Regulating Cancellation Rights with Consumer Experimentation

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Abstract

Embedding consumer experimentation with a product or service into a market environment, we find that unregulated contracts induce too little returns or cancellations, as they do not internalize a pecuniary externality on other firms in the market. Forcing firms to let consumers learn longer by imposing a commonly observed statutory minimum cancellation or refund period is socially efficient only when firms appropriate much of the market surplus, while it backfires otherwise. Interestingly, cancellation rights are a poor predictor of competition, as in the unregulated outcome firms grant particularly generous rights when competition is neither too low nor too high. The overarching theme of our analysis is that both the individual benefits and the welfare consequences of (consumer) experimentation depend crucially on the consumer's reservation value, which is endogenous in a market environment.

Keywords: Consumer experimentation, cancellation rights, market equilibrium, externality, regulation, consumer protection.

JEL Classification: D82, D86, L51.

1 Introduction

Even when firms offer consumers terms of cancellation or return that are bilaterally efficient, we show that unregulated contracts induce too little returns or cancellations as they do not internalize a pecuniary externality on other firms in the market. Whether a mandatory extension of cancellation periods and thus of consumer experimentation is then socially beneficial depends however on whether this increases the likelihood of such cancellation or whether the opposite is the case, to which we refer to either as "bad news learning" or as "good news learning." By appealing to commonly used learning technologies, we can provide a tight characterization on when such "bad news learning" or "good news learning" arises. When consumers' share of total surplus is high, indicating intense competition among firms, policy interference in the form of a statutory minimum cancellation or refund period backfires. Allowing consumers more time for experimentation decreases the likelihood of cancellation. When consumers' share of the market surplus is low, a binding mandatory cancellation period instead is socially beneficial, as there is "bad news learning:" Longer experimentation increases the likelihood of cancellation.

Our results also point to an at first possibly counterintuitive stark difference between imposing a longer cancellation period and granting consumers the right to a higher refund in case of cancellation or return. While the former intervention may backfire, depending on how much of total surplus consumers already appropriate, the latter is always beneficial as it unambiguously increases the probability of a return or cancellation.

Our application to consumer protection and the regulation of cancellation and return policies is timely, given the recent interest in this topic among scholars, business practitioners, and policymakers.[1] Even absent statutory rights, firms frequently grant consumers the right to return a product or to cancel a contract prematurely.[2] These rights do not extend infinitely, such that there is typically a contractually stipulated maximum period for returning the product or revoking the contract. Interestingly, while the level of a refund (or, alternatively, that of a restocking fee) has received considerable scholarly interest, this does

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[1]In the European Union, since 13th June 2014 consumers are given harmonized rights to withdraw from purchases, albeit in many member states similar rights have existed before. The mandatory revocation period applies to purchases made through different channels of distance selling, such as online shops, by phone, or mail order.

[2]Firms may, however, deduct a restocking fee or ask for compensation of any associated costs, which our model also allows for through the choice of the refund level.

not hold with respect to the refund or cancellation period. One set of our results therefore explores also the positive economics of cancellation periods, notably showing that, in a market equilibrium, cancellation rights are particularly attractive only when consumers obtain an intermediate share of total surplus. Firms grant consumers less generous rights either when consumers' share of surplus is low or when it is firms that make very little profits in the market.

We obtain our clear-cut positive and normative implications with respect to the refund period by appealing to several commonly used learning technologies. In particular, when consumers fully exploit the available learning opportunities, e.g., because the prevailing refund period is rather short, which we call "learning until the end," our tight characterization holds whenever allowing for more time to learn results in a single-crossing mean-preserving spread ("rotation") of the ex-ante distribution of a consumer's posterior valuation. Such a rotation property arises naturally with common learning processes, such as Gaussian learning (where the true valuation is drawn from a normal or a binary distribution) and "all-or-nothing" learning (where a perfectly revealing signal arrives stochastically over time). The latter specification also allows for a tractable analysis of situations in which consumers prefer to return or cancel prior to the end of the refund period, as becomes relevant, e.g., with sufficiently high discount rates, showing the robustness of our results.

Related Literature.   We next place our various contributions into the respective literature. Our paper ties into the burgeoning literature on sellers' incentives to provide buyers with match-specific information. Notably, contributions by, for instance, Johnson and Myatt (2006), Bar-Isaac et al. (2010), and Ganuza and Penalva (2010) all assume that more (precise) information leads to a more dispersed distribution of a buyer's expected valuation, often modelled as a rotation of the respective distributions. In our model, with the particular application to "learning until the end," such a rotation arises endogenously from the contractual instrument of providing buyers with a longer time to learn match-specific information.[3] Another key difference to these papers, which arises naturally in our setting, is that a seller can flexibly charge a consumer for a more generous cancellation and return policy, notably through the simultaneous choice of the initial product price and

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[3]This also distinguishes our paper from the literature on information design à la Kamenica and Gentzkow (2011) where a sender commits to an information structure in an unconstrained fashion.

the refund.[4] This also implies that the refund period, as well as the refund level and thus the return decision, will all be bilaterally efficient, albeit an externality on the market is neglected.

Generous money-back guarantees have also long been recognized as ways for sellers to either signal high quality of their products or to mitigate moral-hazard in quality provision (Grossman, 1981; Mann and Wissink, 1990). In Che (1996) consumer risk aversion generates refunds above the salvage value. As noted previously, our main innovations are the consideration of the return or contract revocation period, rather than only the refund level, as well as the consideration of a consumer's option to reenter the market and try another product or service.

In terms of its implications for regulation, our paper is more closely related to Loewenstein et al. (2003) and Inderst and Ottaviani (2013). In a rare exception, as they consider also explicitly the role of time, Loewenstein et al. (2003) argue that a mandatory "cooling-off" period benefits buyers who are unaware that they suffer from a projection bias at the time of purchase. In Inderst and Ottaviani (2013) a minimum refund level is likewise only beneficial when consumers have a behavioral bias: Credulous consumers rely too much on advice and naively overestimate the value of a good or service and thus underestimate the likelihood with which they will return or cancel.[5] In our model with rational consumers and efficient bilateral contracting, scope for regulation arises only when considering the full market equilibrium (rather than in a partial equilibrium analysis).

Lastly, the possibility to return to the market and then experiment with a new product or service relates our paper to the vast literature on search and matching markets, as notably applied in the labour literature. The externality that we isolate in our model is clearly different from congestion and composition externalities typically found in decentralized markets with frictions, e.g., Hosios (1990). Labor market models also consider learning about match-quality when there is uncertainty about the value of a particular worker-firm match (Jovanovic, 1979; Felli and Harris, 1996), albeit there the focus is typically on wage

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[4]Hoffmann and Inderst (2011) allow sellers to charge for costly information provision, albeit theirs is a model of pre-purchase screening (see also Esö and Szentes, 2007). Refunds or restocking fees have been considered in the literature on sequential screening as ways to screen between buyers with different ex-ante valuation (Courty and Li, 2000; Matthews and Persico, 2007; Shulman et al., 2011; Krähmer and Strausz, 2015).

[5]Interestingly, much of the existing regulation is targeted to distance selling, away from sellers' premises, so that the consumer has initially no direct contact with either the product itself or with a salesperson.

dynamics and in particular not on the determination of the contractual parameters that we are interested in.[6] Contributions that consider variations in (regulated) contractual choices, such as temporary and permanent employment contracts and firing taxes, focus more on search and equilibrium unemployment (see, for instance, Alvarez and Veracierto (2012) and the literature discussed there).[7] We further discuss the potential applicability of our modelling and results to other areas, such as labour markets, in our conclusion.

Organization. The rest of this paper is organized as follows. The next Section introduces the baseline model of consumer experimentation and embeds this in a market environment. We identify the positive externality that consumer cancellation or product return have on other firms in the market in Section 3. Sections 4 and 5 contain the main policy results on the regulation of consumers' cancellation and refund period for the cases of "all-or-nothing learning" and "learning until the end" respectively. Section 6 shows the difference to the regulation of the refund level. We consider several alternative market models in Section 7. Section 8 concludes. All proofs are contained in Appendix A. Appendix B provides some additional material.

2   The Model

We start with a (bilateral) contracting problem between a consumer and a firm which will then be embedded in a market environment. The firm can produce a product (or initiate a contract) at cost $c > 0$. The consumer has an unknown time-invariant valuation u which is drawn from a distribution $G(u)$ with finite expectation $E[u] > c$. He can learn about his valuation u in continuous time $t \geq 0$ by experimenting with the product which causes flow costs $z(t) > 0$ to the firm.[8] Further, both the firm and the consumer discount future payoffs at a common rate $r \geq 0$.

Consumer learning is modeled as follows: The consumer learns about his valuation by observing informative signals s_t, defined on a probability space $(S, \mathcal{F}, \mathbb{P})$, which generate a

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[6]Mortensen and Pissarides (1994) as well as Moscarini (2005) also consider evolving idiosyncratic uncertainty in match productivity as the main driver of turnover.

[7]The positive externality that we identify is also related to the negative "business stealing" externality in the labour literature that is identified in Gautier et al. (2010), as there firms posting a vacancy do not take into account the output loss they impose on other firms when they hire an employed worker.

[8]These costs may be interpreted differently depending, in particular, on whether we consider the purchase of a (physical) product or the conclusion of a service contract.

filtration $\{\mathcal{F}_t^s\}$ such that all information collected by observing the signals until time t is contained in the σ-algebra $\mathcal{F}_t^s$. We denote the posterior valuation given time t information by $U_t := E[u|\mathcal{F}_t^s]$ and assume throughout that this conditional expectation exists. The consumer can stop experimenting at any time. Once he has stopped, he has to decide whether to terminate the contract or return the product - or whether to consume it. In the baseline analysis, we specify that the consumer only derives utility in the latter case. In case of cancellation or return, the firm recoups a salvage value or saves further costs of $k < c$, while the consumer realizes the continuation value $\widehat{V}$. This is endogenized subsequently when embedding the bilateral problem into a market environment, where, upon cancellation, a consumer can turn to another firm and experiment with a new product or service.

The bilateral contract between the firm and the consumer specifies, next to the price p, the following two variables: A maximum time T until which the consumer is allowed to experiment and, in case he then decides to cancel or return, a refund q. Note that time independence of the refund q is necessary so as to meaningfully incorporate a refund period T into the analysis (and consider the respective regulation). We return to this discussion later.

In Section 2.1 we, first, formalize the consumer learning and cancellation problem in order to, second, formally state the bilateral contracting problem. Section 2.2 then embeds this into a market environment, which serves as the starting point for our subsequent analysis of regulation. In what follows, it is convenient to simplify the exposition by supposing that the consumer purchases and possibly returns a (physical) product, albeit the analysis also extends to the termination of a (service) contract.

2.1 Bilateral Contracting Problem with Consumer Learning and Contract Cancellation

Once the consumer has accepted the firm's contract and paid the price p, he needs to decide whether to start and when to stop experimenting, as well as, subsequently, whether to return or consume the product given the information at the stopping time $\tau \in [0, T]$.[9] As he realizes a continuation value $\widehat{V}$ when he returns the product, the consumer (weakly) prefers to do so when, at time τ, the expected value $U_\tau = E[u|\mathcal{F}_\tau^s]$ lies below the sum

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[9]Formally, τ is a $\{\mathcal{F}_t^s\}$ measurable stopping time.

of $\widehat{V}$ and the refund q: $U_\tau \leq q + \widehat{V}$. Denoting the given contract by $\gamma = (p, q, T)$, the consumer's initial expected payoff from participation for a given stopping time τ, hence, is given by

$$V = v(\tau, \gamma, \widehat{V}) = E\left[e^{-r\tau} \max\{q + \widehat{V}, U_\tau\}\right] - p, \tag{1}$$

such that the optimal stopping rule τ^* solves $\sup_{0 \leq \tau \leq T} v(\tau, \gamma, \widehat{V})$.

Next, the expected profit of a firm for a given contract γ and stopping time τ equals

$$\Pi = \pi(\gamma, \tau, \widehat{V}) = p - c - (q - k)\beta(q, T, \tau, \widehat{V}) - E\left[\int_0^\tau e^{-rt} z(t)dt\right] \tag{2}$$

where $\beta(q, T, \tau, \widehat{V})$ denotes the *discounted likelihood of cancellation* defined as the present value of a security that pays one when the consumer returns the product:

$$\beta(q, T, \tau, \widehat{V}) := E\left[e^{-r\tau} \mathbf{1}_{\{U_\tau \leq q + \widehat{V}\}}\right].$$

Here $\mathbf{1}_{\{U_\tau \leq q + \widehat{V}\}}$ is an indicator that is equal to one when a return takes place and zero otherwise.[10]

The sum of the two payoffs $V + \Pi$, as defined in (1) and (2), captures the total bilateral surplus and is denoted by $\omega(q, T, \tau, \widehat{V})$. Denote by $\omega^*(\widehat{V}) = \omega(q^*, T^*, \tau^*, \widehat{V})$ the bilateral surplus that results from the firm choosing q^* and T^* to maximize profits, while taking into account the consumer's dynamic optimization of his own surplus, V, as captured by τ^*, for given continuation value $\widehat{V}$, refund payment q^*, and refund period T^*. As utility is fully transferable, namely through adjusting the price p, the choices of q^*, T^*, and then also τ^* are independent of how the joint surplus is shared. In particular, we have the following property:

Bilateral Surplus Maximization (BSM) *The optimal choices q^* and T^* maximize* $\omega(q, T, \tau^*, \widehat{V})$.

For ease of exposition, we assume throughout, that the solution to the bilateral problem exists and is unique (Sections 4 and 5 provide examples where this holds).

2.2   Market Environment and Equilibrium

We now embed the bilateral contracting problem in a market environment. This serves the following objectives. First, we endogenize consumers' continuation value $\widehat{V}$ and thereby

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[10] The discounted likelihood of cancellation is conceptually related to a state price in the Arrow-Debreu model which is the value of a security that generates a cash flow of one in a specific state and time.

fully close the model. The second, related purpose is to analyze subsequently the full implications of regulation. Regulation should have repercussions on the whole market and thus also on consumers' continuation value. In particular, we ask later how the effect of regulation depends on the primitives of the market, such as the prevailing degree of competition.

Our insights are general and applicable to a variety of different market models where consumers have the option to reenter the market and try another product after returning the current one. For these insights to hold a market equilibrium must have the following properties. First, the continuation value $\widehat{V}^*$ that captures the consumer's value of reentering the market after returning a firm's product must be constant across time and firms.

Constant Continuation Value (CCV) *The equilibrium continuation value $\widehat{V}^*$ is time-invariant and constant for all consumers across all firms.*

Second, we require that equilibrium contracts maximize bilateral surplus, i.e., property (BSM) holds also in market equilibrium. As long as these conditions hold, our results do not depend on a particular sharing rule. In Section 7 we present three different market models that satisfy these conditions. First, we consider a market with posted prices and advertising; second, we discuss a model with Nash bargaining where bargaining power determines surplus sharing; and lastly, we present a setting where consumers engage in noisy search. However, in order to make the analysis transparent we focus in the main part on a reduced form market equilibrium. In particular, we take the profits that a single firm can make when matched with a consumer as a primitive and denote this by $\pi \geq 0$. A lower value of π may capture more intense competition and a higher value less intense competition. We streamline the exposition of our baseline setting further by abstracting from any frictions that may result from finding a new match, so that the continuation value equals the expected consumer surplus generated in a match: $\widehat{V}^* = V^* = v(\tau^*, \gamma^*, V^*)$. As we discuss below the market definition extends directly to the case where there are frictions, e.g., hassle costs from returning the product or search costs from finding another firm, so that $\widehat{V}^* \neq V^*$. Still, for the baseline analysis we have:

Surplus Sharing Rule (SSR) *In equilibrium, the bilateral surplus $\omega^*(V^*)$ is shared*

according to

$$\omega^*(V^*) - V^* = \pi \qquad \qquad (3)$$

and p^* is such that $V^* = v(\tau^*, \gamma^*, V^*) \geq 0$.

To summarize, in the main part of our analysis we work with the following (reduced form) definition of a market equilibrium:

Definition 1 (Market Equilibrium) *A market equilibrium is a tuple $(\pi, V^*, \gamma^*, \tau^*)$ that satisfies conditions (BSM), (CCV), and (SSR).*

We close by establishing existence. Define by $\overline{\pi} := \omega^*(0)$ the maximum feasible profit, where $\overline{\pi} > 0$ as $E[u] > c$. When $\pi = \overline{\pi}$, all surplus is extracted by firms, while when $\pi = 0$, consumers extract all surplus. We then have the following result:

Lemma 1 *Suppose that for any value of the consumer outside option $\widehat{V}$ a unique bilaterally optimal contract exists. Then, for any learning technology, there is a $\overline{\pi} > 0$ such that for all $0 \leq \pi \leq \overline{\pi}$ there exists a unique (unregulated) market equilibrium. Equilibrium consumer surplus V^* is strictly decreasing in firm profits π.*

Market with Frictions. It may be argued that realistically the process of returning a product, as well as that of searching for and contracting with a new firm, should involve frictions. Suppose for instance that when a product is returned and a new firm is contacted, this involves disutility $h > 0$ for the respective consumer. To be specific, suppose further that these are hassle costs from returning the product (see also Appendix B). Then, consumers' continuation value is $\widehat{V}^* = V^* - h$ and requirement (3) must then read $\omega^*(\widehat{V}^*) - V^* = \pi$, so that the size of h affects consumers' continuation value and thereby the equilibrium contract. The subtraction of $h > 0$ however does not affect the subsequent results qualitatively.

3   Market Failure and Regulation

Market Externality. In the market equilibrium of Definition 1, the equilibrium contract γ^* is chosen to maximize the joint surplus of any given consumer-firm pair. Recall also that, after returning the product, the consumer will conclude a contract with another firm.

While the consumer's option to take up his continuation value is taken into consideration, the bilaterally efficient contractual choice does not internalize the positive externality that a return exerts on other firms in case $\pi > 0$. We ask whether, in light of this positive externality, more generous return rights can improve social welfare. In particular, we focus on the effects of a statutory minimum refund period, which we, subsequently, compare to the strikingly different implications when regulation imposes a minimum refund payment.[11]

To formalize the preceding discussion, we make use of the equilibrium requirement to compactly express the expected social surplus that is realized with each consumer in market equilibrium as

$$\Omega^* = \omega^*(V^*) + \frac{\beta^*}{1 - \beta^*}\pi, \tag{4}$$

where $\beta^* := \beta(q^*, T^*, \tau^*, \widehat{V}^*)$ is the equilibrium discounted likelihood of cancellation. Expressing the social surplus in this way reveals immediately the aforementioned externality, as captured by the last term in (4), which is only absent when $\pi = 0$. When $\pi > 0$ then, irrespective of the learning technology, the equilibrium likelihood of cancellation and return $\beta^* := \beta(q^*, T^*, \tau^*, \widehat{V}^*)$ in each individual consumer-firm match is too low from the perspective of maximizing total welfare.

Regulating Cancellation Rights.   From the preceeding discussion, scope for regulation arises as, under the bilaterally optimal contract, the equilibrium likelihood of a return will always be too low whenever $\pi > 0$. Regulatory intervention in equilibrium contracts could then target either the refund period or the refund level. We defer discussion of the effects of refund level regulation to Section 6 and for now focus on regulation that prescribes a statutory minimum refund period $\underline{T}$. For this we denote by Γ_R the set of all contracts with $T \geq \underline{T}$ and index variables that are affected by regulation with a subscript R. The definition of a regulated market equilibrium is then completely analogous to that without regulation, with the additional restriction that bilaterally optimal contracts have to satisfy the regulatory constraint $\gamma_R \in \Gamma_R$.

To see the basic mechanics of how such regulation affects social welfare, suppose that

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[11]We note that there further may be scope for competition policy, which in our (reduced form) interpretation of π would lead to a reduction of firm profits. In particular, competition policy affects total welfare through the identified market externality, as there is no deadweight loss in bilateral contracting. This externality is lower when firm profits are lower, so that then the wedge between what is bilaterally efficient and what is socially efficient becomes smaller. A complete analysis of competition policy, however, requires a fully specified market model (see Section 7).

the unregulated refund period is interior, $T^* > 0$. Now, in order to isolate the effect of a changing refund period, we initially consider comparative statics in T holding the refund level q fixed at the unregulated level. We note that while this assumption is taken for illustrative purpose, it does not impose a restriction in any of the concrete specifications of the learning technology analyzed below, where $q_R^* = q^*$ arises endogenously.

When regulation constrains contracts only marginally compared to the unregulated equilibrium outcome, the first-order effect on bilateral surplus and, thus, also on consumers' equilibrium surplus is zero.[12] Hence, the effect on social efficiency is then solely determined by the impact of regulation on the discounted likelihood of cancellation β_R^*. To sign how β_R^* changes in the refund period note, first, that a longer refund period increases at each point in time the value of learning. However, as the set of histories after which the consumer prefers to continue learning increases, the likelihood of a return may both increase as well as decrease, depending on the induced shift in the distribution of posterior expectations. Discounting then complicates the analysis considerably as it matters not only whether the consumer returns but also when. Further, given the restrictions imposed by contracts, in particular the finite horizon T, a tractable analysis of the consumer's optimal stopping problem requires more specific assumptions on the learning technology. Thus, in the first part of our analysis in Section 4 we consider a specific learning technology ("all-or-nothing learning") where a perfectly revealing signal arrives stochastically over time, allowing for an explicit analysis of the discounted likelihood of cancellation under consumer-optimal timing of return. In the second part of our analysis in Section 5 we instead abstract from discounting and consider settings in which consumers fully exploit the refund period ("learning until the end"), as is natural in many relevant applications noting that refund periods usually are rather short.

4   All-or-Nothing Learning

In this section we study all-or-nothing learning, where a perfectly revealing signal arrives stochastically over time. In this setting, at any time t, there either is an event where the consumer learns his true valuation u perfectly or, absent such a fully revealing signal, he retains his prior valuation $E[u]$. The distribution of arrival times is given by a cdf $H(t)$

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[12]For illustration, we assume that both the bilateral surplus ω as well as the discounted likelihood of return β are partially differentiable in T and $\widehat{V}$.

that is endowed with a density $h(t) > 0$ for $t \geq 0$.[13] Besides being analytically tractable, this learning technology appears to be particularly suitable in certain cases. For instance, the consumer may or may not find the time to try out a piece of clothing or to test a service before he has to make his final decision.

We proceed as follows. In Section 4.1 we derive the consumer optimal stopping behavior and use this result to solve for the optimal bilateral contract in Section 4.2, where we also discuss the basic properties of the market equilibrium. Section 4.3 then contains the main policy results on the regulation of consumers' cancellation and refund period.

4.1   Preliminary Results

Consumer Optimal Stopping.   The all-or-nothing learning technology results in a particularly tractable consumer optimal stopping rule. A consumer will stop either because he has learned his valuation u (and therefore there is nothing left to learn), the refund period T is over, or because he has arrived at a point $T' \leq T$ at which waiting for a signal until T is no longer worthwhile. In order to streamline the subsequent analysis we stipulate that if an uninformed consumer starts experimenting at $t = 0$, he continues to do so until he either learns his valuation u or the refund period is over, that is $t = T$. The next Lemma provides a sufficient condition for the (uninformed) consumer to exhaust the available refund period T.

Lemma 2 *When learning is all-or-nothing and when, for a given contract γ and continuation value $\widehat{V}$, for all $0 \leq t \leq T$ it holds that*

$$\frac{h(t)}{1 - H(t)} \frac{E[\max\{q + \widehat{V}, u\}] - \max\{q + \widehat{V}, E[u]\}}{\max\{q + \widehat{V}, E[u]\}} > r, \tag{5}$$

then the uninformed consumer starts learning and continues to wait for a perfectly revealing signal until either such a signal arrives or the refund period T is exhausted.

The intuition for condition (5) is straightforward. If the arrival of information, as captured by the hazard rate $h(t)/(1 - H(t))$, and the information's rate of return are both

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[13]A common example is the Poisson process with a constant arrival rate $\lambda > 0$ and exponential arrival time distribution $H(t) = 1 - e^{-\lambda t}$. In a previous version of this paper we also considered state dependent arrival time distributions $H(t, u)$. As these do not add substantial new economic insights, to streamline exposition, we chose to omit a formal analysis in this version (see however the discussion of how our results extend to more general learning technologies in the Conclusion).

sufficiently high compared to the discount rate r at each point in time $0 \leq t \leq T$, the uninformed consumer prefers to continue learning. Note, however, that the condition in Lemma 2 will not impose a relevant restriction in the subsequent equilibrium analysis, as clearly two contracts with $T'' > T'$, where the consumer stops at T' if he has learnt nothing so far are equivalent under the considered learning technology.

Discounted Likelihood of Cancellation, "Good News Learning", and "Bad News Learning". With all-or-nothing learning we can provide an explicit characterization of the discounted likelihood of cancellation since the distribution of the stopping time τ is now simply a censored version of the signal's arrival time distribution $H(t)$. Note that $\beta(q, T, \tau, \widehat{V})$ is defined for a given contract γ, a continuation value $\widehat{V}$, and under the assumption that the (uninformed) consumer starts and continues experimenting. If the consumer has a strict preference over keeping or returning when uninformed in T, the discounted likelihood of cancellation is given by

$$\beta(q, T, \tau, \widehat{V}) = \begin{cases} \int_0^T e^{-rt} h(t) G(q + \widehat{V}) dt & \text{if } q + \widehat{V} < E[u] \\ \int_0^T e^{-rt} h(t) G(q + \widehat{V}) dt + e^{-rT}(1 - H(T)) & \text{if } q + \widehat{V} > E[u] \end{cases}.$$

For the knife-edge case where the consumer is still uninformed in T and indifferent between consuming or returning the product, $q + \widehat{V} = E[u]$, we allow him to mix, with $\alpha \in [0, 1]$ denoting the probability of a return at T such that $\beta(q, T, \tau, \widehat{V}) = \int_0^T e^{-rt} h(t) G(q + \widehat{V}) dt + \alpha e^{-rT}(1 - H(T))$. Note that while the mixing probability α has no effect on the bilateral surplus in the unregulated case, it has an impact on total social welfare and will be relevant for the existence of a regulated equilibrium. We conclude this subsection with some comparative results:

Lemma 3 *When learning is all-or-nothing, ceteris paribus, the discounted likelihood of cancellation $\beta(q, T, \tau, \widehat{V})$ strictly increases in T for $q + \widehat{V} < E[u]$ and strictly decreases for $q + \widehat{V} > E[u]$.*

Consider an extension of the refund period from T to T'. This increase will have no effect on the rate of cancellations between time 0 and T as an uninformed consumer is prepared to wait until T to obtain a signal. When $q + \widehat{V} < E[u]$, an uninformed consumer at time T prefers to keep the product. Extending the refund period to T' gives him a chance to learn his true valuation u and, if $u < q + \widehat{V}$, to reverse his uninformed decision.

This increases the discounted likelihood of cancellation. Therefore, under all-or-nothing learning we can identify the case with $q + \widehat{V} < E[u]$ as *bad news learning* and the converse case, $q + \widehat{V} > E[u]$, as *good news learning*. In the latter case, the uninformed consumer at time T prefers to return the product and additional opportunities to learn can only increase the likelihood that he keeps the product.

4.2 Optimal Contracting and Market Equilibrium

Given the optimal stopping rule with all-or-nothing learning the consumer's expected surplus reads

$$V = \int_0^T e^{-rt} h(t) E[\max\{q + \widehat{V}, u\}] dt + e^{-rT}(1 - H(T)) \max\{q + \widehat{V}, E[u]\} - p$$

when he starts experimenting as ensured by the conditions in Lemma 2. Otherwise, he obtains $V = E[u] - p$.[14] Accordingly, firm profits are given by

$$\Pi = p - c - (q - k)\beta(q, T, \tau, \widehat{V}) - \int_0^T h(s) \int_0^s e^{-rt} z(t) dt ds - (1 - H(T)) \int_0^T e^{-rt} z(t) dt$$

when the consumer experiments and by $\Pi = p - c$ otherwise. From property (BSM) the optimal choices of q^* and T^* maximize joint surplus and it is again convenient to assume in what follows that these maximizers are unique, albeit our results are independent of this. For completeness, the following Lemma presents an example where the firm's problem indeed has a unique solution.

Lemma 4 *Consider all-or-nothing learning. Bilateral surplus ω is maximized by choosing the refund payment $q^* = k$, uniquely so when $T^* > 0$. Further, when the arrival time is distributed exponentially, $H(t) = 1 - e^{-\lambda t}$, with constant arrival rate $\lambda > 0$, and flow costs are $z(t) = y e^{(\lambda + r)t}$, there exists a unique finite refund period T^*, where $T^* > 0$ when $\underline{u} < k + \widehat{V} < \overline{u}$, $0 < y < \widehat{y}$, and $0 < r < \widehat{r}$ with some uniquely determined thresholds $\widehat{y} > 0$ and $\widehat{r} > 0$. Otherwise, $T^* = 0$.*

Setting the refund level equal to the salvage value, $q^* = k$, is generally optimal as this makes the consumer internalize also the firm's opportunity cost.[15] In contrast, the optimal

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[14]The case $V = \max\{q + \widehat{V}, E[u]\} - p = q + \widehat{V} - p$ is irrelevant in equilibrium as then either the firm would make negative profits or the consumer would not accept the contract in the first place.

[15]We treat the regulation of the refund level in Section 6.

refund period T^* depends, inter alia, on the continuation value $\widehat{V}$, which will be crucial below when considering the overall market equilibrium.

Lemma 5 *With all-or-nothing learning the optimal refund period T^* is increasing in $\widehat{V}$ for $\widehat{V} < \widetilde{V} := E[u] - k$ and decreasing for $\widehat{V} > \widetilde{V}$, both strictly so at points where $T^* > 0$.*

The hump-shaped comparative statics of T^* in Lemma 5 has the following intuition. Recall that, when making his decision whether to return the product, the consumer compares $q^* + \widehat{V} = k + \widehat{V}$ with his current valuation, which equals, depending on whether he has obtained a signal, either $E[u]$ or his true valuation u. The value of information, and thus the magnitude of T^*, depends on how likely it is that information affects the consumer's decision. When $k + \widehat{V}$ is rather low or rather high, relative to $E[u]$, it is relatively unlikely that the consumer changes his (uninformed) decision after learning his true valuation, and, accordingly, information has a rather low value. Instead, when the consumer is ex ante indifferent, $\widehat{V} = \widetilde{V} = E[u] - k$, the value of information and thus also T^* are highest. In market equilibrium, which exists and is unique from Lemma 1, we then have the following result:

Proposition 1 *Consider the market equilibrium when learning is all-or-nothing. Then as firm profits π increase the equilibrium return period T^* changes as follows: There exists a threshold $0 \leq \widetilde{\pi} < \overline{\pi}$ such that T^* is increasing in π for $\pi < \widetilde{\pi}$ and decreasing in π for $\pi > \widetilde{\pi}$.*

The hump-shaped form of T^*, as we vary π, arises when $T^* > 0$ at $V = \widetilde{V}$ and when cost c and the discounted value of flow costs $z(t)$ are not too large, as then $\widetilde{\pi}$ lies strictly between zero and $\overline{\pi}$.[16] This is indeed the more interesting case on which we want to focus the subsequent analysis. Then, starting from low values of π, where most of the surplus goes to consumers, the equilibrium refund period first increases with π. As the share going to firms becomes sufficiently large, however, a further increase in π leads to a less generous refund period. Thus, the refund period is relatively short both when competition is weak and when it is strong (where, as noted before, we presently express variations in

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[16]We note that when the market features frictions $h > 0$, then as these become too high this may constrain the feasible surplus sufficiently so that the case with $0 < \pi < \widetilde{\pi}$ in Proposition 1 is no longer obtained.

competition through variations in π). The generosity of the refund period would therefore *not* provide a good proxy for consumer surplus or the prevailing degree of competition.

The intuition for this comparative result derives directly from the preceding discussion of the value of information. As noted after Lemma 5, this value is lowest when it is a priori relatively likely or relatively unlikely that (additional) information will affect the uninformed consumers' decision, which in turn is the case when consumers' continuation value is relatively high or low. While the refund period thus looks unattractive to consumers when competition is intense, but also when it is weak, we show later that it is only in the latter case that a statutory higher refund period will improve social welfare.

4.3   Regulation of the Refund Period

We now consider the imposition of a statutory minimum refund period $T \geq \underline{T}$. Recall that we denote by Γ_R the set of all contracts satisfying the regulatory constraint. Next, for a given consumer (continuation) value V denote the maximum feasible bilateral joint surplus by $\omega_R^*(V)$, once the restriction is imposed that $\gamma \in \Gamma_R$. While the bilateral surplus maximizing choice of the refund level is generally given by $q_R^* = k$, it is again convenient to stipulate that also the respective optimal choice of the refund period T_R^* is unique, albeit all our results again hold independently of this. The following Lemma provides an Example:

Lemma 6 *Suppose learning is all-or-nothing and consider the imposition of minimum refund period $\underline{T}$. Then, $q_R^* = k$. When, in addition, the arrival time is distributed exponentially, $H(t) = 1 - e^{-\lambda t}$, with constant arrival rate $\lambda > 0$ and flow costs are $z(t) = ye^{(\lambda+r)t}$, we have $T_R^* = \max\{\underline{T}, T^*\}$ where T^* denotes the unregulated optimum.*

Recall that the definition of a market equilibrium is completely analogous to that without regulation, with the additional restriction that bilaterally optimal contracts satisfy $\gamma_R^* \in \Gamma_R$. The following Lemma shows that as long as regulation is not too stringent, we continue to have a unique (regulated) market equilibrium.

Lemma 7 *Consider all-or-nothing learning. For given but not too high minimum refund period $\underline{T}$, so that still $0 < \bar{\pi}_R := \omega_R^*(0)$, a unique regulated market equilibrium exists for any $0 \leq \pi \leq \bar{\pi}_R$.*

We next consider the welfare maximizing choice of the minimum refund period. For this it is instructive to recall the decomposition of the expected social surplus in (4) which, with binding regulatory constraint, reads $\Omega_R^* = \omega_R^*(V_R^*) + \frac{\beta_R^*}{1-\beta_R^*}\pi$. Then, in order to characterize the choice of $\underline{T}$ that maximizes Ω_R^* it is transparent to proceed in two steps. We first consider a marginal increase in $\underline{T}$ above the unregulated market outcome T^*. In a second step we then look at the additional implications when $\underline{T}$ is further increased.

Lemma 8 *Suppose learning is all-or-nothing and consider a regulation requiring $T \geq \underline{T}$. When $\underline{T}$ is only marginally higher than the refund period $T^* > 0$ that would prevail in the unregulated equilibrium, this increases social welfare Ω_R^* when with $\pi > \widetilde{\pi}$, as defined in Proposition 1, competition is low, while it reduces social welfare when with $\pi < \widetilde{\pi}$ competition is high.*

The above Lemma follows from a combination of the following arguments. When we constrain contracts only marginally, compared to the unregulated equilibrium outcome, the first-order effect on bilateral surplus and thus also on consumers' equilibrium surplus V_R^* is zero. The effect on social efficiency is then solely determined by the impact of regulation on the discounted likelihood of of a return β_R^*, for which we can make use of Lemma 3. In particular, when from $\pi > \widetilde{\pi}$ competition is weak, the imposition of a more generous refund period induces bad news learning and, thus, a higher (discounted) likelihood of returning the product. Thereby, the positive externality on other firms in the market is greater. But the opposite holds if $0 < \pi < \widetilde{\pi}$. Then, a more generous refund period leads to more good news learning, which decreases the (discounted) likelihood of returning the product.

When $\underline{T}$ increases further, so that the minimum requirement is more than marginally above the unregulated outcome, this has, by definition of T^*, a negative first-order effect on the (maximized) bilateral surplus. As then V_R^* decreases, this has a negative knock-on effect on the likelihood of return. We can thus summarize results as follows: First, when marginal regulation above the unregulated equilibrium outcome is not beneficial, as is the case with $\pi < \widetilde{\pi}$, this holds a fortiori for a more-than-marginal increase. Second, when such a marginal regulation enhances welfare, as is the case when $\pi > \widetilde{\pi}$, then the described countervailing effects that arise for a further increase in $\underline{T}$ imply the existence of an interior optimum for $\underline{T}$.

Proposition 2 *Imposing a minimum refund period $\underline{T}$ that is strictly higher than the outcome that would prevail in the unregulated equilibrium, $\underline{T} > T^* > 0$, strictly decreases social efficiency under all-or-nothing learning when from $\pi < \widetilde{\pi}$ the degree of competition is high. Instead, when from $\pi > \widetilde{\pi}$ the degree of competition is low, then the socially optimal minimum refund period that a regulator would like to impose is bounded but always satisfies $\underline{T} > T^*$. In the case where the imposition of a binding minimum refund period increases welfare, this leads to more cancellations and returns and thus to more consumer experimentation in the market.*

Contractual Restrictions. As we already noted, so as to consider a refund period (and its regulation) in a meaningful way, we have to restrict the set of feasible contracts. Precisely, apart from the up-front transfer p, contracts can specify a refund level q (optimally set equal to the salvage value k) and a refund period T. We acknowledge that the two parties could increase bilateral efficiency with more general contracts, for instance by still stipulating a refund level $q = k$ but making the buyer pay a time-dependent rent that is equal to the flow costs $z(t)$. One reason why these contracts may not be feasible is that such time-dependent "penalties" would contradict common regulation. Even absent regulation, these contracts may be difficult to communicate to consumers and to subsequently administer. We note however that even with such more complex contracts, the identified externality and thus also the scope for beneficial regulation would persist, albeit a simple minimum return period $\underline{T}$ would clearly lose its bite when the firm could specify an arbitrarily high rental price at points $t < \underline{T}$.

5 Learning Until the End

We now turn to learning technologies where the consumer prefers to always exhaust the available learning opportunities as captured by the refund period T. This is a fitting approximation when the refund period is rather short, which we endogenize by setting the discount rate r to zero. In this setting, what matters for the consumer's decision is his expected valuation at the end of the refund period, $U_T = E[u|\mathcal{F}_T^s]$, and we denote the respective (ex-ante) cdf by $F_T(U)$. In general, an increase in the learning period T, which is equivalent to an increase in information, results in a mean-preserving spread in the distribution of conditional expectations $F_T(U)$. Now, in order to obtain clear-cut

comparative statics results, we put more structure on the respective family of distributions, requiring the mean-preserving spread to be single-crossing:

Definition 2 (Rotation Property) *For all $T' > T \geq 0$ it holds in the interior of the respective supports that*

$$F_{T'}(U) > F_T(U) \text{ for } U < \widetilde{U} \text{ and } F_{T'}(U) < F_T(U) \text{ for } U > \widetilde{U}, \tag{6}$$

where $\widetilde{U} := E[u]$ denotes the rotation point.

We next show that various commonly used learning technologies satisfy this property and defer a discussion of how our key results extend when the mean-preserving spread is not single-crossing to the Conclusion.

Endogenizing the Rotation Property First, consider the prominent case of Gaussian learning where the signal process s_t observed by the consumer while experimenting is a Brownian motion with drift given by his true valuation u:

$$ds_t = u\,dt + \sigma\,dz_t. \tag{7}$$

Here z_t is a standard Brownian motion and $\sigma > 0$ denotes the signal's time-invariant instantaneous variance. With this signal structure the consumer's filtering problem has the following solution:

Lemma 9 *Suppose that the signal s_t evolves according to (7). Then, the consumer's expected valuation conditional on $\mathcal{F}_T^s$ is given by*

$$U_T = E\left[u | \mathcal{F}_T^s\right] = \frac{\int \breve{u} \exp\left(\sigma^{-2}\breve{u}s_T - \frac{1}{2}\sigma^{-2}\breve{u}^2 T\right) dG(\breve{u})}{\int \exp\left(\sigma^{-2}\breve{u}s_T - \frac{1}{2}\sigma^{-2}\breve{u}^2 T\right) dG(\breve{u})}. \tag{8}$$

Note that from (8) the consumer's expected valuation at any point in time T depends only on s_T and, in particular, not on the path that led to this realization. It is this feature that allows for an explicit characterization of the distribution of U_T for two commonly used specifications of the prior distribution $G(u)$. Take first the case where u is drawn from a normal distribution. Then, it is well known that also U_T is normally distributed. The following Lemma restates this result and the immediate implication that then $F_T(U)$ indeed rotates around the prior mean.

Lemma 10 *Suppose a consumer's true valuation u is normally distributed with mean μ and variance ξ^2 and that the observable signal s_t evolves according to (7). Then, the expected valuation U_T, as given by (8), is also normally distributed:*

$$U_T \sim N\left(\mu, \xi^2 \frac{\xi^2}{\xi^2 + \frac{\sigma^2}{T}}\right),$$

so that the respective cdf $F_T(U)$ satisfies the rotation property (6) in T with $\widetilde{U} = \mu$.

A second commonly used specification for the prior distribution in the Gaussian learning setting is the following:

Lemma 11 *Suppose a consumer's valuation is symmetrically Bernoulli distributed, so that $u \in \{u_l, u_h\}$ with $\Pr(u = u_h) = \frac{1}{2}$.[17] Then, the expected valuation U_T, as given by (8), is distributed as follows:*

$$F_T(U) = \frac{1}{2}\Phi\left(\frac{A(U) - \frac{T}{2}(u_h - u_l)}{\sigma\sqrt{T}}\right) + \frac{1}{2}\Phi\left(\frac{A(U) + \frac{T}{2}(u_h - u_l)}{\sigma\sqrt{T}}\right) \quad \text{with}$$

$$A(U) = \frac{\sigma^2}{u_h - u_l}\log\left(\frac{U - u_l}{u_h - U}\right),$$

where $\Phi(\cdot)$ denotes the standard normal cdf. $F_T(U)$ satisfies the rotation property (6) in T with $\widetilde{U} = (u_l + u_h)/2$.

Next, consider the second commonly used learning technology in continuous time, the already introduced "all-or-nothing" learning (now without discounting) where we assume, for convenience, an exponential arrival time distribution.[18] It is easy to show that also this learning technology satisfies the rotation property:

Lemma 12 *Suppose a signal that perfectly reveals a consumer's valuation arrives at rate $\lambda > 0$. Then, the distribution of the expected valuation U_T is given by*

$$F_T(U) = \begin{cases} (1 - e^{-\lambda T})G(U) & \text{for } U < \widetilde{U} \\ (1 - e^{-\lambda T})G(U) + e^{-\lambda T} & \text{for } U \geq \widetilde{U} \end{cases},$$

which satisfies the rotation property (6) in T.

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[17]The extension to non-symmetric Bernoulli priors is straightforward. While the rotation point then shifts with T, all our subsequent results continue to hold qualitatively.

[18]This is also the most common specification found in the literature (cf. Keller et al., 2005). Still it is straightforward to extend our analysis also to cases where the arrival rate is allowed to be time or state dependent.

Analysis We next show that the results obtained for the case of all-or-nothing learning in Section 4 extend to settings with learning until the end. To see how we can also follow the same steps as before, note first that, with learning until the end, the (discounted) likelihood of cancellation is equal to the probability of a return at T, that is $\beta(q, T, \tau, \widehat{V}) = F_T(q + \widehat{V})$,[19] whenever the consumer has a strict preference over keeping or returning the product at the end of the refund period. We again stipulate that, when indifferent, the consumer chooses to return or cancel with probability α, so that in this case we have $\beta(q, T, \tau, \widehat{V}) = F_T^-(q + \widehat{V}) + \alpha \left[F_T(q + \widehat{V}) - F_T^-(q + \widehat{V}) \right]$.[20] While our results do not qualitatively depend on this, to make the analysis more transparent we suppose that $F_T(U)$ is non-degenerate with convex support $[\underline{u}, \overline{u}]$ for $T > 0$ and everywhere continuously differentiable in $T > 0$.

We can now follow the same steps as in the analysis of all-or-nothing learning in Section 4 and it is not surprising that the previously obtained results extend to the case of learning until the end. Hence, in order to avoid repetition, we defer parts of the formal analysis to Appendix B, where we show existence and uniqueness of bilaterally optimal contracts for all common specifications of $F_T(U)$ considered above, which feature $q^* = k$ and a unique refund period T^* that is increasing in the consumer's outside option for $\widehat{V} < E[u] - k$ and decreasing for $\widehat{V} > E[u] - k$. For completeness there we also show the existence of a unique (regulated) market equilibrium under the assumption that the regulatory constraint $T \geq \underline{T}$ is not too stringent. Building on these results, the following Proposition confirms that the main regulatory implications obtained in Section 4 fully extend to the case of learning until the end.

Proposition 3 *Suppose consumers always exhaust their learning opportunities and the distribution of expected valuations conditional on the information provided until T satisfy the rotation property. Then there is a cutoff $0 \leq \widetilde{\pi} < \overline{\pi}$ for firms' equilibrium profits such that imposing a minimum refund period $\underline{T}$ that is strictly higher than the unregulated outcome T^* strictly decreases social welfare when $\pi < \widetilde{\pi}$, whereas the socially optimal level of $\underline{T}$ is strictly larger than T^* but bounded when $\pi > \widetilde{\pi}$.*

The robust intuition underlying the results in Proposition 3 for the case with learning until the end is the same as with all-or-nothing learning (see Proposition 2 and the subse-

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[19]Note that, for convenience, we set $r = 0$.

[20]Note that, analogous to the case of all-or-nothing learning, α has no impact on bilateral surplus.

quent discussion): The effect of binding regulation on the likelihood of a return and, thus, on the positive externality that a return generates for other firms, depends crucially on consumers' outside option which is endogenous in a market environment. If competition is strong, such that consumers' have a generous outside option upon returning the current product, an increase in the refund period leads to more *good news learning*, thus lowering the probability of cancellation which follows directly from the rotation property. Instead, if the outside option is low given weak competition, a more generous cancellation period induces more *bad news learning*, thus, increasing the probability of a return. When regulation constrains contracts only marginally, this is the only effect on social surplus, as, in particular, bilateral surplus is unaffected. The result then follows as more than marginal regulation unambiguously reduces bilateral surplus as well as the likelihood of a return through the resulting decrease in the consumer's outside option.

6 Regulation of the Refund Level

As shown above, without regulation the privately optimal refund level will always be equal to the salvage value, $q^* = k$, as this maximizes the bilateral surplus of any firm-consumer pair.[21] The regulation that we now consider is the imposition of a mandatory refund level $q \geq \underline{q}$. To make this relevant, we restrict consideration to the case where without regulation returns occur with positive probability. Next, we note that the definition of a regulated market equilibrium fully extends, with the only modification that the respective restriction, $\gamma \in \Gamma_R$, is now imposed on q. We can thus also make use of the same notation for equilibrium contracts and payoffs as before. Finally, as in the case where the refund period was regulated, we stipulate for simplicity that optimal contracts are unique and that $\underline{q}$ is not too large such that the market does indeed open up. A sufficient condition is again that $\overline{\pi}_R = \omega_R^*(0) > 0$ and that $0 \leq \pi \leq \overline{\pi}_R$. We then have the following implications of a minimum refund level.

Lemma 13 *Consider the cases of all-or-nothing learning or learning until the end (with rotation of conditional expectations). Suppose now that contracts are restricted by regulation requiring $q \geq \underline{q}$. In a (regulated) market equilibrium, where $\pi \leq \overline{\pi}_R$ and $\overline{\pi}_R > 0$,*

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[21] See Lemma 4 for all-or-nothing learning and Lemma 14 in Appendix B for the learning until the end case.

the equilibrium refund level satisfies $q_R^* = \max\{k, \underline{q}\}$ while there is a unique equilibrium consumer payoff V_R^*.

As the unregulated outcome $q^* = k$ maximizes bilateral surplus, a marginally higher requirement $\underline{q} > k$ has no first-order effect on bilateral surplus and thus also not on the equilibrium payoff of consumers, V_R^*. When $\pi > 0$ holds, however, given that a higher refund level will, ceteris paribus, always lead to a higher (discounted) likelihood of return, there will be a strictly positive effect on welfare. This holds irrespective of the prevailing degree of competition, as expressed by π, as long as this is not perfect so that still $\pi > 0$. As we increase $\underline{q}$ further, in analogy to the discussion of a non-marginal increase in $\underline{T}$, this has two negative effects on overall efficiency: First, bilateral surplus is strictly lower; second, the fact that then also V_R^* decreases has a negative effect on the (discounted) likelihood of a return. The following Proposition summarizes our findings.

Proposition 4 *As long as $\pi > 0$ holds, the socially optimal minimum refund level is bounded but satisfies $\underline{q} > k$ under both all-or-nothing learning and learning until the end with rotation of conditional expectations. The imposition of the socially optimal refund level leads to more cancellations and returns and thus to more consumer experimentation in the market.*

A comparison of Propositions 2 and 3 with Proposition 4 reveals thus a stark difference between regulation that targets the refund period and regulation that targets the refund level. While in our model the latter always enhances welfare, provided that it is not excessive, a minimum statutory refund period is socially beneficial only when competition is not intense, while it reduces welfare otherwise.

7 Market Model Foundations

In this section we discuss three possible foundations for our reduced form approach to competition and contract design, as expressed - without regulation - by the market equilibrium in Definition 1: Posted contracts, noisy consumer search, and bargaining.

Posted Contracts. Here, we suppose that in a first stage, $\tau = 1$, homogeneous firms (of some mass $|I| > 0$, indexed by $i \in I$) simultaneously choose contracts γ_i. A firm can

contract with an arbitrary number of consumers. Firms observe each others' offers. In $\tau = 2$, each firm can decide whether to spend costs $a > 0$ of direct advertising so as to inform any chosen consumer about γ_i, i.e., the firm decides on the respective subset of consumers that it wishes to inform and incurs costs a per consumer. Then, in $\tau = 3$ each consumer chooses to visit one firm.[22] Once contracts have been concluded and products possibly returned, a new round starts. This is again composed of the same three stages $\tau = 1, 2$, and 3, and so on. We show in Appendix B how this model supports an equilibrium outcome in pure strategies where firms do not advertise on-equilibrium (but only off-equilibrium) and $\pi = a$.

Noisy Search. We next discuss a model with noisy consumer search. A consumer searches for offers at cost $h > 0$ and obtains an uncertain number of offers. In particular, he observes i different offers with probability η_i, where $\sum_{i=1}^{\infty} \eta_i = 1$ and $0 < \eta_1 < 1$. Then, he can purchase one of the offers or engage in the search process again. If he chooses the latter, we assume that the offers he obtained become obsolete ("no recall").[23] If he chooses the former, the game follows the description in Section 2. If the consumer decides to return a product as it does not match his preferences, he again engages in noisy search. In this setting, while any firm will prefer to choose q^* and T^* such that the bilateral surplus is maximized, it will randomize over prices since it never knows whether it is the only firm in the consumer's consideration set.[24] We show in Appendix B that this market has a unique equilibrium. A shift in the distribution of the number of observed firms that increases η_1 and (weakly) decreases any other η_i (so that still $\sum_{i=1}^{\infty} \eta_i = 1$) leads to a decrease of the equilibrium consumer surplus V^*, and is thus equivalent to an increase in firm profits π in our original set-up.

Bargaining. Consider next bilateral negotiations between a consumer and a firm. While still restricting ourselves to applications in the field of Industrial Organization, this may be applicable to high-value contracts, e.g., when the buyer is a business. We employ an

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[22]Note that this is not a search model as consumers know the identity of each firm and thus can choose to turn to a firm that did not advertise to them. They have rational expectations about the prevailing contract of all firms, but they can only observe contracts for any given firm if this was advertised directly to them.

[23]In industries like apparel or e-commerce offers may be short-lived so that a product that can be purchased right now will not be available even in the near future.

[24]When $\eta_1 = 0$, firms price at marginal costs, while when $\eta_1 = 1$ the monopoly price prevails.

asymmetric Nash bargaining solution with weight b for the consumer and weight $1-b$ for the firm. Together with transferable utility, this implies that in equilibrium a consumer's payoff is the sum of his reservation value $\widehat{V}^*$ and b times the difference between the maximum feasible surplus, $\omega^*(\widehat{V}^*)$, and the reservation value.[25] So as to obtain a non-trivial solution, where not all of the surplus goes to the consumer, we need to invoke frictions, $h > 0$ (cf. on frictions the discussion at the end of Section 2.2).

Now, with $\widehat{V}^* = V^* - h$ and when h is not too high and b not too low, in an unregulated market equilibrium it must hold that[26]

$$\omega^*(\widehat{V}^*) - \widehat{V}^* = \frac{h}{b}. \tag{9}$$

Condition (9) is analogous to condition (3) (where in the latter we also used $V^* = \widehat{V}^*$ as $h = 0$). Changes in bargaining power affect the equilibrium bilateral surplus only indirectly through the consumer reservation value and the market equilibrium condition (9). Instead of π, now the parameter b is the model's primitive. It is straightforward to show that consequently our previous results map one-to-one into a comparative analysis in b (with an increase in b corresponding to a decrease in π).

8 Conclusion

We identify a positive externality that consumer cancellation or product return have on other firms. From this follows immediately scope for regulatory interference. As we then point out, which is our second contribution, two commonly used interventions, a minimum refund level and a minimum refund period, perform, however, quite differently, as notably the latter may backfire. Our third contribution is consequently to identify when a mandatory extension of consumers' right to experiment and learn increases or decreases social efficiency, which is tightly linked to whether it leads to what we call "good news learning" or "bad news learning." We can finally link this to the size of consumers' continuation or reservation value in the market, as shaped by factors such as the degree of competition.

When consumers fully exploit the available time to experiment, which we termed "learning until the end," we showed a tight connection between the rotation property of the

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[25] Note that as a firm has infinite contracting possibilities, its reservation value is zero.

[26] This is obtained from combining the requirements $V^* - \widehat{V}^* = b\left[\omega^*(\widehat{V}^*) - \widehat{V}^*\right]$ and $\widehat{V}^* = V^* - h$. Note that the requirement that h is not too high and b not too low avoids corner solutions. Otherwise, we would have to replace $\widehat{V}^* = V^* - h$ by $\widehat{V}^* = \max\{0, V^* - h\}$.

distribution of consumers' updated valuations and the hump-shaped relationship between surplus sharing in the market and the prevailing cancellation terms: These are particularly attractive only when consumers obtain an intermediate share of total surplus. Firms grant consumers less generous rights either when consumers' share of surplus is low or when it is firms that make very little profits in the market. However, forcing firms to let consumers learn longer by imposing a commonly observed statutory minimum cancellation or refund period is socially efficient only when firms appropriate much of the market surplus, while it backfires otherwise. We established that the rotation property holds for commonly used learning technologies and showed that our results are robust when allowing for discounting and, thus, consumer-optimal returns prior to the end of the refund period within the tractable setting of "all-or-nothing learning." We note that some of our results extend beyond these standard learning technologies. As is immediate, even when there is no longer a common rotation point, one can still identify a lower and an upper bound on consumers' continuation value (and thus the distribution of surplus in the market), beyond which a mandatory extension of consumers' cancellation period is beneficial or backfires.

As noted in the Introduction, while we do not apply our results outside Industrial Organization, such a future avenue may be fruitful. In other contexts, such as labour markets, it may however be reasonable to suppose that both firms and workers have a positive reservation value, as a firm has only a limited number of vacancies to fill. Both parties may then learn their share of a match-specific value and may then exercise a possible option of early contract termination. Other extensions could allow market participants to control their learning speed at some cost or to search for alternatives while being in a match.

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Appendix A: Proofs

Proof of Lemma 1. We first show that $\omega^*(V)$ is continuous in V. For this observe that $\omega^*(V)$ is increasing in V. Consider $V_l < V_h$ and denote by q_l, q_h, T_l, T_h, τ_l, and τ_h the optimal choices of refunds, refund periods, as well the resulting optimal stopping times, respectively. Then

$$\omega^*(V_h) - \omega^*(V_l)$$
$$= \omega(q_h, T_h, \tau_h, V_h) - \omega(q_l, T_l, \tau_l, V_l)$$
$$\geq \omega(q_l + V_l - V_h, T_l, \tau_l, V_h) - \omega(q_l, T_l, \tau_l, V_l)$$
$$= \beta(q_l, T_l, \tau_l, V_l)(V_h - V_l) \geq 0.$$

The first inequality results from the optimality of q_h, T_h, and consequently τ_h, when the continuation value is V_h and the fact that the consumer's optimal stopping depends on the sum $q + V$ rather than the magnitude of the individual components q and V. The second inequality follows from $0 \leq \beta(\cdot) \leq 1$ as $0 \leq e^{-rt}\mathbf{1}_{\{U_t \leq q + \hat{V}\}} \leq 1$ for $t \geq 0$. As $\omega^*(V)$ is increasing we obtain the following lower and upper bounds for $\omega^*(V_h) - \omega^*(V_l)$:

$$0 \leq \omega^*(V_h) - \omega^*(V_l)$$
$$= \omega(q_h, T_h, \tau_h, V_h) - \omega(q_l, T_l, \tau_l, V_l)$$
$$\leq \omega(q_h, T_h, \tau_h, V_h) - \omega(q_h + V_h - V_l, T_h, \tau_h, V_l)$$
$$= \beta(q_h, T_h, \tau_h, V_h)(V_h - V_l).$$

As V_l approaches V_h the last term approaches 0 and this proves the claim.

Next, we show that $\omega^*(V) - V$ is strictly decreasing in V. For this consider

$$\omega^*(V_h) - V_h - (\omega^*(V_l) - V_l)$$
$$= \omega(q_h, T_h, \tau_h, V_h) - V_h - (\omega(q_l, T_l, \tau_l, V_l) - V_l)$$
$$\leq \omega(q_h, T_h, \tau_h, V_h) - V_h - (\omega(q_h + V_h - V_l, T_h, \tau_h, V_l) - V_l)$$
$$= -\left(1 - \beta(q_h, T_h, \tau_h, V_h)\right)(V_h - V_l).$$

The last expression is strictly negative as $\beta(q_h, T_h, \tau_h, V_h) = 1$ cannot be an equilibrium outcome and the claim follows. Further, note that $\omega^*(V) - V < 0$ for sufficiently large V. Thus, together with $\omega^*(0) > 0$ and the continuity of $\omega^*(V) - V$, we have a value $\overline{V} > 0$

so that $\omega^*(\overline{V}) - \overline{V} = 0$. Taken together, (3) defines a one-to-one mapping from $[0, \overline{V}]$ to $[0, \overline{\pi}]$, such that for any $0 \leq \pi \leq \overline{\pi}$ a unique consumer payoff exists. The comparative statics of V^* in π is implied by the observation that as π increases, V^* must decrease so as to preserve the equality in (3). **Q.E.D.**

Proof of Lemma 2. A consumer, facing a contract γ and a continuation value $\widehat{V}$, who does not know his valuation in $t \leq T$ obtains from waiting until t' with $t \leq t' \leq T$ the expected payoff

$$\int_t^{t'} e^{-r(s-t)} \frac{h(s)}{1 - H(t)} E[\max\{q + \widehat{V}, u\}] ds + e^{-r(t'-t)} \frac{1 - H(t')}{1 - H(t)} \max\{q + \widehat{V}, E[u]\}.$$

Differentiating this payoff with respect to t' yields

$$\frac{e^{-r(t'-t)}}{1 - H(t)} \left[h(t') E[\max\{q + \widehat{V}, u\}] - [r(1 - H(t')) + h(t')] \max\{q + \widehat{V}, E[u]\} \right]$$

which is positive when condition (5) holds. Moreover, when $t' = t$, the payoff equals $\max\{q + \widehat{V}, E[u]\}$. Taken together, waiting until T results in a higher payoff than making immediately an uniformed decision at any point $0 \leq t \leq T$. **Q.E.D.**

Proof of Lemma 3. Partially differentiating $\beta(q, T, \tau, \widehat{V})$ with respect to T yields

$$\frac{\partial \beta}{\partial T} = \begin{cases} e^{-rT} h(T) G(q + \widehat{V}) > 0 & \text{if } q + \widehat{V} < E[u] \\ e^{-rT} h(T)(G(q + \widehat{V}) - 1) - re^{-rT}(1 - H(T)) < 0 & \text{if } q + \widehat{V} > E[u] \end{cases}$$

from which the claim follows. **Q.E.D.**

Proof of Lemma 4. When $T^* > 0$ is optimally chosen we must have from optimality that $\underline{u} < q^* + \widehat{V} < \overline{u}$. That $q^* = k$ must hold uniquely follows from the fact that q affects only whether the consumer starts learning but not his optimal stopping, and as only then the consumer's optimal cutoff rule, $q^* + \widehat{V}$, is also the bilaterally optimal cutoff rule, $k + \widehat{V}$. When $T^* = 0$ all consumers value the product with $E[u]$ when they have to decide whether to keep it. Thus, $q^* = k$ remains optimal, albeit not uniquely as there are other choices of q^* that induce the same decision as $q^* = k$.

When $\underline{u} < k + \widehat{V} < \overline{u}$ and the signal arrives at a constant rate $\lambda > 0$, the bilateral surplus ω is given by

$$\int_0^T \lambda e^{-(\lambda+r)t} E[\max\{k + \widehat{V}, u\}] dt + e^{-(\lambda+r)T} \max\{k + \widehat{V}, E[u]\} - z^d(T) - c,$$

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with

$$z^d(T) = \int_0^T \lambda e^{-\lambda s} \left[\int_0^s e^{-rt} z(t) dt \right] ds + e^{-\lambda T} \int_0^T e^{-rt} z(t) dt$$

being the discounted present value of flow costs $z(t)$. Note also that when these costs are given by $z(t) = y e^{(\lambda + r)t}$, the discounted costs are $z^d(T) = yT$. This allows to solve analytically:

$$T^* = \frac{1}{\lambda + r} \log \left[\frac{\lambda}{y} E \left[\max\{k + \widehat{V}, u\} \right] - \frac{\lambda + r}{y} \max\{k + \widehat{V}, E[u]\} \right],$$

which is strictly positive when $r < \widehat{r}$ and $y < \widehat{y}$ with

$$\widehat{r} = \lambda \frac{E\left[\max\{k + \widehat{V}, u\}\right] - \max\{k + \widehat{V}, E[u]\}}{E\left[\max\{k + \widehat{V}, u\}\right]} \quad \text{and}$$

$$\widehat{y} = \lambda E\left[\max\{k + \widehat{V}, u\}\right] - (\lambda + r)\max\{k + \widehat{V}, E[u]\}.$$

Note that condition (5) holds strictly for all $t > 0$ when $r < \widehat{r}$. The uniqueness of $T^* > 0$ follows from the strict concavity of ω in $T > 0$ as we have

$$\frac{\partial^2 \omega}{\partial T^2} = -(\lambda + r)e^{-(\lambda + r)T}(\lambda E[\max\{k + \widehat{V}, u\}] - (\lambda + r)\max\{k + \widehat{V}, E[u]\})$$

which is strictly negative when $r < \widehat{r}$. When $k + \widehat{V} \le \underline{u}$ or $k + \widehat{V} \ge \overline{u}$, the consumer will not start learning as information about his valuation u has no impact on his decision. In this case, it is optimal to set $T^* = 0$, albeit not uniquely. **Q.E.D.**

Proof of Lemma 5. We show that when optimally $q^* = k$ the bilateral surplus, $\omega = V + \Pi$, has strictly increasing differences in T and $\widehat{V}$ for $\widehat{V} < \widetilde{V} = E[u] - k$ and strictly decreasing differences for $\widehat{V} > \widetilde{V}$. The claim follows then from standard monotone comparative statics results, cf. Theorem 2.3 in Vives (2000). To streamline the exposition we denote with $\omega(T, \widehat{V})$ the bilateral surplus with refund period T and continuation value $\widehat{V}$. Suppose that $\underline{u} - k < \widehat{V}_l < \widehat{V}_h < \overline{u} - k$ and $0 \le T_l < T_h$. Then it holds that

$$\omega(T_h, \widehat{V}_h) - \omega(T_l, \widehat{V}_h) - \left[\omega(T_h, \widehat{V}_l) - \omega(T_l, \widehat{V}_l)\right] \tag{10}$$

$$= \int_{T_l}^{T_h} e^{-rt} h(t) dt \int_{k + \widehat{V}_l}^{k + \widehat{V}_h} G(U) dU + \left[e^{-rT_h}(1 - H(T_h)) - e^{-rT_l}(1 - H(T_l))\right] \tag{11}$$

$$\times \left[\max\{k + \widehat{V}_h, E[u]\} - \max\{k + \widehat{V}_l, E[u]\}\right] \tag{12}$$

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where we apply integration by parts and $E[\max\{k+\widehat{V}, u\}] = G(k+\widehat{V})(k+\widehat{V}) + \int_{q+\widehat{V}}^{\overline{u}} U dG(U)$. This expression is clearly positive if $\underline{u} - k < \widehat{V}_l < \widehat{V}_h < \widetilde{V}$ as then $\max\{k + \widehat{V}_l, E[u]\} = \max\{k + \widehat{V}_h, E[u]\} = E[u]$. When $\widetilde{V} < \widehat{V}_l < \widehat{V}_h < \overline{u} - k$, it holds that $\max\{k + \widehat{V}_h, E[u]\} - \max\{k + \widehat{V}_l, E[u]\} = \widehat{V}_h - \widehat{V}_l$ and expression (12) reads

$$
\begin{aligned}
&\int_{T_l}^{T_h} e^{-rt} h(t) dt \int_{k+\widehat{V}_l}^{k+\widehat{V}_h} G(U) dU + \left[e^{-rT_h}(1 - H(T_h)) - e^{-rT_l}(1 - H(T_l)) \right] \\
&\times \left[\widehat{V}_h - \widehat{V}_l \right] \\
&= \int_{T_l}^{T_h} h(t) \left[e^{-rt} \int_{k+\widehat{V}_l}^{k+\widehat{V}_h} G(U) dU - e^{-rT_l} \left(\widehat{V}_h - \widehat{V}_l \right) \right] dt + (e^{-rT_h} - e^{-rT_l}) \\
&\times (1 - H(T_h)) \left[\widehat{V}_h - \widehat{V}_l \right]
\end{aligned}
$$

which is negative as $e^{-rT_l} \geq e^{-rt}$ for $T_l \leq t \leq T_h$ and since $G(U)$ is a cdf. **Q.E.D.**

Proof of Proposition 1. It follows from Lemma 1 that a unique market equilibrium exists and that V^* is strictly decreasing in π. The comparative statics for $T^* > 0$ in π then follow from Lemma 5 together with the observation that V^* is then strictly decreasing in π. The existence of the cutoff $\widetilde{\pi}$ follows from the one-to-one correspondence between V^* and π. **Q.E.D.**

Proof of Lemma 6. The optimality of $q_R^* = k$ follows the same argument as in the unregulated case. The optimal choice of T_R^* follows immediately from the observation that under this specification the bilateral surplus $\omega(k, T, \tau, \widehat{V})$ is strictly decreasing in $T > T^*$, cf. the proof of Lemma 4. **Q.E.D.**

Proof of Lemma 7. That a regulated market equilibrium exists when $\underline{T}$ is imposed follows by the same arguments as in the proof of Lemma 1 and from the fact that this regulation has no impact on the choice set of the refund level. **Q.E.D.**

Proof of Lemma 8. Given that $T^* > 0$ is interior, it holds that $\partial \omega_R^* / \partial T |_{T=T^*} = 0$. Together with equation (3), this implies $\partial V_R^* / \partial T |_{T=T^*} = 0$. Moreover, Lemma 6 implies $\partial q_R^* / \partial T |_{T=T^*} = 0$. Therefore we have

$$
\left. \frac{d\Omega_R^*}{d\underline{T}} \right|_{\underline{T}=T^*} = \frac{\pi}{(1 - \beta_T^*)^2} \left. \frac{\partial \beta^*}{\partial T} \right|_{T=T^*}
\tag{13}
$$

The term (13) is positive if $\pi > \widetilde{\pi}$ and negative if $\pi < \widetilde{\pi}$, cf. Lemma 3 with Proposition 1. This holds strictly when $\pi > 0$. **Q.E.D.**

Proof of Proposition 2. A regulation $\underline{T} > T^*$ must reduce the bilateral surplus. Thus, in order to preserve condition (3) it must hold that $V^* > V_R^*$. With Lemma 6, we still have $q_R^* = k$. When $\underline{T} > T^*$ and $\pi < \widetilde{\pi}$, we have with Lemma 3 that $\beta_R^* < \beta^*$ which, together with the social surplus definition in (4), proves the claim. The claim for when $\underline{T} > T^*$ and $\pi > \widetilde{\pi}$ follows directly from Lemma 8. **Q.E.D.**

Proof of Lemma 9. Let $(S, \mathcal{F}, \{\mathcal{F}_t\}, P)$ be a filtered probability space. The consumer's true valuation u is drawn from the distribution $G(u)$ at time zero, such that it is a $\mathcal{F}_0$-measurable random variable, and z_t is a $\mathcal{F}_t$-adapted standard Brownian motion. $\mathcal{F}_T^s$ is the σ-algebra generated by the sample paths $\{s_t\}_{t \leq T}$ of (7) and represents the information available at time T. Then, denoting by $E_P[\cdot]$ the expectation with respect to the probability measure P, we have by Bayes rule that

$$E_P[u|\mathcal{F}_T^s] = \frac{E_{Q_T}[uM_T|\mathcal{F}_T^s]}{E_{Q_T}[M_T|\mathcal{F}_T^s]}, \tag{14}$$

for any probability measure Q_T which is absolutely continuous with respect to P, and where M_T is the Radon-Nikodym derivative of P with respect to Q_T. Now take

$$M_T^{-1} = \exp\left(-\int_0^T \frac{u}{\sigma} dz_t - \frac{1}{2}\int_0^T \frac{u^2}{\sigma^2} dt\right) = \exp\left(-\frac{u}{\sigma} z_T - \frac{1}{2}\frac{u^2}{\sigma^2}T\right) = \frac{dQ_T}{dP},$$

such that, by Girsanov's theorem, $\{\sigma^{-1}s_T\}$ is an $\mathcal{F}_T$-adapted Brownian motion under Q_T and, thus, independent of the consumer's true valuation. Next, note that for any function $f(\cdot)$ we have

$$E_{Q_T}[f(u)M_T|\mathcal{F}_T^s] = \underbrace{E_{Q_T}\left[f(u)\exp\left(\sigma^{-2}us_T - \frac{1}{2}\sigma^{-2}u^2T\right)\right]}_{=:g(s_T)}.$$

This follows from $E_{Q_T}[f(u)M_T\mathbf{1}_A] = E_{Q_T}[g(s_T)\mathbf{1}_A]$ for all $A \in \mathcal{F}_T^s$, which holds as u and $\{\sigma^{-1}s_T\}$ are independent under Q_T and thus the joint distribution of u and $(s_T, \mathbf{1}_A)$ can be described by the product measure $Q_T(d\breve{u}) \times Q_T(ds, da)$. Then

$$E_{Q_T}[f(u)M_T\mathbf{1}_A] = \int f(\breve{u})M_T a Q_T(d\breve{u}) \times Q_T(ds, da)$$
$$= E_{Q_T}[g(s_T)\mathbf{1}_A].$$

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Since the law of u is the same under P and Q_T, we can then write

$$E_{Q_T}[f(u)M_T|\mathcal{F}_T^s] = \int f(\check{u}) \exp\left(\sigma^{-2}\check{u}s_T - \frac{1}{2}\sigma^{-2}\check{u}^2 T\right) dG(\check{u}). \tag{15}$$

The result then follows from substituting (15) with $f(u) = u$ and $f(u) = 1$ in (14). **Q.E.D.**

Proof of Lemma 11. From Lemma 9 we have $U_T = u_l + \zeta_T(u_h - u_l)$, with

$$\zeta_T = \left(1 + \exp\left(-s_T \frac{u_h - u_l}{\sigma^2} + \frac{T}{2}\frac{u_h^2 - u_l^2}{\sigma^2}\right)\right)^{-1},$$

such that

$$F_T(U) = \Pr\left(s_T \le A(U) + \frac{T}{2}(u_h + u_l)\right) \quad \text{where}$$

$$A(U) = \frac{\sigma^2}{u_h - u_l} \log\left(\frac{U - u_l}{u_h - U}\right).$$

Note that from an ex-ante perspective, s_T is drawn from an equally weighted mixture of two normal distributions with means Tu_h and Tu_l and a common variance $\sigma^2 T$. This establishes the functional form of $F_T(U)$ in Lemma 11.

Next, we show that this $F_T(U)$ is ordered by a sequence of mean-preserving rotations in T. To see this note that we have

$$\frac{\partial F_T(U)}{\partial T} = \frac{T^{-1.5}}{4\sigma}\left[\phi\left(\frac{A(U) - \frac{T}{2}(u_h - u_l)}{\sigma\sqrt{T}}\right) \cdot \left(-\frac{T}{2}(u_h - u_l) - A(U)\right)\right. \tag{16}$$

$$\left. + \phi\left(\frac{A(U) + \frac{T}{2}(u_h - u_l)}{\sigma\sqrt{T}}\right) \cdot \left(\frac{T}{2}(u_h - u_l) - A(U)\right)\right], \tag{17}$$

where $\phi(\cdot)$ denotes the standard normal density. Now, note that $A(U)$ is strictly increasing in U and let $A^{-1}(\cdot)$ denote the inverse of $A(\cdot)$. Then, from inspection of (17), it clearly holds that $\partial F_T(U)/\partial T > 0$ for $U \le A^{-1}(-\frac{T}{2}(u_h - u_l))$, while for $U \ge A^{-1}(\frac{T}{2}(u_h - u_l))$, we have $\partial F_T(U)/\partial T < 0$. Further, it holds for all T that $\partial F_T(U)/\partial T|_{U=\widetilde{U}} = 0$ with $\widetilde{U} = E[u] = \frac{1}{2}(u_h + u_l)$, where we have used symmetry of the normal density and $A(\widetilde{U}) = 0$. Finally, the unimodality of the normal density implies that $\partial F_T(U)/\partial T$ is strictly decreasing in U over the interval $[A^{-1}(-\frac{T}{2}(u_h - u_l)), A^{-1}(\frac{T}{2}(u_h - u_l))]$, for any given T, and the result follows. **Q.E.D.**

Proof of Proposition 3. The existence of a market equilibrium follows from Lemma 15. Given that $T^* > 0$ is interior, it holds that $\partial \omega_R^*/\partial T|_{T=T^*} = 0$. Together with equation

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(3), this implies $\partial V_R^*/\partial T|_{T=T^*} = 0$. Therefore we have

$$\frac{d\Omega_R^*}{d\underline{T}}\bigg|_{\underline{T}=T^*} = \frac{\pi}{(1-\beta^*)^2}\frac{\partial\beta^*}{\partial T}\bigg|_{T=T^*}.$$

Due to the definition of β^* and as F_T is ordered by rotation this expression is positive if $k + V_R^* < \widetilde{U}$ and negative if $k + V_R^* > \widetilde{U}$. This holds strictly when $\pi > 0$. Then, the claim follows by the same arguments as in the proofs of Propositions 1 and 2. **Q.E.D.**

Proof of Lemma 13. For the determination of q_R^* we distinguish two cases. When $\underline{q} \leq k$, it follows that $q_R^* = k$ from Lemma 4 with all-or-nothing learning and from Lemma 14 with learning until the end. Next, when $\underline{q} > k$, we show that for any given $\widehat{V}$ the bilateral surplus ω is strictly decreasing in $q > k$, from which the claim follows. Take ω as a function of q and T. Let $q_h > q_l > k$ and denote with $T_h > 0$ and $T_l > 0$ the respective optimal refund periods. We obtain under all-or-nothing learning

$$\omega(q_h, T_h) - \omega(q_l, T_l)$$

$$\leq \omega(q_h, T_h) - \omega(q_l, T_h)$$

$$\leq \int_0^T e^{-rt} h(t) dt \left[\int_{q_l+\widehat{V}}^{q_h+\widehat{V}} \left[G(U) - G(q_h + \widehat{V}) \right] dU - (q_l - k)(G(q_h + \widehat{V}) - G(q_l + \widehat{V})) \right] < 0$$

and under learning until the end

$$\omega(q_h, T_h) - \omega(q_l, T_h)$$

$$\leq \int_{(q_l+\widehat{V},q_h+\widehat{V}]} F_{T_h}(U) dU - F_{T_h}^-(q_h + \widehat{V})(q_h - q_l) - (q_l - k)(F_{T_h}^-(q_h + \widehat{V}) - F_{T_h}(q_l + \widehat{V})) < 0.$$

Thus, for either learning technology, the bilateral surplus is decreasing in $q \geq k$, such that when $\underline{q} > k$ it is optimal to set $q_R^* = \underline{q}$.

Now we turn to the existence of a regulated market equilibrium. When regulation is non-binding, $\underline{q} \leq k$, the uniqueness of the consumer payoff V_R^* follows immediately from Propositions 1 and 3. Now consider a binding regulation $\underline{q} > k$ such that $q_R^* = \underline{q}$. It is obvious that $\omega_R^*(V) - V$, the left side of (3), is negative for sufficiently large V.

We show next that this expression is strictly decreasing in V. It holds for $V_h > V_l$, with T_h, T_l, τ_h and τ_l denoting the respective optimal choices of refund periods and stopping

times under regulation, that with all-or-nothing learning

$$\omega(\underline{q}, T_h, \tau_h, V_h) - V_h - \big(\omega(\underline{q}, T_l, \tau_l, V_l) - V_l\big)$$
$$\leq (V_h - V_l) \left[\int_0^{T_h} e^{-rt} h(t) dt + e^{-rT_h}(1 - H(T_h)) - 1 \right]$$
$$- (\underline{q} - k) \big(\beta(\underline{q}, T_h, \tau_h, V_h) - \beta(\underline{q}, T_h, \tau_h, V_l)\big).$$

Since $h(t)$ is a density and $\beta(q, T, \tau, V)$ is, ceteris paribus, increasing in V, it follows that $\omega_R^*(V) - V$ is strictly decreasing with all-or-nothing learning. Under learning until the end we obtain

$$\omega(\underline{q}, T_h, \tau_h, V_h) - V_h - \big(\omega(\underline{q}, T_l, \tau_l, V_l) - V_l\big)$$
$$\leq \int_{(k+V_l, k+V_h]} F_{T_h}(U) dU - (V_h - V_l) - (\underline{q} - k)[F_{T_h}^-(k + V_h) - F_{T_h}(k + V_l)].$$

Since $-(\underline{q} - k)[F_{T_h}^-(k + V_h) - F_{T_h}(k + V_l)]$ is strictly negative, the claim follows here as well.

By the theorem of the maximum $\omega_R^*(V) - V$ is continuous in those points where β_R^* is continuous. At a point V_0 where β_R^* has a discontinuity all values π with $\lim_{V \to V_0^+}[\omega_R^*(V) - V] \leq \pi \leq \lim_{V \to V_0^-}[\omega_R^*(V) - V]$ are still attainable in equilibrium through uniquely determined α_R^*. By the theorem of the maximum, ω_R^* is continuous in $0 \leq \alpha \leq 1$ at V_0. Furthermore, it is strictly decreasing in α: Suppose $\alpha_l < \alpha_h$ and denote with T_h, T_l, τ_h and τ_l the respective optimal choices of refund periods and stopping times. Then, we obtain for both learning technologies

$$\omega_R^*(V_0)\big|_{\alpha_h} - \omega_R^*(V_0)\big|_{\alpha_l} \leq -(\underline{q} - k)\left(\beta(\underline{q}, T_h, \tau_h, V_0)\big|_{\alpha_h} - \beta(\underline{q}, T_h, \tau_h, V_0)\big|_{\alpha_l}\right)$$

which is strictly negative as β is, ceteris paribus, strictly increasing in α at points of discontinuity, cf. the functional form of β in Sections 4.1 and 5 for all-or-nothing learning and learning until the end, respectively. Therefore, for any π for which the corresponding consumer payoff is given by a point of discontinuity V_0, a unique α_R^* is determined. **Q.E.D.**

Proof of Proposition 4. Consider an unregulated equilibrium, under either all-or-nothing learning or learning until the end, where β_R^* is continuous in $q_R^* = k$. Then, it holds that $\partial \omega_R^*/\partial q|_{q=k} = 0$ and by the implicit function theorem we have $\partial V_R^*/\partial q|_{q=k} = 0$ and $\partial T^*/\partial q|_{q=k} = 0$ as $T^* > 0$ is interior. Thus, we have

$$\frac{d\Omega_R^*}{d\underline{q}}\bigg|_{q=k} = \frac{\pi}{(1 - \beta^*)^2} \frac{\partial \beta^*}{\partial q}\bigg|_{q=k} \geq 0$$

which holds strictly when $\pi > 0$. We next turn to the case when β_R^* is discontinuous at $q_R^* = k$. There are still no first-order effects such that any change in social welfare is due to changes in β_R^*. Recall that α_R^* has no effect on the bilateral surplus, cf. Sections 4.1 and 5, and therefore we need to distinguish the following two cases. When $\alpha_R^* = 1$ the right derivative of β_R^* in $\underline{q}$ exists and is strictly positive. When $\alpha_R^* < 1$, β_R^* has a positive jump discontinuity in $\underline{q}$ and the claim follows.

Increasing $\underline{q} > k$ will decrease bilateral surplus $\omega_R^*(V_R)$ for any fixed V_R. Therefore, together with the (regulated) equilibrium requirement in (3), this implies that V_R^* is decreasing as well. Finally, it is immediate that in both cases the socially optimal level of $\underline{q}$ must be bounded. **Q.E.D.**

Appendix B: Additional Material

Learning until the End. The following Lemma shows existence of a unique optimal bilateral contract for all common specifications of $F_T(U)$ considered in Section 5.

Lemma 14 *Consider learning until the end when the cdf of conditional expectations is ordered by rotation in T. Bilateral surplus ω is maximized by setting the refund payment $q^* = k$, uniquely so when $T^* > 0$. Moreover, for all common specifications of $F_T(U)$ considered in Section 5 (Gaussian and all-or-nothing learning without discounting), when $z(t) = y > 0$ and $\underline{U} < k + \widehat{V} < \overline{U}$, there exists a unique finite refund period T^* that maximizes bilateral surplus ω, where $T^* > 0$ for all $0 < y < \widehat{y}$ and $T^* = 0$ for $y > \widehat{y}$, with some uniquely determined (possibly infinite) threshold $\widehat{y} > 0$. The optimal refund period, T^*, is increasing in $\widehat{V}$ for $\widehat{V} < \widetilde{V} := \widetilde{U} - k$ and decreasing for $\widehat{V} > \widetilde{V}$, both strictly so at points where $T^* > 0$.*

Proof. Setting $q^* = k$ is optimal as only then the consumer's optimal cutoff rule, $q^* + \widehat{V}$, is also the bilaterally efficient cutoff rule, $k + \widehat{V}$, cf. also the proof of Lemma 4. We now turn to the optimal refund period and prove the claim for Gaussian learning by inspecting the term

$$B(\gamma, \widehat{V}) = \int_{(k+\widehat{V}, \infty)} \left[U - \left(k + \widehat{V} \right) \right] dF_T(U),$$

so that we have $\omega(q, T, \tau, \widehat{V}) = B(\gamma, \widehat{V}) - yT - c + \left(k + \widehat{V} \right)$ where $yT = \int_0^T z(t) dt$. When $k + \widehat{V} \neq \widetilde{U}$, there exists $T' > 0$ such that $B(T)$ is convex for $0 < T \leq T'$ and concave for $T \geq T'$. Further, $\lim_{T \downarrow 0} B'(T) = \lim_{T \to \infty} B'(T) = 0$ and $B'(T') < \infty$ (cf. Theorem 3 and Corollary 1 in Keppo et al., 2008, as well as Proposition 1 and Theorem 1 in Powell and Frazier, 2010). Therefore, with $z(T) = y$, the solution T^* is unique and a threshold $\widehat{y}$ exists, such that $T^* > 0$ for $0 < y < \widehat{y}$ and $T^* = 0$ for $\widehat{y} \leq y$. It remains to consider the case where $k + \widehat{V} = \widetilde{U}$. There, $B(T)$ is increasing and concave for $T \geq 0$ with $0 < \lim_{T \downarrow 0} B'(T) \leq \infty$ (cf. Theorem 2 in Keppo et al., 2008, as well as Proposition 1 and Corollary 1 in Powell and Frazier, 2010). Therefore, with flow costs $z(t) = y$ a unique maximum exists and the threshold $\widehat{y}$ is given by $\widehat{y} = \lim_{T \downarrow 0} B'(T)$.

With all-or-nothing learning without discounting we can solve analytically the first order condition $\partial \omega / \partial T |_{T=T^*} = 0$ for T^*:

$$T^* = \ln \left[\frac{\lambda}{y} \left(G(k + \widehat{V})(k + \widehat{V}) + \int_{(k+\widehat{V}, \infty)} U dG(U) - \max\{k + \widehat{V}, E[u]\} \right) \right]^{\frac{1}{\lambda}}.$$

T^* is the unique maximum since it holds for all $T \geq 0$ that

$$\frac{\partial^2 \omega}{\partial T^2} = -\lambda^2 e^{-\lambda T} \left[G(k+\widehat{V})(k+\widehat{V}) + \int_{(k+\widehat{V},\infty)} U dG(U) - \max\{k+\widehat{V}, E[u]\} \right] < 0.$$

Lastly, the functional form of T^* implies that the threshold $\widehat{y}$ is given by

$$\widehat{y} = \lambda \left[G(k+\widehat{V})(k+\widehat{V}) + \int_{(k+\widehat{V},\infty)} U dG(U) - \max\{k+\widehat{V}, E[u]\} \right]^{-1}.$$

The comparative statics of the optimal refund period T^* can be shown by applying exactly the same steps as in the proof of Lemma 5. **Q.E.D.**

Next we show existence of a unique (regulated) market equilibrium. For this we maintain the assumption that for any $\widehat{V}$ a unique bilaterally optimal refund level q^* and refund period T^* exists as is the case under the conditions in Lemma 14 above.

Lemma 15 *Suppose the consumer always exhausts the available refund period and $F_T(U)$ satisfies the rotation property. Then, for given but not too high minimum refund period $\underline{T}$ (so that still $\overline{\pi}_R = \omega_R^*(0) > 0$), a unique (regulated) market equilibrium exists.*

Proof. The existence of an unregulated market equilibrium follows immediately from Lemma 1. The existence of a market equilibrium when $T \geq \underline{T}$ is imposed follows by the same arguments as in the proof of Lemma 1 and from the fact that the regulation under consideration has no impact on the choice set of the refund level q. **Q.E.D.**

Market with Frictions. We consider now a market in which returning a product and contacting a new firm generates a disutility $h > 0$ for the respective consumer. Then, the consumer's continuation value is $\widehat{V}^* = V^* - h$ and the market equilibrium condition (3) reads now $\omega^*(\widehat{V}^*) - V^* = \pi$. Now, we redefine $\overline{\pi} := \omega^*(-h)$ and assume that $\overline{\pi} > 0$. A sufficient condition for this is $E[u] - c > 0$.

Proposition 5 *Suppose learning is either all-or-nothing or until the end with rotation. Consider a market where returning a product and contacting a new firm involves a disutility $h > 0$. Then, a unique market equilibrium exists. Further, if costs h and c are not too high and $T^* > 0$ for some V^*, a cutoff $\widetilde{\pi}$ exists such that T^* is increasing in π when $\pi < \widetilde{\pi}$ and decreasing when $\pi > \widetilde{\pi}$. Otherwise, when h or c is too high, T^* is increasing in π.*

Proof. The expression $\omega^*(\widehat{V}) - V = \omega^*(V - h) - V$ is clearly continuous and strictly decreasing in V. It holds further that $\lim_{V \to \overline{U} - k + h}(\omega^*(V - h) - V) < 0$ such that $\overline{V}$ exists with $\omega^*(\overline{V} - h) - \overline{V} = 0$. Thus, $\omega^*(V - h) - V$ continues to be a one-to-one mapping between $[0, \overline{\pi}]$ and $[0, \overline{V}]$.

Next, we show that if c and h are not too high and given that $T^* > 0$ for some V^*, the respective (interior) cutoff $\widetilde{\pi}$ exists. For this we evaluate $\omega^*(V - h) - V$ at $V = E[u] - k + h$:

$$F_{T^*}(\widetilde{U})\widetilde{U} + \int_{(\widetilde{U}, \infty)} U dF_{T^*}(U) - z(T^*) - \widetilde{U} - (c + h) + k.$$

Observe that if $T^* > 0$ for some $\widehat{V}$, it must hold in particular that $T^* > 0$ when $\widehat{V} + h = \widetilde{V}$, cf. Lemma 5 and Lemma 14. Then, by optimality of T^*, for sufficiently small c and h the expression must be strictly positive. Since $\omega^*(V - h) - V$ is strictly decreasing, we have $\overline{V} > E[u] - k + h$. Therefore, together with the one-to-one relationship between π and V^*, the respective cutoff $\widetilde{\pi}$ exists. Otherwise, if c or h are sufficiently high, we have $\overline{V} < E[u] - k + h$, such that for all π it holds that $\widehat{V} < E[u] - k$. **Q.E.D.**

Posted Contracts. We show the following result for the game with posted contracts.

Proposition 6 *Consider the model of posted contracts. A (subgame perfect) equilibrium in pure strategies exists and such a sequence of equilibria leads, for $h \to 0$, to the same contracts and payoffs as characterized in Proposition 5, with $\pi = a$.*

Proof. We first characterize an equilibrium. There, each firm offers the equilibrium contract γ^*. There is no advertising on equilibrium. Each consumers picks a given firm and, for the chosen equilibrium, we specify that each firm is visited by a fraction $M/|I|$ of consumers, where M is the mass of all consumers. For those consumers who then return the product, the game repeats in the next round, where again firms deterministically split the remaining market.

As we noted in the main text, by optimality γ^* must maximize the bilateral surplus. To establish existence of the (candidate) equilibrium, suppose now first a firm deviates at any particular round of the market game and chooses a higher price. This is observed by all other firms, which makes it feasible to advertise their own contracts to the deviating firm's customers. To support the candidate equilibrium, we break the non-deviating firms's

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indifference (given $\pi = a$). Suppose next that a firm deviates at any particular round and chooses a better contract for consumers. The new offer can only attract (additional) consumers when it is advertised to those particular consumers. As this requires to pay a per consumer, by construction of γ^* (with $\pi = a$) this can not be optimal (i.e., it would be loss making). **Q.E.D.**

Noisy Search. We show that an equilibrium exists where the refund level and refund period are bilaterally efficient, firms follow a unique mixed price strategy, and the consumer surplus is uniquely determined. Additionally, we present a comparative result that relates the distribution of the number of observed firms during a search to the equilibrium consumer surplus.

When the consumer has formed his consideration set, he will always choose the offer that gives him the highest expected surplus. Suppose a firm offers a refund level q' and a refund period T' that are not bilaterally optimal and a price that results in profits π' and a consumer surplus V'. If the firm offers instead the bilaterally optimal choices q^* and T^*, the resulting bilateral surplus is higher and the firm can obtain a higher profit $\pi > \pi'$ while keeping the expected consumer surplus - and thus the probability of being chosen - constant. This implies that all firms offer in equilibrium contracts where the refund level q^* and the refund period T^* are bilaterally optimal.

As in this model a firm does not know whether it is the sole firm in a consumer's consideration set it will randomize over prices, see Burdett and Judd (1983). The consumer's optimal strategy is to employ a reservation price rule, that is to stop searching for offers if he observes a price that is below a cutoff $\widehat{p}$ that is defined by

$$h = \int_0^{\widehat{p}} (\widehat{p} - p) dJ(p)$$

where $J(\cdot)$ is the distribution of the lowest price observed in a single noisy search

$$J(p) = \sum_{i=1}^{\infty} \eta_i \left(1 - (1 - K(p))^i\right) \tag{18}$$

with $K(\cdot)$ being the distribution of prices. The effective reservation price $\overline{p}_C$ that is eventually used by consumers reads

$$\overline{p}_C = \min\{\widehat{p}, \omega^*(\widehat{V}^*) + c + z^d(T^*)\} \tag{19}$$

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where $\omega^*(\widehat{V}^*) + c + z^d(T^*)$ with $z^d(T^*) = E\left[\int_0^{\tau^*} e^{-rt}z(t)dt\right]$ equals the gross surplus generated in a single search. Clearly, a consumer would never accept an offer that prices above $\overline{p}_C^*$.

Next, we turn to firm pricing. Denote with $\overline{p}_F$ the reservation price that the firms believe is applied by consumers and with $K(p|\overline{p}_F)$ the resulting mixed price strategy of firms for any p in the support of $K(\cdot|\overline{p}_F)$ that is given by

$$\eta_1(\overline{p}_F - c - z^d(T^*)) = (p - c - z^d(T^*)) \sum_{i=1}^{\infty} i\eta_i(1 - K(p|\overline{p}_F))^{i-1} \tag{20}$$

In equilibrium it must hold that $\overline{p}^* \equiv \overline{p}_F^* = \overline{p}_C^*$. Burdett and Judd (1983) show that, for a given continuation value $\widehat{V}$, a unique market equilibrium in dispersed prices exists. Lastly, the equilibrium consumer surplus V^* satisfies

$$\omega^*(\widehat{V}^*) - V^* = \int_0^{\overline{p}^*} p \, dJ(p|\overline{p}^*) - z^d(T^*) - c \tag{21}$$

where $z^d(T^*) = E\left[\int_0^{\tau^*} e^{-rt}z(t)dt\right]$. The right side of (21) can be interpreted as the expected profit of the firm that is matched with the consumer and thus resembles π from equation (3). It remains to show that a unique consumer surplus V^*, as characterized by (21), exists.

Lemma 16 *With noisy search and any learning technology a unique consumer payoff V^* , as defined by (21), exists.*

Proof. We first show that the right side of (21) is increasing in V. For this we denote by $\overline{p}$ the effective reservation price for a given V and rewrite the right side of (21) as follows

$$\int_0^{\overline{p}} p \, dJ(p|\overline{p}) - c - z^d(T^*) = \int_0^{\overline{\Delta}} \Delta \, dJ^\Delta(\Delta|\overline{\Delta})$$

where $\overline{\Delta} = \overline{p} - c - z^d(T^*)$ and $J^\Delta(\Delta|\overline{\Delta})$ is given by

$$J^\Delta(\Delta|\overline{\Delta}) = \sum_{i=1}^{\infty} \eta_i \left(1 - (1 - K^\Delta(\Delta|\overline{\Delta}))^i\right)$$

with $K^\Delta(\Delta|\overline{\Delta})$ being defined by

$$\eta_1\overline{\Delta} = \Delta \sum_{i=1}^{\infty} i\eta_i(1 - K^\Delta(\Delta|\overline{\Delta}))^{i-1} \tag{22}$$

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when $\underline{\Delta} < \Delta < \overline{\Delta}$ with $\underline{\Delta} \equiv \eta_1 \overline{\Delta}/\sum_{i=1}^{\infty} i\eta_i$, $K^{\Delta}(\Delta|\overline{\Delta}) = 0$ when $\Delta \leq \underline{\Delta}$, and $K^{\Delta}(\Delta|\overline{\Delta}) = 1$ when $\Delta \geq \overline{\Delta}$. From the definition of J^{Δ} and K^{Δ} it is immediate that $\partial J^{\Delta}/\partial\overline{\Delta} < 0$ which implies that the right side of (21) is increasing in $\overline{\Delta}$. From (19) we distinguish two cases. First, when $\omega^*(\widehat{V}) + c + z^d(T^*) < \widehat{p}$ we have $\overline{\Delta} = \omega^*(\widehat{V})$ which is clearly increasing in V. Next, consider the case when $\widehat{p} \leq \omega^*(\widehat{V}) + c + z^d(T^*)$. Recall that $\widehat{p}$ solves

$$\int_0^{\widehat{p}} (\widehat{p} - p)dJ(p|\widehat{p}) = h,$$

that is, $\widehat{p}$ is the reservation price that is applied by consumers and firms believe that consumers use $\widehat{p}$ as their reservation price. Observe that as V varies this has an impact on $z^d(T^*)$ and thus, due to the definition of $K(p|\overline{p})$ in (20), also on $\widehat{p}$. We show that, nonetheless, the resulting $\overline{\Delta} = \widehat{\Delta} \equiv \widehat{p} - c - z^d(T^*)$ is constant in V. For this we show that $\widehat{\Delta}$ solves

$$\int_0^{\widehat{\Delta}} (\widehat{\Delta} - \Delta)dJ^{\Delta}(\Delta|\widehat{\Delta}) = h. \tag{23}$$

Consider the following transformations

$$\int_0^{\widehat{\Delta}} (\widehat{\Delta} - \Delta)dJ^{\Delta}(\Delta|\widehat{\Delta}) = \int_0^{\widehat{p}-c-z^d(T^*)} (\widehat{p} - c - z^d(T^*) - \Delta)dJ^{\Delta}(\Delta|\widehat{\Delta})$$

$$= \int_0^{\widehat{p}} (\widehat{p} - p)\, dJ^{\Delta}(p - c - z^d(T^*)|\widehat{p} - c - z^d(T^*)). \tag{24}$$

We have from the definitions of J and J^{Δ} that $J^{\Delta}(p - c - z^d(T^*)|\widehat{p} - c - z^d(T^*)) = J(p|\widehat{p})$ so that the expression in (24) equals h. Note that from Claim 4 in Burdett and Judd (1983) and the accompanying discussion therein the solution $\widehat{\Delta}$ to (23) exists and is unique. The functional forms of J^{Δ} and K^{Δ} imply that $\widehat{\Delta}$ is independent of V. Thus, the claim follows.

When $V = 0$, firms will mix prices over an interval that is a subset of $[c + z^d(T^*), \omega^*(0) + c + z^d(T^*)]$ so that the right side of (21) is strictly smaller than $\omega^*(0)$. Moreover, the functional forms of J^{Δ} and K^{Δ} and the continuity of $\overline{\Delta}$ in V imply that the right side of (21) is continuous in V. This, together with the arguments from the proof of Lemma 1, proves the claim of the Lemma. **Q.E.D.**

We close by providing a comparative result that relates the equilibrium consumer surplus V^* and expected profits of the matched firm to the probability that the consumer observes only a single firm in a noisy search.

Lemma 17 *With noisy search and any learning technology the consumer payoff V^* decreases and the expected profits of the matched firm increase when η_1 increases while η_i with $i = 2, ...$ either decreases or stays constant so that still $\sum_{i=0}^{\infty} \eta_i = 1$.*

Proof. It is sufficient to show that increasing η_1 marginally while decreasing some η_j by the same amount results in an increase of the right side of (21) for any V. Rewrite $\eta_j = 1 - \eta_1 - \sum_{i=2, i \neq j}^{\infty}$ and recall that the right side of (21) equals $\int_0^{\overline{\Delta}} \Delta dJ^{\Delta}(\Delta|\overline{\Delta})$. First, consider the case when $\omega^*(V) < \widehat{\Delta}$. Then, clearly $\partial \overline{\Delta}/\partial \eta_1 = \partial \omega^*(V)/\partial \eta_1 = 0$. Differentiating (22) with respect to η_1 gives

$$\frac{\partial K^{\Delta}(\Delta|\overline{\Delta})}{\partial \eta_1} = -\frac{\overline{\Delta} - \Delta + \Delta j(1 - K^{\Delta}(\Delta|\overline{\Delta}))^{j-1}}{\sum_{i=2}^{\infty} \eta_i i(i - 1)(1 - K^{\Delta}(\Delta|\overline{\Delta}))^{i-2}}$$

which is clearly negative for Δ in the support of $K^{\Delta}(\cdot|\overline{\Delta})$. This implies that $\partial J^{\Delta}/\partial \eta_1 \leq 0$ so that $\int_0^{\overline{\Delta}} \Delta dJ(\Delta|\overline{\Delta})$ is increasing in η_1.

Next we consider the case when $\widehat{\Delta} \leq \omega^*(V)$ so that $\overline{\Delta} = \widehat{\Delta}$. Consider a distribution $J^{\Delta}(\Delta|\Delta_F)$ and denote by $\widetilde{\Delta}(\Delta_F)$ the solution to

$$\int_0^{\widetilde{\Delta}} \widetilde{\Delta} - \Delta dJ^{\Delta}(\Delta|\Delta_F) = h.$$

Then, it holds that $\widetilde{\Delta}(\widehat{\Delta}) = \widehat{\Delta}$. Burdett and Judd (1983) show that this solution is unique and that $\widetilde{\Delta}(0) = h > 0$. Moreover, it holds that $\widetilde{\Delta}(\omega^*(V)) \leq \omega^*(V)$. Since it holds that $\partial J^{\Delta}/\partial \eta_1 < 0$ we have $\partial \widetilde{\Delta}(\Delta_F)/\partial \eta_1 \geq 0$. This, in turn, implies that $\partial \widehat{\Delta}/\partial \eta_1 \geq 0$. When $\omega^*(V) = \widehat{\Delta}$, this implies that $\overline{\Delta}$ is constant in η_1 and thus it follows that $\int_0^{\overline{\Delta}} \Delta dJ^{\Delta}(\Delta|\overline{\Delta})$ increases in η_1 by the same arguments as in the first case. When $\widehat{\Delta} < \omega^*(V)$ it follows that J^{Δ} decreases first due to an increase in η_1 and second due to an increase in $\overline{\Delta} = \widehat{\Delta}$, cf. the proof of Lemma 16, which results in an increase $\int_0^{\overline{\Delta}} \Delta dJ^{\Delta}(\Delta|\overline{\Delta})$.

This implies that for all V the right side of (21) increases in η_1. This, taken together with the arguments in the proof of Lemma 16 proves the claim. **Q.E.D.**