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DECEPTION AND COMPETITION IN SEARCH MARKETS*

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Abstract

We study the interplay between deception and consumer search in a search market where firms may deceive some naive consumers with inferior products that display hidden (bad) attributes. We derive an equilibrium in which both superior and inferior quality is offered and show that as search frictions vanish, superior goods are entirely driven out of the market. Deception harms sophisticated consumers, as it forces them to search longer to find a superior product. We argue that policy interventions that reduce search frictions such as the standardization of price and package formats may harm welfare. In contrast, reducing the number of naive consumers through transparency policies and education campaigns as well as a minimum quality standard can improve welfare.

Keywords: Deceptive product, Inferior product, Naivete, Consumer Search

JEL Classification No.: D18, D21, D43, D83

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1 Introduction

In many markets, products are complex, and consumers need a basic level of (financial, legal, technical, etc.) expertise to evaluate and compare products. This allows sellers to “deceive” and exploit “naive” consumers who lack this expertise, for example, by hiding (bad) attributes or the true future usage costs of their products.¹ Starting with Gabaix and Laibson (2006) there is by now a large literature that studies firms’ incentives to engage in deception (see the literature review below). In this article, we address a novel issue: the interplay between deception and consumer search. This is motivated by the observation that even for “sophisticated” consumers, who possess the relevant expertise, evaluating and comparing complex products is time-consuming and involves costly search.²

Explicitly accounting for consumer search delivers new and economically relevant insights. Most strikingly perhaps, we show that stronger competition in the form of smaller search frictions may not weaken but strengthen firms’ incentives to engage in inefficient deception. As a consequence, it may be the case that only sophisticated consumers benefit from lower search frictions, whereas naive consumers who are susceptible to deception lose out. When deceptive products are very inefficient, reducing search frictions may even reduce total welfare. We also identify a novel channel through which deception harms sophisticated consumers: they waste time examining irrelevant inferior products. These findings qualify the conventional wisdom about the beneficial effects of policy measures that aim at lowering search frictions such as the harmonization of price or product formats.³ Interventions that improve consumer and overall welfare in our setting are

¹For instance, many financial service products display hidden fees or attributes which some consumers fail to anticipate and account for. Examples include credit cards that display high late-payment and over-limit fees, or steep interest rate changes. Similarly, current accounts feature high overdraft charges, and (adjustable rate) mortgage loans are advertised with low introductory teaser rates, while hiding high reset rates. See, for example, Ausubel (1991), Agarwal et al. (2013), FCA (2014) and Agarwal et al. (2015) on credit cards, the OFT (2008), Stango and Zinman (2009), Armstrong and Vickers (2012), Stango and Zinman (2014) and the CMA (2016) on current accounts, and Woodward and Hall (2010) and Gurun et al. (2016) on mortgages. Other examples are nutritional supplements whose advertised health claims are difficult to ascertain for lay consumers and at times rather weakly founded. More generally, in a range of markets for technical products such as consumer electronics, it is difficult for non-experts to assess whether a product actually exhibits the good reliability and performance it promises.

²Consistent with this observation that searching for a suitable complex product is costly, consumers in many financial service markets often do not engage in substantial search, but visit only very few competitors. For example, according to the CFPB (2015), “almost half of consumers who took out a home purchase mortgage reported that they seriously considered only a single lender or mortgage broker before applying for a loan” and less than 20 % considered more than two. Another example is the UK retail banking market where according to the OFT (2008) only 15 – 20% of all consumers make explicit comparisons across products in terms of interest and rates, although 69% agree that there may be better alternatives available in theory.

³For example, in the context of the UK retail banking market, the Office of Fair Trading discusses the harmonization

minimum quality standards (which, alternatively, correspond to price caps on hidden fees in our model) and transparency policies that seek to educate naive consumers.

To gain insight into the interplay between deception and search, we extend a standard model of consumer search and allow for the presence of naive consumers.⁴ A firm may offer a product of inferior quality which we interpret as a deceptive product with hidden (bad) attributes, opaque usage costs, or surcharge fees concealed in the fine print. Alternatively, a firm may offer a “candid” product of superior quality. Sophisticated consumers engage in costly search to detect the price, the quality, and the idiosyncratic fit of a product. Naive consumers, in contrast, lack the skills to evaluate product quality and fit, and only learn prices when searching. Because they fail to distinguish inferior from superior products, they effectively search for attractive prices only.⁵ Since inferior products are assumed to be socially inefficient, in the benchmark without any naive consumers, competition ensures that all firms are candid and offer superior products.

In contrast, when some consumers are naive, deceptive and candid firms co-exist in equilibrium, entailing market segmentation. Sophisticated consumers never buy deceptive products but naive consumers purchase both products. Intuitively, sophisticated consumers see through deception, avoid purchasing inferior products and search until they find a suitable candid product. Naive consumers, in contrast, search for attractive prices only, and equilibrium prices adjust such that naive consumers consider both candid and deceptive firms’ prices as acceptable. They hence buy from the first firm they visit in equilibrium, be it candid or deceptive. This implies that deceptive and candid firms charge similar prices in equilibrium. Therefore, a firm’s product choice is driven by a trade-off between selling (high cost) candid products at a low mark-up to all consumers and ripping off naive consumers with (low cost), high-margin deceptive products.

Our main insight that lower search frictions may have a detrimental effect on the provision of quality in the market is driven by how lower search frictions affect the relative change in profits in the two segments. In fact, we show that in the extreme case when search costs vanish, candid firms are entirely driven out of the market. The underlying reason is that, as search costs vanish, both naive and sophisticated consumers can compare firms essentially for free. While this puts

of price and product formats in order to address “the difficulties caused by lack of information (which) are exacerbated by the fact that different banks use different terminology and present interest rates and charges in different ways”.

⁴In the absence of naive consumers, our model essentially corresponds to Wolinsky (1986) or Anderson and Renault (1999).

⁵More precisely, we model naive consumers as analogy-based reasoners in the sense of Jehiel (2005). This means that a naive consumer, while having correct beliefs about the marginal distributions of prices and qualities in the market, fails to understand the correlation between price and quality. In particular, he fails to correctly infer on quality from observed prices. Another interpretation which is consistent with our modeling choice is that naive consumers are unaware of hidden product features and consequently make no (correct) inference by assumption.

pressure on all firms' mark-up, this effect is more pronounced for candid firms, and as search costs vanish, candid mark-ups are eliminated whereas, as we show, deceptive mark-ups never erode, ultimately leading all firms to adopt the deceptive business model.⁶ Thus, if only a small fraction of consumers is vulnerable to deception, intense competition in the form of small search costs has the striking consequence that the market will supply almost only inferior quality.

The effect of smaller search costs on the share of candid firms in the market is clear-cut as search costs vanish, because only candid mark-ups entirely erode in this case. For non-negligible search costs, there are counter-veiling effects: On the one hand, as search costs fall, the market becomes more competitive, and both candid and deceptive prices fall (if the share of candid firms does not adjust).⁷ Now, because candid firms operate with larger demand but smaller margins than deceptive firms, a decreasing price level harms the candid business model relatively more than the deceptive one, and makes becoming deceptive relatively more attractive. On the other hand, as search costs fall, the difference in mark-ups between candid and deceptive firms shrinks which makes the candid business model with the lower mark-up more attractive. We identify a necessary and sufficient condition for profits in the candid segment to decrease more than in the deceptive segment in response to a marginal decrease in search costs. In this case, lower search frictions reduce the fraction of candid firms in equilibrium.⁸

These findings imply that stiffer competition may be undesirable from a welfare point of view, because the efficiency losses from the increased consumption of inefficient inferior products by naive consumers may outweigh the gains due to savings on search costs. Even if this is not the case (because efficiency losses are relatively small), the ensuing welfare gains may be unequally

⁶As we explain in the main text, deceptive mark-ups are bounded away from zero since deceptive prices exceed candid prices in equilibrium. Moreover, the above intuition is somewhat incomplete because what matters for sophisticated consumers is not nominal, but effective search costs, that is, the costs it takes to find not any, but a candid firm. If the number of candid firms shrinks, then, all else equal, sophisticated consumer waste time evaluating deceptive products which they will not end up buying. As we show, effective search costs are monotone in nominal search costs and, in fact, go to zero if nominal search costs do, although the share of candid firms vanishes.

⁷The equilibrium effect of a decrease in search costs on the candid mark-up is, however, not clear cut. On the one hand, as in standard search models, as search costs fall, sophisticated consumers search more so that, all else equal, candid firms reduce their prices in response. On the other hand, in our framework, the number of candid firms changes endogenously with search costs which affects the composition of naive and sophisticated customers that make up a firm's demand, possibly resulting in higher or lower prices. As we show, whenever the share of candid firms drops as search costs fall, then equilibrium prices go down. (Prices in our setting may however also increase as search costs fall.)

⁸This result is qualitatively consistent with the results in Ellison and Ellison (2009) which documents that in response to the better search technology (in particular for prices), firms in the online market for memory modules began to "bundle low-quality goods with unattractive contractual terms, like providing no warranty and charging a 20 % restocking fee on all return."

distributed among market participants: Sophisticated consumers gain, because they benefit from lower prices and search costs, whereas naive consumers may lose out. They also benefit from lower prices and search costs, however, they suffer from the higher prevalence of deceptive products.

Our main results thus suggest that policy interventions that aim at reducing search frictions should be viewed with caution, because they may be socially undesirable, or, at least, may not constitute a Pareto improvement from the consumers' point of view. This may include policies that facilitate the comparability of products, such as requiring firms to use standardized price, package, and product information formats, or the promotion of online marketplaces that facilitate the exchange of price and product information. In contrast, we argue that minimum quality standards, which correspond to a price cap on hidden fees in our model, improve consumer welfare, because naive consumer not only pay lower hidden fees but also incur these fees less frequently, as less firms engage in deception in response. More unconventionally, imposing a price floor may improve welfare in our setting, but may primarily benefit firms. Intuitively, a price floor may impede the downward spiral in prices which makes the candid business model unviable.

Our analysis also sheds new light on the effects of information and education campaigns that create consumer awareness and reduce consumer naivete (such as promoting financial literacy in the context of financial service markets). As we show, this increases welfare (under a plausible sufficient condition), because as a result of a shrinking naive customer base, fewer firms adopt the deceptive business model. In our search framework, this benefits also sophisticated consumers, as they waste less time examining inferior products. In this sense, naive consumers exert a negative externality on sophisticated consumers in our framework.

On a related note, we contribute to the debate whether firms themselves have incentives to turn naive into sophisticated consumers ("unshrouding"). We argue that the answer depends crucially on whether firms can educate only those consumers who visit their store or reach and educate all consumers in the market through advertising. In the latter case, unshrouding is profitable for each candid firm, because candid firms steal the entire business from deceptive firms and earn positive profits with any consumer. On the other hand, neither a candid nor a deceptive firm has an incentive to educate naive consumers who visit its store, because this could only trigger them to search for a better firm instead. Moreover, the more consumers are naive, the higher are industry profits so that by unshrouding, a firm may undermine its profits in the "long-run".

Related Literature

Our paper is related to a literature in behavioral industrial organization which studies how firms can exploit boundedly rational consumers. Most closely related are papers which study markets where firms offer goods with (unavoidable) future add-on services which naive consumers

fail to anticipate, and show that deception can occur in equilibrium even under Bertrand competition (see, e.g., Gabaix and Laibson (2006), Spiegler (2006), Armstrong and Chen (2009), Armstrong and Vickers (2012), Heidhues et al. (2017)). We contribute to this literature by adding consumer search to the picture, allowing novel dimensions of comparative statics, in particular with respect to search frictions, which gives rise to new policy implications with respect to the harmonization of price formats and price floors. A prominent theme in the literature is how the number of naive consumers affects (the division) of welfare. Our paper identifies a novel adverse spillover effect from naive to sophisticated consumer welfare: Because the presence of naive consumers reduces the number of candid firms, sophisticated consumers have to search longer to find them. Finally, our analysis of firms' shrouding incentives differs from the literature because in our framework, sophisticated consumers are not subsidized by naive ones and it allows to distinguish different unshrouding strategies.

A closely related paper is also Piccione and Spiegler (2012) whose analysis, like ours, implies that policy interventions that aim at facilitating the comparability of products can turn out to harm consumer welfare. The underlying reason is, however, rather different. In Piccione and Spiegler (2012), a consumer's ability to make a price comparison depends on the price formats chosen by firms. Firms face a trade-off between maximizing sales volume by combining low prices with easily comparable price formats and maximizing mark-ups by combining high prices with onerously comparable formats. Piccione and Spiegler (2012) shows that ("local") interventions that make relatively easily comparable formats even more easily comparable may backfire, because firms are more likely to adopt onerously comparable formats as an equilibrium response. As a result, average prices increase. If, on the other hand, the intervention is "global" and makes all formats more easily comparable, then the intervention is desirable. In contrast, in our model, prices may drop if they become more easily comparable, and what makes the intervention undesirable is that the quality in the market deteriorates, suggesting competing testable predictions. Also, in our framework, interventions that ease comparability (i.e., reduce search costs) are "global" by definition. Finally, the underlying consumer bias is different. Whereas consumers in Piccione and Spiegler (2012) have difficulties in comparing prices, they have difficulties to assess a product's quality in our setting, suggesting different domains of applicability.⁹

The point that stiffer competition (or a pro-competitive policy intervention) may move firms towards greater exploitation of naive consumers appears at various places in the literature.¹⁰ In this literature, competition tends to impair firms' mark-ups from sophisticated consumers only

⁹In a similar framework, Chioveanu and Zhou (2013) shows that a policy intervention which increases the number of competitors may backfire, because firms are more likely to adopt onerous price formats in response.

¹⁰See, among others, Varian (1980), Armstrong and Chen (2009), de Clippel et al. (2014), Murooka and Schwarz (2018), and Johnen (2018).

whereas in our model, the (first order) effect of a decline in search costs is to reduce profits in both segments, giving rise to countervailing effects that may also imply less exploitation.

Our paper also contributes to the literature on consumer search by integrating naive consumers and seller deception into the seminal papers by Wolinsky (1986) and Anderson and Renault (1999). While in this framework, lowering search frictions is typically considered socially desirable,¹¹ our key point is that lower search costs may be detrimental because it enhances a firm's incentives to become deceptive. A related point is made in Kuksov (2004) where lower search costs may give rise to socially excessive product differentiation incentives.¹² Similar to us, Fishman and Levy (2015) study the effects of search costs on firms' quality choices, but high quality comes at a higher fixed (rather than marginal) cost, and might be inefficient. In contrast to our limit result, in Fishman and Levy (2015), a reduction in search costs increases the share of high quality firms only if this share is small.

Our finding that prices may decrease in search costs echoes similar findings in the search literature.¹³ In our model, the price non-monotonicity originates in the fact that the share of candid firms is endogenous which affects both effective search costs and the composition of demand for an individual firm. Somewhat similarly, in Moraga-González et al. (2017), lower search costs also alter the composition of demand, but in their case, this is due to the market entry of less versed consumers whose demand is less elastic.

Our paper is finally related to work which considers consumers who rationally decide to remain "naive" about the quality of the products they purchase. In Gamp (2016b), consumers trade off the risks of a bad buy with the savings on search costs, but in contrast to the current article, always detect larger scams so that, ultimately, equilibria without deception exist. Similarly, in Heidhues et al. (2018), consumers rationally divide their attention between comparing prices across firms ("browsing") and learning the details of few products in depth ("studying"). As our paper, they argue that a minimum quality standard (such as regulating additional prices and secondary product features) improves consumer welfare, among other things, because it leads to more browsing which enhances competition and benefits consumers.

This paper is organized as follows. The next section introduces the model. Section 3 derives equilibrium conditions. Section 4 and 5 perform comparative statics. Section 6 discusses firms'

¹¹See, e.g., Bakos (1997), Anderson and Renault (1999), Cachon et al. (2008), Bar-Isaac et al. (2012), Larson (2013), or Fishman and Levy (2015).

¹²On a related note, it has also been observed that it can be welfare improving if firms may raise search costs through obfuscation (see Gamp (2016a) and Taylor (2017)).

¹³See, e.g., Janssen and Moraga-González (2004), Armstrong and Zhou (2011), Janssen and Shelgia (2015), Moraga-González et al. (2017), or Choi et al. (2017).

incentives to unshroud, and section 7 discusses policy implications. Section 8 discusses extensions and robustness, and section 9 concludes. All proofs are in the appendix.

2 Model

We consider a search market with a unit mass of firms indexed by $k \in [0, 1]$ and a unit mass of consumers indexed by $i \in [0, 1]$. Each consumer seeks to purchase at most one good. Goods are differentiated, and consumer i 's utility from purchasing the good offered by firm k is equal to

$$u_{ik} = q_k + \theta_{ik} - p_k, \quad (1)$$

where q_k is the quality offered, and p_k is the price chosen by firm k . The term θ_{ik} is a consumer-firm specific match-value which represents idiosyncratic product fit. It is common knowledge that for all i, k , the associated random variable θ_{ik} is distributed on the support $[\underline{\theta}, \bar{\theta}]$ with (sufficiently smooth) cumulative density function F and mean zero (hence $\underline{\theta} < 0$), independent and identical across consumers and firms. We assume that the corresponding probability density function f is log-concave with $f(\bar{\theta}) > 0$ which implies that F has an increasing and unbounded hazard rate

$$h(\theta) = \frac{f(\theta)}{1 - F(\theta)}. \quad (2)$$

The objective of our analysis is to study market outcomes when some consumers lack the ability to assess products and can be deceived about the true product characteristics. To capture this, we assume that firms can offer either a good of superior quality \bar{q} or a good of inferior quality $\underline{q} < \bar{q}$. Because some consumers lack the ability to distinguish the two products, we refer to the inferior product as deceptive and to the superior product as candid. More specifically, we may interpret the quality difference $\Delta q = \bar{q} - \underline{q}$ as the result of hidden (bad) attributes such as add-on fees, or opaque contract terms that lower a consumer's utility ex post and whose presence some consumers fail to understand. The quality of a product determines its production costs with $c(q)$ being the marginal costs for producing a good with quality q . Let $\underline{c} = c(\underline{q})$ and $\bar{c} = c(\bar{q})$, with $\underline{c} < \bar{c}$, and $\Delta c = \bar{c} - \underline{c}$. To make the problem interesting, we assume that the superior good is (weakly) socially efficient¹⁴

$$\Delta q \geq \Delta c, \quad (3)$$

and that consumers, net of prices, prefer a superior over an inferior product even if the former displays the worst and the latter the best product fit:

$$\Delta q > \bar{\theta} - \underline{\theta}. \quad (4)$$

¹⁴The case $\Delta c = \Delta q$ corresponds to the case where the inferior product, similarly to Heidhues et al. (2017), comes with an unavoidable add-on price of size Δc which a deceptive firm cashes in ex post.

To purchase a product from a firm, a consumer has to visit it and examine its product which entails a (search) cost. A consumer may examine several products, one at a time and in a random order so as to extend his choice set of products. Prior to examining a product, a consumer is uninformed about its price p_k and its characteristics (q_k, θ_{ik}) . To capture that some consumers are susceptible to deception, we assume that there is a fraction ν^N of naive consumers. A naive consumer differs from a (rational) sophisticated consumer in two respects. First, he lacks the ability to assess products, and upon visiting a firm k , he only learns the price p_k but neither observes the quality q_k nor the product fit θ_{ik} .¹⁵ Second, a naive consumer fails to draw the correct inference from observed prices. More precisely, a naive consumer is an Analogy Based reasoner (Jehiel (2005)) which means that he has correct beliefs about the marginal distribution of qualities, match-values and prices in the market, but (possibly incorrectly) believes that they are independent from each other, thus failing to understand that price and quality might be correlated.¹⁶ As a consequence, firms will be able to deceive naive consumers in the sense that their price choice does not betray their quality. In contrast, a fraction ν^S of sophisticated consumers, upon visiting a firm and examining its product, learns prices and product characteristics, and hence recognizes deceptive products as what they are.

The sequence of events is as follows. At the outset, firms simultaneously, independently, and once and for all set q_k and p_k . Consumers then search (with perfect recall) until they purchase a product. That is, at any point in time, they decide whether to visit an additional firm at random at search costs $s > 0$, or to buy a product from a previously visited firm (and afterwards leave the market).¹⁷

Thus, a strategy for a firm is a quality-price combination, and a strategy for consumers is a search rule that specifies whether to end or continue search contingent on the past search history. We adopt a concept of equilibrium which is essentially Perfect Bayesian equilibrium except that naive consumers are Analogy Based reasoners. That is, we define an equilibrium as a strategy profile where firms and all consumers adopt optimal strategies given their beliefs about the other players' strategies. The beliefs of firms and sophisticated consumers are consistent with the other players' strategies, while naive consumers' beliefs are inconsistent in that they wrongly believe that a firm's quality and price choices are independent from each other. In effect, this means that

¹⁵While we view a naive consumer as a consumer who intrinsically cannot evaluate products, because he lacks the expertise to do so, this is also consistent with an interpretation that naive consumers have prohibitively high costs to learn quality and product fit.

¹⁶Our modeling choice is consistent with an interpretation that naive consumers are unaware of hidden product features and, by assumption, simply believe that all firms in the market offer superior quality (because a too optimistic belief about the average quality offered does not affect the equilibrium outcome as long as naive consumers fail to make correct inferences).

¹⁷We thus implicitly assume that consumers have no interest in leaving the market without purchasing a product.

although naive consumers are aware of firms that pursue a deceptive business model, they fail to identify such firms by evaluation of their products and pricing strategy.¹⁸

In the benchmark without any naive consumers, all firms offer the efficient quality in order to maximize the gains from trade. Hence, all firms offer superior products, as low quality is socially inefficient. In this case, our model essentially corresponds to Wolinsky (1986) or Anderson and Renault (1999) where all firms offer the same quality. In other words, forces other than consumer naivety that may induce firms to offer inefficiently low quality, such as fixed investment costs, are absent from our model.

3 Segmented market equilibrium

In the first step of our analysis, we establish necessary and sufficient conditions for the existence of an equilibrium which displays a segmentation of the market in a candid and deceptive segment.¹⁹ More precisely, we define:

Definition 1 A triple $(\lambda^*, \bar{p}^*, \underline{p}^*) \in (0, 1) \times \mathbb{R}^2$ is a segmented market equilibrium outcome if there is an equilibrium in which

- ◊ a fraction λ^* of firms offers a candid good with $q_k = \bar{q}$ and $p_k = \bar{p}^*$;
- ◊ a fraction $1 - \lambda^*$ of firms offers a deceptive good with $q_k = \underline{q}$ and $p_k = \underline{p}^*$;
- ◊ sophisticated consumers never buy a deceptive good;
- ◊ naive consumers buy candid and deceptive goods.

We refer to an equilibrium which supports a segmented market equilibrium outcome as a segmented market equilibrium.

Next, we establish conditions for the existence of a segmented market equilibrium. We begin by deriving the players' optimal strategies in expectation of a segmented market equilibrium.

Sophisticated consumer search

Because consumers search randomly, it is optimal for any consumer to employ a cutoff search rule

¹⁸An equilibrium in our model thus formally corresponds to an Analogy Based equilibrium of an associated game in which each firm chooses its quality before its price and the analogy class of an Analogy Based reasoner is the set of all quality choices. In such an equilibrium all firms play the same mixed strategy (we thus do not “purify” the mixed equilibrium strategy) and an Analogy Based reasoner therefore believes that each firm's price and quality choice is independent of each other.

¹⁹That candid and deceptive firms co-exist in equilibrium is a novel feature that distinguishes our approach from Gabaix and Laibson (2006) where either all firms or no firm are deceptive.

which is characterized by a reservation utility which is the smallest level of (perceived) utility that a product must supply to induce the consumer to end his search and purchase the product.

As is shown in McCall (1970), the reservation utility is the current utility level that leaves a consumer indifferent between ending search in the current period and visiting a single additional firm. In a segmented market $(\lambda, \bar{p}, \underline{p})$, a sophisticated consumer expects to encounter a candid firm with probability λ and a deceptive firm with probability $1 - \lambda$. Under the hypothesis that a sophisticated consumer does not buy a deceptive product, his reservation utility is therefore given recursively as

$$\hat{U}^S = \lambda \int_{\underline{\theta}}^{\bar{\theta}} \max\{\bar{q} + \theta - \bar{p}, \hat{U}^S\} dF(\theta) + (1 - \lambda) \cdot \hat{U}^S - s. \quad (5)$$

It will often be convenient to work with the reservation match-value which is defined as the smallest match-value $\hat{\theta} \in \mathbb{R}$ at which a sophisticated consumer stops and buys at \bar{p} :

$$\bar{q} + \hat{\theta} - \bar{p} \equiv \hat{U}^S. \quad (6)$$

Define the function $g : \mathbb{R} \rightarrow \mathbb{R}$ by

$$g(z) \equiv \int_{\underline{\theta}}^{\bar{\theta}} \max\{\theta - z, 0\} dF(\theta). \quad (7)$$

Then (5) and (6) yield:

$$g(\hat{\theta}) = \frac{s}{\lambda}. \quad (8)$$

As we show in Lemma A.1 in the appendix, the function g is strictly decreasing on $(-\infty, \bar{\theta}]$ with $g(-\infty) = \infty$ and $g(\bar{\theta}) = 0$. Therefore, equation (8) has a unique solution.

Finally, note that it is indeed optimal for a sophisticated consumer to refrain from a deceptive product for any match-value if $\hat{U}^S \geq \underline{q} + \bar{\theta} - \underline{p}$, or, equivalently:

$$\bar{q} + \hat{\theta} - \bar{p} \geq \underline{q} + \bar{\theta} - \underline{p}. \quad (9)$$

Naive consumer search

To determine a naive consumer's reservation utility \hat{U}^N , we exploit another result in McCall (1970) which says that a consumer's reservation utility is his (perceived) expected utility. Under the hypothesis that a naive consumer buys candid and deceptive products, it is optimal for him to buy at the first firm. Because a naive consumer cannot observe the quality and the match-value of a product and fails to infer its quality from its price, he (wrongly) believes that any firm supplies

him with the average valuation $\mathbb{E}(q + \theta) = \lambda\bar{q} + (1 - \lambda)\underline{q} + 0$. In contrast, he (correctly) believes that any firm charges \bar{p} with probability λ and \underline{p} with probability $1 - \lambda$. His reservation utility is therefore given as

$$\hat{U}^N = \lambda \cdot (\bar{q} - \bar{p}) + (1 - \lambda) \cdot (\underline{q} - \underline{p}) - s. \quad (10)$$

It will be useful to express a naive consumer's search rule in terms of a reservation price $\hat{p} \equiv \hat{U}^N - \mathbb{E}(q)$ which is the maximal price that he is effectively willing to pay for any product given his beliefs in a segmented market equilibrium. With (10), we obtain:

$$\hat{p} = \lambda \cdot \bar{p} + (1 - \lambda) \cdot \underline{p} + s. \quad (11)$$

Finally, note that it is indeed optimal for a naive consumer to buy candid and deceptive products if $\underline{p} \leq \hat{p}$ and $\bar{p} \leq \hat{p}$, or, equivalently:²⁰

$$\bar{p} - \underline{p} \leq \frac{s}{1 - \lambda} \quad \text{and} \quad \underline{p} - \bar{p} \leq \frac{s}{\lambda}. \quad (12)$$

Demand and firm profits

The next lemma states the profits of a firm k that sets q_k and p_k , taking as given that all other firms and consumers adopt the strategy as specified in a (candidate) equilibrium with outcome $(\lambda, \bar{p}, \underline{p})$. To state the lemma, we define for a given \bar{p} and $\hat{\theta}$:

$$\Pi(p, q, c) = \nu^N \cdot (p - c) + \frac{\nu^S}{\lambda} \cdot \frac{1 - F(\hat{\theta} + (p - \bar{p}) - (q - \bar{q}))}{1 - F(\hat{\theta})} \cdot (p - c). \quad (13)$$

Lemma 1 *In a segmented market equilibrium with outcome $(\lambda, \bar{p}, \underline{p})$, the profit of a firm that sets quality q_k and price p_k is given by*

$$\pi(q_k, p_k) = \begin{cases} \Pi(p_k, q_k, c(q_k)) & \text{if } p_k \leq \hat{p}; \\ \Pi(p_k, q_k, c(q_k)) - \nu^N(p_k - c(q_k)) & \text{if } p_k > \hat{p}, \end{cases} \quad (14)$$

where \hat{p} is given by (11). Equilibrium profits are given by

$$\pi(\underline{q}, \underline{p}) = \nu^N \cdot (\underline{p} - \underline{c}) \quad \text{and} \quad \pi(\bar{q}, \bar{p}) = (\nu^N + \frac{\nu^S}{\lambda}) \cdot (\bar{p} - \bar{c}). \quad (15)$$

²⁰Note that although a naive consumer will purchase from the first firm he visits in equilibrium, he will continue search off the path when prices are large. A naive consumer in our setting is therefore different from a Varian (1980) type of "loyal" consumer who always purchases from the first firm he visits no matter what the price. That consumers do not "truly" search and do not visit more than one firm in equilibrium is common if consumers perceive products as homogeneous as, e.g., in Diamond (1971) and Stahl (1989).

To derive some intuition for the lemma, note that the profit of a firm is given as the product of the mass of consumers who visit the firm in its lifetime, the probability that a visitor actually purchases the good, and the firm's mark-up ($p_k - c$).

Now, in equilibrium, all naive consumers visit only one firm, because they buy in the first period and leave the market afterwards. Therefore, each firm attracts a mass ν^N of naive consumers in its lifetime. In contrast, all sophisticated consumers visit in expectation more than one firm, because they search until they find a satisfying superior product. As is shown in the appendix, the mass of sophisticated consumers who visit the firm in its lifetime is $\nu^S / (\lambda(1 - F(\hat{\theta})))$. Upon visiting, a sophisticated consumer actually purchases a firm's product if it supplies more utility than his reservation utility (6), whereas a visiting naive consumer buys the product if the firm charges a price below his reservation price (11), which is reflected in the case distinction in (14). From these observations, profits (14) can be derived straightforwardly.

To see what is behind (15), observe that symmetry implies that the demand ν^N of naive consumers is split equally among the unit mass of all deceptive and candid firms, while the demand ν^S of sophisticated consumers is split equally among only the mass λ of candid firms. Expression (15) therefore simply reflects that a firm's profit is the product of its demand and its mark-up.

Equilibrium

The next lemma characterizes segmented market equilibrium outcomes formally. To ensure that first order conditions are sufficient for profit maximization of candid firms, we focus on the case that $\hat{\theta}^* > \underline{\theta}$ and impose from now on the following regularity condition:

$$\frac{f'(\bar{\theta})}{f(\bar{\theta})} \geq -\frac{\nu^S}{\nu^N} \cdot f(\bar{\theta}). \quad (16)$$

Note that the condition always holds with increasing density.²¹

Lemma 2 *There is a segmented market equilibrium outcome $(\lambda^*, \bar{p}^*, \underline{p}^*)$ if and only if there is $\hat{\theta}^* \in$*

²¹It is standard in the search literature to impose sufficient conditions similar to (16), see, e.g., Anderson and Renault (1999) and Gamp (2016b). Wolinsky (1986) requires (only) the hazard rate of F to be increasing which we also impose in our setting. This is consistent with (16) and the fact that our model coincides with Wolinsky (1986) for $\nu^N = 0$, because if ν^N is sufficiently small, (16) is satisfied under the mild assumption $f(\bar{\theta}) > 0$.

$(\underline{\theta}, \bar{\theta})$, $\hat{p}^* \geq 0$, and $\lambda^* \in (0, 1)$ so that:

$$g(\hat{\theta}^*) = \frac{s}{\lambda^*}, \quad (17)$$

$$\hat{p}^* = \lambda^* \cdot \bar{p}^* + (1 - \lambda^*) \cdot \underline{p}^* + s, \quad (18)$$

$$\bar{p}^* - \bar{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(\hat{\theta}^*)}, \quad (19)$$

$$\underline{p}^* = \hat{p}^*, \quad (20)$$

$$\left(\nu^N + \frac{\nu^S}{\lambda^*} \right) \cdot (\bar{p}^* - \bar{c}) = \nu^N \cdot (\underline{p}^* - \underline{c}). \quad (21)$$

To understand Lemma 2, notice first that (17) restates sophisticated consumers' optimal search rule (8), and second that (18) restates naive consumers' optimal search rule (11). Third, (19) corresponds to the first order condition for \bar{p}^* to be profit maximizing for a candid firm, expressed in terms of the candid mark-up $\bar{p}^* - \bar{c}$. Because of (16), the first order condition is also sufficient. Fourth, condition (20) reflects that a deceptive firm maximizes its profits by charging the naive consumers' reservation price \hat{p}^* , as it only serves naive consumers. Fifth, condition (21) says that candid and deceptive firms make the same profits. Otherwise firms in the less profitable segment would want to move to the more profitable one.

Observe that the deceptive price slightly exceeds the candid price, as inserting (18) into (20) yields:

$$\underline{p}^* = \bar{p}^* + \frac{s}{\lambda^*}. \quad (22)$$

To understand this, note that because a naive consumer cares only about prices, he has strict incentives to buy at a firm which charges the cheapest price in the market. Therefore, it is impossible that a deceptive firm, who only serves naive consumers, charges the cheapest price in equilibrium, as otherwise, it could increase its profits by raising its price without losing any demand.

Lemma 2 implies that equilibrium profits are bounded away from zero, as deceptive (and hence candid) profits exceed $\nu^N \Delta c$. Indeed, a deceptive firm serves a mass ν^N of naive consumers at a mark-up that exceeds Δc , because $\underline{p}^* \geq \bar{p}^*$ by (22) and $\bar{p}^* \geq \bar{c}$. In effect, in choosing their business model, firms face the trade-off to sell deceptive products to only naive consumers at a large mark-up, or to sell candid products to all consumers at a low mark-up.

Equilibrium existence and uniqueness

We now ask when an equilibrium exists. Treating the share of candid firms λ as exogenous for the moment, the equilibrium conditions (17) and (19) pin down candid prices as functions of λ as

$$\bar{p}^*(\lambda) - \bar{c} \equiv \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))}. \quad (23)$$

Inserting this and (22) in firm profits, we obtain the difference between a candid and a deceptive firm's profits as

$$\Delta(\lambda) \equiv \bar{\pi} - \underline{\pi} = \frac{\nu^S}{\lambda} \cdot (\bar{p}^*(\lambda) - \bar{c}) - \nu^N \cdot \left(\frac{s}{\lambda} + \Delta c \right). \quad (24)$$

The equal profit requirement (21) then amounts to the condition $\Delta(\lambda^*) = 0$ and a unique segmented market equilibrium outcome exists if $\Delta(\lambda^*) = 0$ has a solution $\lambda^* \in (-\frac{s}{\underline{\theta}}, 1)$.²² The next proposition establishes that this is the case if search costs are sufficiently small.

Proposition 1 *For sufficiently small search costs, there is a unique segmented market equilibrium outcome. The equilibrium share of candid firms λ^* solves $\Delta(\lambda) = 0$, and equilibrium prices are pinned down by (22) and (23).*

In what follows, we assume that search costs are sufficiently small so that Proposition 1 applies. While this assumption is common in the search literature to ensure that consumers visit more than one firm in expectation (see Footnote 22), in our setting it also guarantees the co-existence of candid and deceptive firms in equilibrium. To see this, suppose first that (almost) all firms are candid. Then, as search costs get close to zero, the mark-up for candid firms gets arbitrarily close to zero,²³ whereas the mark-up for deceptive firms is bounded from below by Δc . Therefore, if (almost) all firms are candid, there is a level of search costs below which candid firms earn lower profits than deceptive firms. On the other hand, if very few firms are candid, then the profit of a candid firm is extremely large, since the demand from sophisticated consumers is divided up between very few firms. As a consequence, there is an intermediate share of candid firms where the profits in the two segments equalize.

It is useful to contrast the equilibrium outcome of our setting with the outcomes in the absence of naive consumers as well as in the absence of search frictions. In the absence of naive consumers, our model essentially coincides with Wolinsky (1986) and all firms offer superior quality. The co-existence of candid and deceptive products in our equilibrium thus reflects the co-existence of naive and sophisticated consumers in our model. In the absence of search frictions, candid and deceptive products co-exist in equilibrium but are sold, in contrast, at the respective marginal production costs. Moreover, the two segments are independent in the sense that naive

²²By Lemma A.1 in the appendix, $\hat{\theta}^*$ is in $(\underline{\theta}, \bar{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Because $\lambda^* \leq 1$, a segmented market equilibrium thus can only exist if search costs are sufficiently small and $-\frac{s}{\underline{\theta}} < 1$, which is a standard condition (for consumer search to take place) in the search literature (see e.g. Wolinsky (1986) and Anderson and Renault (1999)).

²³Formally, as $s/\lambda \rightarrow 0$, equation (17) and the definition of g imply that $\hat{\theta}$ converges to $\bar{\theta}$. It follows that the mark-up $\bar{p} - c$ given by (19) converges to zero, because the hazard rate is unbounded.

consumers purchase exclusively cheap inferior products whereas sophisticated consumers purchase exclusively expensive superior products.²⁴ This implies that only when search is costly, a market outcome emerges which displays low demand but high mark-ups in the deceptive segment (greater than Δc), and high demand but low mark-ups in the candid segment.

Next, we study the comparative statics of segmented market equilibrium outcomes. To do so, we will abuse notation and denote by \bar{p}^* , \underline{p}^* and $\hat{\theta}^*$ both the equilibrium outcomes which depend only on exogenous parameters, as well as the “best-reply” functions $\bar{p}^*(\cdot)$, $\underline{p}^*(\cdot)$ and $\hat{\theta}^*(\cdot)$ which are pinned down by (17-20) as functions of λ when treated as an independent variable. In particular, we use total derivatives to indicate changes of equilibrium outcomes, and partial derivatives to indicate changes of the “best-reply”, taking λ^* as given.²⁵

4 Effects of stiffer competition

This section develops the main insights of our paper that stiffer competition in the form of lower search costs may have detrimental effects in our setting. We begin by showing that lower search costs may deteriorate the quality provision in the market, as it induces firms to engage in more deception. We then argue that this may have adverse welfare consequences. These results will form the basis of our policy implications.

Our first main result says that as search costs vanish, the share of candid firms in the market vanishes.

Proposition 2 *As search costs vanish, the share of candid firms in the market vanishes: $\lim_{s \rightarrow 0} \lambda^* = 0$.*

In other words, when only some consumers are vulnerable to deception, then intense competition has the striking consequence that the market will supply only inferior quality. In a nutshell, the intuition is that vanishing search costs allow consumers to compare prices essentially for free. While this intensifies competition in both segments, the effect is more pronounced in the candid segment where, in fact, mark-ups are eliminated. In contrast, a deceptive firm’s mark-up does not vanish and is bounded away from zero by Δc , because, as we argued above, a candid firm does not charge the lowest market price in equilibrium.

²⁴To be clear, the outcome without search frictions is the limit outcome of a variant of our model with zero search costs and a finite number of firms as the number of firms goes to infinity. Most closely related, Armstrong and Chen (2009) study a variant of this case without horizontal product differentiation and argue that industry profits vanish as the number of firms goes to infinity, as all profits are competed away.

²⁵For example, $d\bar{p}^*/ds$ is the change of the candid equilibrium price with respect to s , whereas $\partial\bar{p}^*/\partial s$ is the change of the candid equilibrium price with respect to s when λ^* does not adjust to the change in s . Therefore: $d\bar{p}^*/ds = \partial\bar{p}^*/\partial s + \partial\bar{p}^*/\partial\lambda \cdot d\lambda^*/ds$.

This intuition hides the subtlety that what matters for sophisticated consumers is not nominal search costs, but the expected search costs to encounter a candid firm. If nominal search costs are one dollar, and among 100 firms there is only a single candid firm, a sophisticated consumer has to spend on average 100 dollars to find a candid firm. Thus, the *effective search costs* that a sophisticated consumer faces in equilibrium are the expected search costs

$$\sigma^* \equiv \frac{s}{\lambda^*} \quad (25)$$

to find a candid firm. A sophisticated consumer can inspect an additional candid firm essentially for free only if effective search costs vanish. In line with this, observe that candid mark-ups depend on search costs only through effective search costs:

$$\bar{p}^* - \bar{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(g^{-1}(\sigma^*))}. \quad (26)$$

In what follows, we will thus interpret candid prices as a function of σ rather than s .

With this in mind, the intuition behind Proposition 2 can be made more precise. If the share of candid firms λ^* did not converge to zero, then effective search costs σ^* would tend to zero as s approaches zero, and candid firms' mark-ups would erode. Because firms make strictly positive profits in equilibrium, the erosion of mark-ups must be offset by an unbounded increase in the demand of a candid firm which, in turn, requires that the share of candid firms shrinks to zero (recall that in equilibrium a candid firm attracts the demand ν^S/λ^* from sophisticated consumers) – a contradiction.²⁶

Proposition 2 shows that only the deceptive business model survives extremely stiff competition where search costs are virtually eliminated. This suggests that, at least when search costs are sufficiently small, lowering search costs induces more firms to become deceptive. We now ask when it is in general the case that more intense competition in the form of lower search costs reduces the share of candid firms in the market, and derive a necessary and sufficient condition for this to be the case. To state the condition, we introduce the elasticity of the candid mark-up with respect to effective search costs:^{27,28}

$$\epsilon(\sigma) \equiv \frac{\partial(\bar{p}^* - \bar{c})/\partial \sigma}{(\bar{p}^* - \bar{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \text{with } \hat{\theta} = g^{-1}(\sigma). \quad (27)$$

²⁶Our model also displays the property that in the zero search cost limit, the market is frictionless in the sense that effective search costs vanish as search costs get small: $\lim_{s \rightarrow 0} \sigma^* = 0$. This also implies that the difference in prices between candid and deceptive firms, which equals σ^* by equation (22), vanishes as search costs go to zero. Formally, we establish that effective search costs vanish as auxiliary result (62) in the proof of Proposition 1.

²⁷The calculation is provided in Lemma A.3 in the appendix.

²⁸To understand why the elasticity of candid mark-ups depends on the steepness h' of the hazard rate, recall from (26) that \bar{p}^* is inversely related to the hazard rate. This reflects that the hazard rate indicates the percentage increase in the mass of sophisticated consumers who will stop to search and buy the firm's product if it marginally

We now show that the share of candid firms increases in search cost if and only if this elasticity is sufficiently large:

Proposition 3 *The share of candid firms increases in s if and only if*

$$\epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c}. \quad (C)$$

Before we explain the intuition behind the result, we shed light on when condition (C) is true. First, the condition is met if $\epsilon(\sigma^*) \geq 1$, that is, an increase in effective search costs of one percent results in an increase in a candid firm's mark-up that exceeds one percent. Moreover, it can be shown that condition (C) holds if (i) $f'(\hat{\theta}^*) \geq 0$, and if (ii) search costs are sufficiently small.²⁹ While (i) is stated in terms of the endogenous cutoff $\hat{\theta}^*$, we can turn it into primitive conditions by requiring it to hold for all θ . For example, (i) is satisfied for the uniform distribution. Finally, condition (C) is more likely to hold if Δc is large so that producing the deceptive quality entails large cost savings.

To see the intuition behind Proposition 3, note that the share of candid firms increases in search costs if, all else equal, candid profits increase by more than deceptive profits as search costs increase, because then more firms will adopt the candid business model in response. We therefore illustrate the economic forces that drive the evolution of profits in both market segments as search costs change.

Consider an increase of search costs by one dollar. This raises effective search costs by $1/\lambda^*$ dollars, all else equal. Because $\underline{p}^* = \bar{p}^* + \sigma^*$, a deceptive firm's price thus increases by $1/\lambda^*$ dollars more than the price of a candid firm. Since deceptive and candid firms have the same number ν^N of naive customers, a deceptive firm thus earns $1/\lambda^*$ marginal dollars more than a candid firm per naive customer, as search costs increase marginally. Reversely, a candid firm earns $\partial \bar{p}^* / \partial \sigma \cdot 1/\lambda^*$ marginal dollars more from any of its additional ν^S / λ^* sophisticated consumers. Hence, as search costs increase, the difference between the change in candid and deceptive profits is

$$\frac{\partial \Delta}{\partial s} = \frac{\partial}{\partial s}(\bar{\pi}^* - \underline{\pi}^*) = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial \bar{p}^*}{\partial \sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}. \quad (28)$$

In what follows, we refer to $\partial \bar{p}^* / \partial \sigma^*$ as the “search effect”. The search effect is unambiguously positive:³⁰ as effective search costs increase, sophisticated consumers become less picky in order

decreases its price. Thus, the larger the hazard rate, the larger is a candid firm's demand elasticity, and the smaller is the equilibrium price. As a consequence, the price a candid firm charges increases sharply in σ if the hazard rate decreases sharply.

²⁹This insight is consistent with Proposition 2 that the share of candid firms actually vanishes as search costs vanish. A formal proof appears in Lemma A.4 in the appendix.

³⁰This feature is analogous to the familiar property that in a search model without naive consumers such as Wolinsky (1986) or Anderson and Renault (1999), prices increase in search costs.

to save on search costs, and the critical match-value $\hat{\theta}^*$ goes down. This renders the demand by sophisticated consumers less elastic (as the hazard rate is increasing) which allows candid firms to increase prices. The last term in (28), in turn, captures that (continued) search becomes more costly also for naive consumers, and as a consequence, deceptive firms can enforce even larger mark-ups against them.

More intuitively, because $\underline{p}^* = \bar{p}^* + \sigma^*$ an increase in search costs entails an increase both in the overall price level as well as in the price difference. The first effect benefits candid firms relatively more than deceptive firms, as they derive more demand, whereas the second effect benefits deceptive firms more.

The intuition behind Proposition 3 follows now from the observation that (28) is positive if the search effect is strong in the sense that $\partial \bar{p}^* / \partial \sigma$ is large. Then, as search costs increase, candid prices increase so sharply that candid profits grow faster than deceptive profits, and the share of candid firms consequently increases in s . Observe that the size of the search effect $\partial \bar{p}^* / \partial \sigma$ is proportional to the elasticity of candid mark-ups, as

$$\frac{\partial \bar{p}^*}{\partial \sigma} = \epsilon \cdot \frac{\bar{p}^* - \bar{c}}{\sigma^*}. \quad (29)$$

Condition (C) is now the necessary and sufficient condition for candid mark-ups, in equilibrium, to be sufficiently elastic so that the search effect is sufficiently strong.

If condition (C) is violated, then the share of candid firms decreases in s . Intuitively, if the candid mark-up elasticity is sufficiently small, then the search effect is so weak that (28) becomes negative. We now give a parameterized example in order to illustrate that this may indeed be the case. The example takes the exponential distribution, which displays a constant hazard rate, and adapts it to our setting with bounded support. (The calculations behind the example are provided in Appendix B.)

Example Let $[\underline{\theta}, \bar{\theta}] = [-1, 1]$, and define

$$f^E(\theta) = \begin{cases} e^{-(\theta+1)} & \theta < 0 \\ e^{-1} & \theta \geq 0. \end{cases} \quad (30)$$

Let $f = f^E$. Then there is an open set of parameters $s, \bar{q}, \underline{q}, \bar{c}, \underline{c}, \nu^N$ so that a segmented market equilibrium exists with $\hat{\theta}^* < 0$ and

$$\lambda^* = \frac{\nu^S - s}{\Delta c - 1}. \quad (31)$$

The example has the property that the equilibrium match-value $\hat{\theta}^*$ is below zero, and that, at $\hat{\theta}^*$, the distribution coincides with an exponential distribution. The hazard rate is thus constant and

hence, the elasticity of candid mark-ups is zero, in violation of condition (C). As a result, the number of candid firms drops as s increases.

Welfare effects

We now argue that stiffer competition can be socially undesirable:

Proposition 4 *Suppose that condition (C) holds. Then total welfare increases in search costs if the efficiency gains from the superior quality, $\Delta q - \Delta c$, are sufficiently large.*

The basic intuition behind the result is that under condition (C) larger search frictions result in a lower share of deceptive products by Proposition 3. If deceptive products are very inefficient, this will outweigh the direct welfare costs associated with higher search costs. To understand the forces behind Proposition 4 in more detail, the following lemma is key.

Lemma 3 *Equilibrium effective search costs are increasing in s : $d\sigma^*/ds \geq 0$.*

Therefore, effective and nominal search costs move in the same direction, and, as search frictions increase, searching for a suitable superior product becomes more tedious for sophisticated consumers, even if the share of candid firms increases in response. In this sense, effective search costs are well-behaved.³¹

Now, because prices only affect the division but not the size of total welfare, total welfare is given as the welfare of sophisticated and naive consumers as stated in (6) and (10) net of prices but accounting for the production costs of firms:³²

$$W = \nu^S \cdot [\bar{q} - \bar{c} + \hat{\theta}^*] + \nu^N \cdot [\lambda^* \cdot (\bar{q} - \bar{c}) + (1 - \lambda^*) \cdot (\underline{q} - \underline{c}) - s]. \quad (32)$$

An increase in search costs affects welfare thus through three channels. First, effective search costs increase by Lemma 3 so that on average sophisticated consumers end up with a product with a lower match-value ($\hat{\theta}^*$ goes down). Second, the single search that naive consumers conduct gets more costly. These two effects resemble the welfare effects in standard search models and are unambiguously detrimental to total welfare. Third, however, search costs also affect the quality

³¹To see the intuition behind the lemma, suppose to the contrary that effective search costs σ^* were decreasing in s . Hence, the share of candid firms λ^* would increase in s . By (26), this, first of all, would trigger an increase in candid prices, and second, because $\underline{p}^* = \bar{p}^* + \sigma^*$, the difference between deceptive and candid prices would shrink. For firms to nevertheless earn the same profit in the two segments after an increase in s , the demand that each candid firm receives from sophisticated consumers (ν^S/λ^*) would have to drop sharply. As it turns out, such a sharp increase of λ^* is impossible in equilibrium.

³²Our model displays the property that a naive consumer's belief about his expected welfare is correct, although he wrongly expects to purchase superior products at \underline{p}^* and deceptive products at \bar{p}^* . The reason is that he holds correct expectations about the average quality that he purchases and the average price that he pays.

provision in the market. Under condition (C), the share of candid firms goes up with search costs so that naive consumers, on average, consume the inefficient deceptive product less often. If the associated efficiency gains are sufficiently large, then an increase in search costs improves total welfare.

Firm profits and consumer welfare

We now take a closer look at the division of total welfare and ask how the welfare gains from an increase in search costs are distributed among consumers and firms. To address this question, it is key to understand how equilibrium prices evolve as search costs change, since it is prices which determine the division of welfare. The next proposition states both the effect of higher search costs on equilibrium prices as well as on individual welfare.

Proposition 5 *Suppose condition (C) holds. Then we have:*

- (o) *Candid and deceptive prices increase in search costs.*
- (i) *Firm profits increase in search costs.*
- (ii) *Sophisticated consumer welfare decreases in search costs.*
- (iii) *(True) naive consumer welfare increase in search costs if the difference in quality between superior and inferior products, Δq , is sufficiently large.*

Parts (i)-(iii) follow straightforwardly from part (o): The profit of a deceptive firm (and hence of any firm) increases, because a deceptive firm's price increases by (o) and its demand remains unchanged (ν^N). Sophisticated consumer welfare (6) unambiguously drops, because a sophisticated consumer suffers from higher candid prices and larger effective search costs so that $\hat{\theta}^*$ goes down (as shown in (90) in the appendix). A naive consumer benefits from the higher likelihood to encounter a candid firm, but suffers from higher market prices, the net effect being positive if the difference in quality between superior and inferior products, Δq , is sufficiently large.³³ Formally, naive consumer welfare (10) can be re-written with $\bar{p}^* = \underline{p}^* - s/\lambda^*$ as

$$\hat{U}^N = \underline{q} + \lambda^* \cdot \Delta q - \underline{p}^*. \quad (33)$$

Hence, \hat{U}^N increases in s if Δq is sufficiently large, because λ^* increases in s by Proposition 3.

While part (o) resembles the price comparative statics in standard search models, we emphasize that condition (C) is critical for it to hold. In fact, we present a counter example below. To

³³Note that (17-21) in Lemma 2 implies that the equilibrium market outcome is independent of Δq . Hence, Δq can be changed without affecting equilibrium variable such as λ^* and \underline{p}^* .

understand the intuition behind part (o), consider first candid prices. By (26), the effect of a change in search costs on candid prices is given as

$$\frac{d\bar{p}^*}{ds} = \frac{\partial \bar{p}^*}{\partial \sigma} \frac{d\sigma^*}{ds} + \frac{\partial \bar{p}^*}{\partial \lambda} \frac{d\lambda^*}{ds}. \quad (34)$$

Recall from (29) that the search effect $\partial \bar{p}^* / \partial \sigma$ is positive. This, together with Lemma 3, immediately implies that the first term in (34) is positive. This captures that in response to an increase in effective search costs, sophisticated consumers become less picky in order to save on search costs, and visit and compare less firms in expectation, which, in turn, softens competition so that the candid price increases. The second term in (34) represents a novel effect which is due to the fact that the share of candid firms is endogenous in our setting. In what follows, we refer to $\partial \bar{p}^* / \partial \lambda$ as the “demand composition” effect. Recall that a candid firm’s total demand is composed of the less elastic demand of naive consumers and the more elastic demand of sophisticated consumers. If the number of candid firms goes up, sophisticated consumers visit in expectation less firms until they find a satisfying superior product to purchase. As a consequence, there are in effect less sophisticated consumers present in the market and the share of naive consumers in a candid firm’s demand increases. Because the demand from naive consumers is less elastic than the demand from sophisticated consumers, prices increase. The demand composition effect is thus positive: $\partial \bar{p}^* / \partial \lambda \geq 0$.³⁴

We conclude that under condition (C), when the share of candid firms increases in s by Proposition 3, the second term on the right hand side of (34) is positive, and we obtain that candid equilibrium prices increase in s . Finally, note that because $\underline{p}^* = \bar{p}^* + \sigma^*$, Lemma 3 implies that if candid prices increase in s , so do deceptive prices.

We conclude this section with a remark on the price comparative statics when condition (C) is violated. In this case, the share of candid firms goes down in s , so that the second term on the right hand side of (34) is negative. We now argue that this may imply that candid prices actually decrease in search costs. As with Proposition 3, the size of the search effect depends on the steepness of the hazard rate and is zero in our example with (locally) constant hazard rate. Because in our example, the share of candid firms decreases in s , we thus infer that candid prices decrease in s .³⁵

³⁴Formally, by (26), we have $\partial \bar{p}^* / \partial \lambda = v^N / (v^S \cdot h(g^{-1}(\sigma^*))) > 0$.

³⁵When candid prices fall in search costs, the question arises whether this may lead to increasing sophisticated consumer welfare, or decreasing deceptive prices and profits. As it turns out, this is not the case even in the exponential example where the price decline of superior products induced by increasing search costs is extreme (because the hazard rate is constant). This suggests that in general, when this price decline is more moderate, welfare does not increase and profits do not decrease in search costs. We were, however, not able to verify this.

Example (ctd.) Let $f = f^E$. Then there is an open set of parameters $s, \bar{q}, \underline{q}, \bar{c}, \underline{c}, \nu^N$ so that a segmented market equilibrium exists with $\hat{\theta}^* < 0$ and

$$\bar{p}^* - \bar{c} = \frac{\Delta c - s \frac{\nu^N}{\nu^S}}{\Delta c - 1}. \quad (35)$$

Thus, candid prices are decreasing in search costs.

5 Effects of changes in naivete

This section addresses the question how (the division of) welfare depends on the number of naive consumers in the market. As outlined in the introduction, this question has received some attention in the literature, and is relevant to evaluate “transparency policies” that aim at fostering educating naive consumers.

Our next result confirms the intuition that as the number of naive consumers drops, the deceptive business model, which targets naive consumers, becomes less attractive so that the share of deceptive firms goes down. This is consistent with the existing literature which shows that the range of equilibria in which deception occurs expands in the number of naive consumers (Gabaix and Laibson (2006), Armstrong and Vickers (2012), Heidhues et al. (2017)). If the deceptive product is sufficiently inefficient, this will have the intuitive implication that welfare improves.

Proposition 6 (i) *The share of candid firms decreases in the fraction of naive consumers.*

(ii) *Total welfare decreases in ν^N if*

- (a) *a superior product is more efficient than a inferior product irrespective of match values: $\Delta q - \Delta c > \bar{\theta} - \underline{\theta}$, or*
- (b) *the efficiency gains from purchasing a superior product exceed the effective search costs of finding one: $\Delta q - \Delta c > \sigma^*$.*

While seemingly obvious, the complete argument for why an increase in the share of naive consumers results in a decrease in the share of candid firms is not entirely straightforward. The reason is that apart from a direct effect (more consumers are susceptible to deception), an indirect price effect works in the opposite direction. Precisely, as ν^N increases, a candid firm serves relatively more naive consumers whose demand is inelastic and, all else equal, the firm thus increases its price. This in turn implies that, all else equal, deceptive firms increase their prices by the same amount, because $\underline{p}^* = \bar{p}^* + \sigma^*$. However, the resulting overall increase in prices benefits candid

firms more than deceptive ones, as candid firms serve more consumers in equilibrium. This indirect effect thus makes offering superior products more attractive. As it turns out, the direct effect dominates the indirect effect, and overall, a firm's incentive to become deceptive indeed increases as ν^N increases.

As to part (ii), observe first that as ν^N goes up, each naive and each sophisticated consumer generates less surplus: A naive consumer is simply more likely to end up with an inefficient deceptive product due to (i). A sophisticated consumer, on the other hand, faces larger effective search costs due to (i) and consequently buys a less suitable product on average ($\hat{\theta}^*$ goes down). Welfare thus decreases in ν^N if, in addition, the surplus an additional naive consumer generates is lower than that generated by the sophisticated consumer he replaces. The sufficient conditions stated in parts (a) and (b) of part (ii) ensure that this is the case.³⁶

A prominent theme in the literature is how the welfare changes in response to a change in the number of naive consumers are distributed among sophisticated and naive consumers.³⁷ A novel negative spillover effect from naive to sophisticated consumers that arises in our framework is that an increase in the number of naive consumers increases the equilibrium effective search costs $\sigma^* = s/\lambda^*$ so that sophisticated consumers have to search longer to find a candid firm. This follows from part (i) of Proposition 6.

Clearly, consumer welfare does not only depend on effective search costs, but also on prices. As explained below, the direction in which equilibrium prices move as the number of naive consumers increases, is, in general, ambiguous. In the next proposition, however, we provide a sufficient condition so that both candid and deceptive prices increase in the number of naive consumers. This condition can be seen as a regularity condition which ensures that candid prices are well-behaved in the sense that candid prices drop when the share of candid firms (exogenously) increases. This, in turn, will imply that candid prices increase in the number of naive consumers and deliver clear-cut effects on individual welfare.

Proposition 7 *Let*

$$\epsilon(\sigma^*) \geq 1. \tag{36}$$

³⁶Notice that by a revealed preference argument, a sophisticated consumer obtains, in equilibrium, a larger utility than a naive consumer. This, however, does not imply that a sophisticated consumer generates more additional surplus, because a naive consumer generates a larger profit for firms.

³⁷In Gabaix and Laibson (2006) and Armstrong and Vickers (2012), sophisticated consumers benefit from being cross-subsidized by naive ones, an effect that is absent in our model. In Heidhues et al. (2017), naive consumers harm sophisticated ones, because their presence pushes up prices and increases deception in the market. In Armstrong and Chen (2009), Kosfeld and Schüwer (2017) and Ispano and Schwardmann (2018) both naive and sophisticated consumer welfare might be non-monotone in the number of naive consumers.

Then, as the share of naive consumers increases,

- (i) candid and deceptive equilibrium prices increase;
- (ii) firm profits increase;
- (iii) sophisticated and naive consumer welfare decreases.

Part (ii) and (iii) follow straightforwardly from part (i) and our previous considerations: As ν^N increases, a deceptive (and hence a candid) firm's profit grows, because it sells deceptive products at a higher price to a larger number of naive consumers. Sophisticated consumer welfare (6) goes down, because a sophisticated consumer suffers from higher candid prices by (i) and larger effective search costs as discussed earlier. Finally, naive consumer welfare (33) falls, because a naive consumer is more likely to end up with a deceptive product and has to pay more.

The more involved part is (i). By (26), the overall effect of an increase in the share of naive consumers on candid prices consists of various effects, and is formally given by

$$\frac{d\bar{p}^*}{d\nu^N} = \frac{\partial\bar{p}^*}{\partial\nu^N} + \frac{d\bar{p}^*}{d\lambda} \cdot \frac{d\lambda^*}{d\nu^N}. \quad (37)$$

To disentangle this expression, observe first that the direct effect $\partial\bar{p}^*/\partial\nu^N$ is unambiguously positive, because the demand of naive consumers is less elastic than the demand of sophisticated ones. Second, $d\lambda^*/d\nu^N < 0$ by Proposition 6. Therefore, (37) is positive if candid prices drop in response to a larger share of candid firms, that is, $d\bar{p}^*/d\lambda \leq 0$. Whereas the sign of $d\bar{p}^*/d\lambda$ is, in general, not clear-cut, the condition that $\epsilon(\sigma^*) \geq 1$ turns out to be a sufficient condition for $d\bar{p}^*/d\lambda \leq 0$.³⁸ Finally, observe that deceptive prices increase in ν^N if candid prices do, because $\underline{p}^* = \bar{p}^* + \sigma^*$ and σ^* increases in ν^N , since λ^* decreases in ν^N by Proposition 6.

6 Shrouding

While the previous results suggest that policies that educate naive consumers improve welfare, this section discusses the question whether firms themselves have incentives to educate naive consumers by unshrouding the hidden (bad) attributes of the deceptive products in the market. A key point in Gabaix and Laibson (2006) and Heidhues et al. (2017) is that equilibria can be sustained in which all firms are deceptive, even when a single deviating firm could educate all

³⁸The underlying reason is that, analogously to the considerations in Proposition 5, an increase in the share of candid firms, first, reduces effective search costs, pushing prices down through the search effect, but, second, affects the composition of the demand of candid firms, as sophisticated consumers visit in expectation less firms, pushing prices up through the demand composition effect. The first effect dominates if ϵ is sufficiently large.

naive consumers to become sophisticated. The reason is that, given the equilibrium pricing of rival firms, a sophisticated consumer can only be attracted when charged below marginal cost. In our framework, if a single firm unshrouds and educates all naive consumers, then the entire demand of otherwise naive consumers is split equally only among candid firms. Hence, each candid firm would want to unshroud, because selling superior goods to sophisticated consumers is profitable.³⁹

In our search framework, however, it seems natural to assume that a firm can only educate those consumers who actually visit the firm. As we argue next, in this case, no firm would educate visiting naive consumers. Suppose that any firm had the opportunity to unshroud so that together with its quality choice, it can also choose whether or not to turn any visiting naive consumer into a sophisticated one by educating him. We may think of two scenarios here. In the first scenario, the firm can only unshroud yet not adjust its price. While it is obvious that no deceptive firm has an interest to unshroud, this is also true for any candid firm,⁴⁰ because naive consumers, in contrast to sophisticated ones, purchase the firm's product in any case at the equilibrium price, that is, irrespective of product fit. A "business stealing effect" that could render unshrouding profitable is thus absent in our model, as unshrouding does not generate any additional demand for candid firms from otherwise naive consumers. This logic carries over to the second scenario in which the firm can unshroud and charge a price different from the equilibrium price. Intuitively, if a candid firm unshrouds so that all its customers are sophisticated, then the firm would optimally lower its price, as the demand from sophisticated consumers is more elastic than the demand from naive consumers. However, given a price lower than the equilibrium price, a candid firm cannot benefit from unshrouding, because at that price a naive consumer, in contrast to sophisticated consumer, would purchase in any case.

On a related note, recall that by Proposition 7 industry profits increase with the number of naive consumers so that by unshrouding a firm may harm its profits in the "long-run". Similarly, Proposition 7 implies that it is in his own best interest for a sophisticated consumer to educate his naive peers, as naive consumers exert a negative externality on him.⁴¹

³⁹In fact, this is a lower bound on the profits that a deviating candid firm could obtain, as it could also change its price in response to its decision to unshroud.

⁴⁰Note that this implies that no deceptive firm has an interest in unshrouding and becoming candid at the same time.

⁴¹We thank Roland Strausz for pointing this out.

7 Policy implications

In this section, we turn to the normative implications of our analysis and discuss policy interventions that may, or may not, improve (consumer) welfare in our framework.

Search costs

Our analysis implies that policy interventions that aim at lowering search costs may backfire. As shown in Proposition 4, the welfare benefits of markets with lower search frictions such as online markets are ambiguous, as they might increase the incentives for firms to become deceptive.

A key measure that aims at lowering search costs in practice is the standardization of price and product information so as to make it easier for consumer to evaluate prices and products. The harmonization of price information is a common regulatory tool in the EU for financial services (see the discussion in Piccione and Spiegler (2012)) or for grocery products (supermarkets are required to quote the price per 100g).

In practice, search frictions are also reduced by measures that promote e-commerce (such as lowering approval standards to register an online business). Moreover, certain information disclosure duties imposed on firms may reduce search costs if they improve consumers' access to product and price information.⁴² Finally, search costs may also be reduced through measures that increase the number of firms in the market, e.g. by reducing the barriers of market entry, because this reduces the time to find (or the distance between) competitors.

Naivete

The welfare implications of policy interventions that aim at reducing the number of naive consumers are described in Proposition 6 and 7. Together with the result in the literature (see footnote 37), our results support a conclusion that such policy interventions are beneficial for total welfare, but their effects on the distribution of individual welfare are context-dependent.

Naivete can be reduced by information or education campaigns (e.g. fostering financial literacy in the context of banking services). If naivete is interpreted more broadly as a form of limited attention at the point of purchase, mandatory product labels which categorize products in eye-catching ways (“red”, “yellow”, “green”) and trigger consumer awareness can be seen as a way to reduce naivete.⁴³

⁴²Information disclosure duties may, however, also make naive consumers aware of the deceptive nature of a product.

⁴³Since boundedly rational consumers may have difficulties to process information correctly, such measures may, however, backfire. For example, categories such “green” or “red” may make consumers excessively negligent resp. diligent. Moreover, product labels are also likely to affect search costs. A comprehensive analysis of these issues requires a more detailed model of consumer naivete and is beyond the scope of this paper.

Price regulation

At a more fundamental level, what ultimately causes welfare losses in our framework is that the presence of naive consumers guarantees positive profits from the deceptive business model which makes price competition dysfunctional. This raises the question whether regulating prices directly may be beneficial in our framework.⁴⁴

To address this question, we consider a situation in which the regulator can impose a price floor \bar{p}_F , and we argue that a price floor exceeding the laissez-faire equilibrium price \bar{p}^* can improve total and consumer welfare. To keep the analysis simple, we consider the effect of an exogenously imposed (marginal) increase of the candid equilibrium price from \bar{p}^* to $\bar{p}_F = \bar{p}^* + dp$ for small $dp > 0$. If such an intervention is welfare improving, then it suggests that it is optimal to impose a price floor above the equilibrium price.⁴⁵ By the equal profit requirement (24), the share of candid firms that balances the profits in the two segments given an arbitrary candid price \bar{p} is

$$\lambda = \frac{\nu^S \cdot (\bar{p} - \bar{c}) - \nu^N \cdot s}{\nu^N \cdot \Delta c}. \quad (38)$$

The equilibrium effect of a price floor on the share of candid firms is then given by the effect of an increase of the equilibrium price \bar{p}^* to $\bar{p}_F = \bar{p}^* + dp$. A price floor thus improves the quality provision in the market, as λ increases in \bar{p} in (38). Intuitively, a candid firm has a larger demand than a deceptive firm so that as prices increase, the candid business model gets more profitable. This increase in the share of candid firms benefits both sophisticated and naive consumers, as the former face lower effective search costs and the latter are more likely to purchase a superior (more efficient) product. A price floor thus improves total welfare, because paid prices are merely a transfer from the point of view of total welfare. Moreover, if the difference between superior and inferior quality, Δq , is sufficiently large, then a price floor also improves total consumer welfare. The intuition is that if Δq is sufficiently large, then the gains of naive consumers (who are more likely to purchase a superior product) outweigh the losses (of all consumers) due to higher prices.

Lemma 4 *Imposing a price floor $\bar{p}_F = \bar{p}^* + dp$ always improves total welfare, and improves consumer welfare if the difference in quality between superior and inferior products, Δq , is sufficiently large.*

Minimum quality standard

⁴⁴We thank Tymon Tatur for raising this question.

⁴⁵It is best to consider our exercise as a first heuristic step of a more rigorous analysis that makes the (realistic) assumption that the regulator lacks relevant knowledge, e.g. about sellers' costs. The regulator then sets a price floor so as to maximize some objective, e.g. expected consumer welfare, anticipating the market outcome that arises under a price floor regulation. Our analysis for a given cost realization suggests that an optimal price floor will be binding for low cost realizations.

If the regulator can observe quality directly, then imposing a minimum quality standard which requires firms to produce the superior product, and thus effectively bans the deceptive product, would (clearly) improve total and consumer welfare. If we interpret an inferior product as a product with a hidden add-on fee, this corresponds to banning this fee and is consistent with some regulatory policies that have been adopted in practice.⁴⁶

However, in practice, a minimum quality standard may sometimes be only effective in preventing the most extreme forms of deception, because firms may, in response, design new deceptive products that satisfy the standard but still provide inefficiently low quality and allow firms to charge relatively high mark-ups to naive consumers. In a worst case scenario, a standard only slightly improves the quality of a deceptive product, but does not improve the efficiency of a deceptive product at all. This raises the issue whether, in this case, a minimum quality standard is still welfare improving.

To explore this issue, suppose that a minimum quality standard $q_M \in (\underline{q}, \bar{q})$ with $dq = q_M - \underline{q} > 0$ is in place and firms can respond by offering either the superior product of quality \bar{q} as before or a deceptive product of quality $q_M > \underline{q}$ which is equally inefficient than the inferior product that has been banned by the standard. That is, production costs for the new deceptive quality q_M are $c_M = \underline{c} + (q_M - \underline{q})$. This guarantees that any welfare enhancing effects of a minimum quality standard do not simply stem from a higher efficiency of deceptive products. Note that if we interpret a deceptive product as a product with a hidden add-on fee, this corresponds to marginally decreasing the (maximal) add-on fee that a firm is allowed to charge (which does not alter the efficiency of the product). The next lemma shows that in this scenario, imposing a minimum quality standard, which is not too large, still improves total welfare and, depending on the mark-up elasticity with respect to search costs, may improve consumer welfare.⁴⁷

Lemma 5 *For sufficiently small $dq = q_M - \underline{q} > 0$, imposing a minimum quality standard $q_M = \underline{q} + dq$, with $c_M = \underline{c} + dq$, always improves total welfare, and improves consumer welfare if*

$$\epsilon(\sigma^*) \geq 1. \quad (39)$$

The intuition behind the lemma is straightforward. A minimum quality standard implies that a deceptive firm can save dq less on marginal production costs by producing the deceptive qual-

⁴⁶For instance, within the EU, consumers are entitled to reimbursement of add-on payments that are not explicitly agreed upon in the initial contract (see Article 22 of the Consumer Rights Directive 2011/83/EU). This can be interpreted as a ban of deception through add-on pricing. Likewise, after the financial crisis, financial supervisory bodies in the EU have been granted the authority to ban “complex” derivatives.

⁴⁷Note however that in a search environment, minimum quality standards or add-on price caps may have adverse effects on consumer welfare, as they may reduce the consumer’s incentives to engage in costly search. See Fershtman and Fishman (1994), or Armstrong et al. (2009).

ity, but since it is still shunned by sophisticated consumers, it cannot attract additional demand through the higher quality. The deceptive business model thus becomes less attractive and the share of candid firms increases as a result of a minimum quality standard. Recall that this improves total welfare, because effective search costs drop and naive consumers are more likely to purchase the superior (more efficient) quality. Moreover, the same reasoning implies that consumer welfare increases if, in addition, candid and deceptive prices fall in response to a minimum quality standard. Now, if candid prices fall, then deceptive prices fall, because $\underline{p}^* = \bar{p}^* + s/\lambda^*$ and λ^* increases in response to a minimum quality standard as argued before. Candid prices in turn fall if they are well-behaved, and the increase in the share of candid firms results in lower candid prices, that is, if $d\bar{p}^*/d\lambda < 0$ holds. As has been argued after Proposition 7, this is the case if $\epsilon(\sigma^*) \geq 1$ holds.

8 Discussion

In this section, we address the robustness of our findings to variations in how consumer naivete is modeled. First, we consider naive consumers who are merely uninformed but otherwise rational and make correct inferences from observed quality. Second, we consider the case that the naive consumers, like in our base model, fail to make correct inferences from observed prices but observe a noisy signal either about product quality or product fit. We will argue that in the first model variation, a segmented market equilibrium fails to exist, highlighting that our results are difficult to reconcile with rational consumer behavior. For the second model variation, we show that our central result that superior products are driven out of the market if search frictions vanish still holds if the signal is sufficiently noisy. Therefore, the fact that naive consumers cannot assess products at all in our base model should be viewed as a simplifying assumption. To establish these results, we assume in this section that inferior products are not only inefficient but strictly inefficient, that is, $\Delta q > \Delta c$.

Rational but uninformed consumers

To study the role of naivete modeled in the sense of analogy based reasoning for the existence of a segmented market equilibrium, suppose that instead of naive consumers there is a fraction of uninformed consumers who can neither observe product quality nor fit but are otherwise fully rational. Then, upon observing a firm's price, an uninformed consumer forms a belief $\hat{\lambda}(p)$ about the probability that the firm offers high quality. In (a Perfect Bayesian) equilibrium, this belief is consistent with the pricing strategies of firms, but can be arbitrary for prices "off the path" which are not chosen by firms in equilibrium. We impose the restriction that $\hat{\lambda}(p)$ is continuous so that

a small change in a product's price does not result in a large change in a consumer's belief.⁴⁸ The following lemma shows that there exists no segmented market equilibrium.

Lemma 6 *Suppose there is a fraction of uninformed (but rational) consumers who hold continuous beliefs. Then there is no segmented market equilibrium.*

The intuition behind the lemma is that a separating equilibrium does not exist, because otherwise uninformed consumers would recognize deceptive products as what they are so that firms would have incentives to offer the efficient quality. On the other hand, a pooling equilibrium does not exist, because in such an equilibrium, uninformed consumers believe that all firms offer the same average quality at the same price. Hence, an uninformed consumer has strict incentives to buy at the current firm instead of continuing to search. As a consequence, because consumer beliefs are continuous, an uninformed consumer would still purchase from the current firm if it slightly increases its price. However, since a deceptive firm only serves uninformed consumers in a segmented market equilibrium, it would therefore be profitable for the firm to slightly increase its price.

Noisy signals about product characteristics

To study the question how our results change when naive consumers have more information about product characteristics, we consider first the case that a naive consumer observes a noisy signal about product quality if he visits a store but neither observes product fit nor makes correct inferences from observed prices. If naive consumer i inspects firm k , then he receives a signal $z_{ik} \in \{\underline{z}, \bar{z}\}$ with $\text{Prob}(\bar{z}|\bar{q}) = \text{Prob}(\underline{z}|\underline{q}) = 1/2 + \varepsilon$ where the parameter $\varepsilon \in [0, 1/2]$ measures the signal's precision. Signals are conditionally independent across consumers, conditional on the firm's quality. The following lemma shows that if ε is sufficiently small, then our central insight from Proposition 2 carries over, that is, as search costs vanish, candid firms are driven out of the market.

Lemma 7 *Suppose that naive consumers observe a noisy signal about product quality. Then, if the signal is sufficiently noisy (ε sufficiently small), the share of candid firms in the market vanishes as search costs vanish: $\lim_{s \rightarrow 0} \lambda^* = 0$.*

Intuitively, if the signal is too informative and (almost) fully reveals the product's quality to a naive consumer, then there does not exist an equilibrium in which firms offer the deceptive product. The reason is that otherwise there are efficiency gains from trading the efficient superior quality which could be split between the firm and its customers, because all consumers recognize products as

⁴⁸Absent any belief restriction, there exists a large number of pooling equilibria which rely on the consumers' belief that a firm offers inferior quality whenever it deviates from the equilibrium price.

what they are.⁴⁹ If the signal about product quality is, however, sufficiently noisy, then some firms offer the deceptive quality but lower their prices slightly (in comparison to the market outcome in our basic model) such that still all naive consumer, including those that receive a bad signal about the product's quality, purchase its product. This slightly reduces prices and profits in the deceptive segment, yet the equilibrium remains to be a segmented market equilibrium. Moreover, deceptive firms sell small volumes at a large mark-up whereas candid firms sell large volumes at small mark-ups. Loosely speaking, this implies that the intuition of Proposition 2 carries over: as search costs vanish, mark-ups in the candid segment erode and must be off-set by an unbounded increase in demand for candid firms, because the profits in the deceptive segment are bounded away from zero.

In a similar way, it can be shown that our main result carries over if naive consumers receive a sufficiently noisy signal about product fit. To see this, note first that Proposition 2 is driven by the fact mark-ups in the deceptive segment remain bounded away from zero as search costs vanish. This remains to be the case if the signal is sufficiently noisy, because a deceptive price must then still exceed the candid price. Otherwise, consider a naive consumer at a deceptive and (market-wide) cheapest firm who has observed the "worst" signal about product fit. The best this naive consumer can hope for by continuing search is to find, again, a cheapest (deceptive) firm, however, with the best signal about product fit. But if the signal is sufficiently noisy, the maximum gain in (expected) product fit is smaller than the costs to conduct this search. Therefore, the consumer would strictly prefer to buy at the current firm, which would induce it to raise its price. Thus, the cheapest price in the market cannot be charged by a deceptive firm, and the mark-up in the deceptive segment is still bounded away from zero as it exceeds Δc .

On a related note, if naive consumers can observe product fit sufficiently well, then two separate markets for naive and sophisticated consumers emerge: Naive consumer search for a suitable (cheap) inferior product and ignore superior products, as they consider these products as too expensive, whereas sophisticated consumers search for a suitable (expensive) superior product and ignore deceptive products, as they recognize these products as what they are. In such a situation, the interaction between the two market segments, which is at the heart of our analysis, is rather limited, as the share of candid firms only influences the respective effective search costs in the two segments.

⁴⁹More precisely, the firm could offer the superior instead of the inferior quality and raise its mark-up by an amount equal to the efficiency gains. As a naive consumer recognizes products as what they are, the firm still offers the same utility to the consumer (and hence loses no demand) but obtains a larger mark-up.

9 Conclusion

In this paper, we integrate consumer naivete in a search market model. The key idea is that the presence of naive consumers prevents competitive forces to unfold, because, simply put, producing inferior products which are purchased by naive consumers is a safe profit haven. In fact, it is precisely in seemingly competitive markets with low search frictions where sellers have strong incentives to adopt the deceptive business model.

In our framework, deception entails that a naive consumer misperceives product differences as excessively small. An interesting avenue for future research is the general question to what extent firms have incentives to manipulate consumer perceptions. This may concern not only their own products but also the products of competitors. For example, it is conceivable that a sales person (of a low quality firm) may emphasize the importance of product fit in order to subsequently convince a naive consumer that the product he offers meets the consumer's need better than any other product in the market. As a result, a naive consumer would misperceive product differences as excessively large. In this example, such a misperception reduces a naive consumer's incentive to search, and deceiving consumers by emphasising spurious product differences would constitute a way to gain market power over naive consumers. In other situations, a sales person may argue that all firms offer more or less identical products in order to discourage a naive consumer from visiting a competitor.

A Appendix

Lemma A.1 (i) g is strictly decreasing on $(-\infty, \bar{\theta})$ with $g(-\infty) = \infty$, $g(\underline{\theta}) = -\underline{\theta}$, and $g(\bar{\theta}) = 0$, and $g(z) = 0$ for all $z \geq \bar{\theta}$. Moreover, $g' = -(1 - F)$.

(ii) The inverse of g , denoted by g^{-1} , is well-defined and strictly decreasing on the domain $(0, \infty)$. By convention, we define $g^{-1}(0) = \bar{\theta}$. Note also that $g^{-1}(-\underline{\theta}) = \underline{\theta}$.

Proof of Lemma A.1 To see (i), note that for all $z < \bar{\theta}$, we have $g'(z) = -(1 - F(z))$, so that g is strictly decreasing on $(-\infty, \bar{\theta})$. The values of g stated in the lemma follow straightforwardly by plugging in the respective arguments. Part (ii) is an immediate consequence of (i). ■

Proof of Lemma 1 The profit of a firm is given as the product of (a) the mass of consumers who visit the firm in its lifetime, (b) the probability that a visitor actually purchases the good, and (c) the firm's mark-up ($p_k - c$).

As to (a), in a segmented market equilibrium, all naive consumers buy in the first period and leave the market afterwards. Thus, the mass of naive consumers who visit a given firm in its lifetime is equal to ν^N .

A sophisticated consumer leaves the market, in a segmented market equilibrium, if he is matched with a candid firm, and the match-value exceeds $\hat{\theta}$. This occurs with probability $\lambda \cdot (1 - F(\hat{\theta}))$. Because the mass of sophisticated consumers who visit a given firm in period t is equal to the mass of sophisticated consumers who have not left the market prior to t , this mass is given as $\nu^S \cdot [1 - \lambda(1 - F(\hat{\theta}))]^t$. Hence, the (expected) mass of sophisticated consumers who visit a given firm in its lifetime is equal to

$$\kappa = \nu^S \sum_{t=0}^{\infty} [1 - \lambda(1 - F(\hat{\theta}))]^t = \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}. \quad (40)$$

As to (b), a visiting sophisticated consumer i buys from firm k if $q_k + \theta_{ik} - p_k \geq \hat{U}^S$. By (6), this is the case if

$$q_k + \theta_{ik} - p_k \geq \bar{q} + \hat{\theta} - \bar{p} \iff \theta_{ik} \geq \hat{\theta} + (p_k - \bar{p}) - (q_k - \bar{q}). \quad (41)$$

Moreover, all visiting naive consumers buy as long as $p_k \leq \hat{p}$. A firm k that sets q_k and $p_k \leq \hat{p}$ therefore earns the profit

$$\pi(q_k, p_k) = \nu^N \cdot (p_k - c(q_k)) + \kappa \cdot [1 - F(\hat{\theta} + (p_k - \bar{p}) - (q_k - \bar{q}))] \cdot (p_k - c(q_k)). \quad (42)$$

Inserting (40) delivers (14). Finally, if a firm deviates to a price $p_k > \hat{p}$, it loses the profits $\nu^N(p_k - c(q_k))$ it would otherwise make from naive consumers.

The expressions for equilibrium profits in (15) follow immediately by inserting equilibrium prices \bar{p} and \underline{p} and the respective qualities \bar{q} and \underline{q} in (14), taking into account that $\underline{p} \leq \hat{p}$ in a segmented market equilibrium. ■

Lemma A.2 *Let $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ and $\lambda \in (0, 1)$ be given, and let*

$$\bar{p} = \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(\hat{\theta})} + c, \quad p_L = (\underline{\theta} - \hat{\theta}) + \bar{p}, \quad p_H = (\bar{\theta} - \hat{\theta}) + \bar{p}. \quad (43)$$

Then the function $\Pi(\cdot, \bar{q}, c)$ as defined in (13) is maximized on the domain (p_L, p_H) by $p^ = \bar{p}$.*

Proof of Lemma A.2 Notice first that $p^* = \bar{p}$ is indeed in (p_L, p_H) as $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$. The necessary first order condition for $p^* = \bar{p}$ to maximize $\Pi(\cdot, \bar{q}, c)$ is

$$0 = \frac{\partial \Pi(p^*, \bar{q}, c)}{\partial p} = \nu^N + \frac{\nu^S}{\lambda} - \frac{\nu^S}{\lambda} \cdot \frac{f(\hat{\theta})}{1 - F(\hat{\theta})} \cdot (p^* - \bar{c}). \quad (44)$$

By (43), $p^* = \bar{p}$ therefore satisfies the first order condition.

To see that $p^* = \bar{p}$ is a global maximizer, we show that $\Pi(\cdot, \bar{q}, c)$ is quasi-concave on (p_L, p_H) . In fact, we show the stronger property that $\Pi(\cdot, \bar{q}, c)$ is log-concave. To see this, let $\tau = \tau(p) = \hat{\theta} + (p - \bar{p})$ and define

$$D(p) \equiv \nu^N + \frac{\nu^S}{\lambda} \cdot \frac{1 - F(\tau)}{1 - F(\hat{\theta})}. \quad (45)$$

Observe that $\Pi(p, \bar{q}, c) = D(p) \cdot (p - c)$, and recall that the product of log-concave functions is log-concave. Because $(p - c)$ is log-concave in p , $\Pi(\cdot, \bar{q}, c)$ is therefore log-concave if D is log-concave. To show this, assume to the contrary that there is a p so that $(\log D)''(p) > 0$. As we show below,

$$(\log D)''(p) > 0 \quad \Rightarrow \quad (\log D)''(p_H) > 0. \quad (46)$$

The right inequality in turn is equivalent to⁵⁰

$$D''(p_H) \cdot D(p_H) - D'(p_H)^2 > 0. \quad (47)$$

With $\alpha \equiv \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}$, we have

$$D'(p) = -\alpha f(\tau), \quad \text{and} \quad D''(p) = -\alpha f'(\tau). \quad (48)$$

Because $D' < 0$ and $D > 0$, dividing (47) by $D'(p_H) \cdot D(p_H)$ and re-arranging terms yields

$$\frac{f'(\bar{\theta})}{f(\bar{\theta})} < -\frac{\alpha f(\bar{\theta})}{\nu^N + \alpha(1 - F(\bar{\theta}))} = -\frac{\alpha f(\bar{\theta})}{\nu^N} < -\frac{\nu^S}{\nu^N} f(\bar{\theta}), \quad (49)$$

⁵⁰Note that $\frac{d^2}{dp_k^2} \log D = \frac{d}{dp_k} (D'/D) = 1/D^2 \cdot (D''D - D'^2)$.

where the final inequality follows from $\alpha > \nu^s$. But this inequality contradicts (16) which establishes that D is log-concave, and hence $\Pi(\cdot, \bar{q}, c)$ is log-concave as desired.

To complete the proof, we have to show (46). By (48) and Footnote 50, we have

$$(\log D)''(p) = \frac{D'(p)}{D(p)} \cdot \left[\frac{f'(\tau)}{f(\tau)} - \frac{D'(p)}{D(p)} \right]. \quad (50)$$

Note that $\frac{D'(p)}{D(p)}$ is negative by (48). Therefore, if the term in the square bracket is decreasing at p whenever $(\log D)''(p)$ is positive, then $(\log D)''(p)$ remains positive once it is positive, and (46) follows. To see that the term in the square brackets is indeed decreasing at p , note that $(\log D)''(p) > 0$ implies that $(\log D)'(p) = \frac{D'(p)}{D(p)}$ is increasing at p , and our assumption that f is log-concave implies that $\frac{f'(\tau)}{f(\tau)}$ is decreasing at p . ■

Proof of Lemma 2 “ \Leftarrow ”: Let $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ and $\lambda^* \in (0, 1)$ be given and suppose (17–21) hold. We first show that consumer behavior is as required in a segmented market equilibrium. By (22), we have $\underline{p}^* = \bar{p}^* + s/\lambda^*$. Condition (12) is thus met. Therefore, naive consumers purchase both superior and deceptive products, and (18) re-states their optimal search rule (11). Also condition (9) is met, because $\underline{p}^* > \bar{p}^*$ by (22) and $\bar{\theta} - \hat{\theta}^* < \Delta q$ where the latter inequality follows from $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ and assumption (4). Therefore, sophisticated consumer indeed prefer not to purchase deceptive products, and (17) re-states their optimal search rule (8).

Consequently, because consumer behavior conforms with a segmented market equilibrium, a firm’s profit function is given by Lemma 1. What remains to be shown is that (\bar{q}, \bar{p}^*) and $(\underline{q}, \underline{p}^*)$ with \bar{p}^* and \underline{p}^* as defined in (19) and (22) are indeed profit maximizing strategies for firms. Notice that by (21) firms earn equal profits in the two segments so that it suffices to show that $p_k = \bar{p}^*$ is profit maximizing given \bar{q} , and $p_k = \underline{p}^*$ is profit maximizing given \underline{q} , as in this case, (21) ensures that a deviation to the other segment is not profitable.

We begin with a candid firm with $q_k = \bar{q}$. By Lemma 1, we have that

$$\pi(\bar{q}, p_k) \leq \Pi(p_k, \bar{q}, \bar{c}) \quad \text{for all } p_k, \quad \text{and} \quad \pi(\bar{q}, \bar{p}^*) = \Pi(\bar{p}^*, \bar{q}, \bar{c}). \quad (51)$$

Hence, Lemma A.2 implies that \bar{p}^* is profit maximizing within the price range $[p_L, p_H]$. Hence, it remains to show that a candid firm cannot improve by choosing a price outside of $[p_L, p_H]$.

Consider first a price $p_k < p_L$. At such a price, any sophisticated and naive consumer for any match-value realization strictly prefers purchasing at the current firm k to visiting the next firm. Thus, the firm’s demand would be locally inelastic, and it would be profitable for the firm to slightly increase its price. Hence, $p_k < p_L$ is not optimal.

Consider next a price $p_k > p_H$. At such a price any sophisticated consumer for any match-value realization strictly prefers visiting the next firm to purchasing at the current firm k , and the firm at most sells to naive consumers. So suppose that a naive consumer indeed purchases a superior

product (and hence also a deceptive product) at p_k such that the firm makes profits of $v^N \cdot (p_k - \bar{c})$. We infer:

$$\pi(\bar{q}, p_k) = v^N \cdot (p_k - \bar{c}) < v^N \cdot (p_k - \underline{c}) = \pi(\underline{q}, p_k) \leq \pi(\underline{q}, \underline{p}^*) = \pi(\bar{q}, \bar{p}^*), \quad (52)$$

where the second inequality follows from $\bar{c} > \underline{c}$, and the third equality follows because at p_k a deceptive firm only derives demand from naive consumers, as a sophisticated consumer, who does not purchase a superior product at p_k , neither purchases a deceptive product at p_k . The fourth inequality follows, as \underline{p}^* is optimal given \underline{q} (which we show next), and the final equality follows from (21). This proves that it is optimal for a candid firm to charge \bar{p}^* .

To see that it is optimal for a deceptive firm to charge \underline{p}^* , consider first a price $p_k > \underline{p}^*$. Because condition (9) is met in (the candidate) equilibrium, a deceptive firm also derives no demand from sophisticated consumers when it charges a larger price $p_k > \underline{p}^*$. Moreover, because $\underline{p}^* = \hat{p}^*$ by (18), $p_k > \underline{p}^*$ implies $p_k > \hat{p}^*$, so that the firm neither derives demand from naive consumers. Consequently, a deviation to $p_k > \underline{p}^*$ is not profitable.

Next, consider $p_k < \underline{p}^*$. Observe that because the demand of naive consumers is inelastic up to the price \hat{p}^* and $\underline{p}^* = \hat{p}^*$ by (18), setting a price $p_k < \underline{p}^*$ can only be profitable if it generates additional demand from sophisticated consumers, that is, if $p_k < p_H - \Delta q$.⁵¹ Now, at very low prices $p_k < p_L - \Delta q$, any visiting naive and sophisticated consumer would purchase the firm's product so that the firm's demand would be perfectly inelastic. Accordingly, it would be profitable to raise prices. Hence, $p_k < p_L - \Delta q$ is not optimal.

Thus, it remains to consider prices $p_k \in [p_L - \Delta q, p_H - \Delta q]$. Because $p_k < \hat{p}^*$, it holds that

$$\pi(\underline{q}, p_k) = \Pi(p_k, \underline{q}, \underline{c}) \leq \Pi(p_k, \underline{q}, \bar{c} - \Delta q), \quad (53)$$

where the second inequality follows, since $\Pi(p, q, c)$ is decreasing in c and $\Delta q \geq \Delta c$ by (3). From definition (13), we infer that

$$\Pi(p_k, \underline{q}, \bar{c} - \Delta q) = \Pi(p_k + \Delta q, \bar{q}, \bar{c}). \quad (54)$$

Note that $p_k + \Delta q \in [p_L, p_H]$, because $p_k \in [p_L - \Delta q, p_H - \Delta q]$. By Lemma A.2, we thus have

$$\Pi(p_k + \Delta q, \bar{q}, \bar{c}) \leq \Pi(\bar{p}^*, \bar{q}, \bar{c}). \quad (55)$$

With (51) and (21), we conclude that

$$\pi(\underline{q}, p_k) \leq \Pi(\bar{p}^*, \bar{q}, \bar{c}) = \pi(\bar{q}, \bar{p}^*) = \pi(\underline{q}, \underline{p}^*). \quad (56)$$

⁵¹By definition, p_H is the largest price that a sophisticated is willing to pay for a superior product that displays the best match-value θ . Therefore, at $p_k \geq p_H - \Delta q$, a deceptive firm derives no demand from sophisticated consumers.

Hence, a deviation from \underline{p}^* to $p_k \in [p_L - \Delta q, p_H - \Delta q]$ is not profitable for a deceptive firm. This completes the proof of the “ \Leftarrow ”- part.

“ \Rightarrow ”: If there is a segmented market equilibrium, then there must exist $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ and $\lambda^* \in (0, 1)$ by definition and, a sophisticated consumer’s optimal search rule satisfies (8) which is the same condition as (17). A naive consumer’s optimal search rule in turn satisfies (11) which is the same condition as (18). In a segmented market equilibrium, a deceptive firm only derives inelastic demand from naive consumers so that it charges the largest price such that naive consumers yet purchase its product. We hence have $\underline{p}^* = \hat{p}^*$ which is the same condition as (20). From (18) and (20) we conclude that $\bar{p}^* < \hat{p}^*$ in any segmented market equilibrium. Because Lemma 1 applies, a candid firm’s profits are thus given by $\pi(\bar{q}, p) = \Pi(p, \bar{q}, \bar{c})$ for p from an open interval around \bar{p}^* , (as $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ and $\bar{p}^* < \hat{p}^*$). Hence, $\pi(\bar{q}, p)$ is differentiable at \bar{p}^* , and the corresponding first order condition for \bar{p}^* to be profit maximizing, stated in (44), implies (19). Finally, candid and deceptive firms must make equal profits, as otherwise, firms in the less profitable segment would move to the other segment, and thus, (21) follows from (15) in Lemma 1. ■

Proof of Proposition 1 The following two claims imply the desired result.

Claim (i): There is a (unique) segmented market equilibrium outcome $(\lambda^, \bar{p}^*, \underline{p}^*)$ if and only if $\Delta(\lambda) = 0$ has a (unique) solution λ^* with $-\frac{s}{\underline{\theta}} < \lambda^* < 1$.*

To see Claim (i), observe that by Lemma 2, it is sufficient to show that (17-21) has a solution $(\hat{\theta}^*, \lambda^*, \hat{p}^*, \bar{p}^*, \underline{p}^*)$ with $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$, $\hat{p}^* \geq 0$ and $\lambda^* \in (0, 1)$ if and only if $\Delta(\lambda^*) = 0$ and $\lambda^* > -\frac{s}{\underline{\theta}}$.

Indeed, it follows by Lemma A.1 that $\hat{\theta}^* = g^{-1}(\frac{s}{\lambda^*})$ is a solution to (17) with $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Note further that with (17), (19) is equivalent to

$$\bar{p}^* - \bar{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda^*}))}. \quad (57)$$

Inserting this and (22) into (21) and re-arranging terms yields that λ^* is a solution to

$$\nu^N \cdot \left(\frac{s}{\lambda} + \Delta c \right) = \frac{\nu^S}{\lambda} \cdot \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))}, \quad (58)$$

which is equivalent to $\Delta(\lambda^*) = 0$, and this completes the proof of claim (i).

Claim (ii): If search costs are sufficiently small, then $\Delta(\lambda) = 0$ has a unique solution λ^ with $-\frac{s}{\underline{\theta}} < \lambda^* < 1$.*

To prove Claim (ii), we first show that a solution exists. Indeed, an intermediate value argument implies that $\Delta(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$ if

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) > 0 \quad \text{and} \quad \Delta(1) < 0. \quad (59)$$

We show that (59) is satisfied if s is sufficiently small.

To see that the left inequality in (59) is met for small s , insert $\lambda = -\frac{s}{\underline{\theta}}$ into Δ to obtain

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) = \left(-\frac{\nu^s \underline{\theta}}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(-\underline{\theta}))} - \nu^N(-\underline{\theta} + \Delta c). \quad (60)$$

Because $\underline{\theta} < 0$, this expression converges to $+\infty$ as $s \rightarrow 0$, as desired.

To see that the right inequality in (59) is met for small s , insert $\lambda = 1$ into Δ to obtain

$$\Delta(1) = \frac{1}{h(g^{-1}(s))} - \nu^N(s + \Delta c). \quad (61)$$

Recall from (7) that $g(\bar{\theta}) = 0$. Therefore, $\lim_{s \rightarrow 0} g^{-1}(s) = \bar{\theta}$, and hence, because $\lim_{\theta \rightarrow \bar{\theta}} h(\theta) = \infty$ by assumption, we obtain that $\lim_{s \rightarrow 0} \Delta(1) = -\nu^N \Delta c < 0$, and this establishes (59).

To show that the solution to $\Delta(\lambda) = 0$ is unique for small s , we show first the auxiliary claim

$$\lim_{s \rightarrow 0} \frac{s}{\lambda^*(s)} = 0 \quad \text{for any solution } \lambda^* = \lambda^*(s) \text{ with } \Delta(\lambda^*) = 0. \quad (62)$$

It is clearly sufficient to show that this holds for $\underline{\lambda}^* = \min\{\lambda^* \mid \Delta(\lambda^*) = 0\}$ (which exists by the continuity of Δ). Indeed, suppose to the contrary that there is a (sub)sequence s_n , $n = 1, 2, \dots$ with $s_n \rightarrow 0$ so that the sequence $s_n / \underline{\lambda}^*(s_n)$ is bounded from below by some $\zeta > 0$, that is, $s_n / \underline{\lambda}^*(s_n) > \zeta$ for all n . This first of all implies that $\lim_{n \rightarrow \infty} \underline{\lambda}^*(s_n) = 0$. Moreover, it implies that there is some $\theta_0 < \bar{\theta}$ so that $g^{-1}\left(\frac{s_n}{\underline{\lambda}^*(s_n)}\right) < \theta_0$ for all n by Lemma A.1. Together with (24), it thus follows that

$$\lim_{n \rightarrow \infty} \Delta(\lambda^*(s_n), s_n) \geq \lim_{n \rightarrow \infty} \left\{ \left(\frac{\nu^s}{\lambda^*(s_n)} + \nu^N \right) \cdot \frac{1}{h(\theta_0)} - \nu^N \left(\frac{s_n}{\lambda^*(s_n)} + \Delta c \right) \right\} = \infty, \quad (63)$$

a contradiction to the assumption that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n , and this establishes (62).

To complete the proof of uniqueness, we show

$$\frac{\partial \Delta(\lambda^*)}{\partial \lambda} < 0 \quad \text{for any solution } \lambda^* \text{ with } \Delta(\lambda^*) = 0 \text{ and } s \text{ sufficiently small.} \quad (64)$$

This is sufficient for uniqueness by (59). Indeed, to compute $\partial \Delta / \partial \lambda$, observe first that by (19)

$$\frac{\partial}{\partial \lambda} (\bar{p}^* - \bar{c}) = \frac{\nu^N}{\nu^s} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^s} \right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{\partial \hat{\theta}^*}{\partial \lambda} \quad (65)$$

$$= \frac{\nu^N}{\nu^s} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^s} \right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{1}{1 - F(\hat{\theta}^*)} \cdot \frac{s}{(\lambda^*)^2}, \quad (66)$$

where in the second line, we have used first that the equilibrium property $g(\hat{\theta}^*) = s / \lambda^*$ implies $\partial \hat{\theta}^* / \partial \lambda^* = -1 / g'(\hat{\theta}^*) \cdot (s / (\lambda^*)^2)$, and second that $g' = -(1 - F)$ by Lemma A.1.

Because the hazard rate is increasing, (66) implies $\frac{\partial}{\partial \lambda}(\bar{p}^* - \bar{c}) \leq \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)}$, and hence

$$\frac{\partial \Delta(\lambda^*)}{\partial \lambda} = \left(-\frac{\nu^S}{(\lambda^*)^2} \right) \cdot (\bar{p}^* - \bar{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial}{\partial \lambda}(\bar{p}^* - \bar{c}) + \nu^N \cdot \frac{s}{(\lambda^*)^2} \quad (67)$$

$$\leq \left(-\frac{\nu^S}{(\lambda^*)^2} \right) \cdot (\bar{p}^* - \bar{c}) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} + \nu^N \cdot \frac{s}{(\lambda^*)^2} \quad (68)$$

$$= -\frac{1}{\lambda^*} \Delta(\lambda^*) - \frac{1}{\lambda^*} \Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} \quad (69)$$

$$= -\frac{1}{\lambda^*} \Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)}, \quad (70)$$

where in the last line, we have used that $\Delta(\lambda^*) = 0$. Now observe that (62) and Lemma A.1 imply that $\hat{\theta}^* = g^{-1}(s/\lambda^*) \rightarrow \bar{\theta}$ as $s \rightarrow 0$. Since the hazard rate is unbounded, $\lim_{\hat{\theta}^* \rightarrow \bar{\theta}} h(\hat{\theta}^*) = \infty$, and expression (70) becomes negative for sufficiently small s . This establishes (64) and completes the proof. \blacksquare

Lemma A.3 *The elasticity of the candid mark-up is given by*

$$\frac{\partial(\bar{p}^*(\sigma) - \bar{c})/\partial \sigma}{(\bar{p}^*(\sigma) - \bar{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \text{with } \hat{\theta} = g^{-1}(\sigma). \quad (71)$$

Proof of Lemma A.3 Differentiating \bar{p}^* as given in (26) with respect to σ and inserting σ^* yields

$$\frac{\frac{\partial}{\partial \sigma}(\bar{p}^* - \bar{c})}{\bar{p}^* - \bar{c}} = -\frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\partial g^{-1}(\sigma^*)}{\partial \sigma}. \quad (72)$$

Observe that $\partial g^{-1}(\sigma^*)/\partial \sigma = 1/g'(g^{-1}(\sigma^*))$. Together with $g' = -(1 - F)$ by Lemma A.1, we obtain:

$$\frac{\frac{\partial}{\partial \sigma}(\bar{p}^* - \bar{c})}{\bar{p}^* - \bar{c}} \cdot \sigma^* = \frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\sigma^*}{1 - F(g^{-1}(\sigma^*))} \quad (73)$$

The desired result follows then from $\hat{\theta}^* = g^{-1}(\sigma^*)$ by (17). \blacksquare

Proof of Proposition 2 Contrary to the claim suppose that there is a (sub)sequence s_n , $n = 1, 2, \dots$ with $s_n \rightarrow 0$ so that $\lambda^*(s_n)$ is bounded from below by some $\underline{\lambda}$, that is, $\lambda^*(s_n) > \underline{\lambda}$ for all n . Then, $\lim_{n \rightarrow \infty} s_n/\lambda^*(s_n) = 0$ holds, and Lemma A.1 implies that $\lim_{n \rightarrow \infty} g^{-1}\left(\frac{s_n}{\lambda^*(s_n)}\right) = \bar{\theta}$. We can then infer that

$$\lim_{n \rightarrow \infty} \Delta(\lambda^*(s_n), s_n) < \left(\frac{\nu^S}{\underline{\lambda}} + \nu^N \right) \cdot \lim_{\theta \rightarrow \bar{\theta}} \frac{1}{h(\theta)} - \nu^N \Delta c < 0, \quad (74)$$

where the last inequality follows from the fact that the hazard rate diverges as θ approaches $\bar{\theta}$. This contradicts the equilibrium condition that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n . \blacksquare

Proof of Proposition 3 By the equilibrium property $\Delta(\lambda^*, s) = 0$, we have that $d\lambda^*/ds = -(\partial\Delta(\lambda^*)/\partial s)/(\partial\Delta(\lambda^*)/\partial\lambda)$. Because $\partial\Delta(\lambda^*)/\partial\lambda < 0$ by (64), it suffices to show that

$$\frac{\partial\Delta(\lambda^*)}{\partial s} \geq 0 \iff \epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c}. \quad (75)$$

As is argued in the text, differentiating (24) with respect to s yields (28):

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\bar{p}^*}{\partial\sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}, \quad (76)$$

which by the definition of ϵ in (27) is equivalent to

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot (\bar{p}^* - \bar{c}) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*}. \quad (77)$$

By (24), $\Delta(\lambda^*) = 0$ is equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot (\bar{p}^* - \bar{c}) = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c \right), \quad (78)$$

and (77) becomes

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c \right) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*} = \frac{\nu^N}{s} \cdot \left[\left(\frac{s}{\lambda^*} + \Delta c \right) \cdot \epsilon(\sigma^*) - \frac{s}{\lambda^*} \right]. \quad (79)$$

Finally, (79) implies (75), which completes the proof. \blacksquare

Lemma A.4 *Condition (C) holds if:*

- (i) $f'(\hat{\theta}^*) \geq 0$, or
- (ii) search costs are sufficiently small.

Proof of Lemma A.4 By (75), it suffices to show that $\partial\Delta(\lambda^*)/\partial s \geq 0$ if (i) $f'(\hat{\theta}^*) \geq 0$, or (ii) search costs are sufficiently small.

As to (i), notice first that a straightforward calculation yields that

$$\frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)} = \frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \quad (80)$$

and $\epsilon(\sigma^*)$ in (27) becomes

$$\epsilon(\sigma^*) = \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \right) \cdot \frac{g(\hat{\theta}^*)}{1 - F(\hat{\theta}^*)} \quad (81)$$

with $\hat{\theta}^* = g^{-1}(\sigma^*)$. Inserting this and \bar{p}^* from (26) in (77) yields

$$\frac{\partial\Delta(\lambda^*)}{\partial s} = \left(\frac{\nu^S}{\lambda^*} + \nu^N \right) \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1 \right) \frac{1}{\lambda^* \cdot (1 - F(\hat{\theta}^*))} - \frac{\nu^N}{\lambda^*}, \quad (82)$$

where we used $g(\hat{\theta}^*)/s = g(g^{-1}(\sigma^*))/s = 1/\lambda^*$. Observe that, as desired, this expression is positive if $f'(\hat{\theta}^*) \geq 0$, because $\lambda^* \cdot (1 - F(\hat{\theta}^*)) < 1$.

As to (ii), note that it suffices to show that (82) is positive as search costs get small. Recall that by Footnote 26, effective search costs σ^* vanish as $s \rightarrow 0$, so that $\hat{\theta}^* \rightarrow \bar{\theta}$ as $s \rightarrow 0$ by Lemma A.1. We hence find that

$$\lim_{s \rightarrow 0} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))} = \lim_{\hat{\theta}^* \rightarrow \bar{\theta}} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))}. \quad (83)$$

Notice that (82) is positive as search costs get small if (83) diverges. The reason why (83) indeed diverges is that, on the one hand, $f'(\hat{\theta}^*)/f(\hat{\theta}^*)$ is bounded from below,⁵² while the hazard rate diverges by assumption, and on the other hand, $1 - F(\hat{\theta}^*)$ tends to zero as $\hat{\theta}^* \rightarrow \bar{\theta}$. ■

Proof of Lemma 3 We have to show that

$$\frac{d\sigma^*}{ds} \geq 0. \quad (84)$$

Recall that $\sigma = s/\lambda$, and define the function

$$\tilde{\Delta}(\sigma) = \Delta\left(\frac{s}{\sigma}\right) = \left(\nu^S \cdot \frac{\sigma}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(\sigma))} - \nu^N(\sigma + \Delta c). \quad (85)$$

The equilibrium condition $\Delta(\lambda^*) = 0$ becomes $\tilde{\Delta}(\sigma^*) = 0$, and hence

$$\frac{d\sigma^*}{ds} = -\frac{\partial \tilde{\Delta}(\sigma^*)/\partial s}{\partial \tilde{\Delta}(\sigma^*)/\partial \sigma}. \quad (86)$$

Differentiating (85) yields that

$$\frac{\partial \tilde{\Delta}(\sigma^*)}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda} \left(-\frac{s}{\sigma^*}\right), \quad (87)$$

and $\partial \tilde{\Delta}(\sigma^*)/\partial \sigma$ is thus positive, because $\partial \Delta(\lambda^*)/\partial \lambda < 0$ by (64). Moreover, by (85):

$$\frac{\partial \tilde{\Delta}(\sigma^*)}{\partial s} = -\nu^S \frac{\sigma^*}{s^2} \cdot \frac{1}{h(g^{-1}(\sigma^*))}. \quad (88)$$

Because this is negative, it follows that $d\sigma^*/ds$ is positive, as desired. ■

Proof of Proposition 4 By (32),

$$\frac{dW}{ds} = \nu^S \cdot \frac{d\hat{\theta}^*}{ds} + \nu^N \cdot \frac{d\lambda^*}{ds} \cdot (\Delta q - \Delta c) - \nu^N. \quad (89)$$

Hence, if Condition (C) holds, so that $d\lambda^*/ds > 0$ by Proposition 2, then (89) is positive if $\Delta q - \Delta c \geq 0$ is sufficiently large (compare Footnote 33).

⁵²By log-concavity of f , f'/f is decreasing and $f'(\bar{\theta})/f(\bar{\theta})$ is bounded from below by (16) and $f(\bar{\theta}) > 0$.

Proof of Proposition 5 The argument is given in the main text, except of the argument for why $\hat{\theta}^*$ is decreasing in s . To see this, note that by (17), we have $g(\hat{\theta}^*) = \sigma^*$, and hence

$$\frac{d\hat{\theta}^*}{ds} = \frac{d\sigma^*}{ds} \frac{1}{g'(\hat{\theta}^*)}, \quad (90)$$

which is negative, because $g' = -(1-F)$ by Lemma A.1 and $d\sigma^*/ds \geq 0$ by Lemma 3. \blacksquare

Proof of Proposition 6 As to (i). Because $d\lambda^*/d\nu^N = -(\partial\Delta(\lambda^*)/\partial\nu^N)/(\partial\Delta(\lambda^*)/\partial\lambda)$ and $\partial\Delta(\lambda^*)/\partial\lambda < 0$ by (64), it is sufficient to show that

$$\frac{\partial\Delta(\lambda^*)}{\partial\nu^N} < 0. \quad (91)$$

Recall that $\Delta = (\nu^N + \nu^S/\lambda^*) \cdot (\bar{p}^* - \bar{c}) - \nu^N \cdot (\bar{p}^* + \sigma^* - \underline{c})$, and $\nu^S = 1 - \nu^N$. Hence,

$$\frac{\partial\Delta(\lambda^*)}{\partial\nu^N} = \left(1 - \frac{1}{\lambda^*}\right) \cdot (\bar{p}^* - \bar{c}) + \left(\nu^N + \frac{\nu^S}{\lambda^*}\right) \cdot \frac{\partial\bar{p}^*}{\partial\nu^N} - (\bar{p}^* + \sigma^* - \underline{c}) - \nu^N \cdot \frac{\partial\bar{p}^*}{\partial\nu^N} \quad (92)$$

$$= \left(1 - \frac{1}{\lambda^*}\right) \cdot (\bar{p}^* - \bar{c}) - (\bar{p}^* + \sigma^* - \underline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\bar{p}^*}{\partial\nu^N} \quad (93)$$

$$= -\frac{1}{\lambda^*} \cdot \frac{1}{\nu^N} \cdot (\bar{p}^* - \bar{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\bar{p}^*}{\partial\nu^N}, \quad (94)$$

where we have inserted the equilibrium condition $\Delta(\lambda^*) = 0$ (which is equivalent to $\bar{p}^* + \sigma^* - \underline{c} = 1/\nu^N \cdot (\nu^N + \nu^S/\lambda^*) \cdot (\bar{p}^* - \bar{c})$) in the second line to obtain the third line. Recall from (26) that

$$\bar{p}^* - \bar{c} = \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{1}{h(\hat{\theta}^*)}, \quad (95)$$

and observe that

$$\frac{\partial\bar{p}^*}{\partial\nu^N} = \frac{\lambda^*}{(1 - \nu^N)^2} \cdot \frac{1}{h(\hat{\theta}^*)}. \quad (96)$$

After inserting (95) and (96) in (94) and simplifying terms, we obtain

$$\frac{\partial\Delta(\lambda^*)}{\partial\nu^N} = -\frac{1}{\lambda^* h(\hat{\theta}^*) \nu^N}, \quad (97)$$

which establishes (91) and completes the proof of part (i).

As to (ii). By (32), a straightforward calculation delivers that

$$\frac{dW}{d\nu^N} = -(\bar{q} - \bar{c} + \hat{\theta}^*) + \nu^S \cdot \frac{d\hat{\theta}^*}{d\nu^N} + \lambda^* \cdot (\Delta q - \Delta c) + \underline{q} - \underline{c} - s + \nu^N \cdot \frac{d\lambda^*}{d\nu^N} \cdot (\Delta q - \Delta c) \quad (98)$$

$$\leq -(1 - \lambda^*) \cdot (\Delta q - \Delta c) - \hat{\theta}^* - s, \quad (99)$$

where the inequality follows from re-arranging terms, the fact that $d\lambda^*/d\nu^N \leq 0$ by part (i), and because $d\hat{\theta}^*/d\nu^N \leq 0$, which in turn follows from

$$\frac{d\theta^*}{d\nu^N} = \frac{\partial\theta^*}{\partial\lambda} \cdot \frac{d\lambda^*}{d\nu^N} = -\frac{s}{(\lambda^*)^2 \cdot g'(\hat{\theta}^*)} \cdot \frac{d\lambda^*}{d\nu^N} \leq 0, \quad (100)$$

because $d\lambda^*/d\nu^N \leq 0$ by part (i) and $g' = -(1-F)$ by Lemma A.1.

Now, a revealed preference argument implies that the equilibrium utility of a sophisticated consumer, $U^S = \bar{q} - \bar{p}^* + \hat{\theta}^*$, is larger than the expected utility that he would obtain if he purchased the first superior product that he encounters, which is $\bar{q} - \bar{p}^* + \mathbb{E}(\theta) - \sigma^*$. Because $\mathbb{E}(\theta) = 0$, this implies that $\sigma^* \geq -\hat{\theta}^*$. Inserting this in (99) yields (recall $s = \sigma^* \cdot \lambda^*$)

$$\frac{dW}{d\nu^N} \leq -(1 - \lambda^*) \cdot (\Delta q - \Delta c + \hat{\theta}^*), \quad (101)$$

and $\Delta q - \Delta c \geq -\hat{\theta}^*$ thus implies $dW/d\nu^N < 0$.

Now, observe that this condition is implied by the conditions (a) and (b) in the statement of the proposition. Indeed, (a) $\Delta q - \Delta c \geq \bar{\theta} - \underline{\theta}$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\hat{\theta}^* \geq \underline{\theta}$ and $\bar{\theta} > 0$, and (b) $\Delta q - \Delta c \geq \sigma^*$ implies $\Delta q - \Delta c \geq -\hat{\theta}^*$, because $\sigma^* \geq -\hat{\theta}^*$ as argued above. ■

Proof of Proposition 7 As argued in the text, it suffices to show that

$$\frac{d\bar{p}^*}{d\lambda} \leq 0 \quad \text{if} \quad \epsilon(\sigma^*) \geq 1. \quad (102)$$

To see this, observe that (26), (27) and a straightforward calculation yield that

$$\frac{d\bar{p}^*}{d\lambda} = \frac{\partial\bar{p}^*}{\partial\lambda} + \frac{\partial\bar{p}^*}{\partial\sigma} \cdot \frac{\partial\sigma^*}{\partial\lambda} = \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} + \epsilon(\sigma^*) \cdot \frac{(\bar{p}^* - \bar{c})}{\sigma^*} \cdot \left(-\frac{s}{(\lambda^*)^2}\right). \quad (103)$$

Inserting (26) and simplifying terms yields that

$$\frac{d\bar{p}^*}{d\lambda} = \left(\frac{\nu^N}{\nu^S} - \epsilon(\sigma^*) \cdot \left(\frac{1}{\lambda^*} + \frac{\nu^N}{\nu^S} \right) \right) \cdot \frac{1}{h(\hat{\theta}^*)}, \quad (104)$$

which establishes (102), as desired. ■

Proof of Lemma 4 The argument why a price floor improves total welfare is given in the main text.

As to consumer welfare, observe that by (6), (22) and (33) consumer welfare is given as

$$W^C = \nu^S \cdot (\bar{q} + \hat{\theta} - \bar{p}) + \nu^N \cdot (\underline{q} + \lambda \cdot \Delta q - (\bar{p} + \frac{s}{\lambda})), \quad (105)$$

where $\hat{\theta}$ and λ are the equilibrium market outcomes given a candid price \bar{p} and are pinned down by (17) and (38). We want to show that

$$\frac{dW^C}{d\bar{p}} > 0 \quad \text{if} \quad \Delta q \text{ is sufficiently large.} \quad (106)$$

By (17) and Lemma A.1, $\hat{\theta}$ increases in λ , and by (38), λ increases in \bar{p} . Therefore, $\hat{\theta}$ increases in \bar{p} , and we infer that

$$\frac{dW^C}{d\bar{p}} \geq -v^S + v^N \cdot \left(\frac{d\lambda}{d\bar{p}} \cdot \Delta q - 1 \right). \quad (107)$$

Differentiating (38) yields an expression for $d\lambda/d\bar{p}$, and, after simplifying terms, we obtain

$$\frac{dW^C}{d\bar{p}} \geq -1 + \frac{v^S}{\Delta c} \cdot \Delta q, \quad (108)$$

which is positive if Δq is sufficiently large, as desired. ■

Proof of Lemma 5 As to total welfare, as argued in the text, it suffices to show that the share of candid firms increases in response to a minimum quality standard. To see this, observe that the difference in candid and deceptive profits as given in (24) is independent of \underline{q} . The effect of a minimum quality standard on the share of candid firms is therefore given by $d\lambda^*/d\underline{c} = -(\partial\Delta(\lambda^*)/\partial\underline{c})/(\partial\Delta(\lambda^*)/\partial\lambda)$. Because $\partial\Delta(\lambda^*)/\partial\lambda < 0$ by (64),

$$\frac{\partial\Delta(\lambda^*)}{\partial\underline{c}} = v^N \quad (109)$$

implies $d\lambda^*/d\underline{c} \geq 0$, as desired.

As to consumer welfare, as is argued in the text, it suffices to show that $d\bar{p}^*/d\lambda \leq 0$ if $\epsilon(\sigma^*) \geq 1$. But this follows immediately from (102). ■

Proof of Lemma 6 First, we show that there is no pooling equilibrium. To the contrary, suppose that there is a segmented market equilibrium such that all firms charge p^* , and the share of candid firms is $\lambda^* \in (0, 1)$. In a pooling equilibrium, an uninformed consumer cannot distinguish between superior and inferior products and purchases from the first firm he visits in equilibrium. Because a consumer's reservation utility equals his expected equilibrium utility, the reservation utility of an uninformed consumer is given by

$$\hat{U}^N = \lambda^* \cdot \bar{q} + (1 - \lambda^*) \cdot \underline{q} - p^* - s. \quad (110)$$

Now, an uninformed consumer expects any product offered at p to supply the utility

$$U(p) = \hat{\lambda}(p) \cdot \bar{q} + (1 - \hat{\lambda}(p)) \cdot \underline{q} - p. \quad (111)$$

Therefore, he purchases a deceptive product offered at $p^* + dp$, because $U(p^* + dp) > \hat{U}^N$ for small dp by consistency and continuity of $\hat{\lambda}$. As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, it would be profitable for it to raise its price marginally – a contradiction of the optimality of prices in equilibrium.

Second, we show that there is no separating equilibrium. Suppose the contrary and consider first the case $\underline{p}^* > \bar{p}^*$. We derive a contradiction by arguing that a candid firm can profitably deviate to a higher price. Indeed, because in a separating equilibrium uninformed consumers learn a firm's quality, for a deceptive firm to derive demand from uninformed consumers in equilibrium, uninformed consumers must prefer purchasing an inferior product at \underline{p}^* to searching for a superior product:

$$(\underline{p}^* - \bar{p}^*) + \Delta q \leq \frac{s}{\lambda^*}. \quad (112)$$

To see the inequality, note that s/λ^* are the expected costs to find a superior product not only for a sophisticated consumer but also for an uninformed consumer, as he recognizes superior products in a separating equilibrium. Now, (112) implies $\Delta q < s/\lambda^*$ because $\underline{p}^* > \bar{p}^*$ by assumption. By assumption (4), we thus have $\bar{\theta} - \underline{\theta} < s/\lambda^*$. Moreover, in a segmented market equilibrium a sophisticated consumer only purchases superior products and, as before, his search rule is characterized by a reservation match-value $\hat{\theta}^*$ that is given by (8). By (8) and Lemma A.1, $\bar{\theta} - \underline{\theta} < s/\lambda^*$ implies $\hat{\theta}^* < \underline{\theta}$. In equilibrium, a sophisticated consumer therefore strictly prefers to purchase a superior product at \bar{p}^* , irrespective of product fit, instead of continuing to search. Note, however, that the same must be true for uninformed consumers, because they purchase in equilibrium the deceptive product which is more expensive and provides a lower quality. Thus, because consumer beliefs $\hat{\lambda}$ are continuous, the entire demand of a candid firm is (locally) inelastic at \bar{p}^* such that it is profitable for a candid firm to increase its price – contradicting that \bar{p}^* is an equilibrium price.

Next consider the case that $\underline{p}^* < \bar{p}^*$. Let ξ be the probability with which an uninformed consumer purchases a product that is offered at \bar{p}^* . Because an uninformed consumer purchases superior products in a segmented market equilibrium, $\xi \in (0, 1]$. If $\xi = 1$, then a deceptive firm could charge \bar{p}^* , and any visiting uninformed consumers would purchase its product. Because $\underline{p}^* < \bar{p}^*$ and because a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, this would constitute a profitable deviation—a contradiction. Therefore, $\xi \in (0, 1)$, and an uninformed consumer is indifferent between purchasing a product offered at \bar{p}^* and continuing to search. Because an uninformed consumer expects a product offered at \bar{p}^* to deliver the utility $\bar{q} - \bar{p}^*$, his reservation utility is:

$$\hat{U}^N = \bar{q} - \bar{p}^*. \quad (113)$$

Moreover, because a naive consumer purchases superior and deceptive products in equilibrium, it is optimal for him to purchase at the first firm he visits. His reservation utility therefore also satisfies

$$\hat{U}^N = \lambda^* \cdot (\bar{q} - \bar{p}^*) + (1 - \lambda^*) \cdot (\underline{q} - \underline{p}^*) - s, \quad (114)$$

and by (113), we have

$$\hat{U}^N = (\underline{q} - \underline{p}^*) - \frac{s}{1 - \lambda^*}. \quad (115)$$

Now, in a separating equilibrium, an uninformed consumer expects a product offered at \underline{p}^* to supply the utility

$$U(\underline{p}^*) = \underline{q} - \underline{p}^*. \quad (116)$$

By (115) and (116), we thus have $U(\underline{p}^*) > \hat{U}^N$, and an uninformed consumer strictly prefers purchasing a product offered at \underline{p}^* to continuing search. As a consequence, because consumer beliefs are continuous in prices, a deceptive firm could raise its price marginally and would not lose any demand from uninformed consumers. As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, such a deviation would be profitable, contradicting the assumption that \underline{p}^* is an equilibrium price. ■

Proof of Lemma 7 First, we argue that there is a segmented market equilibrium with $\underline{p}^* > \bar{p}^*$ if ε and search costs s are sufficiently small. Second, we argue that the share of candid firms vanishes as search costs vanish.

Before, observe that, by Bayes' rule in a segmented market equilibrium, the expected quality of a product, conditional on a good (resp. bad) signal is

$$\mathbb{E}(q|\bar{z}) = \frac{\lambda(\frac{1}{2} + \varepsilon)\bar{q} + (1 - \lambda)(\frac{1}{2} - \varepsilon)\underline{q}}{\lambda(\frac{1}{2} + \varepsilon) + (1 - \lambda)(\frac{1}{2} - \varepsilon)}, \quad \left(\text{resp. } \mathbb{E}(q|z) = \frac{\lambda(\frac{1}{2} - \varepsilon)\bar{q} + (1 - \lambda)(\frac{1}{2} + \varepsilon)\underline{q}}{\lambda(\frac{1}{2} - \varepsilon) + (1 - \lambda)(\frac{1}{2} + \varepsilon)} \right).$$

Observe that $\mathbb{E}(q|\bar{z}) \geq \mathbb{E}(q) \geq \mathbb{E}(q|z)$ and note that for each δ there exists ε such that $\mathbb{E}(q|\bar{z}) - \mathbb{E}(q|z) < \delta$.

Claim For sufficiently small ε and sufficiently small search costs s , there is a solution $\bar{p}^* \geq 0$, $\underline{p}^* \geq 0$, $\hat{p}^* \geq 0$, $\hat{\theta}^* \in (\underline{\theta}, \bar{\theta})$ and $\lambda^* \in (0, 1)$ to the equation system (17), (19-21) and (replacing (18))

$$\hat{p}^* = \lambda^* \cdot \bar{p}^* + (1 - \lambda^*) \cdot \underline{p}^* + s - (\mathbb{E}(q) - \mathbb{E}(q|z)). \quad (117)$$

The solution displays $\underline{p}^* > \bar{p}^*$ and describes a segmented market equilibrium outcome.

To prove the claim, we first show that the equation system (17), (19-21), (117) characterizes the best responses of all agents, given a (candidate) segmented market equilibrium outcome $(\lambda^*, \bar{p}^*, \underline{p}^*)$.

Indeed, as to a naive consumer's optimal best response, (20) and (117) imply

$$\underline{p}^* = \bar{p}^* + \frac{s - (\mathbb{E}(q) - \mathbb{E}(q|z))}{\lambda^*}. \quad (118)$$

For sufficiently small ε , we have $s > \mathbb{E}(q) - \mathbb{E}(q|\underline{z})$, and thus $\bar{p}^* < \underline{p}^*$. Moreover, by (20), we have $\underline{p}^* = \hat{p}^*$. Together, this implies $\bar{p}^* < \underline{p}^* \leq \hat{p}^*$. Now, if it is optimal for a naive consumer to purchase at the first firm he visits (even if he receives a bad signal), then his reservation utility is given by (10). In this case, (117) is a complete characterization of a naive consumer's optimal search rule (his reservation utility), because (117) denotes the maximal price that he is willing to pay for a product upon receiving a bad signal given (10), as can be shown straightforwardly. To see that it is indeed optimal for a naive consumer to purchase at the first firm he visits, it suffices to verify that $\bar{p}^* < \underline{p}^* \leq \hat{p}^*$ holds, which is the case, as was shown before.

As to a sophisticated consumer's optimal best response, as argued in the proof of Lemma 2, $\underline{p}^* > \bar{p}^*$ implies that a sophisticated consumer does not purchase deceptive products, and (17) characterizes his optimal search rule, as before.

As to a candid firm's behavior, for prices below \hat{p}^* , profits of candid firms are given by (14). Because $\underline{p}^* > \bar{p}^*$ and $\underline{p}^* = \hat{p}^*$, (14) is differentiable at \bar{p}^* and (19) is the first order condition for profit maximization of candid firms, as before.

As to a deceptive firm's behavior, for a deceptive firm it is optimal to charge \hat{p} , because its demand from naive consumer is inelastic up to \hat{p} (leaving the demand from sophisticated consumer aside). If a deceptive firm charged a price that exceeds \hat{p} , it would only derive demand from naive consumer who receive a good signal about its quality. At most a deceptive firm could raise its price (and hence its mark-up) by $\mathbb{E}(q|\bar{z}) - \mathbb{E}(q|\underline{z})$, so as to still derive demand from these consumers, whereas it would lose the demand of all naive consumers who receive a bad signal which amounts to a mass of $\nu^N \cdot (1/2 - \varepsilon)$. For ε sufficiently small, a deceptive firm hence loses almost half of its demand, whereas it only increases its mark-up by less than some δ . Because $\underline{p}^* > \bar{p}^*$, the mark-up of a deceptive firm is bounded away from zero in equilibrium, which implies that such a deviation is not profitable for sufficiently small ε . Finally, (21) ensures that firms in the two segments earn the same profits.

A proof why the first order conditions is sufficient for profit maximization for a candid firm and why it is not profitable for a deceptive firm to lower its price by so much that it derives demand from sophisticated consumers follows along the lines of the proof of Lemma 2. Moreover, an argument why there is a solution to the above equation system for sufficiently small search costs follows along the lines of Proposition 1. We omit these details, and this completes the proof of the claim.

As to the second step of the proof, we now show that the share of candid firms vanishes as search costs do. Because $\underline{p}^* > \bar{p}^*$, the deceptive mark-up is bounded away from zero. This implies that deceptive profits are bounded away from zero, because the demand of a deceptive firm is ν^N . Now, suppose that λ^* would not vanish as search costs get small. Then, λ^* would be bounded from below and effective search costs would vanish as search costs get small. This would imply

that a candid firm's mark-up and profits vanish, because, as before, a candid firm's price is given by (19). This contradicts that firms in the two segments earn the same profit. ■

B Appendix

In this appendix, we show that if the distribution f^E of match-values is given by (30), then there is an open set of parameters $s, \bar{q}, \underline{q}, \bar{c}, \underline{c}, \nu^N$ such that there is a segmented market equilibrium with $\hat{\theta}^* < 0$ and

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - 1} \quad \text{and} \quad \bar{p}^* - \bar{c} = \frac{\Delta c - s \frac{\nu^N}{\nu^S}}{\Delta c - 1}. \quad (119)$$

Note that the cumulative distribution function of f^E is

$$F^E(\theta) = \begin{cases} 1 - e^{-(\theta+1)} & \theta < 0 \\ 1 + e^{-1} \cdot (\theta - 1) & \theta \geq 0 \end{cases} \quad (120)$$

and satisfies $h^E(\theta) = 1$ for $\theta \leq 0$.

Moreover, the corresponding function g^E as defined by (7) is given by⁵³

$$g^E(z) = \begin{cases} \frac{1}{2e} \cdot (2e^{-z} - 1) & z < 0 \\ \frac{1}{2e} \cdot (1 - z)^2 & z \geq 0. \end{cases} \quad (124)$$

We now show that provided $\hat{\theta}^* < 0$, the formulae in (119) satisfy the equilibrium conditions

⁵³To see this, note that (7) implies for $z < 0$:

$$g^E(z) = \int_z^0 (\theta - z) e^{-(\theta+1)} d\theta + \int_0^1 (\theta - z) e^{-1} d\theta. \quad (121)$$

Integration by parts yields

$$\int_z^0 (\theta - z) e^{-(\theta+1)} d\theta = -(\theta - z) e^{-(\theta+1)} \Big|_z^0 + \int_z^0 e^{-(\theta+1)} d\theta = (z - 1) e^{-1} + e^{-(z+1)}. \quad (122)$$

Moreover, $\int_0^1 (\theta - z) e^{-1} d\theta = e^{-1} \cdot (\frac{1}{2} - z)$. Inserting this in (121) yields the first line in (124.)

For $z \geq 0$, g^E corresponds to the second line in (124), because by (7):

$$g^E(z) = \int_z^1 (\theta - z) e^{-1} d\theta = e^{-1} \cdot \left(\frac{1}{2} \theta^2 - z\theta \right) \Big|_z^1 = e^{-1} \cdot \left(\frac{1}{2} - z + \frac{1}{2} z^2 \right) = \frac{1}{2e} \cdot (1 - z)^2. \quad (123)$$

(19-21).⁵⁴ In fact, $\hat{\theta}^* < 0$ implies $h^E(\hat{\theta}^*) = 1$, and thus $\Delta(\lambda^*) = 0$ becomes equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) - \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c\right) = 0. \quad (125)$$

Rearranging terms then yields the desired expression for λ^* . Inserting $h^E(\hat{\theta}^*) = 1$ and λ^* from (119) into equation (19) yields

$$\bar{p}^* - \bar{c} = 1 + \frac{1 - s \frac{\nu^N}{\nu^S}}{\Delta c - 1}, \quad (126)$$

which is equivalent to the expression in (119).

To complete the proof, we show that there are parameters so that for λ^* as given in (119), we have that $\lambda^* \in (0, 1)$, and the equilibrium match-value $\hat{\theta}^*$ which is given as $\hat{\theta}^* = (g^E)^{-1}(s/\lambda^*)$ by (17) indeed satisfies $\hat{\theta}^* < 0$.

To see this, let $\nu^S/\nu^N = 1/e$, $s = 1/(2e)$ and $\Delta c = 1 + 1/e$. By a straightforward but tedious calculation, we obtain

$$\lambda^* = \frac{1}{2}, \quad \hat{\theta}^* = -\ln(3/2), \quad \bar{p}^* = \frac{e}{2} + 1, \quad \underline{p}^* = \frac{e}{2} + 1 + \frac{1}{e}. \quad (127)$$

Finally, note that because the equilibrium values are continuous in the parameters, if we perturb the parameters slightly, this does not upset the (strict) inequalities $\lambda^* \in (0, 1)$ and $\hat{\theta}^* < 0$. This establishes the claim. ■

⁵⁴Strictly speaking, it is not a priori clear that with the density f^E the equilibrium conditions (17-21) are actually sufficient for equilibrium existence. The reason is that f^E is not globally log-concave (which we use in our sufficiency proof). However, for the specific case that $f = f^E$, the sufficiency proof still goes through with minor changes. We omit the details.

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