Discussion Paper Series – CRC TR 224

Discussion Paper No. 204
Project B 05

The Tension between Market Shares and Profit under Platform Competition

Paul Belleflamme [1]
Martin Peitz [2]
Eric Toulemonde [3]

August 2020

[1] CORE/LIDAM, Université catholique de Louvain, 34 Voie du Roman Pays, B-1348 Louvain la Neuve, Belgium, Paul.Belleamme@uclouvain.be; also affliated with CESifo.
[2] Department of Economics and MaCCI, University of Mannheim, D-68131 Mannheim, Germany, Martin.Peitz@gmail.com; also affliated with CEPR, CESifo, and ZEW.
[3] Department of Economics and DEFIPP-CERPE, University of Namur, 8 Rempart de la Vierge, B-5000 Namur, Belgium, Eric.Toulemonde@unamur.be; also affliated with IZA.

Funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

The tension between market shares and profit under platform competition*

Paul Belleflamme[†] Université catholique de Louvain

Martin Peitz[‡] University of Mannheim

Eric Toulemonde[§] University of Namur

This version: August 2020

Abstract

We introduce asymmetries across platforms in the linear model of competing two-sided platforms with singlehoming on both sides and fully characterize the price equilibrium. We identify market environments in which one platform has a larger market share on both sides while obtaining a lower profit than the other platform. This platform enjoys a competitive advantage on one or both sides. Our finding raises further doubts on using market shares as a measure of market power in platform markets.

Keywords: Two-sided platforms, market share, market power, oligopoly, network effects, antitrust

JEL-Classification: D43, L13, L86

*Paul Belleflamme gratefully acknowledges financial support by Fédération-Wallonie-Bruxelles through "Action de recherche concertée" (Grant No. 19/24-101). Martin Peitz gratefully acknowledges financial support from Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (project B05).

[†]CORE/LIDAM, Université catholique de Louvain, 34 Voie du Roman Pays, B-1348 Louvain la Neuve, Belgium, Paul.Belleflamme@uclouvain.be; also affiliated with CESifo.

[‡]Department of Economics and MaCCI, University of Mannheim, D-68131 Mannheim, Germany, Martin.Peitz@gmail.com; also affiliated with CEPR, CESifo, and ZEW.

[§]Department of Economics and DEFIPP-CERPE, University of Namur, 8 Rempart de la Vierge, B-5000 Namur, Belgium, Eric.Toulemonde@unamur.be; also affiliated with IZA.

1 Introduction

Competition authorities investigate platform mergers and allegedly anticompetitive practices. A stepping stone is to show that a platform has market power. In this paper we take a look at the link between market share and profit and show that, under some conditions, a multi-sided platform has larger market shares on each side, but obtains a lower profit than its rival.

Why should one care? Competition practice defines thresholds as indicators for market power for antitrust investigations and merger control. Market shares figure prominently as such an indicator. For example, the European Commission writes: “The Commission considers that low market shares are generally a good proxy for the absence of substantial market power. The Commission's experience suggests that dominance is not likely if the undertaking's market share is below 40 % in the relevant market.” (See European Commission, 2009, p. C45/9.)

Economists have been taking a critical view of market share as an indicator of market power for a number of reasons. We point to three issues. (i) To talk about market share, one has to define a market. The market definition exercise is based on the idea to include close substitutes and to keep weak substitutes outside the relevant market. Absent good data on demand substitution, this market definition is somewhat arbitrary. (ii) Leaving the market definition issue aside, market shares are determined across a variety of market environments. But what does a high market share stand for? If products are almost homogeneous then a small cost advantage results in a large market share; however, profit margins are likely to be low. By contrast, if products are more differentiated, a small cost advantage leads to only a slightly higher market share than the ones of its competitors. Due to the lack of very close substitutes, profit margin may well be high. This suggests to be more concerned about high market shares if products are more differentiated. (iii) While one may typically think that a higher market share is related to higher profit margins, this is not necessarily the case. In particular, when vertical product differentiation is present, it may well be the case that a high quality variant caters to a small group of consumers who very much value quality. As a result, profit margin and even firm profit may be larger for the firm with the smaller market share. To the extent that different segments of the car market constitute one relevant market, the car market exemplifies this observation: Premium car brands tend to sell at a higher profit margins and generate higher profits than mass market producers. The former cater to consumers who have a high willingness to pay for certain quality attributes.

While all these points are relevant, we add another complication that is specific to two-sided platforms and relates to (ii) and (iii). Since two-sided platforms offer complementary products to both sides, they are active on two markets. To speak of dominance in one market, is it the share in this market (however it is defined) that matters or should this share always be considered together with the share in the market serving the other side? If cross-group network effects are pronounced in at least one direction, a proper understanding of the competitive situation requires a joint consideration of both markets (see, e.g., Franck and Peitz, 2019). In particular, even if one platform operates as a monopolist on one side, its market power may be very much constrained by competition on the other side. While the platform then enjoys a monopoly margin on one side, its overall profit may be nil, as the surplus extracted on one side may be fully passed through to the other side. However, the result depends in particular on the sign of the cross-group network effects and other market characteristics. This suggests that a large market share on one side may be informative of the degree of market power of a platform vis-a-vis one group or one side of users, but may not allow the platform to sustain high profit margins.

Even if platforms compete on both sides, there is no clear link between market share, the standard profit margin (as the difference between price and marginal cost), and the degree of market power. Even keeping the degree of product differentiation the same on the two markets on which platforms compete, price-cost margins may differ substantially if cross-group network effects differ in size. Platforms then charge different prices on the two sides even if the two sides are otherwise identical. This has been shown by Armstrong (2006) in a symmetric market with singlehoming users on both sides and platforms charging subscription fees.

In this paper, we contribute to the discussion of the link between market share and market power by reconsidering a workhorse model in the two-sided market literature, the two-sided singlehoming duopoly model as proposed by Armstrong (2006). In our extension of this model, we let platforms differ in all relevant parameters (costs, stand-alone benefits and cross-group network effects). In spite of the number of parameters, it is possible to exploit the linearity in this model to solve the two-stage game in which platforms first choose access fees on both sides and, second, users on each side decide simultaneously which platform to join.

The main message is that in some market environments, one of the platforms has a larger market share on both sides, but lower profit than its competitor: We show that under some conditions there are equilibria such that one platform with a 'competitive advantage' (i.e., lower marginal cost, or larger stand-alone benefits, or larger cross-group network effects) on at least one side has a larger market share on both sides but obtains a lower profit than the rival platform. The presence of cross-group network effects (on at least one side) is necessary for this result, as well as different price-sensitivities across the two sides. This result contrasts the finding in standard oligopoly (e.g., the asymmetric Hotelling model) that the more profitable firm must be the larger firm and must have an advantage of some sort. Introducing direct network effects in a Hotelling setting cannot generate the tension between size and profit either: the firm with the higher market share is necessarily the more-profitable firm.[1] This shows that our result is specific to competition between two-sided platforms: market share is a particularly poor measure of market power in markets with two-sided platforms.

A particular market enviroment is one in which one of the two platform enjoys a competitive advantage in both markets. This platform obtains higher market shares in both markets, but under some conditions obtains lower profit than its rival. Through a set of additional examples, we provide other sets of fundamentals that generate a tension between observed markets shares and profit. In particular, it is possible that this tension arises when both firms decide not to

[1]Calculations are available from the authors upon request.

subsidize any users (i.e., they set price above marginal cost).

We are not aware of other work pointing to the tension between markets shares and profit in markets with two-sided platforms. In most of the literature, it is assumed that platforms are symmetric (giving rise to symmetric market shares and profits at equilibrium); this includes Rochet and Tirole (2003, 2006) and Hagiu (2006) among others, as well as, in a model with Hotelling demand, Armstrong (2006) and Armstrong and Wright (2007), and, in an oligopoly model of two-sided singlehoming with more general demand, Tan and Zhou (2020). While some papers allow for asymmetric two-sided platforms (e.g., Viecens, 2006; Njoroge et al. 2009, 2010; Gold 2010; Lin, Li, and Whinston, 2011; Ponce, 2012; Gabszewicz and Wauthy, 2014; Jullien and Pavan, 2019; Anderson and Peitz, 2020), these papers did not investigate the link between market shares and profit margins or profit.

In Section 2, we develop the model. In Section 3, we characterize the equilibrium. In Section 4, we provide conditions under which, in equilibrium, one platform attracts more users on both sides but obtains lower profit than the other platform. We conclude in Section 5.

2 The model

We adapt the models of Armstrong (2006) and Armstrong and Wright (2007). Two platforms are located at the extreme points of the unit interval: platform U (for *Uppercase*, identified hereafter by upper-case letters) is located at 0, while platform l (for *lowercase*, identified by lower-case letters) is located at 1. Platforms facilitate the interaction between two groups of users, noted a and b. Both groups are assumed to be of mass 1 and uniformly distributed on $[0, 1]$. We assume that users of both sides can join at most one platform (so-called 'two-sided singlehoming'); in the real world, singlehoming environments may result from indivisibilities and limited resources or from contractual restrictions.[2]

A user derives a net utility from joining a platform that is defined as the addition of four components: (i) a cross-group external benefit, (ii) a stand-alone benefit, (iii) a 'transportation cost', and (iv) an access fee. The first two components enter the net utility function positively. They correspond to two types of services that a platform offers to its users. Some services facilitate the interaction with the other group; the utility they give is the cross-group external benefit, which is assumed to increase linearly with the number of users of the other group present on the platform.[3] A platform also offers other services that do not relate to the interaction between the groups; these services give a stand-alone benefit to users. The last two components enter the net utility function negatively. In the usual Hotelling fashion, a user incurs a disutility from not being able to use a platform that corresponds to their ideal definition of a platform; this disutility is assumed to increase linearly with the distance separating the user's and the platform's location on the unit line (at a rate that can be interpreted as a measure of the

[2] For a discussion, see Case 22.4 in Belleflamme and Peitz (2010, p. 633).

[3] We focus in this paper on *positive* cross-group network effects; that is, each group positively values the participation of the other group on the platform.

horizontal differentiation between the platforms in the eyes of a particular group of users). Finally, users have to pay a flat fee to access the platform.[4]

We allow all components of the utility function to differ not only across sides but also across platforms. We therefore write the net utility functions for a user of group a and for a user of group b, respectively located at x and $y \in [0,1]$ as:

$$\begin{array}{ll} U_a(x) = E_a N_b + R_a - \tau_a x - P_a & \text{if joining platform } U, \\ u_a(x) = e_a n_b + r_a - \tau_a (1-x) - p_a & \text{if joining platform } l, \\ U_b(y) = E_b N_a + R_b - \tau_b y - P_b & \text{if joining platform } U, \\ u_b(y) = e_b n_a + r_b - \tau_b (1-y) - p_b & \text{if joining platform } l, \end{array}$$

where E_j (resp. e_j) measures the strength of the cross-group network effect that users of group k exert on users of group j on platform U (resp. l),[5] N_j (resp. n_j) is the mass of users of group j that decide to join platform U (resp. l), R_j (resp. r_j) is the valuation of the stand-alone benefit by users of group j on platform U (resp. l), τ_j is the 'transport cost' parameter for group j, and P_j (resp. p_j) is the access fee that platform U (resp. l) sets for users of group j (with j, $k \in \{a,b\}$ and $j \neq k$).

Let $\hat{x}$ (resp. $\hat{y}$) identify the user of group a (resp. b) who is indifferent between joining platform U or platform l; that is, $U_a(\hat{x}) = u_a(\hat{x})$ and $U_b(\hat{y}) = u_b(\hat{y})$. Solving these equalities for $\hat{x}$ and $\hat{y}$ respectively, we have

$$\begin{array}{l} \hat{x} = \frac{1}{2} + \frac{1}{2\tau_a}\left(E_a N_b - e_a n_b + R_a - r_a - (P_a - p_a)\right), \\ \hat{y} = \frac{1}{2} + \frac{1}{2\tau_b}\left(E_b N_a - e_b n_a + R_b - r_b - (P_b - p_b)\right). \end{array}$$

In what follows, we assume that stand-alone and cross-group external benefits are sufficiently large to make sure that all users join one platform. Both sides are then fully covered, so that $N_j + n_j = 1$ $(j = a, b)$. This entails the following equalities: $\hat{x} = N_a = 1 - n_a$ and $\hat{y} = N_b = 1 - n_b$. Using these equalities, we can solve the above systems of equations for N_a and N_b:

$$N_a = \frac{1}{2} + \frac{\tau_b}{2}\frac{\rho_a + \delta_a + p_a - P_a}{\tau_a\tau_b - \sigma_a\sigma_b} + \frac{\sigma_a}{2}\frac{\rho_b + \delta_b + p_b - P_b}{\tau_a\tau_b - \sigma_a\sigma_b}, \tag{1}$$

$$N_b = \frac{1}{2} + \frac{\tau_a}{2}\frac{\rho_b + \delta_b + p_b - P_b}{\tau_a\tau_b - \sigma_a\sigma_b} + \frac{\sigma_b}{2}\frac{\rho_a + \delta_a + p_a - P_a}{\tau_a\tau_b - \sigma_a\sigma_b}, \tag{2}$$

where we have introduced some additional notation (that will prove useful in the rest of the analysis): $\sigma_k \equiv \frac{1}{2}(E_k + e_k)$ is the *sum* of the cross-group external benefits on side k of platforms U and l when the users on the other side split equally; $\delta_k \equiv \frac{1}{2}(E_k - e_k)$ is the *difference* of the cross-group external benefits on side k between platforms U and l when the users on the other side split equally;[6] $\rho_k \equiv R_k - r_k$ is the difference in stand-alone benefits on side k between platforms U and l.

[4]For simplicity and to stay within the well-known model by Armstrong (2006), we do not consider usage fees, nor two-part tariffs. See Reisinger (2014) on platform competition with two-part tariffs.

[5]That is, E_j is the valuation by a group j user of the participation of an extra group k user.

[6]By definition, $E_k = \sigma_k + \delta_k$ and $e_k = \sigma_k - \delta_k$.

To ensure that participation on each side is a decreasing function of the access fee on this side, we assume the following:

$$\tau_a \tau_b > \sigma_a \sigma_b. \tag{3}$$

This assumption, which is common in the analysis of competition between two-sided platforms, says that the strength of cross-group network effects (measured by $\sigma_a \sigma_b$) is smaller than the strength of horizontal differentiation (measured by $\tau_a \tau_b$).

As for platforms, we assume that they face constant costs per user. These costs may also differ across sides and across platforms; we note them C_a and C_b for platform U, and c_a and c_b for platform l. For future reference, we define $\gamma_k \equiv C_k - c_k$ as the difference in costs on side k between platforms U and l $(k = a, b)$. Before solving the model, we introduce one last piece of notation:

$$\Delta_a \equiv \rho_a - \gamma_a + \delta_a \text{ and } \Delta_b \equiv \rho_b - \gamma_b + \delta_b.$$

If positive, Δ_k is a measure of the 'competitive advantage' that platform U has over platform l on side k; this advantage may follow from larger stand-alone benefits ($\rho_a > 0$), smaller costs ($\gamma_a < 0$) or larger cross-group network effects ($\delta_a > 0$); if Δ_k is negative, then it is platform l that has an advantage on side k.

3 Equilibrium of the pricing game

Platforms simultaneously choose their access prices to maximize their profit, given by $\Pi = (P_a - C_a) N_a + (P_b - C_b) N_b$ and $\pi = (p_a - c_a) n_a + (p_b - c_b) n_b$. The first-order conditions require

$$\frac{d\Pi}{dP_a} = \frac{d\Pi}{dP_b} = \frac{d\pi}{dp_a} = \frac{d\pi}{dp_b} = 0,$$

whereas the second-order conditions require

$$\tau_a \tau_b > \sigma_a \sigma_b \text{ and } \tau_a \tau_b > \tfrac{1}{4} (\sigma_a + \sigma_b)^2 .$$

We note that the first condition is equivalent to Assumption (3) and that $\frac{1}{4} (\sigma_a + \sigma_b)^2 - \sigma_a \sigma_b = \frac{1}{4} (\sigma_a - \sigma_b)^2 > 0$, which means that the second condition is more stringent than the first. We thus impose from now on

$$\tau_a \tau_b > \tfrac{1}{4} (\sigma_a + \sigma_b)^2 . \tag{4}$$

We now solve the system of the four first-order conditions. To facilitate the exposition, we define

$$D \equiv 9\tau_a \tau_b - (2\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b),$$

which is positive according to Assumption (4). The equilibrium price of platform U on side a is found as

$$\begin{aligned} P_a^* &= \overbrace{C_a + \tau_a}^{\mathbf{H}} \overbrace{-\sigma_b}^{\mathbf{A}} \overbrace{+\tfrac{1}{3}(\rho_a - \gamma_a)}^{\mathbf{Vs}} \overbrace{+\tfrac{1}{3}\delta_a}^{\mathbf{Vn}} \\ &\quad \underbrace{+ \frac{(\sigma_a - \sigma_b)}{3D} \left[(2\sigma_a + \sigma_b)(\rho_a - \gamma_a + \delta_a) + 3\tau_a (\rho_b - \gamma_b + \delta_b) \right]}_{\mathbf{I}} \end{aligned}$$

We can decompose it as the sum of five components: (i) **H** is the classic *Hotelling* formula (marginal cost + transportation cost); (ii) **A** was identified by *Armstrong* (2006) in a symmetric setting as the price adjustment due to cross-group network effects (the price is decreased by the externality exerted on the other side); (iii) **Vs** is the quality effect in terms of *stand-alone* benefits (or the effect of marginal costs differences);[7] (iv) **Vn** is the quality effect in terms of cross-group *network* effects on the side under review; (v) the last term **I** results from the interplay between vertical differentiation and cross-group network effects. If platforms are symmetric ($\rho_k = \gamma_k = \delta_k = 0$) only **H** and **A** remain; absent external effects ($\sigma_k = \delta_k = 0$), only **H** an **Vs** remain. In the particular case in which cross-group network effects are (on average) the same on the two sides ($\sigma_a = \sigma_b$), all terms but the last remain.

Recalling that $\Delta_k \equiv \rho_k - \gamma_k + \delta_k$, we can rewrite platform U's equilibrium margin on side a as follows:

$$P_a^* - C_a = \tau_a - \sigma_b + \tfrac{1}{3}\Delta_a + \frac{(\sigma_a - \sigma_b)}{3D}\left((2\sigma_a + \sigma_b)\Delta_a + 3\tau_a\Delta_b\right).$$

The other equilibrium margins are found by analogy:

$$\begin{aligned}
P_b^* - C_b &= \tau_b - \sigma_a + \tfrac{1}{3}\Delta_b + \frac{(\sigma_b - \sigma_a)}{3D}\left((2\sigma_b + \sigma_a)\Delta_b + 3\tau_b\Delta_a\right),\\
p_a^* - c_a &= \tau_a - \sigma_b - \tfrac{1}{3}\Delta_a - \frac{(\sigma_a - \sigma_b)}{3D}\left((2\sigma_a + \sigma_b)\Delta_a + 3\tau_a\Delta_b\right),\\
p_b^* - c_b &= \tau_b - \sigma_a - \tfrac{1}{3}\Delta_b - \frac{(\sigma_b - \sigma_a)}{3D}\left((2\sigma_b + \sigma_a)\Delta_b + 3\tau_b\Delta_a\right).
\end{aligned}$$

We can now use the equilibrium prices to compute the equilibrium mass of users of the two groups on the two platforms:

$$\begin{aligned}
N_a^* &= \tfrac{1}{2} + \tfrac{1}{2D}\left(3\tau_b\Delta_a + (\sigma_a + 2\sigma_b)\Delta_b\right),\ n_a^* = 1 - N_a^*,\\
N_b^* &= \tfrac{1}{2} + \tfrac{1}{2D}\left(3\tau_a\Delta_b + (2\sigma_a + \sigma_b)\Delta_a\right),\ n_b^* = 1 - N_b^*.
\end{aligned}$$

To guarantee that the equilibrium mass is strictly positive and lower than unity, we impose the following restrictions on the space of parameters:

$$3\tau_b\Delta_a + (\sigma_a + 2\sigma_b)\Delta_b \text{ and } 3\tau_a\Delta_b + (2\sigma_a + \sigma_b)\Delta_a \in (-D, D). \tag{5}$$

Using the equilibrium values of prices and number of users, we find the equilibrium profits[8]

$$\begin{aligned}
\Pi^* = {} & \frac{1}{2}(\tau_a + \tau_b - \sigma_a - \sigma_b) + \frac{1}{2D}\left(\tau_b\Delta_a^2 + \tau_a\Delta_b^2\right) + \frac{1}{2D}(\sigma_a + \sigma_b)\Delta_a\Delta_b\\
& + \frac{1}{2D}\left(6\tau_a\tau_b + \tau_b(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(2\sigma_a + \sigma_b)\right)\Delta_a\\
& + \frac{1}{2D}\left(6\tau_a\tau_b - \tau_a(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)\right)\Delta_b,
\end{aligned} \tag{6}$$

[7] In this model R_k and C_k play interchangeable roles. What matters is their difference $\rho_k - \gamma_k = (R_k - C_k) - (r_k - c_k)$.

[8] The model has the feature that industry profits increase if one platform obtains a larger competitive advantage starting from such an advantage on both sides (that is, if Δ_a or Δ_b becomes larger if they are positive, or smaller if they are negative):

$$\Pi^* + \pi^* = (\tau_a + \tau_b - \sigma_a - \sigma_b) + \frac{1}{D}\left(\tau_b\Delta_a^2 + \tau_a\Delta_b^2 + (\sigma_a + \sigma_b)\Delta_a\Delta_b\right).$$

$$\pi^* = \frac{1}{2}(\tau_a + \tau_b - \sigma_a - \sigma_b) + \frac{1}{2D}\left(\tau_b\Delta_a^2 + \tau_a\Delta_b^2\right) + \frac{1}{2D}(\sigma_a + \sigma_b)\Delta_a\Delta_b$$
$$-\frac{1}{2D}\left(6\tau_a\tau_b + \tau_b(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(2\sigma_a + \sigma_b)\right)\Delta_a \qquad (7)$$
$$-\frac{1}{2D}\left(6\tau_a\tau_b - \tau_a(\sigma_a - \sigma_b) - (\sigma_a + \sigma_b)(\sigma_a + 2\sigma_b)\right)\Delta_b.$$

4 Contrasting market power indicators

In this section, we show that it is possible to have contrasting views about the relative market power of the two competing platforms depending on the measure that we use. In particular, we find conditions under which, at equilibrium, one platform attracts more users on both sides but achieves lower profit than the other platform.

Without any loss of generality, let platform U be this platform. For platform U to attract more users than platform l on both sides, we need

$$N_a^* - n_a^* = \frac{1}{D}\left[3\tau_b\Delta_a + (\sigma_a + 2\sigma_b)\Delta_b\right] > 0, \qquad (8)$$
$$N_b^* - n_b^* = \frac{1}{D}\left[3\tau_a\Delta_b + (2\sigma_a + \sigma_b)\Delta_a\right] > 0. \qquad (9)$$

For platform U to achieve a *lower* profit than platform l, we must have $\Pi^* - \pi^* < 0$, which is equivalent to

$$\Pi^* - \pi^* = \left[2\tau_a - (\sigma_a + \sigma_b)\right](N_a^* - n_a^*) + \left[2\tau_b - (\sigma_a + \sigma_b)\right](N_b^* - n_b^*) < 0. \qquad (10)$$

This expression highlights the effects of market shares $(N_k^* - n_k^*)$ on platform profits. If conditions (8) and (9) are satisfied, then condition (10) can only be satisfied if either $2\tau_a < (\sigma_a + \sigma_b)$ or $2\tau_b < (\sigma_a + \sigma_b)$. That is, users on one of the two sides must perceive the two platforms as close enough substitutes, in the sense that the 'transportation cost' on that side, τ_k, must be lower than the average cross-group network effects, $(\sigma_a + \sigma_b)/2 = (E_a + e_a + E_b + e_b)/4$. Recall, that to satisfy the second-order conditions (4), this strong substitutability cannot be observed on both sides: $4\tau_a\tau_b > (\sigma_a + \sigma_b)^2$ makes it impossible to have $2\tau_a < (\sigma_a + \sigma_b)$ *and* $2\tau_b < (\sigma_a + \sigma_b)$. An important observation follows from this finding: the three conditions cannot be met jointly if $\sigma_a = \sigma_b = 0$ or $\tau_a = \tau_b$. Furthermore, the larger platform features $N_a^* < N_b^*$ if $\tau_a > \tau_b$.[9]

The next proposition records what we have learned so far.

[9] This is shown as follows. For platform U to have larger market shares yet lower profit than its competitor, we need $2\tau_b - (\sigma_a + \sigma_b) < 0$. By contradiction, suppose that $N_a^* - n_a^* > N_b^* - n_b^*$. Then the left hand side of (10) is larger than

$$\left[2\tau_a - (\sigma_a + \sigma_b)\right](N_b^* - n_b^*) + \left[2\tau_b - (\sigma_a + \sigma_b)\right](N_b^* - n_b^*) = 2\left[\tau_a + \tau_b - (\sigma_a + \sigma_b)\right](N_b^* - n_b^*).$$

By (4) $\tau_a + \tau_b - (\sigma_a + \sigma_b) > \tau_a + \tau_b - 2\sqrt{\tau_a\tau_b} = (\sqrt{\tau_a} - \sqrt{\tau_b})^2 > 0$. Thus the left hand side of (10) is positive, a contradiction: Platform U has larger profit than its competitor. Hence we must have $N_a^* - n_a^* < N_b^* - n_b^*$ if $\tau_a > \tau_b$.

Proposition 1 *For a platform to have larger market shares yet lower profit than its competitor, it is necessary that (i) cross-group network effects exist on at least one side (σ_a and/or $\sigma_b > 0$), (ii) users on one side perceive the two platforms as close enough substitutes (τ_a or $\tau_b < (\sigma_a + \sigma_b)/2$), (iii) the price-sensitivity of demand be different on the two sides ($\tau_a \neq \tau_b$); and (iv) the platform with larger market shares have the largest market share on the side on which the price sensitivity of demand is higher.*

The intuition behind Proposition 1 is the following. The first necessary condition, namely the existence of cross-group network effects, makes a platform's pricing decisions interdependent across the two sides. Suppose instead that cross-group network effects are absent ($\sigma_a = \sigma_b = 0$). Then, each platform can choose its prices for group a and b independently of one another; platforms solve in fact two distinct profit-maximization problems and competition operates separately on each side. On these two parallel 'one-sided' Hotelling markets, it is easy to see that if a firm has a an exogenous advantage (in terms of cost or stand-alone benefits), it achieves both a larger market share and a larger profit at the price equilibrium: there cannot be any contradiction between the two measures of market power.[10]

Things change dramatically in the presence of cross-group network effects. To see why, consider platform U and suppose that $E_a > 0$ (which implies that $\sigma_a = (E_a + e_a)/2 > 0$). As users on side a care about the participation of users on side b, participation on side a is sensitive to changes not only in P_a and p_a, but also to changes in P_b and p_b. It follows that any exogenous advantage that platform U may have on one side (i.e., $\Delta_a > 0$ and/or $\Delta_b > 0$) affects the strategic pricing of both platforms not only on this side but also on the other side.

As Belleflamme and Toulemonde (2018) show, initial asymmetries across platforms induce complex best-response dynamics in the platform pricing decisions, which is responsible for the tension between market shares and profit that we document here. In particular, a platform may suffer (in terms of profit) from a competitive advantage because this advantage triggers an aggressive price reaction from the other platform. Yet, for this to happen, the strategic interaction between the two platforms must be sufficiently strong. This is what condition (ii) captures: platforms must be close enough substitutes in the eyes of the users. However, the substitutability must be larger on one side than on the other, as condition (iii) stipulates; indeed, if the substitutability between the platforms was large on both sides (compared to the strength of the cross-group network effects), one platform would attract all users at equilibrium (and, in that case, this platform would obviously have a larger market share and a larger profit).

Taking a closer look at situations in which one platform has higher market shares but lower profits than its rival, we first consider environments in which one platform has competitive advantage on both sides. We partially characterize such equilibria in Proposition 2 (the proof is relegated to the Appendix) and then provide a numerical example for such an environment.

[10] If $\sigma_a = \sigma_b = 0$, condition (10) is equivalent to $\tau_a (N_a^* - n_a^*) + \tau_b (N_b^* - n_b^*) < 0$, which is clearly impossible if both $N_a^* - n_a^* > 0$ and $N_b^* - n_b^* > 0$.

Proposition 2 *Suppose that a platform has a competitive advantage on both sides (i.e., $\Delta_a > 0$ and $\Delta_b > 0$). For parameter constellations such that this platform has larger market shares yet lower profits than its rival, the equilibrium is such that (i) this platform makes a positive and larger margin than its rival on the side on which users are less price-sensitive, while (ii) it subsidizes users and makes a smaller margin than its rival on the side on which users are more price-sensitive.*

Proposition 2 applies when a platform has a competitive advantage on both sides such that this translates into larger market shares on both sides but lower profits than its rival. Although it makes larger profits on the side on which competition is less intense (because of a larger market share and a larger margin), it makes lower profits on the other side, on which it necessarily offers subsidies (and to more users than the other platform). We provide a numerical example showing that the set of parameters under which Proposition 2 holds is non-empty.

Example 1 *Equal competitive advantages for the same platform*

Let $\Delta_a = \Delta_b = \Delta > 0$. Condition (10) can then be rewritten as

$$\Pi^* < \pi^* \Leftrightarrow 3\left(4\tau_a\tau_b - (\sigma_a + \sigma_b)^2\right) - (\tau_a - \tau_b)(\sigma_a - \sigma_b) < 0.$$

As the first term is positive because of condition (4), we see that $(\tau_a - \tau_b)$ and $(\sigma_a - \sigma_b)$ must have the same sign. Suppose that $\tau_a > \tau_b$, which implies $\sigma_a > \sigma_b$. It is not necessary to assume positive cross-group network effects on both sides: we set $\sigma_b = 0$; take also $\tau_a = 4$ and $\tau_b = 1$. To satisfy condition (4), we need $16 - \sigma_a^2 > 0$ or $\sigma_a < 4$. On the other hand, the above condition becomes $48 - 3\sigma_a - 3\sigma_a^2 < 0$, which is equivalent to $\sigma_a > 3.531$. Setting $\sigma_a = 3.6$ and $\Delta = \frac{1}{4}$, we compute

$$N_a^* - n_a^* = 0.164,\ N_b^* - n_b^* = 0.476, \text{ and } \Pi^* - \pi^* = -0.042.$$

This shows that platform U achieves a lower equilibrium profit than platform l although it attracts more users on both sides. Computing the equilibrium margins, we confirm the result of Proposition 2 on profit margins:

$$P_a^* - C_a = 4.655 > p_a^* - c_a = 3.345 \text{ and } P_b^* - C_b = -2.713 < p_b^* - c_b = -2.487.$$

We observe that what platform l gains less on side a—namely, $(p_a^* - c_a)\, n_a^* - (P_a^* - C_a)\, N_a^* = (3.345)\, 0.418 - (4.655)\, 0.582 = -1.311$—is more than compensated by what it *pays* less on side b—namely $(p_b^* - c_b)\, n_b^* - (P_b^* - C_b)\, N_b^* = (-2.487)\, 0.262 - (-2.713)\, 0.738 = 1.351$. ■

We provide three other illustrative examples. In all these examples, Δ_a and Δ_b are of opposite sign. Thus, it is a priori not clear which of the two platforms should be considered to be the more competitive platform overall. In all these examples, one platform has larger market shares but lower profits. In Example 2, we assume that on average, regardless of the group to which she belongs, a user attaches the same value to the addition of an extra member of the other group ($\sigma_a = \sigma_b = \sigma$). Example 3 envisages a situation where the competitive advantage of platform U

over users of one group is exactly offset by a competitive disadvantage over users of the other group ($\Delta_a = -\Delta_b$). Despite the additional parameter restrictions in both examples, the tension between market shares and profit still applies. Finally, Example 4 shows that subsidization is not necessary as an equilibrium behavior for this tension to arise.

Example 2 *Same average cross-group network effects on both sides*

We suppose here that $\sigma_a = \sigma_b = \sigma > 0$. This means that, on average, cross-group network effects are similar on both sides. Conditions (8) to (10) become, respectively, $\tau_b \Delta_a + \sigma \Delta_b > 0$, $\tau_a \Delta_b + \sigma \Delta_a > 0$ and $\Delta_a + \Delta_b < 0$.[11] For those three conditions to be compatible, one needs $\Delta_k > 0$ and $\Delta_l < 0$: platform U must have a competitive advantage for one group of users and a competitive disadvantage for the other.

One situation that is compatible with this scenario is the following. Platforms are marketplaces linking sellers (group a) and buyers (group b). Sellers sell completely differentiated products. On each platform, each registered buyers makes one transaction with each registered seller; each transaction generates a total value of $2v$. Platforms differ in the way they split the transaction value $2v$ between buyers and sellers: say that sellers gain a larger share than buyers on platform U, while the opposite prevails on platform l. In particular, $E_a = e_b = v + x$ and $E_b = e_a = v - x$, with $0 < x < v$. It follows that $\sigma_a = (E_a + e_a)/2 = \sigma_b = (E_b + e_b)/2 = v$, while $\delta_a = (E_a - e_a)/2 = x$ and $\delta_b = (E_b - e_b)/2 = -x$. If differences in stand-alone benefits and in costs are minor, we will also have that $\Delta_a > 0$ and $\Delta_b < 0$.

Here, we set $\sigma_a = \sigma_b = 2$, $\tau_a = 1$, $\tau_b = 5$, $\Delta_a = 1$ and $\Delta_b = -3/2$. Platform U suffers from a handicap towards users who perceive the platforms as being highly differentiated ($\Delta_b < 0$ and $\tau_b > \tau_a$). We check that in this parameter constellation, platform U has larger market shares, particularly for the users of group a, but it has lower profits: $N_a^* = 5/6$, $N_b^* = 7/12$ and $\Pi^* - \pi^* = -1/3$. Looking at the margins, we find that both platforms subsidize users from group a: $P_a^* - C_a = -2/3 > p_a^* - c_a = -4/3$. Despite a lower subsidy per user, platform U spends more in subsidies than platform l because it attracts more users from group a (it spends $5/9$ in total instead of $2/9$ for platform l). Platform U is trying to attract a large number of users from group a to compensate for its weakness vis-à-vis users from group b, who perceive the two platforms as highly differentiated. Both platforms tax users from group b: $P_b^* - C_b = 5/12 < p_b^* - c_b = 7/12$. Platform l achieves exactly the same revenues as platform U ($35/144$) by setting a higher margin to a lower market share. As a result, platform U earns a lower profit, which nevertheless remains positive ($\Pi^* = 65/72$). ■

Example 3 *Opposite competitive advantages*

We assume now that $\Delta_a = -\Delta_b > 0$. Here, platform U has a competitive advantage vis-à-vis users from group a and an equivalent disadvantage vis-à-vis users of group b. It is readily checked that

$$\Pi^* - \pi^* = \Delta_a (\sigma_a - \sigma_b) [\tau_a + \tau_b - (\sigma_a + \sigma_b)] / D < 0 \iff \sigma_a < \sigma_b.$$

[11] The second-order condition (4) requires $\tau_a \tau_b > \sigma^2$.

This is a general result for cases where $\Delta_a = -\Delta_b > 0$. The platform with a competitive advantage over users who benefit least from the presence of other users has a lower profit than its competitor. Is this result compatible with a situation where platform U also has higher market shares? For conditions (8) and (9) to be met, it is necessary that $3\tau_a < 2\sigma_a + \sigma_b < \sigma_a + 2\sigma_b < 3\tau_b$ where the first inequality corresponds to (9), the second arises from $\sigma_a < \sigma_b$ and the third corresponds to (8).

Let $\sigma_a = 2$, $\sigma_b = 5$, $\tau_a = 2$, $\tau_b = 8$, $\Delta_a = 1$ and $\Delta_b = -1$. In line with our previous results, platform U is expected to be the least profitable because $\sigma_a < \sigma_b$ and $\Delta_a > 0$. Indeed, $\Pi^* - \pi^* = -1/4$. We check that this platform has the largest market shares: $N_a^* = 2/3$, $N_b^* = 13/24$. As far as margins are concerned, it can be shown that $P_a^* - C_a = -11/4 > p_a^* - c_a = -13/4$, while $P_b^* - C_b = 6 = p_b^* - c_b$. Both platforms subsidize the users of group a who perceive the platforms as not very differentiated. The total value of subsidies distributed is higher for platform U despite a lower subsidy per user. Both platforms tax the users of group b by the same amount. It can therefore be noted that platform U attracts more users of group b despite a competitive disadvantage for this type of users and a margin identical to that set by platform l; it achieves this result thanks to the attractiveness it has gained by having attracted a large majority of users of group a. Despite the higher profit it derives from group b users, platform U is unable to compensate for its higher cost of attracting group a users; its profit is therefore lower than that of platform l. ■

Examples 1 to 3 have a common feature: A large market share for users who are subsidized (i.e., each user on one side pays a price below the cost of serving her) is costly and does not play out well for the overall platform profit even if the platform has a larger market share for "paying" users. As we show in a final example, the tension between market share and profit can arise even when users are not subsidized in equilibrium.

Example 4 *No subsidies*

Consider the borderline case in terms of market shares in which platforms have equal market share on one side, $N_b^* - n_b^* = 0$. Expression (9) requires that $\Delta_b = -\Delta_a (2\sigma_a + \sigma_b) / (3\tau_a)$. For this value of Δ_b, one can write $N_a^* - n_a^* = \Delta_a / (3\tau_a)$ which is positive if $\Delta_a > 0$ (and which is smaller than 1 if $\Delta_a < 3\tau_a$). The margins can be written as

$$\begin{aligned} P_a^* - C_a &= \tau_a - \sigma_b + \frac{1}{3}\Delta_a, \qquad p_a^* - c_a = \tau_a - \sigma_b - \frac{1}{3}\Delta_a \\ P_b^* - C_b &= \tau_b - \sigma_a - \frac{1}{3}\frac{\sigma_a}{\tau_a}\Delta_a, \qquad p_b^* - c_b = \tau_b - \sigma_a + \frac{1}{3}\frac{\sigma_a}{\tau_a}\Delta_a \end{aligned}$$

Thus, all parameter constellations with $\sigma_b < \tau_a < \sigma_a < \tau_b$ and Δ_a slightly positive ensure that margins are positive. Moreover, in spite of (slightly) higher market shares than its rival, platform U obtains lower profits than its rival if $\tau_a < (\sigma_a + \sigma_b)/2$ (see expression (10)). Finally, a sufficiently high value for τ_b ensures that the second-order condition (4) is satisfied. Consider the following values: $\sigma_b = 1$, $\tau_a = 2$, $\sigma_a = 4$, $\tau_b = 5$, $\Delta_b = -3\Delta_a/2$ and $\Delta_a < 3/2$. We find

$$\begin{aligned} P_a^* - C_a &= 1 + \frac{1}{3}\Delta_a > 0, \qquad p_a^* - c_a = 1 - \frac{1}{3}\Delta_a > 0, \\ P_b^* - C_b &= 1 - \frac{2}{3}\Delta_a > 0, \qquad p_b^* - c_b = 1 + \frac{2}{3}\Delta_a > 0, \\ N_a^* - n_a^* &= \frac{\Delta_a}{6} \in (0,1), \qquad N_b^* - n_b^* = 0, \, \Pi^* - \pi^* = -\frac{\Delta_a}{6} < 0 \end{aligned}$$

We also verified that the second-order condition is met. Platform U obtains lower profits than its rival despite having at least the same market share. All margins are positive.

Note that to have $N_b^* - n_b^* > 0$, it is sufficient to slightly increase the value of Δ_b, as we show next. We set $\Delta_a = 1$ but leave the value of Δ_b free. We find

$$\begin{aligned} N_a^* - n_a^* &\in (0,1) \iff -\frac{5}{2} < \Delta_b < \frac{7}{2}, \qquad N_b^* - n_b^* \in (0,1) \iff -\frac{3}{2} < \Delta_b < \frac{9}{2} \\ P_a^* - C_a &> 0 \iff \Delta_b > -\frac{19}{2}, \qquad p_a^* - c_a > 0 \iff \Delta_b < \frac{5}{2} \\ P_b^* - C_b &> 0 \iff \Delta_b > -\frac{7}{2}, \qquad p_b^* - c_b > 0 \iff \Delta_b < \frac{17}{2} \\ \Pi^* - \pi^* &< 0 \iff \Delta_b < -\frac{5}{4} \end{aligned}$$

Any value of $\Delta_b \in \left(-\frac{3}{2}, -\frac{5}{4}\right)$ ensures that platform U enjoys larger market shares while earning a lower profit than its rival; all margins are positive.[12] ■

5 Conclusion

In this paper, we revisited one of the workhorse models of platform competition, the linear two-sided singlehoming model proposed by Armstrong (2006). We derive the conditions under which there is a tension between market shares and profits: one platform has higher market shares for both user groups it is catering to, but obtains lower profit. Along the way, we characterize the price equilibrium allowing for asymmetries between platforms and across user groups regarding stand-alone utilities, costs of serving users, and strength of cross-group network effects; furthermore, the price elasticity of demand differs across user groups.

Our result is of interest for oligopoly theory and competition practice. On the theory side, we show that firm asymmetries can lead to surprising equilibrium outcomes. In particular, a firm may have a competitive advantage in serving both groups of users and, thereby (see Proposition 2 and Example 1) a larger market share than its rival for both groups; yet, this platform may at the same time obtain a lower profit than its rival. For competition practice, our result suggests that market share is unsuited as a simple metric to indicate market power. It may actually be the weaker and smaller rival that makes the higher profit.

[12] We can tie our hands further by assuming that $\sigma_b = 0$ and still find parameter constellations that give rise to higher market shares but lower profit for platform U and positive margins for both platforms on both sides. One such constellation is $\sigma_a = 10, \tau_a = 3, \tau_b = 20, \Delta_a = 1, \Delta_b = -2$ and zero costs.

In line with most of the literature, our analysis focused on the properties of the short-run price equilibrium. Our results provide some guidance regarding dynamic effects. If competitive advantages depended positively on past market shares (e.g., because of learning-by-doing), the firm with initially larger market shares, while performing worse than its competitor in the short run (under the conditions established in this paper), would over time increase its competitive advantage and eventually become also the more profitable firm. By contrast, if competitive advantages depended positively on past profit (e.g., because of funding constraints), a firm's competitive advantage would be attenuated over time leading to a more symmetric market outcome in terms of markets shares and profit. These insights apply to myopic firms operating in an evolving industry. A fully dynamic analysis would need to include anticipated future profits in the firms' objective function. Work along these lines would be useful to analyze how market share and long-run profit are related.

6 Appendix: Proof of Proposition 2

Suppose that $\Delta_a > 0$ and $\Delta_b > 0$. Then, conditions (8) and (9) are satisfied: platform U has a larger market share on both sides. Suppose, without loss of generality, that $\tau_a > \tau_b$. As noted above, the satisfaction of conditions (4) and (10) imply that $2\tau_b < \sigma_a + \sigma_b < 2\tau_a$. Substituting the values of $N_a^* - n_a^*$ and $N_b^* - n_b^*$ into condition (10) and developing, we obtain that $\Pi^* - \pi^*$ is equivalent to

$$6\left(\Delta_a + \Delta_b\right)\left(4\tau_a\tau_b - \left(\sigma_a + \sigma_b\right)^2\right) + 2\left[\left(\sigma_a + \sigma_b - 2\tau_b\right)\Delta_a + \left(2\tau_a - \left(\sigma_b + \sigma_a\right)\right)\Delta_b\right]\left(\sigma_b - \sigma_a\right) < 0.$$

As the first term is positive because of condition (4) and as $2\tau_b < \sigma_a + \sigma_b < 2\tau_a$, a necessary condition for the latter inequality to be satisfied is $\sigma_a > \sigma_b$.

Together with $2\tau_b < \sigma_a + \sigma_b < 2\tau_a$, $\sigma_a > \sigma_b$ implies that $\sigma_a > \tau_b$ and $\sigma_b < \tau_a$. Together with condition (8), $\sigma_a > \sigma_b$ also implies that $3\tau_a\tau_b > \sigma_b^2 + 2\sigma_a\sigma_b$. It follows that

$$P_a^* - C_a = \tau_a - \sigma_b + K > p_a^* - c_a = \tau_a - \sigma_b - K \text{ and } P_a^* - C_a > 0,$$

$$\text{with } K \equiv \frac{1}{D}\left[\left(3\tau_a\tau_b - \sigma_b^2 - 2\sigma_a\sigma_b\right)\Delta_a + \tau_a\left(\sigma_a - \sigma_b\right)\Delta_b\right] > 0.$$

This demonstrates statement (ii).

To show statement (i), we rewrite condition (10), $\Pi^* < \pi^*$, as

$$\left(P_a^* - C_a\right)\left(N_a^* - n_a^*\right) + \left[\left(P_a^* - C_a\right) - \left(p_a^* - c_a\right)\right]n_a^* < -\left(P_b^* - C_b\right)\left(N_b^* - n_b^*\right) + \left[\left(p_b^* - c_b\right) - \left(P_b^* - C_b\right)\right]n_b^*.$$

As the LHS is positive, the RHS must also be. This means that if $P_b^* - C_b > 0$, then it must be that $p_b^* - c_b > P_b^* - C_b$. But then, $P_b^* - C_b + p_b^* - c_b = \tau_b - \sigma_a > 0$, which contradicts $\sigma_a > \tau_b$. It follows necessarily that $P_b^* - C_b < 0$, which demonstrates statement (i).

References

[1] Anderson, S., Peitz, M., 2020. Media see-saws: Winners and losers in platform markets. Journal of Economic Theory 186.

[2] Armstrong, M., 2006. Competition in two-sided markets. Rand Journal of Economics 37, 668–691.

[3] Armstrong, M., Wright, J., 2007. Two-sided markets, competitive bottlenecks and exclusive contracts. Economic Theory 32, 353–380.

[4] Belleflamme, P., Peitz, M., 2010. Industrial Organization: Markets and Strategies. Cambridge: Cambridge University Press.

[5] Belleflamme, P., Toulemonde, E., 2018. Tax incidence on competing two-sided platforms. Journal of Public Economic Theory 20, 9-21.

[6] European Commission, 2009. Communication from the Commission – Guidance on the Commission's enforcement priorities in applying Article 82 of the EC Treaty to abusive exclusionary conduct by dominant undertakings. Official Journal of the European Union, 2009/C 45/02.

[7] Franck, J.-U., Peitz, M., 2019. Market definition and market power in the platform economy. CERRE report.

[8] Gabszewicz, J.J., Wauthy, X., 2012. Nesting horizontal and vertical differentiation. Regional Science and Urban Economics 42, 998-1002.

[9] Gabszewicz, J.J., Wauthy, X., 2014. Vertical product differentiation and two-sided markets. Economics Letters 123, 58-61.

[10] Gold, A., 2010. Tipping in two-sided software markets: An investigation of asymmetric cost differences. Mimeo.

[11] Hagiu, A., 2006. Pricing and commitment by two-sided platforms. Rand Journal of Economics 37, 720–737.

[12] Jullien, B., Pavan, A., 2019. Information management and pricing in platform markets. Review of Economic Studies 86, 1666–1703.

[13] Lin, M., Li, S., Whinston, A.B., 2011. Innovation and price competition in a two-sided market. Journal of Management Information Systems 28, 171-202.

[14] Njoroge, P., Ozdaglar, A., Stier-Moses, N., Weintraub, G., 2009. Competition, market coverage, and quality choice in interconnected platforms. Mimeo.

[15] Njoroge, P., Ozdaglar, A., Stier-Moses, N., Weintraub, G., 2010. Investment in two sided markets and the net neutrality debate. Review of Network Economics 12, 355-402.

[16] Ponce, J., 2012. The quality of credit ratings: A two-sided market perspective. Economic Systems 36, 294-306.

[17] Reisinger, M., 2014. Two-part tariff competition between two-sided platforms. European Economic Review 68, 168-180.

[18] Rochet, J.-C., Tirole, J., 2003. Platform competition in two-sided markets. Journal of the European Economic Association 1, 990–1024.

[19] Rochet, J.-C., Tirole, J., 2006. Two-sided markets: a progress report. Rand Journal of Economics 37, 645–667.

[20] Tan, G., Zhou, J., 2020. The effects of competition and entry in multi-sided markets. Review of Economic Studies, forthcoming.

[21] Viecens, M.F., 2006. Two-sided platforms with endogenous quality differentiation. Mimeo.