh (2006), it is assumed that workers have to be paid before production takes place, while returns are realized at the end of the period. This working capital channel (also labeled the *cost channel*) motivates a positive role for external finance. The amount of external finance $L_{j,t}$ demanded by firm j is given by its desired working capital $W_t H_{j,t}$ minus its equity $N_{j,t}$,

$$L_{j,t} = W_t H_{j,t} - N_{j,t}. \tag{4}$$

As in Bernanke et al. (1999), we employ a costly state verification (CSV) approach along the lines of Townsend (1979) for external financing. In the following, we focus on the economic intuition and refer the interested reader to Boehl (2020) for formal details. The central idea of the CSV approach is that the realization of the idiosyncratic productivity shock is private information of the entrepreneur. As a consequence, banks can only observe produced output when paying monitoring costs. The contract that solves this

CSV problem specifies that the interest rate on a loan obtained by an entrepreneur from the intermediary $R^L_{j,t}$ contains an endogenous risk premium on the prevailing real interest rate.[4] This reflects that banks anticipate the possibility that the monitoring fee has to be payed. The risk premium is a credit spread that depends on the individual firm's leverage $LEV_{j,t} = \frac{W_t H_{j,t}}{N_{j,t}}$,

$$R^L_{j,t} = z\left(\frac{W_t H_{j,t}}{N_{j,t}}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t, \tag{5}$$

with $z'(\cdot) > 0$. Intuitively, when the leverage ratio decreases, the premium on external finance falls because more collateral is provided such that the loan becomes less risky. Banks only monitor firms if the entrepreneur defaults, and seize the remaining output as collateral. It can be shown that all entrepreneurs take identical choices in equilibrium, such that Equation (5) also holds in the aggregate,

$$R^L_t = z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t. \tag{6}$$

Since the wholesale sector is assumed to be perfectly competitive, wholesale firms are price takers. Denote by X_t the gross markup that retailers charge over wholesale goods. Equivalently, X_t^{-1} is the relative price of one unit of wholesale goods, which needs to equal marginal costs MC_t. In the aggregate, no-arbitrage requires the rate of return on working capital to equal the rate on external funding. It follows that firms' marginal costs are given by

$$MC_t = X_t^{-1} = W_t R^L_t = W_t\, z\left(\frac{W_t H_t}{N_t}\right)\frac{R_t}{E_t[\Pi_{t+1}]}U_t. \tag{7}$$

We follow a simplified version of Lieberknecht (2019) with respect to equity financing. We assume that entrepreneurs can issue equity in the stock market, which is bought by risk-neutral financial traders associated with the financial intermediaries. Imposing no arbitrage on financial markets and noting that entrepreneurs must be indifferent between external finance and equity finance in equilibrium, the return on assets satisfies

$$E_t[R^A_{t+1}] = R^L_t. \tag{8}$$

Invoking rule-of-thumb behavior from financial traders, it can be shown that the evolution of equity is given by

$$N_t = \Psi\left(Y_t\right), \tag{9}$$

with $\Psi'(\cdot) > 0$, such that equity financing is procyclical with respect to output, which captures the key notion of standard financial accelerator models à la Bernanke et al. (1999).

After wholesale goods have been produced, retailers buy the homogeneous good $Y_{j,t}$ on the wholesale market. After differentiation, they sell it in the monopolistically com-

[4]In line with Bernanke et al. (1999), this solution assumes that all bargaining power accrues to the entrepreneur.

petitive good market. Firms' price setting decisions are subject to nominal rigidities à la Calvo (1983), which gives rise to a classic New Keynesian Phillips curve, either expressed in terms of the markup X_t or in terms of marginal costs MC_t.[5]

2.3 The Central Bank

The central bank follows a standard contemporaneous monetary policy rule for the the notional gross nominal interest rate R_t^n,

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp(v_t). \tag{10}$$

The interest rate on deposits R_t is subject to a ZLB constraint and cannot fall below $\bar{R}$,

$$R_t = \max\left\{\bar{R}, R_t^n\right\}. \tag{11}$$

Lastly, v_t is a monetary policy shock evolving as

$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}. \tag{12}$$

Note that when the ZLB constraint in Equation (11) binds, v_t can be understood as a *forward guidance shock* as it prolongs the expected duration of the ZLB.

2.4 Understanding the Components of Marginal Costs

We linearize the equations characterizing the equilibrium around an efficient steady state in order to gain analytical insights.[6] The full set of equilibrium equations is shown in the Appendix. In the following, we let small-case letters denote variables in log-deviations from the steady state.

In the financial accelerator framework, financial frictions originate in the firm sector and therefore primarily affect the supply side of the economy. In contrast, the behavior of households is standard. The role of financial frictions for marginal costs and, thereby, for inflation dynamics is thus best understood by studying the linearized New Keynesian Phillips curve. The latter can be cast in the familiar textbook form

$$\pi_t = \kappa\, mc_t + \beta E_t[\pi_{t+1}] \tag{13}$$

with slope $\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$.

Hence, financial frictions do not alter the price setting behavior of firms per se, as prices are tied to marginal costs and expectations of future inflation. However, financial frictions determine and affect the components of marginal costs. After linearizing Equation (7), we obtain

$$mc_t = w_t + (r_t - E_t[\pi_{t+1}]) + s_t, \tag{14}$$

[5] See Bernanke et al. (1999) for details on this particular solution.

[6] Steady state subsidies from the government (financed by lump-sum taxes) can correct for the inefficiencies arising from monopolistic competition and the presence of financial frictions. See Lieberknecht (2019) for details.

where s_t denotes the linearized credit spread $s_t = z(lev_t) + u_t$. This highlights that marginal costs consist of three components: first, the costs of hiring production factors, i.e. labor, represented by the real wage. We call this component the *real* marginal costs in the following. The terms in brackets are the costs of external finance, which consist of the risk-free real interest rate and the endogenously determined external finance premium. The former represents a pure cost channel in the spirit of Ravenna and Walsh (2006), whereas the latter constitutes the credit spread arising from informational asymmetries between borrowers and lenders. These costs of external finance are absent from the standard NK model, where marginal costs consist solely of real wages.

Taking a closer look at the components, note that leverage lev_t is (in linearized form) given by

$$lev_t = w_t + h_t - n_t, \tag{15}$$

where we assume $n_t = \psi y_t$, with $\psi = \Psi'(\cdot)$ denoting the elasticity of equity with respect to output. In a financial accelerator economy, the credit spread (also known as the external finance premium) is countercyclical (Bernanke et al., 1999). Entrepreneur leverage is only countercyclical if the procyclicality of net worth outweighs the procyclicality of the loan value. Given Equation (15), this means that entrepreneur net worth must be more procyclical than the wage bill. Using the household's intra-temporal optimality condition and the net worth evolution, we can rewrite the above equation as

$$lev_t = -(\psi - 1 - \sigma - \eta) y_t. \tag{16}$$

For any form of demand-side disturbances like our financial shock or a monetary policy shock, the necessary and sufficient condition for leverage to be countercyclical is thus that the term in brackets is larger than zero. This implies the following parameter restriction:

Assumption 1. *The elasticity of net worth with respect to output satisfies*

$$\psi > 1 + \sigma + \eta. \tag{17}$$

Denote the elasticity of the credit spread with respect to entrepreneur leverage as $z'(\cdot) = \nu$. It can be shown that this elasticity is larger than zero.[7] This implies that the credit spread is a positive function of entrepreneur leverage, such that the countercyclical entrepreneur leverage according to Assumption 1 leads to a countercyclical external finance premium as well.

Using these insights about entrepreneur leverage, marginal costs can be written as

$$mc_t = \gamma\, y_t + (r_t - E_t[\pi_{t+1}]) + u_t, \tag{18}$$

with

$$\gamma \equiv \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{19}$$

capturing the elasticity of marginal costs with respect to output. The marginal factor cost component – the term $\sigma + \eta$ in γ – is procyclical. As output increases, expanding

[7] The parameter ν is a non-linear function of the steady state contract and entrepreneur balance sheet values. In turn, these depend on aggregate (quarterly) default probabilities, the variance of entrepreneurs' idiosyncratic productivity and banks' monitoring costs. See Lieberknecht (2019) for more details.

production requires firms to offer a larger real wage in order to incentivise more labor supply from workers. The finaning cost component $-\nu(\psi - 1 - \sigma - \eta)$ is countercyclical given Assumption 1 as the spread is dominated by the procyclicality of net worth. The financial shock u_t is also countercyclical, as output y_t falls for positive realizations of u_t. The cyclicality of the cost channel (the real interest rate in Equation (18)) is ambiguous and depends on the source of aggregate fluctuations, as this determines the endogenous nominal interest rate reaction by the central bank. As financial shocks are normally deflationary, and as the central bank's response to inflation is dominant over the response to output for most conventional monetary policy rules, the nominal interest rate is thus procyclical following financial shocks.[8]

The three components of marginal costs are thus characterized by opposing cyclicality over the business cycle. For financial shocks, real marginal costs and the pure interest rate channel are procyclical, whereas the external finance premium is countercyclical. Since firms' price setting is tightly connected to their marginal costs, this has important implications for inflation dynamics. In particular, the presence of financial frictions changes the relationship between inflation and output over the business cycle. The extent to which this occurs depends on the relative strength of the various components.

3 Financial Shocks at the Zero Lower Bound

In this section we analyze how a binding ZLB on nominal interest rates affects the transmission of financial shocks in the economy. To this end, we derive closed-form general equilibrium solutions for normal times and for when the economy is at the ZLB. Contrasting the two cases highlights that macroeconomic dynamics at the ZLB may be fundamentally different compared to normal times.

3.1 Financial Shocks in Normal Times

We first consider the macroeconomic effects of financial shocks in normal times, i.e. in the absence of a binding ZLB on nominal interest rates. The whole model can be represented in three equations,[9]

$$\pi_t = \kappa\gamma\, y_t + (\beta - \kappa)E_t[\pi_{t+1}] + \kappa(r_t + u_t), \tag{20}$$

$$y_t = -\sigma^{-1}\left(r_t - E_t[\pi_{t+1}] + u_t\right) + E_t[y_{t+1}], \tag{21}$$

$$r_t = \max\left\{\phi_\pi \pi_t + \phi_y y_t + v_t, \bar{r}\right\}, \tag{22}$$

in addition to the exogenous processes for the financial shock u_t and the monetary policy shock v_t,

$$u_t = \rho\, u_{t-1} + \epsilon_t, \tag{23}$$

$$v_t = \rho_r\, v_{t-1} + \epsilon_{r,t}. \tag{24}$$

Equation (20) again represents the New Keynesian Phillips curve from Equation (13) after plugging in marginal costs from Equation (18). The slope with respect to output is

[8] For technology shocks and other pure supply-side shocks that raise output while being deflationary, the cost channel is countercyclical.

[9] See the Appendix for more details on this particular representation.

given by $\kappa\gamma = \kappa\big[\sigma+\eta-\nu(\psi-1-\sigma-\eta)\big]$ where, as discussed in Section 2.4, the first term captures real marginal costs and the second part represents the endogenous evolution of the external finance premium. The third and fourth term in the Phillips curve reflect the financing cost channel and the purely exogenous markup effect that arises from financial shocks by increasing the credit spread.

Equation (21) is the linearized Euler equation, governing the intertemporal consumption allocation of households as a function of the real interest rate. Equation (22) is the monetary policy rule defining how the central bank sets the (notional) interest rate as a reaction to inflation and output[10], incorporating the ZLB constraint which specifies that the nominal interest rate can not be lower than $\bar{r}$. The latter two equations are, apart from the max-operator, completely standard and identical to the textbook New Keynesian model. Financial frictions thus manifest solely in the New Keynesian Phillips curve, again highlighting that the financial accelerator is a supply-side friction that directly affects inflation dynamics.

We solve the model using the method of undetermined coefficients and postulate that the equilibrium responses of endogenous variables are linear functions of the exogenous financial shock. The results are summarized in the following Proposition:

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are given by*

$$\pi_t = a_0\, u_t, \tag{25}$$
$$y_t = b_0\, u_t, \tag{26}$$

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1)}, \tag{27}$$
$$b_0 = -\frac{1+(\phi_\pi-\rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{28}$$

Proof. See Appendix. ■

In combination with Proposition 1, the following Lemma 1 shows that financial shocks are a particular form of demand shocks. A positive financial shock increases the wedge between the interest rate controlled by the central bank and the return on bonds held by households, thereby reducing current consumption. Thus, a positive financial shock decreases overall output. Via the New Keynesian Phillips curve, inflation decreases as well as marginal factor costs dominate over financing costs.

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \tag{29}$$
$$b_0 < 0, \tag{30}$$

[10] Note that for financial shocks, the responses of output and the output gap are identical: an efficient economy without nominal rigidities and financial frictions does not respond to financial shocks.

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{31}$$

Proof. See Appendix. ■

The analytic solutions from Proposition 1 display precisely the different channels through which the financial shock operates, which allows for a corresponding decomposition. In a_0, the first term in the numerator is the slope of the Phillips curve with respect to output, whereas the second term captures the exogenous markup effect of the financial shock. Following a positive financial shock, real marginal costs decrease, because labor demand falls given the decline in demand (the first part of $\kappa\gamma$). This in turn reduces inflation. At the same time, the financial shock increases the costs of production via the external finance premium, as financial frictions in the firm sector intensify (the second part of $\kappa\gamma$ and the markup effect). This increase in the credit spread partially counteracts the decline in real marginal costs, weakening the overall disinflationary effect.

The cost channel is represented by the last term in the denominator in a_0. This term features a negative sign and is thus – ceteris paribus – disinflationary. Generally, if the central bank reacts stronger (weaker) to fluctuations in inflation and output, the denominator is larger (smaller), such that the overall response of inflation is smaller (larger). However, lower nominal interest rates in reaction to the overall decline in inflation also decrease marginal costs directly. This amplifies the disinflationary response and the cost channel thus weakens the overall stabilizing property of the central bank's interest rate policy.

Following financial shocks, the various components of marginal costs thus move in different directions. Whereas real marginal costs and the pure financial cost channel amplify the disinflationary response, the credit spread channel weakens it. As seen in Lemma 1, the overall inflation response in normal times is negative, as long as the elasticity of the credit spread to entrepreneur leverage is not excessively large.[11] In this case, the real marginal cost channel dominates the price setting of firms, whereas the effects of interest rate channel and credit spread channel approximately level out.

Nevertheless, as summarized in Lemma 2 below, the analytic solutions reveal that Neo-Fisherian effects – an overall increase of inflation following positive financial shocks – are in principle possible. This situation may occur if the credit spread channel dominates both real marginal costs and the pure cost channel because the credit spread sensitivity to leverage is excessively large:

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ZLB on nominal interest rates) is Neo-Fisherian whenever a_0 is positive, i.e.*

[11] Lemma 1 is equivalent to the parameter restriction that guarantees a positive numerator of a_0. As shown in the Appendix, determinacy of the model requires the denominator in a_0 to be positive. Intuitively, the model is only determinate if a stronger central bank reaction to deviations from steady state translates into lower deviations in general equilibrium. The combination of a positive numerator from Lemma 1 and determinacy thus yields $a_0 < 0$ (note the minus in front of the fraction).

if the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{32}$$

Proof. See Appendix. ■

Note that this results directly from the presence of financial frictions linking credit spreads to marginal costs: in the absence of financial frictions, the policy functions in Proposition 1 are unambiguously negative. While the hypothesis that inflation is Neo-Fisherian in normal times cannot be rejected ex-ante (c.f. the discussion in Section 1), we want to focus on the case in which our financial shocks is a classic demand shock to maintain the analogy to the Global Financial Crisis. In the following, we hence generally assume that Equation (32) is not satisfied such that a_0 remains negative:

Assumption 2. *The elasticity of the credit spread to entrepreneur leverage satisfies Condition* (31) *from Lemma 1.*

This implies the natural case of an upwards sloping Phillips curve for financial shocks in normal times, i.e. a positive relationship between inflation and output.

3.2 Financial Shocks at the Zero Lower Bound

We now turn to the case of a binding ZLB on nominal interest rates, and analyze the macroeconomic dynamics following financial shocks. To this end, we assume that a financial shock endogenously brought the economy to the ZLB and makes private agents expect the ZLB to bind for a specific number of periods (often called the *ZLB spell duration*, e.g. Holden, 2019). In this section, we take this ZLB spell duration as given and do *not* adjust agents' expectations on the spell duration to any *additional* shocks, which we discuss in Section 4. This scenario hence focuses on *marginal* effects of (further) financial shocks at the ZLB. While this perspective abstracts from the mapping between shocks and the expected duration of the ZLB, it allows for a straightforward analytical comparison to the case of normal times.

The equilibrium responses of inflation and output can be characterized by recursive policy functions which are conditionally linear given the expected ZLB spell. The following proposition summarizes the results:

Proposition 2. *Suppose that the ZLB on nominal interest rate is expected to bind for $k > 0$ periods. Then, the impact responses of inflation and output to a financial shock are given by*

$$\pi_t = a_k\, u_t, \tag{33}$$

$$y_t = b_k\, u_t, \tag{34}$$

where

$$a_k = \kappa\left(1 - \gamma\sigma^{-1}\right)\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, a_{k-1} + \rho\kappa\gamma\, b_{k-1}, \tag{35}$$

$$b_k = -\sigma^{-1}\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1}\, a_{k-1} + \rho\, b_{k-1}, \tag{36}$$

and $\{a_0, b_0\}$ *as in Proposition 1.*

Proof. See Appendix. ■

To interpret Proposition 2, consider the inflation response for an expected ZLB duration of one quarter ($k = 1$), i.e. a_1, and recall that Assumption 2 guarantees negative policy functions a_0 and b_0. This implies that both the second and third term in Equation (35) are negative. The term in front of a_0 is close to unity for persistent shocks and shows the ZLB's amplification property: the impact response of inflation increases (ceteris paribus) in the expected length of the ZLB. This reflects the inability of the central bank at the ZLB to counteract further contractionary shocks by means of additional conventional monetary stimulus. At the same time, the resulting upward pressure on real interest rates depresses consumption, and accordingly overall output.

However, there is an opposing effect on the overall inflation response, captured by the first term in Equation (35). This term can be positive, such that there is potential for a policy function for inflation that is concave in the expected ZLB spell duration. In other words, it is possible that the disinflationary effect following positive financial shocks is *lower* if the ZLB is expected to bind for a longer period of time. A necessary condition for a concave inflation policy function is that $1 > \gamma\sigma^{-1}$, which is equivalent to the following Lemma 3.

Lemma 3. *The inflation policy function is concave if the elasticity of the credit spread with respect to entrepreneur leverage satisfies*

$$\nu > \frac{\eta}{\psi - 1 - \sigma - \eta}. \tag{37}$$

It thus follows from Lemma 3 that the overall response of inflation following inflationary shocks depends crucially on the elasticity of the credit spread with respect to entrepreneur leverage ν. This can also easily be seen by inspecting the solution for a_k, from which it follows that

$$\frac{\partial a_k}{\partial \nu} = \frac{\partial a_k}{\partial \gamma}\frac{\partial \gamma}{\partial \nu} > 0. \tag{38}$$

The first term in Equation (35) depends negatively on γ. The second depends positively on γ, but following the recursion brings up a_0, which is negative following Assumption 2. The last term is positive in γ as well, while $b_{k-1} < 0$ for all reasonable calibrations. The effect of an increase in ν can hence be traced back unambiguously: a larger elasticity of the credit spread with respect to entrepreneur leverage ceteris paribus increases the inflationary effect of financial shocks.

Intuitively, a concave policy function for inflation hence requires that the credit spread channel (the left-hand side in Equation (37)) dominates both the real marginal cost channel and the financial cost channel (the right-hand side in Equation (37)). As outlined above, the external finance premium rises following contractionary financial shocks, such that marginal costs increase ceteris paribus. This credit channel is stronger, the larger the elasticity of the credit spread with respect to entrepreneur leverage ν. If financial frictions are sufficiently pronounced such that ν is large, credit spreads may dominate the price setting of firms, thereby increasing inflation ceteris paribus. For the following analysis, we capture this scenario via the following assumption:

Assumption 3. *The elasticity of the credit spread with respect to entrepreneur leverage satisfies Condition* (37) *from Lemma 3.*

Note that Assumption 3 is weaker than the counterpart in Assumption 2. A further requirement for a concave policy function is that financial shocks are sufficiently large. This can be seen by inspecting the term $(1 + \frac{\bar{r}}{u_t})$, which is only positive if the following Assumption holds:

Assumption 4. *The financial shock size satisfies*

$$u_t > -\bar{r} = \beta^{-\sigma} - 1. \tag{39}$$

Figure 1 displays the policy functions a_k and b_k under two illustrative calibrations. In the first case, the parameters satisfy Assumptions 1, 2 and 4: the external finance premium is countercyclical, financial shocks have conventional effects in normal times and the shock is relatively large. In the second case, the calibration additionally satisfies Assumption 3. In the first case, the policy functions for inflation and output are strictly decreasing in the expected ZLB spell duration; a longer expected ZLB duration implies a stronger macroeconomic effect of additional financial shocks. In the second case, however, the policy function for inflation is concave, peaking at an expected ZLB duration of six quarters in positive (Neo-Fisherian) territory. In other words, if the ZLB is expected to bind for a longer period of time, the overall inflation response may even turn positive. This illustrates that inflation dynamics following financial shocks may be fundamentally different at the ZLB compared to normal times.

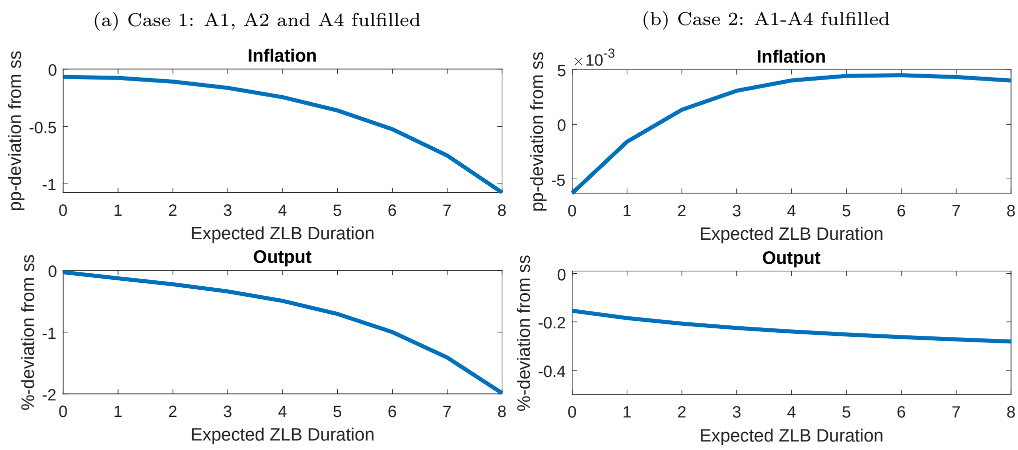

Figure 1: Expected ZLB Duration and Impact Response

4 Numerical Results and the Hockey Stick Phillips Curve

In this section, we supplement our closed-form solutions by a numerical analysis of the full general equilibrium rational expectations solution. While we derived our analytical solutions conditional on the expected ZLB spell k, we now treat it as endogenous. We employ numerical solution methods to present impulse responses to financial shocks and trace out the corresponding observational Phillips curve.

4.1 Calibration and Solution Method

Throughout this section, we fix the model's structural parameters to standard values taken from Woodford (2003), and adjust them to the most recent estimates (up until 2019) from Boehl and Strobel (2020, BS19 henceforth). We set $\beta = 0.99$, representing the standard view of a quarterly model. We calibrate $\sigma = 1$, which is a common assumption in line with a balanced growth path and also backed by BS19. Following the same line of reasoning, we set $\eta = 0.5$. We calibrate the fraction of non-adjusting price setters ζ to the commonly found textbook value of 0.66. This is conspicuously lower than the larger estimates from Smets and Wouters (2007) and BS19, as we want to avoid assuming a flat New Keynesian Phillips curve ex-ante.

For the parameters pertaining to the financial frictions, we fix $\psi = 8$ such that the output effects of financial shocks are amplified by approx. 20% relative to the standard NK model, which is roughly in line with the amplification degree documented by Bernanke et al. (1999). In the following, we regard ν as a free parameter and conduct comparative exercises.

Regarding monetary policy parameters, we set ϕ_π to 1.5 (a commonly used standard prior), and ϕ_y to 0.2. In line with the estimates of BS19, the latter value is large relative to the standard prior mean of 0.125. As the authors argue, this reflects the strong reaction of the Fed to output during the ZLB episode from 2009–2015, during which inflation was close to its target value while the level of output remained persistently depressed. We set $\rho = 0.9$, reflecting a lasting, quite persistent financial shock which resembles a post-2009 scenario.

The analytical solutions shown in the previous section hold for the impact period when the shock occurs, under the assumption that the expected duration of the ELB k is given. However, in general and in the absence of special policy measures such as forward guidance, k is an equilibrium outcome to be determined endogenously at each point in time, given the contemporaneous exogenous disturbances that causes the ELB constraint to bind. To solve the model at the ELB, we use the numerical solution method proposed by Boehl (2021). A brief description of the solution method is outlined in Appendix C.

4.2 Impulse Responses to Financial Shocks

Figure 2 displays impulse responses following contractionary financial shocks of differing size. For the impact responses, these correspond to the analytical policy functions in Proposition 1 and Proposition 2. As the shock size increases, the ZLB spell duration increases. Respectively, the initial response of inflation shifts upwards, in line with the analytical insight from Assumption 4. For a large value of u_t, the initial response of inflation becomes positive. Note that, since the responses of endogenous variables is a simple linear map of u_t and u_t decreases each period by $(1 - \rho)$, the lines are actually the same but shifted outwards by a larger initial shock.

Figure 3 shows impulse responses for different values of the elasticity of the credit spread with respect to entrepreneur leverage ν. In this figure, we consider a weak financial shock that is insufficiently strong to cause a binding ZLB. As a result, the dynamics look conventional, with inflation (and marginal costs) falling in response to the shock.

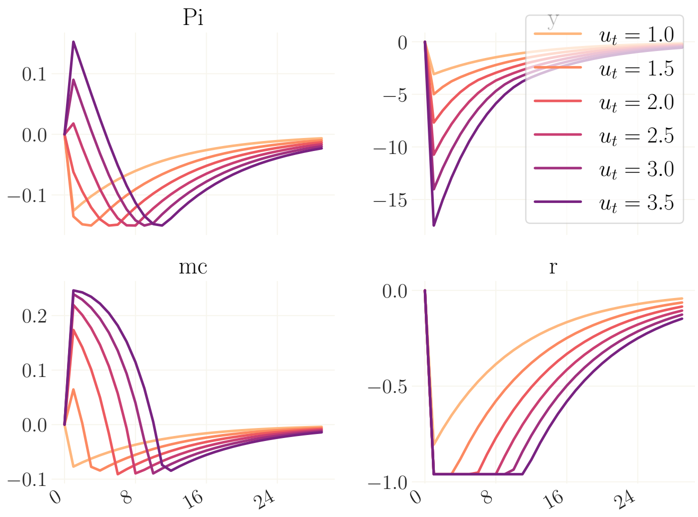

Figure 2: Impulse responses to different financial shocks for $\nu = 0.25$.

An exception is the scenario for $\nu = 0.26$ (not shown in the Figure), in which case inflation rises independently from whether or not the ZLB binds. This extreme case may be interpreted as the *Neo-Fisherian* parameterization outlined in the previous section: financial costs dominate the firms cost structure, and hence financial shocks translate directly to higher prices. It is also equivalent to the case where Assumption 3 and Lemma 1 are not satisfied.

In Figure 4, we consider a large financial shock, pushing the economy to the ZLB, for the same values of ν as in Figure 3. As highlighted by the graphs, the binding ZLB has an elevating effect on marginal costs, which dampens inflation. For $\nu = 0.25$, inflation actually increases, whereas the same calibration yields regular dynamics in the absence of the ZLB (see Figure 3). This corresponds to the standard case outlined in the previous section: the elasticity of the credit spread with respect to entrepreneur leverage is large enough to generate a concave inflation policy function, but not so excessively large such that Neo-Fisherian solutions emerge in normal times.[12]

4.3 The Observational Hockey Stick Phillips Curve

Figure 5 illustrates the main finding of our paper. The figure plots the impulse responses to financial shocks projected into $\{y_t, \pi_t\}$-space. We interpret this as the *observational Phillips Curve*, i.e. the realized values of inflation and output (gap) that

[12]As Proposition 2 suggests, the persistence of financial shocks ρ is another central parameter for inflation dynamics, both at the ZLB and for the rather extreme Neo-Fisherian case. A lower value of ρ yields a more concave inflation policy function (c.f. Equation 35). A lower ρ also implies a stronger discounting and hence a less dominant effect of the anticipated course of the financial shock. We illustrate this in Figure D.3 in the Appendix. We discuss the role of persistence in more detail in Section 5.2.

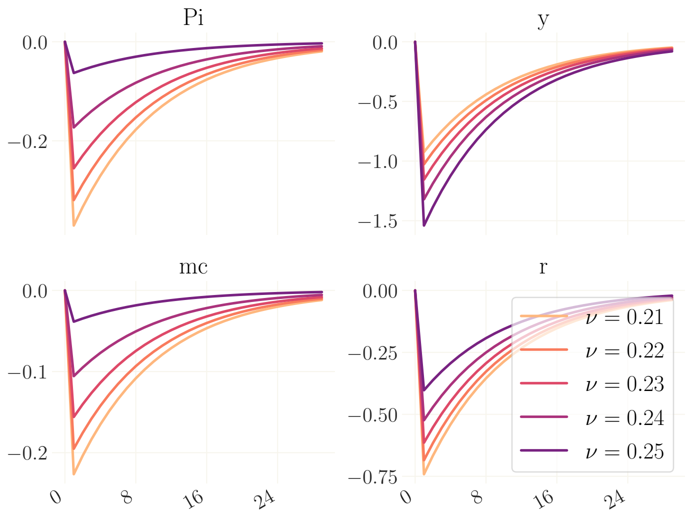

Figure 3: Impulse responses to a 0.5% financial shocks for different values of ν. The shock is not strong enough to cause a binding ZLB, which results in conventional inflation dynamics for most values of ν.

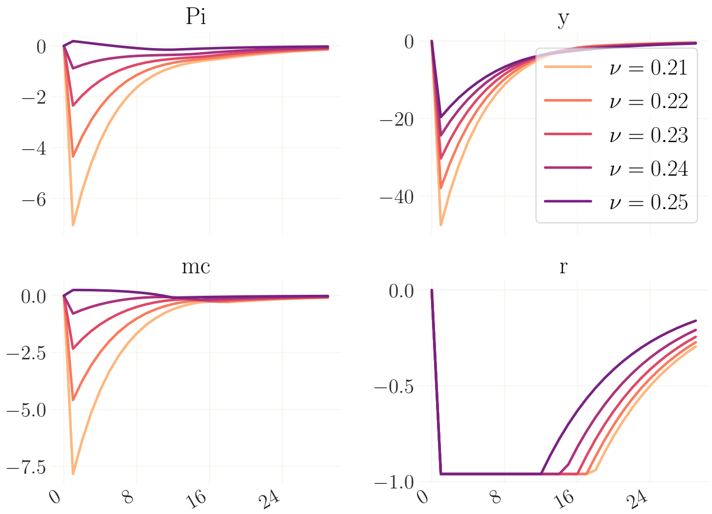

Figure 4: Impulse responses to 3% financial shocks for different values of ν. The shocks are strong enough to cause a binding ZLB, which results in unconventional inflation dynamics via the financial cost channel.

would be observed in general equilibrium.[13] This is in contrast to the theoretical New

[13]Note again that the output response following financial shocks is identical to the output gap response, see Footnote 10. As such, the figure can equivalently be interpret as showing the output gap - inflation

Keynesian Phillips curve – as shown in Equations (13) and (20) – which merely represents firms' price setting under the assumption of nominal price rigidities. The most remarkable observation in Figure 5 is the striking hockey stick shape of the observational Phillips curve. For positive values of output, the observed slope of the Phillips curve is positive, in line with standard theory. However, the observational Phillips curve flattens out at the ZLB, for substantially negative values of output (caused by large financial shocks). For $\nu = 0.24$ the observed slope in the region of -3% output is almost zero, while having a conventional slope in the origin.

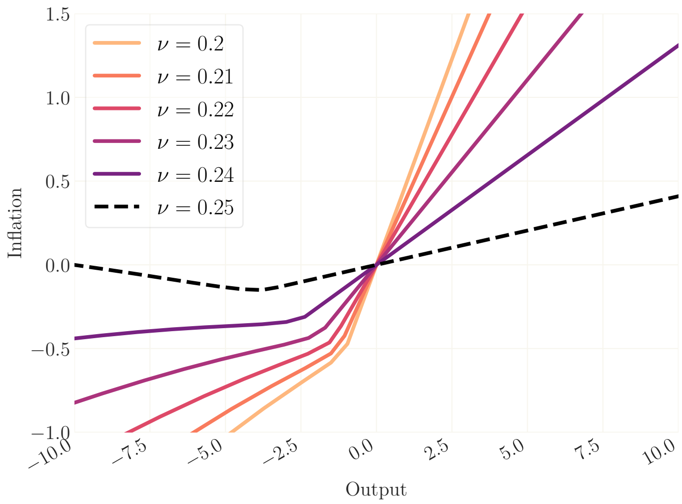

Figure 5: Observed Phillips Curve for an economy facing financial shocks. For each value of ν, we simulate the model for u_t on the interval $[-4, 4]$ and plot the respective combination of π_t and y_t.

In other words, an economic observer aiming to infer the slope of the Phillips curve in times of a binding ZLB and financial frictions would inherently conclude that the Phillips curve is "dead". This observation, however, results from the previously discussed credit spread channel, which may dominate firms' price setting at the ZLB. In contrast to the observational Phillips curve, the New Keynesian Phillips curve is well and alive. This means that the relationship between firms' prices and marginal costs, governed by the Calvo parameter, is intact. Note again that our calibration avoids pre-assuming a flat New Keynesian Phillips curve, with the Calvo parameter $\zeta = 0.66$ being considerably lower than the estimate of $\zeta = 0.85$ in BS19.

As the elasticity of the credit spread to leverage ν increases, the Phillips Curve becomes flatter in both normal times (with active monetary policy) and when the economy is at the ZLB. The hockey stick not only rotates in the origin, but also the ratio of the two slopes decreases. For a value of $\nu = 0.25$, we observe that the credit spread effect at the ZLB is strong enough that inflation actually increases with output, while the Phillips curve is still upwards sloping in normal times. An even larger ν of 0.26 finally shifts

space.

the system towards a fully Neo-Fisherian regime, where the Phillips curve is downward sloping on the full domain of u_t (not shown in Figure 5).

5 Implications for Monetary Policy

We now turn to the implications of our findings for central banks. From a monetary policy perspective, the changing transmission of financial shocks at the ZLB raises a number of challenges that require a different design of monetary policy actions.

5.1 Interpreting the Observational Phillips Curve

Our result about the flat observational Phillips curve at the ZLB means that the correct identification of the relationship between inflation and output is challenging. This is because policymakers need to infer the structural relationship between inflation and output (i.e. the structural New Keynesian Phillips slope and determinants of firms' price setting) using only observed equilibrium values. At the ZLB, this requires estimates of contemporaneous macroeconomic shocks, private sector expectation of the ZLB length and the currently prevailing degree of financial frictions. Acquiring this level of information in real time seems very challenging in practice.

On the bright side, our analysis suggests that the flat observational Phillips curve does not necessarily constitute an additional threat to policymakers per se. At the ZLB, (further) contractionary risk shocks do not lead to a substantial (further) decline of inflation. Given a strong mandate to stabilize inflation, a lower deflationary pressure is equivalent to a lower sense of urgency for monetary policy to counteract. This also means that central banks might not necessarily be forced to resort to unconventional monetary policy instruments at the ZLB. In fact, as Figures 2 and 4 show, a stronger credit spread channel rather leads to an attenuated fall in output.

However, one could argue that the source of inflation is important as well, raising further difficulties for monetary policy. In our theoretical framework, the lower deflationary pressure at the ZLB following financial shocks stems from larger credit spreads. If credit spreads are major determinants of inflation at the ZLB, this also implies that central banks should be predominantly concerned with financial conditions – in particular corporate financing conditions. In such a situation, reducing financial distress directly via appropriate monetary policy operations on financial markets might be the most efficient way to steer inflation. Unfortunately, central banks might find themselves in a catch-22 situation. On the one hand, large credit spreads might reflect substantial distress in the financial sector, thereby constituting a concern from a financial stability perspective. On the other hand, lower credit spreads induced by looser monetary and financial conditions increase the deflationary pressure. Therefore, a financial recovery might not necessarily be associated with (a revival of) inflation. As such, disentangling the role of real marginal costs and credit spreads for overall inflation seems important to design appropriate monetary (and macroprudential) policies.

5.2 Monetary Policy Shocks at the ZLB

The difficulties of interpreting the observational Phillips Curve and identifying the sources of inflation translate into delicate decisions about the appropriate design of monetary policy at the ZLB. To make matters worse, the effects of monetary policy itself are also affected by the presence of financial frictions and the ZLB. We analyze this aspect by first looking at forward guidance shocks, which at the ZLB can be represented by monetary policy shocks v_t (c.f. Section 2).

The first crucial insight regarding monetary policy shocks v_t is that they generate *identical* macroeconomic dynamics as financial shocks u_t in normal times, given the same shock persistence. The associated intuition is straightforward: the three-equation representation from Section 3.1 reveals that monetary policy shocks appear in the same places as financial shocks. Therefore, in this framework and away from the ZLB, monetary policy shocks and financial shocks are observationally equivalent in terms of inflation and output; they are only distinguishable via the response of the interest rate. As a consequence, all results from the previous sections concerning financial shocks in normal times are valid for monetary policy shocks as well. Notably, this includes the closed-form solutions and the possibility of Neo-Fisherian effects of monetary policy shocks in normal times for extreme calibrations. It also follows immediately that the central bank can, in principle, offset financial shocks perfectly in normal times.

The insight that both shocks appear in the same places features major implications for monetary policy at the ZLB, which is the second important contribution of this paper. At the ZLB, monetary policy shocks govern the expectations regarding the future interest rate path, acting like explicit forward guidance by the central bank. Forward guidance hence generates the same macroeconomic dynamics at the ZLB as financial shocks.[14] However, unfortunately for monetary policy, our previous results on the credit channel thus imply that forward guidance at the ZLB might not be particularly effective or even associated with unintended effects on inflation. Notably, this includes the possibility that forward guidance at the ZLB may be *deflationary*, i.e. inducing Neo-Fisherian effects by *decreasing* inflation, while raising output.

Intuitively, forward guidance shocks induce two opposing effects on inflation. First, expected rates are lower, which transmits to the economy via the standard Euler channel and the various channels on expected marginal costs. Second, agents expect that the inversion of the policy function will remain active for more periods. This second effect amplifies the reversal of the inflation response that is induced by the ZLB via the credit channel. As forward guidance unambiguously raises output, this could trigger a drop in inflation. Which of these effects dominates depends crucially on the forward guidance persistence and the degree of financial frictions. This can be seen in Figure 6 and 7, which show impulse responses following forward guidance shocks at the ZLB.

We summarize these considerations in the following lemma:

Lemma 4. *At the ZLB, forward guidance shocks v_t may be associated with Neo-Fisherian effects, such that expansionary forward guidance is disinflationary iff*

$$\rho_r < \rho. \tag{40}$$

Note that the condition in Lemma 4 is a necessary, but not a sufficient condition. To see this, assume a combination (ρ, ν) for which a given shock u_t is deflationary. As the mechanics behind forward guidance and financial shocks are equal, we learn from

[14] At the ZLB, monetary policy shocks v_t and financial shocks u_t are hence not distinguishable, given the same persistence.

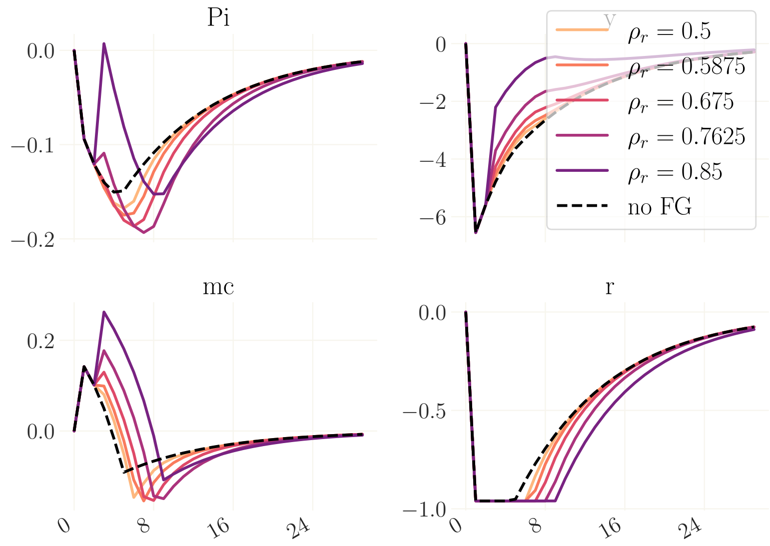

Figure 6: Dashed line: impulse responses to a 2% financial shock for $\nu = 0.25$. Colored lines are the same financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. For many values of ρ_r, the forward guidance shock is deflationary.

Equation (35) in Proposition 2 that a smaller ρ (or here: ρ_r) can reduce the weight on the (negative terminal) second and third term. In that sense, a decrease in ρ has a similar effect as an increase in ν. We show this effect in Figure D.3 in the Appendix.

While it is safe to assume a high persistence of the financial shock, the persistence of the forward guidance shock is to some extent a policy parameter that can in principal be chosen by the central bank. However, it also depends on how successful the central bank is in its communication strategies. As illustrated in Figure 6, a monetary policy shock with low persistence (i.e. low credibility) can hence trigger negative inflation responses because the short-run effect of decreasing financial costs dominates the longer-term effect that works through the household Euler Equation. As such, half-hearted or non-credible forward guidance may be associated with undesirable macroeconomic dynamics.

5.3 Monetary Policy Rules at the ZLB

After investigating monetary policy shocks, we now turn to the systematic behavior of central banks. At first glance, it may seem that these rules are irrelevant at the ZLB. However, they are in fact crucial for macroeconomic dynamics because rational private agents take the monetary policy rule into account when forming expectations about future variables and the remaining ZLB duration. As such, choosing an appropriate monetary policy rule is of central importance for central banks at the ZLB as well. From a policy-making perspective, the minimum requirement that any appropriate rule should satisfy is that it guarantees a determinate equilibrium.

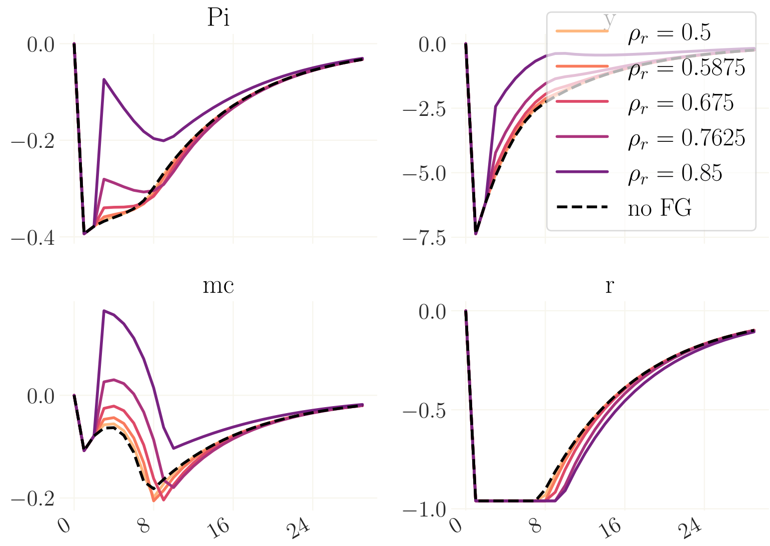

Figure 7: Dashed line: impulse responses to a 2% financial shock for $\nu = 0.24$. Colored lines are the same financial shock combined with a forward guidance shock in period 3. Different colors correspond to different persistences of the forward guidance shock. For this value of ν the forward guidance shock is not deflationary.

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{41}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{42}$$

Proof. See Appendix. ■

Equation (41) may be interpreted as a modified Taylor principle for a financial accelerator economy. If the central bank decides to react to inflation only ($\phi_y = 0$), a necessary condition is that the associated coefficient ϕ_π needs to be larger than unity, as in Taylor (1993). If the central bank reacts to output as well ($\phi_y > 0$), determinacy requires the weighted sum of policy coefficients to be larger than unity. Compared to a standard New Keynesian framework, the key difference is that financial frictions affect the degree of substitutability between reacting to inflation and to output. Under Assumption 1, the slope of the New Keynesian Phillips curve with respect to output (the term $\kappa\gamma$) is lower due to the countercyclical credit spread. At first glance, it thus seems that policy responses to output can *substitute* more effectively for policy responses to inflation in the presence of financial frictions.

However, Equation (42) may constitute additional complications for the design of monetary policy rules. To see this, suppose that $(\sigma^{-1}\gamma - 1) < 0$, which is exactly the condition for a concave policy function of inflation at the ZLB, i.e. Assumption 3. In this case, Equation (42) implies that the responses to inflation and output are *complements* for some combinations of $\{\phi_\pi, \phi_y\}$, or equivalently constitutes a lower bound restriction for the response of output. In other words, a stronger reaction to inflation must be accompanied by a corresponding stronger reaction to output. This clashes with the modified Taylor rule that exhibits the conventional substitutability.

Figure 8 displays this result graphically. As the elasticity of the credit spread with respect to entrepreneur leverage ν increases, a higher value for ϕ_y is necessary to keep the model determined for high values of ϕ_π. For example, in the case of $\nu = 0.2$, $\phi_\pi > 1.76$ requires that $\phi_y > 0$.

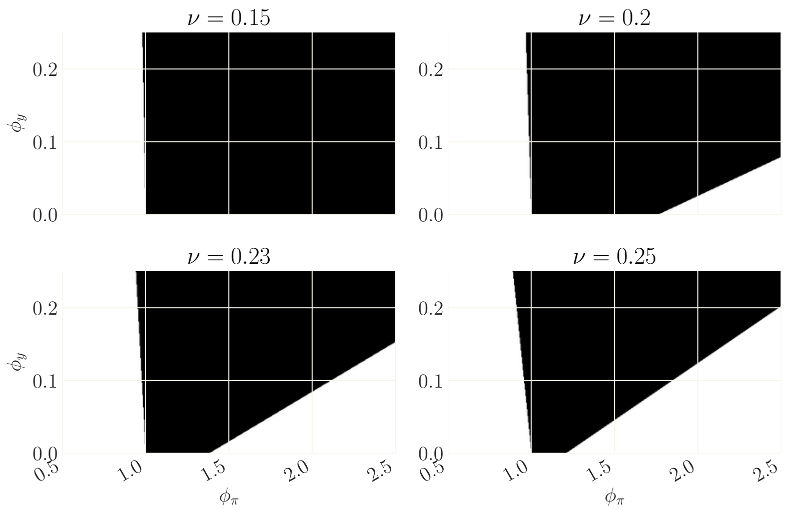

Figure 8: Determinancy regions for different values of ν.

Intuitively, abstracting from financial frictions, inflation can be stabilized by raising nominal interest rates appropriately. Higher nominal interest rates amount to higher real interest rates, decreasing consumption and output. As a consequence, real marginal costs fall, and inflation decreases. Whether the hike of nominal interest rates constitutes a reaction to (positive) deviations of inflation or output is irrelevant. In the presence of financial frictions, however, an interest rate hike as a reaction to output has the additional effect of increasing marginal costs and thus inflation. Depending on the specific characteristics of the economy, the central bank might find itself in a knife-edge scenario where the appropriate window for systematic policy responses to output deviations is quite small.

Overall, the key message emerging from this section is that the conduct of monetary policy in the presence of financial frictions and a binding ZLB may prove difficult. While the hockey stick Phillips curve blurs the relationship between inflation and output at the ZLB, conventional monetary policy wisdoms are abolished: short-lived forward guidance shocks may be associated with Neo-Fisherian inflation effects, and the determinacy conditions may place rather tight restrictions on appropriate monetary policy rules.

6 Conclusion

This paper argues that a binding zero lower bound (ZLB) on nominal interest rates may contribute to an observational disconnect between inflation and economic activity. At the ZLB, the costs of external financing in the form of credit spreads can dominate firms' price setting and thereby generate inflationary pressure. As a result, the Phillips curve features a considerably flatter slope when the ZLB binds compared to normal times. In consequence, the resulting observational Phillips curve is shaped like a hockey stick.

Our results translate into strong implications on the conduct of forward guidance, and provide a potential solution to the forward guidance puzzle: similar to financial shocks, the effects of forward guidance can be decomposed in short-run deflationary effects via the firms' refinancing cost channel, and a longer-term inflationary effect via real marginal costs. For rather short-lived forward guidance impulses, the first deflationary effect may dominate and forward guidance can in fact lower inflation. Accordingly, only forward guidance with a high expected persistence succeeds in fostering inflation and growth.

We view the combination of financial frictions and the ZLB as an additional building block in the quest to explore the recent inflation puzzles. Our theory is complementary to the existing explanations put forward in the related literature. A challenge for future research is to discriminate between these various explanations empirically to assess their relative quantitative importance. This would convey useful insights to policymakers, potentially with a view to enabling better informed (real-time) macroeconomic assessments and (monetary) policy decisions.

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Appendix A Equilibrium Equations

This section lists the full set of equations defining equilibrium. On the household side, we have the inter-temporal Euler equation and the intra-temporal labor-consumption trade-off, Equations (1) and (2) in the main text:

$$C_t^{-\sigma} = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} U_t C_{t+1}^{-\sigma} \right], \tag{A.1}$$

$$H_t^{\eta} = W_t C_t^{-\sigma}. \tag{A.2}$$

On the firm side, we have the aggregate production function, which is obtained by aggregating over the individual production functions displayed in Equation (3):

$$Y_t = \frac{H_t}{v_t^p} \tag{A.3}$$

where v_t^p is a measure of price dispersion defined below. Marginal costs are given by Equation (7):

$$MC_t = W_t R_t^L \tag{A.4}$$

The price setting behavior by firms is defined by the following equations, which are standard for Calvo (1983) pricing and make use of two auxiliary variables f_t^1 and f_t^2:

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \tag{A.5}$$

$$f_t^1 = C_t^{-\sigma} MC_t Y_t + \beta\zeta E_t \left[\Pi_{t+1}^{\varepsilon} f_{t+1}^1 \right] \tag{A.6}$$

$$f_t^2 = C_t^{-\sigma} \Pi_t^* Y_t + \beta\zeta E_t \left[\left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon} \left(\frac{\Pi_t^*}{\Pi_{t+1}^*} \right) f_{t+1}^2 \right] \tag{A.7}$$

$$1 = \zeta \left(\frac{1}{\Pi_t} \right)^{1-\varepsilon} + (1-\zeta) \left(\Pi_t^* \right)^{1-\varepsilon} \tag{A.8}$$

$$v_t^p = \zeta \Pi_t^{\varepsilon} v_{t-1}^p + (1-\zeta) \left(\Pi_t^* \right)^{-\varepsilon} \tag{A.9}$$

The interest rate specified in the credit contract is defined by Equation (6):

$$R_t^L = z \left(\frac{W_t H_t}{N_t} \right) \frac{R_t}{E_t[\Pi_{t+1}]} U_t \tag{A.10}$$

Entrepreneur net worth evolves according to Equation (9):

$$N_t = \Psi\left(Y_t\right), \tag{A.11}$$

The central bank operates according to a monetary policy rule shown in Equation (10)

$$\frac{R_t^n}{R^n} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} exp(v_t), \tag{A.12}$$

The zero lower bound (ZLB) constraint is given by Equation (11):

$$R_t = \max\left\{\bar{R}, R_t^n\right\} \tag{A.13}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t \tag{A.14}$$

These 14 conditions define the equilibrium for the 14 endogenous variables

$$\left(C_t, Y_t, H_t, \Pi_t, \Pi_t^*, W_t, R_t, R_t^L, R_t^n, N_t, MC_t, f_t^1, f_t^2, v_t^p\right), \tag{A.15}$$

together with the evolution of the two exogenous shocks:

$$\ln(U_t) = \rho \ln(U_{t-1}) + \epsilon_t \tag{A.16}$$
$$v_t = \rho_r v_{t-1} + \epsilon_t r, t. \tag{A.17}$$

The linearized equilibrium conditions are as follows:

$$c_t = -\sigma^{-1}\left(r_t + u_t - E_t\pi_{t+1}\right) + E_t[c_{t+1}], \tag{A.18}$$
$$w_t = \eta h_t + \sigma c_t, \tag{A.19}$$
$$y_t = h_t, \tag{A.20}$$
$$mc_t = w_t + r_t^L, \tag{A.21}$$
$$\pi_t = \kappa mc_t + \beta E_t[\pi_{t+1}], \tag{A.22}$$
$$r_t^L = r_t - E_t[\pi_{t+1}] + \nu(w_t + h_t - n_t) + u_t, \tag{A.23}$$
$$n_t = \psi y_t, \tag{A.24}$$
$$r_t^n = \phi_\pi \pi_t + \phi_y y_t + v_t, \tag{A.25}$$
$$r_t = \max\left\{\bar{r}, r_t^n\right\}, \tag{A.26}$$
$$y_t = c_t, \tag{A.27}$$
$$u_t = \rho u_{t-1} + \epsilon_t, \tag{A.28}$$
$$v_t = \rho_r v_{t-1} + \epsilon_{r,t}, \tag{A.29}$$
$$\tag{A.30}$$

where lower-case variables denote log-deviations from steady state.

The three-equation representation shown in Section 3.1 can be obtained by combining Equations (A.19)-(A.24) into one single Phillips curve and using the resource constraint Equation (A.27) to eliminate c_t.

Appendix B Proofs

Proposition 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are given by:*

$$\pi_t = a_0\, u_t, \tag{B.1}$$
$$y_t = b_0\, u_t, \tag{B.2}$$

where

$$a_0 = -\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{(1-\beta\rho)(\sigma(1-\rho)+\phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1)}, \tag{B.3}$$
$$b_0 = -\frac{1 + (\phi_\pi - \rho)a_0}{\sigma(1-\rho)+\phi_y}. \tag{B.4}$$

Proof. The proof relies on the method of undetermined coefficients. We guess that the solution is given by $\pi_t = a_0\, u_t$ and $y_t = b_0\, u_t$. Using this guess, the system of equation can be written as

$$(1 - \kappa\phi_\pi - \rho(\beta - \kappa))a_0 u_t = \kappa u_t + \kappa(\gamma + \phi_y)b_0 u_t, \tag{B.5}$$
$$(1 + \phi_y\sigma^{-1} - \rho)b_0 u_t = -\sigma^{-1}(\phi_\pi - \rho)a_0 u_t - \sigma^{-1}u_t, \tag{B.6}$$

where we replaced the nominal interest rate using the (unconstrained) Taylor rule. Note that expectations of future variables can be replaced by using the law of motion for the financial shocks under rational expectations. The solution is obtained by dividing both equations by u_t, substituting for b_0 in the first equation using the second equation and rearranging. ■

Lemma 1. *The impact responses of inflation and output to a financial shock in normal times (without a binding ZLB on nominal interest rates) are negative, i.e.*

$$a_0 < 0, \tag{B.7}$$
$$b_0 < 0, \tag{B.8}$$

iff the elasticity of the credit spread to entrepreneur leverage satisfies

$$\nu < \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.9}$$

Proof. The proof consists of three parts. First, we show that the model's determinacy conditions imply that the denominator of a_0 is positive. Second, the sign of a_0 then depends on its numerator, which is equivalent to the parameter restriction in the Lemma. Third, the sign of b_0 follows from a_0.

First, let us consider the determinacy conditions. The forward looking components of our model can be expressed as

$$M\mathbf{x}_t = E_t[\mathbf{x}_{t+1}], \tag{B.10}$$

with $\mathbf{x}_t = (y_t, \pi_t)'$. To arrive at this formulation, we can rewrite Equations (21) and (22) (ignoring exogenous innovations and the ELB) as

$$(1 + \sigma^{-1}\phi_y)y_t = -\sigma^{-1}\left(\phi_\pi \pi_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}], \tag{B.11}$$
$$(1 - \kappa\phi_\pi)\pi_t = \kappa(\gamma + \phi_y)y_t + \beta_\kappa E_t[\pi_{t+1}], \tag{B.12}$$

where we define $\beta_\kappa = \beta - \kappa$ for convenience. Then, we can rewrite

$$A\mathbf{x}_t = B\mathbf{x}_{t+1}, \tag{B.13}$$
$$\begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta_\kappa \end{bmatrix} \mathbf{x}_{t+1}. \tag{B.14}$$

It is straightforward that

$$B^{-1} = \frac{1}{\beta_\kappa}\begin{bmatrix} \beta_\kappa & -\sigma^{-1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.15}$$

and hence

$$M = AB^{-1} = \begin{bmatrix} 1 + \sigma^{-1}\phi_y & \sigma^{-1}\phi_\pi \\ -\kappa(\gamma + \phi_y) & 1 - \kappa\phi_\pi \end{bmatrix} \begin{bmatrix} 1 & -\beta_\kappa^{-1}\sigma^{-1} \\ 0 & \beta_\kappa^{-1} \end{bmatrix}, \tag{B.16}$$
$$= \begin{bmatrix} 1 + \sigma^{-1}\phi_y & -\beta_\kappa^{-1}\sigma^{-1}(1 + \sigma^{-1}\phi_y - \phi_\pi) \\ -\kappa(\gamma + \phi_y) & \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) \end{bmatrix}, \tag{B.17}$$
$$= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{B.18}$$

The eigenvalues of the system are given by $|M - \lambda I| = \lambda^2 + p\lambda + q$, where

$$p = -(m_1 + m_4) = -\left(1 + \sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}\kappa(\gamma + \phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi)\right) \tag{B.19}$$

is the negative of the trace and

$$q = m_1 m_4 - m_2 m_3 = \beta_\kappa^{-1}(1 + \sigma^{-1}\phi_y - \kappa\phi_\pi + \sigma^{-1}\phi_\pi\kappa\gamma) \tag{B.20}$$

is the determinant. As there are no endogenous states, determinacy under the conditions by Blanchard and Kahn (1980) requires the modulus of both eigenvalues of M to be larger than zero. We can find a representation of the absolute value of these eigenvalues in terms of the elements of M as

$$|\lambda_{1,2}^r| = \begin{cases} -p/2 + \sqrt{p^2/4 - q} > 1 \\ -p/2 - \sqrt{p^2/4 - q} > 1 \end{cases} \qquad \text{if } p^2/4 \geq q, \tag{B.21}$$
$$|\lambda_{1,2}^i| = \sqrt{p^2/2 - q} > 1 \qquad \text{if } p^2/4 < q. \tag{B.22}$$

$|\lambda_{1,2}^r|$ are the real eigenvalues if the respective condition for the square root is satisfied, $|\lambda_{1,2}^i|$ are corresponding imaginary eigenvalues otherwise. Using the condition in

Equation (B.21) in the second case implies that $-p/2 > 1$, or equivalently

$$p < -2. \tag{B.23}$$

Rearranging the second case in Equation (B.21) also implies

$$1 + p + q > 0. \tag{B.24}$$

Together with Equation (B.23), this implies

$$q > 1. \tag{B.25}$$

Equation (B.25) is also a necessary condition for the case of imaginary eigenvalues. Similarly, one can show that Equation (B.23) and Equation (B.24) imply that Equation (B.22) holds. Therefore, Equations (B.23)-(B.25) are jointly sufficient for both eigenvalues to be larger than one in modulus.

In our model, the three necessary condition $1 + p + q > 0$, $p < -2$ and $q > 1$ thus read

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y > 1, \tag{B.26}$$

$$\sigma^{-1}\phi_y + \beta_\kappa^{-1}\sigma^{-1}(\kappa\gamma + \kappa\phi_y) + \beta_\kappa^{-1}(1 - \kappa\phi_\pi) > 1, \tag{B.27}$$

$$1 + \sigma^{-1}(\kappa\gamma\phi_\pi + \phi_y) - \kappa\phi_\pi > \beta_\kappa. \tag{B.28}$$

As a second step, we can use these determinacy conditions to derive a sign for the denominator of a_0. Let us suppose that the denominator is positive, i.e.

$$(1 - \beta\rho)(\sigma(1-\rho) + \phi_y) + \kappa\gamma(\phi_\pi - \rho) - \kappa\sigma(1-\rho)(\phi_\pi - 1) > 0. \tag{B.29}$$

This can be rearranged to

$$\left(\phi_\pi + \frac{1-\beta}{\kappa\gamma}\phi_y - 1\right) + \frac{1-\rho}{\kappa\gamma}\left(\kappa\gamma + \beta\phi_y + \sigma(1 - \beta\rho - \kappa\rho - \kappa\phi_\pi)\right) > 0. \tag{B.30}$$

The first term in large brackets is positive, which can be seen directly from the necessary condition in Equation (B.26). After some algebraic manipulations, one can show that Equation (B.27) implies that the second term in brackets is also positive. This shows that the denominator of a_0 is positive.

With the denominator being positive, the sign of a_0 depends on the numerator, including the minus in front of the fraction. The condition for $a_0 < 0$ is thus

$$\kappa\gamma - \kappa\sigma(1-\rho) > 0. \tag{B.31}$$

Using the definition of γ, this is equivalent to

$$\sigma + \eta - \nu(\psi - 1 - \sigma - \eta) > \sigma(1-\rho). \tag{B.32}$$

Rearranging yields the parameter restriction in terms of the elasticity of the credit spread to entrepreneur leverage.

As a last step, the sign of b_0 can be determined given the solution for a_0. The denominator of b_0 is positive for conventional parameters, such that the sign is determined by the numerator, including the minus. Inserting a_0, this is given by

$$-1 + (\phi_\pi - \rho)\frac{\kappa\gamma - \kappa\sigma(1-\rho)}{Z}, \tag{B.33}$$

where Z denotes the denominator of a_0. After some algebraic manipulations, this is equivalent to

$$-Z^{-1}\Big((1-\beta\rho)(\sigma(1-\rho)+\phi_y) + (1-\rho)^2\kappa\sigma\Big), \tag{B.34}$$

which is unambiguously negative for $0 \leq \rho \leq 1$. ■

Lemma 2. *The impact response of inflation to a financial shock in normal times (without a binding ZLB on nominal interest rates) is Neo-Fisherian whenever a_0 is positive, i.e. if the elasticity of the credit spread to entrepreneur leverage satisfies*

$$\nu > \frac{\eta + \rho\sigma}{\psi - 1 - \sigma - \eta}. \tag{B.35}$$

Proof. This is the converse case of Lemma 1. As argued in the corresponding proof, determinacy of the model requires the denominator of a_0 to be positive. The condition for $a_0 > 0$ is hence that the numerator (including the minus in front of the fraction) is positive. This is equivalent to

$$\sigma(1-\rho) > \gamma. \tag{B.36}$$

Using the definition of γ to obtain

$$\sigma(1-\rho) > \sigma + \eta - \nu(\psi - 1 - \sigma - \eta) \tag{B.37}$$

and rearranging yields the desired result. ■

Proposition 2. *Suppose that the ZLB on nominal interest rate is expected to bind for $k \geq 1$ periods. Then, the impact responses of inflation and output to a financial shock are given by:*

$$\pi_t = a_k\, u_t, \tag{B.38}$$
$$y_t = b_k\, u_t, \tag{B.39}$$

where

$$a_k = \kappa\left(1 - \gamma\sigma^{-1}\right)\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\left(\beta - \kappa + \kappa\gamma\sigma^{-1}\right)\, a_{k-1} + \rho\kappa\gamma\, b_{k-1}, \tag{B.40}$$
$$b_k = -\sigma^{-1}\left(1 + \frac{\bar{r}}{u_t}\right) + \rho\sigma^{-1}\, a_{k-1} + \rho\, b_{k-1}. \tag{B.41}$$

Proof. Similar to Proposition 1, the proof relies on the method of undetermined coefficients. Suppose that the ZLB on nominal interest is expected to bind for $k \geq 1$ periods.

Denoting the corresponding policy functions for by a_k and b_k, respectively, we can rewrite the system of equations as

$$a_k\, u_t = \kappa\gamma\, b_k u_t + \kappa(\bar{r} + u_t) + (\beta - \kappa)\rho\, a_{k-1}, \tag{B.42}$$

$$b_k\, u_t = -\sigma^{-1}(\bar{r} + u_t) + \rho\sigma^{-1} a_{k-1} + \rho\, b_{k-1}, \tag{B.43}$$

where the central bank interest rate is replaced by the ZLB value. Note that expectations of future variables can be replaced by the corresponding policy functions for the case of an expected ZLB duration of $k-1$ under rational expectations, using the law of motion for the financial shocks. The solution is obtained by dividing both equations by u_t, substituting for b_k in the first equation using the second equation and rearranging. ∎

Proposition 3. *The policy parameters in the central bank's monetary policy rule must satisfy the following conditions to guarantee a determinate solution:*

$$\phi_\pi + \frac{1-\beta}{\kappa\gamma}\,\phi_y > 1, \tag{B.44}$$

$$\kappa(\sigma^{-1}\gamma - 1)\,\phi_\pi + \sigma^{-1}\phi_y > \beta - 1 - \kappa \tag{B.45}$$

Proof. The first equation follows directly from the condition $1 + p + q > 0$, which is required to satisfy the Blanchard and Kahn (1980) conditions. This is Equation (B.26) in the proof for Proposition 1. The second equation can be obtained by rearranging the condition $q > 1$, which is Equation (B.28) above. ∎

Appendix C Numerical Solution Method

For the sake of clarity, we use a different representation of the policy functions to outline the solution procedure. The analytic solutions of the Section 3 are expressed in terms recursive policy functions of u_t. A different, non-recursive way of presenting these policy functions is suggested in Boehl (2021). The simplicity of our model allows to ease the notation therein and express our model with $\mathbf{x}_t = (\pi_t, y_t)'$ in matrix form as

$$\mathbf{x}_t + \mathbf{c}\max\{\mathbf{d}\mathbf{x}_t, \bar{r}\} = \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, \tag{C.1}$$

where $\mathbf{N}$ is the system matrix of the constrained system, $\mathbf{c}$ contains the coefficients that determine how $\mathbf{x}_t$ is affected by r_t (and thereby also by u_t) and $\mathbf{d}$ contains the parameters of the monetary policy rule. $\bar{r} < 0$ is the actual model-implied lower bound of r_t.

Assume again that the economy is at the ELB for k periods. Then

$$\begin{aligned}
\mathbf{x}_t + \mathbf{c}\bar{r} &= \mathbf{N}E_t\mathbf{x}_{t+1} + \mathbf{c}u_t, && \text{(C.2)}\\
E_t\mathbf{x}_{t+1} + \mathbf{c}\bar{r} &= \mathbf{N}E_t\mathbf{x}_{t+2} + \mathbf{c}u_{t+1}, && \text{(C.3)}\\
&\dots \\
E_t\mathbf{x}_{t+k-1} + \mathbf{c}\bar{r} &= \mathbf{N}E_t\mathbf{x}_{t+k} + \mathbf{c}u_{t+k-1}, && \text{(C.4)}\\
E_t\mathbf{x}_{t+k} &= \mathbf{A}(0)u_{t+k}. && \text{(C.5)}
\end{aligned}$$

Recursively inserting (C.5) into (C.4) yields, acknowledging that $E_t u_{t+s} = \rho^s u_t$,

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{N}^k\mathbf{A}(0)\rho^k u_t + \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{c}\rho^i u_t - \sum_{i=0}^{k-1}\mathbf{N}^i\mathbf{b}\bar{r}, && \text{(C.6)}\\
&= \mathbf{A}(k)u_t + \mathbf{a}(k)\bar{r}. && \text{(C.7)}
\end{aligned}$$

Rewriting (C.6) yields

$$\begin{aligned}
\pi_t &= A_\pi(k)u_t + a_\pi(k)\bar{r}, && \text{(C.8)}\\
y_t &= A_y(k)u_t + a_y(k)\bar{r}. && \text{(C.9)}
\end{aligned}$$

In verbal terms, this implies that depending on the expected number of periods at the ELB k, we can express the vector of controls $\mathbf{x}_t$ as a linear map $A_j(k)$ of u_t and the (constant) vector $a_j(k)$. Both terms are nonlinear functions of k defined on $\mathbb{N}_0$. In other words: given k, the policy function is simply a two dimensional linear projection of the scalar u_t.

Definition 1 recapitulates the conditions for k to be an equilibrium value under the assumption that each shock causes the ELB to hold instantly without any transition period.

Definition 1 (equilibrium k)**.** *For each period t, an equilibrium value of $k \in \mathbb{N}_0$ must satisfy that the ELB binds in expectations exactly until period $t + k$. Hence,*

$$\mathbf{d}\mathbf{x}_t > \bar{r} \implies k = 0, \tag{C.10}$$

while for $k > 0$ it must hold that

$$\mathbf{d}E_t\mathbf{x}_{t+k} > \bar{r}, \tag{C.11}$$

and

$$\mathbf{d}E_t\mathbf{x}_{t+k-1} \leq \bar{r}. \tag{C.12}$$

The parsimonious nature of our model allows that, for each u_t, a k can simply be found by iterating over $k \in \mathbb{N}_0$ (where, naturally, k is likely to be small). More sophisticated iteration schemes for a general formulation of the dynamic system can be found in Boehl (2021).

To provide some quantitative impression given our model, for $\nu = 0.2$, a 1% risk premium shock will cause the ELB to initially bind for $k = 2$ periods, a 2% shock will cause $k = 9$ and a 3% shock an endogenous duration of $k = 12$ periods.

In Figure C.1 we show the reduced-form slope of the Phillips Curve, based only on the dynamic effect in response to the risk premium shock. The figure confirms that the slope is considerably high if away from the ELB, but drops once the ELB is reached and remains consistently low as the number of expected durations at the ELB increases.

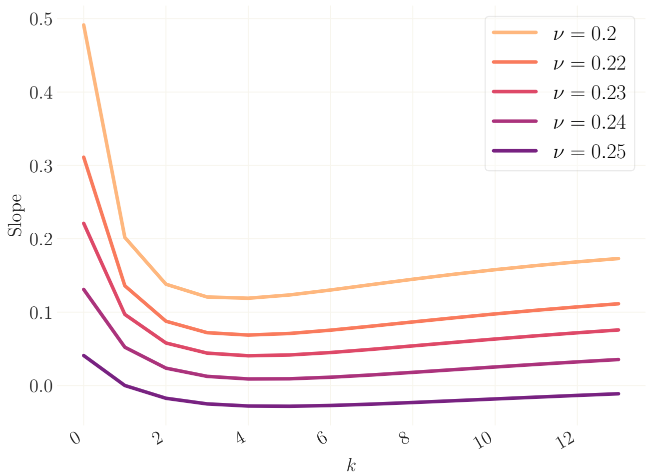

Figure C.1: Theoretical Phillips Curve slope $A_\pi(k)/A_y(k)$. This exercise ignores the static effect of the ELB, that is captured by $a_\pi(k)$ and $a_y(k)$.

Figure C.2 plots the non-recursive policy functions for π_t. For a more moderate value of ν of 0.2, the mapping $A_\pi(k)$ from $u_t \to \pi_t$ decreases with k while the linear part $a_\pi(k)$ increases in about the same fashion. As larger shocks are necessary to cause a higher k, the dynamic effect of the shock dominates the static effect and inflation falls. For $\nu = 0.22$, $A_\pi(k)$ becomes more convex, meaning that the coefficient that translates financial shocks to inflation increases for low expected durations. This effect is not offset by the static effect of a longer anticipated ELB period, which leads to a more muted inflation response. For a value of $\nu = 0.24$, the dynamic response approaches zero while for $\nu = 0.25$, $A_\pi(k)$ turns positve for values of k larger than two. As the static effect is again too weak to counteract, this leads to an increase of inflation on impact, as it is captured in Figure 2.

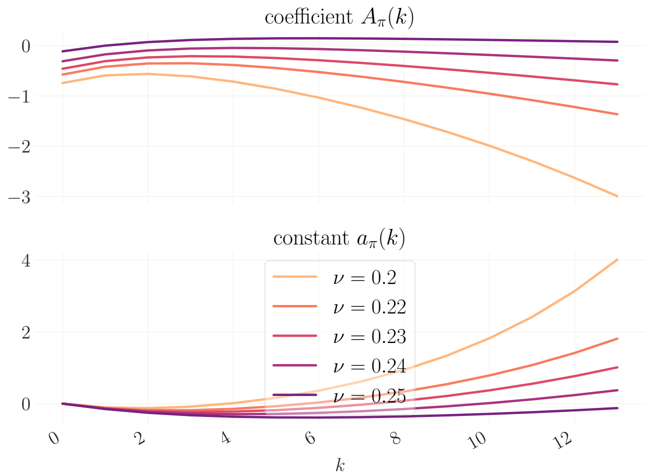

Figure C.2: Expected ZLB Duration and Impact Response

Appendix D Additional Figures

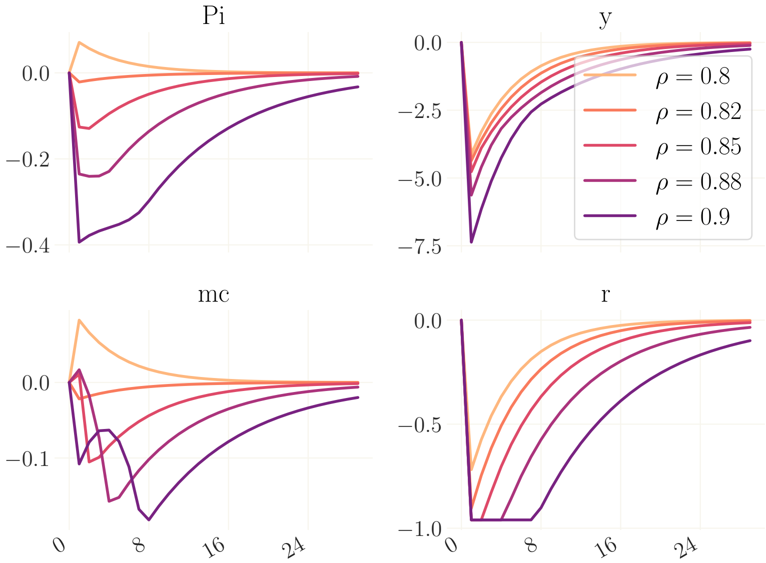

Figure D.3: Impulse responses to 2% risk premium shocks for different values of ρ, given $\nu = 0.24$.