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Shareholder Votes on Sale

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Abstract

This paper examines the effect of vote trading on shareholder activism and corporate governance. We show that vote trading enables hostile activism because voting rights trade at inefficiently low prices even when the activist's motives are transparent. Our results explain empirical findings of low vote prices (Christofferson et al. 2007) and inefficient outcomes (Hu & Black 2006). Though an activist with superior information can facilitate information transmission through vote trading, traditional activist intervention techniques provide the same information transmission without the downsides inherent in vote trading. Our analysis of potential policy measures suggests that adopting simple majority rules and excluding bought votes offer the most promising intervention avenues.

Keywords: blockholder, decoupling techniques, empty voting, hostile activism, shareholder activism, vote trading

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1 Introduction

Shareholder voting is one of the cornerstones of corporate governance. It equips shareholders with the power to enforce their demands, laying the foundation for shareholder activism. Typically, a shareholder's voting rights are determined by her shares on a pro-rata basis—one share, one vote—thereby linking a shareholder's influence to his economic interest. However, activist investors can subvert this principle by acquiring voting rights far in excess of their cash flow claims. While the outright trade of voting rights is illegal in most jurisdictions, financial innovation has created new techniques to decouple voting power and economic exposure—for instance, via the equity lending market. Activist investors were happy to add these new techniques to their toolbox,[1] whereas the decoupling raised eyebrows among policymakers[2] and the press.[3]

In this paper, we analyze how decoupling techniques relate to traditional forms of shareholder activism, and examine the consequences for corporate governance. We focus on the class of decoupling techniques that are economically equivalent to the outright trade of voting rights.[4] In the remainder of the paper, we simply refer to (the usage of) these techniques as *vote trading*. Importantly, this class includes the most common practice of acquiring voting rights by borrowing shares over the record date (Christofferson et al. 2007). Our analysis reveals that vote trading unilaterally benefits hostile activists, and is not needed for friendly activists to guide corporate decision making as they can rely on traditional intervention techniques such as proxy campaigns.[5]

In a first analysis, we build a simple model in which a finite number of shareholders vote on the implementation of a reform. Shareholders know the reform to be value increasing and, thus, support it. In this setting, there is no need for value-increasing activism. Therefore, we concentrate on the case of a hostile activist who derives private benefits from the company sticking with the status quo. Shareholders are fully aware of the activist's motives.

We show that despite the activist's transparent motives, the activist can acquire voting rights at prices close to zero and prevent the value increasing reform. This is the result of a market failure in the market for voting rights. The value of a voting right depends

[1] Hu & Black (2006, 2007, 2008, 2015) document anecdotal evidence of decoupling. We henceforth always reference Hu & Black (2015) as the most recent overview.

[2] Consider, for example, the "SEC Concept Release on the U.S. Proxy System" (July 2010), https://www.sec.gov/rules/concept/2010/34-62495.pdf, the "SEC Staff Roundtable on the Proxy Process" (July 2018), https://www.sec.gov/news/public-statement/statement-announcing-sec-staff-roundtable-proxy-process, or the the ESMA's "Call for evidence on empty voting" (September 2011), https://www.esma.europa.eu/press-news/consultations/call-evidence-empty-voting.

[3] The New York Times, April 26, 2012, "The Curious Case of the Telus Proxy Battle", https://dealbook.nytimes.com/2012/04/26/the-curious-case-of-the-telus-proxy-battle/.

[4] For a structured overview over decoupling techniques, compare Speit & Voss (2020).

[5] Our results imply that activist chooses her intervention method as a function of her motives. Hence, the model explains why studies investigating "traditional" shareholder activism (such as Brav et al. (2008)) find positive effects of activism on shareholder value, whereas the evidence on vote trading suggests adverse effects on shareholder value (Hu & Black 2015).

on the trading and voting decisions of the other market participants: it only bears value if it is decisive (pivotal) in the outcome of the vote, which is unlikely for any individual voting right. Therefore, rational shareholders are willing to sell their voting rights at a price significantly below their individual loss from the blocked reform. This allows the hostile activist to block the value-increasing reform without compensating shareholders.

Competition in the market for voting rights does not fix the market failure and, hence, does not prevent hostile activism. Even if a blockholder is willing to act as a white knight and make a competing offer, he may be at a disadvantage depending on the majority rule. In particular, if the reform requires a supermajority to pass, it may be too expensive for the blockholder to acquire the necessary fraction of voting rights. Therefore, competition reduces the threat of hostile activism, but inefficient outcomes remain.

Our results give a new interpretation of the empirical and anecdotal observations on vote trading. Christofferson et al. (2007) find that voting rights trade at near-zero prices, which they attribute to common interests of investors. On the other hand, Hu & Black (2015) present anecdotal evidence of vote trading which yields—prima facie—inefficient outcomes. We reconcile these two seemingly contradictory findings in that we show that low prices need not be a sign of common interests, and inefficient outcomes do not require hidden motives. Instead, our analysis suggests that low prices are caused by a more fundamental market failure. Further, the competitive advantage of a hostile activist in supermajority decisions delivers an explanation for the disproportionate occurrence of vote trading in these decisions, as documented by Hu & Black (2015).

In a second step, we consider the more complex setup in which the activist possesses superior information about the effect of the reform. We ask the question of whether vote trading may be advantageous for corporate governance by fostering information transmission from the activist to other shareholders,[6] and we compare vote trading to other traditional forms of activist interventions. To this end, we extend the model by an uncertain state that determines whether the reform proposal increases or decreases shareholder value. The activist privately knows the state.

If the activist and shareholders have aligned interests, that is, if the activist's private benefit from the status-quo is negligible, vote trading is not necessary for information transmission: the activist can also communicate her superior information via cheap talk, such as public endorsements.

We focus on the case in which the activist's private benefit from the status quo leads her to oppose the reform in either state, preventing cheap talk. Interestingly, despite the misaligned interests of shareholders and activist, vote trading can facilitate information, and improve firm value in this situation. Shareholders can learn from the activist's vote acquisition: when the activist is endowed with some shares, her willingness to pay for the

[6] The informational advantage of vote trading is stressed by Brav & Mathews (2011) and Eso et al. (2015).

voting rights is correlated with the state. This gives rise to a separating equilibrium in which the activist prevents the reform more often when it is in the shareholders' interest, thereby increasing firm value. However, the ability to improve shareholder value depends on significant prices for voting rights, since those are needed as a costly signal. When shareholdings are dispersed, the emerging low prices prevent an informational benefit. Absent of vote trading, the activist might use other costly signals to achieve the same, or even superior outcomes. Activist investors' traditional methods—the acquisition of a minority stake in the company, or costly proxy fights, for example—can achieve first-best communication, independent of the shareholder structure.

We conclude that vote trading benefits only hostile activists because they cannot rely on traditional forms of activist interventions. As a result, vote trading threatens corporate governance and shareholder value. This is true, even in the (unlikely) best-case scenario in which shareholders are fully informed about the activist's motives. Thus, we advocate the regulation of vote trading.

Because inefficient outcomes from the market for voting rights occur even when motives are transparent, policy measures aimed at increasing transparency are not sufficient to restore efficiency and prevent hostile activism. At the same time, vote trading often emerges as a byproduct, such that banning transactions that may be used for vote trading is costly. Consequently, we recommend policy measures that regulate the eligibility to vote. In particular, we propose regulating entities instead of securities. That is, we argue that any entity who acquires voting rights through vote trading should not be eligible to vote. Further, our analysis reveals that decisions taken by supermajority rule are especially likely to be blocked by hostile activists. Consequently, our model suggests that simple majority voting helps to prevent hostile activism.

1.1 Trading votes for shareholder meetings

In this paper, we analyze the empirically most relevant decoupling techniques,[7] which are the ones that are economically identical to the outright trade of voting rights. For simplicity, we refer to (the usage of) these techniques as *vote trading*. When engaging in vote trading, the activist trades directly with the shareholders, and the economic exposure remains with the shareholders at all times. Only the voting right changes hands for a flat transfer.

In practice, the bulk of vote trading occurs via the equity lending market. Since the possession of a share at the record date suffices to obtain the voting right, an activist investor seeking to acquire voting rights only needs to borrow the shares she wants to vote over the record date. When the lending fee is independent of the share value, as is

[7] Financial innovation has created a multitude of decoupling techniques that diverge in their economic implications depending on the timing, order of transactions, and counterparties. For a more detailed account of the shareholder voting process and an overview over (other) decoupling techniques, compare Speit & Voss (2020).

usually the case, the shareholder (lender) retains the economic exposure, and only sells the voting right. The lending fee captures the cost of acquiring the voting right. Alternatively, the same outcome can be achieved through a repo contract in which the cash-providing side (the activist) obtains the shares for a limited amount of time, before selling it back to the collateral providing side (shareholders) at pre-negotiated terms. Again, the initial shareholder fully retains the economic exposure, whereas the activist only secures her right to vote, at a flat price. Last, it is easy to design synthetic assets that are economically equivalent to vote trading.[8]

1.2 Empirical insights from the equity lending market

Christofferson et al. (2007) provide the first evidence of vote trading via the equity lending market. They find that a significant spike in the volume of share lending over the record date. Kalay et al. (2014) validate this result with a different estimation approach.[9] Hu & Black (2015), collect anecdotal evidence of decoupling between 1988 and 2008. They register over 40 decoupling cases, many of which rely on share lending. In those cases, the additional voting rights were used to influence decisions over diverse issues, ranging from management entrenchment to takeover approvals. The practice continues to be popular with activists, as the recent fight for control of Premier Foods (2018) highlights.[10] Arguably, one of the reasons for this popularity of the equity lending market as a platform for vote trading is its size and liquidity. Within the U.S. stock market, for instance, an average of 20%[11] of a company's shares is available for borrowing (Campello et al. 2019).[12]

Besides providing empirical evidence of an active and sizable market for voting rights, Christofferson et al. (2007) and Kalay et al. (2014) also estimate the market price of voting rights. Christofferson et al. (2007) find no significant prices for voting rights, whereas Kalay et al. (2014) estimate significant but small prices. Christofferson et al. (2007) interpret

[8] For instance, the activist could engage in voting trading by buying synthetic calls, i.e. bundles of shares and a put option, from the shareholders. If the put option is at the money, the activist can exercise it right after the record date, such that she only retains the voting right. In case the activist is hostile and seeks to reduce share value, she will always exercise the option and the economic exposure remains with the shareholders.

[9] Kalay et al. (2014) focus on decoupling techniques that work via the options market and are not equivalent to the outright trade of voting rights (Speit & Voss 2020), i.e. the class of decoupling techniques analyzed in this paper. However, they also use their methodology to analyze data from the equity lending market.

[10] Financial Times, July 15, 2018, "Market reverberates with accusations of empty voting", https://www.ft.com/content/0e28929e-85dd-11e8-a29d-73e3d454535d.

[11] With the continuing growth in popularity of ETF's which use share lending as an integral part of their business model, the size of this market is likely to expand—see, for example, Deutsche Bundesbank Monthly Report, October 2018, https://www.bundesbank.de/resource/blob/766600/2fd3ae4f0593fb2ce465c092ce40888b/mL/2018-10-exchange-traded-funds-data.pdf.

[12] Campello et al. (2019) show that companies try to limit the number of lendable shares with share buybacks, and argue that they do so to limit short-selling opportunities. Our results give another rationale for the buyback—namely that placing a limit on the number of lendable shares limits the number of votes that can be bought via the equity lending market.

their findings as a sign of common values. Because all investors supposedly share the same interests, there is no need to charge positive prices for voting rights. Instead, investors are willing to delegate their voting rights to more informed parties. This argument, however, seems to be at odds the findings of Hu & Black (2015); most of the their cases resulted in—prima facie——inefficient outcomes and reduced shareholder value. While different in detail, the cases share a common feature in that voting rights acquired by a single hostile activist were used to block supermajority decisions.

Our theory reconciles the empirical findings of positive trading volume, low prices, and inefficient outcomes. We show that a market failure in the market for voting rights leads to low prices, and thereby enabling hostile activism. Those inefficient outcomes do not require hidden motives by the activist.[13]

The paper is structured into five sections. Section 2 reviews the literature. In Section 3 we show that vote trading in a symmetric information setting uniquely benefits a hostile activist who can exploit a market failure in the market for voting rights. In Section 4 we investigate the effect of vote trading when the activist has superior information about the correct course of action. We compare vote trading with traditional forms of activist interventions. We draw conclusion from our findings in Sections 5, before developing policy recommendations in Section 6. All proofs are relegated to the appendix.

2 Literature

Decoupled economic interest and voting power has been studied in the context of dual-class share structures and takeovers. Grossman & Hart (1988) as well as Harris & Raviv (1988) provide conditions under which a single share class is optimal. Gromb (1992) proofs that reducing the number of voting shares increases shareholders' likelihood of being pivotal, thereby reducing shareholders' free-riding incentives. Burkart et al. (1998) shows that if private benefits are an endogenous bidder choice after the takeover, reducing the number of voting shares necessary for control can increase welfare. When bidders have private information about their potential value, At et al. (2011) show that dual-class shares can facilitate value-increasing corporate takeovers. For a detailed overview of the theoretical literature on dual-class shares and takeovers compare Burkart & Lee (2008). Adams & Ferreira (2008) summarizes the empirical literature on dual-class shares, stock pyramids or cross-ownership. They find that the value of voting rights differs substantially across countries, time frames, and studies, but can be quite significant. However, trading dual-class shares to decouple voting rights and economic interests is not equivalent to outright trade of voting rights such that it has different economic implications.

[13] Hu & Black (2007) point out that there may be other issues, such as lack of transparency in the market for voting rights and pivotality considerations. We pick up on this issue of pivotality and formalize it.

Burkart & Lee (2015) show how the free-rider problem and asymmetric information can be overcome by the usage of option contracts. In the context of contests for corporate control, Dekel & Wolinsky (2012) find that allowing for vote trading in addition to share trading may increase the probability that an inefficient bidder takes over the company. Neeman & Orosel (2006) consider a repeated control contest among an incumbent manager and a challenger in which vote trading can be used as a signaling device. Blair et al. (1989) analyze the effect of taxation in a takeover contest where shares and votes can be traded separately. In political economy, Dekel et al. (2008) consider a contest between two political party's which can either buy votes or bribe voters. The authors find that overall payments are substantially higher when parties can pay bribes. Dekel et al. (2009) introduce budget constraints in this setting. Their model is related to our competition game, as we discuss in Section 3.3. Casella et al. (2012) demonstrate that the market for voting rights does not have a competitive equilibrium; thus, they introduce an 'ex-ante vote trading equilibrium." They identify conditions under which vote trading fails to aggregate preferences, and generates welfare losses relative to simple majority voting.

Neeman (1999) and Bó (2007) argue that a single buyer can acquire voting rights at zero prices. Neeman (1999) shows that when the number of voters grows large, a zero-price equilibrium is the only pure strategy equilibrium robust to noise voters. Bó (2007) shows that when an activist can write arbitrary, outcome-dependent contracts, she can bribe voters to vote for her at zero cost.

Brav & Mathews (2011) examine the effects of vote trading in the presence of an informed activist who can either buy shares of a company or sell them short. Shareholders are no strategic players, but are noise voters. By assumption, the activist can acquire a certain fraction of their voting rights for free. The activist is more likely to be pivotal when she has aligned interests because additional shares also provide her with additional voting rights. As a consequence, vote trading increases the expected welfare. In Eso et al. (2015) only shareholders with (conditionally) aligned interests participate in the market for voting rights. They use the market as a way to delegate their voting rights to the most informed parties, aiding information aggregation and ensuring that partisans are out-voted.

This paper is also related to the literature on shareholder voting. Yermack (2010) summarizes the empirical literature on shareholder voting in United States based companies, whereas Iliev et al. (2015) present evidence for the importance of shareholder voting in non-U.S. firms. Bar-Isaac & Shapiro (2020) show that a blockholder may optimally abstain from voting with all his shares to not crowd out information of other shareholders. This requires alignment of interests among the blockholder and other shareholders. Levit & Malenko (2011) show that non-binding shareholder voting may fail to aggregate information when interests between management and shareholders only partially align. Malenko & Malenko (2019) study the effect of proxy advisors on information acquisition and voting behavior of shareholders. Levit et al. (2019) analyze the effect of share trading opportunities on shareholder voting, the shareholder base, and the optimal board design.

3 Symmetric information

3.1 Model

We revisit the model of Speit & Voss (2020) but with a finite number of shareholders and an activist who may own shares in the company.

Investors: Consider a public company with $n \in \mathbb{N}$ shares outstanding. Each share consists of a cash-flow claim and a voting right. The company is owned by two types of investors: an activist investor who owns $\alpha n \in \mathbb{N}_0$ shares and $(1-\alpha)n = n_S \geq 3$ ordinary shareholders, who hold a single share each. Henceforth, we will refer to the activist shareholder as *activist*, A, and to the ordinary shareholders as *shareholders*, S, although the activist can be a shareholder herself. All investors are risk neutral.

Shareholder meeting: The company has an upcoming shareholder meeting with a single, exogenously given reform proposal on the agenda. The vote is binding,[14] and the reform is implemented if at least $\lambda n \in \mathbb{N}$ votes are cast in favor of it. Otherwise, the status quo prevails. We assume that $1 - \lambda > \alpha$, such that the activist cannot block the reform unilaterally and that $1 < \lambda n < n_S$ such that an individual shareholder can neither block, nor implement the reform.

Payoffs: If the company sticks with the status quo, the company's total share value remains unchanged at $v > 0$; if the reform is implemented, the company's value increases by $\Delta > 0$ to $v + \Delta$.

Despite the positive effect of the proposed reform on firm value, the activist may oppose it as she gains private benefits $b > 0$ from the status quo. These private benefits can, for instance, stem from other assets in her portfolio.[15] Debt in the same company may reduce the risk appetite, common ownership leading to anti-competitive preferences[16] or different supplier choices. Alternatively, the status-quo may allow the activist to (continue to) extract b at a cost to the firm of Δ. In any case, we take b to be exogenously given and fixed. In summary, the payoffs are

	activist	shareholder
status quo	$\alpha v + b$	$\frac{v}{n}$
reform	$\alpha(v+\Delta)$	$\frac{v+\Delta}{n}$.

[14] In the US binding shareholder voting occurs in the context of by-law amendments, acquisitions, and equity restructuring. In other countries, such as countries of the EU, shareholder decisions are usually binding.

[15] In 2004, during the acquisition of MONY by AXA, bond holdings introduced a wedge in the interest of MONY shareholders, compare https://www.nytimes.com/2004/05/19/business/holders-of-mony-approve-1.5-billion-sale-to-axa.html.

[16] Compare Azar et al. (2018) for empirical evidence on the effects of common ownership.

When $b < \alpha\Delta$, the activist and the shareholders have aligned interests, and both prefer to implement the reform; the activist is *friendly.* If $b > \alpha\Delta$, the activist prefers the company to stick with the status quo, in which case she is *hostile.* Since the friendly activist has no effect on the outcome of the decision under symmetric information, in this section we focus on this case of a hostile activist. Further, we think of the private benefit b as relatively small compared to the overall change in firm value Δ. In particular we assume that $b < \Delta$ such that the reform increases welfare.[17]

3.1.1 Voting stage

As usual, the voting stage has degenerate equilibria in which all investors either vote for the status quo or the reform. When no voter can swing the outcome of the vote unilaterally, voting independent of the own preferences is a best response. However, these strategies are weakly dominated and yield peculiar equilibria, such that we rule them out. We assume that if an investor's voting decision does not affect the outcome of the vote, she votes for her preferred alternative. Hence, the activist casts all of her votes in favor of the status quo and the shareholders in favor of the reform. The outcome of the vote is, thereby, uniquely determined by who owns how many voting rights at the time of the meeting.

In the following, we do not model the voting stage explicitly, but only use that the activist can block the reform if she controls at least $(1-\lambda)n+1$ voting rights. Given that $\alpha < (1-\lambda)$, this means that she needs $m = (1-\lambda-\alpha)n+1$ additional voting rights to prevent the reform. Otherwise, the efficient reform is implemented.

3.2 Vote trading

We now allow the activist to acquire voting rights, for instance by borrowing shares over the record date.

Suppose the activist can make a public take-it-or-leave-it offer $p \in \mathbb{R}_+$ per voting right. The offer is restricted, meaning that the activist can set an upper bound on the number of voting rights she is willing to acquire. If more shareholders sell to her, they are rationed. It is without loss to assume that the activist sets an upper bound at $m = (1-\lambda-\alpha)n+1$ voting rights. Having observed the offer p, shareholders simultaneously decide whether to sell. To capture the anonymity among shareholders, we consider symmetric strategies represented by a response function $q : \mathbb{R}_+ \to [0,1]$, which maps any offer p into an acceptance probability $q(p)$. As a result, the total number shareholders who accept is a binomial random variable $M(n_S, q(p)) \sim Bin(n_S, q(p))$. Since shareholders are rationed when $M(n_S, q(p)) > m$, the activist acquires $\bar{M}(n_S, q(p)) = \min\{M(n_S, q(p)), m\}$ voting rights.

Suppose that the activist offers price p and the shareholders respond by mixing with

[17]If $b \geq \Delta$, the activist could simply take over the company and block the reform, maximizing welfare.

probability $q(p)$. If the activist buys fewer than m votes, the company's value raises to $v+\Delta$. As a result, her payoff is $\alpha(v+\Delta) - pM(n_S, q(p))$. To the contrary, if $M(n_S, q(p)) \geq m$, the firm value remains at v and the activist receives the private benefit b, such that her payoff is $\alpha v + b - pm$. Together, this yields an expected payoff of

$$\Pi_A(p;q) = \alpha(v+\Delta) + \mathbb{P}[M(n_S, q(p)) \geq m](b - \alpha\Delta) - p\mathbb{E}[\bar{M}(n_S, q(p))]. \tag{1}$$

A shareholder's payoff depends on her selling decision, as well as the behavior of the other $n_S - 1$ shareholders. Fix one shareholder, suppose that the activist offers price p and that the other shareholders respond by mixing with probability $q(p)$. If the shareholder decides to sell his voting right, but fewer than $m-1$ other shareholders also sell, the reform passes and the shareholder's payoff is $p + \frac{v+\Delta}{n}$. Conversely, if at least $m-1$ of the other shareholders also sell their voting rights, the reform is blocked and the share value remains at $\frac{v}{n}$. Further, if more than $m-1$ other shareholders sell, i.e. $M(n_S - 1, q(p)) > m - 1$, the shareholder is rationed. In this case, his payoff is

$$p\frac{m}{M(n_S - 1, q(p)) + 1} + \frac{v}{n}.$$

If the shareholder does not sell his voting right, but at least m other shareholders do, the reform is blocked and his payoff is $\frac{v}{n}$. Otherwise, it rises to $\frac{v+\Delta}{n}$. In expectation, this means that a shareholder's payoff is

$$\Pi_S(\text{sell}; p, q) = \frac{v+\Delta}{n} - \mathbb{P}[M(n_S - 1, q(p)) \geq m-1]\frac{\Delta}{n} + p\frac{\mathbb{E}[\bar{M}(n_S, q(p))]}{n_S q(p)}$$

if he sells his voting right and

$$\Pi_S(\text{keep}; p, q) = \frac{v+\Delta}{n} - \mathbb{P}[M(n_S - 1, q(p)) \geq m]\frac{\Delta}{n}$$

if he keeps his voting right. The fraction $\frac{\mathbb{E}[\bar{M}(n_S, q(p))]}{n_S q(p)}$ is the probability not to be rationed.[18]

We consider subgame perfect equilibria.

Proposition 1. *For any n, an equilibrium (p^*, q^*) exists. If $q^*(p^*) > 0$ and thereby $\mathbb{P}[M(n_S, q^*(p^*)) \geq m] > 0$, then*

$$\underbrace{p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))]}_{\mathbb{E}[total\ transfer]} < \underbrace{m\frac{\Delta}{n} \cdot \mathbb{P}[M(n_S, q^*(p^*)) \geq m]}_{\mathbb{E}[loss\ per\ shareholder]}. \tag{2}$$

Further,

[18] Compare (7) in the appendix for an explicit derivation of the expression.

- *there always is an equilibrium in which* $p^* = 0$ *and* $q^*(0) = 1$;
- *as* n *grows large, along any sequence of equilibria,*

$$\lim_{n\to\infty} \mathbb{P}[M(n_S, q^*(p^*)) \geq m] = 1 \quad \textit{and} \quad \lim_{n\to\infty} p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] = 0.$$

Proposition 1 establishes that the activist can obtain the blocking minority without the need to (fully) compensate the shareholders (2). Whenever there is trade,[19] shareholders suffer a strict loss. This is possible because the activist can exploit two inefficiencies, which create a market failure in the market for voting rights.

First, there is the externality of voting. The λ-majority-rule implies that only $(1-\lambda)n+1$ votes have to be cast against the reform to block it. This blocking minority does not internalize the effect of their behavior on the rest of the shareholders. As a result, it would suffice if the activist compensated m shareholders for their individual loss of $\frac{\Delta}{n}$.

However, the activist can do even better and pays less than m times the expected loss of a shareholder (2). A shareholder's valuation for her voting right depends on the selling decisions of the other shareholders. The voting right is only valuable if it is decisive or pivotal in the vote– that is, if exactly $m-1$ other shareholders sell their voting rights. Therefore, any shareholder compares the expected payment the activist offers with the expected loss from the offer, but weighs the expected loss with the probability to be pivotal. In particular, if the activist offers p and the other shareholders sell with probability $q(p)$, a shareholder prefers to sell if $\Pi_S(\text{sell}; p, q) \geq \Pi_S(\text{keep}; p, q)$, which rearranges to

$$\underbrace{p\frac{\mathbb{E}[\bar{M}(n_S, q(p))]}{n_S q(p)}}_{\mathbb{E}[payment]} \geq \underbrace{\mathbb{P}[M(n_S - 1, q(p)) = m-1]}_{\mathbb{P}[pivotal]} \underbrace{\frac{\Delta}{n}}_{loss}. \tag{3}$$

As $m = (1-\lambda-\alpha)n + 1 \in \{2, ..., n_S - 1\}$, the probability of being pivotal is always strictly smaller than 1.[20] Hence, there is a dilution of control and the activist can acquire the voting rights at a discount.

The proof of Proposition 1 further shows that as the population of shareholders grows, the probability that any single shareholder is pivotal quickly converges to zero.[21] Therefore, any equilibrium outcome approaches the most extreme one in which every shareholder sells his voting rights to the activist for free, and the activist always blocks the reform.

When the number of shareholders is sufficiently large, the market failure that creates inefficient outcomes occurs across all symmetric equilibria, such that our result does not rely

[19] Whenever n and b are sufficiently small, there may also be an equilibrium in which $p^* = 0$ and $q^*(p^*) = 0$.

[20] If $q \in \{0, 1\}$ such that every other or no other shareholder sells, $\mathbb{P}[pivotal] = 0$ and the shareholder sells at any positive price. For all $q \in (0, 1)$, every or no shareholder sells with strictly positive probability, such that $\mathbb{P}[pivotal] < 1$.

[21] Suppose for instance that $\alpha = 0$, $n = n_S = 101$ and $\lambda = \frac{51}{101}$. Then $m = 51$ and $\mathbb{P}[M(100, q^*(p)) = 50] \leq \mathbb{P}[M(100, 0.5) = 50] < 0.08$.

on an equilibrium selection. Further, Neeman (1999) shows that the zero-price equilibrium is the only asymmetric equilibrium robust to noise voters; this highlights the robustness of our results.

3.2.1 Conditional or unrestricted offers

Restricted offers are natural since an activist only needs to acquire a fraction of the voting rights. Further, shareholders correctly anticipate the possibility to be rationed (left side of (3)), and demand a higher price to compensate for the possibility. Thereby, the restriction has no effect on the transfers, and Proposition 1 is completely driven by the shareholders' pivotality considerations. If we were to consider unrestricted offers, the results would remain unchanged for large n. For small n and large b the activist may choose a price that gives her, in expectation, more than m voting rights, to guarantee that she can block the reform. As a result, when there are few shareholders, the total transfer can exceed $m\frac{\Delta}{n}$. In the alternative case in which the activist can restrict the offer and condition it on the event that at least m shareholders agree to sell their voting right, the result of Proposition 1 is strengthened: for any n only the zero-price equilibrium survives. We prove the results in Lemmas 7 and 8 in the appendix.

3.3 Competing offers

We now investigate how the market failure and the resulting threat of hostile activism react to competition by a friendly blockholder. To that end, suppose that there is such a blockholder B who owns $\beta n \in \mathbb{N}$ shares but $\beta < \lambda$ such that he cannot implement the reform unilaterally. The number of ordinary shareholders is $n_S = (1-\alpha-\beta)n \in \mathbb{N}$. As before, activist A first makes an offer p_A for $m_A = (1-\lambda-\alpha)n+1$ voting rights. After observing A's offer, blockholder B, acting as a white knight who wants to implement the reform, jumps in and makes a counteroffer p_B for up to $m_B = (\lambda-\beta)n = n_S - m_A + 1$ voting rights. Thus, B's strategy is a function $p_B : \mathbb{R}_+ \to \mathbb{R}_+$ which maps any offer p_A into a counteroffer $p_B(p_A)$.

Note that for the shareholders, selling the voting rights to the blockholder dominates holding onto them. Thus, every shareholder (tries to) sell his voting right to either the activist or the blockholder. The symmetric best response function of shareholders is given by $q : \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$, where q is the probability that shareholders sell to A and $1-q$ the probability that they sell to B. Further, define $\bar{M}_A = \max\{M(n_S, q), m_A\}$ and $\bar{M}_B = \max\{n_S - M(n_S, q), m_B\}$ as the random number of shares A and B actually acquire. Again, we consider subgame perfect equilibria.

Proposition 2. *For any n, an equilibrium (p_A^*, p_B^*, q^*) exists.*

1. *If* $\frac{b-\alpha\Delta}{1-\lambda-\alpha} > \frac{\beta\Delta}{\lambda-\beta}$ *and* n *is sufficiently large, the reform is always blocked,* $q^*(p_A^*, p_B^*(p_A^*)) = 1$. *Further,*

$$p_A^* \mathbb{E}[\bar{M}_A(n_S, q^*(p_A^*, p_B^*(p_A^*)))] = p_A^* m_A < \frac{1-\lambda-\alpha}{\lambda-\beta}\beta\Delta,$$

but $\lim_{n\to\infty} p_A^* \mathbb{E}[\bar{M}_A(n_S, q^*(p_A^*, p_B^*(p_A^*)))] = \frac{1-\lambda-\alpha}{\lambda-\beta}\beta\Delta$.

2. *If* $\frac{b-\alpha\Delta}{1-\lambda-\alpha} < \frac{\beta\Delta}{\lambda-\beta}$, *as* n *grows large, along any sequence of equilibria, the reform becomes certain,* $\lim_{n\to\infty} \mathbb{P}[M_A(n_S, q^*(p_A^*, p_B^*(p_A^*)) \geq m_A] = 0$, *and transfers converge to zero,* $\lim_{n\to\infty} p_A^* n_S = \lim_{n\to\infty} p_B^*(p_A^*) n_S = 0$.

When the shareholdings are dispersed, i.e. n and m_A, m_B are large, no individual shareholder has a substantial probability of being pivotal. Thus, he simply sells to the investor who offers the higher expected payment, anticipating the different probabilities to be rationed. How much A and B are willing to offer depends on their willingness to pay, $b - \alpha\Delta$ and $\beta\Delta$, as well as the number of shares they have to acquire, $(1-\lambda-\alpha)n + 1$ and $(\lambda - \beta)n$. In particular, the activist has a comparative advantage when she has to acquire fewer shares than the blockholder—that is, when $\frac{(1-\lambda-\alpha)n+1}{(\lambda-\beta)n} \approx \frac{1-\lambda-\alpha}{\lambda-\beta} < 1$. Note that this is true whenever λ is large, such that competition is unlikely to deter hostile activism in supermajority decisions. Further, the compensation shareholders receive when the activist blocks the reform is decreasing in λ. Surprisingly, when λ is large, the total transfer from the activist to the shareholders can be substantially below the expected loss of the blockholder. When the hostile activist succeeds and blocks the reform, welfare is reduced, although small shareholders may be (partially) compensated.

If the blockholder deters the activist from making an offer, vote prices in our model are close to zero. On the other hand, if the blockholder cannot deter the activist, the activist has to pay a strictly positive transfer. The analysis by Dekel et al. (2009) suggest that strictly positive prices may be the result of the offer structure. Dekel et al. (2009) show that the unique trading price is zero if the activist and the blockholder can sequentially adjust their offer upwards, and if there is a continuum of shareholders.[22] Therefore, (close to) zero prices and positive trade volumes are compatible with competition in the market for voting rights.

3.4 Discussion

In markets for standard assets without externalities, voluntary trade produces Pareto improvements. We show that this intuition cannot be transferred to the market for voting

[22] Dekel et al. (2009) analyze a game with a continuum of voters in which the two contestants make alternating, increasing offers until one stops. By an unraveling argument, the loser does not compete, because while she would acquire a strictly positive fraction of the voting rights at a positive price, she would acquire too few voting rights to change the outcome of the vote.

rights. Not only does voting create an externality of the majority on the minority, but there is a market failure in the voting right market that goes beyond the externality of voting. The activist does not even compensate m shareholders for their loss; she pays close to zero compensation. This market failure is the result of the relative value of a voting right, which depends entirely on the other investors' trading and voting decisions, and is close to zero when the shareholdings are dispersed. Importantly, it does not depend on hidden motives by the activist, or the details of the modeling approach.[23] As long as shareholders do not believe that they are pivotal with probability one, the voting rights trade at inefficiently low prices.

As we show further, competition in the market for voting rights does not eliminate the market failure, and, by extension, cannot solve the problem of hostile activism. The threat of competition by a blockholder may deter hostile activists without raising voting right prices, but relies on the blockholder's willingness to pay as well as the number of voting rights he and the activist must acquire.

As pointed out previously, we do not consider a friendly activist in this section since she would not change the outcome of the vote. When the optimal decision is common knowledge, an activist plays a role only if she has misaligned interests, i.e. is hostile. Hence, in a symmetric information setting, vote trading uniquely aids hostile activists. In Section 4, we investigate the situation with asymmetric information.

Empirical predictions: Our model jointly explains low prices for voting rights (Christofferson et al. (2007), Kalay et al. (2014)) and inefficient outcomes caused by hostile activists which engage in vote trading (Hu & Black 2015).

Moreover, we show that active blockholders may deter hostile activists from acquiring voting rights, such that it is less likely to occur in companies with large, active blockholders. Interestingly, the competition does not need to increase prices in order to deter vote trading. Hence, the observed low prices in the market for voting rights do not necessarily indicate a lack of competition.

Last, our results imply that supermajority decisions are particularly likely to be targeted by hostile activists. In addition to market frictions, decisions that require a supermajority for approval give her a distinct advantage over any potential competitor. This fits the anecdotal evidence of Hu & Black (2015) showing that most incidents of vote trading occurred when a hostile activist blocked a reform that required a supermajority.

4 Asymmetric information

In the previous section, we established that in a symmetric information setting vote trading promotes hostile activism, threatening corporate governance and shareholder value. Cer-

[23] Casella et al. (2012) show that a competitive equilibrium does not exist. Instead, they consider a novel equilibrium concept, and show that vote trading can reduce (expected) welfare.

tainly, activism can also be put to good use.[24] Shareholders often rely on activist investors for their professional insights and analysis to identify value-increasing reforms. However, this mutually beneficial relationship is hindered by ulterior motives of the activist, which can make it hard for her to communicate with the shareholders. To solve this problem, activists engage in proxy fights and disclose their share position to convince shareholders of their best intentions.

In this section, we investigate the possibilities of vote trading to improve corporate governance under asymmetric information.[25] To this end, we consider a version of the model in which the activist possesses private information about the optimal reform decision. We compare vote trading with traditional forms of intervention, which we identify by their potential to (credibly) communicate the information. The analysis is split into two cases: when the activist and the shareholders have common interests (friendly activist), and when the activist always wants to block the reform (hostile activist).

4.1 Model

States and payoffs: We extend the model by introducing an uncertain state $\omega \in \{Q, R\}$ with prior probability $\rho \in (0, \frac{1}{2})$ that the state is Q. The activist investor, A, knows the state, the shareholders, S, do not. Throughout Section 4, the activist has a strictly positive share endowment, $\alpha > 0$.

Again, the activist obtains private benefits whenever the status quo remains. The reform, however, is not uniformly beneficial for shareholders. In state Q, the reform reduces firm value by Δ, such that shareholders also prefer the status quo over the reform; in state R the reform raises firm value by Δ. As a result, the payoffs are

Q	activist	shareholder	R	activist	shareholder
status quo	$\alpha v + b$	$\frac{v}{n}$	status quo	$\alpha v + b$	$\frac{v}{n}$
reform	$\alpha(v - \Delta)$	$\frac{v-\Delta}{n}$	reform	$\alpha(v + \Delta)$	$\frac{v+\Delta}{n}$.

4.1.1 Voting stage

Shareholders try to maximize their (expected) share value by matching the state. Let ξ be the shareholders' belief that the state is Q at the time of the vote. As before, we ignore degenerate equilibria where voters play weakly dominated strategies. This means that if $\xi < \frac{1}{2}$ shareholders vote for the reform, and if $\xi > \frac{1}{2}$ they vote for the status quo. Absent of any additional information $\xi = \rho < \frac{1}{2}$, meaning that shareholders vote for the reform. The activist knows the state and matches the state whenever $b < \alpha\Delta$, but she always votes in

[24] Compare Brav et al. (2008, 2015) for an empirical analysis of the effects of hedge fund activism.

[25] Brav & Mathews (2011) and Eso et al. (2015) stress the positive effect of vote trading on information transmission and aggregation.

favor of the status quo if $b > \alpha\Delta$. As noted before, we refer to these two cases as a *friendly activist* and *hostile activist*, respectively.

4.2 Friendly activist, $b < \alpha\Delta$

When the activist has superior information valuable to shareholders, she can potentially improve corporate decision making. Therefore, we also need to analyze the friendly activist, who did not change the outcome of the decision in the symmetric information case.

4.2.1 Vote trading

Suppose the activist can make a public take-it-or-leave-it offer $p \geq 0$ for up to m voting rights. Alternatively, the activist may make no offer, which we denote by $\emptyset$.[26] Since the activist's offer depends on the state, her strategy becomes $p : \{Q, R\} \to \mathbb{R}_+ \cup \emptyset$. Having observed the offer, any individual shareholder updates her belief to $\xi(p)$ and sells with probability $q(p) \in [0, 1]$.

Because the activist votes for the firm-value maximizing decision, shareholders benefit from selling their voting right to her. The activist, on the other hand, tries to acquire the voting rights or steer the decision at the lowest possible cost. The payoffs are stated explicitly in the proof of Lemma 1.

We solve the game for perfect Bayesian equilibria.

Lemma 1. *An equilibrium $(p^*, q^*; \xi^*)$ exists. In any equilibrium,*

- *the activist offers $p^*(\omega) = 0$ in at least one state $\omega \in \{h, \ell\}$;*
- *the reform is implemented in state R and the status quo remains in state Q.*

By Lemma 1, vote trading increases the probability that the state is matched from $1 - \rho$ to 1. Since this is in the best interest of both shareholders and the activist, welfare rises from $v + (1 - 2\rho)\Delta$ to $v + (1 - \rho)\Delta + \rho b$. This improvement is achieved through one of two types of equilibria. In the "delegation equilibrium," the activist acquires all voting rights for $p^*(Q) = p^*(R) = 0$. Shareholders know that the activist has aligned interests and that she implements the correct decision, such that they cede their voting rights to her.[27] In a "signaling equilibrium," the friendly activist only offers to purchase the voting rights in one state. Therefore, the presence (or lack) of an offer reveals the state to the shareholders and they vote in favor of the correct decision.

[26] Such an action would be (weakly) dominated by offering zero in the symmetric information game.

[27] Observe that while such an equilibrium also exists in the game with a hostile activist, the rationale here is different. Shareholders benefit from delegating their voting rights, such that they strictly prefer to do so, independent of pivotality considerations.

4.2.2 Costless communication

Whenever the activist is friendly, there are other forms of activist interventions by which she can ensure that the correct decision is implemented. She just has to communicate the optimal decision to the shareholders.

Formally, suppose that the activist cannot acquire voting rights, but communicates with the shareholders before the meeting by sending a message from $\{0, 1\}$. Thus, a strategy for the activist is a mapping from the state into the binary message space $\mu : \{Q, R\} \to \{0, 1\}$. Having observed $\mu(\omega)$, shareholders form posterior $\xi(\mu(\omega))$ and vote for the status quo if $\xi(\mu(\omega)) > \frac{1}{2}$, and vote for the reform if $\xi(\mu(\omega)) < \frac{1}{2}$. We consider perfect Bayesian equilibria.

Lemma 2. *There is an equilibrium $(\mu^*; \xi^*)$ in which the activist sends $\mu^*(Q) \neq \mu^*(R)$, such that shareholders learn the state. Thereby, the reform is implemented in state R and the status quo remains in state Q.*

Since shareholders and the friendly activist have aligned interests, they follow her recommendation, such that the correct decision is taken, and welfare is maximized. Thus, vote trading does not have a unique upside when the activist is friendly.

In practice, means of (cheap talk) communication are readily available and there is a long-standing tradition of activist investors endorsing company policies or publicly venting their discontent with management, be it through public statements, interviews, or 13D attachments. Further, the internet significantly simplifies the communication among shareholders, and regulatory authorities have deliberately removed legal obstacles to foster communication. For example, proxy rule amendments made in 2007 by the U.S. Securities and Exchange Commission (SEC) encourage electronic shareholder forums with this in mind. Christopher Cox, who served as SEC chairman at that time, summarized the reform, saying,[28]

"Today's action is intended to tap the potential of technology to help shareholders communicate with one another and express their concerns to companies in ways that could be more effective and less expensive. The rule amendments are intended to remove legal concerns, such as the risk that discussion in an online forum might be viewed as a proxy solicitation, that might deter shareholders and companies from using this new technology."

Ultimately, there is another channel by which the correct decision can be implemented by the friendly activist: delegation. Uniformed shareholders have an incentive to give a proxy to the informed, friendly activist free of charge. This allows the friendly activist to implement the correct decision in their place, resulting in the same Pareto improvement that vote trading offers.

[28] SEC press release from November 28, 2007, http://www.sec.gov/news/press/2007/2007-247.htm.

4.3 Hostile activist, $b > \alpha\Delta$

As we have seen in the last section, vote trading as well as other forms of costless communication or delegation can improve corporate governance and shareholder value when the activist is friendly. When the activist is hostile, however, she always wants to block the reform, such that she cannot transmit information to shareholders via cheap talk, and shareholders are unwilling to delegate their voting rights. However, we show in this section that vote trading might still improve corporate governance and the expected firm value. We then investigate whether traditional forms of intervention can have similar benefits.

4.3.1 Vote trading

Again, the activist can make a public take-it-or-leave-it offer p for up to m voting rights. Shareholders update their belief to $\xi(p)$ and decide with which probability to sell, $q(p)$. Thus, strategies are $p : \{Q, R\} \to \mathbb{R}_+$ and $q : \mathbb{R}_+ \to [0, 1]$.

The shareholders' posterior belief about the state, $\xi(p)$, affects their expected loss when the activist blocks the reform. When $\xi(p) > \frac{1}{2}$, shareholders actually prefer the status quo, fixing the firm value at v. In this case, shareholders' incentives are aligned with those of the activist and selling to the activist does not change the outcome of the vote, such that there is no expected loss in firm value when the activist blocks the reform. On the other hand, when $\xi(p) < \frac{1}{2}$, shareholders prefer the reform since it increases the expected firm value to $v + (1 - 2\xi(p))\Delta$. Thus, when the activist blocks the reform, shareholders incur a loss of $(1 - 2\xi(p))\frac{\Delta}{n}$.[29]

The activist's payoff is also influenced by the shareholders' belief, $\xi(p)$, because it determines their voting behavior. Suppose that $\xi(p) < \frac{1}{2}$, such that shareholders who do not sell their voting right, vote for the reform. In state R, the activist's payoff is given by equation (1), whereas in state Q, it is

$$\Pi_A(p; q, \xi, Q) = \alpha(v - \Delta) + \mathbb{P}[M(n_S, q(p)) \geq m](b + \alpha\Delta) - p\mathbb{E}[\bar{M}(n_S, p(q))].$$

If $\xi(p) \geq \frac{1}{2}$ and shareholders who do not sell their voting right vote against the reform, the activist's payoff is $\alpha v + b - p\mathbb{E}[\bar{M}(n_S, p(q))]$, independent of the state.

Since $\alpha > 0$, the activist's willingness to pay for the voting rights is higher in state Q than in state R. As a result, there are separating perfect Bayesian equilibria in which vote trading can be welfare increasing. The following exemplary equilibrium illustrates this effect.

[29] Fully spelled out, this means that

$$\Pi_S(\text{sell}; p, q, \xi) = \frac{v}{n} + (1 - \mathbb{P}[M(n_S - 1, q(p)) \geq m - 1]) \max\{0, 1 - 2\xi(p)\} \frac{\Delta}{n} + p\frac{\mathbb{E}[\bar{M}(n_S, q(p))]}{n_S q(p)},$$
$$\Pi_S(\text{keep}; p, q, \xi) = \frac{v}{n} + (1 - \mathbb{P}[M(n_S - 1, q(p)) \geq m]) \max\{0, 1 - 2\xi(p)\} \frac{\Delta}{n}.$$

Example: Suppose there are $n = 4$ shares and that the activist and three other shareholders each own one share. The reform changes the firm value by $\Delta = 1$, whereas the status quo provides the activist with a private benefit of $b = \frac{1}{2}$. The prior probability of state Q is $\rho = \frac{1}{4}$, such that, in expectation, the shareholders benefit from the reform. The activist, on the other hand, wants to block the reform in either state. The reform requires a simple majority; in case of a tie, it is implemented as well. Thus, the activist needs to acquire $m = 2$ voting rights to prevent the reform.

There is an equilibrium in which $p^*(Q) = \frac{1}{8}, p^*(R) = 0, q^*(p^*(Q)) = 1$ and $q^*(p^*(R)) = 0$. In this separating equilibrium, the reform is implemented only in state R, and welfare is maximized. Figure 1 illustrates the equilibrium strategies (p^*, q^*).

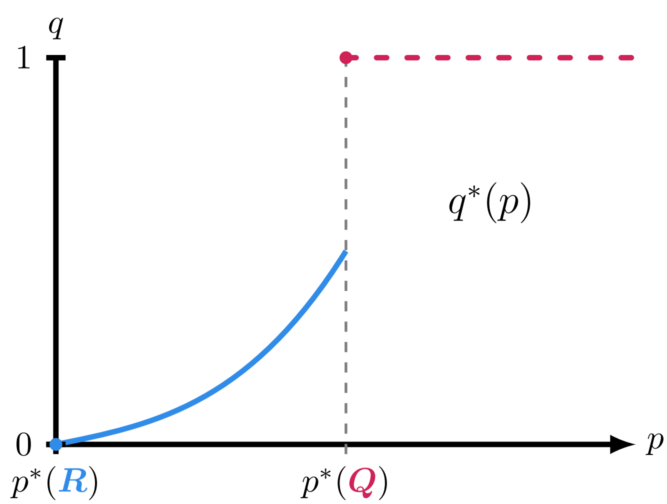

Figure 1: Example of a fully separating equilibrium.

To construct this equilibrium, suppose that $\xi^*(p) = 0$ for all $p \in [0, p^*(Q))$. Given this belief, let $q^*(p)$ be the smallest solution to the condition that shareholders are indifferent between selling and retaining their voting right

$$\underbrace{2(1-q)q}_{\mathbb{P}[M(n_S-1,q)=m-1]} \frac{\Delta}{n} = p \underbrace{\left[(1-q)^2 + 2q(1-q) + q^2\frac{2}{3}\right]}_{\frac{\mathbb{E}[\bar{M}(n_S,q)]}{n_S q}}.$$

For all $p \geq p^*(Q)$, let $\xi^*(p) = 1$, such that it is strictly optimal for shareholders to sell, $q^*(p) = 1$. Naturally, the resulting q^* is a best response given their belief ξ^*.

When shareholders respond with q^*, in state R, the activist is indifferent between $p^*(R) = 0$ and $p^*(Q) = \frac{1}{8}$, $\Pi_A(p^*(R); q^*, \xi^*, R) = \Pi_A(p^*(Q); q^*, \xi^*, R) = \frac{1}{4}$. Further, we show in Appendix B.8, that all prices except 0 and $\frac{1}{8}$ are dominated. Thus, $p^*(R)$ is a best response. In state Q, the activist's payoff from blocking the reform is higher than in state R, such that $p^*(Q) = \frac{1}{8}$ is the unique best response.

By construction, all investors play best responses and the beliefs are consistent, such that the proposed strategies and beliefs form a perfect Bayesian equilibrium.

As the next proposition shows, a separating perfect Bayesian equilibrium always exists, but fails to improve expected firm value, when n is large.

Proposition 3. *There always exists a separating equilibrium* $(p^*, q^*; \xi^*)$*, i.e. an equilibrium in which* $p^*(Q) \neq p^*(R)$*, such that shareholders learn the state. Further,*

1. *in any separating equilibrium* $p^*(R) < p^*(Q)$*, and* $q^*(p^*(R)) < q^*(p^*(Q)) = 1$*;*
2. *as* n *grows large, along any sequence of equilibria and for* $\omega \in \{Q, R\}$*,*

$$\lim_{n\to\infty} \mathbb{P}[M(n_S, q^*(p^*(\omega))) \geq m] = 1 \quad \textit{and} \quad \lim_{n\to\infty} p^*(\omega)\mathbb{E}[\bar{M}(n_S, q^*(p^*(\omega)))] = 0.$$

When the number of shareholders is small, the separating equilibrium can, as Example 1 demonstrates, raise the probability that the correct decision is implemented beyond the ex-ante probability $1 - 2\rho$. Thus, vote trading can increase welfare, even with a hostile activist, and even if the private benefit does not suffice to make up for the expected loss in firm value when the reform is blocked, $b < (1 - \rho)\Delta - \rho\Delta = (1 - 2\rho)\Delta$.

This effect, however, utilizes vote trading as a costly signal, which can only work in case the voting rights are sufficiently expensive. As established by Proposition 1, vote prices quickly converge to zero when the firm is owned by more shareholders; if shareholdings are dispersed, the activist can (and will) acquire a blocking minority of voting rights at negligible cost and block the reform in either state. As a result, the expected firm value converges to $v < \rho(v - \Delta) + (1 - \rho)(v + \Delta) = v + (1 - 2\rho)\Delta$, while the expected transfer converges to zero. When $b < (1-2\rho)\Delta$, overall welfare is reduced compared to the situation without vote trading.

4.3.2 Costly communication

Vote trading may improve communication by acting as a costly signal, but so does any traditional form of costly intervention, yielding (weakly) superior outcomes.

To formalize the idea, suppose that, instead of vote trading, the activist can spend spend amount $\kappa \in \mathbb{R}_+$, for example, on running a costly but non-informative public proxy campaign. Thus, her strategy is $\kappa : \{Q, R\} \to \mathbb{R}_+$. Shareholders observe κ, form posterior $\xi(\kappa)$, and vote for the status quo if $\xi(\kappa) > \frac{1}{2}$; they vote for the reform if $\xi(\kappa) < \frac{1}{2}$. Again, we consider perfect Bayesian equilibria.

Proposition 4.

1. *There is an equilibrium* $(\kappa^*; \xi^*)$ *in which the activist spends* $\kappa^*(Q) = b - \alpha\Delta$ *and* $\kappa^*(R) = 0$. *Shareholders learn the state, block the reform in state* Q, *and implement the reform in state* R.

2. *In every (other) equilibrium, the state is matched with probability of at least* $1 - \rho$.

Proposition 4 shows that a costly signal can also be used to credibly communicate that the state is Q. In any separating equilibrium, the activist needs to spend at least $\kappa^*(Q) = b - \alpha\Delta$ to signal that the state is Q, and to block the reform. At $\kappa^*(Q) = b - \alpha\Delta$ the activist in state R is exactly indifferent between spending $\kappa^*(Q)$ and remaining passive, $\kappa^*(R) = 0$: both yield her a payoff of v. In state Q, the activist strictly benefits from spending $\kappa^*(Q)$, because $\alpha v + b - \kappa^*(Q) > \alpha(v - \Delta)$.

Different from vote trading, in any separating equilibrium of the costly communication game, the first-best firm value is attained. In case the costly signal is not wasteful, this implies that welfare is maximized. Further, costly signaling can never reduce shareholder value relative to the pure voting benchmark. It therefore circumvents the risks of hostile activism inherent to vote trading.

Traditional forms of costly intervention include public proxy campaigns or the public acquisition of shares. Our results generate two new insights regarding the usage of these tools. First, even if the activist cannot provide evidence of her claims during the proxy fight, the fact that she is willing to engage in a costly proxy fight can suffice as a credible signal. Proxy fights are valuable not because they directly transmit information but because the associated costs give credence to the activist. Further, the public acquisition of shares not only aligns the activist and the shareholders' interests by raising α, but can be a credible signal that the activist wants to maximize shareholder value. Hence, the public disclosure of these acquisitions—through regulatory filings, for instance—serves an important function in the communication between investors.

5 Conclusion

Financial innovation has created manifold new ways to exchange voting rights; most notably using the equity lending market. Vote trading became a new force in shareholder activism, raising the question whether regulators should embrace or worry about vote trading. Our results show that regulators have reason to be concerned.

Vote trading does not yield Pareto improvements, but renders shareholders vulnerable to hostile activism—even in a best-case environment with transparent motives by the activist. It is true that when the activist has private information about the optimal decision vote trading can be beneficial, despite the activist's ulterior motives. Nevertheless, compared with traditional forms of intervention such as public endorsements, proxy campaigns, or share acquisitions, vote trading creates inferior outcomes. Note that we even consider a lower bound on the efficiency of these traditional forms of interventions by reducing them to their capacity to act as a costless or costly signal. Further, we analyze models of non-verifiable information only. In practice, activist investors not only suggest certain courses of action but also (try to) provide evidence for their claims, which can be scrutinized by shareholders and outside analysts alike.

In conclusion, claims of more efficient corporate governance via vote trading seem unconvincing, when compared with the traditional forms of intervention by activist investors. Instead, vote trading threatens shareholder value by enabling hostile activism. This goes to show that the long-standing tradition of outlawing the outright trade of voting rights in most countries is well founded. To prevent the new, indirect ways of vote trading, regulation has to be updated. We discuss some salient policy proposals in the final section.

6 Policy implications

6.1 Transparency measures

The market failure in the market for voting rights does not depend on hidden motives of the activist. As a result, policies aimed at increasing transparency—such as extended disclosure requirements[30] or rules of informed consent—do not suffice to prevent inefficient market outcomes and hostile activism. Nevertheless, additional transparency rules might be helpful, to prevent problems of asymmetric information, and to monitor the extent of vote trading.

6.2 Self-regulation by shareholders

Because shareholders *collectively* bear the cost of vote trading, they have an incentive to self-regulate. In this spirit, large asset managers such as BlackRock claim to recall shares in case of an "economically relevant vote."[31] Further, non-binding regulations such as stewardship codes have extended asset managers' "best practice" recommendations in the same direction. However, without some form of commitment, none of these self-imposed rules or "shareholder-cartels" are stable. Since it is *individually* optimal for shareholders to sell their voting rights if others do not, there can be no collective abstention from vote trading.

[30] Compare Hu & Black (2006) for a discussion of disclosure requirements with the SEC.

[31] See https://www.ft.com/content/0e28929e-85dd-11e8-a29d-73e3d454535d.

6.3 Forced recalls

Regulatory authorities could force shareholders to recall their shares for the record date by changing the collateral of a repo, or by canceling lending agreements, for example. While such measures would prevent the most relevant forms of vote trading, they would also come at a substantial cost. For instance, such regulation would imply a temporary shutdown of the equity lending market, thereby preventing (non-naked) short sales over the record date.

6.4 Excluding bought votes

One way to substantially reduce the scope for vote trading would be to suspend the voting rights of shares that were acquired in a way that can be exploited for vote trading. Shares borrowed or posted as collateral would thus lose their voting right until they were returned or resold to a third party.[32] This would leave the equity lending and repo markets unaffected in terms of their capacity to enable short selling or financing. However, this exclusion would not be a comprehensive solution since a hostile activist with a positive share endowment could still obtain control. When owning $\alpha > 0$ shares, the activist could borrow a fraction $\sigma > \frac{1-\alpha-\lambda}{1-\lambda}$ of the shares, implicitly voting σ as abstentions, thereby blocking the reform.

6.5 Excluding vote buyers

A more reliable solution than excluding bought votes would be to exclude the vote buyer from voting any of her shares. This solution not only has the same upsides as excluding borrowed votes but also prevents the acquisition of voting rights to void them.

6.6 Share blocking, lead time of the record date

Prior to 2007, it was common in many EU countries that shares, when voted on, were blocked from trading before the meeting.[33] This was done in an effort to prevent investors from voting shares they no longer owned, aligning the economic interest and voting power. However, the class of decoupling techniques discussed in this paper is unaffected by such measures. In the case of vote trading via the equity lending market, for example, share blocking would only require the activist to borrow the shares for the whole lead time of the record date. The economic exposure would still remain with the initial shareholders whereas the activist would only receive the voting right.

Similarly, the lead time of the record date has no effect on the economic forces of vote trading and, thereby, the possibility to use vote trading for hostile activism. Consider, for

[32] If a borrowed share would not regain its voting right, share lending would endogenously create non-voting shares, leading to additional problems.

[33] See European Commission Staff Working Document SEC(2006) 181, https://ec.europa.eu/transparency/regdoc/rep/2/2006/EN/2-2006-181-EN-1-0.Pdf.

instance, the most extreme case, in which the voting and the record date coincide. Such an arrangement would not prevent the activist from borrowing shares before the record/voting date and returning them afterwards, yielding the same outcome as the current practice.

6.7 Majority rules

The anecdotal evidence of Hu & Black (2015) suggests that decisions that require a supermajority are particularly vulnerable to hostile activism via vote trading. In Section 3.3, we give one reason for this effect: if the reform requires a supermajority, a blockholder is not able to deter a hostile activist because he is at a disadvantage relative to the activist, and the transfers from the activist to shareholders is particularly low. In addition to that, though the depth of the equity lending market may be sizeable, it is still limited. For both reasons, reducing the required majority towards a simple majority will help to deter hostile activism.

A Identities

Lemma 3. $\mathbb{P}[M(n_S-1,q)=m-1]<1$ *and* $\lim_{n\to\infty}\mathbb{P}[M(n_S-1,q)=m-1]=0$.

Proof. The first assertion follows because $0<m<n_S-1$ such that $1=\sum_{i=0}^{n_S-1}\mathbb{P}[M(n_S-1,q)=i]>\mathbb{P}[M(n_S-1,q)=m-1]$.

For the second, note that $\mathbb{P}[M(n_S-1,q)=m-1]=\binom{n_S-1}{m-1}q^{m-1}(1-q)^{n_S-m}$ is maximized if

$$\begin{aligned}
&0=\binom{n_S-1}{m-1}q^{m-2}(m-n_Sq+q-1)(1-q)^{-m+n_S-1}\\
\iff\ &q=\frac{m-1}{n_S-1}.
\end{aligned}$$

Thus,

$$\mathbb{P}[M(n_S-1,q)=m-1]\leq\binom{n_S-1}{m-1}(\frac{m-1}{n_S-1})^{m-1}(\frac{n_S-m}{n_S-1})^{n_S-m}. \tag{4}$$

Using Stirling's formula, $\binom{a}{b}=(1+o(1))\sqrt{\frac{a}{2\pi(a-b)b}}\frac{a^a}{(a-b)^{a-b}b^b}$, the right side of (4) becomes

$$=(1+o(1))\sqrt{\frac{n_S-1}{2\pi(n_S-m)(m-1)}}=(1+o(1))\sqrt{\frac{1}{2\pi(1-\eta)(n_S-1)\eta}}, \tag{5}$$

with $\eta=\frac{m-1}{n_S-1}$ (implying that $\eta\approx\frac{1-\lambda-\alpha}{1-\alpha}$). When n, n_S, and $m\to\infty$, the second result follows. $\square$

Lemma 4.

$$\sum_{i=m-1}^{n_S-1}\mathbb{P}[M(n_S-1,q)=i]\frac{m}{i+1}=\mathbb{P}[M(n_S,q)\geq m]\frac{m}{n_Sq}. \tag{6}$$

$$\sum_{i=0}^{m-2}\mathbb{P}[M(n_S-1,q)=i]+\sum_{i=m-1}^{n_S-1}\mathbb{P}[M(n_S-1,q)=i]\frac{m}{i+1}=\frac{\mathbb{E}[\bar{M}(n_S,q)]}{n_Sq}. \tag{7}$$

$$\mathbb{P}[M(n_S-1,q)=m-1]=\frac{m}{n_Sq}\mathbb{P}[M(n_S,q)=m]. \tag{8}$$

Proof.

$$\begin{aligned}
\sum_{i=m-1}^{n_S-1} \mathbb{P}[M(n_S-1,q)=i]\frac{m}{i+1} &= \sum_{i=m-1}^{n_S-1} \binom{n_S-1}{i} q^i (1-q)^{n_S-1-i} \frac{m}{i+1} \\
&= \sum_{i=m-1}^{n_S-1} \frac{1}{n_S q} \binom{n_S}{i+1} q^{i+1} (1-q)^{n_S-(i+1)} m \\
&= \sum_{k=m}^{n_S} \frac{1}{nq} \binom{n_S}{k} q^k (1-q)^{n_S-k} m \\
&= \frac{m}{n_S q} \cdot \mathbb{P}[M(n_S,q) \geq m].
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}[\bar{M}(n_S,q)] &= \mathbb{P}[M(n_S,q)\geq m]m + \sum_{i=0}^{m-1} \mathbb{P}[M(n_S,q)=i]i \\
&= \mathbb{P}[M(n_S,q)\geq m]m + \sum_{i=1}^{m-1} \binom{n_S}{i} q^i (1-q)^{n_S-i} i \\
&= \mathbb{P}[M(n_S,q)\geq m]m + \sum_{i=1}^{m-1} \binom{n_S-1}{i-1} n_S \cdot q \cdot q^{i-1} (1-q)^{n_S-i} \\
&= \mathbb{P}[M(n_S,q)\geq m]m + \sum_{k=0}^{m-2} \binom{n_S-1}{k} n_S \cdot q \cdot q^{k} (1-q)^{n_S-1-k} \\
&= n_S q\Big(\mathbb{P}[M(n_S,q)\geq m]\frac{m}{n_S q} - \mathbb{P}[M(n_S-1,q)\leq m-2]\Big),
\end{aligned}$$

and plugging (6) into the equation, (7) follows.

$$\begin{aligned}
\mathbb{P}[M(n_S-1,q)=m-1] &= \binom{n_S-1}{m-1} q^{m-1}(1-q)^{n_S-m} \\
&= \frac{(n_S-1)!}{(n_S-m)!(m-1)!} q^{m-1}(1-q)^{n_S-m} \\
&= \frac{(n_S)!}{(n_S-m)!(m)!} \frac{m}{n_S q} q^{m}(1-q)^{n_S-m} \\
&= \frac{m}{n_S q} \mathbb{P}[M(n_S,q)=m].
\end{aligned}$$

□

Lemma 5.

$$\phi(q) = \frac{\mathbb{P}[M(n_S-1,q)=m-1]n_S q}{\mathbb{E}[\bar{M}(n_S,q)]}$$

is continuous, strictly concave, with a unique maximum $\bar{\phi} < 1$ *and* $\phi(0) = \phi(1) = 0$*. Also,*

$\lim_{n\to\infty} \phi(q) = 0$ *for all* q*. Further, there are two continuous functions* $q_-(\phi), q_+(\phi)$ *on* $[0, \bar{\phi}]$*, of which* q_- *is strictly increasing and* q_+ *is strictly decreasing. For all* $\phi \in [0, \bar{\phi})$ *it holds that* $q_-(\phi) < q_+(\phi)$ *but* $\bar{\phi} = \phi(q_-) = \phi(q_+)$*. In particular* $q_-(0) = 0$ *and* $q_+(0) = 1$*.*

Proof.

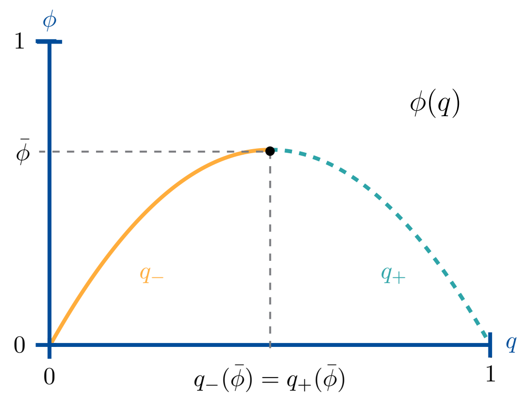

Figure 2: Form of $\phi(q)$ and definition of q_- and q_+.

Using (8), $\frac{1}{\phi(q)}$ can be rewritten as

$$\iff \frac{1}{\phi(q)} = \frac{\sum_{i=0}^{m} \mathbb{P}[M(n_S,q) = i]i + \mathbb{P}[M(n_S,q) > m]m}{m\mathbb{P}[M(n_S,q) = m]}$$

$$\iff \frac{1}{\phi(q)} = \frac{\sum_{i=1}^{m} \binom{n_S}{i} q^i (1-q)^{n_S-i} i + \sum_{i=m+1}^{n_S} \binom{n_S}{i} q^i (1-q)^{n_S-i} m}{m\binom{n_S}{m} q^m (1-q)^{n_S-m}}$$

$$\iff \frac{1}{\phi(q)} = \frac{1}{m\binom{n_S}{m}} [\sum_{i=1}^{m} \binom{n_S}{i} q^{i-m} (1-q)^{m-i} i + \sum_{i=m+1}^{n_S} \binom{n_S}{i} q^{i-m} (1-q)^{m-i} m]$$

$$\iff \frac{1}{\phi(q)} = \frac{1}{m\binom{n_S}{m}} [\sum_{i=1}^{m} \binom{n_S}{i} (\frac{q}{1-q})^{i-m} i + \sum_{i=m+1}^{n_S} \binom{n_S}{i} (\frac{q}{1-q})^{i-m} m].$$

Both summands are strictly convex in q such that $\frac{1}{\phi(q)}$ is strictly convex in q. Further, $\lim_{q\to 0} \frac{1}{\phi(q)} = \lim_{q\to 1} \frac{1}{\phi(q)} = \infty$, such that $\frac{1}{\phi(q)}$ is U-shaped. Since $\frac{1}{\phi(q)} \geq 0$, it follows that ϕ is hump-shaped, with $\phi(0) = \phi(1) = 0$ and a unique maximum $\bar{\phi}$. Further, because

$$\phi(q) = \frac{\mathbb{P}[M(n_S-1,q) = m-1]n_S q}{\mathbb{E}[\min\{m, M(n_S,q)\}]} < \frac{\mathbb{P}[M(n_S-1,q) = m-1]n_S q}{n_S q},$$

Lemma 3 implies that $\bar{\phi} < 1$ and $\lim_{n\to\infty} \phi(q) = 0$.

Last, since ϕ is hump-shaped, with $\phi(0) = \phi(1) = 0$ and a unique maximum $\bar{\phi}$, for all $p < \bar{\phi}$ there are exactly two functions $q_-(p) < q_+(p)$ such that $p = \phi(q_-(p)) = \phi(q_+(p))$.

Since ϕ is continuous, so are q_- and q_+. $\square$

B Proofs

B.1 Proof of Proposition 1

Note that $\Pi_S(\text{sell}; p, q) = \Pi_S(\text{keep}; p, q)$ rearranges to

$$\mathbb{P}[M(n_S - 1, q(p)) = m - 1]\frac{\Delta}{n} = p\frac{\mathbb{E}[\bar{M}(n_S, q(p))]}{n_S q(p)}. \tag{9}$$

Step 1. *There is always an equilibrium in which $p^* = 0$ and $q^*(0) = 1$.*

Since $1 < m < n_S$ and $n_S \geq 3$, if $q^*(0) = 1$, no shareholder is pivotal and selling the voting right is a best response. Since this is the lowest possible price, the activist has no profitable deviation.

Step 2. *If $q^*(p^*) > 0$ and thereby $\mathbb{P}[M(n_S, q^*(p^*)) \geq m] > 0$, then it has to hold that $p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] < m\frac{\Delta}{n}\mathbb{P}[M(n_S, q^*(p^*)) \geq m]$.*

If $q^*(p^*) \in (0, 1)$, then (9) holds with equality. Further, by (8), (9) can restated as

$$p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] = \mathbb{P}[M(n_S, q^*(p^*)) = m]m\frac{\Delta}{n} < \mathbb{P}[M(n_S, q^*(p^*)) \geq m]m\frac{\Delta}{n}.$$

Now suppose that $q^*(p^*) = 1$. Using Lemma 5, let $\bar{p} = \max_q \phi(q)\frac{\Delta}{n} < \frac{\Delta}{n}$. At any $p > \bar{p}$, (9) cannot hold with equality, such that $q^*(p) = 1$. It follows that if $q^*(p^*) = 1$, then $p^* \leq \bar{p}$, otherwise a deviation to a price $\frac{\bar{p}+p^*}{2}$ would be strictly profitable. Thereby, $p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] < \frac{\Delta}{n}m = \frac{\Delta}{n}\mathbb{P}[M(n_S, q^*(p^*)) \geq m]$.

Step 3. $\lim_{n\to\infty}\mathbb{P}[M(n_S, q^*(p^*)) \geq m] = 1$ *and* $\lim_{n\to\infty} p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] = 0$.

Suppose to the contrary that one of the statements was violated. In this case

$$\alpha(v + \Delta) + \mathbb{P}[M(n_S, q^*(p^*)) \geq m](b - \alpha\Delta) - p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))] < \alpha v + b$$

for n arbitrary large. Using Lemma 5, let $\bar{p} = \max_q \phi(q)\frac{\Delta}{n}$, and consider a deviation to $p' = \bar{p} + \frac{\epsilon}{m}$. Since $n \cdot \bar{p} \to 0$ and $q^*(p') = 1$, it follows that

$$\lim_{n\to\infty}\alpha(v + \Delta) + \mathbb{P}[M(n_S, q^*(p')) \geq m](b - \alpha\Delta) - p'\mathbb{E}[\bar{M}(n_S, q^*(p'))] = \alpha v + b - \epsilon,$$

such that the deviation is profitable when ϵ is small and n is large.

Step 4. *When b and n are small, there are equilibria in which there is no trade.*

Using Lemma 5, there is a best response $q^*(p) = q_-(p)$ which is continuous and strictly increasing on $[0, \bar{p}]$ with $\bar{p} = \max_q \phi(q)\frac{\Delta}{n}$. Further, $q^*(0) = 0$ and $q^*(p) = 1$ for all $p > \bar{p}$.

Suppose that the activist offers a price $p^* \in (0, \bar{p})$ such that $q^*(p^*) \in (0, 1)$ and equality (9) holds. Since p^* is a best response, $\Pi_A(p^*; q^*) \geq \Pi_A(0; q^*) = \alpha(v + \Delta)$. Plugging (9) into $\Pi_A(p; q^*)$ and using (8) this can be rearranged to

$$\alpha(v + \Delta) - \mathbb{P}[M(n_S, q) = m]\Delta\frac{m}{n} + \mathbb{P}[M(n, q) \geq m](b - \alpha\Delta) \geq \alpha(v + \Delta)$$
$$\iff -\Delta\frac{m}{n} + \frac{\mathbb{P}[M(n_S, q) \geq m]}{\mathbb{P}[M(n_S, q) = m]}(b - \alpha\Delta) > 0.$$

The likelihood ratio

$$\frac{\mathbb{P}[M(n_S, q) \geq m]}{\mathbb{P}[M(n_S, q) = m]} = \frac{\sum_{i=m}^{n_S} \binom{n_S}{i} q^i (1-q)^{n-i}}{\binom{n_S}{m} q^m (1-q)^{n_S - m}} = \frac{1}{\binom{n_S}{m}} \sum_{i=m}^{n_S} \binom{n_S}{i} (\frac{q}{1-q})^{i-m} \tag{10}$$
$$= \frac{1}{\binom{n_S}{m}} \binom{n_S}{m} (\frac{q}{1-q})^0 + \sum_{i=m+1}^{n_S} \binom{n_S}{i} (\frac{q}{1-q})^{i-m} \overset{q \to 0}{\to} 1.$$

Thus, for p (and, hence, $q^*(p)$) sufficiently low, $\Pi_A(p; q^*) < \Pi_A(0; q^*)$ when $-\Delta\frac{m}{n} + \frac{\mathbb{P}[M(n_S,q) \geq m]}{\mathbb{P}[M(n_S,q)=m]}(b - \alpha\Delta) \approx b - (1 - \lambda)\Delta - \frac{1}{n}\Delta < 0$. Further, any price above $\frac{b}{m}$ is dominated by offering $p = 0$ and not trading. If b is sufficiently small, this means that we found a contradiction and $p = 0$ is the unique best response.

B.2 Proof of Proposition 2

To enhance clarity, we prove equilibrium existence separately in Lemma 6 and characterize the equilibrium first.

Suppose the activist offers p_A, the blockholder p_B, and shareholders mix with probability $q(p_A, p_B)$. Then, an individual shareholder (weakly) prefers to sell to A if and only if

$$\mathbb{P}[M(n_S - 1, q(p_A, p_B)) < m_A - 1]\frac{\Delta}{n} + p_A\frac{\mathbb{E}[\bar{M}_A(n_S, q(p_A, p_B))]}{n_S q(p_A, p_B)}$$
$$\geq \mathbb{P}[M(n_S - 1; q(p_A, p_B)) < m_A]\frac{\Delta}{n} + p_B\frac{\mathbb{E}[\bar{M}_B(n_S, q(p_A, p_B))]}{n_S(1 - q(p_A, p_B))}$$
$$\iff p_A\frac{\mathbb{E}[\bar{M}_A(n_S, q)]}{n_S q} - \mathbb{P}[M(n_S - 1; q) = m_A - 1]\frac{\Delta}{n} \geq p_B\frac{\mathbb{E}[\bar{M}_B(n_S, q)]}{n_S(1 - q)}. \tag{11}$$

The expected payoffs for the activist and blockholder are

$$\begin{aligned}
&\Pi_A(p_A; p_B, q) \\
&= \alpha(v+\Delta) + \mathbb{P}[M(n_S, q(p_A, p_B)) \geq m_A](b - \alpha\Delta) - p_A \mathbb{E}[\bar{M}_A(n_S, q(p_A, p_B))], \\
&\Pi_B(p_B; p_A, q) \\
&= \beta(v+\Delta) + \mathbb{P}[M(n_S, q(p_A, p_B)) \geq m_A](-\beta\Delta) - p_B \mathbb{E}[\bar{M}_B(n_S, q(p_A, p_B))].
\end{aligned}$$

In an effort to keep notation cleaner, we henceforth drop the explicit reference to the shareholders' strategy q.

For any n, let $p_{A;n}$ and $p_{B;n}$ be any two prices and let q_n^* be a best responses. Given q_n^*, let $p_{B;n}^*$ be a best response, and, given q_n^* and $p_{B;n}^*$, let $p_{A;n}^*$ be an equilibrium price. We take converging (sub)sequences of prices and probabilities as needed.

Step 0. *Suppose that* $\lim p_{A;n} n > 0$ *and/or* $\lim p_{B;n} n > 0$.

1. *If* $\lim \frac{p_{A;n}}{p_{B;n}} > \frac{1-\alpha-\beta}{1-\lambda-\alpha}$ *then* $q_n^*(p_{A;n}, p_{B;n}) = 1$ *when* n *is sufficiently large;*
2. *If* $\lim \frac{p_{A;n}}{p_{B;n}} > 1$ *but* $\lim \frac{p_{A;n}}{p_{B;n}} \leq \frac{1-\alpha-\beta}{1-\lambda-\alpha}$, *then* $\lim q_n^*(p_{A;n}, p_{B;n}) = \lim \frac{p_{A;n}}{p_{B;n}} \frac{1-\lambda-\alpha}{1-\alpha-\beta}$ *and* $\lim \mathbb{P}[M(n_S, q_n^*(p_{A;n}, p_{B;n})) \geq m_A] = 1$*;*
3. *If* $\lim \frac{p_{A;n}}{p_{B;n}} = 1$, *then* $\lim q_n^*(p_{A;n}, p_{B;n}) = \frac{1-\lambda-\alpha}{1-\alpha-\beta}$ *as well as* $\lim \mathbb{P}[M(n_S, q_n^*(p_{A;n}, p_{B;n})) \geq m_A] = \frac{1}{2}$*;*
4. *If* $\lim \frac{p_{A;n}}{p_{B;n}} < 1$ *but* $\lim \frac{p_{A;n}}{p_{B;n}} \geq \frac{\lambda-\beta}{1-\alpha-\beta}$, *then* $\lim q_n^*(p_{A;n}, p_{B;n}) = 1 - \lim \frac{p_{B;n}}{p_{A;n}} \frac{\lambda-\beta}{1-\alpha-\beta}$ *and* $\lim \mathbb{P}[M(n_S, q_n^*(p_{A;n}, p_{B;n})) \geq m_A] = 0$*;*
5. *If* $\lim \frac{p_{A;n}}{p_{B;n}} < \frac{\lambda-\beta}{1-\alpha-\beta}$, *then* $q_n^*(p_{A;n}, p_{B;n}) = 0$ *when* n *is sufficiently large.*

For ease of notation, let $q_n^* = q^*(p_{A,n}, p_{B,n})$.

By Lemma 3, for any q, $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$. Further, by the LLN, if $\lim q_n^* > \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, then $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] = 1$, $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{n_S q_n^*} = \lim \frac{1-\lambda-\alpha}{q_n^*(1-\alpha-\beta)}$, and $\lim \frac{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]}{n_S(1-q_n^*)} = 1$. If, on the other hand, $q_n^* < \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, then $\lim \mathbb{P}[M(n_S, q_n^*) < m_A] = 1$, $\lim \frac{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]}{n_S(1-q_n^*)} = \lim \frac{\lambda-\beta}{(1-q_n^*)(1-\alpha-\beta)}$, and $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{n_S q_n^*} = 1$. Last, if $\lim q_n^* = \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, then $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{n_S q_n^*} = \lim \frac{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]}{n_S(1-q_n^*)} = 1$ and $\lim \mathbb{P}[M(n_S, q_n^*(p_{A;n}, p_{B;n})) \geq m_A] = \frac{1}{2}$.

If $q_n^* = 1$ and n is arbitrary large, then inequality (11), $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$, and $\lim p_{A;n} n > 0$ or $\lim p_{B;n} n > 0$ imply that $\lim \frac{p_{A;n}}{p_{B;n}} \geq \frac{\lambda-\beta}{1-\alpha-\beta}$. If $q_n^* = 0$ for n arbitrary large, the inequality of (11) reverses. Since $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$, and $\lim p_{A;n} n > 0$ or $\lim p_{B;n} n > 0$, it follow that that $\lim \frac{p_{A;n}}{p_{B;n}} \leq \frac{\lambda-\beta}{1-\alpha-\beta}$.

Suppose that $\lim \frac{p_{A;n}}{p_{B;n}} = \gamma > 1$. When $q_n^* < 1$ s.th. (11) holds with equality, $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$, and $\lim p_{A;n} n > 0$ or $\lim p_{B;n} n > 0$, it follows that $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]} \frac{1-q_n^*}{q_n^*} = \frac{1}{\gamma}$. By our earlier observation, this means that $\lim q_n^* > \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, such that equality (11) implies that $\lim q_n^* = \gamma \frac{1-\lambda-\alpha}{1-\alpha-\beta} = \lim \frac{p_{A;n}}{p_{B;n}} \frac{1-\lambda-\alpha}{1-\alpha-\beta}$. If $\gamma > \frac{1-\alpha-\beta}{1-\lambda-\alpha}$, equality (11) cannot hold when n is large, such that $q_n^* = 1$. In either case $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] = 1$. This proves properties *1* and *2*.

Next, consider the case in which $\lim \frac{p_{A;n}}{p_{B;n}} = \gamma < 1$. When $q_n^* > 0$ s.th. (11) holds with equality, $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$, and $\lim p_{A;n} n > 0$ or $\lim p_{B;n} n > 0$, it follows that $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]} \frac{1-q_n^*}{q_n^*} = \frac{1}{\gamma}$. By our earlier observation, this means that $\lim q_n^* < \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, such that equality (11) implies that $\lim 1 - q_n^* = \lim \frac{p_{B;n}}{p_{A;n}} \frac{\lambda-\beta}{1-\alpha-\beta}$. If $\gamma < \frac{\lambda-\beta}{1-\alpha-\beta}$, equality (11) cannot hold when n is large, such that $q_n^* = 0$. In either case $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] = 0$. This proves properties *4* and *5*.

Last, if $\lim \frac{p_{A;n}}{p_{B;n}} = 1$, then equality (11), $\lim \mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1] = 0$, and $\lim p_{A;n} n > 0$ or $\lim p_{B;n} n > 0$ imply that $\lim \frac{\mathbb{E}[\bar{M}_A(n_S, q_n^*)]}{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]} \frac{1-q_n^*}{q_n^*} = 1$. By our observation, this is the case if and only if $\lim q_n^* = \frac{1-\lambda-\alpha}{1-\alpha-\beta}$. This proves property *3*.

Step 1. *If* $\frac{b-\alpha\Delta}{1-\lambda-\alpha} > \frac{\beta\Delta}{\lambda-\beta}$ *and* n *is sufficiently large, then* $q_n^*(p_{A;n}^*, p_{B;n}^*(p_{A;n}^*)) = 1$. *Further,* $p_{A;n}^* \mathbb{E}[\bar{M}_A(n_S, q_n^*(p_{A;n}^*, p_{B;n}^*(p_{A;n}^*)))] = p_{A;n}^* m_A < \frac{1-\lambda-\alpha}{\lambda-\beta} \beta\Delta$, *but* $\lim_{n\to\infty} \mathbb{E}[\bar{M}_A(n_S, q_n^*(p_{A;n}^*, p_{B;n}^*(p_{A;n}^*)))] p_{A;n}^* = \frac{1-\lambda-\alpha}{\lambda-\beta} \beta\Delta$.

Suppose to the contrary that $q_n^*(p_{A,n}^*, p_{B,n}^*(p_{A,n}^*)) < 1$ even when n is arbitrary large. When there is no room for confusion, we employ the convention that $q_n^* = q_n^*(p_{A,n}^*, p_{B,n}^*(p_{A,n}^*))$ and $p_{B,n}^* = p_{B,n}^*(p_{A,n}^*)$.

First, we consider the case in which $\lim q_n^* > \frac{1-\lambda-\alpha}{1-\alpha-\beta}$. Observe that

$$\lim \frac{1 - q_n^*}{\sum_{i=0}^{m_A - 1} \mathbb{P}[M(n_S, q_n^*) = i]} = \infty.$$

For $\lim q_n^* < 1$, this follows directly, when $\lim q_n^* = 1$, we apply L'Hopital[34] to receive

$$\lim \frac{1 - q_n^*}{\sum_{i=0}^{m_A - 1} \mathbb{P}[M(n_S, q_n^*) = i]} = \lim \frac{1}{\mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1]} = \infty.$$

[34]
$$\begin{aligned}
&\frac{\partial}{\partial q} \sum_{i=0}^{m_A - 1} \mathbb{P}[M(n_S, q_n^*) = i] \\
&= \sum_{i=1}^{m_A - 1} \binom{n_S}{i} [i (q_n^*)^{i-1} (1 - q_n^*)^{n_S - i}] - \sum_{i=0}^{m_A - 1} \binom{n_S}{i} [(q_n^*)^i (1 - q_n^*)^{n_S - i - 1} (n_S - i)] \\
&= \sum_{i=0}^{m_A - 2} \mathbb{P}[M(n_S - 1, q_n^*) = i] n_S - \sum_{i=0}^{m_A - 1} \mathbb{P}[M(n_S - 1, q_n^*) = i] n_S = -\mathbb{P}[M(n_S - 1, q_n^*) = m_A - 1].
\end{aligned}$$

Since $\lim q_n^* > \frac{1-\lambda-\alpha}{1-\alpha-\beta}$ and $\lim \sum_{i=m_A}^{n_S-1} \mathbb{P}[M(n_S-1, q_n^*) = i] = 1$, this means that

$$
\begin{aligned}
&\frac{\mathbb{E}[\bar{M}_B(n_S, q_n^*)]}{n(1-\mathbb{P}[M(n_S,q_n^*) \geq m_A])} \\
=&\frac{\sum_{i=0}^{m_A-1} \mathbb{P}[M(n_S,q_n^*) = i] m_B + \sum_{i=m_A}^{n_S} \mathbb{P}[M(n_S,q_n^*) = i](n_S - i)}{n \sum_{i=0}^{m_A-1} \mathbb{P}[M(n_S,q_n^*) = i]} \\
=&\frac{\sum_{i=0}^{m_A-1} \mathbb{P}[M(n_S,q_n^*) = i] m_B + \sum_{i=m_A}^{n_S-1} \binom{n_S-1}{i} (q_n^*)^i (1-q_n^*)^{n_S-1-i} (1-q_n^*) n_S}{n \sum_{i=0}^{m_A-1} \mathbb{P}[M(n_S,q_n^*) = i]} \\
=&\frac{m_B}{n} + (1-\alpha-\beta) \sum_{i=m_A}^{n_S-1} \mathbb{P}[M(n_S-1, q_n^*) = i] \frac{1-q_n^*}{\sum_{i=0}^{m_A-1} \mathbb{P}[M(n_S,q_n^*) = i]}
\end{aligned}
$$

grows without bound. This growth implies that $\lim p_{B;n}^* n = 0$, because when $\lim p_{B;n}^* n > 0$ and n is large

$$
\begin{aligned}
&\beta(v+\Delta) + \mathbb{P}[M(n_S,q_n^*) \geq m_A](-\beta\Delta) - p_{B;n}\mathbb{E}[\bar{M}_B(n_S,q_n^*)] < \beta v \\
\iff\ &\beta\Delta < \frac{\mathbb{E}[\bar{M}_B(n_S,q_n^*)]}{n(1-\mathbb{P}[M(n_S,q_n^*) \geq m_A])} p_{B;n} n,
\end{aligned}
$$

such that a deviation by B to $p_B = 0$ is strictly profitable. If $\lim p_{B;n}^* n = 0$, then $\lim q_n^* \geq \frac{1-\lambda-\alpha}{1-\alpha-\beta}$ and Step 0 imply that $\lim p_{A;n}^* n = 0$. This means that when n is sufficiently large, B has an incentive to deviate to $p'_{B;n} = p_{A;n}^* + \frac{\epsilon}{n}$. By Step 0, when n is sufficiently large, $q_n^*(p_{A;n}^*, p'_{B;n}) = 0$, implying that

$$
\Pi_B^n(p'_{B;n}; p_{A;n}^*) = \beta(v+\Delta) - n(\lambda-\beta)(p_{A;n}^* + \frac{\epsilon}{n}),
$$

which is obviously larger than $\Pi_B^n(p_{B;n}^*; p_{A;n}^*)$ when ϵ is sufficiently small and n is large. Consequently, it cannot be that $q_n^* < 1$ for n arbitrary large, but $\lim q_n^* > \frac{1-\lambda-\alpha}{1-\alpha-\beta}$.

In a second step, suppose that $\lim q_n^* = \frac{1-\lambda-\alpha}{1-\alpha-\beta}$. If $\lim p_{A;n}^* n > 0$ or $\lim p_{B;n}^* n > 0$, then Step 0 implies that $\lim p_{A;n}^* n = \lim p_{B;n}^* n$ and $\lim \mathbb{P}[M(n_S,q_n^*) \geq m_A] = \frac{1}{2}$, such that

$$
\lim \Pi_B^n(p_{B;n}^*; p_{A;n}^*) = \beta v + \frac{1}{2}\beta\Delta - \lim p_{B;n} n(\lambda-\beta).
$$

Now consider a deviation by B to $p'_{B;n} = p_{B;n}^* + \frac{\epsilon}{n}$ which, by Step 0, guarantees that $\lim \mathbb{P}[M(n_S, q^*(p_{A;n}, p'_{B;n})) \geq m_A] = 0$ and, hence, yields

$$
\lim \Pi_B^n(p'_{B;n}; p_{A;n}^*) = \beta v + \beta\Delta - \lim p_{B;n}^* n(\lambda-\beta) - \epsilon(\lambda-\beta).
$$

When n is sufficiently large and ϵ sufficiently small, such a deviation is always profitable. When $\lim p_{A;n}^* n = \lim p_{B;n}^* n = 0$, the same deviation is profitable.

Last, suppose that $\lim q_n^* < \frac{1-\lambda-\alpha}{1-\alpha-\beta}$. Then $\lim \mathbb{P}[M(n_S,q_n^*) \geq m_A] = 0$, such that

$\lim \Pi^n_A(p^*_{A;n}; p^*_{B;n}) \leq \alpha(v + \Delta)$. Now consider a deviation by A to $p'_{A;n} = \frac{\beta\Delta}{n(\lambda-\beta)}$ and B's possible responses. If B offers $p^*_{B;n}(p'_A)$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} < 1$, then, by Step 0, $\lim \mathbb{P}[M(n_S, q^*_n(p'_{A;n}, p^*_{B;n}(p'_{A;n}))) \geq m_A] = 0$, and because $\lim p^*_{B;n}(p'_{A;n})n > \frac{\beta\Delta}{(\lambda-\beta)}$, it follows that

$$\lim \Pi^n_B(p^*_{B;n}(p'_{A;n}); p'_{A;n}) < \beta(v + \Delta) - (\lambda - \beta)\frac{\beta\Delta}{(\lambda - \beta)} = \beta v,$$

which is dominated by $p_B = 0$ when n is sufficiently large. If B offers $p^*_{B;n}(p'_{A;n})$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} = 1$, then, by our observation above, B has a strict incentive to deviate upwards. Thus, B has to respond by offering $p^*_{B;n}(p'_{A;n})$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} > 1$. As a result, $\lim \mathbb{P}[M(n_S, q^*(p'_{A;n}, p^*_{B;n}(p'_{A;n}))) \geq m_A] = 1$ and, in the limit, the deviation yields A the payoff

$$\lim \Pi^n_A(p'_{A;n}; p^*_{B;n}(p'_{A;n})) = \alpha v + b - (1 - \lambda - \alpha)\frac{\beta\Delta}{(\lambda - \beta)},$$

which is larger than $\alpha(v + \Delta)$ by assumption. Hence, the deviation is profitable for A when n is sufficiently large. This proves that $q^*_n = 1$ when n is sufficiently large.

When $q^*_n = 1$ and $p^*_{A;n}m_A = p^*_{A;n}\mathbb{E}[\bar{M}_A(n_S, q^*(p^*_{A;n}, p^*_{B;n}))] \geq \frac{1-\lambda-\alpha}{\lambda-\beta}\beta\Delta$, then $p^*_{A;n} \geq \frac{\beta\Delta}{n(\lambda-\beta)} - \frac{\beta\Delta}{m_A n(\lambda-\beta)}$. Suppose A chooses or deviates to $p'_{A;n} = \frac{\beta\Delta}{n(\lambda-\beta)} - \frac{\beta\Delta}{m_A n(\lambda-\beta)}$. If B offers $p^*_{B;n}(p'_{A;n})$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} < 1$, then $\lim p^*_{B;n}(p'_{A;n})n > \lim \frac{\beta\Delta}{(\lambda-\beta)}$ and by Step 0, it follows that $\lim \mathbb{P}[M(n_S, q^*_n(p'_{A;n}, p^*_{B;n}(p'_{A;n}))) \geq m_A] = 0$. However, in this case,

$$\lim \Pi^n_B(p^*_{B;n}(p'_{A;n}); p'_{A;n}) < \beta(v + \Delta) - (\lambda - \beta)\frac{\beta\Delta}{(\lambda - \beta)} = \beta v,$$

such that $p^*_{B;n}(p'_{A;n})$ is dominated by $p_B = 0$ when n is sufficiently large. If B offers $p^*_{B;n}(p'_{A;n})$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} = 1$, then, by our observation above, B would have a strict incentive to deviate upwards. This means that B has to choose a $p^*_{B;n}(p'_{A;n})$ such that $\lim \frac{p'_{A;n}}{p^*_{B;n}(p'_{A;n})} > 1$, which implies, by our previous argument, that $q^*_n(p'_{A;n}, p^*_{B;n}(p'_{A;n})) = 1$ when n is large. Thereby, the deviation to $p'_{A;n}$ is profitable for A when n is sufficiently large. Further, because all expressions are continuous and inequalities strict, the same can be achieved with a $p'_{A;n}$ marginally below $\frac{\beta\Delta}{n(\lambda-\beta)} - \frac{\beta\Delta}{m_A n(\lambda-\beta)}$, meaning that $p'_{A;n}m_A = p'_{A;n}\mathbb{E}[\bar{M}_A(n_S, q^*(p'_{A;n}, p^*_{B;n}(p'_{A;n})))] < \frac{1-\lambda-\alpha}{\lambda-\beta}\beta\Delta$.

Last, if $\lim p^*_A\mathbb{E}[\bar{M}_A(n_S, q^*(p^*_A, p^*_B))] < \frac{1-\lambda-\alpha}{\lambda-\beta}\beta\Delta$ this means that $p^*_{A;n} < \frac{\beta\Delta}{n(\lambda-\beta)} - \frac{\epsilon}{n}$ for some $\epsilon > 0$ and any n sufficiently large. In this case, however, B could deviate to $p'_{B;n} = \frac{\beta\Delta}{n(\lambda-\beta)} - \frac{\epsilon}{2n}$. As a result, $\lim \mathbb{P}[M(n_S, q^*_n(p^*_{A;n}, p'_{B;n})) \geq m_A] = 0$ and

$$\lim \Pi^n_B(p'_{B;n}; p^*_{A;n}) = \beta(v + \Delta) - \beta\Delta + \epsilon\frac{\beta\Delta}{2(\lambda - \beta)} > \beta v,$$

such that the deviation is profitable when n is sufficiently large.

Step 2. *If* $\frac{b-\alpha\Delta}{1-\lambda-\alpha} < \frac{\beta\Delta}{\lambda-\beta}$, *as* n *grows large, along any sequence of equilibria,* $\lim_{n\to\infty}\mathbb{P}[M_A(n_S, q_n^*(p_{A;n}^*, p_{B;n}^*(p_{A;n}^*))) \geq m_A] = 0$ *and* $\lim_{n\to\infty} p_{A;n}^* n_S = \lim_{n\to\infty} p_{B;n}^*(p_{A;n}^*) n_S = 0$.

For ease of notation, let $q_n^* = q_n^*(p_{A,n}^*, p_{B,n}^*(p_{A,n}^*))$. When there is no room for confusion, we employ the convention that $p_{B,n}^* = p_{B,n}^*(p_{A,n}^*)$.

First, suppose to the contrary that $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] > 0$. Since $\Pi_A^n(p_{A;n}^*; p_{B;n}^*) \geq \Pi_A^n(0; p_{B;n}^*(0)) \geq \alpha(v + \Delta)$, it follow that

$$\begin{aligned}\Pi_A^n(p_{A;n}^*; p_{B;n}^*) &= \alpha(v + \Delta) + (b - \alpha\Delta)\mathbb{P}[M(n_S, q_n^*) \geq m_A] - p_{A;n}^*\mathbb{E}[\bar{M}_A(n_S, q_n^*)] \\ &\geq \alpha(v + \Delta).\end{aligned}$$

Since $\frac{\mathbb{E}[\bar{M}_A(n_S,q_n^*)]}{\mathbb{P}[M(n_S,q_n^*)\geq m_A]} \geq m_A$ and $m_A = n(1 - \lambda - \alpha) + 1$, it follows that in the limit

$$\lim p_{A;n}^* n \leq \frac{b - \alpha\Delta}{1 - \lambda - \alpha}.$$

Now consider a deviation by B from $p_{B;n}^*$ to $p'_{B;n} = p_{A;n}^* + \frac{\epsilon}{n}$. Because $\lim q_n^*(p_{A;n}^*, p'_{B;n}) > \frac{1-\lambda-\beta}{1-\alpha-\beta}$, it follows that $\lim \mathbb{P}[M(n_S, q_n^*(p_{A;n}^*, p'_{B;n})) \geq m_A] = 0$. Such deviation is profitable when $\epsilon > 0$ is small and n is large because

$$\begin{aligned}&\lim \Pi_B^n(p'_{B;n}; p_{A;n}^*) - \Pi_B^n(p_{B;n}^*; p_{A;n}^*) \\ &\geq \lim(1 - \mathbb{P}[M(n_S, q_n^* \geq m_A])[\beta\Delta - (\lambda - \beta)np_{A;n}^*] - \epsilon,\end{aligned}$$

where

$$\beta\Delta - (\lambda - \beta)np_{A;n}^* \geq \beta\Delta - (\lambda - \beta)\frac{b - \alpha\Delta}{1 - \lambda - \alpha} > 0.$$

This establishes that $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] = 0$.

We now show that $\lim p_{A;n}^* n = \lim p_{B;n}^* n = 0$. First, suppose to the contrary that $\lim p_{A;n}^* n > 0$. In this case, it has to hold that $\lim q_n^* > 0$. Assume this was not true either, that is $\lim p_{A;n}^* n > 0$ and $\lim q_n^* = 0$. Then, there is a small $\epsilon > 0$ such that $\lim \frac{p_{A;n}^*}{p_{B;n}^* - \frac{\epsilon}{m_B}} \in (\frac{\lambda-\beta}{1-\alpha-\beta}, 1)$, which still implies that $\lim q_n^*(p_{A;n}^*, p_{B;n}^* - \frac{\epsilon}{m_B}) < \frac{1-\lambda-\alpha}{1-\alpha-\beta}$, and, thereby,

$$\lim \Pi_B^n(p_{B;n}^* - \frac{\epsilon}{m_B}; p_{A;n}^*) - \lim \Pi_B(p_{B;n}^*; p_{A;n}^*) = (\lambda - \beta)\epsilon,$$

making it a profitable deviation when n is large. Now, if $\lim q_n^* > 0$ and $\lim p_{A;n}^* n > 0$ but $\lim \mathbb{P}[M(n_S, q_n^*) \geq m_A] = 0$, then

$$\lim \Pi_A^n(p_{A;n}^*; p_{B;n}^*) = \alpha(v + \Delta) - \lim p_{A;n}^* q_n^* n < \alpha(v + \Delta) \leq \lim \Pi_A^n(0; p_{B;n}^*),$$

such that A would have profitable deviation to 0. Last, if $\lim p_{A;n}^* n = 0$, then $\lim p_{B;n}^* n = 0$. Otherwise, a deviation to $\frac{p_{B;n}^*}{2}$ would always be profitable for B and when n is sufficiently large.

Lemma 6. *The competition game always has an equilibrium* (p_A^*, p_B^*, q^*).

Proof. We are going to show existence by construction. Fix some p_A. Then, shareholders are indifferent between selling to A and B if $p_B = \psi(q; p_A)$ where

$$\psi(q; p_A) = (p_A \frac{\mathbb{E}[\bar{M}_A(n_S, q)]}{n_S q} - \mathbb{P}[M(n_S - 1; q) = m_A - 1]\frac{\Delta}{n})\frac{n_S(1-q)}{\mathbb{E}[\bar{M}_B(n_S, q)]}$$

is a polynomial of q and strictly increasing and continuous in p_A. For later use, we further note that the slope of $\psi(q; p_A)$ with respect to p_A is decreasing in q ($-\psi$ is supermodular): for any $p_A < p'_A$ and $q < q'$, it holds that

$$\psi(q; p'_A) - \psi(q; p_A) > \psi(q'; p'_A) - \psi(q'; p_A).$$

We can use $\psi(q; p_A)$ to define a best response for shareholders as

$$q^*(p_A, p_B) = \begin{cases} 1 \text{ for } p_B < \psi(1; p_A) \\ \min\{q \colon \psi(q; p_A) = p_B\} \text{ for } \psi(1; p_A) \le p_B < \psi(0; p_A) \\ 0 \text{ for } p_B \ge \psi(0; p_A). \end{cases}$$

By construction, q^* is (weakly) decreasing and right-continuous in p_B. Note that $q^*(p_A, \psi(q; p_A)) \le q$.

Step 1. *Given any offer* p_A*, B has at least one best response* $p_B^*(p_A)$.

Since q^* is (weakly) decreasing and right-continuous in p_B and all expressions are bounded, B's problem has at least one solution. We denote an arbitrary one by $p_B^*(p_A)$.

Step 2. *B's problem can be restated as*

$$\begin{aligned} &\operatorname*{arg\,max}_{q \in \operatorname{supp} q^*(p_A, \cdot)} \hat{\Pi}_B(q; p_A) \\ = &\operatorname*{arg\,max}_{q \in \operatorname{supp} q^*(p_A, \cdot)} \beta(v + \Delta) - \mathbb{P}[M(n_S, q) \ge m]\beta\Delta - \psi(q; p_A)\mathbb{E}[\bar{M}_B(n_S, q)]. \end{aligned}$$

If $\hat{\Pi}_B(q; p_A) \ge \beta v$*, and* $q' < q$ *s.th.* $\psi(q; p_A) = \psi(q'; p_A)$*, then* $\hat{\Pi}_B(q'; p_A) > \hat{\Pi}_B(q; p_A)$.

The first restatement follows directly from the definition of ψ and q^*. For the second, note that $\hat{\Pi}_B(q; p_A) \ge \beta v$ can be rearranged to

$$\begin{aligned} &(1 - \mathbb{P}[M(n_S, q) \ge m_A])\beta\Delta - \psi(q; p_A)\mathbb{E}[\bar{M}_B(n_S, q)] \ge 0 \\ \iff &\mathbb{P}[M(n_S, q) < m_A](\beta\Delta - \psi(q; p_A)\frac{\mathbb{E}[\bar{M}_B(n_S, q)]}{\mathbb{P}[M(n_S, q) < m_A]}) \ge 0. \end{aligned} \tag{12}$$

We want to show that the left side of (12) is strictly decreasing in q. Since $\mathbb{P}[M(n_S,q) < m_A]$ is strictly decreasing in q and (12) is positive, it suffices to show that $\frac{\mathbb{E}[\bar{M}_B(n_S,q)]}{\mathbb{P}[M(n_S,q)<m_A]}$ is strictly increasing in q. Note that

$$\begin{aligned}\frac{\mathbb{E}[\bar{M}_B(n_S,q)]}{\mathbb{P}[M(n_S,q) < m_A]} &= \frac{\mathbb{P}[M(n_S,q) < m_A]m_B + \sum_{i=0}^{m_B-1}\mathbb{P}[M(n_S,1-q)=i]i}{\mathbb{P}[M(n_S,q) < m_A]} \\ &= m_B + \frac{\sum_{i=0}^{m_B-1}\mathbb{P}[M(n_S,1-q)=i]i}{\sum_{i=m_B}^{n_S}\mathbb{P}[M(n_S,1-q)=i]} \\ &= m_B + \frac{\sum_{i=0}^{m_B-1}\binom{n_S}{i}i(1-q)^i q^{n_S-i}i}{\sum_{i=m_B}^{n_S}\binom{n_S}{i}i(1-q)^i q^{n_S-i}} \\ &= m_B + \frac{\sum_{i=0}^{m_B-1}\binom{n_S}{i}i(\frac{1-q}{q})^{i-(m_B-1)}i}{\sum_{i=m_B}^{n_S}\binom{n_S}{i}(\frac{1-q}{q})^{i-(m_B-1)}},\end{aligned}$$

where the numerator is increasing in q for all $i \in (0,...,m_B-1)$, and the denominator is strictly decreasing in q for all $i \in (m_B,...,n_S)$. Thereby, the assertion follows.

Step 3. *Any best response $p_B^*(p_A)$ is such that $q(p_A,p_B^*(p_A))$ is nondecreasing in p_A.*

Suppose to the contrary that $p'_A > p_A$, but $q' = q^*(p'_A,p_B^*(p'_A)) < q = q^*(p_A,p_B^*(p_A))$. If $q \in \operatorname{supp} q^*(p'_A,\cdot)$ and $q' \in \operatorname{supp} q^*(p_A,\cdot)$, then, by revealed preferences

$$\hat{\Pi}_B(q';p'_A) \geq \hat{\Pi}_B(q;p'_A) \quad \textit{and} \quad \hat{\Pi}_B(q;p_A) \geq \hat{\Pi}_B(q';p_A). \tag{13}$$

Suppose that $q \notin \operatorname{supp} q^*(p'_A,\cdot)$ but $\hat{\Pi}_B(q;p'_A) \geq \beta v$. Then, $q^*(p'_A,\psi(q;p'_A)) < q$ and revealed preferences imply that $\hat{\Pi}_B(q';p'_A) \geq \hat{\Pi}_B(q^*(p'_A,\psi(q;p'_A));p'_A) > \hat{\Pi}_B(q;p'_A)$. If $\hat{\Pi}_B(q;p'_A) < \beta v$, then $\hat{\Pi}_B(q';p'_A) \geq \hat{\Pi}_B(q^*(0;p_A),p'_A) \geq \beta v$ implies that $\hat{\Pi}_B(q';p'_A) \geq \hat{\Pi}_B(q;p'_A)$. The argument for q' follows symmetrically, such that (13) holds.

Rearranging (13) using Step 2 gives

$$\begin{aligned}&(\mathbb{P}[M(n_S,q) \geq m_A] - \mathbb{P}[M(n_S,q') \geq m_A])\beta\Delta \\ &\qquad\geq \mathbb{E}[\bar{M}_B(n_S,q')]\psi(q';p'_A) - \mathbb{E}[\bar{M}_B(n_S,q)]\psi(q;p'_A), \\ &(\mathbb{P}[M(n_S,q) \geq m_A] - \mathbb{P}[M(n_S,q') \geq m_A])\beta\Delta \\ &\qquad\leq \mathbb{E}[\bar{M}_B(n_S,q')]\psi(q';p_A) - \mathbb{E}[\bar{M}_B(n_S,q)]\psi(q;p_A).\end{aligned}$$

Combined, these yield

$$\begin{aligned}&\mathbb{E}[\bar{M}_B(n_S,q')]\psi(q';p_A) - \mathbb{E}[\bar{M}_B(n_S,q)]\psi(q;p_A) \\ &\qquad\geq \mathbb{E}[\bar{M}_B(n_S,q')]\psi(q';p'_A) - \mathbb{E}[\bar{M}_B(n_S,q)]\psi(q;p'_A),\end{aligned}$$

which rearranges to

$$\mathbb{E}[\bar{M}_B(n_S, q')](\psi(q'; p'_A) - \psi(q'; p_A)) \leq \mathbb{E}[\bar{M}_B(n_S, q)](\psi(q; p'_A) - \psi(q; p_A)).$$

Now, because $q' < q$, it follows that $\mathbb{E}[\bar{M}_B(n_S, q')] > \mathbb{E}[\bar{M}_B(n_S, q)]$ and since $\psi(q; p_A)$ is more increasing for lower q, $\psi(q'; p'_A) - \psi(q'; p_A) \geq \psi(q; p'_A) - \psi(q; p_A)$, such that (13) is violated. This completes the contradiction.

Step 4. *Without loss, $q^*(p_A, p^*_B(p_A))$ is right-continuous in p_A. Since $q^*(p_A, p^*_B(p_A))$ is nondecreasing in p_A (Step 3), A's maximization problem has at least one solution and an equilibrium exists.*

Suppose to the contrary that there exists a decreasing sequence $(p_{A;n})_{n\mathbb{N}}$ with $\lim p_{A;n} = p_A$, and that $\lim q^*(p_{A;n}, p^*_B(p_{A;n})) = q^+$, but $q^+ > q^*(p_A, p^*_B(p_A)) = q^-$.

We argue that it has to hold that

$$\hat{\Pi}_B(q^-; p_A) \geq \hat{\Pi}_B(q^*(p_A, \psi(q^+; p_A)), p_A) \geq \hat{\Pi}_B(q^+; p_A)$$
$$\hat{\Pi}_B(q^*(p_{A;n}; p^*_B(p_{A;n})); p_{A;n}) \geq \hat{\Pi}_B(q^*(p_{A;n}, \psi(q^-; p_{A;n})), p_{A;n}) \geq \hat{\Pi}_B(q^-; p_{A;n}).$$

By construction of q^*, for any q it is true that $q^*(\psi(q, p_A), p_A)$ is in the support of $q^*(p_A, \cdot)$ and $q^*(\psi(q, p_A), p_A) \leq q$. Thereby, the first inequality of either line is a result of p^*_B being a best response of B and the second inequality follows by Step 2.

Since ψ and, thereby, $\hat{\Pi}_B$ are continuous in p_A and q, and because $q^*(p_{A;n}, p^*_B(p_{A;n}))$ as well as $p_{A;n}$ converge, it follows that $\hat{\Pi}_B(q^-; p_A) = \hat{\Pi}_B(q^+; p_A)$. Therefore, it's without loss to change B's response function at p_A to $p^*_B(p_A) = \psi(q^+; p_A)$ and $q^*(p_A, p^*_B(p_A)) = q^+$.

Since $q^*(p_A, p^*_B(p_A))$ is nondecreasing and right-continuous in p_A and all expressions are bounded, $\Pi_A(p^*_A; p^*_B, q)$ always has at least one maximizer such that an equilibrium exists. □

B.3 Proof of Lemma 1

When the activist makes no offer, $\emptyset$, no shareholder can sell, $q(\emptyset) = 0$.

In state Q, the activist's payoff is

$$\Pi_A(p; q, \xi, Q) = \alpha v + b - p\mathbb{E}[\bar{M}(n_S, q(p))]$$

if $\xi(p) \leq \frac{1}{2}$ and shareholders vote against the reform, and

$$\Pi_A(p; q, \xi, Q) = \alpha(v - \Delta) + \mathbb{P}[M(n_S, q(p)) \geq m](b + \alpha\Delta) - p\mathbb{E}[\bar{M}(n_S, q(p))]$$

when $\xi(p) \geq \frac{1}{2}$ and shareholders vote for the reform.

In state R, the activist's payoff is

$$\Pi_A(p;q,\xi,R) = \alpha v + b + \mathbb{P}[M(n_S,q(p)) \geq m](\alpha\Delta - b) - p\mathbb{E}[\bar{M}(n_S,q(p))]$$

in case $\xi(p) \leq \frac{1}{2}$ and shareholders vote against the reform, and

$$\Pi_A(p;q,\xi,R) = \alpha(v+\Delta) + b - p\mathbb{E}[\bar{M}(n_S,q(p))]$$

if $\xi(p) \geq \frac{1}{2}$ and shareholders vote for the reform.

When $\xi(p) \geq \frac{1}{2}$ and shareholders block the reform, the firm value is v; if the activist dictates the outcome of the vote, it rises in expectation by $(1-\xi(p))\Delta$. If $\xi(p) \leq \frac{1}{2}$ and shareholders implement the reform, the expected firm value is $v + (1-2\xi(p))\Delta$, and rises in expectation by $\xi(p)\Delta$ when the activist dictates the outcome of the vote. Therefore, the shareholders' payoffs can be written as

$$\begin{aligned}
\Pi_S(\text{sell};p,q,\xi) &= \frac{v}{n} + \max\{0, 1-2\xi(p)\}\frac{\Delta}{n} \\
&\quad + \mathbb{P}[M(n_S-1,q(p)) \geq m-1]\min\{\xi(p), 1-\xi(p)\}\frac{\Delta}{n} + p\frac{\mathbb{E}[\bar{M}(n_S,q(p))]}{n_S q(p)}, \\
\Pi_S(\text{keep};p,q,\xi) &= \frac{v}{n} + \max\{0, 1-2\xi(p)\}\frac{\Delta}{n} \\
&\quad + \mathbb{P}[M(n_S-1,q(p)) \geq m]\min\{\xi(p), 1-\xi(p)\}\frac{\Delta}{n}.
\end{aligned}$$

Step 1. *There cannot be an equilibrium with $p^*(\omega) > 0$ in either state $\omega \in \{Q, R\}$.*

If A offers any price $p > 0$, all shareholders sell because they know that the friendly activist matches the state. Thus, if $p^*(\omega) > 0$, the activist has a profitable deviation to any $p' \in (0, p^*)$ because it reduces her transfer.

Step 2. *There cannot be an equilibrium where $p^*(\omega) \neq 0$ in both states $\omega \in \{Q, R\}$.*

Suppose A never offers $p^* = 0$. By Step 1, $p^*(Q) = p^*(R) = \emptyset$. Thus, shareholders do not learn from the activists action and implement the reform. In state Q, this means that the activist's payoff is $\alpha(v - \Delta)$. Consider a deviation to $\frac{\epsilon}{m} > 0$ in state Q. Being offered this positive price, all shareholders sell because they know that the friendly activist matches the state. Thus, the activist's payoff is $b + \alpha v - \epsilon$, such that the deviation is profitable when ϵ i sufficiently small. By Step 1 and Step 2, it follows that the activist offers $p^*(\omega) = 0$ in at least one state $\omega \in \{h, \ell\}$.

Step 3. *In any equilibrium, the reform is implemented in state R, but status quo remains in state Q.*

Given Step 1 and 2, there are two possibilities. If $p^*(Q) = p^*(R) = 0$, shareholders do not learn from the offer, $\xi^*(p^*(Q)) = \xi^*(p^*(R)) = \rho$. If they do not sell and implement the reform, they choose the wrong action with probability $1-\rho$. Since the friendly activist always matches the state, if $q^*(0) > 0$ and shareholders are pivotal with positive probability, it is strictly optimal for them to sell. In case $q^*(0) = 0$, the activist has a profitable deviation in state Q by offering a small positive price $\frac{\epsilon}{m}$, securing all voting rights, and blocking the reform (compare Step 2).

When $p^*(Q) = 0$ and $p^*(R) = \emptyset$, or $p^*(R) = 0$ and $p^*(Q) = \emptyset$, shareholders learn the state from the offer, and vote for the reform in state R and for the status quo in state Q. The activist also matches the state. Thus, when $p^*(\omega) = 0$, shareholders are indifferent between voting themselves or delegating their voting right to activist. Since the firm value is maximized and the activist has no cost, there are no profitable deviations.

B.4 Proof of Lemma 2

Suppose that $\mu^*(Q) = 1$ and $\mu^*(R) = 0$. Conditional on observing the message, the shareholders learn the state, $\xi^*(0) = 0$ and $\xi^*(1) = 1$, implement the reform in state R and block it in state Q. Since this maximizes firm value and the activist has aligned incentives, no investor has an incentive to deviate.

B.5 Proof of Proposition 3

Step 1. *There always exists a separating equilibrium.*

We construct an equilibrium of the following form:

- The activist offers $p^*(Q) > p^*(R) \geq 0$.
- Shareholders sell with probability $q^*(p) \begin{cases} = 1 & \text{if } p \geq p^*(Q) \\ < 1 & \text{if } p < p^*(Q). \end{cases}$
- On path beliefs are correct, $\xi^*(p^*(Q)) = 1$ and $\xi^*(p^*(R)) = 0$. Off-path beliefs are $\xi^*(p) = 0$ for all $p < p^*(Q)$ (shareholders believe that the state is R), and $\xi^*(p) = 1$ for all $p > p^*(Q)$.

Let $q^*(p) = q_-(p)$ as defined by (9) and Lemma 5 for all $p < \bar{p} = \max_q \phi(q)\frac{\Delta}{n}$ (where $\xi^*(p) = 0$), and $q^*(p) = 1$ for all $p \geq \bar{p}$.

If $\bar{p} \notin \arg\max_p \Pi_A(p; q^*, R)$, reduce $\bar{p}$ and modify q^* till it is. This has to be possible, because $\Pi_A(\bar{p}; q^*, R) = \alpha v + b - m\bar{p}$ is continuous and strictly decreasing in $\bar{p}$, whereas for any $p < \bar{p}$ it holds that $\Pi_A(p; q^*, R) \leq \alpha(v + \Delta) + \mathbb{P}[M(n_S, q^*(p)) \geq m](b - \alpha\Delta)$ and $\mathbb{P}[M(n_S, q^*(p)) \geq m]$ is bounded away from one.

When $\bar{p} \in \arg\max \Pi_A(p; q^*, R)$, select a p' and set $q^*(p') = q_+(p')$ as defined in Lemma 5, such that $\Pi_A(\bar{p}; q^*, R) = \Pi_A(p'; q^*, R)$. Such a p' has to exist, because $q_+(p')$ is continuous and strictly decreasing in p' with $q_+(0) = 1$, and $\Pi_A(p; q, R)$ is continuous in both, p and q. Notice that $p' < \bar{p}$ and $q^*(p') < 1 = q^*(\bar{p})$.

Let $p^*(R) = p'$, which, by construction, is a best response. Further, let $p^*(Q) = \bar{p}$ and notice that

$$\begin{aligned}\Pi_A(p; q^*, Q) &= \alpha(v - \Delta) + \mathbb{P}[M(n_S, q^*(p)) \geq m](b + \alpha\Delta) - p\mathbb{E}[\bar{M}(n_S, q^*(p))] \\ &= \Pi_A(p; q^*, R) - 2(1 - \mathbb{P}[M(n_S, q^*(p)) \geq m])\alpha\Delta \\ &< \Pi_A(\bar{p}; q^*, R) = \alpha v + b - \bar{p}m = \Pi_A(\bar{p}; q^*, Q),\end{aligned}$$

for all $p \neq \bar{p}$. All prices above $\bar{p}$ are dominated by $\bar{p}$. Thus, the activist has no profitable deviation in either state.

Last, shareholders do not want to deviate. If the price is $p > \bar{p}$, then $q^*(p) = 1$, such that no shareholder is pivotal and selling is a best response. At any price below $\bar{p}$, shareholders play a best response given their belief that the state is R. When the price is $p^*(R)$, this belief is correct.

Step 2. *In any separating equilibrium $p^*(R) < p^*(Q)$ and $q^*(p^*(R)) < q^*(p^*(Q)) = 1$.*

Suppose to the contrary that $p^*(R) \neq p^*(Q)$, but $q^*(p^*(R)) \geq q^*(p^*(Q))$. In any separating equilibrium, after observing $p^*(Q)$, shareholders know that the activist has aligned interests.

If $p^*(Q) > 0$, shareholders sell with probability $q^*(p^*(Q)) = 1$. Thus, the claim can only be violated if $q^*(p^*(R)) = 1$. However, this contradicts the separation, $p^*(R) \neq p^*(Q)$, because the lower price dominates the higher price, such that the activist would want to deviate in one state.

If $p^*(Q) = 0$, shareholders either sell or vote to block the reform. In either case, the reform does not pass, such that any $p^*(R) > 0$ is dominated by $p^*(Q) = 0$, which contradicts the separation. Thereby, $q^*(p^*(R)) < q^*(p^*(Q))$.

If $p^*(R) \geq p^*(Q)$, then $p^*(Q)$ dominates $p^*(R)$, because $q^*(p^*(R)) < q^*(p^*(Q))$. Thereby, $p^*(R) < p^*(Q)$, completing the proof.

Step 3. *As n grows large, along any sequence of equilibria and for $\omega \in \{Q, R\}$,*

$$\mathbb{P}[M(n_S, q^*(p^*(\omega))) \geq m] \to 1 \quad \textit{and} \quad p^*(\omega)\mathbb{E}[\bar{M}(n_S, q^*(p^*(\omega)))] \to 0.$$

In the proof of Proposition 1, we derived that there is a price $\bar{p}$ such that $q^*(p) = 1$ for all $p > \bar{p}$, even when shareholders believe the state is R, $\xi^*(\bar{p}) = 0$, such that their expected loss is maximal. Further, $n\bar{p} \to 0$. Without loss, suppose that $q^*(\bar{p}) = 1$ as well. Then, $\lim \Pi_A(\bar{p}; q^*, \xi^*, \omega) = \alpha v + b$ in both state $\omega \in \{h, \ell\}$.

Suppose the assertion was violated and consider a sequence of separating equilibria. By Step 2, it suffices to show that $p^*(Q)\mathbb{E}[\bar{M}(n_S, q^*(p^*(Q)))] \to 0$ and $\mathbb{P}[M(n_S, q^*(p^*(R))) \geq m] \to 1$. In a separating equilibrium, $\xi^*(p^*(Q)) = 1$ such that shareholders vote for the status quo and

$$\Pi_A(p^*(Q); q^*, \xi^*, Q) = \alpha v + b - p^*(Q)\mathbb{E}[\bar{M}(n_S, q^*(p^*(Q)))].$$

If $p^*(Q)\mathbb{E}[\bar{M}(n_S, q^*(p^*(Q)))] \not\to 0$, a deviation to $\bar{p}$ is profitable when n is sufficiently large. In state R, the belief is $\xi^*(p^*(R)) = 0$ such that shareholders vote for the reform and

$$\begin{aligned}\Pi_A(p^*(R); q^*, \xi^*, R) = \alpha(v + \Delta) + \mathbb{P}[M(n_S, q^*(p^*(R))) \geq m](b - \alpha\Delta) \\ - p^*(R)\mathbb{E}[\bar{M}(n_S, q^*(p^*(R)))].\end{aligned}$$

If $\mathbb{P}[M(n_S, q^*(p^*(R))) \geq m] \not\to 0$, a deviation to $\bar{p}$ is profitable when n is sufficiently large.

Next, consider a sequence of pooling equilibria, where $p^*(Q) = p^*(R) = p^*$, such that $\xi^*(p^*) = \rho$ and shareholders vote for the reform. Then,

$$\Pi_A(p^*; q^*, \xi^*, Q) = \alpha(v - \Delta) + \mathbb{P}[M(n_S, q^*(p^*)) \geq m](b + \alpha\Delta) - p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))]$$
$$\Pi_A(p^*; q^*, \xi^*, R) = \alpha(v + \Delta) + \mathbb{P}[M(n_S, q^*(p^*)) \geq m](b - \alpha\Delta) - p^*\mathbb{E}[\bar{M}(n_S, q^*(p^*))].$$

When either assertion is violated, then $\Pi_A(p^*; q^*, \xi^*, \omega) < \alpha v + b$, for n arbitrary large, such that a deviation to $\bar{p}$ is profitable.

B.6 Proof of Proposition 4

The equilibrium is supported by off-path beliefs where $\xi^*(\kappa) < \frac{1}{2}$ for any $\kappa \in (0, b - \alpha\Delta)$ and the correct on-path belief $\xi^*(0) = 0$. Thus, after any $\kappa < b - \alpha\Delta$, the reform is implemented, such that $\kappa = 0$ dominates all $\kappa < b - \alpha\Delta$. After observing $\kappa = b - \alpha\Delta$, the shareholders believe that the state is Q, $\xi^*(b - \alpha\Delta) = 1$, and the reform is blocked. Above $\kappa = b - \alpha\Delta$, the off-path beliefs are arbitrary. Thus, any $\kappa > b - \alpha\Delta$ is also dominated by either $\kappa = 0$ or $\kappa = b - \alpha\Delta$.

In state Q, the activist has an incentive to spent $\kappa = b - \alpha\Delta$ yielding a payoff of $b + \alpha v - \kappa = b + \alpha v - (b - \alpha\Delta) = \alpha(v + \Delta)$ instead of spending $\kappa = 0$ which yields her a profit of $\alpha(v - \Delta)$. In state R, the activist spends $\kappa = 0$ and receives $\alpha(v + \Delta)$ which yields the same payoff as spending $\kappa = b - \alpha\Delta$. Hence, $\kappa^*(R) = 0$ and $\kappa^*(Q) = b - \alpha\Delta$ is optimal for the activist and the on-path beliefs are consistent.

There cannot be an equilibrium in which the state is matched with probability strictly smaller than $(1 - \rho)$. In any separating equilibrium, shareholders learn the state and therefore the probability of matching the state is one. In any pooling equilibrium, shareholders vote according to their prior and implement the reform, such that the probability of matching the state is $(1 - \rho)$.

B.7 Unrestricted and conditional offers

Lemma 7. *When the activist cannot set a restriction, there are equilibria in which* $\mathbb{P}[M(n_S, q^*(p^*)) \geq m] > 0$ *but*

$$p^* \mathbb{E}[M(n_S, q^*(p^*))] > m\frac{\Delta}{n}\mathbb{P}[M(n_S, q^*(p^*)) \geq m].$$

Proof. Suppose that there is no restriction, such that the activist has to buy from all shareholder who sell to her. Given offer p and response $q(p)$, shareholders are willing to sell, if

$$p \geq \mathbb{P}[M(n_S - 1, q(p)) = m - 1]\frac{\Delta}{n}. \tag{14}$$

We prove the result by an example.

Suppose that $\alpha = 0$, $n = 11$ and $m = 2$. Further, $\Delta = 1$ and $b = \frac{3}{4}$. In this case

$$\mathbb{P}[M(n_S - 1, q(p)) = m - 1] \leq \mathbb{P}[M(10, 0.1) = 1] = 0.38742.$$

Solving for $q(p)$ when (14) holds with equality, there is a continuous, strictly increasing best response q^* with $q^*(p) < 0.1$ for all $p < 0.38742\frac{\Delta}{n}$ and $q^*(p) = 1$ for all $p \geq 0.38742\frac{\Delta}{n}$.

It now follows that $p^* = 0.38742\frac{\Delta}{n}$ because for all $p < p^*$

$$\begin{aligned}\Pi_A^{\text{nr}}(p; q^*) &< b * \mathbb{P}[M(n_S, 0.1) \geq 2] \\ &= b * 0.302643 < b - n * 0.38742\frac{\Delta}{n} = b - 0.38742 = \Pi_A^{\text{nr}}(p^*; q^*).\end{aligned}$$

Any $p > p^*$ is dominated by p^*. Further, $\mathbb{E}[M(n_S, q^*(p^*))]p^* = 0.38742 > \frac{2}{11}\Delta$, completing the proof. ☐

Lemma 8. *When the activist can condition her restricted offer on success, in the unique equilibrium* $p^* = 0$ *and* $q^*(p^*) = 1$.

Proof. As in the case without the condition, $p^* = 0$ and $q^*(0) = 1$ constitute an equilibrium. We show that there is no other equilibrium.

Given any q and the conditional restricted offer p, a shareholder is indifferent between selling the and retaining the share if

$$p \sum_{i=m-1}^{n_S - 1} \mathbb{P}[M(n_S - 1, q) = i]\frac{m}{i+1} = \frac{\Delta}{n}\mathbb{P}[M(n_S - 1, q) = m - 1]. \tag{15}$$

With (6) and (8) this rearranges to

$$
\begin{aligned}
p\mathbb{P}[M(n_S,q)\geq m]\frac{m}{qn_S} &= \frac{\Delta}{n}\mathbb{P}[M(n_S-1,q)=m-1] \\
\iff p\mathbb{P}[M(n_S,q)\geq m] &= \frac{\Delta}{n}\mathbb{P}[M(n_S,q)=m] \\
\iff p &= \frac{\Delta}{n}\frac{\mathbb{P}[M(n_S,q)=m]}{\mathbb{P}[M(n_S,q)\geq m]}.
\end{aligned}
$$

We now note that by (10), $\frac{\mathbb{P}[M(n_S,q)=m]}{\mathbb{P}[M(n_S,q)\geq m]}$ is monotonically decreasing in q with

$$
\lim_{q\searrow 0}\frac{\mathbb{P}[M(n_S,q)=m]}{\mathbb{P}[M(n_S,q)\geq m]}=1 \qquad \lim_{q\nearrow 1}\frac{\mathbb{P}[M(n_S,q)=m]}{\mathbb{P}[M(n_S,q)\geq m]}=0.
$$

By offering $p>0$, either $q^*(p)=1$, or q^* is determined by (15). In either case, for any $p>0$ and any $\epsilon>0$ there is a price $p_\epsilon<\epsilon$ such that $\frac{q^*(p_\epsilon)}{q^*(p)}\geq 1-\epsilon$. Hence, a profitable deviation always exists. This means that in equilibrium, it has to hold that $p^*=0$ and $q^*(0)=1$. □

B.8 Proof of the example

Most of the proof can be found in the body of the text. What remains to be shown is that in state R, the activist does not want to deviate from 0 to any $p\in(0,\bar{p})$.

At any $p\in(0,\bar{p})$, the shareholders' belief is $\xi^*(p)=0$, and because $q^*(p)\in(0,1)$, q^* is determined by the shareholders' indifference condition (9). In state R, the activist's payoff function is given by (1). Plugging in (9), and using (8) gives

$$
\begin{aligned}
&\Pi_A(p;q^*,\xi^*,R) \\
&=\alpha(v+\Delta)+\mathbb{P}[M(n_S,q^*(p))\geq m](b-\alpha\Delta)-m\mathbb{P}[M(n_S,q^*(p))=m]\frac{\Delta}{n} \\
&=\alpha(v+\Delta)+\mathbb{P}[M(n_S,q^*(p))\geq m]((b-\alpha\Delta)-m\frac{\mathbb{P}[M(n_S,q^*(p))=m]}{\mathbb{P}[M(n_S,q^*(p))\geq m]}\frac{\Delta}{n}),
\end{aligned}
$$

for all $p\in(0,\bar{p})$.

Since $\mathbb{P}[M(n_S,q)\geq m]$ is increasing in q, $\frac{\mathbb{P}[M(n_S,q)=m]}{\mathbb{P}[M(n_S,q)\geq m]}$ is decreasing in q (cf. equation (10)), and q^* is strictly increasing in p, there can be no interior optimum $p^*\in(0,\bar{p})$. Since every $p>\bar{p}$ is also dominated by $\bar{p}$, it follows that 0 and $\bar{p}$ are the only two non-dominated actions. Since the activist is indifferent between 0 and $\bar{p}$ when the state is R, there can be no profitable deviations.

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