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Abstract

We consider two-sided platforms with the feature that some users on one or both sides of the market lack information about the price charged to participants on the other side of the market. With positive cross-group external effects, such lack of price information makes demand less elastic. A monopoly platform does not benefit from opaqueness and optimally reveals price information. By contrast, in a two-sided singlehoming duopoly, platforms benefit from opaqueness and, thus, do not have an incentive to disclose price information. In competitive bottleneck markets, results are more nuanced: if one side is fully informed (for exogenous reasons), platforms may decide to inform users on the other side either fully, partially or not at all, depending on the strength of cross-group external effects and the degree of horizontal differentiation.

Keywords: price transparency, two-sided markets, competitive bottleneck, platform competition, price information, strategic disclosure

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1 Introduction

In markets with two-sided platforms, cross-group external effects make the individually optimal participation decision on one side dependent on how many users are active on the other. In markets with a lot of turnover of market participants, this decision has to be based on expected participation on the other side. The level of expected participation depends on market characteristics and, if observable, on the actions of platform providers—in particular, their pricing decisions.

This paper focuses on the disclosure of prices. In contrast to most of the existing literature, we do not impose that users know *all* prices. In particular, we would argue that on many two-sided platforms, information about the price charged to the other side is not universally known. Possibly, platforms can strategically decide whether and to what extent they provide price information to the group of users to whom the respective price does not apply. This is an issue that is specific to two-sided platforms in contrast to networks or platforms with only one group of users.

To illustrate what we have in mind, consider a simplified model of video game markets that abstracts from non-linear pricing and lock-in. Game developers know the fees charged by platforms to end users but the reverse is often not the case. Platforms sometimes inform the market about reductions in the costs of developing games for them. For instance, Sony announced a cut in the price for developers in 2007 and in 2009. In 2009 it released the statement that it "will deploy various measures to further reinforce game development for PS3 and will continue to expand the platform to offer attractive interactive entertainment experiences only available on PS3," and informed the public that it reduced the price of the development kit from US$ 10,000 to US$ 2,000. This announcement was not restricted to the developer community, but spread more widely to users. Thus, Sony's information policy arguably affected the information available to gamers and, therefore, their expectation about the availability of games on the platform.

This paper formally investigates the incentive of platform operators to disclose price information to the other side of the market. We use standard models of competition between two-sided platforms to obtain equilibrium predictions on the pricing behavior for given disclosure rules and then endogenize the disclosure decision. If some users on one side are not informed about the price charged to users on the other side, they cannot infer the intensity of usage on the other side from the observation of actual prices. Instead, these uninformed users have to form expectations about participation on the other side without knowing the prices that platforms charge on that side. We assume that these expectations are constant (that is, users hold passive beliefs), and are confirmed in perfect Bayesian equilibrium. Clearly, users with constant expectations about participation on the other side do not stop to participate when the price on the other side is increased. This makes market demand less price elastic. Consequently, the decision to widely disclose price information tends to lead to lower prices.

If the platform is in a monopoly position and if users move simultaneously on both sides, we find that the platform fully discloses prices. Why is that so? If users do not observe the price on the other side, the platform has an incentive to raise this price too much for its own good. This is similar to the classic opportunism problem (Hart and Tirole, 1990) and generalizes the result by Hagiu and Halaburda (2014) who allow for no or full disclosure on each side of the market.[1]

By contrast, with competing platforms (and positive cross-group external effects on both sides), full disclosure does not necessarily obtain, as higher prices may benefit both platforms. In particular, under two-sided singlehoming, we show that at equilibrium, platforms do not disclose any information to users on one side about the price they charge to the other side—the outcome under strategic disclosure here is the same as when platforms coordinate their disclosure decisions. Thus, while a monopoly platform always chooses to disclose information fully on both sides of the market, competing platforms decide not to disclose information whatsoever.

In competitive bottlenecks (i.e., a market in which users on one side singlehome, while users on the other side can multihome) results are more nuanced. Because the analysis becomes more involved in this setting, we focus on situations in which users on one side are fully informed; platforms must then decide the extent to which they want to inform users on the other side. In case singlehomers are fully informed, we find that platforms choose to disclose no information to multihomers (about the price they charge to singlehomers) for a large range of parameters. However, if the horizontal differentiation between the platform is very low (i.e., close to the limit under which only one platform would survive at equilibrium), then full disclosure and no disclosure may coexist at equilibrium, or full disclosure may be the unique equilibrium (if the multihoming side exerts much larger cross-group external effects than the singlehoming side). Here, a firm attracts more users on the singlehoming side through full disclosure, and this increase in market share overcompensates the loss in revenue per single-homing users (accounting for revenues on both sides of the market). In the other case, in which multihomers are fully informed, the information that platforms give to singlehomers depend again on the parameters of the model: If multihomers exert sufficiently larger cross-group external effects than singlehomers, then platforms find it optimal to inform a fraction of the singlehomers, or even all of them (if multihomers exert even proportionately larger external effects and platforms are not too differentiated); otherwise, platforms do not inform singlehomers whatsoever. In all these models of platform competition, platforms would not inform any users if they could coordinate their disclosure decisions.

Related literature. The early literature on network effects has considered alternative specifications about output information. In particular, in their seminal paper, Katz and Shapiro (1985) contrast two models. In the first model (developed in the main text), they assume that users first form expectations about output levels (i.e., network sizes) and next, on the basis of their expectations and observed prices, they make their consumption decisions; to tie down equilibrium predictions, expectations are required to be self-fulfilling. In the second model (sketched in the appendix), the authors assume alternatively that firms can commit to output levels, which allows them to directly influence consumer expectations. In more recent work, Griva and Vettas

[1] This result holds if cross-group external effects are positive on both sides and also if they are positive on one side but negative on the other side. In an extension, Hagiu and Halaburda (2014) allow for partial disclosure on one side; our generalization is to allow for partial disclosure on both sides.

(2011) and Hurkens and Lopez (2014) further explore the effect of user expectations on market outcomes. In a two-sided market setting, Gabszewicz and Wauthy (2014) consider two versions of expectation formation to show that two ex ante non-differentiated platforms can coexist and make positive profits when the strength of cross-group external effects is heterogeneous across users.[2]

Within the two-sided platform literature, we follow Armstrong (2006) in postulating that users on both sides are heterogeneous with respect to their opportunity cost of joining a platform and suppose that platforms set membership fees.[3] While a substantial literature has evolved focusing on pricing implications, several contributions have introduced additional strategic variables including Belleflamme and Peitz (2010) studying sellers' ex ante investment incentives in two-sided markets. In particular, Jullien and Pavan (2019) analyze the information management of platforms that affects the users' ability to predict participation decisions on the other side. In this paper, we add to this line of research an alternative strategic variable of platforms, namely the platforms' decisions whether to disclose prices on the other side of the market, which directly affects the ability of users to predict participation decisions on the other side.

As far as we know, the only other paper to explore the impact of price information on equilibrium outcomes in markets with two-sided platforms is Hagiu and Halaburda (2014). We follow Hagiu and Halaburda (2014) in postulating that users observe the price they have to pay, but possibly not the price users on the other side have to pay and that uninformed users have passive beliefs.[4] Our monopoly result shows that the result of Hagiu and Halaburda (2014) is robust to allowing partial disclosure on both sides. Under platform competition, we analyze strategic disclosure, whereas Hagiu and Halaburda (2014) consider coordinated decisions. In their competitive bottleneck model, both or none of the two platforms are assumed to fully disclose or to not disclose at all on the singlehoming side;[5] all users on the multihoming side are assumed to be fully informed. They find that platforms jointly decide not to inform users on the singlehoming side. This finding generalizes to the other models of platform competition analyzed in our paper: if platforms can coordinate their disclosure decision they do not inform users. By contrast, we consider the platforms' strategic disclosure decision (either on the singlehoming or on the multihoming side) and show that, depending on the setting and the parameter constellation, platforms fully, partially or not at all disclose information to users.

While we analyze a static setting, expectations about participation decision also matter in dynamic markets with installed user base. Cabral (2019) provides a theoretical contribution to the dynamics of two-sided market; Tucker and Zhang (2010) provide an empirical investigation into a platform's ability to affect the expectation about participation decisions on the other side of the market.

[2]A similar endogenous differentiation result is obtained by Halaburda et al. (2018).

[3]Rochet and Tirole (2003) consider heterogeneity in usage costs and consider the setting of usage fees. Hagiu (2006) considers sequential participation decisions by the two sides of the platform. Armstrong and Wright (2007) and Belleflamme and Peitz (2019) explore the effects of multihoming.

[4]In an extension, Hagiu and Halaburda show that their qualitative findings also hold under wary beliefs.

[5]In an extension, they allow that some but not all singlehoming users are informed about prices on the other side of both platforms.

Our paper also connects to the literature on price transparency. Furthermore, there is a scant literature on the disclosure of price and product information in oligopoly, which analyzes the competitive effects of informing only a fraction of consumers (see, e.g., Schultz, 2004, 2009).

Finally, the disclosure decision can be seen as an instance of informative advertising. The literature has considered oligopoly environments in which firms advertise the existence of a product, its price and product characteristics (including the contributions by Butters, 1977, and Grossman and Shapiro, 1984). In our paper, we presume that product and price are known, but that the price on the other side is not necessarily known. Since the price on the other side affects participation on the other side, and participation on the other side generates an external effect, our model captures a situation in which platforms can disclose information that affects the quality perception.

The rest of the paper is organized as follows. In Sections 2 and 3, we analyze in turn the cases of a monopoly platform and of two competing platforms, distinguishing between environments with singlehoming on both sides or with potential multihoming on one side. In Section 4, we make some concluding remarks.

2 Monopoly platform

In this section, we set up a simple monopoly platform model and examine the incentives of the platform operator to disclose price information on the two sides of the market. We examine the model in which the platform sets membership fees on both sides and users on both sides simultaneously decide whether or not to join the platform.

2.1 Model

Consider a monopoly platform serving two distinct groups of users. Each group $i = a, b$ comprises a mass of users v_i who decide whether to join the platform. The platform charges (possibly different) membership fees for the two groups, A and B. Below we use the terms 'membership fee' and 'price' interchangeably. The constant marginal cost of attracting users on the platform is normalized to zero. A user of group i enjoys the following net utility when interacting on the platform with users of the other group:

$$U_i = \begin{cases} u_a + \gamma_a n_b^e - A & \text{if } i = a \\ u_b + \gamma_b n_a^e - B & \text{if } i = b \end{cases},$$

where u_i is the intrinsic value of being on the platform, γ_i measures the cross-group external effect provided by an additional member of side j on each member of side i, n_j^e is the expected number of members of side j on the platform. We assume that u_i is drawn from a uniform distribution on $[0, v_i]$ and that there is a mass of v_i of potential users on side i. As for cross-group external effects, we assume that they are positive for at least one side.[6]

[6]It is indeed hard to imagine a two-sided platform connecting two groups that dislike each other.

Facing a membership fee A and expecting participation n_b^e on the other side, a user of side a decides to join the platform if $u_a \geq A - \gamma_a n_b^e$. Correspondingly, a user on side b faces a membership fee B and decides to join if $u_b \geq B - \gamma_b n_a^e$. Hence, the number of participating users on side i is computed as

$$\begin{cases} n_a = v_a + \gamma_a n_b^e - A, \\ n_b = v_b + \gamma_b n_b^e - B. \end{cases} \tag{1}$$

Concerning the users' information about membership fees, we assume that on side a, all users observe the fee that the platform charges on their own side (A) but only have a probability α to observe the membership fee charged on the other side (side B), and correspondingly on side b where β denotes the probability that users observe the membership fee charged on side A; the probabilities α and β are common knowledge.

From the platform's point of view, these assumptions imply that when the platform changes its membership fee for side i, all users on side i incorporate this change of price in their decision, by taking into account that only a share of users on the other side will be aware of these modifications. We call the couple (α, β) the 'information structure'. In what follows, we assume that the platform can commit to this information structure, but, at the ex ante stage, is able to modify it at zero cost.

We analyze the following three-stage game: (stage 1) the platform chooses the information structure; (stage 2) the platform sets the membership fees A and B; (stage 3) users in the two groups simultaneously decide whether or not to join the platform. We look for a perfect Bayesian equilibrium with passive beliefs of this game.

2.2 Analysis

2.2.1 Participation decisions

Our first task is to determine the number of users that will join the platform on each side, given the information structure (α, β). On side i, with some probability, a user is informed of the membership fee charged on the other side and, therefore, is able to anticipate correctly the number of users that will join on the other side: $n_j^e = n_j$; with the remaining probability, a user is not informed of the membership fee charged on the other side. We assume that such an uninformed user holds passive beliefs about participation on the other side; that is, the expected number of participants on the other side is taken as constant $n_j^e = x_j$. Note that such a user on side i knows that a fraction of users on the other side j do observe the membership fee and, thus, could make her participation decision dependent on the price on side i. Thus, uninformed users hold the belief that the number of participants is equal to the equilibrium number and independent of the price on their own side.[7] We mean to speak of perfect Bayesian equilibrium with passive beliefs whenever we refer to an equilibrium.

[7] That is, we assume that if both prices change, x_a and x_b remain unchanged. Alternatively, we could assume that, even though a user only observes A, she could nevertheless adjust her beliefs x_b by inferring that those users on side b that observe A will adjust their decision, knowing that participation on side a changes. We leave the study of alternative beliefs to further research.

Using expression (1), we can thus write:

$$\begin{cases} n_a = v_a + \gamma_a \left(\alpha n_b + (1-\alpha) x_b \right) - A, \\ n_b = v_b + \gamma_b \left(\beta n_a + (1-\beta) x_a \right) - B. \end{cases}$$

The solution to this system is

$$\begin{cases} n_a = \frac{v_a + \gamma_a x_b - A + \gamma_a \alpha (v_b - x_b - B + \gamma_b (1-\beta) x_a)}{1-\alpha\beta\gamma_a\gamma_b}, \\ n_b = \frac{v_b + \gamma_b x_a - B + \gamma_b \beta (v_a - x_a - A + \gamma_a (1-\alpha) x_b)}{1-\alpha\beta\gamma_a\gamma_b}. \end{cases}$$

Demand n_a is decreasing in A and n_b in B if $\alpha\beta\gamma_a\gamma_b < 1$. This condition is satisfied for all information structures if $\gamma_a\gamma_b < 1$.

2.2.2 Membership fees

The platform chooses A and B to maximize its profits $\Pi = An_a + Bn_b$. To assure an interior maximum, cross-group external effects cannot be too large (Assumption 1)[8] and, in case of a negative cross-group external effect, an additional parameter restriction must be satisfied (Assumption 2).

Assumption 1. $4 > (\gamma_a + \gamma_b \beta)(\gamma_b + \gamma_a \alpha)$.

Assumption 2. $2v_a + (\gamma_a + \gamma_b\beta) v_b > 0$ *and* $2v_b + (\gamma_b + \gamma_a\alpha) v_a > 0$.

Under these conditions, we obtain the following lemma (we relegate all the proofs to the appendix).

Lemma 1. *For a given information structure, the platform chooses the equilibrium membership fees*

$$A^* = \frac{(2 - \gamma_b\beta(\gamma_b + \gamma_a\alpha)) v_a + (\gamma_a - \gamma_b\beta) v_b}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}, B^* = \frac{(2 - \gamma_a\alpha(\gamma_a + \gamma_b\beta)) v_b + (\gamma_b - \gamma_a\alpha) v_a}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}$$

which yield equilibrium participation levels

$$n_a^* = \frac{2v_a + (\gamma_a + \gamma_b\beta) v_b}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}, n_b^* = \frac{2v_b + (\gamma_b + \gamma_a\alpha) v_a}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}.$$

Note that Assumptions 1 and 2 guarantee positive participation on both sides. We also note that $A^* = n_a^* - \beta\gamma_b n_b^*$ and $B^* = n_b^* - \alpha\gamma_a n_a^*$.

2.2.3 Information structure

We now examine how the equilibrium responds to a change in α and β. In the appendix, we prove the following result.

Proposition 1. *A monopoly platform chooses to inform all users at equilibrium:* $\alpha^* = \beta^* = 1$.

[8] It can be checked that Assumption 1 implies that $\gamma_a\gamma_b < 1$, so that participation on each side is a decreasing function of the price set on that side.

To understand this result, let us decompose the effects on the platform's equilibrium profit of a change in the information on side a:

$$\frac{d\Pi^*}{d\alpha} = \frac{\partial A^*}{\partial\alpha} n_a^* + \frac{\partial n_a^*}{\partial\alpha} A^* + \frac{\partial B^*}{\partial\alpha} n_b^* + \frac{\partial n_b^*}{\partial\alpha} B^*. \tag{2}$$

We compute

$$\begin{aligned} \partial A^*/\partial\alpha &= (\gamma_a - \beta\gamma_b)\,\gamma_a K_a, \\ \partial n_a^*/\partial\alpha &= (\gamma_a + \beta\gamma_b)\,\gamma_a K_a, \\ \partial B^*/\partial\alpha &= -\left(2 - \gamma_b\left(\gamma_a + \beta\gamma_b\right)\right)\gamma_a K_a, \\ \partial n_b^*/\partial\alpha &= 2\gamma_a K_a, \end{aligned} \tag{3}$$

with

$$K_a \equiv \frac{2v_a + (\gamma_a + \gamma_b\beta)\, v_b}{\left(4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)\right)^2} > 0 \text{ by Assumption 2.}$$

Consider first a platform that exhibits *positive cross-group external effects on both sides.* For instance, a gaming platform with developers on side a and gamers on side b: developers' profits increase with the number of gamers ($\gamma_a > 0$) and gamers' utilities increase with the availability of games ($\gamma_b > 0$). By raising α, the platform makes side-a users more aware of –and so more sensitive to–price changes on side b. Other things equal, this gives the platform larger incentives to lower B^* because the leverage effect of attracting more side-b users (so as to attract more side-a users) increases. One expects thus an increase in α to be followed by a decrease in B^*, resulting in an increase in n_b^* and n_a^*; one also expects the platform to raise A^* so as to take advantage of the increase in n_a^* induced by the decrease in B^*. We observe in expression (3) that the increase in n_b^* is unambiguous, as is the increase in n_a^* when γ_b is positive.[9] As for the change in prices (namely a decrease in B^* and an increase in A^*), they are as expected as long as $\beta\gamma_b$ is not too large with respect to γ_a; if not (i.e., if gamers exert sufficiently stronger external effects than developers, and if gamers users are sufficiently informed of the fee charged to developers), then the platform may adjust its price structure differently (it may reduce A^* and even raise B^*).[10] We thus see that changing information on one side can have opposite effects on participation and on fees. To assess the net effect, we use the fact that $A^* = n_a^* - \beta\gamma_b n_b^*$ and $B^* = n_b^* - \alpha\gamma_a n_a^*$. This allows us to rewrite expression (2) as

$$\frac{d\Pi^*}{d\alpha} = \gamma_a K_a \left(2\gamma_a (1-\alpha)\, n_a^* + \gamma_b (1-\beta)(\gamma_a + \beta\gamma_b)\, n_b^*\right), \tag{4}$$

which is clearly positive when $\gamma_a, \gamma_b > 0$.

Consider now a *platform combining positive and negative cross-group external effects.* An example is a media platform that links advertisers to readers: advertisers welcome a larger audience whereas readers dislike more advertising. Suppose first that advertisers are on side a, so that $\gamma_a > 0 > \gamma_b$. In that case, it is clear that $\partial A^*/\partial\alpha > 0$ and $\partial n_b^*/\partial\alpha > 0$: increasing the advertisers' information about the readers' fee yields the platform to charge more to advertisers

[9]The same result applies for the users' net values. We have indeed that $n_i = v_i + \gamma_i n_j^e - M_i = U_i + v_i - u_i$ (where $M_a \equiv A$ and $M_b \equiv B$).

[10]For instance, if all gamers know about the fee charged to developers ($\beta = 1$), then $\partial A^*/\partial\alpha < 0$ for $\gamma_b > \gamma_a$ and $\partial B^*/\partial\alpha > 0$ for $\gamma_b > (\sqrt{\gamma_a^2 + 8} - \gamma_a)/2$, which is compatible with Condition (1) for $\gamma_b > \gamma_a$.

and to increase readers' participation. If $\gamma_a < -\beta\gamma_b$ (i.e., if advertisers value less an extra reader than readers dislike an extra advertiser), the effect on the readers' fee is ambiguous but it is clear that $\partial n_a^*/\partial\alpha < 0$: better information on the advertisers' side leads the platform to decrease advertisers' participation. Also, if $\gamma_a < -\beta\gamma_b$, expression (4) reveals that the net effect on profit is positive.

Let us now reverse the roles by taking readers on side a, so that $\gamma_b > 0 > \gamma_a$. Clearly, $\partial A^*/\partial\alpha < 0$ and $\partial n_b^*/\partial\alpha < 0$: increasing the readers' information about the advertisers' fee yields the platform to charge less to readers and to decrease advertisers' participation. If $\beta\gamma_b < -\gamma_a$ (i.e., if advertisers value less an extra reader than readers dislike an extra advertiser), we have that $\partial n_a^*/\partial\alpha > 0$ and $\partial B^*/\partial\alpha > 0$: better information on the readers' side leads the platform to raise readers' participation and the advertisers' fee. Again, in that case, expression (4) reveals that the net effect on profit is positive.

3 Competing platforms

In this section we turn to platform competition. Suppose that there are two platforms, located at the extremes of the unit interval (platform 1 at 0 and platform 2 at 1). There is a mass 1 of users on the two sides of the market (noted a and b). We denote by m_i the mass of users of side i ($i = a, b$) who join platform 1 and by n_i the mass of users of side i who join platform 2. We also denote by A_k and B_k the membership fees charged by platform k ($k = 1, 2$) on side a and b respectively.

We contrast two settings. In the first setting, called "two-sided singlehoming", users on both sides on the platform are supposed to singlehome; that is, they can be present on at most one platform. In the second setting, called "competitive bottlenecks", users on side a still singlehome while users on side b have the possibility to multihome (or, equivalently, each platform can be accessed by separate subsets of side-b users).

3.1 Two-sided singlehoming

Users on both sides have to choose to visit either platform 1 or platform 2. We make the following assumptions about the users' ability to observe prices. First, all users observe the membership fees charged by the two platforms on their own side (i.e., users of side a observe A_1 and A_2, and users of side b observe B_1 and B_2). Second, on side a (resp. b), each user has a probability α_k (resp. β_k) to be exposed to the membership fee charged by platform k on the other side of the market. Third, these probabilities are common knowledge.

We analyze the following three-stage game. In stage 1, the two platforms choose the share of users they want to inform about prices; that is, platform 1 (resp. 2) chooses α_1 and β_1 (resp. α_2 and β_2); we continue to assume that no cost is associated to these decisions. In stage 2, the two platforms set their membership fees; that is, platform 1 (resp. 2) chooses A_1 and B_1 (resp. A_2 and B_2). Finally, in stage 3, users on both sides decide which platform to visit. We solve the game for perfect Bayesian equilibria with passive beliefs.

3.1.1 Participation decisions

A user of type a located at $x \in [0,1]$ compares the net value she gets from joining either platform 1 or platform 2; that is, respectively $u_a + \gamma_a m_b^e - A_1 - \tau_a x$, and $u_a + \gamma_a n_b^e - A_2 - \tau_a (1-x)$, where u_a is the stand-alone utility, γ_a measures the cross-group external effects that side b exerts on side a, τ_a is the unit transportation cost on side a, and m_b^e and n_b^e are the expected mass of users of side b who will join, respectively, platforms 1 and 2. The indifferent user of type a is thus identified as

$$\bar{x}_a = \frac{1}{2} - \frac{A_1 - A_2}{2\tau_a} + \frac{\gamma_a}{2\tau_a}\left(m_b^e - n_b^e\right).$$

We define similarly the indifferent user of type b as

$$\bar{x}_b = \frac{1}{2} - \frac{B_1 - B_2}{2\tau_b} + \frac{\gamma_b}{2\tau_b}\left(m_a^e - n_a^e\right).$$

We focus here on the case in which $\gamma_a > 0$ but we do not place any restriction on γ_b, which can be positive or negative. As for the stand-alone utilities u_a and u_b, we assume that they are large enough so that all users join one or the other platform at equilibrium.[11]

Using the identities of the indifferent users, platforms can compute the share of users on each side that will react or not to a modification of their membership fees. This can be modeled in the following way. Take side a. Platform 1 knows that with probability α_1, the user observes B_1 and can correctly anticipate the number of type b users that will join platform 1; that is, for such a user, $m_b^e = m_b$. In contrast, with probability $(1-\alpha_1)$, the user does not observe B_1 and forms a passive expectation about the number of type b users that will join platform 1; that is, for such a user, $m_b^e = y_b$, with y_b being some constant (we require, however, that expectations be fulfilled at equilibrium: $y_b = m_b$ at equilibrium). Similarly for platform 2: with probability α_2, $n_b^e = n_b$ and with probability $(1-\alpha_2)$, $n_b^e = z_b$.

Making similar definitions on side b, we can thus write the following:

$$\begin{aligned} m_b^e - n_b^e &= \alpha_1 m_b + (1-\alpha_1) y_b - \alpha_2 n_b - (1-\alpha_2) z_b \\ &= (\alpha_1 + \alpha_2)(m_b - y_b) + 2y_b - 1. \end{aligned}$$

where, in the second line, we make use of the fact that, because of full participation on both sides, $m_i = 1 - n_i$ and $y_i = 1 - z_i$. Correspondingly, we find $m_a^e - n_a^e = (\beta_1 + \beta_2)(m_a - y_a) + 2y_a - 1$. Under full participation, we also have that $m_i = \bar{x}_i$. To find the number of users joining the two platforms as a function of the four membership fees, we must then solve the following system of equations:

$$\begin{cases} m_a = \frac{1}{2} - \frac{A_1 - A_2}{2\tau_a} + \frac{\gamma_a}{2\tau_a}\left((\alpha_1+\alpha_2)(m_b - y_b) + 2y_b - 1\right), \\ m_b = \frac{1}{2} - \frac{B_1 - B_2}{2\tau_b} + \frac{\gamma_b}{2\tau_b}\left((\beta_1+\beta_2)(m_a - y_a) + 2y_a - 1\right). \end{cases}$$

We define $h_a \equiv \gamma_a(\alpha_1+\alpha_2)/2$ and $h_b \equiv \gamma_b(\beta_1+\beta_2)/2$ as the as "effective" intensity of the external effects given the information structure. In the full-information case, $h_i = \gamma_i$. We note that in the two-sided singlehoming case all our results depend on this effective intensity of the

[11] More precisely, we require $2u_a \geq 3\tau_a - \gamma_a - (\beta_1+\beta_2)\gamma_b$ and $2u_b \geq 3\tau_b - \gamma_b - (\alpha_1+\alpha_2)\gamma_a$.

external effect.[12] We can write the solution as

$$m_a = \frac{F_a - \tau_b (A_1 - A_2) - h_a (B_1 - B_2)}{2(\tau_a \tau_b - h_a h_b)} \text{ and } m_b = \frac{F_b - \tau_a (B_1 - B_2) - h_b (A_1 - A_2)}{2(\tau_a \tau_b - h_a h_b)}, \text{ with}$$

$$\begin{aligned} F_a &\equiv 2h_a (\gamma_b - h_b) y_a + 2\tau_b (\gamma_a - h_a) y_b + (\tau_b - \gamma_b) h_a - (\gamma_a - \tau_a) \tau_b, \\ F_b &\equiv 2h_b (\gamma_a - h_a) y_b + 2\tau_a (\gamma_b - h_b) y_a + (\tau_a - \gamma_a) h_b - (\gamma_b - \tau_b) \tau_a. \end{aligned}$$

3.1.2 Membership fees

Platform 1 chooses A_1 and B_1 to maximize $\Pi_1 = A_1 m_a + B_1 m_b$, while platform 2 chooses A_2 and B_2 to maximize $\Pi_2 = A_2 (1 - m_a) + B_2 (1 - m_b)$. Solving for the system of four first-order conditions (and assuming $4\tau_a \tau_b > (h_a + h_b)^2$ to satisfy the second-order conditions), we obtain values of the membership fees that we use to compute the values of m_a and m_b. The next step consists in imposing fulfilled expectations, i.e., $y_a = m_a$ and $y_b = m_b$. Replacing and solving, one finds that the unique fixed point is $m_a = m_b = 1/2$. Substituting $y_a = m_a$ and $y_b = m_b$ into the expressions of F_a and F_b allows us to complete the characterization of the equilibrium (see the appendix). We can then state the following results:

Lemma 2. *In the two-sided singlehoming setting, for a given information structure, the platform chooses the equilibrium membership fees*

$$A_1^* = A_2^* = \tau_a - \tfrac{1}{2} (\beta_1 + \beta_2) \gamma_b \text{ and } B_1^* = B_2^* = \tau_b - \tfrac{1}{2} (\alpha_1 + \alpha_2) \gamma_a,$$

which yield equilibrium participation levels $m_a^* = n_a^* = m_b^* = n_b^* = 1/2$*, and equilibrium profits*

$$\Pi_1^* = \Pi_2^* = \tfrac{1}{2} (\tau_a + \tau_b) - \tfrac{1}{4} ((\alpha_1 + \alpha_2) \gamma_a + (\beta_1 + \beta_2) \gamma_b). \quad (5)$$

3.1.3 Information structure

We observe from expression (5) that $\partial \Pi_i^* / \partial \alpha_i$ has the opposite sign of γ_a and $\partial \Pi_i^* / \partial \beta_i$ has the opposite sign of γ_b. We therefore conclude:

Proposition 2. *In the two-sided singlehoming case with symmetric platforms and positive cross-group external effects on both sides, it is a dominant strategy for each platform to disclose no information to users in some group about the membership fee they charge to the other group.*

Note that the information structure chosen by the platforms at the equilibrium of the game is completely at odds with the preferences of the users. The equilibrium net values for the users are indeed computed as: $U_a^* = \frac{1}{2}\gamma_a + \frac{1}{2} (\beta_1 + \beta_2) \gamma_b - \tau_a$ and $U_b^* = \frac{1}{2}\gamma_b + \frac{1}{2} (\alpha_1 + \alpha_2) \gamma_a - \tau_b$. Thus, $\partial \Pi_i^* / \partial \alpha_i$ and $\partial U_b^* / \partial \alpha_i$ have opposite signs, and so do $\partial \Pi_i^* / \partial \beta_i$ and $\partial U_a^* / \partial \beta_i$. So, as cross-group external effects are positive on both sides, we see that the users benefit from more

[12] As we show below, this does not hold in the competitive bottlenecks model because, in this model, user behavior differs on the two sides.

observation of prices, while the platforms suffer from it (and will therefore choose to disclose no information at equilibrium).

Importantly, we observe that the introduction of a competing platform completely reverses the conclusion that we drew in the monopoly case: *while a monopoly platform always chooses to disclose information fully on both sides of the market, competing platforms decide to disclose no information whatsoever when users singlehome on both sides.* To understand this result, note that the logic that we described in the monopoly case still applies: by disclosing information to more users on one side, a platform increases the leverage it can gain on this side by lowering its fee on the other side. This is understood by all market participants in stages 2 and 3. Consider the deviation of a platform from symmetric disclosure policies. A platform that discloses information to more users on one side than its competitor will increase the number of users on this side and obtain a larger market share on this side. Such a deviation tends to be profit-increasing, taking equilibrium prices under symmetric disclosure policies as given. However, since the competing platform best-responds, more disclosure heats up competition and, therefore, a platform may not have an incentive to further disclose information. In this setting, with singlehoming and full participation on both sides, competition is particularly intense (each user gained by platform i is a user lost by platform j, thereby generating a positive feedback loop on platform i but a negative feedback loop on platform j). What Proposition 2 shows is that this competition effect is so strong that platforms prefer not to disclose any information at equilibrium and strategic disclosure gives the same outcome as coordinated disclosure.

3.2 Competitive bottlenecks

We now examine whether no disclosure is still observed at equilibrium when users on one side have the possibility to multihome. As multihoming may relax competition between platforms (for a systematic analysis, see Belleflamme and Peitz, 2019), the balance between the two effects that we just outlined is likely to be affected.

Suppose that users on side a still singlehome while users on side b have the possibility to multihome. (Equivalently, users on side b could be split into two separate subsets, with one subset being able to access platform 1 only and the other subset being able to access platform 2 only.)[13] We analyze the same three-stage game as in the previous section: (1) choice of information structure, (2) choice of membership fees, (3) participation decisions. As a word of caution, in the two propositions in this section (Propositions 3 and 4), at stage 1, we characterize the solutions to the system of first-order conditions and check that second-order conditions are satisfied locally. In this sense, we characterize "local" equilibria.

3.2.1 Participation decisions

A user of type a located at $x \in [0,1]$ compares the net value she gets from joining either platform 1 or platform 2; that is, respectively $\gamma_a m_b^e - A_1 - \tau_a x$ and $\gamma_a n_b^e - A_2 - \tau_a (1-x)$, where m_b^e

[13]With this interpretation in mind, it could be possible to examine the case where each subset may not observe the fee charged to the other subset. We leave this issue for further research.

and n_b^e are the expected mass of users of side b who will join, respectively, platforms 1 and 2.[14] The indifferent user of type a is thus identified as

$$\bar{x}_a = \frac{1}{2} - \frac{A_1 - A_2}{2\tau_a} + \frac{\gamma_a}{2\tau_a}\left(m_b^e - n_b^e\right).$$

A user of type b located at $x \in [0,1]$ decides to join platform 1 if $\gamma_b m_a^e - B_1 - \tau_b x \geq 0$ and to join platform 2 if $\gamma_b n_a^e - B_2 - \tau_b (1-x) \geq 0$. That is, the marginal type b users joining platforms 1 and 2 are identified by

$$\hat{x}_1 = \frac{1}{\tau_b}\left(\gamma_b m_a^e - B_1\right) \text{ and } \hat{x}_2 = 1 - \frac{1}{\tau_b}\left(\gamma_b n_a^e - B_2\right).$$

We make the same assumptions as before regarding the users' ability to observe prices. It follows that $m_b^e = \alpha_1 m_b + (1-\alpha_1)\, y_b$, $n_b^e = \alpha_2 n_b + (1-\alpha_2)\, z_b$, $m_a^e = \beta_1 m_a + (1-\beta_1)\, y_a$ and $n_a^e = \beta_2 n_a + (1-\beta_2)\, z_a$. We assume full participation on the singlehoming side, so that $m_a + n_a = 1$ and $y_a + z_a = 1$. Moreover, $m_a = \bar{x}_a$. On side b, we have that $m_b = \hat{x}_1$ and $n_b = 1 - \hat{x}_2$. Combining these equalities and solving, we obtain the following numbers of users as functions of the membership fees and the information structure:

$$\begin{array}{l} m_a = \frac{G - \tau_b(A_1 - A_2) - \alpha_1\gamma_a B_1 + \alpha_2\gamma_a B_2}{2\tau_a\tau_b - \gamma_a\gamma_b(\alpha_1\beta_1 + \alpha_2\beta_2)}, \\ m_b = \frac{\gamma_b}{\tau_b}\left(\beta_1 \frac{G - \tau_b(A_1 - A_2) - \alpha_1\gamma_a B_1 + \alpha_2\gamma_a B_2}{2\tau_a\tau_b - \gamma_a\gamma_b(\alpha_1\beta_1 + \alpha_2\beta_2)} + (1-\beta_1)\, y_a\right) - \frac{B_1}{\tau_b}, \\ n_b = \frac{\gamma_b}{\tau_b}\left(\beta_2\left(1 - \frac{G - \tau_b(A_1 - A_2) - \alpha_1\gamma_a B_1 + \alpha_2\gamma_a B_2}{2\tau_a\tau_b - \gamma_a\gamma_b(\alpha_1\beta_1 + \alpha_2\beta_2)}\right) + (1-\beta_2)(1-y_a)\right) - \frac{B_2}{\tau_b}, \end{array} \tag{6}$$

where

$$\begin{aligned} G \;=\; & \tau_a\tau_b - \alpha_2\gamma_a\gamma_b + \gamma_a\gamma_b\left(\alpha_1(1-\beta_1) + \alpha_2(1-\beta_2)\right) y_a \\ & + \gamma_a\tau_b(1-\alpha_1)\, y_b - \gamma_a\tau_b(1-\alpha_2)\, z_b. \end{aligned}$$

As solving the full problem appears to be cumbersome, we study two specific cases. First, we consider the case where the singlehoming side is aware of prices on the multihoming side but that users on the multihoming side are initially uninformed (but may become so as the result of the disclosure decisions by platforms). Then, we consider the opposite case. To ensure the existence of a perfect Bayesian equilibrium with passive beliefs in both cases, we impose the following restriction on the parameters:

$$t \equiv \tau_a\tau_b > \tfrac{1}{6}\left(\gamma_b^2 + \gamma_a^2 + 4\gamma_a\gamma_b\right) \equiv t_{\min}, \tag{7}$$

where t stands for $\tau_a\tau_b$ and can be seen as a measure of the degree of horizontal differentiation between the two platforms.

3.2.2 All singlehoming users are informed

We assume here that users on side a (where users singlehome) observe the membership fees that the platforms set on the other side but that the reverse may not be true; that is, α_1 and α_2 are exogenously set to unity, but the platforms choose $\beta_1, \beta_2 \in [0,1]$.

[14] To ease the exposition, we assume here that users do not receive any stand-alone benefits when joining a platform (i.e., we set $u_a = u_b = 0$).

Membership fees. Platform 1 chooses A_1 and B_1 to maximize $\Pi_1 = A_1 m_a + B_1 m_b$, while platform 2 chooses A_2 and B_2 to maximize $\Pi_2 = A_2(1-m_a) + B_2 n_b$. In the next lemma, we characterize the equilibrium.

Lemma 3. *In the competitive bottlenecks case in which all singlehoming users are informed while some multihoming users are uninformed, the platforms set the following membership fees:*

$$\begin{aligned} A_1^* &= \tfrac{4\tau_a\tau_b-\beta_1\gamma_b^2-(\beta_1+2\beta_2)\gamma_a\gamma_b}{2\tau_b} m_a^*,\ B_1^* = \tfrac{\gamma_b-\gamma_a}{2} m_a^*, \\ A_2^* &= \tfrac{4\tau_a\tau_b-\beta_2\gamma_b^2-(2\beta_1+\beta_2)\gamma_a\gamma_b}{2\tau_b}\left(1-m_a^*\right),\ B_2^* = \tfrac{\gamma_b-\gamma_a}{2}\left(1-m_a^*\right), \end{aligned}$$

with

$$\begin{aligned} m_a^* &= \tfrac{1}{2} - \tfrac{\gamma_b(\gamma_a-\gamma_b)(\beta_1-\beta_2)}{2\left(12\tau_a\tau_b-\left(2\gamma_a^2+(\beta_1+\beta_2)\gamma_b^2+(2+3\beta_1+3\beta_2)\gamma_a\gamma_b\right)\right)}, n_a^* = 1-m_a^* \\ m_b^* &= \tfrac{1}{2\tau_b}(\gamma_a+\gamma_b) n_a^*,\ n_b^* = \tfrac{1}{2\tau_b}(\gamma_a+\gamma_b) m_a^*. \end{aligned}$$

The resulting equilibrium profits are

$$\begin{aligned} \Pi_1^* &= \tfrac{8\tau_a\tau_b-\gamma_a^2+(1-2\beta_1)\gamma_b^2-2(\beta_1+2\beta_2)\gamma_a\gamma_b}{4\tau_b}\left(m_a^*\right)^2, \\ \Pi_2^* &= \tfrac{8\tau_a\tau_b-\gamma_a^2+(1-2\beta_2)\gamma_b^2-2(2\beta_1+\beta_2)\gamma_a\gamma_b}{4\tau_b}\left(1-m_a^*\right)^2. \end{aligned}$$

Regarding the equilibrium fees, we observe that if the singlehoming side exerts larger (resp. smaller) cross-group external effects than the multihoming side (i.e., $\gamma_a > \gamma_b$), then the fees charged on the multihoming side are clearly negative (i.e., below marginal cost), while condition (7) implies that the fees charged on the singlehoming side are positive (whatever the values of β_1 and β_2). If the opposite prevails (i.e., $\gamma_a < \gamma_b$), then platforms set positive fees to multihomers, while the sign of the singlehomers' fees cannot be ascertained.

Regarding the equilibrium participation on the single-homing side, we observe that the platform that discloses to more users on the single-homing side, obtains more participation than the competitor if and only if the multi-homing side is subject to stronger network effects than the single-homing side (i.e., $\gamma_a < \gamma_b$).

Information structure. Platforms simultaneously choose their value of β_i in $(0,1)$. We look for a symmetric equilibrium. We compute

$$\left.\frac{\partial \Pi_i^*}{\partial \beta_i}\right|_{\beta_1=\beta_2=\beta} = \gamma_b \frac{4\gamma_a\gamma_b(3\gamma_a+\gamma_b)\beta-\left(4t(5\gamma_a+\gamma_b)-(\gamma_a+\gamma_b)\left(3\gamma_a^2+\gamma_b^2\right)\right)}{16\left(6t-\gamma_a^2-\gamma_a\gamma_b-\beta\gamma_b^2-3\beta\gamma_a\gamma_b\right)\tau_b}. \tag{8}$$

Under condition (7), the denominator is positive, which implies that the derivative is increasing in β. There are thus three possible situations according to the sign of the derivative at $\beta = 0$ and $\beta = 1$. We compute

$$\begin{aligned} \left.\frac{\partial \Pi_i^*}{\partial \beta_i}\right|_{\beta_1=\beta_2=0} &> 0 \Leftrightarrow t < \frac{(\gamma_a+\gamma_b)\left(3\gamma_a^2+\gamma_b^2\right)}{4(5\gamma_a+\gamma_b)} \equiv t_1, \\ \left.\frac{\partial \Pi_i^*}{\partial \beta_i}\right|_{\beta_1=\beta_2=1} &> 0 \Leftrightarrow t < \frac{(\gamma_a+\gamma_b)\left(3\gamma_a^2+\gamma_b^2\right)}{4(5\gamma_a+\gamma_b)} + \frac{\gamma_a\gamma_b(3\gamma_a+\gamma_b)}{(5\gamma_a+\gamma_b)} \equiv t_2. \end{aligned}$$

As $t_1 < t_2$, the derivative in (8) is positive for all values of β if $t < t_1$, negative for small values of β and positive for large values of β if $t_1 < t < t_2$, and negative for all values of β if $t > t_2$. We have thus proved the following result (which we illustrate in Figure 1).

Proposition 3. *Consider the competitive bottleneck setting in which platforms have to decide to which extent they inform the multihoming side about the fees charged on the singlehoming side. At the symmetric perfect Bayesian equilibrium with passive beliefs, the platforms' choice of information structure features: (i) full disclosure ($\beta_1 = \beta_2 = 1$) if $t_{\min} < t < t_1$; (ii) full disclosure or no disclosure ($\beta_1 = \beta_2 \in \{0, 1\}$) if $\max\{t_{\min}, t_1\} < t < t_2$; (iii) no disclosure ($\beta_1 = \beta_2 = 0$) if $t > \max\{t_{\min}, t_2\}$.*

We observe in Figure 1 that for a large parameter range, firms will choose at equilibrium to disclose no information, which is reminiscent of what was observed in the two-sided singlehoming case. This is certainly so if the singlehoming side exerts stronger cross-group external effects than the multihoming side ($\gamma_a > \gamma_b$, which implies $t_{\min} > t_2$). Yet, if the latter condition is reversed (i.e., if the multihoming side generates the stronger external effects) and platforms are not too differentiated ($t < t_2$), then full disclosure can be another equilibrium, or even the unique equilibrium, of the game. However, for full disclosure to be the unique equilibrium of the game, the multihoming side must exert much stronger cross-group external effects than the singlehoming side ($t_1 > t_{\min}$ if and only if $\gamma_b > 17\gamma_a$). It is also important to note that when the two equilibria coexist (i.e., for $\max\{t_{\min}, t_1\} < t < t_2$), platforms strictly prefer no disclosure ($\beta_1 = \beta_2 = 0$) to full disclosure ($\beta_1 = \beta_2 = 1$). We have indeed

$$\Pi_i^*|_{\beta_1=\beta_2=0} - \Pi_i^*|_{\beta_1=\beta_2=1} = \tfrac{8t-\gamma_a^2+\gamma_b^2}{16\tau_b} - \tfrac{8t-\gamma_a^2-6\gamma_a\gamma_b-\gamma_b^2}{16\tau_b} = \gamma_b \tfrac{3\gamma_a+\gamma_b}{8\tau_b} > 0.$$

Hence, in this parameter range, if platforms could coordinate on the preferred equilibrium, they would not disclose prices on the other side.

Keeping the parameters for the cross-group external effects constants and varying t, we see that for full disclosure to be an equilibrium outcome, platform differentiation cannot be too large (and requires $\gamma_a < \gamma_b$). The reason is that, for $\gamma_a < \gamma_b$, if a platform discloses to more single-homing users, its market share on the single-homing side increases (as follows from Lemma 3). This makes disclosure particularly attractive when t is small, as demand is more sensitive.

Restricting strategies to $\beta_i \in \{0, 1\}$, in the special case $\gamma_a = 0$, we obtain simple expressions for profits in each of the four subgames after stage 1. Denoting profits at stage 1 by $\Pi_i^*(\beta_1, \beta_2)$, these profits are $\Pi_i^*(0,0) = \frac{8t+\gamma_b^2}{16\tau_b}$, $\Pi_i^*(1,1) = \frac{8t-\gamma_b^2}{16\tau_b}$,

$$\begin{aligned}
\Pi_1^*(1,0) &= \tfrac{8t-\gamma_b^2}{4\tau_b}\left(\tfrac{1}{2} + \tfrac{\gamma_b^2}{2(12t-\gamma_b^2)}\right)^2 = \Pi_2^*(0,1), \text{ and} \\
\Pi_1^*(0,1) &= \tfrac{8t+\gamma_b^2}{4\tau_b}\left(\tfrac{1}{2} - \tfrac{\gamma_b^2}{2(12t-\gamma_b^2)}\right)^2 = \Pi_2^*(1,0).
\end{aligned}$$

Hence, we find that $\min\{\Pi_1^*(0,0), \Pi_1^*(1,0)\} > \Pi_i^*(1,1)$ and $\Pi_i^*(0,0) > \Pi_1^*(0,1)$. Depending on the parameters, $\Pi_i^*(1,0)$ may be smaller or larger than $\Pi_1^*(0,0)$. If $\Pi_1^*(1,0) > \Pi_1^*(0,0)$, no disclosure is not an equilibrium. Since one can show that $\Pi_1^*(1,0) > \Pi_1^*(0,0)$ implies $\Pi_1^*(1,1) >$

$\Pi_1^*(0,1)$, full disclosure is the unique equilibrium. The game at stage 1 then has the structure of a prisoner's dilemma. Otherwise, either no disclosure is the unique equilibrium or full and no disclosure are equilibria.

More generally, returning to continuous strategies at stage 1, for any admissible parameters, using the profit formulas in Lemma 3, we obtain that $d\Pi_i^*(\beta,\beta)/d\beta < 0$. Thus, if platforms could coordinate their disclosure decisions, no information would be disclosed. Proposition 3 shows that the preferred no disclosure outcome is supported as equilibrium of the strategic disclosure game if $t > \max\{t_{\min}, t_1\}$.

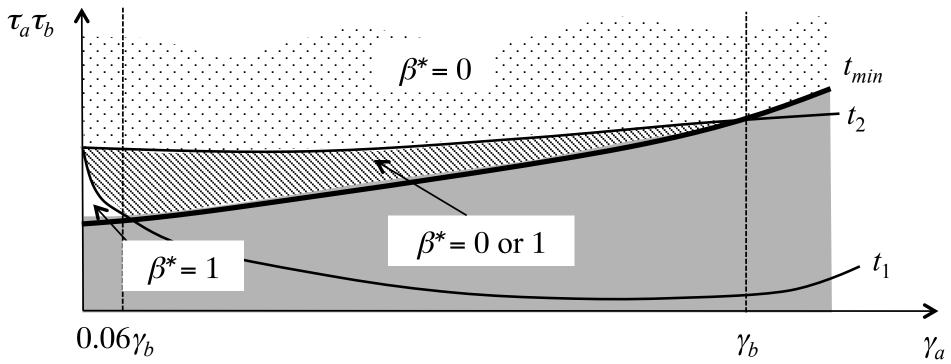

Figure 1: Price disclosure in competitive bottlenecks (all singlehomers are informed)

3.2.3 All multihoming users are informed

We assume now that all multihoming users (i.e., side b users) are informed, so that $\beta_1 = \beta_2 = 1$, and some singlehoming users may be uninformed, so that α_1 and α_2 range between 0 and 1.[15]

Membership fees. Platform 1 chooses A_1 and B_1 to maximize $\Pi_1 = A_1 m_a + B_1 m_b$, while platform 2 chooses A_2 and B_2 to maximize $\Pi_2 = A_2(1-m_a) + B_2 n_b$. We characterize the equilibrium in the next lemma.

Lemma 4. *In the competitive bottleneck case in which all multihoming users are informed while some singlehoming users are uninformed, the platforms set the following membership fees:*

$$A_1^* = \frac{4\tau_a\tau_b - \gamma_b(\gamma_b + (\alpha_1+2\alpha_2)\gamma_a)}{2\tau_b} m_a^*, \; B_1^* = \frac{\gamma_b - \alpha_1\gamma_a}{2} m_a^*,$$
$$A_2^* = \frac{4\tau_a\tau_b - \gamma_b(\gamma_b + (2\alpha_1+\alpha_2)\gamma_a)}{2\tau_b}(1-m_a^*), \; B_2^* = \frac{\gamma_b - \alpha_2\gamma_a}{2}(1-m_a^*).$$

The resulting equilibrium profits are

$$\Pi_1^* = \frac{8\tau_a\tau_b - \left(\gamma_b^2 + \alpha_1^2\gamma_a^2 + 2(\alpha_1+2\alpha_2)\gamma_a\gamma_b\right)}{4\tau_b}(m_a^*)^2,$$
$$\Pi_2^* = \frac{\left(8\tau_a\tau_b - \left(\gamma_b^2 + \alpha_2^2\gamma_a^2 + 2(2\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}{4\tau_b}(1-m_a^*)^2,$$

[15]This seems to be a realistic assumption in many environments in which sellers are on the multihoming side and buyers on the singlehoming side. Coordinated disclosure decisions in this setting have been analyzed by Hagiu and Halaburda (2014); they find that equilibrium profits are largest under no disclosure.

where

$$\begin{aligned} m_a^* &= \tfrac{1}{2} + \tfrac{\gamma_a(\gamma_a-\gamma_b)(\alpha_1-\alpha_2)}{2(12\tau_a\tau_b-(2\gamma_b(\gamma_a+\gamma_b)+(\alpha_1+\alpha_2)(\gamma_a^2+3\gamma_a\gamma_b)))}, \\ m_b^* &= \tfrac{\gamma_b+\alpha_1\gamma_a}{2\tau_b} m_a^*,\ n_b^* = \tfrac{\gamma_b+\alpha_2\gamma_a}{2\tau_b}\left(1-m_a^*\right). \end{aligned}$$

Regarding the signs of the optimal fees, all we can ascertain here is that $\gamma_a > \gamma_b$ (resp. $\gamma_b > \gamma_a$) implies that singlehomers (resp. multihomers) pay positive fees; otherwise, the signs depend on the share of informed singlehomers.

Information structure. Platforms simultaneously choose their value of α_i in $[0, 1]$. To characterize the symmetric equilibrium of this game, we evaluate $\partial\Pi_i^*/\partial\alpha_i$ at $\alpha_1 = \alpha_2 = \alpha$. We obtain

$$\left.\frac{\partial\Pi_i^*}{\partial\alpha_i}\right|_{\alpha_1=\alpha_2=\alpha} = \frac{1}{16\tau_b}\frac{\gamma_a}{6t-\left(\alpha\gamma_a^2+\gamma_b^2+(1+3\alpha)\gamma_a\gamma_b\right)}P(\alpha),$$

with

$$P(\alpha) \equiv \gamma_a^2(\gamma_a+7\gamma_b)\alpha^2 + 2\gamma_a\left(7\gamma_b^2-6t-\gamma_a\gamma_b\right)\alpha + \gamma_b^2(\gamma_a+3\gamma_b) + 4t(2\gamma_a-5\gamma_b).$$

Condition (7) implies that the derivative has the same sign as $P(\alpha)$. Because $P(\alpha)$ is a second-order polynomial, there are potentially three symmetric equilibria: full disclosure ($\alpha^* = 1$), no disclosure ($\alpha^* = 0$), or partial disclosure ($0 < \alpha^* < 1$). In the latter case, the level of disclosure is given by the smaller root of $P(\alpha)$, denoted by α_p. It can be shown that $\alpha_p < \frac{1}{2}$. In the proof of Proposition 4 in the Appendix we derive the following three thresholds:

$$\begin{aligned} t_3 &\equiv \frac{1}{9}\left(3\gamma_a\gamma_b+\gamma_a^2-7\gamma_b^2+\sqrt{(\gamma_a-\gamma_b)(\gamma_a+7\gamma_b)\left(\gamma_a^2+2\gamma_b^2\right)}\right), \\ t_4 &\equiv \frac{\gamma_b^2(\gamma_a+3\gamma_b)}{4(5\gamma_b-2\gamma_a)}, \\ t_5 &\equiv \frac{\gamma_a^3+5\gamma_a^2\gamma_b+15\gamma_a\gamma_b^2+3\gamma_b^3}{4(\gamma_a+5\gamma_b)}. \end{aligned}$$

In the appendix, we show that $P(\alpha)$ has no real roots and is everywhere positive for $t > t_3$, while $P(0) < 0$ if $2\gamma_a < 5\gamma_b$ and $t > t_4$. In the following proposition, we provide a partial equilibrium characterization.

Proposition 4. *Consider the competitive bottleneck setting in which platforms have to decide to which extent they inform the singlehoming side about the fees charged on the multihoming side. At the symmetric perfect Bayesian equilibrium with passive beliefs, the platforms' choice of information structure $\alpha_1 = \alpha_2 = \alpha^*$ is as follows:*

$$\begin{cases} \alpha^* = 0 & \text{if } t > \max\{t_{\min}, t_4\}, \\ \alpha^* = \alpha_p & \text{if } \max\{t_{\min}, t_3\} < t < t_4, \\ \alpha^* = 1 & t_{\min} < t < t_5. \end{cases}$$

As shown in the proof in the Appendix, we have identified two parameter regions in which two candidate equilibria coexist. First, for $\max\{t_{\min}, t_4\} < t < t_5$ (which implies $2\gamma_a < 5\gamma_b$),

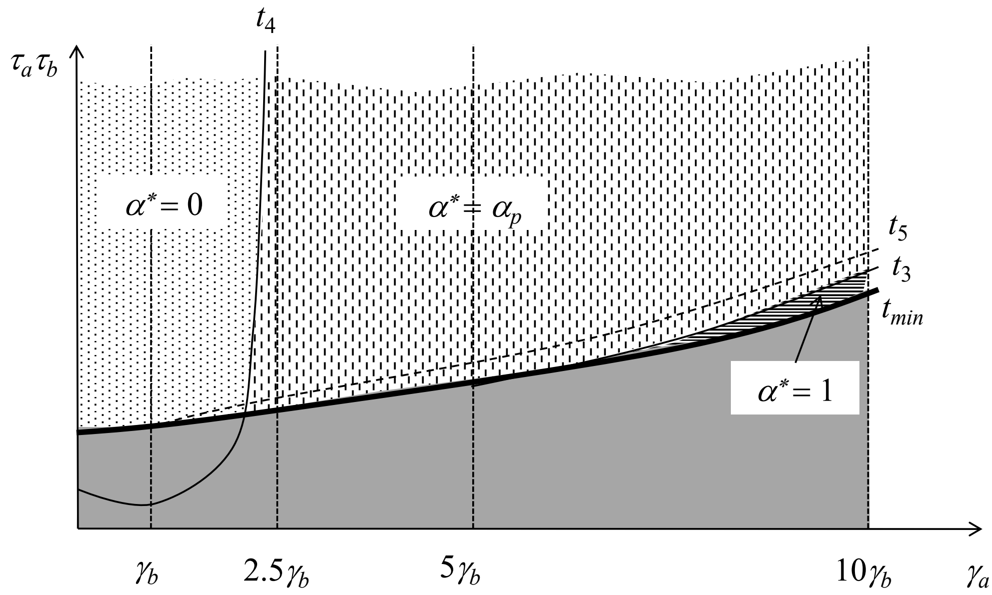

Figure 2: Price disclosure in competitive bottlenecks (all multihomers are informed)

we find that full and no disclosure are equilibrium candidates. Second, for $\max\{t_{\min}, t_3\} < t < \min\{t_4, t_5\}$, full and partial disclosure are equilibrium candidates. The following remark (formally shown in the Appendix), provides a partial answer to the question as to which equilibrium candidates survive global deviations.[16]

Remark 1. *For $\gamma_a \in [\gamma_b, 5\gamma_b]$, we have that $t_{\min}$ is the lower bound of the parameter region with multiple equilibrium candidates. At this lower bound, the equilibrium candidate with full disclosure is not a global maximizer, while the other equilibrium candidate gives a higher profit than the maximal deviation.*

By continuity, the result also holds in the vicinity of the lower bound. We illustrate our findings in Figure 2 in which, for simplicity, we do not show the full disclosure equilibrium candidate within the parameter range with multiple equilibrium candidates—such an equilibrium candidate exists in the region between $t_{\min}$ and t_5.

We observe from Figure 2 that the equilibrium information structure crucially depends on the relative strength of the cross-group external effects on the two sides. If multihomers exert sufficiently larger cross-group external effects than singlehomers (i.e., γ_a being larger than about $2.5\gamma_b$), then, in equilibrium, platforms choose to inform at least a fraction of the singlehomers. Full disclosure is the only equilibrium candidate if multihomers exert a sufficiently larger external effects than singlehomers and platforms are not too differentiated. Intuitively, a relatively large γ_a generates strong positive revenue effects on the singlehoming side from gaining additional multihoming users; this gives incentives to platforms to inform at least some users on the

[16]The analysis is partial insofar as the complexity of the problem prevents us from checking the second-order conditions globally on the whole parameter range.

singlehoming side about the price charged to the multihoming side. Otherwise, platforms decide not to inform singlehomers.

For further illustration, consider two special cases. First, suppose that multihoming and singlehoming users are subject to the same marginal external benefit; i.e., $\gamma_a = \gamma_b \equiv \gamma$. Then, for any (α_1, α_2), $m_a^* = n_a^* = 1/2$ and, thus, more disclosure does not shift demand when prices are endogenous. Platform i's equilibrium profit at stage 2 is

$$\Pi_i^* = \frac{\tau_a}{2} - \frac{1}{16\tau_b}(1 + \alpha_i^2 + 2\alpha_i + 4\alpha_j)\gamma^2.$$

Thus, at stage 1, platforms do not disclose information (i.e., $\alpha_1^* = \alpha_2^* = 0$) because disclosure increases the intensity of competition on the singlehoming side. The outcome is the same as in an environment in which they coordinated their disclosure decisions (as in Hagiu and Halaburda, 2014).

Second, suppose that multihoming users are not subject to any external benefit; thus, participation decisions on side a do not affect multihomers directly; i.e. $\gamma_b = 0$. In this case, participation on multihoming side is $-B_i/\tau_b$. We obtain that, at stage 2, for given prices on the singlehoming side A_i, platform i's profit-maximizing price on the multihoming side is $B_i = -\frac{a_i \gamma_a}{4\tau_a} A_i$. In equilibrium, the platforms set the following membership fees:

$$\begin{array}{l} A_1^* = 2\tau_a m_a^*,\ B_1^* = -\frac{\alpha_1\gamma_a}{2} m_a^*, \\ A_2^* = 2\tau_a\left(1 - m_a^*\right),\ B_2^* = -\frac{\alpha_2\gamma_a}{2}\left(1 - m_a^*\right), \end{array}$$

where $m_a^* = \frac{1}{2} + \frac{\gamma_a^2(\alpha_1 - \alpha_2)}{2(12\tau_a\tau_b - (\alpha_1+\alpha_2)\gamma_a^2)}$. Evaluated at $\alpha_1 = \alpha_2$, by increasing α_i, more users on side a will learn about the price charged to users on side b, and this gives incentives to platform i at stage 2 to further subsidize users on side b; i.e., $\partial B_i^*/\partial\alpha_i|_{\alpha_1=\alpha_2} < 0$. In other words, for larger α_i, the price on the multihoming side, B_i, becomes a more effective instrument to attract users on the singlehoming side. Equilibrium profits at stage 2 are

$$\begin{array}{rcl} \Pi_1^* & = & \frac{8\tau_a\tau_b - \alpha_1^2\gamma_a^2}{4\tau_b}\left(m_a^*\right)^2, \\ \Pi_2^* & = & \frac{8\tau_a\tau_b - \alpha_2^2\gamma_a^2}{4\tau_b}\left(1 - m_a^*\right)^2. \end{array}$$

We have that

$$\left.\frac{\partial\Pi_i^*}{\partial\alpha_i}\right|_{\alpha_1=\alpha_2=\alpha} = \frac{\gamma_a^2}{16\tau_b}\frac{\gamma_a^2\alpha^2 - 12\alpha t + 8t}{6t - \alpha\gamma_a^2}.$$

Evaluated at $\alpha = 0$, this derivative is $\frac{\gamma_a^2}{12\tau_b} > 0$, which shows that platforms choosing no disclosure cannot occur in equilibrium. Setting a larger α_i than the competitor gives platform i an advantage, but competition becomes more intense. Each platform has to balance these countervailing forces, given the behavior of the competing platform. Therefore, for some parameter range, the equilibrium features partial disclosure $\alpha_1^* = \alpha_2^* \in (0, 1)$, which is given by $\alpha_i^* = 2t\left(3 - \sqrt{9 - (2\gamma_a^2/t)}\right)/\gamma_a^2$. By contrast, if platforms coordinated their decisions (as in Hagiu and Halaburda, 2014), both platforms would implement no disclosure.

Comparing the results of Propositions 3 and 4, we notice a couple of common patterns in the two cases that we analyzed. First, we see that in both setting, *platforms choose to disclose no* *information at equilibrium when the informed users exert stronger cross-group external effects than the initially uninformed users* (i.e., $\gamma_a > \gamma_b$ when all singlehomers are informed or $\gamma_b > \gamma_a$ when all multihomers are informed). This result is easily seen for symmetric cross-group effects ($\gamma_a = \gamma_b$). When all singlehomers are informed, a change in β_i at the first stage only affects the equilibrium fee on the singlehoming side at the second stage (the other fee and the participation levels are constant; see Lemma 3); as $\partial A_i^*/\partial \beta_i < 0$, it follows that platforms prefer opaqueness ($\beta_i = 0$). When all multihomers are informed, an increase in α_i leaves participation on side a unaffected, and increases participation on side b; yet, as both fees decrease (see Lemma 4), the net effect on profit is negative and, again, platforms prefer opaqueness ($\alpha_i = 0$).

Second, there exist parameter constellations such that platform fully or partially inform the initially uninformed side. *Partial or full disclosure of information to the uninformed side requires that this side exerts proportionately larger cross-group external effects. For platforms to inform fully the initially uninformed side at equilibrium, this side must exert substantially stronger cross-group external effects than the other side* (namely, $\gamma_b > 17\gamma_a$ when side a is informed, and $\gamma_a > 5\gamma_b$ when side b is informed). Moreover, we observe that partial disclosure may take two different forms: when singlehomers are informed, partial disclosure may result from a mixed-strategy equilibrium (when both full and no disclosure are pure-strategy equilibria); in contrast, when all multihomers are informed, an interior equilibrium in pure strategies may emerge. This stands in contrast to a setting in which platforms can coordinate their disclosure decision; with such coordination, platforms would not inform users.

4 Discussion and Conclusion

How are market outcomes affected if platforms do not inform all users about the prices charged to users on the other sides of the platform? Market outcomes then depend on user expectations about participation levels. These expectations are independent of the actual price on the other side if this price is not observed, but may depend on the price set on the own side. In this paper, we characterize perfect Bayesian equilibria with passive beliefs, i.e., users who only observe the price on their own side expect that the participation on the other side is given by its equilibrium level.

While a monopoly always has an incentive to inform all participants about prices, the result is reversed under platform competition with strategic disclosure two-sided singlehoming. In markets that feature competitive bottlenecks, results are more nuanced. Suppose first that all singlehomers are informed about the prices on the other side of the market. Then, no disclosure is an equilibrium for a large range of parameters; yet, full disclosure is an equilibrium if the horizontal differentiation between the platform is very low and if multihomers exert stronger cross-group external effects than singlehomers. Suppose second that all multihomers are informed about the prices on the other side of the market. If multihomers exert sufficiently larger cross-group external effects than singlehomers, then platforms find it optimal to inform a fraction of the singlehomers, or even all of them (if multihomers exert even proportionately larger external effects and platforms are not too differentiated); otherwise, no disclosure is again the equilibrium. If, instead, platforms could coordinate their disclosure decision, platform competition would always feature no disclosure.

In our analysis, we assumed that uninformed users hold passive beliefs. An alternative is to consider wary beliefs—Hagiu and Halaburda (2014) show that their results qualitatively carry over to this alternative belief formation. We leave it for future research to investigate the robustness of our results with respect to these alternative beliefs.

While in many contexts some users do not have information on how much the platform charges the other side, it may well be true that also some users have to make an adoption decision before they learn the price they have to pay themselves. An intermediate case is the situation in which consumers make a participation decision for multiple periods but only know the current price. We note that there is a link between disclosure and commitment when participation decisions are lumpy. The ability to commit to future prices is akin to disclosing those prices. Future work may want to look at price disclosure decisions in such environments.

More generally, not only prices charged to the other side, but also some other platform choices that affect participation of users on the other may be unknown. For instance, users of video game platforms may well not know the extent to which platforms provide tools to game developers. Our analysis can easily be extended to cover such non-price instruments. In this sense, our model should be seen as a particular instance in which a platform affects expected participation decision and, thus, expected quality, through disclosure decisions.

In previous work (Belleflamme and Peitz, 2019) we compared the two-sided singlehoming model to the competitive bottlenecks under full disclosure and we endogenized the homing decision. A natural extension is to combine our present setting with one in which also homing decisions are affected by platform strategy (namely, the imposition of exclusivity). In light of the rich results and sometimes tedious expressions obtained in isolation (either endogenizing price information or endogenizing the homing decision), we leave this issue for further research.

5 Appendix

5.1 Proof of Lemma 1

The first-order conditions yield

$$\begin{aligned}
2A + (\gamma_a\alpha + \gamma_b\beta)\, B &= v_a + \gamma_a \alpha v_b + \alpha\gamma_a\gamma_b\,(1-\beta)\, x_a + \gamma_a\,(1-\alpha)\, x_b, \\
(\gamma_a\alpha + \gamma_b\beta)\, A + 2B &= v_b + \gamma_b \beta v_a + \gamma_b\,(1-\beta)\, x_a + \gamma_a\gamma_b\beta\,(1-\alpha)\, x_b.
\end{aligned}$$

The Hessian matrix is computed as

$$\begin{matrix}
\frac{2}{\alpha\beta\gamma_a\gamma_b-1} & \frac{\alpha\gamma_a+\beta\gamma_b}{\alpha\beta\gamma_a\gamma_b-1} \\
\frac{\alpha\gamma_a+\beta\gamma_b}{\alpha\beta\gamma_a\gamma_b-1} & \frac{2}{\alpha\beta\gamma_a\gamma_b-1}
\end{matrix}$$

It is a negative definite matrix for all admissible values of A and B provided that the following two conditions are met:

$$\begin{cases}
\gamma_a\alpha\gamma_b\beta < 1, \\
(2 - (\gamma_a\alpha + \gamma_b\beta))\,(2 + (\gamma_a\alpha + \gamma_b\beta)) > 0.
\end{cases} \tag{9}$$

Under conditions (9), we have thus global maxima. Note that the first condition is automatically satisfied if cross-group external effects are negative for one side (as this implies $\gamma_a \alpha \gamma_b \beta < 0$). If $\gamma_a, \gamma_b > 0$, then it is easy to show that the second condition is more stringent than the first.

Solving for A and B, one finds

$$\begin{aligned} A &= \frac{(2-\gamma_b\beta(\gamma_a\alpha+\gamma_b\beta))v_a+(\gamma_a\alpha-\gamma_b\beta)v_b}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} + \frac{\gamma_a(1-\alpha)(2-\gamma_b\beta(\gamma_a\alpha+\gamma_b\beta))x_b-(1-\beta)\gamma_b(\gamma_a\alpha-\gamma_b\beta)x_a}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} \\ B &= \frac{(2-\gamma_a\alpha(\gamma_a\alpha+\gamma_b\beta))v_b-(\gamma_a\alpha-\gamma_b\beta)v_a}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} + \frac{\gamma_b(1-\beta)(2-\gamma_a\alpha(\gamma_a\alpha+\gamma_b\beta))x_a-(1-\alpha)\gamma_a(\gamma_a\alpha-\gamma_b\beta)x_b}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} \end{aligned}$$

Plugging these values into the expression for n_a and n_b, we get:

$$\begin{aligned} n_a &= \frac{2v_a+(\gamma_a\alpha+\gamma_b\beta)v_b+\gamma_b(1-\beta)(\gamma_a\alpha+\gamma_b\beta)x_a+2\gamma_a(1-\alpha)x_b}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} \\ n_b &= \frac{2v_b+(\gamma_a\alpha+\gamma_b\beta)v_a+\gamma_a(1-\alpha)(\gamma_a\alpha+\gamma_b\beta)x_b+2\gamma_b(1-\beta)x_a}{(2-(\gamma_a\alpha+\gamma_b\beta))(2+(\gamma_a\alpha+\gamma_b\beta))} \end{aligned}$$

We now impose fulfilled expectations: $x_a = n_a$ and $x_b = n_b$. Solving for n_a and n_b, we find

$$\begin{aligned} n_a^* &= \frac{2v_a + (\gamma_a + \gamma_b\beta)\, v_b}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}, \\ n_b^* &= \frac{2v_b + (\gamma_b + \gamma_a\alpha)\, v_a}{4 - (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)}. \end{aligned}$$

It seems logical to impose that n_i^* increase with v_i, which requires $4 > (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)$. The latter condition follows from the second conditions in (9) if $(\gamma_a\alpha + \gamma_b\beta)^2 > (\gamma_a + \gamma_b\beta)(\gamma_b + \gamma_a\alpha)$, which is equivalent to

$$-\alpha(1-\alpha)\gamma_a^2 - \beta(1-\beta)\gamma_b^2 - \gamma_a\gamma_b(1-\alpha\beta) > 0.$$

As this inequality may not satisfied (take, e.g., $\gamma_a, \gamma_b > 0$ and $\alpha, \beta < 1$), we need Condition 1. Furthermore, n_a^* and n_b^* have to be non-negative, which justifies Condition (2).

Finally, substituting n_a^* and n_b^* for x_a and x_b in the above expressions of A and B, we obtain A^* and B^* as expressed in the lemma.

5.2 Proof of Proposition 1

The platform's equilibrium profit is computed as

$$\Pi^*(\alpha,\beta) = \frac{(2v_a+(\gamma_a+\gamma_b\beta)v_b)((2-\gamma_b\beta(\gamma_b+\gamma_a\alpha))v_a+(\gamma_a-\gamma_b\beta)v_b)+(2v_b+(\gamma_b+\gamma_a\alpha)v_a)((2-\gamma_a\alpha(\gamma_a+\gamma_b\beta))v_b+(\gamma_b-\gamma_a\alpha)v_a)}{(4-(\gamma_a+\gamma_b\beta)(\gamma_b+\gamma_a\alpha))^2}.$$

Evaluated at $(\alpha, \beta) = (1, 1)$, we find

$$\Pi^*(1,1) = \frac{v_a^2 + v_b^2 + \gamma_a v_a v_b + \gamma_b v_a v_b}{(2 - \gamma_a - \gamma_b)(2 + \gamma_a + \gamma_b)},$$

where the denominator is positive given Condition 1. We now prove that $\Pi^*(1,1) > \Pi^*(\alpha,\beta)$ for all $0 \le \alpha < 1$ and $0 \le \beta < 1$. The inequality $\Pi^*(1,1) > \Pi^*(\alpha,\beta)$ is equivalent to

$$K_1 \left(\frac{v_a}{v_b}\right)^2 + K_2 \frac{v_a}{v_b} + K_3 > 0, \text{ where} \tag{10}$$

$$
\begin{aligned}
K_1 &\equiv (4-(\gamma_a+\gamma_b\beta)(\gamma_b+\gamma_a\alpha))^2 \\
&\quad +(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2)((\gamma_b+\alpha\gamma_a)(\gamma_b-\alpha\gamma_a)-2\beta\gamma_b(\gamma_b+\alpha\gamma_a)+4), \\
K_2 &\equiv (\gamma_a+\gamma_b)(4-(\gamma_b+\alpha\gamma_a)(\gamma_a+\beta\gamma_b))^2 \\
&\quad +\left(4-(\gamma_a+\gamma_b)^2\right)\left(\gamma_a^2(\gamma_a+\beta\gamma_b)\alpha^2+\gamma_a\gamma_b(\beta+1)(\gamma_a+\beta\gamma_b)\alpha\right) \\
&\quad +\left(4-(\gamma_a+\gamma_b)^2\right)\left(\beta\gamma_b^2(\gamma_a+\beta\gamma_b)-4\gamma_b-4\gamma_a\right), \\
K_3 &\equiv (4-(\gamma_b+\alpha\gamma_a)(\gamma_a+\beta\gamma_b))^2 \\
&\quad +(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2)((\gamma_a+\beta\gamma_b)(\gamma_a-\beta\gamma_b)-2\alpha\gamma_a(\gamma_a+\beta\gamma_b)+4).
\end{aligned}
$$

Because of Condition 1, we have that

$$K_2^2-4K_1K_3=\gamma_a^2\gamma_b^2(1-\alpha)^2(1-\beta)^2(4-(\gamma_a+\beta\gamma_b)(\gamma_b+\alpha\gamma_a))^2(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2)<0.$$

It follows that the polynomial (10) has the sign of K_3. Regrouping terms, we can write

$$K_3=L_1\alpha^2+L_2\alpha+L_3, \text{ where} \tag{11}$$

$$
\begin{aligned}
L_1 &\equiv \gamma_a^2(\gamma_a+\beta\gamma_b)^2, \\
L_2 &\equiv 2\gamma_a(\gamma_a+\beta\gamma_b)\left(\beta\gamma_b^2-\gamma_a^2-\gamma_b^2-\gamma_a\gamma_b\right), \\
L_3 &\equiv (4-\gamma_b(\gamma_a+\beta\gamma_b))^2 \\
&\quad +((\gamma_a+\beta\gamma_b)(\gamma_a-\beta\gamma_b)+4)(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2).
\end{aligned}
$$

Because of Condition 1, we have that

$$L_2^2-4L_1L_3=4\gamma_a^2\gamma_b^2(1-\beta)^2(\gamma_a+\beta\gamma_b)^2(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2)<0.$$

It follows that the polynomial (11) has the sign of L_3. Regrouping terms, we can write

$$L_3=M_1\beta^2+M_2\beta+M_3, \text{ where} \tag{12}$$

$$
\begin{aligned}
M_1 &\equiv \gamma_b^2\left(4-\left(\gamma_a^2+2\gamma_a\gamma_b\right)\right), \\
M_2 &\equiv 2\gamma_b^2(\gamma_a\gamma_b-4), \\
M_3 &\equiv 4\gamma_b^2+2\gamma_a^2\gamma_b^2+2\gamma_a^3\gamma_b+\gamma_a^4.
\end{aligned}
$$

Because of Condition 1, we have that

$$M_2^2-4M_1M_3=4\gamma_a^2\gamma_b^2(\gamma_a+\gamma_b)^2(\gamma_a+\gamma_b-2)(\gamma_a+\gamma_b+2)<0.$$

It follows that the polynomial (12) has the sign of M_3. Regrouping terms, we can write

$$M_3=N_1\gamma_b^2+N_2\gamma_b+N_3, \text{ where} \tag{13}$$

$$N_1\equiv 2\left(2+\gamma_a^2\right), N_2\equiv 2\gamma_a^3, N_3=\gamma_a^4.$$

We compute $N_2^2-4N_1N_3=-4\gamma_a^4\left(\gamma_a^2+4\right)<0$. Hence, the polynomial (13) has the sign of $N_3=\gamma_a^4$, which is positive. It follows that the previous polynomials are all positive as well, which completes the proof.

5.3 Proof of Lemma 2

We derive here the equilibrium at stage 2 of the game with two-sided singlehoming. Platform 1 chooses A_1 and B_1 to maximize $\Pi_1 = A_1 m_a + B_1 m_b$, while platform 2 chooses A_2 and B_2 to maximize $\Pi_2 = A_2(1 - m_a) + B_2(1 - m_b)$, with

$$m_a = \frac{F_a - \tau_b (A_1 - A_2) - h_a (B_1 - B_2)}{2(\tau_a \tau_b - h_a h_b)} \text{ and } m_b = \frac{F_b - \tau_a (B_1 - B_2) - h_b (A_1 - A_2)}{2(\tau_a \tau_b - h_a h_b)}$$

$$\begin{aligned} F_a &\equiv 2h_a(\gamma_b - h_b) y_a + 2\tau_b (\gamma_a - h_a) y_b + (\tau_b - \gamma_b) h_a - (\gamma_a - \tau_a) \tau_b, \\ F_b &\equiv 2h_b(\gamma_a - h_a) y_b + 2\tau_a (\gamma_b - h_b) y_a + (\tau_a - \gamma_a) h_b - (\gamma_b - \tau_b) \tau_a, \end{aligned}$$

$$h_a \equiv \tfrac{1}{2}\gamma_a (\alpha_1 + \alpha_2), \; h_b \equiv \tfrac{1}{2}\gamma_b (\beta_1 + \beta_2).$$

The first-order conditions yield, respectively,

$$\begin{aligned} 2\tau_b A_1 + (h_a + h_b) B_1 &= \tau_b A_2 + h_a B_2 + F_a \\ (h_a + h_b) A_1 + 2\tau_a B_1 &= h_b A_2 + \tau_a B_2 + F_b. \end{aligned} \tag{14}$$

$$\begin{aligned} 2\tau_b A_2 + (h_a + h_b) B_2 &= \tau_b A_1 + h_a B_1 + (2\tau_a \tau_b - 2h_a h_b - F_a) \\ (h_a + h_b) A_2 + 2\tau_a B_2 &= h_b A_1 + \tau_a B_1 + (2\tau_a \tau_b - 2h_a h_b - F_b) \end{aligned} \tag{15}$$

The second-order conditions require: $4\tau_a \tau_b > (h_a + h_b)^2$. Solving the system of equations (14)-(15) gives values of A_1, A_2, B_1 and B_2 such that

$$\begin{aligned} A_1 - A_2 &= 2\frac{3\tau_a F_a - (2h_a + h_b) F_b - (3\tau_a - 2h_a - h_b)(\tau_a \tau_b - h_a h_b)}{9\tau_a \tau_b - (2h_a + h_b)(h_a + 2h_b)} \\ B_1 - B_2 &= 2\frac{3\tau_b F_b - (h_a + 2h_b) F_a - (3\tau_b - h_a - 2h_b)(\tau_a \tau_b - h_a h_b)}{9\tau_a \tau_b - (2h_a + h_b)(h_a + 2h_b)} \end{aligned}$$

Substituting these expressions into m_a and m_b and replacing F_a and F_b by their respective values gives

$$\begin{aligned} m_a &= \frac{1}{2}\frac{2(h_a+2h_b)(\gamma_b-h_b)y_a+6\tau_b(\gamma_a-h_a)y_b+9\tau_a\tau_b-2h_a^2-3\gamma_a\tau_b+3\tau_b h_a-\gamma_b h_a-2\gamma_b h_b-4h_a h_b}{9\tau_a\tau_b-(2h_a+h_b)(h_a+2h_b)}, \\ m_b &= \frac{1}{2}\frac{2(2h_a+h_b)(\gamma_a-h_a)y_b+6\tau_a(\gamma_b-h_b)y_a+9\tau_a\tau_b-2h_b^2-3\tau_a\gamma_b+3\tau_a h_b-2\gamma_a h_a-\gamma_a h_b-4h_a h_b}{9\tau_a\tau_b-(2h_a+h_b)(h_a+2h_b)}. \end{aligned}$$

The next step consists in imposing fulfilled expectations, i.e., $y_a = m_a$ and $y_b = m_b$. Replacing and solving, one finds that the unique fixed point is $m_a = m_b = \frac{1}{2}$. Substituting $y_a = m_a = \frac{1}{2}$ and $y_b = m_b = \frac{1}{2}$ into the expressions of F_a and F_b allows us to compute the equilibrium membership fees and profits as:

$$A_1^* = A_2^* = \tau_a - \tfrac{1}{2}(\beta_1 + \beta_2)\gamma_b \text{ and } B_1^* = B_2^* = \tau_b - \tfrac{1}{2}(\alpha_1 + \alpha_2)\gamma_a,$$

$$\Pi_1^* = \Pi_2^* = \tfrac{1}{2}(\tau_a + \tau_b) - \tfrac{1}{4}((\alpha_1 + \alpha_2)\gamma_a + (\beta_1 + \beta_2)\gamma_b).$$

5.4 Proof of Lemma 3

In stage 3, users make their participation decisions according to (6), which simplifies to

$$\begin{aligned} m_a &= \tfrac{G-\tau_b(A_1-A_2)-\gamma_a B_1+\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\beta_1+\beta_2)}, \\ m_b &= \tfrac{\gamma_b}{\tau_b}\left(\beta_1 \tfrac{G-\tau_b(A_1-A_2)-\gamma_a B_1+\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\beta_1+\beta_2)} + (1-\beta_1)\, y_a\right) - \tfrac{B_1}{\tau_b}, \\ n_b &= \tfrac{\gamma_b}{\tau_b}\left(\beta_2\left(1-\tfrac{G-\tau_b(A_1-A_2)-\gamma_a B_1+\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\beta_1+\beta_2)}\right) + (1-\beta_2)(1-y_a)\right) - \tfrac{B_2}{\tau_b}. \end{aligned}$$

where

$$G = \tau_a\tau_b - \gamma_a\gamma_b + \gamma_a\gamma_b(2-\beta_1-\beta_2)\, y_a.$$

In stage 2, the four first-order conditions of profit maximization can be written as

$$\begin{cases} 2\tau_b A_1 + (\gamma_a+\beta_1\gamma_b)\, B_1 - \tau_b A_2 - \gamma_a B_2 = G \\ \tau_b(\gamma_a+\beta_1\gamma_b)\, A_1 + 2(2\tau_a\tau_b - \beta_2\gamma_a\gamma_b)\, B_1 - \beta_1\tau_b\gamma_b A_2 - \beta_1\gamma_a\gamma_b B_2 \\ = \gamma_b(\beta_1 G + (1-\beta_1)(2\tau_a\tau_b - (\beta_1+\beta_2)\gamma_a\gamma_b)\, y_a) \\ \tau_b A_1 + \gamma_a B_1 - 2\tau_b A_2 - (\gamma_a+\beta_2\gamma_b)\, B_2 = G - (2\tau_a\tau_b - (\beta_1+\beta_2)\gamma_a\gamma_b) \\ \beta_2\tau_b\gamma_b A_1 + \beta_2\gamma_a\gamma_b B_1 - \tau_b(\gamma_a+\beta_2\gamma_b)\, A_2 - 2(2\tau_a\tau_b - \beta_1\gamma_a\gamma_b)\, B_2 \\ = \gamma_b(\beta_2 G - (1-(1-\beta_2)\, y_a)(2\tau_a\tau_b - (\beta_1+\beta_2)\gamma_a\gamma_b)). \end{cases}$$

As for the second-order conditions, it can be checked that they become more restrictive as β_1 and β_2 increase. Setting $\beta_1 = \beta_2 = 1$, we have thus the following sufficient conditions: (i) $\tau_a\tau_b > \gamma_a\gamma_b$ and (ii) $\tau_a\tau_b > \frac{1}{8}\left(\gamma_a^2 + \gamma_b^2 + 6\gamma_a\gamma_b\right)$. It is easily seen that the latter condition is more stringent than the former.

Following the same procedure as in the proof of Lemma 4 (see below), we derive the equilibrium membership fees, participations and profits (the intermediary results are tedious and therefore omitted):

$$\begin{aligned} m_a^* &= \frac{1}{2} - \frac{\gamma_b(\gamma_a-\gamma_b)(\beta_1-\beta_2)}{2\left(12\tau_a\tau_b - \left(2\gamma_a^2 + (\beta_1+\beta_2)\gamma_b^2 + (2+3\beta_1+3\beta_2)\gamma_a\gamma_b\right)\right)}, \\ m_b^* &= \frac{1}{2\tau_b}(\gamma_a+\gamma_b)\, n_a^*,\ n_b^* = \frac{1}{2\tau_b}(\gamma_a+\gamma_b)(1-n_a^*), \end{aligned}$$

$$\begin{aligned} A_1^* &= \tfrac{4\tau_a\tau_b - \beta_1\gamma_b^2 - (\beta_1+2\beta_2)\gamma_a\gamma_b}{2\tau_b} m_a^*,\ B_1^* = \tfrac{\gamma_b-\gamma_a}{2} m_a^*, \\ A_2^* &= \tfrac{4\tau_a\tau_b - \beta_2\gamma_b^2 - (2\beta_1+\beta_2)\gamma_a\gamma_b}{2\tau_b}(1-m_a^*),\ B_2^* = \tfrac{\gamma_b-\gamma_a}{2}(1-m_a^*), \end{aligned}$$

$$\begin{aligned} \Pi_1^* &= \tfrac{8\tau_a\tau_b - \gamma_a^2 + (1-2\beta_1)\gamma_b^2 - 2(\beta_1+2\beta_2)\gamma_a\gamma_b}{4\tau_b}(m_a^*)^2, \\ \Pi_2^* &= \tfrac{8\tau_a\tau_b - \gamma_a^2 + (1-2\beta_2)\gamma_b^2 - 2(2\beta_1+\beta_2)\gamma_a\gamma_b}{4\tau_b}(1-m_a^*)^2. \end{aligned}$$

Note that we have $0 < m_a^* < 1$ for all values of β_1 and β_2 provided that $\tau_a\tau_b \geq \frac{1}{6}\left(\gamma_b^2 + \gamma_a^2 + 4\gamma_a\gamma_b\right)$, which is condition (7).[17]

[17] We check again that with $\beta_1 = \beta_2 = 1$, we recover the expressions of Belleflamme and Peitz (2010).

5.5 Proof of Lemma 4

In stage 3, users make their participation decisions according to (6), which simplifies to

$$\begin{aligned} m_a &= \tfrac{G-\tau_b(A_1-A_2)-\alpha_1\gamma_a B_1+\alpha_2\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\alpha_1+\alpha_2)}, \\ m_b &= \tfrac{\gamma_b}{\tau_b}\tfrac{G-\tau_b(A_1-A_2)-\alpha_1\gamma_a B_1+\alpha_2\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\alpha_1+\alpha_2)} - \tfrac{B_1}{\tau_b}, \\ n_b &= \tfrac{\gamma_b}{\tau_b}\left(1-\tfrac{G-\tau_b(A_1-A_2)-\alpha_1\gamma_a B_1+\alpha_2\gamma_a B_2}{2\tau_a\tau_b-\gamma_a\gamma_b(\alpha_1+\alpha_2)}\right) - \tfrac{B_2}{\tau_b}. \end{aligned}$$

with

$$G \equiv \tau_a\tau_b - \alpha_2\gamma_a\gamma_b + \gamma_a\tau_b\,(1-\alpha_1)\,y_b - \gamma_a\tau_b\,(1-\alpha_2)\,z_b.$$

In stage 2, the four first-order conditions of profit maximization can be written as

$$\begin{cases} 2\tau_b A_1 + (\gamma_b + \alpha_1\gamma_a)\,B_1 - \tau_b A_2 - \alpha_2\gamma_a B_2 = G \\ \tau_b\,(\gamma_b + \alpha_1\gamma_a)\,A_1 + 2\,(\tau_a\tau_b - \alpha_2\gamma_a\gamma_b)\,B_1 - \tau_b\gamma_b A_2 - \alpha_2\gamma_a\gamma_b B_2 = \gamma_b G \\ \tau_b A_1 + \alpha_1\gamma_a B_1 - 2\tau_b A_2 - (\alpha_2\gamma_a + \gamma_b)\,B_2 = G - 2\tau_a\tau_b + (\alpha_1+\alpha_2)\,\gamma_a\gamma_b \\ \tau_b\gamma_b A_1 + \alpha_1\gamma_a\gamma_b B_1 - \tau_b\,(\gamma_b + \alpha_2\gamma_a)\,A_2 - 2\,(2\tau_a\tau_b - \alpha_1\gamma_a\gamma_b)\,B_2 \\ = \gamma_b\,(G - 2\tau_a\tau_b + (\alpha_1+\alpha_2)\,\gamma_a\gamma_b)\,. \end{cases}$$

As for the second-order conditions, it can be checked that they become more restrictive as α_1 and α_2 increase. Setting $\alpha_1 = \alpha_2 = 1$, we have thus the following sufficient conditions: (i) $\tau_a\tau_b > \gamma_a\gamma_b$ and (ii) $\tau_a\tau_b > \frac{1}{8}\left(\gamma_a^2 + \gamma_b^2 + 6\gamma_a\gamma_b\right)$. It is easily seen that the latter condition is more stringent than the former.

The solution to the above system of four equations is

$$\begin{aligned} A_1 &= \tfrac{\left(4\tau_a\tau_b-\gamma_b^2-(\alpha_1+2\alpha_2)\gamma_a\gamma_b\right)\left(4\tau_a\tau_b+2G-\gamma_b^2-\alpha_2^2\gamma_a^2-2\alpha_1\gamma_a\gamma_b\right)}{2\tau_b\left(12\tau_a\tau_b-\left(2\gamma_b^2+\left(\alpha_1^2+\alpha_2^2\right)\gamma_a^2+4(\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}, \\ B_1 &= \tfrac{(\gamma_b-\alpha_1\gamma_a)\left(4\tau_a\tau_b+2G-\gamma_b^2-\alpha_2^2\gamma_a^2-2\alpha_1\gamma_a\gamma_b\right)}{2\left(12\tau_a\tau_b-\left(2\gamma_b^2+\left(\alpha_1^2+\alpha_2^2\right)\gamma_a^2+4(\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}, \\ A_2 &= \tfrac{\left(4\tau_a\tau_b-\gamma_b^2-(2\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\left(8\tau_a\tau_b-2G-\gamma_b^2-\alpha_1^2\gamma_a^2-2(\alpha_1+2\alpha_2)\gamma_a\gamma_b\right)}{2\tau_b\left(12\tau_a\tau_b-\left(2\gamma_b^2+\left(\alpha_1^2+\alpha_2^2\right)\gamma_a^2+4(\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}, \\ B_2 &= \tfrac{(\gamma_b-\alpha_2\gamma_a)\left(8\tau_a\tau_b-2G-\gamma_b^2-\alpha_1^2\gamma_a^2-2(\alpha_1+2\alpha_2)\gamma_a\gamma_b\right)}{2\left(12\tau_a\tau_b-\left(2\gamma_b^2+\left(\alpha_1^2+\alpha_2^2\right)\gamma_a^2+4(\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}. \end{aligned}$$

We can now replace these values in the expressions for m_a, m_b and n_b. We also impose fulfilled expectations: $y_b = m_b$ and $z_b = n_b$, so that $G = \tau_a\tau_b - \alpha_2\gamma_a\gamma_b + (1-\alpha_1)\,\gamma_a\tau_b m_b - (1-\alpha_2)\,\gamma_a\tau_b n_b$. Solving for m_a, m_b and n_b, we find:

$$\begin{aligned} m_a^* &= \frac{1}{2} + \frac{\gamma_a\,(\gamma_a-\gamma_b)\,(\alpha_1-\alpha_2)}{2\,(12\tau_a\tau_b - (2\gamma_b\,(\gamma_a+\gamma_b) + (\alpha_1+\alpha_2)\,(\gamma_a^2 + 3\gamma_a\gamma_b)))}, \\ m_b^* &= \frac{\gamma_b + \alpha_1\gamma_a}{2\tau_b} m_a^*,\ n_b^* = \frac{\gamma_b + \alpha_2\gamma_a}{2\tau_b}\,(1-m_a^*)\,. \end{aligned}$$

Note that we have $0 < m_a^* < 1$ for all values of α_1 and α_2 provided that $\tau_a\tau_b \geq \frac{1}{6}\left(\gamma_b^2 + \gamma_a^2 + 4\gamma_a\gamma_b\right)$. It is readily checked that the latter condition is more stringent that the most restrictive of the two second-order conditions. This explains why we need to impose condition (7)

We can now compute the equilibrium prices and profits:[18]

$$\begin{aligned} A_1^* &= \tfrac{4\tau_a\tau_b-\gamma_b(\gamma_b+(\alpha_1+2\alpha_2)\gamma_a)}{2\tau_b} m_a^*,\ B_1^* = \tfrac{\gamma_b-\alpha_1\gamma_a}{2} m_a^*, \\ A_2^* &= \tfrac{4\tau_a\tau_b-\gamma_b(\gamma_b+(2\alpha_1+\alpha_2)\gamma_a)}{2\tau_b}\,(1-m_a^*)\,,\ B_2^* = \tfrac{\gamma_b-\alpha_2\gamma_a}{2}\,(1-m_a^*)\,, \end{aligned}$$

[18]We check that with $\alpha_1 = \alpha_2 = 1$, we recover the expressions of Belleflamme and Peitz (2010): $A_1^* = A_2^* = \tau_a - \gamma_b\,(3\gamma_a + \gamma_b)\,/\,(4\tau_b)$, $B_1^* = B_2^* = (\gamma_b - \gamma_a)\,/4$, $\Pi_1^* = \Pi_2^* = \left(8\tau_a\tau_b - 6\gamma_a\gamma_b - \gamma_a^2 - \gamma_b^2\right)/\,(16\tau_b)$.

$$\Pi_1^* = \frac{8\tau_a\tau_b-\left(\gamma_b^2+\alpha_1^2\gamma_a^2+2(\alpha_1+2\alpha_2)\gamma_a\gamma_b\right)}{4\tau_b}(m_a^*)^2,$$
$$\Pi_2^* = \frac{\left(8\tau_a\tau_b-\left(\gamma_b^2+\alpha_2^2\gamma_a^2+2(2\alpha_1+\alpha_2)\gamma_a\gamma_b\right)\right)}{4\tau_b}(1-m_a^*)^2.$$

5.6 Proof of Proposition 4

Platforms simultaneously choose their value of α_i in $[0,1]$. To characterize the symmetric equilibrium of this game, we evaluate $\partial\Pi_i^*/\partial\alpha_i$ at $\alpha_1 = \alpha_2 = \alpha$. We obtain

$$\left.\frac{\partial\Pi_i^*}{\partial\alpha_i}\right|_{\alpha_1=\alpha_2=\alpha} = \frac{\gamma_a}{16\left(6t-\gamma_b^2-\gamma_a\gamma_b-\alpha\gamma_a^2-3\alpha\gamma_a\gamma_b\right)\tau_b}P(\alpha),$$

with

$$\begin{aligned} P(\alpha) &\equiv A\alpha^2+B\alpha+C,\\ A &\equiv \gamma_a^2(\gamma_a+7\gamma_b)>0,\\ B &\equiv 2\gamma_a\left(7\gamma_b^2-6t-\gamma_a\gamma_b\right),\\ C &\equiv \gamma_b^2(\gamma_a+3\gamma_b)+4t(2\gamma_a-5\gamma_b).\end{aligned}$$

Condition (7) and our assumption that $\gamma_a > 0$ together imply that the derivative has the same sign as $P(\alpha)$. Because $P(\alpha)$ is a second-order polynomial, there are potentially three types of symmetric equilibria: full disclosure ($\alpha^* = 1$), no disclosure ($\alpha^* = 0$), or partial disclosure ($0 < \alpha^* < 1$). In the last case, the level of disclosure is given by the smaller root of $P(\alpha)$,

$$\alpha_p \equiv \frac{6t+\gamma_a\gamma_b-7\gamma_b^2-2\sqrt{9t^2-2\left(\gamma_a^2-7\gamma_b^2+3\gamma_a\gamma_b\right)t+\gamma_b^3(7\gamma_b-6\gamma_a)}}{\gamma_a(\gamma_a+7\gamma_b)}.$$

We examine first the conditions for these three types of equilibria to emerge as the unique equilibrium of the game. We consider next the configurations of parameters for which two candidate equilibria coexist.

We first note that $P(\alpha)$ has two real roots provided that $B^2-4AC>0$, which is equivalent to

$$16\gamma_a^2\left(9t^2+2\left(7\gamma_b^2-3\gamma_a\gamma_b-\gamma_a^2\right)t+\gamma_b^3(7\gamma_b-6\gamma_a)\right)>0.$$

Our assumption that $t > t_{\min}$ implies that this expression is larger than

$$\frac{4}{3}\gamma_a^2(5\gamma_b-\gamma_a)\left(\gamma_a^3+23\gamma_b^3+15\gamma_a\gamma_b^2+9\gamma_a^2\gamma_b\right).$$

So, $\gamma_a < 5\gamma_b$ is a sufficient condition for $P(\alpha)$ to have two real roots. If $\gamma_a \geq 5\gamma_b$, then t must be large enough, namely

$$t > \frac{1}{9}\left(3\gamma_a\gamma_b+\gamma_a^2-7\gamma_b^2+\sqrt{(\gamma_a-\gamma_b)(\gamma_a+7\gamma_b)\left(\gamma_a^2+2\gamma_b^2\right)}\right)\equiv t_3,$$

with $t_3 \geq t_{\min}$. We can therefore conclude that $P(\alpha)$ has no real root for $\gamma_a \geq 5\gamma_b$ and $t < t_3$. In that case $P(\alpha)$ has the same sign as $C = \gamma_b^2(\gamma_a+3\gamma_b)+4t(2\gamma_a-5\gamma_b)$, which is positive here as $\gamma_a \geq 5\gamma_b$. As $P(\alpha) > 0$ for all $\alpha \in [0,1]$, full disclosure is then the unique symmetric equilibrium of the game.

Suppose now that $t > t_3$ (which is implied by $t > t_{\min}$ for $\gamma_a < 5\gamma_b$). Given that $A > 0$, the two roots are then such that

$$\alpha_p \equiv \frac{-B - \sqrt{B^2 - 4AC}}{2A} < \alpha^+ \equiv \frac{-B + \sqrt{B^2 - 4AC}}{2A}.$$

It is also easily shown that $P(\alpha) > 0$ for $\alpha < \alpha_p$ and for $\alpha > \alpha^+$, while $P(\alpha) < 0$ for $\alpha_p < \alpha < \alpha^+$. Hence, if a symmetric equilibrium with partial disclosure exists, it is such that both platforms choose α_p. Existence requires that $0 < \alpha_p < 1$. To have $\alpha_p > 0$, we need $B < 0$ and $C > 0$; it can be shown that for $t > t_{\min}$, $C > 0$ implies that $B < 0$. For $\gamma_a \leq 10\gamma_b$ (which we assume here), we need $A + C < -B$ to have $\alpha_p < 1$. Developing the latter two conditions, we can state that partial disclosure (with $\alpha^* = \alpha_p$) is the unique symmetric equilibrium if

$$t > \frac{\gamma_a^3 + 5\gamma_a^2\gamma_b + 15\gamma_a\gamma_b^2 + 3\gamma_b^3}{4(\gamma_a + 5\gamma_b)} \equiv t_5 \text{ and } \begin{cases} \text{either} & 2\gamma_a \geq 5\gamma_b \\ \text{or} & 2\gamma_a < 5\gamma_b \text{ and } t < \frac{\gamma_b^2(\gamma_a+3\gamma_b)}{4(5\gamma_b-2\gamma_a)} \equiv t_4 \end{cases}$$

Finally, no disclosure is the unique symmetric equilibrium if $P(\alpha) < 0$ for all $\alpha \in [0,1]$. This is so provided that $C < 0$ and $\alpha^+ > 1$. From the previous case, we know that $C < 0$ requires $2\gamma_a < 5\gamma_b$ and $t < t_4$. It can be shown that if $C > 0$, $t > t_5$ is a sufficient condition for $\alpha^+ > 1$. We note that $t_5 > t_{\min}$ for $\gamma_a > \gamma_b$. We can thus conclude that no disclosure is the unique symmetric equilibrium if $t > \max\{t_{\min}, t_4, t_5\}$.

Combining the previous results, we can identify two regions of parameters where two candidate equilibria coexist. First, for $\max\{t_{\min}, t_4\} < t < t_5$ (which implies $2\gamma_a < 5\gamma_b$), we have both $P(0) < 0$ and $P(1) > 0$, showing that both full and no disclosure are equilibrium candidates. Second, for $\max\{t_{\min}, t_3\} < t < \min\{t_4, t_5\}$, both $\alpha = \alpha_p$ and $\alpha = 1$ are equilibrium candidates, as $0 < \alpha_p < 1$ and $P(1) > 0$.

5.7 Proof of Remark 1

We compare stage-2 equilibrium profits at the boundary where $t = t_{\min}$. The platforms' profits are given by

$$\begin{aligned} \Pi_1^*(\alpha_1, \alpha_2; t_{\min}) &= \frac{\left(4\gamma_a^2+\gamma_b^2-3\alpha_1^2\gamma_a^2+16\gamma_a\gamma_b-6\alpha_1\gamma_a\gamma_b-12\alpha_2\gamma_a\gamma_b\right)\left(\gamma_a+3\gamma_b-2\alpha_1\gamma_b-\alpha_2\gamma_a-\alpha_2\gamma_b\right)^2}{12\tau_b(2-\alpha_1-\alpha_2)^2(\gamma_a+3\gamma_b)^2}, \\ \Pi_2^*(\alpha_1, \alpha_2; t_{\min}) &= \frac{\left(4\gamma_a^2+\gamma_b^2-3\alpha_2^2\gamma_a^2+16\gamma_a\gamma_b-12\alpha_1\gamma_a\gamma_b-6\alpha_2\gamma_a\gamma_b\right)\left(\gamma_a+3\gamma_b-\alpha_1\gamma_a-\alpha_1\gamma_b-2\alpha_2\gamma_b\right)^2}{12\tau_b(2-\alpha_1-\alpha_2)^2(\gamma_a+3\gamma_b)^2}. \end{aligned}$$

Consider first parameters such that $\gamma_b < \gamma_a < 2.2373\gamma_b$; here, $\alpha_p < 0$, $P(0) < 0$, $P(1) > 0$, so that both $\alpha^* = 0$ and $\alpha^* = 1$ could be an equilibrium. We compute

$$\begin{aligned} \Pi_1^*(0,0;t_{\min}) - \Pi_1^*(1,0;t_{\min}) &= \gamma_b \frac{5\gamma_b^3+102\gamma_a\gamma_b^2+45\gamma_a^2\gamma_b-8\gamma_a^3}{48\tau_b(\gamma_a+3\gamma_b)^2} > 0, \\ \Pi_1^*(0,1;t_{\min}) - \Pi_1^*(1,1;t_{\min}) &= \gamma_a\gamma_b^2 \frac{\gamma_a+2\gamma_b}{\tau_b(\gamma_a+3\gamma_b)^2} > 0. \end{aligned}$$

The first line suggests that $\alpha_1 = 0$ is not only a local maximizer but also a best-response to $\alpha_2 = 0$, while the second line establishes that $\alpha_1 = 1$ is not a best-response to $\alpha_2 = 1$.

Consider next parameters such that $2.2373\gamma_b < \gamma_a < 5\gamma_b$; here, $0 < \alpha_p < 1$, $P(0) > 0$, and $P(1) > 0$, so that both $\alpha^* = \alpha_p$ and $\alpha^* = 1$ could be an equilibrium. We find that $\Pi_1^*(\alpha_p, 1; t_{\min}) - \Pi_1^*(1, 1; t_{\min})$ is equal to

$$\gamma_b^2 \frac{\gamma_a^4+29\gamma_b^4+90\gamma_a^2\gamma_b^2+248\gamma_a\gamma_b^3+16\gamma_a^3\gamma_b+\left(2\gamma_a^2+2\gamma_b^2+12\gamma_a\gamma_b\right)\sqrt{3(5\gamma_b-\gamma_a)\left(\gamma_a^3+23\gamma_b^3+15\gamma_a\gamma_b^2+9\gamma_a^2\gamma_b\right)}}{3\tau_b(\gamma_a+3\gamma_b)^2(\gamma_a+7\gamma_b)^2} > 0,$$

which implies that $\alpha_1 = 1$ is not a best-response to $\alpha_2 = 1$.

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