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# Incentive-Compatibility, Limited Liability and Costly Liquidation in Financial Contracting \*

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## Abstract

We characterize an optimal financial contract when the firm's realized cash flow is unobservable to the investor and the firm's collateral can only be liquidated partially by resorting to the services of a costly third party. An optimal contract may exhibit a piecewise structure and vary with the liquidation cost and the firm's actual liquidity shortage. Partial liquidation and wholesale transfers of collateral can coexist in an optimal contract. In contrast to part of the literature, the incentive-compatibility constraint incorporates the firm's limited liability, and may be slack at the optimum. Allowing the firm to overcome an ex-post liquidity shortage by borrowing surreptitiously from a third party may reduce the firm's ex-ante expected utility.

**Keywords:** Financial contracting, incentive-compatibility, limited liability, indivisible collateral, costly liquidation.

**JEL Classification:** D86, G33

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# 1 Introduction

Information asymmetry is one of the most important impediments to efficient interactions in financial markets. In corporate finance firms usually have an informational advantage after the outcomes of projects are materialized, because it is often difficult for investors to observe these outcomes directly. Similar problems are prevalent in consumer finance, in particular the housing market, where fluctuations in the incomes of borrowers are not easily observed by lenders. Such adverse selection in the post-contracting stage is one of the main concerns in the theory of financial contracting. In a typical corporate financial contracting environment with adverse selection, the firm borrows from an investor to invest in a risky project from which it will receive returns that are only observable to itself. As a response to such adverse selection ex-post, the parties ex-ante use instruments such as collateral or threats of bankruptcy agreed upon in an incentive-compatible contract to secure the truthful execution of their agreement ex-post.

Most of the literature in this agenda assumes that the threats and promises underlying these contracts can be arbitrarily finely adjusted and costlessly executed. This can be because the firm's collateral is infinitely divisible (e.g., Faure-Grimaud 2000; Townsend 1979; Gale and Hellwig 1985; Hart and Moore 1998), the penalties imposed by the investor can be applied without cost (e.g., Faure-Grimaud 2000; Lacker 2001), or the firm ex-post has enough liquidity to offer repayments that would otherwise lead to liquidation (e.g., Faure-Grimaud 2000; Povel and Raith 2004b; Hege and Hennessy 2010). In this paper, we move away from these assumptions, and ask what happens if the firm's assets are not perfectly divisible, the ex-post execution of threats can be costly, or the firm cannot resort to ex-post payments to avoid liquidation threats. We do this by means of a simple model in the tradition of Hart and Moore 1998 and Faure-Grimaud 2000 that focuses on the problem of indivisible collateral. In practice, most collateral comes in indivisible units, and even when these units (such as small production units) can eventually be divided into smaller pieces, this requires technical or legal expertise to disentangle the multiple links that make the unit cohere in the first place. And if the contracting parties agree to play a lottery in order to achieve a division in expected terms, they may have different opinions on the credibility of the randomization devices or need an independent auditor for their use. Moreover, the contracting parties may need an outsider to verify firm's cash repayment if partial liquidation is contingent on it. All these procedures are costly.

In this paper, we therefore study a financial contracting problem with ex-post adverse selection and indivisible collateral. We allow for partial liquidation, but assume that it can only be implemented by a third party who has technical or legal expertise in dividing the

collateral or implementing a randomization scheme. Using a third party is costly to the firm, but it is the only way to avoid the transfer of its collateral as a whole.

Our approach is less drastic than Lacker (2001), who argues that randomization mechanisms in debt contracts cannot be enforced and therefore that they can only be used if they are ex-post optimal. As a consequence, he shows that with indivisible collateral the optimal contract takes a bang-bang structure: The firm is obligated to repay a certain amount of cash to the investor. If it fails to repay this amount, its collateral will be seized as a whole, and the firm must make a smaller payment. If the costs of relying on a third party is high, we recover his prediction as a special case of our model.

We characterize a specific type of optimal contracts and discuss how it varies with the liquidation cost and the firm's actual liquidity shortage. The optimal contract derived in our model takes a piecewise linear structure; we name this specific type of contracts *piecewise debt-like contracts* (PDC). We show that the optimal PDC is indeed an optimal contract, and our discussion focuses on the structure and comparative statics of the optimal PDC.

The intuition for the piecewise structure of the optimal PDC is as follows. When the firm's cash flow is sufficiently high, it is required to repay a fixed amount of cash to the investor without any liquidation. When the firm is unable to repay this fixed amount, it may want to repay all the realized cash to the investor and seek for a third party to perform partial liquidation. However, when the firm's cash flow is sufficiently low, it has to liquidate a large fraction of the collateral even with full cash repayment and partial liquidation. Thus the firm would rather transfer all of its collateral to the investor without involving a third party and save the liquidation cost.

The comparative statics of the optimal PDC is straightforward. When the liquidation cost is sufficiently large, the firm would never want to introduce a third party to perform partial liquidation, so it will transfer the collateral to the investor whenever its cash flow falls short. Our optimal contract thus resembles the one derived in Lacker (2001). Similarly, when the liquidation cost is sufficiently small, the firm would always prefer partial liquidation when it is insolvent. In this case, our optimal contract resembles the debt contract in Faure-Grimaud (2000).

We also discuss some important features of the optimal PDC. First, unlike part of the literature in this agenda (e.g., Faure-Grimaud 2000; Povel and Raith 2004b; Hege and Hennessy 2010), in our model, the IC constraint may not bind in the optimal PDC. The firm loses its collateral when its cash flow is sufficiently low, but its valuation of the collateral could be higher than the fixed cash repayment when there is no liquidation. In other words, the firm is punished more severely when it is insolvent. The reason is that the firm cannot misreport a state that necessitates a cash repayment higher than its realized cash flow, i.e.,

the firm cannot exaggerate its cash holdings. That means our IC constraint is specified consistently with the firm's limited liability. On the contrary, some papers assumed that the firm can repay the requested amount of cash for other states irrespective of its realized cash flow, meaning that the firm must be punished uniformly across the state space. Hence, our IC constraint is weaker than assumed in part of the literature and may be slack at the optimum.

Second, we discuss an extension of the model where the firm can surreptitiously liquidate its collateral to an outside investor after the realization of its cash flow, and before the execution of the contractual repayments. This assumption relaxes the firm's limited liability and enables it to exaggerate its cash flow ex-post. We show that our piecewise structure is robust to this extension when liquidation to the new investor is less efficient than liquidation to the incumbent investor in the contractual relationship. When liquidation to the new investor is sufficiently efficient, the optimal contract is a debt contract without bankruptcy, i.e., the firm will always liquidate to the new investor to repay the debt, and avoid any liquidation to the incumbent investor. Moreover, we show that allowing for surreptitious liquidation may hurt the firm, as it makes the IC constraint stronger by relaxing the firm's limited liability.

This paper can be viewed as a synthesized framework of financial contracting with ex-post adverse selection and indivisible collateral. When the cost of partial liquidation is small, our model becomes similar to the classical model of Costly State Verification (CSV) established by Townsend (1979) and Gale and Hellwig (1985). When the cost of partial liquidation is infinity, there are some common features that our model shares with the one in Hart and Moore (1998), where the firm's cash flow cannot be verified by a third party. Moreover, this paper can also be interpreted as a theory of bankruptcy, if one views the bankruptcy court as a trusted third party. When the firm is insolvent, it can either turn to a bankruptcy court which verifies its cash repayment, divides its collateral, and transfers part of it to the investor, or it can walk away from the contract, keep its remaining cash, and leave the assets wholesale to the creditor. This latter option resembles the strategy often chosen by defaulting homeowners in the real estate market. Our model thus provides a benchmark for studying the firm's trade-off between these two choices and shows that they can endogenously arise within the same contract.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents our main results. Section 4 provides the example of one single indivisible asset to be used as collateral. Section 5 discusses the literature and extends our benchmark model to allow for surreptitious liquidation. Section 6 concludes. All the proofs are relegated to the Appendix.

## 2 Model

Consider a financial contracting environment between a firm and an investor. The firm is endowed with  $I_0 \geq 0$  units of cash (numeraire), and fixed assets of which the measure is normalized to 1, with a specific future value  $V > 0$  to itself and a market value of  $W < V$ . The investor is assumed to have deep pockets.

The firm has the opportunity to use its assets and expertise to undertake a risky project. The project needs  $I > I_0$  units of cash to initiate, and will generate some cash flow  $\omega$ , which is stochastic with c.d.f.  $F$  and p.d.f.  $f$  over  $[0, \bar{\omega}]$ .<sup>1</sup> Moreover, the distribution of  $\omega$  is common knowledge, while the realization of  $\omega$  can be only observed freely by the firm. Thus  $\omega$  is the firm's private information, to which we will sometimes refer as its (ex-post) type. Assume that  $f(\omega) \in [f, \bar{f}]$  for any  $\omega \in [0, \bar{\omega}]$ , and  $I < E(\omega)$ , meaning that the investment is profitable. Since  $I > I_0$ , the firm has to obtain external funds from the investor to start the project. We let  $B$  denote the firm's initial borrowing, and allow it to borrow more than its actual liquidity shortage, i.e.,  $B \geq I - I_0$ . After the realization of  $\omega$ , the firm can use the cash or liquidate part of its assets to repay the investor. Due to the information asymmetry between the two parties, both cash repayment and liquidation can only be contingent on the firm's report of the state, which is denoted by  $\hat{\omega}$ .

We suppose that the firm's cash repayment cannot exceed its ex-post cash holdings, i.e., the firm is protected by limited liability. If we denote by  $R(\hat{\omega})$  the firm's cash repayment when it reports  $\hat{\omega}$ , then limited liability requires that

$$R(\hat{\omega}) \leq I_0 + B - I + \omega \quad \text{for any } \omega, \hat{\omega} \in [0, \bar{\omega}]. \quad (\text{LL})$$

By assuming limited liability, we rule out the firm's ability to borrow after the realization of  $\omega$ . One may argue that ex-post the firm can obtain additional finance from an outside investor by pledging some of its remaining assets. However, if refinancing is costly, it will not be in the firm's interest to issue new debt. In Section 5 we show this in a variation of our model in which ex-post borrowing is allowed and takes the form of surreptitious liquidation.

Furthermore, we denote by  $X(\hat{\omega})$  the fraction of assets to be liquidated when the firm reports  $\hat{\omega}$ . Since the investor and other market participants have the same valuation of the firm's assets, which is lower than that of the firm, it does not matter whether the firm liquidates the assets and pays the liquidation value to the investor or whether the firm transfers the assets to the investor. For notational convenience, we denote  $W/V = \alpha < 1$ . Hence, as in Hart and Moore (1998), assets should optimally remain in the possession of the

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1. Our analysis will go through if  $\omega$  is distributed over  $[0, +\infty)$ .

firm; liquidation is socially inefficient.<sup>2</sup>

We assume that assets consist of finitely many indivisible units and that, without further action, each such unit can only be transferred as a whole. Hence, the firm's assets are what we call partially divisible and thus can be divided into finitely many parts. In other words, the firm can only choose  $X(\hat{\omega})$  from a finite set, denoted by  $\mathcal{X} = \{x_0, x_1, \dots, x_n\}$ , with  $n \geq 1$ . We further assume that  $0 = x_0 < x_1 < \dots < x_n = 1$ , implying that no liquidation and full liquidation can always be chosen. The case  $n = 1$  of a single indivisible asset (think of a house as collateral for a mortgage) will serve as a lead example in Section 4; we analyze the general case  $n \geq 1$  in Section 3.

While the firm's asset base as such is only partially divisible, it can be continuously liquidated if the parties bring in a third party. This costs  $c \geq 0$  and makes it possible to liquidate the assets in any desired way. This may either be through technical and legal expertise with respect to the assets or by using and certifying an appropriate stochastic mechanism.<sup>3</sup> The third party can be interpreted as a court, which supervises the partial liquidation procedure or which implements stochastic liquidation, but since we do not model institutional details here, it can also be interpreted as a private out-of-court mediator.<sup>4</sup> Liquidation supported by the third party can take any value in the closed interval  $[0, 1]$ . Let  $\psi$  be a function of  $\hat{\omega}$  that indicates whether the third party should be introduced when the firm reports  $\hat{\omega}$ :  $\psi(\hat{\omega}) = 1$  if yes, otherwise  $\psi(\hat{\omega}) = 0$ . Therefore, the liquidation decision  $X(\hat{\omega})$  is affected by  $\psi(\hat{\omega})$ . We will formally refer to this constraint as the feasibility constraint, i.e.,

$$\text{If } \psi(\hat{\omega}) = 0 \text{ then } X(\hat{\omega}) \in \mathcal{X}. \text{ If } \psi(\hat{\omega}) = 1 \text{ then } X(\hat{\omega}) \in [0, 1]. \quad (\text{FC})$$

Varying values of  $c$  correspond to different cases in the literature. If  $c = 0$ , the firm's assets are perfectly divisible as in the models of Hart and Moore (1998), Faure-Grimaud (2000), and others. If  $c$  is sufficiently large, the asset is completely indivisible, such as in Lacker (2001). If  $c$  is positive but small, this is as in the models of costly state verification of Townsend-Gale-Hellwig. For simplicity, we assume the cost  $c$  is monetary and borne by the investor.

A contract now specifies: (1) the firm's initial borrowing  $B$ ; (2) the cash repayment  $R$  from the firm to the investor; (3) the fraction of assets to be liquidated  $X$ ; and (4) a function  $\psi$  indicating whether the third party is called upon. (2), (3), and (4) are all functions of  $\hat{\omega}$ .

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2. In Hart and Moore (1998), the cash flow is observable by both parties but not verifiable by outsiders, in order to simplify the problem of ex-post renegotiation.

3. Hence, there is continuous liquidation in expected value, which is all that matters in our risk-neutral setting. Stochastic liquidation has been discussed, e.g., by Mookherjee and Png (1989).

4. See, e.g., Aghion, Hart, and Moore (1992) or von Thadden, Berglöf, and Roland (2010) for fuller models of bankruptcy and bargaining.

In the following analysis, we will denote the contract by  $\Gamma$ , i.e.,  $\Gamma = (B, R, X, \psi)$ .

In addition, we assume that both contracting parties are risk-neutral. Given the realized cash flow  $\omega$  and the firm's report  $\hat{\omega}$ , the utility functions are

$$u_I(\Gamma, \omega, \hat{\omega}) = -B + R(\hat{\omega}) + X(\hat{\omega})W - \psi(\hat{\omega})c$$

for the investor, and

$$u_F(\Gamma, \omega, \hat{\omega}) = I_0 + B - I + \omega - R(\hat{\omega}) + [1 - X(\hat{\omega})]V$$

for the firm.

We assume that the parties can commit to the ex-post execution of the contract ex-ante. Therefore, by the revelation principle, we can without loss of generality focus on direct mechanisms. In these mechanisms, the firm must have incentives to truthfully report its cash flow. That leads to the incentive-compatibility constraint

$$u_F(\Gamma, \omega, \omega) \geq u_F(\Gamma, \omega, \hat{\omega}) \quad \text{for any } \omega, \hat{\omega} \text{ such that } R(\hat{\omega}) \leq I_0 + B - I + \omega. \quad (\text{IC})$$

Note that we only require that each (ex-post) type of the firm has no incentive to choose the repayment/liquidation combinations designed for other types among those it can afford in state  $\omega$ . Hence, the set of deviations possible for each type depends on the (endogenous) contract. This formulation of the incentive-compatibility constraint has been overlooked in the literature up to now<sup>5</sup> and makes the analysis more complicated than under the standard approach (which ignores the qualification of feasible deviations). However, the qualification is essential in a model built on the very notion of limited liability.

Finally, the investor should at least break even if she accepts the contract, which implies her individual rationality constraint

$$E_\omega u_I(\Gamma, \omega, \omega) \geq 0. \quad (\text{IR})$$

Hence, a full statement of the contracting problem is

$$\begin{aligned} & \max_{\Gamma} E_\omega u_F(\Gamma, \omega, \omega), \\ & \text{subject to} \quad (\text{LL}), (\text{FC}), (\text{IC}) \text{ and } (\text{IR}). \end{aligned}$$

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5. See, e.g., Faure-Grimaud 2000; Povel and Raith 2004b; Hege and Hennessy 2010 and our discussion in Section 5.

We say that a contract  $\Gamma$  is *optimal* if it is a solution to this problem.

### 3 Analysis

We start with some concepts and notations. First, for any two contracts,  $\Gamma$  and  $\hat{\Gamma}$ , we say that  $\Gamma$  is *weakly dominated* by  $\hat{\Gamma}$  if the following two inequalities hold:

$$E_\omega u_I(\Gamma, \omega, \omega) \leq E_\omega u_I(\hat{\Gamma}, \omega, \omega), \quad (1)$$

$$E_\omega u_F(\Gamma, \omega, \omega) \leq E_\omega u_F(\hat{\Gamma}, \omega, \omega). \quad (2)$$

Furthermore,  $\Gamma$  is *strictly dominated* by  $\hat{\Gamma}$  if (1) and (2) hold, and at least one of them is strict.

Then, we use  $\Phi(\omega)$  to represent the firm's ex-post total payout under the contract  $\Gamma$  as valued by the firm, i.e.,

$$\Phi(\omega) = R(\omega) + X(\omega)V.$$

By (IC), the firm can always understate its cash flow, so  $\Phi(\omega)$  must be nonincreasing.

Finally, we describe a specific form of contract that is defined piecewisely, and in each of these pieces, the contract resembles a debt.

**Definition 1.**  $\Gamma$  is a *piecewise debt-like contract (PDC)* if there exists a sequence of triples,  $\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}$ , such that:

(a)  $0 = r_n^w \leq r_{n-1}^p \leq r_{n-1}^w \leq \dots \leq r_0^p \leq r_0^w \leq \bar{\omega}$ ;

(b) For any  $j = 0, 1, \dots, n-1$ ,

$$I_0 + B - I + r_j^w + x_j V \leq \phi_j \leq I_0 + B - I + r_{j+1}^w + x_{j+1} V; \quad (3)$$

(c) For any  $j = 0, 1, \dots, n$ ,

(c.1) if  $\omega \in [r_j^w, r_{j-1}^p)$  for  $j \geq 1$ , or  $\omega \geq r_0^w$  for  $j = 0$ , then  $R(\omega) = I_0 + B - I + r_j^w$ ,  
 $X(\omega) = x_j$ ,  $\psi(\omega) = 0$ ;

(c.2) if  $\omega \in [r_j^p, r_j^w)$ , then  $R(\omega) = I_0 + B - I + \omega$ ,  $X(\omega) \in [0, 1]$ ,  $\Phi(\omega) = \phi_j$ ,  $\psi(\omega) = 1$ .

Figure 1 gives us a graphical illustration of a PDC. In a PDC, the firm is supposed to perform wholesale transfers of a fraction  $x_{j+1}$  of the assets when its realized cash flow is higher than, and close to,  $r_{j+1}^w$ , i.e., when  $\omega \in [r_{j+1}^w, r_j^p)$ . When the firm's cash flow is much higher than  $r_{j+1}^w$ , but still less than  $r_j^w$ , the firm will need the third party and perform partial liquidation. In this case the firm's total payout is determined by  $\phi_j$ . Here the superscript  $w$  stands for "wholesale transfers", i.e.,  $r_j^w$  is the left endpoint of an interval with wholesale

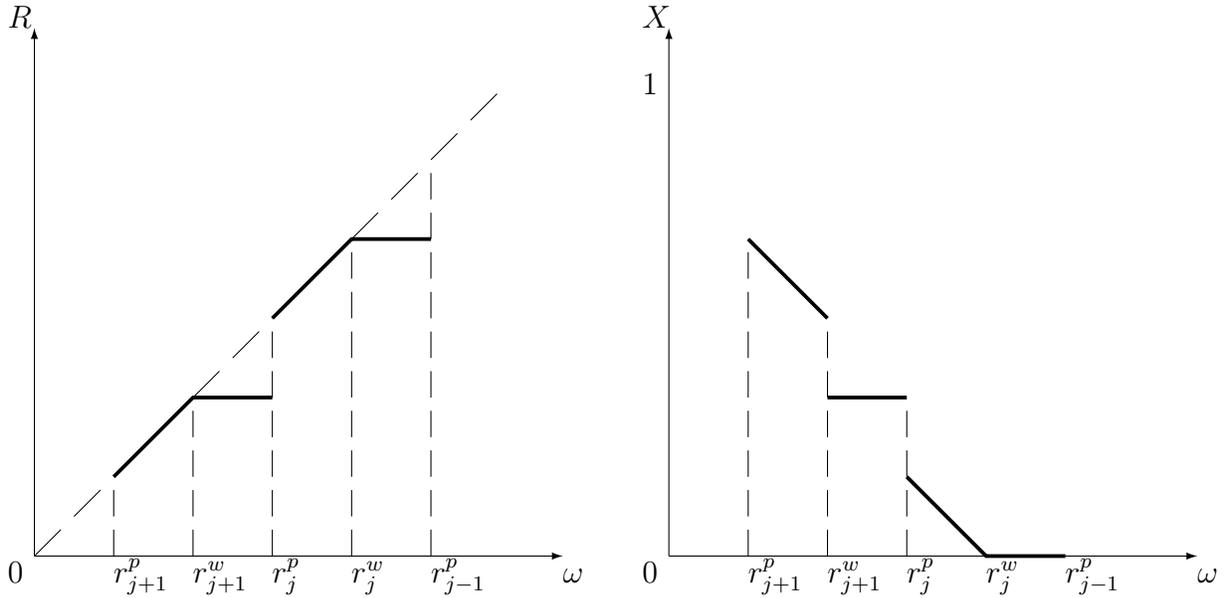


Figure 1: The structure of a PDC.

transfers. Similarly, the superscript  $p$  stands for “partial liquidation”, i.e.,  $r_j^p$  is the left endpoint of an interval with partial liquidation.

In what follows, we will show that any contract that satisfies all the constraints of the contracting problem is weakly dominated by a PDC; moreover, any PDC satisfies these constraints. Hence, the optimal PDC must be an optimal contract of the firm’s problem.

**Proposition 1.** *If  $\Gamma$  is a PDC, then it satisfies (LL), (FC) and (IC).*

The proof of Proposition 1 is centered around verifying the incentive constraint. Since by (b) of the definition of PDC, the firm’s payout  $\Phi(\omega)$  is nonincreasing, it is not profitable for the firm to understate its cash flow. Moreover, when  $\omega \in [r_j^p, r_j^w)$  for some  $j$ , the firm cannot exaggerate its cash flow, because a higher cash flow leads to a higher cash repayment, which will certainly exceed its realized cash flow  $\omega$ . When  $\omega \in [r_j^w, r_{j-1}^p)$  for some  $j$ , the firm can exaggerate its cash flow, but its payout is then the same as that with a truthful report.

Proposition 2 tells us that, if a contract satisfies (LL), (FC) and (IC), then it is weakly dominated by a PDC. Moreover, in such a PDC the firm borrows exactly the same amount as its actual liquidity shortage. This result echos the notion of maximum equity participation in Gale and Hellwig (1985).

**Proposition 2.** *If  $\Gamma$  satisfies (LL), (FC) and (IC), then it is weakly dominated by a PDC  $\hat{\Gamma}$  with  $\hat{B} = I - I_0$ .*

The proof of Proposition 2 involves several steps. First, for any contract that satisfies (LL), (FC) and (IC), whenever there is an  $\omega_j$  such that the firm liquidates  $x_j$  at  $\omega_j$  with a wholesale transfer, we find the smallest possible state that can afford the cash repayment  $R(\omega)$ , and make it the definition of  $r_j^w$ . It can be shown that all the states between  $r_j^w$  and  $\omega$  share the same payout, i.e.,  $\Phi(\omega)$  is constant for any  $\omega \in [r_j^w, \omega_j]$ . Second, consider an alternative contract that is defined as follows: When  $\omega$  is sufficiently close to  $r_j^w$ , and  $\omega \geq r_j^w$ , the firm liquidates  $x_j$  at  $\omega$  with a wholesale transfer, otherwise, the firm liquidates as little as it can with partial liquidation. At the same time, we make the firm's expected payout identical between the two contracts. The key point in this step is to find the cutoff that determines whether the firm in state  $\omega$  should use partial liquidation. Roughly speaking, the benefit of using partial liquidation is that it enables the firm to minimize its liquidation for a given  $\omega$ , therefore such benefit decreases with  $x_j - X(\omega)$ . The cost of using partial liquidation is the constant  $c$ , which ultimately is borne by the firm. The cutoff is then pinned down by resolving this tradeoff. Finally, we can verify that this new contract is a PDC that weakly dominates the initial contract.

Note that while any contract that satisfies (LL), (FC) and (IC) is weakly dominated by a PDC, it does not mean that PDCs are the only possible structure of optimal contracts. To see this, suppose that  $\Gamma$  is an optimal PDC. By Definition 1, a PDC has  $\Phi(\omega) = \phi_j$  for any  $\omega \in [r_j^p, r_j^w)$ , which means  $X(\omega)$  has a slope of  $-1$ , as shown in Figure 1. Consider another decreasing function  $\hat{X}(\omega)$  defined for any  $\omega \in [r_j^p, r_j^w)$ . Let  $\hat{X}(\omega)$  have a slope smaller than  $-1$  and satisfy the following:

$$E_\omega[X(\omega)|r_j^p \leq \omega < r_j^w] = E_\omega[\hat{X}(\omega)|r_j^p \leq \omega < r_j^w].$$

Then, suppose that

$$\hat{\Gamma} = \begin{cases} (B, R, \hat{X}, \psi) & \text{if } \omega \in [r_j^p, r_j^w), \\ \Gamma & \text{otherwise.} \end{cases}$$

We can prove that  $\hat{\Gamma}$  is also an optimal contract, but clearly it is not a PDC. In other words,  $\hat{\Gamma}$  generates the same expected payoffs for two contracting parties as  $\Gamma$ , but has a steeper liquidation function on one of its intervals with partial liquidation; this feature still makes  $\hat{\Gamma}$  an incentive-compatible contract.

Based on this result, we know that the firm's problem can be solved by finding an optimal PDC, a problem that can be solved using the standard Lagrangian method. Therefore, if we denote the upper endpoint  $r_{-1}^p = \bar{\omega}$ , the problem of finding the optimal PDC can be

rewritten as

$$\min_{\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}} \sum_{j=0}^n (r_j^w + x_j V) [F(r_{j-1}^p) - F(r_j^w)] + \sum_{j=0}^{n-1} \phi_j [F(r_j^w) - F(r_j^p)]$$

subject to the constraints that are imposed by the PDC structure:

$$0 = r_n^w \leq r_{n-1}^p \leq r_{n-1}^w \leq \dots \leq r_0^p \leq r_0^w \leq \bar{\omega}, \quad (4)$$

$$r_0^w \leq \phi_0 \leq r_1^w + x_1 V \leq \phi_1 \dots \leq \phi_{n-1} \leq V, \quad (5)$$

and the investor's participation constraint:

$$-(I - I_0) + \sum_{j=0}^n (r_j^w + x_j W) [F(r_{j-1}^p) - F(r_j^w)] + \sum_{j=0}^{n-1} \int_{r_j^p}^{r_j^w} [\omega + \alpha(\phi_j - \omega) - c] dF(\omega) \geq 0. \quad (6)$$

Proposition 3 shows that the optimal PDC has a potentially rich structure that depends on the size of the liquidation cost  $c$  and the firm's funding need  $I - I_0$ . In particular, when  $I - I_0$  is sufficiently small, the optimal PDC only has partial liquidation; when  $c$  exceeds a certain threshold and  $I - I_0$  is not too large, the optimal PDC has only wholesale transfers and no partial liquidation.

**Proposition 3.** *There exist two cutoffs,  $\underline{c}$  and  $\bar{c}$ , and two functions,  $\underline{I}(c)$  and  $\bar{I}(c)$ , such that:*

- (a) *If  $I - I_0 < \underline{I}(c)$ , then any optimal PDC with  $B = I - I_0$  has only partial liquidation, moreover, when  $c > \underline{c}$ ,  $\underline{I}(c) = 0$ ;*
- (b) *If  $I - I_0 < \bar{I}(c)$ , then any optimal PDC with  $B = I - I_0$  has only wholesale transfers; moreover, when  $c < \bar{c}$ ,  $\bar{I}(c) = 0$ .*

The results in Proposition 3 are depicted in Figure 2.

Proposition 3 is proved by exploiting the Lagrangian. Let  $L$  be the Lagrangian for the firm's problem, and  $\lambda$  be the multiplier for the investor's participation constraint. We first assume that there are wholesale transfers in the optimal PDC. By first-order necessary conditions, we can compute a lower bound for  $\lambda$ . Then, we show that a larger  $\lambda$  is associated with a larger  $I - I_0$ , so there is an upper bound for  $I - I_0$ , which in turn serves as a sufficient condition for partial liquidation, i.e., the result stated in (a) of Proposition 3. Following the same procedure, we assume that there is partial liquidation in the optimal PDC to get the sufficient condition in (b) of Proposition 3.

Proposition 4 characterizes the optimal PDC without partial liquidation. It provides a necessary condition that allows us to compute  $r_j^w$  based on  $r_{j-1}^w$  and  $r_{j+1}^w$ .

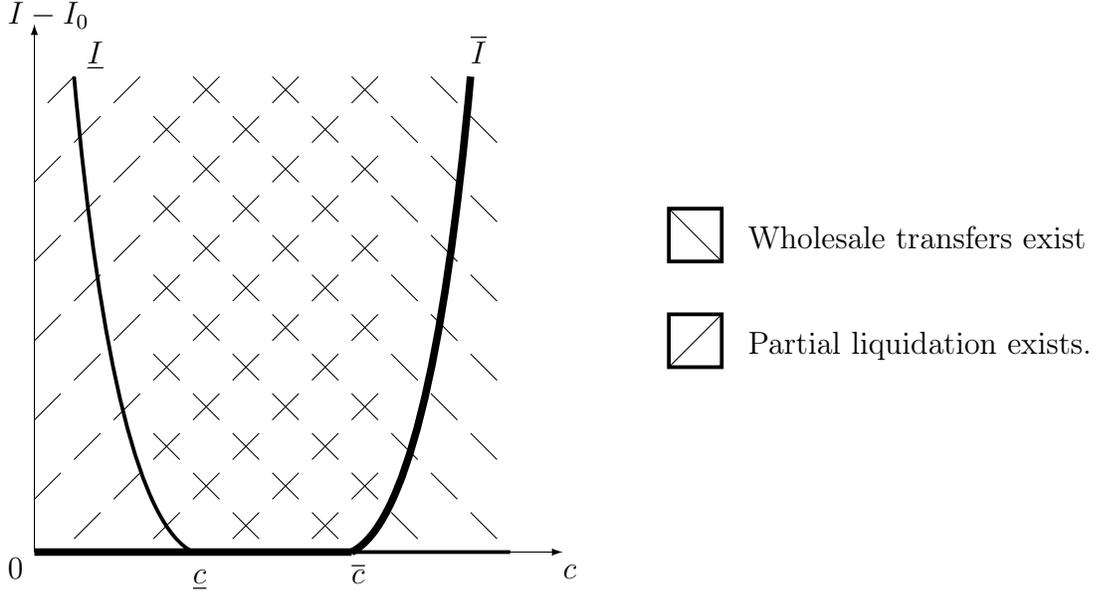


Figure 2: Wholesale transfers and partial liquidation in the optimal PDC.

**Proposition 4.** *If  $\Gamma$  is an optimal PDC without partial liquidation, then there exists  $\mu \leq \alpha$  such that for any  $j = 0, 1, \dots, n - 1$ , if further  $\Phi(r_{j-1}^w) < \Phi(r_j^w) < \Phi(r_{j+1}^w)$ , then*

$$r_j^w - r_{j+1}^w - \mu(x_{j+1} - x_j)V = \frac{F(r_{j-1}^w) - F(r_j^w)}{f(r_j^w)}. \quad (7)$$

To understand the intuition behind (7), consider the problem of choosing the optimal  $r_j^w$  on the interval  $[r_{j+1}^w, r_{j-1}^w]$ . By definition, the firm whose type is lower than  $r_j^w$  has to repay  $r_{j+1}^w$  units of cash, and liquidate a fraction  $x_{j+1}$  of its assets to the investor, while the firm with a type higher than  $r_j^w$  has to repay  $r_j^w$  and liquidate  $x_j$ . Note that  $r_{j+1}^w < r_j^w$ , and  $x_{j+1} > x_j$ . Therefore, it is as if the investor is selling  $x_{j+1} - x_j$  of the assets to the firm whose ability to pay is distributed over  $[r_{j+1}^w, r_{j-1}^w]$ . Standard theory of optimal pricing tells us that the price net of the cost should be equal to the inverse of the conditional hazard rate, i.e.,

$$\underbrace{r_j^w - r_{j+1}^w}_{\text{Price}} - \underbrace{\mu(x_{j+1} - x_j)V}_{\text{Shadow cost}} = \underbrace{\frac{F(r_{j-1}^w) - F(r_j^w)}{f(r_j^w)}}_{\text{Inverse of the conditional hazard rate}}.$$

In other words, the firm whose type is higher than  $r_j^w$  repays extra  $r_j^w - r_{j+1}^w$  units of cash, and retains a fraction  $x_{j+1} - x_j$  of its assets. Thus, by choosing  $r_j^w$ , the investor actually sets a “price” that equals  $r_j^w - r_{j+1}^w$ , and her “shadow cost” is  $x_{j+1} - x_j$  multiplied by  $\mu V \leq W$ , which indexes the efficiency loss from liquidation. The investor’s marginal revenue is determined

by the inverse of the conditional hazard rate. Hence, (7) comes from setting the price equal to the marginal revenue. Moreover, since (7) holds for any  $j$ , the multiplier before “shadow cost”, which is  $\mu$ , should be constant for all  $j$  in an optimal contract. This means that the marginal efficiency loss from liquidation is constant for all  $j$  at the optimum.

Figure 3 depicts the optimal PDC characterized in Proposition 4. This contract can be implemented by a sequential repurchase agreement: For any  $j$ , the contracting parties set a price  $r_j^w - r_{j+1}^w$  for a fraction  $x_{j+1} - x_j$  of the assets; the firm can repurchase its assets if it can afford the corresponding price after the realization of  $\omega$ . Moreover, the firm must repurchase sequentially: It should pay for  $x_{j+1} - x_j$  before  $x_j - x_{j-1}$ .

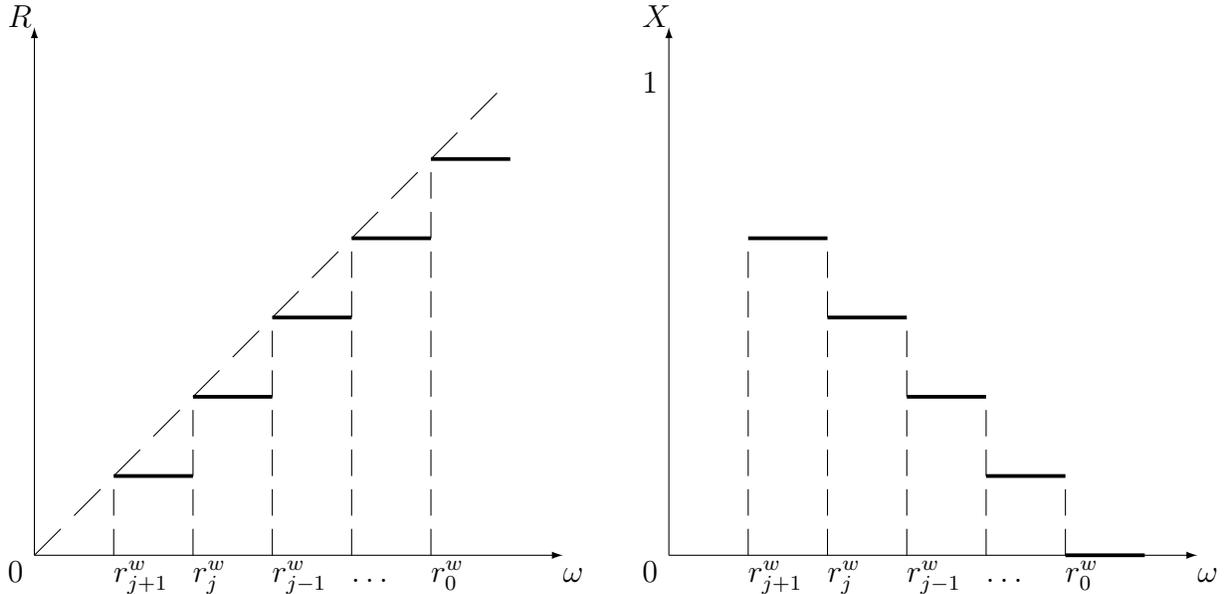


Figure 3: Optimal contract without partial liquidation.

The following Proposition 5 answers the following question: How frequently would wholesale transfers be used in an optimal PDC? In other words, what is the ex-ante probability for the assets to be transferred without outside intervention? Note that in a PDC, wholesale transfers are implemented only when  $\omega \in [r_j^w, r_{j-1}^w)$ . Proposition 5 tells us that the length of such interval is bounded above by a linear function of  $c$ .

**Proposition 5.** *If  $\Gamma$  is an optimal PDC, then for any  $j = 0, 1, \dots, n - 1$ ,*

$$r_j^p - r_{j+1}^w \leq \frac{c}{1 - \alpha}. \quad (8)$$

To prove Proposition 5, we only need to use the fact that if  $r_j^p > r_{j+1}^w$ , then  $\partial L / \partial r_j^p$  must be nonpositive, otherwise one can reduce  $r_j^p$  to increase the value of  $L$ .

Corollary 1 directly comes from Proposition 5. In the optimal PDC, the intervals implementing wholesale transfers shrink as  $c$  converges to zero. In the extreme case with  $c = 0$ , the firm can always implement partial liquidation.

**Corollary 1.** *If  $c = 0$ , then the optimal PDC has the following feature: There exists a cutoff  $r_0^w$  such that when  $\omega \geq r_0^w$ ,  $R(\omega) = r_0^w$ ,  $X(\omega) = 0$ ; when  $\omega < r_0^w$ ,  $R(\omega) = \omega$ ,  $X(\omega) = (r_0^w - \omega)/V$ .*

In this case, the optimal PDC becomes similar to the ones derived in the literature with continuous liquidation, such as Diamond (1984) and Faure-Grimaud (2000).

## 4 An Illustrative Example

In this section, we assume that assets are fully indivisible, i.e.,  $n = 1$ , and graphically illustrate the structure of the optimal PDC under this simplified assumption. Moreover, we show how this structure varies with  $c$  and  $I - I_0$ . According to Proposition 2, when  $n = 1$ , the optimal PDC is pinned down by only three parameters:  $r_0^p$ ,  $r_0^w$ , and  $\phi_0$ .

By Proposition 3, if  $c$  and  $I - I_0$  are sufficiently small, liquidation in the optimal contract is fully continuous, which is shown in Figure 4.

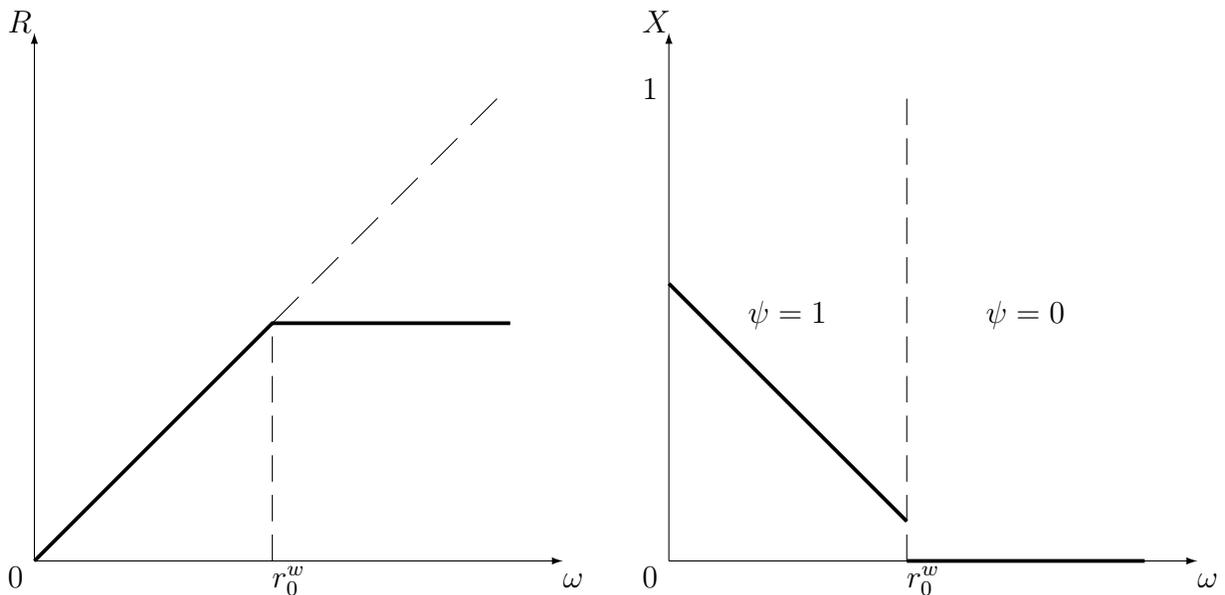


Figure 4: Optimal PDC with indivisible assets ( $c$  is small).

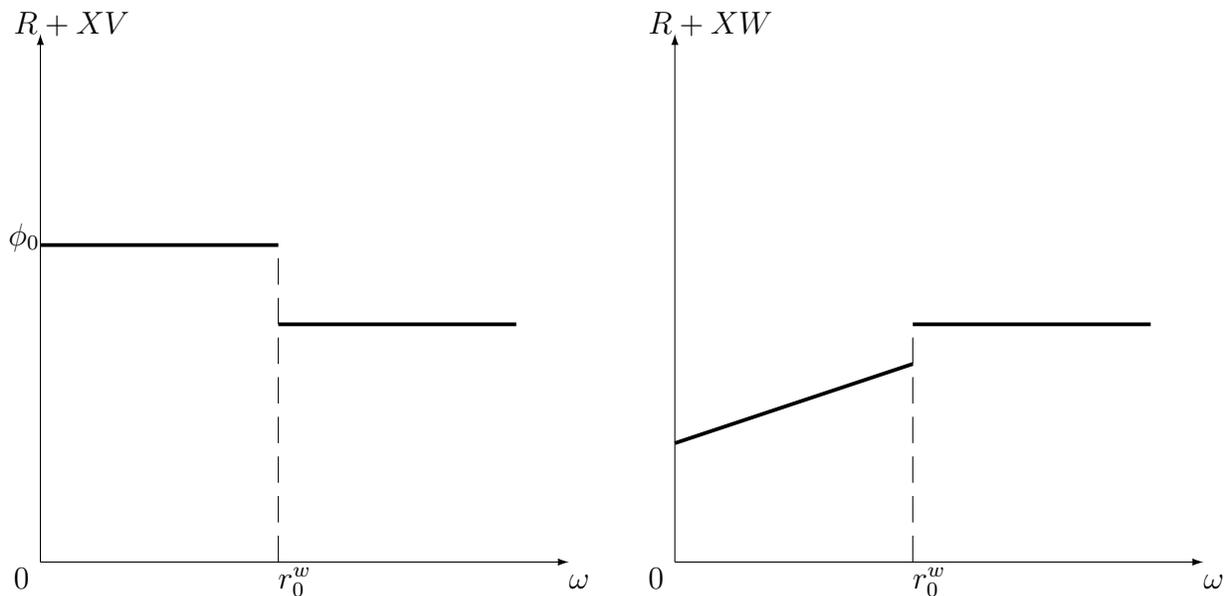


Figure 5: Valuation of contractual transfers.

It is also instructive to study the valuation of the contractual transfers from both contracting parties' perspectives. As a complement to Figure 4, the total transfers from the firm as valued by the firm,  $R(\omega) + X(\omega)V$ , and as valued by the investor,  $R(\omega) + X(\omega)W$ , are depicted in Figure 5.

The contract in Figure 4 differs from the optimal contract in Faure-Grimaud (2000) in an important aspect: The firm's expected payout on  $[0, r_0^w)$  is determined by  $\phi_0$ , which may exceed the "face value"  $r_0^w$ . In other words, while the firm needs to pay  $r_0^w$  to settle its debt, it may be punished by more than this amount if it defaults.

According to Gale and Hellwig (1985), a standard debt contract is defined by the following three properties: (1) the firm pays a fixed amount of cash when it is solvent; (2) the firm is declared bankrupt if this fixed cash payment cannot be met; (3) the investor can recoup as much of the debt as possible from the firm's assets. From Figure 5, the optimal PDC satisfies the first two properties but violates the third one. When the firm goes bankrupt, the market value of the assets liquidated is not enough to make up for the gap between the firm's cash payment and the fixed repayment. However, the firm still possesses some assets after bankruptcy. In other words, the investor suffers a loss when the firm is insolvent, while the firm is punished more than required in bad states due to (IC). On the other hand, a standard debt contract would lead to excessive liquidation, i.e., excessive deadweight loss, and thus is not optimal in our framework. Therefore, we call the optimal PDC in Figure 4 a debt-like contract.

Turning to Figure 6, if  $I - I_0$  is relatively large, the firm may have to liquidate a large fraction of assets through the third party when its cash flow is far below the cutoff  $r_0^w$ . When this fraction is sufficiently large, the relative benefit from implementing partial liquidation becomes small and the borrower would rather transfer the asset fully than pay the cost of partial liquidation. Partial liquidation and wholesale transfers thus coexist in the optimal PDC in an interesting way: if realized cash is small, the borrower keeps the cash and walks away from the contract, leaving the asset wholly with the creditor. If the cash is intermediate, liquidation does not have to be that large, and the borrower prefers to repay what she has and liquidate partially. This contract resembles actual behavior in the US housing market, and to the best of our knowledge has not been identified in the contracting literature so far.

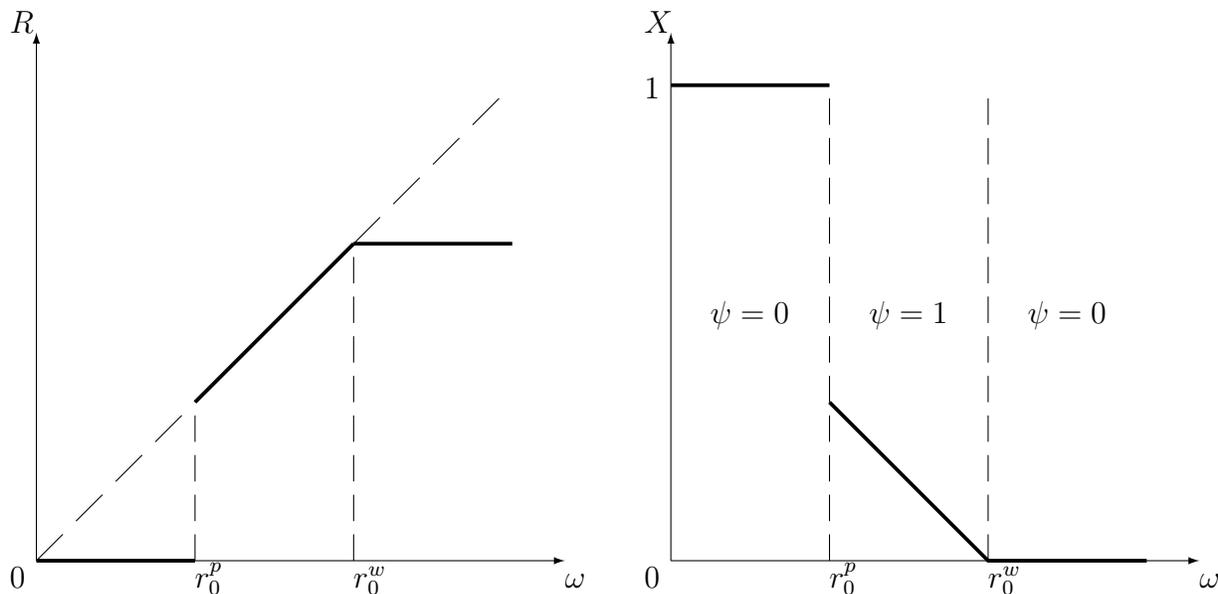


Figure 6: Optimal PDC with indivisible assets ( $c$  is medium).

If, on the other hand,  $c$  is large, then by Proposition 3, when  $c \geq (1 - \alpha)V$ , it is possible that the firm will never implement partial liquidation. The optimal PDC in this case is a pure “walk away contract”, similar to the one derived in Lacker (2001) and is depicted in Figure 7.

Hence, even in the simple case of assets composed of only one indivisible unit, the optimal PDCs can take a number of different forms that resolve the underlying tradeoff between the costs of defaulting in an interesting way.

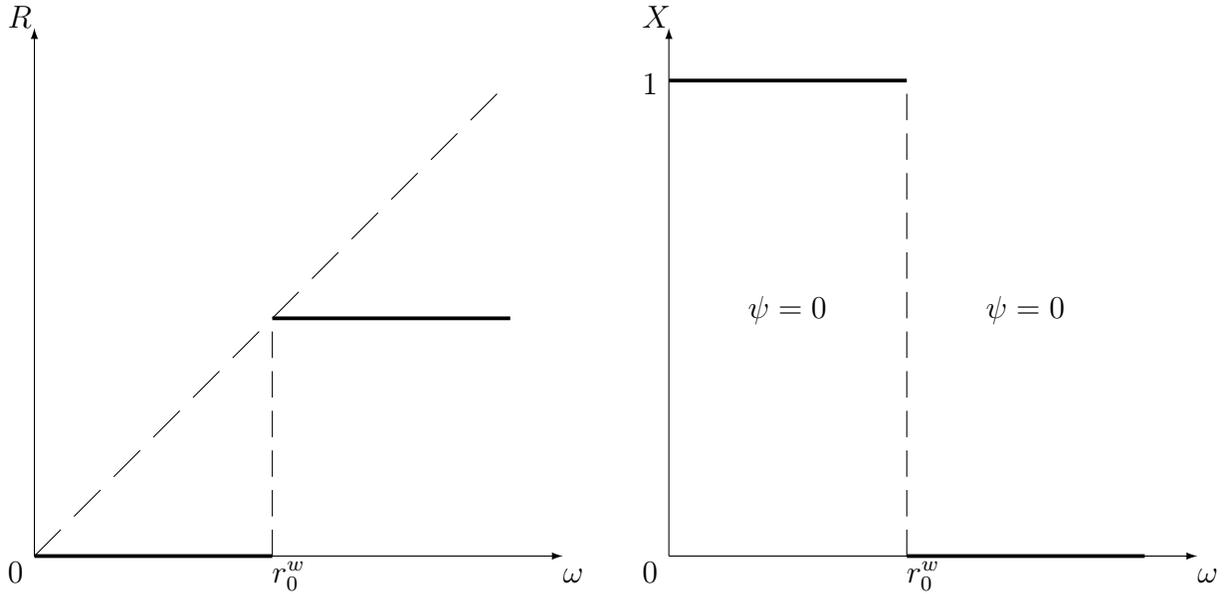


Figure 7: Optimal PDC with indivisible assets ( $c$  is large).

## 5 Discussion

**Nonbinding incentive-compatibility constraint.** As discussed in Section 1, our model belongs to the literature of financial contracting with ex-post adverse selection. While this approach has proved to be highly insightful, most of the literature has employed a shortcut with respect to the IC constraint that is inconsistent with the very assumption underlying the problem, namely the informed party's limited liability. Taking Faure-Grimaud (2000) as an example, in its benchmark model, the incentive-compatibility constraint is specified as

$$u_F(\Gamma, \omega, \omega) \geq u_F(\Gamma, \omega, \hat{\omega}) \quad \text{for any } \omega, \hat{\omega}. \quad (\text{IC}')$$

This means that the firm can deviate to announce any payoff realization, regardless of whether the deviation entails feasible transfers. In other words, the constraint ignores the limited liability constraint (LL) off equilibrium.

Clearly, the correct constraint (IC) is weaker than (IC'), because under the correct constraint a type- $\omega$  firm cannot mimic states  $\hat{\omega}$  with  $R(\hat{\omega}) > \omega$ . Thus, there are fewer deviations rule out, which means that (IC) is less restrictive than (IC').

Following Faure-Grimaud (2000), other important papers have used this restricted approach, including (but may not be restricted to) Faure-Grimaud and Mariotti (1999), Povel and Raith (2004a), Hege and Hennessy (2010), Khanna and Schroder (2010), Arve (2014),

and Tamayo (2015).<sup>6</sup> Clearly this approach simplifies the analysis considerably, because (IC') immediately implies that

$$R(\omega) + X(\omega)V$$

is constant across  $\omega$  and that the incentive-compatibility constraint and the investor's participation constraint therefore bind.

This analysis is incomplete as long as one does not show that the solution under the restricted approach is also optimal in the larger admissible set of the full approach. Since the case  $c = 0$  in our model is mathematically equivalent to the case of continuous liquidation of the previous literature, our analysis in Section 3 fills this gap. However, it also shows that when  $c > 0$  the incentive constraint may cease to be binding, which invalidates the restricted approach. In fact, whether (IC) binds in the optimal PDC actually depends on the cutoffs  $\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}$  resulting from the firm's optimization problem. An extreme example is that when  $c$  is sufficiently large, the contract in Figure 7 is optimal, but (IC) binds only when  $r_0^w = V$ .

Moreover, even if  $c$  is sufficiently small, and the optimal PDC only has partial liquidation, as depicted in Figure 4, (IC) may not be binding. Proposition 6 provides a necessary condition for a binding (IC) when the optimal PDC has the structure displayed in Figure 4.

**Proposition 6.** *If  $\Gamma$  is an optimal PDC with only partial liquidation, and  $\Gamma$  has a binding (IC), then*

$$\frac{1 - F(r_0^w)}{f(r_0^w)} > c,$$

where  $r_0^w$  is determined by a binding (IR).

The condition stated in Proposition 6 is satisfied by  $c = 0$ . When  $c > 0$ , this condition depends on the hazard rate of the distribution of  $\omega$ . The key intuition for the nonbinding (IC) is as follows. When partial liquidation is costly, the firm's objective function becomes two-dimensional: It not only wants to minimize the ex-ante probability of being liquidated, but also aims to save the cost of partial liquidation. In fact, its objective would be a linear combination of the expectation of the two variables,  $X$  and  $\psi$ . Therefore, compared to the model with  $c = 0$  and a binding (IC), the firm may reduce  $r_0^w$  and make the incentive constraint slack in order to reduce the expected liquidation cost.

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6. See (IC) in Faure-Grimaud and Mariotti (1999), (3) in Faure-Grimaud (2000), (A.1) in Povel and Raith (2004a), (27) in Hege and Hennessy (2010), Section 1.1 in Khanna and Schroder (2010), (11) in Arve (2014), and (P.3) in Tamayo (2015). All of these papers ignore the role of limited liability in specifying the incentive-compatibility constraints, although their results may not be affected by the misspecification.

**Surreptitious liquidation.** In the model discussed up to now, the firm’s limited liability constraint referred to its cash flow and nothing more. However, the firm can possibly generate liquidity by liquidating some of its assets privately and thus mimic types with higher cash flow. In this subsection, we provide a simple example to discuss how the structure of the optimal contract varies if we allow for private surreptitious liquidation.

We take all the model settings in Section 2 as given, and further allow the firm to liquidate its assets right after the realization of  $\omega$ , and before the execution of the contractual payment. If being liquidated secretly, the assets would generate  $W_p$  units of cash for the firm, where  $0 < W_p < V$ . Therefore, if  $W < W_p$ , then being liquidated by an outsider is more efficient than being liquidated by the incumbent investor, and vice versa.

To simplify our analysis, we assume that assets are indivisible, i.e.,  $n = 1$ . Let  $Y$  denote the quantity of assets being liquidated secretly, then  $Y \in \{0, 1\}$ . We assume that the incumbent investor is aware of the possibility that assets may be liquidated secretly by the firm, but she cannot observe how much of the assets are liquidated.<sup>7</sup> In other words,  $Y$  is also the firm’s private information, and it is a function of  $\omega$ . Moreover, we rule out partial liquidation by assuming that  $c$  is sufficiently large. This is a relatively strong assumption, but it suffices to make the point. The general case is similar, because the secret extension of ex-post cash is limited and comes at a cost that the contracting parties anticipate and therefore try to avoid in the optimal contract. Finally, we rule out the firm’s choice on initial borrowing and take  $B = I - I_0$  as given.

Therefore, in this simplified model, the firm’s utility function is given by:

$$u_F(\Gamma, \omega, \hat{\omega}) = \omega + Y(\omega)W_p - R(\hat{\omega}) + [1 - X(\hat{\omega}) - Y(\omega)]V.$$

The limited liability constraint, the feasibility constraint, and the incentive-compatibility constraint are given by

$$R(\hat{\omega}) \leq \omega + Y(\omega)W_p; \tag{LL}_p$$

$$0 \leq X(\hat{\omega}) \leq 1 - Y(\omega); \tag{FC}_p$$

$$u_F(\Gamma, \omega, \omega) \geq u_F(\Gamma, \omega, \hat{\omega}) \quad \text{for any } \omega, \hat{\omega} \text{ such that} \tag{IC}_p$$

$$R(\hat{\omega}) \leq \omega + Y(\omega)W_p \text{ and } 0 \leq X(\hat{\omega}) \leq 1 - Y(\omega).$$

Here the subscript  $s$  stands for “surreptitious liquidation”. Note that even if we allow for surreptitious ex-post liquidation, the firm is still subject to an ex-post resource constraint,

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7. If the incumbent investor is unaware of the possibility of secrete liquidation, the model becomes similar to the costly state falsification model studied in Lacker and Weinberg (1989).

i.e., the total amount that can be used to repay its loan is bounded. Thus, our main point that (IC) must incorporate the firm's affordability constraint will not be affected.

Proposition 7, with its graphical illustration in Figure 8, characterizes the optimal contract.

**Proposition 7.** *Suppose that  $n = 1$ ,  $c$  is large, and surreptitious liquidation is allowed.*

- (a) *If  $W_p < \max\{W, I - I_0\}$ , then there exist  $r_0^w$ ,  $r_1^w$ , and an optimal contract such that when  $\omega < r_0^w$ ,  $R(\omega) = r_1^w \leq 0$ ,  $X(\omega) = 1$ ; when  $\omega \geq r_0^w$ ,  $R(\omega) = r_0^w$ ,  $X(\omega) = 0$ ; moreover, either  $r_1^w = 0$ , or  $r_1^w = r_0^w - W_p$ , where  $r_0^w$  is determined by a binding (IR).*
- (b) *If  $W_p \geq \max\{W, I - I_0\}$ , then there exists an optimal contract such that  $R(\omega) = I - I_0$ ,  $X(\omega) = 0$  for any  $\omega$ .*

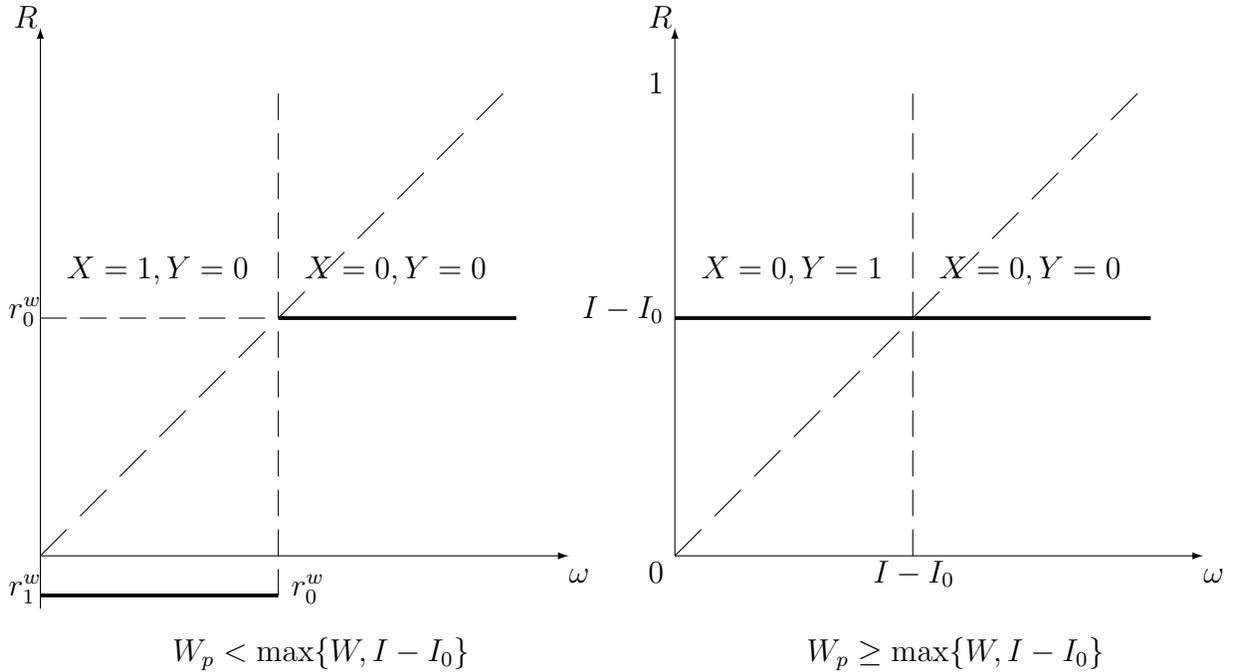


Figure 8: Optimal contract with indivisible assets and surreptitious liquidation.

There are several important remarks regarding Proposition 7 and Figure 8. First, note that when  $W_p < \max\{W, I - I_0\}$ , the optimal contract characterized in Proposition 7 has no surreptitious liquidation, and when  $W_p \geq \max\{W, I - I_0\}$ , it has no liquidation to the incumbent investor. There is no need to have a mixture of the two forms of liquidation because the firm can shift all its liquidation to either of the two investors, whichever is more efficient.

Second, while the optimal contract does not use any surreptitious liquidation when  $W_p < \max\{W, I - I_0\}$ , it is still different from the optimal contract depicted in Figure 7 when secret liquidation is not possible. According to Proposition 7, the firm's cash repayment

$r_1^w$  may be negative, i.e., it may receive reimbursements from the incumbent investor when there is bankruptcy. However, from Definition 1, the cash repayment in a PDC should be nonnegative. This is purely due to the revised incentive constraint ( $IC_p$ ). Since the incentive constraint only applies to deviations that are affordable to the firm, allowing for surreptitious liquidation makes it possible for the firm to exaggerate its cash flow and deviate to another state with a cash repayment higher than  $\omega$ . Therefore, if  $r_0^w$  is small, the firm at state  $\omega < r_0^w$  may still be able to repay  $r_0^w$  by surreptitious liquidation. This possibility entails a reimbursement from the incumbent investor to the firm when  $\omega < r_0^w$ .

Third, there are still cases in which ( $IC_p$ ) is slack in the optimal contract. When  $W_p < \max\{W, I - I_0\}$ , either  $r_1^w = r_0^w - W_p$ , or  $r_1^w = 0$  with  $r_0^w$  determined by a binding (IR). In the former case ( $IC_p$ ) is slack, while in the latter case whether ( $IC_p$ ) binds depends on  $I - I_0$ . Therefore, even if we allow for surreptitious liquidation, the firm still faces some liquidity constraint, because it is impossible for the firm to liquidate as much as it wants without any cost. Hence, the incentive constraint should still be specified consistently with the firm's limited liability, and moreover, the misspecified incentive constraint in the previous part of this section cannot be used as a shortcut.

Our final observation is that allowing for surreptitious liquidation may sometimes reduce the firm's expected utility. This result is formally stated in Proposition 8.

**Proposition 8.** *Suppose that  $n = 1$ ,  $c$  is large, and surreptitious liquidation is allowed.*

- (a) *If  $W_p < \max\{W, I - I_0\}$ , then the firm is weakly worse off compared to the model without surreptitious liquidation; moreover, if  $I - I_0 < W$ , then the firm is strictly worse off.*
- (b) *If  $W_p \geq I - I_0 \geq W$ , then the firm is weakly better off compared to the model without surreptitious liquidation; moreover, if either  $I - I_0 > W$  or  $W_p > W$ , then the firm is strictly better off.*

On the one hand, allowing for surreptitious liquidation relaxes the firm's liquidity constraint, so it could be beneficial to the firm when such liquidation is not too costly, i.e., when  $W_p$  is large. On the other hand, since the incentive constraint interacts with the firm's limited liability, surreptitious liquidation enables the firm to make a cash repayment higher than its realized cash flow. That is, it enlarges the scope to which the incentive constraint applies. Therefore, the revised incentive constraint ( $IC_p$ ) is stronger than (IC). As a result, when  $W_p$  is small, the firm may receive reimbursements in the optimal contract, which implies that the optimal contract in the model without surreptitious liquidation is ruled out by ( $IC_p$ ). In this case, the firm is strictly worse off.

**Related literature.** Our paper extends the theory of financial contracting with adverse selection, with a special focus on the tradeoff between partial liquidation and wholesale transfers.<sup>8</sup> The literature starting from Townsend (1979) and then developed by Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990), and Faure-Grimaud (2000) proved that the optimal contract with costless partial liquidation resembles a debt. Border and Sobel (1987) and Mookherjee and Png (1989) showed that debts may be Pareto dominated when random auditing is allowed. In Krasa and Villamil (2000), whether the optimal contract is stochastic and partial liquidation is performed depends on the informed party’s commitment power. However, in none of these papers costly partial liquidation and wholesale transfers can coexist in the optimal contract. This is in sharp contrast to our results.

The paper perhaps closest to the one presented here is by Lacker (2001). In that paper, the author provided a necessary and sufficient condition for a debt contract to be optimal, and also discussed indivisible assets with stochastic liquidation. That paper also assumed that the contracting parties cannot credibly pre-commit to randomize their future actions, while we make a stronger assumption that allows the firm to commit to introduce the third party, but possibly at a cost. Our paper is distinct from Lacker (2001) in several aspects. The key difference is that in our model the third party does something more than a randomization device: he may also inspect the firm’s cash repayment, which provides the basis of state-contingent partial liquidation. In this sense, our model builds a bridge between several different approaches in the field of financial contracting, such as the CSV model (Townsend 1979; Gale and Hellwig 1985) and the Hart-Moore model (Hart and Moore 1998). Moreover, the main question addressed in Lacker (2001) is “When would a debt contract be optimal”, while in our paper the question is “What is an optimal contract”. Finally, in our model, assets can be partially divisible, and can also be surreptitiously liquidated to a third party; these topics are not covered in Lacker (2001). There are also other papers adopting the assumption of indivisible collateral, but they did not focus on how the indivisibility affects the structure of the optimal contract (e.g., Yao and Zhang 2005; Gorton and Ordonez 2014).

Our paper is further related to the literature on optimal bankruptcy design (e.g., Aghion, Hart, and Moore 1992; Berkovitch and Israel 1999; Bris, Welch, and Zhu 2006; von Thadden, Berglöf, and Roland 2010; Gennaioli and Rossi 2013). When facing financial distress, the firm can either go through the bankruptcy procedure and have its assets liquidated by the decision of the court or reach an agreement with the investor that includes a transfer of asset ownership. Our paper provides a new perspective to study the firm’s tradeoff between

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8. There is another stream of literature that studies the moral hazard problem due to the separation between ownership and control. See, e.g., Innes (1990) and more generally Tirole (2010).

going through the bankruptcy procedure and reaching an out-of-court settlement, and how the firm's choice varies with the realized states.

## 6 Conclusion

In this paper, we present a model of financial contracting where the firm's realized cash flow is its private information, and thus entails an ex-post adverse selection problem. We show that an optimal contract may have a piecewise structure when collateral is indivisible and partial liquidation is costly. Moreover, partial liquidation and wholesale transfers of collateral may coexist in the optimal PDC. This feature is in contrast to the existing literature which emphasizes the optimality of standard debt.<sup>9</sup> It is also consistent with observed behavior in the real estate market, where borrowers frequently walk away from mortgage contracts without going through bankruptcy or repaying all their cash. We also discuss how the structure of the optimal PDC varies with the liquidation cost and the firm's actual liquidity shortage. In addition, our model can be extended to allow for secret ex-post liquidation, which is shown to sometimes reduce the firm's expected payoff.

This paper can be viewed as a contribution to the theory of corporate restructuring. In practice, the firm faces the tradeoff between going through bankruptcy procedure and reaching an agreement directly with the investor. Our model provides a synthesized framework explaining the coexistence of these two alternatives. Also, this paper is related to the literature on optimal security design. In the optimal PDC, the firm will take two different actions, depending on the realized performance of its project. This feature is similar to the optimal security derived in Schmidt (2003) which allows the holder to take a state-dependent action. We believe that our approach of making the different costs of resolving ex-post asymmetric information in contracting frameworks explicit can have a wide range of applications in other related fields, like the theory of auditing and optimal taxation.

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9. For an exception, see Stenzel (2018).

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# A Appendix

## A.1 Proof of Proposition 1

Let  $\Gamma$  be a PDC. By definition,  $\Gamma$  satisfies (LL) and (FC). To prove (IC), note that the monotonicity of  $\Phi(\omega)$  ensures that the firm does not want to understate its cash flow, therefore we only need to rule out the firm's incentive to exaggerate.

Suppose that  $\omega_1 < \omega_2$ , and  $\Phi(\omega_1) > \Phi(\omega_2)$ . If  $\omega_2 \in [r_j^w, r_{j-1}^p)$  for some  $j = 1, 2, \dots, n$ , or  $\omega_2 \geq r_0^w$  for  $j = 0$ , then  $\omega_1 < r_j^w$ , otherwise  $\Phi(\omega_1) = \Phi(\omega_2)$ . However,  $\omega_1 < r_j^w$  implies that

$$I_0 + B - I + \omega_1 < I_0 + B - I + r_j^w = R(\omega_2).$$

If  $\omega_2 \in [r_j^p, r_j^w)$  for some  $j = 0, 1, \dots, n$ , then (LL) binds at  $\omega_2$ , meaning that a type- $\omega_1$  firm cannot misreport  $\omega_2$  due to its inability to repay  $R(\omega_2)$ . Hence,  $\Gamma$  satisfies (IC).

## A.2 Proof of Proposition 2

Let  $\Gamma$  be a contract that satisfies (LL), (FC), and (IC). Our goal is to find a PDC  $\hat{\Gamma}$  that satisfies the conditions listed in the proposition and weakly dominates  $\Gamma$ . We will first construct  $\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}$  and  $r$  in three steps.

**Step 1.** For any  $j = 0, 1, \dots, n$ , if  $X(\omega_j) = x_j$  and  $\psi(\omega_j) = 0$  for some  $\omega_j$ , then

$$r_j^w = R(\omega_j) - (I_0 + B - I).$$

Moreover, when  $x_j < c/(1 - \alpha)V$ ,

$$r_{j-1}^p = \min\{\sup\{\omega : \Phi(\omega) = \Phi(\omega_j)\}, r_{j-1}^w\};$$

when  $x_j \geq c/(1 - \alpha)V$ ,

$$r_{j-1}^p = \min\{\sup\{\omega : \Phi(\omega) = \Phi(\omega_j)\}, r_j^w + \frac{c}{1 - \alpha}, r_{j-1}^w\}.$$

If there does not exist any  $\omega_j$  that satisfies  $X(\omega_j) = x_j$  and  $\psi(\omega_j) = 0$ , then  $r_j^w = r_j^p$  for  $j \geq 1$ , and  $r_j^w = \bar{\omega}$  for  $j = 0$ .

It can be verified that  $\{r_j^w\}_{j=0,1,\dots,n}$  is nonincreasing: Suppose that  $X(\omega_j) = x_j$ ,  $X(\omega_k) = x_k$ , and  $\psi(\omega_j) = \psi(\omega_k) = 0$  for some  $j, k \in \{0, 1, \dots, n\}$  and  $\omega_j, \omega_k$ , with  $j < k$ . From (IC) and  $x_j < x_k$ , we know that  $R(\omega_j) > R(\omega_k)$ , so  $r_j^w > r_k^w$ . Also, for any  $\omega$ ,  $R(\omega) \leq I_0 + B - I + \bar{\omega}$ ,

which means  $r_j^w \leq \bar{\omega}$ . Moreover, note that

$$\sup\{\omega : \Phi(\omega) = \Phi(\omega_j)\} \geq \omega_j \geq R(\omega_j) - (I_0 + B - I) = r_j^w.$$

Therefore, for any  $j = 1, 2, \dots, n$ ,  $r_j^w \leq r_{j-1}^p \leq r_{j-1}^w$ , meaning that  $\{(r_j^w, r_j^p)\}_{j=0,1,\dots,n}$  satisfies condition (a) of Definition 1 except that  $r_n^w$  may be negative.

**Step 2.** For any  $j = 0, 1, \dots, n-1$ , if  $r_j^p < r_j^w$ , then let  $\phi_j$  be the solution for

$$\int_{r_j^p}^{r_j^w} \phi_j dF(\omega) = \int_{r_j^p}^{r_j^w} [\Phi(\omega) - (I_0 + B - I)] dF(\omega).$$

If  $r_j^w = r_j^p$ , then  $\phi_j$  is irrelevant to the contract. Since  $\Phi(\omega)$  is nonincreasing, we know that  $\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}$  satisfies condition (b) of Definition 1.

Now we are ready to construct  $\hat{\Gamma}$ . Let  $\hat{B} = I - I_0$ . For any  $j = 0, 1, \dots, n$ , if  $\omega \in [r_j^w, r_{j-1}^p)$  for  $j \geq 1$ , or  $\omega \geq r_0^w$  for  $j = 0$ , then

$$\hat{R}(\omega) = r_j^w, \quad \hat{X}(\omega) = x_j, \quad \hat{\psi}(\omega) = 0.$$

If  $\omega \in [r_j^p, r_j^w)$ , then

$$\hat{R}(\omega) = \omega, \quad \hat{X}(\omega) = \frac{\phi_j - \omega}{V}, \quad \hat{\psi}(\omega) = 1.$$

Note that by the definition of  $\phi_j$ ,

$$\begin{aligned} \Phi(r_j^w) - (I_0 + B - I) &\leq \phi_j \leq \omega + V \\ \Rightarrow r_j^w + x_j V &\leq \phi_j \leq \omega + V. \end{aligned}$$

Therefore when  $\omega \in [r_j^p, r_j^w)$ ,  $\hat{X}(\omega) \in [0, 1]$ . Finally, if  $r_n^w < 0$ , one can increase  $r_n^w$  and reduce  $r_0^w$  to save the firm's liquidation, thus it is without loss to let  $r_n^w = 0$ . Hence,  $\hat{\Gamma}$  is a PDC by construction. Besides, we also need the following lemma.

**Lemma A.1.** *For any  $j = 0, 1, \dots, n$ , if  $\omega \in [r_j^w, r_{j-1}^p)$  for  $j \geq 1$ , or  $\omega \geq r_0^w$  for  $j = 0$ , then  $\Phi(\omega) = I_0 + B - I + r_j^w + x_j V$ .*

*Proof.* Suppose that  $\omega \in [r_j^w, r_{j-1}^p)$  for some  $j \geq 1$ . Then, there exists some  $\omega_j$  such that  $X(\omega_j) = x_j$  and  $\psi(\omega_j) = 0$ . By our construction,  $r_j^w = R(\omega_j) - (I_0 + B - I) \leq \omega_j$ , meaning that the firm at state  $r_j^w$  and state  $\omega_j$  can mimic each other. From (IC),  $\Phi(r_j^w) = \Phi(\omega_j)$ . Since  $\Phi(\omega)$  is nonincreasing, it must be constant on  $[r_j^w, r_{j-1}^p)$ ; this again comes from our definition of  $r_{j-1}^p$ . Therefore  $\Phi(\omega) = \Phi(\omega_j) = I_0 + B - I + r_j^w + x_j V$ . Our analysis can be extended to the case when  $\omega \geq r_0^w$ , so the lemma is proved.  $\square$

Our remaining task is to prove that  $\Gamma$  is weakly dominated by  $\hat{\Gamma}$ . By Lemma A.1,  $\Phi(\omega) = \hat{\Phi}(\omega) + (I_0 + B - I)$  for any  $\hat{\psi}(\omega) = 0$ . Furthermore, for any  $j = 0, 1, \dots, n-1$ , our construction of  $\phi_j$  implies that  $E_\omega[\Phi(\omega)|[r_j^p, r_j^w]] = E_\omega[\hat{\Phi}(\omega) + I_0 + B - I|[r_j^p, r_j^w]]$ . Therefore,

$$\begin{aligned} E_\omega u_F(\Gamma, \omega, \omega) &= I_0 + B - I + V + E_\omega[\omega - \Phi(\omega)] \\ &= V + E_\omega[\omega - \hat{\Phi}(\omega)] \\ &= E_\omega u_F(\hat{\Gamma}, \omega, \omega). \end{aligned}$$

The last equality comes from the fact that  $\hat{B} = I - I_0$ . In other words, the firm's expected payout is the same in the two contracts, so we only need to show that the investor is weakly better off.

Suppose that  $\omega \in [r_j^w, r_{j-1}^p)$  for some  $j = 1, 2, \dots, n$ , or  $\omega \geq r_0^w$  for  $j = 0$ . Then according to our construction,  $\hat{\psi}(\omega) = 0$ . If  $\psi(\omega) = 0$ , then the firm must liquidate some  $x_{j'}$  with  $j' \geq j + 1$  in the original contract. Therefore,

$$\begin{aligned} R(\omega) &\leq I_0 + B - I + r_j^w = I_0 + B - I + \hat{R}(\omega); \\ R(\omega) + X(\omega)W &= \alpha\Phi(\omega) + (1 - \alpha)R(\omega) \\ &= \alpha[I_0 + B - I + \hat{\Phi}(\omega)] + (1 - \alpha)R(\omega) \\ &\leq I_0 + B - I + \hat{R}(\omega) + \hat{X}(\omega)W. \end{aligned}$$

If  $\psi(\omega) = 1$  and  $x_j < c/(1 - \alpha)V$ , then

$$\begin{aligned} R(\omega) + X(\omega)W - c &\leq \Phi(\omega) - c = I_0 + B - I + r_j^w + x_jV - c \\ &\leq I_0 + B - I + r_j^w + x_jW. \end{aligned}$$

If  $\psi(\omega) = 1$  and  $x_j \geq c/(1 - \alpha)V$ , then

$$\begin{aligned} R(\omega) &\leq I_0 + B - I + \omega \leq I_0 + B - I + r_j^w + \frac{c}{1 - \alpha}, \\ R(\omega) + X(\omega)W - c &= \alpha\Phi(\omega) + (1 - \alpha)R(\omega) - c \\ &\leq I_0 + B - I + r_j^w + x_jW. \end{aligned}$$

From the three cases above, it can be concluded that for any  $\omega$  such that  $\hat{\psi}(\omega) = 0$ ,

$$R(\omega) + X(\omega)W - \psi(\omega)c \leq I_0 + B - I + \hat{R}(\omega) + \hat{X}(\omega)W. \quad (9)$$

Suppose that  $\omega \in [r_j^p, r_j^w)$  for some  $j = 0, 1, \dots, n-1$ . Then  $\hat{\psi}(\omega) = 1$ . If  $\psi(\omega) =$

0, then the firm must liquidate some  $x_{j'}$  with  $j' \geq j + 1$  in the original contract, thus,  $R(\omega) = I_0 + B - I + r_{j'}^w$ , and  $X(\omega) = x_{j'} \geq c/(1 - \alpha)V$ . However, Lemma A.1 and the monotonicity of  $\Phi(\omega)$  imply that  $\Phi(\omega) \leq I_0 + B - I + r_{j+1}^w + x_{j+1}V$ . Therefore, we must have  $\Phi(\omega) = I_0 + B - I + r_{j+1}^w + x_{j+1}V$ , and  $\omega \geq r_{j+1}^w + c/(1 - \alpha)$ . Thus,

$$\begin{aligned} \hat{R}(\omega) &= \omega, \quad \hat{X}(\omega) = \frac{\phi_j - \omega}{V}; \\ R(\omega) + X(\omega)W &= \alpha\Phi(\omega) + (1 - \alpha)R(\omega) \\ &\leq \alpha\Phi(\omega) + (1 - \alpha)(I_0 + B - I + r_{j+1}^w) \\ &\leq \alpha\Phi(\omega) + (1 - \alpha)(I_0 + B - I + \omega - \frac{c}{1 - \alpha}) \\ &= \alpha\Phi(\omega) + (1 - \alpha)[I_0 + B - I + \hat{R}(\omega)] - c. \end{aligned} \quad (10)$$

If  $\psi(\omega) = 1$ , then

$$\begin{aligned} R(\omega) + X(\omega)W - c &= \alpha\Phi(\omega) + (1 - \alpha)R(\omega) - c \\ &\leq \alpha\Phi(\omega) + (1 - \alpha)[I_0 + B - I + \hat{R}(\omega)] - c. \end{aligned} \quad (11)$$

Therefore, from (10)–(11),

$$\begin{aligned} E_\omega[R(\omega) + X(\omega)W - \psi(\omega)c | \hat{\psi}(\omega) = 1] \\ &\leq E_\omega[\alpha\Phi(\omega) + (1 - \alpha)(I_0 + B - I + \hat{R}(\omega)) - c | \hat{\psi}(\omega) = 1] \\ &= E_\omega[I_0 + B - I + \alpha\hat{\Phi}(\omega) + (1 - \alpha)\hat{R}(\omega) - c | \hat{\psi}(\omega) = 1] \\ &= E_\omega[I_0 + B - I + \hat{R}(\omega) + \hat{X}(\omega)W - c | \hat{\psi}(\omega) = 1]. \end{aligned}$$

This inequality, together with (9), implies that

$$\begin{aligned} E_\omega u_I(\Gamma, \omega, \omega) &= -B + E_\omega[R(\omega) + X(\omega)W - \psi(\omega)c] \\ &\leq -\hat{B} + E_\omega[\hat{R}(\omega) + \hat{X}(\omega)W - \hat{\psi}(\omega)c] \\ &= E_\omega u_I(\hat{\Gamma}, \omega, \omega). \end{aligned}$$

Hence,  $\Gamma$  is weakly dominated by  $\hat{\Gamma}$ , and our proof of the proposition is completed.

### A.3 Proof of Proposition 3

By standard arguments, if  $\{(r_j^w, r_j^p, \phi_j)\}_{j=0,1,\dots,n}$  minimize the firm's expected payout subject to the investor's participation constraint, then there exists a constant  $\lambda$  such that they also

minimize the following Lagrangian:

$$L = E_\omega[R(\omega) + X(\omega)V] + \lambda\{I - I_0 - E_\omega[R(\omega) + X(\omega)W - \psi(\omega)c]\} \quad (12)$$

subject to all the constraints listed in the reformulated problem. Moreover, if  $\lambda \leq 1$ ,  $L$  becomes a nondecreasing function of  $E_\omega R(\omega)$  and  $E_\omega X(\omega)$ , meaning that  $L$  is minimized when  $E_\omega R(\omega) = E_\omega X(\omega) = 0$ , a contradiction to (IR). Hence, we must have  $\lambda > 1$  and a binding (IR) in any optimal PDC. Moreover, we have the following Lemma.

**Lemma A.2.**  $\lambda$  is nondecreasing in  $I - I_0$ .

*Proof.* For any  $j = 1, 2$ , suppose that  $\Gamma_j$  is the optimal PDC when  $I - I_0 = I_j$ . Then for any  $\Gamma_j$ , let  $R_j = E_\omega R(\omega)$  and  $X_j = E_\omega X(\omega)$ . By optimality, we have

$$\begin{aligned} R_1 + X_1V &\leq R_2 + X_2V + \lambda_1(I_1 - I_2); \\ R_2 + X_2V &\leq R_1 + X_1V + \lambda_2(I_2 - I_1). \end{aligned}$$

These two inequalities jointly imply that

$$(\lambda_1 - \lambda_2)(I_1 - I_2) \geq 0.$$

Therefore, if  $I_1 > I_2$ , then  $\lambda_1 \geq \lambda_2$ . □

**Proof of part (a).** Suppose that  $\Gamma$  is an optimal PDC with wholesale transfers. Let  $x_j$  be the smallest quantity of assets transferred as a whole, then  $j \geq 1$ , and  $r_j^w < r_{j-1}^p$ .

If  $r_{j-1}^p < r_0^w$ , then there is only partial liquidation when  $\omega \in [r_{j-1}^p, r_0^w)$ . If we denote by  $L(a, b)$  the value of the Lagrangian conditional on the interval  $(a, b)$ , then  $L(r_j^w, \bar{\omega})$  is given by

$$\begin{aligned} L(r_j^w, \bar{\omega}) &= (r_j^w + x_jV)[F(r_{j-1}^p) - F(r_j^w)] + \phi_0[F(r_0^w) - F(r_{j-1}^p)] + r_0^w[1 - F(r_0^w)] \\ &\quad - \lambda\{(r_j^w + x_jW)[F(r_{j-1}^p) - F(r_j^w)] + \int_{r_{j-1}^p}^{r_0^w} [\omega + \alpha(\phi_0 - \omega) - c]dF(\omega) \\ &\quad + r_0^w[1 - F(r_0^w)]\}. \end{aligned}$$

First-order conditions are given by

$$\begin{aligned}\frac{\partial L}{\partial r_0^w} &= [(1 - \alpha\lambda)(\phi_0 - r_0^w) + c\lambda]f(r_0^w) - (\lambda - 1)[1 - F(r_0^w)]; \\ \frac{\partial L}{\partial r_{j-1}^p} &= [(1 - \alpha\lambda)(r_j^w + x_jV - \phi_0) + \lambda(1 - \alpha)(r_{j-1}^p - r_j^w) - c\lambda]f(r_{j-1}^p); \\ \frac{\partial L}{\partial \phi_0} &= (1 - \alpha\lambda)[F(r_0^w) - F(r_{j-1}^p)].\end{aligned}$$

When  $1 - \alpha\lambda > 0$ ,  $\partial L/\partial \phi_0 > 0$ , meaning that  $\phi_0 = r_0^w$ . Moreover,  $r_{j-1}^p < r_0^w$  implies that  $\partial L/\partial r_0^w \leq 0$ , which means

$$\lambda[1 - c\frac{f(r_0^w)}{1 - F(r_0^w)}] \geq 1.$$

$c$  must be sufficiently small so that the left-hand side is positive. Actually there are two necessary conditions:

$$c < \frac{1 - F(r_0^w)}{f(r_0^w)} \leq \underline{f}, \quad (13)$$

$$\lambda \geq \frac{1 - F(r_0^w)}{1 - F(r_0^w) - cf(r_0^w)} \geq \frac{1}{1 - c\underline{f}} = \lambda_1. \quad (14)$$

If  $r_{j-1}^p = r_0^w$ , then

$$\begin{aligned}L(r_j^w, \bar{\omega}) &= (r_j^w + x_jV)[F(r_0^w) - F(r_j^w)] + r_0^w[1 - F(r_0^w)] \\ &\quad - \lambda\{(r_j^w + x_jW)[F(r_0^w) - F(r_j^w)] + r_0^w[1 - F(r_0^w)]\}.\end{aligned}$$

The first-order condition is given by

$$\frac{\partial L}{\partial r_0^w} = [(\lambda - 1)(r_0^w - r_j^w) + (1 - \alpha\lambda)x_jV]f(r_0^w) - (\lambda - 1)[1 - F(r_0^w)].$$

Similarly, we have  $\partial L/\partial r_0^w \leq 0$ , which means

$$r_0^w - r_j^w + \frac{1 - \alpha\lambda}{\lambda - 1}x_jV \leq \frac{1 - F(r_0^w)}{f(r_0^w)} \leq \underline{f}.$$

A necessary condition is

$$\lambda \geq \frac{1 + x_jV\underline{f}}{1 + \alpha x_jV\underline{f}} = \lambda_2. \quad (15)$$

From (13) and (15), we know that  $r_j^w < r_{j-1}^p$  implies either  $\lambda \geq \lambda_1$ , or  $\lambda \geq \lambda_2$ . We also

know that  $c < 1/\underline{f}$  is necessary for  $r_{j-1}^p < r_0^w$ . Put differently,  $\lambda < \min\{\lambda_1, \lambda_2\}$  is sufficient for  $r_j^w = r_{j-1}^p$ , and  $c \geq 1/\underline{f}$  is sufficient for  $r_{j-1}^p = r_0^w$ . Hence, from Lemma A.2, there exists a cutoff  $\underline{I}$  such that  $I - I_0 < \underline{I}$  is sufficient for that  $\Gamma$  has only partial liquidation. Moreover, when  $c \geq 1/\underline{f}$ ,  $\Gamma$  cannot have partial liquidation, so it is without loss to make  $\underline{I} = 0$  when  $c \geq 1/\underline{f} = \underline{c}$ .

**Proof of part (b).** Suppose that  $\Gamma$  is an optimal PDC with partial liquidation, and  $c \geq (1 - \alpha)V$ . Let  $r_j^p$  be the lower bound of the states that induce partial liquidation, then  $r_j^p < r_j^w$ .

If  $r_j^p > 0$ , then  $j \leq n - 1$  and  $r_{j+1}^w \geq 0$ . The Lagrangian conditional on  $(r_{j+1}^w, r_{j-1}^p)$  is given by

$$\begin{aligned} L(r_{j+1}^w, r_{j-1}^p) &= (r_{j+1}^w + x_{j+1}V)[F(r_j^p) - F(r_{j+1}^w)] + \phi_j[F(r_j^w) - F(r_j^p)] \\ &\quad + (r_j^w + x_jV)[F(r_{j-1}^p) - F(r_j^w)] - \lambda\{(r_{j+1}^w + x_{j+1}W)[F(r_j^p) - F(r_{j+1}^w)] \\ &\quad + \int_{r_j^p}^{r_j^w} [\omega + \alpha(\phi_0 - \omega) - c]dF(\omega) + (r_j^w + x_jW)[F(r_{j-1}^p) - F(r_j^w)]\}. \end{aligned}$$

The first-order conditions are given by

$$\frac{\partial L}{\partial r_j^p} = [(1 - \alpha\lambda)(r_{j+1}^w + x_{j+1}V - \phi_j) + \lambda(1 - \alpha)(r_j^p - r_{j+1}^w) - c\lambda]f(r_j^p); \quad (16)$$

$$\frac{\partial L}{\partial \phi_j} = (1 - \alpha\lambda)[F(r_j^w) - F(r_j^p)]. \quad (17)$$

When  $1 - \alpha\lambda < 0$ ,  $\partial L/\partial \phi_j < 0$ , meaning that  $\phi_j = r_{j+1}^w + x_{j+1}V$ . Moreover,  $r_j^p < r_j^w$  implies that  $\partial L/\partial r_j^p \geq 0$ , which means

$$\lambda(1 - \alpha)(r_j^p - r_{j+1}^w - \frac{c}{1 - \alpha})f(r_j^p) \geq 0.$$

A necessary condition is

$$\begin{aligned} c &\leq (1 - \alpha)(r_j^p - r_{j+1}^w) < (1 - \alpha)(r_j^w - r_{j+1}^w) \leq (1 - \alpha)(x_{j+1} - x_j)V \\ &\leq (1 - \alpha)V. \end{aligned} \quad (18)$$

However, this contradicts our assumption that  $c \geq (1 - \alpha)V$ .

If  $r_j^p = 0$ , then the Lagrangian conditional on  $[r_j^p, r_{j-1}^p)$  is given by

$$L(r_j^p, r_{j-1}^p) = \phi_j[F(r_j^w) - F(r_j^p)] + (r_j^w + x_j V)[F(r_{j-1}^p) - F(r_j^w)] \\ + \int_{r_j^p}^{r_j^w} [\omega + \alpha(\phi_0 - \omega) - c]dF(\omega) + (r_j^w + x_j W)[F(r_{j-1}^p) - F(r_j^w)].$$

The first-order conditions are given by

$$\frac{\partial L}{\partial r_j^w} = [(1 - \alpha\lambda)(\phi_j - r_j^w - x_j V) + c\lambda]f(r_j^w) - (\lambda - 1)[F(r_{j-1}^p) - F(r_j^w)]; \\ \frac{\partial L}{\partial \phi_j} = (1 - \alpha\lambda)F(r_j^w).$$

Similarly, when  $1 - \alpha\lambda > 0$ ,  $\phi_j = r_j^w + x_j V$ .  $r_j^p < r_j^w$  implies that  $\partial L/\partial r_j^w \leq 0$ , which means

$$\lambda[1 - c \frac{f(r_j^w)}{F(r_{j-1}^p) - F(r_j^w)}] \geq 1.$$

Two necessary conditions are

$$c < \frac{F(r_{j-1}^p) - F(r_j^w)}{f(r_j^w)} \leq \frac{1}{\underline{f}}, \quad (19)$$

$$\lambda \geq \frac{F(r_{j-1}^p) - F(r_j^w)}{F(r_{j-1}^p) - F(r_j^w) - cf(r_j^w)} \geq \frac{1}{1 - c\underline{f}} = \lambda_1. \quad (20)$$

Hence, there exists a cutoff  $\bar{I}$  such that  $I - I_0 < \bar{I}$  is sufficient for that  $\Gamma$  has only wholesale transfers. Moreover, since all the analysis in this part is conducted under the assumption that  $c \geq (1 - \alpha)V$ , it is without loss to make  $\bar{I} = 0$  when  $c < (1 - \alpha)V = \bar{c}$ .

#### A.4 Proof of Proposition 4

If  $\Phi(r_{j-1}^w) < \Phi(r_j^w) < \Phi(r_{j+1}^w)$ , then  $r_{j+1}^w < r_j^w < r_{j-1}^w$ . Following the proof of Proposition 3, the first-order derivative of  $L$  with respect to  $r_j^w$  is given by

$$\frac{\partial L}{\partial r_j^w} = [(\lambda - 1)(r_j^w - r_{j+1}^w) + (1 - \alpha\lambda)(x_{j+1} - x_j)V]f(r_j^w) - (\lambda - 1)[F(r_{j-1}^w) - F(r_j^w)].$$

Since (IC) is slack at  $r_j^w$  and  $r_{j-1}^w$ , we must have  $\partial L/\partial r_j^w = 0$ . Simplifying this equality, and letting  $\mu = (\alpha\lambda - 1)/(\lambda - 1)$  will give us (7).

## A.5 Proof of Proposition 5

If  $r_j^p \in (r_{j+1}^w, r_j^w)$ , then the first-order derivatives of  $L$  with respect to  $r_j^p$  and  $\phi_j$  are given by (16) and (17). By (17), when  $1 - \alpha\lambda < 0$ ,  $\phi_j = r_{j+1}^w + x_{j+1}V$ ; when  $1 - \alpha\lambda \geq 0$ ,  $\phi_j \leq r_{j+1}^w + x_{j+1}V$ . In both cases, we have

$$\frac{\partial L}{\partial r_j^p} \geq \lambda(1 - \alpha)(r_j^p - r_{j+1}^w - \frac{c}{1 - \alpha})f(r_j^p).$$

Since  $\frac{\partial L}{\partial r_j^p} \leq 0$ , we immediately have (8). A similar analysis will go through if  $r_j^p = r_j^w > r_{j+1}^w$ .

## A.6 Proof of Proposition 6

$\Gamma$  has only partial liquidation implies that  $r_0^p = 0$ . The first-order derivative of  $L$  is given by

$$\frac{\partial L}{\partial r_0^w} = c\lambda f(r_0^w) - (\lambda - 1)[1 - F(r_0^w)].$$

That means

$$\frac{1 - F(r_0^w)}{f(r_0^w)} = \frac{\lambda c}{\lambda - 1} > c.$$

## A.7 Proof of Proposition 7

Since it is too costly to use partial liquidation, in the optimal contract the firm only has three possible ways to deal with its assets:  $X = Y = 0$  (no liquidation),  $X = 0, Y = 1$  (surreptitious liquidation), or  $X = 1, Y = 0$  (full liquidation). By incentive-compatibility, there are at most three distinct cash repayment choices in the contract that correspond to the three actions on the assets. Let  $\omega_{xy}$  be a state with  $X = x \in \{0, 1\}$  and  $Y = y \in \{0, 1\}$ , and  $r_{xy}$  be the cash repayment specified for state  $\omega_{xy}$ . We proceed the proof by showing a sequence of lemmas.

**Lemma A.3.**  $r_{10} < r_{01} = r_{00}$ .

*Proof.* Suppose that  $r_{10} \geq r_{01}$ . The firm at state  $\omega_{10}$  can misreport state  $\omega_{01}$  without any form of liquidation, which is a violation of  $(IC_p)$ . Thus,  $r_{10} < r_{01}$ . Suppose that  $r_{01} < r_{00}$ . By a similar logic, the firm at state  $\omega_{00}$  can misreport state  $\omega_{01}$  without surreptitious liquidation, which violates  $(IC_p)$ . Suppose that  $r_{01} > r_{00}$ . The firm at state  $\omega_{01}$  can mimic state  $\omega_{00}$  with surreptitious liquidation, but it would pay less cash by reporting  $\omega_{00}$ . This also contradicts  $(IC_p)$ , so  $r_{01} = r_{00}$ . The lemma is thereby proved.  $\square$

From Lemma A.3 and (LL<sub>p</sub>), we must have  $r_{10} \leq 0$ . Since any state can deviate to repay  $r_{10}$  with full liquidation, from (IC<sub>p</sub>) we also have  $r_{00} \leq r_{10} + V$ . Finally, since the firm's payout function should be nonincreasing in  $\omega$ , we can prove that any contract that satisfies (LL<sub>p</sub>), (FC<sub>p</sub>) and (IC<sub>p</sub>) is weakly dominated by a contract with a cutoff structure: There exists  $r_{00}^w$  such that when  $\omega < r_{00}^w$ ,  $R(\omega) = r_{10}$ ,  $X(\omega) = 1$ ; when  $\omega \geq r_{00}^w$ ,  $R(\omega) = r_{00}$ ,  $X(\omega) = 0$ . Note that the firm would surreptitiously liquidate its assets only when  $\omega \in [r_{00}^w, r_{00})$ . In what follows, we will focus on the set of contracts with such cutoff structure.

**Lemma A.4.** *There exists an optimal contract with either  $r_{00}^w = 0$  or  $r_{00}^w = r_{00}$ .*

*Proof.* Let  $\Gamma$  be a contract that satisfies (LL<sub>p</sub>), (FC<sub>p</sub>) and (IC<sub>p</sub>). Suppose that  $r_{00}^w > 0$  and  $r_{00}^w < r_{00}$ , i.e.,  $\Gamma$  has both surreptitious liquidation and liquidation by the investor. By (IC<sub>p</sub>),  $r_{00} \leq r_{10} + W_p$ , otherwise the firm at state  $\omega \in [r_{00}^w, r_{00})$  would have its assets fully liquidated by the investor. However, the firm at state  $\omega_{10}$  can deviate to repay  $r_{00}$  by surreptitious liquidation. Again by (IC<sub>p</sub>), we have  $r_{00} = r_{10} + W_p$ .

Given this equality, we can improve the investor's payoff from  $\Gamma$  based on the relationship between  $W_p$  and  $W$ . If  $W_p \leq W$ , the firm can liquidate its assets to the investor and induce cash repayment  $r_{10}$  on  $[r_{00}^w, r_{00})$ . Similarly, if  $W_p > W$ , the firm can liquidate its assets to the third party and repay  $r_{00}$  to the investor on  $[0, r_{00}^w)$ . In both cases, (IC<sub>p</sub>) is satisfied by the revised contract, the firm's utility is unchanged, and the investor is weakly better off. Moreover, in both cases the revised contract has either  $r_{00}^w = 0$  or  $r_{00}^w = r_{00}$ , so the lemma is proved.  $\square$

If there exists an optimal contract with  $r_{00}^w = r_{00}$ , then by (IC<sub>p</sub>),  $r_{10} + W_p \leq r_{00}$ . However, when  $r_{10} < 0$ , the firm can always increase  $r_{10}$  and reduce  $r_{00}$  to minimize its probability of being liquidated. Therefore, if  $r_{10} < 0$ , then we must have  $r_{10} + W_p = r_{00}$ . In this case the contract is purely determined by  $r_{10}$ , which is in turn determined by a binding (IR), i.e.,

$$(W + r_{10})F(r_{00}) + r_{00}[1 - F(r_{00})] = I - I_0.$$

Hence, the firm's expected utility is given by

$$\begin{aligned} E_\omega u_F(\Gamma, \omega, \omega) &= \int_0^{r_{00}} (\omega - r_{10})dF(\omega) + \int_{r_{00}}^{\bar{\omega}} (\omega + V - r_{00})dF(\omega) \\ &= \int_0^{\bar{\omega}} \omega dF(\omega) - r_{10}F(r_{00}) + (V - r_{00})[1 - F(r_{00})] \\ &= \int_0^{\bar{\omega}} \omega dF(\omega) + V - (I - I_0) - (V - W)F(r_{00}). \end{aligned} \quad (21)$$

If there exists an optimal contract with  $r_{00}^w = 0$ , then it must have  $r_{00} = I - I_0$ . That is, the firm repays a fixed amount of cash irrespective of  $\omega$ . In this case, the firm's expected utility is given by

$$\begin{aligned} E_\omega u_F(\Gamma, \omega, \omega) &= \int_0^{I-I_0} [\omega + W_p - (I - I_0)] dF(\omega) + \int_{I-I_0}^{\bar{\omega}} [\omega + V - (I - I_0)] dF(\omega) \\ &= \int_0^{\bar{\omega}} \omega dF(\omega) + V - (I - I_0) - (V - W_p)F(I - I_0). \end{aligned} \quad (22)$$

Comparing (21) and (22), we can conclude that there exists an optimal contract with  $r_{00}^w = r_{00}$  if  $W_p \leq W$ , or  $W < W_p \leq I - I_0$ . Similarly, there exists an optimal contract with  $r_{00}^w = 0$  if  $W_p \geq W$ , and  $W_p \geq I - I_0$ . After changing some notations we can get the results stated in the proposition.

## A.8 Proof of Proposition 8

When surreptitious liquidation is not allowed, by Proposition 2 and Proposition 3, the optimal PDC has the following form: When  $\omega < r_0^w$ ,  $R(\omega) = 0$ ,  $X(\omega) = 1$ ; when  $\omega \geq r_0^w$ ,  $R(\omega) = r_0^w$ ,  $X(\omega) = 0$ , where  $r_0^w$  is determined by a binding (IR). That is,

$$WF(r_0^w) + r_0^w[1 - F(r_0^w)] = I - I_0. \quad (23)$$

Therefore the firm's expected utility is given by

$$\begin{aligned} E_\omega u_F(\Gamma, \omega, \omega) &= \int_0^{r_0^w} \omega dF(\omega) + \int_{r_0^w}^{\bar{\omega}} (\omega + V - r_0^w) dF(\omega) \\ &= \int_0^{\bar{\omega}} \omega dF(\omega) + V - (I - I_0) - (V - W)F(r_0^w). \end{aligned} \quad (24)$$

When surreptitious liquidation is allowed, and  $W_p < \max\{W, I - I_0\}$ , there is no surreptitious liquidation in the optimal contract, so the first statement of (a) is straightforward. To prove the second statement of (a), note that the optimal contract characterized in Proposition 7 also has a binding (IR). That is,

$$(W + r_{10})F(r_{00}) + r_{00}[1 - F(r_{00})] = I - I_0.$$

Here we use  $r_{10}$  to replace  $r_1^w$ , and  $r_{00}$  to replace  $r_0^w$  to avoid confusion. Therefore,  $r_{10} < 0$  if and only if  $I - I_0 < W$ , meaning that the firm is strictly worse off.

When surreptitious liquidation is allowed, and  $W_p \geq \max\{W, I - I_0\}$ , the firm's expected

utility is given by (22). If further we have  $I - I_0 \geq W$ , then by (23),  $r_0^w \geq I - I_0$ . We can see that the firm is weakly better off by comparing (22) and (24). Similarly, the firm is strictly better off if we have either  $I - I_0 > W$ , or  $W_p > W$ .