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The Tension between Market Shares and Profit Under Platform Competition

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The tension between market shares and profit under platform competition*

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Abstract

We introduce asymmetries across platforms in the linear model of competing two-sided platforms with singlehoming on both sides and fully characterize the price equilibrium. We identify market environments in which one platform has a larger market share on both sides while obtaining a lower profit than the other platform. This is compatible with higher price-cost margins on one or both sides, noting that in the latter case one margin must be negative. Our finding raises further doubts on using market shares as a measure of market power in platform markets.

Keywords: Two-sided platforms, market share, market power, oligopoly, network effects, antitrust

JEL-Classification: D43, L13, L86

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1 Introduction

The definition and measurement of market power are central issues in the theory of industrial organization. It is customary to define market power as a firm's ability to profitably set a price above marginal cost. As a consequence, the absolute price-cost margin (or, more common, the Lerner index as the ratio of price-cost margin over price) gives a *direct* measure of market power. On the other hand, market share and profit can be seen as *indirect* measures of market power. In industries that can be represented by standard oligopoly models of price competition with horizontally differentiated products (e.g., the asymmetric Hotelling duopoly and the logit demand oligopoly), these measures can be used interchangeably as there is a positive association between them: Within an industry, a larger firm also has a larger absolute price-cost margin and higher profit (see Remark 1 in Section 3). Therefore, in such industries, market power is clearly defined and the various indicators proposed to measure it can be used interchangeably.

In industries with two-sided platforms, defining and measuring market power proves more problematic. Two-sided platforms offer products or services to two distinct groups of users and are thus active on two markets. The presence of cross-group network effects creates an interrelation between the prices that platforms set on these two markets. This interrelation may also drive platforms to decrease the price on one side – possibly even below the marginal cost – to increase revenues on the other side. Defining market power as the ability to raise price above marginal cost is therefore inappropriate in these industries: First, several prices – and not just one – need to be jointly considered; second, *reducing* price *below* marginal cost may increase profitability. Besides this problem surrounding the *definition* of market power, which has already been amply discussed in the literature,[1] we argue in this paper that industries with two-sided platforms also raise serious issues regarding the *measurement* of market power of different firms within an industry.

The main problem is that the three measures of market power – price-cost margins, market shares, and profit – do not always go hand-in-hand in industries with two-sided platforms. A platform with higher profits than its rival can be seen as reflecting a high degree of "platform market power" considering the two markets in conjunction. We show that *a two-sided platform may have larger market shares on both sides, while obtaining a lower profit than its rival.* Moreover, in such cases, the price-cost margins of the larger platform must be higher on at least one side – when one margin is negative, margins may be higher on both sides. It follows that market shares and also price-cost margins cannot be taken as adequate measures of platform market power if this is associated with high profitability.

To establish these results, we reconsider a workhorse model in the two-sided platform literature, namely the two-sided singlehoming duopoly model with access fees proposed by Armstrong (2006). Market shares on each side correspond to the fraction of users that join a platform and platforms incur a cost for each active user.[2] In this model, platforms simultaneously choose

[1] For early discussions, see Evans (2003) and Wright (2004).

[2] This is different from platforms that monetize through taxing transactions on the platform and that incur costs that depend on the transaction volume.

first their access fees on both sides and, next, users on each side decide simultaneously which platform to join. Contrary to Armstrong (2006), we do not postulate symmetry but we allow platforms to differ in all relevant parameters (costs, stand-alone benefits for each user group and cross-group network effects enjoyed by the two user groups). In spite of the large number of parameters, exploiting the linearity in this model makes it possible to reduce the parameter space and to fully characterize the subgame-perfect Nash equilibrium of the two-stage game.

Within this richer framework, we show that under some conditions, there are equilibria such that one platform with a "competitive advantage" (that is, lower marginal cost, larger stand-alone benefits, or larger cross-group network effects) on at least one side has a larger market share on both sides but obtains a lower profit than the rival platform. The presence of cross-group network effects (on at least one side) is necessary for this result, as well as different price-sensitivities across the two sides.[3] This result sharply contrasts with the finding mentioned before that, in standard oligopoly, the more profitable firm must be the larger firm, must enjoy higher price-cost margins, and must have an advantage of some sort. Introducing direct network effects in a Hotelling setting cannot generate the tension between size and profit either: The firm with the higher market share is necessarily the more-profitable firm.[4] This shows that our result is specific to competition between two-sided platforms: Market share can be a particularly poor measure of market power or success in markets with two-sided platforms. In particular, we show that the tension between market shares and profit can arise when both firms decide not to subsidize any user (that is, they set prices above marginal cost).

Related literature. We are not aware of previous work addressing the relationship between market shares, price-cost margins and profits in markets with two-sided platforms. In most of the literature, it is assumed that platforms are symmetric (giving rise to symmetric market shares and profits at equilibrium); this includes Rochet and Tirole (2003, 2006) and Hagiu (2006) among others, as well as, in a model with Hotelling demand, Armstrong (2006) and Armstrong and Wright (2007),[5] and, in an oligopoly model of two-sided singlehoming with more general demand, Tan and Zhou (2021). While some papers allow for asymmetric two-sided platforms (e.g., Viecens, 2006; Ambrus and Argenziano, 2009; Njoroge et al., 2009, 2010; Gold, 2010; Lin, Li, and Whinston, 2011; Ponce, 2012; Gabszewicz and Wauthy, 2014; Belleflamme and Toulemonde, 2018; Jullien and Pavan, 2019; Anderson and Peitz, 2020), these papers did not investigate the link between market shares and price-cost margins or profit. The only paper – to the best of our knowledge – that assesses the link between market shares and profits is Sato

[3]What is *not* necessary for this result is that platforms choose prices on *both* sides. We also show that under some conditions the same result holds in settings in which platforms are forced to set a zero price on one side, which deprives them of one pricing instrument.

[4]Calculations are available from the authors upon request.

[5]Armstrong (2006) also considers a model in which users on one side singlehome and users on the other side multihome; for a comparison of single- and multihoming on one side, see Belleflamme and Peitz (2019). Bakos and Halaburda (2020) consider the model in which both sides can multihome and some users on each side do so in equilibrium.

(2021), which replicates our main finding under logit demand – that is, if a platform has larger market shares than its rival, this does not necessarily imply that it makes higher profits.

On the empirical side, perhaps closest is Argentesi and Filistrucchi (2007) who assess the market power of Italian newspapers by estimating a structural oligopoly model of competing platforms under the assumption that both sides – readers and advertisers – singlehome and that readers are not affected by the level of advertising. In particular, they find that readers are subsidized.[6]

In Section 2, we develop the model. In Section 3, we characterize the equilibrium. In Section 4, we provide conditions under which, in equilibrium, one platform attracts more users on both sides but obtains lower profit than the other platform and characterize possible outcomes with respect to market shares, price-cost margins, and profit. In Section 5, we discuss the policy implications of our findings. We conclude in Section 6. Several proofs and the derivations of the numerical examples are relegated to the Appendix.

2 The model

We adapt the models of Armstrong (2006) and Armstrong and Wright (2007). Two platforms are located at the extreme points of the unit interval: Platform 1 is located at 0, while platform 2 is located at 1. Platforms facilitate the interaction between two groups of users, noted a and b. Both groups are assumed to be of mass 1 and uniformly distributed on $[0, 1]$. We assume that users of both sides can join at most one platform (i.e., there is "two-sided singlehoming"); in the real world, singlehoming environments may result from indivisibilities and limited resources or from contractual restrictions.[7]

A user derives a net utility from joining a platform that is defined as the addition of four components: (i) A stand-alone benefit, (ii) a cross-group network benefit, (iii) a "transportation cost", and (iv) an access fee. The first two components enter the net utility function positively. They correspond to two types of services that a platform offers to its users. Some services facilitate the interaction with the other group; the utility they give is the cross-group network benefit, which is assumed to increase linearly with the number of users of the other group present on the platform.[8] A platform also offers other services that do not relate to the interaction between the groups; these services give a stand-alone benefit to users. The last two components enter the net utility function negatively. In the usual Hotelling fashion, a user incurs a disutility from not being able to use a platform that corresponds to their ideal definition of a platform; this disutility is assumed to increase linearly with the distance separating the user's and the platform's location on the unit line (at a rate that can be interpreted as a measure of the

[6]Another empirical paper with two-sided single-homing is Kaiser and Wright (2006) – they focus on the price structure in a number of duopoly markets for magazines in Germany.

[7]For a discussion, see Case 22.4 in Belleflamme and Peitz (2010, p. 633).

[8]We focus in this paper on *positive* cross-group network effects; that is, each group positively values the participation of the other group on the platform.

horizontal differentiation between the platforms in the eyes of a particular group of users). Finally, users have to pay a flat fee to access the platform.[9]

We allow all components of the utility function to differ not only across sides but also across platforms. We therefore write the net utility functions for a user of group a and for a user of group b, respectively located at x_a and $x_b \in [0,1]$ as:

$$\begin{array}{ll} U_a^1(x_a) = r_a^1 + \beta_a^1 n_b^1 - \tau_a x_a - p_a^1 & \text{if joining platform 1,} \\ U_a^2(x_a) = r_a^2 + \beta_a^2 n_b^2 - \tau_a (1-x_a) - p_a^2 & \text{if joining platform 2,} \\ U_b^1(x_b) = r_b^1 + \beta_b^1 n_a^1 - \tau_b x_b - p_b^1 & \text{if joining platform 1,} \\ U_b^2(x_b) = r_b^2 + \beta_b^2 n_a^2 - \tau_b (1-x_b) - p_b^2 & \text{if joining platform 2,} \end{array}$$

where β_j^i measures the value that a user of group j attaches to interacting with an extra user of the other group on platform i,[10] n_j^i is the mass of users of group j that decide to join platform i, r_j^i is the valuation of the stand-alone benefit by users of group j on platform i, τ_j is the "transportation cost" parameter for group j, and p_j^i is the access fee that platform i sets for users of group j (with $j \in \{a,b\}$ and $i \in \{1,2\}$).

Let $\hat{x}_j$ identify the user of group j who is indifferent between joining platform 1 or platform 2; that is, $U_a^1(\hat{x}_a) = U_a^2(\hat{x}_a)$ and $U_b^1(\hat{x}_b) = U_b^2(\hat{x}_b)$. Solving these equalities for $\hat{x}_a$ and $\hat{x}_b$ respectively, we have:

$$\begin{array}{l} \hat{x}_a = \frac{1}{2} + \frac{1}{2\tau_a}\left(\beta_a^1 n_b^1 - \beta_a^2 n_b^2 + r_a^1 - r_a^2 - \left(p_a^1 - p_a^2\right)\right), \\ \hat{x}_b = \frac{1}{2} + \frac{1}{2\tau_b}\left(\beta_b^1 n_a^1 - \beta_b^2 n_a^2 + r_b^1 - r_b^2 - \left(p_b^1 - p_b^2\right)\right). \end{array}$$

In what follows, we assume that stand-alone and cross-group network benefits are sufficiently large to make sure that all users join one platform. Both sides are then fully covered, so that $n_j^1 + n_j^2 = 1$ $(j = a,b)$. This entails the following equalities: $\hat{x}_a = n_a^1 = 1 - n_a^2$ and $\hat{x}_b = n_b^1 = 1 - n_b^2$. We introduce some additional notation that will prove useful in the rest of the analysis. Supposing that users on both sides split equally across platforms and considering the realized cross-group network benefits exerted on users of group j, we denote the sum of these effects across platforms as $\Sigma b_j \equiv (\beta_j^1 + \beta_j^2)/2$, and their difference between platforms 1 and 2 as $\Delta b_j \equiv (\beta_j^1 - \beta_j^2)/2$.[11] We also define $\Delta r_j \equiv r_j^1 - r_j^2$ as the difference in stand-alone benefits on side j between platforms 1 and 2. Using these equalities and notation, we can solve the above systems of equations for n_a^1 and n_b^1:

$$n_a^1 = \frac{1}{2} + \frac{\tau_b}{2}\frac{\Delta r_a + \Delta b_a + p_a^2 - p_a^1}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)} + \frac{\Sigma b_a}{2}\frac{\Delta r_b + \Delta b_b + p_b^2 - p_b^1}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)}, \tag{1}$$

$$n_b^1 = \frac{1}{2} + \frac{\tau_a}{2}\frac{\Delta r_b + \Delta b_b + p_b^2 - p_b^1}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)} + \frac{\Sigma b_b}{2}\frac{\Delta r_a + \Delta b_a + p_a^2 - p_a^1}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)}. \tag{2}$$

[9] For simplicity and to stay within the well-known model by Armstrong (2006), we do not consider usage fees, nor two-part tariffs. See Reisinger (2014) on platform competition with two-part tariffs.

[10] Equivalently, β_j^i measures the intensity of the cross-group network effects that the other group exerts on users in group j on platform i.

[11] By definition, $\beta_j^1 = \Sigma b_j + \Delta b_j$ and $\beta_j^2 = \Sigma b_j - \Delta b_j$.

To ensure that participation on each side is a decreasing function of the access fee on this side, we assume the following:

$$\tau_a \tau_b > (\Sigma b_a)(\Sigma b_b). \tag{3}$$

This assumption, which is common in the analysis of competition between two-sided platforms, says that the strength of cross-group network effects – measured by $(\Sigma b_a)(\Sigma b_b)$ – is smaller than the strength of horizontal differentiation – measured by $\tau_a \tau_b$.[12]

As for platforms, we assume that they face constant costs per user. These costs may also differ across sides and across platforms; we note them c_j^i for group $j \in \{a, b\}$ and platform $i \in \{1, 2\}$. For future reference, we define $\Delta c_j \equiv c_j^1 - c_j^2$ as the difference in costs on side j between platforms 1 and 2. Before solving the model, we introduce one last piece of notation (with $j \in \{a, b\}$ and $i \in \{1, 2\}$):

$$s_j^i \equiv r_j^i - c_j^i + \tfrac{1}{2}\beta_j^i \text{ and } \Delta s_j \equiv s_j^1 - s_j^2 = \Delta r_j - \Delta c_j + \Delta b_j,$$

where s_j^i is the surplus generated from participating on side j of platform i when users split equally across platforms (gross of transportation costs), and Δs_j, if positive, can be seen as a measure of the "competitive advantage" that platform 1 has over platform 2 on side j; this advantage may follow from larger stand-alone benefits ($\Delta r_j > 0$), smaller costs ($\Delta c_j < 0$) or larger cross-group network effects ($\Delta b_j > 0$); if Δs_j is negative, then it is platform 2 that has an advantage on side j.

3 Equilibrium of the pricing game

Platforms simultaneously choose their access prices to maximize their profit, given by $\Pi^1 = (p_a^1 - c_a^1)n_a^1 + (p_b^1 - c_b^1)n_b^1$ and $\Pi^2 = (p_a^2 - c_a^2)n_a^2 + (p_b^2 - c_b^2)n_b^2$.

Remark 1 *As a backdrop, consider the setting without cross-group network effect. Here, we can allow for more general demand in which the location of the marginal user of group j depends only on the price difference $\Delta p_j \equiv p_j^2 - p_j^1$. Thus, the pricing problems for the two user groups are independent and it is sufficient to solve for the equilibrium when each firm maximizes group-j profit $\Pi_j^i = (p_j^i - c_j^i)n_j^i$ with respect to p_j^i. The first-order conditions are $-(p_j^i - c_j^i)\frac{d\hat{x}_j}{d(\Delta p_j)} + n_j^i = 0$. In equilibrium we must have:*

$$\frac{p_j^1 - c_j^1}{n_j^1} = \frac{1}{d\hat{x}_j / d(\Delta p_j)} = \frac{p_j^2 - c_j^2}{n_j^2}$$

and thus

$$\frac{p_j^1 - c_j^1}{p_j^2 - c_j^2} = \frac{n_j^1}{n_j^2}.$$

[12]This assumption also ensures that both platforms are active at the unique equilibrium of the game. Hence, it rules out the possibility of multiple equilibria, with either platform attracting all users on both sides. Otherwise (when the market is said to have "tipped"), the tension we study would not be observed as the platform that corners the market would trivially have a larger market share and a larger profit than the platform that does not manage to attract any user.

This shows that the firm with a higher price-cost margin must have a larger market share (in terms of units sold). This firm is obviously the more profitable firm. Hence, within an industry, price-cost margins, market shares, and profits are positively associated.[13]

We note that, in general, there is no clear association between the Lerner index $(p_j^i - c_j^i)/p_j^i$ *and market share. However, restricting attention to symmetric costs* ($c_j^1 = c_j^2$, $j \in \{a, b\}$), *the firm with the higher Lerner index has the larger price-cost margin and thus the larger market share. Hence, with symmetric costs, also Lerner index and market share are positively associated.*

In the model with cross-group network effects, we are looking for a solution to the first-order conditions:

$$\frac{d\Pi^1}{dp_a^1} = \frac{d\Pi^1}{dp_b^1} = \frac{d\Pi^2}{dp_a^2} = \frac{d\Pi^2}{dp_b^2} = 0.$$

The second-order conditions require:

$$\tau_a \tau_b \geq (\Sigma b_a)(\Sigma b_b) \text{ and } \tau_a \tau_b \geq \tfrac{1}{4}(\Sigma b_a + \Sigma b_b)^2 .$$

To satisfy both Assumption (3) and the second-order conditions, we impose from now on:

$$\tau_a \tau_b > \tfrac{1}{4}(\Sigma b_a + \Sigma b_b)^2 . \tag{4}$$

We next solve the system of the four first-order conditions. To facilitate the exposition, we define:

$$D \equiv 9\tau_a \tau_b - (2\Sigma b_a + \Sigma b_b)(\Sigma b_a + 2\Sigma b_b),$$

which is positive according to Assumption (4). The equilibrium price of platform 1 on side a is found as:

$$\begin{aligned} p_a^{1*} &= \overbrace{c_a^1 + \tau_a}^{\mathbf{H}} \overbrace{-\Sigma b_b}^{\mathbf{A}} \overbrace{+\tfrac{1}{3}(\Delta r_a - \Delta c_a)}^{\mathbf{Vs}} \overbrace{+\tfrac{1}{3}\Delta b_a}^{\mathbf{Vn}} \\ &\quad \underbrace{+ \frac{\Sigma b_a - \Sigma b_b}{3D}\left[(2\Sigma b_a + \Sigma b_b)\Delta s_a + 3\tau_a \Delta s_b\right]}_{\mathbf{I}}. \end{aligned}$$

We can decompose it as the sum of five components: (i) **H** is the classic *Hotelling* formula (marginal cost + transportation cost); (ii) **A** was identified by Armstrong (2006) in a symmetric setting as the price adjustment due to cross-group network effects (the price is decreased by the externality exerted on the other side); (iii) **Vs** is the quality effect in terms of *stand-alone* benefits (or the effect of marginal costs differences);[14] (iv) **Vn** is the quality effect in terms of cross-group *network* effects on the side under review; (v) the last term **I** results from the interplay between vertical differentiation and cross-group network effects. If platforms are symmetric

[13]This result carries over to n-firm oligopoly with logit demand; see Anderson, de Palma, and Thisse (1992, equation 7.24) and Anderson and de Palma (2001). The latter also state that the result holds when introducing an outside option.

[14]In this model r_j^i and c_j^i play interchangeable roles. What matters is their difference on each platform: $r_j^i - c_j^i$, with $\Delta r_j - \Delta c_j = (r_j^1 - c_j^1) - (r_j^2 - c_j^2)$.

($\Delta r_j = \Delta c_j = \Delta b_j = 0$) only **H** and **A** remain; absent network effects ($\Sigma b_j = \Delta b_j = 0$), only **H** an **Vs** remain. In the particular case in which cross-group network effects are (on average) the same on the two sides ($\Sigma b_a = \Sigma b_b$), all terms but the last remain.

Recalling that $\Delta s_j \equiv \Delta r_j - \Delta c_j + \Delta b_j$, we can rewrite platform 1's equilibrium margin on side a as follows:

$$p_a^{1*} - c_a^1 = \tau_a - \Sigma b_b + \tfrac{1}{3}\Delta s_a + \frac{\Sigma b_a - \Sigma b_b}{3D}\left(\left(2\Sigma b_a + \Sigma b_b\right)\Delta s_a + 3\tau_a \Delta s_b\right).$$

The other equilibrium margins are found by analogy:

$$\begin{aligned}
p_b^{1*} - c_b^1 &= \tau_b - \Sigma b_a + \tfrac{1}{3}\Delta s_b + \frac{\Sigma b_b - \Sigma b_a}{3D}\left(\left(2\Sigma b_b + \Sigma b_a\right)\Delta s_b + 3\tau_b \Delta s_a\right), \\
p_a^{2*} - c_a^2 &= \tau_a - \Sigma b_b - \tfrac{1}{3}\Delta s_a - \frac{\Sigma b_a - \Sigma b_b}{3D}\left(\left(2\Sigma b_a + \Sigma b_b\right)\Delta s_a + 3\tau_a \Delta s_b\right), \\
p_b^{2*} - c_b^2 &= \tau_b - \Sigma b_a - \tfrac{1}{3}\Delta s_b - \frac{\Sigma b_b - \Sigma b_a}{3D}\left(\left(2\Sigma b_b + \Sigma b_a\right)\Delta s_b + 3\tau_b \Delta s_a\right).
\end{aligned}$$

We can now use the equilibrium prices to compute the equilibrium mass of users of the two groups on the two platforms:

$$\begin{aligned}
n_a^{1*} &= \tfrac{1}{2} + \tfrac{1}{2D}\left(3\tau_b \Delta s_a + \left(\Sigma b_a + 2\Sigma b_b\right)\Delta s_b\right), \; n_a^{2*} = 1 - n_a^{1*}, \\
n_b^{1*} &= \tfrac{1}{2} + \tfrac{1}{2D}\left(3\tau_a \Delta s_b + \left(2\Sigma b_a + \Sigma b_b\right)\Delta s_a\right), \; n_b^{2*} = 1 - n_b^{1*}.
\end{aligned}$$

To guarantee that the equilibrium mass is strictly positive and lower than unity, we impose the following restrictions on the space of parameters:

$$3\tau_b \Delta s_a + \left(\Sigma b_a + 2\Sigma b_b\right)\Delta s_b \text{ and } 3\tau_a \Delta s_b + \left(2\Sigma b_a + \Sigma b_b\right)\Delta s_a \in (-D, D). \tag{5}$$

Using the equilibrium values of prices and number of users, we find the equilibrium profits:[15]

$$\begin{aligned}
\Pi^{1*} = {} & \frac{1}{2}\left(\tau_a + \tau_b - \Sigma b_a - \Sigma b_b\right) + \frac{1}{2D}\left(\tau_b\left(\Delta s_a\right)^2 + \tau_a\left(\Delta s_b\right)^2\right) + \frac{1}{2D}\left(\Sigma b_a + \Sigma b_b\right)\left(\Delta s_a\right)\left(\Delta s_b\right) \\
& + \frac{1}{2D}\left(6\tau_a\tau_b + \tau_b\left(\Sigma b_a - \Sigma b_b\right) - \left(\Sigma b_a + \Sigma b_b\right)\left(2\Sigma b_a + \Sigma b_b\right)\right)\Delta s_a \\
& + \frac{1}{2D}\left(6\tau_a\tau_b - \tau_a\left(\Sigma b_a - \Sigma b_b\right) - \left(\Sigma b_a + \Sigma b_b\right)\left(\Sigma b_a + 2\Sigma b_b\right)\right)\Delta s_b,
\end{aligned} \tag{6}$$

$$\begin{aligned}
\Pi^{2*} = {} & \frac{1}{2}\left(\tau_a + \tau_b - \Sigma b_a - \Sigma b_b\right) + \frac{1}{2D}\left(\tau_b\left(\Delta s_a\right)^2 + \tau_a\left(\Delta s_b\right)^2\right) + \frac{1}{2D}\left(\Sigma b_a + \Sigma b_b\right)\left(\Delta s_a\right)\left(\Delta s_b\right) \\
& - \frac{1}{2D}\left(6\tau_a\tau_b + \tau_b\left(\Sigma b_a - \Sigma b_b\right) - \left(\Sigma b_a + \Sigma b_b\right)\left(2\Sigma b_a + \Sigma b_b\right)\right)\Delta s_a \\
& - \frac{1}{2D}\left(6\tau_a\tau_b - \tau_a\left(\Sigma b_a - \Sigma b_b\right) - \left(\Sigma b_a + \Sigma b_b\right)\left(\Sigma b_a + 2\Sigma b_b\right)\right)\Delta s_b.
\end{aligned} \tag{7}$$

[15]The model has the feature that industry profits increase if one platform obtains a larger competitive advantage starting from such an advantage on both sides (that is, if Δs_a or Δs_b becomes larger if they are positive, or smaller if they are negative):

$$\Pi^{1*} + \Pi^{2*} = \left(\tau_a + \tau_b - \Sigma b_a - \Sigma b_b\right) + \frac{1}{D}\left(\tau_b\left(\Delta s_a\right)^2 + \tau_a\left(\Delta s_b\right)^2 + \left(\Sigma b_a + \Sigma b_b\right)\Delta s_a \Delta s_b\right).$$

One remark is necessary before moving on to the analysis of market power indicators. Without loss of generality, let us consider side a. The determinants of the equilibrium variables remain unchanged if we modify c_a^1 and r_a^1 such that Δs_a is not affected. Therefore, this change of parameters leaves the equilibrium values of prices, absolute price-cost margins, quantities and profits unchanged. By contrast, it affects the Lerner index calculated on side a while not changing the Lerner index on side b. This observation makes the use of Lerner index values dubious for assessing the market power of the platform as a whole.

4 Contrasting market power indicators

In this section, we show that it is possible to have contrasting views about the relative market power of the two competing platforms depending on the measure that we use. In particular, we find conditions under which, at equilibrium, one platform attracts more users on both sides but achieves a lower profit than the other platform.

Without any loss of generality, let platform 1 be this platform. For platform 1 to attract more users than platform 2 on both sides, we need:

$$n_a^{1*} - n_a^{2*} = \frac{1}{D}\left[3\tau_b \Delta s_a + (\Sigma b_a + 2\Sigma b_b)\,\Delta s_b\right] > 0, \tag{8}$$

$$n_b^{1*} - n_b^{2*} = \frac{1}{D}\left[3\tau_a \Delta s_b + (2\Sigma b_a + \Sigma b_b)\,\Delta s_a\right] > 0. \tag{9}$$

For platform 1 to achieve a *lower* profit than platform 2, we must have $\Pi^{1*} - \Pi^{2*} < 0$, which is equivalent to:

$$\Pi^{1*} - \Pi^{2*} = \left[2\tau_a - (\Sigma b_a + \Sigma b_b)\right]\left(n_a^{1*} - n_a^{2*}\right) + \left[2\tau_b - (\Sigma b_a + \Sigma b_b)\right]\left(n_b^{1*} - n_b^{2*}\right) < 0. \tag{10}$$

This expression highlights the effects of market shares $(n_j^{1*} - n_j^{2*})$ on platform profits. If conditions (8) and (9) are satisfied, then condition (10) can only be satisfied if either $2\tau_a < \Sigma b_a + \Sigma b_b$ or $2\tau_b < \Sigma b_a + \Sigma b_b$. That is, users on one of the two sides must perceive the two platforms as close enough substitutes, in the sense that the transportation cost parameter on that side, τ_j, must be lower than the average cross-group network effects (across sides), $(\Sigma b_a + \Sigma b_b)/2 = (\beta_a^1 + \beta_a^2 + \beta_b^1 + \beta_b^2)/4$. Recall, that to satisfy the second-order conditions (4), this strong substitutability cannot be observed on both sides: $4\tau_a\tau_b > (\Sigma b_a + \Sigma b_b)^2$ makes it impossible to have $2\tau_a < \Sigma b_a + \Sigma b_b$ *and* $2\tau_b < \Sigma b_a + \Sigma b_b$. An important observation follows from this finding: The three conditions cannot be met jointly if $\Sigma b_a = \Sigma b_b = 0$ or $\tau_a = \tau_b$. Furthermore, the larger but less profitable platform features $n_a^{1*} < n_b^{1*}$ if $\tau_a > \tau_b$.[16]

[16] This is shown as follows. For platform 1 to have larger market shares yet lower profit than its competitor, we need $2\tau_b - (\Sigma b_a + \Sigma b_b) < 0$. By contradiction, suppose that $n_a^{1*} - n_a^{2*} > n_b^{1*} - n_b^{2*}$. Then the left hand side of (10) is larger than:

$$\left[2\tau_a - (\Sigma b_a + \Sigma b_b)\right]\left(n_b^{1*} - n_b^{2*}\right) + \left[2\tau_b - (\Sigma b_a + \Sigma b_b)\right]\left(n_b^{1*} - n_b^{2*}\right) = 2\left[\tau_a + \tau_b - (\Sigma b_a + \Sigma b_b)\right]\left(n_b^{1*} - n_b^{2*}\right).$$

By (4), $\tau_a + \tau_b - (\Sigma b_a + \Sigma b_b) > \tau_a + \tau_b - 2\sqrt{\tau_a\tau_b} = (\sqrt{\tau_a} - \sqrt{\tau_b})^2 > 0$. Thus the left hand side of (10) is positive, a contradiction: Platform 1 has larger profit than its competitor. Hence, we must have $n_a^{1*} - n_a^{2*} < n_b^{1*} - n_b^{2*}$ if $\tau_a > \tau_b$.

The next proposition records what we have learned so far.

Proposition 1 *For a platform to have larger market shares yet lower profit than its competitor, it is necessary that (i) cross-group network effects exist on at least one side (Σb_a and/or $\Sigma b_b > 0$), (ii) users on one side perceive the two platforms as close enough substitutes (τ_a or $\tau_b < (\Sigma b_a + \Sigma b_b)/2$), (iii) the price-sensitivity of demand be different on the two sides ($\tau_a \neq \tau_b$); and (iv) the platform with larger market shares have the largest market share on the side on which the price sensitivity of demand is higher.*

The intuition behind Proposition 1 is the following. The first necessary condition, namely the existence of cross-group network effects, makes a platform's pricing decisions interdependent across the two sides. Suppose instead that cross-group network effects are absent ($\Sigma b_a = \Sigma b_b = 0$). Then, each platform can choose its prices for group a and b independently of one another; platforms solve in fact two distinct profit-maximization problems and competition operates separately on each side, as seen in Remark 1. If a firm has an exogenous advantage (in terms of cost or stand-alone benefits), it receives both a larger market share and a larger profit in equilibrium and the two measures of market power are aligned.[17]

Things change dramatically in the presence of cross-group network effects. To see why, consider platform 1 and suppose that $\beta_a^1 > 0$ (which implies that $\Sigma b_a = (\beta_a^1 + \beta_a^2)/2 > 0$). As users on side a care about the participation of users on side b, participation on side a is sensitive to changes not only in p_a^1 and p_a^2, but also to changes in p_b^1 and p_b^2. It follows that any exogenous advantage that platform 1 may have on one side (i.e., $\Delta s_a > 0$ and/or $\Delta s_b > 0$) affects the strategic pricing of both platforms not only on this side but also on the other side.

As Belleflamme and Toulemonde (2018) show, initial asymmetries across platforms induce complex best-response dynamics in the platform pricing decisions, which is responsible for the tension between market shares and profit that we document here. In particular, a platform may suffer (in terms of profit) from a competitive advantage because this advantage triggers an aggressive price reaction from the other platform. Yet, for this to happen, the strategic interaction between the two platforms must be sufficiently strong. This is what condition (ii) captures: platforms must be close enough substitutes in the eyes of the users. However, the substitutability must be larger on one side than on the other, as condition (iii) stipulates; indeed, if the substitutability between the platforms was large on both sides (compared to the strength of the cross-group network effects), one platform would attract all users at equilibrium (and, in that case, this platform would obviously have a larger market share and a larger profit).

Taking a closer look at situations in which one platform has higher market shares but lower profits than its rival, we first consider environments in which one platform has competitive advantage on both sides. We partially characterize such equilibria in Proposition 2 (the proof is relegated to the Appendix) and then provide a numerical example for such an environment.

[17] If $\Sigma b_a = \Sigma b_b = 0$, condition (10) is equivalent to $\tau_a(n_a^{1*} - n_a^{2*}) + \tau_b(n_b^{1*} - n_b^{2*}) < 0$, which is clearly impossible if both $n_a^{1*} - n_a^{2*} > 0$ and $n_b^{1*} - n_b^{2*} > 0$.

Proposition 2 *Suppose that a platform has a competitive advantage on both sides (i.e., $\Delta s_a > 0$ and $\Delta s_b > 0$). For parameter constellations such that this platform has larger market shares yet lower profits than its rival, the equilibrium is such that (i) this platform makes a positive and larger margin than its rival on the side on which users are less price-sensitive, while (ii) it subsidizes users and makes a smaller margin than its rival on the side on which users are more price-sensitive.*

Proposition 2 applies when a platform has a competitive advantage on both sides such that this translates into larger market shares on both sides but lower profits than its rival. Although it makes larger profits on the side on which competition is less intense (because of a larger market share and a larger margin), it makes lower profits on the other side, on which it necessarily offers subsidies (and to more users than the other platform). We provide a numerical example showing that the set of parameters under which Proposition 2 holds is non-empty. (For this example and the following ones, we explain in Appendix 7.4 how we selected the values of the parameters.)

Example 1 *Equal competitive advantages for the same platform*

Considering the following values

τ_a	τ_b	Δs_a	Δs_b	Σb_a	Σb_b
4	1	0.25	0.25	3.6	0

we find:

	$p_a^{i*} - c_a^i$	$p_b^{i*} - c_b^i$	n_a^{i*}	n_b^{i*}	$\left(p_a^{i*} - c_a^i\right) n_a^{i*}$	$\left(p_b^{i*} - c_b^i\right) n_b^{i*}$	Π^{i*}
$i = 1$	4.655	-2.713	0.582	0.738	2.708	-2.003	0.706
	$\vee$	$\wedge$	$\vee$	$\vee$	$\vee$	$\wedge$	$\wedge$
$i = 2$	3.345	-2.487	0.418	0.262	1.399	-0.651	0.747

Observations. What platform 2 gains less on side a – namely, $(p_a^{2*} - c_a^2)n_a^{2*} - (p_a^{1*} - c_a^1)n_a^{1*} = 1.399 - 2.708 = -1.309$ – is more than compensated by what it *pays* less on side b – namely $(p_b^{2*} - c_b^2)n_b^{2*} - (p_b^{1*} - c_b^1)n_b^{1*} = -0.651 - (-2.003) = 1.352$. ■

We provide three other illustrative examples. Let us preview what these examples will show. In all these examples, Δs_a and Δs_b are of opposite sign. Thus, it is a priori not clear which of the two platforms should be considered to be the more competitive platform overall. In all these examples, one platform has larger market shares on both sides but lower profits. In Example2, we assume that on average, regardless of the group to which she belongs, a user attaches the same value to the addition of an extra member of the other group ($\Sigma b_a = \Sigma b_b \equiv \Sigma b$). Example 3 envisages a situation where the competitive advantage of platform 1 over users of one group is exactly offset by a competitive disadvantage over users of the other group ($\Delta s_a = -\Delta s_b$). Despite the additional parameter restrictions in both examples, the tension between market shares and profit still applies. Example 4 shows that subsidization is not necessary as an equilibrium behavior (i.e., all price-cost margins are positive) for the tension between market shares and profit to arise.

Examples 1, 2, and 4 share the feature that platform 1 enjoys a larger price-cost margin than platform 2 on one side, and a smaller margin on the other. Thus, using price-cost margins as direct measures of market power does not give a clear indication as to which platform enjoys more market power. In example 3, platform 1 has larger market shares on both sides *and* higher price-cost margins on both sides. Thus, both the direct and the indirect measures of market power point towards platform 1. Yet, platform 1 obtains lower profits.

Example 2 *Same average cross-group network effects on both sides*[18]

Considering the following values

τ_a	τ_b	Δs_a	Δs_b	Σb_a	Σb_b
1	5	1	-1.5	2	2

we find:

	$p_a^{i*} - c_a^i$	$p_b^{i*} - c_b^i$	n_a^{i*}	n_b^{i*}	$\left(p_a^{i*} - c_a^i\right) n_a^{i*}$	$\left(p_b^{i*} - c_b^i\right) n_b^{i*}$	Π^{i*}
$i = 1$	-0.667	2.5	0.833	0.583	-0.556	1.458	0.903
	$\vee$	$\wedge$	$\vee$	$\vee$	$\wedge$	$\Vert$	$\wedge$
$i = 2$	-1.333	3.5	0.167	0.417	-0.222	1.458	1.236

Observations. Platform 1 suffers from a handicap towards users who perceive the platforms as being highly differentiated ($\Delta s_b < 0$ and $\tau_b > \tau_a$). Looking at the margins, we find that both platforms subsidize users from group a. Despite a lower subsidy per user, platform 1 spends more in subsidies than platform 2 because it attracts more users from group a (it spends 0.556 in total instead of 0.222 for platform 2). Platform 1 is trying to attract a large number of users from group a to compensate for its weakness vis-à-vis users from group b, who perceive the two platforms as highly differentiated. Both platforms tax users from group b. Platform 2 achieves exactly the same revenues as platform 1 (1.458) by setting a higher margin to a lower market share. As a result, platform 1 earns a lower profit, which nevertheless remains positive ($\Pi^{1*} = 0.903$). ■

[18] One situation that is compatible with this scenario is the following. Platforms are marketplaces linking sellers (or advertisers, group a) and buyers (group b). Sellers sell completely differentiated products. On each platform, each registered buyer makes one transaction with each registered seller; each transaction generates a total value of $2v$. Platforms differ in the way they split the transaction value $2v$ between buyers and sellers: say that sellers gain a larger share than buyers on platform 1, while the opposite prevails on platform 2. In particular, $\beta_a^1 = \beta_b^2 = v + \kappa$ and $\beta_b^1 = \beta_a^2 = v - \kappa$, with $0 < \kappa < v$. It follows that $\Sigma b_a = (\beta_a^1 + \beta_a^2)/2 = \Sigma b_b = (\beta_b^1 + \beta_b^2)/2 = v$, while $\Delta b_a = (\beta_a^1 - \beta_a^2)/2 = \kappa$ and $\Delta b_b = (\beta_b^1 - \beta_b^2)/2 = -\kappa$. If differences in stand-alone benefits and in costs are minor, we will also have that $\Delta s_a > 0$ and $\Delta s_b < 0$.

Example 3 *Opposite competitive advantages*

Considering the following values

τ_a	τ_b	Δs_a	Δs_b	Σb_a	Σb_b
2	8	1	-1	2	5

we find:

	$p_a^{i*} - c_a^i$	$p_b^{i*} - c_b^i$	n_a^{i*}	n_b^{i*}	$\left(p_a^{i*} - c_a^i\right) n_a^{i*}$	$\left(p_b^{i*} - c_b^i\right) n_b^{i*}$	Π^{i*}
$i = 1$	-2.75	6	0.667	0.542	-1.833	3.250	1.417
	$\vee$	$\Vert$	$\vee$	$\vee$	$\wedge$	$\vee$	$\wedge$
$i = 2$	-3.25	6	0.333	0.458	-1.083	2.750	1.667

Observations. Both platforms subsidize the users of group a, who perceive the platforms as not very differentiated. The total value of subsidies distributed is higher for platform 1 despite a lower subsidy per user. Both platforms tax the users of group b by the same amount. It can therefore be noted that platform 1 attracts more users of group b despite a competitive disadvantage for this type of users and a margin identical to that set by platform 2; it achieves this result thanks to the attractiveness it has gained by having attracted a large majority of users of group a. Despite the higher profit it derives from group b users, platform 1 is unable to compensate for its higher cost of attracting group a users; its profit is therefore lower than that of platform 2. This example has the feature that one platform has higher market shares and (weakly) higher price-cost margins on both sides, yet makes lower profits. If profits are the ultimate guide of market power (at least in a static environment), watching out for high market shares or high price-cost margins goes the wrong way in this example. While the price-cost margin is the same across firms on one side of the market, the example is easily modified to generate strictly higher price-cost margins and strictly higher market shares yet lower profit.[19] Clearly, a necessary property for profits to be misaligned with market shares and price-cost margins on both sides is that one of those margins is negative. In this case, a higher price cost margin on one side can lead to higher losses on that side because the respective firm has a larger market share, and profit accumulated over both sides may even be lower. ■

Examples 1 to 3 have a common feature: A large market share for users who are subsidized (i.e., each user on one side pays a price below the cost of serving her) is costly and does not play out well for the overall platform profit even if the platform has a larger market share for "paying" users. As we show in a final example, the tension between market share and profit can arise even when users are not subsidized in equilibrium.

[19] We use the same parameter value except that we set $\tau_b = 7$ instead of 8. In this modified example, we find that $p_a^{1*} - c_a^1 = -\frac{17}{6} > p_a^{2*} - c_a^2 = -\frac{19}{6}$, $p_b^{1*} - c_b^1 = \frac{31}{6} > p_b^{2*} - c_b^2 = \frac{29}{6}$ and $n_a^{1*} = \frac{3}{4}$, $n_b^{1*} = \frac{7}{12}$, while $\Pi^{1*} = \frac{8}{9} < \Pi^{2*} = \frac{11}{9}$.

Example 4 *No subsidies*

Consider the following values for the parameters

τ_a	τ_b	s_a	s_b	b_a	b_b
2	5	1	-1.5	4	1

we find:

	$p_a^{i*} - c_a^i$	$p_b^{i*} - c_b^i$	n_a^{i*}	n_b^{i*}	$\left(p_a^{i*} - c_a^i\right) n_a^{i*}$	$\left(p_b^{i*} - c_b^i\right) n_b^{i*}$	Π^{i*}
$i = 1$	1.333	0.333	0.583	0.5	0.778	0.167	0.944
	$\vee$	$\wedge$	$\vee$	$\Vert$	$\vee$	$\wedge$	$\wedge$
$i = 2$	0.667	1.667	0.417	0.5	0.278	0.833	1.111

Observations. We consider here the borderline case in terms of market shares in which platforms have equal market share on one side, $n_b^{1*} - n_b^{2*} = 0$. This assumption allows us to pin down the relationship between Δs_a and Δs_b and build the example by focusing on Δs_a. We manage to find parameter values such that platform 1 has a lower profit that its rival in spite of its dominance in terms of market shares, and both platforms have positive margins at equilibrium. We see that platform 2 has a larger margin than platform 1 on side b, on which platforms equally share the market. This allows platform 2 to make up for the disadvantage it faces on side a (where it has both a smaller market share and a smaller margin than platform 1). In Appendix 7.4, we show that we can obtain similar results with $n_b^{1*} > n_b^{2*}$. ■

Focusing on situations in which market shares as indirect measures of market power point towards platform 1, we have seen in the examples that price-cost margins as direct measures of market power may point in the same direction or provide a mixed picture. Yet, in all examples, the larger platform obtains lower profits.

Is it possible that the larger platform obtains lower price-cost margins on both sides? As stated in the next remark (and proved in Appendix 7.2), our linear model does not admit such an outcome. However, we conjecture that it could emerge as an equilibrium configuration in a more general setting.

Remark 2 *In the linear two-sided single-homing model, if one of the two platforms obtains larger market shares on both sides, then this platform cannot have lower price-cost margins on both sides in equilibrium.*

The different outcomes that we have exemplified are summarized in Table 1 (in which Example 3 includes the modified parameter constellation). Thus, the following proposition has been proven by example (together with Remark 2):

Proposition 3 *In markets with competing platforms, market shares, price-cost margins, and profits are not necessarily positively aligned. In particular, a platform with larger market shares on both sides may obtain lower profits. In addition, the following outcome obtains:*

- *Either the platform with larger market shares on both sides obtains a higher price-cost margin than its competitor on one side and a lower on the other, or*
- *the larger platform with larger market shares on both sides obtains a higher price-cost margin than its competitor on both sides.*

		Market shares	**Profits**	**Price-cost margins**
Example 1	Side a	$n_a^{1*} > n_a^{2*}$	$\Pi^{1*} < \Pi^{2*}$	$p_a^{1*} - c_a^1 > p_a^{2*} - c_a^2 > 0$
	Side b	$n_b^{1*} > n_b^{2*}$		$0 > p_b^{2*} - c_b^2 > p_b^{1*} - c_b^1$
Example 2	Side a	$n_a^{1*} > n_a^{2*}$	$\Pi^{1*} < \Pi^{2*}$	$0 > p_a^{1*} - c_a^1 > p_a^{2*} - c_a^2$
	Side b	$n_b^{1*} > n_b^{2*}$		$p_b^{2*} - c_b^2 > p_b^{1*} - c_b^1 > 0$
Example 3	Side a	$n_a^{1*} > n_a^{2*}$	$\Pi^{1*} < \Pi^{2*}$	$0 > p_a^{1*} - c_a^1 > p_a^{2*} - c_a^2$
	Side b	$n_b^{1*} > n_b^{2*}$		$p_b^{1*} - c_b^1 \geq p_b^{2*} - c_b^2 > 0$
Example 4	Side a	$n_a^{1*} > n_a^{2*}$	$\Pi^{1*} < \Pi^{2*}$	$p_a^{1*} - c_a^1 > p_a^{2*} - c_a^2 > 0$
	Side b	$n_b^{1*} > n_b^{2*}$		$p_b^{2*} - c_b^2 > p_b^{1*} - c_b^1 > 0$

Table 1: Summary of results in the four examples

So far, we assumed that platforms have two price instruments, namely a subscription or participation fee on each side. As is often observed on B2C platforms (with side b representing buyers), platforms set zero prices on side b (i.e., $p_b^1 = 0$ and $p_b^2 = 0$). This may be the result of technological constraints or a binding non-negativity constraint. Thus, side b is the "free" side, whereas side a is the "money" side. What about the association between market shares, price-cost margins and profits in such a setting? The following remark shows that the tension between market shares and profit remains an issue – for the derivation and a numerical example, we refer to Appendix 7.3.

Remark 3 *In the linear two-sided single-homing model in which platforms can charge users on only one side, the larger platform (with larger market shares on both sides) may have a higher price-cost margin on the "money" side, but lower profits than its competitor. Clearly, if the larger platform makes lower profits, its price minus average cost (which includes costs on the free side)* $p_a^1 - c_a^1 - (c_b^1 n_b^1)/n_a^1$ *must be lower.*

5 Implications for competition policy

Competition practice traditionally looks at indicators for market power for antitrust investigations and merger control. Market shares and price-cost margins figure prominently as such indicators. As Baker and Bresnahan (1992, p. 745) put it, "to infer the existence and magnitude of market power, antitrust today relies routinely on market share ... Accounting measures of markup or profits have also been employed in this task." For example, the European Commission writes: "The Commission considers that low market shares are generally a good proxy for the absence of substantial market power. The Commission's experience suggests that dominance is not likely if the undertaking's market share is below 40 % in the relevant market." (See European Commission, 2009, p. C45/9.)

Competition scholars have exposed the limitations of these indicators. As far as market shares are concerned, one issue in practice is that measuring market share requires the definition of a relevant market, which can be a tricky task.[20] Market shares are also particularly problematic when making comparisons *across* markets, because of the impact that product differentiation exerts on the link between cost differences, market shares, and profits.[21] As for price-cost margins, empirical work often has to deal with the problem that marginal costs are not directly observable and economic profits often differ from accounting profits.[22]

Markets with two-sided platforms challenge the usual indicators of market power even further. As already noted, two-sided platforms are active on two interrelated markets. If cross-group network effects are pronounced in at least one direction, a proper understanding of the competitive situation requires a joint consideration of both markets, which complicates the definition of the relevant market – see, e.g., Franck and Peitz (2021) who discuss contrasting views on market definition and whether markets can be considered in isolation. In particular, even if one platform operates as a monopolist on one side, its market power may be very much constrained by competition on the other side. While the platform then enjoys a monopoly margin on one side, its overall profit may be nil, as the surplus extracted on one side may be fully passed through to the other side. However, the result depends in particular on the sign of the cross-group network effects and other market characteristics. This suggests that a large market share on one side may be informative of the degree of market power of a platform vis-a-vis one group or one side of users, but may not allow the platform to be very profitable overall.[23]

It is also well-known that there is no clear link between market share and the price-cost margin across the two sides. Keeping the degree of product differentiation the same on the two markets on which platforms compete, price-cost margins on the two sides may differ substantially

[20]The market definition exercise is based on the idea that the relevant market should include close substitutes while keeping weak substitutes out.

[21]If products are almost homogeneous, then a small cost advantage results in a large market share; however, profit margins are likely to be low and the firm cannot raise price much above the competitive level. By contrast, if products are more differentiated, a small cost advantage leads to only a slightly higher market share than the ones of its competitors. Due to the lack of very close substitutes, the price-cost margin may well be high (and so may profits). This suggests to be more concerned about high market shares if products are more differentiated.

[22]For a discussion, see, e.g., Baker and Bresnahan (1992). In most empirical work, the Lerner index and not the absolute price-cost margin is used. In our model, we can adjust stand-alone utilities such that marginal costs are the same across firms. With this adjustment in place, qualitative finding regarding the Lerner index are the same as the ones regarding absolute price-cost margins.

[23]Law scholars are aware of the difficulty in using market shares as measures of market power in markets with platforms, as well as the importance of considering the interaction between the markets linked through cross-group network effects. For instance, Hovenkamp (2021, p. 525) writes: "Direct measures of market power on platforms are probably superior for most purposes. For both direct and indirect measures, however, effects on the other must be taken into account."

if cross-group network effects differ in size (while market shares are the same) – this has been shown by Armstrong (2006) in a setting with symmetric platforms with singlehoming users on both sides and platforms charging subscription fees.

We have taken a different perspective in this paper by comparing the same measure across firms within the same market(s). Our results inform competition policy in markets with two-sided platforms by uncovering additional reasons for discarding market shares (and price-cost margins) as the lead indicators of market power. If high profit is the main concern, neither price-cost margins nor market shares provide, on their own, any clear indication as to which firm in an industry one should be most concerned about.

6 Conclusion

In this paper, we revisit one of the workhorse models of platform competition, the linear two-sided singlehoming model proposed by Armstrong (2006). We derive the conditions under which there is some tension between market shares, price-cost margins and profits. In particular, we identify conditions under which one platform has higher market shares for both user groups it is catering to, but obtains lower profit. When one margin is negative, it is even possible that one firm has higher market shares and price-cost margins on both sides and yet makes lower profit. Along the way, we characterize the price equilibrium allowing for asymmetries between platforms and across user groups regarding stand-alone utilities, costs of serving users, and strength of cross-group network effects.

Our result is of interest for oligopoly theory and competition practice. On the theory side, we show that firm asymmetries can lead to equilibrium outcomes that differ qualitatively from those in "standard" oligopoly theory. In particular, a firm may have a competitive advantage in serving both groups of users and, thereby a larger market share than its rival for both groups; yet, this platform may at the same time obtain a lower profit than its rival (see Proposition 2 and Example 1).[24]

For competition practice, our result suggests that market share is unsuited as a simple metric for indicating market power, in contrast to standard oligopoly models of price competition with differentiated products. It may actually be the smaller rival that makes the higher profit. Furthermore, the smaller rival may even have lower price-cost margins and still be more profitable (see Example 3). This requires that one side is subsidized by both platforms – then, having a larger market share on the side that is subsidized becomes less attractive and may more than offset the positive effect of higher price-cost margins and the larger market share on the profitable

[24] We restrict attention to market environments in which there is "competition in the market." In this case, stronger network effects intensify price competition, as is well-known in symmetric models from Armstrong (2006). If we consider market environments instead in which network effects are sufficiently strong such that the market "tips" and all trade takes place on one platform, this "winning" platform trivially has higher market shares, larger price-cost margins and higher profits than its competitor. For moderate asymmetries between platforms there are multiple equilibria and it could be the platform with a competitive advantage or a competitive disadvantage that attracts all users.

side. Such a situation can also arise in platform markets in which platforms only charge one side of the market (see Appendix 7.3). Then, using price-cost margins as an indicator of market power is also misleading.

In line with most of the literature, our analysis focuses on the short-run price equilibrium. Our results also provide some guidance regarding dynamic effects. If competitive advantages depended positively on past market shares (e.g., because of learning-by-doing), the firm with initially larger market shares, while performing worse than its competitor in the short run (under the conditions established in this paper), would over time increase its competitive advantage and eventually become also the more profitable firm. By contrast, if competitive advantages depended positively on past profit (e.g., because of funding constraints), a firm's competitive advantage would be attenuated over time leading to a more symmetric market outcome in terms of market shares and profit. These insights apply to myopic firms operating in an evolving industry. A fully dynamic analysis would need to include anticipated future profits in the firms' objective function. Work along these lines would be useful to analyze how market share and long-run profit are related.

Also in line with a large part of the literature, our analysis is restricted to prices as the platforms' strategic variables. In light of our results, it is a natural question to ask whether a platform has an incentive to degrade quality on one or both sides. To shed some light on this issue, one may want to look at comparative static results in the quality parameters of one platform taking the quality choices of the other platform as given. One could also endogenize platforms' quality by adding quality choice as an additional initial stage to the game that we have analyzed. Endogenous platform quality is a topic of independent interest and we leave such extensions for future work.

7 Appendix

7.1 Proof of Proposition 2

Suppose that $\Delta s_a > 0$ and $\Delta s_b > 0$. Then, conditions (8) and (9) are satisfied: Platform 1 has a larger market share on both sides. Suppose, without loss of generality, that $\tau_a > \tau_b$. As noted above, the satisfaction of conditions (4) and (10) imply that $2\tau_b < \Sigma b_a + \Sigma b_b < 2\tau_a$. Substituting the values of $n_a^{1*} - n_a^{2*}$ and $n_b^{1*} - n_b^{2*}$ into condition (10) and developing, we obtain that $\Pi^{1*} - \Pi^{2*}$ is equivalent to:

$$\begin{aligned}
&6\left(\Delta s_a + \Delta s_b\right)\left(4\tau_a\tau_b - \left(\Sigma b_a + \Sigma b_b\right)^2\right) \\
&+2\left[\left(\Sigma b_a + \Sigma b_b - 2\tau_b\right)\Delta s_a + \left(2\tau_a - \left(\Sigma b_a + \Sigma b_b\right)\right)\Delta s_b\right]\left(\Sigma b_b - \Sigma b_a\right) < 0.
\end{aligned}$$

As the first term is positive because of condition (4) and as $2\tau_b < \Sigma b_a + \Sigma b_b < 2\tau_a$, a necessary condition for the latter inequality to be satisfied is $\Sigma b_a > \Sigma b_b$.

Together with $2\tau_b < \Sigma b_a + \Sigma b_b < 2\tau_a$, $\Sigma b_a > \Sigma b_b$ implies that $\Sigma b_a > \tau_b$ and $\Sigma b_b < \tau_a$. Together with condition (8), $\Sigma b_a > \Sigma b_b$ also implies that $3\tau_a\tau_b > \left(\Sigma b_b\right)^2 + 2\left(\Sigma b_a\right)\left(\Sigma b_b\right)$. It follows that:

$$p_a^{1*} - c_a^1 = \tau_a - \Sigma b_b + K > p_a^{2*} - c_a^2 = \tau_a - \Sigma b_b - K \text{ and } p_a^{1*} - c_a^1 > 0,$$
$$\text{with } K \equiv \frac{1}{D}\left[\left(3\tau_a\tau_b - \left(\Sigma b_b\right)^2 - 2\left(\Sigma b_a\right)\left(\Sigma b_b\right)\right)\Delta s_a + \tau_a\left(\Sigma b_a - \Sigma b_b\right)\Delta s_b\right] > 0.$$

This demonstrates statement (ii).

To show statement (i), we rewrite condition (10), $\Pi^{1*} < \Pi^{2*}$, as:

$$\begin{aligned}
&\left(p_a^{1*} - c_a^1\right)\left(n_a^{1*} - n_a^{2*}\right) + \left[\left(p_a^{1*} - c_a^1\right) - \left(p_a^{2*} - c_a^2\right)\right] n_a^{2*} \\
&< -\left(p_b^{1*} - c_b^1\right)\left(n_b^{1*} - n_b^{1*}\right) + \left[\left(p_b^{2*} - c_b^2\right) - \left(p_b^{1*} - c_b^1\right)\right] n_b^{2*}.
\end{aligned}$$

As the LHS is positive, the RHS must also be. This means that if $p_b^{1*} - c_b^1 > 0$, then it must be that $p_b^{2*} - c_b^2 > p_b^{1*} - c_b^1$. But then, $p_b^{1*} - c_b^1 + p_b^{2*} - c_b^2 = \tau_b - \Sigma b_a > 0$, which contradicts $\Sigma b_a > \tau_b$. It follows necessarily that $p_b^{1*} - c_b^1 < 0$, which demonstrates statement (i).

7.2 Proof of Remark 2

We will show that if one platform has a larger market share on both sides, then it cannot have lower price-cost margins on both sides. Suppose platform 1 has a larger market share on both sides. Using the definition of $D = 9\tau_a\tau_b - \left(2\Sigma b_a + \Sigma b_b\right)\left(\Sigma b_a + 2\Sigma b_b\right)$ and the equilibrium market shares, we have:

$$\begin{aligned}
n_a^{1*} - n_a^{2*} &= \frac{1}{D}\left[3\tau_b\Delta s_a + \left(\Sigma b_a + 2\Sigma b_b\right)\Delta s_b\right], \\
n_b^{1*} - n_b^{2*} &= \frac{1}{D}\left[3\tau_a\Delta s_b + \left(2\Sigma b_a + \Sigma b_b\right)\Delta s_a\right].
\end{aligned}$$

We can therefore write:

$$\Delta s_a = 3\tau_a\left(n_a^{1*} - n_a^{2*}\right) - \left(\Sigma b_a + 2\Sigma b_b\right)\left(n_b^{1*} - n_b^{2*}\right), \tag{11}$$
$$\Delta s_b = 3\tau_b\left(n_b^{1*} - n_b^{2*}\right) - \left(2\Sigma b_a + \Sigma b_b\right)\left(n_a^{1*} - n_a^{2*}\right). \tag{12}$$

Using the equilibrium market shares, we can write the differences in the price-cost margins as:

$$\begin{aligned}\left(p_a^{1*}-c_a^1\right)-\left(p_a^{2*}-c_a^2\right) &= \frac{2}{3}\Delta s_a+\frac{2}{3}\left(\Sigma b_a-\Sigma b_b\right)\left(n_b^{1*}-n_b^{2*}\right),\\ \left(p_b^{1*}-c_b^1\right)-\left(p_b^{2*}-c_b^2\right) &= \frac{2}{3}\Delta s_b-\frac{2}{3}\left(\Sigma b_a-\Sigma b_b\right)\left(n_a^{1*}-n_a^{2*}\right).\end{aligned}$$

Using (11) and (12) those differences can be written as:

$$\begin{aligned}\left(p_a^{1*}-c_a^1\right)-\left(p_a^{2*}-c_a^2\right) &= 2\tau_a\left(n_a^{1*}-n_a^{2*}\right)-2\Sigma b_b\left(n_b^{1*}-n_b^{2*}\right),\\ \left(p_b^{1*}-c_b^1\right)-\left(p_b^{2*}-c_b^2\right) &= 2\tau_b\left(n_b^{1*}-n_b^{2*}\right)-2\Sigma b_a\left(n_a^{1*}-n_a^{2*}\right).\end{aligned}$$

Suppose that platform 1 has a lower price cost margin on side a – i.e., the inequality $\left(p_a^{1*}-c_a^1\right)<\left(p_a^{2*}-c_a^2\right)$. This is the case if and only if $\left(n_a^{1*}-n_a^{2*}\right)<\Sigma b_b\left(n_b^{1*}-n_b^{2*}\right)/\tau_a$. If so, it must be the case on side b that:

$$\left(p_b^{1*}-c_b^1\right)-\left(p_b^{2*}-c_b^2\right)>\frac{2}{\tau_a}\left(\tau_a\tau_b-\Sigma b_a\Sigma b_b\right)\left(n_b^{1*}-n_b^{2*}\right)>0.$$

The second inequality follows from the fact that we postulated that $n_b^{1*}>n_b^{2*}$ and we have that $\tau_a\tau_b>\Sigma b_a\Sigma b_b$ (which, as we recall, is implied by inequality (4) in the main text). A lower price-cost margin on side a thus implies that the larger platform has a higher margin on side b. Correspondingly, a lower price-cost margin on side b implies that the larger platform has a higher margin on side a.

7.3 Proof of Remark 3

We consider situations in which platforms set zero prices on side b (i.e., $p_b^1=0$ and $p_b^2=0$). Platform i's profit is then written as:

$$\Pi^i=\left(p_a^i-c_a^i\right)n_a^i-c_b^i n_b^i.$$

Note that a necessary condition for positive profit is $p_a^i-c_a^i>0$, $i\in\{1,2\}$.

Equilibrium prices on side a are given by the solution to:

$$\begin{aligned}\frac{d\Pi^1}{dp_a^1} &= \frac{d}{dp_a^1}\left(\left(p_a^1-c_a^1\right)n_a^1-c_b^1 n_b^1\right)=0,\\ \frac{d\Pi^2}{dp_a^2} &= \frac{d}{dp_a^2}\left(\left(p_a^2-c_a^2\right)\left(1-n_a^1\right)-c_b^2\left(1-n_b^1\right)\right)=0,\end{aligned}$$

where

$$\begin{aligned}n_a^1 &= \frac{1}{2}+\frac{\tau_b}{2}\frac{\Delta r_a+\Delta b_a+p_a^2-p_a^1}{\tau_a\tau_b-\left(\Sigma b_a\right)\left(\Sigma b_b\right)}+\frac{\Sigma b_a}{2}\frac{\Delta r_b+\Delta b_b}{\tau_a\tau_b-\left(\Sigma b_a\right)\left(\Sigma b_b\right)},\\ n_b^1 &= \frac{1}{2}+\frac{\tau_a}{2}\frac{\Delta r_b+\Delta b_b}{\tau_a\tau_b-\left(\Sigma b_a\right)\left(\Sigma b_b\right)}+\frac{\Sigma b_b}{2}\frac{\Delta r_a+\Delta b_a+p_a^2-p_a^1}{\tau_a\tau_b-\left(\Sigma b_a\right)\left(\Sigma b_b\right)}.\end{aligned}$$

The system of linear equations has a unique solution with platform 1's price being equal to:

$$\begin{aligned}p_a^{1*} &= c_a^1+\frac{\Sigma b_b}{\tau_b}c_b^1+\tau_a-\frac{\left(\Sigma b_a\right)\left(\Sigma b_b\right)}{\tau_b}\\ &\quad+\frac{1}{3}\left(\Delta r_a-\Delta c_a-\frac{\Sigma b_b}{\tau_b}\Delta c_b+\Delta b_a\right)+\frac{\Sigma b_a}{3}\frac{\Delta r_b+\Delta b_b}{\tau_b},\end{aligned}$$

where $c_a^1 + \frac{\Sigma b_b}{\tau_b} c_b^1$ is the marginal cost of attracting one extra a-user. It directly costs c_a^1 and it induces $\frac{\Sigma b_b}{\tau_b}$ extra b-users to join (see the definition of $\hat{x}_b$): Increasing marginally n_a^1 – and thus decreasing marginally n_a^2 – increases $\hat{x}_b$ by $\left(\beta_b^1 + \beta_b^2\right) / (2\tau_b) = \Sigma b_b / \tau_b$), which costs $\frac{\Sigma b_b}{\tau_b} c_b^1$. Hence, the difference in marginal costs on side a becomes $\Delta c_a + \frac{\Sigma b_b}{\tau_b} \Delta c_b$.

Similarly,

$$\begin{aligned} p_a^{2*} &= c_a^2 + \frac{\Sigma b_b}{\tau_b} c_b^2 + \tau_a - \frac{(\Sigma b_a)(\Sigma b_b)}{\tau_b} \\ &\quad - \frac{1}{3}\left(\Delta r_a - \Delta c_a - \frac{\Sigma b_b}{\tau_b}\Delta c_b + \Delta b_a\right) - \frac{\Sigma b_a}{3}\frac{\Delta r_b + \Delta b_b}{\tau_b}. \end{aligned}$$

The equilibrium number of users on the each side is:

$$\begin{aligned} n_a^{1*} &= \frac{1}{2} + \frac{1}{6}\frac{\tau_b\left(\Delta r_a - \Delta c_a - \frac{\Sigma b_b}{\tau_b}\Delta c_b + \Delta b_a\right) + \Sigma b_a(\Delta r_b + \Delta b_b)}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)}, \\ n_b^{1*} &= \frac{1}{2} + \frac{1}{6\tau_b}\frac{\Sigma b_b\tau_b\left(\Delta r_a - \Delta c_a - \frac{\Sigma b_b}{\tau_b}\Delta c_b + \Delta b_a\right) + (3\tau_a\tau_b - 2(\Sigma b_a)(\Sigma b_b))(\Delta r_b + \Delta b_b)}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)}. \end{aligned}$$

As mentioned above, the perceived marginal cost on side a is $c_a^1 + \frac{\Sigma b_b}{\tau_b} c_b^1$ and the corresponding difference in marginal costs across platforms is thus $\Delta c_a + \frac{\Sigma b_b}{\tau_b}\Delta c_b$. Let us define, in an asymmetric way, a platform's "competitive advantage" on the two sides as follows:

$$\begin{aligned} Z_a &= \Delta r_a - \Delta c_a - \frac{\Sigma b_b}{\tau_b}\Delta c_b + \Delta b_a, \\ Z_b &= \Delta r_b + \Delta b_b. \end{aligned}$$

On side a, we account for perceived marginal costs, but we do not include any costs on side b. While this advantage is clearly beneficial for platform 1 on side a, it has negative implications for profits on the free side, as being more attractive on side b tends to attract more users, which increases losses. With this new notation in place, we can write equilibrium outcomes as:

$$\begin{aligned} p_a^{1*} - c_a^1 &= \frac{\Sigma b_b}{\tau_b} c_b^1 + \tau_a - \frac{(\Sigma b_a)(\Sigma b_b)}{\tau_b} + \frac{1}{3} Z_a + \frac{1}{3}\frac{\Sigma b_a}{\tau_b} Z_b, \\ p_a^{2*} - c_a^2 &= \frac{\Sigma b_b}{\tau_b} c_b^2 + \tau_a - \frac{(\Sigma b_a)(\Sigma b_b)}{\tau_b} - \frac{1}{3} Z_a - \frac{1}{3}\frac{\Sigma b_a}{\tau_b} Z_b, \\ n_a^{1*} &= \frac{1}{2} + \frac{1}{6}\frac{\tau_b Z_a + \Sigma b_a Z_b}{\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b)}, \\ n_b^{1*} &= \frac{1}{2} + \frac{\Sigma b_b \tau_b Z_a + (3\tau_a\tau_b - 2(\Sigma b_a)(\Sigma b_b)) Z_b}{6\tau_b(\tau_a\tau_b - (\Sigma b_a)(\Sigma b_b))}. \end{aligned}$$

The difference in profits is:

$$\Pi^1 - \Pi^2 = \left(p_a^1 - c_a^1\right) n_a^1 - c_b^1 n_b^1 - \left(\left(p_a^2 - c_a^2\right)\left(1 - n_a^1\right) - c_b^2\left(1 - n_b^1\right)\right).$$

The question we ask is: Can this difference be negative in equilibrium if n_a^{1*} and n_b^{1*} are both larger than $1/2$? The difference in equilibrium profits is:

$$\Pi^{1*} - \Pi^{2*} = \frac{1}{6\tau_b}\left(4\tau_b Z_a + \left(4\Sigma b_a - 3\left(c_b^1 + c_b^2\right)\right) Z_b - 3\left(c_b^1 - c_b^2\right)(\tau_b - \Sigma b_b)\right).$$

This expression can be negative if Z_a and Z_b are positive. Note that n_a^1 and n_b^1 are both larger than 1/2 if Z_a and Z_b are positive. In particular, an increase in Z_b increases n_a^1 and n_b^1 but is detrimental to the profit if Σb_a is lower than $3\left(c_b^1 + c_b^2\right)/4$ (that is, if users of group a do not value users in group b very much, while users in group b are costly to serve).

To take a concrete numerical example, consider the following parameter constellation: $\Delta r_a - \Delta c_a = \Delta r_b = 1/2$, $c_b^1 = c_b^2 = 3$, $\tau_a = \tau_b = 2$, $\Sigma b_a = \Sigma b_b = 1$ and $\Delta b_a = \Delta b_b = 0$ (i.e., $\beta_a = \beta_b = 1$). In this case, we obtain that platform 1 enjoys a higher price-cost margin ($p_a^{1*} - c_a^1 = 13/4 > 11/4 = p_a^{2*} - c_a^2$) and serves more users on both sides of the market ($n_a^{1*} = 7/12 > 1/2$ and $n_b^{1*} = 2/3 > 1/2$), but obtains lower profits: $\Pi^{1*} - \Pi^{2*} = -1/4$. In this example, under symmetric costs, platform 1 offers a higher gross surplus to users on both sides ($\Delta r_a = \Delta r_b = 1/2$). While this leads to higher price-cost margins on the money side and higher market shares on both sides, it leads to lower profits. Under asymmetric costs on the free side, by construction, the larger platform has either a lower or a higher margin than its competitor on this side.

7.4 Building of examples

7.4.1 Example 1

In this example, platform 1 has equal competitive advantages on both sides; that is, $\Delta s_a = \Delta s_b = \Delta s > 0$. Condition (10) can then be rewritten as:

$$\Pi^{1*} < \Pi^{2*} \Leftrightarrow 3\left(4\tau_a\tau_b - (\Sigma b_a + \Sigma b_b)^2\right) - (\tau_a - \tau_b)(\Sigma b_a - \Sigma b_b) < 0.$$

As the first term is positive because of condition (4), we see that – to satisfy the above inequality – the terms $(\tau_a - \tau_b)$ and $(\Sigma b_a - \Sigma b_b)$ must have the same sign. Suppose that $\tau_a > \tau_b$, which implies $\Sigma b_a > \Sigma b_b$. It is not necessary to assume positive cross-group network effects on both sides: We set $\Sigma b_b = 0$; take also $\tau_a = 4$ and $\tau_b = 1$. To satisfy condition (4), we need $16 - (\Sigma b_a)^2 > 0$ or $\Sigma b_a < 4$. On the other hand, the above condition becomes $48 - 3\Sigma b_a - 3(\Sigma b_a)^2 < 0$, which is equivalent to $\Sigma b_a > 3.531$. For instance, we set $\Sigma b_a = 3.6$ and $\Delta s = 0.25$.

7.4.2 Example 2

We suppose the same average cross-group network effects on both sides; that is, $\Sigma b_a = \Sigma b_b = \Sigma b > 0$. This means that, on average, cross-group network effects are similar on both sides. Conditions (8) to (10) become, respectively, $\tau_b\Delta s_a + \Sigma b\Delta s_b > 0$, $\tau_a\Delta s_b + \Sigma b\Delta s_a > 0$ and $\Delta s_a + \Delta s_b < 0$. The second-order condition (4) requires $\tau_a\tau_b > (\Sigma b)^2$. For those conditions to be compatible, one needs $\Delta s_j > 0$ and $\Delta s_k < 0$: Platform 1 must have a competitive advantage for one group of users (j) and a competitive disadvantage for the other ($k \neq j \in \{a, b\}$). Here, we set $\Sigma b_a = \Sigma b_b = 2$, $\tau_a = 1$, $\tau_b = 5$, $\Delta s_a = 1$ and $\Delta s_b = -3/2$.

7.4.3 Example 3

We assume opposite competitive advantages; that is, $\Delta s_a = -\Delta s_b > 0$. Here, platform 1 has a competitive advantage vis-à-vis users from group a and an equivalent disadvantage vis-à-vis users of group b. It is readily checked that:

$$\Pi^{1*} - \Pi^{2*} = \Delta s_a \left(\Sigma b_a - \Sigma b_b\right) \left[\tau_a + \tau_b - \left(\Sigma b_a + \Sigma b_b\right)\right] / D < 0 \Leftrightarrow \Sigma b_a < \Sigma b_b.$$

This is a general result for cases where $\Delta s_a = -\Delta s_b > 0$. The platform with a competitive advantage over users who benefit least from the presence of other users has a lower profit than its competitor. Is this result compatible with a situation where platform 1 also has higher market shares? For conditions (8) and (9) to be met, it is necessary that $3\tau_a < 2\Sigma b_a + \Sigma b_b < \Sigma b_a + 2\Sigma b_b < 3\tau_b$ where the first inequality corresponds to (9), the second arises from $\Sigma b_a < \Sigma b_b$ and the third corresponds to (8). The following set of values satisfies the previous requirements: $\Sigma b_a = 2$, $\Sigma b_b = 5$, $\tau_a = 2$, $\tau_b = 8$, $\Delta s_a = 1$ and $\Delta s_b = -1$. In line with our previous results, platform 1 is expected to be the least profitable because $\Sigma b_a < \Sigma b_b$ and $\Delta s_a > 0$.

7.4.4 Example 4

We build an example in which platforms do not offer subsidies to any user. Consider the borderline case in terms of market shares in which platforms have equal market share on one side, say $n_b^{1*} - n_b^{2*} = 0$. This assumption allows us to pin down the relationship between Δs_a and Δs_b and build the example by focusing on Δs_a:

$$n_b^{1*} - n_b^{2*} = \frac{1}{D}\left[3\tau_a \Delta s_b + \left(2\Sigma b_a + \Sigma b_b\right) \Delta s_a\right] = 0 \Leftrightarrow \Delta s_b = -\frac{\Delta s_a \left(2\Sigma b_a + \Sigma b_b\right)}{3\tau_a}.$$

For this value of Δs_b, one can write $n_a^{1*} - n_a^{2*} = \Delta s_a / \left(3\tau_a\right)$ which is positive if $\Delta s_a > 0$ (and which is smaller than 1 if $\Delta s_a < 3\tau_a$). The margins can be written as:

$$\begin{aligned} p_a^{1*} - c_a^1 &= \tau_a - \Sigma b_b + \frac{1}{3}\Delta s_a, \qquad p_a^{2*} - c_a^2 = \tau_a - \Sigma b_b - \frac{1}{3}\Delta s_a, \\ p_b^{1*} - c_b^1 &= \tau_b - \Sigma b_a - \frac{1}{3}\frac{\Sigma b_a}{\tau_a}\Delta s_a, \qquad p_b^{2*} - c_b^2 = \tau_b - \Sigma b_a + \frac{1}{3}\frac{\Sigma b_a}{\tau_a}\Delta s_a. \end{aligned}$$

Thus, all parameter constellations with $\Sigma b_b < \tau_a < \Sigma b_a < \tau_b$ and Δs_a slightly positive ensure that margins are positive. Moreover, in spite of (slightly) higher market shares than its rival, platform 2 obtains lower profits than its rival if $\tau_a < \left(\Sigma b_a + \Sigma b_b\right)/2$ (see expression (10)). Finally, a sufficiently high value for τ_b ensures that the second-order condition (4) is satisfied. Consider the following values: $\Sigma b_a = 4$, $\Sigma b_b = 1$, $\tau_a = 2$, $\tau_b = 5$, $\Delta s_a < 3/2$, and $\Delta s_b = -3\Delta s_a/2$. We find:

$$\begin{aligned} p_a^{1*} - c_a^1 &= 1 + \frac{1}{3}\Delta s_a > 0, \qquad p_a^{2*} - c_a^2 = 1 - \frac{1}{3}\Delta s_a > 0, \\ p_b^{1*} - c_b^1 &= 1 - \frac{2}{3}\Delta s_a > 0, \qquad p_b^{2*} - c_b^2 = 1 + \frac{2}{3}\Delta s_a > 0, \\ n_a^{1*} - n_a^{2*} &= \frac{\Delta s_a}{6} \in (0,1), \qquad n_b^{1*} - n_b^{2*} = 0,\ \Pi^{1*} - \Pi^{2*} = -\frac{\Delta s_a}{6} < 0. \end{aligned}$$

We also verified that the second-order condition is met. Platform 1 obtains lower profits than its rival despite having at least the same market share. All margins are positive. In the example presented in the text, we set $\Delta s_a = 1$ and $\Delta s_b = -3/2$.

Note that to have $n_b^{1*} - n_b^{2*} > 0$, it is sufficient to slightly increase the value of Δs_b, as we show next. We set $\Delta s_a = 1$, but now leave the value of Δs_b free. We find:

$$
\begin{aligned}
n_a^{1*} - n_a^{2*} &\in (0,1) \Leftrightarrow -\frac{5}{2} < \Delta s_b < \frac{7}{2}, \qquad n_b^{1*} - n_b^{2*} \in (0,1) \Leftrightarrow -\frac{3}{2} < \Delta s_b < \frac{9}{2}, \\
p_a^{1*} - c_a^1 &> 0 \Leftrightarrow \Delta s_b > -\frac{19}{2}, \qquad p_a^{2*} - c_a^2 > 0 \Leftrightarrow \Delta s_b < \frac{5}{2}, \\
p_b^{1*} - c_b^1 &> 0 \Leftrightarrow \Delta s_b > -\frac{7}{2}, \qquad p_b^{2*} - c_b^2 > 0 \Leftrightarrow \Delta s_b < \frac{17}{2}, \\
\Pi^{1*} - \Pi^{2*} &< 0 \Leftrightarrow \Delta s_b < -\frac{5}{4}.
\end{aligned}
$$

Any value of $\Delta s_b \in \left(-\frac{3}{2}, -\frac{5}{4}\right)$ ensures that platform 1 enjoys larger market shares while earning a lower profit than its rival; all margins are positive.[25]

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