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The Effects of Trend Inflation on Aggregate Dynamics and Monetary Stabilization

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Abstract

I derive a set of new analytic results for the effects of trend inflation on aggregate price and output dynamics in menu cost models. I find that positive trend inflation: (1) induces asymmetry in price and output responses to monetary shocks, (2) leads to price overshooting after large shocks, and (3) destroys the monetary neutrality result for large shocks. Under positive trend inflation, large expansionary monetary interventions lead to output contractions, and smaller expansionary interventions have substantially reduced potency. Using U.S. sectoral data, I provide supporting evidence for these model predictions. Calibrating a general equilibrium model to the U.S. economy, I find sizable effects of trend inflation on monetary stabilization policy. Raising the inflation target from 2% to 4% increases the economy's sensitivity to an adverse markup shock and worsens the stabilization trade-off.

Keywords: trends, asymmetry, trend inflation, aggregate dynamics
JEL Codes: E32, E52

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1 Introduction

Does the level of trend inflation matter for the economy's responses to monetary shocks? Alvarez, Le Bihan, and Lippi (2016) showed that the answer is no, as long as shocks are marginal and trend inflation is small. I show that the result is overturned if one considers shocks of any size instead. This has important implications for the effectiveness of monetary stabilization policy and, more generally, provides new insights into aggregate dynamics in economies with lumpy adjustments.

A set of recent papers provides analytic results for aggregate dynamics in lumpy economies.[1] The existing literature, however, focuses on *marginal* aggregate shocks in *driftless* economies, which in the context of pricing corresponds to the case of zero trend inflation. Two reasons motivate this approach. Firstly, the assumption of zero drift substantially increases analytic tractability. Secondly, if shocks are marginal, aggregate dynamics under small trend inflation can be well approximated by the dynamics of an economy with zero trend inflation. Given that most developed countries have very low inflation levels, the zero drift approximation appears to be both convenient and useful, provided one is interested in the effects of marginal shocks.

Extending the analysis beyond marginal shocks is important for two reasons. Firstly, recent economic developments have vividly illustrated that both exogenous shocks to the economy and subsequent policy measures can sometimes be hardly characterized as marginal. Secondly, theoretical and empirical results in the literature show that economy's responses to shocks and policy interventions are highly non-linear in the shock size, which renders first order approximations insufficient.

To address these issues, I study price and output responses to aggregate shocks of *any size* in economies with (small) non-zero trend inflation. I show analytically that the two key statistics of aggregate price and output dynamics – the impact effect and the cumulative impulse response, are both affected by trend inflation to first order, in contrast to the results obtained for marginal shocks. Most importantly, trend inflation induces *qualitative* changes in the two statistics, generating asymmetry in aggregate dynamics and leading to unintended effects of monetary interventions. This challenges the approach of approximating small inflation economies with a zero inflation benchmark. Using U.S. sectoral data, I show that the new analytic predictions are supported empirically. In addition, I find that the effect of trend inflation is sizable in a general equilibrium model calibrated to the U.S. economy.

The results have important implications for the monetary stabilization policy. The prolonged period of low interest rates revitalized the debate on increasing the inflation target to minimize the risk of hitting the zero lower bound. My results show that a higher

[1] See Caballero and Engel (2007), Alvarez, Le Bihan, and Lippi (2016), Alvarez and Lippi (2019) and Baley and Blanco (2020).

inflation target impedes the ability of a monetary authority to counteract adverse shocks at times when the zero lower bound is not binding.

New Analytic Results. I use the workhorse menu cost model of price dynamics, although the results can easily be extended to other applications, including lumpy capital or labor adjustment, durable good consumption, and others. In the model, firms face an exogenously given optimal price determined by common trend inflation (drift) and idiosyncratic shocks. Flow profit is maximized when the actual price is equal to the optimal one, and price adjustment comes at a cost. Because of the cost, firms keep their prices fixed most of the time and adjust infrequently, which results in gaps between actual and optimal prices. These price gaps are the key variable in the model and the evolution of the price gap distribution determines aggregate price and output dynamics.

Following the literature, I consider an unexpected permanent one-time monetary shock. The shock generates aggregate price and output responses, which can be summarized by two statistics: the impact effect and the cumulative impulse response. The impact effect is the aggregate price change at the instant when the shock arrives. It depends on the mass of adjusting firms and the sizes of their adjustments. The cumulative impulse response (CIR) reflects an overall, cumulated over time, effect of the shock on aggregate output. It is determined by both the strength and speed of the price response.

The analysis requires an analytic characterization of the CIR for non-zero levels of trend inflation. Alvarez, Le Bihan, and Lippi (2016) show that in order to compute an infinite-horizon cumulative impulse response in an economy with no drift, it is sufficient to track agents until the first time of adjustment, as subsequent paths net out to zero in expectation. Alvarez and Lippi (2019, 2020) and Baley and Blanco (2020) suggest that this approach is also valid in economies with non-zero drift. However, as I show in this paper, non-zero drift introduces an additional term, which is related to paths after the first time of adjustment. I refer to this new term as a 'tail' term, since it emerges from the tail of a finite horizon cumulative impulse response, when letting the finite horizon grow to infinity. I propose two ways of computing the tail term analytically, each being better suited for different applications. My results apply to settings well beyond the scope of this paper, including other types of aggregate shocks and functions of interest. Surprisingly, the new term is independent of the shock and thus acts as an intercept: it shifts the entire CIR curve but does not affect its shape. While the term is relevant for shocks of any size, I show that it is of particular importance for large shocks, cumulative responses to which consist entirely of the tail term. Equipped with the new approach to computing cumulative responses in economies with drift, I study how the drift affects aggregate impulse responses.

If there is no drift, impulse responses are symmetric, meaning that positive shocks trigger responses of the same magnitude as negative shocks. The key property of trend inflation is that it affects responses to positive and negative shocks *asymmetrically.* In particular, positive trend inflation amplifies price responses to positive monetary shocks and mitigates price responses to negative monetary shocks, compared to the case of zero trend inflation. Since the strength of output responses is inversely related to the magnitude of aggregate price responses, the effect of trend inflation on output is reversed. Under positive trend inflation, output becomes less sensitive to positive monetary shocks and more sensitive to negative monetary shocks. Note that these results require analyzing shocks of any size, in contrast to marginal shocks, as responses to the latter are symmetric by definition.

There are two channels through which drift creates asymmetry in aggregate dynamics: the optimal policy of firms and the price gap distribution. Firstly, if trend inflation is positive, firms expect higher prices in the future and are eager to increase them once a positive shock arrives, despite the adjustment cost. For the same reason, firms are reluctant to decrease their prices after a negative shock because it induces additional adjustment costs in the future. Secondly, positive trend inflation erodes relative prices and leads to a higher concentration of price gaps at the bottom of the price gap distribution. Thus, positive shocks trigger more firms to adjust compared to negative shocks. Both of these channels work in the same direction and result in asymmetric aggregate price and output responses.

The asymmetry, induced by trend inflation, creates two additional effects: price overshooting and output contractions after sufficiently large positive monetary shocks. To avoid ambiguity, I now distinguish between intended and realized effects of monetary shocks. I label shocks as 'positive' or 'negative' when referring to the intended effects (e.g., interest rate cuts vs. hikes). I label shocks as 'expansionary' or 'contractionary' when referring to the realized effects (output increases vs. decreases). In a driftless economy, the price level has at most a one-to-one reaction to monetary shocks and output always weakly increases after positive shocks. I show that in economies with positive trend inflation, positive shocks may cause price overreaction and a contraction in output. Firms prefer to overshoot when adjusting upward, as they anticipate relative price erosion due to positive trend inflation. Overshooting at the aggregate level occurs if the mass of adjusters is large enough, which highlights a special effect of the drift on aggregate responses to large shocks.

A shock is considered 'large' if it forces all firms to adjust their prices. In an economy with zero trend inflation, large shocks are neutral – aggregate price responds one-to-one, and output does not move. When trend inflation is positive, large positive shocks become contractionary. Interestingly, a shock does not have to force all firms to adjust to have such an effect – even smaller positive shocks can cause a decline in output. Therefore, the overall ability of a monetary authority to stimulate output deteriorates as trend inflation rises: moderate shocks cause weaker responses compared to the driftless benchmark, and larger shocks become counterproductive.

Empirical Evidence. In the second part of the paper, I provide empirical evidence consistent with the new analytic results. In order to ensure enough variation in the level of trend inflation, I use U.S. sectoral data on the Producer Price Index (PPI) and industrial production (IP). I compute trend inflation for each sector as the average PPI growth rate and split sectors into two groups: with trend inflation above and below the median. I then estimate non-linear impulse responses to identified monetary shocks within each group and compare the results between the two groups. Even though the results can not be interpreted in a causal sense, they show that many of the model predictions are in line with the data.

Firstly, I find that trend inflation is strongly related to asymmetry in PPI and production responses to monetary shocks. Price responses in sectors with high trend inflation exhibit primarily positive asymmetry, i.e., prices rise more after positive shocks than they fall after negative shocks. On the contrary, the asymmetry of price responses is negative in sectors with low trend inflation. Responses of industrial production are generally negatively asymmetric, meaning that production contracts more after negative shocks than it rises after positive shocks. However, the asymmetry is much more negative in sectors with higher trend inflation, where positive shocks have almost no impact on production, and negative shocks cause substantial responses. The model does not always match the *level* of asymmetry in the data, but it correctly predicts the *relationship* between trend inflation and asymmetry.

Secondly, I find that production responses to positive shocks have an inverse U-shape, meaning that the maximum stimulative effect on production is achieved for moderate shock sizes. In addition, sufficiently large positive shocks cause a reverse effect, leading to a contraction in production. As predicted by the model, these reverse effects are strongly related to the level of trend inflation: the size of a positive shock that leads to a zero production response is substantially smaller in sectors with higher trend inflation. The results suggest that monetary policy is not only less effective in stimulating output in sectors with higher trend inflation, but also has much less 'room' for doing so.

Finally, I provide evidence for the mechanism linking trend inflation and asymmetries in aggregate responses to monetary shocks. I use daily item-level price data provided by the Billion Prices Project to study the relationship between trend inflation and asymmetry in individual price adjustments. I find that a one percentage point increase in monthly trend inflation is associated with a 5% increase in the ratio between the magnitudes of positive and negative adjustments. This relationship between trend inflation and micro-level asymmetries matches the model predictions and is an important channel leading to aggregate asymmetries in responses to monetary shocks, as observed in the sectoral data.

Implications for Monetary Policy. In the last section of the paper, I show that the effects of trend inflation are sizable and relevant for policy. I embed the analytic framework into a general equilibrium model, calibrated to the U.S. economy. I consider a negative markup shock and study the ability of a policymaker to stabilize the economy in the baseline model with a 2% inflation target (trend inflation). I then compare the results with a counterfactual economy, in which the inflation target is set to 4%.

The analysis is positive, as I do not consider optimal policy, but assume a simple stabilization objective instead. A markup shock suits this purpose well, as it amplifies inefficiency stemming from price dispersion but does not affect the efficient allocation, which rationalizes the stabilization objective. In addition, the shock increases prices and depresses consumption, introducing a trade-off for the monetary authority, as it can not stabilize consumption and prices simultaneously.

I find that raising the inflation target from 2% to 4% has two adverse effects. Firstly, it amplifies the initial reaction to the markup shock, causing larger consumption and price deviations. Secondly, it worsens the stabilization trade-off. A policymaker must sacrifice more consumption when stabilizing prices and tolerate larger price deviations when stimulating consumption. These effects are due to weaker upward price rigidity and stronger downward price rigidity, caused by a higher inflation target. Increasing the inflation target leads to more price flexibility exactly when it is desirable to have rigid prices and makes prices stickier exactly when flexibility is needed. In addition, the effects of trend inflation are more pronounced for larger shocks, in accordance with the analytic results.

Relation to the Literature. The effect of drift on individual and aggregate behavior has previously drawn the attention of many researchers. Several early theoretical contributions (Sheshinski and Weiss (1977), Mankiw and Ball (1994), Tsiddon (1993)) have shown that trend inflation can affect the magnitude of individual price adjustments and the mass of adjusting firms after aggregate nominal shocks. My work closely relates to the subsequent research, which has focused on analytic characterization of aggregate dynamics in economies with lumpy adjustments (Caballero and Engel (2007), Alvarez, Le Bihan, and Lippi (2016), Alvarez and Lippi (2019) and Baley and Blanco (2020)). This strand of literature has either restricted its attention to marginal aggregate shocks or considered driftless economies. I contribute to the literature by simultaneously allowing for non-zero trend inflation and providing analytic results for shocks of any size.

The empirical results of this paper provide new insights into price and output responses to monetary policy shocks. Several studies have tested whether aggregate impulse responses exhibit state dependence (Lo and Piger (2005), Auerbach and Gorodnichenko (2012), Ramey and Zubairy (2014)), asymmetries with respect to positive and negative shocks (Long and Summers (1988), Cover (1992), Angrist, Jordà, and Kuersteiner (2018)) and non-linearities with respect to the shock size (Tenreyro and Thwaites (2016), Ascari and

Haber (2020)). I add to the literature by showing that both asymmetry and non-linearity of aggregate impulse responses are tightly linked to trend inflation. In addition, I provide evidence for the mechanism behind this link and show that trend inflation affects the asymmetry of price adjustments at the micro level, even if trend inflation is low. This result complements the work of Alvarez et al. (2019) who find evidence for this relationship only in a high inflation environment.

The last section of this paper contributes to the ongoing discussion on the role of trend inflation for the effectiveness of monetary stabilization policy. The proposal of raising inflation targets to gain sufficient policy 'room' from the zero lower bound (see Blanchard, Dell'Ariccia, and Mauro (2010) and Ball (2013)) drew the attention of researchers to the consequences of higher trend inflation. Ascari and Sbordone (2014) discuss the implications of a higher inflation target for price dispersion, stability of inflation expectations and macroeconomic volatility. L'Huillier and Schoenle (2020) argue that a higher inflation target increases the frequency of price adjustments, lowers the potency of monetary policy, and thus provides an effective extra room smaller than the nominal one. Blanco (2020) points out that a higher inflation target increases downward price rigidity, which mitigates recessions at the zero lower bound. I show that higher trend inflation has an additional adverse effect on the efficacy of monetary policy away from the zero lower bound, particularly for shocks that introduce a stabilization trade-off. Although I do not study the optimal level of trend inflation, my results have implications for the normative analysis (see Coibion, Gorodnichenko, and Wieland (2012), Adam and Weber (2019), Blanco (2020), and Diercks (2017) for a review).

Since the setting considered in this paper lies at the heart of numerous models that feature adjustment costs, most analytic results apply to problems of lumpy capital and labor adjustment, durable good consumption and others. In many of these fields, previous studies have noted the role of drift in shaping responses to aggregate shocks. In investment models with capital adjustment costs (e.g., Khan and Thomas (2008) and Bachmann, Caballero, and Engel (2013)), the distribution of mandated investment is highly skewed, and responses to aggregate shocks are asymmetric. In this setting, capital depreciation plays the role of drift because it erodes capital stock. Similarly, depreciation of durable goods generates asymmetry in consumption responses to fiscal stimuli over the cycle in Berger and Vavra (2015). Jaimovich and Siu (2020) show empirically that employment in routine occupations in the U.S. falls over time and predominantly during recessions, whereas employment in non-routine occupations is increasing and does not contract in recessions. These findings provide another example of the relationship between drift and cyclical behavior and are in line with my analytic results.

2 Theoretical Results

I consider the simplest version of a two-sided sS model with a quadratic objective and fixed costs of adjustment. This framework serves as an approximation to numerous applications, including models of capital, labor or price adjustment, portfolio or inventory management, as well as durable good consumption. In the following, I outline the model setup and briefly review the benchmark case of a driftless economy. I then move to economies with non-zero drift and highlight the main qualitative differences.

2.1 Problem of a Firm

I consider a model of a firm that sets its price subject to a menu cost, given the optimal price target.[2] The instantaneous profits of the firm are given by $\pi(z) = -z^2$, where $z = \ln p - \ln p^*$ is the log deviation of the current price p from its frictionless optimum p^*. The optimal price p^* maximizes the instantaneous profits of the firm and follows a geometric Brownian motion with drift μ:

$$d \ln p^*(t) = \mu dt - \sigma dW(t)$$

where $\sigma > 0$ and $W(t)$ is a Wiener process. In this setup, the drift μ corresponds to trend inflation and is the key parameter of interest. Price adjustment comes at a fixed cost $\kappa > 0$, so that the firm keeps its price p constant most of the time and adjusts it infrequently. In the absence of price adjustment, the price gap z evolves as follows:

$$dz(t) = -\mu dt + \sigma dW(t)$$

Whenever the firm intervenes and changes its price by $\Delta \ln p$, the price gap z jumps by the same amount and in the same direction. The profits are discounted at rate $\rho > 0$, and the firm's objective is to maximize discounted stream of profits subject to the adjustment costs it pays upon each intervention. Its problem can be formulated entirely in terms of the price gaps z, with $\{\tau_i\}_{i=1}^{\infty}$ denoting the sequence of times when the firm adjusts and $\{\Delta z_i\}_{i=1}^{\infty}$ being the sequence of adjustments:

$$v(z) = \max_{\{\tau_i, \Delta z_i\}_{i=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t} \pi(z(t)) dt - \sum_{i=1}^{\infty} e^{-\rho \tau_i} \kappa \;\middle|\; z(0) = z \right]$$

$$\text{s.t.} \quad z(t) = z(0) - \mu t + \sigma W(t) + \sum_{i=1}^{N(t)} \Delta z_i$$

[2] Here, I take the optimal price as given. In a standard setting, the price target is typically determined by a markup over marginal costs, which in turn depend both on aggregate and individual states.

where $N(t)$ is the number of adjustments that occurred until t. This constitutes a standard impulse control problem, the solution to which are boundaries of inaction region $(\underline{z}, \overline{z})$ and a return point $\hat{z}$. Whenever $z(t) \in (\underline{z}, \overline{z})$, the firm keeps its current price and lets the price gap evolve stochastically. As soon as $z(t)$ reaches one of the boundaries, the firm pays an adjustment cost κ and sets $z(t) = \hat{z}$. At any intervention time τ_i, the adjustment is given by $\Delta z_i = \hat{z} - \lim_{t \uparrow \tau_i} z(t)$, where $\lim_{t \uparrow \tau_i} z(t)$ is the value of z right before the adjustment and is equal to either $\underline{z}$ or $\overline{z}$.

The value function $v(z)$ satisfies the following Hamilton–Jacobi–Bellman equation for any $z \in (\underline{z}, \overline{z})$:

$$\rho v(z) = -z^2 - \mu v'(z) + \frac{\sigma^2}{2} v''(z)$$

together with smooth pasting conditions $v'(\underline{z}) = v'(\overline{z}) = 0$, optimality of return point $v'(\hat{z}) = 0$ and boundary conditions $v(\underline{z}) = v(\overline{z}) = v(\hat{z}) - \kappa$. These conditions constitute a system of equations, which implicitly defines the solution triplet $\{\underline{z}(\mu), \hat{z}(\mu), \overline{z}(\mu)\}$. I highlight the dependence of optimal policy on trend inflation by explicitly stating μ as its argument, even though it also depends on other model parameters.

2.2 Aggregate Dynamics

Assume that the economy is populated by a continuum of ex-ante identical firms that face the same drift in optimal price μ but experience idiosyncratic shocks. Firms follow the same policy $\{\underline{z}(\mu), \hat{z}(\mu), \overline{z}(\mu)\}$ and the economy has a stationary distribution of price gaps z with a cumulative distribution function $F(z, \mu)$. The corresponding density is denoted by $f(z, \mu)$ and solves the following Kolmogorov forward equation:

$$0 = \mu f_z(z, \mu) + \frac{\sigma^2}{2} f_{zz}(z, \mu)$$

together with boundary conditions $f(\underline{z}(\mu), \mu) = f(\overline{z}(\mu), \mu) = 0$, unit mass condition $\int_{\underline{z}(\mu)}^{\overline{z}(\mu)} f(z, \mu) dz = 1$ and continuity at $z = \hat{z}(\mu)$.[3]

Following the literature, I consider an unexpected permanent nominal shock that changes the optimal (log) price $\ln p^*$ by δ simultaneously for all firms. The shock shifts the stationary distribution of price gaps in the opposite direction (because $z = \ln p - \ln p^*$), as illustrated in Figure 1 for $\delta > 0$. The stationary distribution of the price gaps is depicted by the dashed blue line, whereas the solid red line shows the density immediately after the shock has arrived, but before firms responded to it. This distribution is referred to as the initial distribution and is denoted by $F_0(z, \mu)$. For this type of shock, the initial distribution is a shifted version of the stationary distribution $F(z, \mu)$, such that $F_0(z, \mu) = F(z + \delta, \mu)$.

[3]The density $f(z, \mu)$ is non-differentiable at the return point $\hat{z}$ and the Kolmogorov forward equation does not hold at this point.

Figure 1: Aggregate shock δ

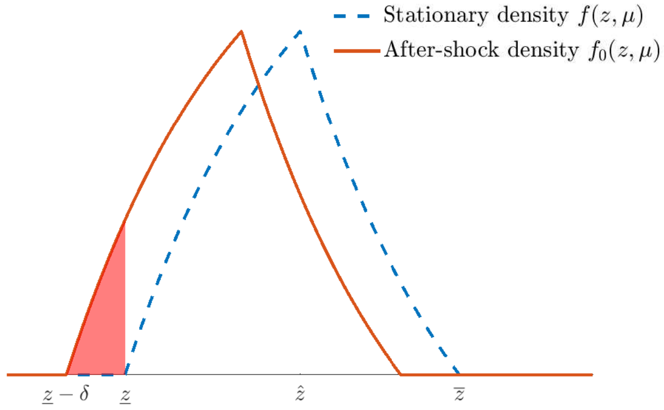


Illustration of a positive shock $\delta > 0$ in an economy with positive drift ($\mu > 0$). The dashed blue line is the stationary density of price gaps $f(z,\mu)$, whereas the solid red line is the density immediately after the shock and before firms adjust, $f_0(z,\mu)$. The shaded triangle corresponds to the mass of firms that adjust on impact.

The shaded area corresponds to the mass of firms that are pushed outside the inaction region and adjust immediately on impact. Their adjustment results in an immediate change of the aggregate price level, denoted by $\Theta(\delta,\mu)$ and commonly referred to as the impact effect. Formally, for a positive shock $\delta > 0$, $\Theta(\delta,\mu)$ is given by the following expression:

$$\Theta(\delta,\mu) = \int_{\underline{z}(\mu)-\delta}^{\underline{z}(\mu)} \big(\hat{z}(\mu) - z\big) f(z+\delta,\mu)dz \tag{2.1}$$

which is simply the sum of all adjustments $\big(\hat{z}(\mu) - z\big)$ weighed with the initial density. The resulting distribution of price gaps then gradually converges to the stationary one, inducing a path for aggregate variables. These dynamics are summarized in Figure 2.

The solid red line shows the realized path of the aggregate log-price $P(t)$, whereas the dashed red line corresponds to its hypothetical path absent of any shock $\bar{P}(t)$. The price impulse response at any time t is given by the difference between $P(t)$ and $\bar{P}(t)$. The shock arrives at $t = 0$ and triggers the impact effect $\Theta(\delta,\mu)$, given by the initial vertical jump of $P(t)$. Another statistic, commonly studied in the literature, is the cumulative impulse response (CIR), which is shown as the shaded area on the graph and denoted by $M(\delta,\mu)$:

$$M(\delta,\mu) = \int_0^\infty \big[\delta - \big(P(t) - \bar{P}(t)\big)\big]dt$$

This statistic summarizes the strength and speed of the price response, although in a reversed way. The stronger and faster firms react to the shock, the smaller $M(\delta,\mu)$ is. For example, if the immediate price response $\Theta(\delta,\mu)$ is equal to δ, then the shaded area in Figure 2 collapses and $M(\delta,\mu) = 0$, provided that there are no further fluctuations in $P(t)$ around the trend. In addition, $M(\delta,\mu)$ is of special use in a certain class of general

Figure 2: Dynamics after an aggregate shock δ

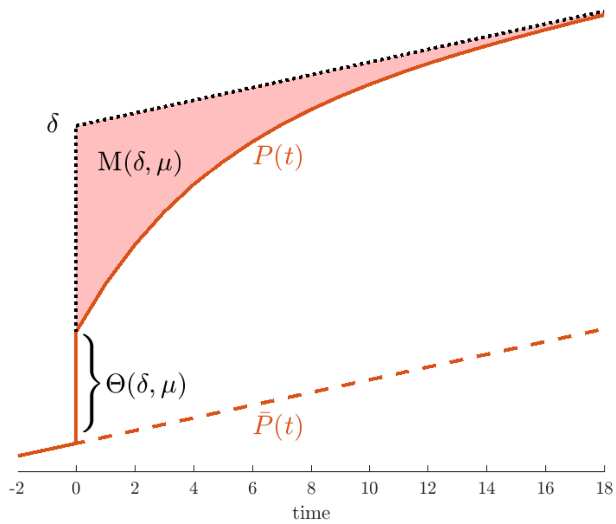


Illustration of aggregate dynamics after positive shock $\delta > 0$ in economy with positive drift ($\mu > 0$). The solid red line is the realized path of the aggregate log-price $P(t)$, the dashed red line is its hypothetical path absent of shock $\bar{P}(t)$. The initial vertical segment of $P(t)$ shows the impact effect $\Theta(\delta, \mu)$. The shaded area corresponds to the cumulative impulse response $M(\delta, \mu)$.

equilibrium models (e.g. Golosov and Lucas (2007)), as it measures the cumulative output response to a nominal shock δ. Under logarithmic preferences, the output response at any time t is given by $Y(t) = \delta - \big(P(t) - \bar{P}(t)\big)$, meaning that output absorbs the part of the shock that was not captured by the price response.[4] Cumulating output responses over time recovers the expression for $M(\delta, \mu)$, which provides an immediate mapping from price to output responses and characterizes the real effects of nominal shocks.

Because aggregate price dynamics are determined by the dynamics of the price gap distribution, one can express $M(\delta, \mu)$ in the following way:[5]

$$M(\delta, \mu) = -\int_{\underline{z}(\mu)}^{\bar{z}(\mu)} \mathbb{E}\left(\int_0^\infty \big(z(t) - \bar{x}(\mu)\big)dt \;\middle|\; z(0) = z\right) dF_\delta(z, \mu)$$

where $\bar{x}(\mu)$ is the average gap in the steady state $\left(\bar{x}(\mu) = \int_{\underline{z}(\mu)}^{\bar{z}(\mu)} z dF(z, \mu)\right)$. The outer integrand is the expectation of the cumulated deviations of the price gap $z(t)$ from its steady state average $\bar{x}(\mu)$, given a particular starting value $z(0) = z$. The integrand is then averaged across all starting values z, using the distribution of price gaps immediately after the shock has arrived and firms outside of the inaction region have adjusted, which is denoted by $F_\delta(z, \mu)$. This distribution is equal to the stationary one, shifted by δ and truncated to the inaction region, together with a mass point at $\hat{z}(\mu)$ due to a positive mass of firms that adjust on impact.

[4] Relaxing logarithmic preferences to a more general case of CES preferences makes output responses proportional to $\delta - \big(P(t) - \bar{P}(t)\big)$.

[5] See Appendix A.1 for details.

There are several limitations of this framework. Firstly, the quadratic profit function serves as a second-order approximation to a more general one, e.g., the one resulting from a CES demand function. Secondly, I ignore any general equilibrium feedback effects from aggregate dynamics to the optimal policy of firms to ensure that firms follow the steady state policy along the transition path.[6] Both assumptions are crucial for analytic tractability and are relaxed in Section 4, where I calibrate a general equilibrium model to the U.S. economy.

The primary interest of this paper is the sensitivity of the impact price effect $\Theta(\delta, \mu)$ and the cumulative output response $M(\delta, \mu)$ to changes in trend inflation μ, particularly for shocks that are not marginal. I briefly review the main properties of $\Theta(\delta, \mu)$ and $M(\delta, \mu)$ in a benchmark driftless setting, and then discuss economies with non-zero drift.

2.3 Driftless Benchmark

Driftless economies have been well studied in the literature and serve as an important benchmark for economies with small drift. In a recent study, Alvarez and Lippi (2019) characterize the entire impulse response to any initial disturbance for economies without drift, whereas full characterization with non-zero drift is still a challenge. To allow for comparability between the setups, I keep the focus on the impact effect and the cumulative impulse response, and review their main properties in economies without drift.

The absence of drift in the optimal price coupled with a quadratic profit function results in a symmetric optimal policy $\{\underline{z}(0), \hat{z}(0), \overline{z}(0)\} = \{-\overline{z}_0, 0, \overline{z}_0\}$. The return point $\hat{z}(0)$ is set to zero and the lower boundary of inaction region $\underline{z}(0)$ is the negative of the upper boundary $\overline{z}(0)$, denoted by $\overline{z}_0$ to ease notation. Stationary density $f(z, 0)$ becomes a piecewise linear function with a kink at zero. Solving for the impact effect $\Theta(\delta, 0)$ of a positive shock $\delta > 0$ yields the following result (derivation is provided in Appendix A.2):

$$\Theta(\delta, 0) = \begin{cases} \frac{1}{6\overline{z}_0^2}\delta^2(3\overline{z}_0 + \delta), & \text{for } \delta < \overline{z}_0 \\ \frac{1}{6\overline{z}_0^2}\left[\delta(6\overline{z}_0^2 + 3\delta\overline{z}_0 - \delta^2) - 4\overline{z}_0^3\right], & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ \delta, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

While Alvarez and Lippi (2014) characterize $\Theta(\delta, 0)$ given small ($\delta \leq \overline{z}_0$) and large ($\delta \geq 2\overline{z}_0$) values of the shock, I also derive an expression for intermediate values. Three key features of impact effect under zero drift should be highlighted. Firstly, due to symmetries in optimal policy and stationary density, the impact effect is symmetric for positive and negative shocks, i.e., $\Theta(-\delta, 0) = -\Theta(\delta, 0)$. Secondly, the impact response of aggregate price never exceeds the shock: $|\Theta(\delta, 0)| \leq |\delta|$ for all δ. Finally, when a shock is large ($\delta \geq 2\overline{z}_0$), the price level responds one-to-one, meaning that $\Theta(\delta, 0) = \delta$. In general, a

[6] Alvarez and Lippi (2014) show that such a setting, these effects are of second order only.

shock is considered 'large' if it pushes all firms outside of the inaction region, forcing all of them to adjust. Therefore, the shock must be larger than the width of the inaction region $\overline{z}(\mu) - \underline{z}(\mu)$, which in the driftless case is equal to $2\overline{z}_0$. Both the average size of price adjustments $\mathbb{E}(|\Delta \ln p|)$ and the standard deviation of adjustments $Std(\Delta \ln p)$ are equal to $\overline{z}_0$, so that δ is large if it is twice as big as the average adjustment size or exceeds two standard deviations.

Alvarez, Le Bihan, and Lippi (2016) show that to compute the cumulative output response $M(\delta, 0)$, one does not have to consider the entire path of price gap deviations, as it is enough to keep track of each firm until the first adjustment. Because of zero drift, the expected price gap deviation is always zero after the first adjustment. Furthermore, the steady state average gap $\bar{x}(0)$ is also zero, which gives the following expression for the CIR:

$$M(\delta, 0) = -\int_{-\overline{z}_0}^{\overline{z}_0} \mathbb{E}\left(\int_0^{\tau} z(t)dt \mid z(0) = z\right) dF_{\delta}(z, 0)$$

where τ is the first time of adjustment. I provide an expression for $M(\delta, 0)$ in Appendix A.2 and briefly review its main properties below.

Identical to the impact effect, there are three key features to note: (1) CIR is symmetric around zero in the sense that $M(-\delta, 0) = -M(\delta, 0)$; (2) The cumulative output response is non-negative for all $\delta > 0$, meaning that positive nominal shocks either increase output or are neutral; (3) Large shocks are neutral ($M(\delta, 0) = 0$), because aggregate price adjusts one-to-one to these shocks on impact and $\Theta(\delta, 0) = \delta$.

As I show in subsequent sections, none of the main properties of the impact effect and the cumulative output response are valid in economies with non-zero drift.

2.4 Introducing Drift

I now study economies with non-zero drift: $\mu \neq 0$. The optimal inaction region of a firm is no longer symmetric and the return point is not zero. Given that the problem is well characterized for zero drift, I consider a first-order approximation of the key statistics around the zero drift point:[7]

$$\Theta(\delta, \mu) = \Theta(\delta, 0) + \frac{\partial \Theta(\delta, 0)}{\partial \mu}\mu + o(\mu^2)$$
$$M(\delta, \mu) = M(\delta, 0) + \frac{\partial M(\delta, 0)}{\partial \mu}\mu + o(\mu^2)$$

This approach is novel, as I compute the first derivatives of aggregate responses with respect to the drift μ for shocks of any size. To date, the literature has only considered the effect of drift on responses to marginal shocks, given by cross-derivatives $\frac{\partial^2 \Theta(\delta, \mu)}{\partial \delta \partial \mu}\big|_{\delta=0,\mu=0}$ and

[7] I use short-hand notation $\frac{\partial X(\delta,0)}{\partial \mu}$ for $\frac{\partial X(\delta,\mu)}{\partial \mu}\big|_{\mu=0}$.

$\frac{\partial^2 M(\delta,\mu)}{\partial\delta\partial\mu}\Big|_{\delta=0,\mu=0}$. Alvarez, Le Bihan, and Lippi (2016) show that these cross-derivatives are equal to zero due to the symmetry properties of the model and the assumed differentiability of $\Theta(\delta,\mu)$ and $M(\delta,\mu)$ with respect to μ. This result does not require characterizing $\Theta(\delta,\mu)$ and $M(\delta,\mu)$ for non-zero levels of drift. On the contrary, such characterization is crucial for my approach and introduces two challenges.

Firstly, drift μ affects $\Theta(\delta,\mu)$ and $M(\delta,\mu)$ by altering the stationary distribution of price gaps and changing the optimal policy (which also feeds into the gap distribution via boundary conditions). Thus, understanding the effects of drift on aggregate dynamics requires a characterization of its effects on the optimal policy $\left\{\frac{\partial \underline{z}(0)}{\partial\mu}, \frac{\partial \hat{z}(0)}{\partial\mu}, \frac{\partial \overline{z}(0)}{\partial\mu}\right\}$ and stationary density $\left(\frac{\partial f(z,0)}{\partial\mu}\right)$.

Secondly, as I show later, non-zero drift introduces an additional term into the expression for the cumulative output response $M(\delta,\mu)$, which is not captured when tracking firms until the first time of adjustment. I provide a way of computing this new term and generalize the approach of characterizing cumulative impulse responses, introduced by Alvarez, Le Bihan, and Lippi (2016), to economies with drift and asymmetries.

2.5 Optimal Policy of Firms under Non-Zero Drift

To approximate the optimal policy for the case of non-zero drift, I apply implicit function theorem to the system of equations that characterize the solution of the firm's problem, as discussed in Section 2.1. Proposition 1 states the result and the proof is provided in Appendix B.

Proposition 1. *Let $\sigma, \rho, \kappa > 0$. Then:*

$$\frac{\partial \underline{z}(\mu)}{\partial\mu}\Big|_{\mu=0} = \frac{\partial \overline{z}(\mu)}{\partial\mu}\Big|_{\mu=0} > 0 \qquad \text{(i)}$$

$$\frac{\partial \hat{z}(\mu)}{\partial\mu}\Big|_{\mu=0} > \frac{\partial \overline{z}(\mu)}{\partial\mu}\Big|_{\mu=0} \qquad \text{(ii)}$$

The first line states that that boundaries of the inaction region move in parallel to the right as trend inflation increases. This implies that the width of the inaction region $\big(\overline{z}(\mu) - \underline{z}(\mu)\big)$ is insensitive to trend inflation at $\mu = 0$. The second line states that the return point moves in the same direction, but stronger than the boundaries. Both effects are due to the desire of the firm to stay close to the profit maximizing zero price gap for as long as possible. With a positive drift in the optimal price, the price gaps are expected to fall over time. Therefore, firms move the return point to the right to increase the total time spent in the vicinity of zero. Firms also tolerate larger positive gaps and delay adjusting because the gaps are expected to fall on their own. For the same reason, firms adjust 'sooner' for negative price gaps, as these are not expected to rise over time. Note that by taking the limit as $\rho \to 0$, I recover expressions for the no-discounting case considered in

Alvarez et al. (2019).

The uneven shift of the return point and boundaries leads to asymmetry in individual adjustments. Denote the size of positive adjustments by $\Delta^+(\mu) := \hat{z}(\mu) - \underline{z}(\mu)$, and of negative adjustments by $\Delta^-(\mu) := \overline{z}(\mu) - \hat{z}(\mu)$. An immediate implication of Proposition 1 is that $\frac{\partial \Delta^+(0)}{\partial \mu} = -\frac{\partial \Delta^-(0)}{\partial \mu} > 0$, meaning that positive adjustments become larger as drift increases, whereas negative adjustments become smaller. Finally, defining asymmetry in individual adjustments by $A_I(\mu) = \frac{\Delta^+(\mu)}{\Delta^-(\mu)}$, obtains:

$$\frac{\partial A_I(0)}{\partial \mu} = \frac{2}{\overline{z}(0)} \frac{\partial \Delta^+(0)}{\partial \mu} > 0$$

This implies that asymmetry in individual price adjustments increases with trend inflation in the sense that positive adjustments become larger relative to negative adjustments. While the result might not appear surprising, it is not immediate. When exposed to a small positive drift, a firm may adjust its behavior entirely via the relative frequency of price increases and decreases, while keeping adjustments symmetric. Instead, because of a forward-looking behavior, it chooses to increase its positive adjustments in anticipation of price erosion due to trend inflation and decrease its negative adjustments for the same reason. The concerns of firms regarding future dynamics are key here: stronger discounting weakens the asymmetry, and it is entirely gone in a static model, which is the limit case as $\rho \to \infty$.

2.6 Impact Effect

Before stating the results for the impact effect, it is instructive to outline the channels through which drift influences the impact response of the price level. Let us decompose the impact effect of a positive shock δ, given in equation (2.1), using the definition of positive adjustments $\Delta^+(\mu) = \hat{z}(\mu) - \underline{z}(\mu)$ and performing variable substitution $z \to x := \underline{z}(\mu) - z$:

$$\Theta(\delta, \mu) = \underbrace{\Delta^+(\mu) F(\underline{z}(\mu) + \delta, \mu)}_{\text{Minimal adjustment}} + \underbrace{\int_0^\delta x f(\underline{z}(\mu) + \delta - x, \mu) dx}_{\text{Additional adjustment}} \tag{2.2}$$

If there is a positive shock of size δ, then a total mass $F(\underline{z}(\mu) + \delta, \mu)$ of agents adjust immediately, with each of them adjusting by $\Delta^+(\mu)$ at least. This is reflected in the first term and denoted by 'minimal adjustment'. Because agents are shifted strictly outside of the inaction region, their actual adjustment is larger. This 'additional adjustment' component depends on the position of the agent prior to the shock, is denoted by x and is captured by the second term. Differentiating $\Theta(\delta, \mu)$ and evaluating at $\mu = 0$ provides the following expression:

$$\frac{\partial \Theta(\delta,0)}{\partial \mu} = \underbrace{\frac{\partial \Delta^{+}(0)}{\partial \mu} F(\underline{z}(0)+\delta,0)}_{\text{Intensive margin}} + \underbrace{\Delta^{+}(0)\frac{dF(\underline{z}(0)+\delta,0)}{d\mu} + \int_0^{\delta} x \frac{df(\underline{z}(0)+\delta-x,0)}{d\mu}dz}_{\text{Extensive margin}}$$

The effect of trend inflation on the immediate price response can be decomposed into two terms. The first is the effect on the minimal adjustment size, labeled 'intensive margin' and driven purely by changes in optimal policy. The second is the effect on the mass of adjusting agents, labeled as 'extensive margin' and driven by changes in the stationary distribution. Note that stationary density depends on optimal policy, and thus the latter will indirectly affect the extensive margin as well.[8] I provide an expression for $\frac{\partial \Theta(\delta,0)}{\partial \mu}$ in Appendix A.5 and Proposition 2 states that this derivative is always positive.

Proposition 2. *Let $\sigma, \rho, \kappa > 0$. Then for any $\delta \neq 0$:*

$$\left.\frac{\partial \Theta(\delta,\mu)}{\partial \mu}\right|_{\mu=0} > 0$$

The result implies that a small positive trend amplifies the responses to positive shocks and mitigates the responses to negative shocks. Importantly, trend has a first-order effect on $\Theta(\delta,\mu)$, irrespective of shock size. Define asymmetry for impact effect analogously to individual adjustments: $A_\Theta = \frac{\Theta(\delta,\mu)}{-\Theta(-\delta,\mu)}$. It follows that:

$$\frac{\partial A_\Theta(\delta,0)}{\partial \mu} = \frac{2}{\Theta(\delta,0)}\frac{\partial \Theta(\delta,0)}{\partial \mu} > 0$$

Therefore, asymmetry in the impact price responses goes up as trend inflation rises, in the sense that the magnitude of responses to positive shocks increases relative to the magnitude of responses to negative shocks.

Interestingly, the effect of drift on asymmetry in aggregate price responses does not vanish as shock size goes to zero:

$$\lim_{\delta \to 0} \frac{\partial A_\Theta(\delta,0)}{\partial \mu} = \frac{2\overline{z}_0}{\sigma^2}$$

This is because the impact effect and its derivative with respect to trend inflation are of the same order for small shocks.[9] Combining a first-order approximation with respect to

[8] Derivatives of F and f are total (not partial) since both of their arguments depend on μ in (2.2).

[9] Alvarez and Neumeyer (2019) also mention that trend affects the coefficient in front of δ^2 in a second-order approximation of $\Theta(\delta,\mu)$ with respect to δ. Here, I provide an explicit expression for this interaction term for small values of μ.

drift μ with a second-order approximation with respect to shock δ gives:

$$\Theta(\delta,\mu) \approx \begin{cases} \left(1+\frac{\overline{z}_0}{\sigma^2}\mu\right)\Theta(\delta,0) & \text{for } \delta>0 \\ \\ \left(1-\frac{\overline{z}_0}{\sigma^2}\mu\right)\Theta(\delta,0) & \text{for } \delta<0 \end{cases}$$

This shows that drift has a multiplicative effect on the impact response. For a small positive drift, the impact effect of a positive shock is increased by $100 \cdot \frac{\overline{z}_0}{\sigma^2}\mu$ percent, whereas the response to a negative shock is decreased in the same proportion. Therefore, if a shock is small, ignoring the effect of drift produces an error of the same order as simply setting the impact response to zero.

One can also compare asymmetry at individual and aggregate levels. At the micro-level, firms react to shocks as soon as the inaction region boundaries are reached, therefore, it would be fair to compare the trend effect on individual asymmetry ($A_I(\mu)$, introduced earlier) with the trend effect on aggregate asymmetry $\left(\frac{\partial A_\Theta(\delta,0)}{\partial\mu}\right)$ for an aggregate shock δ approaching zero. The comparison yields:

$$\lim_{\delta\to 0}\frac{\partial A_\Theta(\delta,0)}{\partial\mu} > \frac{\partial A_I(0)}{\partial\mu}$$

This follows because the trend effect on aggregate asymmetry consists of extensive and intensive margins, whereas individual asymmetry is only driven by the latter. These work in the same direction, amplifying asymmetry at the aggregate level even further.

2.7 Cumulative Impulse Response

An extremely useful result for cumulative impulse responses in driftless economies is that one only has to keep track of price gaps until the first adjustment. Unfortunately, this result does not hold in economies with non-zero drift. Explicitly writing an infinite-horizon CIR as a limit of a finite-horizon CIR reveals that cumulative responses until finite horizon t have an additional 'tail' term, which represents cumulative deviations between the time of the last adjustment and period t. This tail term does not vanish in the limit and is not equal to zero in expectation. In the following, I derive an extension of the CIR formula for economies with non-zero drift in a more general setting, which might be useful for purposes beyond the scope of this paper.

Following Alvarez and Lippi (2019), I let $z(t)$ be an individual process on $Z = [\underline{z}, \overline{z}]$, endowed with the strong Markov property and a stationary distribution $F(z)$. I denote by $g : Z \to \mathbb{R}$ a bounded, Borel-measurable function of interest. Suppose the economy is in a steady state. In period $t = 0$, an aggregate shock distorts the distribution of z, such that distribution in $t = 0$ is given by $F_0(z)$. One can express the impulse response t periods after as follows:

$$IRF(t,F_0)=\int_{\underline{z}}^{\overline{z}}\mathbb{E}\Big(g(z(t))-\bar{g}\mid z(0)=z\Big)dF_0(z)\quad\text{where}\quad\bar{g}=\int_{\underline{z}}^{\overline{z}}g(z)dF(z)$$

Therefore, $IRF(t,F_0)$ is the period t economy-wide average deviation of g from its steady state average $\bar{g}$ if z was initially distributed according to $F_0(z)$. Denote by $CIRF(t,F_0)$ the cumulative impulse response up to period t:

$$CIRF(t,F_0)=\int_0^t IRF(s,F_0)ds$$

Switching the order of the integration and taking the expectation operator out of the inner integral yields:

$$CIRF(t,F_0)=\int_{\underline{z}}^{\overline{z}}\mathbb{E}\Big(\int_0^t(g(z(s))-\bar{g})ds\mid z(0)=z\Big)dF_0(z)$$

One can first compute expected cumulative deviation of g from its steady state until t for each starting value z and then average across all starting values using the initial distribution function F_0. The statistic of interest is the infinite-horizon cumulative IRF:

$$CIRF(F_0)=\lim_{t\to\infty}CIRF(t,F_0)=\int_{\underline{z}}^{\overline{z}}\mathbb{E}\Big(\int_0^\infty(g(z(s))-\bar{g})ds\mid z(0)=z\Big)dF_0(z)$$

2.7.1 Cumulative Impulse Responses in Impulse Control Models

Now consider a special case for the process $z(t)$, namely the one resulting from an impulse control problem with a fixed return point $\hat{z}$, as in the model considered in this paper. The next proposition characterizes $CIRF(F_0)$ as a sum of two terms: the familiar expected deviation until the first adjustment and the new tail term.

Proposition 3. *Denote by $m(z)$ the expected cumulative deviation of g from its steady state $\bar{g}$ until the time of the first adjustment τ, conditional on the initial value $z(0)=z$:*

$$m(z)=\mathbb{E}\left(\int_0^\tau(g(z(s))-\bar{g})ds\;\middle|\;z(0)=z\right)$$

Let $n(t)$ be the number of adjustments between time 0 and t, so that $\tau_{n(t)}$ denotes the time of the last adjustment before t. Then:

$$CIRF(F_0)=\int_{\underline{z}}^{\overline{z}}m(z)dF_0(z)+\lim_{t\to\infty}\mathbb{E}\left(\int_{\tau_{n(t)}}^t(g(z(s))-\bar{g})ds\right)$$

Function $m(z)$ is similar to the one used to compute the cumulative IRF under zero drift. This function provides the expected cumulative deviation of g from its steady state

$\bar{g}$ until the time of the first adjustment and can typically be defined with an ordinary differential equation.

To understand the new term, consider the cumulative response until some large finite time t. Each agent will have a certain number of adjustments, made until that period, denoted by $n(t)$, and the cumulative response can be split into periods before the first adjustment, in between adjustments and after the last adjustment:

$$\int_0^t (g(z(s))-\bar{g})ds = \int_0^{\tau_1} (g(z(s))-\bar{g})ds + \sum_{i=2}^{n(t)} \int_{\tau_{i-1}}^{\tau_i} (g(z(s))-\bar{g})ds + \int_{\tau_{n(t)}}^t (g(z(s))-\bar{g})ds$$

The idea of the proof is that when taking expectations and letting $t \to \infty$, the first term becomes the function $m(z)$, terms in the middle vanish as shown by Baley and Blanco (2020), and the last term converges to some number, which is not necessarily zero. Deviations sum up to zero in expectation if they are considered strictly in between adjustment times, as in the middle terms. This does not apply to the last term, which cumulates deviations between an adjustment time and some arbitrary t, given that the next adjustment occurs after t.

In fact, for the model considered in this paper and $g(z) = z$, this tail term is equal to zero if and only if $\mu = 0$. This is because if $\mu \neq 0$, then the return point $\hat{z}$ is not equal to the average gap $\bar{x} = \int_{\underline{z}}^{\overline{z}} z dF(z)$, which implies that the expected cumulative deviation between the last time of adjustment $\tau_{n(t)}$ and arbitrary t is not equal to zero. For example, if t is very close to $\tau_{n(t)}$, then for any time $s \in (\tau_{n(t)}, t)$, $z(s)$ is very close to the return point $\hat{z}$ in expectation, and thus relatively far from the average gap $\bar{x}$, so that the expected cumulative deviation $\mathbb{E}\big(\int_{\tau_{n(t)}}^t (z(s) - \bar{x})ds\big)$ is non-zero.

2.7.2 Computing the Tail Term

Unlike the cumulative response until the first adjustment, the tail term does not allow for an immediate characterization. However, there are at least two ways of dealing with this issue. The first one relies on the fact that the new term does not depend on the initial distribution F_0, as it considers paths after the first adjustments. These paths are independent of the initial condition due to the strong Markov property of $z(t)$. Note that setting the initial distribution F_0 equal to the stationary distribution F results in zero impulse response by definition:

$$CIRF(F) = \int_{\underline{z}}^{\overline{z}} m(z)dF(z) + \lim_{t\to\infty} \mathbb{E}\left(\int_{\tau_{n(t)}}^t (g(z(s)) - \bar{g})ds\right) = 0$$

This allows to obtain an expression for the limit term and express the cumulative response as follows:

Corollary 1.

$$CIRF(F_0) = \int_{\underline{z}}^{\overline{z}} m(z)dF_0(z) - \int_{\underline{z}}^{\overline{z}} m(z)dF(z)$$

When there is no drift, the inaction region is symmetric ($\underline{z} = -\overline{z}$), $F(z)$ is a symmetric distribution, and $m(z)$ exhibits negative symmetry ($m(-z) = -m(z)$). These imply that $\int_{\underline{z}}^{\overline{z}} m(z)dF(z) = 0$ and $CIRF(F_0) = \int_{\underline{z}}^{\overline{z}} m(z)dF_0(z)$. Therefore, one only needs to track each agent until the first time of adjustment to compute the entire cumulative response. However, if drift is non-zero, one can still consider paths until the first adjustment, but must additionally subtract the average cumulated deviation under the stationary distribution.

The second approach uses the notion of a discounted cumulative impulse response:

$$DCIRF(r, F_0) = \lim_{t\to\infty} DCIRF(r, t, F_0) = \int_0^\infty e^{-rs} IRF(s, F_0)ds$$

which allows to eliminate the tail term for any $r > 0$. Then, the usual CIR can be expressed as a limit case of its discounted counterpart:

$$CIRF(F_0) = \lim_{r\to 0} DCIRF(r, F_0)$$

This approach substitutes the inconvenient tail term with a more tractable one, and the next proposition provides the result.

Proposition 4. *Denote by $m(r, z)$ the expected discounted cumulative deviation of g from its steady state until the time of the first adjustment τ, conditional on initial value $z(0) = z$:*

$$m(r, z) = \mathbb{E}\left(\int_0^\tau e^{-rs}(g(z(s)) - \bar{g})ds \;\middle|\; z(0) = z\right)$$

Then:

$$CIRF(F_0) = \int_{\underline{z}}^{\overline{z}} m(0, z)dF_0(z) + \frac{1}{\mathbb{E}\left(\tau \mid z(0) = \hat{z}\right)} \lim_{r\to 0} \frac{m(r, \hat{z})}{r}$$

Here, the first term is the same as before, since trivially $m(0, z) = m(z)$, whereas the second term provides an alternative way of computing the tail term from Proposition 3.

The first approach from Corollary 1 uses all familiar objects but requires computing the integral of $m(z)$ twice – under initial and stationary distributions. Using the second approach from Proposition 4, one needs to compute an additional function $m(r, z)$, which can typically be defined with an ordinary differential equation, similar to $m(z)$. Therefore, each approach may be more or less preferable, depending on the application. For example, if one deals with shocks that shift the stationary distribution (as in this paper), then the first approach provides a much easier way of computing CIR because F_0 inherits the shape of F. On the other hand, if the initial distribution is not related to the stationary one, it might be more convenient to analyze CIR using the second approach, as it only requires computing the integral under F_0 (although F is still required to compute the steady state average $\bar{g}$).

To determine whether the tail term is qualitatively important and whether one can omit it for simplicity, recall that it neither depends on the initial distribution F_0 nor interacts with it. This implies that it corrects for the level of the cumulative response, acting as an intercept. Therefore, omitting it not only changes the CIR value, but may also flip its sign if the true value is sufficiently close to zero. In the next section, I discuss the special importance of the tail term for cumulative responses to δ shocks considered in this paper.

2.7.3 Application to δ Shocks

A δ shock considered in this paper shifts the stationary distribution F in parallel. The initial distribution F_0 is given by the stationary distribution F, shifted by δ and truncated to the inaction region, together with a mass point at $\hat{z}$, which is due to firms that adjust on impact. As noted earlier, it is most convenient in this situation to use Corollary 1 for computing the cumulative response, so that for $\delta > 0$ it is given by:

$$CIRF(\delta) = \int_{\underline{z}}^{\overline{z}-\delta} m(z)dF(z+\delta) + \underbrace{m(\hat{z})F(\underline{z}+\delta)}_{=0} - \int_{\underline{z}}^{\overline{z}} m(z)dF(z) \tag{2.3}$$

where $m(\hat{z}) = 0$, as shown in Baley and Blanco (2020). Note that the tail term does not affect the *slope* of CIRF(δ), which is entirely determined by cumulative deviations until the first adjustment, captured in the first term. Instead, it shifts the entire function, which has special importance for very small and very large shocks. If $\delta = 0$, then ignoring the tail term would give that $CIRF(0) = \int_{\underline{z}}^{\overline{z}} m(z)dF(z)$, which may not be equal to zero, implying a 'response' despite the absence of a shock. If δ is large, so that $\delta \geq (\overline{z} - \underline{z})$, then the first term in (2.3) vanishes, and the CIR is entirely determined by the tail term. Omitting the tail term would imply that $CIRF(\delta) = 0$ for all $\delta \geq (\overline{z} - \underline{z})$, whereas it might be different from zero.

2.8 Sensitivity of the Cumulative Impulse Response to Drift

I now use results from the previous section to study the sensitivity of the cumulative output response to drift μ. Recall from section 2.2 that it is given by:

$$M(\delta, \mu) = -\int_{\underline{z}(\mu)}^{\overline{z}(\mu)} \mathbb{E}\left(\int_0^\infty (z(t) - \bar{x}(\mu))\, dt \;\middle|\; z(0) = z \right) dF_\delta(z, \mu)$$

where $\bar{x}(\mu) = \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} z dF(z,\mu)$. Using Corollary 1 and writing $M(\delta,\mu)$ as in (2.3) yields:

$$M(\delta,\mu) = \int_{\underline{z}(\mu)}^{\overline{z}(\mu)-\delta} m(z,\mu) f(z+\delta,\mu) dz - \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} m(z,\mu) f(z,\mu) dz$$

where

$$m(z,\mu) = -\mathbb{E}\left(\int_0^\tau (z(t) - \bar{x}(\mu)) dt \;\middle|\; z(0) = z \right)$$

Function $m(z,\mu)$ solves the following differential equation: $z - \bar{x}(\mu) = -\mu m_z(z,\mu) + (\sigma^2/2) m_{zz}(z,\mu)$ with boundary conditions $m(\underline{z}(\mu),\mu) = m(\overline{z}(\mu),\mu) = 0$. Proposition 5 states that drift has a first-order effect on the cumulative output response, irrespective of the shock size.

Proposition 5. *Let $\sigma, \rho, \kappa > 0$. Then for any $\delta \neq 0$:*

$$\left.\frac{\partial M(\delta,\mu)}{\partial \mu}\right|_{\mu=0} < 0$$

In accordance with the results on impact effect $\Theta(\delta,\mu)$, trend inflation amplifies price responses to positive shocks and thus mitigates responses of output, which is reflected by the negative sign of the derivative. The reverse is true for negative shocks, as in this case output responses are amplified. Note that the tail term is crucial for this result, without it, the derivative is positive for small shocks, zero for large shocks and negative for intermediate values.

Define asymmetry in CIR as a difference in magnitudes of responses to positive and negative shocks: $A_M(\delta,\mu) = M(\delta,\mu) - (-M(-\delta,\mu))$. Here, I am using difference instead of ratio in order to ensure that asymmetry is well-defined for shocks of any size because $M(\delta,0) = 0$ for all δ such that $|\delta| > 2\overline{z}_0$. It is, however, also possible to define it as a ratio, provided $M(\delta,\mu) > 0$ and $M(-\delta,\mu) < 0$. It follows immediately that:

$$\frac{\partial A_M(\delta,0)}{\partial \mu} = 2\frac{\partial M(\delta,0)}{\partial \mu} < 0$$

Therefore, cumulative output responses to positive shocks become smaller relative to cumulative output responses to negative shocks as trend inflation increases.

To determine whether the drift effect is sizable, I combine a first-order approximation of $M(\delta,\mu)$ with respect to μ and a second-order approximation with respect to δ:

$$M(\delta,\mu) \approx \begin{cases} \left(1 - \frac{|\delta|}{\sigma^2}\mu\right) M(\delta,0) & \text{for } \delta > 0 \\ \left(1 + \frac{|\delta|}{\sigma^2}\mu\right) M(\delta,0) & \text{for } \delta < 0 \end{cases}$$

Cumulative output response is amplified by $100 \cdot \frac{|\delta|}{\sigma^2}\mu$ percent if a shock is negative and mitigated in the same proportion for a positive shock. Thus, for small shocks, the drift effect is negligible, but it becomes more important as the shock size increases. For large shocks, the drift might not only amplify or mitigate output responses, but also change their sign. This result is discussed and formalized in the following section.

2.9 Large Shocks

The drift effect is of particular importance for large shocks. As noted previously, in the driftless case, the price level reacts one-to-one to a large nominal shock on impact, which results in monetary neutrality, i.e., output is not affected by the shock. These results break down when trend inflation is non-zero.

To see why this occurs, consider a positive shock δ that is large in the sense that it shifts the entire distribution outside of the inaction region ($\delta \geq \overline{z}(\mu) - \underline{z}(\mu)$). The impact effect for this shock is given by:

$$\Theta(\delta, \mu) = \delta + \hat{z}(\mu) - \bar{x}(\mu) \quad \text{where } \; \bar{x}(\mu) = \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} z f(z, \mu) dz$$

The entire distribution of price gaps is initially shifted to the left by δ, and the mean price gap immediately after the shock and before the adjustment becomes $\bar{x}(\mu) - \delta$. Because all firms are pushed outside the inaction region, the aggregate adjustment equals $\hat{z}(\mu) - (\bar{x}(\mu) - \delta)$, which gives the impact effect. When $\mu = 0$, both the average gap $\bar{x}(0)$ and the return point $\hat{z}(0)$ are zero, and thus the impact effect is equal to the shock, meaning that the aggregate price responds one-to-one. If trend inflation is positive ($\mu > 0$), then $\hat{z}(\mu) > 0$ and $\bar{x}(\mu) < 0$, which leads to price overreaction: $\Theta(\delta, \mu) > \delta$. Note that if the shock is negative and $\delta \leq -|\overline{z}(\mu) - \underline{z}(\mu)|$, then $|\Theta(\delta, \mu)| < |\delta|$ and there is no overreaction. All of the above has implications for the cumulative output response $M(\delta, \mu)$. Proposition 6 formalizes the results.

Proposition 6. *Let $\sigma, \rho, \kappa > 0, \mu > 0$ and sufficiently small.*

Then there exist $\delta_\Theta(\mu), \delta_M(\mu) > 0$ such that $\delta_\Theta(\mu), \delta_M(\mu) < \overline{z}(\mu) - \underline{z}(\mu)$ and:

$$\begin{aligned} \Theta(\delta, \mu) > \delta \quad & \textit{for all} \;\; \delta > \delta_\Theta(\mu) \\ M(\delta, \mu) < 0 \quad & \textit{for all} \;\; \delta > \delta_M(\mu) \end{aligned}$$

Proposition 6 states that if trend inflation is positive and small, then there exist thresholds $\delta_\Theta(\mu)$ and $\delta_M(\mu)$ such that price overreacts on impact to any positive shock larger than $\delta_\Theta(\mu)$ and the cumulative output response is negative for any positive shock larger than $\delta_M(\mu)$.[10] The latter implies that positive shocks eventually become contractionary

[10] Alvarez and Neumeyer (2019) show that price response exceeds the shock on impact if $\mu \to \infty$ and provide numerical examples when this happens for finite values of μ.

if trend inflation is positive. Crucially, the thresholds are both strictly smaller than the width of the inaction region $\overline{z}(\mu) - \underline{z}(\mu)$. Therefore, a positive shock does not have to shift the entire distribution outside of the inaction region to induce price overshooting and a decline in output when trend inflation is positive, as these effects are already achieved for smaller shocks.

2.10 Summary and Comparison with the Driftless Case

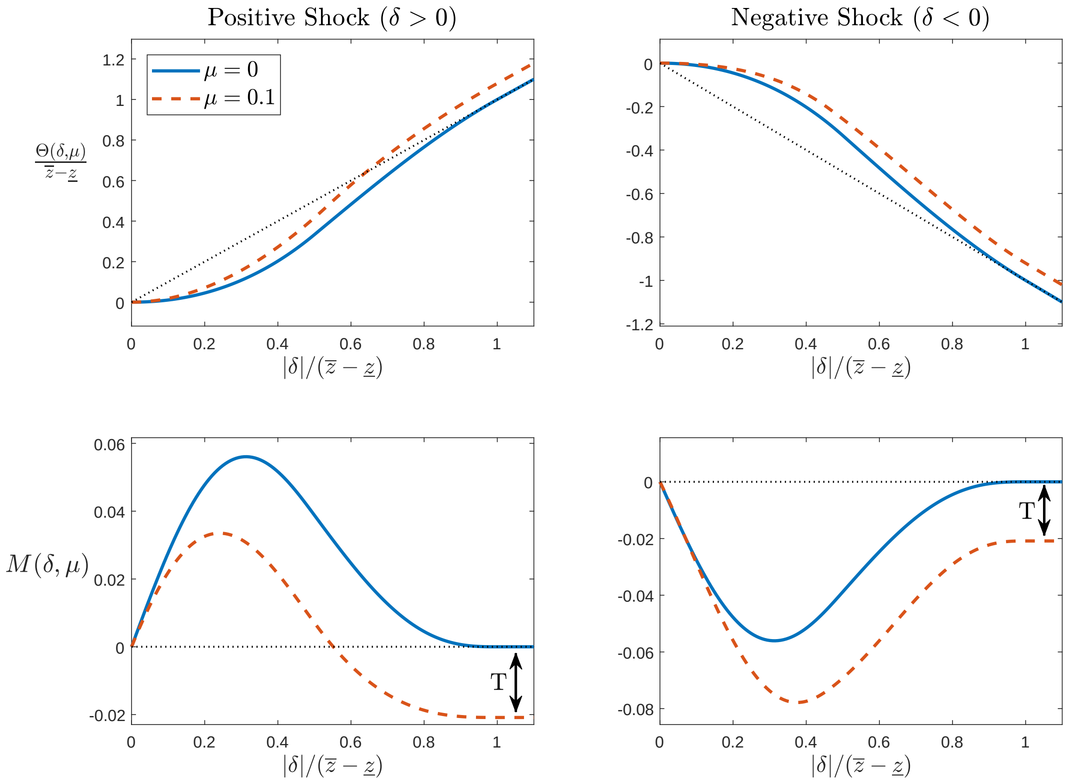
I conclude this section by illustrating the main analytical results of the paper. Figure 3 plots $\Theta(\delta, \mu)$ (top row) and $M(\delta, \mu)$ (bottom row) against the absolute shock values normalized by the width of the inaction region $\left(\frac{|\delta|}{\overline{z}-\underline{z}}\right)$ for positive (left column) and negative (right column) shocks.[11] The solid blue lines correspond to the driftless case $\mu = 0$, and the red dashed lines correspond to $\mu = 0.1$. Recall the three properties of the impact effect and the cumulative output response under zero trend inflation, discussed in section 2.3: (1) both statistics are symmetric, (2) the size of the impact effect $\Theta(\delta, 0)$ is always weakly smaller than the shock size and cumulative output response $M(\delta, 0)$ does not switch its sign, and (3) if $|\delta|/(\overline{z} - \underline{z})$ is equal to one or larger, then $\Theta(\delta, 0) = \delta$ and $M(\delta, 0) = 0$. All these properties are illustrated in Figure 3 by the solid blue lines.

Now consider the case of positive trend inflation. The responses to positive and negative shocks are asymmetric, so that the first property does not hold anymore. The impact price responses to positive shocks are amplified, whereas the responses to negative shocks are mitigated compared to the driftless benchmark. This can be seen in the top row, as the dashed red lines lie above the solid blue ones. The opposite is true for the cumulative output responses, which become stronger after negative shocks and weaker after positive ones as trend inflation rises. This is depicted in the bottom row, where the dashed red lines lie substantially below the solid blue ones.

Furthermore, large positive shocks lead to price overreaction on impact and cause negative cumulative output responses, which invalidates both the second and third properties. Proposition 6 states that there are two thresholds, $\delta_\Theta(\mu)$ and $\delta_M(\mu)$, such that shocks larger than these thresholds cause price overshooting and output contraction, respectively. The former is determined by the intersection of the red dashed line and a 45° line on the top left panel of Figure 3, where $\Theta(\delta, \mu) = \delta$. The latter threshold corresponds to the point where the dashed red line crosses zero on the bottom left panel of Figure 3, so that $M(\delta, \mu) = 0$. Numerical computation yields that $\delta_\Theta(0.1) = 0.64(\overline{z} - \underline{z})$ and $\delta_M(0.1) = 0.55(\overline{z} - \underline{z})$. Therefore, when $\mu = 0.1$, any shock δ larger than 64% of the width of the inaction region causes price overshooting and any shock larger than 55% of the width of the inaction region leads to a contraction in output. The higher the trend inflation, the harder it is to stimulate output, as even medium-size positive nominal shocks

[11] Both statistics are exact values and not first-order approximations with respect to drift μ.

Figure 3: Impact and Cumulative Responses



Top row: impact effect $\Theta(\delta, \mu)$, bottom row: cumulative response $M(\delta, \mu)$. Left column: positive shocks $\delta > 0$, right column: negative shocks $\delta < 0$. X-axis: absolute value of shock δ normalized by the width of the inaction region $\overline{z} - \underline{z}$. Impact effect is normalized by $\overline{z} - \underline{z}$ for comparability of x- and y-axes. Solid blue lines: $\mu = 0$, dashed red lines: $\mu = 0.1$. The tail term is denoted by T in the bottom row. Rest parameter values: $\sigma^2 = 0.05$, $\rho = 0.05$, $\kappa = 0.05$. Threshold values: $\delta_\Theta(0.1) = 0.64(\overline{z} - \underline{z})$ and $\delta_M(0.1) = 0.55(\overline{z} - \underline{z})$.

have a reversed effect. Interestingly, the threshold for the reversed effect on output is smaller than the threshold for price overshooting. Thus, there is a range of shocks (55% - 64% of the width of the inaction region) for which cumulative output response is negative, even though price level responds less than one-to-one on impact. On the contrary, negative shocks never lead to an expansion in output and price overshooting. The price level always underreacts to negative shocks on impact and the cumulative output response is always negative if trend inflation is positive.

I lastly note the role of the tail term that appears in the expression for the cumulative output response when trend inflation is non-zero. As discussed in section 2.7, the CIR is entirely determined by this term if $|\delta| \geq (\overline{z} - \underline{z})$. Therefore, one can directly see the tail term in Figure 3, where it is denoted by T in the bottom row. Not only is it quantitatively important, but it is also the only source of difference between the cases of positive and zero trend inflation.

3 Empirical Evidence

In this section I test several predictions of the model derived above. I start with the effect of trend inflation on aggregate responses to shocks, as these are the main focus of the paper. However, a substantial part of the mechanism operates via changes in firm behavior induced by the presence of drift. Therefore, I also provide evidence for the relationship between drift and asymmetry in micro-level adjustments. As I show, many of the micro- and macro-level implications of the theory are supported by the data.

3.1 Drift and Asymmetry at the Macro Level

I start by testing whether trend inflation affects asymmetry in aggregate responses to monetary shocks. I use monthly sectoral data on Producer Price Index (PPI) provided by the Bureau of Labor Statistics, as well as data on Industrial Production (IP) provided by the Federal Reserve System. To estimate impulse responses I use local projections as in Jordà (2005). This approach has been widely utilized in the literature to test for asymmetries, non-linearities and state-dependence of impulse responses (Auerbach and Gorodnichenko (2012), Ramey and Zubairy (2014), Tenreyro and Thwaites (2016)). The main advantage of local projections is the ease of inclusion of non-linear terms, which are of central interest in this paper. The baseline shock measure is the one computed by Jarociński and Karadi (2020) using high frequency identification and separating monetary policy shocks from central bank information shocks. In Appendix C.2 I show that results are generally robust to alternative shock measures.

The central idea is to exploit cross-sectoral heterogeneity in trend inflation to see whether it relates to asymmetry in production and price responses. To ensure that impulse responses for every subset of industries are estimated on the same set of shocks, I use a balanced panel. The sample spans between February 1990 and January 2013 and consists of 52 industries.[12]

3.1.1 Asymmetric Responses

The simplest way of introducing asymmetry is estimating piecewise linear impulse responses with a kink at zero by including positive and negative shocks separately in the regression. To avoid ambiguity, I will refer to interest rate cuts (monetary easing) as 'positive' shocks, whereas to interest rate hikes (monetary tightening) as 'negative' shocks. Thus, the sign of a shock corresponds to the intended effect on output, which provides the following

[12]Even though interest rates have stayed low in $2009 - 2013$, this period is still informative as it features both positive and negative monetary shocks (see Figure 11 in Appendix C.1). Considering the period until June 2008 does not alter the main results.

non-linear panel local projection:

$$y_{i,t+h} - y_{i,t-1} = \alpha_{i,h} + \beta_h^P \max(\varepsilon_t, 0) + \beta_h^N \min(\varepsilon_t, 0) + \gamma_h' \mathbf{x}_{i,t} + \nu_{i,t+h} \tag{3.1}$$

where $y_{i,t+h} - y_{i,t-1}$ is the growth rate of the dependent variable (IP or PPI) between $t-1$ and $t+h$ in industry i, $\alpha_{i,h}$ is an industry fixed effect, ε_t is the monetary policy shock, and $\mathbf{x}_{i,t}$ is a vector of controls. This specification directly estimates cumulative impulse responses. The monetary shocks are scaled and normalized such that positive values correspond to interest rate cuts and a shock of size one represents a one standard deviation shock. Here, β_h^P provides the impulse response to a one standard deviation positive shock h periods after impact, and $(-\beta_h^N)$ is the response to a negative shock of the same size. Note that the standard theory predicts that $\beta_h^P > 0$ and $\beta_h^N > 0$. The set of controls includes a time trend, contemporaneous and lagged growth rates of aggregate industrial production and of a commodity price index, as well as lags of the monetary shock, effective federal funds rate, and industry-specific growth rates of IP and PPI. I set the lag length to 6 months and also include contemporaneous industry-specific growth rate of IP in the PPI projection and vice versa.[13] Finally, I smooth impulse responses with a 5-month centered moving average when plotting them, in order to ease comparisons.[14]

The preferred measure of asymmetry is the ratio between the magnitudes of responses to positive and negative shocks, given by β_h^P/β_h^N, because it controls for the size of an average (linear) response. Values below one indicate that positive shocks have a smaller effect relative to negative shocks, and larger deviations from one correspond to stronger degrees of asymmetry. However, this measure is only meaningful if both β_h^P and β_h^N are positive. Whenever this condition is violated, I have to use an alternative measure, defined as a difference in magnitudes $(\beta_h^P - \beta_h^N)$. In this case, negative values indicate that monetary tightening has stronger effects compared to monetary easing.

As a first step, I estimate (3.1) on the entire sample. Figure 4 plots the impulse responses of industrial production (top row) and PPI (bottom row) to one standard deviation monetary shock,[15]. The dashed red lines depict responses to negative shocks $(-\beta_h^N)$, whereas the solid blue lines show the negatives of responses to positive shocks $(-\beta_h^P)$ to ease comparison. In the right column, I plot asymmetries in responses to positive and negative shocks. I employ the preferred measure of asymmetry (ratio) for industrial production, but have to use the alternative (difference) for PPI because these responses switch signs.

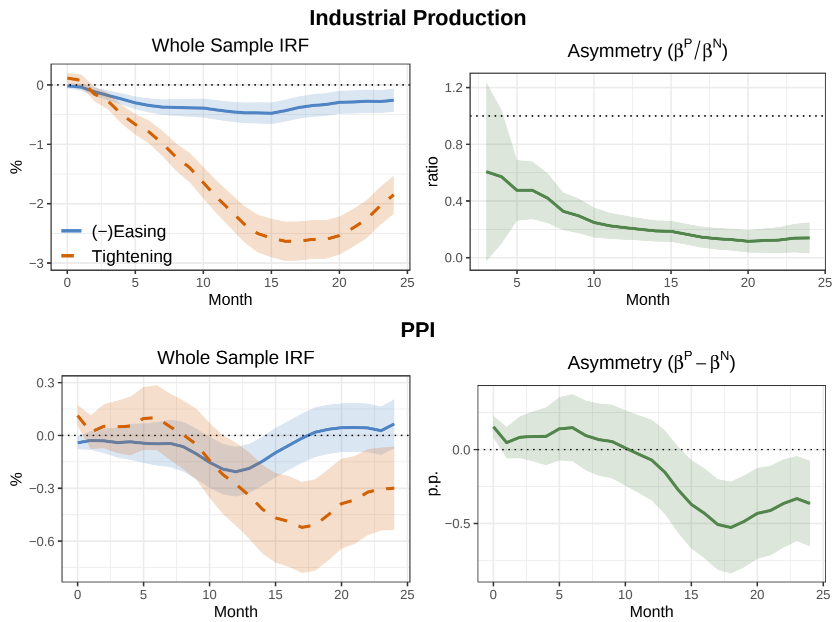
[13] The set of controls is standard, and I consider a much smaller set of controls as a robustness check in Appendix C.2.6. The commodity price index is the one produced by the Commodity Research Bureau and used in Coibion (2012) (data taken from the website of Valerie Ramey https://econweb.ucsd.edu/∼vramey/research.html#data)

[14] This is a common practice in the literature, it does not affect the results, and the unsmoothed plots are presented in Appendix C.2.6.

[15] In the sample, monetary shocks have a standard deviation of 4.8 basis points.

Industrial production exhibits a strong and significant degree of asymmetry, with negative shocks having a much larger effect on IP than positive shocks. At the horizon of 12 months, a one standard deviation negative shock has a five times stronger effect on production than a positive shock of the same size. There is less evidence for asymmetry in PPI responses, although at longer horizons negative shocks tend to cause larger responses than positive ones. The previous literature has focused on asymmetries at the aggregate level, and similar patterns have been documented by Angrist, Jordà, and Kuersteiner (2018) and Tenreyro and Thwaites (2016), among others. Results in Figure 4 suggest that asymmetries found in aggregate data are also present at the sectoral level. I now turn to the interaction between trend inflation and asymmetry in responses.

Figure 4: Piecewise Linear Cumulative Impulse Responses, Entire Sample



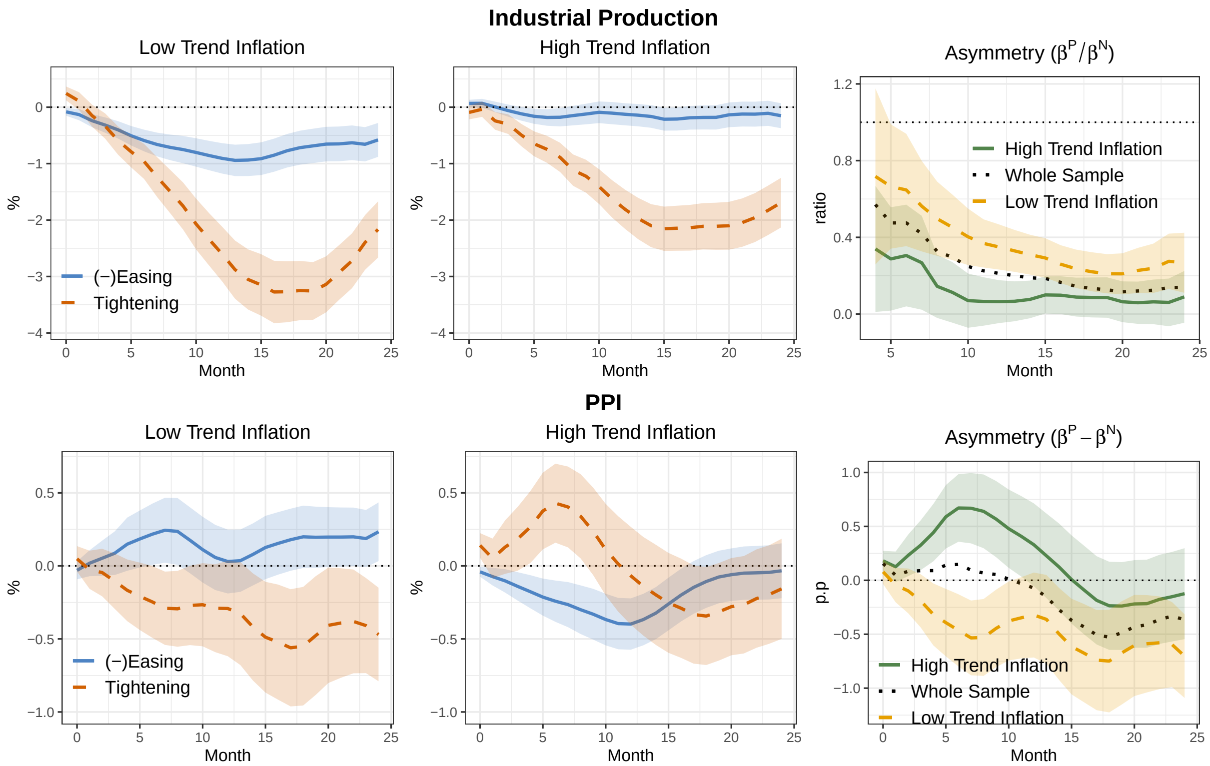
Impulse responses of industrial production (top row) and PPI (bottom row) to one standard deviation monetary shock, estimated on the entire sample. Piecewise linear specification as in (3.1). Dashed red lines: responses to a negative shock, solid blue lines: negatives of responses to a positive shock. Right column: asymmetry in responses, measured as the ratio of magnitudes for IP (positive over negative) and as the difference in magnitudes for PPI (positive minus negative). The shaded areas correspond to 68% confidence intervals, based on Newey-West standard errors. The standard errors for asymmetry are computed by the delta method.

To determine whether asymmetry is affected by trend inflation, I compute trend inflation for each industry as an average PPI growth rate over the entire period, and split the sample into two groups: industries with trend inflation above and below the median.[16] The 'low' inflation group has an average (median) trend inflation of 1.79% (1.85%) p.a., whereas for the 'high' inflation group the numbers are 3.44% and 3.22% respectively.

[16]In addition, I omit the top and bottom 2.5% of industries in terms of trend inflation from the original sample in order to control for outliers. Results are robust to a more conservative trimming, as well as to using the entire sample (see Appendix C.2.4).

For the next step, I estimate (3.1) separately for each group. Figure 5 summarizes the results for industrial production (top row) and PPI (bottom row). The first column provides responses of industries with trend inflation below the median, whereas the second column shows those with trend inflation above the median. As before, I plot negatives of responses to positive shocks to ease comparison. The third column compares the asymmetry between responses to positive and negative shocks in the two groups. Again, I employ the preferred measure (ratio) for industrial production and have to use an alternative (difference) for PPI. The solid green lines correspond to the high trend inflation group, the dashed yellow lines represent the low trend inflation group, and the dotted black lines show asymmetry in the entire sample.

Figure 5: Piecewise Linear Cumulative Impulse Responses for Low and High Trend Inflation Industries



Impulse responses of Industrial Production (top row) and PPI (bottom row) to one standard deviation monetary shock in industries with trend inflation below the median (left column) and above the median (central column). Piecewise linear specification as in (3.1). Dashed red lines: responses to a negative shock, solid blue lines: negatives of responses to a positive shock. Third column: asymmetry in responses, measured as the ratio of magnitudes for IP (positive over negative) and as the difference in magnitudes for PPI (positive minus negative). Solid green lines: industries with trend inflation above the median, dashed yellow lines: below the median, dotted black lines: entire sample. The shaded areas correspond to 68% confidence intervals, based on Newey-West standard errors. The standard errors for asymmetry are computed by the delta method.

Firstly, PPI in the low inflation industries exhibits negative asymmetry, whereas in the high inflation industries asymmetry is predominantly positive. Sectors with low trend inflation do not raise prices after positive shocks, but decrease them substantially after negative ones. The opposite is observed for industries with high trend inflation: in the first year after impact, positive shocks have a much larger effect than negative shocks, whereas at longer horizons effects are not significantly different. These findings are in line with the theoretical predictions of this paper: higher trend inflation amplifies price responses to positive shocks and mitigates reaction to negative shocks.

Secondly, there is a substantial difference in the asymmetry of industrial production responses between the two groups. In the low inflation sectors, positive shocks have a strong and significant effect on IP, whereas among sectors with high trend inflation their effect is more than halved and barely significant. Negative shocks also have a smaller but nevertheless pronounced and significant effect in the latter sample. This indicates that higher trend inflation is related to overall weaker effects of monetary shocks on industrial production, which is also found by Ascari and Haber (2020) in aggregate data, who use time variation in trend inflation. However, this drop in overall policy potency is disproportionately split between positive and negative shocks, as shown in the third panel, depicting asymmetry.

In both groups, the ratio between the magnitudes of responses to positive and negative shocks lies below one, but is much smaller for industries with high trend inflation. For example, compare the asymmetries at a 12-month horizon. In the low inflation group, interest rate cuts cause a three times weaker response than interest rate hikes. In the high inflation group, the impact of positive shocks is more than 10 times weaker than the impact of negative shocks. Although the difference between the two groups is significant only at medium term horizons, there is a clear and consistent distance between the point estimates at all horizons. This is in line with the theoretical prediction of the model, which states that trend inflation reduces the relative strength of positive shocks on output.

3.1.2 Asymmetry and Shock Size

So far I estimated piecewise linear impulse responses, focusing on asymmetry in reactions to positive and negative shocks, irrespective of their size. The theoretical results, however, highlight the importance of non-linearities and their interactions with trend inflation. The size of a shock is especially important for output responses. While price reaction in the model is always increasing in the shock size, the output response is non-monotonic, i.e., it grows for small shocks and falls when shocks are large. In the latter case, trend inflation plays a special role, as large positive shocks may lead to contractions in output under positive trend inflation. To determine whether this holds empirically, I now add non-linear terms to local projections for industrial production.

To allow for non-monotonicity of production impulse responses for both positive and negative shocks, at least a third-order polynomial is required.[17] I estimate the following

[17] In addition, a second-order polynomial would always result in larger degrees of asymmetry for larger

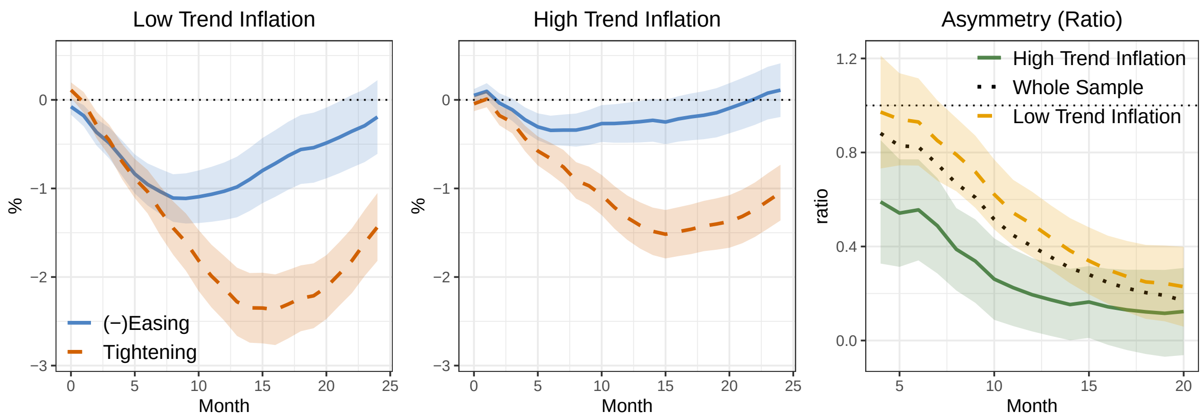
non-linear panel local projection:

$$IP_{i,t+h} - IP_{i,t-1} = \alpha_{i,h} + \beta_{1h}\varepsilon_t + \beta_{2h}\varepsilon_t^2 + \beta_{3h}\varepsilon_t^3 + \gamma_h'\mathbf{x}_{i,t} + \nu_{i,t+h} \tag{3.2}$$

where $IP_{i,t+h} - IP_{i,t-1}$ is the growth rate of industrial production between $t-1$ and $t+h$, $\alpha_{i,h}$ is an industry fixed effect, ε_t is the monetary policy shock and $\mathbf{x}_{i,t}$ is a vector of controls, which is the same as before.

Firstly, I plot the impulse responses to one standard deviation positive and negative shocks in Figure 6 to determine whether the findings of the previous section are robust to an alternative projection specification. The results closely resemble those obtained using piecewise linear local projections, depicted in Figure 5.

Figure 6: Non-Linear Cumulative Impulse Responses of Industrial Production



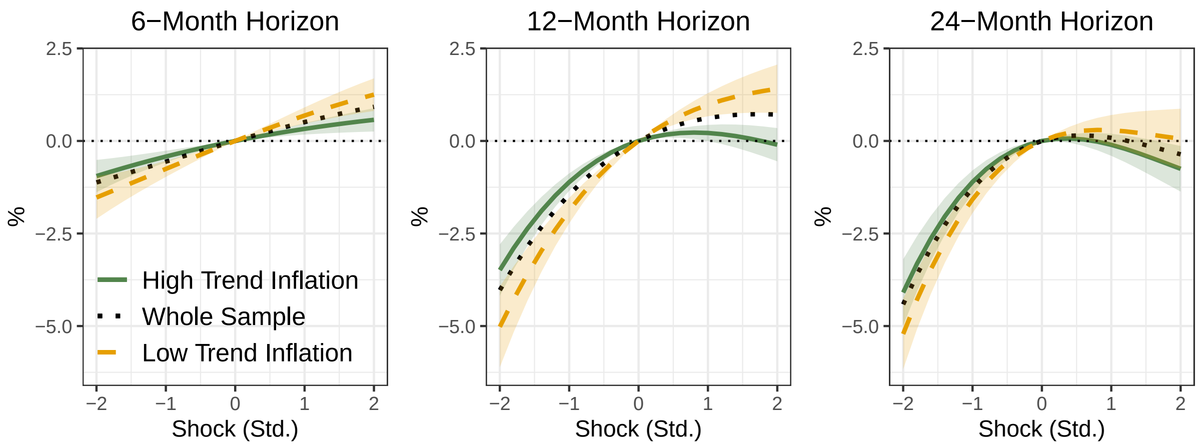
Impulse responses of industrial production to one standard deviation monetary shock in industries with trend inflation below the median (left panel) and above the median (central panel). Non-linear specification as in (3.2). Dashed red lines: responses to a negative shock, solid blue lines: negatives of responses to a positive shock. Third panel: asymmetry in responses, measured as the ratio of magnitudes (positive over negative). Solid green line: industries with trend inflation above the median, dashed yellow line: below the median, dotted black line: entire sample. The shaded areas correspond to 68% confidence intervals, based on Newey-West standard errors. The standard errors for asymmetry are computed by the delta method.

Secondly, making use of non-linearity, I plot the impulse responses for 6-, 12- and 24-months horizons for different shock values in Figure 7. The x-axis corresponds to the shock values between -2 and 2 standard deviations. The y-axis shows the impulse responses as functions of the shock value. The left panel depicts the impulse responses 6 months after impact, which are close to linear for both groups, but already exhibit small asymmetry. At the 12-months horizon, asymmetry strengthens, especially for the high inflation group. In these industries, positive shocks have a very small impact on production, whereas negative shocks lead to substantial responses. Therefore, production responses to positive shocks estimated on the entire sample are almost entirely driven by industries with low trend

shocks. In Appendix C.2.5 I show that the results are robust to including higher order terms.

inflation. In addition, the curve in the high inflation group bends toward zero as positive shocks become larger, which does not happen among sectors with low trend inflation.

Figure 7: Non-Linear Cumulative Impulse Responses of Industrial Production



Impulse responses of industrial production at 6-, 12-, and 24-months horizons. Shock values are on the x-axis, measured in standard deviations. Solid green lines: industries with trend inflation above the median, dashed yellow lines: below the median, dotted black lines: entire sample. The shaded areas correspond to 68% confidence intervals, based on Newey-West standard errors, computed by the delta method.

At the 24-months horizon, production falls after large positive shocks in the high inflation sectors, but its response remains positive in the low inflation group. In contrast, negative shocks always lead to output contractions in both groups, even though the polynomial permits reversals for both positive and negative shocks simultaneously. This shape of the impulse response curve persists as I increase polynomial order, allowing for more flexibility, and is a robust feature of the data.

Altogether, the results show that trend inflation is more strongly related to asymmetry in responses to large shocks, than to small ones. Furthermore, I find evidence for the reverse effects of large positive shocks on production, as predicted by the model. Most importantly, these reversals are affected by trend inflation, i.e., the size of a positive shock leading to zero production response is substantially smaller in industries with high trend inflation than in those with low trend inflation. Even though these results can not be interpreted in a causal sense, they show that many of the model predictions are in line the data.

3.1.3 Robustness

I show that findings discussed above are robust to several important deviations from the baseline strategy considered so far. I briefly outline the alternatives and provide the results in Appendix C.2.

Alternative shock measures. In the baseline specification I use a measure of monetary policy shocks, computed by Jarociński and Karadi (2020) using high frequency identification and separating monetary policy shocks from central bank information shocks using sign restrictions. I show that the results are generally robust to alternative shocks measures, commonly used in the literature. Firstly, I consider Jarociński and Karadi (2020) shock series based on a simpler separating procedure, the so called 'poor man's sign restrictions', as well as the original shock series, computed by Gertler and Karadi (2015). Secondly, I employ other widely used high-frequency identified shocks, estimated by Barakchian and Crowe (2013) and Nakamura and Steinsson (2018). Figure 12 shows that the main results of the paper are in general robust to these alternative shock measures.

Measurement error in trend inflation. I estimate trend inflation at the sector level by an average PPI growth rate, which can be contaminated by a measurement error. However, I only use trend inflation to classify sectors into below and above median groups. Thus, the only way measurement error might affect the results is by distorting the ordering of sectors by trend inflation and leading to misclassification. To address this issue, I omit the middle 40% of sectors and compare the top 30% with the bottom 30%. Because sectors with trend inflation that is close to the median are much more likely to be misclassified, excluding them alleviates the problems associated with measurement error. Figures 13 and 14 show that results are robust to such a split.

Great Recession and ZLB. The baseline sample spans the period between February 1990 and January 2013, which includes the apex of the Great Recession and the subsequent period of low interest rates. As a robustness check, I consider a shorter sample period ending in June 2008. Figures 15 and 16 show that excluding the Great Recession and the ZLB period only strengthens the main results of paper.

Trimming the data. In the baseline scenario I omit the top and bottom 2.5% of sectors in terms of trend inflation from the original sample to control for potential outliers. This choice does not affect the results of the paper, with Figures 17 - 20 showing that the main findings are robust to trimming the top and bottom 15%, as well as to using the entire sample.

Polynomial degree. When testing for non-linearity of industrial production responses, I use a third order polynomial because it is the minimal degree that allows for non-monotonicity of impulse responses with respect to both positive and negative shocks. As a robustness check, I provide the results for the 4th, 5th and 6th order polynomials in Figure 21, which shows that the effect of trend inflation on responses to large shocks does not depend on the degree of a shock polynomial.

Other. Finally, I set the number of lags to 3 and 12 (baseline specification has 6 lags) and reduce the set of controls, only keeping a time trend and lags of the dependent variable, monetary shock, and effective federal funds rate. In addition, I provide the unsmoothed impulse responses. Figure 22 shows that the main findings remain unchanged.

3.2 Drift and Micro Level Asymmetry

In this section I show that trend inflation induces asymmetry in individual price adjustments, as follows from Proposition 1.[18] Working with price adjustments is a challenge because only continuous tracking of the price of an item can ensure that one observes adjustments as opposed to growth rates, which can consist of multiple adjustments. In addition, growth rates are functions of both adjustment size and frequency, so that any observed asymmetry in growth rates can be driven by the asymmetry in adjustment frequencies.

To address these issues, I use scraped daily data from the Billion Prices Project by Cavallo (2018). Under the assumption that prices do not change more than once a day, daily data provides the desired price adjustments. This assumption is much milder compared to those required for monthly or even weekly data. I focus on U.S. supermarket data (store 1), as it provides the longest time series, and consider items with at least 2 years of observations and at least 10 price adjustments. In addition, I exclude items that have adjustments larger than 50% to control for the outliers. The sample period is between May 2008 and July 2010, and the total number of items used in the analysis is 1924, with 28808 observed price adjustments.

I compute asymmetry for each item i as the ratio between average sizes of positive and negative adjustments. Drift μ_i is recovered as the average price growth rate over the entire period. Baley and Blanco (2020) show that it can also be computed as the ratio between average adjustment and average time between adjustments, so I use their approach as a robustness check. The two approaches converge as the sample size increases, but can produce different estimates in finite samples. The baseline regression has the following form:

$$\log \frac{\Delta^+ p_i}{\Delta^- p_i} = \alpha_c + \beta\mu_i + \gamma' \mathbf{x}_i + \varepsilon_i$$

where $\Delta^+ p_i$ is the average positive price adjustment of item i, $\Delta^- p_i$ – average negative price adjustment, μ_i is the drift, $\mathbf{x}_i$ – a vector of controls and α_c is a category fixed effect. Items in the data are grouped into narrowly-defined categories, corresponding to the URLs where the items are found on the website. These categories are narrower than the COICOP groups and there are seven items in each category on average.[19] Including category fixed effects controls for many unobservables such as category-specific demand, adjustment costs,

[18] Alvarez et al. (2019) work with Argentinian micro-level price data and study the effect of inflation on price behavior. The main distinction of my work is that I focus on the cross-sectional variation in item-level trend inflation, whereas they use time variation in aggregate levels of inflation. Alvarez et al. (2019) find that asymmetry in adjustments is insensitive to inflation at low inflation rates, but is positively related at high levels of inflation. I work with U.S. data and find evidence for the positive relationship even at low levels of trend inflation. A potential reason for the differences in our findings is that I consider trend inflation, i.e., the long-term growth rate of the price level, whereas Alvarez et al. (2019) focus on period-specific actual inflation, i.e., log-difference in price levels between two consecutive periods.

[19] I exclude categories with less than 3 items to allow for enough within-category variation.

or other characteristics that may simultaneously affect both the drift and the asymmetry. The set of controls includes the frequency and standard deviation of adjustments, as well as the variance of idiosyncratic shocks σ_i^2, computed following Baley and Blanco (2020). All statistics are normalized to monthly frequency. The first three columns of Table 1 show the results from an OLS regression with standard errors clustered at the category level. Columns (1) and (2) employ the baseline estimates of μ_i as an average price growth rate, and an alternative measure for μ_i (as in Baley and Blanco (2020)) is used in column (3).

Table 1: Micro-level Asymmetry

	Dependent variable:				
	Asymmetry $\left(\log \frac{\Delta^+ p_i}{\Delta^- p_i}\right)$				
	OLS			IV	
	(1)	(2)	(3)	(4)	(5)
Drift μ	4.969***	4.966***		11.407**	
	(1.830)	(1.873)		(5.448)	
Drift μ (alt.)			4.552***		36.707
			(1.745)		(25.146)
σ^2		−0.899	−0.890		
		(3.173)	(3.175)		
Frequency		0.114	0.113		
		(0.151)	(0.151)		
Std. Dev.		0.028	0.024		
		(0.730)	(0.730)		
Observations	1,924	1,924	1,924	1,376	1,376
R^2	0.458	0.460	0.460	0.483	0.413

Note: *p<0.1; **p<0.05; ***p<0.01. All specifications include category FE. Standard errors are clustered at category level.

Table 1 suggests that higher trend inflation is positively related to asymmetry in individual adjustments, independent of the way drift μ is computed. The inclusion of controls does not alter this result. The coefficient in the first column is interpreted in the following way: a one percentage point increase in monthly trend inflation is associated with a 5% increase in the size of positive adjustments relative to the size of negative adjustments. Note that a 1 p.p. increase in trend inflation is not a very large change at the item level: standard deviation of the drift μ_i is 0.8 p.p. in the cross-sectional distribution, so that the drift effect is sizable.

As noted previously, the positive relationship between the average growth rate and the asymmetry in adjustments may not be too surprising, but it is not immediate either. A higher trend may be purely driven by more frequent positive adjustments and less frequent negative adjustments, however, this option is not supported by the data.

One potential drawback of the baseline OLS specification is the fact that drifts and asymmetries are computed using the same item-level time series. This may lead to spurious results in a short sample because a large positive adjustment simultaneously increases the estimates of drift and asymmetry. To resolve this issue, I split the sample into two equal parts for each item. I use the drift in the first subsample as an instrument for the drift in the second subsample. I then compute asymmetry in the second subsample and regress it onto the instrumented drift. Thus, the drifts and asymmetries are effectively estimated on different samples, which helps addressing this issue. The results are presented in columns (4) and (5) of Table 1. The coefficient in front of the drift increases, and so do the standard errors.[20] The baseline estimate of the drift remains significant and the alternative specification becomes marginally insignificant with p-value = 0.14. Overall, I conclude that the results are robust and provide supporting evidence for the model predictions regarding the link between trend inflation and asymmetry at the level of individual price adjustment.

4 Monetary Policy in General Equilibrium

The analytic results of this paper provide new insights into the efficacy of monetary policy and its dependence on trend inflation. However, these results are obtained in a rather restrictive environment. Firstly, I assume that firms follow the steady state optimal policy along the transition path. Secondly, I use a second-order approximation of the profit function, which ensures symmetry and substantially contributes to analytic tractability. Finally, I consider monetary interventions in isolation, whereas this policy instrument is often used as a counteractive measure to mitigate the effects of other disturbances. Therefore, monetary policy is often implemented outside of an economy's steady state, in contrast to the assumption imposed in the analytic framework.

I now address all these issues and embed the analytic framework into a standard general equilibrium model, calibrated to the U.S. data. I consider a transitory adverse markup shock, which leads to an increase in prices and a drop in consumption. Firms now correctly anticipate the economy dynamics and follow the appropriate optimal policy. I then compare the ability of a monetary authority to stabilize the economy under the baseline 2% inflation target and a counterfactual 4% inflation target. A markup shock is well-suited for this exercise, as it only increases a wedge in the economy stemming from price dispersion, without affecting the efficient allocation. This provides a rationale for the imposed stabilization objective of the monetary authority. Because the markup shock depresses consumption and increases prices, it introduces a trade-off for the monetary authority, as it can not stabilize consumption and prices simultaneously.

I find that increasing the inflation target imposes two negative effects on the ability of

[20] The standard errors increase due to the instrumenting procedure and a smaller sample as I additionally restrict attention to items with at least 5 adjustments in the second subsample.

a policymaker to stabilize the economy after such a shock. Firstly, higher trend inflation amplifies the initial effect of the markup shock, leading to larger price and consumption deviations. Secondly, it worsens the trade-off between price and consumption stabilization. Both effects are sizable and arise due to the impact of trend inflation on the asymmetry of price and output responses. The results relate to the ongoing discussion on increasing the inflation target, highlighting adverse implications for stabilization policy away from the zero lower bound, in particular for the type of shocks that exhibit the 'cost-push' property of moving prices and output in opposite directions.

4.1 General Equilibrium Setup

4.1.1 Households

I embed the analytic model from Section 1 into a general equilibrium setting, similar to those in Nakamura and Steinsson (2010) and Karadi and Reiff (2019). Representative households maximize the present discounted value of their utility, given by

$$\int_0^\infty e^{-\rho t} \left(\log C_t - \alpha L_t\right) dt$$

where C_t denotes consumption of a composite good, L_t is the household's labor supply, ρ is the discount rate, and α is the disutility of labor. The household's budget constraint is as follows:

$$P_t C_t + \dot{B}_t = R_t B_t + W_t L_t + \Pi_t$$

where P_t is the aggregate price level, B_t are the holdings of a bond with nominal gross return R_t, W_t is the wage and Π_t are the firms' profits. Consumption C_t is composed of a continuum of differentiated goods and is given by

$$C_t = \left[\int \left(A_t(i) C_t(i)\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where $C_t(i)$ is consumption of a good produced by firm i, $A_t(i)$ is its quality, and θ is the elasticity of substitution. The aggregate price level is $P_t = \left[\int \left(P_t(i)/A_t(i)\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ and cost minimization yields the following demand for good i:

$$C_t(i) = A_t(i)^{\theta-1} \left[\frac{P_t(i)}{P_t}\right]^{-\theta} C_t$$

First-order conditions imply that wage W_t is proportional to nominal aggregate consumption $P_t C_t$, and nominal interest rate is determined by the growth rate of nominal consumption:

$$W_t = \alpha P_t C_t$$
$$R_t = \rho + \frac{\dot{(P_t C_t)}}{P_t C_t}$$

4.1.2 Firms

There is a continuum of firms producing differentiated goods, indexed by $i \in [0, 1]$. Firms demand labor $L_t(i)$ and set prices $P_t(i)$. Production technology is given by $Y_t(i) = L_t(i)/A_t(i)$, so that higher quality goods are more costly to produce. Firms' profits are given by $\Pi_t(i) = P_t(i)Y_t(i) - W_t L_t(i)$. To adjust its price at time t, a firm must hire additional labor and the total cost of adjustment is given by $\kappa P_t(i)Y_t(i)$. In addition, firms receive an opportunity to adjust for free at rate λ. Such a setup is typically referred to in the literature as a 'CalvoPlus' model because it nests both the standard menu cost model and the Calvo (1983) setting. Each firm maximizes the expected discounted stream of profits:

$$\mathbb{E}\left[\int_0^\infty Q_t \Pi_t(i) dt - \kappa \sum_{i=1}^{\infty} Q_{\tau_i} P_{\tau_i}(i) Y_{\tau_i}(i)\right]$$

where $Q_t = \frac{\alpha e^{-\rho t}}{W_t}$ is the discount factor implied by the household's problem and τ_i are the adjustment times when a firm pays adjustment costs. The goods quality $A_t(i)$ evolves as a geometric Brownian motion with no drift: $d \log A_t(i) = \sigma dW_t(i)$. Using the household's first-order conditions and the fact that firms face consumers' demand function ($Y_t(i) = C_t(i)$), one can rewrite the firm's profit and cost functions as:

$$\Pi_t(i) = \alpha^{-\theta} W_t \left(\frac{\theta C_t}{\theta - 1}\right)^{1-\theta} \overbrace{e^{-\theta z_t(i)}\left(e^{z_t(i)} - \frac{\theta - 1}{\theta}\right)}^{\pi(z_t(i))}$$
$$\kappa P_t(i) Y_t(i) = \kappa \alpha^{-\theta} W_t \left(\frac{\theta C_t}{\theta - 1}\right)^{1-\theta} \underbrace{e^{(1-\theta) z_t(i)}}_{c(z_t(i))}$$

where $z_t(i)$ is the price gap, given by $z_t(i) = \log P_t(i) - \log P_t^*(i)$, and $P_t^*(i)$ is the frictionless optimal price, given by $P_t^*(i) = \frac{\theta}{\theta - 1} W_t A_t(i)$. Note that W_t cancels out in the firm's objective function, so that in a stationary equilibrium the firm's problem does not depend on any aggregate state, as constant aggregate consumption may be taken out of the problem.

4.1.3 Monetary Authority and Stationary Equilibrium

Following Nakamura and Steinsson (2010) and Midrigan (2011), I assume that the monetary authority is in full control of the nominal output $M_t = P_t C_t$, which in the steady state grows at a constant rate μ: $d\log M_t = \mu dt$. This assumption is common in the literature and can be rationalized by a binding cash-in-advance constraint. Given that μ is set exogenously in this model, I will refer to it both as 'trend inflation' and 'inflation target'.

Household's first-order conditions imply that the equilibrium nominal interest rate is constant and equal to $\bar{R} = \rho + \mu$, and the wage follows the law of motion of nominal output: $d\log W_t = \mu dt$. This creates a drift in the firm's optimal price $P_t^*(i)$, and thus in the price gaps $z_t(i)$. In the absence of action, price gaps evolve as $dz_t(i) = -\mu dt + \sigma dW_t(i)$. The firm's problem becomes almost identical to the one considered in the analytic section, with a few exceptions: (1) the profit function is no longer symmetric, (2) adjustment costs depend on the price gap at the time of adjustment, and (3) firms receive costless adjustment opportunities at rate λ. The solution to the firm's problem is characterized by a triplet $\{\underline{z}, \hat{z}, \overline{z}\}$ where $\underline{z}$ and $\overline{z}$ are the lower and upper boundaries of inaction region, and $\hat{z}$ is the return point. The value function satisfies the following Hamilton–Jacobi–Bellman equation in the inaction region:

$$(\rho + \lambda)v(z) = \pi(z) + \lambda v(\hat{z}) - \mu v'(z) + \frac{1}{2}\sigma^2 v''(z)$$

where $\pi(z) = e^{-\theta z}\left(e^z - \frac{\theta-1}{\theta}\right)$ and $\hat{z}$ is the optimal return point. The boundary conditions are $v(\underline{z}) = v(\hat{z}) - c(\underline{z})$ and $v(\overline{z}) = v(\hat{z}) - c(\overline{z})$, where $c(z) = e^{(1-\theta)z}$. Optimality and smooth pasting require $v'(\hat{z}) = 0$, $v'(\underline{z}) = (\theta - 1)c(\underline{z})$ and $v'(\overline{z}) = (\theta - 1)c(\overline{z})$. The density of the stationary price gap distribution $f(z)$ is determined by a Kolmogorov forward equation:

$$\lambda f(z) = \mu f'(z) + \frac{1}{2}\sigma^2 f''(z)$$

Aggregate consumption, price level and employment can be computed using the stationary price gap distribution as follows:

$$\begin{aligned}
C_t = \bar{C} &= \frac{\theta-1}{\alpha\theta}\left[\int_{\underline{z}}^{\overline{z}} e^{(1-\theta)z} f(z)dz\right]^{\frac{1}{\theta-1}} \\
P_t &= \frac{\alpha\theta}{\theta-1} M_t \left[\int_{\underline{z}}^{\overline{z}} e^{(1-\theta)z} f(z)dz\right]^{\frac{1}{1-\theta}} \\
L_t = \bar{L} &= \bar{C}^{1-\theta}\left(\frac{\alpha\theta}{\theta-1}\right)^{-\theta} \int_{\underline{z}}^{\overline{z}} e^{-\theta z} f(z)dz \ + \ \Gamma \\
\Gamma &= \kappa\alpha^{-\theta}\left(\frac{\theta\bar{C}}{\theta-1}\right)^{1-\theta} \left[\gamma^+ e^{(1-\theta)\underline{z}} + \gamma^- e^{(1-\theta)\overline{z}}\right]
\end{aligned}$$

where Γ is the total labor hired for price adjustment per unit of time and γ^+ and γ^- are the masses of firms adjusting at a cost upward or downward, respectively, per unit of time. Finally, bond holdings B_t are in zero net supply, so that in equilibrium $B_t = 0$.

4.2 Calibration

I set the discount rate ρ to 0.04 in annual terms and trend inflation μ to 0.02, roughly matching the average annual inflation in the U.S. over the last two decades.[21] The elasticity of substitution θ is set to 5, which is an intermediate value among those considered in the literature.[22] The remaining parameters, namely the disutility of labor α, the variance of idiosyncratic shocks σ^2, the adjustment cost κ, and the rate at which firms receive free adjustment opportunities λ, are calibrated internally. I target equilibrium employment of 1/3 and three moments of the distribution of price adjustments: frequency, average size, and kurtosis. All three moments are informative of aggregate responses to shocks and are a typical choice for calibration targets. Alvarez, Le Bihan, and Lippi (2016) show analytically that in a wide class of menu cost models the ratio of kurtosis to frequency is a sufficient statistic for the cumulative effect of a marginal monetary shock on output. In the first section of this paper I show that the effect of trend inflation on aggregate price and output responses depends on the average size of adjustment.

I target values of frequency, average size and kurtosis, reported in the literature. I set the frequency of price changes to 10% per month, the average size of adjustment to 10%, and the kurtosis of the distribution of price adjustments to 4. The first two values are standard, as many studies report very similar estimates using different data sets.[23] The estimates of kurtosis are much more dispersed: Alvarez, Lippi, and Oskolkov (2020) report values close to 2, Midrigan (2011): 3.15, Alvarez, Le Bihan, and Lippi (2016): 4, Vavra (2014): 6.4. I use an intermediate value of 4, obtained by Alvarez, Le Bihan, and Lippi (2016) from the weekly scanner data of the Dominick's dataset, accounting for heterogeneity and measurement errors. The calibrated values in annual terms for σ, κ, and λ are 0.148, 0.11, and 1.126 respectively, and the model matches the targeted statistics exactly. In Appendix D.3 I use an alternative calibration, targeting the kurtosis of the price adjustment distribution of 3. This affects the overall non-neutrality of monetary policy, but the main findings remain qualitatively unchanged.

4.3 MIT Markup Shock

I now consider an unexpected shock that increases steady state optimal markup $\left(\frac{\theta}{\theta-1}\right)$ by 3% and then gradually reverts to zero in AR(1) fashion. Formally, the dynamics of the

[21] When calibrating a continuous time model, the period length is innocuous, as it only scales certain parameters up or down.

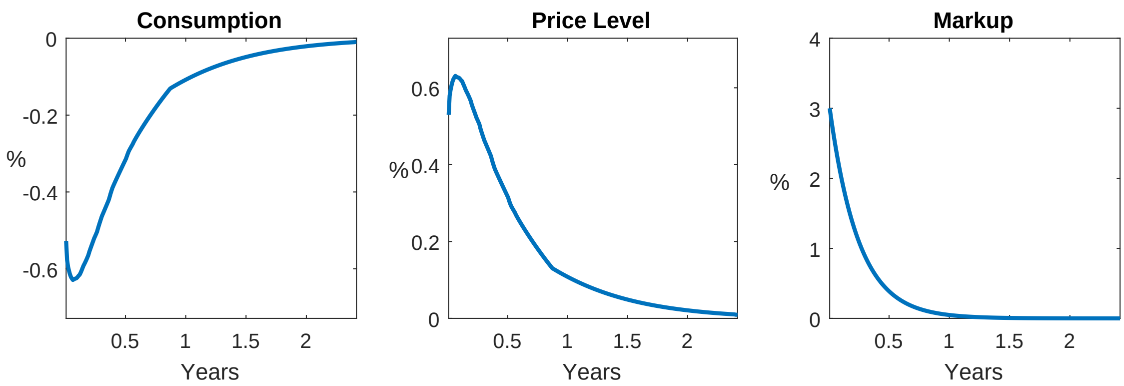
[22] Midrigan (2011) sets θ to 3, Nakamura and Steinsson (2010): $\theta = 4$, Karadi and Reiff (2019): $\theta = 5$, Golosov and Lucas (2007) use $\theta = 7$.

[23] Frequency: Nakamura and Steinsson (2008): 10.8%, Nakamura and Steinsson (2010): 8.7%, Vavra (2014): 10.9%. Average size of adjustment: Nakamura and Steinsson (2008): 8.5%, Kehoe and Midrigan (2015): 11%, Vavra (2014): 7.7%. For the average size of adjustment, the mean and median estimates are usually similar, whereas the mean frequency is typically higher than the median. I use the median frequency estimates, as this is the preferred choice for single-sector models (see Nakamura and Steinsson (2010)).

shock ε_t are governed by an Ornstein-Uhlenbeck process, so that $\varepsilon_t = 0.03 \cdot e^{-\eta t}$, where η determines the speed of convergence and is set to generate a half-life of two months. The shock sets the economy on a deterministic transition path, increasing the aggregate price and depressing consumption. I defer the description of the non-stationary equilibrium conditions to Appendix D.1 and plot the dynamics of consumption and prices on Figure 8.

The price level response is plotted in terms of percent deviations from the trend, whereas consumption and markup responses are in terms of percent deviations from the steady state. The markup shock raises the firms' optimal prices, leading to an increase in the actual price level. Because the nominal output stays constant and prices increase, consumption falls. Integrating the area under the lines, one obtains cumulative impulse responses, which are given by $\int_0^\infty (p_t - \bar{p}_t)dt = 0.44\%$ for the price level and $\int_0^\infty (c_t - \bar{c})dt = -0.44\%$ for consumption, where $\bar{p}_t$ is the trend of the aggregate log-price and $\bar{c}$ is the steady state log-consumption.[24]

Figure 8: Markup Shock



Model-generated impulse responses of consumption, price level and markup to a 3% markup shock. Consumption and markup responses are in terms of percent deviations from the steady state, whereas price level responses are in terms of percent deviations from the trend.

The shock is purely inefficient in the sense that it increases the wedge between the actual and efficient level of output, without affecting the efficient allocation.[25] Thus, it would be desirable to 'undo' its consequences by means of policy. To capture this in a simple way, I assume that the policymaker dislikes negative deviations of consumption from its efficient level and values price stability (dislikes any deviations from the trend).[26]

[24] A 1% negative cumulative response of consumption is equivalent to a scenario when consumption is held at 1% below its steady state for one year.

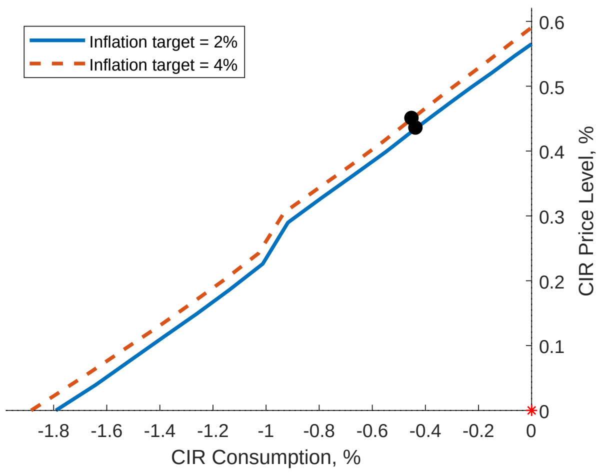
[25] Efficient output is achieved under zero price dispersion and is given by $C_t^* = L_t$. Due to price dispersion, $C_t = \left[\int (P_t(i)/(A_t(i)P_t))^{-\theta} di\right]^{-1} C_t^*$ (see Yun (1996)). An increase in optimal markup lowers θ and increases the inefficiency stemming from price dispersion.

[26] Such an objective is different from an optimal policy that considers welfare, which depends on the level of consumption, the degree of price dispersion and the volume of adjustment costs paid by the firms. A meaningful study of optimal policy with respect to trend inflation would require additional model components, e.g. heterogeneity in individual price trends as in Adam and Weber (2020).

I also assume that monetary interventions follow the same dynamics as the markup shock, and the only choice of the policymaker is the level of monetary intervention. Formally, monetary intervention δ_t is proportional to the markup shock: $\delta_t = \delta\varepsilon_t$, where $\delta \in \mathbb{R}$ and is chosen by the monetary authority. A stimulus ($\delta > 0$) mitigates the negative response of consumption, but raises prices even further (see Figure 23 in Appendix D.2). A contraction ($\delta < 0$) creates an opposite effect, stabilizing prices and amplifying the drop in consumption. The policymaker thus faces a trade-off, as it is impossible to stabilize consumption and prices simultaneously.

I do not assign any weights to these objectives, but rather consider the whole possibility frontier of the policymaker, given the initial markup shock and the restrictions on policy outlined above. By varying the sign and size of the monetary intervention δ, the policymaker achieves different combinations of cumulative consumption and price responses. The resulting frontier depends, among other parameters, on trend inflation μ. I now compare these frontiers for the baseline level of trend inflation of 2% per year and a counterfactual value of 4%. Figure 9 shows the results.

Figure 9: Frontiers, Small Shock



Feasible combinations of cumulative responses of consumption (x-axis) and price level (y-axis) after a 3% markup shock. The solid blue line corresponds to the baseline economy with a 2% trend inflation, and the dashed red line represents a counterfactual economy with a 4% trend inflation. Consumption responses are in terms of percent deviations from the steady state, whereas price level responses are in terms of percent deviations from the trend. Black dots show the outcomes if the monetary authority does not intervene.

On the x-axis I plot cumulative consumption responses, on the y-axis – cumulative responses of the price level.[27] The curves show feasible outcomes for the baseline economy with trend inflation of 2% (solid blue line) and a counterfactual economy with a 4% trend inflation (dashed red line), given the initial 3% markup shock. The red asterisk corresponds

[27] I consider cumulative deviations of the price level rather than inflation, as in this case it is impossible to completely neutralize the effect of the shock on inflation due to the imposed restriction on monetary policy. However, the findings of the paper remain unchanged if I substitute the price level CIR with the CIR of inflation, as shown in Appendix D.4.

to a (0, 0) scenario, where the effect of the markup shock is completely neutralized. Black dots on the curves correspond to scenarios when the monetary authority does not intervene ($\delta = 0$). Stimulative policy ($\delta > 0$) moves an economy along its frontier to the right, contractionary measures ($\delta < 0$) move it to the left.

Firstly, note that in the economy with a 4% trend inflation the black dot is further away from the (0, 0) point, which means that the negative effects of the markup shock are on their own stronger if trend inflation is higher. Under a 4% inflation target, the markup shock leads to a 3.4% stronger increase in prices and a 3.4% stronger drop in consumption, compared to the baseline economy with an inflation target of 2%. When trend inflation is higher, prices exhibit less upward rigidity and have a stronger response to the markup shock, which also results in a larger consumption drop if the monetary authority keeps the nominal output constant. Importantly, this result is not driven by changes in the overall frequency of price adjustments because it remains virtually constant as I vary the level of trend inflation. Instead, the result is due to changes in the relative frequencies and sizes of positive and negative price adjustments.

Secondly, higher trend inflation worsens the trade-off between consumption and price stabilization. This latter effect is less apparent on the graph, but can be seen when calculating the curvature of the frontiers. I measure the curvature as a ratio between the slopes of stimulative and contractionary interventions. The slope of stimulative policy α_S reflects the rate at which the policymaker gains consumption and loses price stability, when conducting stimulative policy ($\delta > 0$). Graphically, it is the slope of a straight line, passing through an economy's initial point (black dot) and the intersection of the frontier with the y-axis. The slope of contractionary policy α_C reflects the rate at which the policymaker gains price stability and loses consumption, when conducting contractionary policy ($\delta < 0$). Graphically, it is the slope of a straight line, passing through an economy's initial point (black dot) and the intersection of the frontier with the x-axis. The curvature is then measured as a ratio between the slopes: α_S/α_C.[28] A higher curvature indicates that the stimulative slope becomes steeper, whereas the contractionary slope flattens out. Therefore, the monetary authority must sacrifice more consumption when stabilizing prices and tolerate larger price deviations when restoring consumption. Thus, the higher the curvature, the worse the stabilization trade-off is.

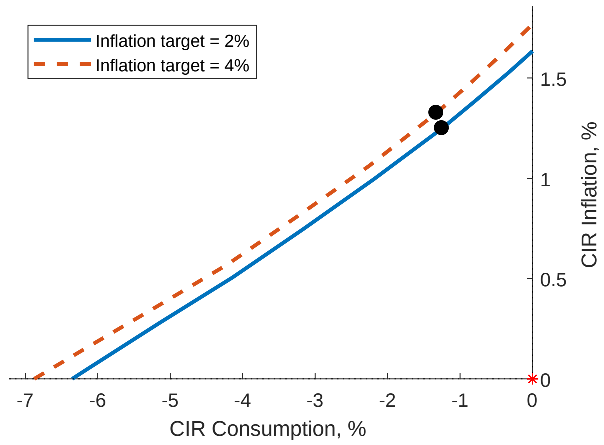
For the baseline economy with a 2% inflation target the curvature is equal to 0.93, whereas under a 4% inflation target it increases by by 7.5% to 1.0. Under higher trend inflation, the policymaker must sacrifice more consumption when stabilizing prices, and must tolerate larger price responses when stimulating consumption. This is again caused by the effect of trend inflation on the asymmetry of price and consumption responses. As

[28]This measure is not ideal, as it assigns a unique value to the entire frontier, whereas the degree of curvature may vary along the frontier. However, it summarizes the overall trade-off, considering two extreme points of achieving zero consumption or zero price CIRs.

the inflation target rises, prices become more sensitive to stimulative shocks and it becomes harder for the monetary authority to stimulate consumption. Simultaneously, prices become less sensitive to contractionary shocks, which impedes the ability of policymakers to stabilize prices. Higher trend inflation increases price flexibility exactly when it is desirable to have rigid prices, and makes them stickier exactly when flexibility is needed.

Both of the effects of a higher inflation target are amplified if the initial markup shock is larger. Figure 10 plots the same frontiers for a 10% markup shock. The economy with a 4% trend inflation now has a 6.1% stronger response to the initial markup shock and an 11% higher curvature, compared to the economy with a 2% trend inflation.

Figure 10: Frontiers, Large Shock



Feasible combinations of cumulative responses of consumption (x-axis) and price level (y-axis) after a 10% markup shock. The solid blue line corresponds to the baseline economy with a 2% trend inflation, and the dashed red line represents a counterfactual economy with a 4% trend inflation. Consumption responses are in terms of percent deviations from the steady state, whereas price level responses are in terms of percent deviations from the trend. Black dots show the outcomes if the monetary authority does not intervene.

Overall, the results show that trend inflation affects the ability of a policymaker to stabilize the economy after an adverse markup shock. Higher trend inflation decreases upward price stickiness and leads to stronger price and consumption responses to the initial markup shock. In addition, higher trend inflation amplifies the asymmetry of price and consumption responses to positive and negative monetary shocks, which worsens the policymaker's trade-off when stabilizing the economy. I finally note that these results are of greater importance for large shocks, as the effects of trend inflation become more pronounced.

In both scenarios I considered a shock that temporarily increases monopolistic power of firms. A shock that decreases firms' monopolistic power and drives optimal markups down would have two distinct effects. Firstly, because the steady state markup is positive and price dispersion is non-zero, a fall in markups would decrease the inefficiency in the economy and bring consumption closer to its efficient level. Such a shock would increase consumption and decrease prices, so that any subsequent expansionary monetary policy would lead to price stabilization and further consumption growth, thus inducing no trade-off. Secondly, because higher trend inflation increases downward price rigidity, the initial response to the shock would be larger in the baseline economy with a 2% inflation target than in the counterfactual with a 4% inflation target. In addition, it will be easier for the monetary authority to stabilize prices and consumption under higher trend inflation, again due to lower upward price rigidity and higher downward rigidity. Therefore, all results are 'mirrored' if the markup shock is of the opposite sign, and higher trend inflation would be beneficial from a pure stabilization perspective.[29] It follows that the overall potency of monetary stabilization policy would depend on which types of shocks prevail in the economy.

5 Summary

In this paper I show that trend inflation matters for economy's responses to aggregate shocks and monetary policy interventions. I derive a set of new analytic results for the effect of trend inflation on aggregate dynamics in a standard menu cost model. The main contribution is that I consider monetary shocks of any size in an environment with non-zero drift. This approach reveals several new properties of aggregate dynamics, especially for large shocks.

The key characteristic of trend inflation is that it affects aggregate responses to positive and negative shocks asymmetrically. In the presence of adjustment costs, prices are more sensitive to shocks that push them in the same direction as the trend, and are less sensitive to shocks the push them in the opposite direction. Under positive trend inflation, larger price flexibility in responses to positive monetary shocks leads to weaker output increases, whereas smaller price flexibility in responses to negative shocks leads to stronger output declines. These effects are especially pronounced for large shocks that force all firms to update prices. While positive large shocks are neutral in the driftless case, they cause output contractions in economies with positive trend inflation.

The empirical analysis shows that the new analytic predictions of the model are not rejected by the data. I find that sectors with a higher PPI growth rate exhibit stronger price responses to positive monetary shocks and weaker responses to negative shocks, compared to sectors with a lower growth rate of PPI. I also find that aggregate output expansions after positive monetary shocks are almost entirely driven by sectors with a low PPI growth rate, whereas output contractions are distributed more equally. In addition, production responses are generally non-linear and large positive shocks may lead to a decline in output. This holds for sectors with both low and high levels of trend inflation, however the size of a positive shock that causes an output contraction is smaller for sectors

[29] See also the discussion in Blanco (2020) on the effects of higher trend inflation on the likelihood of hitting the zero lower bound.

with a larger level of trend inflation.

My results have important implications for monetary stabilization policy and contribute to the ongoing discussion on the necessity to raise the inflation target. Using a general equilibrium model calibrated to the U.S. data, I find that higher trend inflation has a sizable effect on the ability of a policymaker to stabilize the economy after an adverse markup shock. Raising the inflation target from 2% to 4% amplifies the initial response to the markup shock and worsens the stabilization trade off. A policymaker has to sacrifice more consumption when stabilizing prices and has to tolerate larger price deviations when stimulating consumption. Thus, a higher inflation target impedes the ability of a monetary authority to counteract adverse shocks that move output and prices in opposite directions.

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A Miscellaneous results

A.1 Expressing $M(\delta, \mu)$ in terms of price gaps

Alvarez and Lippi (2014) show that impulse response of aggregate price level can be approximated as:

$$P(t) - \bar{P}(t) \approx \delta + \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} z dF_t(z,\mu) - \bar{x}(\mu) \quad \text{with} \quad \bar{x}(\mu) = \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} z dF(z,\mu)$$

where $P(t)$ is the aggregate log-price t periods after shock δ, $\bar{P}(t)$ is the hypothetical price in absence of shock, $F_t(z,\mu)$ is the period t distribution of price gaps, $F(z,\mu)$ is the stationary distribution of price gaps and $\bar{x}(\mu)$ is the average price gap in steady state. Note that instead of evolution of gap distribution ($F_t(z,\mu)$), one can consider conditional evolution of gaps given initial after-shock distribution $F_\delta(z,\mu)$:

$$P(t) - \bar{P}(t) \approx \delta + \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} \mathbb{E}\left(z(t) - \bar{x}(\mu) \;\middle|\; z(0) = z \right) dF_\delta(z,\mu)$$

where $\bar{x}(\mu)$ is taken inside integral and expectation. Finally, switching the order of integration, one obtains:

$$M(\delta,\mu) = \int_0^\infty \left[\delta - \left(P(t) - \bar{P}(t)\right)\right] dt \approx - \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} \mathbb{E}\left(\int_0^\infty \left(z(t) - \bar{x}(\mu)\right) dt \;\middle|\; z(0) = z \right) dF_\delta(z,\mu)$$

A.2 Driftless Benchmark

Suppose $\mu = 0$. Firms' value function satisfies the following HJB:

$$\rho v(z) = -z^2 + \frac{\sigma^2}{2} v''(z)$$

The general solution to which is:

$$v(z) = A(e^{\alpha z} + e^{-\alpha z}) - \frac{1}{\rho} z^2 - \frac{\sigma^2}{\rho^2}$$

where $\alpha = \sqrt{2\rho/\sigma^2}$ and A is the unknown coefficient that depends on boundary conditions. These are given by $v(\underline{z}(0)) = v(\overline{z}(0)) = v(\hat{z}(0)) - \kappa$. Due to symmetry, $\underline{z}(0) = -\overline{z}(0)$ and $\hat{z}(0) = 0$. Denote $\overline{z}_0 = \overline{z}(0)$ to ease notation. Using the expression for $v(z)$ and combining one of the boundary conditions with smooth pasting condition $v'(\overline{z}_0) = 0$, one gets:

$$A = \frac{2\overline{z}_0}{\alpha\rho(e^{\alpha\overline{z}_0} - e^{-\alpha\overline{z}_0})}$$

$$\overline{z}_0^2 = A\rho(e^{\alpha\overline{z}_0} + e^{-\alpha\overline{z}_0} - 2) + \rho\kappa$$

which implicitly defines solution triplet $\{-\overline{z}_0, 0, \overline{z}_0\}$.

Stationary density is defined by Kolmogorov forward equation $(\sigma^2/2)f_{zz}(z,0) = 0$ with boundary conditions $f(\overline{z}_0,0) = f(-\overline{z}_0,0) = 0$, integration to one $\int_{-\overline{z}_0}^{\overline{z}_0} f(z,0)dz = 1$ and continuity at $z = 0$. It is thus given by:

$$f(z,0) = \frac{\overline{z}_0 - |z|}{\overline{z}_0^2}$$

for all $z \in [-\overline{z}_0, \overline{z}_0]$ and is zero otherwise. For completeness, cumulative distribution function $F(z,0)$ is then given by:

$$F(z,0) = \begin{cases} \frac{(\overline{z}_0+z)^2}{2\overline{z}_0^2}, & \text{for } z < 0 \\ 1 - \frac{(\overline{z}_0-z)^2}{2\overline{z}_0^2}, & \text{for } z \geq 0 \end{cases}$$

Consider a positive shock $\delta > 0$. Impact effect in driftless economy is:

$$\Theta(\delta,0) = -\int_{-\overline{z}_0-\delta}^{-\overline{z}_0} zf(z+\delta,0)dz$$

and due to a kink in $f(z,0)$ is computed separately for smaller ($\delta \leq \overline{z}_0$) and larger ($\delta \geq \overline{z}_0$) shocks. A direct computation of the integral provides:

$$\Theta(\delta,0) = \begin{cases} \frac{1}{6\overline{z}_0^2}\delta^2(\delta + 3\overline{z}_0), & \text{for } \delta < \overline{z}_0 \\ \frac{1}{6\overline{z}_0^2}\left[\delta(6\overline{z}_0^2 + 3\delta\overline{z}_0 - \delta^2) - 4\overline{z}_0^3\right], & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ \delta, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

The last line follows since for any $\delta \geq 2\overline{z}_0$:

$$\begin{aligned} \Theta(\delta,0) &= -\int_{-\overline{z}_0-\delta}^{-\overline{z}_0} zf(z+\delta,0)dz = -\int_{-\overline{z}_0}^{-\overline{z}_0+\delta} (z-\delta)f(z,0)dz \\ &= -\int_{-\overline{z}_0}^{\overline{z}_0} (z-\delta)f(z,0)dz = \delta - \bar{x}(0) = \delta \end{aligned}$$

where $\bar{x}(0) = \int_{-\overline{z}_0}^{\overline{z}_0} zf(z,0)dz$ is the average gap. First equality is due to variable substitution, second follows from the fact that $f(z,0) = 0$ for $z \geq \overline{z}_0$ and third one is immediate. Finally, $\bar{x}(0) = 0$ due to symmetry of $f(z,0)$.

Consider now cumulative impulse response ($\delta > 0$):

$$M(\delta,0) = -\int_{-\overline{z}_0}^{\overline{z}_0} \mathbb{E}\left(\int_0^{\tau} z(t)dt \mid z(0) = z\right) dF_\delta(z,0)$$

Define $m(z,0)$ to be the expected cumulated price gap until first adjustment, so that $m(z,0) = \mathbb{E}\left(\int_0^\tau z(t)dt \mid z(0) = z\right)$. The second argument of the function highlights that it is evaluated under $\mu = 0$. This function is characterized by $z + (\sigma^2/2)m_{zz}(z,0) = 0$ together with boundary conditions $m(\overline{z}_0, 0) = m(-\overline{z}_0, 0) = 0$, which implies that:

$$m(z,0) = \frac{\overline{z}_0^2 z - z^3}{3\sigma^2}$$

Given that shock shifts the entire distribution in parallel and some firms adjust immediately, distribution $F_\delta(z,0)$ is the shifted stationary distribution, so that $F_\delta(z,0) = F(z+\delta,0)$ for all $z \in [\underline{z}, \overline{z}-\delta]$ and $F_\delta(z,0) = 1$ for all $z \in (\overline{z}-\delta, \overline{z}]$. In addition, there is a mass point at $z = 0$ due to firms that adjust immediately, equal to $F(-\overline{z}_0+\delta, 0)$. $M(\delta,0)$ is then given by:

$$M(\delta,0) = -\int_{-\overline{z}_0}^{\overline{z}_0-\delta} m(z,0)f(z+\delta,0)dz + m(0,0)F(-\overline{z}_0+\delta,0)$$

where the second term can be ignored since $m(0,0) = 0$. Again, due to a kink in $f(z,0)$, the integral has to be considered separately for smaller ($\delta \leq \overline{z}_0$) and larger ($\delta \geq \overline{z}_0$) shocks. A direct computation yields:

$$M(\delta,0) = \begin{cases} \frac{1}{180\sigma^2\overline{z}_0^2}[3\delta^5 + 15\delta^4\overline{z}_0 - 40\delta^3\overline{z}_0^2 + 30\delta\overline{z}_0^4], & \text{for } \delta < \overline{z}_0 \\ \frac{1}{180\sigma^2\overline{z}_0^2}[-3\delta^5 + 15\delta^4\overline{z}_0 - 20\delta^3\overline{z}_0^2 + 16\overline{z}_0^5], & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ 0, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

The last line is trivial since if $\delta \geq 2\overline{z}_0$, then the integral in $M(\delta,0)$ is taken over the interval $[\overline{z}_0-\delta, -\overline{z}_0]$, where $F_\delta(z,0)$ has no mass.

A.3 Optimal policy under non-zero drift

Recall that firm's value function solves the following HJB equation for any $z \in [\underline{z}(\mu), \overline{z}(\mu)]$:

$$\rho v(z) = -z^2 - \mu v'(z) + \frac{\sigma^2}{2}v''(z)$$

General solution to $v(z)$ is thus given by:

$$v(z) = C_1 e^{R_1 z} + C_2 e^{R_2 z} - \frac{1}{\rho}z^2 + \frac{2\mu}{\rho^2}z - \left(\frac{\sigma^2}{\rho^2} + \frac{2\mu^2}{\rho^3}\right)$$

where $R_1 = \frac{\mu-\sqrt{\mu^2+2\sigma^2\rho}}{\sigma^2}$, $R_2 = \frac{\mu+\sqrt{\mu^2+2\sigma^2\rho}}{\sigma^2}$. Coefficients C_1 and C_2 are unknown and determined by boundary conditions $v(\underline{z}) = v(\overline{z}) = v(\hat{z}) - \kappa$, where I drop the argument μ in policy variables in order to ease notation. In addition $v(z)$ satisfies smooth pasting conditions $v'(\underline{z}) = v'(\overline{z}) = 0$ and optimality condition $v'(\hat{z}) = 0$. Altogether this results in a system of equations:

$$C_1R_1e^{R_1\underline{z}} + C_2R_2e^{R_2\underline{z}} - \frac{2}{\rho}\underline{z} + \frac{2\mu}{\rho^2} = 0 \tag{h_1}$$

$$C_1R_1e^{R_1\hat{z}} + C_2R_2e^{R_2\hat{z}} - \frac{2}{\rho}\hat{z} + \frac{2\mu}{\rho^2} = 0 \tag{h_2}$$

$$C_1R_1e^{R_1\overline{z}} + C_2R_2e^{R_2\overline{z}} - \frac{2}{\rho}\overline{z} + \frac{2\mu}{\rho^2} = 0 \tag{h_3}$$

$$C_1(e^{R_1\underline{z}} - e^{R_1\hat{z}}) + C_2(e^{R_2\underline{z}} - e^{R_2\hat{z}}) - \frac{1}{\rho}(\underline{z}^2 - \hat{z}^2) + \frac{2\mu}{\rho^2}(\underline{z} - \hat{z}) + \kappa = 0 \tag{h_4}$$

$$C_1(e^{R_1\overline{z}} - e^{R_1\hat{z}}) + C_2(e^{R_2\overline{z}} - e^{R_2\hat{z}}) - \frac{1}{\rho}(\overline{z}^2 - \hat{z}^2) + \frac{2\mu}{\rho^2}(\overline{z} - \hat{z}) + \kappa = 0 \tag{h_5}$$

Let ψ denote the vector of unknowns: $\psi = [\underline{z}, \hat{z}, \overline{z}, C_1, C_2]$. Then the above system of equations can be summarized as:

$$H(\mu, \psi) = \mathbf{0} \tag{A.1}$$

where $H : \mathbb{R} \times \mathbb{R}^5 \to \mathbb{R}^5$ and each row of $H(\mu, \psi)$ corresponds to one of the equations (h_1) – (h_5). Given μ, equation (A.1) implicitly defines solution triplet $\{\underline{z}, \hat{z}, \overline{z}\}$ and coefficients C_1 and C_2. Applying Implicit Function Theorem yields:

$$\left.\frac{\partial \psi}{\partial \mu}\right|_{\mu=0} = -\left[\left.\frac{\partial H}{\partial \psi}\right|_{\mu=0}\right]^{-1} \left.\frac{\partial H}{\partial \mu}\right|_{\mu=0}$$

provided $\left.\frac{\partial H}{\partial \psi}\right|_{\mu=0}$ has full rank. Recall from Appendix A.2 that under $\mu = 0$ solution to (A.1) is $\psi_0 = [-\overline{z}_0, 0, \overline{z}_0, A, A]$, where $\overline{z}_0$ and A satisfy:

$$A = \frac{2\overline{z}_0}{\alpha\rho(e^{\alpha\overline{z}_0} - e^{-\alpha\overline{z}_0})}$$
$$\overline{z}_0^2 = A\rho(e^{\alpha\overline{z}_0} + e^{-\alpha\overline{z}_0} - 2) + \rho\kappa$$

with $\alpha = \sqrt{2\rho/\sigma^2}$. Let $w_1 = (e^{\alpha\overline{z}_0} - e^{-\alpha\overline{z}_0})$, $w_2 = (e^{\alpha\overline{z}_0} + e^{-\alpha\overline{z}_0})$, $\gamma = \frac{2\alpha\overline{z}_0 w_2 - 2w_1}{\rho w_1}$ and $\beta = \frac{4\alpha\overline{z}_0 - 2w_1}{\rho w_1}$. Then a direct computation provides:

$$\left.\frac{\partial H}{\partial \psi}\right|_{\mu=0} = \begin{bmatrix} \gamma & 0 & 0 & -\alpha e^{\alpha\overline{z}_0} & \alpha e^{-\alpha\overline{z}_0} \\ 0 & \beta & 0 & -\alpha & \alpha \\ 0 & 0 & \gamma & -\alpha e^{-\alpha\overline{z}_0} & \alpha e^{\alpha\overline{z}_0} \\ 0 & 0 & 0 & e^{\alpha\overline{z}_0} - 1 & e^{-\alpha\overline{z}_0} - 1 \\ 0 & 0 & 0 & e^{-\alpha\overline{z}_0} - 1 & e^{\alpha\overline{z}_0} - 1 \end{bmatrix}$$

This matrix can be inverted as:

$$\left[\frac{\partial H}{\partial \psi}\bigg|_{\mu=0}\right]^{-1} = \begin{bmatrix} \gamma^{-1} & 0 & 0 & \frac{\alpha(w_2+1)}{\gamma w_1} & \frac{\alpha}{\gamma w_1} \\ 0 & \beta^{-1} & 0 & \frac{\alpha}{\beta w_1} & -\frac{\alpha}{\beta w_1} \\ 0 & 0 & \gamma^{-1} & -\frac{\alpha}{\gamma w_1} & -\frac{\alpha(w_2+1)}{\gamma w_1} \\ 0 & 0 & 0 & \frac{w_1+w_2-2}{2w_1(w_2-2)} & \frac{w_1-w_2+2}{2w_1(w_2-2)} \\ 0 & 0 & 0 & \frac{w_1-w_2+2}{2w_1(w_2-2)} & \frac{w_1+w_2-2}{2w_1(w_2-2)} \end{bmatrix}$$

The derivative of $H(\mu, \psi)$ with respect to μ evaluated at $\mu = 0$ is:

$$\frac{\partial H}{\partial \mu}\bigg|_{\mu=0} = \begin{bmatrix} \frac{\alpha \bar{z}_0 w_2 + \alpha^2 \bar{z}_0^2 w_1 + 2w_1}{\rho^2 w_1} \\ 2\frac{\alpha \bar{z}_0 + w_1}{\rho^2 w_1} \\ \frac{\alpha \bar{z}_0 w_2 + \alpha^2 \bar{z}_0^2 w_1 + 2w_1}{\rho^2 w_1} \\ -\frac{\alpha^2 \bar{z}_0^2 w_2 + 2\alpha \bar{z}_0 w_1}{\alpha \rho^2 w_1} \\ \frac{\alpha^2 \bar{z}_0^2 w_2 + 2\alpha \bar{z}_0 w_1}{\alpha \rho^2 w_1} \end{bmatrix}$$

Multiplying and collecting terms yields:

$$\begin{aligned} \frac{\partial \underline{z}}{\partial \mu}\bigg|_{\mu=0} = \frac{\partial \bar{z}}{\partial \mu}\bigg|_{\mu=0} &= \frac{4\alpha^2 \bar{z}_0^2 + \alpha \bar{z}_0 w_1 w_2 - 2w_1^2}{2\rho(\alpha \bar{z}_0 w_1 w_2 - w_1^2)} \\ \frac{\partial \hat{z}}{\partial \mu}\bigg|_{\mu=0} &= \frac{\alpha^2 \bar{z}_0^2 w_2 + \alpha \bar{z}_0 w_1 - w_1^2}{\rho(2\alpha \bar{z}_0 w_1 - w_1^2)} \end{aligned}$$

In order to recover the no-discounting case of Alvarez et al. (2019), use expressions from Appendix B.1 and expand numerators up to 6th degree and denominators up to 4th degree.

A.4 Stationary density under non-zero drift

Both the impact and cumulative impulse responses depend on stationary density. In this section I provide derivatives of the stationary density function $f(z, \mu)$ with respect to drift μ, evaluated at $\mu = 0$.

Recall that stationary density satisfies the following Kolmogorov forward equation:

$$0 = \mu f_z(z, \mu) + \frac{\sigma^2}{2} f_{zz}(z, \mu)$$

together with boundary conditions $f(\underline{z}(\mu), \mu) = f(\bar{z}(\mu), \mu) = 0$, unit mass condition $\int_{\underline{z}(\mu)}^{\bar{z}(\mu)} f(z, \mu) dz = 1$ and continuity at $z = \hat{z}(\mu)$. Note that density depends on drift μ both directly as it appears in KFE, and indirectly as it also appears in boundary conditions via policy variables. For the purpose of derivation, it is thus convenient to include policy variables explicitly as arguments with some abuse of notation: $f(z;\mu,\underline{z},\hat{z},\overline{z})$, so that $f(z,\mu)=f(z;\mu,\underline{z}(\mu),\hat{z}(\mu),\overline{z}(\mu))$. Stokey (2009) shows that stationary density is given by[30]:

$$f(z;\mu,\underline{z},\hat{z},\overline{z})=\begin{cases}\frac{e^{\eta(\mu)\hat{z}}-e^{\eta(\mu)\overline{z}}+e^{\eta(\mu)(\overline{z}+\underline{z}-z)}-e^{\eta(\mu)(\hat{z}+\underline{z}-z)}}{(\overline{z}-\underline{z})e^{\eta(\mu)\hat{z}}-(\overline{z}-\hat{z})e^{\eta(\mu)\underline{z}}-(\hat{z}-\underline{z})e^{\eta(\mu)\overline{z}}} & \text{for } z<\hat{z}\\[2ex] \frac{e^{\eta(\mu)\hat{z}}-e^{\eta(\mu)\underline{z}}+e^{\eta(\mu)(\overline{z}+\underline{z}-z)}-e^{\eta(\mu)(\hat{z}+\overline{z}-z)}}{(\overline{z}-\underline{z})e^{\eta(\mu)\hat{z}}-(\overline{z}-\hat{z})e^{\eta(\mu)\underline{z}}-(\hat{z}-\underline{z})e^{\eta(\mu)\overline{z}}} & \text{for } z\geq\hat{z}\end{cases}$$

where $\eta(\mu)=2\mu/\sigma^2$. To ease notation and simplify later derivations, define $v(z,\mu)=e^{\eta(\mu)z}$, so that:

$$f(z;\mu,\underline{z},\hat{z},\overline{z})=\begin{cases}\frac{v(\hat{z},\mu)-v(\overline{z},\mu)+v(\overline{z}+\underline{z}-z,\mu)-v(\hat{z}+\underline{z}-z,\mu)}{(\overline{z}-\underline{z})v(\hat{z},\mu)-(\overline{z}-\hat{z})v(\underline{z},\mu)-(\hat{z}-\underline{z})v(\overline{z},\mu)} & \text{for } z<\hat{z}\\[2ex] \frac{v(\hat{z},\mu)-v(\underline{z},\mu)+v(\overline{z}+\underline{z}-z,\mu)-v(\hat{z}+\overline{z}-z,\mu)}{(\overline{z}-\underline{z})v(\hat{z},\mu)-(\overline{z}-\hat{z})v(\underline{z},\mu)-(\hat{z}-\underline{z})v(\overline{z},\mu)} & \text{for } z\geq\hat{z}\end{cases}$$

Partial derivative of $f(z,\mu)$ with respect to drift is the total derivative of $f(z,\mu,\underline{z},\hat{z},\overline{z})$:

$$\begin{aligned}\left.\frac{\partial f(z,\mu)}{\partial\mu}\right|_{\mu=0} &= \left.\frac{df(z;\mu,\underline{z}(\mu),\hat{z}(\mu),\overline{z}(\mu))}{d\mu}\right|_{\mu=0,\underline{z}=\underline{z}(0),\hat{z}=\hat{z}(0),\overline{z}=\overline{z}(0)}\\ &= \left.\left(\frac{\partial f(z;\cdot)}{\partial\mu}+\frac{\partial f(z;\cdot)}{\partial\underline{z}}\frac{\partial\underline{z}}{\partial\mu}+\frac{\partial f(z;\cdot)}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial\mu}+\frac{\partial f(z;\cdot)}{\partial\overline{z}}\frac{\partial\overline{z}}{\partial\mu}\right)\right|_{\mu=0,\underline{z}=\underline{z}(0),\hat{z}=\hat{z}(0),\overline{z}=\overline{z}(0)}\end{aligned}$$

the first component is the direct effect of μ on the shape of stationary density, whereas the latter three are indirect effects through optimal policy. Recall that $\hat{z}(0)=0$ and note that due to symmetry of density around $\mu=0$, $f(-z,\mu)=f(z,-\mu)$ and thus $\frac{df(-z,0)}{d\mu}=-\frac{df(z,0)}{d\mu}$, so it suffices to calculate the derivative for $z<0$ only.

The three derivatives $\left\{\frac{\partial f(z;\cdot)}{\partial\underline{z}},\frac{\partial f(z;\cdot)}{\partial\hat{z}},\frac{\partial f(z;\cdot)}{\partial\overline{z}}\right\}$ are straightforward to obtain given the formula for $f(z;0,\underline{z},\hat{z},\overline{z})$:

$$f(z;0,\underline{z},\hat{z},\overline{z})=\begin{cases}2\frac{z-\underline{z}}{(\hat{z}-\underline{z})(\overline{z}-\underline{z})} & \text{for } z<\hat{z}\\[2ex] 2\frac{\overline{z}-z}{(\overline{z}-\hat{z})(\overline{z}-\underline{z})} & \text{for } z\geq\hat{z}\end{cases}$$

Differentiating $f(z;0,\underline{z},\hat{z},\overline{z})$ with respect to $\underline{z}$, $\hat{z}$ and $\overline{z}$, and evaluating at the optimal policy $\{\underline{z},\hat{z},\overline{z}\}=\{-\overline{z}_0,0,\overline{z}_0\}$ yields:

$$\left.\left\{\frac{\partial f(z;\cdot)}{\partial\underline{z}},\frac{\partial f(z;\cdot)}{\partial\hat{z}},\frac{\partial f(z;\cdot)}{\partial\overline{z}}\right\}\right|_{\mu=0,\underline{z}=-\overline{z}_0,\hat{z}=0,\overline{z}=\overline{z}_0}=\left\{\frac{3z+\overline{z}_0}{2\overline{z}_0^3},-\frac{z+\overline{z}}{\overline{z}_0^3},-\frac{z+\overline{z}}{2\overline{z}_0^3}\right\}\text{ for } z<0$$

The derivative with respect to μ is somewhat more complicated. First, set policy variables

[30] See Chapter 5. The formula is obtained as $f(z)=L(z)/\tau$, where $L(z)$ is the expected local time at z and τ is the average length between adjustments.

to their optimal values under $\mu = 0$:

$$f(z;\mu,-\overline{z}_0,0,\overline{z}_0) = \frac{1 - v(\overline{z}_0,\mu) + v(-z,\mu) - v(-\overline{z}_0 - z,\mu)}{2\overline{z}_0 - \overline{z}_0 v(-\overline{z}_0,\mu) - \overline{z}_0 v(\overline{z}_0,\mu)} \text{ for } z < 0$$

Second, denote the numerator by $N(\mu)$ and denominator by $D(\mu)$, so that:

$$\frac{\partial f(z;\mu,-\overline{z}_0,0,\overline{z}_0)}{\partial \mu} = \frac{N'(\mu)D(\mu) - D'(\mu)N(\mu)}{D(\mu)^2} \text{ for } z < 0 \tag{A.2}$$

Third, note that $v_\mu(z,\mu) = \frac{2}{\sigma^2} z v(z,\mu)$ and thus derivatives of numerator and denominator are given by:

$$N^k(\mu) = \frac{2^k}{\sigma^{2k}}\Big(- \overline{z}_0^k v(\overline{z}_0,\mu) + (-z)^k v(-z,\mu) - (-\overline{z}_0 - z)^k v(-\overline{z}_0 - z,\mu)\Big)$$
$$D^k(\mu) = \frac{2^k}{\sigma^{2k}}\Big((-\overline{z}_0)^{k+1} v(-\overline{z}_0,\mu) - \overline{z}_0^{k+1} v(\overline{z}_0,\mu)\Big)$$

Since $v(z,0) = 1$, evaluating at $\mu = 0$ yields:

$$N^k(0) = \begin{cases} \frac{2^k}{\sigma^{2k}}\Big(- \overline{z}_0^k - z^k + (z+\overline{z}_0)^k\Big) & \text{for } k \text{ odd} \\ \frac{2^k}{\sigma^{2k}}\Big(- \overline{z}_0^k + z^k - (z+\overline{z}_0)^k\Big) & \text{for } k \text{ even} \end{cases}$$
$$D^k(0) = \begin{cases} 0 & \text{for } k \text{ odd} \\ \frac{2^k}{\sigma^{2k}}\Big(- 2\overline{z}_0^{k+1}\Big) & \text{for } k \text{ even} \end{cases}$$

Note that $N(0) = D(0) = 0$ and it follows that evaluating (A.2) at $\mu = 0$ directly is not possible since both the numerator and the denominator converge to zero as $\mu \to 0$. Applying L'Hospital's rule four times yields:

$$\left.\frac{\partial f(z;\mu,\overline{z}_0,0,\overline{z}_0)}{\partial \mu}\right|_{\mu=0} = -\frac{z^2 + \overline{z}_0 z}{\sigma^2 \overline{z}_0^2} \qquad \text{for } z < 0$$

Finally, collecting all the terms:

$$\left.\frac{\partial f(z,\mu)}{\partial \mu}\right|_{\mu=0} = -\frac{z^2 + \overline{z}_0 z}{\sigma^2 \overline{z}_0^2} + \frac{z}{\overline{z}_0^3}\frac{\partial \overline{z}(0)}{\partial \mu} - \frac{z + \overline{z}_0}{\overline{z}_0^3}\frac{\partial \hat{z}(0)}{\partial \mu} \quad \text{for } z < 0$$

which provides derivative of density with respect to drift μ at any point $z \in [-\overline{z}_0, 0)$. Density is non-differentiable at $z = 0$ and for positive values $z \in (0, \overline{z}_0]$ derivative of density is given by $\frac{\partial f(z,0)}{\partial \mu} = -\frac{\partial f(-z,\mu)}{\partial \mu}$.

A.5 Impact effect under non-zero drift

Recall that for a positive shock $\delta > 0$, impact effect is given by:

$$\Theta(\delta, \mu) = \int_{\underline{z}(\mu)-\delta}^{\underline{z}(\mu)} (\hat{z}(\mu) - z) f(z + \delta, \mu) dz$$

and its derivative with respect to μ is:

$$\frac{\partial \Theta(\delta, \mu)}{\partial \mu} = \frac{\partial \underline{z}(\mu)}{\partial \mu} \Delta^+(\mu) f(\underline{z}(\mu){+}\delta, \mu) + \int_{\underline{z}(\mu)-\delta}^{\underline{z}(\mu)} \left(\frac{\partial \hat{z}(\mu)}{\partial \mu} f(z{+}\delta, \mu) + (\hat{z}(\mu) - z) \frac{\partial f(z{+}\delta, \mu)}{\partial \mu} \right) dz$$

where $\Delta^+(\mu) = \hat{z}(\mu) - \underline{z}(\mu)$ and I have used the fact that $f(\underline{z}(\mu), \mu) = 0$. Evaluating at $\mu = 0$ yields:

$$\begin{aligned} \frac{\partial \Theta(\delta, 0)}{\partial \mu} &= \overline{z}_0 \frac{\partial \underline{z}(0)}{\partial \mu} f(-\overline{z}_0{+}\delta, 0) + \int_{-\overline{z}_0-\delta}^{-\overline{z}_0} \left(\frac{\partial \hat{z}(\mu)}{\partial \mu} f(z{+}\delta, 0) - z \frac{\partial f(z{+}\delta, 0)}{\partial \mu} \right) dz \\ &= \overline{z}_0 \frac{\partial \underline{z}(0)}{\partial \mu} f(-\overline{z}_0{+}\delta, 0) + \frac{\partial \hat{z}(\mu)}{\partial \mu} F(-\overline{z}_0{+}\delta) - \int_{-\overline{z}_0-\delta}^{-\overline{z}_0} z \frac{\partial f(z{+}\delta, 0)}{\partial \mu} dz \end{aligned}$$

Previous sections of Appendix provide expressions for all terms in the above equation. Note that as long as $\delta < \overline{z}_0$, the integral in the last term is well defined, however if $\delta \geq \overline{z}_0$, then it has to be split into two integrals since $f(z, 0)$ is not differentiable at $z = 0$:

$$\int_{-\overline{z}_0-\delta}^{-\overline{z}_0} z \frac{\partial f(z{+}\delta, 0)}{\partial \mu} dz = \int_{-\overline{z}_0-\delta}^{-\delta} z \frac{\partial f(z{+}\delta, 0)}{\partial \mu} dz + \int_{-\delta}^{-\overline{z}_0} z \frac{\partial f(z{+}\delta, 0)}{\partial \mu} dz$$

A direct computation provides the following result:

$$\left. \frac{\partial \Theta(\delta, \mu)}{\partial \mu} \right|_{\mu=0} = \begin{cases} \frac{\delta^2(6\overline{z}_0^2 - \delta^2 - 2\delta\overline{z}_0)}{12\sigma^2\overline{z}_0^2} - \frac{\delta^3}{6\overline{z}_0^3} \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta < \overline{z}_0 \\ \frac{\delta^2(\delta^2 - 2\delta\overline{z}_0 - 6\overline{z}_0^2) + \overline{z}_0^3(16\delta - 6\overline{z}_0)}{12\sigma^2\overline{z}_0^2} - \frac{\delta^3 - 12\delta\overline{z}_0^2 + 12\overline{z}_0^3}{6\overline{z}_0^3} \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ \frac{\overline{z}_0^2}{6\sigma^2} + \frac{2}{3} \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

A.6 Cumulative Impulse Response under non-zero drift

It is convenient to split $M(\delta,\mu)$ into several smaller parts. First, note that function $m(z,\mu)$ can be written as:

$$\begin{aligned}
m(z,\mu) &= -\mathbb{E}\left(\int_0^\tau (z(s)-\bar{x}(\mu)ds \;\middle|\; z(0)=z\right) \\
&= \underbrace{-\mathbb{E}\left(\int_0^\tau z(s)ds \;\middle|\; z(0)=z\right)}_{\hat{m}(z,\mu)} + \bar{x}(\mu)\underbrace{\mathbb{E}\left(\tau \;\middle|\; z(0)=z\right)}_{\tau(z,\mu)} \\
&= \hat{m}(z,\mu) + \bar{x}(\mu)\tau(z,\mu)
\end{aligned}$$

Function $\hat{m}(z,\mu)$ is now the expected cumulative gap until first adjustment and is defined by the following ODE:

$$z = -\mu\hat{m}_z(z,\mu) + \frac{\sigma^2}{2}\hat{m}_{zz}(z,\mu) \tag{A.3}$$

with boundary conditions $\hat{m}(\underline{z}(\mu),\mu) = \hat{m}(\overline{z}(\mu),\mu) = 0$. Function $\tau(z,\mu)$ is the expected time of first adjustment conditional on $z(0)=z$, and is also defined by ODE:

$$1 = -\mu\tau_z(z,\mu) + \frac{\sigma^2}{2}\tau_{zz}(z,\mu)$$

and boundary conditions $\tau(\underline{z}(\mu),\mu) = \tau(\overline{z}(\mu),\mu) = 0$. Solution to (A.3) is:

$$\hat{m}(z,\mu) = C_1 + C_2 e^{\frac{2\mu}{\sigma^2}z} - \frac{1}{2\mu}z^2 - \frac{\sigma^2}{2\mu^2}z \tag{A.4}$$

where C_1 and C_2 are determined by boundary conditions. Solution to $\tau(z,\mu)$ is provided in Chapter 5.5 of Stokey (2009).

Using this notation, express $M(\delta,\mu)$ as follows:

$$\begin{aligned}
M(\delta,\mu) &= \int_{\underline{z}(\mu)}^{\overline{z}(\mu)-\delta} m(z,\mu)f(z+\delta,\mu)dz \; - \int_{\underline{z}(\mu)}^{\overline{z}(\mu)} m(z,\mu)f(z,\mu)dz \\
&= \underbrace{\int_{\underline{z}(\mu)}^{\overline{z}(\mu)-\delta} \hat{m}(z,\mu)f(z+\delta,\mu)dz}_{\hat{M}(\delta,\mu)} \; + \bar{x}(\mu)\underbrace{\int_{\underline{z}(\mu)}^{\overline{z}(\mu)-\delta} \tau(z,\mu)f(z+\delta,\mu)dz}_{T(\delta,\mu)} \\
&\quad - \underbrace{\int_{\underline{z}(\mu)}^{\overline{z}(\mu)} \hat{m}(z,\mu)f(z,\mu)dz}_{\hat{M}(0,\mu)} \; - \bar{x}(\mu)\underbrace{\int_{\underline{z}(\mu)}^{\overline{z}(\mu)} \tau(z,\mu)f(z,\mu)dz}_{T(0,\mu)} \\
&= \hat{M}(\delta,\mu) - \hat{M}(0,\mu) \; + \; \bar{x}(\mu)\big[T(\delta,\mu) - T(0,\mu)\big]
\end{aligned}$$

and thus:

$$\frac{\partial M(\delta,0)}{\partial\mu} = \frac{\partial\hat{M}(\delta,0)}{\partial\mu} - \frac{\partial\hat{M}(0,0)}{\partial\mu} + \frac{\partial\bar{x}(0)}{\partial\mu}\big[T(\delta,0) - T(0,0)\big]$$
$$+\underbrace{\bar{x}(0)}_{=0}\big[\frac{\partial T(\delta,0)}{\partial\mu} - \frac{\partial T(0,0)}{\partial\mu}\big]$$

where $\bar{x}(0) = 0$ due to symmetry of $f(z,0)$. Derivatives of $\hat{M}(\delta,\mu)$ and $\bar{x}(\mu)$ are given by:

$$\frac{\partial\hat{M}(\delta,0)}{\partial\mu} = \frac{\partial\overline{z}(0)}{\partial\mu}\hat{m}(\overline{z}(0)-\delta,0)\overbrace{f(\overline{z}(0),0)}^{=0} - \frac{\partial\underline{z}(0)}{\partial\mu}\overbrace{\hat{m}(\underline{z}(0),0)}^{=0}f(\underline{z}(0)+\delta,0)$$
$$+\int_{\underline{z}(0)}^{\overline{z}(0)-\delta}\frac{\partial\hat{m}(z,0)}{\partial\mu}f(z+\delta,0)dz + \int_{\underline{z}(0)}^{\overline{z}(0)-\delta}\hat{m}(z,0)\frac{\partial f(z+\delta,0)}{\partial\mu}dz$$

$$\frac{\partial\bar{x}(0)}{\partial\mu} = \frac{\partial\overline{z}(0)}{\partial\mu}\underbrace{f(\overline{z}(0),0)}_{=0} - \frac{\partial\underline{z}(0)}{\partial\mu}\underbrace{f(\underline{z}(0),0)}_{=0} + \int_{\underline{z}(0)}^{\overline{z}(0)}\frac{\partial f(z,0)}{\partial\mu}dz$$

where derivatives of integration boundaries are zero due to boundary conditions of $\hat{m}(z,0)$ and $f(z,0)$. Note that integrals have to split accordingly since stationary density $f(z,0)$ is not differentiable at $z=0$. Derivative of stationary density $f(z,\mu)$ with respect to drift is provided in Appendix A.4. A direct computation yields:

$$\frac{\partial\bar{x}(0)}{\partial\mu} = \frac{2}{3}\frac{\partial\overline{z}(0)}{\partial\mu} + \frac{1}{3}\frac{\partial\hat{z}(0)}{\partial\mu} - \frac{\overline{z}_0^2}{6\sigma^2}$$

Stokey (2009) shows that $\tau(z,0) = \frac{\overline{z}_0^2 - z^2}{\sigma^2}$ and thus computing $T(\delta,0)$ gives:

$$T(\delta,0) = \begin{cases} \frac{1}{12\sigma^2\overline{z}_0^2}\big[\delta^4 + 4\delta^3\overline{z}_0 - 12\delta^2\overline{z}_0^2 + 10\overline{z}_0^4\big], & \text{for } \delta < \overline{z}_0 \\ \frac{1}{12\sigma^2\overline{z}_0^2}\big[-\delta^4 + 4\delta^3\overline{z}_0 - 16\delta\overline{z}_0^3 + 16\overline{z}_0^4\big], & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ 0, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

Computation of $\frac{\partial\hat{M}(\delta,0)}{\partial\mu}$ requires knowledge of $\frac{\partial\hat{m}(z,0)}{\partial\mu}$ and $\hat{m}(z,0)$. The latter solves (A.3) for $\mu = 0$ and is given by:

$$\hat{m}(z,0) = \frac{z^3 - \overline{z}_0^2 z}{3\sigma^2}$$

It remains to characterize derivative of $\hat{m}(z,\mu)$ with respect to μ and then derivative of CIR can be computed. First, note that $\hat{m}(z,\mu)$ depends on μ both directly as can be seen in (A.4), as well as indirectly through boundaries of inaction region that appear in expressions for C_1 and C_2:

$$C_2 = \frac{1}{v(\overline{z}(\mu),\mu) - v(\underline{z}(\mu),\mu)} \left[\frac{1}{2\mu}\big(\overline{z}(\mu)^2 - \underline{z}(\mu)^2\big) + \frac{\sigma^2}{2\mu^2}\big(\overline{z}(\mu) - \underline{z}(\mu)\big)\right]$$

$$C_1 = \frac{1}{2\mu}\overline{z}(\mu)^2 + \frac{\sigma^2}{2\mu^2}\overline{z}(\mu) - C_2 v(\overline{z}(\mu),\mu)$$

where $v(z,\mu) = e^{\frac{2\mu}{\sigma^2}z}$. It is thus convenient to include the boundaries explicitly as arguments of $\hat{m}(z,\mu)$, so that $\hat{m}(z,\mu) = \hat{m}(z;\mu,\overline{z}(\mu),\underline{z}(\mu))$. Then:

$$\begin{aligned}
\left.\frac{\partial \hat{m}(z,\mu)}{\partial \mu}\right|_{\mu=0} &= \left.\frac{d\hat{m}(z;\mu,\overline{z},\underline{z})}{d\mu}\right|_{\mu=0,\overline{z}=\overline{z}_0,\underline{z}=-\overline{z}_0} \\
&= \left.\left(\frac{\partial \hat{m}(z;\cdot)}{\partial \mu} + \frac{\partial \hat{m}(z;\cdot)}{\partial \overline{z}}\frac{\partial \overline{z}(\mu)}{\partial \mu} + \frac{\partial \hat{m}(z;\cdot)}{\partial \underline{z}}\frac{\partial \underline{z}(\mu)}{\partial \mu}\right)\right|_{\mu=0,\overline{z}=\overline{z}_0,\underline{z}=-\overline{z}_0}
\end{aligned}$$

Derivatives with respect to boundaries $\underline{z}$ and $\overline{z}$ are relatively easy to obtain. Set $\mu = 0$, then:

$$\hat{m}(z;0,\overline{z},\underline{z}) = \frac{z^3 - \underline{z}^3}{3\sigma^2} - \frac{\overline{z}^3 - \underline{z}^3}{3\sigma^2(\overline{z} - \underline{z})}(z - \underline{z})$$

$$\left.\frac{\partial \hat{m}(z;\mu,\overline{z},\underline{z})}{\partial \overline{z}}\right|_{\mu=0,\overline{z}=\overline{z}_0,\underline{z}=-\overline{z}_0} = -\frac{\overline{z}_0 z + \overline{z}_0^2}{3\sigma^2}$$

$$\left.\frac{\partial \hat{m}(z;\mu,\overline{z},\underline{z})}{\partial \underline{z}}\right|_{\mu=0,\overline{z}=\overline{z}_0,\underline{z}=-\overline{z}_0} = \frac{\overline{z}_0 z - \overline{z}_0^2}{3\sigma^2}$$

Obtaining derivative of $\hat{m}(z,\mu)$ is somewhat more involved. Setting $\overline{z} = \overline{z}_0, \underline{z} = -\overline{z}_0$:

$$\hat{m}(z;\mu,\overline{z}_0,-\overline{z}_0) = \frac{2\overline{z}_0\sigma^2\beta(z,\mu) + \mu\gamma(\mu)(\overline{z}_0^2 - z^2) + \sigma^2\gamma(\mu)(\overline{z}_0 - z)}{2\mu^2\gamma(\mu)}$$

where $\gamma(\mu) = v(\overline{z}_0,\mu) - v(-\overline{z}_0,\mu)$ and $\beta(z,\mu) = v(z,\mu) - v(\overline{z}_0,\mu)$. Differentiating with respect to μ and collecting terms:

$$\left.\frac{\partial \hat{m}(z;\mu,\overline{z},\underline{z})}{\partial \mu}\right|_{\mu=0,\overline{z}=\overline{z}_0,\underline{z}=-\overline{z}_0} = \tag{A.5}$$

$$= \frac{2\overline{z}_0\sigma^2\big(\mu\gamma(\mu)\beta'_\mu(z,\mu) - 2\gamma(\mu)\beta(z,\mu) - \mu\gamma'(\mu)\beta(z,\mu)\big) - \mu\gamma(\mu)^2(\overline{z}_0^2 - z^2) - 2\sigma^2\gamma(\mu)^2(\overline{z}_0 - z)}{2\mu^3\gamma(\mu)^2}$$

Note that as $\mu \to 0$, $\gamma(\mu) \to 0$ and $\beta(z,\mu) \to 0$. In addition, derivatives of $\gamma(\mu)$ and

$\beta(z, \mu)$ with respect to μ evaluated at $\mu = 0$ are given by:

$$\gamma^k(0) = \begin{cases} \frac{2^{k+1}}{\sigma^{2k}} \overline{z}_0^k & \text{for } k \text{ odd} \\ \\ 0 & \text{for } k \text{ even} \end{cases}$$

$$\beta_\mu^k(z, 0) = \frac{2^k}{\sigma^{2k}} (z^k - \overline{z}_0^k)$$

This implies that evaluating (A.5) at $\mu = 0$ is not possible as both denominator and numerator are zero at $\mu = 0$. Applying L'Hospital's rule five times provides the result:

$$\left. \frac{\partial \hat{m}(z; \mu, \overline{z}, \underline{z})}{\partial \mu} \right|_{\mu=0, \overline{z}=\overline{z}_0, \underline{z}=-\overline{z}_0} = \frac{(\overline{z}_0^2 - z^2)^2}{6\sigma^4}$$

Collecting all the terms gives the derivative of interest:

$$\left. \frac{\partial \hat{m}(z, \mu)}{\partial \mu} \right|_{\mu=0} = \frac{(\overline{z}_0^2 - z^2)^2}{6\sigma^4} - \frac{2\overline{z}_0^2}{3\sigma^2} \frac{\partial \underline{z}(\mu)}{\partial \mu}$$

Now all necessary ingredients for the derivative of cumulative impulse response with respect to drift μ are collected and direct computation yields:

$$\left. \frac{\partial M(\delta, \mu)}{\partial \mu} \right|_{\mu=0} = \begin{cases} \frac{1}{360\sigma^4 \overline{z}_0^2} \left[-4\delta^6 - 18\delta^5 \overline{z}_0 + 45\delta^4 \overline{z}_0^2 + 20\delta^3 \overline{z}_0^3 - 60\delta^2 \overline{z}_0^4 \right] & \\ -\frac{1}{180\sigma^2 \overline{z}_0^3} \left[3\delta^5 + 10\delta^4 \overline{z}_0 \right] \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta < \overline{z}_0 \\ \\ \frac{1}{360\sigma^4 \overline{z}_0^2} \left[4\delta^6 - 18\delta^5 \overline{z}_0 + 15\delta^4 \overline{z}_0^2 + 20\delta^3 \overline{z}_0^3 - 48\delta \overline{z}_0^5 + 10\overline{z}_0^6 \right] & \\ -\frac{1}{180\sigma^2 \overline{z}_0^3} \left[3\delta^5 - 10\delta^4 \overline{z}_0 + 80\delta \overline{z}_0^4 - 60\overline{z}_0^5 \right] \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \in [\overline{z}_0, 2\overline{z}_0) \\ \\ -\frac{\overline{z}_0^4}{60\sigma^4} - \frac{\overline{z}_0^2}{5\sigma^2} \frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

B Proofs

B.1 Some useful expressions

I provide several expressions which will be used later. Let $w_1 = e^x - e^{-x}$ and $w_2 = e^x + e^{-x}$ where $x > 0$. Using Tailor expansion one obtains following results:

$$w_1 = 2 \sum_{i=1,3,5...}^{\infty} \frac{x^i}{i!} > 0 \tag{B.1}$$

$$w_2 = 2 + 2 \sum_{i=2,4,6...}^{\infty} \frac{x^i}{i!} > w_1 \tag{B.2}$$

$$w_1 w_2 = e^{2x} - e^{-2x} = 2 \sum_{i=1,3,5...}^{\infty} \frac{2^i x^i}{i!} \tag{B.3}$$

$$w_1^2 = e^{2x} + e^{-2x} - 2 = 2 \sum_{i=2,4,6...}^{\infty} \frac{2^i x^i}{i!} \tag{B.4}$$

$$w_2^2 = e^{2x} + e^{-2x} + 2 = 4 + 2 \sum_{i=2,4,6...}^{\infty} \frac{2^i x^i}{i!} \tag{B.5}$$

$$w_1^3 = e^{3x} - e^{-3x} - 3w_1 = 2 \sum_{i=1,3,5...}^{\infty} \frac{3^i x^i}{i!} - 3w_1 \tag{B.6}$$

$$w_1^2 w_2 = e^{3x} + e^{-3x} - w_2 = 2 + 2 \sum_{i=2,4,6...}^{\infty} \frac{3^i x^i}{i!} - w_2 \tag{B.7}$$

B.2 Proof of Proposition 1

First, let's show that $\frac{\partial \overline{z}}{\partial \mu}\big|_{\mu=0} > 0$. Denote $x := \alpha \overline{z}_0$:

$$\left.\frac{\partial \overline{z}}{\partial \mu}\right|_{\mu=0} = \frac{4x^2 + x w_1 w_2 - 2w_1^2}{2\rho(x w_1 w_2 - w_1^2)}$$

Firstly, using expressions from Appendix B.1, one can show that denominator is positive:

$$2\rho(x w_1 w_2 - w_1^2) > 0 \iff x w_2 - w_1 > 0 \iff \sum_{i=3,5,7...}^{\infty} \frac{x^i}{(i-1)!} - \sum_{i=3,5,7...}^{\infty} \frac{x^i}{i!} > 0$$

where last inequality is trivially satisfied. Secondly, similar logic applies to the numerator:

$$\begin{aligned} 4x^2 + x w_1 w_2 - 2w_1^2 > 0 &\iff 4x^2 + \sum_{i=2,4,6...}^{\infty} \frac{2^i x^i}{(i-1)!} - \sum_{i=2,4,6...}^{\infty} \frac{2^{i+2} x^i}{i!} > 0 \\ &\iff \sum_{i=6,8,10...}^{\infty} \frac{2^i x^i}{(i-1)!} - \sum_{i=6,8,10...}^{\infty} \frac{2^{i+2} x^i}{i!} > 0 \end{aligned}$$

where last line follows since $\frac{2^i}{(i-1)!} > \frac{2^{i+2}}{i!}$ for all $i > 4$. Thus $\frac{\partial \bar{z}}{\partial \mu}\big|_{\mu=0} > 0$ which concludes the proof of the first part of Proposition 1.

Now let's show that $\frac{\partial \hat{z}}{\partial \mu}\big|_{\mu=0} > \frac{\partial \bar{z}}{\partial \mu}\big|_{\mu=0}$. Using expressions for these derivatives and the same substitution ($x := \alpha \bar{z}_0$) this amounts to showing:

$$\frac{2(x^2 w_2 + x w_1 - w_1^2)(x w_2 - w_1) - (4x^2 + x w_1 w_2 - 2w_1^2)(2x - w_1)}{2\rho w_1 (2x - w_1)(x w_2 - w_1)} > 0$$

Note that denominator is negative since $2x - w_1 < 0$ (trivial) and $x w_2 - w_1 > 0$ (shown above). Thus it remains to show that numerator (Num) is also negative. Opening the brackets, collecting terms and dividing by x yields:

$$Num = 2x^2 w_2^2 - w_1^2 w_2 + 2w_1^2 - 8x^2 - 2x w_1 w_2 + 4x w_1$$

Plugging expressions from Appendix B.1:

$$\begin{aligned}
Num &= 2x^2\left(4 + 2\sum_{i=2,4,6\ldots}^{\infty}\frac{2^i x^i}{i!}\right) - \left(2 + 2\sum_{i=2,4,6\ldots}^{\infty}\frac{3^i x^i}{i!}\right) + \left(2 + 2\sum_{i=2,4,6\ldots}^{\infty}\frac{x^i}{i!}\right) \\
&\quad + 4\sum_{i=2,4,6\ldots}^{\infty}\frac{2^i x^i}{i!} - 8x^2 - 4x\sum_{i=1,3,5\ldots}^{\infty}\frac{2^i x^i}{i!} + 8x\sum_{i=1,3,5\ldots}^{\infty}\frac{x^i}{i!} \\
&= 8x^2 + \frac{16}{2}x^4 + \sum_{i=6,8,10\ldots}^{\infty}\frac{2^i x^i}{(i-2)!} - 2\frac{9}{2}x^2 - 2\frac{81}{24}x^4 - 2\sum_{i=6,8,10\ldots}^{\infty}\frac{3^i x^i}{i!} \\
&\quad + 2\frac{1}{2}x^2 + 2\frac{1}{24}x^4 + 2\sum_{i=6,8,10\ldots}^{\infty}\frac{x^i}{i!} + 4\frac{4}{2}x^2 + 4\frac{16}{24}x^4 + 4\sum_{i=6,8,10\ldots}^{\infty}\frac{2^i x^i}{i!} - 8x^2 \\
&\quad - 2\frac{4}{1}x^2 - 2\frac{16}{6}x^4 - 2\sum_{i=6,8,10\ldots}^{\infty}\frac{2^i x^i}{(i-1)!} + 8x^2 + 8\frac{1}{6}x^4 + 8\sum_{i=6,8,10\ldots}^{\infty}\frac{x^i}{(i-1)!} \\
&= \sum_{i=6,8,10\ldots}^{\infty}\left[\frac{2^i}{(i-2)!} - 2\frac{3^i}{i!} + \frac{2}{i!} + \frac{2^{i+2}}{i!} - \frac{2^{i+1}}{(i-1)!} + \frac{8}{(i-1)!}\right]x^i
\end{aligned}$$

Thus if $\frac{2}{i!}\left(2^{i-1}i(i-1) - 3^i + 1 + 2^{i+1} - 2^i i + 4i\right) \leq 0$ for all $i = \{6, 8, 10, \ldots\}$ and inequality is strict for some i, then $Num < 0$. Define:

$$q(i) = 2^{i-1}i(i-1) - 3^i + 1 + 2^{i+1} - 2^i i + 4i$$

and note that $q(6) = 0$ and $q(8) < 0$. Now split $q(i)$ into two parts:

$$
\begin{aligned}
q_1(i) &= 2^{i-1} i(i-1) + 2^{i+1} - 3^i \\
q_2(i) &= 1 + 4i - 2^i i \\
q(i) &= q_1(i) + q_2(i)
\end{aligned}
$$

It is easy to see that $q_2(i) < 0$ for all $i > 3$. Let's show by induction that $q_1(i)$ is negative for all $i \geq 10$. First note that $q_1(10) < 0$. Now assume $q_1(i) < 0$. Rearranging, this implies:

$$
i(i-1) < 2\left[\left(\frac{3}{2}\right)^i - 2\right]
$$

Then for $i+1$ it holds:

$$
(i+1)i = i(i-1)\frac{i+1}{i-1} < 2\left[\left(\frac{3}{2}\right)^i - 2\right]\frac{i+1}{i-1} < 2\left[\left(\frac{3}{2}\right)^{i+1} - 2\right]
$$

where the first inequality is due to induction assumption and the last one is true for all $i > 5$. Thus $q_1(i) < 0$ for all $i \geq 10$ and same applies to $q(i)$, which concludes the proof.

B.3 Lemma 1

Let $\rho, \kappa, \sigma > 0$. Let $\Delta^+(\mu) = \hat{z}(\mu) - \underline{z}(\mu)$. Then:

$$
\frac{\partial \Delta^+(0)}{\partial \mu} < \frac{1}{10}\frac{\overline{z}_0^2}{\sigma^2}
$$

Proof. Using expressions for $\frac{\partial \underline{z}(0)}{\partial \mu}$ and $\frac{\partial \hat{z}(0)}{\partial \mu}$, and denoting $x := \alpha \overline{z}_0$, the above expressions can be written as:

$$
\frac{10(4x^2 + xw_1w_2 - 2w_1^2)(2x - w_1) - 20(x^2w_2 + xw_1 - w_1^2)(xw_2 - w_1) + w_1(xw_2 - w_1)(2x - w_1)x^2}{20\rho w_1(2x - w_1)(xw_2 - w_1)} > 0
$$

Given that denominator is negative (as shown in proof of Proposition 1), it is required to show that numerator is positive. Opening the brackets, collecting terms and dividing by x, gives that numerator (Num) is negative if:

$$
Num = 80x^2 - 20w_1^2 + 10w_1^2w_2 + x(20w_1w_2 - 40w_1 + w_1^3) - x^2(20w_2^2 + 2w_1^2 + w_1^2w_2) + 2x^3w_1w_2 < 0
$$

Using expressions from Appendix B.1:

$$\begin{aligned}
Num &= 80x^2 - 40\sum_{i=2,4,6...}^{\infty}\frac{2^i x^i}{i!} + 20\sum_{i=2,4,6...}^{\infty}\frac{3^i x^i}{i!} - 20\sum_{i=2,4,6...}^{\infty}\frac{x^i}{i!} \\
&\quad + x\left(40\sum_{i=1,3,5...}^{\infty}\frac{2^i x^i}{i!} + 2\sum_{i=1,3,5...}^{\infty}\frac{3^i x^i}{i!} - 86\sum_{i=1,3,5...}^{\infty}\frac{x^i}{i!}\right) \\
&\quad - x^2\left(80 + 44\sum_{i=2,4,6...}^{\infty}\frac{2^i x^i}{i!} + 2\sum_{i=2,4,6...}^{\infty}\frac{3^i x^i}{i!} - 2\sum_{i=2,4,6...}^{\infty}\frac{x^i}{i!}\right) + 4x^3\sum_{i=1,3,5...}^{\infty}\frac{2^i x^i}{i!} \\
&= 80x^2 - 40\frac{4}{2}x^2 - 40\sum_{i=4,6,8...}^{\infty}\frac{2^i x^i}{i!} + 20\frac{9}{2}x^2 + 20\sum_{i=4,6,8...}^{\infty}\frac{3^i x^i}{i!} - 20\frac{1}{2}x^2 - 20\sum_{i=4,6,8...}^{\infty}\frac{x^i}{i!} \\
&\quad + 40\frac{2}{1}x^2 + 20\sum_{i=4,6,8...}^{\infty}\frac{2^i x^i}{(i-1)!} + 2\frac{3}{1}x^2 + 2\sum_{i=4,6,8...}^{\infty}\frac{3^{i-1} x^i}{(i-1)!} - 86x^2 - 86\sum_{i=4,6,8...}^{\infty}\frac{x^i}{(i-1)!} \\
&\quad - 80x^2 - 11\sum_{i=4,6,8...}^{\infty}\frac{2^i x^i}{(i-2)!} - 2\sum_{i=4,6,8...}^{\infty}\frac{3^{i-2} x^i}{(i-2)!} + 2\sum_{i=4,6,8...}^{\infty}\frac{x^i}{(i-2)!} + 2\sum_{i=4,6,8...}^{\infty}\frac{2^{i-2} x^i}{(i-3)!} \\
&= \sum_{i=4,6,8...}^{\infty}\frac{2x^i}{i!}\left[\underbrace{10(3^i-2^{i+1}-1)+i(10\cdot 2^i+3^{i-1}-43)+i(i-1)(1-11\cdot 2^{i-1}-3^{i-2})+i(i-1)(i-2)2^{i-2}}_{q(i)}\right]
\end{aligned}$$

Thus if $q(i) \leq 0$ for all $i \in \{4, 6, 8...\}$ and $q(i) < 0$ for some $i \in \{4, 6, 8...\}$, then $Num < 0$ and Lemma 1 is proven. A direct computation gives that $q(4) = q(6) = q(8) = 0$ and $q(10) < 0$. Let's show that $q(i) < 0$ for all $i \geq 12$. Note that $q(i) < 0$ if and only if:

$$\underbrace{10(3^i-2^{i+1}-1)}_{q_1(i)} + \underbrace{i(10\cdot 2^i+3^{i-1}-43)}_{q_2(i)} + \underbrace{i(i-1)(i-2)2^{i-2}}_{q_3(i)} < \underbrace{i(i-1)(3^{i-2}+11\cdot 2^{i-1}-1)}_{q_4(i)}$$

Let's establish relations between these terms:

- $q_1(i) < \frac{1}{2}q_4(i)$:

$$\begin{aligned}
q_1(i) < \frac{1}{2}q_4(i) &\iff 20(3^i-2^{i+1}-1) < i(i-1)(3^{i-2}+11\cdot 2^{i-1}-1) \\
&\iff \underbrace{3^{i-2}(180-i(i-1))+i(i-1)}_{<0 \text{ for } i\geq 12}\ \underbrace{-2^{i-1}(80+11i(i-1))-20}_{<0} < 0
\end{aligned}$$

  Term in second bracket is trivially negative. To see why term in the first bracket is negative as well, consider a proof by induction. If $i = 12$, then $3^{i-2}(180 - i(i-1)) + i(i-1) < 0$. Suppose now that for some i, $3^{i-2}(180 - i(i-1)) < -i(i-1)$.

Consider $i+1$:

$$3^{i-1}(180-(i+1)i) < 3^{i-1}(180-i(i-1)) = 3\cdot 3^{i-2}(180-i(i-1))$$
$$< \underbrace{-3i(i-1) < -i(i+1)}_{\text{for } i>2}$$

where the second line follows from the induction assumption and the last one inequality is true for all $i > 2$. As a result, $q_1(i) < \frac{1}{2}q_4(i)$ for all $i \geq 12$.

- $q_2(i) < \frac{1}{4}q_4(i)$:

$$q_2(i) < \frac{1}{4}q_4(i) \iff 4(10\cdot 2^i + 3^{i-1} - 43) < (i-1)(3^{i-2} + 11\cdot 2^{i-1} - 1)$$
$$\iff \underbrace{3^{i-2}(13-i)+i}_{<0 \text{ for } i\geq 14} < \underbrace{2^{i-1}(11i-91)}_{>0 \text{ for } i\geq 9} + 173$$

The right hand side is trivially positive for $i \geq 9$. To see why term on the left hand side is negative, consider $i \geq 14$ and rewrite it as:

$$3^{i-2}(13-i)+i < 0 \iff 3^{i-2} > \frac{i}{i-13}$$

Here, $\frac{i}{i-13}$ is a decreasing function of i, whereas 3^{i-2} is increasing. In addition, the inequality is true for $i = 14$ and thus it is true for all $i \geq 14$. Finally, direct computation shows that $q_2(i) < \frac{1}{4}q_4(i)$ for $i = 12$ and, as a result, $q_2(i) < \frac{1}{4}q_4(i)$ for all $i \geq 12$.

- $q_3(i) < \frac{1}{5}q_4(i)$:

$$q_3(i) < \frac{1}{5}q_4(i) \iff 2^{i-2}(5i-22) < 3^{i-2} - 1$$

It suffices to show that $2^{i-2}5i < 3^{i-2} - 1$, which can be proven by induction. First, it holds for $i = 14$. Now assume that it holds for some i and consider $i+1$:

$$2^{i-1}5(i+1) = 2^{i-2}5i\frac{2(i+1)}{i} < \underbrace{(3^{i-2}-1)\frac{2(i+1)}{i} < 3^{i-1} - 1}_{\text{for } i\geq 2}$$

where the first inequality follows from induction assumption and the second one can be seen by multiplying both sides with i and collecting terms, so that it is equivalent to $3^{i-2}(2-i) < i+2$ which holds trivially if $i \geq 2$. Finally, direct computation shows that $q_3(i) < \frac{1}{5}q_4(i)$ for $i = 12$ and so $q_3(i) < \frac{1}{5}q_4(i)$ for all $i \geq 12$.

It follows that $q_1(i) + q_2(i) + q_3(i) < \frac{19}{20}q_4(i) < q_4(i)$ for all $i \geq 12$ and thus $q(i) < 0$ for all $i \geq 8$, which concludes the proof.

B.4 Lemma 2

Let $\rho, \kappa, \sigma > 0$. If μ is small and non-zero, then the average price gap in the steady state $\bar{x}(\mu)$ is not equal to zero.

Proof. Due to the symmetry of the stationary distribution under zero drift, $\bar{x}(0) = 0$. It thus suffices to show that the derivative $\frac{\partial \bar{x}(0)}{\partial \mu}$ is not equal to zero. Recall from Appendix A.6 that:

$$\begin{aligned}\frac{\partial \bar{x}(0)}{\partial \mu} &= \frac{2}{3}\frac{\partial \overline{z}(0)}{\partial \mu} + \frac{1}{3}\frac{\partial \hat{z}(0)}{\partial \mu} - \frac{\overline{z}_0^2}{6\sigma^2} \\ &= \frac{\partial \overline{z}(0)}{\partial \mu} + \frac{1}{3}\frac{\partial \Delta^+(0)}{\partial \mu} - \frac{\overline{z}_0^2}{6\sigma^2}\end{aligned}$$

where $\Delta^+(\mu) = \hat{z}(\mu) - \underline{z}(\mu)$. From Lemma 1 it follows:

$$\begin{aligned}\frac{\partial \bar{x}(0)}{\partial \mu} &< \frac{\partial \overline{z}(0)}{\partial \mu} + \frac{1}{30}\frac{\overline{z}_0^2}{\sigma^2} - \frac{\overline{z}_0^2}{6\sigma^2} \\ &= \frac{\partial \overline{z}(0)}{\partial \mu} - \frac{2}{15}\frac{\overline{z}_0^2}{\sigma^2}\end{aligned}$$

Let me now show that $\frac{\partial \overline{z}(0)}{\partial \mu} - \frac{2}{15}\frac{\overline{z}_0^2}{\sigma^2} < 0$. Using the expression for $\frac{\partial \overline{z}(0)}{\partial \mu}$, rearranging terms and denoting $x := \alpha \overline{z}_0$, it is equivalent to showing that:

$$\frac{60x^2 + 15xw_1w_2 - 30w_1^2 - 2x^3w_1w_2 + 2x^2w_1^2}{30\rho(xw_1w_2 - w_1^2)} < 0$$

Note that the denominator is positive, as shown in the proof of Proposition 1. It thus suffices to show that the numerator is negative:

$$Num = 60x^2 + 15xw_1w_2 - 30w_1^2 - 2x^3w_1w_2 + 2x^2w_1^2 < 0$$

Using the expansion formulas from Appendix B.1, rewrite the numerator as:

$$\begin{aligned}Num &= 60x^2 + 30x \sum_{i=1,3,5\ldots}^{\infty} \frac{2^i x^i}{i!} - 60 \sum_{i=2,4,6\ldots}^{\infty} \frac{2^i x^i}{i!} - 4x^3 \sum_{i=1,3,5\ldots}^{\infty} \frac{2^i x^i}{i!} + 4x^2 \sum_{i=2,4,6\ldots}^{\infty} \frac{2^i x^i}{i!} \\ &= 60x^2 + 15 \sum_{i=2,4,6\ldots}^{\infty} \frac{2^i x^i}{(i-1)!} - 60 \sum_{i=2,4,6\ldots}^{\infty} \frac{2^i x^i}{i!} - \sum_{i=4,6,8\ldots}^{\infty} \frac{2^{i-1} x^i}{(i-3)!} + \sum_{i=4,6,8\ldots}^{\infty} \frac{2^i x^i}{(i-2)!} \\ &= 60x^2 + 60x^2 - 60\frac{4}{2}x^2 + 15 \sum_{i=4,6,8\ldots}^{\infty} \frac{2^i x^i}{(i-1)!} - 60 \sum_{i=4,6,8\ldots}^{\infty} \frac{2^i x^i}{i!} - \sum_{i=4,6,8\ldots}^{\infty} \frac{2^{i-1} x^i}{(i-3)!} + \sum_{i=4,6,8\ldots}^{\infty} \frac{2^i x^i}{(i-2)!} \\ &= \sum_{i=4,6,8\ldots}^{\infty} \frac{2^{i-1} x^i}{i!} \Bigg[\underbrace{30i - 120 - i(i-1)(i-2) + 2i(i-1)}_{q(i)} \Bigg]\end{aligned}$$

If $q(i) \leq 0$ for all $i \in \{4, 6, 8...\}$ and $q(i) < 0$ for some of these i, it would follow that $Num < 0$ and Lemma 2 is proven.

Note first that $q(4) = q(6) = 0$, whereas $q(8) < 0$ and $q(10) < 0$. Let me prove by induction that $q(i) + 120 < 0$ for any $i \geq 10$. Suppose that for some i, $q(i) + 120 < 0$. Consider $i + 1$:

$$q(i+1) + 120 = 30(i+1) - (i+1)i(i-1) + 2(i+1)i < 0 \iff \\ 30 - i(i-1) + 2i < 0 \iff 30 - i(i-3) < 0$$

The last inequality is trivially satisfied for any $i \geq 10$, which concludes the proof.

B.5 Proof of Proposition 2

First, for convenience, denote $\hat{\Theta}(\delta) = \frac{\partial \Theta(\delta,\mu)}{\partial \mu}\Big|_{\mu=0}$. Consider $\delta > 0$. Note that:

$$\hat{\Theta}(\delta) = \begin{cases} 0, & \text{for } \delta = 0 \\ \frac{\overline{z}_0^2}{4\sigma^2} - \frac{1}{6}\frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta = \overline{z}_0 \\ \frac{\overline{z}_0^2}{6\sigma^2} + \frac{2}{3}\frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

And thus $\hat{\Theta}(\overline{z}_0) > 0$ by Lemma 1, and $\hat{\Theta}(\delta) > 0$ for all $\delta \geq 2\overline{z}_0$ since $\frac{\partial \Delta^+(0)}{\partial \mu} > 0$ by Proposition 1.

Consider now $\delta \in (0, \overline{z}_0)$. For such δ:

$$\begin{aligned} \hat{\Theta}'(\delta) &= \delta \left[\frac{6\overline{z}_0^2 - 2\delta^2 - 3\delta\overline{z}_0}{6\sigma^2\overline{z}_0^2} - \frac{\delta}{2\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\right] > \delta \left[\frac{6\overline{z}_0^2 - 2\overline{z}_0^2 - 3\overline{z}_0^2}{6\sigma^2\overline{z}_0^2} - \frac{\overline{z}_0}{2\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\right] \\ &= \delta \left[\frac{1}{6\sigma^2} - \frac{1}{2\overline{z}_0^2}\frac{\partial \Delta^+(0)}{\partial \mu}\right] > 0 \end{aligned}$$

where first inequality is due to $\delta < \overline{z}_0$ and second one due to Lemma 1. It follows that $\hat{\Theta}(\delta)$ is strictly increasing over $(0, \overline{z}_0)$ and since $\hat{\Theta}(0) = 0$ it follows that $\hat{\Theta}(\delta) > 0$ for all $\delta \in (0, \overline{z}_0]$.

Consider now $\delta \in (\overline{z}_0, 2\overline{z}_0)$. For such δ:

$$\hat{\Theta}'(\delta) = \frac{2\delta^3 - 3\delta^2\overline{z}_0 - 6\delta\overline{z}_0^2 + 8\overline{z}_0^3}{6\sigma^2\overline{z}_0^2} - \frac{\delta^2 - 4\overline{z}_0^2}{2\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}$$

so that $\lim\limits_{\delta \downarrow \overline{z}_0} \hat{\Theta}'(\delta) = \frac{\overline{z}_0}{6\sigma^2} + \frac{3}{2\overline{z}_0}\frac{\partial \Delta^+(0)}{\partial \mu} > 0$. Given that $\hat{\Theta}(\overline{z}_0), \hat{\Theta}(2\overline{z}_0) > 0$, the only case when

$\hat{\Theta}(\delta)$ is negative for some $\delta \in (\overline{z}_0, 2\overline{z}_0)$ is if its derivative $\hat{\Theta}'(\delta)$ becomes negative and then again positive, i.e. switches its sign at least twice. To see if that is the case, consider second and third derivatives:

$$\hat{\Theta}''(\delta) = \frac{\delta^2 - \delta\overline{z}_0 - \overline{z}_0^2}{\sigma^2 \overline{z}_0^2} - \frac{\delta}{\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}$$

$$\hat{\Theta}'''(\delta) = \frac{2\delta - \overline{z}_0}{\sigma^2 \overline{z}_0^2} - \frac{1}{\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu} > \frac{\overline{z}_0}{\sigma^2 \overline{z}_0^2} - \frac{1}{\overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu} > 0$$

where first inequality follows since $\delta > \overline{z}_0$ and second one from Lemma 1. Third derivative is strictly positive for all $\delta \in (\overline{z}_0, 2\overline{z}_0)$ and thus second derivative is monotonic and can only cross zero at most once. It follows that first derivative $\hat{\Theta}'(\delta)$ can switch its sign at most once and thus $\hat{\Theta}(\delta)$ is strictly positive for all $\delta \in (\overline{z}_0, 2\overline{z}_0)$. Given previous results, it follows that $\frac{\Theta(\delta,\mu)}{\partial \mu}\Big|_{\mu=0} > 0$ for all $\delta > 0$. Noting that impact effect is symmetric around zero drift ($\Theta(-\delta, \mu) = -\Theta(\delta, -\mu)$) provides that $\frac{\Theta(-\delta,0)}{\partial \mu} = \frac{\Theta(\delta,0)}{\partial \mu} > 0$ which concludes the proof.

B.6 Proof of Proposition 3

Let $\hat{m}(z,t)$ denote the expected cumulative deviation of g from its steady state until time t, conditional on initial value $z(0) = z$:

$$\hat{m}(z,t) = \mathbb{E}\left(\int_0^t (g(z(s)) - \bar{g})ds \middle| z(0) = z\right)$$

Denote $\hat{m}(z) = \lim\limits_{t\to\infty} \hat{m}(z,t)$ and thus:

$$CIRF(F_0) = \int_{\underline{z}}^{\overline{z}} \hat{m}(z) dF_0(z)$$

Let τ_i be the i-th adjustment and let $t_a \wedge t_b = min\{t_a, t_b\}$. Fix a staring value z and consider the cumulated deviation of g form its steady state until $t > 0$, writing all the random variables explicitly as a function of the underlying outcome ω:

$$\int_0^t (g(z(s,\omega)) - \bar{g})ds = \int_0^{\tau_1(\omega)\wedge t} (g(z(s,\omega)) - \bar{g})ds + \sum_{i=1}^{N-1} \int_{\tau_i(\omega)\wedge t}^{\tau_{i+1}(\omega)\wedge t} (g(z(s,\omega)) - \bar{g})ds$$
$$+ \int_{\tau_N(\omega)\wedge t}^{t} (g(z(s,\omega)) - \bar{g})ds$$

for some fixed $N \geq 1$. Take the limit of the above expression as $N \to \infty$. For a fixed horizon t and outcome ω there will be $n(t,\omega)$ adjustments between time 0 and t. Let

$N(t,\omega) = \max\{1, n(t,\omega)\}$. Then:

$$\int_0^t (g(z(s,\omega)) - \bar{g})ds = \int_0^{\tau_1(\omega)\wedge t} (g(z(s,\omega)) - \bar{g})ds + \sum_{i=1}^{N(t,\omega)-1} \int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} (g(z(s,\omega)) - \bar{g})ds$$
$$+ \int_{\tau_{N(t,\omega)}(\omega)\wedge t}^{t} (g(z(s,\omega)) - \bar{g})ds$$

Applying conditional expectation ($\mathbb{E}_z(\cdot) = \mathbb{E}(\cdot|z(0,\omega) = z)$) yields an expression for $\hat{m}(z,t)$:

$$\begin{aligned}\hat{m}(z,t) = \mathbb{E}_z &\left(\int_0^{\tau_1(\omega)\wedge t} (g(z(s,\omega)) - \bar{g})ds\right)\\ &+ \sum_{i=1}^{\infty} \mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} (g(z(s,\omega)) - \bar{g})ds \middle| N(t,\omega) \geq i+1\right) \mathbb{P}_z\left(N(t,\omega) \geq i+1\right)\\ &+ \mathbb{E}_z\left(\int_{\tau_{N(t,\omega)}(\omega)\wedge t}^{t} (g(z(s,\omega)) - \bar{g})ds\right)\end{aligned}$$

Where $\mathbb{P}_z\left(N(t,\omega) \geq i+1\right)$ is the probability that number of adjustments until t exceeds $i+1$ conditional on $z(0,\omega) = z$. Note that once we take expectation with respect to ω, the finite sum from the previous expression becomes infinite. That is due to the fact that for any $t > 0$ and any M there exists ω such that $N(t,\omega) > M$, which follows from the fact that increments of $z(t)$ are normally distributed. Each summand i is the expected cumulated deviation between i-th and $(i+1)$-th adjustment, conditional on there being at least $i+1$ adjustments, and weighted with corresponding probability.

Finally, take the limit as $t \to \infty$. For every z and every $i \in R_+$, $\mathbb{P}_z\left(N(t,\omega) \geq i+1\right)$ converges to one and the conditional expectation in second line converges to unconditional one. Also $N(t,\omega)$ converges to $n(t,\omega)$, $\tau_1(\omega) \wedge t$ converges to $\tau_1(\omega)$ and $\tau_{N(t,\omega)}(\omega) \wedge t \to \tau_{n(t,\omega)}(\omega)$. As has been shown in Baley and Blanco (2020), $\mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)}(g(z(s,\omega)) - \bar{g})ds\right)$ $= 0$ for all i, and thus:

$$\hat{m}(z) = \lim_{t\to\infty} \hat{m}(z,t) = m(z) + \tilde{m}(z)$$

where

$$m(z) = \mathbb{E}\left(\int_0^{\tau_1(\omega)} (g(z(s,\omega)) - \bar{g})ds \middle| z(0,\omega) = z\right)$$
$$\tilde{m}(z) = \lim_{t\to\infty} \mathbb{E}\left(\int_{\tau_{n(t,\omega)}(\omega)}^{t} (g(z(s,\omega)) - \bar{g})ds \middle| z(0,\omega) = z\right)$$

Note that due to Markov property, $\tilde{m}(z)$ does not depend on z since after the first adjustment initial condition does not matter and expectation becomes unconditional, so that $\tilde{m}(z) = \tilde{m} = \lim_{t\to\infty} \mathbb{E}\left(\int_{\tau_{n(t,\omega)}(\omega)}^{t} (g(z(s,\omega)) - \bar{g})ds\right)$. Thus:

$$CIRF(F_0) = \int_{\underline{z}}^{\overline{z}} m(z) dF_0(z) + \tilde{m}$$

which concludes the proof.

B.7 Proof of Proposition 4

Let $\hat{m}(r, z, t)$ denote the expected discounted cumulative deviation of g from its steady state until time t, conditional on initial value $z(0) = z$:

$$\hat{m}(r, z, t) = \mathbb{E}\left(\int_0^t e^{-rs}(g(z(s)) - \bar{g})ds \middle| z(0) = z\right)$$

with $r > 0$. Denote $\hat{m}(r, z) = \lim_{t\to\infty} \hat{m}(r.z, t)$ so that discounted cumulative impulse response is given by:

$$DCIRF(r, F_0) = \int_{\underline{z}}^{\overline{z}} \hat{m}(r, z) dF_0(z)$$

Let τ_i be the i-th adjustment and let $t_a \wedge t_b = min\{t_a, t_b\}$. Fix a staring value z and consider the discounted cumulated deviation of g form its steady state until $t > 0$, writing all the random variables explicitly as a function of the underlying outcome ω:

$$\int_0^t e^{-rs}(g(z(s,\omega)) - \bar{g})ds = \int_0^{\tau_1(\omega)\wedge t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds + \sum_{i=1}^{N-1} \int_{\tau_i(\omega)\wedge t}^{\tau_{i+1}(\omega)\wedge t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds$$
$$+ \int_{\tau_N(\omega)\wedge t}^{t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds$$

for some fixed $N \geq 1$. Take the limit of the above expression as $N \to \infty$. For a fixed horizon t and outcome ω there will be $n(t, \omega)$ adjustments between time 0 and t. Let $N(t, \omega) = \max\{1, n(t, \omega)\}$. Then:

$$\int_0^t e^{-rs}(g(z(s,\omega)) - \bar{g})ds = \int_0^{\tau_1(\omega)\wedge t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds + \sum_{i=1}^{N(t,\omega)-1} \int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds$$
$$+ \int_{\tau_{N(t,\omega)}(\omega)\wedge t}^{t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds$$

Applying conditional expectation ($\mathbb{E}_z(\cdot) = \mathbb{E}(\cdot|z(0,\omega) = z)$) yields an expression for $\hat{m}(z,t)$:

$$\begin{aligned}
\hat{m}(r,z,t) &= \mathbb{E}_z\left(\int_0^{\tau_1(\omega)\wedge t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right) \\
&\quad + \sum_{i=1}^{\infty} \mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds \middle| N(t,\omega) \geq i+1\right) \mathbb{P}_z\left(N(t,\omega) \geq i+1\right) \\
&\quad + \mathbb{E}_z\left(\int_{\tau_{N(t,\omega)}(\omega)\wedge t}^{t} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right)
\end{aligned}$$

Where $\mathbb{P}_z\left(N(t,\omega) \geq i+1\right)$ is the probability that number of adjustments until t exceeds $i+1$ conditional on $z(0,\omega) = z$. Note that once we take expectation with respect to ω, the finite sum from the previous expression becomes infinite. That is due to the fact that for any $t > 0$ and any M there exists ω such that $N(t,\omega) > M$, which follows from the fact that increments of $z(t)$ are normally distributed. Each summand i is the expected cumulated deviation between i-th and $(i+1)$-th adjustment, conditional on there being at least $i+1$ adjustments, and weighted with corresponding probability.

Finally, take the limit as $t \to \infty$. For every z and every $i \in R_+$, $\mathbb{P}_z\left(N(t,\omega) \geq i+1\right)$ converges to one and the conditional expectation in second line converges to unconditional one. Also $N(t,\omega)$ converges to $n(t,\omega)$, $\tau_1(\omega) \wedge t$ converges to $\tau_1(\omega)$ and $\tau_{N(t,\omega)}(\omega) \wedge t \to \tau_{n(t,\omega)}(\omega)$. Due to $r > 0$, the last summand converges to zero and thus:

$$\begin{aligned}
\hat{m}(r,z) = \lim_{t\to\infty} \hat{m}(r,z,t) &= \overbrace{\mathbb{E}_z\left(\int_0^{\tau(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right)}^{m(r,z)} \\
&\quad + \sum_{i=1}^{\infty} \mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right)
\end{aligned}$$

Because of discounting, expected deviations between adjustments are not zero anymore. However one can still characterize them. First, consider some $i \geq 1$ and rewrite as:

$$\mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right) = \mathbb{E}_z\left(e^{-r\tau_i(\omega)} \int_0^{\tau_{i+1}(\omega)-\tau_i(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds\right)$$

Note that due to strong Markov property, expectation of the integral does not depend on i or z, whereas expectation of $e^{-\tau_i(\omega)}$ depends on both. Thus the two terms are independent and we can split the expectation:

$$\mathbb{E}_z\left(e^{-r\tau_i(\omega)}\int_0^{\tau_{i+1}(\omega)-\tau_i(\omega)} e^{-rs}(g(z(s,\omega))-\bar{g})ds\right) = \mathbb{E}_z\left(e^{-r\tau_i(\omega)}\right)\cdot \underbrace{\mathbb{E}_{\hat{z}}\left(\int_0^{\tau(\omega)} e^{-rs}(g(z(s,\omega))-\bar{g})ds\right)}_{m(r,\hat{z})}$$

Now second term is the expectation of cumulated deviations until first adjustment conditional on starting at the return point: $z(0)=\hat{z}$. Denote the first term for $i=1$ by $q(r,z)=\mathbb{E}_z\left(e^{-r\tau(\omega)}\right)$. Then for any $i\geq 1$:

$$\begin{aligned}
\mathbb{E}_z\left(e^{-r\tau_i(\omega)}\right) &= \mathbb{E}_z\left(e^{-r\tau_1(\omega)}\cdot e^{-r(\tau_2(\omega)-\tau_1(\omega))}\cdots e^{-r(\tau_i(\omega)-\tau_{i-1}(\omega))}\right)\\
&= \mathbb{E}_z\left(e^{-r\tau_1(\omega)}\right)\cdot \mathbb{E}_z\left(e^{-r(\tau_2(\omega)-\tau_1(\omega))}\right)\cdots \mathbb{E}_z\left(e^{-r(\tau_i(\omega)-\tau_{i-1}(\omega))}\right)\\
&= \underbrace{\mathbb{E}_z\left(e^{-r\tau(\omega)}\right)}_{q(r,z)}\cdot\underbrace{\mathbb{E}_{\hat{z}}\left(e^{-r\tau(\omega)}\right)}_{q(r,\hat{z})}\cdots\underbrace{\mathbb{E}_{\hat{z}}\left(e^{-r\tau(\omega)}\right)}_{q(r,\hat{z})}\\
&= q(r,z)q(r,\hat{z})^{i-1}
\end{aligned}$$

where second and third lines follow due to strong Markov property of $z(t)$. Because of this property, times between adjustments are independent (2nd line) and initial condition $z(0)=z$ is irrelevant once there was an adjustment (3rd line). Thus:

$$\sum_{i=1}^{\infty}\mathbb{E}_z\left(\int_{\tau_i(\omega)}^{\tau_{i+1}(\omega)} e^{-rs}(g(z(s,\omega))-\bar{g})ds\right) = \sum_{i=1}^{\infty} q(r,z)q(r,\hat{z})^{i-1}m(r,\hat{z}) = \frac{q(r,z)}{1-q(r,\hat{z})}m(r,\hat{z})$$

and so:

$$DCIRF(r,F_0) = \int_{\underline{z}}^{\overline{z}} m(r,z)dF_0(z) + \frac{m(r,\hat{z})}{1-q(r,\hat{z})}\int_{\underline{z}}^{\overline{z}} q(r,z)dF_0(z)$$

Now in order to obtain undiscounted CIRF, it remains to take the limit as $r\to 0$. Note that $\lim_{r\to 0} m(r,z)=m(z)$ where $m(z)=\mathbb{E}\left(\int_0^{\tau}(g(z(s))-\bar{g})ds \middle| z(0)=z\right)$ and $\lim_{r\to 0} q(r,z)=1$. This implies that second integral converges to 1. In addition, since $m(\hat{z})=0$, as shown in Baley and Blanco (2020), $\lim_{r\to 0} m(r,\hat{z})=0$, and the coefficient in front of the second integral converges to some finite number. We can further simplify the expression by noting that:

$$q(r,z)=\mathbb{E}_z\left(e^{-r\tau(\omega)}\right)=1-r\mathbb{E}_z\left(\int_0^{\tau(\omega)} e^{-rs}ds\right)$$

and since $\lim_{r\to 0}\mathbb{E}_z\left(\int_0^{\tau(\omega)} e^{-rs}ds\right)=\mathbb{E}_z\left(\tau(\omega)\right)$, for small values of r, $1-q(r,\hat{z})$ behaves like $r\mathbb{E}_z\left(\tau(\omega)\right)$. Thus CIRF can be expressed as:

$$CIRF(F_0)=\lim_{r\to 0} DCIRF(r,F_0) = \int_{\underline{z}}^{\overline{z}} m(z)dF_0(z) + \frac{1}{\mathbb{E}\left(\tau(\omega)\middle| z(0,\omega)=\hat{z}\right)}\lim_{r\to 0}\frac{m(r,\hat{z})}{r}$$

where

$$m(z) = \mathbb{E}\left(\int_0^{\tau(\omega)} (g(z(s,\omega)) - \bar{g})ds \,\middle|\, z(0,\omega) = z\right)$$
$$m(r,\hat{z}) = \mathbb{E}\left(\int_0^{\tau(\omega)} e^{-rs}(g(z(s,\omega)) - \bar{g})ds \,\middle|\, z(0,\omega) = \hat{z}\right)$$

which concludes the proof.

B.8 Proof of Proposition 5

First, for convenience, denote $\hat{M}(\delta) = \frac{\partial M(\delta,\mu)}{\partial \mu}\Big|_{\mu=0}$ and consider $\delta > 0$. Note that:

$$\hat{M}(\delta) = \begin{cases} 0, & \text{for } \delta = 0 \\ -\frac{17\overline{z}_0^4}{360\sigma^4} - \frac{13\overline{z}_0^2}{180\sigma^2}\frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta = \overline{z}_0 \\ -\frac{\overline{z}_0^4}{60\sigma^4} - \frac{\overline{z}_0^2}{5\sigma^2}\frac{\partial \Delta^+(0)}{\partial \mu}, & \text{for } \delta \geq 2\overline{z}_0 \end{cases}$$

so that $\hat{M}(\overline{z}_0) < 0$ and $\hat{M}(\delta) < 0$ for all $\delta \geq 2\overline{z}_0$ since $\frac{\partial \Delta^+(0)}{\partial \mu} > 0$ by Proposition 1.

Now let's show that $\hat{M}(\delta) < 0$ for any $\delta > 0$. First, consider $\delta \in (0, \overline{z})$ and denote by $\hat{M}^k_-(\overline{z}_0)$ the limit of k-th derivative of $\hat{M}(\delta)$ for $\delta \uparrow \overline{z}_0$. The proof consists of five claims. To ease exposition, proofs of the claims are provided at the end of this section.

1a. $\hat{M}^V(0) < 0$ and $\hat{M}^V_-(\overline{z}) < 0$. In addition, $\hat{M}^V(\delta)$ is linear for $\delta \in (0, \overline{z}_0)$ and thus $\hat{M}^V(\delta) < 0$ for all $\delta \in (0, \overline{z}_0)$. This implies that $\hat{M}^{IV}(\delta)$ is strictly decreasing for $\delta \in (0, \overline{z}_0)$.

2a. $\hat{M}^{IV}(0) > 0$ and $\hat{M}^{IV}_-(\overline{z}) < 0$. Together with (1a) this implies that $\hat{M}^{IV}(\delta)$ crosses zeros once in $(0, \overline{z}_0)$ and thus $\hat{M}^{III}(\delta)$ is strictly concave and singe-peaked in $(0, \overline{z}_0)$.

3a. $\hat{M}^{III}(0) > 0$ and $\hat{M}^{III}_-(\overline{z}) < 0$. Together with (2a) it implies that $\hat{M}^{III}(\delta)$ crosses zeros once in $(0, \overline{z}_0)$ so that $\hat{M}^{II}(\delta)$ first increases and then decreases as δ goes from 0 to $\overline{z}_0$.

4a. $\hat{M}^{II}(0) < 0$ and $\hat{M}^{II}_-(\overline{z}) > 0$. Together with (3a) it implies that $\hat{M}^{II}(\delta)$ crosses zero once in $(0, \overline{z}_0)$ and thus $\hat{M}^{I}(\delta)$ first decreases and then increases as δ goes from 0 to $\overline{z}_0$.

5a. $\hat{M}^{I}(0) = 0$. Together with (4a) this implies that $\hat{M}^{I}(\delta)$ crosses zero *at most* once in $(0, \overline{z}_0)$.

Finally, since $\hat{M}(0) = 0$, $\hat{M}(\overline{z}_0) < 0$, $\hat{M}^I(0) = 0$, $\hat{M}^{II}(0) < 0$ and $\hat{M}^I(\delta)$ crosses zero at most once in $(0, \overline{z}_0)$, it follows that $\hat{M}(\delta) < 0$ for all $\delta \in (0, \overline{z}_0)$. In order to have $\hat{M}(\delta) \geq 0$ for some $\delta \in (0, \overline{z}_0)$, it must be the case that $\hat{M}^I(\delta)$ crosses zero at least twice, which contradicts (5a).

Now consider $\delta \in (\overline{z}_0, 2\overline{z}_0)$ and denote by $\hat{M}^k_+(\overline{z}_0)$ the limit of k-th derivative of $\hat{M}(\delta)$ for $\delta \downarrow \overline{z}_0$. The proof consists of five claims, proofs of which are also delegated to the end of this section.

1b. $\hat{M}^V_+(\overline{z}_0) > 0$ and $\hat{M}^V(2\overline{z}_0) > 0$. In addition, $\hat{M}^V(\delta)$ is linear for $\delta \in (\overline{z}_0, 2\overline{z}_0)$ and thus $\hat{M}^V(\delta) > 0$ for all $\delta \in (\overline{z}_0, 2\overline{z}_0)$. This implies that $\hat{M}^{IV}(\delta)$ is strictly increasing for $\delta \in (\overline{z}_0, 2\overline{z}_0)$.

2b. $\hat{M}^{IV}_+(\overline{z}_0) < 0$ and $\hat{M}^{IV}(2\overline{z}_0) > 0$. Together with (1b) this implies that $\hat{M}^{IV}(\delta)$ crosses zeros once in $(\overline{z}_0, 2\overline{z}_0)$ and thus $\hat{M}^{III}(\delta)$ is strictly convex in $(\overline{z}_0, 2\overline{z}_0)$.

3b. $\hat{M}^{III}_+(\overline{z}_0) < 0$ and $\hat{M}^{III}(2\overline{z}_0) > 0$. Together with (2b) it implies that $\hat{M}^{III}(\delta)$ crosses zeros once in $(\overline{z}_0, 2\overline{z}_0)$ so that $\hat{M}^{II}(\delta)$ first decreases and then increases as δ goes from $\overline{z}_0$ to $2\overline{z}_0$.

4b. $\hat{M}^{II}_+(\overline{z}_0) > 0$ and $\hat{M}^{II}(2\overline{z}_0) = 0$. Together with (3b) it implies that $\hat{M}^{II}(\delta)$ crosses zero once in $(\overline{z}_0, 2\overline{z}_0)$ and thus $\hat{M}^I(\delta)$ first increases and then decreases as δ goes from $\overline{z}_0$ to $2\overline{z}_0$.

5b. $\hat{M}^I(2\overline{z}_0) = 0$. Together with (4b) and (3b) this implies that $\hat{M}^I(\delta)$ crosses zero *at most* once in $\overline{z}_0$ to $2\overline{z}_0$.

Finally, since $\hat{M}(\overline{z}_0) < 0$, $\hat{M}(2\overline{z}_0) < 0$, $\hat{M}^I(2\overline{z}_0) = 0$, $\hat{M}^{II}(2\overline{z}_0) = 0$, $\hat{M}^{III}(2\overline{z}_0) > 0$ and $\hat{M}^I(\delta)$ crosses zero at most once in $(\overline{z}, 2\overline{z})$, it follows that $\hat{M}(\delta) < 0$ for all $\delta \in (\overline{z}, 2\overline{z})$. In order to have $\hat{M}(\delta) \geq 0$ for some $\delta \in (\overline{z}, 2\overline{z})$, it must be the case that $\hat{M}^I(\delta)$ crosses zero at least twice, which contradicts (5b).

Altogether, this implies that $\hat{M}(\delta) < 0$ for all $\delta > 0$. Note that $M(\delta, \mu)$ is symmetric in the sense that $M(-\delta, \mu) = -M(\delta, -\mu)$, so that $\frac{\partial M(-\delta,0)}{\partial \mu} = \frac{\partial M(\delta,0)}{\partial \mu} < 0$, which concludes the proof. Below I prove claims used above.

Consider $\delta > 0$. Since $\hat{M}(\delta)$ is a polynomial of degree 6, it follows that $\hat{M}^V(\delta)$ is a linear function. Direct computation yields:

$$\begin{aligned}
\hat{M}^V(0) &= -\frac{6}{\sigma^4 \overline{z}_0} - \frac{2}{\sigma^2 \overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^V_-(\overline{z}_0) &= -\frac{14}{\sigma^4 \overline{z}_0} - \frac{2}{\sigma^2 \overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^V_+(\overline{z}_0) &= \frac{2}{\sigma^4 \overline{z}_0} - \frac{2}{\sigma^2 \overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^V(2\overline{z}_0) &= \frac{10}{\sigma^4 \overline{z}_0} - \frac{2}{\sigma^2 \overline{z}_0^3}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{IV}(0) &= \frac{3}{\sigma^4} - \frac{4}{3\sigma^2 \overline{z}_0^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{IV}_-(\overline{z}_0) &= -\frac{7}{\sigma^4} - \frac{10}{3\sigma^2 \overline{z}_0^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{IV}_+(\overline{z}_0) &= -\frac{1}{\sigma^4} - \frac{2}{3\sigma^2 \overline{z}_0^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{IV}(2\overline{z}_0) &= \frac{5}{\sigma^4} - \frac{8}{3\sigma^2 \overline{z}_0^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{III}(0) &= \frac{\overline{z}_0}{3\sigma^4}\\
\hat{M}^{III}_-(\overline{z}_0) &= -\frac{\overline{z}_0}{\sigma^4} - \frac{7}{3\sigma^2 \overline{z}_0}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{III}_+(\overline{z}_0) &= -\frac{\overline{z}_0}{3\sigma^4} + \frac{1}{3\sigma^2 \overline{z}_0}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{III}(2\overline{z}_0) &= \frac{\overline{z}_0}{\sigma^4} - \frac{4}{3\sigma^2 \overline{z}_0}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{II}(0) &= -\frac{\overline{z}_0^2}{3\sigma^4}\\
\hat{M}^{II}_-(\overline{z}_0) &= \frac{\overline{z}_0^2}{6\sigma^4} - \frac{1}{\sigma^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{II}_+(\overline{z}_0) &= \frac{\overline{z}_0^2}{6\sigma^4} + \frac{1}{3\sigma^2}\frac{\partial \Delta^+(0)}{\partial \mu}\\
\hat{M}^{II}(2\overline{z}_0) &= 0\\
\hat{M}^{I}(0) &= 0\\
\hat{M}^{I}(2\overline{z}_0) &= 0
\end{aligned}$$

Inequalities in (1a - 5a) and (1b - 5b) follow either trivially, or due to Proposition 1 or due to Lemma 1.

B.9 Proof of Proposition 6

Let $\mu > 0$ and small. First, consider impact effect $\Theta(\delta, \mu)$. Its first order approximation with respect to drift is given by:

$$\Theta(\delta, \mu) = \Theta(\delta, 0) + \frac{\partial \Theta(\delta, 0)}{\partial \mu}\mu$$

Since for $\delta \geq 2\overline{z}_0$, $\Theta(\delta, 0) = \delta$ and $\frac{\partial \Theta(\delta,0)}{\partial \mu} > 0$ by Proposition 2, it follows that:

$$\Theta(\delta, \mu) - \delta > 0 \quad \text{for} \quad \delta \geq 2\overline{z}_0$$

Since both $\Theta(\delta, 0)$ and $\frac{\partial \Theta(\delta,0)}{\partial \mu}$ are second order in δ for small shocks, it follows that:

$$\Theta(\delta, \mu) - \delta < 0 \quad \text{for some small } \delta > 0$$

Thus, due to continuity of $\Theta(\delta, \mu)$, there exists $\delta_\Theta(\mu) \in (0, 2\overline{z}_0)$ such that $\Theta(\delta_\Theta(\mu), \mu) - \delta = 0$ and $\Theta(\delta, \mu) - \delta > 0$ for all $\delta > \delta_\Theta(\mu)$. Finally, since width of inaction region $\overline{z}(\mu) - \underline{z}(\mu)$ does not change with μ to first order (Proposition 1), and $\delta_\Theta(\mu) < 2\overline{z}_0$, it follows that $\delta_\Theta(\mu) < \overline{z}(\mu) - \underline{z}(\mu)$ if μ is sufficiently small.

Now, consider cumulative response $M(\delta, \mu)$. Its first order approximation with respect to drift is given by:

$$M(\delta, \mu) = M(\delta, 0) + \frac{\partial M(\delta, 0)}{\partial \mu}\mu$$

Since for $\delta \geq 2\overline{z}_0$, $M(\delta, 0) = 0$ and $\frac{\partial M(\delta,0)}{\partial \mu} < 0$ by Proposition 5, it follows that:

$$M(\delta, \mu) < 0 \quad \text{for} \quad \delta \geq 2\overline{z}_0$$

Since $M(\delta, 0)$ is first order and $\frac{\partial M(\delta,0)}{\partial \mu}$ is second order in δ for small shocks, it follows that:

$$M(\delta, \mu) > 0 \quad \text{for some small } \delta > 0$$

Thus, due to continuity of $M(\delta, \mu)$, there exists $\delta_M(\mu) \in (0, 2\overline{z}_0)$ such that $M(\delta_M(\mu), \mu) = 0$ and $M(\delta, \mu) < 0$ for all $\delta > \delta_M(\mu)$. Similar logic as before leads to $\delta_M(\mu) < \overline{z}(\mu) - \underline{z}(\mu)$ if μ is sufficiently small.

B.10 Proofs of several results regarding $\Theta(\delta, \mu)$ and $M(\delta, \mu)$

- **Result 1**

$$\lim_{\delta \to 0} \frac{\partial A_\Theta(\delta, 0)}{\partial \mu} = \frac{2\overline{z}_0}{\sigma^2}$$

Recall that for a small shock ($\delta < \overline{z}_0$):

$$\Theta(\delta, 0) = \frac{1}{6\overline{z}_0^2}\delta^2(\delta + 3\overline{z}_0)$$

$$\frac{\partial\Theta(\delta,0)}{\partial\mu} = \frac{\delta^2(6\overline{z}_0^2 - \delta^2 - 2\delta\overline{z}_0)}{12\sigma^2\overline{z}_0^2} - \frac{\delta^3}{6\overline{z}_0^3}\frac{\partial\Delta^+(0)}{\partial\mu}$$

So that:

$$\frac{\partial A_\Theta(\delta,0)}{\partial\mu} = \frac{2}{\Theta(\delta,0)}\frac{\partial\Theta(\delta,0)}{\partial\mu} = 2\left[\frac{(6\overline{z}_0^2 - \delta^2 - 2\delta\overline{z}_0)}{2\sigma^2(\delta + 3\overline{z}_0)} - \frac{\delta}{\overline{z}_0(\delta + 3\overline{z}_0)}\frac{\partial\Delta^+(0)}{\partial\mu}\right]$$

Taking the limit as $\delta \to 0$ provides the result.

- **Result 2**

$$\Theta(\delta,\mu) \approx \begin{cases} (1 + \frac{\overline{z}_0}{\sigma^2}\mu)\Theta(\delta,0) & \text{for } \delta > 0 \\ (1 - \frac{\overline{z}_0}{\sigma^2}\mu)\Theta(\delta,0) & \text{for } \delta < 0 \end{cases}$$

First order approximation of $\Theta(\delta,\mu)$ with respect to drift μ is given by:

$$\Theta(\delta,\mu) \approx \Theta(\delta,0) + \frac{\partial\Theta(\delta,0)}{\partial\mu}\mu$$

Now approximate each term to second order with respect to positive shock $\delta > 0$:

$$\Theta(\delta,0) \approx \frac{\delta^2}{2\overline{z}_0}$$
$$\frac{\partial\Theta(\delta,0)}{\partial\mu} \approx \frac{\delta^2}{2\sigma^2}$$

Then:

$$\Theta(\delta,\mu) \approx \frac{\delta^2}{2\overline{z}_0} + \frac{\delta^2}{2\sigma^2}\mu = \left(1 + \frac{\overline{z}_0}{\sigma^2}\mu\right)\frac{\delta^2}{2\overline{z}_0} \approx \left(1 + \frac{\overline{z}_0}{\sigma^2}\mu\right)\Theta(\delta,0)$$

The result for $\delta < 0$ can be shown analogously, with the only difference that second order approximation of $\Theta(\delta,0)$ is given by: $\Theta(\delta,0) \approx -\frac{\delta^2}{2\overline{z}_0}$.

- **Result 3**

$$\lim_{\delta\to 0}\frac{\partial A_\Theta(\delta,0)}{\partial\mu} > \frac{\partial A_I(0)}{\partial\mu}$$

Using expressions for asymmetries, above relation is equivalent to:

$$\frac{2\overline{z}_0}{\sigma^2} > \frac{2}{\overline{z}_0}\frac{\partial\Delta^+(0)}{\partial\mu} \iff \frac{\overline{z}_0^2}{\sigma^2} > \frac{\partial\Delta^+(0)}{\partial\mu}$$

which follows from Lemma 1.

- **Result 4**

$$M(\delta,\mu) \approx \begin{cases} (1-\frac{|\delta|}{\sigma^2}\mu)M(\delta,0) & \text{for } \delta>0 \\ \\ (1+\frac{|\delta|}{\sigma^2}\mu)M(\delta,0) & \text{for } \delta<0 \end{cases}$$

First order approximation of $M(\delta,\mu)$ with respect to drift μ is given by:

$$M(\delta,\mu) \approx M(\delta,0) + \frac{\partial M(\delta,0)}{\partial \mu}\mu$$

Now approximate each term to second order with respect to shock δ:

$$\begin{aligned} M(\delta,0) &\approx \frac{\overline{z}_0^2\delta}{6\sigma^2} \\ \frac{\partial M(\delta,0)}{\partial \mu} &\approx -\frac{\overline{z}_0^2\delta^2}{6\sigma^4} \end{aligned}$$

Then for $\delta>0$:

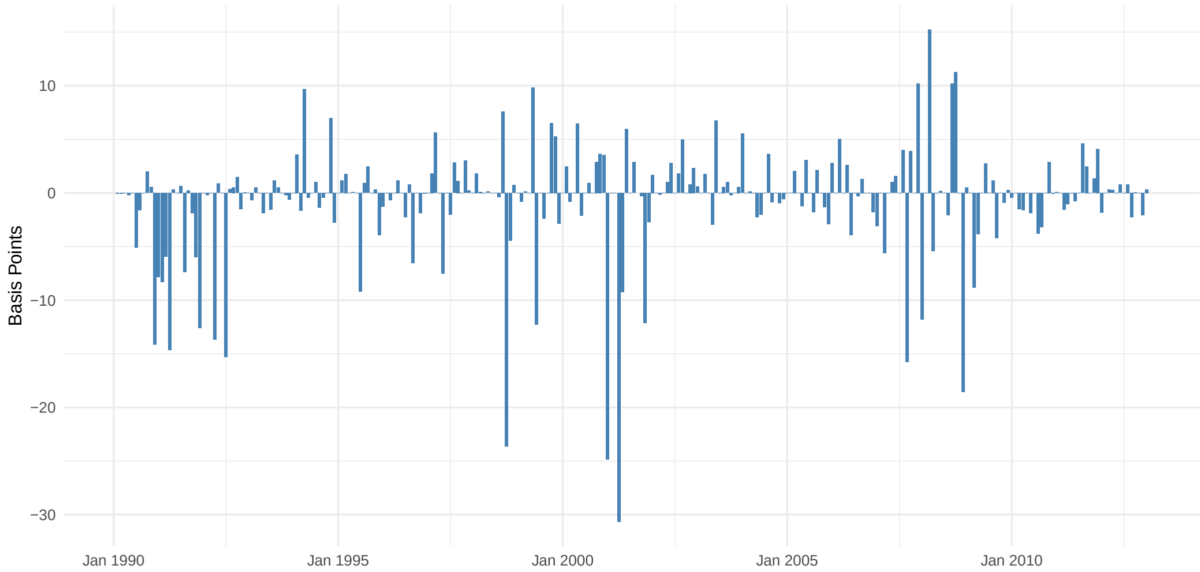
$$M(\delta,\mu) \approx \frac{\overline{z}_0^2\delta}{6\sigma^2} - \frac{\overline{z}_0^2\delta^2}{6\sigma^4}\mu = \left(1-\frac{|\delta|}{\sigma^2}\mu\right)\frac{\overline{z}_0^2\delta}{6\sigma^2} \approx \left(1-\frac{|\delta|}{\sigma^2}\mu\right)M(\delta,0)$$

The result for $\delta<0$ is analogous and immediate.

C Empirics

C.1 Monetary Policy Shocks

Figure 11: Monetary Policy Shocks



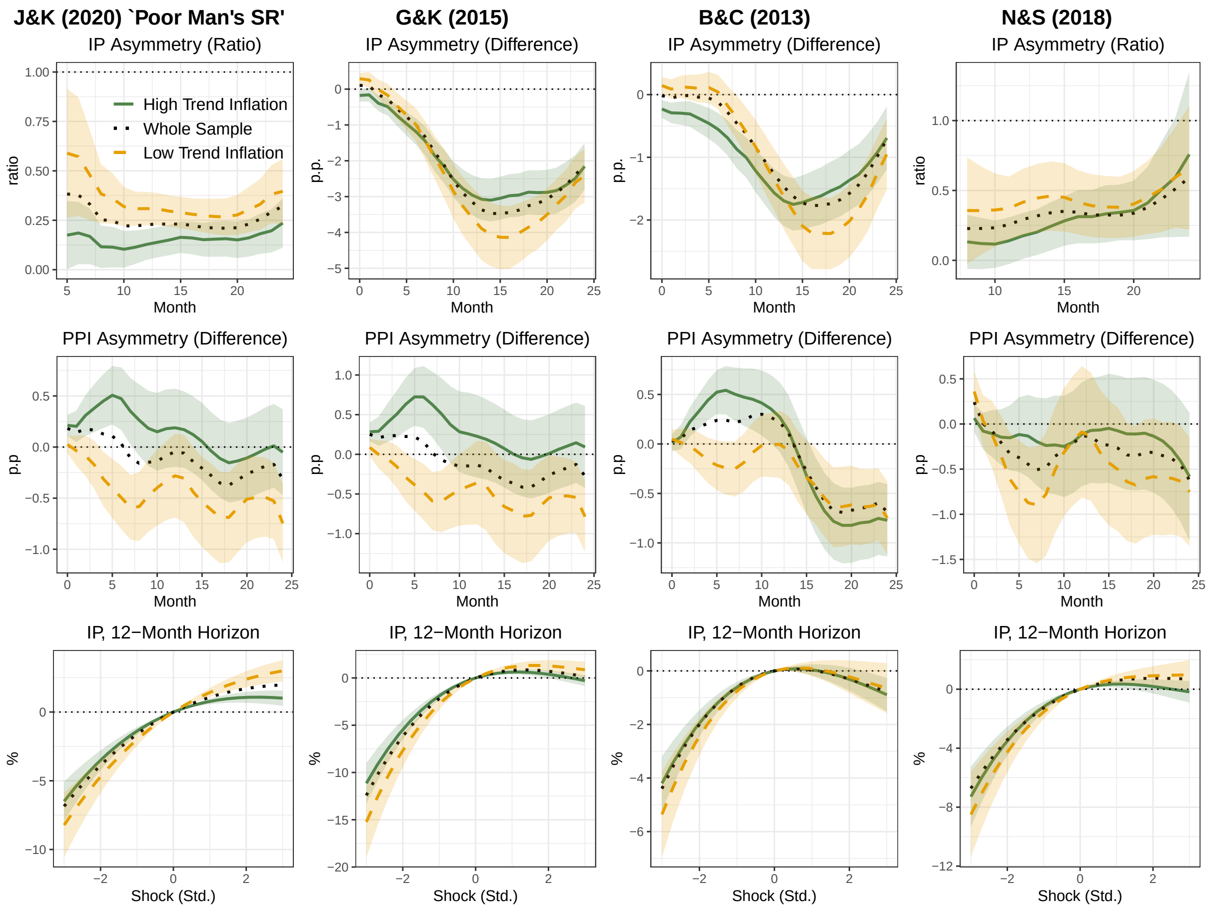
Monetary policy shocks, as estimated by Jarociński and Karadi (2020)

C.2 Robustness Checks

C.2.1 Alternative Shock Measures

To ease comparison, I only plot asymmetries (1st row: industrial production, 2nd row: PPI) and non-linear IP responses at a 12-months horizon (3rd row) for each alternative shock measure by column: (1) Jarociński and Karadi (2020) 'poor man's sign restrictions', (2) Gertler and Karadi (2015), (3) Barakchian and Crowe (2013), (4) Nakamura and Steinsson (2018). Whenever possible, I use the preferred measure of asymmetry (ratio), otherwise I compute asymmetry as a difference. The main results of the paper remain generally valid. Asymmetry in the IP responses relates negatively to trend inflation, although results are weaker when measuring asymmetry as a difference. Asymmetry in the PPI responses relates positively to trend inflation. Large positive shocks tend to cause contractions in IP when trend inflation is high, but not so much when trend inflation is low.

Figure 12: Main Results under Alternative Shock Series



C.2.2 Measurement Error

Figure 13: Asymmetry for a Top-30% / Bottom-30% Split

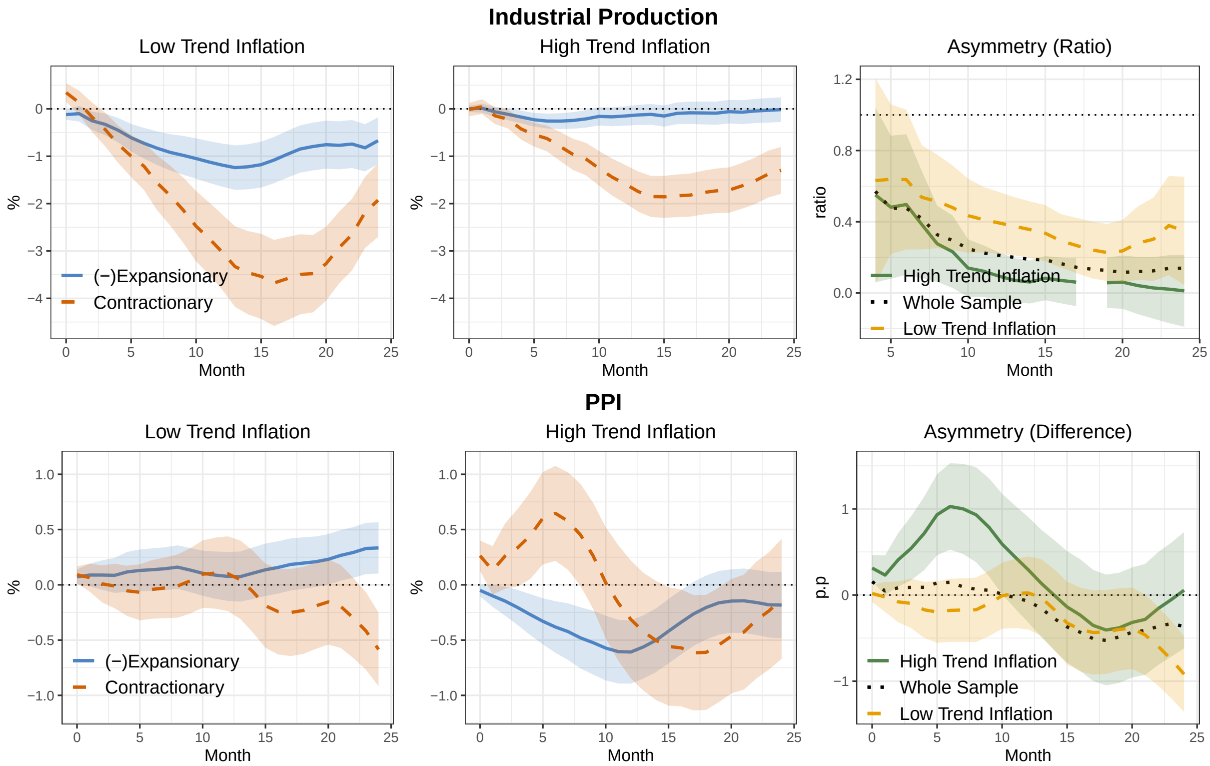
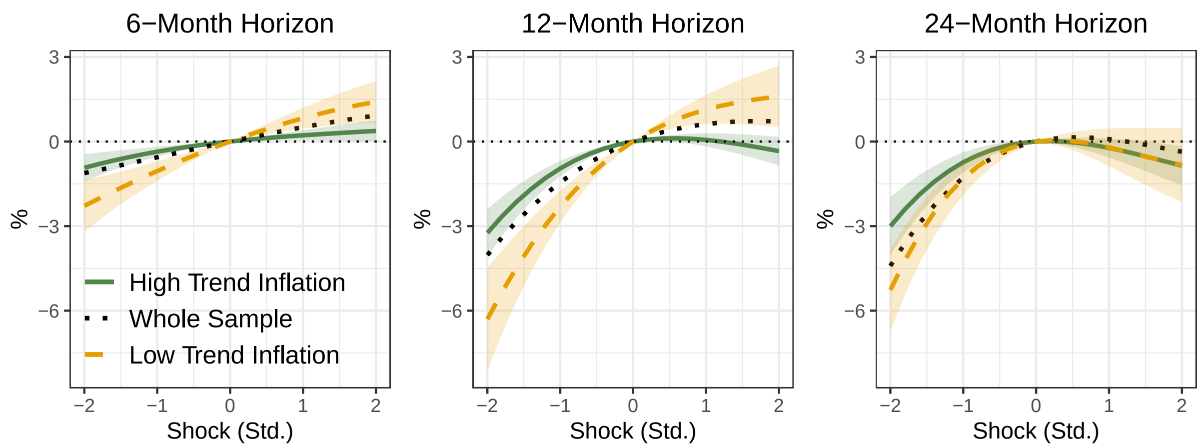


Figure 14: Non-Linearity of Industrial Production Responses for a Top-30% / Bottom-30% Split



C.2.3 Excluding Great Recession and ZLB period

Figure 15: Asymmetry, Sample Until June 2008

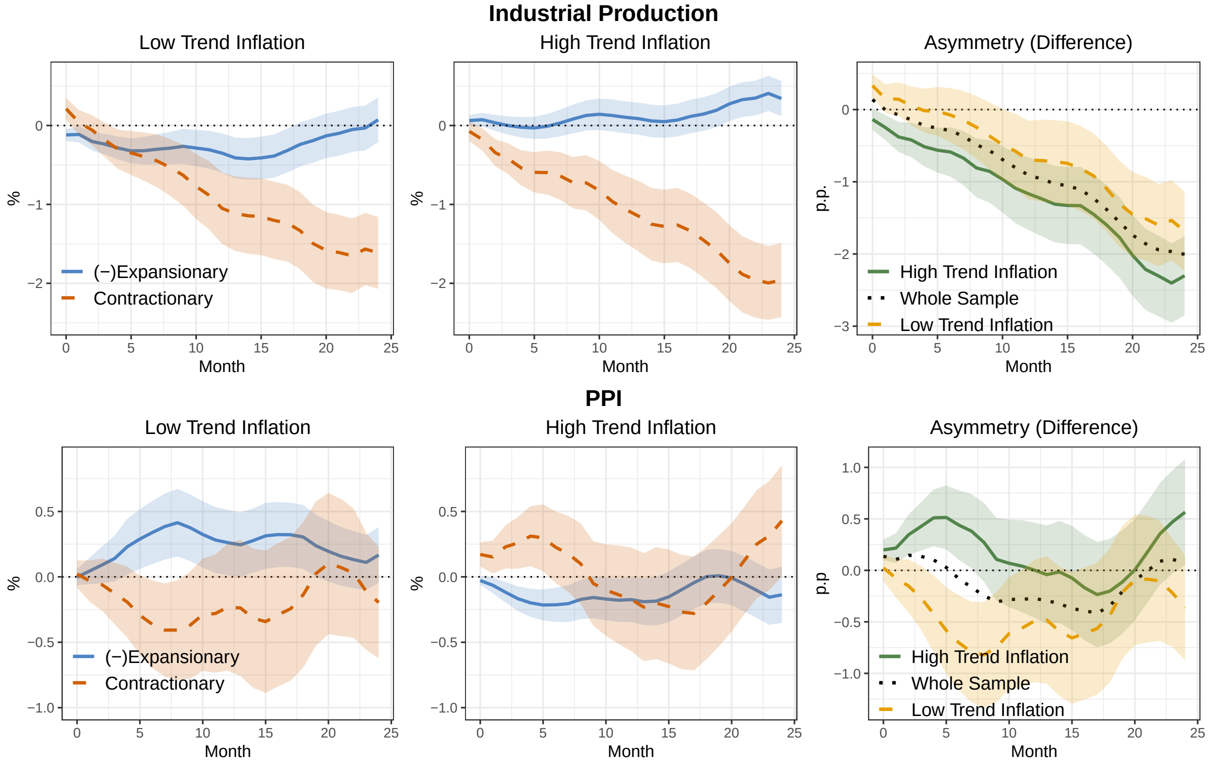
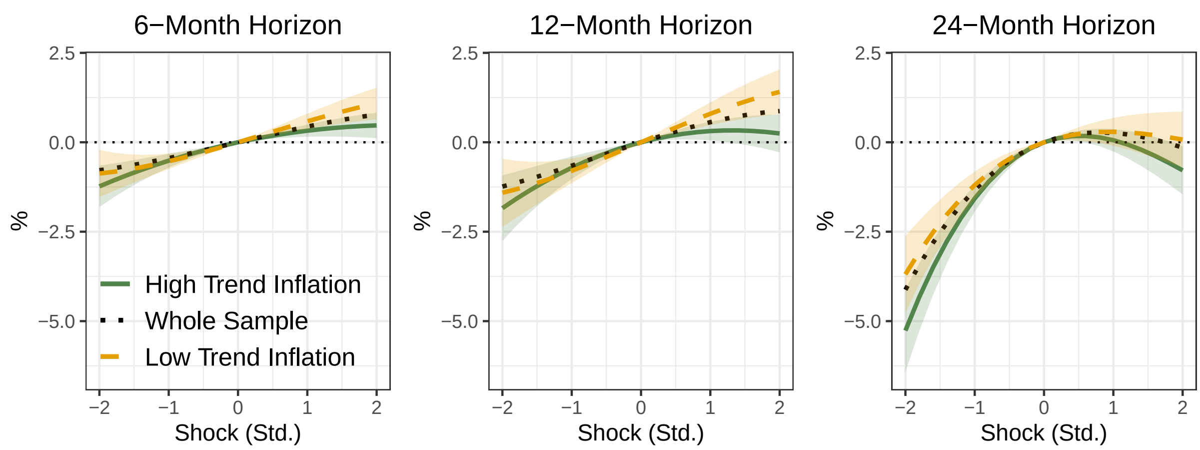


Figure 16: Non-Linearity of Industrial Production Responses, Sample Until June 2008



C.2.4 Alternative Trimming

Figure 17: Asymmetry, Trimming Top and Bottom 15%

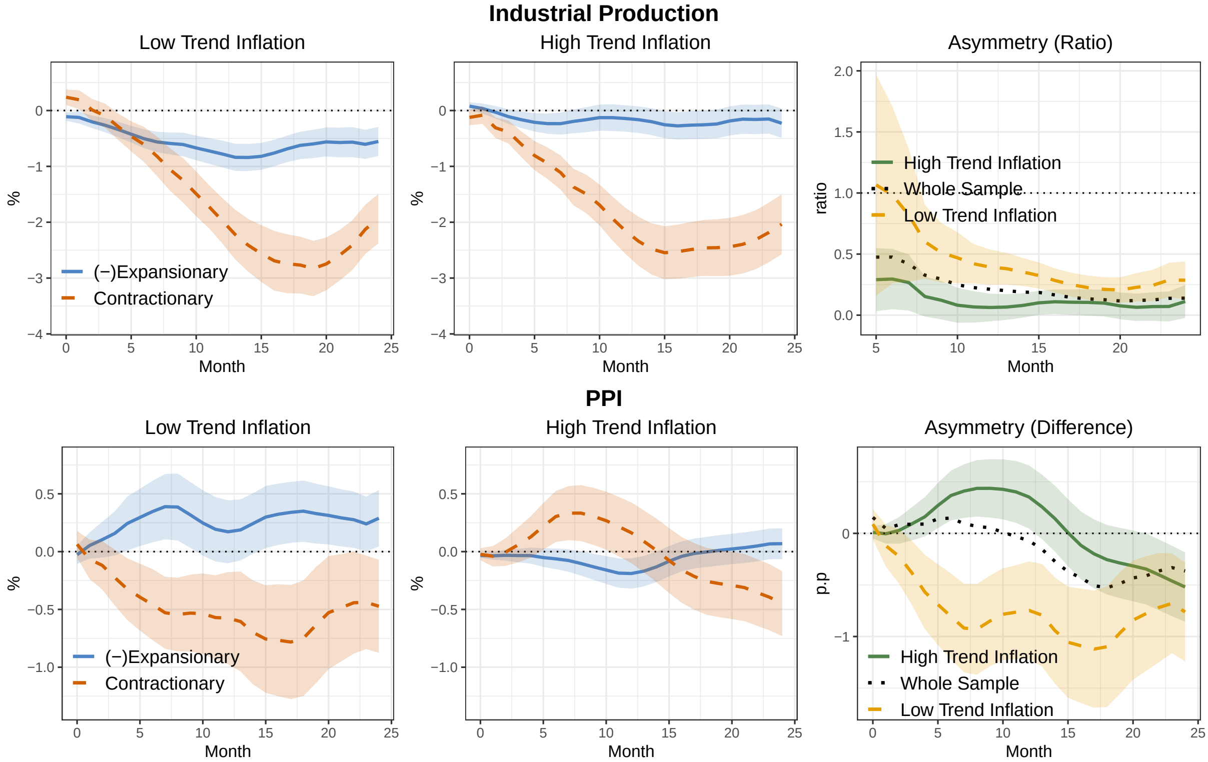


Figure 18: Non-Linearity of Industrial Production Responses, Trimming Top and Bottom 15%

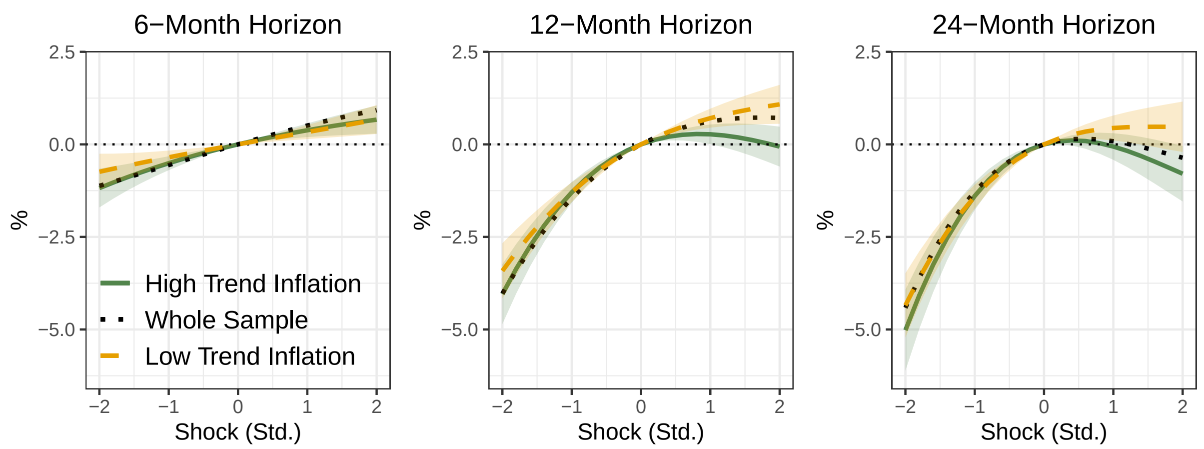


Figure 19: Asymmetry, No Trimming

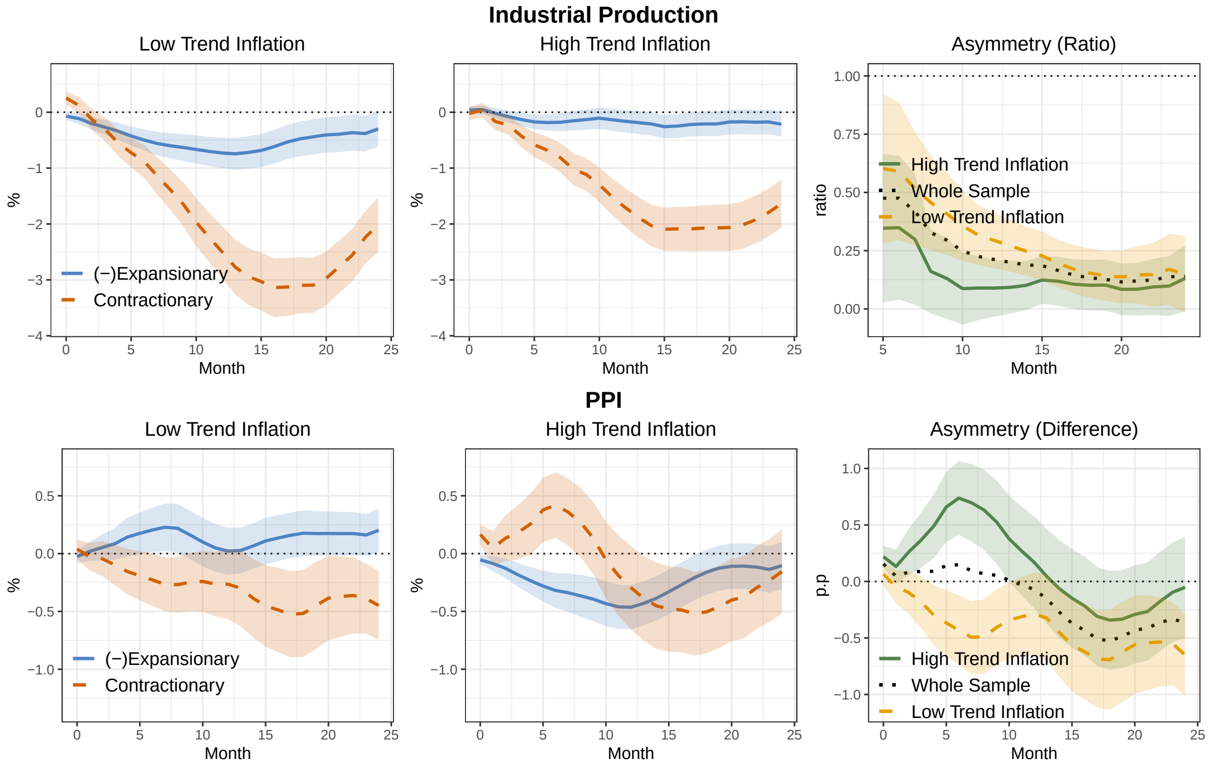
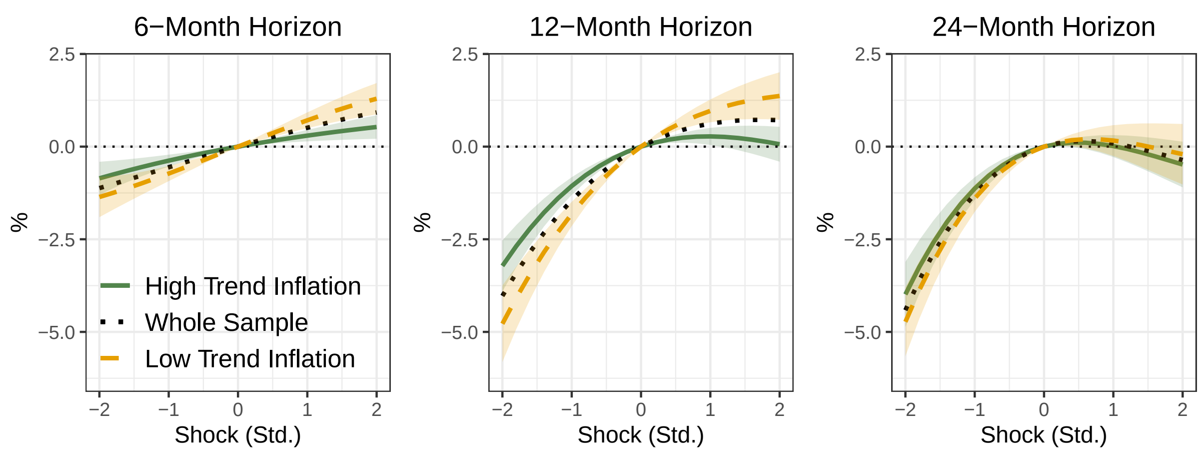
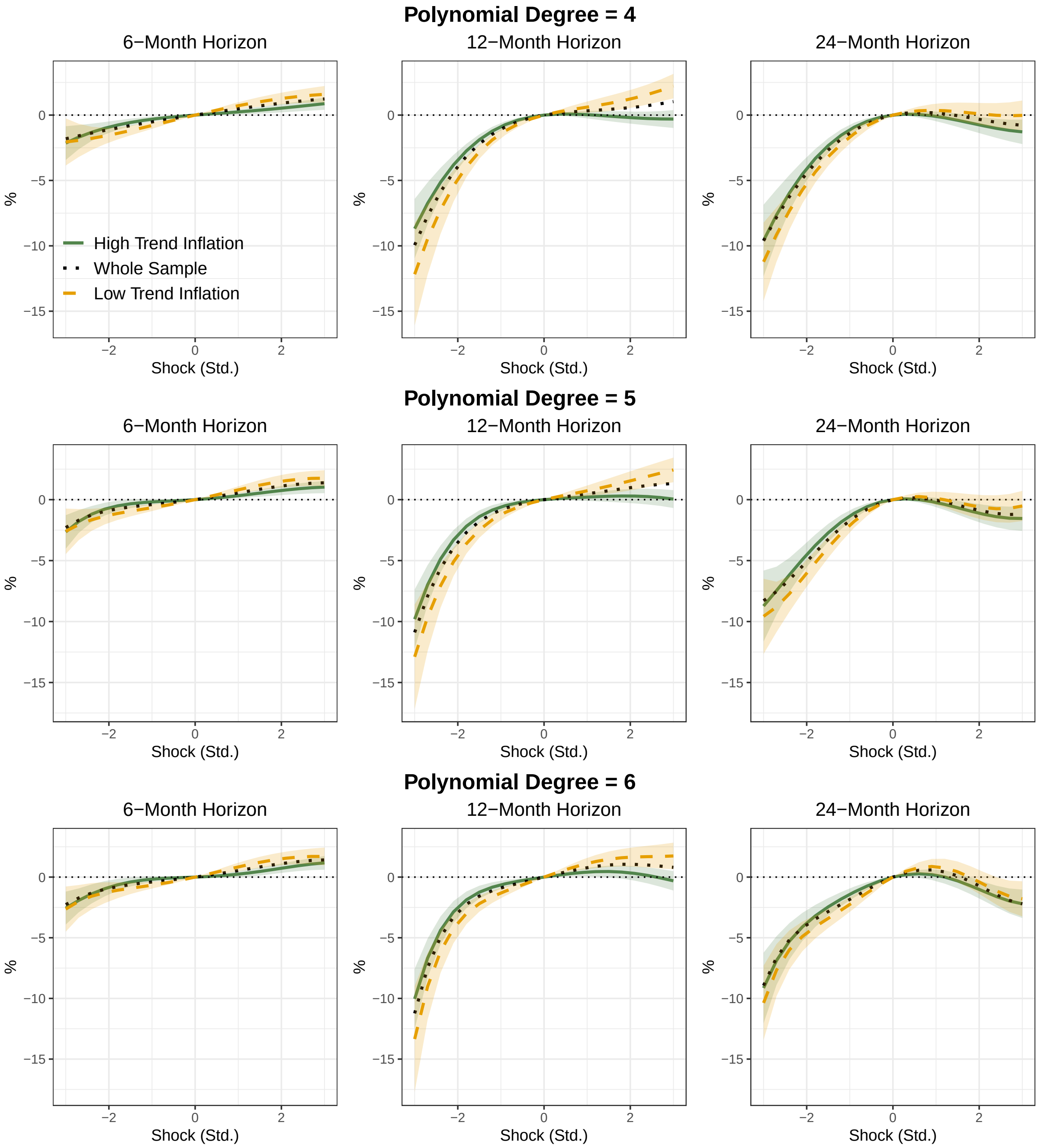


Figure 20: Non-Linearity of Industrial Production Responses, No Trimming



C.2.5 Varying Polynomial Degree

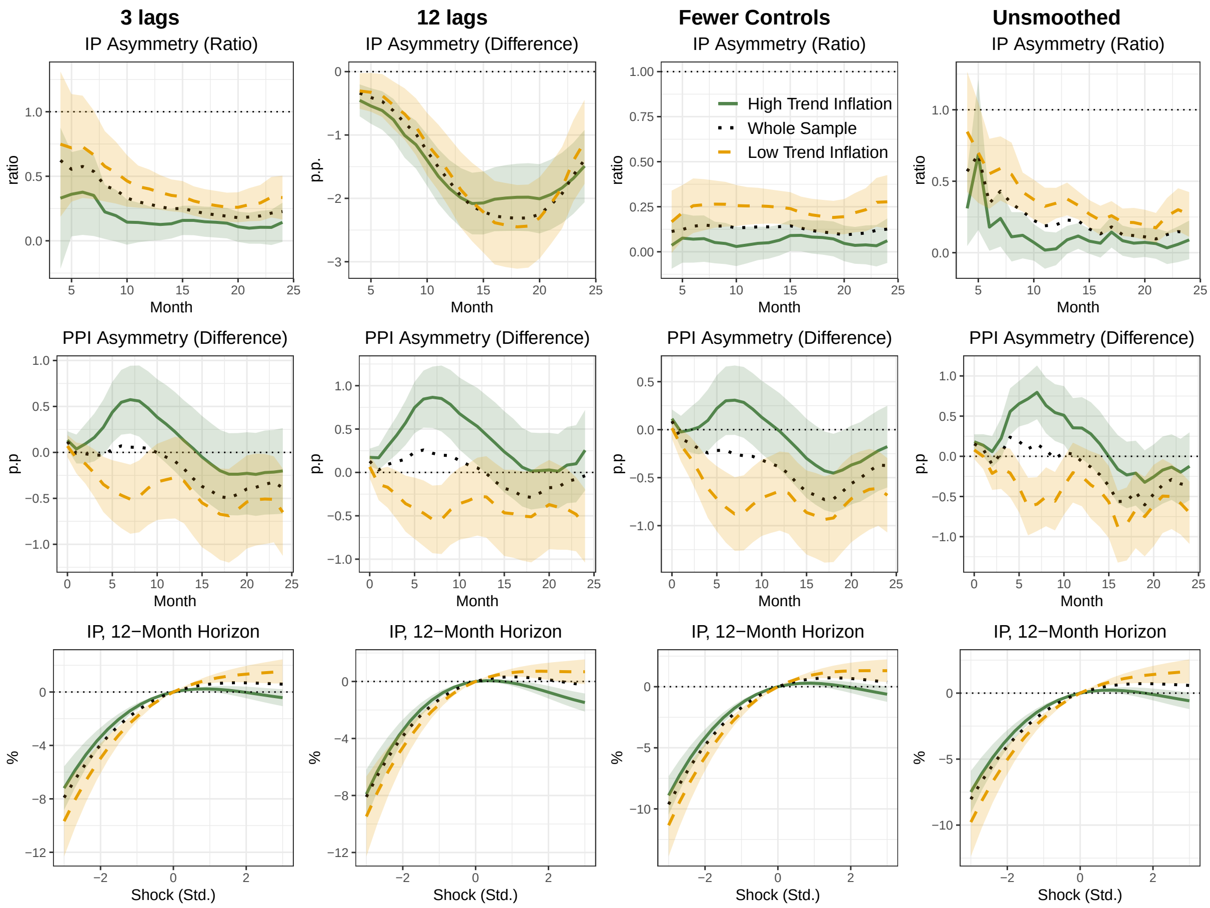
Figure 21: Non-Linearity of Industrial Production Impulse Responses



C.2.6 Other

To ease comparison, I only plot asymmetries (1st row: industrial production, 2nd row: PPI) and non-linear IP responses at a 12-months horizon (3rd row) for each alternative specification by column: (1) number of lags is set to 3, (2) number of lags is set to 12, (3) set of controls consists of a time trend and lags of the dependent variable, monetary shock and effective federal funds rate, (4) unsmoothed impulse responses. Whenever possible, I use the preferred measure of asymmetry (ratio), otherwise I compute asymmetry as difference. The main results of the paper are unchanged.

Figure 22: Other Robustness Checks



D General Equilibrium

D.1 Equilibrium along the transition path

Deterministic dynamics after transitory shocks, considered in this paper, introduce three changes relative to the stationary equilibrium. Firstly, the changing markup makes firms profits time dependent. Secondly, the drift in firms optimal price is also affected by the moving markups and the nominal wage. Finally, aggregate consumption can not be omitted from the firms problem, as it is no longer constant.

Denote the time-dependent drift in firms optimal price by $\mu_t = \left(d\log M_t + d\log\left(\frac{\theta_t}{\theta_t - 1}\right)\right)/dt$. The value function of a firm becomes time-dependent:

$$(\rho+\lambda)v(z,t) = \pi(z,t) + \lambda v(\hat{z},t) - \mu_t v_z(z,t) + \frac{1}{2}\sigma^2 v_{zz}(z,t) + v_t(z,t)$$

as well as the distribution of price gaps:

$$f_t(z,t) = \mu_t f_z(z,t) + \frac{1}{2}\sigma^2 f_{zz}(z,t) - \lambda f(z,t)$$

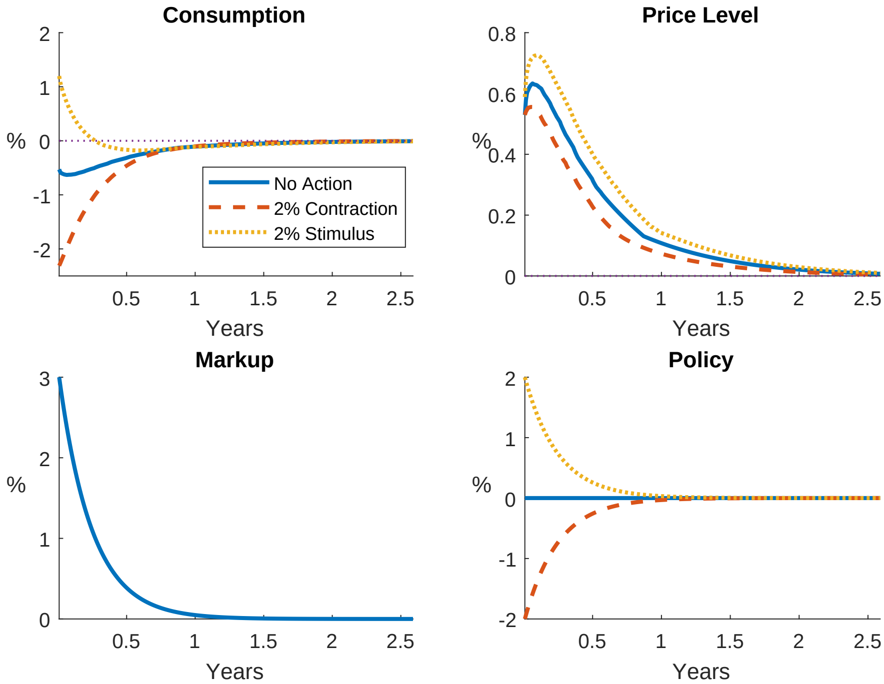
Time-dependent profit and cost functions are:

$$\pi(z,t) = \left(\frac{\alpha\theta_t C_t}{\theta_t - 1}\right)^{1-\theta_t} e^{-\theta_t z}\left(e^z - \frac{\theta_t - 1}{\theta_t}\right)$$
$$c(z,t) = \kappa\left(\frac{\alpha\theta_t C_t}{\theta_t - 1}\right)^{1-\theta_t} e^{(1-\theta_t)z}$$

All other equilibrium objects can be computed as before, substituting constant variables with time-dependent ones. When solving for the transition dynamics, I follow the numerical approach of Achdou et al. (2017).

D.2 Dynamics after Policy Interventions

Figure 23: Markup Shock and Policy Response



Impulse responses of consumption, price level and markup to a 3% markup shock and policy intervention. Solid blue lines correspond to a zero monetary response, dashed red lines – to a 2% contraction, dotted yellow lines – to a 2% expansion. Consumption and markup responses are in terms of percent deviations from the steady state, price level responses are in terms of percent deviations from the trend.

D.3 Alternative Calibration

I now target the same values of price adjustment frequency and average size of adjustment, but consider a lower target for kurtosis, setting it to 3. The calibrated values in annual terms for σ, κ and λ are now 0.142, 0.048 and 1.03. The next two figures show policymaker's frontiers after 3% and 10% markup shocks, same as those considered under the baseline calibration.

Figure 24: Frontiers, Small Shock

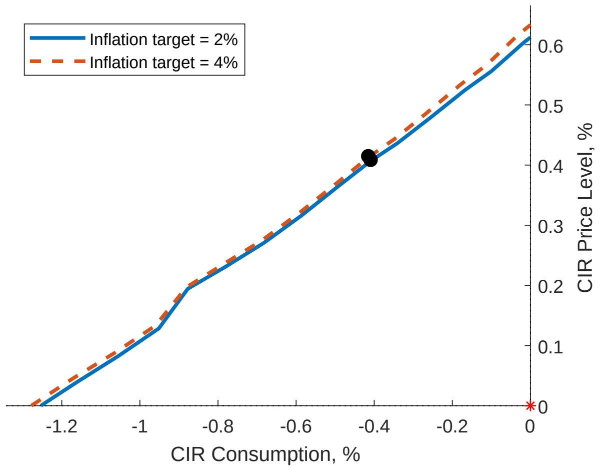
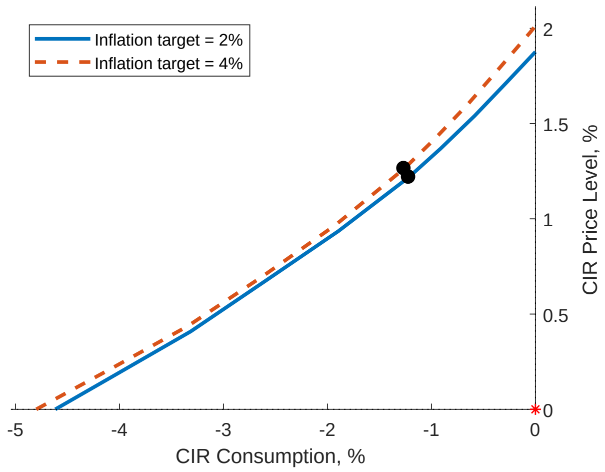


Figure 25: Frontiers, Large Shock



Increasing inflation target from 2% to 4% amplifies the response to the markup shock by 1.2% when the shocks is small (2%) and by 4.1% when the shocks is large (10%). At the same time, the curvature of the frontier increases by 7.4% for the small shock and by 10.2% for the large shock. Thus, the effect of trend inflation remains quantitatively sizable under an alternative calibration.

D.4 Using Inflation CIR

Here I consider an alternative frontier of the monetary authority, defined in terms of the usual consumption CIR and a cumulative response of absolute values of inflation: $\int_0^\infty |\pi_t - \mu| dt$. The next two figures show policymaker's frontiers after 3% and 10% initial markup disturbances, same as those considered under the baseline specification.

Figure 26: Frontiers, Small Shock

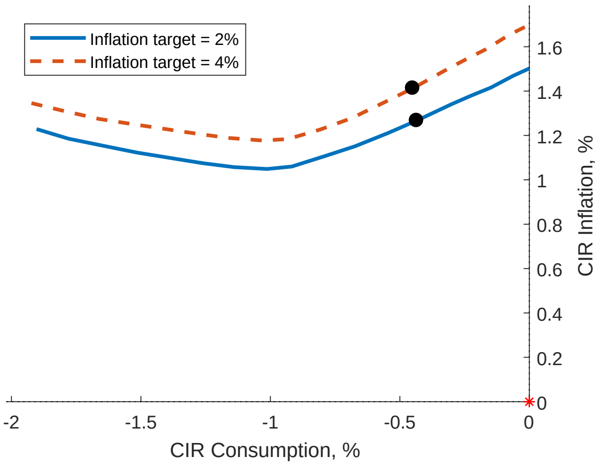
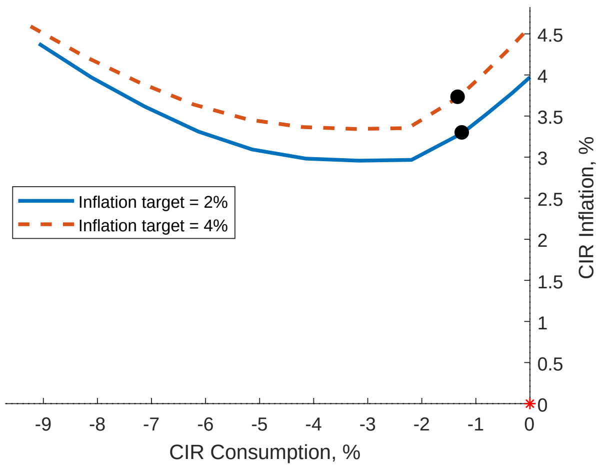


Figure 27: Frontiers, Large Shock



In both cases a higher trend inflation leads to a larger initial response to the markup shock in terms of both consumption and inflation deviations. In addition, monetary authority becomes more constrained in stabilizing inflation, as some levels of inflation CIR become infeasible (the red dashed lines lie above the blue ones).