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Aggregate and Intergenerational Implications of School Closures: A Quantitative Assessment

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Aggregate and Intergenerational Implications of School Closures: A Quantitative Assessment*

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Abstract

A majority of governments around the world unprecedentedly closed schools in response to the COVID-19 pandemic. This paper quantitatively investigates the macroeconomic and distributional consequences of school closures through intergenerational channels in the medium- and long-term. The model economy is a dynastic overlapping generations general equilibrium model in which schools, in the form of public education investments, complement parental investments in producing children's human capital. We calibrate the stationary equilibrium of the model to the U.S. economy and compute the equilibrium responses following unexpected school closure shocks. We find that school closures have moderate long-lasting adverse effects on macroeconomic aggregates such as output. In addition, we find that school closures reduce intergenerational mobility, especially among older children. Finally, we find that lower substitutability between public and parental investments induces larger damages in the aggregate economy and overall lifetime incomes of the affected children, while mitigating negative impacts on intergenerational mobility. In all findings, heterogeneous parental responses to school closures play a key role. Our results provide a quantitatively relevant dimension to consider for policymakers assessing potential costs of school closures.

Keywords: Intergenerational mobility, lifetime income, parental investments, aggregate loss, substitutability

JEL codes: E24, I24, J22

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1 Introduction

In early 2020, a majority of governments around the world unprecedentedly decided to close daycares, pre-schools, and primary and secondary schools nationwide in response to the COVID-19 pandemic. Interestingly, the extent to which governments engage in or maintain school closures varies significantly over time across countries.[1] The key to such decisions is understanding the benefits and costs of school closures during the pandemic. In this regard, there has been relatively active research on the short-run consequences of school closures, such as the benefit of keeping parents to be involved in economic activities (e.g., Alon et al. 2020) and the epidemiological risk associated with the reopening of schools (e.g., Isphording et al. 2020). However, there have been few studies that quantify and enhance the understanding of various factors behind the longer-term consequences of school closures. This line of research is important for policymakers who assess the relative costs and benefits of school closures, not only today but also as related to potential pandemics in the future.

In this paper, we quantitatively investigate the medium- and long-term aggregate and distributional consequences of school closures through intergenerational channels.[2] Specifically, we use a dynastic overlapping generations general equilibrium model where parents are linked to children through multiple transmission channels to study how study how school closures affect aggregate dynamics, inequality, and intergenerational mobility over time and across cohorts. The model economy combines a standard heterogeneous-agent incomplete-markets framework with production (Aiyagari 1994) with the model of altruistic dynasties in the tradition of Becker and Tomes (1986), while endogenizing several additional key ingredients relevant to our research questions. These include multi-stage human capital production technology for children (Cunha and Heckman 2007), where inputs include not only parental financial and time investments but also schools in the form of public investments that complement parental investments. Children become young adults with human capital and assets shaped by their parents and make their own college decisions that affect their future life-cycle wage profiles. Aggregate production combines skilled and unskilled workers along with capital to produce final outputs.

We calibrate the stationary equilibrium of the model to the U.S. economy in normal times. The stationary equilibrium of our model is consistent with various empirical features such as the increasing importance of parental financial investments over children's age, the income quintile transition matrix, and the rising income inequality over the life cycle, all of which are important for the main analysis of school closures effects. For the main quantitative analysis, we model the

[1] The United Nations Educational, Scientific and Cultural Organization (UNESCO) provides a daily map showing the global status on school closures caused by COVID-19 at https://en.unesco.org/covid19/educationresponse.

[2] We note that we do not attempt to quantify the overall effects of school closures induced by COVID-19. Instead, our study focuses on the consequences of school closures on intergenerational human capital transmission through the human capital production function, while abstracting their potential effects on parents' income or parents' human capital accumulation, among others. This is not because we believe that the other effects are irrelevant. Rather, it is because COVID-19 induced various drastic measures in addition to school closures, which makes it very difficult to empirically disentangle the partial effects of school closures on parents.

school closure shock as an unexpected temporary decline in public investments in the child human capital production (Fuchs-Schündeln et al. 2020). We then investigate the economy over the full transition equilibrium paths.[3] In particular, our rich framework naturally enables us to answer how the effects of school closures differ across child cohorts of different ages at the time of the school closure and what role the substitutability between public and private investments plays in determining the consequences of school closures.

Our first finding on aggregate consequences is that school closures have moderate yet long-lasting adverse effects on the aggregate economy. For instance, the closure of the year-long closure (including vacations) would lead to up to 0.3% decline in outputs over a number of decades to follow. In the short term, as parents' incentive to substitute for the reduced public inputs increases, aggregate capital accumulation is negatively affected, which in turn affects aggregate output negatively. More importantly, as the children directly affected by the school closure shocks enter the labor market, the decreased human capital accumulated during their childhood contributes negatively to outputs persistently in the following decades. On the other hand, we find that the adverse effects of school closures on college attainment and cross-sectional inequality are too negligible to be economically meaningful over time. We show that general equilibrium plays a very important quantitative role in mitigating the above aggregate effects. Specifically, when we fix the prices at the stationary equilibrium level, we find that output effects are overstated by 50%, and college-educated labor falls by twice as much.

We then investigate the implications of school closures for intergenerational mobility. Unlike the negligible effects on inequality, we find that the school closure shocks strengthen the extent to which income distribution is associated between parents and children. Specifically, a 1-year school closure would lower the probability of children born into the bottom income quintile moving up to the top quintile by 2-3%. We also find a significant loss (around 1%) in average lifetime income for the affected cohorts. In particular, these adverse effects on relative mobility (measured by intergenerational elasticities and the upward mobility rate) and absolute mobility (measured by average lifetime income changes) are found to be larger among older children. This is due to the temporary nature of the school closure shock. We show that although young children are more negatively affected on impact, they recover due to the equalizing effect of public education (Fernandez and Rogerson 1998) over time without school closures. We further show that both the direct impact of the school closures on the child human capital production function as well as the endogenous parental responses, featuring positive income gradients especially in financial investments for older children, underlie the above findings.

Finally, we also systematically analyze the role of substitutability between public and parental investments in producing children's human capital. Motivated by the possibility that this elasticity of substitution could vary across countries, depending on the relative importance of private versus

[3] We also confirm that our model-generated data following the school shocks are reasonably in line with the causal evidence of school closures on test scores in the Netherlands (Engzell et al. 2020) as well as time-use evidence in Germany (Grewenig et al. 2020).

public education system, we consider an alternative model economy with a lower elasticity (1.5 versus 3.0 in the baseline economy).[4] We find that although the alternative economy is able to match the important target statistics equally well, it results in school closure effects that differ substantially as compared to the baseline economy. Specifically, it generates substantially larger declines in aggregate output and lifetime income of the affected children, whereas it reduces intergenerational mobility much less. As public investments are harder to substitute, children experience greater losses in human capital during childhood, which is amplified by the weaker parental motive to compensate for the fall in human capital. This muted incentive to respond also implies a smaller parental background role, thereby generating much weaker impacts on intergenerational mobility.

Following a seminal study by Restuccia and Urrutia (2004), the literature increasingly investigates intergenerational economic persistence in quantitative macroeconomic models with heterogeneous households where the distribution of income across generations is endogenously determined. The steady-state version of our general equilibrium model herein builds on the model in Yum (2020) by allowing flexible substitutability between public and private investments – a departure from most existing papers in the literature that assume that public and parental investments are perfectly substitutable (e.g., Holter 2015, Lee and Seshadri 2019, Daruich 2020).[5]

A recent paper by Fuchs-Schündeln et al. (2020) also studies the implications of school closures in a rich two-generations lifecycle model. Although both studies share similar emphasis on the importance of parental income and children's age, as is generally the case in the literature, we view our study as complementary to theirs since the focus is different. Specifically, while they focus on implications of school closures for affected children's welfare and inequality, we focus on the implications for macroeconomic aggregates and intergenerational mobility and on the role of substitutability between public and parental investments.[6]

To the best of our knowledge, this paper is the first to conduct analysis on aggregate effects of school closures in a dynamic general equilibrium model with endogenous parental decisions. Most of such existing estimates are based on the back-of-the-envelope calculations relying on estimates of short-term learning losses and returns to education (e.g., Hanushek and Woessmann 2020). Therefore, they do not account for various factors we model in this paper, such as endogenous parental investment responses, dynamic effects on human capital, and general equilibrium considerations, among others. Overall, our estimates of the negative effects on the aggregate economy are more conservative, but are still highly relevant given that these output declines last for many decades to

[4] For example, East Asian countries generally have large private education markets, which are believed to be very good substitutes for public education. By contrast, in Scandinavian countries, where public educations play a huge role, parental education investments are less likely to be an adequate substitute.

[5] Unlike most existing studies that focus on steady-state comparisons, our quantitative exercise provides one of the few numerical implementations of the equilibrium paths over the perfect foresight transition in general equilibrium models with endogenous intergenerational human capital transmission (e.g., Lee and Seshadri 2019, Daruich 2020).

[6] Unlike Fuchs-Schündeln et al. (2020), our paper focuses on the aggregate implications that require an overlapping-generations general equilibrium framework as a natural laboratory. In Section 4, we indeed confirm that general equilibrium effects are quantitatively important for our research question.

follow.[7]

The empirical education and economics literature has shown that school interruptions can have negative consequences for children's learning and skills (e.g., Cooper et al. 1996, Meyers and Thomasson 2017). A number of papers explore learning losses in terms of test scores during summer breaks, but the evidence is somewhat mixed in terms of magnitudes (see Atteberry and McEachin (2020) and references therein). There is a growing body of empirical literature that estimates how the COVID-19 pandemic has affected parental responses using real-time data (e.g., Adams-Prassl et al. 2020, Bacher-Hicks et al. forthcoming, Chetty et al. 2020). For example, Chetty et al. (2020) find that during the school closures, children, especially those who live in low income areas, experienced reductions in math learning, measured by online Zearn Math participation. There are also empirical studies, such as Engzell et al. (2020) and Grewenig et al. (2020), which estimate these effects on learning losses and parental responses in European countries, which we discuss more extensively in Section 5. These empirical findings are broadly in line with the key mechanisms in our quantitative theory; that is, that school closures induce human capital losses, especially among children from low income families, and that parents try to compensate for these losses. Our quantitative theoretical results could help better understand the underlying sources of these empirical observations.

This paper is organized as follows. Section 2 presents the model economy and defines the equilibrium. Section 3 describes the calibration strategy and the properties of the stationary equilibrium of the calibrated model economy. Section 4 presents the main quantitative analysis of school closures along the full equilibrium transitional paths. Section 5 concludes the paper.

2 Model Economy

We begin by describing the model economy used for the quantitative analysis. It is based on the model in Yum (2020), which builds on a standard incomplete-markets general equilibrium framework in a production economy (Aiyagari 1994) while following the tradition of Becker and Tomes (1986) for intergenerational transmissions. Parents face the identical multi-period human capital production technology but are heterogeneous in assets and productivity. To enrich the analysis of school closures, our model allows the elasticity of substitution between parental and public investments to be less than perfect. In our equilibrium model with altruistic parents, parental choices such as parental investments and inter-vivos transfers take into account parents' expectations of the future paths of the economy following unexpected school closures today.

Time (t) is discrete, and a model period corresponds to five years. Our analysis not only considers steady states but also transitional dynamics across steady states. We now describe the model environments in more details.

[7] For instance, Hanushek and Woessmann (2020) computes that the year-long closure would lead to 4.3% lower GDP on average for the remainder of the century. In our model, the persistent reductions in GDP following the same closure peaks at approximately -0.3%.

Table 1: Timeline of life-cycle events for a parent-child pair

						Parent						
Age	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*	*50-54*	*55-59*	*60-64*	*65-69*	*70-74*	
Model age j	1	2	3	4	5	6	7	8	9	10	11	
	← – – – – – – – – – – – – – – – – Consumption-savings – – – – – – – – – – – – – – – – →											
	← – – – – – – – – – – – – – – – Labor supply – – – – – – – – – – – – – →									Retirement		
	College		← – Parental – → investments			Inter-vivos						
						Child						
Age			*0-4*	*5-9*	*10-14*	*15-19*	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	...
Model age j			← –– Childhood –– →				1	2	3	4	5	...
							← – – – – – – Consumption-savings – – – →					
							← – – – – – – – – –– Labor supply – – – →					
							College		← – Parental – → investments			

2.1 Households

There is a continuum (measure one) of overlapping generations in the economy. A household always includes an adult but it can also include a child. As summarized in Table 1, an adult lives for eleven model periods (age 20-74) as an active decision maker. Specifically, in the first model age $j = 1$, an agent chooses whether or not to obtain a college education. Once this higher education choice is made, the adult agent supplies labor from $j = 1$ until retirement at the beginning of $j = 10$ (age 65). The agent then lives for two more periods as a retiree and dies at the end of period $j = 11$ (age 75). In all periods, the agent makes a standard consumption-savings decision.

An important building block of our model is the intergenerational transmission. This initially happens at the beginning of $j = 3$ (age 30) when the adult is endowed with a child. In addition to the stochastic ability draw for the child, the parent invests time and money in their children in multiple periods $j = 3, 4, 5$ while taking into account the presence of public education. Before the child becomes independent, the parent decides the amount of inter-vivos transfers to give in $j = 6$. Then, the child, now an adult, forms a new household when the parent enters $j = 7$, and faces the same lifetime structure, described above.

All households share identical preferences over consumption c and hours worked n, represented by a standard separable utility function:

$$\frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi}, \tag{1}$$

where $\sigma > 0$ and $\chi > 0$ capture the curvatures and $b > 0$ is the disutility constant.

In all working-age periods ($j = 1, 2, ..., 9$), labor earnings y are subject to progressive taxation.

Specifically, after-tax earnings with respect to pre-tax earnings y are given by:

$$\lambda_j \left(y/\bar{y}\right)^{-\tau_j} y, \tag{2}$$

following a simple, yet widely used, parametric form (Benabou 2002; Heathcote, Storesletten and Violante 2014). Note that τ_j shapes the degree of progressivity, λ_j captures the scale of taxation and $\bar{y}$ denotes average earnings. We allow τ_j and λ_j to depend on age in order to capture differences in labor taxation across the family structure (Guner, Kaygusuz and Ventura 2014; Holter, Krueger and Stepanchuk 2019).

In all periods, capital income is subject to a tax rate of τ_k if the capital income is positive. Households receive lump-sum transfers T and are allowed to borrow up to the borrowing limit (Aiyagari 1994).

We now present the household's decision problems sequentially starting with the first adult age $j = 1$.

Model Age 1 In period t, a child who forms a new household in the model age $j = 1$ (20 years old) begins their adult life with individual state variables such as age j, a human capital stock of h_t, a level of asset holdings a_t, the childhood learning ability ϕ, and the aggregate state variable of the distribution of households in the economy $\boldsymbol{\pi}_t$. The two individual state variables, h_t and a_t, are endogenously shaped by the parent of the agent during childhood. Although childhood ability does not enter adults' economic decisions directly, it is still a state variable because it determines the learning ability of their own child later in $j = 3$. The distribution of households in period t, $\boldsymbol{\pi}_t$, is an aggregate state variable because equilibrium prices depend on the equilibrium distribution.

Given the state variables, the agent first decides whether or not to obtain a college education. The value of not going to college ($\kappa = 1$) is given by:

$$N(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi, \boldsymbol{\pi}_{t+1}) \right\} \tag{3}$$

$$\begin{aligned}
\text{subject to} \quad c_t + a_{t+1} &\leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + P_t + T_t \\
P_t &= \left(1 + r_t(\boldsymbol{\pi}_t)\right) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} \\
h_{t+1} &= \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\
\kappa &= 1 \\
\boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t),
\end{aligned}$$

where $w_{\kappa,t}(\boldsymbol{\pi}_t)$ is the rental price of human capital for skill type κ per unit hours of work, $r_t(\boldsymbol{\pi}_t)$ is the real interest rate, and P_t is the initial assets given by the parents (i.e., inter-vivos transfers). Human capital increases at the gross growth rate of $\gamma_{j,\kappa}$, which is allowed to depend on age j and education κ to capture the empirical age-profile of wage for each education type. Human capital is subject to the idiosyncratic shock z, which follows an independent and identically distributed (i.i.d.) normal distribution with mean zero and the standard deviation of σ_z. We assume a standard incomplete-markets structure by assuming that the idiosyncratic shock z is not fully insurable as a is not a state-contingent asset. $\Gamma(\boldsymbol{\pi}_t)$ captures the law of motion for the distribution of households as perceived by households, which should be consistent with the actual evolution of the distribution in equilibrium. Because h_{t+1} is uncertain in period t, households form expectation regarding the next period's value.

An alternative choice is to go to college and become a skilled worker. College education is costly and requires the agent to pay a stochastic fixed cost where ξ follows an i.i.d. log normal distribution with a mean of μ_ξ and a standard deviation of σ_ξ. The value of going to college after the realization of ξ is given by:

$$C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

$$\begin{aligned} \text{subject to} \quad c_t + a_{t+1} + \xi &\leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + P_t + T_t \\ P_t &= (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\ \kappa &= 2 \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned} \tag{4}$$

The above conditional decision problem illustrates how college education could benefit households in the model. First, college educated workers are in the skilled labor market ($\kappa = 2$), which gives $w_{\kappa,t}(\boldsymbol{\pi}_t)$. Second, college-educated workers experience a life cycle profile of wages that differs from that of their counterparts without a college degree through $\gamma_{j,\kappa}$.

Given the above two conditional value functions, households make a discrete college choice after observing a draw of ξ. The expected value at the beginning of $j = 1$ is:

$$V_1(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \mathbb{E}_\xi \max\left\{N(h_t, a_t, \phi; \boldsymbol{\pi}_t), C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t)\right\}. \tag{5}$$

Model Age 2 In $j = 2$, households face a standard life cycle problem with consumption-savings and labor supply decisions, represented by the following:

$$V_2(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1},\phi'|\phi} V_3(h_{t+1}, a_{t+1}, \kappa, \phi'; \boldsymbol{\pi}_{t+1}) \right\}$$

$$\text{subject to} \quad c_t + a_{t+1} \leq \lambda_2 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_2} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t$$

$$h_{t+1} = \exp(z_{t+1}) \gamma_{2,\kappa} h_t$$

$$\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t).$$

The higher education decision made in $j = 1$ shows up as a state variable κ. Because a child is going to be born in the next period, households take expectation over the ability of the new child to be born (ϕ'). We assume that it is correlated across generations, following an AR(1) process in logs

$$\log \phi' = \rho_\phi \log \phi + \epsilon_\phi \tag{6}$$

where $\epsilon_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$. This form of the exogenous source of a positive correlation of human capital across generations is standard in the literature (e.g., Restuccia and Urrutia 2004; Lee and Seshadri 2019; Yum 2020), capturing any intergenerational persistence, such as genetic transmission, not endogenously explained by the model.

Model Ages 3-5 At the beginning of $j = 3$, a child is born with learning ability ϕ. Building on the childhood skill formation literature (Cunha and Heckman 2007, Caucutt and Lochner 2020), human capital formation is modeled as a multi-stage process that takes place in $j = 3, 4, 5$, featuring parental inputs in different periods that are complementary and parental investments that are more effective for those who have higher current human capital stock. In addition, we also introduce public investments in different stages, which are complementary inputs to parental investments, to capture the effects of schools (Fuchs-Schündeln et al. 2020).

The structure is similar to those in Lee and Seshadri (2019) and Yum (2020). Specifically, let I_j denote the total investment inputs in period j, aggregated following the two nested constant elasticity of substitution (CES) technology:

$$I_j = \left\{ \left(\theta_j^x \left(\frac{x_j}{\bar{x}} \right)^{\zeta_j} + (1 - \theta_j^x) \left(\frac{e_j}{\bar{e}} \right)^{\zeta_j} \right)^{\frac{\psi}{\zeta_j}} + \left(\frac{g_j}{\bar{g}} \right)^{\psi} \right\}^{\frac{1}{\psi}}, \tag{7}$$

where x_j denotes parental time investments, e_j is parental monetary investments, g_j denotes public education investment, and $\theta_j^x \in (0, 1)$ captures the relative share of time investments in period j. Each input is entered after being normalized by its unconditional mean. The first CES aggregation is about parental time and money inputs. The elasticity of substitution between parental time and money investments depends on the stage j and is given by $1/(1 - \zeta_j)$, where $\zeta_j \leq 1$. The second CES aggregation is about the aggregated parental inputs and public investments. There, we allow the elasticity of substitution to be less than perfect, which is given by $1/(1 - \psi)$, where $\psi \leq 1$. Although this departure from perfect substitutability is relatively unexplored, we are going to show that this elasticity is highly relevant to the implications of school closures in various dimensions, as analyzed systematically in Section 5.

The aggregated inputs in different periods $j = 3, 4, 5$ shape the child's human capital at the end of $j = 5$. In other words, $h_{c,6}$, is given by the technology f :

$$h_{c,6} = \phi f(I_3, I_4, I_5). \tag{8}$$

As is standard in the literature, we assume unit elasticity of substitution across periods and constant returns to scale (e.g., Lee and Seshadri 2019, Fuchs-Schündeln et al. 2020, Yum 2020). This is captured by the following recursive formulation:

$$\begin{aligned} h_{c,j+1} &= \phi I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I}, \quad \text{if } j = 5; \\ &= I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I}, \quad \text{if } j = 3, 4, \end{aligned} \tag{9}$$

where $\theta_j^I \in (0, 1)$. Note that this technology features two properties highlighted by Cunha and Heckman (2007) and Caucutt and Lochner (2020): (i) dynamic complementarity, meaning that a higher $h_{c,j}$ increases the productivity of investments in period j ($\frac{\partial^2 f}{\partial I_i \partial h_{c,j}} > 0$) and (ii) self-productivity, meaning that a higher $h_{c,j}$ increases human capital in the next period $h_{c,j+1}$. The initial human capital h_c in $j = 3$ when a child is just born is set to 1 as we allow for heterogeneity in learning ability ϕ (Lee and Seshadri 2019).

We now incorporate the above technology into the decision problem of parents. The following functional equation summarizes a parent's problem in $j = 3$:

$$V_3(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t, e_t \geq 0;\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_4(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

$$\begin{aligned} \text{subject to} \quad & c_t + a_{t+1} + e_t \leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ & x_t + n_t \leq 1 \\ & h_{t+1} = \exp(z_{t+1}) \gamma_{3,\kappa} h_t \\ & h_{c,t+1} = \left\{ \left(\theta_3^x \left(x_t/\bar{x}\right)^{\zeta_3} + (1-\theta_3^x)\left(e_t/\bar{e}\right)^{\zeta_3} \right)^{\frac{\psi}{\zeta_3}} + (g_3/\bar{g})^{\psi} \right\}^{\frac{\theta_3^I}{\psi}} h_{c,t}^{1-\theta_3^I} \qquad (10) \\ & \boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t). \qquad (11) \end{aligned}$$

We assume that the child shares the household consumption c, captured by the household equivalence scale q. (10) is obtained by combining (7) and (9). Parents decide how much time and money to invest, while taking into account the returns to such investments, according to the production technology (8), the associated costs in terms of utility φ, and the reduced income available for consumption and savings.

The parent's decision problems in $j = 4, 5$ are similarly given by:

$$V_j(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t, e_t \geq 0;\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_{j+1}(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

$$\begin{aligned}
\text{subject to} \quad & c_t + a_{t+1} + e_t \leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
& x_t + n_t \leq 1 \\
& h_{t+1} = \exp(z_{t+1}) \gamma_{j,\kappa} h_t \\
& h_{c,t+1} = \left\{ \left(\theta_4^x \left(x_t/\bar{x}\right)^{\zeta_4} + (1-\theta_4^x) \left(e_t/\bar{e}\right)^{\zeta_4} \right)^{\frac{\psi}{\zeta_4}} + (g_4/\bar{g})^{\psi} \right\}^{\frac{\theta_4^I}{\psi}} h_{c,t}^{1-\theta_4^I} \quad \text{if } j = 4 \qquad (12) \\
& \qquad\;\; = \phi \left\{ \left(\theta_5^x \left(x_t/\bar{x}\right)^{\zeta_5} + (1-\theta_5^x) \left(e_t/\bar{e}\right)^{\zeta_5} \right)^{\frac{\psi}{\zeta_5}} + (g_5/\bar{g})^{\psi} \right\}^{\frac{\theta_5^I}{\psi}} h_{c,t}^{1-\theta_5^I} \quad \text{if } j = 5 \qquad (13) \\
& \boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t).
\end{aligned}$$

where state variables further include the child's human capital level at the beginning of the period h_c.

Model Age 6 At the end of $j = 6$, the child leaves the original household and forms a new household. The asset level of the newly formed household is shaped by the parents' decision on inter-vivos transfers a_c. Holding other things constant, this would facilitate the child's college decision indirectly by alleviating the financial burden of college. The decision problem in $j = 6$ is summarized by:

$$V_6(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1] \\ a_c' \geq [0, \bar{a}_c]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} \left[V_7(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) + \eta V_1(h_c', a_c', \phi; \boldsymbol{\pi}_{t+1}) \right] \right\} \qquad (14)$$

$$\begin{aligned}
\text{subject to} \quad & c_t + a_{t+1} + a_c' \leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
& h_{t+1} = \exp(z_{t+1}) \gamma_{6,\kappa} h_t \\
& h_c' = \gamma_c h_{c,t} \\
& \boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t).
\end{aligned}$$

Note that the continuation value now includes the initial value function of the child V_1, defined above in (5), discounted by the degree of altruism $\eta > 0$. This continuation value clearly shows our dynastic set-up, where parents care about their child's utility, which in turn depends on the following generations' utilities in the spirit of Becker and Tomes (1986). Note also that parents cannot borrow from their child's future income since a_c' cannot be negative.

Model Ages 7-11 In periods $j = 7$ and onwards, the state variables do not include h_c and ϕ because there is no need to keep track of these after the child leaves the original household. Until they retire in $j = 10$, households make consumption-savings and labor supply decisions. Hence, the household's problems in $j = 7, 8, 9$ are standard:

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_{j+1}(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\}, \quad \text{if } j = 7, 8, 9 \tag{15}$$

$$\begin{aligned} \text{subject to} \quad & c_t + a_{t+1} \leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T \\ & h_{t+1} = \exp(z_{t+1}) \gamma_{j,\kappa} h_t \\ & \boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

After retirement, households receive social security pension benefits Ω_t. The value functions in the retirement periods ($j = 10, 11$) are given by:

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{c_t \geq 0;\ a_{t+1} \geq \underline{a}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V_{j+1}(h_t, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\} \tag{16}$$

$$\begin{aligned} \text{subject to} \quad & c_t + a_{t+1} \leq (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t + \Omega_t \\ & \boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t), \end{aligned}$$

and $V_{j=12}(\cdot) = 0$.

2.2 Firm's Problem and Government

There is a representative firm that produces output with technology featuring constant returns to scale and nested CES specifications. Specifically, we assume that output is given by the Cobb-Douglas function:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \tag{17}$$

where K_t is the aggregate capital stock, H_t is the aggregate labor input, and $\alpha \in (0,1)$. The aggregate labor input H is then aggregated under the CES technology following:

$$H_t = \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}}, \tag{18}$$

where $\rho < 1$ shapes the elasticity of substitution $(1/(1-\rho))$ between skilled workers H_2 and unskilled workers H_1.

Given the above production technology, the representative firm in competitive markets maximizes profits. One can easily show that the optimality conditions are given by:

$$\alpha K_t^{\alpha-1} H_t^{1-\alpha} = r + \delta \tag{19}$$

$$(1-\alpha) K_t^{\alpha} H_t^{-\alpha} \frac{1}{\rho} \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}-1} \nu \rho H_{1,t}^{\rho-1} = w_{1,t} \tag{20}$$

$$(1-\alpha) K_t^{\alpha} H_t^{-\alpha} \frac{1}{\rho} \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}-1} (1-\nu) \rho H_{2,t}^{\rho-1} = w_{2,t}, \tag{21}$$

where δ is the capital depreciation rate.

The government collects taxes from households through (progressive) labor income taxation and capital income taxation. These tax revenues are spent on four categories: (i) social security pension Ω to retirees; (ii) lump-sum transfers T to all households, (iii) public education expenditures $\{g_j\}_{j=3}^{5}$; and (iv) government spending $G \geq 0$ that is not valued by households. We assume that the government balances its budget each period j.

2.3 Equilibrium

Let us denote by $x_{j,t} \in X_j$ a vector of individual state variables at age j in period t in the household's recursive problems described in the previous subsection. Given an initial distribution $\boldsymbol{\pi}_{-T} \equiv (\pi_{j,-T})_{j=1}^{11}$, a competitive general equilibrium is a sequence of factor prices $\{w_{1,t}(\boldsymbol{\pi}_t), w_{2,t}(\boldsymbol{\pi}_t), r_t(\boldsymbol{\pi}_t)\}_{t=-T}^{\infty}$, the household's decision rules, value functions $\left\{\{V_j(x_{j,t}, \boldsymbol{\pi}_t)\}_{j=1}^{11}\right\}_{t=-T}^{\infty}$, government policies including $\{(g_{j,t})_{j=3}^{5}\}_{t=-T}^{\infty}$, and distributions $\{(\pi_{j,t}(\cdot))_{j=1}^{11}\}_{t=-T}^{\infty}$ over $x_{j,t}$ such that:

1. given the government policies and factor prices, household decision rules solve the associated household's life cycle problems in the previous subsection, and $V_j(x_{j,t}, \boldsymbol{\pi}_t)$ are the associated value functions;

2. factor prices are competitively determined according to (19), (20), and (21);

3. market clears;

$$K_t = \sum_{j=1}^{11} \int a_{j,t} d\pi_{j,t}(x_{j,t})$$

$$H_{s,t} = \sum_{j=1}^{11} \int h_{j,t} n_{j,t}(x_{j,t}, \boldsymbol{\pi}_t) d\pi_{j,t}(x_{j,t}|\kappa = s), s = 1, 2;$$

4. the government budget is balanced for each period: the sum of transfers payments, social security pension payments, public education spending, and government spending is equal to the sum of labor income tax revenues and capital income tax revenues for each period;

5. the evolution of the distribution $\boldsymbol{\pi}_t$ is given by $\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t)$, which is consistent with the household optimal choices and the exogenous probability distributions.

Note that this competitive equilibrium nests its stationary version of equilibrium where market-clearing prices and aggregate quantities are constant over time.

3 Calibrating the Model Economy in Stationary Equilibrium

Before we evaluate the aggregate and intergenerational implications of school closures using numerical experiments in the next section, we discuss how we calibrate the model economy. Our approach is to calibrate the model in stationary equilibrium to U.S. data.

We consider two model economies in which the elasticity of substitution between public and parental investments differs. There is limited evidence of this in the literature. A number of papers assume perfect substitutability whereas a few papers estimate that this elasticity of substitution is less than perfect.[8] Given that there is no clear consensus on this parameter that could be useful for understanding the theoretical mechanisms we study here, we consider two different values. Specifically, the baseline economy uses $\psi = 2/3$, implying that the elasticity of substitution is 3. This implies that public and parental investments are highly substitutable, close to a common assumption of perfect substitutability in the literature, yet are less than perfect substitutes.[9] In addition, we also consider an alternative model economy with $\psi = 1/3$, implying a lower value of the elasticity of substitution (1.5). This alternative model would enable us to investigate the role of the elasticity of substitution between public and private investments, which could differ across countries.

We first discuss the parameter values that are commonly set across the two model economies. Then, we explain the remaining parameters that are internally calibrated to match the relevant

[8] In the literature, it is common to assume that parental and public investments are perfect substitutes. For example, see Restuccia and Urrutia (2004), Holter (2015), Lee and Seshadri (2019), Daruich (2020), Yum (2020) among others. On the other hand, there are lower estimates of this elasticity of substitution, such as 1.92 by Blankenau and Youderian (2015) and 2.43 by Kotera and Seshadri (2017).

[9] This value is similar to the one in Fuchs-Schündeln et al. (2020).

target statistics in the U.S. We then present the properties of the baseline model economy in stationary equilibrium before we conduct numerical experiments on school closures in the next section.

3.1 Common Parameters

We adopt a standard approach to match relevant U.S. statistics externally and internally. We first discuss the first set of parameters that are calibrated externally. These are also commonly set across the two model economies that vary in terms of the elasticity of substitution between public and parental investments.

First, for preference parameters, we set the value of σ equal to 1.5 such that the intertemporal elasticity of substitution for consumption is 2/3 and set the value of χ equal to 4/3 such that the Frisch elasticity is 0.75 (Chetty et al. 2013). Because our model frequency is five years, the relevant margin of labor supply adjustments includes both intensive and extensive margins. The value of q, which determines how consumption enters into utility in the presence of a child in the household is set to 1.59, based on the OECD equivalence scale. Next, we set $\bar{a}_c$ to be 25% of average income, which is close to the exemption limit of gift tax in the U.S.

The life cycle wage profiles for high- and low-skilled workers are governed by the gross growth rates of human capital during adulthood $\{\gamma_{j,\kappa}\}_{j=1}^{8}$. These values are computed based on Rupert and Zanella's (2015) estimates from the Panel Study of Income Dynamics (PSID). As reported in Table $A1$, these estimates show two notable patterns: (i) for each education group, the growth rates are higher in the early adult periods and then decline with age, and (ii) college-educated workers experience much higher growth rates.

We now discuss parameters related to government. Recall that the degree of progressivity in labor taxation differs based on household structure in the model. As reported in Table $A2$, progressivity tends to be higher for households with a child. The capital income tax rate τ_k is set to 0.36. These taxation-related parameters are based on the estimates by Holter et al. (2019). The next parameters are related to the public education expenditures. Here, we follow the approach used by Restuccia and Urrutia (2004) and Holter (2015): education expenditures by state and federal governments are defined as public investments, while those by local government are considered to be private investments. This is because education in the U.S. is financed locally depending on where people live. Using the information in 2016 from the Education at Glance published by the OECD, we compute the g_j in $j = 3, 4, 5$ relative to steady-state output per capita to be 0.060, 0.098 and 0.111, respectively.[10] A key feature of g_j is that it increases as achild progresses through education stages. Next, following Lee and Seshadri (2019), the value for government lump-sum transfers T is set to 2% of steady-state output per capita to capture welfare programs, and the borrowing limit is set as $\underline{a} = -T/(1+r)$, where r is the real interest rate in the steady state. The value of Ω is set

[10] These values are in line with estimates by Holter (2015) and Lee and Seshadri (2019), the latter of who use micro data (PSID-CDS).

Table 2: Internally calibrated parameters and target statistics for the baseline model economy

Parameter		Target statistics	Data	Model
β	.972	Equilibrium real interest rate (annualized)	0.04	0.04
b	23.3	Mean hours of work in $j = 3, ..., 9$	.287	.287
η	.323	Mean inter-vivos transfers/GDP per-capita	.056	.057
θ_3^x	.753	Mean parental time investments in $j = 3$	.061	.060
θ_4^x	.397	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.259	Mean parental time investments in $j = 5$	.020	.020
θ_3^I	.571	Mean parental monetary investments in $j = 3$	.098	.098
θ_4^I	.349	Mean parental monetary investments in $j = 4$	.113	.111
θ_5^I	.229	Mean parental monetary investments in $j = 5$	.128	.125
ζ_3	-1.78	Educational gradients in parental time in $j = 3$ (%)	20.9	20.5
ζ_4	-0.19	Educational gradients in parental time in $j = 4$ (%)	14.8	15.1
ζ_5	-0.12	Educational gradients in parental time in $j = 5$ (%)	20.2	20.5
ν	.529	Fraction with a college degree (%)	34.2	34.2
μ_ξ	.244	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.732	Observed college wage gap (%)	75.0	85.8
ρ_ϕ	.103	Intergenerational corr of percentile-rank income	.341	.342
σ_ϕ	.590	Gini wage	.370	.369
σ_z	.140	Slope of variance of log wage from $j = 2$ to $j = 8$	.180	.184

to imply that the social security replacement rate is 40%.

Finally, we discuss parameters related to the production sector. We set $\alpha_K = 0.36$ to be consistent with the capital share in the aggregate US data. The five-year capital depreciation rate δ is based on 2.5% of the quarterly depreciation rate. These values are standard in the literature. We set $\rho = 1/3$, implying that the elasticity of substitution between skilled and unskilled workers is 1.5 (Ciccone and Peri 2005).

3.2 Parameters Calibrated Internally

We now discuss the parameters that are calibrated internally by matching the relevant target statistics in U.S. data, given the value of ψ. The discussion herein focuses on the baseline economy with $\psi = 2/3$, as summarized in Table 2, and the Appendix provides the calibrated parameters for the model economy with $\psi = 1/3$. These parameter values are determined as minimizers of the squared sum of the distance between the relevant statistics from the data and those from the model-generated data. Although there is a relatively large number of parameters and targets, each parameter is connected to its corresponding target quite well. We now explain these relationships. All target statistics reported in Table 2 are constructed and discussed in details by Yum (2020).

The first parameter in Table 2 is β, which captures the household's discount factor. Its relevant target is chosen to be the annual interest rate of 4%. The next parameter b is the disutility constant for labor supply. Its relevant target is chosen to be the mean hours worked by those aged between 30 and 65 (or $j = 3, ..., 9$). Assuming that the weekly feasible time endowment is $105(= 15 \times 7)$ hours, excluding sleeping time and basic personal care, this statistic in the data yields $30.16/105 = 0.287$ as a target. There is a disutility parameter φ for parental time investments. This parameter is linked to B such that the marginal disutility of parental time investment is given by the marginal disutility of work evaluated at the mean hours worked.[11] Next, η governs the degree of altruism and is calibrated to match the mean inter-vivos transfers. Because inter-vivos transfers in the model are meant to capture financial help for college, we choose the total parental transfers made for children during the college years.[12] As a result, we obtain a target statistic of 0.056 – the ratio of the mean parental financial transfers to the five-year GDP per-capita.

We now discuss parameters related to the child human capital production functions. Recall that in each j, there are three parameters—θ_j^x, θ_j^I and ζ_j—in (10), (12) and (13). We calibrate these parameters by exploiting the clear linkages between each of these parameters and its corresponding target moment in the model economy. Specifically, θ_j^x captures the relative importance of parental time investments (vs. parental financial investments), and it clearly increases the mean parental time investments in period j, which are used as target statistics. Statistics on parental time investments are obtained from the 2003-2017 American Time Use Survey (ATUS) only with educational, interactive activities that require the presence of both a parent and a child in a common space.[13] A key feature of these moments is that the mean time investment is highest in the earliest period $j = 3$ (0.061 in the model or 6.4 hours per week) and it decreases with children's age. The next parameter θ_j^I increases overall parental investments in period j. Hence, we use the mean private education spending in each period as a target moment for θ_j^I. As discussed above, the mean private education expenditure in the data is constructed as the sum of private spending and local government spending because public schools are largely funded locally in the U.S. Consequently, we obtain the target statistics of $0.098, 0.113$ and 0.128 for $j = 3, 4$ and 5, respectively. Note that, unlike the parental time inputs, parental financial inputs increase with children's education stage, in line with the existing evidence. Finally, ζ_j shapes the elasticity of substitution between time and money in period j. These are calibrated to match the salient facts in the U.S. that more educated parents spend more time with children (Guryan et al. 2008; Ramey and Ramey 2010). Specifically, we allow our model to replicate the fact that parents who are college-educated spend around 20 percent more time with their children than those without a college degree.[14] In particular, we allow the elasticity of substitution to be j-dependent since the same elasticity of substitution would lead to a lower educational gradient in early periods (Yum 2020). As a result, our calibration leads to a lower elasticity of substitution in $j = 3$ (0.36) than in later periods (0.84 and 0.89 in $j = 4$ and 5,

[11]Specifically, φ is given by $b\bar{n}^{\chi}$. We calibrate θ_j^x to match the mean parental time investments in j, as described below.

[12]Specifically, we sum the money from parents and college transfers from age 18 to 26, reported in Table 4 of Johnson (2013), while accounting for the fraction of recipients.

[13]Such activities include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities.

[14]To be precise, the education gradient is defined as the percentage difference in mean parental time investments between education groups while controlling for parental observables. See the Appendix for details.

respectively), implying that parental time and monetary investments are especially complementary to each other when children are very young. Although not reported in Table 2, the parameter γ_c that maps childhood human capital to adulthood human capital is also internally calibrated to be 3.03 such that the steady-state output per capita is normalized to 1.

The next parameters are related to college education. In the aggregate production function (18), ν is calibrated to match the fraction of people with a college degree (34.2%). The mean of college costs is determined by μ_ξ, which naturally gives a target statistic: the equilibrium ratio of the mean (tuition and non-tuition) expenses after financial aid to per capita GDP. According to detailed procedures explained by Yum (2020), this statistic (relative to the five-year GDP) is 0.140. The next parameter is is related to the variance of the college costs. Note that as σ_ξ increases, the observed wage premium would decline since college decisions are more strongly shaped by costs relative to pre-college human capital. Therefore, its relevant target is set to be the observed college wage premium of 75% (Heathcote et al. 2010).

Next, ρ_ϕ determines the persistence of exogenous ability across generations. We set its relevant target as the rank correlation of family income of 0.341 (Chetty et al. 2014). Note that Chetty et al. (2014) estimate intergenerational persistence using a proxy income variable instead of lifetime income due to the data limitation, as is common in the literature. Therefore, our target statistic from the model also uses proxy income.[15] The last two parameters in Table 2 govern the variability of wages in different ways. Although either would increase the overall wage inequality in the model, the variability of the idiosyncratic shocks to adult human capital σ_z also shapes the rising lifecycle inequality. Therefore, the two target statistics are the Gini coefficient of wage and the difference between the variance of log wage at age 55-59 ($j = 2$) and that of log wage at age 25-29 ($j = 8$), as reported in Table 2 (Heathcote et al. 2010).

The alternative model with a lower elasticity of substitution between public and parental investments ($\psi = 1/3$) is calibrated using the same calibration strategy. The calibration results are reported in the Appendix.[16]

3.3 Properties of the Baseline Model in Stationary Equilibrium

In this subsection, we present the properties of the baseline model in stationary equilibrium before we conduct the main quantitative analysis on school closures.

We first evaluate the intergenerational mobility implied by the model. Specifically, we measure the model-implied intergenerational mobility in three ways and compared them to the data counterparts. The data counterparts are from Chetty et al. (2014) who use administrative data.[17] As

[15] Specifically, Chetty et al. (2014) measure a child's income at around 30 years old, averaged over two years. The parent's income is averaged over five years when parents' ages are around 45 years. Equivalently, our model-based proxy income is measured for parents in $j = 6$, and for children in $j = 3$.

[16] The most notable difference, compared to the baseline model with $\pi = 2/3$, is that the elasticity of substitution between parental time and money investments is lower (0.22, 0.58 and 0.57 for $j = 3, 4$, and 5, respectively).

[17] Specifically, parental income is defined as the average five-year pre-tax income per parent, which is either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability

Table 3: Intergenerational persistence estimates

	U.S. data	Model	
	Chetty et al. (2014)	Proxy income	Lifetime income
IGE: log-log slope	0.344	.309	.375
Rank corr: rank-rank slope	0.341	.342	.361

mentioned above, income in the model is the five-year per parent sum of labor earnings, interest income, and social security benefits.

The first measure is the intergenerational elasticity (IGE), obtained from the following log-log equation:

$$\mathcal{Y}_{child} = \rho_0 + \rho_1 \mathcal{Y}_{parent} + \varepsilon, \tag{22}$$

where $\mathcal{Y}$ is log permanent income. This is a conventional way to measure the degree of intergenerational persistence in the empirical literature. Its interpretation is straightforward: a 1% increase in parental permanent income is associated with a ρ_1% increase in their children's permanent income. The second measure is to use a rank-rank specification instead of a log-log specification (Chetty et al. 2014). This can be estimated when $\mathcal{Y}$ is the percentile rank of income. This slope coefficient (or the rank correlation) tells us that a one percentage point increase in parent's percentile rank is associated with a ρ_1 percentage point increase in their children's percentile rank. In the model, we estimate these slopes using both proxy income, which is defined equivalently as its empirical counterpart, and the lifetime income, which is constructed as present-value lifetime income discounted according to the interest rate (Haider and Solon 2006) in stationary equilibrium.

Table 3 reports the two slope estimates from the data and the model. Recall that we directly targeted to match the rank correlation using proxy income. Although data limitation prevents researchers from investigating the lifetime income, it is possible to estimate the mobility measures using the lifetime income in the model. As is well known in the literature, we can see that the estimate of the IGE using lifetime income (0.376) is substantially larger than the counterpart using proxy income (0.309) because the short-term income may not represent the long-term lifetime income (Haider and Solon 2006). Interestingly, this attenuation bias is smaller in the rank correlation (0.342 versus 0.361).

The above slope estimates are easy to interpret and convenient, but they do not fully describe how income distribution persists across generations. The income quintile transition matrix provides a richer description of how economic status is transmitted across generations.[18] We now compare

benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits. For children's income, they use a short horizon (2-year average) due to data availability.

[18] An income quintile transition matrix is a 5 by 5 matrix where the (a, b) element provides the conditional probability that a child's lifetime income is in the b-th quintile, conditional on the parent's income belonging to the a-th

Table 4: Income quintile transition matrices: data vs. model

Unit: %	U.S. data					Model									
	Chetty et al. (2014)					Proxy income					Lifetime income				
Parent	Child quintile					Child quintile					Child quintile				
quintile	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	35.2	25.8	17.7	14.1	7.2	35.9	27.0	16.1	14.3	6.8
2nd	24.2	24.2	21.7	17.6	12.3	24.5	22.2	21.7	18.1	13.5	25.1	22.1	21.7	18.2	12.9
3rd	17.8	19.8	22.1	22.0	18.3	18.9	19.2	22.6	20.6	18.7	19.4	18.5	23.6	20.0	18.5
4th	13.4	16.0	20.9	24.4	25.4	14.0	18.1	20.2	22.4	25.3	13.4	17.6	21.6	21.9	25.6
5th	10.9	11.9	17.0	23.6	36.5	7.3	14.8	17.7	24.9	35.3	6.4	14.9	16.9	25.5	36.3

the quintile transition matrix from the model-generated data to the empirical quintile transition matrix (Chetty et al. 2014). Because calibration does not directly target any elements in the income quintile transition matrix, this is a natural way of evaluating how successful a model is as a quantitative theory of intergenerational mobility (Yum 2020).[19]

Table 4 reports the transition matrices, obtained from U.S. (Chetty et al. 2014) and model-generated data. The data shows that the probability of children remaining in the bottom quintile when their parents' income is also in the bottom quintile is 33.7%. Similarly, the probability of staying in the top income quintile is quite high at 36.5%. A particularly interesting one is the probability of moving up from the bottom quintile to the top quintile, namely upward mobility. In the data, the upward mobility rate is 7.5%. The middle panel of Table 4 displays the quintile transition matrix from the model when the equivalent measure of proxy income is used. The model successfully replicates the empirical patterns noted above. In particular, the upward mobility rate in the model is 7.2%, which is very close to the data counterpart.

Table 4 also reports the quintile transition matrix using lifetime income. Compared to the one with proxy income, we can see that the diagonal elements are generally higher, which is consistent with lower intergenerational mobility measured by the slope coefficients in Table 3. The upward mobility rate in terms of lifetime income is slightly lower at 6.8%. In the following numerical experiments, we use the intergenerational mobility measures based on lifetime income because the mobility measures based on proxy income are subject to attenuation biases (Haider and Solon 2006) as also confirmed by the model-generated data in stationary equilibrium.

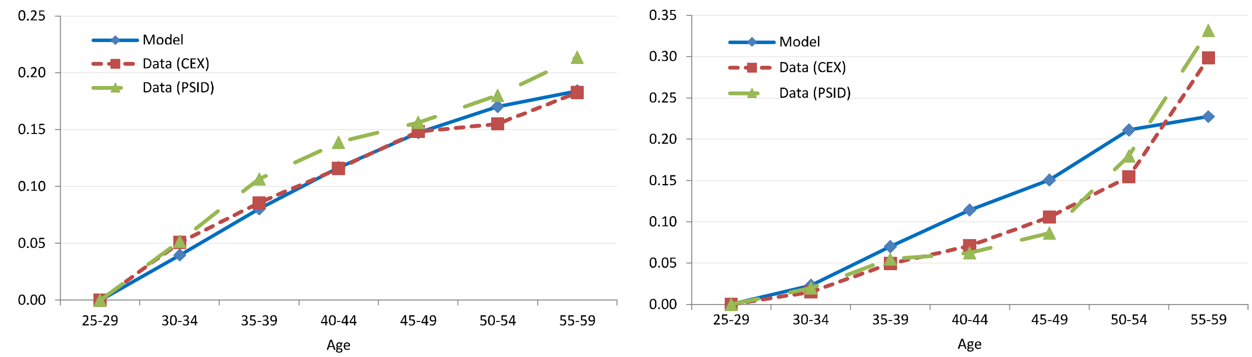
As is well known, cross-sectional inequality in labor market variables tends to increase over the lifecycle in the data (e.g., Heathcote et al. 2010). As Figure 1 shows, the model replicates the increasing dispersion in wages (left) and earnings (right) quite well.[20] We note that these features are important because a higher dispersion in income among relatively older parents would

quintile. Quintiles are based on their own generation.

[19] Note that the same correlation of income across generations can be consistent with different quintile transition matrices. This is similar to the fact that the same Gini coefficient can be consistent with different shapes of income distributions.

[20] Note that this is disciplined mainly by the calibrated dispersion in idiosyncratic shocks to adult human capital.

Figure 1: Inequality over the life cycle



Note: The left figure shows the variance of log wage by age relative to age 25-29. The right figure shows the variance of log wage by age relative to age 25-29. US data is from Heathcote et al. (2010).

be transmitted into the extent to which parents with different permanent incomes afford additional parental investments in response to school closures.

4 Quantitative Analysis of School Closures

We now move on to the main analysis of this paper on the implications of school closures. This requires us to compute the equilibrium away from the steady state. We first explain how we conduct the numerical experiments and then briefly discuss empirical consistency with the best existing evidence on the short-run effects of school closures. Afterwards, our main analyses on the medium- and long-run effects follow.

4.1 Computational Experiment Design

In this section, we analyze transitional dynamics following unexpected school closure shocks. In the simulation, in each period, the economy consists of 11 cohorts, and each cohort is composed of 500,000 household units. Thus, the total number of households is 5,500,000 in each period t. We first simulate the model economy for sufficiently long periods until it reaches the stationary equilibrium.[21] The economy is in stationary equilibrium at $t = ..., -2, -1, 0$, and school closures unexpectedly take place at the beginning of $t = 1$. Our baseline exercise considers universal, nationwide school closures where all schools are closed for the same period of time.[22] As in Fuchs-

[21] Specifically, we simulate 55 periods to reach the steady state from a given initial distribution and drop the first 50 periods. We keep the five periods of the steady state economy to keep information about parents whose children directly experience school closures.

[22] In the Appendix, we also examine the effects of *partial* school closures where there is a stochastic difference in closure lenghs across households. This exercise reflects the fact that there could be regional variations in the effective length of school closures, caused by the uncertain local pandemic progress and political factors not modeled herein.

Schündeln et al. (2020), we represent these school closures by reducing the size of public investments in the child human capital production according to the closure length. For example, if a school closure lasts for one year, we reduce one-fifth the public investments in $t = 1$. We consider three different lengths of school closures: 0.5, 1 and 1.5 years. We note that our notion of school closure length should be interpreted in terms of academic years (AY), and should be mapped to the actual days of school closures with caution due to the presence of breaks, even in normal times.[23] In $t = 2, 3, ...$, there are no further shocks and the economy returns to the original stationary equilibrium.[24] We compute the transitional equilibrium paths under perfect foresight.

In addition to the consequences of school closures on macro aggregates such as output, our analysis also focuses on heterogeneous impacts on children of different ages in which the school closure shock hits the economy. Therefore, we will also present the results for three child cohorts that directly experience the school closure in different ages: the cohort aged between 0 and 4 (*Cohort 1* or C1) at the school closure; that aged between 5 and 9 (*Cohort 2* or C2); and that aged between 10 and 14 (*Cohort 3* or C3). We also keep track of parents matched to these children to examine intergenerational implications.

4.2 Quantitative Results

Consistency with short-run evidence on school closures Since most governments (including the U.S. government) closed schools in early 2020 in response to the COVID-19 pandemic, there has been limited empirical evidence on the direct effects of such closures on the general child performance even in the short run.[25] Although there has been suggestive evidence to indicate significant drops in the amount of learning (Chetty et al. 2020), the lack of data prevents researchers from investigating the negative consequences of learning loss in a broader setting with causal interpretations. Ideally, we would need to have observations on a large number of representative students whose academic progress (e.g., in terms of test scores) in multiple points within a year is observed, not only in the regular year but also during the pandemic period when schools were almost universally closed.

An exception is Engzell et al. (2020) who use a rich nationally representative data set from the Netherlands. Their data set satisfies all of the ideal settings mentioned above, thereby allowing them

[23] For example, as 4-5 months of vacation already exist in normal years, the school closure of 1-year length would correspond to the actual days of closure for 7-8 months (including weekends). One might think that 1.5 AY is not realistic, but given that the current forecast projects that the vaccine is going to be widely available only in 2022, we think that it is still worth considering as an extreme case. Further, it helps us to investigate potential nonlinearity in the effects. Finally, we note that, although school closures reduce g_j for children's human capital production, they are not changed in the government budget because these shocks are meant to capture non-permanent school closures.

[24] Although shocks are temporary and relatively small, it is important to run the model economy long enough for several reasons. First, as our key variable is lifetime income, we need to generate the whole life-cycle for the youngest cohort that directly experienced the school closures. In addition, as we show below, school closure shocks have long-lasting effects. In our exercises, we use $t = 30$.

[25] The empirical literature on the learning loss during summer break (Cooper et al. 1996, Atteberry and McEachin 2020) could be useful, although it might be nontrivial to apply the summer break effects to the effects of closing during regular school periods, especially at longer horizons.

to conduct a different-in-difference estimation. According to their estimates based on composite scores aggregating math, reading and spelling scores for the students aged 7-11, they estimate a learning loss of about 3.1 percentile points or 0.08 standard deviations during the lockdown which induced school closures of 2 to 2.5 months. Although child human capital in the model does not exactly correspond to the observed test scores, it is useful to compare how school closures affect human capital loss in the model. In our model, we find that a 0.5-year closure leads to a human capital loss of 2.4 percentile points or 0.07 standard deviations.[26] In addition, we also find a larger fall in children's human capital with lower parental permanent income (Figure 6), in line with their findings that parental education is the only significant factor shaping the negative impacts. This comparison shows that our model generates reasonable magnitudes of negative impacts on the children's outcomes.[27]

As discussed below, parental responses to school closures are an important channel that not only mitigates the aggregate effects but also impacts intergenerational effects. A recent paper by Grewenig et al. (2020) provides interesting results related to our findings. They use a survey in Germany with detailed time use information and find that children reduced their daily learning time significantly during school closures. More interestingly, they also find that the reduction in learning time was not statistically different by parental education or income. This is in fact consistent with our finding below that the positive income gradients in parental responses materialize in terms of money, not in terms of time (Figure 5).[28]

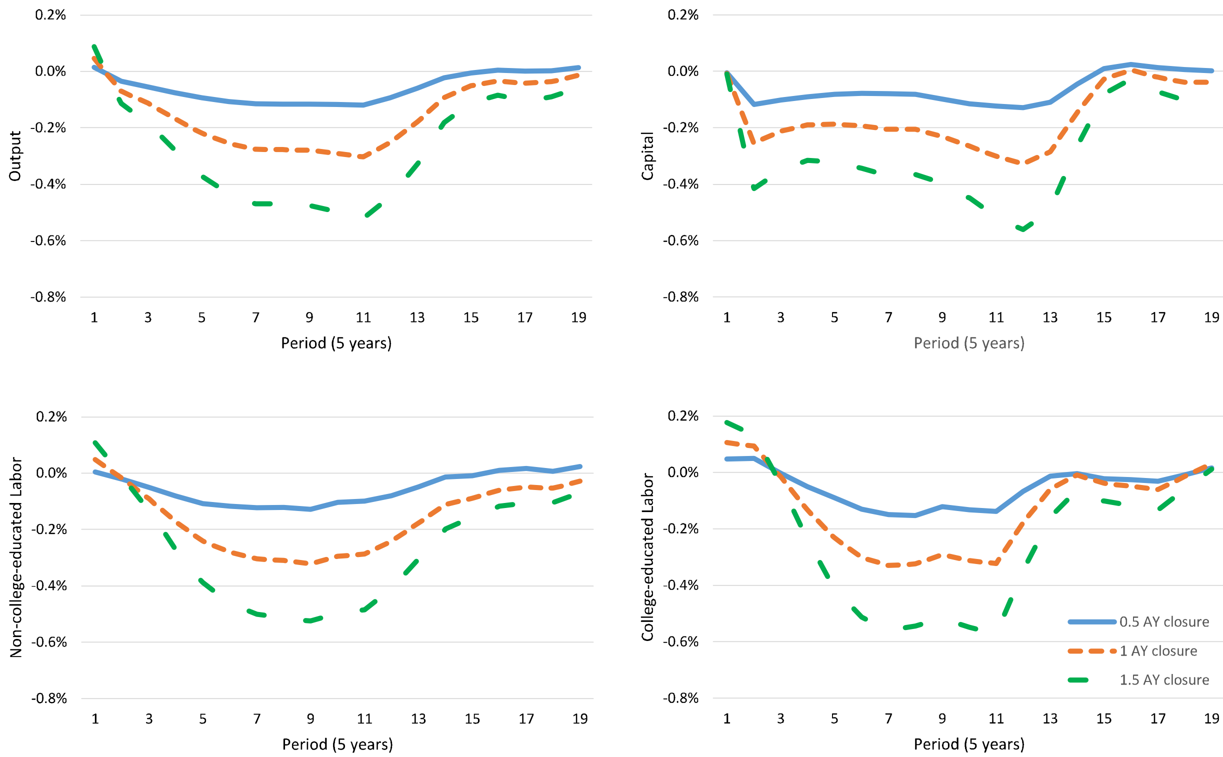
Aggregate Implications We now present the main systematic results from the quantitative exercises. Figure 2 plots the dynamics of output, capital, efficiency units of labor for non-college and college graduates following unexpected school closures of different lengths in $t = 1$. Overall, the changes of these aggregate variables are small yet persistent. The top-left panel shows that the aggregate output declines gradually over time, and this decline continues until period 11. The top-right panel implies that the initial drop in output is due to dissaving to increase parental investments. This reduction in capital is amplified over time by lower human capital formations of those who experienced the school closures during their childhood. The bottom panels suggest that parents increase their labor supply to earn more income, thus raising parental investments to counter school closures. The aggregate efficiency unit of labor for each skill type starts to decrease when the cohorts, experiencing these school closures during childhood, enter the labor market with lower levels of human capital. This reduction in the aggregate labor continues to decline until $t = 11$ and gradually recovers afterward.

[26] Note that the 0.5-year-closure in the model should approximately correspond to 3.5 months closure net of summer breaks and holidays.

[27] Recall that human capital in our model is supposed to be a broader concept than test scores and that our model allows any compensatory parental investments within a model period of five years, which might dampen the very short-run loss right after school closure shocks.

[28] For example, parents could spend more on better tablets or online resources of higher quality (Bacher-Hicks et al. forthcoming), which would increase the efficiency of learning, but not necessarily the time spent on these activities.

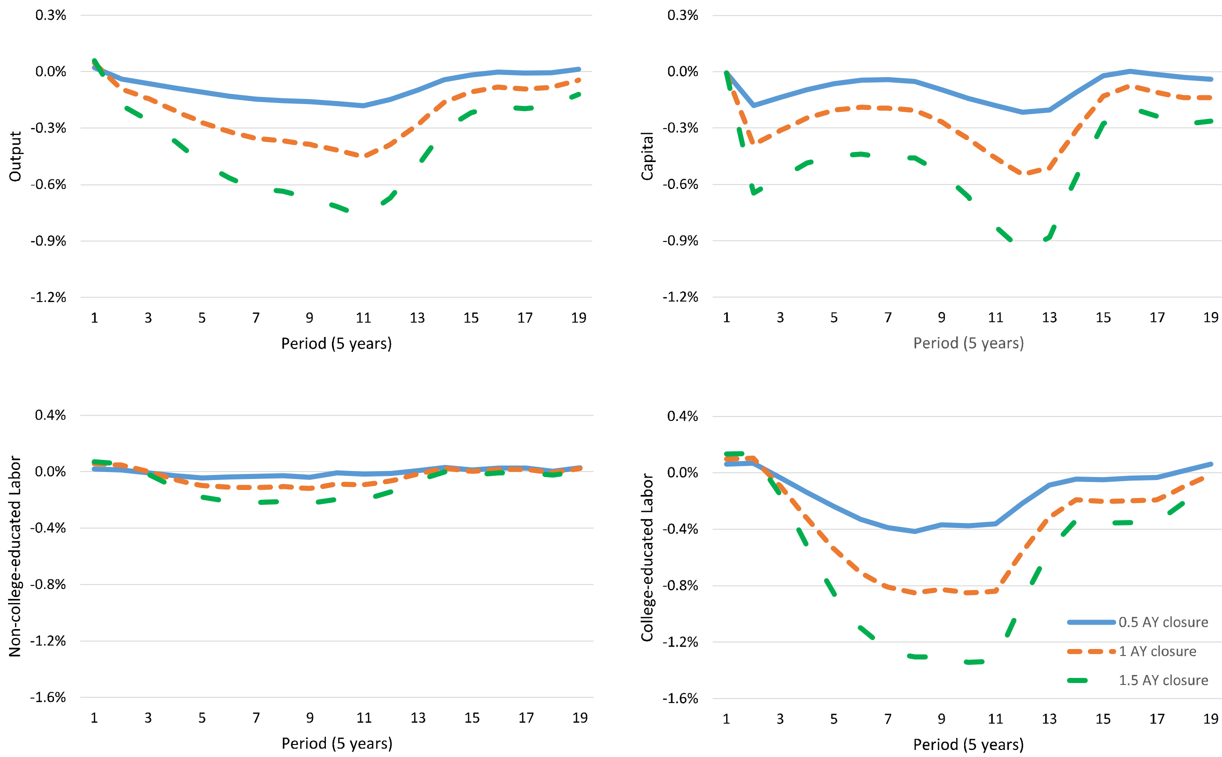
Figure 2: Evolution of macroeconomic aggregates



Another noticeable feature is that the responses of the aggregate variables are non-linear to the length of school closures. The top-left panel of Figure 2 demonstrates that in period 11, while the 0.5-year-closure reduces output by less than 0.1%, the 1.5-year-closure decreases output by more than 0.4%. The top-right panel shows that the 1.5-year-closure reduces capital three times more than does the 0.5-year-closure. These non-linear responses also appear in the aggregate labor responses, as shown in the bottom panels. These findings imply that, since school closures would have small, non-linear, and long-lasting impacts on the aggregate economy, these impacts might be difficult to forecast empirically in a reduced-form way.

Another noticeable feature we highlight is the general equilibrium effects that play a role in adjusting the magnitude of the responses of these aggregate variables to the school closures. In particular, as revealed by a comparison between Figures 2 and 3, these general equilibrium effects tend to balance the responses of the efficiency units of labor between college and non-college graduates. Specifically, Figure 3 shows that when prices are fixed at their stationary equilibrium levels, aggregate labor for college graduates is more significantly reduced in response to these school closures. A change in efficiency units of labor in each education group can be driven by (i) the fraction of the skill group relative to population (extensive margin), (ii) hours worked conditional

Figure 3: Evolution of macroeconomic aggregates without GE feedback



Note: Factor prices are fixed at stationary equilibrium levels

on working (intensive margin), and (iii) the quality of the work force (human capital). The large reduction in the efficiency units of labor for the college-educated individuals that materializes gradually over time is due to the direct loss of human capital and a relatively noticeable decrease in college attainment indirectly driven by lower child human capital, both of which were caused by school closures. By contrast, we see that labor efficiency for non-college graduates does not decline as much because the reduction in human capital is offset by an increased number of people who do not go to college in the case where prices are exogenously fixed. However, in general equilibrium, the decrease in college attainment tends to increase the relative premium of college graduates, thereby dampening the reductions in the efficiency units of labor for college graduates and amplifying those for non-college graduates. Similarly, general equilibrium effects mitigate the reductions in aggregate capital by increasing the risk-free interest rate. Consequently, general equilibrium effects moderate the overall responses of output to these school closure shocks.

We now move on to the distributional changes over time. Table 5 reports the effects of school closures on three cross-sectional inequality measures, demonstrating that school closure shocks bring about negligible changes in cross-sectional inequalities. In the 0.5-year-closure scenario, there is almost no change in the Gini coefficient of current income for the first three periods and there is

Table 5: Distributional changes over time

		Time (1 period: 5 years)				
		1	2	3	4	5
	Steady state	**% change** rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.341	0.0	0.0	0.0	0.1	0.1
Bottom 20% inc (%)	7.73	-0.0	-0.0	-0.1	-0.1	-0.0
Share of college (%)	34.2	0.0	0.0	0.0	-0.0	-0.0
Closure length: 1 AY						
Gini income	.341	0.0	-0.0	0.1	0.1	0.1
Bottom 20% inc (%)	7.73	-0.0	0.0	-0.1	-0.1	-0.0
Share of college (%)	34.2	0.0	0.1	0.0	-0.1	-0.1
Closure length: 1.5 AY						
Gini income	.341	0.0	-0.1	0.1	0.2	0.2
Bottom 20% inc (%)	7.73	-0.1	0.0	-0.1	-0.2	-0.1
Share of college (%)	34.2	0.0	0.1	-0.0	-0.1	-0.2

an increase of at most 0.1% in the last two periods. Just as the income share held by the lowest 20 percent shows no significant change for five periods, so does the share of college graduates. Although longer school closures result in stronger impacts on cross-sectional inequalities, the magnitude is still insignificant. Compared to the steady state, the economy with the 1.5-year-closure generates differences in the absolute value of the Gini income coefficient near 0.1% until $t = 3$ and at most 0.2% in the last two periods. This little difference also appears in both the income share held by the lowest 20 percent and the share of college graduates. However, this finding does not necessarily imply that school closures have limited impacts on changes in inequality across generations, which we investigate next.

Intergenerational Implications Table 6 reports that the school closure shocks reduce intergenerational mobility quite substantially. Compared to the steady state, the 0.5-year-closure increases the IGE by 0.5 to 1.0% and the rank correlation by 0.4 to 0.9%, while decreasing the upward mobility by 0.9 to 1.6% across cohorts. These changes are amplified by the length of school closures. Across cohorts, the 1.5-year-closure generates increases in the IGE and rank correlation three times as large as the 0.5-year-closure. Likewise, the 1.5-year-closure reduces the upward mobility two and a half to three times more than does the 0.5-year-closure.

Note that the school closure effects on intergenerational mobility are quantitatively heterogeneous across cohorts: the older cohorts are, the more reduced intergenerational mobility is. While

Table 6: Effects on intergenerational mobility of lifetime income

	IGE			Rank cor.			Upward Mobility		
Steady state	.375			.361			6.8%		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.5	0.8	1.0	0.4	0.7	0.9	-0.9	-1.3	-1.6
1.0 AY	1.0	1.6	2.0	0.9	1.5	1.8	-1.6	-2.6	-2.7
1.5 AY	1.5	2.5	3.0	1.3	2.2	2.8	-2.4	-4.0	-4.3

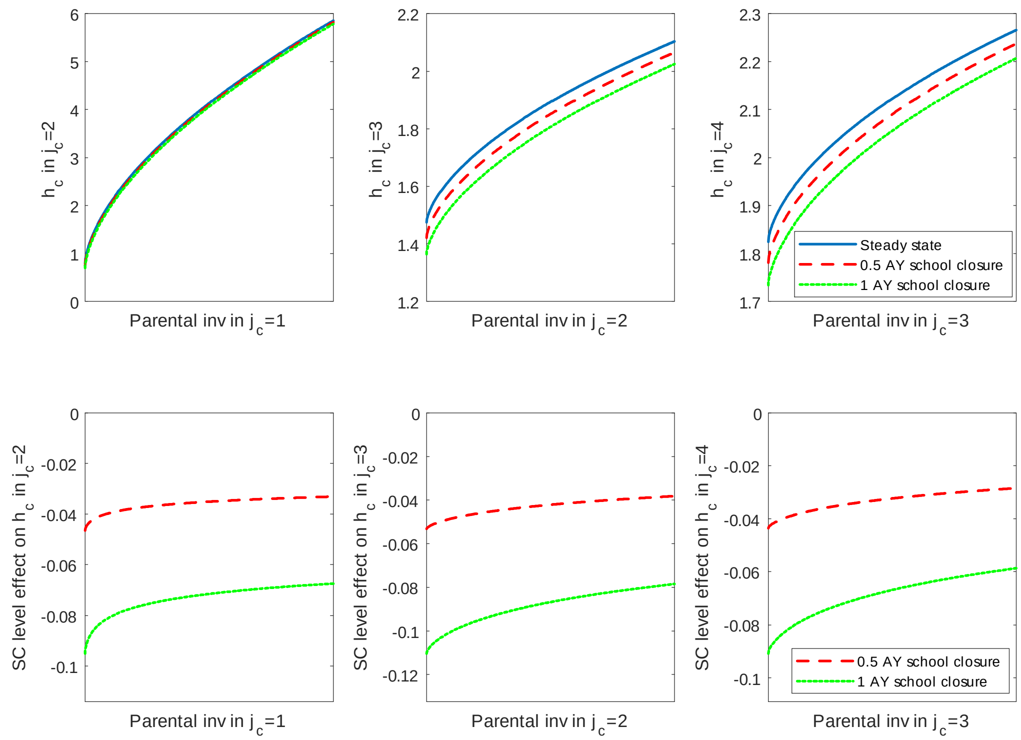
the 1-year-closure increases the IGE by 1% for C1, it does so by 2% for C3. The rank correlation also has similar differences across cohorts. In response to the 1-year school closure, the rank correlation increase for C3 is three times that for C1. Similarly, given a school closure, older cohorts suffer from a greater reduction in upward mobility. The 1-year-closure decreases C1's upward mobility rate by 1.6% but C3's by 2.7%. These patterns are preserved regardless of the length of school closures. Both the 0.5-year-closure and the 1.5-year-closure lead older cohorts to experience greater reductions in upward mobility and larger increases in IGE and rank correlation.

To understand these intergenerational implications, it is useful to first examine the *direct* effects of school closures on the human capital production function. For this purpose, Figure 4 plots the effects of changes in g_j on the level of human capital produced for an average child as a function of parental investments aggregated from time and money with the calibrated parameters. Note that because parental investments are largely shaped by income, the horizontal axis can be interpreted as the parental socioeconomic status (SES).

There are several noticeable features. First, longer school closures bring about greater reductions in child human capital. Second, within a cohort, parents with low SES experience greater reductions in child human capital. Since the portion of public investment g_j is greater for lower SES parents, they are more adversely affected by school closures. In addition, this parental SES gap tends to increase with the child's age. These features suggest that overall damages to child human capital increase with the length of school closures; children from parents with low SES experience larger damages than those with high SES parents and this difference is greater for older cohorts.

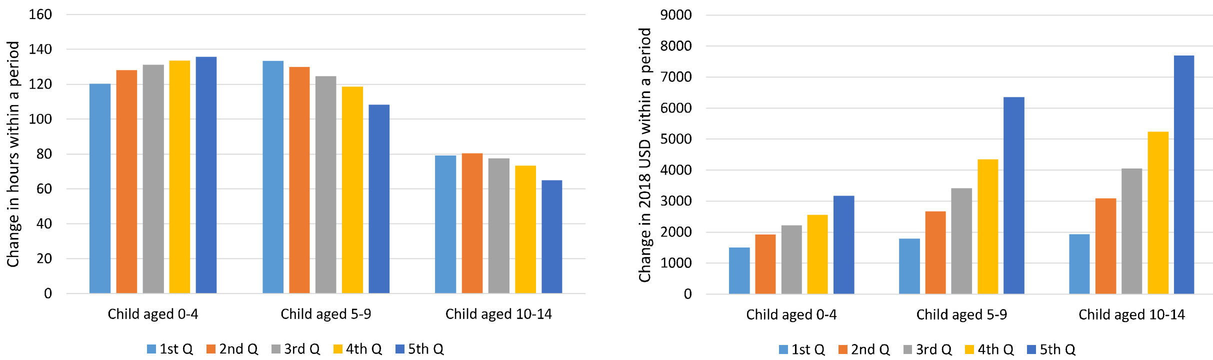
In addition to the direct effects of school closures, the other important mechanism is related to endogenous parental responses: parents have incentives to respond to this reduced child human capital following school closures by increasing their parental investments. Note that parental investment behaviors are different according to their children's age. As shown in Table 2, the importance of financial relative to time investments increases with children's age, in line with estimates by Del Boca et al. (2014). These calibration results imply that parental investments in time

Figure 4: Direct effects of school closures on the child human capital production function



Note: The figures plot the j_c-period human capital output for children with average human capital in j_c in stationary equilibrium and $\phi = 1$. The range of parental investments in the horizontal axis are a composite of parental investments when parental time and money vary from 0 to twice the mean values in stationary equilibrium. The upper panels show production levels with different school closure lengths. The bottom panels plot changes in level relative to the case without school closures.

Figure 5: Parental responses by parental permanent income



Note: A set of five bars plots changes in parental investments by the quintile of parent's permanent income for each cohort, ordered by the child's age during the 1-year school closure. The left shows time investment responses and the right shows monetary investment responses.

are more crucial in forming human capital in the very early childhood period (C1), but parental financial investments become more important in later periods (C2 and C3). In addition, the degree of complementarity between time and monetary investments is much stronger in C1 than in C2 and C3.

This age-dependent human capital production technology brings about differences in the composition of parental investments according to the child's age. Figure 5 presents the parental responses to the 1-year-closure by parental lifetime income (or permanent income). Although the average time investment response is smaller, the monetary one is larger for older children (C2 and C3). Note that when children are aged between 0 and 4, richer parents invest in time more than poorer parents do, but this gap is small because time constraints are more equally distributed across parents than budget ones. The richer parents cannot easily compensate financially for the lack of time investments, as monetary investments are not as effective as or easily substitutable for time investments for children in the early period. In the later periods, as financial investments become more important, richer parents substitute time between the two more than poor parents do. Further, note that financial investments can better substitute time investments for older children (due to lower elasticities of substitution) and that parents' income dispersion increases with age, which would show up as greater dispersions in financial investments for older children, as demonstrated in Figure 1. These jointly result in substantial positive income gradients in monetary investment responses for the older children cohorts (C2 and C3).

These heterogeneous parental investments play an important role in generating disparities in child human capital formations. Recall that, as demonstrated by Figure 4, all three cohorts experience a reduction in human capital due to the direct effects of school closures, but children with low-income parents are disproportionately affected. In addition, the heterogeneous parental

Table 7: Effects on inequality and loss of lifetime income

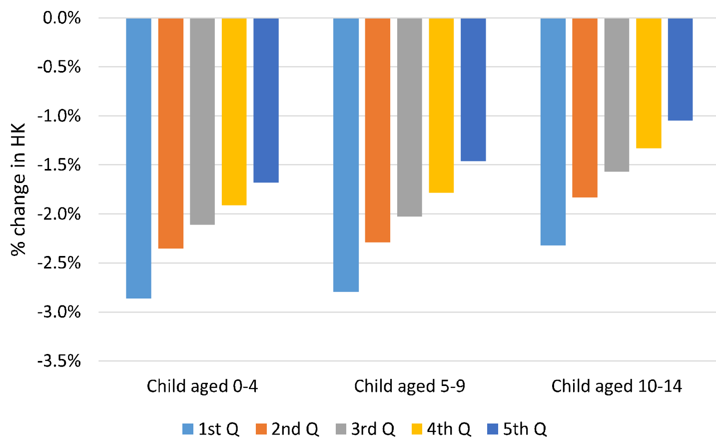
	Lifetime income						Fraction of		
	Gini index			Average			College-educated		
Steady state	.293			4.15			.342		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.0	0.1	-0.4	-0.7	-0.6	-0.2	-0.4	-0.4
1.0 AY	0.1	0.1	0.2	-0.9	-1.3	-1.3	-0.3	-0.7	-0.8
1.5 AY	0.1	0.2	0.2	-1.3	-2.0	-2.0	-0.4	-1.0	-1.3

investments discussed above amplify these differences. For the older cohorts (C2 and C3), larger differences in parental monetary investments lead to greater disparities in the changes of human capital across parental income groups, which in turn shapes intergenerational mobility. As a result, intergenerational mobility decreases more in the older cohorts (C2 and C3).

Next, we investigate how school closures influence the overall economic status (or absolute mobility) by cohort and the dispersion of lifetime income within cohorts. Table 7 reports the effects of school closures on the average and inequality of lifetime income. While these school closures have small impacts on lifetime income inequality, the average reveals substantial losses. Specifically, the 0.5-year-closure increases the lifetime income Gini coefficient by less than 0.1% over cohorts. The effect on lifetime inequality is still negligible for the longer school closures. The two longer school closures increase the Gini coefficient by 0.1 to 0.2%, while resulting in sizeable lifetime income losses. The 0.5-year-closure reduces average lifetime income by 0.4 to 0.7%; the magnitude increases with the length of school closures.

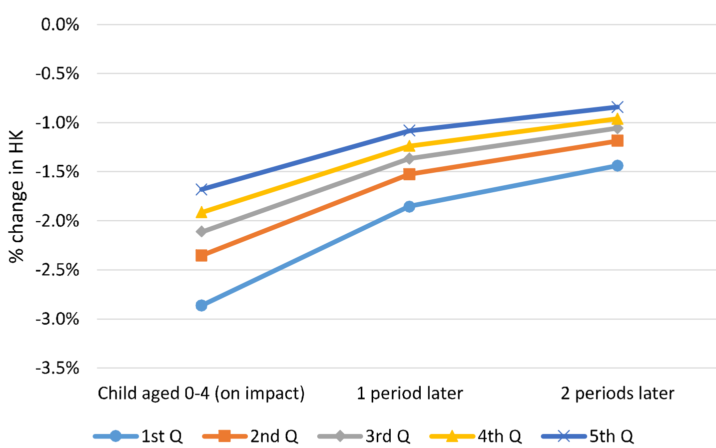
Note that school closures bring about more significant losses in lifetime income for older children (C2 and C3) than for the youngest cohort (C1). This is indeed not necessarily inconsistent with the literature highlighting the importance of early childhood in human capital formation (Heckman 2008). On impact, school closures have the strongest substantial impacts on child human capital for the youngest pre-school-aged cohort (C1), as shown in Figure 6. Although this might seem inconsistent with the above results on lifetime income loss, it becomes clearer when we recall that the school shocks are temporary. As can be seen in Figure 7, differences in losses in human capital for C1 become narrower over time, as the public investments play an equalizing role (Fernandez and Rogerson 1998). In fact, this narrowing gap in school closures' negative consequences is closely related to the direct effect of school closures on the human capital production function that is heterogeneous across parental SESs. Specifically, Figure 4 shows that, although child human capital with a lower parental SES is more adversely affected, corresponding marginal productivity of

Figure 6: Child human capital in the next period by parental permanent income



Note: A set of five bars plots percent changes in the next period human capital on impact by the quintile of parent's permanent income for each cohort, ordered by the child's age during the 1-year school closure.

Figure 7: Effects of school closures on child human capital (initially aged 0-4) over time, by parental permanent income



Note: A model period corresponds to five years. This figure plots percent changes in the next period human capital (relative to the case without school closure shocks) of children who are affected by school closures when $j_c = 1$ (age 0-4).

aggregate investments, measured by the slope of the below graph becomes greater. While these young children experience gains in the next coming years, children from lower parental SES benefits the most, thereby narrowing the gap over time. This results in differences in the changes of college attainment across cohorts. The fraction of college-educated is reduced more in C2 and C3 than in C1.

4.3 The role of the elasticity of substitution between public and private investments

To examine the role of the elasticity of substitution between public and private investments, we consider an alternative model economy with a lower elasticity of substitution (1.5 or $\psi = 1/3$) than the baseline economy (3.0 or $\psi = 2/3$) and recalibrate the model to match the same set of target statistics presented in Table 2. We note that this elasticity of substitution, $1/(1-\psi)$, could reflect education systems that vary across countries. For example, it is likely that Scandinavian countries where public services play a major role in education would have a lower elasticity of substitution than East Asian countries, such as South Korea, where private education is prevalent and large in market size (Kim et al. 2020). Therefore, our analysis herein intends to provide useful considerations for different countries with different approaches to public and private education.

Figure 8 shows the aggregate level evolution of output, capital, efficiency units of labor for non-college and college graduates in the case with a lower elasticity of substitution. As shown in a comparison of Figure 2, although all these aggregate variables fall as in the case with a higher elasticity of substitution, the magnitudes are greater in the case with a lower elasticity of substitution. While the 1-year-closure decreases the aggregate output by up to 0.3% in the case with a higher elasticity of substitution (Figure 2), it does so by around 0.5% in the case with a lower elasticity of substitution. A decrease in aggregate capital in $t = 12$ in the case with a lower elasticity of substitution is double that of the baseline economy. Effective labor units for both non-college and college graduates also show two times greater reductions. These results suggest that school closures bring greater declines in aggregate variables for countries wherein public educational investment is difficult to substitute with private educational investment, such as Scandinavian countries.

Table 8 shows intergenerational mobility of lifetime income in the case with a lower elasticity of substitution between public and parental investments. As revealed by a comparison with Table 6, as the degree of complementarity increases (lower ψ), the effects of school closures become weaker on intergenerational mobility. In all cases with three different closure lengths, increases in the IGEs in the case with a lower elasticity of substitution is half as large as in the baseline model. Likewise, increases in the rank correlation in the case with a lower elasticity of substitution is less than those in the case with a higher elasticity of substitution. The upward mobility also displays similar patterns: the declines in the upward mobility rate in the case with a lower elasticity of substitution are smaller than those in the baseline model.

As demonstrated previously with Figures 5 and 6, for C2 and C3, the substitution of time into

Figure 8: Evolution of macroeconomic aggregates with a lower elasticity of substitution between public and parental investments

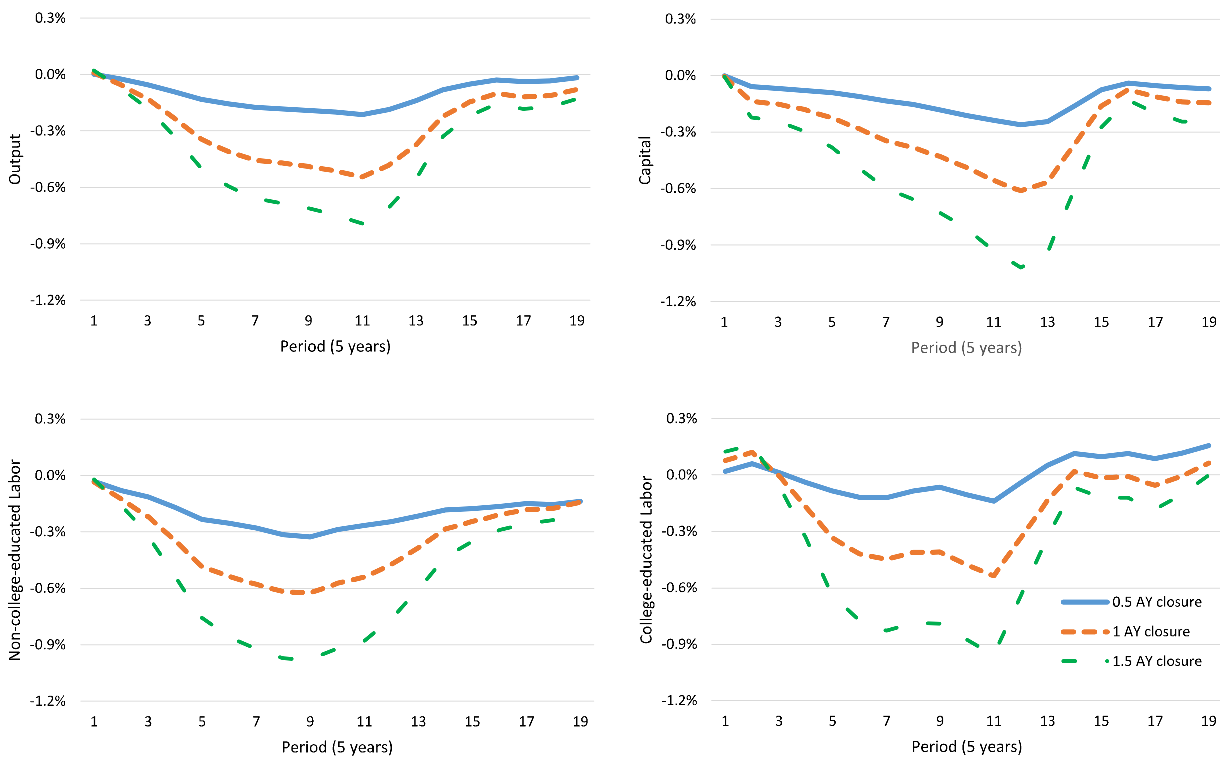
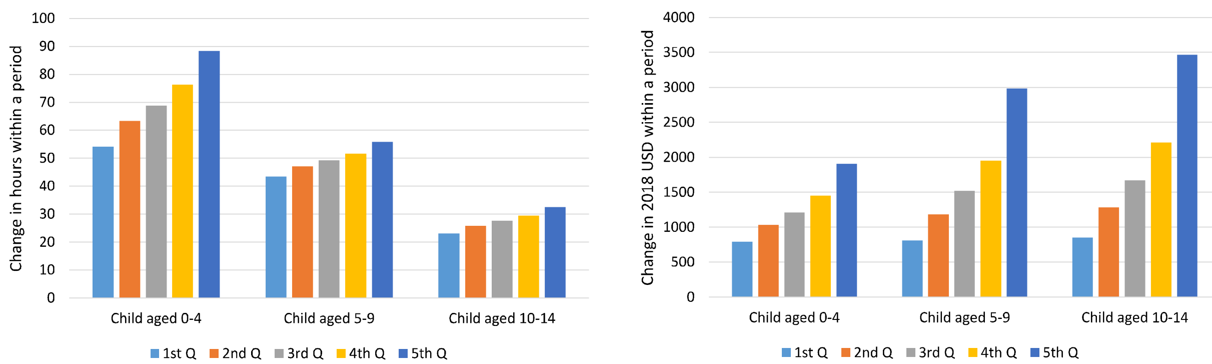


Table 8: Effects on intergenerational mobility of lifetime income with a lower elasticity of substitution between public and parental investments

	IGE			Rank cor.			Upward Mobility		
Steady state	.372			.359			6.7%		
	% **change** rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.3	0.4	0.4	0.3	0.4	0.4	-0.6	-0.5	-0.8
1.0 AY	0.7	0.9	0.9	0.6	0.7	0.9	-0.9	-0.9	-1.2
1.5 AY	1.2	1.4	1.4	0.9	1.2	1.3	-1.4	-1.7	-1.8

Figure 9: Parental responses by parental permanent income with a lower elasticity of substitution between public and parental investments



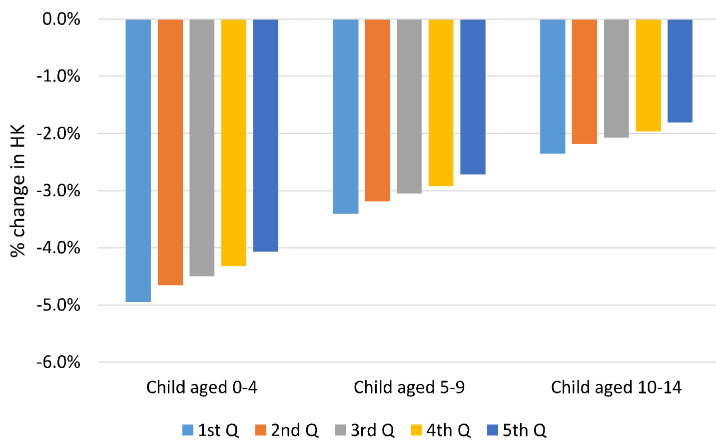
Note: A set of five bars plots changes in parental investments by the quintile of parent's permanent income for each cohort, ordered by the child's age during the 1-year school closure. The left shows time investment responses and the right shows monetary investment responses.

financial investments plays a role in increasing differences in the responses of child human capital to school closures across parental permanent income groups, thereby reducing intergenerational mobility. These differences in the case with a lower elasticity of substitution are smaller than those in the case with a higher elasticity of substitution because this lower elasticity weakens parents' incentive to compensate for school closures. As a result, Figure 9 shows that, on average, parental responses are substantially weaker in the model with a lower elasticity of substitution as compared to the baseline model. These more muted responses in parental investments result in smaller gaps in child human capital changes, as shown in a comparison between Figures 6 and 10. These findings imply that in countries where public investments are crucial and irreplaceable, school closures could have smaller impacts on intergenerational mobility.

Finally, Table 9 shows the responses of the average and inequality of lifetime income to school closures in the case with a lower elasticity of substitution. As in the baseline model (Table 7), these school closures have little impact on lifetime inequality. However, school closures generally induce larger losses in lifetime income in this model. As mentioned previously, under a lower elasticity of substitution between public and private investments, it is difficult to compensate for the lack of public investments with parental financial investments, thus increasing overall loss of child human capital, as shown in the comparison between Figures 6 and 10. Therefore, this greater reduction in child human capital leads to a larger decrease in overall college attainment and a larger drop in average lifetime income. Another interesting difference from the baseline model is that the average lifetime income loss for C1 becomes disproportionately larger, mainly caused by the much more significant initial impact on human capital, as shown in Figure 10.

To summarize, a decrease in the elasticity of substitution between public and private invest-

Figure 10: Child human capital by parental permanent income with a lower elasticity of substitution between public and parental investments



Note: A set of five bars plots percent changes in the next period human capital on impact by the quintile of parent's permanent income for each cohort, ordered by the child's age during the 1-year school closure.

Table 9: Effects on inequality and loss of lifetime income with a lower elasticity of substitution between public and parental investments

	Lifetime income						Fraction of		
	Gini			Average			College-educated		
Steady state	.291			4.15			.342		
				% **change** rel. to no school closure, by cohort					
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.1	0.1	0.1	-0.9	-0.9	-0.8	-0.3	-0.3	-0.2
1.0 AY	0.1	0.1	0.1	-1.8	-1.9	-1.7	-0.9	-0.9	-0.7
1.5 AY	0.2	0.2	0.2	-2.8	-3.0	-2.7	-1.4	-1.5	-1.3

ments leads to a larger reduction in the aggregate variables and average lifetime income but a lesser reduction in intergenerational mobility. These results are driven by reduced substitutions by parental financial investments, generating overall greater but less heterogeneous changes in child human capital across parental permanent income groups.

5 Conclusion

In this paper, we have investigated how school closures affect the aggregate economy, inequality, and intergenerational mobility through intergenerational human capital transmissions in the medium and long term. Using a dynastic overlapping generations general equilibrium model wherein altruistic parents invest in the children's human capital, which complements public schooling, we have found three main results. First, school closures bring about moderate yet long-lasting adverse effects on the aggregate economy. General equilibrium effects play a substantial role in reshaping aggregate variables' dynamics. Second, school closures reduce the average lifetime income and intergenerational mobility of directly affected children, and these reductions are more severe for older children cohorts. These results are driven mainly by parental investment responses that differ by a child's age and parental income. Finally, we have shown that substitutability between public and private investment shapes school closure costs in a non-trivial way. While a lower elasticity of substitution induces more significant damages in the aggregate economy and overall lifetime incomes of the affected children, it mitigates a reduction in intergenerational mobility. The key underlying mechanism for both changes is the dampened parental motives to compensate for the lack of public investments in the presence of lower substitution possibility.

Given these clear, interesting differences driven by substitutability between public and parental investments, we believe that school closure shocks could provide ideal opportunities to estimate the elasticity of substitution between public and private investments, which could vary across countries. The availability of more data in the near future would make it possible to perform such analysis and contribute to the literature in which there is limited empirical evidence for this substitution elasticity that we have shown to matter quantitatively not only for child human capital formations but also for aggregate dynamics. Likewise, our model framework would be useful for studying unexplored interesting research topics as data become more available and more accessible. For example, an interesting normative question is how to optimally make up for losses from school closures dynamically by adjusting the length of school operations in the near future. This more short-run oriented question would require the model to have a high frequency such as 6 months, but probably not additional forces such as general equilibrium, which could balance the computational burden. We leave these interesting related questions for future work.

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A Appendix

A.1 Calibration details

Most calibration targets are based on samples from the 2003-2017 waves of the ATUS, combined with the Current Population Survey (Yum 2020). Table $A1$ reports the estimation results that are used to compute the educational gradients in parental time investments. The sample is restricted to households who have any number of children and aged between 21 and 55 (inclusive), as in Guryan

Table A1: Education gradients in parental time investments

	$j=3$	$j=4$	$j=5$
College-educated	1.342	.561	.416
	(.133)	(.109)	(.091)
Sex	-2.62	-1.51	-1.20
	(.123)	(.101)	(.083)
Age	-.041	.016	.023
	(.009)	(.007)	(.006)
Married	-.911	-.318	-.102
	(.085)	(.064)	(.053)
R^2	.023	.014	.017
Average x	6.43	3.78	2.06

Notes: Numbers in parentheses are standard errors. The dependent variable is parental time investments (weekly hours). These estimates are from Yum (2020).

Table A2: Gross growth rates of human capital by age and education

$j=$	1	2	3	4	5	6	7	8
$\gamma_{j,1}$	1.231	1.052	1.017	1.004	0.998	0.995	0.994	0.994
$\gamma_{j,2}$	1.317	1.152	1.101	1.063	1.032	1.004	0.975	0.942

Notes: The reported values are based on the estimates from the PSID samples in Rupert and Zanella (2015).

et al. (2008). The three periods in the model ($j = 3, 4, 5$) correspond to the youngest children's age bands: ages 0-4, ages 5-9, and ages 10-14, respectively. The coefficient on the dummy college variable, divided by the corresponding average, captures the educational gradient while controlling for parents' sex, age, and marital status. We note that the college coefficients are quite stable regardless of control variables, in line with the evidence in Guryan et al. (2008).

Table $A2$ reports the gross growth rates of human capital by age and education. These are computed based on the estimates from the PSID samples in Rupert and Zanella (2015).

Table $A3$ reports the estimates of τ_j and λ_j in labor taxation by age, obtained from Holter et al. (2019). We use the estimates for single households for $j = 1, 2$, and the estimates for married households for the later periods (either with a child for $j = 3, ..., 6$ or without children for $j = 7, 8, 9$). Table $A3$ also reports the estimates of g_j. The public and private education investments are based on the 2016 information in the 2019 Education at a Glance by the OECD. We consider pre-primary as $j = 3$, primary as $j = 4$, and secondary as $j = 5$ in the model. We treat state and federal government spending as public investments while local government spending is included in the private investments (Restuccia and Urrutia 2004; Holter 2015). See Yum (2020) for more

Table A3: Parameter values for progressive taxation and public education investments

	τ_j	λ_j		g_j
$j = 1, 2$	.1106	.8177	$j = 3$	0.060
$j = 3, ..., 6$	.1585	.9408	$j = 4$	0.098
$j = 7, 8, 9$	.1080	.8740	$j = 5$	0.111

Notes: τ_j and λ_j are based on the estimates in Holter et al. (2019). Public education investments g_j are based on 2019 Education at a Glance (OECD).

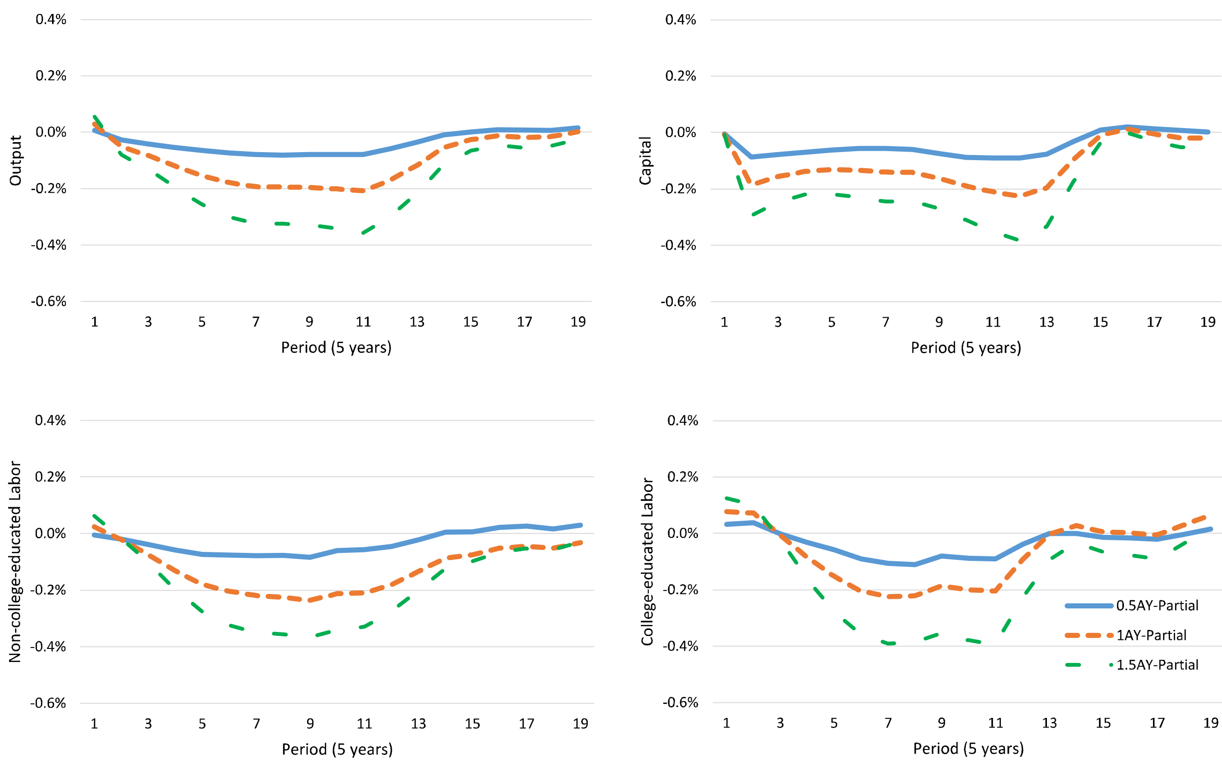
details. Although this method does not exploit micro-level data directly, it is reassuring that these estimates are generally in line with those by Lee and Seshadri (2019) who use a micro data set (PSID-CDS).

A.2 Partial (stochastic) closures

We also consider additional experiments based on partial school closures. Specifically, we assume that school closures are still unexpected but there is another dimension of uncertainty: half of the agents still experience full closures, but the other half experience a school closure of limited intensity. This within-period variation could capture additional closures due to local outbreaks of COVID-19 cases even after re-opening nationwide. This could also capture the variability of effectiveness of online substitute teaching by schools. The results reported below are based on a partial intensity of 50%. Our findings suggest that the main findings are generalizable in terms of the relationship between average school closure length and the corresponding aggregate effects. But they also suggest that partial closures induce additional variations that happen within each cohort.

A.3 Additional figures and tables

Figure A1: Evolution of macroeconomic aggregates



Note: A half of agents experience full closures whereas the other agents experience partial closures, the intensity of which is given by 50%.

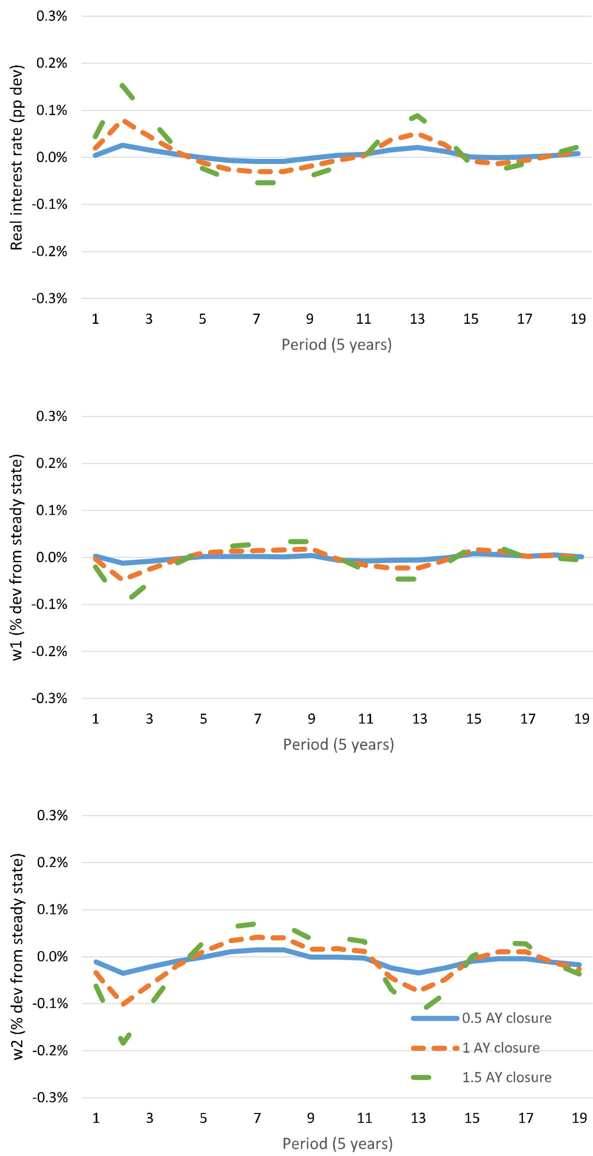
Table A4: Effects on intergenerational mobility of lifetime income: Partial closures

	IGE			Rank cor.			Upward Mobility		
Steady state	.375			.361			6.8%		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
	All children								
0.5 AY	0.4	0.6	0.7	0.3	0.6	0.7	-0.7	-1.2	-1.0
1.0 AY	0.7	1.2	1.5	0.6	1.1	1.4	-1.2	-2.2	-2.2
1.5 AY	1.1	1.8	2.2	1.0	1.7	2.1	-1.8	-3.1	-3.3
	Children who experienced full closure								
0.5 AY	0.5	0.8	1.0	0.4	0.7	0.9	-0.7	-1.5	-1.0
1.0 AY	1.0	1.6	2.0	0.9	1.5	1.8	-1.5	-2.8	-2.5
1.5 AY	1.5	2.4	3.0	1.3	2.2	2.8	-2.4	-4.3	-4.1
	Children who experienced 50% closure								
0.5 AY	0.2	0.4	0.5	0.2	0.4	0.6	-0.4	-0.9	-0.9
1.0 AY	0.5	0.8	1.0	0.4	0.7	0.9	-0.9	-1.5	-1.9
1.5 AY	0.7	1.2	1.5	0.6	1.1	1.3	-1.2	-2.0	-2.3

Table A5: Effects on inequality and loss of lifetime income: Partial closures

	Lifetime income Gini			Lifetime income Average			Fraction of College-educated		
Steady state	.293			4.15			.342		
Closure length	**% change** rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
				All children					
0.5 AY	0.0	0.0	0.1	-0.3	-0.5	-0.5	-0.2	-0.3	-0.3
1.0 AY	0.1	0.1	0.1	-0.7	-1.0	-1.0	-0.2	-0.5	-0.6
1.5 AY	0.1	0.1	0.2	-1.0	-1.5	-1.5	-0.3	-0.8	-0.9
				Children who experienced full closure					
0.5 AY	0.1	0.2	0.2	-0.4	-0.7	-0.6	-0.3	-0.5	-0.5
1.0 AY	0.1	0.1	0.1	-0.9	-1.3	-1.3	-0.5	-0.8	-1.0
1.5 AY	0.1	0.2	0.2	-1.3	-2.0	-2.0	-0.7	-1.3	-1.5
				Children who experienced 50% closure					
0.5 AY	0.1	0.1	0.1	-0.2	-0.3	-0.3	0.0	-0.1	-0.1
1.0 AY	0.1	0.1	0.1	-0.4	-0.6	-0.6	0.1	-0.1	-0.2
1.5 AY	0.1	0.1	0.1	-0.6	-1.0	-1.0	0.1	-0.3	-0.1

Figure A2: Evolution of equilibrium prices in the baseline model



Note: The top panel shows the equilibrium interests over the transition. The middle panel shows the equilibrium wages for non-college workers, and the bottom panel shows the equilibrium wages for college-educated workers over the transition

Table A6: Internally calibrated parameters and target statistics for the alternative model economy with a lower elasticity of substitution between public and parental investments

Parameter		Target statistics	Data	Model
β	.972	Equilibrium real interest rate (annualized)	0.04	0.04
b	23.4	Mean hours of work in $j = 3, ..., 9$	.287	.287
η	.322	Mean inter-vivos transfers/GDP per-capita	.056	.056
θ_3^x	.895	Mean parental time investments in $j = 3$	.061	.061
θ_4^x	.374	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.168	Mean parental time investments in $j = 5$	.020	.020
θ_3^I	.623	Mean parental monetary investments in $j = 3$	.098	.098
θ_4^I	.348	Mean parental monetary investments in $j = 4$	.113	.112
θ_5^I	.224	Mean parental monetary investments in $j = 5$	.128	.127
ζ_3	−3.55	Educational gradients in parental time in $j = 3$ (%)	20.9	20.3
ζ_4	−0.73	Educational gradients in parental time in $j = 4$ (%)	14.8	14.8
ζ_5	−0.74	Educational gradients in parental time in $j = 5$ (%)	20.2	20.1
ν	.530	Fraction with a college degree (%)	34.2	34.2
μ_ξ	.245	Average college expenses/GDP per-capita	.140	.140
δ_ξ	.732	Observed college wage gap (%)	75.0	84.9
ρ_ϕ	.141	Intergenerational corr of percentile-rank income	.341	.338
σ_ϕ	.590	Gini wage	.370	.370
σ_z	.141	Slope of variance of log wage from $j = 2$ to $j = 8$	.180	.181

Table A7: Distributional changes over time: Partial closures

		Time (1 period: 5 years)				
		1	2	3	4	5
	Steady state	**% change** rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.341	0.0	0.0	0.0	0.1	0.0
Bottom 20% inc (%)	7.73	-0.0	-0.0	-0.0	-0.0	-0.0
Share of college (%)	34.2	0.0	0.0	0.0	-0.0	-0.0
Closure length: 1 AY						
Gini income	.341	0.0	0.0	0.1	0.1	0.1
Bottom 20% inc (%)	7.73	-0.0	0.0	-0.1	-0.1	-0.0
Share of college (%)	34.2	0.0	0.1	0.0	-0.0	-0.1
Closure length: 1.5 AY						
Gini income	.341	0.0	-0.0	0.1	0.2	0.1
Bottom 20% inc (%)	7.73	-0.0	0.0	-0.1	-0.1	-0.0
Share of college (%)	34.2	0.0	0.1	-0.0	-0.1	-0.1

Table A8: Distributional changes over time with a lower elasticity of substitution between public and parental investments

$\xi = 1/3$		Time (1 period: 5 years)				
		1	2	3	4	5
	Steady state	**% change** rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.340	0.0	0.0	0.1	0.1	0.1
Bottom 20% inc (%)	7.73	-0.0	-0.0	-0.1	-0.1	-0.0
Share of college (%)	34.2	0.1	0.1	0.1	0.1	0.1
Closure length: 1 AY						
Gini income	.340	0.0	-0.0	0.1	0.2	0.2
Bottom 20% inc (%)	7.73	-0.0	0.0	-0.1	-0.1	-0.1
Share of college (%)	34.2	0.1	0.2	0.1	0.1	0.0
Closure length: 1.5 AY						
Gini income	.340	0.0	-0.1	0.2	0.3	0.3
Bottom 20% inc (%)	7.73	-0.0	0.0	-0.2	-0.2	-0.1
Share of college (%)	34.2	0.1	0.2	0.1	-0.0	-0.1