Communication with Forgetful Liars*

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Abstract

I consider cheap talk environments in which the informed party sends twice a message about the state and does not remember which message was previously sent in case of lie. I characterize the equilibria with forgetful liars in such settings assuming that the expectation about the past message after a lie coincides with the aggregate distribution of lies over all possible realizations of the states. The approach provides a simple rationale as to why multi-rounds of communication may be helpful at disciplining the informed party to tell the truth and why inconsistency may be harmful.

Keywords: forgetful liars, analogy-based expectations, cheap talk

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1 Introduction

A common tactic in criminal investigations consists in letting the suspects repeatedly expose their views and take advantage of potential inconsistencies in the expressed allegations to obtain admissions of guilt (or at least increase the belief of guilt in the mind of third parties such as jurors).

Game theory has devoted significant and successful effort in the modelling of strategic communication when the objectives of the informed and the uninformed parties may be misaligned as is the case in criminal investigations. In situations in which it would not be possible to verify (in a reasonable time-scale) what the informed party says, the corresponding literature -referred to as the literature on cheap talk- establishes that information transmission would be partial. That is, some information held by the informed party would not be passed to the uninformed party, and in the case of sufficiently strong conflicts of interest (which may be relevant for criminal investigations) there would be no information transmission at all (see Crawford and Sobel (1982) or Green and Stokey (2007)).

While the early and most of the subsequent literature on cheap talk has considered one-round communication protocols, some scholars (in particular, Forges (1990), Aumann and Hart (2003) or Krishna and Morgan (2004)) have explored the possibility of multi-round communication and noted that more information could then, in general, be transmitted. Yet, the fundamental reason why multi-round communication may help in such approaches is unrelated to the idea of tracking inconsistencies in the communication strategy of the informed party. It follows because more complex communication protocols allow to implement a larger spectrum of the communication equilibria that could be obtained through the use of a mediator as compared with the smaller set of Nash equilibria that can be implemented with one round of direct communication between the two parties.

While the prediction of the game theoretic literature on cheap talk may fit the observation that some suspects tend to remain silent in the investigation phase, it fails to explain the common tactic reported above aimed at tracking communication inconsistencies and taking advantage of these. I explore this question by making the assumption that when a person lies at some point in time, the exact lie may not be perfectly remembered later by this person. And, to sharpen the analysis, I make the extreme assumption that there is no memory at all of what the lie was in this case. I note that the idea that lies may be hard to remember appears in many places and even in the popular culture. For example, it is subtly expressed by Mark Twain as *"When you tell the truth you do not have to remember anything,"* which implicitly but clearly suggests a memory asymmetry whether you tell the truth or you lie.[1]

I consider the following communication setting. As usual, the informed party I communicates about what he knows s (the state) where s can take discrete values in $S \subseteq [0,1]$, and party U makes a decision (chooses an action) based on the messages that are transmitted to her. To fix ideas, I assume that party U optimally chooses the action that matches the expectation of the state given what she observes from the communication stage. Communication does not take place at just one time. Instead, I assume that two messages are being sent by party I at two different times $t = 1, 2$ (assumed to be sufficiently far apart to make the imperfect memory assumption more plausible). If party I with information s tells the truth and communicates $m_1 = s$ at time $t = 1$, he remembers it, but if he lies by saying $m_1 \neq s$, he does not remember at time $t = 2$ what message was sent at time $t = 1$. He is always assumed to know the state s though. That is, the imperfect memory is only about the message sent at time $t = 1$, not about the state. Party U is assumed to make the optimal decision given what she is told. When two identical messages $m_1 = m_2 = m$ are being sent by party I at $t = 1$ and 2, party U observes the message m, but when $m_1 \neq m_2$ are being sent, I assume that party U is only informed that $m_1 \neq m_2$.[2]

A key modelling choice concerns the expectation at time $t = 2$ of party I in state s about the message m_1 sent at time $t = 1$ when party I made a lie at $t = 1$. I will be assuming that in equilibrium party I then believes that the message sent at time $t = 1$ is distributed according to the aggregate (equilibrium) distribution of lies made across all

[1] Mark Twain's quote has sometimes been used to motivate that explicit lies (as opposed to lies by omission) may be costly (see, for example, Hart, Kremer and Perry (2017)). To the best of my knowledge, it has not been modeled as an asymmetry in memory whether truthful messages or lies are reported, which is the subject of this paper.

[2] As a result, the action chosen in the case of inconsistent messages while optimal in expectation (over events in which $m_1 \neq m_2$) cannot be fine tuned to the exact realization of (m_1, m_2). The assumption that only inconsistency is observed by party U when $m_1 \neq m_2$ allows me to simplify some of the analysis. I also believe it fits with a number of applications in which the decision maker does not directly participate in the hearings but is presented with a summary of those.

possible states at $t = 1$.[3] I think of such an expectation as being guided by a coarse access to past lies in similar interactions allowing party I to have a good sense of the aggregate distribution of lies but not of how lies are distributed as a function of the state. Such a modelling is in the spirit of the analogy-based expectation equilibrium (Jehiel (2005) or Jehiel and Koessler (2008)), and it will be contrasted with the more standard modelling of imperfect recall (in the spirit of either of the multi-self approaches defined in Piccione and Rubinstein (1997)) in which after a lie, party I in state s has the correct expectation about m_1 conditional on the state s.

All other expectations are assumed to be correct, implying that party I at time $t = 1$ has the correct expectation about what he may be doing at time $t = 2$ whatever the message he sends at time $t = 1$, and party I (whatever the time t and the state s) has the correct expectation about what action is made by party U as a function of what she observes from the communication stage. Strategies are required to be best-responses to the expectations as usual. The corresponding equilibria are referred to as equilibria with forgetful liars.

I characterize the equilibria with forgetful liars in the communication setting just described adding the perturbation that with a tiny probability party I always communicates the truth and party I incurs a tiny extra cost when lying (so that party I would consider lying only if it is strictly beneficial).[4] The main findings are as follows.

I first consider pure persuasion situations in which party I 's objective is the same for all states and consists in inducing a belief about s in party U's mind as high as possible. For such specifications, the equilibria in pure strategies take the following form.

[3] So if there is only one lie that is being made in equilibrium, party I would believe after a lie that he sent as a message at time $t = 1$ this unique lie (even if in truth he may have sent another lie if a deviation is considered), or if in equilibrium party I in states $s \in S_A$ makes the lie A and party I in states $s \in S_B$ makes the lie B while party I in any other state tells the truth, party I with information s would believe at time $t = 2$ after a lie made at time $t = 1$ that he sent message A with probability $\frac{\lambda_A}{\lambda_A+\lambda_B}$ and message B with probability $\frac{\lambda_B}{\lambda_A+\lambda_B}$ where λ_A (resp. λ_B) stands for the probability that s belongs to A (resp. B).

[4] Such perturbations while natural allow to pin down the beliefs of party U after hearing any consistent message that corresponds to an existing state. It should be noted that with no memory issues, such perturbations when small would not affect the substance of the communication outcome. In particular, in pure persuasion games, there would be no information transmission (the analysis of such perturbations in one shot versions of the game appears in Chen (2011), and it can be shown that, as the perturbations go to 0, the limiting equilibrium outcome corresponds to one of the equilibria in the communication game without the perturbations.

Inconsistent messages are necessary detrimental and party I whatever s avoids sending inconsistent messages. There is at most one lie s^* made in equilibrium. Party I in states $s \leq s_*$ makes the consistent lie $m_1 = m_2 = s^*$, party I in all other states tells the truth, and (s_*, s^*) are such that $E(s \in S,\ s \leq s_* \text{ or } s = s^*)$ is in between s_* and the state s in S just above s_*. Moreover, when considering the fine grid case in which two consecutive types are close to each other, I show that all equilibria with forgetful liars whether in pure or in mixed strategies lead approximately to the first-best in which party U infers the state whatever s and chooses the action $a = s$ accordingly.

I next extend the analysis to more general situations in which the objective of the informed party I may depend on the state s as in classic cheap talk games. I note in that extension that sometimes multiple lies can arise in equilibrium, but again, in the fine grid case, the equilibrium outcome must be close to the first-best.

To sum up, the analysis developed here shows why multi-round communications may be very effective at eliciting information efficiently when it is hard to remember one own's past lies (and there are always sufficiently nearby states- the fine grid aspect). It also establishes why inconsistencies must lead to detrimental outcomes (this was derived as an equilibrium property) and why such a cost of inconsistency disciplines potential liars to engage into lying only if they can be sure to stick to their original lies (through their cognitive reasoning) later on.

Related Literature

The above findings can be related to different strands of literature. First, the equilibria with forgetful liars turn out to be similar to the Perfect Bayesian Nash equilibria that would arise in certification games in which all types but those corresponding to the lies could certify all what they know (see Grossman and Hart (1980), Grossman (1981), Milgrom (1981), Dye (1985) or Okuno-Fuijiwara and Postlewaite (1990) for some key references in the certification literature). In particular, when there is one lie s^* as in the case of pure persuasion games, the equilibrium outcome is similar to that in Dye (1985)'s model identifying type s^* with the type that cannot be certified (the uninformed type) in Dye. Of course, a key difference is that, in this analogy, the set of types that cannot be certified is not exogenously given in the present context, as it is determined by the set of lies made in equilibrium, which are endogenously determined.

Second, the analysis would be substantially altered if considering equilibria in the vein of Piccione and Rubinstein (1997) in which forgetful liars would have access to the distribution of lies conditional on the state. With such a modelling, the mere knowledge of s would allow liars to have a much more precise expectation about the past lie even if lies cannot be physically remembered, thereby allowing to sustain equilibria with many more lies (even in pure persuasion games) that would remain bounded away from the first-best even in the fine grid case (see subsection 3.3 for elaborations). Thus, the sharp characterization of equilibria and the link to first-best in the fine grid case require that the informed party I has only access from past database to the marginal distribution of lies and not to the joint distribution of lies and states (which I would argue is a plausible feature in a number of applications).

Third, it may be interesting to compare the results obtained here with those obtained when explicit costs to lying (possibly determined by the distance between the state and the lie) are added to the standard cheap talk game (see, in particular Kartik (2003)). In the case of lying costs, every type has an incentive to inflate his type and there is some pooling at the highest messages, which sharply contrasts with the pooling that was obtained in the equilibria with forgetful liars in pure persuasion situations and that concerned the low types making the same lie.

The rest of the paper is organized as follows. Section 2 describes the model and solution concept. Section 3 analyzes pure persuasion games. Section 4 considers alternative specifications of payoffs for the informed party. Section 5 offers a discussion, in particular adding a mechanism design perspective.

2 The Model

There are n possible states $s_1 < s_2 < ...s_n$ distributed between $s_1 = 0$ and $s_n = 1$ and $S = \{s_k\}_{k=1}^n$ will denote the state space. The ex ante probability that state s arises is $p(s)$. There are two parties, an informed party I and an uninformed party U. The informed party knows the realization of the state s, the uninformed party does not.

Party I first communicates about s according to a protocol to be described shortly. At the end of the communication phase, party U has to choose an action $a \in [0, 1]$. The objective of party U takes the quadratic form $-(a-s)^2$ so that whenever she has to make a decision, party U chooses the action that corresponds to the expected value of s given what she believes about the distribution of s.

Party I cares about the action a chosen by U and possibly (but not necessarily) about the state s. Ignoring the messages sent during the communication phase, party I's payoff can be written as $u(a, s)$.

I will start the analysis with the case -sometimes referred to in the literature as pure persuasion- in which $u(a, s) = a$ for all s so that party I wishes a to be as large as possible whatever the state. I will next discuss how the analysis should be modified for the more general specification $u(a, s) = -(a - b(s))^2$ where $b(s)$ -assumed to be strictly increasing- represents the ideal action of party I now allowed to vary with s.

A simple interpretation of the pure persuasion specification is that party I is a financial institution knowing the quality s of its asset, party U is an independent observer communicating with I and making some public announcement a about what she thinks the quality of i's asset is (and reputation considerations would lead party U to correctly announce what she thinks about s). More elaborate specifications of party I's objectives with state-dependent preferences allow to accommodate situations in which the informed party while willing the public announcement to be good would prefer it to be not too disconnected from reality.

Communication game.

In standard communication games à la Crawford and Sobel (1982), party I sends a message m to party U who then makes a decision a. Message m need not have any exogenous meaning in that approach. That is, the message space M need not be related to the state space S.

I consider the following modifications. I explicitly let all the states $s \in S$ be possible messages, that is $S \subseteq M$. When message $m = s$ is sent, it can be thought of as party I saying "The state is s.". I also allow party I to send messages outside S such as "I do not know the state" when everybody knows that I knows s, that is $M \setminus S \neq \varnothing$. Moreover, party I sends two messages m_1, $m_2 \in M$ one after the other at times $t = 1$ and 2 (that should be thought as being sufficiently far apart). When the two messages are the same $m_1 = m_2 = m$, party U is informed of m. When they are inconsistent in the sense that

$m_1 \neq m_2$, party U is only informed that $m_1 \neq m_2$. In all cases, party U chooses her action a based on what she is told about the communication phase. That is, $a(m)$ if $m_1 = m_2 = m$, and a_{inc} if $m_1 \neq m_2$.[5]

Having party I send two messages instead of one would make no difference if after sending message m_1, party I always remembered what message m_1 he previously sent and if both parties I and U were fully rational as usually assumed. While party U will be assumed to be rational, I consider situations in which party I at time $t = 2$ has imperfect memory about the message m_1 sent at time $t = 1$. More precisely, I assume that when party I with type s tells the truth at time $t = 1$, i.e. says $m_1 = s$, he remembers that $m_1 = s$ at $t = 2$, but when he lies and says $m_1 \neq s$ he does not remember what message m_1 he previously sent (he may still think that he sent $m_1 = s$ as I do not impose in the basic approach that he is aware that he lied). More sophisticated forms of imperfect memory can be considered, but the crucial feature here is the asymmetric nature of memory whether party I tells the truth or lies at $t = 1$. I will describe shortly how party I at time $t = 2$ forms his expectation about the message sent at $t = 1$ when he lied lie at $t = 1$, but before that let me complete the description of the communication game, which is perturbed as follows.

For some exogenous parameters $\varepsilon_1 > \varepsilon_2$ assumed to be small, party I's payoff as a function of the state (s, a, m_1, m_2) is represented as:

$$U_I(s, a, m_1, m_2) = u(a, s) - \varepsilon_1 1_{m_1 \neq s} - \varepsilon_2 1_{m_2 \neq s}. \quad (1)$$

That is, party I is assumed to have a slight preference for truth-telling and more so for the first time he communicates. It should be noted that the assumption $\varepsilon_1 < \varepsilon_2$ simplifies the analysis but is not needed for the derivation of equilibria employing pure strategies.[6] It can be motivated whenever the first message is (slightly) more likely than the second message to be made public, thereby making a lie at $t = 1$ a bit more costly than a lie at

[5] I have formulated the communication phase as one in which party U would not be present and would only be informed of some aspects of it (i.e. when $m_1 \neq m_2$). An alternative interpretation is that party U would be constrained to choose the same action when $m_1 \neq m_2$, which can be motivated on the ground that outside parties who judge party U would only be informed that there were inconsistencies in such a case (while being informed of the sent messages when consistent).

[6] In appendix, I show that the same equilibria in pure strategies arise when $\varepsilon_2 > \varepsilon_1$ (as when $\varepsilon_1 > \varepsilon_2$) in pure persuasion situations.

$t = 2$ for reputational reasons.

Moreover, with probability ε (again assumed to be small), party I in every state s tells the truth twice $m_1 = m_2 = s$ while optimizing on his communication strategy otherwise, i.e. with probability $1 - \varepsilon$.

I note that similar perturbations have been considered in the literature on strategic communication (see in particular Chen (2011) in a cheap talk context or Hart et al. (2017) in a certification context). While I would say such perturbations are natural, technically, they are used to rule out unnatural interpretations of unused messages that would correspond to a possible state (the truth-telling perturbation), and to ensure that unless party I sees a net material gain in lying he would refrain from doing so. I will discuss later where the assumption $\varepsilon_1 > \varepsilon_2$ simplifies the analysis.

In the analysis, I will present the results for the limiting case in which ε tends to 0 while keeping ε_1, ε_2 small but fixed. It is worth mentioning that if parties had perfect recall and were rational, one would get equilibrium outcomes identical to those arising in the strategic communication game of Crawford and Sobel (1982) in the limit as $\varepsilon, \varepsilon_1, \varepsilon_2$ go to 0. In particular, in the pure persuasion game scenario, there would be no information transmission, and the action a would be the mean value of s, $E(s)$, whatever the state. Departures from the standard cheap talk predictions will thus be caused by the imperfect memory of party I.

Solution concept.

To define the concept with forgetful liars, think of the state s as a type for party I, and envision party I with type s at times $t = 1$ and 2 as two different players $I_1(s)$ and $I_2(s)$ having the same preferences given by (1). To model the belief of a forgetful liar, let $\mu_1(m; s)$ denote the (equilibrium) probability with which message $m_1 = m$ is sent at $t = 1$ by party I with type s. Assuming that at least one type s lies with positive probability at $t = 1$, i.e. $\mu_1(m; s) > 0$ for at least one (m, s) with $m \neq s$, one can define the distribution of lies at $t = 1$ aggregating over all possible realizations of s. The corresponding probability of message m is

$$\sum_{s,\ s \neq m} \mu_1(m; s)p(s) / \sum_{(m',s'),\ m' \neq s'} \mu_1(m'; s')p(s'). \tag{2}$$

I assume that player $I_2(s)$ at time $t = 2$ after player $I_1(s)$ lied at time $t = 1$ believes that player $I_1(s)$ sent m with a probability given by (2). If no lie is ever made at time $t = 1$ in equilibrium, the belief after a lie can be arbitrary.[7] When a truthful message is sent by $I_1(s)$, player $I_2(s)$ remembers (or observes) that $m_1 = s$.

Apart from the (equilibrium) belief of $I_2(s)$ about m_1 after $I_1(s)$ sent $m_1 \neq s$ which was just defined, in equilibrium all expectations are correct and all players are requested to choose a best-response to their beliefs given their preferences as described above. In particular party I whether at time $t = 1$ or $t = 2$ has a perfect understanding of the action $a(m)$ chosen by party U when $m_1 = m_2 = m$ and the action a_{inc} chosen when $m_1 \neq m_2$. Given that party U is rational, $a(m)$ and a_{inc} correctly represent $E(s \mid m_1 = m_2 = m)$ and $E(s \mid m_1 \neq m_2)$ respectively, taking expectations with respect to the distribution of (m_1, m_2, s) as dictated by $p(s)$, $\mu_1(m; s)$ and $\mu_2(m; s, m_1)$ where $\mu_2(m; s, m_1)$ stands for the equilibrium probability with which player $I_2(s)$ chooses m at $t = 2$ when of type s after player $I_1(s)$ has sent m_1. If it turns out that no inconsistent messages $m_1 \neq m_2$ are sent in equilibrium, a_{inc} can be set arbitrarily.[8] I will also assume that when faced with the same belief and the same preference, which happens for all $I_2(s)$ after a lie at $t = 1$ in the pure persuasion case, the strategy is the same.[9]

In the next Sections, I characterize such equilibria that I refer to as equilibria with forgetful liars.

Comments.

1. I think of the belief of a liar as resulting from his access to a record of how lies are distributed in similar interactions without being told what the underlying state was when each specific lie was made. This is meant to capture situations in which one has a good sense of how typical the various possible lies are but no more precise information concerning the joint distribution of lies and states (for example, interpreting the state s as quality, one may have a good sense that most lies take either the form "the quality is this much" or "I don't know the quality" in equal proportion, without having a precise

[7] One could refine this by considering small trembles, but this is not required for the characterization of equilibria.

[8] Alternatively, one may request that a_{inc} is obtained as the limiting outcome of some perturbed strategy profile. I will consider such a scenario when considering state-dependent preferences for party I.

[9] One possible way to rationalize this is to add a stochastic payoff perturbation to the payoffs that would be state-independent (say it would depend only on the message).

view about the proportions of these two lies being made as a function of the true quality).

2. To elaborate on comment 1, the modelling of the belief of a forgetful liar is in the spirit of the analogy-based expectation equilibrium (Jehiel (2005), and Jehiel and Koessler (2008)) to the extent that the considered belief is the distribution of messages sent when there is a lie aggregating over all possible states. It is distinct from the more "rational" modelling in the spirit of Piccione and Rubinstein (1997) in which there would be no aggregation over states, and accordingly player $I_2(s)$ after player $I_1(s)$ lied would consider that m was sent with probability

$$\mu_1(m;s)/\sum_{m'\neq s}\mu_1(m';s). \tag{3}$$

To illustrate the difference, suppose that all players employ pure strategies. In this case, when player $I_1(s)$ lies at $t=1$, one would have $\mu_1(m;s)=1$ for one $m\neq s$ and $\mu_1(m';s)=0$ for all other $m'\neq m$. In the rational approach to forgetfulness, player $I_2(s)$ after $I_1(s)$ lied would know perfectly which lie was sent at $t=1$ even if not physically remembering the lie. This is because the mere knowledge of the state s together with the equilibrium inference allowed by the rational approach would leave no doubt to player $I_2(s)$ about how he (player $I_1(s)$) lied at $t=1$. By contrast, if several different lies are made by different types s in equilibrium, a forgetful liar would be doubtful in the approach considered here (as implied by (2)). I will indicate how the insights are changed when considering the rational approach to forgetful liars as opposed to the one defined above.

3. In the approach developed above, I assume that player $I_2(s)$ when a lie was sent by $I_1(s)$ is not aware that $I_1(s)$ lied and accordingly can assign positive probability to $m_1=s$ in his belief as defined in (2) if it turns out that $m_1=s$ is a lie made with positive probability by some type $s'\neq s$. If such a player $I_2(s)$ is aware he made a lie, it is then natural that he would rule out $m_1=s$ and a new definition of belief (just deleting all the lies such that $m_1=s$ in the aggregate distribution of lies should be considered instead). The equilibria characterized below with the above approach would remain equilibria with this modification. Whether there could be other equilibria in this variant should be the subject of further research.

3 Pure Persuasion Situations

In this Section, I assume that for all a and s, $u(a,s) = a$. That is, whatever the state s, party I wants the belief held by party U about the expected value of s to be as high as possible.

A simple class of strategies.

I consider the following family of communication strategies for party I referred to as (s_*, s^*)-communication strategies. Party I sends twice the same message $m_1(s) = m_2(s)$ whatever $s \in S$. There are two type $s^*, s_* \in S$ with $s^* > s_*$ such that all types $s \leq s_*$ lie and say s^* , i.e. $m_1(s) = m_2(s) = s^*$, and all types, $s > s_*$ say twice the truth, i.e. $m_1(s) = m_2(s) = s$.

If party I follows the (s_*, s^*)- strategy:

1) The aggregate distribution of lie at $t = 1$ is a mass point on s^*.

2) The best-response of party U is to choose $a(s) = s$ whenever $m_1 = m_2 = s \neq s^*$ and (approximately as ε goes to 0) $a(s^*) = a^E(s_*, s^*) = E(s \mid s \leq s_* \text{ or } s = s^*)$ when $m_1 = m_2 = s^*$. This follows because the action just defined corresponds to the true expected value of s after two consistent messages $m_1 = m_2 \in S$ (where this expectation for $m_1 = m_2 \leq s_*$ is pinned down by the trembling hand assumption stipulating that any type s sends a truthful message twice with probability ε).

3) If party I with type $s \leq s_*$ were to tell the truth $m_1 = m_2 = s$, he would induce action $a = s$ instead of $a^E(s_*, s^*)$. So a necessary condition for the communication strategy (s_*, s^*) to be part of an equilibrium is that (s_*, s^*) satisfies $a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_*$.

4) If party I with type $s > s_*$ were to lie at time $t = 1$, he would believe at time $t = 2$ that he said $m_1 = s^*$ according to the proposed solution concept. By lying and saying $m_1 = s^*$ at time $t = 1$, party I with type s could ensure to get $a^E(s_*, s^*)$ just assuming party $I_2(s)$ wants to avoid that inconsistent messages are being sent (it will be shown to be a necessary requirement in equilibrium). Thus, letting $s_*^+ = \min\{s_k \text{ such that } s_k > s_*\}$, another necessary condition for the strategy (s_*, s^*) to be part of an equilibrium is that (s_*, s^*) satisfies $s_*^+ \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2$.

Proposition 1 *Any equilibrium with forgetful liars in pure strategies either takes the form that no lie is being made by any type or it requests that party* I *uses an* (s_*, s^*)*-communication strategy for some* (s_*, s^*) *satisfying* $s_*^+ \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_*$.

To see that truth-telling can be part of an equilibrium, set $a_{inc} = 0$ and specify the belief of any $I_2(s)$ after a lie has been made at $t = 1$ to be that $m_1 = 0$ was sent with probability 1. It is readily verified that such specifications together with $a(s) = s$ for every $s \in S$ is part of an equilibrium with forgetful liars (noting that $I_1(s)$ by lying would expect to get $a = 0$ which is clearly no better than s).[10] Observe that in a truth-telling equilibrium, party U obtains the first-best, choosing $a = s$ when the state is s.

Turning to the second class of equilibria and to complete the proof that any (s_*, s^*)-communication strategy with $s_*^+ \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_*$ can be part of an equilibrium, specify that in case inconsistent messages would be sent $m_1 \neq m_2$,[11] some action $a_{inc} < a^E(s_*, s^*)$ would be chosen by party U so that sending inconsistent messages for party I would be worse than any action that can be taken on the equilibrium path. With such a specification of a_{inc}, party I would make any effort to avoid sending inconsistent messages, and thus after a lie of $I_1(s)$ at time $t = 1$, $I_2(s)$ would pick $m_2 = s^*$ at time $t = 2$, as he would believe in such an event that message $m_1 = s^*$ was sent with probability 1. Given the correct anticipation of $I_1(s)$' behavior, player $I_1(s)$ with $s \neq s^*$ would either lie and say $m_1 = s^*$ at time $t = 1$ or he would tell the truth $m_1 = s$: Given that $\left\{s \in S \text{ such that } a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s\right\}$ coincides with $\{s \in S \text{ such that } s \leq s_*\}$ whenever $s_*^+ \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_*$ the proposed strategies define an equilibrium.

It should be noted that when $s_* = s_1 = 0$ and $s^* = s_2$, we have that $s_*^+ = s_2 \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_* = s_1$ whenever $E(s = s_1 \text{ or } s_2) > \varepsilon_1 + \varepsilon_2$. Thus, there is always an equilibrium with party I using an (s_*, s^*)-communication strategy when ε_1 and ε_2 are small enough.

It should be noted that the above strategies would still define equilibria with forgetful liars in the case in which there would be no perturbations, i.e. $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0$. The

[10] It may be noted that such an equilibrium would not be robust to some trembling-hand perturbations in the communication strategy as such perturbations would make it impossible that I with type $s = 0$ is better off by telling the truth.

[11] Observe that no inconsistent messages are sent in the proposed communication strategy so that a_{inc} can be set freely.

perturbations will allow me to rule out other potential equilibria as I now explain. To this end, let me focus on equilibria in which there would be some lies (as equilibria with no lies trivially result in the first-best in which $a = s$ when the state is s). I first establish a few results that apply to all such equilibria whether in pure or in mixed strategies.

Lemma 1 *Suppose $I_1(s)$ tells the truth. Then party I with type s gets* $\max(a(m_1 = m_2 = s), a_{inc} - \varepsilon_2)$.

Proof. After $m_1 = s$, $I_2(s)$ would choose $m_2 = s$ if $a(m_1 = m_2 = s) \geq a_{inc} - \varepsilon_2$ and $m_2 \neq s$ otherwise yielding the result. ♣

Lemma 2 *Suppose m is a lie made with positive probability at time $t = 1$ by some $I_1(s)$, $s \neq m$. Then $a(m_1 = m_2 = m) \geq a_{inc} + \varepsilon_1$.*

Proof. Suppose by contradiction that $a(m) < a_{inc} + \varepsilon_1$ and $m_1(s) = m \neq s$. By saying $m_1 = m$, $I(s)$ gets $\max(a(m) - \varepsilon_1 - \varepsilon_2, a_{inc} - \varepsilon_1)$. By saying $m_1 = s$, $I(s)$ gets $\max(a(m_1 = m_2 = s), a_{inc} - \varepsilon_2)$ (see lemma 1). which is strictly larger than $a_{inc} - \varepsilon_1$ because $\varepsilon_1 > \varepsilon_2$. Thus, $I_1(s)$ cannot choose $m_1 = m$ providing the desired result. ♣

Lemma 3 *If $I_1(s)$ says $m_1 = m \neq s$ with positive probability, it must be that $I_2(s)$ finds it (weakly) best to say $m_2 = m$ so that $m_2 = m$ is said with positive probability.*

Proof. If $I_2(s)$ does not choose $m_2 = m$, $a_{inc} - \varepsilon_1$ would be obtained at best by $I_1(s)$, and by lemma 1, $I_1(s)$ would be strictly better off saying $m_1 = s$. ♣

The next lemma is specific to equilibria employing strategies.

Lemma 4 *In an equilibrium with forgetful liars employing pure strategies, there can be at most one lie.*

Proof. Suppose $I(s)$ lies and says $m_1 = m \neq s$ and $I(s')$ lies and says $m_1 = m' \neq s'$. By lemma 3, the same lie must be repeated at $t = 2$. $I(s)$ by saying $m_1 = m_2 = m$ gets $a(m) - \varepsilon_1 - \varepsilon_2$. If $m' \neq s$ and $I_1(s)$ says $m_1 = m'$, he must find $m_2 = m'$ optimal (as $I_2(s')$ finds $m_2 = m'$ optimal). Thus, he must pick $m_2 = m'$ (as does $I_2(s')$)[12] so that one

[12] This makes use of the requirement that $I_2(s)$ and $I_2(s')$ having the same preferences over $m_2 \neq s, s'$ should choose the same best-response.

should have $a(m) - \varepsilon_1 - \varepsilon_2 \geq a(m') - \varepsilon_1 - \varepsilon_2$. If $m' = s$ and $I_1(s)$ says $m_1 = m'$, then $I(s)$ gets at least $a(m')$. Thus, in all cases, $a(m) \geq a(m')$. By a symmetric argument, one should also have $a(m') \geq a(m)$, and thus $a(m) = a(m')$. I next observe that it cannot be that m' is equal to s as otherwise, $I(s)$ would strictly prefer telling the truth rather than saying $m_1 = m$. Thus, m and m' are both different from s and s' and $I_2(s)$ and $I_2(s')$ should thus pick the same m_2,[13] leading to a contradiction (since $I_2(s)$ should be saying m and $I_2(s')$ should be saying m'). ♣

Following lemma 4, I let m^* denote the unique lie made in an equilibrium with forgetful liars employing pure strategies.

The next lemma establishes that there can be no inconsistent messages in a pure strategy equilibrium.

Lemma 5 *There can be no inconsistent messages in equilibria employing pure strategies.*

Proof. Assume by contradiction that inconsistent messages can be sent in an equilibrium in pure strategy and call $S_{inc} = \{s \in S \text{ such that } m_1(s) \neq m_2(s)\}$. One should have $a_{inc} = E(s \in S_{inc})$ by the optimality of party U' strategy. Because lies are costly and more so at $t = 1$, if $s \in S_{inc}$, one should have $m_1(s) = s$. Moreover by lemma 2, $a(m^*) \geq a_{inc} + \varepsilon_1$, and thus, if $m^* \in S$, party I with type m^* after the truth being told at $t = 1$ would strictly prefer telling the truth at $t = 2$, thereby implying that $m^* \notin S_{inc}$. We thus have that $m^* \notin S_{inc}$ (whether or not $m^* \in S$). Consider $s_{inc}^{\max} = \max S_{inc}$. By telling the truth twice, $I(s_{inc}^{\max})$ gets $s_{inc}^{\max}$ (since $s_{inc}^{\max} \neq m^*$ and thus $a(s_{inc}^{\max})$ is pinned down by the truth-telling trembling behavior of $I(s_{inc}^{\max})$). Since $\max S_{inc} > E(s \in S_{inc}) - \varepsilon_2$, it must be that $I(s_{inc}^{\max})$ strictly prefers telling the truth, thereby implying the absurd conclusion $s_{inc}^{\max} \notin S_{inc}$. ♣

By lemma 2, it should be that $a(m^*)$ satisfies $a(m^*) \geq a_{inc} + \varepsilon_1$. Given that there is one lie m^*, the belief of $I_2(s)$ if $I_1(s)$ lies must be that $m_1 = m^*$ was sent with probability 1 at $t = 1$. Given that $a(m^*) \geq a_{inc} + \varepsilon_1 > a_{inc} + \varepsilon_2$, $I_2(s)$ would then find it strictly optimal to say $m_2 = m^*$. Given the expectation that when $I_1(s)$ lies, $I_2(s)$ says $m_2 = m^*$ and given that $a_{inc} < a(m^*)$, $I_1(s)$ if he lies, says $m_1 = m^*$. So for any s, either $I_1(s)$ tells

[13] This is again using the assumption that with the same preferences and the same beliefs, choices should be the same.

the truth $m_1 = s$ expecting to get $\max(a(m_1 = m_2 = s), a_{inc} - \varepsilon_2)$ by lemma 1 or lies and says $m_1 = m^*$ expecting to get $a(m^*) - \varepsilon_1 - \varepsilon_2$.

To sum up, for any $s \neq m^*$, the choice of $I(s)$ is between truth-telling resulting in $a(s) = s$ (because s is not in the support of equilibrium lie) or lying twice according to $m_1 = m_2 = m^*$ resulting in $a(m^*) - \varepsilon_1 - \varepsilon_2$ where $a(m^*) = E(s \in S^*)$ with $S^* = \{s \in S \text{ such that } I_1(s) \text{ says } m_1 = m^*\}$ (using the best-response of party U). And $I(m^*)$ can do no better than telling the truth (as a lie at $t = 1$ would result in $a_{inc} - \varepsilon_1$ and $a_{inc} - \varepsilon_1 \leq a(m^*)$). I let s^* denote $\max\{s \in S^*\}$.

If S^* consist only of s^* then $s^* = 0$ as otherwise any $s < s^* = E(s \in S^*)$ would strictly prefer to lie as does s^* contradicting the premise that S^* is a singleton. But when $s^* = 0$, party I with type s^* would strictly prefer telling the truth due to the $-\varepsilon_1 1_{m_1 \neq s^*} - \varepsilon_2 1_{m_2 \neq s^*}$ terms, violating the premise that $I(s^*)$ is lying.

If S^* contains at least two s_k and if $s^* \neq m^*$, then party I with type s^* would strictly prefer $a = s^*$ to $a(m^*) = E(s \in S^*)$ leading him to tell the truth rather m^* at $t = 1$ in contradiction with the equilibrium assumption.

Thus, it must be that S^* contains at least two states and the lie m^* is the maximal element s^*. The requirement that for $s \neq s^*$, $I(s)$ lies and says m^* whenever $a(m^*) - \varepsilon_1 - \varepsilon_2 > s$ ensures that S^* takes the form $\{s \in S, s \leq s_*\} \cup \{s^*\}$ for some s_* with s_* being the largest s in S such that $s < E(s \in S^*) - \varepsilon_1 - \varepsilon_2$. This precisely corresponds to the (s_*, s^*)-communication strategy with $s_*^+ \geq a^E(s_*, s^*) - \varepsilon_1 - \varepsilon_2 \geq s_*$, as required to complete the proof of the Proposition.

Comment. I have used $\varepsilon_1 > \varepsilon_2$ to ensure that if inconsistent messages are being sent by $I(s)$ in a pure strategy equilibrium, a truthful message ($m_1 = s$) would be sent at $t = 1$. Using also that there can be only one consistent lie in equilibrium, this has allowed me to obtain using a standard unravelling argument, that there can be no inconsistent lie in equilibrium. In Appendix, I consider the case in which $\varepsilon_1 < \varepsilon_2$ and observe that no other pure strategy equilibrium can be obtained.

3.1 Mixed strategy equilibria

Suppose several lies m_k^*, $k = 1, ..K$, are made in equilibrium. Let a_k denote $a(m_k^*)$ and μ_k the probability with which m_k^* is sent at $t = 1$ conditional on a lie being sent then

$(m_1 \neq s)$. By lemma 3 it should be that when $I_1(s)$ lied at $t = 1$ and said m_k^*, $I_2(s)$ finds it optimal to say m_k^*. This implies that:

Lemma 6 *m_k cannot be one of one of the state $s \in S$ such that $I_1(s)$ lies with strictly positive probability.*

Proof. Assume by contradiction that $I_1(s)$ lies and says m_k^* with positive probability, $I_1(s')$ lies and says $m_{k'}^*$ with positive probability and $m_{k'}^* = s$. For $I_2(s')$ to find it optimal to say $m_2 = m_{k'}^*$, one should have $\mu_{k'} a_{k'} + (1 - \mu_{k'}) a_{inc} \geq \mu_k a_k + (1 - \mu_k) a_{inc}$. But, then $I_2(s)$ would strictly prefer saying $m_2 = m_{k'}^*$ so as to save the $\varepsilon_2 1_{m_2 \neq s}$ obtained when $m_2 = m_k^*$. As a result, $I_2(s)$ would never find it optimal to say $m_2 = m_k^*$, violating lemma 3. ♣

The optimality to repeat the same lie also imposes that $\mu_k a_k + (1 - \mu_k) a_{inc}$ be the same for all k. Let a^* denote this constant. Let also denote by μ_k^2 the common probability of saying m_k^* at $t = 2$ when a lie was made at $t = 1$.[14] Given that all lies m_k^* must be chosen at $t = 1$ with positive probability, this imposes that $\mu_k^2 a_k + (1 - \mu_k^2) a_{inc}$ should be the same for all k which together with the constraint that $\sum_k \mu_k = \sum_k \mu_k^2 = 1$ imposes that $\mu_k^2 = \mu_k$ for all k.

It is readily verified that $m_k^* \in S$ as otherwise party I with the maximum type $\overline{s}_k^*$ among those who send $m_1 = m_k^*$ with positive probability at $t = 1$ would strictly prefer telling the truth (this makes use of the fact that $a_k \leq \overline{s}_k^*$).

Moreover, take any s other than m_k^* for $k = 1, ..K$. If $s < a^* - \varepsilon_1 - \varepsilon_2$, $I_1(s)$ would strictly prefer saying any m_k^* expecting to get $a^* - \varepsilon_1 - \varepsilon_2$ rather than the truth that would only yield s. If $s > a^* - \varepsilon_1 - \varepsilon_2$, $I_1(s)$ would strictly prefer telling the truth rather than lying. On the other hand, any $I_1(m_k^*)$ would go for telling the truth (using the ε_1 preference for truth telling at $t = 1$ and the observation that a lie would not induce a higher expected action). Moreover, for all k, one must have $m_k^* > a^*$ as otherwise lemma 6 would be violated.

The above properties imply that an equilibrium in mixed strategy would take the following form.

[14] That μ_k^2 is common follows from the requirement that with identical preferences and identical beliefs, the strategy should be the same.

Proposition 2 *Any mixed strategy equilibrium with forgetful liars is characterized by. For some* a^*, $m_k^* > a^*$, $k = 1...K$, *and* $\mu_k > 0$, *with* $\sum_k \mu_k = 1$
$\mu_k a_k + (1 - \mu_k)a_{inc} = a^*$
$a_{inc} = E(s \in S, s < a^*)$
$a_k = \frac{\mu_k \Pr(s \in S, s < a^*) a_{inc} + p(m_k^*) m_k^*}{\mu_k \Pr(s \in S, s < a^*) + 1}$
$a(m_1 = m_2 = s) = s$ *for* $s \in S$, $s \neq m_k^*$, $k = 1, ...K$.
$I_t(s)$ *with* $s < a^* - \varepsilon_1 - \varepsilon_2$ *says* m_k^* *with probability* μ_k *for* $t = 1, 2$
$I_t(s)$ *with* $s > a^* - \varepsilon_1 - \varepsilon_2$ *says the truth at* $t = 1, 2$.

Observe that when $K = 1$, the conditions shown in Proposition 2 boil down to those in Proposition 1. Moreover, unlike for the equilibria in pure strategies, there are inconsistent messages being sent in mixed strategy equilibria explaining why a_{inc} is pinned down in such equilibria.

Remarks: 1) Mixed strategy equilibria can be constructed for any a^* and m_k^* small enough whenever ε_1 and ε_2 are small enough. 2) If preferences were perturbed to say that more distant lies are less preferred than less distant ones, the mixed strategy equilibria shown in Proposition 2 would disappear in contrast to the pure strategy equilibria shown in Proposition 1. Whether one can obtain other mixed strategy equilibria in this case would depend on the exact form of the perturbation. 3) Such equilibria remain equilibria when $\varepsilon_1 < \varepsilon_2$. Whether other mixed strategy equilibria can be sustained in this case requires additional analysis.[15]

3.2 Approximate first-best with fine grid

So far, types s_k could be distributed arbitrarily on $[0, 1]$. What about the case when consecutive types are close to each other and all types have a comparable ex ante probability? I show that in such a case, all equilibria are close to the truth-telling equilibrium, resulting in the approximate first-best outcome for party U.

To show this formally, let me simplify the above setting by assuming that $p(s_k) = 1/n$ for all k (extension to the case in which $p(s_k)/p(s_{k'})$ would remain bounded uniformly over all sequences indexed by n would raise no conceptual difficulties).

[15] I have used $\varepsilon_1 > \varepsilon_2$ to be able to apply Lemma 3.

Definition 1 *A state space $S^n = \{s_1, ...s_n\}$ satisfies the n-fine grid property if for all k, $\frac{1}{2n} < s_{k+1} - s_k < \frac{2}{n}$.*

Proposition 3 *Consider a sequence $(S^n)_{n=\underline{n}}^{\infty}$ of state spaces such that, for each n, S^n satisfies the n-fine grid property. Consider a sequence $(\sigma^n)_{n=\underline{n}}^{\infty}$ of equilibria with forgetful liars associated with S^n. For any $\widehat{a} > 0$, there exists $\overline{n}$ such that for all $n > \overline{n}$, the equilibrium action of party U after a lie prescribed by σ^n is smaller than $\widehat{a}$. As n approaches ∞, the expected utility of party U approaches the first-best (i.e. converges to 0).*

To prove Proposition 3, I make use of the characterization results of Propositions 1 and 2. If the action a^* after two consistent lies (in an equilibrium either in pure or in mixed strategies) is significantly away from 0, say bigger than $\widehat{a}$ assumed to be strictly positive, then under the fine grid property the expectation of s over the set of s that either lie below a^* or else $s = 1$ must be significantly below a^* (at a distance at least $\frac{2}{n}$) but then some s_k strictly below a^* would strictly prefer telling the truth rather than lying undermining the construction of the equilibrium (that requires party $I(s)$ with $s < a^*$ to be lying). This argument shows that the action a^* after two consecutive lies must get close to 0 as n approaches ∞, thereby paving the way to prove Proposition 3. The detailed argument appears in the appendix.

3.3 Link to other approaches

3.3.1 Equilibria with standard approach

I wish to investigate how the analysis of equilibria would be affected if considering the (more standard) approach to forgetfulness according to which a forgetful liar would know the distribution of lies conditional on the state (and not just in aggregate over the various states as assumed above, see expression (3)).

While the equilibria arising with the main proposed approach can still arise with this alternative approach, the main observation is that many more equilibrium outcomes can be sustained now. In particular, even in the fine grid case, equilibrium outcomes significantly away from the first-best can now be supported. To illustrate this, I focus on equilibria employing pure strategies. Consider a setup with an even number n of states and a pairing of states according to $S_k = \{\underline{s}_k, \overline{s}_k\}$ with $(S_k)_k$ being a partition of the state space and $\underline{s}_k < \overline{s}_k$ for all k. I claim that with the standard approach, one can support an equilibrium in which for every k, $I_t(\underline{s}_k)$ lies and sends twice $m_t = \overline{s}_k$ while $I(\overline{s}_k)$ tells the truth. To complete the description of the equilibrium, one should specify that $a(m_1 = m_2 = \overline{s}_k) = E(s \in S_k)$, for example require that $a_{inc} = 0$ so that party I whatever his type is not tempted to send inconsistent messages, and that the belief of $I_2(\overline{s}_k)$ if $I_1(\overline{s}_k)$ were to lie is that message 0 was sent at $t = 1$ (many other specifications would do, as explained below).

The key reason why such an equilibrium can arise is that under the considered expectation assumptions, when $I_1(\underline{s}_k)$ lies at $t = 1$, player $I_2(\underline{s}_k)$ believes that player $I_1(\underline{s}_k)$ said $m_1 = \overline{s}_k$ given that this is the only lie made by $I_1(\underline{s}_k)$ in equilibrium. As a result, player $I_2(\underline{s}_k)$ after a lie at $t = 1$ would find it optimal to say $m_2 = \overline{s}_k$ as any other message would be perceived to trigger a_{inc}, and $a_{inc} < E(s \in S_k)$. Given that $I_1(\underline{s}_k)$ has correct expectation about $I_2(\underline{s}_k)$' strategy, $I_1(\underline{s}_k)$ would either lie and say $m_1 = \overline{s}_k$ or he would tell the truth. Given that $E(s \in S_k) > \underline{s}_k$, he strictly prefers lying (whenever ε_1, ε_2 are small enough, i.e. $\varepsilon_1 + \varepsilon_2 < E(s \in S_k) - \underline{s}_k$), thereby showing the optimality of $I_t(\underline{s}_k)$' strategy for $t = 1, 2$. Showing the optimality of $I_t(\overline{s}_k)$' strategy is easily obtained using the off-path beliefs proposed above.

One may be willing to refine the off-path beliefs of $I_2(\overline{s}_k)$ in the above construction for example by requiring that a lie $m_1 = 1$ (instead of $m_1 = 0$) is more likely to occur when $I_1(\overline{s}_k)$ lied (and $\overline{s}_k \neq 1$). Note that the above proposed strategies would remain part of an equilibrium with this extra perturbation, assuming that $\{0, 1\}$ is one of the pairs S_k and $E(s = 0 \text{ or } 1)$ takes the smallest value among all $E(s \in S_k)$ (think of assigning sufficient weight on the state being $s = 0$). Indeed, in such a scenario, if $I_1(\overline{s}_k)$ were to lie, he would say $m_1 = 1$ anticipating that $I_2(\overline{s}_k)$ would say $m_2 = 1$ next, and this would be worse than truth-telling.

The key reason why multiple lies can be sustained now and not previously is that the belief of $I_2(\underline{s}_k)$ after a lie at $t = 1$ now depends on $\underline{s}_k$ given that the mere memory of the state $\underline{s}_k$ together with the equilibrium knowledge of the strategy of $I_1(\underline{s}_k)$ allows player $I_2(\underline{s}_k)$ to recover the lie made by $I_1(\underline{s}_k)$ even if he does not directly remember m_1.

It is also readily verified that such equilibrium outcomes can lead party U to get payoffs bounded away from the first-best, even in the fine grid case as the number of states gets large (think for example, of the limit pairing of s and $1-s$ in the uniform case that would result in party U choosing approximately action $a = \frac{1}{2}$ in all states, which corresponds to what she would do in the absence of any communication).

3.3.2 Models of Certification

Classic models of persuasion assume that the informed party I can only lie by omission in which case the so called unravelling argument yields that in any Perfect Bayesian Equilibrium (PBE), party I with type $s > 0$ tells all what he knows. Such models have been extended by Dye (1985) to allow for the possibility that party I would be uninformed and that party I would not be able to prove (certify) to be uninformed. In such a scenario, sufficiently bad types (i.e. types below a threshold determined through a fixed point argument in equilibrium) have an incentive to pretend they are uninformed while other informed types disclose their type.

The equilibrium analysis of Proposition 1 bears some similarity with the equilibrium appearing in Dye's analysis, interpreting $s = s^*$ in the (s_*, s^*)-communication strategy as the type in S that cannot be certified whereas all other types could be certified. Of course, a key difference is that no type in the above setting is a priori assumed to be able to certify what he knows, but the same (s_*, s^*)-communication strategy as shown in Proposition 1 would arise as a PBE of the one shot communication game in which all types $s \neq s^*$ can be certified and type s^* cannot. Hence the memory asymmetry between liars and truth-tellers lead endogenously some types to be unable to certify their type (those types which correspond to lies in the equilibrium) while those types who would not correspond to any lie would be able to prove their type simply by telling the truth twice (the perturbation according to which any type tells the truth with probability ε is being used here). The additional insights of Proposition 1 are that 1) There can only be at most one lie in an equilibrium using pure strategies (making the analysis close to that of Dye in which party I cannot certify his type only when he is uninformed) and 2) Endogenize what this unique lie can possibly be in equilibrium.

4 State-dependent objectives

I consider now alternative specifications of party I's preferences in which I's blisspoint action may now depend on the state. Specifically, $u(a,s) = -(a - b(s))^2$ where $b(s)$ is assumed to be increasing with s, and $b(s_k) > \frac{s_k + s_{k+1}}{2}$ for every $k < N$. I wish to characterize the equilibria with forgetful liars as defined above restricting attention to pure strategy equilibria (so as to avoid getting into intricate technicalities).

The main observation is that with such specifications, multiple lies may arise in equilibrium, as long as the various states are not too close to each other (i.e., the non-fine grid case). The key reason why multiple equilibria may arise is that even though party I is still willing to avoid sending inconsistent messages, because the objective of party I is state-dependent, party I at $t = 2$ may end up choosing different messages as a function of the state, even if exposed to the same belief about what the first message was in case of lie at $t = 1$. This in turn allows party I at $t = 1$ to engage in different lies as a function of the state, thereby leading to equilibria with multiple lies. Beyond the observation that multiple lies can be sustained, I also provide a characterization of all equilibria with forgetful liars employing pure strategies, which happen to have a simple structure (somehow borrowing features both from the analysis of cheap talk games and of the analysis of certification games).

To simplify the analysis, I will assume that for any two distinct pairs (N_1, N_1') and (N_2, N_2') such that N_1, N_1', N_2, N_2' are subsets of $N = \{1, ..., n\}$, it is not the case that $p(N_1)E(s_k,\ k \in N_1') = p(N_2)E(s_k,\ k \in N_2')$ where $p(N_i)$ consists of the sum of p_k for $k \in N_i$ (such a condition is satisfied for generic specifications of the type space). I will also further perturb the description of the communication game as defined above by assuming that with a tiny probability ε_0, party I with type $s = 0$ randomizes over all possible messages in an independent way at $t = 1$ and 2, which will allow me to pin down the equilibrium value of a_{inc}, the action chosen by party U when inconsistent messages are being sent (no essential qualitative features of the equilibria shown below depend on this extra perturbation). The other perturbations parameterized by $\varepsilon, \varepsilon_1 > \varepsilon_2$ are maintained, and I will be concerned in the rest of this Section with describing the set of pure strategy equilibria in the limit in which ε, ε_0, ε_1, ε_2 (with $\varepsilon_1 > \varepsilon_2$) as well as $\varepsilon_0/\varepsilon$ go to 0.

Consider a pure strategy equilibrium with forgetful liars. As for pure persuasion games,

$\varepsilon_1 > \varepsilon_2$ guarantees that if party $I(s)$ is to engage into sending inconsistent messages, he would first tell the truth and then lie. Let m_k^* denote a consistent lie made by at least one type $s \neq m_k^*$, i.e. party I with type s sends twice the message m_k^*, and assume there are K different such lies in equilibrium. Define then L_k as the set of types s such that party I with type s sends twice m_k^*, i.e. $m_1 = m_2 = m_k^*$ (this includes those types who lie and say consistently m_k^* and possibly type $s = m_k^*$ if this type tells the truth), and let $L = (L_k)_k$. Clearly, in such an equilibrium, after the message m_k^* has been sent twice, party U would (approximately as ε goes to 0) choose $a_k = E(s \in L_k)$. I let $\overline{s}_k$ denote $\max L_k$ and observe that $\overline{s}_k$ should be one of the consistent lies m_r^* for $r = 1..., K$:

Lemma 7 *For all k, $\overline{s}_k = \max L_k$ should be a consistent lie.*

Proof. Suppose this is not the case. Then party I with type $\overline{s}_k$ would induce action $a = \overline{s}_k$ by telling twice the truth. This would be strictly better than what he obtains by saying twice m_k^*, which gives action $a_k = E(s \in L_k) \leq \overline{s}_k = \max L_k$ (and inflicts an extra $\varepsilon_1 + \varepsilon_2$ penalty for not telling the truth - this is needed to take care of the case in which L_k would consist of $\overline{s}_k$ only). ♣

A simple implication of lemma 7 is:

Corollary 1 *There is a bijection between $\{L_1, ...L_K\}$ and $\{\overline{s}_1, ...\overline{s}_K\}$.*

Another observation similar to that obtained in pure persuasion games is:

Lemma 8 *There can be no (voluntary) inconsistent lie in equilibrium made by any type $s \neq 0$, which in turn implies that $a_{inc} = 0$.*

Proof. Assume by contradiction that there are voluntary inconsistent lies in equilibrium made by at least one type $s \neq 0$. As already noted, party I with such a type s would first tell the truth $m_1 = s$ and then lie to $m_2 \neq s$. Consider $\overline{s}_{inc} = \max\{s \text{ such that } m_1(s) \neq m_2(s)\}$. $\overline{s}_{inc}$ is not one of the m_k^* because $\overline{s}_{inc}$ is none of the $\overline{s}_k$ and Corollary 1 holds. It follows that party I with type $\overline{s}_{inc}$ would be strictly better off by telling twice the truth rather than by sending inconsistent messages (this makes use of the perturbation $-\varepsilon_2 1_{m_2 \neq s}$ when there is only one s sending inconsistent messages), thereby leading to a contradiction. That $a_{inc} = 0$ follows then from the perturbation that was assumed on the communication strategy of $s = 0$. ♣

Let μ_k denote the overall probability (aggregating over all s) with which m_k^* is sent at $t = 1$ conditional on a lie being sent then ($m_1 \neq s$). Without loss of generality reorder the k so that $\mu_k a_k$ increases with k. The single crossing property of $u(a, s)$ implies that:

Lemma 9 *For any $k_1 < k_2$, if in equilibrium $I(s)$ makes the consistent lie $m_{k_1}^*$ and $I(s')$ makes the consistent lie $m_{k_2}^*$, it must be that $s < s'$. Moreover, for every k, it must be that the consistent lie m_k^* in L_k coincides with* $\max L_k$, *i.e.* $\overline{s}_k = m_k^*$.

Proof. For the first part, note that after a lie, player $I_2(s)$ would say $m_2 = m_{k(s)}^*$ where

$$\begin{aligned} k(s) &= \arg\max_k v(k, s) \text{ and} \\ v(k, s) &= -\mu_k (a_k - b(s))^2 - (1 - \mu_k)(a_{inc} - b(s))^2. \end{aligned}$$

Given that $a_{inc} = 0$, and $\mu_1 a_1 < \mu_2 a_2 ... < \mu_K a_K$ (they cannot be equal by the genericity assumption), it is readily verified that for any $s_1 < s_2$, and $k_1 < k_2$, if $v(k_2, s_1) > v(k_1, s_1)$ then $v(k_2, s_2) > v(k_1, s_2)$.[16]

Thus if party I with type s_2 finds lie $m_{k_2}^*$ optimal, he must find it better than $m_{k_1}^*$ and thus by the property just noted, party I with any type $s > s_2$ must also find $m_{k_2}^*$ better than $m_{k_1}^*$, making it impossible that he finds $m_{k_1}^*$ optimal.

To show the second part ($\overline{s}_k = m_k^*$), I make use of Corollary 1 to establish that if it were not the case there would exist an increasing sequence $k_1 < k_2 ... < k_J$ such that type $\overline{s}_{k_j}$ would lie and say $\overline{s}_{k_{j+1}}$ for $j < J$ and $\overline{s}_{k_J}$ would lie and say $\overline{s}_{k_1}$, which would violate the property just established. ♣

To complete the description of equilibria, let $L_k^- = L_k \backslash \{m_k^*\}$ where $m_k^* = \overline{s}_k = \max L_k$; $p(L_k^-)$ denote the probability that $s \in L_k^-$; $\mu_k(L) = \frac{p(L_k^-)}{\sum_r p(L_r^-)}$ the probability that the lie m_k^* is made at $t = 1$ in the aggregate distribution of lies at $t = 1$; $k(s) = \arg\max_k v(k, s)$ where $v(k, s) = -\mu_k (a_k - b(s))^2 - (1 - \mu_k)(b(s))^2$ and $a_k(L) = E(s \in L_k)$. Realizing

[16]This makes use of $(v(k_2, s_2) - v(k_1, s_2)) - (v(k_2, s_1) - v(k_1, s_1)) = 2(\mu_{k_2} a_{k_2} - \mu_{k_1} a_{k_1})(b(s_2) - b(s_1))$ noting that $b(s_2) > b(s_1)$.

that party I with a type s that lies outside $\{m_1^*, ...m_K^*\}$ will either tell the truth or lie and say $m_{k(s)}^*$ depending on what he likes best and that by Lemma 9 party I with type $\overline{s}_k = m_k^*$ should prefer telling the truth to lying by saying $m_{k(\overline{s}_k)}^*$, the following proposition characterizes the equilibria with forgetful liars in pure strategies.

Proposition 4 *There always exists an equilibrium with forgetful liars in pure strategies and any such equilibrium is characterized by the following conditions. There is a disjoint family of lie sets $L = (L_k)_{k=1}^K$, with $L_1^- < \cdots < L_K^-$, $\mu_1(L)a_1(L) < \cdots < \mu_K(L)a_K(L)$, and $|a_k(L) - b(\overline{s}_k)| \leq \left|a_{k(\overline{s}_k)}(L) - b(\overline{s}_k)\right|$ such that 1) Party I with type $s \in S \setminus \{\overline{s}_1, ...\overline{s}_K\}$ lies twice by saying $m_1 = m_2 = m_{k(s)}^*$ whenever $\left|a_{k(s)}(L) - b(s)\right| < |s - b(s)|$ and tells the truth twice otherwise; 2) Party I with type $s = \overline{s}_k$ tells twice the truth; 3) Party U when hearing inconsistent messages chooses $a_{inc} = 0$; when hearing $m_1 = m_2 = m_k^*$ chooses $a = a_k(L)$; when hearing $m_1 = m_2 = s \in S \setminus \{m_1^*, ...m_K^*\}$ chooses $a = s$; and when hearing any other consistent messages chooses $a = 0$.*

Proof. That equilibria with some consistent lies must take the form shown in the Proposition follows from the arguments that precede its statement together with the observation that with the considered strategies and the ε_0-perturbation of the strategy of party I with type $s = 0$, if consistent messages not in S are received, they must come from type $s = 0$, thereby leading party U to choose $a = 0$ in such events.

To show that there exists an equilibrium in pure strategies with some consistent lies, think of having a unique lie set, $K = 1$, and set $L_1 = \{s_1, s_2\}$ with the lie being $m_1^* = s_2$, and consider the strategies as specified in the proposition. It is readily verified that all the required conditions for equilibrium are satisfied.

That there can be no equilibrium with no consistent lie follows from the observation that in such a case (due to the ε_0 perturbation of the strategy of party I with type $s = 0$), the support of equilibrium consistent lies would assign equal probability to all messages and player $I_1(s)$ would then strictly prefer lying to the message that corresponds to the type $s_k \in S$ that is closest to $b(s)$ anticipating that player $I_2(s)$ will make the same lie (I am using here that $b(s_k) > \frac{s_k + s_{k+1}}{2}$ to ensure that every type would like to be confused with a higher type if possible). ♣

The existence of equilibria with forgetful liars in pure strategies was obtained exhibiting an equilibrium with just one lie. A question not addressed in the above Proposition is whether it is possible to sustain equilibria with multiple lies, which is the subject of the next example.

An example with multiple lies.

Consider the two lie scenario with $b(s) = s+\beta$ (β not too large), $L_1 = (S \cap [0, s_*]) \cup \{\overline{s}_1\}$ with $s_* = E(s \in L_1)$, $L_2 = (S \cap [\underline{s}, \overline{s}]) \cup \{\overline{s}_2\}$ with $\overline{s} = E(s \in L_2)$ and $\underline{s} = \overline{s} - 2\beta$ (so that $|\underline{s} - b(\underline{s})| = |\overline{s} - b(\underline{s})|$). Clearly, if β is not too large, $p(L_1^-)$ and $p(L_2^-)$ are not too far from each other and $\overline{s}$ can be chosen large enough so that no type in L_1 would be tempted by the lie $m_2^* = \overline{s}_2$ and no type in L_2 would be tempted by the lie $m_1^* = \overline{s}_1$ (this imposes that $\overline{s}_1 < \underline{s}$). The remaining conditions ensure that every type chooses what is best for him between his most preferred lie from $\{m_1^*, m_2^*\}$ and truth-telling. It is readily verified that one can adjust s_k , p_k, and β so that the above conditions are satisfied, and thus one can construct such an equilibrium with two lies. Multiple lies can arise because even though party I is willing to avoid making inconsistent lies, the mere dependence of I's objective in s ensures that when s is small enough, only the lie $\overline{s}_1$ would be considered (both at $t = 1$ and 2) and when s is large enough, only the lie $\overline{s}_2$ would be considered. Making this construction possible requires that the two lie sets be sufficiently far apart so that no type in one lie set would be tempted by making the lie corresponding to the other lie set.

First-best with fine grid.

While multiple lies can arise as shown through the previous example, in the fine grid case with uniform distribution over the s_k in S (as defined in the pure persuasion case), it is not possible to sustain equilibria with multiple lies. In the context of the example just constructed, the problem is that in the fine grid case, it would not be possible to ensure that $\overline{s} = E(s \in L_2)$ while $\underline{s} = \overline{s} - 2\beta$ because $L_2 = (S \cap [\underline{s}, \overline{s}]) \cup \{\overline{s}_2\}$ would contain too many states below $\overline{s}$ and only one above (with sparse state space, this is however possible).

More generally, coming back to the general characterization shown in Proposition 4, in the fine grid case, all a_k must be approaching 0 as otherwise too many s_k smaller than a_k would be willing to join the lie $m_k^* = \overline{s}_k$ making it impossible to have that $a_k = E(s \in L_k)$.

As a result, in the fine grid case, assuming that $b(s) \geq s + \beta$ for some $\beta > 0$, there can only be one lie in a pure strategy equilibrium, and the first-best for party U is being approached in the limit. This is similar to what was obtained in the pure persuasion case.

Comment. When multiple lies m_k^* can be sustained, on can view the corresponding equilibrium as being analogous to the PBE that would arise in the one shot communication game in which all types except those corresponding to lies m_k^* could be certified whereas types m_k^* could not be certified. Such a richer certification setup falls in the general framework defined in Green and Laffont (1986) or Okuno-Fujiwara and Postlewaite (1990) with again the observation that here what a type can certify depends on the equilibrium strategies of lies and is thus endogenously determined.

5 Discussion

Mechanism design and commitment

The above communication game had no commitment component. Could the uninformed party U benefit from committing to some courses of actions that would be contingent on what happens in the communication stage? This shifts the perspective to that of mechanism design. I note that in a word with forgetful liars, the revelation principle does not apply. Party U can do better than just asking party I to send a report about the state s no matter what she commits to. Indeed, the resulting outcome in such (one-shot) mechanisms would never approach the first-best when there are conflicts of interest.[17] By contrast, in the fine grid case, it was observed that the first-best could be approximated in all equilibria with forgetful liars of the above two-round communication game. The idea that it may help to increase the size of the message space (as results from multi-round communications) also appears in Deneckere and Severinov (2017) who assume the informed party incurs a cost by misrepresenting his information. Not remembering one own's lie together with the harmful outcome in case of inconsistency results in a cost of misrepresentations explaining the link between the two approaches. The idea that it be-

[17] Any such outcome can be thought of as resulting from a delegation game in which party I chooses the action from some subset of S optimally chosen by party U. No matter what this subset is, one cannot be close to the first-best.

comes more complex to lie when asked several times about the same information can also be related to Glazer and Rubinstein (2014) who investigate how complex questionnaires may help the uninformed principal extract more information from the informed agent assumed to be boundedly rational. But the constraints imposed on the cognitive ability of the informed party and the response of the uninformed party in terms of making the communication more complex are of a different nature here and in Glazer and Rubinstein (2014), making the comparison between the two works not so clear.

Even if one cannot apply the revelation principle due the imperfect memory of liars, it may still be of interest to explore whether commitment may be helpful or not (this parallels a question addressed in a context with certification by Hart, Kremer and Perry (2017) or Ben-Porath et al. (2017) following Glazer and Rubinstein (2004)).

In the context of two round communications with two consecutive messages m_1 and m_2 to be sent by party I, a simple idea that comes to mind is that party U commits to a detrimental action (say $a = 0$) if $m_1 \neq m_2$ and to some action $a(m)$ in case $m_1 = m_2 = m$ where now $a(m)$ can be freely chosen ex ante. In the pure persuasion context, I note that if $a(1) = 1$ (which is the action arising in the equilibria shown above when $s = 1$), an equilibrium with forgetful liars in the induced game is that whatever s, $I(s)$ sends twice $m_1 = m_2 = 1$ resulting in action $a = 1$ for all states (and such an equilibrium arises also in the fine grid case). The resulting outcome is very bad for party U, and it could not possibly arise in the context of the communication game studied above. It arises now and not before because party I whatever the state $s < 1$ is better off with the lies $m_1 = m_2 = 1$ rather than with any other strategy (this was no so for large s in the main communication game). Considering the outcome of the main communication game in the fine grid case (where approximate first-best was obtained) illustrates that by not committing to specific choices of actions, party U is able to rule out most of the undesirable equilibrium outcomes that would arise if party U had pre-committed to a fixed schedule of actions as a function of the message profiles. This seems to offer a novel perspective on the strong implementation agenda when considering forgetful liars, as considered in this paper.[18]

[18]This seems different from what happens in a standard certification setting in which the equilibrium outcome can always be replicated as the only possible outcome in a mechanism in which party U would pre-commit to playing as in the original equilibrium. The fundamental difference here is that whether a

Partial memory of lies

In the above analysis, I have assumed that when party I lies at $t = 1$, he has no memory at all of m_1 at $t = 2$. As a natural extension, one may consider a less extreme situation in which party I would have at $t = 2$ a partial memory of m_1. This could be modelled by assuming that in case of lie, party I at $t = 2$ receives a noisy signal η about m_1. Party I at time $t = 2$ after a lie at $t = 1$ would then form an updated belief about m_1 taking both into account the signal η and the aggregate distribution of lies (serving here the role of the prior). In such a situation, if there are several lies being made in equilibrium, then it may well be for some signals η that after a lie m_1^* at $t = 1$, party I is led at time $t = 2$ to believe that he is more likely to have sent another lie m_2^*. If one considers a setting in which party U can commit to very detrimental actions in case of inconsistency, this would lead party I at time $t = 2$ to choose $m_2 = m_2^*$ then, which would result for party I in a poor expected consequence of engaging into the lie m_1^* at time $t = 1$. Under natural specifications of the signal structure, such considerations imply that it would not be possible to support multiple lies in equilibrium even in the payoff specification considered in Section 4. Clearly, the equilibria with only one lie obtained in the main analysis are unaffected by the possibility of partial memory of the lie, as in the one lie case, the signal η is not needed to know what the lie was.

Inconsistencies in richer contexts

In the above setting, I have assumed that the information concerning s held by party I did not change over time. In some applications, it may be natural to consider situations in which party I could either learn more about the state with time or forget some aspects of s as time passes. Such extensions would deserve further research, but one can already indicate that any of these extensions would call for considering more nuanced notions of inconsistency, for example identifying two messages m_1 and m_2 at times $t = 1, 2$ as inconsistent only if it would not be possible to explain them through a change of I's information (or memory) about the state s.

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type can be certified or not endogenously depends on the equilibrium distribution of lies.

Appendix

Pure strategy equilibria in pure persuasion games when $\varepsilon_1 < \varepsilon_2$

Consider a pure strategy equilibrium. Let $S_{inc} = \{s \text{ such that } m_1(s) \neq m_2(s)\}$ and $s^*_{inc} = \max S_{inc}$. The main issue is to show that $S_{inc} = \varnothing$ from which it is easy to proceed as in the main text to show that the equilibria in pure strategy when $\varepsilon_1 < \varepsilon_2$ are the same as those shown in Proposition 1 (when $\varepsilon_1 > \varepsilon_2$). This is established using the observation that if a type engages into inconsistent messages he should first lie and then tell the truth as well as the next lemma

Lemma 10 *s^*_{inc} cannot be a consistent lie, i.e. there is no $s \neq s^*_{inc}$ such that $I(s)$ sends $m_1 = m_2 = s^*_{inc}$.*

Proof. Suppose that $I(s)$ sends $m_1 = m_2 = s^*_{inc}$. One should have

$$a(s^*_{inc}) - \varepsilon_1 - \varepsilon_2 \geq a_{inc} - \varepsilon_1$$

for $I(s)$ not to prefer sending inconsistent messages, and

$$a_{inc} - \varepsilon_1 \geq a(s^*_{inc})$$

for $I(s^*_{inc})$ not to prefer telling the truth. These two conditions are incompatible. ♣

The above lemma implies that $a(s^*_{inc}) = s^*_{inc}$. Given that $a_{inc} = E(s \in S_{inc})$, this implies that $a_{inc} - \varepsilon_1 < a(s^*_{inc})$ and thus, $s^*_{inc} \notin S_{inc}$ yielding a contradiction.

Proof of Proposition 3

Let a^*_n denote the equilibrium action after a lie in σ^n. Suppose by contradiction that for some $\widehat{a}$ and all $n > \overline{n}$, $a^*_n > \widehat{a}$. There must be at least $n\widehat{a}/2$ states s_k smaller than a^*_n in S_n. Moreover $\frac{1}{2n} < s_{k+1} - s_k$ implies that $E(s \in S_n,\, s < a^*_n) < a^*_n - a^*_n/8$ for n large enough. The condition $\mu_k a_k + (1-\mu_k)a_{inc} = a^*_n$ with $a_k = \frac{\mu_k \Pr(s \in S, s < a^*_n) a_{inc} + p(m^*_k) m^*_k}{\mu_k \Pr(s \in S, s < a^*_n) + 1}$ and $a_{inc} = E(s \in S_n,\, s < a^*_n)$ in the mixed strategy shown in Proposition 2 cannot be satisfied for every k given that $E(s \in S_n,\, s < a^*_n) < a^*_n - \widehat{a}/8$; μ_k must be bounded away from 0 irrespective of n (to ensure that $\mu_k a_k + (1-\mu_k)a_{inc} = a^*_n$); and when μ_k is bounded away from 0, $\mu_k \Pr(s \in S, s < a^*_n)/p(m^*_k)$ grows arbitrarily large with n so that a_k approaches

a_{inc} in the limit. This leads to inconsistent conditions, thereby showing the desired result. ♣

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