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Network Goods, Price Discrimination, and Two-sided Platforms

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Network goods, price discrimination, and two-sided platforms^{*}

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Abstract

A monopolist selling a network good to heterogeneous users is shown to become a twosided platform if it can condition prices on some user characteristics or if it cannot but induces user self-selection by offering screening contracts. This shows that the availability of sophisticated pricing instruments is essential to make a platform two-sided, not the ability to distinguish separate user groups. The use of freemium strategies (which consists of offering a base version at zero price and a premium version at a positive price) emerges as a special case of versioning.

Keywords: Network goods, two-sided platforms, platform pricing, group pricing, versioning, freemium

JEL-Classification: D21, D42, L12, L14

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1 Introduction

In an increasing number of markets, users enjoy benefits that depend on the decisions of other users and a firm takes actions that contribute to shaping these benefits and allocating them among users. In the parlance of industrial economics, the interaction among users is said to exhibit "network effects" and the firm that orchestrates the interaction is called a "platform".

Platforms are often seen as two-sided or multi-sided in that they cater to two or more audiences, whose decisions are interdependent because of cross-side network effects. For instance, using a non-technical and broad definition, Evans and Schmalensee (2016, p. 210) define a multisided platform as "a business that operates a physical or virtual place (a platform) to help two or more different groups find each other and interact. The different groups are called "sides" of the platform."¹ A for-profit multi-sided platform uses price and non-price strategies to make profits and, at the same time, manages network effects. A focus of the early literature is on the characterization of pricing strategies in various market environments (mostly with a single or two two-sided platforms); see, e.g., Rochet and Tirole (2003, 2006) and Armstrong (2006).

A well-known example of two-sided platform is Airbnb. The two groups are accommodation owners ("hosts") and travelers ("guests"). There exist positive cross-side network effects in the sense that participation becomes more valuable for users of one group as soon as participation intensifies in the other group: guests enjoy more participation from hosts because it widens their choice set and hosts enjoy more participation from guests because it expands their potential demand. By setting fees and by choosing governance rules (for instance, cancellation policies), Airbnb affects the benefits that users in each group obtain when interacting on the platform.

However, many platforms – like social networks and messaging applications – cater mostly to a single audience.² Network effects are also present on these platforms as their users are keener to join them the larger the participation of other users (as this is synonymous of more communication possibilities). Yet, network effects are of a different nature: they occur within the same side of the platform and not across distinct sides. Are then the formal models by which we analyze "one-sided platforms" different from those for two- or multi-sided platforms? It depends. In our view, the focus should not be put on the presence of different audiences, but on the set of instruments that the intermediary can use. Our first key message in this paper is indeed that the definition of multi-sided platforms should not rest on easily distinguishable groups of users, but on the ability to target groups with different prices; that is, to be able to employ group pricing (or, in other words, third-degree price discrimination). To show this, we consider the market of a network good in which all users care about the overall level of participation. At first sight, this looks different from a two-sided platform. However, as soon as we allow the platform to condition prices on user characteristics, the profit-maximizing prices are determined in a very similar way as in the case of a multi-sided platform (like in the setting of Armstrong,

¹Other authors have provided similar definitions; see e.g. Rysman (2009) and, being a case of the pot calling the kettle black, Belleflamme and Peitz (2010, Chapter 22).

 $^{^{2}}$ As these platforms grow, advertisers are sometimes added as a second audience for monetization purposes.

 $2006).^{3}$

While our first message relies on the platform to be able to observe some user characteristics, our second key message is that this insight generalizes when this information is not available to the platform, but when it can offer different versions of the network good. Here, in line with the literature on versioning (or, in other words, second-degree price discrimination), the platform has to respect the incentive constraints of the users to ensure self-selection. In our setting, these constraints will be slack and, as a result, the outcome under versioning resembles the one under group pricing.

We then consider special cases in which the profit-maximizing strategy features a base version of the network good being offered at a price of zero (used by some) and a premium version being offered at a strictly positive price (purchased by others). We interpret this as a true freemium strategy (there are no revenues directly associated with the users of the free version). This shows that the presence of network effects may make it optimal for the platform to offer a base version for free.

The paper proceeds as follows. In Section 2, we describe the model, discuss the various ways in which it can be interpreted, and solve it for the case of uniform pricing. In Section 3, we solve the model for the case of group pricing; that is, we assume that the platform can identify users according to the group they belong to and charge them differential prices accordingly. We analyze the profit-maximizing price structure to understand why some users end up paying more than other users to participate in the platform. In Section 4, we assume instead that the platform cannot tell users apart, so that group pricing is not feasible. We show then that the platform can, nevertheless, design a versioning scheme that allows it to achieve the same profit as under group pricing. We also show that a freemium strategy – whereby users can access the platform's basic services for free but must pay to access some premium services – can be seen as a special case of this versioning scheme. In each of these two sections, we put our results into perspective with the existing literature. We conclude in Section 5.

2 The model

2.1 Setting

We consider a model in which a monopoly platform offers interaction possibilities to a set of users, who can be categorized into different "groups". In the first stage, we consider two scenarios. In the group pricing scenario, the monopoly platform can identify to which group a user belongs to; it sets then an access price conditional on this observation. In the versioning

³Another observation is that same-side network effects are also often present on multi-sided platforms. On Airbnb, for instance, the review system becomes more informative for any guest as more guests leave reviews; there is thus a positive network effect on the guests side. Also, a new host joining the platform is likely to steal business from neighboring hosts, a form of negative same-side network effect. Thus, the early models on multi-sided platforms that that focus on cross-side network effects need to be augmented to allow for same-side network effects. For a discussion, see e.g. Belleflamme and Peitz (2021).

scenario, the platform is aware of the existence of various groups of users but cannot tell users apart; it then offers a menu of contracts that contains price-quality pairs. In the second stage, users simultaneously make their participation decision after observing all participation prices and anticipating the decisions of other users in the various groups. We solve for subgame perfect equilibrium.

To simplify the exposition, we assume that there are two groups of users, which we note 1 and 2.⁴ Each user's valuation of the platform depends on two additively separable components: a "stand-alone value", which the user enjoys irrespective of other users' decisions, and a "network value", which linearly increases with the number of participating users in the two groups. Users possibly differ in two dimensions. First, users of different groups can have different valuations for the stand-alone and/or network components of their valuation. Second, within each group, individual users differ by their opportunity cost of joining the platform. Accordingly, we formulate the utility of a user in group *i* as follows (with $i, j \in \{1, 2\}, j \neq i$):

$$u_i = r_i + \beta_i n_j + \gamma_i n_i - p_i, \tag{1}$$

where r_i is the stand-alone benefit, β_i the marginal valuation of interacting with users of group j ($j \neq i \in \{1, 2\}$), γ_i the marginal valuation of interacting with users of group i, n_i (n_j) the number of participating users in group i (j), and p_i the price paid when accepting the platform service (or a particular version thereof).

2.2 Interpretations

As the title of the paper indicates, our analysis provides a link between three strands of literature: the literatures on network goods, two-sided markets, and price discrimination (and multi-product pricing more generally). As we document in Section 3.3, these three strands have remained mostly distinct so far. However, we do not claim to be the first to recognize the interconnection between them. First, Rochet and Tirole (2006, p. 646) point to the link of two-sided platforms (they use the term two-sided markets) to multi-product pricing and network effects:

"Conceptually, the theory of two-sided markets is related to the theories of network externalities and of (market or regulated) multi-product pricing. From the former (...) it borrows the notion that there are noninternalized externalities among end-users. From the latter, it borrows the focus on price structure and the idea that price structures are less likely to be distorted by market power than price levels. The multi-product pricing literature, however, does not allow for externalities in the consumption of different products (...) The starting point for the theory of two-sided markets, by contrast, is that an end-user does not internalize the welfare impact of his use of the platform on other end-users."

Second, in early work, Jullien (2001, p. 2) foresaw the link between the literature on the provision of a network good to the two-sided market literature:

⁴In the appendix, we show that most of our results carry through with an arbitrary number of groups.

"... one feature of modern networks that has not received considerable attention is that network effects are often not isotropic: members may join for different reasons and value both the service and the participation of others in very different ways. In conjunction with that, and partly because of that, most suppliers of network goods practice some form of price discrimination."

Finally, Jullien (2011) studies divide-and-conquer strategies in a Stackelberg duopoly. As one of his two applications, he considers the setting with a network good being sold to different groups of users. He writes (p. 203) that:

"there is only one side with standard network effects but the platform can price discriminate between several groups of users. This shows that it is not crucial to have effectively several sides to generate multi-sided market effects. The ability to price discriminate along with the presence of network effects suffices."

We now systematize the previous views by showing that the utility formulation (1) is compatible with several interpretations of what the platform offers and who the users are.

Two-sided platform. We can see the groups as being the two sides of a two-sided platform. Users of different groups play different roles on the platform, like hosts and guests on Airbnb. Network benefits mainly stem from the interaction among users of different groups. Most of the literature on two-sided platforms only considers such cross-side network effects. In expression (1), this amounts to set $\gamma_1 = \gamma_2 = 0$. Assuming $\beta_1 > 0$ and $\beta_2 > 0$ fits platforms (like Airbnb) on which all users enjoy interacting with users of the other group.⁵ Alternatively, assuming $\beta_1 > 0$ and $\beta_2 < 0$ account for platforms on which group 1 enjoys interacting with group 2 but the feeling is not mutual; media platforms are a case in point with advertisers being group 1 and content users, group 2. Yet, users can also care about participation on their own side.⁶ We have then $\gamma_i > 0$ or $\gamma_i < 0$ according to whether users in group *i* exert positive or negative same-side network effects on one another.

One network good. Network goods exhibit "direct network effects" insofar as each user directly benefits from the participation of any other user. For instance, the more a messaging app is adopted, the larger the communication possibilities that this app offers to all its users. Similarly, a larger social network provides its users with more value as more content is shared and more links can be formed. As users essentially play the same role on the platform, it is of no use to talk of "sides" and distinguish between cross-side and same-side network effects. Users may, nevertheless, belong to different groups that value or generate direct network effects

 $^{{}^{5}}$ This is the linearized version of the analysis in Armstrong (2006); it has been analyzed, e.g., in Belleflamme and Peitz (2018).

⁶For papers explicitly considering this possibility, see, for instance, Nocke, Peitz, and Stahl (2007), Hagiu (2009), Belleflamme and Toulemonde (2009), Belleflamme and Peitz (2019), Karle, Peitz, and Reisinger (2020), and Teh (2022).

in different ways. For example, on a fintech platform, group 1 consists of expert investors who do not rely on information provided from other users of the platform, whereas group 2 consists of amateur investors who care about the presence of expert investors. We then have that $\beta_2 > 0$ and $\gamma_1 = \beta_1 = \gamma_2 = 0$. Differently, on a messaging app, group 1 can be made of professional users and group 2, of individual users. Suppose that all users equally enjoys communicating with any other user, professional or individual, but that professional users attach a larger value to any comunication than individual users do. We have then that $\gamma_1 = \beta_1 > \gamma_2 = \beta_2 > 0$. Fashion goods are another example that fits this situation: They generate direct network effects (because of conspicuous consumption) and total network size matters but some consumers – fashionistas - attach a larger value to the consumption externalities that the goods generate. In the previous two examples, all users generate network effects of similar strengths but some users value these effects more than others. Alternatively, it could be that some users generate stronger network effects than others, while all users value network effects from different orgins in the same way. Think for instance of a social network with influencers (group 1) and regular users (group 2). If users in both groups agree that interacting with an influencer is more valuable than interacting with a regular user, we have that $\gamma_1 = \beta_2 > \beta_1 = \gamma_2 > 0$.

Another application is to consider user groups arriving sequentially over time. Our analysis is then directly applicable if the platform can publicly commit to the price path, users cannot delay participation, and only the final overall participation level is relevant for users. Another way to think about a platform evolving is that all users are present from the start, but decide about staying or leaving the platform at each point in time. The platform obtains larger profits if it can offer different prices depending on the user's characteristics. Moreover, as the platform becomes more apt at using multiple price instruments, user groups can become more granular over time.

Two network goods. Formulation (1) can also reflect situations in which the groups are made of users of two separate network goods, say, two messaging apps. The key question here is to which extent the users of one app can communicate with the users of the other app. There are three typical cases. First, if apps are incompatible, communication is only possible among users of the same app. In that case, we have $\beta_i = 0$ and $\gamma_i > 0$. Second, apps can be partially compatible in the sense that users can communicate if they use different apps but not as well as if they all use the same app (for instance because some features – like sharing pictures – are specific to each app). In that case, we have $\gamma_i > \beta_i > 0$. Finally, in the case of full compatibility, the quality of communication is the same irrespective of the app that the users use; this means that $\gamma_i = \beta_i > 0$, which brings us back to the case of a single network good.

In what follows, we assume that all network effects are positive: $\beta_i > 0$ and $\gamma_i > 0$, i = 1, 2. Thereby, our general analysis can encompass the three interpretations.

2.3 Participation decisions

At stage 2, users of both groups simultaneously decide whether to participate in the platform or not, given the observed prices p_1 and p_2 . A user decides to participate if and only they prefer the service proposed by the platform over their outside option, which has value x, that is, if $u_i \ge x$. We assume for simplicity that in each group, the value of the outside option, x, is uniformly and independently distributed on [0, X], with X sufficiently large. Assuming that an increase of net utility of $\Delta u_i = 1$ leads to an increase in the number of users of mass 1, the number of participating users in the two groups are implicitly given by $n_1 = r_1 + \beta_1 n_2 + \gamma_1 n_1 - p_1$ and $n_2 = r_2 + \beta_2 n_1 + \gamma_2 n_1 - p_2$. Solving this system of equations, we find the the number of participating users as a function of the observed prices p_1 and p_2 :

$$\begin{cases} n_1(p_1, p_2) = \frac{(1 - \gamma_2)(r_1 - p_1) + \beta_1(r_2 - p_2)}{(1 - \gamma_1)(1 - \gamma_2) - \beta_1\beta_2}, \\ n_2(p_1, p_2) = \frac{(1 - \gamma_1)(r_2 - p_2) + \beta_2(r_1 - p_1)}{(1 - \gamma_1)(1 - \gamma_2) - \beta_1\beta_2}. \end{cases}$$
(2)

For the rest of the analysis, we assume that (i) $\beta_i, \gamma_i < 1$ and (ii) $(1 - \gamma_1)(1 - \gamma_2) > \beta_1\beta_2$. Under these assumptions, demand for participation in group *i* decreases with p_i and p_j (as we ssume $\beta_i > 0$).

2.4 Uniform pricing

As a benchmark, we analyze the platform's problem in stage 1 in the case where the platform is constrained to charge the same price to both groups. If $p_1 = p_2 = p$, then the platform sets p to maximize $\Pi = p [n_1(p, p) + n_2(p, p)]$.⁷ Using expressions (2) and solving for the first-order condition, we find the uniform profit-maximizing price as:

$$p^{u} = \frac{r_{1} + r_{2}}{4} - \frac{1}{4} \left(r_{1} - r_{2} \right) \frac{\left(\beta_{1} - \gamma_{1}\right) - \left(\beta_{2} - \gamma_{2}\right)}{2 + \beta_{1} + \beta_{2} - \gamma_{1} - \gamma_{2}}.$$
(3)

At this price, the platform achieves a profit equal to:

$$\Pi^{u} = \frac{1}{4} \frac{\left[\left(1 + \beta_{2} - \gamma_{2}\right) r_{1} + \left(1 + \beta_{1} - \gamma_{1}\right) r_{2}\right]^{2}}{\left(2 + \beta_{1} + \beta_{2} - \gamma_{1} - \gamma_{2}\right) \left[\left(1 - \gamma_{1}\right) \left(1 - \gamma_{2}\right) - \beta_{1}\beta_{2}\right]}$$

We see that the profit-maximizing uniform price can be independent of the strength of the network effects in two special case: when the two groups share either the same stand-alone benefits $(r_1 = r_2)$ or the same relative valuation of the network benefits $(\beta_1 - \gamma_1 = \beta_2 - \gamma_2)$ or, equivalently, $\beta_1 - \beta_2 = \gamma_1 - \gamma_2$).⁸ This is an artifact of the linear formulation of the utility functions. Network effects, however, affect the equilibrium number of participants and, hence, the platform's profit.

⁷We assume throughout that the marginal cost for the platform of serving a user of either group is equal to zero. It can be checked that if the marginal cost for group *i* is constant and equal to $c_i > 0$, then we can recover all our results by substituting $\rho_i \equiv r_i - c_i$ for r_i in all expressions.

⁸The latter case fits the interpretation of a single network good with all users either generating similar network effects ($\beta_i = \gamma_i$) or valuing network effects in a similar way ($\beta_i = \gamma_j$). It also fits the interpretation of two equivalent network goods with partial two-way compatibility ($\gamma_1 = \gamma_2 > \beta_1 = \beta_2$).

3 Group pricing

We assume here that the platform is able to tell users apart and, thereby, to charge users a different price according to the group they belong to. This assumption is rather natural for twosided platforms insofar as users of different groups (or sides) can be easily identified through the different ways in which they participate in the platform. Think of buyers and sellers on a marketplace, employers and workers on a job board, or drivers and riders on a ride-hailing platform. Even if some platforms allow their users to be active on either side, they invite them to choose which role they want to play for a given transaction. For instance, on peer-to-peer platforms organizing the exchange of secondhand items, users must choose, each time they use the platform, whether they do so to buy or sell an item. On the other hand, the fact that all users of a network good use it in a similar way does not necessarily prevent the operator of the good from identifying groups of users with specific characteristics. For instance, in the example of a messaging app, professional users can be identified by asking their company number at the registration process.

We start by deriving the profit-maximizing prices. Next, we ask why a group ends up paying less to access the platform than the other group. That is, we study how the exogenous differences between the two groups shape the profit-maximizing "price structure" (as referred to by Rochet and Tirole, 2003).

3.1 Profit maximum

Under group pricing, the platform chooses p_1 and p_2 to maximize $\Pi = p_1 n_1 (p_1, p_2) + p_2 n_2 (p_1, p_2)$. The first-order condition with respect to p_1 is:

$$\frac{d\Pi}{dp_1} = n_1 (p_1, p_2) + p_1 \frac{dn_1 (p_1, p_2)}{dp_1} + p_2 \frac{dn_2 (p_1, p_2)}{dp_1} = 0$$

$$\Leftrightarrow \frac{(1 - \gamma_2) (r_1 - p_1) + \beta_1 (r_2 - p_2) - (1 - \gamma_2) p_1 - \beta_2 p_2}{(1 - \gamma_1) (1 - \gamma_2) - \beta_1 \beta_2} = 0$$

$$\Leftrightarrow 2 (1 - \gamma_2) p_1 + (\beta_1 + \beta_2) p_2 = (1 - \gamma_2) r_1 + \beta_1 r_2.$$
(4)

Similarly, the first-order condition with respect to p_2 yields: $(\beta_1 + \beta_2) p_1 + 2(1 - \gamma_1) p_2 = \beta_2 r_1 + (1 - \gamma_1) r_2$. The second-order conditions require:

$$4(1 - \gamma_1)(1 - \gamma_2) > (\beta_1 + \beta_2)^2.$$
(5)

Solving the system of first-order conditions, we find the profit-maximizing group prices as:

$$p_{1}^{g} = \frac{\left[2\left(1-\gamma_{1}\right)\left(1-\gamma_{2}\right)-\beta_{2}\left(\beta_{1}+\beta_{2}\right)\right]r_{1}+\left(1-\gamma_{1}\right)\left(\beta_{1}-\beta_{2}\right)r_{2}}{4\left(1-\gamma_{1}\right)\left(1-\gamma_{2}\right)-\left(\beta_{1}+\beta_{2}\right)^{2}},$$

$$p_{2}^{g} = \frac{\left[2\left(1-\gamma_{1}\right)\left(1-\gamma_{2}\right)-\beta_{1}\left(\beta_{1}+\beta_{2}\right)\right]r_{2}+\left(1-\gamma_{2}\right)\left(\beta_{2}-\beta_{1}\right)r_{1}}{4\left(1-\gamma_{1}\right)\left(1-\gamma_{2}\right)-\left(\beta_{1}+\beta_{2}\right)^{2}}.$$
(6)

Substituting these values back in the participation numbers and platform's maximal profit, we find:

$$n_{1}^{g} = \frac{2(1-\gamma_{2})r_{1} + (\beta_{1}+\beta_{2})r_{2}}{4(1-\gamma_{1})(1-\gamma_{2}) - (\beta_{1}+\beta_{2})^{2}}, n_{2}^{g} = \frac{2(1-\gamma_{1})r_{2} + (\beta_{1}+\beta_{2})r_{1}}{4(1-\gamma_{1})(1-\gamma_{2}) - (\beta_{1}+\beta_{2})^{2}},$$
$$\Pi^{g} = \frac{(1-\gamma_{2})r_{1}^{2} + (1-\gamma_{1})r_{2}^{2} + (\beta_{1}+\beta_{2})r_{1}r_{2}}{4(1-\gamma_{1})(1-\gamma_{2}) - (\beta_{1}+\beta_{2})^{2}}.$$

As the profit-maximizing prices in (6) can be computed for any combination of $(\beta_1, \gamma_1, \beta_2, \gamma_2)$ that satisfies condition (5), they can be applied to any of the interpretations that we discussed below. We can thus record the following lesson.

Remark 1 The definition of multi-sided platforms should not rest on easily distinguishable groups of users, but on the ability to target groups with different prices.

3.2 Price structure

To understand how a platform chooses its price structure, we evaluate the profit-maximizing group prices against the benchmark of the profit-maximizing uniform price. Comparing expressions (3) and (6), we see that the group price for users in group 1 is below the uniform price $(p_1^g < p^u)$ if and only if:

$$\beta_1 - \beta_2 < \frac{(1 - \gamma_1)(\beta_2 - \beta_1) - \beta_1(\beta_1 + \beta_2) + 2(1 - \gamma_1)(1 - \gamma_2)}{2 + \beta_1 + \beta_2 - \gamma_1 - \gamma_2} \frac{r_2 - r_1}{r_1}.$$
(7)

It is immediately seen from the previous condition that if $r_1 = r_2$, then $p_1^g < p^u \Leftrightarrow \beta_1 < \beta_2$. Second, consider the case in which $\beta_1 = \beta_2 = \beta$. Then, condition (7) becomes: $((1 - \gamma_1)(1 - \gamma_2) - \beta^2)(r_2 - r_1) > 0$. As for the second-order condition (5), it becomes: $(1 - \gamma_1)(1 - \gamma_2) > \beta^2$. It follows that $p_1^g < p^u \Leftrightarrow r_1 < r_2$. We have thus established the following result:

Remark 2 Suppose that a platform can practice group pricing. Other things being equal, the platform reduces the price below the uniform price for the group that enjoys lower stand-alone benefits and/or generates larger network effects for the other group.

This result was already stressed in the seminal contributions to the literature on two-sided platforms. Rochet and Tirole (2003, 2006) and Armstrong (2006) explain that the price structure on two-sided platforms is likely to be "skewed" for the two reasons that we just exposed. Intuitively, if the two groups (or 'sides' in this interpretation) enjoy the same stand-alone benefits, then the group that is charged less is the one that exerts the larger "attraction" on the other group. To see this, recall that β_2 measures the marginal valuation for a user in group 2 to interact with a user in group 1. So, the larger β_2 , the larger the attraction that users of group 1 exerts on users on group 2. The motivation for the platform is clear. Other things being equal, the platform can generate more revenues from the other group when it reduces the price for group 1 rather than for group 2. On the other hand, if the two groups value the interaction with the other group in the same way (that is, here, if cross-side network effects are equivalent), the price structure only depends on the stand-alone benefits: The platform charges a lower price to the group that enjoys smaller stand-alone benefits (that is, the group whose users value relatively more their outside option).

Interestingly, our analysis shows that this result applies more broadly. First, in the context of two-sided platforms, we note that the two borderline cases hold irrespective of the strength of the same-side network effects (γ_1 and γ_2). Only if $r_1 \neq r_2$ and $\beta_1 \neq 2$, is it possible that the strength of the same-side network effects are decisive as to which side pays the lower price. Second, we can rephrase the insights of 2 in the context of network goods. For instance, if users benefit from similar stand-alone benefits, then a platform that is able to segment users into distinct groups will sell the network good at a lower price to the user group that exerts a stronger attraction on other users. For an illustration, think of social media platforms paying "influencers" (that is, people who build a reputation for their expertise on a specific topic and, thereby, attract a large crowd of followers).

As we mentioned earlier, our model can also be interpreted as a simplified representation of a model in which groups of users arrive sequentially over time on the market. Here, the common wisdom is that a monopoly platform has an incentive to charge lower prices to early users so as to build its network and, thereby, raise the willingness-to-pay of future users. The idea behind this so-called "seeding strategy" is that the revenues that are foregone initially will be more than compensated by the increased revenues that can be reaped at a later stage. In our model, think of the platform operating a social media. Group 1 users post content on the platform in period 1, while group 2 users do so in period 2. Assume that the content posted by group 1 users can still be enjoyed by group 2 users in period 2. If each user supplies one unit of content and all users value a unit of content at $\gamma > 0$, then the utilities of the two groups of users are respectively given by: $u_1 = r_1 + \gamma n_1 - p_1$ and $u_2 = r_2 + \gamma (n_1 + n_2) - p_2$. Developing condition (7) for this specific case, we find:

$$p_1^g < p^u \Leftrightarrow \frac{\gamma}{1-\gamma} > \frac{r_1 - r_2}{r_1}$$

This inequality is clearly satisfied if stand-alone benefits are valued more in group 2 than in group 1 $(r_2 > r_1)$. Yet, the opposite case seems more likely as early adopters are usually those with a larger valuation of stand-alone benefits $(r_1 > r_2)$ In that case, early adopters pay a lower price if and only if network effects are strong enough, namely: $\gamma > (r_1 - r_2) / (2r_1 - r_2)$.

3.3 Related literature

As already mentioned, the seminal contributions to the literature on two-sided platforms focused on pricing strategies, taking group pricing as a given. In contrast, the earlier literature on network goods does not consider group pricing, either because users are assumed to be homogenous (as in Rohlfs, 1974) or because the focus is on the competition between two network goods (as in Katz and Shapiro, 1985, 1986, or in Farrell and Saloner, 1985, 1986).

Price discrimination becomes an issue for network goods when dynamic pricing is considered. If groups of users arrive sequentially on the market, how does the sequence of prices chosen by a monopolist look like? Garcia and Resende (2011) survey the literature on dynamic network effects, both in monopoly and oligopoly markets. A common theme in this literature is that a network good provider sets its pricing strategies by trading off profits across periods along the life cycle of the good. In particular, in the case of backward network effects (earlier users generate network benefits for later users), a monopolist has an incentive to lower its current price to enhance both its present and future sales. This justifies the type of "seeding" or "introductory pricing" strategies that we hinted at earlier.

Closer to our paper, let us mention the work by Hashizume, Ikeda, and Nariu (2021). They consider a monopolist selling a network good in two markets. In each market, there is a continuum of users that differ in their relative valuation of the stand-alone benefits with respect to their outside option. They assume that network effects have a different intensity if they happen "within a market" rather than "between markets" (but the valuation of these two types of network effects is common across markets). With our notation, they consider the particular case in which $r_1 \neq r_2$, $\beta_1 = \beta_2$, and $\gamma_1 = \gamma_2$. They analyse the welfare effects of going from uniform to group pricing. Their main result is that group pricing deteriorates total consumer surplus with contrasted impacts on the two groups (if $r_1 > r_2$, group 1 users are worse off while group 2 users are better off).

The literature on network economics, which follows the 'social networks approach', also contains works that relate to our paper but with a different angle. Fainmesser and Galeotti (2016) study how a monopoly seller of a network good adapts its pricing strategy when more information becomes available about network effects across users.⁹ In contrast with the approach generally taken in industrial organization (our paper being no exception), network effects are not a simple function of the number of users making the same choice but are modeled as a deterministic graph (network effects can then be heterogeneous across users as they depend on their position in the network). In this setting, the authors show that the monopoly seller price discriminates by offering discounts to users generating stronger network effects (that is, having a larger 'influence') and premia to users receiving network effects more intensively (that is, being more 'susceptible to influence'). Using different approaches to modeling the architecture of network effects, Candogan, Bimpikis, and Ozdaglar (2012) and Bloch and Quérou (2013) find similar profit-maximizing pricing structures.¹⁰

⁹Fainmesser and Galeotti (2020) extend the analysis to competing providers of a network good – this paper also provides a useful entry guide to the emerging literature on profit-maximizing marketing strategies in the presence of network effects, which spans the fields of economics, marketing, and computer science. Following the two-sided markets approach, Jullien (2011) and Aoyagi (2018) also analyze competing non-differentiated providers of network goods and make a connection to the 'social networks approach'; for analyses with differentiated network providers, see Tan and Zhou (2021) and Peitz and Sato (2023).

¹⁰Leduc, Jackson, and Johari (2017) consider a monopolist introducing a new product about which potential consumers can learn from their own experience or by learning from others. Potential consumers are connected by a network and the monopolist can provide incentives by offering price discounts to become early adopters allowing others to learn. Absent other incentives provided by the monopolist, it would be consumers with few friends who tend to be early adopters. If the monopolist rewards early adopters for purchases by friends in the second period, the monopolist encourages consumers with many friends to adopt early. As the authors show, the

4 Versioning

In contrast with the previous case, we assume here that the platform does not observe (or is not able to verify) to which group users belong to. The platform, therefore, offers a menu of options with the aim to make users self-select among the options and, thereby, reveal their characteristic. Arguably, this situation is much more likely to be observed in the case of a single network good as all users participate in the platform in a similar way. We focus thus, from now on, on the case in which $\gamma_1 = \beta_1$ and $\gamma_2 = \beta_2$. To simplify the exposition, we also assume that both groups enjoy the same stand-alone benefits ($r_1 = r_2 = r$). The two groups differ thus in their valuation of the network benefits: we posit that for a given network size ($n_1 + n_2$), group 1 users have a larger utility than group 2 users: $\beta_1 > \beta_2$.¹¹

Under these assumptions, we can adapt expression (1) to express the net utilities that group 1 and group 2 users obtain on the platform:

$$\begin{cases} u_1 = r + \beta_1(n_1 + n_2) - p_1, \\ u_2 = r + \beta_2(n_1 + n_2) - p_2. \end{cases}$$
(8)

Applying the analysis of Section 3, we find the profit-maximizing prices that the platform would set *if it could identify users*:

$$p_1^g = \frac{(2-\beta_1-3\beta_2) - (\beta_1-\beta_2)^2}{4(1-\beta_1-\beta_2) - (\beta_1-\beta_2)^2}r \text{ and } p_2^g = \frac{(2-3\beta_1-\beta_2) - (\beta_1-\beta_2)^2}{4(1-\beta_1-\beta_2) - (\beta_1-\beta_2)^2}r.$$

The corresponding participation levels and profit would then be equal to:

$$n_1^g = \frac{2 + (\beta_1 - \beta_2)}{4(1 - \beta_1 - \beta_2) - (\beta_1 - \beta_2)^2} r, \ n_2^g = \frac{2 - (\beta_1 - \beta_2)}{4(1 - \beta_1 - \beta_2) - (\beta_1 - \beta_2)^2} r,$$
$$\Pi^g = \frac{2}{4(1 - \beta_1 - \beta_2) - (\beta_1 - \beta_2)^2} r^2.$$

4.1 Relationship between versioning and group pricing

We show here that it is possible for the platform to design a versioning scenario whereby the analysis under group pricing is replicated (and we obtain Π^g for appropriate versions targeted to the two groups). Suppose that the platform offers a base and premium version of the network good. The platform has designed the two versions in such a way that they provide the two groups of users with differentiated network effects, as reported in Table 1.

profit-maximizing strategy of the monopolist may feature such referral rewards with the result that consumers with few and consumers with many friends adopt early while those with an intermediate number of friends adopt later. Here, the monopolist's pricing policy is used as a screening device on the consumer characteristics regarding their connectedness to other consumers.

¹¹In the appendix, we assume instead that groups differ in their valuation of the stand-alone benefits but value network effects in the same way: $r_1 \neq r_2$ and $\beta_1 = \beta_2$. We obtain qualitatively the same results in this case.

	Base	Premium
Group 1	$\beta_1 - \delta_1$	β_1
Group 2	β_2	$\beta_2 + \delta_2$

Table 1: Valuation of network effects

We assume that $0 \leq \delta_2 < \delta_1 < \beta_1$, so that, other things being equal, both groups value more the premium version than the base version $(\delta_1, \delta_2 > 0)$, users in group 1 value the upgrade from base to premium more than users in group 2 $(\delta_1 > \delta_2)$, and the base version provides users of group 1 with positive network effects $(\delta_1 < \beta_1)$.

The platform's goal is to induce users to self-select so that users in group 1 buy the premium version (sold at price p_1), while users in group 2 buy the base version (sold at price p_2). Supposing for now that the platform manages to do so, we can write the users' net utilities as $u_1 =$ $r + \beta_1 (n_1 + n_2) - p_1$ and $u_2 = r + \beta_2 (n_1 + n_2) - p_2$. As these utilities are exactly the same as in expression (8), the platform would maximize its profit by setting $p_1 = p_1^g$ and $p_2 = p_2^g$, which would generate the participation levels n_1^g and n_2^g , and would lead to a profit of Π^g . All this, of course, is conditional of users self-selecting as intended. That is, the following two incentive-compatibility constraints must be satisfied:

$$\begin{aligned} r + \beta_1 \left(n_1^g + n_2^g \right) - p_1^g & \geq \quad r + (\beta_1 - \delta_1) \left(n_1^g + n_2^g \right) - p_2^g \Leftrightarrow \delta_1 \left(n_1^g + n_2^g \right) \geq p_1^g - p_2^g, \\ r + \beta_2 \left(n_1^g + n_2^g \right) - p_2^g & \geq \quad r + (\beta_2 + \delta_2) \left(n_1^g + n_2^g \right) - p_1^g \Leftrightarrow p_1^g - p_2^g \geq \delta_2 \left(n_1^g + n_2^g \right). \end{aligned}$$

The first constraint imposes that any group 1 user be better off buying the base rather than the premium version; the second applies to any group 2 user (who must prefer the premium version over the base one). Note that the decision by an individual user has a negligible impact in our model; that is, if a user switches versions, this decision leaves the participation levels n_1 and n_2 unaffected.

A few lines of computations establish that $(p_1^g - p_2^g) / (n_1^g + n_2^g) = (\beta_1 - \beta_2)/2$, which is positive as we assume $\beta_1 > \beta_2$. We can then rewrite the incentive-compatibility constraints in the following compact way:

$$\delta_2 \le \left(\beta_1 - \beta_2\right)/2 \le \delta_1. \tag{9}$$

If the two versions are designed to meet the requirements of condition (9), the platform sets the same prices as under group pricing even though it is not able to observe the group characteristics (under the constraint that under group pricing, only the base version were available to group 2). We have thus proven by example:

Proposition 1 When users of a network good benefit from network size to a varying degree according to the version of the network good they consume, a monopolist can profitably introduce two versions of the same network good inducing user self-selection. Under some conditions, the profit-maximizing strategy coincides with the strategy chosen by a two-sided platform that identifies the two groups of consumers, but can not provide the premium version to one of the two groups.

The intuition behind this result can be explained as follows. Note first that because we consider technologies featuring constant marginal costs, the supply side does not generate any difference or link between the groups of users. It is thus only demand characteristics that, at the same time, separate groups (groups value network benefits differently) and link them (network effects make all users value total participation). We compare group pricing (when the monopolist can observe the users' characteristics) to versioning (when it cannot but has the option to offer a menu of versions of the network good). In the absence of network effects, the monopoly problem under group pricing becomes a set of independent pricing problems for each group while, under versioning, it becomes a set of pricing problems that are independent or linked through binding incentive-compatibility constraints. When network effects are present, the pricing problem under group pricing becomes akin to the pricing problem of a two-sided platform (as we made it clear in Section 2). As for versioning, Proposition 1 tells us that the incentive-compatibility constraint may be slack and, as a result, only participation constraints matter. In our simple model, this is achieved by additional heterogeneity among users (which always remains their private information): users value differently the outside option.¹² Then, with network effects, the pricing problem under versioning is also akin to the pricing problem of a two-sided platform. This suggests that understanding the pricing of two-sided platforms deserves managerial attention outside platform contexts and applies broadly when firms formulate their marketing strategy in the presence of network effects.

Finally note that under our construction group pricing does not give the same profit as versioning. A platform being able to use group pricing without any constraint would offer the premium version to both user groups.

4.2 Freemium

We now consider a freemium strategy as a particular case of the versioning strategy that we just described. The freemium strategy combines a base version of the network good that is offered at a price of zero and a premium version that is offered at a strictly positive price. As, in our context, there are no revenues directly associated with the users of the free version, this is a true freemium strategy indeed. For the base version to be offered for free, it must be $p_2^g = 0$, which is equivalent to $\beta_1 = \beta_2 - \frac{3}{2} + \frac{1}{2}\sqrt{17 - 16\beta_2}$.¹³

To understand better what a freemium strategy involves, we develop a specific example by setting $\beta_1 = \beta + \delta$, $\beta_2 = \beta$, $\delta_1 = \delta$, and $\delta_2 = 0$. In this example, the base version is valued

¹²Absent network effects, Rochet and Stole (2002) obtain a similar result in a more general setting by introducing independent randomness into the agents' outside options.

¹³This may appear as a knife-edge case as a small pertubation of β_1 or β_2 may lead to a non-zero price. An alternative – and more general – interpretation could then be that the platform would find it optimal to set $p_2^g < 0$ but cannot set a negative price (for some legal or technical reason). This would be so, here, for any values of the parameters that satisfy the following inequality: $\beta_1 > \beta_2 - \frac{3}{2} + \frac{1}{2}\sqrt{17 - 16\beta_2}$. If so, $p_2^g = 0$ would emerge as the corner solution of the profit-maximization problem of the platform, given the non-negativity constraint on prices. As the same constraint should logically be binding as well when the platform can identiy users, the optimality of the freemium strategy continues to hold.

equally by users in both groups with the strength of the network effect β . As for the premium version, it does not provide group 2 users with any stronger network effect but provides group 1 users with a stronger network effect $\beta + \delta$, with $\delta > 0$. Under these assumptions, we have that:

$$p_2^g = \frac{2 - 4\beta - 3\delta - \delta^2}{4 - 8\beta - 4\delta - \delta^2} r = 0 \Leftrightarrow 4\beta = 2 - 3\delta - \delta^2.$$

Imposing $0 < \delta < 0.561$, we can find positive values of β that satisfy the latter equality.¹⁴ We can, therefore, use this equality to compute the prices of the premium and free versions, the participation levels, and the platform's profit under the freemium strategy:

$$p_1^F = \frac{2}{2+\delta}r, \, p_2^F = 0, \, n_1^F = \frac{1}{\delta}r, \, n_2^F = \frac{2-\delta}{\delta(2+\delta)}r, \, \text{and} \, \Pi^F = p_1^F n_1^F = \frac{2}{\delta(2+\delta)}r^2.$$

Even if group 2 does not generate any direct revenue, it does so indirectly by raising, through its participation, the network benefits and, thereby, the willingness to pay of group 1 users. To see this, we compare the results we just derived with a hypothetical situation in which group 2 users would not exist. In that case, $n_1 = r + (\beta + \delta) n_1 - p_1$; solving for n_1 , we have $n_1 = (r - p_1)/(1 - \beta - \delta)$. As the platform maximizes n_1p_1 , we find the profit-maximizing price as $\tilde{p}_1 = r/2$. It follows that $\tilde{n}_1 = \frac{1}{2(1-\beta-\delta)}r$ and $\tilde{n}_1\tilde{p}_1 = \frac{1}{4(1-\beta-\delta)}r^2$. We check that:¹⁵

$$\Pi^{F} - \tilde{n}_{1}\tilde{p}_{1} = \frac{2}{\delta(2+\delta)}r^{2} - \frac{1}{4(1-\beta-\delta)}r^{2} = \frac{8-10\delta-\delta^{2}-8\beta}{4\delta(\delta+2)(1-\beta-\delta)}r^{2} > 0.$$

This means that the platform does indeed achieve a larger profit when offering a menu in which users can obtain the basic version for free. The reason is that giving away the basic version for free leads some group 2 users to join; this makes participation more attractive for group 1 users, and leads to higher platform profits. The freemium strategy also dominates any strategy with a uniform price that leads some group 2 users to join.¹⁶ We record thus the following result.

Remark 3 Because of the presence of network effects, a platform providing a network good may find it optimal to adopt a freemium strategy, whereby it lets user self-select between a base version that is available for free and a premium version that is sold at a positive price.

For an illustration, consider the recent price changes that a number of social networks announced or introduced in 2022–23. Social networks have traditionally offered their service for free to users and relied instead on advertising and data exploitation to monetize their activities. Yet, as the global economic conditions caused a decline of these revenues, several social networks started to propose paid plans that allow users to unlock some features that are not included in the free version. These features are, for instance, verification tools (as the 'blue tick' proposed

¹⁴We also check that this equality is compatible with the second-order conditions, which impose, in the present case, that $4\beta < 2 - 2\delta - (\delta^2/2)$.

¹⁵Recall that the SOC impose $8\beta < 4-4\delta-\delta^2$. This implies that $8\beta < 8-10\delta-\delta^2$ as $8-10\delta-\delta^2-(4-4\delta-\delta^2) = 2(2-3\delta) > 0$ for $\delta < 0.561$.

¹⁶It is readily found that the profit-maximizing uniform price is equal to r/2. For $4\beta = 2 - 3\delta - \delta^2$, the corresponding profit is equal to $\Pi^u = r^2/(\delta(1+\delta)) < \Pi^F = 2r^2/(\delta(2+\delta))$.

by Twitter, Instagram, or Facebook), ability to pin 'special' contacts (as on Snapchat+), custom avatar (as on Reddit Premium).¹⁷ As these features improve the quality of any interaction on the social network, they increase the valuation of the network effects for the users who choose to pay for the premium version. It is also reasonable to assume that for many users, these features do not add any value whatsoever. Note that because the features of the "premium" versions used to be free in the past, one can consider that social networks resort to a "damaged good" strategy (Deneckere and Preston McAfee, 1996): The low-quality version (free) is obtained by damaging – that is, by removing features from – the high-quality version (premium).

4.3 Related literature

Versioning and two-sided platforms. Jeon, Kim and Menicucci (2022) consider versioning by a two-sided monopoly platform. In their baseline model, users on side A (the "value creation side") may be of two types, while users on side B (the "value capture side") are homogenous; the type of a side-A user conditions not only their stand-alone benefit but also the cross-side network effect that they exert on side-B users. A key feature of the model is that the dual nature of users' types may cause tension between the menu of versions that the platform can implement (which is determined by the incentive compatibility constraints) and the menu that maximizes its profits (through the internalization of network effects). If so, the platform may prefer not to discriminate by offering a single 'quality' at a uniform price. Such issues do not arise in our setting because we consider a single group of users (meaning that the platform creates and captures value for and from the same users).

Böhme (2016) also considers versioning by a two-sided platform; users on each side can be of two types, which differ in their valuation of both the stand-alone utility and the crossgroup network effects. The author shows that a two-sided platform that practices versioning (incomplete information) reduces the quantity for high-type users compared to what it would do under group pricing (complete information). It is the presence of cross-group network effects that explains this violation of the 'no-distortion-at-the-top' result, which holds in the textbook versioning problem: the platform cannot fully extract the willingness to pay of high-type users on one side because of the network effects that these users exert on both types of users on the other side (and, thereby, on the menu of options that can be offered to them). Relatedly, Lin (2020) studies versioning by a media platform in which buyers experience a negative crossgroup network effect from more sellers (advertising nuisance) and may be offered a version with premium content and another version with basic content. Also in this model, the "nodistortion-at-the-top" principle is violated. Different from Jeon, Kim, and Menicucci (2021), in Lin's setting price discrimination on one side can strengthen the incentive to discriminate on the other.

Gomes and Pavan (2016) examine versioning by matching platforms. In their model, users differ in both their interest to interact with agents from the other side and in their attractiveness.

 $^{^{17}}$ See, e.g., Mehta (2023).

In their setting, the platform chooses not only a menu of prices but also a matching rule; they show how cross-group network effects entail subsidization across sides, which shapes both the price and the matching schedules. Gomes and Pavan (forthcoming) consider the case where users' preferences are both vertically and horizontally differentiated. In that setting, they study the interaction between versioning and group pricing, and examine the effects of banning group pricing (thereby forcing the platform to set a uniform price for users on one side).¹⁸ In our paper, we do not allow for type-specific matches (and an endogenous matching rule); we provide a simple setting that allows us to compare versioning to group pricing.

Versioning and network goods. Closer to our work are papers that examine differential pricing of goods exhibiting direct network effects. Regarding versioning, Hahn (2004) and Csorba and Hahn (2006) consider a vertical differentiation model à la Mussa and Rosen (1978) with network effects. They show that versioning by introducing a damaged good may be profitable and welfare-enhancing. In a similar vein, Jing (2007) studies the case of monopolist that produces two fully compatible versions of a network good, assuming that the high-quality version generates stronger direct network effects than the low-quality version. The main result is that the presence of network effects make versioning profitable as, absent network effects, the monopolist's optimal conduct would be to market a single version of the good. Sundararajan (2004) describes the optimal non-linear pricing scheme for the monopoly provider of a network good. In his setting, users are heterogeneous in both stand-alone and network benefits, can use the good in continuously varying quantities, and enjoy direct network effects from total consumption. Csorba (2008) considers the monopoly provision of a network good with a finite number of vertically differentiated user types. In his setting, all neighboring downward incentive compatibility constraints are binding and the profit-maximizing menu features social underprovision. Veiga (2018) also considers the selling of different versions of a network good; however, in his setting, the network benefit only accrues to the users of a specific version and not the total number of users across all versions; therefore, his finding that a monopolist never wants to use versioning stands in contrast to the finding in our paper.

Our analysis also bears a connection with the mechanism design literature that considers situations with one principal and multiple agents exerting externalities on one another. We can indeed draw a parallel between our monopoly platform offering different versions of a network good to its users and a principal presenting a menu of contracts to its agents. An important contribution in this literature is Segal (2003), who shows how the principal's optimal strategy depends on the types of externalities among agents, and the principal's ability to price discriminate and to coordinate agents on a given outcome. In Segal's setting, the principal is an incumbent trying to deter entry and the agents are customers; by signing an exclusionary contract with any customer, the incumbent makes entry less profitable for the entrant; this means that by accepting to trade with the principal, an agent exerts a negative externality on the other agents (as the market becomes less competitive). In our setting, a user trading with the

¹⁸For a short exposition of these works, see Jullien, Pavan, and Rysman (2021).

platform (that is, purchasing some version of the network good) exerts a positive externality on the other users. Winter (2004) considers incentive schemes in organizations. He shows that in the presence of externalities among agents, optimal incentive mechanisms may require differential rewards even if agents are homogeneous; in particular, in environments in which an agent's incentive to exert effort increases with the number of other agents who do so, agents should be provided with differential incentives. Bernstein and Winter (2012) explore this issue further by examining the problem of a principal that tries to coordinate the participation of agents when there exist heterogeneous externalities among them; one example related to platforms is a mall owner (the principal) whose success depends on the participation of a group of brand stores of various sizes (the heterogenous agents).

Although all these papers bring useful insights regarding the profitability and the impacts of versioning for network gods, they do not compare versioning to group pricing, neither do they make any link with the literature on two-sided platforms. Hence, they do not address the two key issues developed in this paper.

Freemium. Shi, Zhang, and Srinivasan (2019) share our approach of the freemium strategy as long as the free version does not generate any form of complementary revenues (like advertising or data exploitation revenues). The main result of their analysis is that the freemium strategy is not profitable unless all users value more the network effects for the premium than for the free version. In contrast, in our model, the two groups of users must value differently the network effects generated by the two versions.¹⁹ These seemingly contrasting results follow from different modeling choices. Shi, Zhang, and Srinivasan (2019) extend the traditional model of vertical differentiation à la Mussa and Rosen (1978), with users being heterogenous along a single dimension (their relative valuation of the stand-alone benefits with respect to their outside option). It is thus as though users belonged all to the same group whereas, in our setting, we assume that users belong to different groups and, on top, are heterogenous in the way they value network benefits (and/or stand-alone benefits). Behind these differences, the two papers convey the same message: The potential profitability of free users stems from the network effects that they generate, allowing the platform to extract more value from premium users. Interestingly, this explanation of the profitability of freemium differs from the one that is proposed in the earlier management and marketing literature, namely that the free version allows its users to sample the good, thereby reducing their uncertainty about the benefit of the premium version (see, for example, Deng, Lambrecht, and Liu, 2018, and the references therein).

The literature on end-user piracy of content goods also relates to our finding that the freemium strategy may be profit-maximizing. In a model of vertical differentiation à la Mussa and Rosen (1978) with network effects, Takeyama (1994) shows that a firm may be better off when a base product is available at a price of zero (the pirated product) than when this version

¹⁹This is so if they share the same valuation of the stand-alone benefits. In the appendix, we analyse the case in which groups differ in their valuation of the stand-alone benefits but not of the network benefits. The freemium strategy is shown to be profitable in this case as well.

is not available. King and Lampe (2003) provide a related analysis; for a survey, see Belleflamme and Peitz (2012). These results speak to the profitability of the freemium strategy; however, in a piracy setting, the presence of the free version is assumed rather than derived as part of the profit-maximizing strategy.

5 Conclusion

This paper establishes that price discrimination with a network good can be interpreted as pricing of a multi-sided platform. More specifically, we show this result in a linear setting, which allows us to provide closed-form solutions. This insight generalizes when the platform cannot observe users' characteristics, but can offer different versions of its service. Due to different sources of heterogeneity across users, versioning may allow the monopolist to achieve the same profit as under group pricing – when this is the case, the monopolist does not gain from observing and verifying the users' characteristics. These insights neither rely on the linear structure nor the monopoly setting and, therefore, apply more generally to non-linear specifications of utilities and settings with competing platforms. Regarding the latter point, we submit that specific models of price discrimination in oligopoly with network goods can be translated into models of competing two-sided platforms. For example, such a setting could allow us to obtain a special case of the model analyzed by Tan and Zhou (2021). The general conceptual point we wanted to make is that the concept of multi-sided platforms is useful to analyze price discrimination with network goods.

A possible extension is to consider versioning with a menu of *non-monetary* payments. For instance, a social network may offer a portfolio of privacy choices or data-sharing agreements. These could be tied to different functionalities or services provided by the social network. A take-away from our analysis in this paper is that public interventions concerning one particular option targeted toward one user group affect the overall menu and, thus, have repercussions for other user groups – this may happen even when incentive constraints are not binding and be entirely driven by network effects.

The observation in this paper that price discrimination with network effects resembles pricing on a two-sided platform has implications for market definition in competition practice (on market definition in the context of two-sided platforms, see Katz and Sallet, 2018, and Franck and Peitz, 2021). As a platform uses different price (and possibly non-price) instruments to cater to heterogeneous groups of users who are subject to network effects, separate antitrust markets can be identified for each of those groups; however, the interaction between these markets thanks to network effects is essential for an understanding of the business strategy.

Appendix

We extend here the analysis of a monopoly provider of a network good by considering an arbitrary number of groups of users.

5.1 Participation decisions (Stage 2)

A user of group i obtains the utility

$$u_i = r_i + \beta_i N - p_i, \tag{10}$$

where r_i is the stand-alone benefit, β_i the strength of the network benefit, N the total number of users, and p_i the price a user pays conditional on belonging to group *i* when accepting the platform service (or a particular version thereof). The outside option of a user has value x. Thus, the user with characteristics (i, x) prefers a proposed service over the outside option if and only if $r_i + \beta_i N - p_i \ge x$.

In what follows, we assume for simplicity that there are $I \ge 2$ realizations of i, which are present with equal shares in the population.²⁰ Under the same assumptions as in the model with two groups, we have that the number of users of group i as a function of the price p_i is implicitly given by

$$n_i = r_i + \beta_i N - p_i. \tag{11}$$

We can rewrite this by expressing the number of participating users in one group as a function of the price charged to that group and the number of participating users in the other groups: $n_i = (r_i + \beta_i N_{-i} - p_i) / (1 - \beta_i)$ where $N \equiv \sum_{i=1}^{I} n_i$ and $N_{-i} \equiv N - n_i$ is the total number of users in all groups but group *i*. Assuming that $\beta_i < 1$, we observe that network effects make participation in any two groups complementary with one another.

For given prices, users play an anonymous game and we solve for the Nash equilibrium of this game; each user makes their participation decision given the participation decision of all other users and these decisions must be mutually consistent. At the Nash equilibrium, we obtain user participation as a function of all prices.²¹ To ease the exposition, we introduce the following notation:

$$R \equiv \sum_{i=1}^{I} r_i, \ B \equiv \sum_{i=1}^{I} \beta_i, \ P \equiv \sum_{i=1}^{I} p_i.$$

Summing equation (11) over all types i = 1, ..., I gives N = R + BN - P. Solving for N and assuming that B < 1, we find N = (R - P) / (1 - B). Plugging this value into (11) yields the

²⁰In what follows, we refer to these realizations as 'types' or 'groups' interchangeably.

²¹In general, for each group with characteristic *i*, there is a conditional demand $n_i(\mathbf{p})$ that depends on the price vector **p**. Suppose that *i* is distributed according to *F* over some space \mathcal{I} , and *x* according to *G* on the positive reals. The demand system $n_i(\mathbf{p})$ is the solution to $n_i = G(r_i + \beta_i N - p_i)$ and $N = \int n_i dF(i)$. The monopoly platform with constant per-user costs *c* then solves the maximization problem $\max_{\mathbf{p}} \int [(p_i - c)n_i(\mathbf{p})]dF(i)$. This resembles the maximization problem of a multi-product monopolist. Because of network effects, demand for the different offers are interdependent; monopoly price p_i^* and/or monopoly quantity n_i^* may depend on parameters that determine the valuation of users of group (j, x) with $j \neq i$.

demand for participation in group i as a function of all prices:

$$n_i = r_i - p_i + \beta_i \frac{R - P}{1 - B}.$$
 (12)

As B < 1, participation is complementary across groups (that is, n_i is a decreasing function af all prices).

5.2 Pricing (Stage 1)

5.2.1 Uniform pricing

We first look at the benchmark case of uniform pricing. If the network good is sold at the same price p to all users (irrespective of the group they belong to), then equation (12) becomes:

$$n_i = r_i - p + \beta_i \frac{R - Ip}{1 - B}.$$

So, the platform chooses p to maximize:

$$\Pi = p \sum_{i=1}^{I} n_i = p \sum_{i=1}^{I} \left(r_i - p + \beta_i \frac{R - Ip}{1 - B} \right) = p \left(R - Ip + B \frac{R - Ip}{1 - B} \right) = p \frac{R - Ip}{1 - B}$$

The profit-maximizing price is easily found as:

$$p^u = \frac{R}{2I} = \frac{1}{2}\bar{r},$$

where $\bar{r} \equiv R/I$ is the mean of the r_i 's. The corresponding participation levels are:

$$n_i^u = r_i - \frac{R}{2I} + \frac{\beta_i}{1-B}\frac{R}{2} = r_i - \frac{1}{2}\frac{1-B-I\beta_i}{I(1-B)}R.$$

To make sure that the monopoly platform chooses to serve all groups of users under uniform pricing, we assume that:

$$r_i > \frac{1 - B - I\beta_i}{2(1 - B)}\bar{r}$$
 for all $i = 1, \dots, I$.

The platform's profit at its maximum is then computed as:

$$\Pi^u = \frac{R^2}{4I\left(1-B\right)}.$$

5.2.2 Group pricing

Assume now that the platform has enough information to tell users apart according to the group they belong to. It thus sells the network good at price p_i to users of group i, with i = 1, ..., I. That is, the platform has the following maximization program:

$$\max_{p_1,\dots,p_I} \Pi = \sum_{i=1}^{I} p_i n_i = \frac{1}{1-B} \sum_{i=1}^{I} p_i \left[(1-B) \left(r_i - p_i \right) + \beta_i \left(R - P \right) \right].$$

The first-order condition with respect to p_i yields:²²

$$(1-B)(r_i - p_i) + \beta_i (R-P) - (1-B + \beta_i) p_i - \sum_{k=1, k \neq i}^{I} \beta_k p_k = 0,$$

or, equivalently,

$$(1-B)r_i - 2(1-B)p_i + \beta_i(R-P) - P_\beta = 0,$$
(13)

where $P_{\beta} \equiv \sum_{i=1}^{I} \beta_i p_i$.

Computing the difference between the first-order conditions with respect to prices p_i and p_j and rearranging terms, we find:

$$p_i - p_k = \frac{1}{2} (r_i - r_k) + \frac{R - P}{2(1 - B)} (\beta_i - \beta_k).$$

Extending the results that we obtained for the case of two groups in Section 3, we observe that the platform chooses $p_i > p_k$ if either $r_i > r_k$ and $\beta_i = \beta_k$, or $\beta_i > \beta_k$ and $r_i = r_k$. That is, a platform that practices group pricing charges higher prices to groups that enjoy larger stand-alone or network benefits.

Summing equation (13) over all *i* and solving for *P*, we find $P = (R - IP_{\beta})/(2 - B)$. Plugging this expression into (13) and solving for p_i gives:

$$p_i = \frac{r_i}{2} + \frac{\beta_i \left(1 - B\right) R - \left(2 - B - \beta_i I\right) P_\beta}{2 \left(1 - B\right) \left(2 - B\right)}.$$
(14)

Multiplying on both sides by β_i , summing over all *i*, and solving for P_{β} , we obtain:

$$P_{\beta} = \frac{(1-B)B_2}{(2-B)^2 - IB_2}R + \frac{(1-B)(2-B)}{(2-B)^2 - IB_2}R_{\beta},$$

where $R_{\beta} \equiv \sum_{i=1}^{I} \beta_i r_i$ and $B_2 \equiv \sum_{i=1}^{I} \beta_i^2$. Inserting the latter expression into equation (14), we derive the profit-maximizing price for users of type *i*, denoted by p_i^g , as:

$$p_i^g = \frac{r_i}{2} + \frac{1}{2} \frac{(2-B)R + IR_\beta}{(2-B)^2 - IB_2} \beta_i - \frac{1}{2} \frac{B_2R + (2-B)R_\beta}{(2-B)^2 - IB_2}.$$
(15)

Summing over all types i, we compute:

$$P^{g} = \frac{R}{2} + \frac{1}{2} \frac{(2-B)R + IR_{\beta}}{(2-B)^{2} - IB_{2}} B - \frac{1}{2} \frac{B_{2}R + (2-B)R_{\beta}}{(2-B)^{2} - IB_{2}} I.$$
 (16)

We can now derive the number of participating users of type i by plugging expressions (15) and (16) into expression (12):

$$n_i^g = \frac{r_i}{2} + \frac{1}{2} \frac{(2-B)R + IR_\beta}{(2-B)^2 - IB_2} \beta_i + \frac{1}{2} \frac{B_2R + (2-B)R_\beta}{(2-B)^2 - IB_2}.$$
(17)

Finally, we compute the platform's profit under group pricing as:

$$\Pi^{g} = \sum_{i=1}^{I} p_{i}^{g} n_{i}^{g} = \sum_{i=1}^{I} \left(\frac{r_{i}}{2} + \frac{1}{2} \frac{(2-B)R + IR_{\beta}}{(2-B)^{2} - IB_{2}} \beta_{i} \right)^{2} - \left(\frac{1}{2} \frac{B_{2}R + (2-B)R_{\beta}}{(2-B)^{2} - IB_{2}} \right)^{2}.$$
 (18)

 $^{^{22}}$ It can be checked that assumption (A1) guarantees that the second-order conditions are satisfied.

5.2.3 Versioning

We focus here on the case in which all groups have a common valuation of the network effects: $\beta_i = \beta$ for all *i*. Suppose that the platform can offer *I* versions of the network good, with version *k* providing users of group *i* with the following stand-alone utility (i, k = 1, ..., I):

$$r_i(k) = \begin{cases} r + (k-1)\Delta & \text{if } k \le i, \\ r + (i-1)\Delta & \text{if } k > i, \end{cases}$$

where $r, \Delta > 0$. In words, we assume that users in group *i* care about upgrades up to version k = i, but not about further upgrades. We see indeed that group 1 is indifferent between all versions as it values them all (in terms of stand-alone utility) at *r*. Moving to group 2, we see that it also values version 1 at *r*, while it values all other versions at $r + \Delta$. As for group 3, it values versions 1 at *r*, version 2 at $r + \Delta$, and all other versions from version 3 on at $r + 2\Delta$, and so on so forth. We can thus see version 1 as the basic version (as it provides all users with the lowest stand-alone utility of *r*), and group 1 as the group with the lowest willingness to pay (as all versions provides its users with the same stand-alone utility of *r*).

If the contract menu is such that group *i* users choose version *i*, we have $u_i = r_i(i) + \beta N - p_i$ with $r_i(i) = r + (i-1)\Delta$, $\beta < 1/I$, and i = 1, ..., I. These are the exact same utilities as in expression (10) above, but with a different interpretation.²³ Hence, if the incentive constraints of group i = 1, ..., I - 1 are not binding (that is, if group *i* users are strictly better off when they buy version *i* instead of any other version), then the analysis carried out under group pricing still holds. In particular, the profit-maximizing prices are $p_i^* = r_i(i)/2 = (r + (i - 1)\Delta)/2$.

Let us then check that the incentive constraints are slack as postulated. We start with group I. Its users prefer version I over any version k = 1, ..., I - 1 if and only if

$$r_{I}(I) - p_{I}^{*} > r_{I}(k) - p_{k}^{*} \Leftrightarrow \frac{r + (I-1)\Delta}{2} > \frac{r + (k-1)\Delta}{2} \Leftrightarrow I > k_{I}$$

which is clearly satisfied. Moving to group I - 1, the following conditions must be met: $r_{I-1}(I-1) - p_{I-1}^* > r_{I-1}(I) - p_I^*$ and $r_{I-1}(I-1) - p_{I-1}^* > r_{I-1}(k) - p_k^*$ for $k = 1, \ldots, I-2$. The first condition is equivalent to $(r + (I-2)\Delta)/2 > (r + (I-2)\Delta) - (r + (I-1)\Delta)/2$ or $(I-1)\Delta > (I-2)\Delta$; the other conditions are equivalent to $(r + (I-2)\Delta)/2 > (r + (k-1)\Delta)/2$ or I-1 > k; they are all clearly satisfied. Recursively, we check that no incentive constraint is binding for any group.

We can thus conclude that in these particular circumstances, the platform achieves the same profit under versioning as under group pricing if the platform were constrained to offering the corresponding version to each group. We have thus proven by example a result that is qualitatively similar to the one of Proposition 1 (the difference here is that there may be more than groups of users and groups differ according to the stand-alone – and not network – benefits that their users enjoy).

²³If $r_i(i)$ is group *i*'s valuation of the stand-alone utility of a unique network good, and if the monopolist sets a uniform price, then all groups are served provided that assumption (A2) is satisfied; under this formulation, condition (A2) becomes $r/\Delta > (1 - 2I\beta)(I - 1)/2$.

5.2.4 Freemium

Limiting the number of groups to I = 2, we extend the previous result to design a specific freemium strategy in the case where users are heterogeneous in terms of stand-alone benefits. Setting r = 0, the basic version is sold at a price of zero and, thus, group 2 users obtain the good for free. As for the premium version, it is sold at $\Delta/2$. The equilibrium numbers of users in the two groups are then computed as:

$$n_1^* = \frac{1-\beta}{2(1-2\beta)}\Delta \text{ and } n_2^* = \frac{\beta}{2(1-2\beta)}\Delta.$$

As for the platform's profit, it is equal to:

$$\Pi^* = n_1^* p_1^* = \frac{1 - \beta}{4(1 - 2\beta)} \Delta^2.$$

Even if group 2 does not generate any direct revenue, it does so indirectly by raising, through its participation, the network benefits and, thereby, the willingness to pay of group 1 users. To see this, we compare the results we just derived with a hypothetical situation in which group 2 users would not exist. In that case, $n_1 = r_1 + \beta n_1 - p_1$; solving for n_1 and replacing r_1 by Δ , we have $n_1 = (\Delta - p_1)/(1 - \beta)$. As the platform maximizes n_1p_1 , we find the profit-maximizing price as $\tilde{p}_1 = \Delta/2$. It follows that $\tilde{n}_1 = \frac{1}{2(1-\beta)}\Delta$ and $\tilde{n}_1\tilde{p}_1 = \frac{1}{4(1-\beta)}\Delta^2$. We check that:

$$\Pi^* - \tilde{n}_1 \tilde{p}_1 = \frac{1 - \beta}{4(1 - 2\beta)} \Delta^2 - \frac{1}{4(1 - \beta)} \Delta^2 = \frac{\beta^2}{4(1 - 2\beta)(1 - \beta)} \Delta^2 > 0,$$

meaning that the platform does indeed achieve a larger profit when offering a menu in which users can obtain the basic version for free.

References

- Aoyagi, Masaki. (2018). Bertrand Competition under Network Externalities. Journal of Economic Theory 178, 517–550.
- [2] Armstrong, M. (2006). Competition in Two-Sided Markets. Rand Journal of Economics 37, 668–691.
- [3] Belleflamme, P. and E. Toulemonde (2009). Negative Intra-Group Externalities in Two-Sided Markets. International Economic Review, 50/1, 245–272.
- [4] Belleflamme, P. and M. Peitz (2010). Industrial Organization: Markets and Strategies. Cambridge: Cambridge University Press.
- [5] Belleflamme, P. and M. Peitz (2012). Digital Piracy: Theory. In: M. Peitz and J. Waldfogel (eds.), The Oxford Handbook of the Digital Economy, Oxford University Press.
- [6] Belleflamme, P. and M. Peitz (2018). Platforms and Network Effects. In: L. Corchon and M. Marini (eds.), Handbook of Game Theory and Industrial Organization, vol. II, Edward Elgar, 286–317.
- [7] Belleflamme, P. and M. Peitz (2019). Managing Competition on a Two-Sided Platform. Journal of Economics & Management Strategy 28, 5–22.
- [8] Belleflamme, P. and M. Peitz (2021). The Economics of Platforms: Concepts and Strategy. Cambridge: Cambridge University Press.
- [9] Bernstein, S. and E. Winter (2012). Contracting with Heterogeneous Externalities, American Economic Journal: Microeconomics 4, 50–76.
- [10] Bloch, F. and N. Quérou (2013). Pricing in Social Networks. Games and Economic Behavior 80, 243–261.
- [11] Böhme, E.(2016). Second-Degree Price Discrimination on Two-Sided Markets. Review of Network Economics 15, 91–115.
- [12] Candogan, O., K. Bimpikis, and A. Ozdaglar (2012). Optimal Pricing in Networks With Externalities. Operations Research 60, 883–905.
- [13] Csorba, G. (2008). Screening Contracts in the Presence of Positive Network Effects. International Journal of Industrial Organization 26, 213–226.
- [14] Csorba, G. and J.-H. Hahn (2006). Functional Degradation and Asymmetric Network Effects. Journal of Industrial Economics 54, 253–268.
- [15] Deneckere, R.J. and R. Preston McAfee (1996). Damaged Goods. Journal of Economics & Management Strategy 5, 149–174.

- [16] Deng, Y, A. Lambrecht, and Y. Liu (2022). Spillover Effects and Freemium Strategy in the Mobile App Market. Management Science (ahead of print).
- [17] Evans, D. and R. Schmalensee (2016), Matchmakers: The New Economics of Multisided Platforms, Boston: Harvard Business Review Press.
- [18] Fainmesser, I.P. and A. Galeotti (2016). Pricing Network Effects. Review of Economic Studies 83, 165–198.
- [19] Fainmesser, I.P. and A. Galeotti (2020). Pricing Network Effects: Competition. American Economic Journal: Microeconomics 12, 1–32.
- [20] Farrell, J. and G. Saloner (1985). Standardization, Compatibility and Innovation. Rand Journal of Economics 16, 70–83.
- [21] Farrell, J. and G. Saloner (1986). Installed Base and Compatibility: Innovation, Product Preannouncement, and Predation. American Economic Review 76, 940–955.
- [22] Franck, J.-U. and M. Peitz (2021). Market Definition in the Platform Economy. Cambridge Yearbook of European Legal Studies (CYELS) 23, 91–127.
- [23] Garcia, F. and J. Resende. (2011). Dynamic Games of Network Effects. In Peixoto, M.M., A.A. Pinto, and D.A. Rand (editors), Dynamics, Games and Science II (Vol 2). Springer. 323-342.
- [24] Gomes, R. and A. Pavan (2016). Many-to-Many Matching and Price Discrimination. Theoretical Economics 11, 1005–1052.
- [25] Gomes, R. and A. Pavan (forthcoming). Price Customization and Targeting in Matching Markets. Mimeo. Rand Journal of Economics.
- [26] Hagiu, A. (2009). Two-Sided Platforms: Product Variety and Pricing Structures. Journal of Economics & Management Strategy 18, 1011–1043.
- [27] Hahn, J.H. (2004). The Welfare Effect of Quality Degradation in the Presence of Network Externalities. Information Economics and Policy 16, 535–552.
- [28] Hashizume, R., T. Ikeda, and T. Nariu, (2021). Price Discrimination with Network Effects: Different Welfare Results from Identical Demand Functions. Economics Bulletin 41(3), 1807–1812.
- [29] Jeon, D.-S., B.-C. Kim, and D. Menicucci (2022). Second-Degree Price Discrimination by a Two-Sided Monopoly Platform. American Economic Journal: Microeconomics 14, 322–369.
- [30] Jing, B. (2007). Network Externalities and Market Segmentation in a Monopoly. Economics Letters 95, 7–13.

- [31] Jullien, B. (2001). Competing in Network Industries: Divide and Conquer. Mimeo. IDEI and GREMAQ, University of Toulouse.
- [32] Jullien, B. (2011). Competition in Multi-Sided Markets: Divide and Conquer. American Economic Journal: Microeconomics 3, 186–219.
- [33] Jullien, B., A. Pavan, and M. Rysman (2021). Two-Sided Markets, Pricing, and Network Effects. In: K. Ho, A. Hortaçsu, and A. Lizzeri (eds.). Handbook of Industrial Organization, vol. 4, Amsterdam: North Holland.
- [34] Karle, H., M. Peitz, and M. Reisinger (2020). Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers. Journal of Political Economy 128, 2329–2374.
- [35] Katz, M. and C. Shapiro (1985). Network Externalities, Competition and Compatibility. American Economic Review 75, 424–440.
- [36] Katz, M. and C. Shapiro (1986). Technology Adoption in the Presence of Network Externalities. Journal of Political Economy 95, 822–841.
- [37] Katz, M. L. and J. Sallet (2018). Multisided Platforms and Antitrust Enforcement. Yale Law Journal 127, 2142–2175.
- [38] King, S. and R. Lampe (2003). Network Externalities, Price Discrimination and Profitable Piracy. Information Economics and Policy 15, 271–290.
- [39] Leduc, M., M. Jackson, and R. Johari (2017). Pricing and Referrals in Diffusion on Networks. Games and Economic Behavior 104, 568–594.
- [40] Lin, S. (2020). Two-Sided Price Discrimination by Media Platforms, Marketing Science 39, 317–338.
- [41] Mehta, I. (2023). Here's how every social media company is adopting subscriptions. TechCrunch (February 27). Available at https://techcrunch.com/2023/02/27/ social-media-apps-adopting-subscription-models/.
- [42] Mussa, M. and S. Rosen (1978). Monopoly and Product Quality. Journal of Economic Theory 18, 301–317.
- [43] Nocke, V., M. Peitz, and K. Stahl (2007). Platform Ownership. Journal of the European Economic Association 5, 1130–1160.
- [44] Peitz, M. and S. Sato (2023). Asymmetric Platform Oligopoly. Unpublished manuscript.
- [45] Rochet, J.-C. and L.A. Stole (2002). Nonlinear Pricing with Random Participation. Review of Economic Studies 69, 277–311.

- [46] Rochet, J.-C. and J. Tirole (2003). Platform Competition in Two-sided Markets. Journal of the European Economic Association 1, 990–1024.
- [47] Rochet, J.-C. and J. Tirole (2006). Two-sided Markets: A Progress Report. Rand Journal of Economics 37, 645–667.
- [48] Rohlfs, J. (1974). A theory of interdependent demand for a communications service. The Bell Journal of Economics and Management Science 5(1), 16–37.
- [49] Rysman, M. (2009). The Economics of Two-Sided Markets. Journal of Economic Perspectives 23, 125–143.
- [50] Segal, I. (2003). Coordination and Discrimination in Contracting with Externalities: Divide and Conquer? Journal of Economic Theory 113, 147–181.
- [51] Shi, Z., K. Zhang, and K. Srinivasan (2019). Freemium as an Optimal Strategy for Market Dominant Firms. Marketing Science 38(1), 150–169.
- [52] Sundararajan, A. (2004). Nonlinear Pricing and Type-Dependent Network Effects. Economics Letters 83, 107–113.
- [53] Takeyama, L. (1994). The Welfare Implications of Unauthorized Reproduction of Intellectual Property in the Presence of Network Externalities. Journal of Industrial Economics 42, 155–166.
- [54] Tan, G. and J. Zhou (2021). The Effects of Competition and Entry in Multi-sided Markets. Review of Economic Studies 88, 1002–1030.
- [55] Teh, T.-H. (2022). Platform Governance. American Economic Journal: Microeconomics 14, 213–254.
- [56] Varian, H. (1989). Price Discrimination. In: R. Schmalensee and R.D. Willig (eds.) Handbook of Industrial Organization, vol. 1. Amsterdam: North Holland.
- [57] Veiga, A. (2018). A Note on How to Sell a Network Good. International Journal of Industrial Organization 59, 114–126.
- [58] Wicklow, M., P. Belleflamme, and M. Peitz (2020). Who Pays What on Spotify? IPdigIT (18 May 2020). Available at http://www.ipdigit.eu/2020/05/who-pays-what-on-spotify/ (last accessed on June 17, 2021).
- [59] Winter, E. (2004). Incentives and Discrimination. American Economic Review 94, 764–773.