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Heterogeneity, Transfer Progressivity and Business Cycles

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Abstract

This paper studies how transfer progressivity influences aggregate fluctuations when interacted with household heterogeneity. Using a simple static model of the extensive margin labor supply, we analytically characterize how transfer progressivity influences differential labor supply responses to aggregate conditions across heterogeneous households. We then build a quantitative dynamic general equilibrium model with both idiosyncratic and aggregate productivity shocks and show that the model delivers moderately procyclical average labor productivity and a large cyclical volatility of aggregate hours relative to output. Counterfactual exercises indicate that redistributive policies have very different implications for business cycle fluctuations, depending on whether tax progressivity or transfer progressivity is used. Finally, we provide empirical evidence on the heterogeneity of employment responses across the wage distribution, which supports the key mechanism of our model.

Keywords: Progressivity, government transfers, extensive margin labor supply, business cycles, redistributive policies

JEL codes: E32, E24, H31, H53, E21

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1 Introduction

A large literature in recent years has investigated the macroeconomic implications of the progressive nature of tax and transfers that are prevalent in many developed countries.[1] A natural yet relatively unexplored question is how the progressivity of tax and transfers affects aggregate fluctuations (see McKay and Reis, 2016 for an exception). In particular, given that the size of various welfare programs–e.g., cash transfers and food, medical and childcare support to low-income households–in the United States has been steadily growing since the 1970's (Ben-Shalom, Moffitt and Scholz, 2011), it would be timely and relevant to enhance the understanding of how transfer progressivity influences business cycles.[2]

The key question for this paper is how the existence of progressive transfers alter the way aggregate shocks are transmitted to the macroeconomy in the presence of heterogeneous households. We ask whether this channel is important for the dynamics of macroeconomic aggregates in terms of not only volatility (McKay and Reis, 2016), but also comovement with output. We begin with a simple tractable model of the extensive margin labor supply showing analytically that more progressive transfers induce the labor supply of low productivity households to be disproportionately more responsive to aggregate conditions.[3] We show that this heterogeneity of labor supply responses leads to less procyclical average labor productivity through changes in the composition of workers (Bils, 1985) as well as a greater volatility of aggregate hours worked. We then construct a dynamic general equilibrium incomplete-markets model with both idiosyncratic and aggregate shocks to quantitatively evaluate the role of this channel in shaping aggregate fluctuations. We find that our baseline model delivers moderately procyclical average labor productivity and a large cyclical volatility of aggregate hours relative to output, both of which are known to be difficult to explain by standard real business cycle models. Using impulse response functions at the disaggregated level, we show that the key to our quantitative results is the heterogeneity of labor supply responses at the micro level in the presence of progressive transfers, in line with both our analytical results from the simple

[1] Research questions range from normative ones such as optimal progressivity to positive ones such as the role of progressivity in explaining macro outcomes. For example, see Conesa, Kitao and Krueger (2009), Heathcote, Storeletten and Violante (2014), Bick and Fuchs-Schündeln (2018), and Guner, Kaygusuz and Ventura (2019) among others.

[2] Interestingly, tax progressivity has no clear trend during the post war period (Ferriere and Navarro, 2018).

[3] Because about three quarters of fluctuations of total hours are along the extensive margin (e.g., Hansen, 1985; King and Rebelo, 1999), our model focusing on business cycle fluctuations features the extensive margin of labor supply.

model and the microeconomic evidence we document using the Panel Study of Income Dynamics (PSID).

Our simple tractable model of the extensive margin labor supply builds on Doepke and Tertilt (2016). In this static model, there are two productivity types and the continuous distribution of wealth featuring a large fraction of households with low wealth. In this environment, we consider higher transfer progressivity as a variation of the transfer schedule such that low productivity households receive more than the high type while holding the average transfers constant. We show that higher transfer progressivity induces the labor supply of low productivity households to respond more strongly to a change in aggregate conditions whereas it weakens the response of high productivity households. Consequently, higher transfer progressivity leads to a lower cyclicality of average labor productivity, and potentially a greater volatility of aggregate hours driven by low productivity households. Although the analytical results in our simple static model are useful for illustrating the mechanism, they are derived in a somewhat restricted static environment without considering risk. As the existence of transfers could also affect the distribution of wealth in a dynamic setting where risk-averse agents face risk, it is a quantitative question whether this mechanism would be relevant in a more realistic model environment.

Therefore, we next consider a quantitative, dynamic general equilibrium model, based on a standard incomplete markets framework with heterogeneous households who make consumption-savings and extensive-margin labor supply decisions in the presence of both idiosyncratic productivity risk and aggregate risk, a model pioneered by Chang and Kim (2006, 2007).[4] Our model features progressive tax and transfers, captured by two separate parsimonious yet flexible nonlinear functions. We calibrate our model economy to match salient features in the micro-level data, including the degree of progressivity in the welfare programs, as seen in the Survey of Income and Program Participation (SIPP) data. To better understand the underlying mechanisms, we also consider several nested versions of the baseline model that abstracts from the transfers entirely (a model similar to Chang and Kim, 2007), or transfer progressivity (a model similar to Chang, Kim and Schorfheide, 2013), or household heterogeneity (a model similar to Hansen, 1985).

We find that our baseline model features aggregate labor market dynamics that differ considerably

[4] Their model in turn builds on the workhorse incomplete-markets general equilibrium model without aggregate risk, developed by Huggett (1993) and Aiyagari (1994).

from its nested versions, improving standard business cycle statistics on aggregate labor market fluctuations. First, our baseline model, with both heterogeneity and progressive transfers, generates considerably lower correlations of average labor productivity with output (0.55 vs. 0.30 in the data) than all nested versions that generate much higher values.[5] These results point to the importance of the interplay between household heterogeneity and progressive transfers. At the same time, in our baseline model, the cyclical volatility of aggregate hours relative to output is rather high at 0.84 (vs. 0.98 in the data), compared to 0.55 and 0.61 in the nested heterogeneous-agent models. This volatility of hours in our baseline model is very close to the value obtained from its representative-agent counterpart, 0.83, which is often considered to be the upper bound due to the stand-in household's utility function having the lowest curvature in labor supply.

The existence of progressive transfers in our baseline quantitative model plays two major roles underlying its more successful aggregate fluctuations, in contrast to the heterogeneous-agent model without transfers that in fact generates aggregate dynamics rather similar to those from the representative-agent model.[6] The first relates to transfer progressivity, the role of which is already highlighted in our simple static model. We show by the impulse responses at a disaggregated level that low productivity households are more responsive to the fall in the total factor productivity in our baseline model, relative to the model without transfer progressivity. On top of this, the second role arises due to risk and market incompleteness. In the absence of transfers, wealth-poor households who are likely to hit the borrowing constraint in the near future tend to keep working irrespective of aggregate state changes. Our nested model specifications allow us to disentangle these two roles of progressive transfers in our baseline model and show that both channels are quantitatively important in delivering the quantitative success in our baseline model.

We also use the quantitative model to conduct counterfactual exercises. Specifically, we explore how a change in the progressivity of transfers, or of taxes, would affect both the steady state and aggregate fluctuations. Most importantly, we find that increasing transfer progressivity further reduces the correlation of average labor productivity with output and raises the volatility of total

[5] All nested models generate a correlation of average labor productivity with output greater than 0.8.

[6] This finding also suggests that the presence of household heterogeneity per se is not sufficient to alter aggregate dynamics in an incomplete markets framework (e.g., see Krusell and Smith, 1998), thereby explaining why the standard heterogeneous-agent model of Chang and Kim (2007; 2014) is unable to deliver our main quantitative results. Note that Chang and Kim (2007)'s original quantitative results have been corrected in Chang and Kim (2014), following Takahashi (2014)'s comments.

hours, in line with the results from the simple model. On the other hand, we find that higher tax progressivity actually does the opposite, raising the cyclicality of average labor productivity and reducing the volatility of hours. A novel policy implication is that redistributive policies that are meant to be more progressive may have very different consequences for business cycle fluctuations, depending on the policy tool used to achieve it (tax vs. transfers).

Finally, we use micro data from the PSID to empirically explore the heterogeneity of labor supply responses–a key underlying force at work in our model.[7] We document two key empirical findings using two different approaches: the first approach uses individual-level flow data, and the second uses short-run employment rate changes shaped by aggregate factors.[8] First, we find that the individual-level probability of adjusting labor supply along the extensive margin is significantly higher among low-wage workers, and this probability tends to decrease with wages. Second, we document that during the recent recessions, the employment rate has fallen most sharply in the first wage quintile, compared to the other wage quintiles. Although the above two approaches capture labor supply adjustments over different time horizons, shaped by different forcing variables (idiosyncratic vs. aggregate factors), we find robustly that lower-wage workers adjust labor supply along the extensive margin more elastically, supporting our key model mechanism.

There has been great interest in incorporating rich micro-level heterogeneity into macroeconomic models in recent decades. Clearly, it is essential to incorporate household or firm heterogeneity when studying distributional issues within a macroeconomic framework. However, it is less clear whether heterogeneity at the micro level matters for aggregate business cycle dynamics at the macro level. Although extensive studies show the importance of heterogeneity in accounting for macroeconomic aggregates and equilibrium prices in the absence of aggregate risk (e.g., Huggett, 1993; and Heathcote, 2005 among others), the literature that employs a model with aggregate uncertainty has suggested that incorporating micro-level heterogeneity may have only limited impacts on the business cycle fluctuations of macroeconomic aggregates (e.g., Krusell and Smith, 1998; Khan and Thomas, 2008; and Chang and Kim, 2014). Noting that our main result suggests that household heterogeneity at the micro level can be important for the dynamics of macroeconomic variables, our paper is broadly in

[7] There is limited empirical evidence on heterogeneity in labor supply responses at the extensive margin. See Kydland (1984) and Juhn, Murphy and Topel (1991) for earlier evidence.

[8] We use the panel structure of the PSID, which allows us to keep track of the same individuals over time.

line with recent papers such as Krueger, Mitman and Perri (2016) and Ahn, Kaplan, Moll, Winberry and Wolf (2017), both of which find that heterogeneity at the micro level is important in shaping the impact of aggregate shocks on macroeconomic variables. Although the distribution of wealth plays an important role both in these studies and in our study, it is important to note that Krueger et al. (2016) and Ahn et al. (2017) focus on aggregate consumption via savings decisions, whereas our paper focuses on aggregate labor market dynamics via labor supply choices.

Weak correlations of average labor productivity with output and hours, often referred to as the Dunlop–Tarshis observation, are known to be difficult to explain by standard real business cycle models that, instead, generate very strongly procyclical average labor productivity. The literature has suggested various quantitative theoretical mechanisms to dampen strongly positive correlations of average labor productivity with output and hours. The earlier literature relies on the introduction of additional shocks to representative-agent models such as home-production technology shocks (Benhabib, Rogerson and Wright, 1991), government spending shocks (Christiano and Eichenbaum, 1992), and income tax shocks (Braun, 1994). Recently, Takahashi (2018) incorporates uncertainty shocks that play a role of shocks to labor supply into a standard heterogeneous-agent model (Chang and Kim, 2007) to reduce correlation of average labor productivity with hours. Our result is distinct from the existing literature because our mechanism relies on the existence of institutional features leading to hetergeneous responses.

Our quantitative incomplete markets model highlights the role of government transfers in affecting the precautionary behavior of poor households. An earlier paper by Hubbard et al. (1995) shows that social insurance discourages precautionary savings among low-income households. In a similar manner, using an incomplete-markets model without aggregate uncertainty, Yum (2018) finds that government transfers help bring the employment rate of wealth-poor households closer to the data, influencing the precautionary employment motive of wealth-poor households. This in turn is shown to have important implications for the long-run employment effects of labor taxes. Our present results suggest that the existence of progressive transfers in incomplete-markets environments, interacted with household heterogeneity, has important implications for the dynamics of macroeconomic aggregates over the business cycle as well.

The paper is organized as follows. Section 2 presents a simple model of extensive margin labor supply analytic results on the key mechanism of this paper. Section 3 describes the model environment of equilibrium business cycle models, defines equilibrium, and discusses numerical solution methods. In Section 4, we describe how the parameters are calibrated and show the properties of the quantitative models in stationary equilibrium. Section 5 presents the main quantitative results. Section 6 presents empirical evidence on the heterogeneity of labor supply responses using the panel structure of the PSID. Section 7 concludes the paper.

2 A model of the extensive margin labor supply with heterogeneity

In this section, we present a simple model of the extensive margin labor supply to study how progressive transfers could influence aggregate labor market fluctuations. For analytical tractability and clear illustrations, we consider a static environment by taking the distribution of wealth as given in this section.[9] In the following sections for the quantitative analysis, we will relax this assumption and consider a dynamic environment where the distribution of wealth is endogenously determined by households' consumption-savings decisions.

There is a continuum of agents in the unit interval. We assume that there are two types of productivity. That is, individual productivity x_i can be either low or high: $x_i \in \{x_l, x_h\}$. The mass of each type is denoted by π_l and π_h satisfying $\pi_l + \pi_h = 1$. A subscript $i \in \{l, h\}$ denotes the type of the agent throughout this section. Agents are allowed to differ also in their level of asset holdings, a. Because our focus is on the extensive margin, the agent can choose to either work full-time or not at all: $n_i \in \{0, 1\}$.

The decision problem of each type i is given by

$$\max_{c_i \geq 0, n_i \in \{0,1\}} \{\log c_i - bn_i\}$$

subject to

$$c_i \leq zx_in_i + a + T_i,$$

[9] Our analytical framework in this section builds on the theoretical framework in Doepke and Tertilt (2016). Since the focus of the analysis is different, their model is based on two gender types and continuous preference heterogeneity whereas our model is based on two productivity types and continuous asset heterogeneity. Moreover, our results cover not only the labor supply elasticity but also average labor productivity.

where c denotes consumption, n is the employment choice, a is the level of assets, and $b > 0$ captures the disutility of work. We use z to denote aggregate productivity. Finally, we introduce a productivity-dependent public insurance scheme $T_i \geq 0$. We assume that T_l is greater than T_h, implying that it is progressive.[10]

The above maximization problem characterizes the optimal decision for the discrete employment choice. Specifically, comparing the utility conditional on working to not working, the agent chooses to work if

$$\log\left(zx_i + T_i + a\right) - b \geq \log\left(T_i + a\right).$$

Note that this can be equivalently written as

$$b \leq \log\left(\frac{zx_i + T_i + a}{T_i + a}\right) = \log\left(1 + \frac{zx_i}{T_i + a}\right),$$

or

$$a \leq zx_i - T_i$$

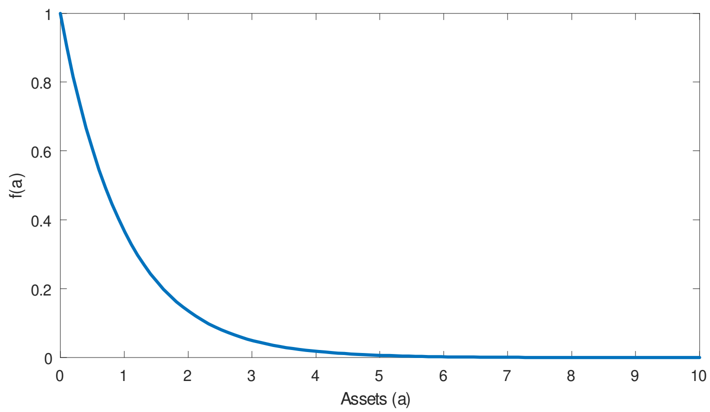
where we assume that the constant b is equal to $\log(2) > 0$ without loss of generality. This decision rule shows that the agent is more likely to choose to work if the aggregate condition z or individual productivity x is higher. Also, note that the agent is less likely to choose to work if the size of transfers is higher.

In our model with the extensive margin labor supply, aggregate employment is shaped by both the decision rule and the distribution. Let $F_i(a)$ be the conditional (differentiable) distribution function of assets with its probability density being $f_i(a) = F_i'(a)$. Specifically, we use the exponential function for our following results: for $a \geq 0$,

$$F_i(a) = 1 - \exp(-a),$$
$$f_i(a) = F_i'(a) = \exp(-a).$$

[10] Individual types are likely to be unobservable to government. In this section, we assume that transfers depend on productivity for analytical tractability. However, our quantitative exercises in the following sections does not maintain this assumption.

Figure 1: Probability density function of assets in the static model



This density function has the mode at 0 and is strictly decreasing in a, as shown in Figure 1. In addition, it generates a long right tail of asset distribution, as in the data. Put differently, the key feature of the distribution function is that a large fraction holds low wealth and the density function becomes thinner with higher asset levels.[11]

Given the density function and the work decision rule, the fraction of agents working (i.e., the employment rate) for each type is given by

$$N_i = F\left(\tilde{a}_i\right) = 1 - \exp(-\tilde{a}_i)$$

where

$$\tilde{a}_i = zx_i - T_i. \tag{1}$$

In other words, the employment rate of the type i, N_i, is the integral of the type i agents whose asset level is lower than the threshold level $\tilde{a}_i$. We now present some theoretical results based on this model. All proofs are provided in Appendix A.

Proposition 1 *Let ε_i be the labor supply elasticity of the type i.*

$$\varepsilon_i \equiv \frac{\partial N_i}{\partial z}\frac{z}{N_i}.$$

[11] Our results below can be also obtained with alternative tractable distributions featuring these properties.

Assume $T_i = 0$. The labor supply elasticity of the low type is greater than that of the high type. That is, $\varepsilon_l > \varepsilon_h$.

This shows that our model naturally delivers heterogeneity of the labor supply elasticity. The feature of the wealth distribution described above plays a key role for this result. To see this, note that the threshold asset level of employment for the low-type agents is lower than that for the high-type agents: $\tilde{a}_l < \tilde{a}_h$. As shown in Figure 1, the thickness of the distribution decreases with a. Therefore, since there are more marginal households around $\tilde{a}_l$, the same change in z, which shifts the employment thresholds to the same degree, has a larger impact on the employment rate of the low type.

We now consider the role of government transfers and how they interact with heterogeneity. To simplify the algebra, we impose symmetry. Specifically, we assume that $\pi_l = \pi_h = 0.5$. In addition, $x_h = 1 + \lambda$ and $x_l = 1 - \lambda$ where $\lambda \in [0, 1]$ measures the cross-sectional dispersion of productivity.

To study the effect of progressivity in the transfer schedule, T_i is assumed to be determined by

$$T_l = T\left(1 + \omega\lambda\right)$$
$$T_h = T\left(1 - \omega\lambda\right)$$

where $T \in [0, z]$ is the scale of transfers, and $\omega \in [0, \frac{1}{\lambda}]$ captures progressivity. A changes in progressivity ω does not affect the aggregate amount of transfers.[12]

Given the above assumptions, we can derive the type-specific employment rate:

$$N_l = 1 - \exp(-\tilde{a}_l)$$
$$N_h = 1 - \exp(-\tilde{a}_h)$$

where $\tilde{a}_l = \max\left\{0, z(1-\lambda) - T - T\omega\lambda\right\}$ and $\tilde{a}_h = \max\left\{0, z(1+\lambda) - T + T\omega\lambda\right\}$.

Proposition 2 *Greater transfer progressivity increases the labor supply elasticity of the low-type agents, while it decreases the labor supply elasticity of the high-type agents. That is, $\frac{\partial \varepsilon_l}{\partial \omega} > 0$ and $\frac{\partial \varepsilon_h}{\partial \omega} < 0$.*

[12] Note that $\sum \pi_i T_i = \pi_l T\left(1 + \omega\lambda\right) + \pi_h T\left(1 - \omega\lambda\right) = T$.

Intuitively, greater transfer progressivity (or a higher ω) shifts $\tilde{a}_l$ to the left where the distribution of assets is denser. There, the same change in z would induce more agents to change their employment decision, thereby leading to an even larger elasticity of the low type. In contrast, greater transfer progressivity shifts $\tilde{a}_h$ to the right around which the distribution of assets is scarcer. A fewer number of agents around the new employment threshold implies that the elasticity of the high type agents would become smaller.

Proposition 3 *Let N denote the aggregate employment rate: $N = \pi_l N_l + \pi_h N_h$. Let ε be the aggregate labor supply elasticity:*

$$\varepsilon \equiv \frac{\partial N}{\partial z}\frac{z}{N}.$$

The aggregate labor supply elasticity is higher with greater transfer progressivity. That is, $\frac{\partial \varepsilon}{\partial \omega} > 0$.

The key for this result is that an increase in the elasticity of low types should be large enough to outweigh the opposing effects from a decrease in the elasticity of high types. Given the shape of the density function that declines at an increasing rate, this is easily satisfied. This result suggests the potential role of transfer progressivity in generating a large volatility of aggregate hours, as observed in the data.

Finally, we consider the implications for the cyclicality of average labor productivity. Define average labor productivity as output divided by aggregate hours,

$$\chi \equiv \frac{\sum_{j\in\{l,h\}} \pi_i \left(z x_i N_i\right)}{\sum_{j\in\{l,h\}} \pi_i N_i}.$$

Note that it can be rewritten as

$$\chi = z\frac{\sum_{j\in\{l,h\}} \pi_i \left(x_i N_i\right)}{\sum_{j\in\{l,h\}} \pi_i N_i} \equiv z\chi_0,$$

where we define the second term as χ_0. Here, we can clearly see that aggregate productivity z would directly make average labor productivity procyclical through the first term z, as in real business cycle models. The second term χ_0 captures the effects of worker composition on average labor productivity. Note that χ_0 also depends indirectly on z through type-specific employment responses. The following two propositions focus on the second term.

Proposition 4 *The effect of z on average labor productivity through worker composition effects is negative. That is, $\frac{\partial \chi_0}{\partial z} < 0$.*

Proposition 5 *Average labor productivity becomes less procyclical with greater transfer progressivity. That is, $\frac{\partial}{\partial \omega}\left(\frac{\partial \chi_0}{\partial z}\right) < 0$.*

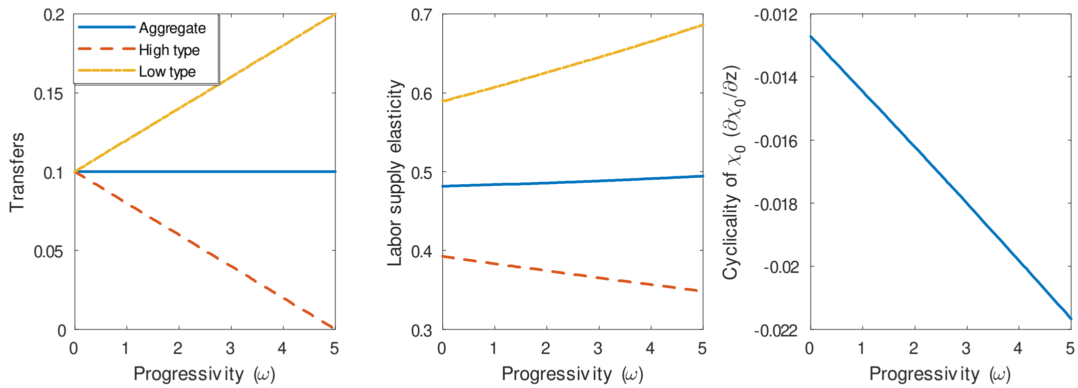
Proposition 5 tells us that transfer progressivity shapes the cyclicality of average labor productivity through worker composition effects. Specifically, it implies that greater progressivity makes average labor productivity *less* procyclical, as illustrated in a numerical example in the right panel of Figure 2. Propositions 1 and 2 provide the intuition behind this result. Note that the positive impact of progressivity on the aggregate labor supply elasticity in Proposition 3 (as depicted in the middle panel of Figure 2) is driven by the low type having a stronger labor supply response to a change in z. If a fall in z (e.g., in a recession) generates a large fall in the labor supply of the low type (especially relative to the high type), it would then tend to raise average labor productivity, while output falls. This force dampens the tight positive link between z and average labor productivity.

It is worth discussing our assumption that $F_l(a) = F_h(a)$. In other words, the conditional distribution of assets for each type is assumed to be identical. In fact, it should be noted that this assumption is quite conservative. For example, if we assume that $F_i(a) = 1 - \exp(-\frac{1}{\mu_i}a)$ and $\mu_l < \mu_h$ (as is consistent with the data), the distribution of assets for the low type would become more packed around the threshold asset level, which in turn would strengthen the above results (due to the even more elastic labor supply of the low-type). Furthermore, this points to the potential importance of the persistence of idiosyncratic shocks in a dynamic environment where the distribution of assets is endogenously determined because higher persistence would tend to widen the gap between μ_l and μ_h endogenously.[13]

It is also worth discussing what this simple model would imply for the role of progressive tax in the transmission of aggregate shocks to the macroeconomy. Now let earnings be $(1 - \tau_i)zx_in_i$. Tax progressivity increases when τ_h increases along with a lower τ_l. In the budget constraint, one can easily see that more redistribution can be achieved by raising relative tax rates for high type households (i.e., *higher* τ_h/τ_l) or alternatively, by raising transfers to low type households (i.e., *lower*

[13] As an extreme example, consider the case where the idiosyncratic shock has zero persistence (i.i.d.). In this case, the equilibrium asset distribution for each type would become identical, as in this section.

Figure 2: Impact of progressivity on aggregate labor market



Note: A numerical example of Propositions 1-5, based on $b = 2$, $\lambda = 0.2$, $T = 0.1$, and $z = 1.53$ to match the aggregate employment rate of 75% when $\omega = 0$.

T_h/T_l). An interesting observation arises in terms of their implications for aggregate labor market fluctuations. One can easily show that the asset thresholds for employment for each type i derived in (1) are now replaced by

$$\tilde{a}_i = (1 - \tau_i)zx_i - T_i$$

in the presence of taxes. Then, we can see that the two ways of increasing progressivity (either by decreasing T_h/T_l or by increasing τ_h/τ_l) would have exactly opposite effects on these employment thresholds $\tilde{a}_i$ because $\frac{\partial \tilde{a}_i}{\partial \tau_i}$ and $\frac{\partial \tilde{a}_i}{\partial T_i}$ have the same sign for both types. We revisit this point in our quantitative analysis in Section 5.

The main purpose of the analysis in this section has been to illustrate the role of transfer progressivity for aggregate fluctuations. By construction, we made several strong assumptions, some of which could be potentially important for this mechanism to work. Most notably, the model is static, and takes the distribution of wealth as fixed with respect to changes in fiscal policy. The distribution of wealth in a dynamic setting is expected to change as well. In particular, this relationship can be more complicated in a setting where risk-averse agents with different asset holdings face risk. Therefore, it is a quantitative question whether this mechanism would be relevant in a more realistic dynamic model environment where the simplifying assumptions made here are relaxed. In the next sections, we pursue exactly this.

3 Quantitative business cycle models

In this section, we introduce the economic environment of the quantitative, dynamic general equilibrium models with aggregate risk.

3.1 Baseline model with heterogeneity and progressive tax and transfers

Our baseline quantitative model builds on a standard incomplete markets framework with both idiosyncratic productivity risk and aggregate risk (e.g., Krusell and Smith, 1998). In particular, heterogeneous households make a consumption-savings choice, which endogenizes the distribution of wealth, and a labor supply decision at the extensive margin, as is consistent with the static model in Section 2.

Households The model economy is populated by a continuum of infinitely lived households. It is convenient to describe the infinitely lived household's decision problem recursively. At the beginning of each period, households are distinguished by their asset holdings a and productivity x_i. We assume that x_i takes a finite number of values N_x and follows a Markov chain with transition probabilities π^x_{ij} from the state i to the state j. In addition to the individual state variables, a and x_i, there are aggregate state variables, including the distribution of households $\mu(a, x_i)$ over a and x_i and aggregate total factor productivity shocks z_k. We also assume that z_k takes a finite number of values N_z following a Markov chain with transition probabilities π^z_{kl} from the state k to the state l. We assume that these Markov processes of individual productivity x and aggregate productivity z capture the following continuous AR(1) processes in logs.

$$\log x' = \rho_x \log x + \varepsilon'_x \tag{2}$$

$$\log z' = \rho_z \log z + \varepsilon'_z \tag{3}$$

where $\varepsilon_x \sim N(0, \sigma_x^2)$ and $\varepsilon_z \sim N(0, \sigma_z^2)$. A variable with a prime denotes its value in the next period. Finally, we assume competitive markets; in other words, households take as given the wage rate per efficiency unit of labor $w(\mu, z_k)$ and the real interest rate $r(\mu, z_k)$, both of which depend on the aggregate state variables. Households also take government policies as given.

The dynamic decision problem of households can be written as the following functional equation:

$$V(a, x_i, \mu, z_k) = \max\left\{V^E(a, x_i, \mu, z_k), V^N(a, x_i, \mu, z_k)\right\}$$

where

$$V^E(a, x_i, \mu, z_k) = \max_{\substack{a' \geq \underline{a}, \\ c \geq 0}} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \mu', z_l') \right\} \tag{4}$$

subject to

$$\begin{aligned} c + a' &\leq \left(\lambda_s \left(e/\bar{e}\right)^{-\lambda_p} - \tau\right) e + (1 + r(\mu, z_k))a + T \\ e &= w(\mu, z_k) x_i \bar{n} \\ T &= T_1 + T_2(m) \\ m &= e + r(\mu, z_k) \max\{a, 0\} \\ \mu' &= \Gamma(\mu, z_k). \end{aligned} \tag{5}$$

and

$$V^N(a, x_i, \mu, z_k) = \max_{\substack{a' \geq \underline{a}, \\ c \geq 0}} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \mu', z_l') \right\} \tag{6}$$

subject to

$$\begin{aligned} c + a' &\leq (1 + r(\mu, z_k))a + T \\ T &= T_1 + T_2(m) \\ m &= r(\mu, z_k) \max\{a, 0\} \\ \mu' &= \Gamma(\mu, z_k). \end{aligned} \tag{7}$$

Households maximize utility by choosing optimal consumption c, asset holdings in the next period a', and labor supply n. Households face a borrowing limit $\underline{a} \leq 0$. The labor supply decision is discrete: $n \in \{0, \bar{n}\}$. The disutility of work is captured by $B > 0$. Households understand that the expected future value, discounted by a discount factor β, is affected by stochastic processes for individual productivity x' and aggregate productivity z' as well as the whole distribution μ'. The budget constraints state that the sum of spending should be less than or equal to the sum of income. The evolution of μ is governed by the law of motion $\mu' = \Gamma(\mu, z_k)$.

As shown in the budget constraints, our model incorporates progressive tax and transfers, captured by two separate nonlinear functions. First, as is standard in the recent quantitative literature, earnings e are subject to both progressive taxation and a flat payroll tax rate $\tau \geq 0$ (e.g., Holter, Krueger and Stepanchuk, 2019). For those who have earnings e, progressive taxation leads to the after-tax earnings:

$$\left(\lambda_s \left(e/\bar{e}\right)^{-\lambda_p}\right) e, \tag{8}$$

following the parametric form of Benabou (2002) and Heathcote, Storeletten and Violante (2014). Note that $\lambda_p \geq 0$ captures the degree of progressivity and $\lambda_s \geq 0$ controls the scale of taxation (inversely). As the input to the progressive tax schedule is earnings normalized by average earnings $\bar{e}$ (Guner, Kaygusuz and Ventura, 2014), a change in λ_p tilts the tax schedule around the average earnings, thereby affecting tax progressivity strongly yet having little effects on the size of taxation.

On top of the progressive tax schedule, we also introduce progressive transfers separately. Specifically, following Krusell and Rios-Rull (1999), we assume that transfers T consist of two components. The first component T_1 is given to all households equally whereas the second component T_2 captures the income security aspect of transfers. In the U.S., there are various means-tested programs such as food stamps, the Supplemental Nutrition Assistance Program and the Temporary Assistance for Needy Families (formerly the Aid to Families with Dependent Children). As shown in Section 4, these programs lead to the observation that the amount of transfers is negatively associated with income. We assume that T_2 depends on total household income m to replicate the progressivity observed in the U.S. data using the following functional form (Yum, 2018):

$$T_2(m) = \omega_s (1+m)^{-\omega_p}. \tag{9}$$

This parametric assumption adds two parameters. First, $\omega_s \geq 0$ is a scale parameter, which determines the overall size of progressive part of government transfers (i.e., T_2). The next parameter $\omega_p \geq 0$ governs the degree of progressivity: a higher ω_p would make T_2 decrease faster with income.

Firm and government Aggregate output Y is produced by a representative firm. The firm maximizes its profit

$$\max_{K,L} \{z_k F(K,L) - (r(\mu, z_k) + \delta)K - w(\mu, z_k)L\} \tag{10}$$

where $F(K,L)$ captures a standard neoclassical production technology in which K denotes aggregate capital, L denotes aggregate efficiency units of labor inputs, and δ is the capital depreciation rate. As is standard in the literature, we assume that the aggregate production function follows a Cobb-Douglas function with constant returns to scale:

$$F(K,L) = K^{\alpha} L^{1-\alpha}. \tag{11}$$

The first-order conditions for K and L give

$$r(\mu, z_k) = z_k F_1(K,L) - \delta, \tag{12}$$

$$w(\mu, z_k) = z_k F_2(K,L). \tag{13}$$

The government in this economy collects labor taxes from households and use the tax revenue to finance total transfers to households. The remaining tax revenue is spent as government spending G, which is not valued by households.

Equilibrium A recursive competitive equilibrium is a collection of factor prices $r(\mu, z_k), w(\mu, z_k)$, household decision rules $g_a(a, x_i, \mu, z_k)$, $g_n(a, x_i, \mu, z_k)$, government policy variables $\tau, G, T(\cdot)$, a value function $V(a, x_i, \mu, z_k)$, a distribution of households $\mu(a, x_i)$ over the state space, the aggregate capital and labor $K(\mu, z_k), L(\mu, z_k)$ and the aggregate law of motion $\Gamma(\mu, z_k)$ such that

1. Given factor prices $r(\mu, z_k), w(\mu, z_k)$ and government policy $\tau, G, T(\cdot)$, the value function $V(a, x_i, \mu, z_k)$ solves the household's decision problems defined above, and the associated household decision rules are

$$a'^{*} = g_a(a, x_i, \mu, z_k) \tag{14}$$

$$n^{*} = g_n(a, x_i, \mu, z_k). \tag{15}$$

2. Given factor prices $r(\mu, z_k), w(\mu, z_k)$, the firm optimally chooses $K(\mu, z_k)$ and $L(\mu, z_k)$ following (12) and (13).

3. Markets clear:

$$K(\mu, z_k) = \sum_{i=1}^{N_x} \int_a a d\mu \tag{16}$$

$$L(\mu, z_k) = \sum_{i=1}^{N_x} \int_a x_i g_n(a, x_i, \mu, z_k) d\mu. \tag{17}$$

4. Government balances its budget: the sum of government spending G and total transfers to households is equal to the total tax revenue.

5. The law of motion for the distribution of households over the state space $\mu' = \Gamma(\mu, z_k)$ is consistent with individual decision rules and the stochastic processes governing x_i and z_k.

3.2 Alternative model specifications

In addition to the baseline model just introduced, we also consider alternative specifications to illustrate the importance of the interplay between household heterogeneity and government transfers.[14]

The first alternative model specification, denoted as Model (HA-N), is simply a nested specification of the baseline model by shutting down the entire government transfers (i.e., $T_1 = \omega_s = 0$) while keeping household heterogeneity. This model roughly corresponds to a standard incomplete-markets real business cycle model with household heterogeneity and endogenous labor supply at the extensive margin in Chang and Kim (2007).[15]

The second alternative model specification also keeps household heterogeneity but removes transfer progressivity. We denote this model specification by Model (HA-F), which is obtained as a nested model by making transfers flat, independent of income (i.e., $\omega_p = 0$). Chang et al. (2013) consider a business cycle model that is close to this model specification. Note that this form of transfers (flat

[14] We have also considered a specification which shuts down tax progressivity only. Because its quantitative role is minimal, we place the results in Appendix as a sensitivity check. In Section 5.3, we consider a counterfactual exercise where we alter tax progressivity using the baseline model specification.

[15] A noticeable difference between Model (HA-N) in our paper and the model in Chang and Kim (2007) is that our model has the progressive taxation whereas theirs does not. However, as shown in Section 5 and Appendix, the business cycle properties of the model are barely affected by the existence of progressive taxation except for output volatility.

lump-sum) is very broadly used in quantitative macroeconomics literature.

The final alternative specification shuts down household heterogeneity. Given the indivisible labor assumption, our representative-agent version of the model is essentially the business cycle model studied in Hansen (1985) augmented with tax and transfers. The key feature of this model specification is that the aggregation of Rogerson (1988) under certain assumptions such as employment lotteries and consumption insurance leads to the stand-in household whose disutility from work is linear–a powerful mechanism to generate a large volatility of aggregate hours, as observed in U.S. data (Hansen, 1988). We consider the decentralized competitive equilibrium given distortionary labor taxation. Appendix includes the detailed model environment and equilibrium definition.

3.3 Solution method

We solve the models numerically. Several key features make the numerical solution method for the heterogeneous models nontrivial. First, key decision variables in our model are a discrete employment choice and a consumption-savings choice in the presence of a borrowing constraint. Therefore, our solution method is based on the nonlinear method (i.e., the value function iteration) applied to the recursive representation of the problem described above. Second, the aggregate law of motion and state variables involve an infinite-dimensional object: the distribution μ. Therefore, we solve the model by approximating the distribution of wealth by the mean of the distribution (Krusell and Smith, 1998). In addition, since market-clearing is nontrivial in our model with endogenous labor, our solution method incorporates a step to find market-clearing prices in each period when simulating the model.

We describe the solution method briefly herein, but more details can be found in Appendix. Following Krusell and Smith (1998), we assume that households use a smaller object that approximates the infinite-dimensional distribution when they forecast the future state variables to make current decisions. More precisely, we approximate $\mu(a, x_i)$ by its mean of the asset distribution K. Also, the next period's aggregate capital K', real wage rate w and real interest rate r are assumed to be functions of (K, z_k) instead of (μ, z_k). We impose the parametric assumptions to approximate the aggregate law of motion $K' = \Gamma(K, z_k)$ and $w = w(K, z_k)$ following

$$\hat{K}' = \hat{\Gamma}(K, z_k) = \exp\left(a_0 + a_1 \log K + a_2 \log z_k\right) \tag{18}$$

$$\hat{w} = \hat{w}(K, z_k) = \exp\left(b_0 + b_1 \log K + b_2 \log z_k\right), \tag{19}$$

as in Chang and Kim (2006, 2007). Based on these forecasting rules, households obtain a forecasted $\hat{r}$ implied by the first-order conditions of firm's profit maximization problem.

Given the above forecasting rules, the model is solved in the two steps. First, we solve for the individual policy functions given the forecasting rules using the value function iterations (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual policy functions (*the outer loop*). As noted above, it is important to note that, since our model environment with endogenous labor supply involves non-trivial factor market clearing, we incorporate a step to find market-clearing factor prices in the outer loop (Chang and Kim, 2014; Takahashi, 2014). We repeat this procedure until the coefficients in the forecasting rules converge.

It is more straightforward to solve the representative-agent version of the model. Due to the distortionary tax, we solve the decentralized competitive equilibrium. For the purpose of comparison, we keep the same assumptions on the discretization of the aggregate productivity process as in the heterogeneous-agent model. The steady-state equilibrium can be obtained analytically. For solutions with aggregate uncertainty, we use the policy function iteration method.

4 Calibration and model properties in steady state

All model specifications are calibrated to U.S. data. A period in the model is a quarter, as is standard in the business cycle literature.

Calibrating the baseline model: Model (HA-T) We first describe how we calibrate the baseline specification: Model (HA-T). There are two sets of parameters for the baseline calibration. The first set of parameters is calibrated externally, in line with the business cycle literature. These parameter values are commonly set in all model specifications. The second set of parameters is calibrated to match the same number of relevant target statistics.

We begin with describing the first set of externally calibrated parameters. Most of these parameters are commonly used in the real business cycle literature. The capital share, α, is chosen to be consistent with the capital share of 0.36. The quarterly depreciate rate, δ, is 2.5 percent. In our model specifications with a binary labor supply choice, the level of hours worked, $\bar{n}$, can be arbitrarily set since it simply determines the scale of the calibrated disutility parameter B. We set it to 1/3, implying that working individuals spend a third of their time endowment on working. The borrowing limit $\underline{a}$ is set to be $-T_1/(1+r)$ where r is the equilibrium interest rate in steady state.[16] The payroll tax rate is set to $\tau = 0.0765$ (Holter et al., 2019).

In the literature, tax progressivity λ_p has been estimated using the same functional form we use. As noted by Holter et al. (2019), the estimate of λ_p varies quite a lot from 0.05 to 0.18, depending on the degree of completeness of government transfers in the data used by researchers. Because we model progressive transfers explicitly in addition to progressive taxes, taxation parameters in (8) should ideally capture tax progressivity only. As IRS income tax data used by Guner et al. (2014) do not include welfare transfers, we use their estimate for $\lambda_p = 0.053$ and $\lambda_s = 0.911$.[17] As discussed below, we then use micro data on the distribution of welfare transfers across households to calibrate the transfer function in (9).

The broad goal of this paper is to study how progressive transfers would alter the transmission of aggregate shocks to the macroeconomy. As a first step, we consider the most standard one–total factor productivity shocks (Kydland and Prescott, 1982)–as aggregate risk, and employ standard values of $\rho_z = 0.95$ and $\sigma_z = 0.007$ (Cooley and Prescott, 1995). Note that these values are useful as we can easily compare our results to those from recent related papers such as Chang and Kim (2007) and Takahashi (2018) who also use the same aggregate productivity estimates.[18]

Finally, the parameter ρ_x captures the persistence of idiosyncratic risk in the productivity of households. We estimate the persistence of idiosyncratic risk using the PSID following a standard

[16] Note that T_1 is calibrated internally, as described below. In Appendix, we report a version of the model with an alternative level of the borrowing limit. The main results found in this paper are barely affected.

[17] This is the estimate when EITC is included because we do not consider it in our calibration of welfare transfers. We have also considered alternative values for λ_p, which do not affect our quantitative results substantially. This quantitative insignificance of tax progressivity can be also seen explicitly in our counterfactual exercise in Section 5.3.

[18] An interesting exercise to be followed in the future is to investigate how our results would carry over in the presence of other types of aggregate shocks on top of the standard TFP shocks. The estimation of multiple aggregate shocks within a model with heterogeneous agents and nonconvexities is an important yet difficult task at this stage due to the computational costs.

Table 1: Parameter values chosen internally

Parameters			Target statistics		
Values		Description	Model	Data	Description
$B =$	.548	Disutility of work	.781	.782	Employment rate
$\beta =$	.986	Subject discount factor	.010	.010	Real interest rate
$\sigma_x =$	.130	S.D. of innovations to $\ln x$	.357	.359	Wage Gini index
$T_1 =$	.120	Overall transfer size	.101	.102	Ratio of Avg $(T_1 + T_2)$ to output
$\omega_s =$	.114	Progressive transfer scale	.0197	.0201	Ratio of Avg T_2 to output
$\omega_p =$	3.40	Progressivity of transfers	3.06	3.06	$E(T_2 \mid \text{1st income quintile})/E(T_2)$

Note:

method in the literature (e.g., Heathcote et al. 2010), as discussed in Appendix. The quarterly value based on the estimate leads to $\rho_x = 0.9847$. The variability of the idiosyncratic risk is calibrated internally and is explained below. Note that we keep the same values for these two parameters, ρ_x and σ_x, for all nested model specifications with heterogeneous agents in order to control for the underlying idiosyncratic risk present in the model.

The second set of parameters is jointly calibrated. As shown in Table 1, six parameters are calibrated by matching the same number of target statistics. We now explain how each parameter is linked to the target statistics.

The first parameter is B, which captures the disutility of work. The most relevant target moment is the employment rate of 78.2 percent in our SIPP sample.[19] The next parameter β captures the discount factor of households. As is standard in the literature, it is targeted to match the quarterly interest rate of 1 percent. The next parameter, σ_x, governs the variability of idiosyncratic labor productivity. We calibrate this parameter to match the overall wage dispersion captured by the Gini index of worker wages. The target statistic is chosen to be 0.359, which is the average Gini wage in 2000 (Heathcote, Perri and Violante, 2010).[20]

The last three parameters, T_1, ω_s and ω_p, govern statistics regarding transfers. Recall that T_1 determines the size of universal transfers and ω_s determines the scale of progressive transfers (T_2). Therefore, the first target statistic regarding transfers is set as the total transfers-output ratio of

[19] This value is higher than the employment-population ratio around 60% used in the previous literature (e.g., Chang and Kim, 2007). Note that our sample restricts age to be 23–65.

[20] Inequality has been steadily rising in the U.S. In Appendix, we also consider different values of this target.

10.2%, which is obtained as the time-series average of the ratio of transfers to output over the years 1961-2016 according to the BEA data. The target statistics include the average of income-security related government expenditures on social benefits (Table 3.12) over the years 1961-2016 (2.0% of output) in the BEA data.[21] Next, note that ω_p shapes the degree of progressivity in government transfers. Our calibration strategy is to let the model to replicate an empirically reasonable degree of transfer progressivity through ω_p (given the value of ω_s). For this purpose, we measure the degree of progressivity in the U.S. transfer programs using the SIPP data. We construct a broad measure of government transfers, including means-tested programs and social insurance (as detailed in Appendix). Since these welfare programs are highly relevant for the poor households, we choose the ratio of the average means-tested transfers received by the first income quintile to its unconditional mean (3.06) as a target statistic (Yum, 2018).

Calibrating alternative model specifications Having explained the calibration strategy of our baseline model, we now describe how we calibrate the nested model specifications–Model (HA-N), Model (HA-F) and Model (RA). In general, it would be ideal to minimize the number of parameters to be recalibrated to make sure that our comparison across different model specifications are not driven by different calibrated parameters. These include the parameters governing the idiosyncratic productivity risk, ρ_x and σ_x, which we hold constant across the nested specifications. However, a subset of the internally calibrated parameters is necessarily required to be recalibrated to make sure that different model specifications are similar in terms of the target statistics (e.g., the employment rate and equilibrium interest rate in steady state equilibrium).

More specifically, consider Model (HA-N). Because it abstracts from transfers ($T_1 = \omega_s = 0$), ω_p is irrelevant. We only recalibrate B and β to match the employment rate of 78.2% and the 1% real interest rate. This leads to $B = 0.933$ and $\beta = 0.9836$. Next, consider Model (HA-F), which shuts down transfer progressivity. With $\omega_p = 0$, the distinction between T_1 and ω_s is unnecessary. Therefore, we calibrate the sum of T_1 and ω_s to match the total transfers-output ratio of 10.2%. Aside from this, we again recalibrate $B = 0.714$ and $\beta = 0.9848$ in the same manner. Finally, unlike the heterogeneous-agent models, Model (RA) can be calibrated analytically, as shown in Appendix.

[21] We select the components to be consistent with our measure of transfers in the SIPP data, as describe below. Our classification of transfers is similar to Krusell and Rios-Rull (1999). See Appendix for details.

As for the tax and transfers, we use the average tax rate and transfers because progressivity is irrelevant in Model (RA) without heterogeneity.

Steady state properties Table 1 reports that the baseline model does a good job of matching the target statistics. In fact, the other nested model specifications do a great job of matching a smaller number of targets as well. This does not necessarily mean that the model can account for other relevant statistics. We thus present (non-targeted) distributional aspects of the model economy in steady state. First, Table 2 summarizes the share of wealth, employment rates by wealth quintile from both the model and the data. Overall, all heterogeneous-agent model specifications do a good job of accounting for the share of wealth by wealth quintile. A closer look reveals that Model (HA-T) does a moderately better job of accounting for the wealth concentration at the top of the wealth distribution.[22] Specifically, the relative share of the top quintile is closer to the data in Model (HA-T) (75.2%) compared to Model (HA-N) (73.9%) and Model (HA-F) (74.2%). Because the presence of government transfers reduces households' incentive to save (Hubbard, Skinner and Zeldes, 1995), the relative share of wealth by households in the top quintile becomes relatively larger in Model (HA-T) with progressive government transfers.

When we look at the employment rate by wealth quintile reported also in Table 2, we can clearly see that Model (HA-T) does a significantly better job of accounting for the cross-sectional employment-wealth relationship. In the U.S., the employment rate of the first wealth quintile is relatively low (70.0%) and then it is relatively flat across with wealth quintiles. This weakly inverse-U shape of the employment rates across wealth quintiles in the data are well captured in Model (HA-T). On the other hand, Model (HA-N) predicts that employment falls sharply with wealth, consistent with the findings in Chang and Kim (2007). In this class of the incomplete markets framework, the existence of transfers mitigates the excessively strong precautionary motive of employment among the poor households who expect to be near the borrowing limit in the near future (Yum, 2018). Interestingly, Model (HA-F) mitigates the high employment rate of the second wealth quintile but not the first wealth quintile, suggesting the importance of transfer progressivity for matching the

[22] The table also presents the statistics on wealth distribution obtained from the 1992-2007 Survey of Consumer Finances (SCF), as reported in Yum (2018). The statistics from the SCF shows a greater concentration of wealth in the top wealth quintile as it better captures the top of the wealth distribution by over-sampling the rich.

Table 2: Characteristics of wealth distribution

	Wealth quintile				
	1st	2nd	3rd	4th	5th
Share of wealth (%)					
U.S. Data (SIPP)	-2.2	1.2	6.8	18.4	76.3
U.S. Data (SCF)	-0.4	1.2	5.1	13.6	80.5
Model (HA-T)	-0.1	0.7	5.2	19.0	75.2
Model (HA-N)	-0.2	0.3	5.1	20.8	73.9
Model (HA-F)	-0.2	0.5	5.1	20.4	74.2
Employment rate (%)					
U.S. Data (SIPP)	70.0	77.9	80.9	82.5	79.7
Model (HA-T)	75.7	81.6	88.3	74.6	70.2
Model (HA-N)	100.0	96.7	76.5	65.9	51.6
Model (HA-F)	99.4	79.1	82.6	72.4	57.6

Note: U.S. data are based on the 2001 Survey of Income and Program Participation (SIPP) and the 1992-2007 Survey of Consumer Finances (SCF) (Yum, 2018). Model (HA-T) is the baseline specification: a heterogeneous-agent model with government transfers. Model (HA-N) shuts down government transfers but keeps household heterogeneity. Model (HA-F) shuts down transfer progressivity but keeps household heterogeneity.

relatively low employment rate of the bottom wealth quintile.

Lastly, Table 3 shows the joint relationship between income and transfers in steady state equilibrium. Specifically, the reported numbers are the ratio of average progressive-component transfers in each income quintile to the unconditional mean progressive-component transfers. In the U.S., there is a clear negative relationship between the amount of income-security transfers and income. Note that, in the model, this is a complicated equilibrium object, which is shaped not only by the parametric assumption on the nonlinear transfer schedule (9) but also by the endogenous household choices such as consumption-saving and labor supply. Despite the relatively simple functional form (9), we can see that our baseline model does an excellent job of replicating the degree of the transfer progressivity in the U.S. Note that since transfer progressivity is shut down by construction in Model (HA-F), this ratio is 1 for all income quintiles.

Table 3: Progressivity of income-security transfers

	Income quintile				
	1st	2nd	3rd	4th	5th
Conditional mean/unconditional mean					
U.S. Data	3.06	0.99	0.52	0.26	0.17
Model (HA-T)	3.06	1.06	0.56	0.25	0.07
Model (HA-F)	1.00	1.00	1.00	1.00	1.00

Note: The source of U.S. data is the Survey of Income and Program Participation 2001.

5 Quantitative analysis

In this section, we report the main business cycle results and illustrate the mechanism underlying the main quantitative results.

5.1 Business cycle properties

We first compare business cycle statistics of key macroeconomic variables from simulations of the models to those from the data. We filter all the series using the Hodrick-Prescott filter with a smoothing parameter of 1600. The U.S. data statistics are computed using the aggregate data from 1961Q1 to 2016Q4 (see Appendix for more details). Table 4 summarizes the cyclical volatility of the key aggregate variables: Y is output, C is consumption, I is investment, L is aggregate efficiency unit of labor, H is aggregate hours, and Y/H is average labor productivity. The volatility is measured by the percentage standard deviation. As is standard in the business cycle literature, our discussion focuses on the relative volatility, computed as the absolute volatility of each variable divided by that of output.

The most notable finding in Table 4 is that the high volatility of aggregate hours relative to output observed in U.S. data ($\sigma_H/\sigma_Y = 0.98$) is well accounted for by Model (HA-T). This finding is notable for several reasons. First, note that standard real business cycle models are known to have difficulties in generating a large relative volatility of hours without relying on a low curvature of the utility function (or a high Frisch elasticity). In Model (RA), recall that the stand-in household's disutility is linear in aggregate hours. When the utility function features zero curvature in the labor supply, we can see that the model indeed generates a substantial relative volatility of hours (0.83),

Table 4: Volatility of aggregate variables

		Model			
	U.S. data	(HA-T)	(HA-N)	(HA-F)	(RA)
σ_Y	1.50	1.19	1.49	1.31	1.92
σ_C/σ_Y	0.58	0.24	0.26	0.25	0.22
σ_I/σ_Y	2.96	2.69	2.88	2.83	2.97
σ_L/σ_Y	-	0.40	0.66	0.51	-
σ_H/σ_Y	0.98	0.84	0.54	0.61	0.83
$\sigma_{Y/H}/\sigma_Y$	0.52	0.62	0.54	0.60	0.22

Note: See Table 2 for the description of the model specifications. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Volatility is measured by the percentage standard deviation of each variable. The U.S. statistics are based on aggregate time-series from 1961Q1 to 2016Q4.

as also shown in Hansen (1985). It is striking that our baseline model, Model (HA-T), delivers a comparably high volatility of hours (0.84).

In addition, it is important to note the finding of Chang and Kim (2006, 2007) that a large relative volatility of hours obtained through indivisible labor (Rogerson, 1988) in Hansen (1985) may not be robust to incomplete-markets economies with heterogeneous households. We can see this point also in the performance of Model (HA-N), which delivers a substantially smaller volatility of hours (0.54). However, our result from Model (HA-T) suggests that once progressive government transfers are incorporated, the heterogeneous-agent incomplete-markets model can perform as well as the Hansen-Rogerson economy in terms of a large relative volatility of hours over the business cycle.[23]

The performance of Model (HA-F) reveals that introducing flat transfers into the model helps obtain a larger relative volatility of hours (0.61). However, it is still quite far from its counterpart (0.83) in Model (HA-T), suggesting the importance of transfer progressivity. This finding is in line with our analytical results (Proposition 3) in Section 2 that greater transfer progressivity increases the degree to which aggregate hours fluctuates with respect to aggregate conditions.

Having highlighted the most notable difference across the four model specifications, we also note that there are also interesting differences in the volatility of macroeconomic aggregates. For instance, the volatility of consumption and average labor productivity over the business cycle tends to be more

[23] Introducing flat transfers into the model helps obtaining a large relative volatility of hours

Table 5: Cyclicality of aggregate variables

		Model			
	U.S. data	(HA-T)	(HA-N)	(HA-F)	(RA)
$Cor(Y, C)$	0.81	0.85	0.83	0.84	0.81
$Cor(Y, I)$	0.90	0.99	0.99	0.99	0.99
$Cor(Y, L)$	-	0.91	0.93	0.93	-
$Cor(Y, H)$	0.86	0.78	0.92	0.83	0.99
$Cor(Y, Y/H)$	0.30	0.55	0.92	0.82	0.81
$Cor(H, Y/H)$	-0.23	-0.08	0.70	0.35	0.70

Note: See Table 2 for the description of the model specifications. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Cyclicality is measured by the correlation of each variable with output. The statistics are based on aggregate time-series from 1961Q1 to 2016Q4.

consistent with the data in the heterogeneous-agent models, compared to the representative-agent model.[24] Another observation is that the presence of government transfers tends to reduce the volatility of consumption over the business cycle. This suggests the role of government transfers as stabilizers, effectively providing insurance against aggregate risk (e.g., see McKay and Reis, 2016).

We now move on to the cyclicality of macroeconomic variables, a key focus of this paper. The first five rows of Table 5 show correlations of output with other aggregate variables considered in Table 5. The last row shows the correlation between aggregate hours and labor productivity. As is well known in the literature (e.g., King and Rebelo, 1999), most macroeconomic variables such as consumption, investment, and aggregate hours are highly procyclical in the U.S. Table 5 shows that the strongly positive correlations with output are fairly well replicated in all model specifications, regardless of the presence of heterogeneity or institutional details. Therefore, one might conclude that heterogeneity or government transfers could be irrelevant, at least in regard to cyclicality of macroeconomic variables over the business cycle.

However, this conclusion is premature when we consider the comovement of average labor productivity and output. In the U.S., average labor productivity does not feature strong procyclicality (i.e., $Cor(Y, Y/H) = 0.30$). A related observation is that the correlation between hours and average labor productivity is even weakly negative (-0.23), often terms as the Dunlop–Tarshis observation (Christiano and Eichenbaum, 1992). In contrast, canonical real business cycle models generate highly

[24] Note that in the representative-agent model, average labor productivity is proportional to consumption.

procyclical average labor productivity, and thus fail to replicate the cyclicality of average labor productivity, as is well known in the literature. High correlation between output and average labor productivity in Models (RA) (0.81) manifests this weakness as well.[25]

The most notable finding in Table 5 is that strong procyclicality of average labor productivity is considerably muted (0.55) in our baseline model (i.e., Model (HA-T)), closer to the data (0.30). In contrast to the existing literature, which tends to rely on the introduction of additional exogenous shocks (e.g., Benhabib, Rogerson, and Wright, 1991; Christiano and Eichenbaum, 1992; Braun, 1994; and Takahashi, 2018), the key to our result is the interaction between household heterogeneity and transfer progressivity, which in turn generates heterogeneous labor supply behavior across households as highlighted in Section 2. The importance of the interplay between household heterogeneity and transfers can be seen by the performance of the nested model specifications. Once we abstract from either heterogeneity or transfer progressivity, the model generates highly procyclical average labor productivity. In particular, Model (HA-N), which abstracts from the entire transfers (e.g., Chang and Kim, 2006, 2007) generates a very high correlation of 0.92, implying that heterogeneity per se cannot dampen highly procyclical average labor productivity in real business cycle models.

5.2 Impulse responses

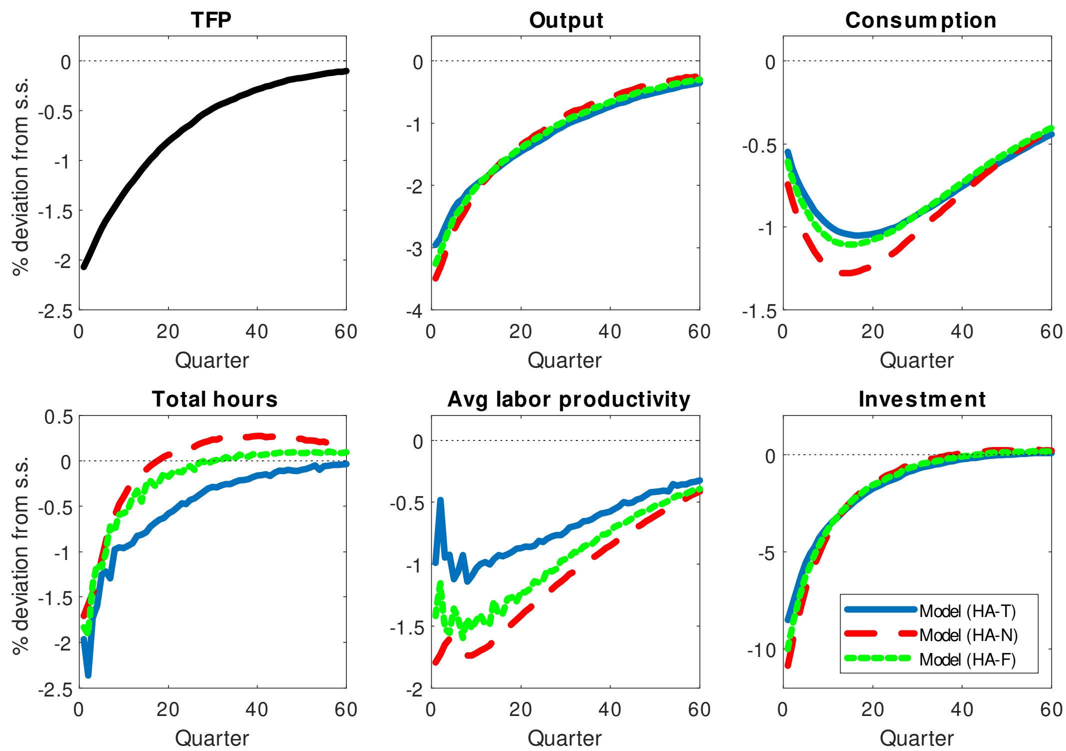
We now investigate the mechanism underlying our quantitative success through impulse response functions. Figure 3 shows the impulse responses of the key aggregate variables such as output, consumption, aggregate hours, average labor productivity, and investment following a persistent negative 2% shock to z (or TFP) for each heterogeneous-agent model specifications. We follow the simulation-based methodology developed by Koop, Pesaran and Potter (1996) (see also Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry, 2018).

The impulse response of aggregate hours clearly confirms that Model (HA-T) (solid line) delivers a larger fall in hours than the nested models with heterogeneity–Model (HA-N) (dashed line) and Model (HA-F) (dotted line)–despite the fact that output declines least strongly on impact in our baseline model, Model (HA-T).[26] These are consistent with the business cycle result on volatility in Table 4.

[25] These correlations would become even higher in models without indivisibility of labor (Hansen, 1985) or in the absence of labor taxes.

[26] There are wiggles in impulse responses involving hours. These naturally arise in our model with a discrete choice

Figure 3: Impulse responses of macroeconomic aggregates



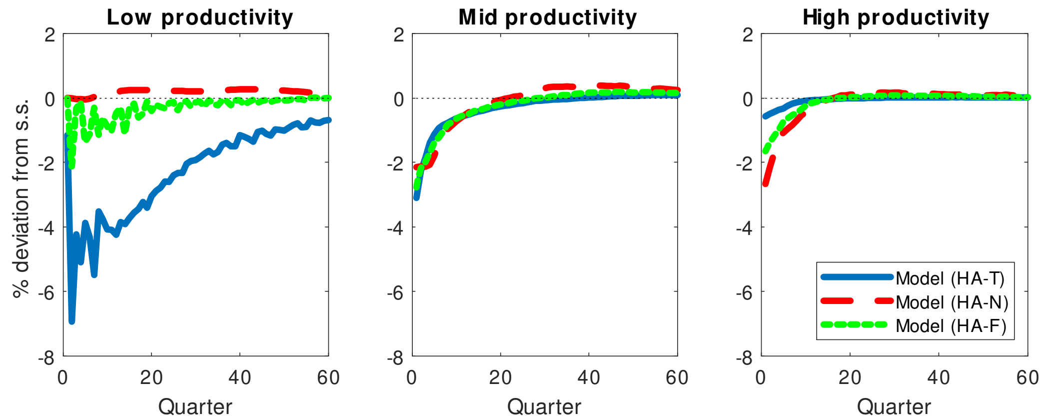
Note: TFP denotes the total factor productivity (or aggregate productivity shocks). The figures display the IRFs of macroeconomic aggregates to a negative 2 percent TFP shock with persistence ρ_z.

Another important difference to note is the impulse responses of average labor productivity. In our heterogeneous-agent model without transfers (i.e., Model (HA-N)), the dynamics of average labor productivity closely follows the pattern of output as the average labor productivity falls quite sharply on impact. This explains a very high correlation of Y/H with Y in Table 5. When flat transfers are present in Model (HA-F), we can see that the overall decrease in average labor productivity becomes mitigated. In our baseline model with progressive transfers (Model (HA-T)), this tendency becomes even stronger. In particular, the magnitude of the fall in average labor productivity is considerably smaller.

To understand the underlying source of the differences in aggregate dynamics, it is useful to in-

of labor supply.

Figure 4: Impulse responses of hours by productivity



Note: Households are grouped into low productivity (below median), mid productivity (median), and high productivity (above median). The figures display impulse responses for employment in each group to a negative 2 percent TFP shock with persistence ρ_z.

vestigate the impulse responses at a more disaggregated level.[27] Figure 4 plots the impulse responses of total hours by productivity. Specifically, we categorize households into three groups such that they are most evenly distributed: (i) the low productivity group $\{x_i\}_{i=1}^{4}$; (ii) the mid productivity group $\{x_i\}_{i=5}^{6}$; and (iii) the high productivity group $\{x_i\}_{i=7}^{10}$.

There are several important patterns worth noting. First, note that there is essentially no difference in labor supply responses among the mid productivity group across different model specifications. On the other hand, we see that the response of the high productivity group is weakest in our baseline model, compared to the model without transfer progressivity (i.e., Model (HA-F)). The difference is even more prominent when compared to Model (HA-N), which abstracts from the entire transfers. This pattern is consistent with a key insight of the simple model that transfer progressivity reduces the elasticity of higher productivity agents, as shown in Proposition 2.

Proposition 1 from our simple model implies that households with higher productivity tend to be less elastic in their labor supply. In fact, this pattern clearly applies to our baseline model that generates greater magnitudes of the decrease in labor supply among lower productivity groups. This

[27] An obvious candidate is the dynamics of equilibrium prices. Figure $A1$ displays the changes in market-clearing wage and interest rates following the same negative TFP shock for each model specification. It appears that the difference across the model specifications is not substantial among the heterogeneous agent models, which suggests that our main results are not driven mainly by the difference in equilibrium price dynamics.

heterogeneity of labor supply responses explains why Model (HA-T) is able to reduce the cyclicality of average labor productivity.

However, this monotone relationship between the elasticity and productivity breaks down at the low productivity group in Model (HA-N). This exceptionally inelastic employment response is related to the finding in Yum (2018), who shows that the absence of public insurance in incomplete markets models raises the precautionary motive for employment among wealth-poor households who lack self-insurance. When the precautionary motive is too high, this motive dominates the intertemporal substitution motive, thereby weakening the responses of hours with respect to a persistent fall in wages. This inelastic labor supply among the low productivity group provides a key reason for both lower volatility of total hours and highly procyclical average labor productivity in Model (HA-N), illustrating why heterogeneity per se is not sufficient for our key results in incomplete markets environment. The impulse responses from Model (HA-F) that only shuts down transfer progressivity indeed show that the low productivity group now become responsive to aggregate state changes although its magnitude is still not as sizeable as that of the baseline model.

In summary, the above findings suggest that the presence of progressive transfers in our baseline model plays a dual role. On the one hand, the progressivity of transfers induces low productivity workers to be more responsive to aggregate productivity shocks, as illustrated in the simple model as well. On the other hand, in an incomplete markets environment, the mere presence of transfers helps relax the precautionary motive among wealth-poor households, most of who are low productivity workers. Both roles turn out to be quantitatively important for the dynamics of aggregate hours and average labor productivity.

5.3 Progressivity and the macroeconomy

We now use the baseline model to conduct counterfactual exercises. Given that there are two separate ways of adjusting the degree of progressivity in the tax and transfers system in our baseline model (tax progressivity vs. transfer progressivity), we explore the implications of each tool for the macroeconomy. In particular, because our model features aggregate shocks, we investigate their effects not only on steady states but also on aggregate fluctuations. To control for the strength of each policy reform, we make sure that each policy makes the difference between Gini income before

Table 6: Effects of progressivity on the steady-state economy and aggregate fluctuations

	Baseline Model (HA-T)	Counterfactuals Higher progressivity Transfers	Tax
	Steady state		
Employment rate (%)			
Overall	78.1	72.4	79.4
By wealth quintile			
1st	75.7	46.4	80.4
2nd	81.6	83.7	82.9
3rd	88.3	85.5	88.5
4th	74.6	75.5	75.3
5th	70.2	70.9	69.9
- Cond. mean/uncond. mean of T_2 by income quintile			
1st	3.06	3.64	3.00
2nd	1.06	0.87	1.09
3rd	0.56	0.34	0.58
4th	0.25	0.12	0.26
5th	0.07	0.02	0.07
	Business cycles		
σ_Y	1.19	1.24	1.19
σ_H/σ_Y	0.84	1.06	0.77
$Cor(Y, Y/H)$	0.55	0.30	0.64
$Cor(H, Y/H)$	-0.08	-0.43	0.04

Note: Each counterfactual exercise leads to the same Gini of after-tax-and-transfer income using each policy instrument.

tax and transfers and Gini income after tax and transfers to be 2 percentage point larger, compared to the baseline model.[28]

Table 6 reports the results. It reveals that the steady state effects of higher transfer progressivity differ from those of higher tax progressivity. Higher transfer progressivity reduces the employment rate of wealth-poor households quite substantially yet increases that of wealth-rich households mildly. As higher transfer progressivity raises transfers to low-income households, relatively stronger income effects on labor supply can easily explain this result. On the other hand, higher tax progressivity

[28]Specifically, tax progressivity is increased by raising λ_p by 79% (or $\lambda_p = 0.095$). A higher λ_p tends to raise the overall tax revenue, which works for more redistribution. As for transfer progressivity, we adjust both ω_p and ω_s simultaneously because a higher ω_p tends to reduce the overall size of transfers, which works against more redistribution. The required percentage increase is 28%.

decreases the labor supply of wealth-rich households whereas it increases the employment rate of the wealth poor. This happens because higher progressivity raises the tax rate faced by high-wage households while it reduces the tax rate for low-wage households.

Having checked the steady state effects, we now move on to their business cycle implication, which is probably not as straightforward as the steady state implications. Table 6 shows that more progressive transfers would reduce the cyclicality of average labor productivity and raise the volatility of total hours. These are exactly in line with the main results derived using the simple model in Section 2. On the other hand, higher tax progressivity actually does the opposite, although quantitative magnitudes are weaker, compared to the effects of transfer progressivity: Average labor productivity becomes more procyclical and the volatility of hours declines. These results are in line with discussion in Section 2 that the impact of higher transfer progressivity on the employment policy functions can be exactly opposite to that of higher tax progressivity. These results have potentially important policy implications: policy makers who attempt to pursue more redistributive policies may face very different business cycle consequences, depending on the fiscal tool (tax vs. transfers).

6 Microeconomic evidence on heterogeneity in the extensive margin labor supply responses

As shown in the previous sections, the key element of our model is the existence of heterogeneous labor supply responses. More precisely, low-wage workers are considerably more elastic in adjusting labor supply at the extensive margin, which weakens a highly procyclical average labor productivity and, at the same time, enlarges the volatility of aggregate hours worked over the business cycle. In this section, we empirically document heterogeneity in labor supply responses to verify whether our key model mechanism exists in the micro data.

Specifically, we exploit the panel structure of the PSID to explore whether extensive margin labor supply responses differ by the hourly wage. The panel structure is useful because we can keep track of the same people whose labor supply decisions are observed over time. Because labor supply changes can be measured in different ways and can be shaped by forces at a different level (idiosyncratic vs. aggregate), we consider two approaches. The first approach focuses on identifying the probability of

Table 7: Probability of extensive margin adjustment, by wage quintile

	Length of tracking each individual T		
	5 years	10 years	15 years
Wage quintile in base year			
1st	0.097	0.078	0.068
2nd	0.052	0.044	0.041
3rd	0.039	0.036	0.035
4th	0.035	0.033	0.033
5th	0.040	0.039	0.038
Base years	1969-1993 ($J = 25$)	1969-1988 ($J = 20$)	1969-1983 ($J = 15$)
Avg. no. obs in base years	1,743	1,281	927
Total no. obs.	43,580	25,619	13,911
Avg. age in total sample	43.0	43.7	44.4

Note: See text for the definition of the switching probability reported in this table. Numbers in parentheses show the number of base years.

the extensive margin labor supply adjustment for each individual and illustrating how it differs by wage. On the other hand, the second approach focuses on differences in the magnitude of employment rate changes across wage groups during the last six recessions. We present each empirical analysis in more detail.

As mentioned above, the key object of interest in the first approach is the probability of the extensive margin adjustment for each individual. Note that it requires us to have relatively long time-series observations for each individual to obtain a consistent estimate of the adjustment probability, based on the individual-level flow data.[29] Let us fix a year at j. Let i denote an individual index and t denote the year when the individual is observed. We define the extensive margin adjustment based on full-time employment, E, consistent with the previous sections. In other words, an individual i in year t is full-time employed (i.e., $E_{i,t} = 1$) if the annual hours worked are greater than 1,000 hours.[30] Then, we define a binary switching variable, $S_{i,t}$, such that $S_{i,t} = 1$ if $E_{i,t} \neq E_{i,t-1}$ and $S_{i,t} = 0$ otherwise. We exclude the transition from $E_{i,t-1} = 1$ to $E_{i,t} = 0$ if the individual has a non-zero

[29] Since the frequency of the PSID survey has been annual until 1997 and became biannual since 1999, we use the samples observed annually from the 1969-1997 waves.

[30] The results in this section is quite robust to alternative threshold values around $1,000$ for the full-time employment.

unemployment spell in period t to rule out transitions caused by layoffs.

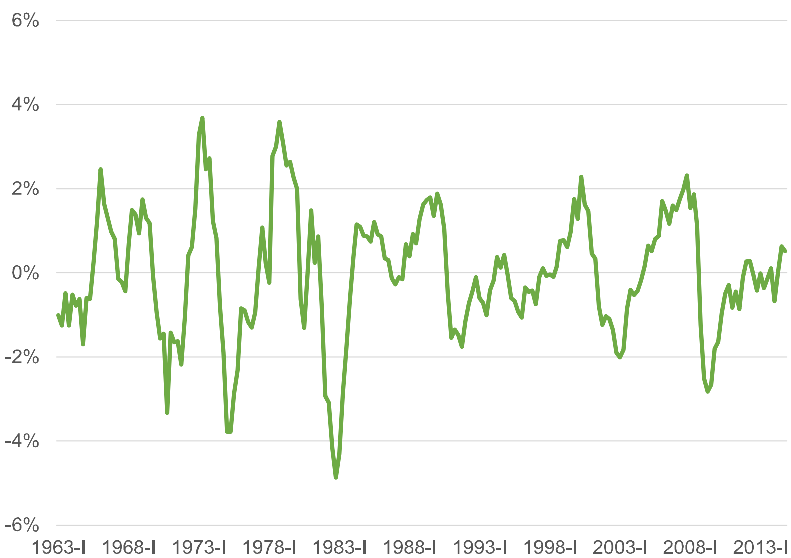
Note that, given the length of tracking each individual, T, there are $(T-1)$ numbers of $S_{i,t}$ for each individual i. Once we take the average over time, we obtain the individual-specific probability of extensive-margin adjustment at an annual frequency (i.e., $p_{i,j} \equiv \frac{1}{T-1}\sum_{t=j+1}^{j+T-1} S_{i,t}$). As we are interested in differences across wage distribution, we compute p_j^q, defined as the conditional mean of $p_{i,j}$ for each individual's wage quintile bin $q \in \{1, 2, ..., 5\}$ determined in the base year j.

We consider three different values for the length of tracking each individual: $T \in \{5, 10, 15\}$ because a different value of T entails a trade-off. On the one hand, a larger number is beneficial because we are more likely to have a consistent estimate of the adjustment probability at the individual level. On the other hand, a longer time of tracking implies a stricter restriction on samples (because we keep only samples that are observed for T *consecutive* years). Given the value of T, we compute the estimates of $\{p_j^q\}_{q=1}^5$ by changing the base year j. That way, we attempt to mitigate variations due to the difference in the initial distribution of wage, which is potentially affected by business cycle fluctuations. The reported values in Table 7 are the mean switching probabilities by wage quintile, averaged across the base years, $p^q \equiv \frac{1}{J}\sum p_j^q$ where J is the number of base years (J is reported in parentheses in Table 7).

Table 7 reveals a clear pattern: the individual-level probability of adjusting the labor supply along the extensive margin is significantly higher among low-wage workers. For instance, when $T = 5$, the probability of switching to/from full-time employment among the first wage quintile is 9.7% at the annual frequency. In particular, we can see that this probability tends to decrease with wage. For the third to fifth quintiles, this probability is relatively flat at approximately 4%. When T increases, we also find that the key pattern of extensive margin adjustment probabilities across wage quintiles is still present. Because the samples become slightly older and T becomes longer, however, we also see that the switching probabilities become generally lower.

The above exercise is based on long-run information on the labor market flow at the individual level. The next empirical exercise, on the other hand, uses the differences in magnitude of the employment level changes across wage groups during the recessions. More specifically, we choose six recessions, as evident from Figure 6, which plots the cyclical component of quarterly real GDP per

Figure 5: Cyclical component of real GDP per capita



Note: A quarterly series of real GDP per capita is detrended using HP filter with a smoothing parameter of 1,600.

capita.[31] For each recession, we choose a peak year and a trough year, guided by Figure 6. Note that our definition of peak and trough years is limited by the frequency of the PSID because the PSID data set is available annually (until 1997) or biannually (since 1999). Therefore, our choice is also based on the aggregate employment declines in each recession event, according to our micro samples in the PSID. The resulting year combinations for each recession are shown in Table 8.

For each recession, we compute the conditional mean of full-time employment by wage quintile in the peak year: $\frac{1}{N^q_{peak}} \sum_i E^q_{i,peak}$ where N^q_{peak} is the number of observations in the wage quintile bin q in the peak year. Then, we measure percentage point changes in the employment rate by wage quintile in the corresponding trough year: that is, $\frac{1}{N^q_{peak}} \sum_i \left(E^q_{i,trough} - E^q_{i,peak} \right)$. It is important to note that we keep the set of households in each wage group fixed by assigning a wage quintile to each household in the peak year. That way, our measured changes in employment by wage quintile are not affected by compositional changes but are rather based on decisions by the same households.

Table 8 clearly shows that the employment rate fell most sharply in the first and second wage

[31] A quarterly series of real GDP per capita is detrended using HP filter with a smoothing parameter of 1,600.

Table 8: Employment changes in recessions, by wage quintile

	Recession					
	1969-71	1973-76	1980-83	1990-92	2000-02	2006-10
Wage quintile in peak year						
1st	-6.0	-13.9	-11.2	-7.4	-9.8	-17.7
2nd	-6.9	-7.6	-5.3	-8.2	-5.2	-12.9
3rd	-4.6	-7.4	-4.8	-5.3	-3.4	-10.8
4th	-5.4	-4.9	-6.6	-5.4	-4.5	-10.3
5th	-1.7	-5.4	-5.3	-4.4	-1.7	-6.3
No. obs.	1,749	1,838	2,095	2,201	2,970	2,873

Note: The year ranges denote the peak and trough years of each recession. Reported values are percentage point changes in the employment rate by wage quintiles (in the peak year of each recession) using the same set of individuals.

quintiles in all of the recessions. Furthermore, the magnitude of the decrease in employment tends to be smaller as the wage quintile increases. For example, in the last recession (i.e., the Great Recession), the employment rate among the first wage quintile fell by 17.7 percentage points, whereas the employment rate among the fifth wage quintile fell by only 6.3 percentage points. This pattern of employment changes by wage quintiles is quite robust across different recessions despite variations in the overall changes in the employment rate.[32]

One may be concerned about the possibility that the wage gradient of employment changes found in Table 8 is driven mostly by the firms' demand channel, which may affect household employment status differentially across the wage distribution. To address this concern, we utilize the information about the unemployment spell in the PSID data.[33] More precisely, we exclude samples that experience any unemployment spells over the whole survey years belonging to the range of each recession. That way, we attempt to rule out the effects caused by a differential layoff probability across the wage distribution. Because we impose the additional sample restriction, the number of observations in each recession decreases.

Table 9 summarizes the results. In general, we see that the magnitudes of employment changes are

[32] Note that the overall magnitude of the fall in employment is relatively stronger in the recessions of 1973-76, 1980-83 and 2006-10. This finding is, in fact, consistent with relatively larger amplitudes of these recessions, as shown in Figure 6, providing some external validity for our micro samples.

[33] This information is available since the 1976 wave.

Table 9: Employment changes in recessions excluding samples with unemployment spells, by wage quintile

	Recession				
	1973-76	1980-83	1990-92	2000-02	2006-10
Wage quintile in peak year					
1st	-14.6	-11.5	-7.0	-5.9	-10.8
2nd	-6.6	-2.4	-6.7	-3.6	-7.7
3rd	-7.2	-5.1	-4.4	-2.2	-5.9
4th	-5.8	-7.1	-3.7	-3.7	-6.8
5th	-4.0	-6.7	-4.2	-1.3	-4.6
No. obs.	1,469	1,387	1,689	2,511	2,244

Note: See Table 8 for the basic description. The only difference relative to Table 9 is that we exclude samples that experienced unemployment spells in either the peak year or the trough year. The results for the first recession is omitted because the unemployment information is available only since the 1976 wave (i.e., since the year of 1975).

somewhat weaker, implying that the demand channel played a role in reducing aggregate employment rates in most recessions. However, we note that our key findings in Table 8 still appears even when we exclude samples that experienced any unemployment spells. First, low-wage workers experience the largest fall in employment across the wage distribution. Furthermore, we still see the general pattern that the magnitude of the fall is negatively related to the wage quintiles in all the recessions. Therefore, we conclude that the salient findings in Table 8 on heterogeneity in employment changes during the recessions remain robust even after accounting for the firms' demand channel.

Although the above two approaches are designed to capture different aspects of labor supply adjustments, they yield consistent results, which demonstrates the robustness of our empirical result that lower-wage workers adjust the labor supply along the extensive margin more elastically. More importantly, both of these empirical findings are consistent with the pattern of heterogeneity in labor supply responses in our model economy, thereby supporting our key mechanism of the heterogeneous-agent model with progressive government transfers.

7 Conclusion

In this paper, we have explored the interplay of household heterogeneity and progressive government transfers in shaping the dynamics of macroeconomic aggregates over the business cycle. We first presented the key insights using analytical results obtained from a stylized static model of the extensive margin labor supply with heterogeneous households. We then constructed a full general equilibrium business cycle model with household heterogeneity. We have shown that in our baseline heterogeneous-agent model, micro-level heterogeneity shapes the dynamics of aggregate labor market variables substantially when household heterogeneity interacts with progressive government transfers. In particular, our baseline model delivers moderately procyclical average labor productivity unlike standard real business cycle models. At the same time, it retains the success of the canonical representative-agent indivisible labor model in generating a large volatility of aggregate hours without the assumptions of lotteries and perfect consumption insurance (Rogerson, 1988). Our counterfactual analysis shows that greater transfer progressivity, but not tax progressivity, lowers the correlation of average labor productivity with output (or hours).

Using the panel structure of the PSID, we have also documented that the individual-level probability of adjusting the labor supply along the extensive margin is significantly higher among low-wage workers. Furthermore, we have shown that the magnitude of the decline in the employment rate is considerably larger among low-wage workers during the last six recessions. This microeconomic evidence on the heterogeneous responses of labor supply along the extensive margin supports the key mechanism of our heterogeneous-agent model with progressive government transfers.

There are several future research questions naturally following our study. An interesting novel result we highlight in this paper is that the effects of higher transfer progressivity can be even qualitatively different from those of higher tax progressivity on both steady state and business cycles, although both can potentially achieve the same degree of additional redistribution. A straightforward application relevant to this result would be how to design a tax and transfer system while taking into account the welfare costs of business cycles in the presence of heterogeneous agents (e.g., Krusell, Sahin, Mukoyama and Smith, 2009). Next, although the current paper has focused on the transmission of aggregate shocks to the macroeconomy, it would be interesting to explore how other kinds of aggregate shocks such as monetary policy shocks would be transmitted differently in our framework. Finally, our paper provides some theoretical and quantitative mechanisms suggesting that transfer progressivity might be behind the vanishing procyclicality of average labor productivity, as documented in Galí and van Rens (2019).[34] Formal empirical investigations of these relationship are out of the scope of the current paper but would be highly valuable. These interesting questions are left for future work.

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A Appendix

A.1 Proofs in Section 2

Proof of Proposition 1 Assume $T_i = 0$. Then, we can rewrite

$$\tilde{a}_i = zx_i.$$

Therefore,

$$N_i = 1 - \exp(-zx_i)$$

Given this, note that

$$\begin{aligned}
\varepsilon_i \equiv \frac{\partial N_i}{\partial z}\frac{z}{N_i} &= x_i \exp(-zx_i)\frac{z}{1-\exp(-zx_i)} \\
&= \frac{zx_i \exp(-zx_i)}{1-\exp(-zx_i)}
\end{aligned}$$

For expositional convenience, assume that x is continuous for now.

$$\varepsilon(x) = \frac{zx\exp(-zx)}{1-\exp(-zx)}$$

$$\begin{aligned}
\frac{\partial \varepsilon(x)}{\partial x} &= \frac{\left[z\exp(-zx) - z^2 x\exp(-zx)\right]\left[1-\exp(-zx)\right] - zx\exp(-zx)\left[z\exp(-zx)\right]}{\left[1-\exp(-zx)\right]^2} \\
&= \frac{\exp(-zx)z\left[1-zx\right]\left[1-\exp(-zx)\right] - z^2 x\exp(-zx)\left[\exp(-zx)\right]}{\left[1-\exp(-zx)\right]^2} \\
&= \frac{z\exp(-zx)\left\{1-zx-\exp(-zx)\right\}}{\left[1-\exp(-zx)\right]^2}
\end{aligned}$$

Since $\exp(-zx) < 1$ for all $z, x > 0$,

$$\begin{aligned}
\frac{\partial \varepsilon(x)}{\partial x} &= \frac{z\exp(-zx)\left(1-zx-\exp(-zx)\right)}{\left[1-\exp(-zx)\right]^2} < \frac{z\exp(-zx)\left(1-zx-1\right)}{\left[1-\exp(-zx)\right]^2} \\
&= \frac{z\exp(-zx)\left(-zx\right)}{\left[1-\exp(-zx)\right]^2} < 0.
\end{aligned}$$

Proof of Proposition 2 Since

$$\frac{\partial N_l}{\partial z} = \exp(-\tilde{a}_l)(1-\lambda),$$
$$\frac{\partial N_h}{\partial z} = \exp(-\tilde{a}_h)(1+\lambda).$$

we have

$$\frac{\partial}{\partial \omega}\left(\frac{\partial N_l}{\partial z}\right) = \exp(-\tilde{a}_l)(1-\lambda)T\lambda > 0,$$
$$\frac{\partial}{\partial \omega}\left(\frac{\partial N_h}{\partial z}\right) = -\exp(-\tilde{a}_h)(1+\lambda)T\lambda < 0.$$

Also, note that

$$\frac{\partial N_l}{\partial \omega} = -\exp(-\tilde{a}_l)T\lambda < 0$$
$$\frac{\partial N_h}{\partial \omega} = \exp(-\tilde{a}_l)T\lambda > 0.$$

Proof of Proposition 3 Since

$$\begin{aligned}\varepsilon &\equiv \frac{\partial N}{\partial z}\frac{z}{N} \\ &= \left(\pi_l \frac{\partial N_l}{\partial z} + \pi_h \frac{\partial N_h}{\partial z}\right)\frac{z}{\pi_l N_l + \pi_h N_h}\end{aligned}$$

the aggregate labor supply elasticity is given by

$$\varepsilon = z\frac{\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)}$$

where

$$\tilde{a}_l = z(1-\lambda) - T - T\omega\lambda$$

$$\tilde{a}_h = z(1+\lambda) - T + T\omega\lambda.$$

Then, we have

$$\frac{\partial \varepsilon}{\partial \omega} = z\frac{\begin{array}{c}[\exp(-\tilde{a}_l)(1-\lambda)(-1)(-T\lambda)+\exp(-\tilde{a}_h)(1+\lambda)(-1)T\lambda]\left[2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)\right] \\ -\left[\exp(-\tilde{a}_l)(1-\lambda)+\exp(-\tilde{a}_h)(1+\lambda)\right]\left[-\exp(-\tilde{a}_l)(-1)(-T\lambda)-\exp(-\tilde{a}_h)(-1)T\lambda\right]\end{array}}{\left[2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)\right]^2}$$

$$= zT\lambda\frac{\begin{array}{c}[\exp(-\tilde{a}_l)(1-\lambda)-\exp(-\tilde{a}_h)(1+\lambda)]\left[2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)\right] \\ +\left[\exp(-\tilde{a}_l)(1-\lambda)+\exp(-\tilde{a}_h)(1+\lambda)\right]\left[\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)\right]\end{array}}{\left[2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)\right]^2}$$

The sign of $\frac{\partial \varepsilon}{\partial \omega}$ is equal to that of the numerator, which can be rewritten as

$$\begin{aligned}
\text{Numerator} &= 2(1-\lambda)\exp(-\tilde{a}_l) - (1-\lambda)\exp(-2\tilde{a}_l) - (1-\lambda)\exp(-\tilde{a}_h-\tilde{a}_l) \\
&\quad - 2(1+\lambda)\exp(-\tilde{a}_h) + (1+\lambda)\exp(-\tilde{a}_h-\tilde{a}_l) + (1+\lambda)\exp(-2\tilde{a}_h) \\
&\quad + (1-\lambda)\exp(-2\tilde{a}_l) - (1-\lambda)\exp(-\tilde{a}_h-\tilde{a}_l) \\
&\quad + (1+\lambda)\exp(-\tilde{a}_h-\tilde{a}_l) - (1+\lambda)\exp(-2\tilde{a}_h) \\
&= 2\left[(1-\lambda)\exp(-\tilde{a}_l) - (1+\lambda)\exp(-\tilde{a}_h) + 2\lambda\exp(-\tilde{a}_h-\tilde{a}_l)\right].
\end{aligned}$$

Letting $\theta = \frac{(1-\lambda)}{(1+\lambda)}$, we can rewrite

$$\begin{aligned}
&2(1+\lambda)\left[\frac{(1-\lambda)}{(1+\lambda)}\exp(-\tilde{a}_l) - \exp(-\tilde{a}_h) + \frac{2\lambda}{(1+\lambda)}\exp(-\tilde{a}_h-\tilde{a}_l)\right] \\
&= 2(1+\lambda)\left[\theta\exp(-\tilde{a}_l) + (1-\theta)\exp(-\tilde{a}_h-\tilde{a}_l) - \exp(-\tilde{a}_h)\right].
\end{aligned}$$

Since $\exp(-x)$ is convex, we know

$$\begin{aligned}
\theta\exp(-\tilde{a}_l) + (1-\theta)\exp(-\left(\tilde{a}_h+\tilde{a}_l\right)) &> \exp\left(-\left\{\theta\tilde{a}_l + (1-\theta)\left(\tilde{a}_h+\tilde{a}_l\right)\right\}\right) \\
&= \exp\left(-\left\{(1-\theta)\,\tilde{a}_h + \tilde{a}_l\right\}\right).
\end{aligned}$$

Applying this inequality, we have

$$\begin{aligned}\text{Numerator} &= 2(1+\lambda)\left[\theta \exp(-\tilde{a}_l) + (1-\theta)\exp(-\tilde{a}_h - \tilde{a}_l) - \exp(-\tilde{a}_h)\right] \\ &> 2(1+\lambda)\left[\exp\left(-\left\{(1-\theta)\,\tilde{a}_h + \tilde{a}_l\right\}\right) - \exp(-\tilde{a}_h)\right] \geq 0\end{aligned}$$

if and only if

$$\begin{aligned}(1-\theta)\,\tilde{a}_h + \tilde{a}_l &\leq \tilde{a}_h \\ \tilde{a}_l &\leq \theta\tilde{a}_h \\ (1+\lambda)\left[z(1-\lambda) - T - T\omega\lambda\right] &\leq (1-\lambda)\left[z(1+\lambda) - T + T\omega\lambda\right] \\ z(1+\lambda)(1-\lambda) - (1+\lambda)T - (1+\lambda)T\omega\lambda &\leq z(1+\lambda)(1-\lambda) - (1-\lambda)T + (1-\lambda)T\omega\lambda \\ -(1+\lambda) - (1+\lambda)\omega\lambda &\leq -(1-\lambda) + (1-\lambda)\omega\lambda \\ -1 &\leq \omega\end{aligned}$$

which is always satisfied.

Proof of Proposition 4 Note that

$$\begin{aligned}\chi_0 &= \frac{(1-\lambda)\,(1-\exp(-\tilde{a}_l)) + (1+\lambda)\,(1-\exp(-\tilde{a}_h))}{2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)} \\ &= \frac{1-\lambda-\exp(-\tilde{a}_l)+\lambda\exp(-\tilde{a}_l)+1+\lambda-\exp(-\tilde{a}_h)-\lambda\exp(-\tilde{a}_h)}{2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)} \\ &= \frac{2-(1-\lambda)\exp(-\tilde{a}_l)-(1+\lambda)\exp(-\tilde{a}_h)}{2-\exp(-\tilde{a}_l)-\exp(-\tilde{a}_h)}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\frac{\partial \chi_0}{\partial z} &= \frac{\left[(1-\lambda)^2 \exp(-\tilde{a}_l) + (1+\lambda)^2 \exp(-\tilde{a}_h)\right] [2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)]}{(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h))^2} \\
&\quad - \frac{[2 - (1-\lambda)\exp(-\tilde{a}_l) - (1+\lambda)\exp(-\tilde{a}_h)]\,[\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)]}{(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h))^2} \\
&= \frac{1}{(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h))^2} \left\{ \begin{array}{c}
2(1-\lambda)^2 \exp(-\tilde{a}_l) + 2(1+\lambda)^2 \exp(-\tilde{a}_h) \\
-(1-\lambda)^2 \exp(-2\tilde{a}_l) - (1+\lambda)^2 \exp(-\tilde{a}_h - \tilde{a}_l) \\
-(1-\lambda)^2 \exp(-\tilde{a}_h - \tilde{a}_l) - (1+\lambda)^2 \exp(-2\tilde{a}_h) \\
-2(1-\lambda)\exp(-\tilde{a}_l) - 2(1+\lambda)\exp(-\tilde{a}_h) \\
+(1-\lambda)^2 \exp(-2\tilde{a}_l) + (1+\lambda)(1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) \\
+(1+\lambda)(1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) + (1+\lambda)^2 \exp(-2\tilde{a}_h)
\end{array} \right\} \\
&= \frac{2\lambda(\lambda-1)\exp(-\tilde{a}_l) + 2\lambda(\lambda+1)\exp(-\tilde{a}_h) - 4\lambda^2 \exp(-\tilde{a}_h - \tilde{a}_l)}{(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h))^2} \\
&= \frac{2\lambda\left\{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)\right\}}{(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h))^2} < 0.
\end{aligned}$$

Proof of Proposition 5 Define

$$\Phi(\omega) \equiv \log\left(\frac{\partial \chi_0}{\partial z}\right).$$

Since the log transformation preserves monotonicity, it suffices to show that $\Phi'(\omega) < 0$. As

$$\begin{aligned}
\Phi(\omega) &= \log 2\lambda + \log\left\{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)\right\} \\
&\quad - 2\log\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)
\end{aligned}$$

we have

$$
\begin{aligned}
\Phi'(\omega) &= \frac{-T\lambda(\lambda-1)\exp(-\tilde{a}_l) + T\lambda(\lambda+1)\exp(-\tilde{a}_h)}{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)} \\
&- 2\frac{T\lambda\exp(-\tilde{a}_l) - T\lambda\exp(-\tilde{a}_h)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)} \\
&= \frac{\underbrace{T\lambda(1-\lambda)\exp(-\tilde{a}_l) + T\lambda(\lambda+1)\exp(-\tilde{a}_h)}_{\text{positive}}}{\underbrace{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)}_{\text{negative}}} \\
&- 2\frac{\underbrace{T\lambda\left[\exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]}_{\text{positive}}}{\underbrace{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)}_{\text{positive}}} \\
&< 0.
\end{aligned}
$$

A.2 Representative-agent (RA) model

We first describe the model environment. At the beginning of each period, the stand-in household has the current period's assets k. The aggregate state variables are the aggregate capital K and the aggregate productivity z_k. The aggregate productivity follows the same stochastic process as in the baseline model. Taking the wage rate $w(K, z_k)$ and the real interest rate $r(K, z_k)$, as well as the aggregate law of motion $\Gamma(K, z_k)$ as given, the dynamic decision problem of the representative household can be written as the following functional equation:

$$
V(k, K, z_k) = \max_{\substack{k' \geq 0, c \geq 0 \\ n \in [0,1]}} \left\{ \log c - Bn + \beta \sum_{l=1}^{N_z} \pi^z_{kl} V(k', K', z'_l) \right\}
$$

$$
\begin{aligned}
\text{subject to} \quad & c + k' \leq (1-\tau_l)w(K, z_k)n + (1 + r(K, z_k))k + T \\
& K' = \Gamma(K, z_k)
\end{aligned}
$$

The household maximize utility by choosing optimal consumption c, the next period's capital k' and labor supply n. The stand-in household's utility is linear in employment n due to the aggregation theory in Rogerson (1988). The budget constraint states that the sum of consumption c and the next period's capital k' should be less than or equal to the sum of net-of-tax labor income $(1-\tau_l)w(K, z_k)n$, current capital k, capital income $r(K, z_k)k$ and government transfers T.

Government collects taxes on labor earnings $\tau_l wn$ to finance transfers T and government expenditure G. We keep the same assumptions on the firm side, as in the heterogeneous-agent models. The resulting first-order conditions for K and L are the same as those in (12) and (13).

A recursive competitive equilibrium is a collection of factor prices $r(K, z_k)$, $w(K, z_k)$, household decision rules $g_k(k, K, z_k)$, $g_n(k, K, z_k)$, government policy variables τ_l, G, T, the household value function $V(k, K, z_k)$, the aggregate labor $L(K, z_k)$ and the aggregate law of motion for aggregate capital $\Gamma(K, z_k)$ such that

1. Given factor prices $r(K, z_k)$, $w(K, z_k)$ and government policy τ_l, G, T, the value function $V(k, K, z)$ solves the household's decision problem, and the associated decision rules are

$$\begin{aligned} k'^{*} &= g_k(k, K, z_k) \\ n^{*} &= g_n(k, K, z_k). \end{aligned}$$

2. Prices $r(K, z_k)$, $w(K, z_k)$ are competitively determined following (12) and (13).

3. Government balances its budget:

$$G + T = \tau_l w(K, z_k) L(K, z_k).$$

4. Consistency is satisfied: for all K,

$$\begin{aligned} K' = \Gamma(K, z_k) &= g_k(K, K, z_k) \\ L(K, z_k) &= g_n(K, K, z_k). \end{aligned}$$

It is straightforward to calibrate the parameters of the representative-agent model using the steady state equilibrium equations. First, β is directly obtained by

$$\beta = (1+r)^{-1}$$

Then, given the target of $T/Y = 0.102$, $L = 0.782$ and $\tau_l = 0.1874$, B is obtained by

$$B = \frac{(1-\tau_l)(1-\alpha)}{\left(1 - \delta\frac{K}{Y} - \frac{G}{Y}\right) L}$$

where

$$\begin{aligned} \frac{K}{Y} &= \frac{\alpha}{r+\delta} \\ \frac{G}{Y} &= \tau(1-\alpha) - \frac{T}{Y}. \end{aligned}$$

Finally, since $\frac{Y}{K} = \left(\frac{K}{L}\right)^{\alpha-1}$, we can obtain $\frac{K}{L}$, which in turn gives K and thus Y. Then, T is obtained using $T/Y = .102$. The resulting calibrated values are $\beta = 0.9901$, $B = 0.9174$ and $T = 0.2959$.

A.3 Aggregate data

The business cycle statistics are based on the aggregate time-series data covering from 1961Q1 to 2016Q4. As for output, we use "Real Gross Domestic Product (millions of chained 2012 dollars)" in Table 1.1.6 of the Bureau of Economic Analysis (BEA). As for consumption, we use expenditures in non-durable goods and services reported in Table 2.3.5 of the BEA (Personal Consumption Expenditure). Investment is constructed as the sum of expenditures in durable goods (Table 2.3.5) and private fixed investment in Table 5.3.5. The real values of consumption and investment are calculated using the price index for Gross Domestic Product in Table 1.1.4. Data on total hours worked are obtained from Cociuba et al. (2018). We modified all of the raw time series into those per capita by dividing the raw data by quarterly population in Cociuba et al.(2018).

A target statistic regarding the size of income-security transfers is based on the aggregate data obtained from the BEA. Specifically, we use Supplemental Nutrition Assistance Program (SNAP), Supplemental security income, temporary disability insurance, medical care (Medicaid, general medical assistance and state child health care programs), supplemental security income in Table 3.12

on Government Social Benefits. Note that we do not include large programs such as Medicare, unemployment insurance and veterans' benefits.

A.4 Micro data

For the statistics obtained at the micro level, we use data from the Survey of Income and Program Participation (SIPP). This data set is representative of the non-institutionalized U.S. population. The survey period is in a monthly basis. The SIPP covers a wide range of information on income, wealth, and participation in various transfer programs. We choose the samples from the first wave to the ninth wave of the SIPP in 2001, covering from 2001 to 2003. The original data set is composed of a main module and several topical modules. While the main module contains monthly information on income and transfers, variables such as wealth are reported quarterly in the topical modules. We combine both modules on a quarterly basis.

We construct variables at household level. Data sets in the SIPP contain not only household variables but also individual variables. To generate a household variable from its corresponding individual variable, we take the following steps. First, we identify households with sample unit identifier (SSUID) and household address id in sample unit (SHHADID). Second, we add up the values of a variable for all members in a household. The government transfers that is used to infer the degree of progressivity is based on a broad range of transfer programs including Supplemental Security Income (SSI), Temporary Assistant for Needy Family (TANF), Supplemental Nutrition Assistance Program (SNAP), Supplemental Nutrition Program for Women, Infants, and Children (WIC), childcare subsidy and Medicaid. We do not include age-dependent programs such as Social Security and Medicare. We construct a variable of household income broadly; it consists of labor income, income from financial investments, and property income. We consider households whose head's age is between 23 and 65. We convert all of their nominal values to the values in 2001 US dollar using the CPI-U.

A.5 Estimation of the persistence of idiosyncratic productivity risk

We estimate the persistence of idiosyncratic productivity risk in the U.S. using the PSID data, following Heathcote et al. (2010). We choose samples for the period of 1969-2010. Our measure of labor productivity is defined as a worker's relative hourly wage to other individuals. To avoid the oversampling of low-income household heads, we exclude households from the Survey of Economic Opportunity. We consider household heads whose age is between 18 and 70 and whose wages are observed at least for four consecutive periods.[35] To focus on full-time workers, we drop the samples whose annual hours worked is less than $1,000$ hours. We also drop the samples whose wage is below a half of the minimum wage. The nominal values are converted into the value of US dollar in 2001 with the CPI-U.

We run the ordinary least square regression of the log of the productivity (hourly wages) on a dummy for male, a cubic polynomial in potential experience (age minus years of education minus five), a time dummy, and a time dummy interacted with a college education dummy. We take its residual, $x_{i,j}$, as an idiosyncratic productivity that contains a wide range of individual abilities in the labor market. This stochastic process is composed of the summation of a persistent, $\eta_{i,j}$ and a transitory process, $\nu_{i,j}$:

$$\begin{aligned} x_{i,j} &= \eta_{i,j} + \nu_{i,j}, \nu_{i,j} \sim N(0, \sigma_\nu^2) \qquad (20) \\ \eta'_{i,j} &= \rho_\eta \eta_{i,j-1} + \epsilon'_{i,j}, \epsilon'_{i,j} \sim N(0, \sigma_\epsilon^2) \end{aligned}$$

We use a Minimum Distance Estimator to estimate the parameters of the process. The mechanism is to find parameters that minimizing the distance between empirical and theoretical moments. We take the covariance matrix of the residual $x_{i,j}$ as our moments. Denote θ by a vector of $(\rho_\eta, \sigma_v, \sigma_\epsilon)$. Let $m_{j,j+n}(\theta)$ be the covariance of the labor productivity between age j and $j+n$ individuals. $\hat{m}_{j,j+n}$ is defined as the empirical counterpart of $m_{j,j+n}(\theta)$. Then,

$$E\left[\hat{m}_{j,j+n} - m_{j,j+n}(\theta)\right] = 0 \qquad (21)$$

where

$$\hat{m}_{j,j+n} = \frac{1}{N_{j,j+n}} \sum_{i=1}^{N_{j,j+n}} x_{i,j} \cdot x_{i,j+n}$$

[35] This is somewhat wider than the restriction imposed in SIPP for a larger number of samples. Note that we impose stricter restrictions on wages and hours, which would naturally remove irrelevant samples such as retirees.

The moments can be represented by as an upper triangle matrix:

$$\bar{m}(\theta)=\begin{bmatrix} m_{0,0}(\theta) & m_{0,1}(\theta) & \cdots & \cdots & m_{0,J-1}(\theta) & m_{0,J}(\theta) \\ 0 & m_{1,1}(\theta) & \cdots & \cdots & m_{1,J-1}(\theta) & m_{1,J}(\theta) \\ 0 & 0 & m_{2,2}(\theta) & \cdots & m_{2,J-1}(\theta) & m_{2,J}(\theta) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & m_{J-1,J-1}(\theta) & m_{J-1,J}(\theta) \\ 0 & 0 & 0 & \cdots & 0 & m_{J,J}(\theta) \end{bmatrix}$$

We denote a vector of $\bar{M}(\theta)$ by vectorizing $\bar{m}(\theta)$ with length $(J+1)(J+2)/2$. To estimate parameters θ, we solve

$$\min_{\theta}\left[\hat{\bar{M}}-\bar{M}(\theta)\right]' W\left[\hat{\bar{M}}-\bar{M}(\theta)\right]$$

where the weighting matrix W is set to be an identity matrix.[36]

A.6 More on numerical methods for the heterogeneous-agent models

A.6.1 Solving for the equilibrium with aggregate risk

The models with aggregate risk are solved in the following two steps. First, we solve for the individual policy functions given the forecasting rules (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual policy functions (*the outer loop*). We iterate the two steps until the forecasting rules converge. That is, the difference between the old forecasting rule used in the inner loop and the new forecasting rule generated in the outer loop is small enough.

Inner loop In the inner loop, we solve for the value functions: $V(a,x_i,K,z_k)$, $V^E(a,x_i,K,z_k)$ and $V^N(a,x_i,K,z_k)$, as defined below. These value functions are stored on a non-evenly spaced grid for a and an evenly spaced grid for K with the number of grid points $n_a=400$ and $n_K=5$, respectively. Unlike Chang and Kim (2006, 2007) and Takahashi (2014), we discretize stochastic processes x_i and z_k by using the Rouwenhorst (1995) method. We find that the approximation of continuous AR(1) processes with our estimate featuring very high persistence is considerably better with the

[36]Using the identity matrix has been common in the literature since Altonji and Segal (1996) show that the optimal weighting matrix generate severe small sample biases.

Rouwenhorst method given the same number of grid points.[37] Our baseline results are based on $n_x = 10$ and $n_z = 5$, both of which replicate the true parameters of the continuous AR(1) processes very precisely.

To obtain $V(a, x_i, K, z_k) = \max\left[V^E(a, x_i, K, z_k), V^N(a, x_i, K, z_k)\right]$, we solve the following problems

$$V^E(a, x_i, K, z_k) = \max_{\substack{a' \geq \underline{a}, \\ c \geq 0}} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{22}$$

subject to

$$\begin{aligned} c + a' \leq & \left(\lambda_s \left(\hat{w}(K, z_k) x_i \bar{n}/\bar{e}\right)^{-\lambda_p} - \tau\right) e + (1 + \hat{r}(K, z_k))a \\ & + T_1 + T_2(\hat{w}(K, z_k) x_i \bar{n} + \hat{r}(K, z_k) \max\{a, 0\}) \end{aligned}$$

and

$$V^N(a, x_i, K, z_k) = \max_{\substack{a' > \underline{a}, \\ c > 0}} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{23}$$

$$c + a' \leq (1 + \hat{r}(K, z_k))a + T_1 + T_2(\hat{r}(K, z_k) \max\{a, 0\})$$

To evaluate the functional value of the expected value function on $(a', \hat{K}')$ which are not on the grid points, we use the piecewise-linear interpolation. By solving these problems, we obtain the individual policy function for work $g_n(a, x_i, K, z_k)$ based on comparing $V^E(a, x_i, K, z_k)$ and $V^N(a, x_i, K, z_k)$. We also obtain conditional policy functions for the optimal $a' : g_a^E(a, x_i, K, z_k)$ as the maximizer of the problem (22) and $g_a^N(a, x_i, K, z_k)$ as the maximizer of the problem (23).

Outer loop In the outer loop, we simulate the model economy based on the information obtained in the inner loop. We note that a key step is to find the market-clearing prices in each period during the simulation. Although this is computationally burdensome, we find that the results without the market-clearing step are substantially misleading, as is consistent with Takahashi (2014) and Chang and Kim (2014).

The measure of households $\mu(a, x_i)$ is approximated by a finer (non-evenly spaced) grid on a than

[37] Specifically, we use the simulated data from Rouwenhorst and Tauchen's methods and estimate the persistence and the standard deviation of error terms in the AR(1) processes for both aggregate productivity shocks and idiosyncratic shocks (available upon request).

that in the inner loop (Rios-Rull, 1999) with the number of grid points equal to $1,000$. K is constructed by aggregating individual asset holdings over the measure of households: $\int_a \sum_{i=1}^{N_x} a\mu(da, x_i)$. Following Takahashi (2014), in each simulation period, we use a bisection method to obtain the equilibrium factor prices as follows:

1. Set an initial range of (w_L, w_H) and calculate the aggregate labor demand $L^d = (1-\alpha)^{\frac{1}{\alpha}}(z_k/w)^{\frac{1}{\alpha}}K$ implied by the firm's FOC for each w. Note that r is obtained by using the relationship $r = z_k^{\frac{1}{\alpha}}\alpha\left(\frac{w}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$, implied jointly by (12) and (13).

2. Calculate the aggregate efficiency unit of labor supply L^s at each w and make sure that the excess labor demand $(L^d - L^s)$ is positive at w_L and it is negative at w_H.

3. Compute $\tilde{w} = \frac{w_L+w_H}{2}$ and obtain $L^d - L^s$ at $\tilde{w}$. If $L^d - L^s > 0$, set $w_L = \tilde{w}$; otherwise, set $w_H = \tilde{w}$.

4. Continue updating (w_L, w_H) until $|w_L - w_H|$ is small enough.

Taking the measure of households $\mu(a, x_i)$, the aggregate state (K, z_k), and factor prices w and r as given, we compute the aggregate efficiency unit of labor supply $L^s(K, z_k)$ by using the threshold asset $a^*(x_i, K, z_k)$ for each individual productivity. Specifically, we solve (22) and (23) given the expected value function in the next period using interpolation. Note that we use the valued function obtained in the inner loop and the forecasting rule (18) for $\hat{K}' = \Gamma(K, z_k)$ which is not on the grid points of K. Then, the individual household decision rules are given by

$$n = g_n(a, x_i, K, z_k) = \begin{cases} \bar{n} & \text{if } V^E(a, x_i, K, z_k) > V^N(a, x_i, K, z_k), \\ 0 & \text{otherwise.} \end{cases}$$

Having $n = g_n(a, x_i, K, z_k)$ for each grid point (a, x_i) on μ at hand, the aggregate efficiency unit of labor supply is obtained by $L^s(K, z_k) = \int_a \sum_{i=1}^{N_x} x_i g_n(a, x_i, K, z_k)\mu(da, x_i)$. After finding the market-clearing prices, we update the measure of households in the next period by using

$$a' = g_a(a, x_i, K, z_k) = \begin{cases} g^E(a, x_i, K, z_k) & \text{if } V^E(a, x_i, K, z_k) > V^N(a, x_i, K, z_k), \\ g^N(a, x_i, K, z_k) & \text{otherwise,} \end{cases}$$

and the stochastic process for x_i. We simulate the economy for $10,000$ periods, as in Khan and Thomas (2008).

Finally, the coefficients $(a_0, a_1, a_2, b_0, b_1, b_2)$ in the forecasting rules

$$\log K' = a_0 + a_1 \log K + a_2 \log z, \tag{24}$$

$$\log w = b_0 + b_1 \log K + a_2 \log z, \tag{25}$$

are updated by ordinary least squares with the simulated sequence of $\{K', w, K, z\}$. Our parametric assumption on the forecasting rules are the same as those in Chang and Kim (2007; 2014) and Takahashi (2014; 2019). We repeat the whole procedure of the inner and outer loops until the coefficients in the forecasting rules converge.

As is clear in the forecasting rules (24) and (25), households predict prices and the future distributions of capital only with the mean capital stock. Therefore, it is important to check whether the equilibrium forecast rules are precise or not. We summarize results for the accuracy of the forecasting rules for the future mean capital stock K' and for the wage w in Table $A1$. It is clear that all R^2 are very high in all specifications. We also check the accuracy statistic proposed by Den Haan (2010). Since our dependent variables are in logs, we multiply the statistics by 100 to interpret them as percentage errors. We find that mean errors are sufficiently small (considerably less than 0.1%) and maximum errors are also reasonably small, not exceeding 0.7 percent for all specifications.

Table A1: Estimates and accuracy of forecasting rules

Model	Dependent variable	Coefficient Const.	$\log K$	$\log z$	R^2	Den Haan (2010) error Mean (%)	Max (%)
(HA-T)	$\log K'$	0.1116	0.9591	0.0832	0.99999	0.056	0.293
Baseline	$\log w$	-0.2125	0.3968	0.8538	0.99904	0.065	0.594
(HA-N)	$\log K'$	0.1450	0.9448	0.1091	0.99995	0.089	0.535
	$\log w$	-0.4892	0.5154	0.6944	0.99909	0.069	0.467
(HA-F)	$\log K'$	0.1285	0.9520	0.0945	0.99997	0.082	0.370
	$\log w$	-0.3386	0.4498	0.7922	0.99895	0.068	0.679

A.6.2 Impulse response functions

There is no generally accepted way to calculate conditional impulse responses in nonlinear models. In this paper, to compute impulse response functions, we follow the simulation-based procedure developed by Koop et al. (1996) (see also Bloom et al., 2018):

- Draw $i = 1, ..., N_{sim}$ sets of exogenous random variables for aggregate productivity, each of which have $t = 1, ..., T_{sim}$ periods.[38]
- For each set of i, simulate two sequences, one is from a shock economy and the other is from no shock economy.

    1. In the shock economy, simulate all interested variables X_{it}^{shock} for $t = 1, ..., T_{shock} - 1$ as normal (as we do in the outer loop). Then, in period T_{shock} impose a disturbance to aggregate productivity so that it takes an extreme value (say the lowest one z_1). Simulate the economy as normal for the rest of the periods $t = T_{shock} + 1, ..., T_{sim}$.[39]
    2. In the no shock economy, simulate all interested variables $X_{it}^{noshock}$ for all the periods without any restrictions. The two economies are different in terms of only imposing the extreme shock in period T_{shock}.

- The effect of the disturbance on X is given by the average percentage (or percentage point) difference between the two sequences:

$$\hat{X}_t = 100 \times \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} \log\left(X_{it}^{shock} / X_{it}^{noshock}\right) \quad \text{(percentage difference)}$$

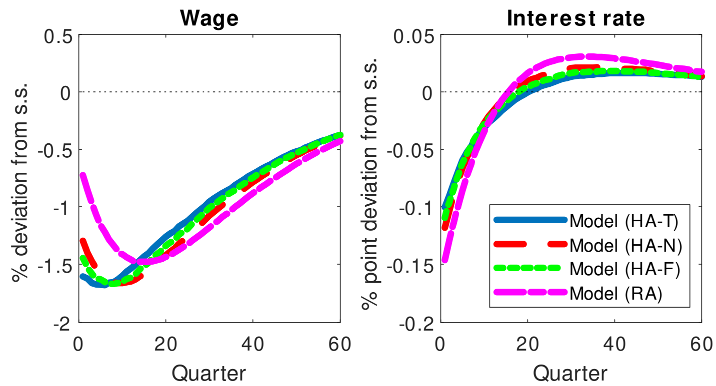
$$\hat{X}_t = 100 \times \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} \left(X_{it}^{shock} - X_{it}^{noshock}\right) \quad \text{(percentage point difference)}$$

The results are based on $N_{sim} = 2,000$ simulations and each simulation has two sequences of the interested variables for $T_{sim} = 150$ periods. The responses are equal to zero before T_{shock} by

[38] We use a random sampling with Markov chains. That is, taking the index for today's aggregate productivity i and the conditional distribution for tomorrow's productivity $\{\pi_{ij}^z\}_{j=1}^{N_z}$ (i.e., the i-th row of the Markov chain) as given, we draw a random variable $u \sim U[0,1]$ to pick up tomorrow's shock index j. We do so by choosing the highest j satisfying $u < \sum_{k=1}^{j} \pi_{ik}^z$.

[39] Note that the effect of the disturbance is *persistent* as we sample aggregate productivity using the conditional distribution of the Markov chain.

Figure A1: Impulse responses of equilibrium prices



Note: The figures display equilibrium market-clearing price responses to a negative 2 percent TFP shock with persistence ρ_z.

construction. The disturbance hits the economy at period $T_{shock} = 50$ which we label as the first period in figures.

A.6.3 Additional figures and tables

Table $A2$ reports business cycle results for several alternative model recalibrated to match the same target statistics as Table 1. First, we replace the progressive taxation in (8) with a linear taxation while keeping the average tax constant. This is helpful for understanding how important the presence of progressive taxation is for business cycles while controlling for transfer progressivity. We find that its impact is very minimal for business cycle fluctuations. The second sensitivity check is about the borrowing limit. The third column reports the results when we reduce the size of $\underline{a}$ approximately by 50% from the baseline model. The results show that aggregate fluctuations are barely affected by this change. Next, we consider a change in target statistics regarding the variability of idiosyncratic shocks. Recall that the baseline model targets the Gini wage of 0.36. We find that although its impact is not sizable, a higher wage variation tends to lower the cyclicality of average labor productivity and raise the relative volatility of hours.

Table A2: Sensitivity checks

	Baseline	Linear Taxation	$\underline{a} = -0.5T_1/(1+r)$	Gini wage = 0.35	Gini wage = 0.37
σ_Y	1.19	1.15	1.19	1.21	1.15
σ_C/σ_Y	0.24	0.26	0.24	0.24	0.25
σ_I/σ_Y	2.69	2.69	2.69	2.71	2.67
σ_L/σ_Y	0.40	0.35	0.41	0.42	0.37
σ_H/σ_Y	0.84	0.84	0.86	0.78	0.91
$\sigma_{Y/H}/\sigma_Y$	0.62	0.69	0.63	0.58	0.70
$Cor(Y,C)$	0.85	0.87	0.85	0.85	0.86
$Cor(Y,I)$	0.99	0.99	0.99	0.99	0.99
$Cor(Y,L)$	0.91	0.90	0.91	0.92	0.90
$Cor(Y,H)$	0.78	0.73	0.78	0.81	0.74
$Cor(Y,Y/H)$	0.55	0.56	0.53	0.63	0.47
$Cor(H,Y/H)$	-0.08	-0.15	-0.13	0.06	-0.25

Note: Each alternative model is recalibrated to match the same target statistics as in the baseline model.