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The Deposit Base – Multibanking and Bank Stability

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The Deposit Base — Multibanking and Bank Stability*

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Abstract: To provide maturity transformation, banks need a *deposit base* – deposits that could be, but are not, withdrawn most of the time and are, thus, used for long-term lending. In a global-games environment, we show that a higher deposit base protects banks against panic runs. As depositors become more flexible in their bank relations, keeping *multiple accounts* at different institutions, the deposit base of banks changes. We analyze the impact of multi-banking on bank stability and show that in an economy with specialized institutions, households allocate too few funds to maturity-transforming institutions (banks). A policy-maker should support the banks, even though they are more fragile. If only some institutions are protected by deposit insurance, the deposit base moves away from the unprotected institutions, leaving them *more prone to runs*.

Keywords: Bank Runs, Deposit Base, Global Games, Financial Stability

JEL-Classification: G21, G28, H23,

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1 Introduction

Banks transform maturity. They borrow short-term and lend long-term. That way, firms receive long-term funds for illiquid investments while households benefit from the liquidity of their deposits. But this maturity transformation exposes banks to illiquidity risk: the probability to default due to a run on short-term liabilities when the bank would have been solvent otherwise. During normal times, the liquidity risk from short-term deposits is low: deposit withdrawals are linked to the depositors' idiosyncratic liquidity needs and are, therefore, predictable at the aggregate level. This stack of continuously prolonged deposits – the *deposit base* – is a cheap funding source for long-term investments by the bank. We show that a declining deposit base increases the inherent illiquidity risk of a bank.

Our model builds on Diamond and Dybvig (1983, DD in the following) and on its global-games version Goldstein and Pauzner (2005, GP in the following). Our only modification is a risk-neutral utility function where depositors, if they are impatient, do not necessarily want to withdraw their complete investment.¹ Formally, our impatient consumers receive heterogeneous consumption shocks that create a high marginal utility up to a particular consumption target.² In other words, it is not the fact whether there is a liquidity shock that is uncertain ex-ante, but the size of the liquidity shock. Then, if depositors have deposited at more than one bank, they may withdraw their deposits in some endogenous order. As a consequence, the individual deposit base of the banks differs. This modification allows us discuss the determinants and consequences of the allocation of the deposit base in the financial system. To our knowledge, this is the first theoretical paper that concentrates on the deposit base and analyzes its effect on financial stability.

Around the world, households have become more and more flexible in their financial relations. In contrast to the traditional life-time bank-customer relationship, where a single bank provides all financial services, households have started to keep multiple accounts at different financial institutions that have different tax statuses and include both, mutual funds and direct lending or equity investments (Campbell, 2006). In 2014, 62% of German consumers used more than one bank, as did 40% of Italians, 43% of French consumers and 49% of the Americans (Evans, 2015). As it becomes easier to manage financial products

¹Our modification is also more realistic. Some consumers want to withdraw their complete deposit, some consume only a part of their deposited savings. Some consumers may become sick and need to spend their entire savings on an expensive therapy, or they might find an interesting consumption opportunity, or they have to pay the college tuition for their kids. Other consumers may also have immediate but smaller consumption desires. They might need to fix their broken car or buy some medicine to recover. Each immediate consumption gives the particular consumer an exceptionally high utility, but only up to the level that is needed to satisfy the consumption shock.

²This yields a kinked linear function for impatient depositors, where the quasi-concavity of the kink replaces the assumption of a high relative risk aversion in DD.

from multiple providers, consumers are less relying on a bank account to bundle all their financial services, but go on to buy other financial products from a much broader range of providers. Even though traditional banks remain the primary provider of financial services, the 40% of retail customers expressed a decreased dependence on their bank as their primary financial services provider and stated that they use non-bank providers for financial services (EY, 2016).

Given the fixed, aggregate liquidity need of households, the investment into traditional bank deposits alongside with other products from different institutions changes the composition of the banks' deposit base. If households, instead of leaving their entire liquid savings at the bank, use their current account only for their most frequent liquidity needs and other products for the less frequently needed liquidity, the proportion of the deposit base at the bank decreases while the competing institutions enjoy a greater proportion of constantly prolonged short-term debt. This uneven and more volatile allocation of the deposit base in the banking sector has consequences for the individual and systemic banking stability. In particular, we show that an uneven allocation of the deposit base destabilizes the financial sector as a whole.

The theoretical literature on bank runs has identified many potential causes for depositor runs and potential remedies. However, little research is done on the decision of households to deposit at banks in the first place. Peck and Setayesh (2019) represent a remarkable exception. Their paper investigates how much depositors optimally invest at banks when they face the opportunity to invest also into a non-bank sector. They show that the less households overall deposit at banks, the more likely patient consumers are withdrawing their deposits early. In equilibrium, depositors trade-off more depositing and therefore more stable banks with higher returns. Our model predicts a similar trade-off when depositors can invest into non-bank funds. In equilibrium too little is invested in the banking sector, making it more fragile to bank runs. Our paper complements and extends this insight by showing that bank stability not only depends on how much is invested but also how much and in what order is withdrawn from each institution.

To our knowledge, however, this paper is the first to analyze the impact of the *deposit base* on the stability of the bank and the banking system. This is somewhat surprising, as the fact that demandable deposits provide an attractive, relatively stable source of long-term funds for banks, is well established among practitioners: "The value of stable deposit funding is now higher than ever before" said Anju Jain³ in 2013 after winning the majority control over the local rival Deutsche Postbank, which almost doubled Deutsche Bank's German retail deposit customer base.⁴ As the local retail deposit base is quasi-fixed in size, (Billett and Garfinkel, 2004; Flannery, 1982) bank mergers and acquisitions

³At that time, Jain was head of Deutsche Bank's investment bank and later Deutsche's Co-CEO.

⁴Expanding Deutsche's retail base was the declared key goal of its CEO Josef Ackermann in taking over Postbank, see <https://www.ft.com/content/601c6480-f94b-11df-a4a5-00144feab49a>.

are often the only way to increase the retail deposit base of a bank in order to enhance the funding stability.⁵

The idea that a more stable deposit base increases banking stability is also supported by empirical evidence. Marques-Ibanez *et al.* (2011) show that a solid customer deposit base is effective in reducing distress from individual banks. Han and Melecky (2013) find empirical evidence that a greater aggregate deposit base can increase systemic stability of a financial system. Martin *et al.* (2018) find connections between account characteristics and deposit stability of financially distressed banks.

The liquidity problems of numerous financial institutions during the 07/08 financial crisis have also encouraged regulation to explicitly account for a stable deposit base. The Net Stable Funding Ratio (NSFR) in the Basel III liquidity requirement is designed to limit funding risk arising from maturity mismatches between bank assets and liabilities and explicitly considers the immanent stability of demandable deposits.⁶

This paper develops a systematic analysis of the stability of a bank's *deposit base* and the externalities between heterogeneous institutions when depositors can split their savings among the institutions to benefit from different services of these institutions. In our model, households prefer demand deposit contracts because they do not know ex-ante when, and how much they will want to consume. But only a proportion of a bank's demand deposits are actually withdrawn in each period to satisfy consumption needs. A considerable stock of deposits remains at the bank. This *deposit base* is used by the banks for long-term investments. The greater such a deposit base of a bank is, the higher are the returns it can offer to its depositors and the less prone a bank is to a run by its depositors. A smaller deposit base, therefore, decreases the long run returns a bank can offer and, thus, increases the probability of a bank run. Because of the strategic complementarity among depositors, this increase is disproportionately high.

In the model, the deposit base is determined by long-term deposit rates. In reality, the complete package of deposit rates, services, payment functions, and the like plays, a role. Depositors will first withdraw from banks that they find less attractive in the long run or that offer better short-term transaction conditions, and keep deposits at the more long-term attractive banks. Hence, a structure emerges that resembles a pecking order. This

⁵Increasing the deposit base was a driving motivation for the big bank merger wave in the 1990s. The Chase Manhattan merger with Chemical Bank in 1996 increased the deposit base by 88% (\$75 billion). In the same year, Wells Fargo increased its deposit base even by 94% (\$36.8 billion) by several acquisitions Dymski (2016).

⁶Article 421(3) introduces a weighting scheme for retail deposits according to their run-off rates that are minimum floors of anticipated withdrawals. Retail deposits are divided into "stable" and "less stable" with minimum run-of rates for each category. Moreover, individual jurisdictions are allowed to establish higher run-off rates to capture depositor behavior in a period of stress in certain jurisdiction.

pecking order affects the deposit base of institutions. At more senior institutions the deposit base increases, at junior institutions, where depositors withdraw first, it shrinks.

We first concentrate on households that deposit at two otherwise identical banks, but for exogenous reasons choose to withdraw from one bank first, creating a difference in the deposit base. This allows us to focus on the consequences for financial stability. We then show that, the more even the deposit base is allocated in the system, the better for financial stability. Finally, we endogenize the optimal household investment decision when households can choose between banks that provide maturity transformation services and non-banks that focus on higher long-term returns.

In DD-style models bank run equilibria occur as self-fulfilling beliefs that are unrelated to fundamentals, without explaining the determinants and likelihood of each equilibrium.⁷ To analyze the impact of the deposit base on the stability of financial institutions, we use a global games approach. This approach was developed by Carlsson and van Damme (1993) and allows a tighter analysis of panic-based models. Our model is based on GP, who apply the global games approach to a bank-run setting. This allows us to analyze the relationship between the deposit base and the likelihood of inefficient bank runs.

In the model setup, we focus on uninsured deposits as an extreme cause of bank fragility. As in all run-based models, the bank fragility would disappear if a government could credibly guarantee to pay any liability at each point in time either by perfect deposit insurance or with bail-out policies. Yet, our setup is based on a macroeconomic shock that affects the economy as a whole and, thus, also the entire banking sector. Therefore, an interbank based liquidity insurance would not mitigate the run incentives, since all banks suffer low expected returns and therefore higher bank run probabilities. Likewise, a macro-shock also affects the tax income of a government. Therefore, also the government might not be able to rescue the entire banking sector in case of a severe recession.⁸ Therefore, even with state guarantees and deposit insurance, we get the same qualitative results as long as guarantees and insurance are not perfectly covering the liabilities from depositors after a macro shock. For simplicity we focus on the extreme case without any guarantee or deposit insurance. The impact of an imperfect deposit insurance and state guarantees is discussed in Section 6. Moreover, Iyer and Puri (2012) show that full deposit insurance is only partially effective in preventing depositors from withdrawing their deposits.

⁷We base our analysis on DD, but instead could have used other models that justify the existence of short-term lending and bank crisis. Diamond and Rajan (2001) argue that the link between short-term lending and bank fragility has a reverse causality. Banks that want to provide liquidity and loans to risky borrowers have to borrow short-term because the threat of a bank run from short-term borrowing prevents banks from renegotiating contracting terms. Another justification for demand deposits is given by Gorton and Pennacchi (1990) who argue that banks optimally offer demand deposits to uninformed agents because they are risk-less; their value does not depend on the information known only by informed agents.

⁸An anecdotal example that governments are not able to fully bail out a banking system after a macro shock can be found in the bail-in in Cyprus that included senior unsecured debt and even deposits.

In fact, even with a perfectly effective deposit insurance in place, not all funds are insured. Banks fund considerable amounts of their investments with uninsured wholesale funding (Feldman and Schmidt, 2001; Oura *et al.*, 2013). Since the deposit base affects also the yield curve of wholesale funding, a decreased deposit base would likewise increase the run on repo or wholesale funding probability, which we elaborate in the discussion section.

2 The Model Setup

Our model builds on Goldstein and Pauzner (2005, GP in the following) but modifies the utility function of depositors to allow for heterogeneous sizes of deposit withdrawals.

Consider an economy with one good and three dates ($t = 0, 1, 2$), a continuum of consumers of mass 1, and a continuum of banks.

Consumers. Consumers are endowed with one unit of a consumption-investment good at date 0, which they can deposit at a bank in order to consume at date 1 or date 2. Each consumer can be of three types. With probability $1 - \lambda - \eta$ he is of type 0 and wants to consume nothing at date 0, with probability λ he is of type L and has a consumption target of $L > 0$ at date 0, and with probability η he is of type H and has a consumption target of $H > \max\{L; 1\}$ at date 0. We capture the idea of a target consumption with the date-1 utility function

$$u_{1,\sigma}(c) = \begin{cases} \mu c & \text{for } c \leq \sigma, \\ \mu \sigma + (c - \sigma) & \text{for } c > \sigma. \end{cases},$$

where c is the level of consumption, σ is the type, $\sigma \in \{0, L, H\}$, and $\mu > 1$ is a measure of the urgency of early consumption. Assume that $\eta H + \lambda L < 1$, implying that immediate liquidation needs in the economy can be satisfied by liquidating the productive asset.

For all types, the utility of consuming nothing is normalized to $u_{1,\sigma}(0) = 0$. For type L , the marginal utility is $\mu > 1$ below the threshold L , above the threshold it is again 1. This means that type L has a preference to consume at least L at date 1; his *consumption target* is L . For type H , the consumption target is H . For the patient type, the marginal utility is always 1, or in other words, his consumption target is zero. For date 2, the utility function for all types is $u_{2,x}(c) = c$, the marginal utility of consumption is always 1.

In DD, the optimal contract results from the assumption that consumers have a relative risk aversion that is greater than the liquidation value of the asset larger than one. This implies that the optimal contract should offer impatient consumers more than the technologies are able to produce at $t = 1$. Our assumption of $H > 1$ reflects the very same idea: Impatient consumers optimally require more than the technology is able to produce.

In other words, we simplify the restrictive assumption on the curvature of the utility function in DD to a simple restriction on the kink of the quasi-concave utility function. Even though our structure deviates from the classic DD model, deposit banks are beneficial for consumers compared to markets if μ is large enough, see Lemma 1.

The deposit contract offered by a bank increases welfare if the short-term consumption utility is greater than the expected long-term return. However, this increase in welfare comes at the cost of destabilizing the bank, i.e., the optimal contract makes a bank vulnerable to runs. Like in GP, this intrinsic fragility lowers the value of the bank deposit contract to consumers but makes it still valuable if the run probability is not too high.

Investment. Like in GP, there is a single risky investment technology. Per unit of investment at date 0, the investment technology returns $R > 1$ at date 2 with probability $p(\vartheta)$ increasing in the state variable ϑ , otherwise it returns zero. The investment can be liquidated at date 1, in which case it returns 1. Partial liquidation is possible.

The variable ϑ is uniformly distributed on the unit interval, $\vartheta \sim [0, 1]$ and represents the state of the economy. Assume that $p''(\vartheta) > 0$, such that good and normal states are the norm, and bad states are the exception. In line with the literature, to get an upper dominance region, we also assume that if $\vartheta \approx 1$, the investment project becomes so profitable that early withdrawal is never optimal.

Information. The state of the economy ϑ is realized at date 1, but does not become public information. Instead, each consumer gets a private signal

$$x_i = \vartheta + \varepsilon_i, \tag{1}$$

where ε_i is a stochastically independent private error term that is uniformly distributed over the interval $[-\varepsilon, \varepsilon]$ with $\varepsilon > 0$. Like in GP, consumers observe the signal, then decide whether or not to withdraw their deposit from the bank. Like in GP and the ensuing literature, we assume that ε is infinitesimally small. To be more precise, we consider a small but positive ε , and then let it converge to zero in our proofs.

3 One Representative Bank

We first concentrate on symmetric equilibria. We can, thus, consider a single representative bank under perfect competition. At date 0, the bank offers a deposit contract that promises some fixed $r_1 \in [1; H]$ per unit of investment at date 1, and r_2 per unit of non-withdrawn investment at date 2. A short-term rate of $r_1 > H$ cannot be optimal, because it does not give consumers a benefit in the short run, but wastes benefits from the long-term

investment. A rate $r_1 < 1$ cannot be optimal, it is dominated by the market solution. Banks are assumed to operate under perfect competition and therefore distribute their complete revenues at date 2. Therefore, r_2 will depend on whether the project is successful, and on the fraction of depositors that have already withdrawn at date 1. The bank invests all collected deposits in the risky technology. For now, assume that μ is high enough such that maturity transformation enhances ex-ante expected utility. We will derive a condition in Lemma 1.

At date 1 consumers learn their type. In the absence of a bank run H -consumers withdraw their entire deposit and receive r_1 . L -consumers withdraw only the fraction L/r_1 of their deposit. Patient consumers, as long as $E(r_2) > H$ holds such that incentive compatibility is given, do not withdraw their endowments and simply wait until they receive r_2 .

The Deposit Base. Let n denote the fraction of deposits that is *actually* withdrawn at date 1, and let n_{\min} denote the amount that is withdrawn with certainty to satisfy consumption needs of the impatient consumers. So n is known only after the withdrawal decisions of depositors at date 1, but n_{\min} is known ex ante. Because a fraction η are H -consumers and withdraw everything, and a fraction λ are L -consumers and withdraw a fraction L/r_1 of their deposit, the minimum withdrawal is $n_{\min} = \eta + \lambda \cdot L/r_1$. The amount of deposits that are generally not withdrawn (only in a bank run) are called the *deposit base* $\beta(r_1) := 1 - r_1 n_{\min} = 1 - r_1 (\eta + \lambda L/r_1)$. We will show that banks optimally set $r_1 = H$, such that the deposit base becomes $\beta := 1 - \eta H - \lambda L$, which depends only on the consumers' preferences. If there are several institutions, the aggregate deposit base will still be invariant.

Because of competition, the bank must distribute all long-term returns from investment to its remaining depositors. 1 has been invested, but $1 - \beta(r_1)$ has been liquidated, such that $\beta(r_1)$ long-term investments remains, absent a run. Hence in case of a successful investment project (probability $p(\vartheta)$), the bank promises to pay

$$r_2 = \frac{1 - \eta r_1 - \lambda L}{1 - \eta - \lambda L/r_1} R = \frac{\beta(r_1)}{1 - (1 - \beta(r_1))/r_1} R \quad (2)$$

on deposits that have not yet been withdrawn. Because $r_1 > 1$, the long-term return r_2 increases in the deposit base β .

In addition to withdrawals due to liquidity shocks, depositors may withdraw strategically, depending on the private information they receive and their corresponding higher order beliefs on the behavior of the other depositors. This behavior can be an equilibrium strategy since the maturity transformation of the bank makes the bank fragile to illiquidity risk: If the promised repayment r_1 is greater than liquidation value 1, which will be optimal if μ is high enough, the bank is not able to repay all depositors, if all depositors withdraw their entire deposits at date 1. Moreover, whenever at least $1/r_1$ depositors withdraw

their deposits the bank would have to liquidate all their assets in order to pay r_1 . In that case, consumers who do not withdraw at date 1 will receive nothing later at date 2. This strategic complementarity between the withdrawal behavior of depositors facilitates self-fulfilling, panic based, bank runs.

In addition to panic bank runs, our model exhibits fundamental bank runs: There are realizations of the fundamental variable that are so low (high) that no matter of what other depositors do, it is the dominant strategy of each depositor to withdraw (not withdraw) its deposits. These regions of realizations of the fundamental variable are called lower (upper) dominance region.

The Lower Dominance Region. When fundamentals (ϑ) are bad, the expected return at date 2 can fall below the certain return of a date 1 withdrawal. In contrast to GP we have two types of patient consumers to consider: patient consumers and the L -consumers that have to decide to fully or only partially withdraw their deposit.

Consider the patient consumers. Let $p(\underline{\vartheta})$ denote the realized success probability that solves $r_1 = p(\underline{\vartheta}) r_2$, such that it is a dominant strategy for patient consumers to withdraw if $\vartheta < \underline{\vartheta}$. Using (2) for the repayments and solving for $p(\underline{\vartheta})$, we get

$$p(\underline{\vartheta}) = \frac{1}{R} \frac{r_1 (1 - \eta) - \lambda L}{1 - r_1 \eta - \lambda L} = \frac{1}{R} \frac{r_1 - (1 + \beta(r_1))}{\beta(r_1)}. \quad (3)$$

Consider now the lower dominance region of L -consumers. The decision to withdraw the fraction L/r_1 is non-strategic, i. e., it is independent of the behavior of other depositors: L -consumers want to consume $r_1 \frac{L}{r_1} = L$ at date 1. The remaining share of deposits is $(1 - L/r_1)$. Only the withdrawal decision for this remaining share depends on the behavior of other depositors.

The critical success probability for the lower dominance region is implicitly defined by

$$\mu L + \left(1 - \frac{L}{r_1}\right) r_1 = \mu L + p(\underline{\vartheta}) r_2. \quad (4)$$

L -consumers receive utility μL from their private investment opportunity no matter if they withdraw their residual deposit or not. The strategic decision to withdraw only concerns the residual deposit fraction $(1 - \frac{L}{r_1})$. Solving for $p(\underline{\vartheta})$, we get the same critical value as in (3). The strategic part of the decision to withdraw is the same for all consumers that form the deposit base.

No matter what other consumers do, patient and L -consumers will therefore always find it optimal to withdraw their entire deposit if they observe a signal $x_i < \underline{\vartheta} - \varepsilon$ where $\underline{\vartheta}$ is implicitly defined by (3). If the realization of the state variable is sufficiently bad: $\vartheta < \underline{\vartheta} - 2\varepsilon$, there is a bank run for sure.

The Upper Dominance Region. The upper dominance region corresponds to realizations of the fundamental variable for which it is never optimal for patient L -consumers to withdraw the deposits that are not necessary for private investments early. Like GP, we have assumed that such an upper dominance region exists in the range $(\bar{\vartheta}, 1]$, and that the liquidation value increases in these very good states.

The Intermediate Region. In the intermediate region, the consumer's private information is crucial for the decision to withdraw. Consider a patient consumer who thinks about withdrawing early. For L -consumers, who have the choice between withdrawing partially or completely, the decision is identical, with the only difference that he *strategically* decides on withdrawing only the remaining $1 - L/r_1$ instead of the complete deposit. There are two fundamentally different cases. If $n > 1/r_1$, the bank must liquidate its investment completely; it is insolvent. Consequently, if the depositor withdraws, he still gets $1/n$ in expected terms, leading to a utility of $1/n$. If he does not withdraw, he gets nothing, leading to an expected utility of 0. The analysis is more interesting for the case where the bank remains liquid. If the depositor withdraws, he gets r_1 . If he does not withdraw, he gets his share of the final outcome. Because n depositors have already withdrawn, the bank has to liquidate $n r_1$, the remaining $1 - n r_1$ lead to a return of $(1 - n r_1) R$ with probability $p(\vartheta)$. Because there are $1 - n$ consumers left who have not yet claimed a repayment, the consumer gets an expected amount of $\frac{1 - n r_1}{1 - n} R$ with probability $p(\vartheta)$, leading to an expected utility of

$$p(\vartheta) \frac{1 - n r_1}{1 - n} R. \quad (5)$$

In equilibrium, there is a critical level x^* such that all consumers with private signals $x_i < x^*$ withdraw, those with $x_i \geq x^*$ leave their deposit at the bank. This x^* is defined such that the critical consumer (that with private information $x_i = x^*$) is indifferent: the expected utility when withdrawing the deposit equals the expected utility when leaving the deposit at the bank. As the depositor's posterior distribution of ϑ is uniform, the indifferent agent who receives $x_1 = x^*$ expects the proportion of running depositors n to be uniformly distributed between n^{min} and 1. Such that we can write the indifference condition as

$$0 = \int_{n_{min}}^{1/r_1} \left(p(\vartheta^*) \frac{1 - n r_1}{1 - n} R - r_1 \right) dn + \int_{1/r_1}^1 \left(0 - \frac{1}{n} \right) dn, \quad (6)$$

Calculating the integrals in (6) and solving for $p(\vartheta)$ yields our first proposition.

Proposition 1 (Extension of Theorem 1 in Goldstein and Pauzner, 2005)

The model has a unique equilibrium, in which patient consumers run (withdraw) if they

observe a signal below threshold x^* , and do not run above. In the limit $\varepsilon \rightarrow 0$, x^* is equal to ϑ^* as defined by

$$p(\vartheta^*) = \frac{1 - n_{\min} r_1 + \log r_1}{1 - n_{\min} r_1 - (r_1 - 1) \log(r_1 (1 - n_{\min}) / (r_1 - 1))} \frac{1}{R}. \quad (7)$$

We show in the appendix that this term is always positive. For $p(\vartheta^*)$ to be an interior solution, we have to assume that R is sufficiently high.

Lemma 1 *For every parameter constellation, there is a critical $\bar{\mu}$ such that whenever $\mu > \bar{\mu}$ the optimal short-term interest payment is $r_1 = H$.*

From now on, assume that $\mu > \bar{\mu}$, such that $r_1 = H$. We can rewrite the critical success probability that determines the bank run probability in terms of the deposit base in the economy β :

$$p(\vartheta^*) = \frac{1}{R} \frac{\beta + \log(H)}{\beta - (H - 1) \log\left(\frac{(H-1)+\beta}{(H-1)}\right)}. \quad (8)$$

Lemma 2 (Theorem 2 in Goldstein and Pauzner, 2005) *Given a fixed deposit base, the probability of a bank run increases in the promised short-term repayment r_1 .*

A higher short-term repayment reduces the payment for patient agents in $t = 2$ and therefore increases their incentive to run. The strategic complementarity among patient agents amplifies this incentive to run as the higher willingness to run of other agents further decreases the expected long-run return.

A similar logic applies to the deposit base. We can therefore summarize our first main insight,

Lemma 3 *The probability of a bank run decreases in the deposit base β : $\partial p(\vartheta^*) / \partial \beta < 0$.*

An increase in β increases the long-term investment of the institution, and thereby, the long run return r_2 that the institution is able to offer. Consumers have an additional incentive not to run, which reduces the states that trigger a bank run. Intuitively, an agent that had originally observed a signal that made him indifferent between running and not running, will now find it optimal to wait, if the long run return slightly increases.

Lemma 4 *A reduction in the deposit base increases the probability of a bank run over-proportionally, i. e., $\partial^2 p(\vartheta^*) / \partial \beta^2 > 0$.*

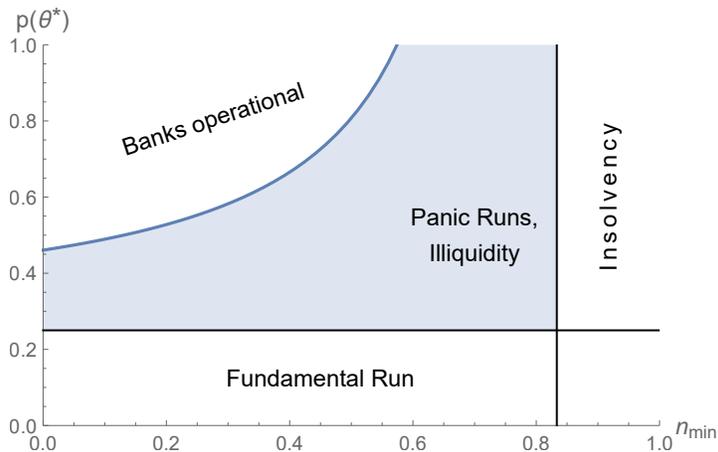


Figure 1: Critical Success Probability $p(\vartheta^*)$ of the Investment Project

Figure 1 shows how the critical success probability $p(\vartheta^*)$ depends on n_{\min} . Because $n_{\min} = (1-\beta)/r_1$, the figure immediately contains lemmas 3 and 4. Banks fail whenever the signal on the state variable ϑ falls short of the blue curve. Note that the figure is not just an illustration but based on a numerical simulation. The parameters used for the simulation are $H = 1.2$, $L = 0.8$, $\eta = \lambda = 1/4$, and $R = 4$.

The strategic complementarity among agents amplifies the initial increase in the bank run probability caused by a lower deposit base. As a lower deposit base increases the agents' incentives to run, more agents withdraw their entire deposits early, such that even less long-term investments are left for agents that wait. This effect is more severe, the more deposits are withdrawn in the first place.

4 Multibanking with Homogeneous Banks

In contrast to the existing literature we now allow depositors to split their deposit investment among several banks. Instead of relying on one bank account to bundle all financial services we allow all agents to diversify their initial investment in order to benefit from potentially different returns, services or business models. In this section, banks are still assumed to be homogenous, in the sense that they have access to the same investment technology. Their business strategy can differ only in the contracts that banks offer to depositors.

The Liquidity Pecking Order In this section, we show that asymmetric deposit contracts have two effects. The first effect is discrete. At date $t = 1$, patient consumers

will not withdraw at all, and H -consumers will withdraw their complete investment. L -consumers, however, withdraw only part of their investment: from the bank with the lower deposit rate for the second period. Consequently, there is a *pecking order* between banks. The bank with a smaller fraction r_2/r_1 has a discretely lower deposit base – even for only tiny differences. The second effect is continuous. For a fixed pecking order, the deposit base of each bank depends continuously on the short-term rate. We show that consumers prefer a symmetric deposit base, and consequently identical deposit rates, which can also be implemented by a single institution. In other words, if the investment opportunities of banks do not differ, consumers dislike multibanking.

4.1 Symmetric Contracts with a Liquidity Pecking Order

Assume for now that all banks offer $r_1 = H$ but for some exogenous reason consumers have a liquidity pecking order: they all withdraw from one bank first. For illustration, think of two banks that are situated along a road and that depositors always pass by one bank first. This bank, however, has a limited investment volume s . Assume that depositors with a liquidity need withdraw from this bank first, and only if they need more money, they withdraw also from the bank further down the road, which has an investment volume limited to $1 - s$. Note that, because of the strategic complementarities, given that a consumer knows that others will withdraw from one bank first, it is optimal for him to also withdraw from this bank first. Hence the described behavior constitutes an equilibrium. Note also that the bank further away is in a better position because less money is withdrawn in the short run, and the long-term expected return is thus higher.

In reality, the closer bank can be thought of as a bank that is used more for transaction payments, whereas the bank further away is used for long-term investment. In the next section, we endogenize the optimal investment volume for institutions with different business models. In particular, we assume that the second “bank” does not provide maturity transformation but specializes on creating long-term returns.

Let us call the first institution with volume s a “bank”, and the second institution with volume $1 - s$ the “fund”. As before, we assume that funds and banks operate under perfect competition, they do not make profits. Because consumers want to be able to consume H at date 1, the short deposit rate in a symmetric equilibrium is $r_1^f = r_1^b = H$.⁹

Depositors in need for cash withdraw first from a bank, then from a fund. There is, thus, a pecking order between banks. If a consumer wants to consume, it is first the bank that must provide the consumer with the money. Formally, patient consumers do not need to

⁹There is a continuum of deposit rates that could be optimal for consumers $sr_1^b + (1 - s)r_1^f = H$. However, those asymmetric contracts do not maximize the depositors’ expected utility as proven under proposition 2.

consume, they withdraw only for strategic (panic) reasons. H -consumers need to withdraw their complete deposit, hence they withdraw everything from both funds and bank.

For L -consumers there are two cases. *First*, a relatively small fund, or a large bank with size s that satisfies $L < sr_1 = sH$. Hence, withdrawing from the bank is sufficient to satisfy the desired consumption L . At date 1, L -consumers have sH at the bank and $(1-s)H$ at the fund and withdraw L from the bank and nothing from the fund. *Second*, if $L > sH$, withdrawing only from the bank is not sufficient to consume L . At date 1, the consumer withdraws sH from the bank, and the remaining $L - sH$ from the fund. The analysis of both cases is very similar. We concentrate on the case of a large bank s , i. e., the first case; the second case is treated in proofs. It yields similar results but has a slightly different algebra.

At date 1, patient consumers must decide whether they want to withdraw their deposits.¹⁰ The same applies to L -consumers, who must decide whether they want to withdraw the remaining deposit. Importantly, the rational decision may differ between fund and bank. The risk in both banks is driven by the same fundamental ϑ , hence the information on both banks is the same. However, the liquidity and solvency situation differs, because more deposits are withdrawn from the bank than from the fund at date 1. We need to analyze both separately, starting with the fund.

Runs on the Fund. Consider a late consumer who thinks about withdrawing early. In order to better be able to re-use former results, let us re-normalize his deposit to 1 (although, of course, he has deposited only $1-s$ at the fund). The lower bound of withdrawals is $n_{\min} = \eta$. H -consumers withdraw, L -consumers cover the required amount $L \leq sH$ by withdrawing from the bank. Therefore, L -consumers and patient consumers withdraw only for strategic reasons from the fund. Consequently, (6) adjusts to

$$0 = \int_{\eta}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1-nH}{1-n} R - H \right) dn + \int_{\frac{1}{H}}^1 \left(0 - \frac{1}{n} \right) dn. \quad (9)$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1}{R} \frac{1 - \eta H + \log H}{1 - \eta H + (H - 1) \log \frac{H-1}{H(1-\eta)}}. \quad (10)$$

The deposit base of the funds increases because L -consumers withdraw their target consumption L only from the bank leaving the deposit base at the fund unaffected. The increased deposit base allows the fund to pay a higher expected return per unit of non-withdrawn investment at date 2. The prospect of higher expected date 2 return makes consumers with lower signals willing to leave their deposit at the fund. The increased

¹⁰The lower dominance regions change for each institution. The exact values are given in the proofs.

deposit base shifts the critical consumer's signal down. Only consumers that receive a lower signal than the decreased critical signal are willing to run the fund. Therefore the probability of a bank run decreases when the deposit base of the fund is increased.

Runs on the Bank. Each consumer deposits s at the bank. The promised repayment at date 1 is sH . Now H -consumers withdraw the complete sH . L -consumers withdraw L from the bank and nothing from the fund. Hence, the minimum withdrawal of L -consumers from the bank is $\frac{L}{sH}$. For $s = 1$, we get back the factor L/H from Section 3.

Summing up, $n_{\min} = \eta + \frac{L}{sH} \lambda$. Consequently, (6) adjusts to

$$0 = \int_{\eta + \frac{L}{sH} \lambda}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1 - nH}{1 - n} R - H \right) dn + \int_{\frac{1}{H}}^1 \left(0 - \frac{1}{n} \right) dn. \quad (11)$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1}{R} \frac{1 - (\eta + \frac{L}{sH} \lambda) H + \log H}{1 - (\eta + \frac{L}{sH} \lambda) H + (H - 1) \log \frac{H-1}{H(1 - \eta - \frac{L}{sH} \lambda)}}. \quad (12)$$

A liquidity pecking order between two banks that offer the same contract terms, affects the bank run probabilities of both banks dis-proportionally.

Proposition 2 *A Liquidity Pecking Order increases the aggregate bank-run probability of a homogeneous banking system.*

The intuition is straightforward: as the bank run thresholds $p(\vartheta^*)$ of each institution are convex functions of the according β s, any linear combination of minimum withdrawals in the banking system results in a higher aggregate bank-run probability in the banking sector compared to the symmetric banking sector.

4.2 Optimal Deposit Contracts

Allowing depositors to split their investment among homogeneous institutions allows for asymmetric equilibria: With many identical banks offering deposit contracts, it may be optimal for single banks to deviate from the symmetric contract or a set of banks could deviate and offer a set of contracts that in combination could be preferable to consumers, potentially leading to an asymmetric equilibrium.

In an asymmetric equilibrium, the aggregate degree of liquidity that a consumer wants to have in his portfolio is always H . The aggregate liquidity provision by banks is, thus,

constant but distributed asymmetrically over the banking system. Consequentially, banks that provide more liquidity are over-proportionally prone to runs, whereas banks that provide less liquidity are less prone to runs, but not in proportion. Interestingly, this argument does not hold in DD. In DD, a consumer is exactly indifferent between depositing his complete endowment at a bank that offers e.g. $r_1 = 10\%$, or half of his endowment at a bank that offers 9% (and a higher long-term return, accordingly), and the other half at a bank that offers 11%. This is a technical point, but we have never seen a discussion in the broad literature on DD.

Due to the convexity of the bank run probability in the deposit base, any linear combination of results in a higher aggregate probability of a bank run. Therefore, a consumer who chooses to split his liquidity over multiple asymmetric contracts faces, ex-ante, a higher chance of liquidation losses due to bank runs.

To show this formally, assume that there are two banks, A and B . Bank A offers $r_A > H$ and bank B offers $r_B < H$ accordingly. Bank B offers a lower short-term rate, to complement bank A 's higher rate, and is therefore also able to offer a higher long-term return than bank A . In the following, we show that any pair of contracts will result in a strictly lower expected utility than the symmetric equilibrium contract.

Facing the choice of return rates, the depositor has to invest at both banks in order to secure a repayment of H in case he receives the high shock. Let s denote the volume invested at bank A , and $1 - s$ the investment at bank B . Taking the promised short-term rates as given, we must have $s r_A + (1 - s) r_B = H$ and hence

$$s = \frac{H - r_B}{r_A - r_B}.$$

The depositor would not want to invest more at bank A , as this would result in losing higher long-term rates from bank B . Similarly, a lower investment at bank A would result in missing the consumption target H at date 1.

Once the consumption shock realizes, H -consumers withdraw their entire investment from both banks. More interestingly, L -consumers now can choose from which bank to withdraw how much in order to satisfy their consumption need. In the absence of a bank run, L -consumers only withdraw the amount $L < 1$ necessary to benefit from consumption opportunity μ . In order to benefit from higher long-term rates at bank B , L -consumers withdraw first from bank A and leave the share of deposits that is not needed to satisfy consumption needs at bank B . This affects the deposit base of both banks: bank A 's deposit base shrinks, while bank B 's deposit base increases, which translates into a higher bank run fragility of bank A and a safer bank B .

Let us analyze the consequences of asymmetric deposit rates on the deposit bases of the two banks. There are two effects: a discrete and a continuous effect. Even if r_A is just

infinitesimally above H , and r_B infinitesimally below, the deposit base will shift discretely. The reason is that bank B will offer a higher long-term return. Depositors invest s such that they can satisfy their high consumption target. Consumers with low consumption shocks, however, can now decide from which bank to withdraw more to satisfy consumption target L . Because yields between dates 1 and 2 are higher at bank B , they withdraw their maximum amount from A and only the residual amount needed from bank B , instead of withdrawing proportionally to their original investment from both banks. Hence, even due to a tiny difference in deposit rates, bank B gets the senior position in the deposit pecking order. Then if deposit rates change more, the deposit bases will also shift, but the deposit pecking order is constant, so there is no further discrete shift.

The aggregate withdrawals from bank A are $n_{\min,A} = \eta + \lambda \frac{L}{s r_A}$ if $s r_A \geq L$, or otherwise $n_{\min,A} = \eta + \lambda$. For bank B , the calculation is slightly more complex. If $s r_A \geq L$, the L -consumer can reach his consumption target withdrawing from bank A only, thus $n_{\min,B} = \eta$. If $s r_A < L$, there is still a gap of $L - s r_A$. To reach the target of L , he must withdraw a fraction $\frac{L - s r_A}{(1-s)r_B}$ of his deposit at bank B . Hence for bank B , $n_{\min,B} = \eta + \lambda \frac{L - s r_A}{(1-s)r_B}$. If $s r_A \geq L$, the deposit bases are

$$\beta_A = 1 - r_A \left(\eta + \lambda \frac{L}{s r_A} \right) \quad \text{and}$$

$$\beta_B = 1 - r_B \eta.$$

If $s r_A < L$, they are

$$\beta_A = 1 - r_A (\eta + \lambda), \quad \text{and}$$

$$\beta_B = 1 - r_B \left(\eta + \lambda \frac{L - s r_A}{(1-s)r_B} \right).$$

In both cases, the weighted sum of the deposit bases is $\beta = 1 - \eta H + \lambda L$. The distribution of the deposit base is affected by the choice of deposit rates, but the aggregate deposit base is constant. Second, if both r_A and r_B converge to H , either $\beta_B \rightarrow 1 - \eta H$ or $r_A \rightarrow 1 - (\eta + \lambda) H$. The allocation of deposit bases does not converge to the symmetric case, which would be $1 - \eta H - \lambda L$. This is the discrete jump in the deposit base, due to the seniority of B to A in the deposit pecking order.

Lemma 5 *Multibanking with $r_A > H > r_B$ implies that $\beta_A > \beta > \beta_B$. Consequently, bank A becomes more fragile to bank runs, while B becomes more stable $p_A(r_A, r_B) > p(\vartheta^*) > p_B(r_A, r_B)$.*

Splitting the endowment between two homogeneous banks that offer different contract terms, affects the bank run probabilities of both banks. While the bank that provides more maturity transformation becomes more fragile to bank runs, the other becomes safer. These changes in the bank stability do not cancel out but increase the expected liquidity risk.

Proposition 3 *The symmetric contract $r_A = r_B = H$ maximizes the expected utility of depositors. Consequently, in equilibrium with homogeneous banks, $r_A = r_B = H$.*

From Lemma 4 we know that the increase in fragility to a bank run is disproportionate. Therefore, banks have no incentive to deviate from the symmetric equilibrium by offering higher or lower deposit rates because the expected utility of depositors is maximized with $r_A = r_B = H$. This allocation can also be implemented with only one single bank.

5 Heterogeneous Institutions and Endogenous Investment

We have shown that with homogeneous institutions, depositors would want to allocate the deposit base evenly in the banking system. In other words, if all banks offer equal services, it is optimal for depositors to leave their entire liquid savings at just one bank. However, if institutions fundamentally differ in their business model, it might be optimal for investors to use their bank account only for their most frequent liquidity needs and other institutions for long-term savings.

The heterogeneity between institutions is reflected in the long-term technology. We assume that one kind of institutions (banks) has a comparative advantage in the liquidation technology, whereas the other kind (funds) can produce a higher long-term return. We can show that households in such a setting use banks to transform maturity, and prefer funds to do no maturity transformation whatsoever. Funds therefore have a large deposit base, because all the investment is meant not to be withdrawn. Accordingly, banks have a small deposit base and are relatively fragile. We show that in equilibrium households allocate too much money to the funds. Optimally, they should allocate more money to banks, even though they are fragile, in order to strengthen their deposit base and avoid inefficient runs.

5.1 The Model Extension

Consider a continuum of banks that are specialized in maturity transformation with an investment technology that yields R at date 2 with probability $p(\vartheta)$, or, if liquidated, 1 at date 1 for each unit of investment. Additionally, there is a continuum of institutions that we call funds, with a return of $\bar{R} > R$ at date 2 with probability $p(\vartheta)$, or, if liquidated, $\ell < 1$ at date 1. Hence, banks have a comparative advantage in offering short-term liquidity, but also lower long-term returns than funds. This difference could reflect the banks' specialization in relationship lending that allows the bank to increase the liquidity of long-term assets as specifically modeled in (Diamond and Rajan, 2001). However, the cost of providing this service reduces the long-term profits of the banks. In contrast,

the funds do not specialize in relationship lending and therefore cannot create the same liquidity value out of long-term assets. This saves costs and increases the returns that can be provided to the long-term depositors. Note that both institutions provide a valuable service: banks have a comparative advantage in maturity transformation while funds have an advantage in creating long-term returns.

Let us focus on parameters such that households use both institutions, banks and funds. Otherwise, we would be back in the single-institution case of section 3. The aggregate short-term return from both institutions must be H , to secure that H -consumers can reach their target consumption under the assumption that $\mu > \bar{\mu}$ defined in 1. If s denotes the amount of money that a household deposits at a bank, and $1 - s$ the investment into a fund, then

$$s r_b + (1 - s) r_f = H, \quad (13)$$

where r_b is the short-term rate from a bank, and r_f is the short-term rate from a fund. The long-run return of an institution is given by the according budget constraint. Now funds have a comparative advantage in long-run investments, and banks have a comparative advantage in the short-term investment. This implies that funds choose the lowest possible short-term rate, to let households participate from their relatively high long-run returns. Banks should choose a higher short-term rate, and accordingly a lower long-term rate, to let households participate from the relatively high short-term investment returns. However, from the bank's perspective, there is a trade-off. If a bank's short-term rate is high, and the long-term rate low, then the probability of a run on the bank grows. This trade-off does not exist for a fund. As a consequence, funds offer the highest possible long-term rate, which implies that they do not provide any maturity transformation. The optimal short-term rate is ℓ , and the long-term rate is \bar{R} . Hence, funds as institutions become irrelevant if households can directly invest $1 - s$ into the funds' technology. Without funds as intermediaries, the outcome is the same. As a financial intermediary, only banks are of interest. Without loss of generality, let us thus assume that households can either invest themselves, giving them a short-term rate of l or alternatively a long-term rate of $\bar{R} > R$ with probability $p(\vartheta)$, or deposit their money at a bank. A bank has access to an investment technology that gives it either a short-term return of 1, or alternatively a long-term return or R .

Banks. There is a continuum of potential banks, each endowed with the same relationship banking investment technology. Hence, there is perfect competition between banks. Households will deposit their money at the bank that delivers the best trade-off between stability, liquidity, and expected return.

The bank deposit s , in absence of a bank run, returns a contracted repayment $r > 1$ per unit invested at $t = 1$ and $\frac{1-nr}{1-n}R$ with probability $p(\vartheta)$ at time $t = 2$, where n is

the number of withdrawing depositors. If a bank run occurs, i.e., if $\vartheta < \vartheta^*$ immediate withdrawal returns 1 and zero at $t = 2$. Following our previous discussion, this results in a critical success probability for a bank run $p(\vartheta^*)$ as in (7). From Lemma 3 we know that $p(\vartheta^*)$ increases in n^{\min} , from Lemma 2 we know that $p(\vartheta^*)$ increases in r_b .

Assume now that, due to competition, banks consider to offer different short-term rates r_b among which the depositors can choose. Depositors anticipate that a higher promised short-term rate, reduces the promised long-term rate, and leads, ex-ante, to higher run probabilities in the form of higher critical success probabilities $p(\vartheta^*(r'_b)) > p(\vartheta^*(r_b))$ if $r'_b > r_b$. Yet, a higher short-term rate allows the depositor to deposit more at the fund and benefit of higher long-term returns there. Banks will have to offer a short-term rate r_b that, in combination with the possibility to invest directly, maximizes the depositors' expected utility.

Direct Investment. Alternatively, investors can directly invest $1 - s$, which gives a high long-term return \bar{R} . In case of liquidation, the direct investment returns $1 - s$ at $t = 1$ and nothing later on.

The critical success probability for a liquidation of the direct investment is simply defined by the fundamental break even condition and not dependent on the liquidated amount. The expected long-term returns should not be smaller than the short-term return.

$$p(\vartheta_F^*) = \frac{\ell}{\bar{R}}. \quad (14)$$

5.2 The Minimum Withdrawal

Before learning their actual type, depositors have to decide how much of their endowment to place at a bank and at which rate. If μ is sufficiently high, depositors will want an aggregate short-term return of H , such that (13) holds. Thus, mechanically,

$$s = \frac{H - l}{r_b - l} \quad (15)$$

The higher r_b , the flatter the yield curve of the bank, and the less the household deposits at the bank. The depositor chooses the r_b that balances his expected payoffs from becoming each type of consumer, given the outside investment opportunity of the fund.

At date $t = 1$, given the investment decision s and the chosen r_1 , the minimum withdrawal at the bank is

$$n_{\min} = \eta + \lambda \min \left\{ \frac{L}{s r_b}; 1 \right\} = \eta + \lambda \min \left\{ \frac{L}{r_b} \frac{r_b - l}{H - l}; 1 \right\} \quad (16)$$

due to (15). Note that n_{\min} increases in r_b . The higher the short-term rate r_b , the higher the minimum withdrawal from the bank per dollar invested of the bank at $t = 1$. This is because a higher short-term rate allows for lower ex-ante investment at the bank. In turn, the higher direct investment increases long-term returns.

5.3 The Optimal Allocation of the Endowment

At $t = 0$ all depositors have to choose a repayment offer from a bank. Anticipating that a higher repayment implies higher fragility of a run, the depositor balances the trade-off of higher chances of not getting the payment from the bank due to a bank run with the benefit of enjoying higher long-term returns from direct investment. Under the constraint, that he receives H in the absence of a bank run, he chooses a bank offer that maximizes the expected payoffs.

While the liquidation threshold of the direct investment is independent of the bank rate and minimum withdrawals, the bank run threshold depends on n^{\min} as a function of r_b . The individual depositor maximizes his expected utility by choosing r_b and the according s without taking into account that his personal choice has in equilibrium an effect on n^{\min} .

He thus maximizes the expected return with no run ($1 - \Pr[\vartheta < \vartheta^*(r_b, s)]$), a bank run only ($\Pr[\vartheta < \vartheta^*(r_b, s)] - \Pr[\vartheta < \vartheta^F]$), and a run both on the bank and the fund ($\Pr[\vartheta < \vartheta^F]$).

Taking n^{\min} as given, the depositor only considers

$$\frac{\partial p(\vartheta^*(r_b, n^{\min}))}{\partial r_b}, \quad (17)$$

which is positive (see Lemma 2).

We now want to analyze the socially optimal choice of r_b , and as a consequence of s . A social planner takes into account that, as r_b increases, also n^{\min} will be affected, and thus the deposit base of banks. From the planner's perspective, the derivative is

$$\frac{dp(\vartheta^*(r_b, n^{\min}))}{dr_b} = \frac{\partial p(\vartheta^*(r_b, n^{\min}))}{\partial n^{\min}} \left(\frac{\partial n^{\min}}{\partial s} \frac{\partial s}{\partial r_b} + \frac{\partial n^{\min}}{\partial r_b} \right) + \frac{\partial p(\vartheta^*(r_b, n^{\min}))}{\partial r_b}. \quad (18)$$

instead of (17). Clearly, the last term is identical to (17), but there are new components that stem from the fact that the planner takes the change in the deposit bases into account. We can show that the new addend is positive, and thus

Proposition 4 *Depositors allocate too little investment to the banking sector.*

The planner sees a steeper reaction of risk to the deposit rate, and this implies that the planner prefers a larger banking sector s . Funds compete away the deposit base from the banking sector. This reduces the stability of the banking sector. This externality is not taken into account by depositors.

This proposition is highly relevant for policy making. Banks are more risky than either funds or direct investments because they provide maturity transformation. At first sight, a policy maker might, thus, be induced to reduce the volume of the banking sector. However, if consumers value maturity transformation, as we assume in our model, more transformation will take place with a smaller deposit volume. In other words, the deposit base erodes. This destabilizes the banking sector. Proposition 4 implies that the destabilization is so strong that it more than outweighs the smaller size of the banking sector. Hence, a policy maker should facilitate bank deposits. The question which policy instrument to facilitate bank deposits is, admittedly outside the current model. Possible instruments that would all reach the same allocation in our model include, but are not restricted to: a regulation of investment volumes, a tax on investment funds, and a subsidy for banks. The introduction of deposit insurance, possibly under-priced, would go in the same direction and is discussed in the next section.

6 Extensions

Deposit Insurance. Many models in the tradition of Diamond and Dybvig (1983) are subject to the criticism that they become irrelevant if banks are covered by deposit insurance. This criticism holds if deposit insurance is perfect, if it covers deposits completely and immediately. The strategic complementarity among depositors then disappears, and so does the foundation of the global game. In our framework, one can ask a number of more specific questions. First, how does the introduction of deposit insurance affect the impact of the deposit base on the run probability? For exposition, let us assume that in case of default, deposit insurance steps in with probability γ and pays depositors their claims. The case of no deposit insurance is embedded for $\gamma = 0$, and the case of perfect deposit insurance is embedded for $\gamma = 1$. Then, the probability of a run decreases in γ . The role of the deposit base is reduced quantitatively, but the qualitative results remain intact. Especially, in an economy with symmetric investment opportunities, consumers prefer the symmetric equilibrium. In an economy with inherently different investment opportunities, consumers allocate too little capital to the maturity-transforming institution.

A second question that can be discussed in our framework is, what happens if the policy maker implements deposit insurance only for a fraction of the banking system? The reason could be, for example, that some banks are based abroad, so they do not fall under the regional safety net. They could also be financial institutions without a bank licence, such

as investment funds, life insurers, or shadow banks. In the model, the deposit insurance will then shift part of the deposit base to the insured part of the system. At date 1, L -consumers prefer to keep their savings at the insured institutions, which means that they withdraw from those that are not covered by deposit insurance. These institutions are destabilized. We can show that, if γ is small, the aggregate effect of deposit insurance is negative. If $\gamma = 1$, it is positive.

This result is a feature of the externalities stemming from shifts in the deposit base. In a model \tilde{A} la GP, the introduction of deposit insurance is always unambiguously positive, even if it does not cover the complete financial system, and even if it is not perfectly credible. In our model with an endogenous deposit base, there are negative externalities on the uninsured institutions, and they can even outweigh all benefits.

Subsidized Long-Term Savings. In many countries, the government subsidizes long-term investment products. In some cases, the illiquidity of these products are a prerequisite for the subsidy. But even if not, these products can offer a higher long-term return than traditional banks, and thus acquire the senior position in the liquidity pecking order. Consequently, their ability to offer high long-term returns is reinforced.

From the perspective of our model, such subsidies are questionable. Proposition 4 shows that households put too little money into the maturity-transforming sector. So the policy maker should not subsidize the long-term investment industry, vice versa.

The Banks' Investment Decision. The deposit base of a bank influences not only its fragility to bank runs but also its investment decision. For simplicity, we have assumed that banks can only invest in a single liquid, but risky, asset. We can extend our model to introduce an endogenous investment choice. Assume an investment set identical to the classical Diamond and Dybvig (1983) model. Banks can choose between a safe and liquid investment opportunity (storage) and a risky and illiquid investment opportunity (risky investment). The risky investment is illiquid because its liquidation value ℓ at date 1 is lower than unity. In contrast to our model, liquidation of the risky asset now become costly because of $\ell < 1$. In order to avoid costly liquidation, the banks now have to store some of their investment in order to be able to serve impatient consumers withdrawals. The basic mechanism is that banks can invest their deposit base in the long run productive technology and store the amount they expect depositors to withdraw at date 1. In this framework, a reduction in the deposit base also influences the investment decision of banks, i. e., it decreases the bank's ability to invest in the long run productive technology. However, the main insights of the asymmetry among banks and the results for the stability of the banking system remain unchanged.

Ring-Fencing. Our model can also be used to analyze uneven seniority of investment withdrawals within one single financial institution. Our results apply if individual banks are forced to separate parts of their businesses and funding sources from each other by a ring-fence. One example are the considerations to separate deposit funded operations from the other activities of a bank. On the one hand, households may want to invest some of their endowment in liquid bank accounts and the residual part in less liquid investments that offer higher long-term interest rates but can be liquidated only with a penalty. As long as the bank can cross-fund liquidity needs within the institution, the bank-run probability is similar to our example of the symmetric banking sector. However, if the bank has to ring-fence the deposit funds from its residual business, the deposit base of the deposit-funded bank is much lower than the average deposit base of the bank. Overall, the bank may become less stable because of the separation of the different liquidity sources.

A similar argument can be made for subsidized saving products. Consider a government decides to allow all banks to offer a special bank account that is subsidized and refinanced by taxes. The amount collected in this bank account can be distributed to risky long-term investments by the banks. However, if the bank has liquidity needs from its daily business, it is not or only partly allowed to use the liquidity from the special account. In this case, the same mechanisms that we discussed for banking systems apply for a single financial institution. Households would invest their entire savings at the one bank but distribute as much as possible in the subsidized account and only the residual in a normal bank account at the bank. In case of a consumption shock, the short depositors would first withdraw their deposits in the normal bank account and only the residual in case of need from the subsidized account in order to maximize subsidy payments.

This decreases the deposit base for the bank's normal business while increasing the deposit base of the special account. In the case of ring-fencing, between the two types of accounts, the normal bank business becomes more fragile and the special account safer than the single unsubsidized bank. However, as shown above: in aggregate, the bank would become more fragile to panic-based runs even as a single institution.

Save Havens. We have shown that any asymmetry in the deposit base increases the aggregate probability of a run, or more precisely, the volume-weighted probability of a run. Note, however, that because all banks invest into the same technology, runs are triggered by the same macro-variable ϑ . Only the threshold differs. Hence, if there are two types of banks, there are three states of nature: one in which no bank fails, one in which both types of banks fail, and one in which only the more risky bank fails.

Consider a policy-maker that wants to minimize the probability that both banks fail. In our model, this is equivalent to the creation of a safe haven, a bank that is as stable as possible. The deposit base would still play a role, but now the policy maker would prefer an asymmetric solution with a liquidity pecking order. (7) implies that $\lim_{r_1 \rightarrow 1} p(\vartheta^*) = 1/R$,

which means that banks have an efficient level of liquidation if (and only if) they emulate the market solution and do not engage in maturity transformation. The remaining bank has an accordingly higher run probability, depending on its size. The larger it is, the smaller its run probability, but the smaller the size of the safe haven. This trade-off determines the optimal size of the banking sector.

Contagion. The same arguments may apply if we consider contagion among banks and the resulting systemic risk. If there is a risk that bank-runs spread from one bank to another bank there are two effects. On the one hand, it would be optimal to minimize the overall probability of bank-runs. Therefore, subsidies would be harmful. On the other hand, a banking system with many banks and symmetrically allocated liquidity might be vulnerable already to small liquidity shocks that spread from one bank to the other. Therefore, the creation of safer banks could be beneficial, if those banks would be able to survive liquidity shock that would already trigger a bank panic in an even banking system.

7 Conclusion

In this paper, we analyze the effect of the deposit base on bank stability. We show that a decrease in the deposit base over-proportionally increases the probability of a panic based runs. Using this insight, we analyze how the more and more popular multi-banking behavior of depositors may affect the stability of banks. We discuss how exogenous policies can introduce a pecking order in the withdrawal behavior of depositors. As a result of this pecking order, the deposit base is unevenly allocated in the banking system such that the banking sector as a whole becomes more prone to panic-based bank runs. This additional effect is special in banking because it results from the role of banks as liquidity insurers.

In our opinion, more than proving a number of specific propositions, the contribution of this paper is to draw the attention of the role of the deposit base on financial stability. The aggregate deposit base in an economy is defined by preferences, and thus is relatively constant. Its distribution in the economy, however, is endogenous. The deposit base of a bank depends not only on its balance sheet structure and the yield curve that it offers to its depositor. If depositors use multiple financial institutions and contracts, the deposit base of a certain institution depends on the rank it gets in the deposit pecking order relative to its competitors. This externality is a driving factor for bank stability.

A Proofs

Proof of Proposition 1. The first part of the proposition is equivalent to Theorem 1 in GP, modelling the differences in the consumers' utility function. The second part is equivalent to Proposition 1 in Allen *et al.* (2013). In our special case, equation (5) in that paper simplifies to

$$0 = \int_{n_{min}}^{\frac{1}{r_1}} \left(p(\vartheta^*) \frac{1 - n r_1}{1 - n} R - r_1 \right) dn + \int_{\frac{1}{r_1}}^1 \left(0 - \frac{1}{n} \right) dn$$

$$0 = (p(\vartheta^*) R - 1) (1 - n_{min} r_1) - \log r_1 - p(\vartheta^*) R (r_1 - 1) \log \left(\frac{r_1 (1 - n_{min})}{r_1 - 1} \right).$$

Solving for $p(\vartheta^*)$ yields (7). If $1 > 1/r_1 > n_{min}$ we can integrate and both logarithms are positive. We therefore have to show that the denominator of equation (7) is positive. At the upper limit as $\lim_{n_{min} \rightarrow 1/r_1}$, the denominator approaches zero,

$$\lim_{n_{min} \rightarrow 1/r_1} = (1 - 1) - (r_1 - 1) \log \left(\frac{r_1 - 1}{r_1 - 1} \right) = 0. \quad (19)$$

The term's first order derivative with respect to n_{min} is negative: $\frac{n_{min} r_1 - 1}{1 - n_{min}} - r_1 < 0 \forall n_{min} < \frac{1}{r_1}$ such that the denominator is a strictly decreasing function over the domain and it approaches zero from above at the upper limit of β . ■

Proof of Lemma 1. The creation of maturity transformation in the form of liquid bank deposits increases welfare as it allows consumers to benefit from their entire consumption opportunities at $t = 1$. Yet, this increase in welfare comes at the cost of the bank becoming vulnerable to bank runs. If μ is high enough, this welfare enhancing effect outweighs the losses from inefficient bank runs. To define this critical utility we need to compare the expected utility created by bank deposits with the expected utility of households in the absence of banks. We show that households are better off when they pool their savings at a bank compared to trading their claims in a financial market.¹¹

In an economy without a bank, all households invest their unit endowment into the risky investment technology that produces expected return $\bar{p}R \geq 1$ at date 2. At date 1, a market to trade claims on investments opens, whereby P units of the good at date 1 are exchanged against the promise to receive 1 unit of the good at date 2.

At date 1 impatient consumers sell at the market or liquidate their investment if they can pursue their private investment.

¹¹As we assume liquidation is costless, the social welfare in autarky is the same as under financial market trading.

In an arbitrage free market the equilibrium price for a claim on the risky investment at date 1 is $P = \frac{1}{\bar{p}R}$. By selling one claim at date 1, the household can obtain $P\bar{p}R = 1$. Again, H -consumers, cannot consume H because $1 < H$. L -consumers sell a proportion of L bonds on their risky investments. They obtain L to consume and retain $1 - L$ as risky asset investment.

At date 2 risky investment returns realize. All households consume. Ex ante, the expected utility of households with a financial market is

$$u(c_0, c_1, c_2) = \mu(\eta + \lambda L) + \bar{p}R(1 - (\eta + \lambda L)). \quad (20)$$

A deposit account, in contrast, allows impatient households to satisfy their immediate consumption needs to full scale but only if there is no run on the bank. In case of a bank run, H -consumers can only consume μ , L -consumers can consume $\mu L + (1 - L)$ and the residual patient consumers can consume 1 if they take part in the bank run.

If no bank run occurs, H -consumers consume μH , L -consumers consume μL and, together with the residual patient consumers, they consume the long-term return the invested deposit base $\bar{p}R(1 - \beta)$.

Let $\Delta = \eta(H - 1)$ the amount shifted from long-term technology to short-term consumption opportunities in order to allow H -consumers to meet their target consumption. It denote the difference between the short-term liquidity needs of the economy $1 - \beta$ and the achievable short-term consumption without maturity transformation, e.g., in case of liquidation.

Assuming that ϑ is uniformly distributed, we can replace $d\vartheta$ with $dp(\vartheta)$ and write the expected utility of households with bank deposits as

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0, \bar{\vartheta} \rightarrow 1} E[u] &= \int_0^{p(\vartheta^*)} (\mu(1 - (\beta + \Delta)) + (\beta + \Delta)) dp(\vartheta) + \int_{p(\vartheta^*)}^1 (\mu(1 - \beta) + p(\vartheta)R\beta) dp(\vartheta) \\ &= p(\vartheta^*)(\mu(1 - (\beta + \Delta)) + (\beta + \Delta)) + (1 - p(\vartheta^*))\mu(1 - \beta) + \frac{(1 - p(\vartheta^*))^2}{2} R\beta \end{aligned} \quad (21)$$

In the same way, we can summarize the expected utility of the market solution for a uniformly distributed p as

$$\begin{aligned} E[u] &= \int_0^1 (\mu(1 - (\beta + \Delta)) + p(\vartheta)R(\beta + \Delta)) dp(\vartheta) \\ &= \frac{1}{2} R(\beta + \Delta) + \mu(1 - (\beta + \Delta)) \end{aligned} \quad (22)$$

This allows us to define the minimum μ that guarantees that demand deposit contracts with banks increase surplus even though they also increase the probability of early liquidation,

$$\mu > \bar{\mu} := \frac{\frac{R}{2}(\Delta + \beta p(\vartheta^*)^2) - p(\vartheta^*)(\beta + \Delta)}{\Delta(1 - p(\vartheta^*))}. \quad \blacksquare$$

Proof of Lemma 2 We concentrate on the limit $\varepsilon \rightarrow 0$. For a general proof, see GP. For brevity we use here $n = n_{min}$ and $r = r_1$. We have to show that $\frac{\partial p(\vartheta^*)}{\partial r} > 0 \forall n \in [0, \frac{1}{r}]$. The closed form is:

$$\frac{\partial p(\vartheta^*)}{\partial r} = \frac{rn \log(r) + \log\left(\frac{1-n}{r-1}\right) + r(\log(r) - n) \log\left(\frac{r(1-n)}{r-1}\right)}{rR \left(rn + (r-1) \log\left(-\frac{r(n-1)}{r-1}\right) - 1 \right)^2} \quad (23)$$

This is positive, iff the numerator is positive. Denote the numerator as function

$$f(x, r) = rn \log(r) + \log\left(\frac{1-n}{r-1}\right) + r(\log(r) - n) \log\left(\frac{r(1-n)}{r-1}\right).$$

The numerator is decreasing in $n \in [0, \frac{1}{r}] \forall r \geq 1$, i. e.,

$$\frac{\partial f(n, r)}{\partial n} = -\frac{1 - rn + rn \log(r)}{1 - n} - r \log\left(\frac{r(1-n)}{r-1}\right) < 0$$

At the upper limit the second term in the numerator approaches zero:

$$\lim_{n \rightarrow \frac{1}{r}} f\left(\frac{1}{r}, r\right) = \log(r) - \log(r) + r(\log(r) - \frac{1}{r}) \log\left(\frac{r-1}{r-1}\right) = 0. \quad (24)$$

As the numerator is a strictly decreasing function over the domain it approaches zero from above at the upper limit of n . It, therefore, must be positive in the domain $n \in [0, \frac{1}{r}]$, such that $\frac{\partial p(\vartheta^*)}{\partial r} > 0 \forall x \in [0, \frac{1}{r}]$. \blacksquare

Proof of Lemma 3 For brevity we use here $n = n_{min}$ and a fixed $r = r_1$. The generalized critical probability of success that defines the run threshold is then defined as:

$$p(\vartheta^*) = \frac{(1 - rn) + \log(r)}{R \left((1 - rn) - (r-1) \log\left(\frac{r(1-n)}{r-1}\right) \right)} \quad (25)$$

We have to show that $\frac{\partial p(\vartheta^*)}{\partial n} > 0 \forall n \in [0, \frac{1}{r}]$. The closed form is:

$$\frac{\partial p(\vartheta^*)}{\partial n} = \frac{(1 - nr) \log(r) + (r-1) \left(r(1-n) \log\left(\frac{r(1-n)}{r-1}\right) - (1 - nr) \right)}{R(1-n) \left((r-1) \log\left(\frac{r(1-n)}{r-1}\right) - (1 - nr) \right)^2} \quad (26)$$

For $r > 1$ and $n \in [0, \frac{1}{r}]$ it must hold that $1 - n > 0$ and $1 - nr > 0$. This implies that the denominator and the first term of the numerator is unambiguously positive in the domain. Hence, it is sufficient to show that the second term of the numerator is positive.

At the upper limit the second term in the numerator approaches zero:

$$\lim_{n \rightarrow \frac{1}{r}} \left(r(1-n) \log\left(\frac{r(1-n)}{r-1}\right) - (1 - nr) \right) = 0. \quad (27)$$

The first derivative of the term is: $-r \log\left(\frac{r(1-n)}{r-1}\right)$, which is negative if $\left(\frac{r(1-n)}{r-1}\right) > 1$. This holds for all $n \in [0, \frac{1}{r})$. The second term in the numerator is a decreasing function that approaches zero from above. It is therefore positive in the domain $n \in [0, \frac{1}{r})$. All terms of the first derivative are positive, such that $\frac{\partial p(\vartheta^*)}{\partial n} > 0 \forall n \in [0, \frac{1}{r})$. Since we defined the deposit base as a decreasing linear transformation of n_{min} , in particular $\beta = 1 - n_{min} * r$ such that $n_{min} = \frac{1-\beta}{r}$ it is straightforward that $\frac{\partial p(\vartheta^*)}{\partial \beta} = \frac{\partial p(\vartheta^*)}{\partial n} \frac{\partial n}{\partial \beta} > 0$. The higher the deposit base, the higher the critical threshold probability of a success, which decreases the ex-ante probability of a bank run. ■

Proof of Lemma 4 To simplify the notation we write $p(\vartheta^*(\beta)) := p^*(\beta)$. We have to show that, for a given r , the threshold $p^*(\beta)$ is a convex function of β in the domain $\beta \in [0, 1)$:

$$\frac{\partial^2 p^*(\beta)}{\partial \beta^2} > 0 \quad (28)$$

As we defined $\beta = 1 - n_{min} * r$ we first show that $p^*(\beta)$ is convex in n_{min} , which we denote n for brevity.

Because the closed form is quite complex, we consider the nominator and the denominator separately. Consider first the denominator,

$$R(1-n)^2 \left((1-rn) - (r-1) \log\left(\frac{r(1-n)}{r-1}\right) \right)^3 \quad (29)$$

It approaches zero for $\lim_{n \rightarrow \frac{1}{r}}$. Its first derivative $-\frac{1-rn}{1-n}$ is negative. Therefore, the denominator of $p^{*''}(n)$ is positive. The numerator of the $p^{*''}(n)$ is more complicated,

$$\begin{aligned} & (rn-1)((2rn+r-3)\log(r) - 3(r-1)(rn-1)) \\ & - (r-1)((rn-1)(r(2n-3)+1) - (r-1)\log(r)) \log\left(\frac{r(1-n)}{r-1}\right) \end{aligned} \quad (30)$$

It also approaches zero for $\lim_{n \rightarrow \frac{1}{r}}$. We, thus, have to show that the numerator is a decreasing function of n , i. e., that its first derivative is negative in the domain. This first derivative is

$$(r(4n-3)-1) \left(-\frac{(rn-1)(r-\log(r)-1)}{n-1} + (r-1)r \log\left(\frac{r(1-n)}{r-1}\right) \right). \quad (31)$$

It contains two coefficients. The *first* coefficient is an increasing function of n , which approaches $\lim_{n \rightarrow \frac{1}{r}} (r(4n-3)-1) = (3-3r)$ which is negative for $r > 1$. It is therefore negative in the domain $n \in [0, \frac{1}{r})$. The *second* coefficient approaches zero at $\lim_{n \rightarrow \frac{1}{r}}$.

Its derivative $\frac{(r-1)(1-rn+\log(r)-1)}{(1-n)^2} < 0$ is negative in the domain $n \in [0, \frac{1}{r})$. The second coefficient is therefore positive. The derivative of the numerator has a negative and a positive coefficient, it is therefore negative.

The numerator is a decreasing function of n that approaches zero at the upper limit of the domain. It is therefore positive over the domain.

In summary, we find that the denominator and numerator of $p^{*''}(n)$ are positive for $n \in [0, \frac{1}{r})$, which implies that $\frac{\partial^2 p^*(n)}{\partial n^2} > 0$. The bank-run probability is a convex function of n . As we defined $\beta = 1 - nr \leftrightarrow n = \frac{1-\beta}{r}$ the deposit base is a linear transformation of n , i.e., $\frac{\partial^2 p^*(\beta)}{\partial \beta^2} = \frac{1}{r^2} \frac{\partial^2 p^*(n)}{\partial n^2} > 0$: the critical threshold success probability is a convex function of the deposit base.

Taking the convexity of p in β we have to show that ϑ is convex in β as well. By assumption $p'(\vartheta) > 0$ and $p''(\vartheta) < 0$. The inverse function $\vartheta(p)$ must be convex: If $\vartheta(p)$ is the inverse function of $p(\vartheta)$ it must hold that $\vartheta(p(\beta)) = \beta$. Derivation with respect to β yields $\vartheta'(p(\beta))p'(\beta) = 1$ or

$$\vartheta'(p(\beta)) = \frac{1}{p'(\beta)} \quad (32)$$

Taking the second derivative with respect to x yields,

$$p'(\beta)^2 \vartheta''(p(\beta)) + \vartheta'(p(\beta))p''(\beta) = 0 \quad (33)$$

Using equation (32) we get

$$\vartheta''(p(\beta)) = -\frac{p''(\beta)}{p'(\beta)^3}, \quad (34)$$

which is negative for $p'(\beta) > 0$ and $p''(\beta) < 0$. Hence, ϑ as a function of p is convex in p . This implies that $\vartheta(p(\beta))$ is convex in β . ■

Proof of Proposition 2. For uneven financial institutions the lower dominance region changes due to the changes in the deposit base.

The Lower Dominance Region for Banks. Let $p(\underline{\vartheta}^C)$ denote the realized success probability that solves

$$(1-s)H = p(\vartheta)(1-s)\left(\frac{1-(\eta+\lambda)H}{1-(\eta+\lambda)}R\right) \quad (35)$$

$$p(\underline{\vartheta}^C) = \frac{H(1-(\eta+\lambda))}{R(1-(\eta+\lambda)H)}. \quad (36)$$

The Lower Dominance Region for Funds. Let $p(\underline{\vartheta}^S)$ denote the realized success probability that solves

$$sH = p(\vartheta)s\left(\frac{1-\eta H}{1-\eta}R\right) \quad (37)$$

$$p(\underline{\vartheta}^S) = \frac{H(1-\eta)}{R(1-\eta H)}. \quad (38)$$

We also have to consider the second case, that $L > (1 - s)H$. Withdrawing only from the bank is not sufficient to consume L . At date 1, the L -consumer therefore withdraw all deposits $(1 - s)H$ from the bank, and the remaining $L - (1 - s)H$ from the fund.

Runs on Funds. In addition to the proportion of η H -consumers that withdraw all deposits from the subsidized bank, now also λ L -consumers withdraw the fraction $\frac{L-(1-s)H}{sH}$ of their deposits from the fund, such that

$$s \left(\frac{L - (1 - s)H}{sH} \right) H + (1 - s)H = L. \quad (39)$$

Consequently, (6) adjusts to

$$0 = \int_{\eta + \frac{L-(1-s)H}{sH}}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1-nH}{1-n} R - H \right) dn + \int_{\frac{1}{H}}^1 \left(0 - \frac{1}{n} \right) dn. \quad (40)$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1 - \left(\eta + \frac{L-(1-s)H}{sH} \lambda \right) H + \log H}{\left(1 - \left(\eta + \frac{L-(1-s)H}{sH} \lambda \right) H + (H - 1) \log \frac{H-1}{H \left(1 - \left(\eta + \frac{L-(1-s)H}{sH} \lambda \right) \right)} \right)} R. \quad (41)$$

Runs on Banks. In this case both impatient consumers withdraw all their deposits from the bank. Consequently, (6) adjusts to

$$0 = \int_{\eta + \lambda}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1-nH}{1-n} R - H \right) dn + \int_{\frac{1}{H}}^1 \left(0 - \frac{1}{n} \right) dn. \quad (42)$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1 - (\eta + \lambda)H + \log H}{\left(1 - (\eta + \lambda)H + (H - 1) \log \frac{H-1}{H(1-(\eta+\lambda))} \right)} R. \quad (43)$$

Aggregate Bank Run Probability. Now we can analyze the impact of the unevenly allocated deposit base on the aggregate bank-run probability. The bank-run probability in a symmetric banking sector can be written as

$$p^* \left(\eta + \frac{L}{H} \lambda \right). \quad (44)$$

For $s \leq \frac{H-L}{H}$ the aggregate bank-run probability of an asymmetric banking sector can be written as

$$s p^* (\eta) + (1 - s) p^* \left(\eta + \frac{L}{(1-s)H} \lambda \right). \quad (45)$$

For $s > \frac{H-L}{H}$ the aggregate bank-run probability of an asymmetric banking sector is

$$s p^* \left(\eta + \frac{L - (1-s)H}{sH} \lambda \right) + (1-s) p^* (\eta + \lambda). \quad (46)$$

Note that

$$\begin{aligned} \eta + \frac{L}{H} \lambda &= s \eta + (1-s) \left(\eta + \frac{L}{(1-s)H} \lambda \right) \\ &= s \left(\eta + \frac{L - (1-s)H}{sH} \lambda \right) + (1-s) (\eta + \lambda). \end{aligned} \quad (47)$$

The withdrawals in uneven banking systems are a mean preserving spread of the minimum withdrawals in the symmetric banking system.

The convexity of $p^*(\beta)$ in $\beta \in [0, \frac{1}{H})$ therefore implies:

$$p^* \left(s \eta + \frac{L}{H} \lambda \right) < s p^* (\eta) + (1-s) p^* \left(\eta + \frac{L}{(1-s)H} \lambda \right) \quad (48)$$

for all small $\forall s \in \left(0, \frac{H-L}{H}\right]$ and

$$p^* \left(s \eta + \frac{L}{H} \lambda \right) < s p^* \left(\eta + \frac{L - (1-s)H}{sH} \lambda \right) + (1-s) p^* (\eta + \lambda) \quad (49)$$

for all high $s \in \left[\frac{H-L}{H}, 1\right)$. The aggregate bank-run probability increases as the spread of withdrawals increases. ■

Proof of Lemma 5. At time $t = 1$, when the consumption shocks realize, H -consumers withdraw their entire endowment from both banks and receive H . L -consumers only withdraw any amount necessary to satisfy $c_1 = L$. There are two cases to be considered: If $sr_A > L$ the L -consumers can satisfy their consumption need by withdrawing only from bank A . In particular, L -consumers withdraw an amount $\frac{L}{sr_A}$ from bank A and nothing from bank B in order to satisfy their consumption shock. As a result, out of the s deposited endowment units at bank A a proportion of $s(\eta + \frac{L}{sr_A}\lambda)$. The proportional certain withdrawal is therefore $\eta + \frac{L}{sr_A}\lambda > \eta + \frac{L}{H}\lambda$. The reason is that the withdrawal $\eta + \frac{L}{sr_A}\lambda > \eta + \frac{L}{H}$ translates into $H > sr_A$ which must hold for $s = \frac{H-r_B}{r_A-r_B}$ for all $r_A > H$.

Bank A 's deposit base $\beta_A < \beta$ is lower than the deposit base of households in the economy, as $1 - \eta r_A - \lambda \frac{L}{s} < 1 - \eta H - \lambda L$ and $r_A > H$ by assumption, which results in $s < 1$. As L -consumers can satisfy their consumption need by withdrawing a proportion from bank A , only H -consumers withdraw their endowment from bank B for sure. Therefore, the deposit base at bank B is $\beta_B = 1 - \eta r_B > \beta$. We can derive similar results for the second case, where $sr_A \leq L$. In this case, both, H and L -consumers withdraw their entire

investment from bank A such that the proportional withdrawal at bank A is $\eta + \lambda > \eta + \frac{L}{H}\lambda$. The deposit base of bank A shrinks as $\beta_A = 1 - \eta r_A - \lambda r_A < \beta$ for $r_A > H$. At bank B , H -consumers withdraw their entire investment, L -consumers withdraw only a share to satisfy their consumption need in order to benefit from the higher long-term return. This results in a certain withdrawal at bank B of $\beta_B = 1 - \eta r_B - \frac{L - sr_A}{(1-s)}\lambda < \beta$. This requires $L(1-s)r_B > H(L - sr_A)$. Substituting $s = \frac{H - r_B}{r_A - r_B}$ and simplifying, yields $H > L$, which always holds under our assumptions.

Using Lemma 2 and Lemma 3 it follows directly for both cases that $p(\vartheta(\beta_A)) > p(\vartheta^*(\beta)) > p(\vartheta(\beta_B))$ since $\beta_A > \beta > \beta_B$ and $r_A > H > r_B$. ■

Proof of Proposition 3. The bank deposit contract, that allows to satisfy the target consumption of each consumption type ($r_1 = H$) generates an expected utility of

$$E[u] = \int_0^{p(\vartheta^*)} \mu(1 - \beta - \Delta) + (\beta + \Delta) d\vartheta + \int_{p(\vartheta^*)}^1 (\mu(1 - \beta) + p(\vartheta)R\beta) d\vartheta.$$

The expected utility is strictly decreasing in $p(\vartheta^*)$ as a higher bank run threshold puts more mass from the optimal consumption (right integral) to the inefficient consumption caused by a bank run (left integral). Formally we get

$$\frac{\partial E[u(\vartheta^*)]}{\partial \vartheta^*} = -\beta(p(\vartheta^*)R - 1) - \Delta(\mu - 1) < 0 \forall \vartheta^* > \underline{\vartheta}.$$

As the lower dominance region is defined by $\beta p(\underline{\vartheta})R > \beta + (r_1 - 1)$ the first term in brackets must be positive for any $\vartheta^* > \underline{\vartheta}$ with $r_1 > 1$ in equilibrium. The second term in brackets is positive by the definition of $\mu > 1$. From Lemma 5 we know that any deviating multi-banking contract that allows all types to reach their target consumption in the absence of a bank run results in a higher aggregate bank run probability and must therefore result in a lower expected utility. Utility is maximized by the symmetric contract $r_A = r_B = H$. ■

Proof of Proposition 4. Total differentiation yields

$$\frac{dp(\vartheta^*(r_b, n^{\min}))}{dr_b} = \underbrace{\frac{\partial p(\vartheta^*(r_b, n^{\min}))}{\partial n^{\min}}}_{+} \left(\underbrace{\frac{\partial n^{\min}}{\partial s}}_{-} \frac{\partial s}{\partial r_b} + \underbrace{\frac{\partial n^{\min}}{\partial r_b}}_{-} \right) + \underbrace{\frac{\partial p(\vartheta^*(r_b, n^{\min}))}{\partial r_b}}_{+}. \quad (50)$$

The last term is identical to (17), but because the single household takes n^{\min} as given, the other terms do not appear in (17).

We have already put the signs of the derivatives on the terms. The sign of the term in the brackets looks ambiguous. We have

$$\left(\frac{\partial n^{\min}}{\partial s} \frac{\partial s}{\partial r_b} + \frac{\partial n^{\min}}{\partial r_b} \right) = \frac{L\lambda}{r_b^2 s^2} \left(r_b \frac{H-l}{(r_b-l)^2} - s \right) = \frac{lL\lambda}{(H-l)r_b^2}, \quad (51)$$

which is positive. ■

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