Nonlinear Occupations and Female Labor Supply Over Time

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Abstract

High hours worked and higher returns to longer hours worked are common in many occupations, namely nonlinear occupations (Goldin 2014). Over the last four decades, both the share and relative wage premium of nonlinear occupations have been rising. Females have been facing rising experience premiums especially in nonlinear occupations. To quantitatively explore how these changes affected female labor supply over time, we build a quantitative, dynamic general equilibrium model of occupational choice and labor supply at both extensive and intensive margins. A decomposition analysis finds that the rising returns to experience, especially in nonlinear occupations, and technical change biased towards nonlinear occupations are important to explain the intensive margin of female labor supply that keeps rising even in the recent period during which female employment stagnates. Finally, a counterfactual experiment suggests that if the nonlinearities were to be gradually vanishing, female employment could have been higher at the expense of significantly lower intensive margin labor supply.

Keywords: Female labor supply, occupational choice, Roy model, experience premium

JEL codes: E2, J2, J1

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1 Introduction

High hours worked and higher returns to long hours worked are prevalent in many occupations (Goldin 2014; Erosa, Fuster, Kambourov, and Rogerson 2020). As these occupations reward individuals who work longer hours more and penalize those who work shorter hours, earnings increase nonlinearly with hours worked. We document that there have been significant changes regarding these nonlinear occupations.\(^1\) The fraction of people, especially of women, working in these nonlinear occupations have been increasing over the last four decades while positive wage premiums for these occupations rose steadily, suggesting that the relative demand for the nonlinear occupations have been rising. Further, large gender gaps in experience premiums that existed four decades ago have been narrowed significantly, especially in nonlinear occupations.

What are the implications of these changes in the relative demand for nonlinear occupations and the disproportionately rising experience premium in nonlinear occupations for the recent evolution of female labor supply? In particular, we ask if these changes help account for ever-rising average hours per female worker (i.e., the intensive margin) that differs sharply from the stagnating employment rate (i.e., the extensive margin) in recent years.\(^2\) At first glance, given that there are increasingly more women than men in these nonlinear occupations that generally pay higher, it may be expected that both changes in nonlinear occupations have contributed to the shrinking gap in the wages of women relative to men, which could in turn induce greater female labor supply at both margins. On the other hand, high nonlinearities could hinder the participation of many women who would be unwilling to work long hours. We address this quantitative question using a version of the neoclassical growth model with heterogeneous agents in which occupational differences arise endogenously.

Specifically, we build upon the Erosa et al. (2020) model of occupational choice (Roy 1951) and endogenous labor supply where two occupations differ by the degree of nonlinearities. Our model is essentially a dynamic version of their model, based on a standard heterogeneous-

\(^1\)This terminology of nonlinear occupations (versus linear occupations) as well as the classification of occupations based on mean hours at the occupation level follow a recent paper by Erosa et al. (2020), who in turn are motivated by Goldin (2014).

\(^2\)The sharp and steady increase in female labor force participation—one of the most remarkable changes in the US labor market during the postwar period—has been stagnating in recent decades (Moffitt 2012; Blau and Kahn 2013). The literature provides potential explanations for the stalling extensive margin (e.g., Fogli and Veldkamp 2011; Fernandez 2013; Albanesi and Prados 2019), but these are at odds with the still-rising intensive margin.
agent incomplete markets framework (Huggett 1993; Aiyagari 1994)—a workhorse macroeconomic model to study issues related to distributions.\(^3\) Compared to Erosa et al. (2020), our dynamic environment is advantageous as it allows us to specify origins of the nonlinearities. In our model, nonlinearities are shaped by dynamic returns to working long hours and part-time penalties. Specifically, in each occupation, a worker can be upgraded stochastically if she worked in the same occupation in the previous period and worked more than or equal to the (occupation-specific) threshold hours.\(^4\) Once a worker is upgraded, she earns the (occupation-specific) experience premium. Part-time penalties also vary by occupations, modeled as a proportional tax on earnings for those who work short hours. These differences across occupations along with individual state variables such as idiosyncratic productivity, comparative advantages, experience, and household assets affect occupational choice and hours of work conditional on occupational choice in the model.\(^5\)

We calibrate the model to US data in a standard way by matching the relevant statistics, obtained from the Current Population Survey (CPS) during the initial period of 1976-1985.\(^6\) Following Erosa et al. (2020), we categorize occupations in the data into two groups, based on mean hours worked at the occupational level.\(^7\) Without assuming any further condition on this categorization, our calibration result distinguishes the occupations with higher mean hours from those with lower mean hours along three dimensions: (i) threshold hours for one to be eligible for potential upgrade are higher; (ii) experience premiums are higher; and (iii) part-time penalties are higher. Since nonlinear occupations would compensate more for working longer hours while not favoring shorter hours (Goldin 2014), our estimation and model calibration confirm that the former is more nonlinear (i.e., nonlinear occupations) than the latter group (i.e., linear occupations). We further corroborate the properties of our model in delivering the salient facts across occupations that nonlinear occupations have greater mean wages and higher

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\(^3\)More precisely, our model framework builds on a standard general equilibrium incomplete-markets framework with production (Aiyagari, 1994) augmented with endogenous labor supply at the intensive margin (e.g., Pijoan-Mas, 2006) as well as the extensive margin (e.g., Chang and Kim, 2006).

\(^4\)In other words, those who worked less than the threshold hours are not counted as valid career experience.

\(^5\)Agents are also allowed to choose to work or not (i.e., an operative extensive margin).

\(^6\)We start from 1976 because of the data availability regarding occupational information. Because we are interested in long-run trends, we divide our sample period into four decades based on CPS years: 1976-1985; 1986-1995; 1996-2005; and 2006-2015.

\(^7\)Specifically, occupations are first ranked by mean hours worked by men and then divided into two weighted by both men and women (Erosa et al. 2020).
dispersion of wages, as also documented by Erosa et al. (2020).

We then use the model to quantify the role of changes in nonlinear occupations in explaining the evolution of female labor supply. We are particularly interested in the factors responsible for the ever-rising intensive margin labor supply, in sharp contrast to the stagnating employment rates. While keeping the calibrated parameters, we consider changes in selected driving forces. These include not only the key interests of our paper—experience premiums and nonlinear-occupation biased technical change—but also factors that are known to be important for determining female labor supply such as relative wage changes (Heathcote, Storesletten and Violante 2010; Jones, Manuelli and McGrattan 2015; Bick, Brüggemann, Fuchs-Schündeln and Paule-Paludkiewicz 2019) and preference shifts (Fernández, Fogli and Olivetti, 2004, Fogli and Veldkamp 2011; Fernandez 2013) to allow the model to replicate the observed changes in gender wage gaps and aggregate hours.

The first notable finding in our decomposition analysis is that rising experience premiums especially in nonlinear occupations are quantitatively important in accounting for the rising intensive margin of female labor supply. Specifically, the model predicts that the model-implied increment of 144 annual hours per worker from 1976-1985 to 2006-2015 would have been nearly 40% lower in the absence of the observed increases in experience premiums. We find that this is particularly driven by the significant increase in hours per worker in nonlinear occupations. That is, the model predicts that nearly 57% of the 166-hour increase in annual hours per worker in nonlinear occupations from 1976-1985 to 2006-2015 would disappear when experience premiums were held fixed at the baseline period (1976-1985). Next, our calibration implies that there have been noticeable changes in demand factors, favoring nonlinear occupations increasingly more over time. We find that this change naturally increases the share of women working in nonlinear occupations but reduces average hours worked in each occupation. However, as nonlinear occupations have higher hours worked (relative to linear occupations), the induced increase in the share of women in nonlinear occupations raises average hours per worker, thereby contributing to the rising intensive margin labor supply. We also find that wage changes for women are very powerful in shifting more women to work (extensive margin) but are not as important as experience premiums in raising the intensive margin.
Finally, we conduct a counterfactual analysis asking what would have happened to female labor supply trends if the nonlinearities had been gradually vanishing. This is motivated by Goldin (2014) who argues that these nonlinearities are an important source of the gender wage gap as they prevent women from working in nonlinear occupations that pay higher wages on average. In this experiment, we adjust nonlinearities by (i) reducing the threshold hours relevant for upgrade or (ii) decreasing the threshold hours relevant for part-time penalties, while keeping the changes in experience premiums.\(^8\) We find that female employment indeed could have been significantly higher especially when the requirement of working long hours is eliminated, leading to a 9-percentage point higher employment rate in 2006-2015. Because experience premiums have been rising especially in nonlinear occupations, the model predicts a significantly more prominent increase in the number of women working in nonlinear occupations. However, we also find that this change is accompanied with significantly lower intensive margin labor supply (11% lower in 2006-2015), thereby resulting in rather lower total hours worked. Our exercise suggests that while the nonlinearities are indeed a quantitatively important barrier for many women out of the labor force, they also play an important role of providing an incentive for women to supply long hours.

A large literature has investigated the determinants of changes in female labor supply over time, as reviewed recently by Doepke and Tertilt (2016) and Greenwood, Guner, and Vandenbroucke (2017). Our results are closely related to Olivetti (2006) and Attanasio, Low and Sánchez-Marcos (2008), both of who consider returns to experience as a main determinant. Relative to these papers, our paper differs in that we consider experience premiums separately in different occupations categorized by nonlinearities, document differential trends, and investigate their implications for occupational choice—a channel that also shapes intensive margin labor supply.\(^9\) Moreover, our findings shed light on the seemingly conflicting findings between Olivetti (2006) and Attanasio et al. (2008): the latter finds that returns to experience are unlikely to be a quantitatively important factor in explaining rising female labor supply in contrast to the former who finds a substantial role of experience premiums. Note that in our model incorporating

\(^8\)We keep the rising experience premiums because they are a beneficial feature of nonlinearities unlike the threshold hours that are essentially a form of friction.

\(^9\)As noted by Heathcote, Perri and Violante (2010), relatively little attentions have been paid to the rising trends in experience premiums.
both intensive and extensive margins, we find that the effects of experience premiums mostly work through the intensive margin only, having only limited effects on the extensive margin. Therefore, our result suggests that a model without the intensive margin as in Attanasio et al. (2008) may understate the importance of the experience premiums in accounting for female labor supply changes. At the same time, relative to Olivetti (2006), in our model that incorporates idiosyncratic uncertainty and higher model frequency (annual vs. 10 years), we find that the role of experience premiums in explaining overall female labor supply is quantitatively not as strong, as also pointed out by Attanasio et al. (2008).

We highlight that our novel findings on the role of nonlinear occupations and experience premiums are based on the model where the effects of the other factors are quantitatively in line with the existing literature. In particular, in line with theory highlighting the role of learning in shifting women’s disutility of work (Fogli and Veldkamp 2011; Fernandez 2013), our decomposition exercise finds that the role of preference shift in explaining overall increases in female labor supply became increasingly important until 1996-2005 and then became much weaker in 2006-2015.

The rest of the paper is organized as follows. The next section presents the stylized facts about how labor supply and occupations (nonlinear vs. linear) have evolved over the last four decades using data from the CPS. Section 3 presents the model economy and defines the equilibrium. Section 4 explains how the model is calibrated and presents some properties of the baseline economy. Section 5 presents a decomposition analysis showing how different factors affect the observed trends in labor supply and gender wage gaps. Section 6 conducts a counterfactual experiment to quantify what would have happened to labor supply trends if nonlinearities were to gradually disappear. Section 7 concludes. Appendix includes more details about data and the model and provides additional results about sensitivity.

2 Trends in labor supply and nonlinear occupations

In this section, we present the stylized facts about the evolutions of labor supply and occupational choice. As in Erosa et al. (2020), we use the IPUMS-CPS files to take information on
the 1976-2015 Current Population Survey (CPS). The CPS provides information not only on demographic characteristics but also on labor market outcomes, such as the number of hours worked per week, usual hours worked per week, total labor income, and occupations. We use the occupational classification of Author and Dorn (2013) to generate occupational codes consistent over the whole sample period. Because the focus of this paper is long-term changes, we divide the sample period into four 10-year intervals: 1976-1985 (benchmark), 1986-1995, 1996-2005, 2006-2015. We restrict our samples to households in which a male head and a female spouse coexist because prominent changes in labor supply have been observed in married women (Jones et al. 2015). We consider the samples aged between 22 and 64.

As for the definition of nonlinear occupations, we take the following steps. First, in the baseline period (1976-1985), we rank occupations according to the average working hours for males, using personal-level weights, at the occupation level. Second, we compute the size of occupations using personal-level weights for both male and female workers at the occupational level. Third, reflecting the size of occupations, we evenly split occupations into two groups according to the occupational ranking, denoting the top and bottom 50 percent of occupations as nonlinear and linear occupations, respectively. Finally, we apply this occupational grouping over the whole period.

Figure 1 shows the trend of the U.S. labor supply: the total hours worked, the extensive margin, and the extensive margin by gender. The top panel of Figure 1 shows that although the male total hours worked remain stable over the period, the female total hours worked have an upward trend until the 1996-2005 period and then level off in the following period. Consequently, the speed at which the gender gap in total hours worked converge has slowed down significantly in recent periods.

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10 We follow most data-related procedures in Erosa et al. (2020). Appendix also covers more details related to data.

11 Despite the same criterion used to rank occupations, the timing differs from Erosa et al. (2020). Specifically, Erosa et al. (2020) compute the average working hours for males at the occupational level for each period, thus updating the ranking of occupations in every period. Meanwhile, we choose the 1976-1985 period as our benchmark period; compute the ranking of occupations according to the average working hours for males at the occupational level in the benchmark; apply this occupational ranking over the whole period. As our paper is interested in changes in occupation composition over time unlike Erosa et al. (2020), we use the time-invariant occupation categorization applied consistently to the whole period.

12 A few occupations are newly observed after the benchmark period. When these occupations are first observed in the data, we categorize them according to the threshold working hours of the occupational grouping in the benchmark period; keep using it for the substantial periods. More details are addressed in Appendix.
Figure 1: Trends in female labor supply in the US

(i) Total hours worked

(ii) Extensive margin

(iii) Intensive margin
It is useful to decompose whether this slowdown in convergence is due to the evolution of hours per worker (the intensive margin) or of employment (extensive margin). The second and last panels of Figure 1 show that the evolution of labor supply differs sharply between the extensive and intensive margins, especially for females. Specifically, although the female extensive margin has an upward trend until the 1996-2005 period and becomes stagnant afterwards (e.g., Moffitt 2012; Blau and Kahn 2013), the female intensive margin shows an ever-rising trend over the whole period. To explain these observations jointly, it would be necessary to delve into factors that could drive differential dynamics between the extensive and intensive margins of labor supply. We argue that such candidates are in the evolution of nonlinear occupations, which we now investigate next.

Figure 2 presents the trends of the relative quantity and price of nonlinear occupations by gender. It is clear to notice steady increases in both the relative quantity and relative price of nonlinear occupations. The top panel of Figure 2 shows that the proportion of workers in nonlinear occupations out of all workers had increased in both men and females. However, this increase in the share of workers in nonlinear occupations is gender biased. Although the relative quantity of nonlinear occupations has an upward trend in both genders, this trend is much steeper for females. Specifically, from the 1976-1985 period to the 2006-2015 period, while the relative quantity of nonlinear occupations for males increased near by 4 percentage points, that for females increased near by 15 percentage points.

This increase in the relative quantity could be driven by both an increase in the relative demand for the nonlinear occupations and an increase in relative labor supply such as rising college-educated workers who are more likely to be in nonlinear occupations.\textsuperscript{13} The middle panel of Figure 2, however, shows that the rising relative quantity was accompanied by an increase in the relative wage of nonlinear occupations, pointing to a significant role of demand-driven technological change biased toward nonlinear occupations. As nonlinear wage premiums could be due to factors such as education and selection as well, we also compute residual premiums by controlling for age, education, race, industry, and the number of children under age five. The

\textsuperscript{13}In fact, Table A2 in Appendix shows that although nonlinear occupations have relatively more college-educated workers within each period, the share of college educated workers have been rising in both nonlinear and linear occupations for each gender in a parallel manner.
Figure 2: Trends in the relative quantity and price of nonlinear occupations

(i) Share working in NL occupations, by gender

(ii) Nonlinear wage premium, by gender

(iii) Nonlinear (residual) wage premium, by gender

Note: Nonlinear vs. linear occupations are defined based on occupation-level mean hours following Erosa et al. (2020) in the base years of 1976-1985. We keep using the base-year occupation categorization for the following periods (1986-1995, 1996-2005, 2006-2015). The second figure is based on observed raw wage data whereas the bottom figure is based on residual wages after controlling for age, education, race, industry, and the number of children under 5.
bottom panel of Figure 2 shows that although a quite significant portion of the observed nonlinear wage premiums can be explained by observables in each period, the rising trend is still clearly observed for both men and women. Intuitively, the rising demand for nonlinear occupations would increase the relative share of women working in nonlinear occupations. Moreover, it could also have interesting implications for the evolution of the intensive margin drive differential dynamics between the extensive and intensive margins. Since nonlinear occupations have higher hours worked, this change could induce increasing labor supply at the intensive margin through compositional change.

Having documented the trends in nonlinear occupations, we now discuss how experience premiums evolved over the same periods in nonlinear occupations relative to linear occupations. As Heathcote, Perri and Violante (2010) point out, relatively little attentions have been paid to the rising trends in experience premiums. These are particularly relevant for our analysis in this paper because they could shape the observed changes in wage premiums for the nonlinear occupations by affecting occupational choice and labor supply, especially how long one would like to work.

We compute occupation-specific experience premiums for females and males separately, based on the wage differences between age 45-55 and 25-35, as in Heathcote, Perri and Violante (2010) and Erosa et al. (2020), but with some modifications. As a first step, for each occupation group and each period, we regress log wages on age and age squared while controlling for education, the number of children under age five, race and industry. It is particularly important to control for education because within each period, education is systematically lower among older people due to the rising educational level over time. Further, as the number of children under five is a strong predictor of female labor supply, it could mitigate biases that may arise due to selection. In the second step, we use the estimated coefficients for age and age squared to predict the age profile. Then, we use these residual age profiles to compute mean wages for age 45-55 and 25-35, which are used to compute the experience premiums.

Table 1 reports the estimates. First of all, we find that nonlinear occupations tend to have greater experience premiums regardless of gender. Second, it shows that women used to have

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14 We indeed find that this variable is quantitatively relevant for women.
15 Although our methodology cannot exploit panel structures as in Kambourov and Manovskii (2009b) who
Table 1: Experience premiums over time, by gender and occupation

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>.0942</td>
<td>.1365</td>
<td>.1544</td>
<td>.2403</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>.0575</td>
<td>.1173</td>
<td>.1532</td>
<td>.1556</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>.3370</td>
<td>.3502</td>
<td>.3203</td>
<td>.3166</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>.1768</td>
<td>.2522</td>
<td>.2307</td>
<td>.2244</td>
</tr>
</tbody>
</table>

much lower experience premiums relative to men in both occupations, qualitatively consistent with Heathcote et al. (2010). Most importantly, experience premiums for women have been increasing sharply, especially in nonlinear occupations, although those for men are quite stable over time. Specifically, in nonlinear occupations, the 24.3 percentage point of the gender gap in the experience premium has narrowed substantially to a 7.6 percentage point difference from 1976-1985 to 2006-2015. By contrast, in linear occupations, the 11.9 percentage point of the initial gap has become a 6.9 percentage point difference over the same period.

To sum up, when it comes to female labor supply, we highlight that while the extensive margin became stagnant in the 2006-2015 period, the intensive margin kept rising steadily. We then document the evolutions related to nonlinear occupations. First, the relative quantity and price of nonlinear occupations have been increasing, suggesting technological change biased towards nonlinear occupations. Moreover, we document that the gender difference in experience premiums that used to be quite sizeable have been shrinking sharply, especially in nonlinear occupations. In the next sections, we explore the implications of these changes regarding nonlinear occupations for female labor supply over time.

3 Model economy

In this section, we describe the model that we use for the quantitative analysis. In an incomplete markets environment (Aiyagari 1994), our model features adjustments along the intensive and
extensive margins of female labor supply with occupational choice in the spirit of Roy (1951). The main decisions such as labor supply and occupational choice in our model are for women while we simplify male counterparts.\textsuperscript{16} A model period is annual in line with the frequency of the data in our empirical analysis.

### 3.1 Households

There is a continuum of households. A household is composed of married couples in accordance with the empirical analysis in Section 2.\textsuperscript{17} Households face an exogenous positive probability of dying. When a household dies, it is replaced by a new household. We denote the household state vector by \((a, z, x, j, η, j_m)\) where \(a\) is assets, \(z\) is female idiosyncratic productivity, and \(x \in \{0, 1\}\) is the indicator of occupational experience (e.g., Kambourov and Manovskii 2009). \(j \in \{0, 1, 2\}\) denotes the female occupational history in the last period, being either no relevant history \((j = 0)\) or the nonlinear occupation history \((j = 1)\) or the linear occupation history \((j = 2)\). \(η \in \mathbb{R}\) is Roy comparative advantage (which we explain below) following a normal distribution \(\mathcal{N}(0, σ_η^2)\), and \(j_m \in \{0, 1, 2\}\) is the type of male occupation where 0 means non-employed, 1 is the nonlinear occupation, and 2 denotes the linear occupation.

At the beginning of each period, a female chooses to work or not (extensive margin labor supply), which is summarized by the following equation:

\[
V(a, z, x, j, η, j_m) = \max\{N(a, z, η, j_m), W(a, z, x, j, η, j_m) - ξ I_{j_m \neq 0}\}
\]  

(1)

where \(N(a, z, η, j_m)\) is the value of not working and \(W(a, z, x, j, η, j_m)\) is the value of working. Participation costs \(ξ \in \mathbb{R}\) are incurred if both spouses work (e.g., Cho and Rogerson 1988; Guner Kaygusuz and Ventura 2012; and Erosa, Fuster and Kambourov 2016). We first explain the value of working, and then describe the value of non-working.

\textsuperscript{16} As mentioned in Section 2, male labor supply and occupational choice have been relatively more stable.

\textsuperscript{17} It is common to model married women only for the studies on female labor supply as most prominent variations in female labor supply exist among married women (e.g., Heathcote, Storesletten and Violante 2010; Jones et al. 2015; Bick and Fuchs-Schündeln 2018; Erosa et al. 2020 among others).
The value of working involves another discrete choice about occupation in the current period:

\[ W(a, z, x, j, \eta, j_m) = \max \left\{ J_1(a, z, x, j, \eta, j_m), J_2(a, z, x, j, \eta, j_m) \right\} \]  

(2)

where \( J_j \) is the value of working in occupation \( j \) at the beginning of the period before actually working in the period. When occupational choice \( j \) in this period is the same as that in the last period \( \hat{j} \), she is eligible for the upgrade of experience. That is, with a probability of \( \pi \), she becomes experienced \((x' = 1)\), and with a probability of \( 1 - \pi \), her experience does not change. If the current occupation choice \( j \) is different from \( \hat{j} \) (e.g., switched occupation or did not work in the previous period), their occupational experience is set to 0 (unexperienced). Formally, the value of occupation \( j \) is defined as

\[
J_j(a, z, x, \hat{j}, \eta, j_m) = \begin{cases} 
\pi P_j(a, z, 1, \eta, j_m) + (1 - \pi) P_j(a, z, x, \eta, j_m) & \text{if } j = \hat{j} \\
\pi P_j(a, z, 0, \eta, j_m) & \text{if } j \neq \hat{j}
\end{cases}
\]

(3)

where \( P_j \) is the interim period value of working in occupation \( j \) in the current period after the realization of the upgrade uncertainty.

Conditional on occupational choice and after the realization of experience relevant for this period, households choose how many hours to work in the occupation \( j \). The value of working in occupation \( j \) at this moment, \( P_j \), is given by

\[
P_j(a, z, x, \eta, j_m) = \max_{c, \alpha' \geq 0, h \in [h, 1 - n_p]} \left\{ u(c_f, n_p + h) + u(c_m, h_{j_m}) + \beta \begin{bmatrix} q \mathbb{E}_{z'|z} V(a', z', x, j', \eta, j_m) \\ (1 - q) \mathbb{E}_{z'|z} V(0, z', 0, \eta', j'_m) \end{bmatrix} \right\}
\]

(4)
subject to

\[ c_f + c_m + a' \leq w_{jf} z_j (1 + \mathbb{I}_{x=1} \chi_j - \mathbb{I}_{x<F} \tau_j) h + w_{jm} e_{jm} h_{jm} + (1 + r)a + T \]  \hspace{1cm} (5)

\[ z_j = \exp(\eta) z \quad \text{if } j = 1 \]
\[ = z \quad \text{if } j = 2 \]
\[ j' = j \mathbb{I}_{h \geq U_j}. \]  \hspace{1cm} (6)

where \( c_f \) is female consumption, \( c_m \) is male consumption, \( n_p \) denotes homework hours when participating in the labor market, \( a' \) is asset holdings in the next period, \( \beta \) is the discount rate, and \( r \) is the real interest rate. The expected values in the next period show that households survive with a probability of \( q \) and die with a probability of \( 1 - q \) in the next period. \( T \) refers to accidental bequest transfers redistributed from assets of the dying households, as in Conesa, Kitao and Krueger (2009). The same measure is replaced by new households with zero assets and new draws of \( z' \), \( \eta' \) and husband \( j'_m \in \{0,1,2\} \sim F_m(j_m) \). The new households have no female occupational experience, \( x = 0 \), no female previous occupational career history, \( \dot{j} = 0 \).

Female labor income depends on the female market wage in occupation \( j \), \( w_{jf} \), labor productivity in occupation \( j \), \( z_j \), occupation-specific experience premium \( \chi_j \) (available only for the experienced), part-time penalty \( \tau_j \) (applied to those who work less than the full-time threshold hours \( F \)), and hours worked \( h \). Unlike female labor income that involves various endogenous objects, male labor income is simply given. For the husband \( j_m \), his labor income is exogenously determined by male market wage in occupation \( j_m \) (\( w_{jm} \)), the efficient unit of husband who work in occupation \( j_m \) (\( e_{jm} \)) and his hours worked (\( h_{jm} \)).

We now explain how we model labor productivity in occupation \( j \), \( z_j \), especially compared to a closely related paper by Erosa et al. (2020) which our model framework builds upon. They consider occupation-specific ability \( a_j \) with different means and variances in a static Roy model. There, those who draw a high \( a_1 \) compared to \( a_2 \) have comparative advantage in occupation 1. In our model, \( z_1 \) is equal to \( \exp(\eta) z \) where a positive \( \eta \) (or \( \exp(\eta) > 1 \)) implies comparative advantage in occupation 1, and the degree of this advantage increases with \( \eta \). In addition, as our model is dynamic, we allow idiosyncratic productivity to be stochastically evolving through
\( z \), which follows a standard AR(1) process in logs:

\[
\log z' = \rho z + \epsilon', \quad \epsilon' \sim \mathcal{N}(0, \sigma^2_z).
\] (7)

Next, it is also important to discuss how nonlinearity of occupations is captured in our model.\(^{18}\) We note that we explicitly capture two salient underlying forces of nonlinearity. The first concerns differential dynamic returns to working long hours. As shown in (5), the experience premium \( \chi_j \) differs across occupations. In our model, these differential dynamic returns are only eligible for those who work hours long enough (above the occupation-specific upgrade thresholds hours \( U_j \)). These threshold hours essentially capture frictional aspects of nonlinearity by prohibiting those who work relatively short hours from advancing their career. And these are allowed to differ across occupations. The second is about differential penalties for working part-time. As shown in (5), part-time penalties are modeled as a tax on earnings \( \tau_j \), which differs between occupations, for those who choose to work less than the full-time threshold \( F \). This could capture the tendency that firms with nonlinear occupations may not favor short hours if \( \tau_1 > \tau_2 \) (Goldin 2014). Also note that \( F \) also captures the degree of frictional aspects of nonlinearity because fewer people would be subject to part-time penalties if \( F \) decreases.

Finally, the value of non-working, which shares a number of elements that are also present in the value of working is given by:

\[
N(a, z, \eta, j_m) = \max_{c, a' \geq 0} \left\{ u(c_f, n_n) + u(c_m, h_{jm}) + \beta \begin{bmatrix} q\mathbb{E}_{z', \eta} V(a', z', 0, 0, \eta, j_m) \\
(1 - q)\mathbb{E}_{z', \eta'} V(0, z', 0, 0, \eta', j'_m) \end{bmatrix} \right\}
\] (8)

subject to

\[
c_f + c_m + a' \leq w_{jm} e_{jm} h_{jm} + (1 + r)a + T
\]

where \( n_n \) denotes homework hours when not working. When women do not work, experience in

\(^{18}\)In the static model of Erosa et al. (2020), nonlinearity of occupation \( j \) is captured by labor earnings being nonlinear in hours:

\[
\omega h^{1+\theta_j}
\]

where \( \theta_j \geq 0 \) captures nonlinearity of occupation \( j \), and \( \omega \) denotes some constant capturing productivity and market wages (potentially occupation specific as well). There, nonlinear occupations are those with a higher \( \theta \) whereas linear occupations are those with a lower \( \theta \).
the next period is set to zero \((x' = 0)\), capturing the negative aspects of career disruptions.

### 3.2 Firm

The economy has a representative firm. The representative firm solves:

\[
\max_{L_{1f}, L_{1m}, K} \left( Y - w_{1f}L_{1f} - w_{1m}L_{1m} - w_{2f}L_{2f} - w_{2m}L_{2m} - (r + \delta)K \right)
\]

(9)

where

\[
Y = AK^\alpha L^{1-\alpha}
\]

\[
L = \left[ \nu L_1^\psi + (1 - \nu)L_2^\psi \right]^{\frac{1}{\psi}}
\]

\[
L_j = \lambda_j L_{jf} + (1 - \lambda_j)L_{jm}
\]

\(Y\) is the aggregate output, and \(L_{jf}\) (\(L_{jm}\)) is the female (male) aggregate labor in occupation \(j\). \(\delta\) is the depreciation rate of capital. \(A\) is the total factor productivity, \(K\) is the aggregate capital, and \(\alpha\) is the capital share. \(\nu\) captures the relative demand for nonlinear occupations, and \(\psi\) shapes the elasticity of substitution between occupations. Finally, \(\lambda_j\) captures the gender-biased demand in occupation \(j\).

The first-order conditions yield the following equations characterizing factor demands

\[
[K]: r + \delta = A\alpha K^{\alpha - 1} L^{1 - \alpha}
\]

(10)

\[
[L_{1f}]: w_{1f} = A(1 - \alpha)K^\alpha L^{-\alpha}\frac{\partial L}{\partial L_1}\frac{\partial L}{\partial L_{1f}}
\]

(11)

\[
[L_{1m}]: w_{1m} = A(1 - \alpha)K^\alpha L^{-\alpha}\frac{\partial L}{\partial L_1}\frac{\partial L}{\partial L_{1m}}
\]

(12)

\[
[L_{2f}]: w_{2f} = A(1 - \alpha)K^\alpha L^{-\alpha}\frac{\partial L}{\partial L_2}\frac{\partial L}{\partial L_{2f}}
\]

(13)

\[
[L_{2m}]: w_{2m} = A(1 - \alpha)K^\alpha L^{-\alpha}\frac{\partial L}{\partial L_2}\frac{\partial L}{\partial L_{2m}}
\]

(14)
Note that one can easily derive

\[
\frac{w_{1f}}{w_{1m}} = \frac{\lambda_1}{1 - \lambda_1},  \quad (15) \\
\frac{w_{2f}}{w_{2m}} = \frac{\lambda_2}{1 - \lambda_2},  \quad (16)
\]

which show that the relative wages between females and males are shaped by gender-biased demand parameter \( \lambda_j \), as in Heathcote, Storesletten and Violante (2010) and Cerina, Moro and Rendall (2018).\(^{19}\) Note that we allow the gender-biased demand, \( \lambda_j \), to differ by occupation.

Similarly, the relative market wage of nonlinear occupations can be obtained as

\[
\frac{w_{1f}}{w_{2f}} = \frac{\partial L}{\partial L_1} = \frac{\nu L_1^{\psi-1}}{(1 - \nu)L_2^{\psi-1}}.  \quad (17)
\]

showing that the wage premiums for nonlinear occupations would tend to increase with \( \nu \).

### 3.3 Equilibrium

We now define a stationary recursive competitive equilibrium. We define a measure space to describe the stationary equilibrium. Let us denote \( S = A \times Z \times X \times J \times E \times J_m \) as the state space of households such that \( (a, z, x, \hat{j}, \eta, \hat{j}_m) \in S \). Then, a probability measure \( F(\cdot) \) is defined on the Borel \( \sigma \)-algebra \( \mathbb{B}(S) \) such that \( F(\cdot) : \mathbb{B}(S) \rightarrow [0, 1] \). \( F(B) \) represents the measure of households whose state lies in \( B \in \mathbb{B}(S) \) as a proportion of all households.

A stationary recursive equilibrium is a set of factor prices \( (r, w_{1f}, w_{2f}, w_{1m}, w_{2m}) \); a set of female decision rules \( \left( g_a(a, z, x, \hat{j}, \eta, \hat{j}_m, \xi), g_o(a, z, x, \hat{j}, \eta, \hat{j}_m), \{g_{a,j}(a, z, x, \eta, \hat{j}_m)\}_{j=0}^2 \right) \); a set of value functions \( \left( V(a, z, x, \hat{j}, \eta, \hat{j}_m), N(a, z, \eta, \hat{j}_m), W(a, z, x, \hat{j}, \eta, \hat{j}_m), \{P_j(a, z, x, \eta, \hat{j}_m)\}_{j=1}^2 \right) \); the aggregate capital \( K \), the aggregate labor \( L \), and the aggregate labor by gender and occupation \( L_{1f}, L_{2f}, L_{1m}, L_{2m} \); the distribution of households \( F(\cdot) \) such that

1. Given factor prices \( (r, w_{1f}, w_{2f}, w_{1m}, w_{2m}) \), the value functions \( V(a, z, x, \hat{j}, \eta, \hat{j}_m), N(a, z, \eta, \hat{j}_m), W(a, z, x, \hat{j}, \eta, \hat{j}_m), \{P_j(a, z, x, \eta, \hat{j}_m)\}_{j=1}^2 \) solve the associated problems defined above, the

\(^{19}\)In practice, this is similar to gender-specific taxes in Jones et al. (2015) without the redistribution of the tax revenue.
associated decision rules are

\[ g_n(a, z, x, j, \eta, j_m, \xi) = \arg \max \{ N(a, z, \eta, j_m), W(a, z, x, j, \eta, j_m) - \xi I_{j_m \neq 0} \} \]  
(18)

\[ g_0(a, z, x, j, \eta, j_m) = \arg \max \left\{ J_1(a, z, x, j, \eta, j_m), J_2(a, z, x, j, \eta, j_m) \right\} \]  
(19)

\[ \alpha^* = g_{a,j}(a, z, x, j, \eta, j_m); : j \in \{0, 1, 2\} \]  
(20)

\[ h^* = g_{h,j}(a, z, x, j, \eta, j_m); : j \in \{1, 2\}. \]  
(21)

2. Given factor prices \( r, w_1, w_2, w_1, w_2, \) the representative firm optimally chooses \( K, L_{1,f}, L_{2,f}, L_{1,m}, \) and \( L_{2,m} \) following (10)-(14)

3. Markets clear:

\[ K = \int a F(d(a, z, x, j, \eta, j_m)) \]  
(22)

\[ L_{j,f} = \int \left[ \int_{\xi} I_{\{g_n(s, \xi) = W\}} P(d\xi) \cdot z_j \cdot \left( I_{\{j = g_n(a) = j\}} \cdot \left( \pi \cdot (1 + \chi_j) \cdot g_{h,j}(a, z, x^\prime = 1, \eta, j_m) \right) 
+ (1 - \pi) \cdot (1 + \chi_j \cdot I_{x=1}) \cdot g_{h,j}(a, z, x, \eta, j_m) \right) 
+ I_{\{j = g_n(a) \neq j\}} \cdot g_{h,j}(a, z, x^\prime = 0, \eta, j_m) \right] F(d(a, z, x, j, \eta, j_m)); : j \in \{1, 2\} \]  
(23)

\[ L_{j,m} = \int e_{j,m} h_{j,m} F(d(a, z, x, j, \eta, j_m)); : j \in \{1, 2\} \]

where \( s = (a, z, x, j, \eta, j_m) \in S \) and \( P(\cdot) \) is the cumulative distribution function of \( \xi \).

4. The household distribution \( F(\cdot) \) is consistent with the household optimal choices defined
above. Specifically, for any $B \in \mathbb{B}(S)$,

$$F(B) = q \cdot \sum_{j=1}^{2} \left\{ \int_{\{(a,z,x,j,j'_m)| (a,z,x,j,j'_m) \in B\}} \left[ \pi_{z'}|z \cdot \int_{\xi} I_{\{g_n(s,\xi)=N\}} P(d\xi) \right] F(d(a,z,x,j),\eta',j'_m) \right. \\
+ q \cdot \sum_{j=1}^{2} \left\{ \int_{\{(a,z,x,j,j'_m)| (a,z,x,j,j'_m) \in B\}} \left[ \pi_{z'}|z \cdot \int_{\xi} (1- I_{\{g_n(s,\xi)=N\}}) P(d\xi) \right] \\
\cdot \pi \cdot I_{\{g_n(s)>\mathcal{F}_j\}} \cdot I_{\{x=0\}} + I_{\{g_n(s)>\mathcal{F}_j\}} \cdot I_{\{x=1\}} \right. \left. \left( \pi_{z'}|z \cdot \int_{\xi} (1- I_{\{g_n(s,\xi)=N\}}) P(d\xi) \right) \\
\cdot (1- \pi) \cdot I_{\{g_n(s)>\mathcal{F}_j\}} \cdot I_{\{x=0\}} \right\} F(d(a,z,x,j),\eta',j'_m) \right\} \\
+ q \cdot \sum_{j=1}^{2} \left\{ \int_{\{(a,z,x,j,j'_m)| (a,z,x,j,j'_m) \in B\}} \left[ \pi_{z'}|z \cdot \int_{\xi} (1- I_{\{g_n(s,\xi)=N\}}) P(d\xi) \right] \\
\cdot I_{\{g_n(s)=j\}} \cdot I_{\{g_n(s)\leq\mathcal{F}_j\}} \right\} F(d(a,z,x,j),\eta',j'_m) \right\} \\
+ (1-q) \cdot \int_{\{(a,z,x,j,j,j'_m)| (0,0,j,j'_m) \in B\}} \pi_{z'}|z \cdot \pi_{\eta'}|z \cdot \pi_{j'_m} F(d(a,z,x,j,\eta,j'_m))$$

where $s = (a,z,x,j,\eta',j'_m) \in S$, $\pi_{z'}|z$ is the transitional probability from $z$ to $z'$, and $P(\cdot)$ is the cumulative distribution function of $\xi$. $\pi_{z'}|z$, $\pi_{\eta'}|z$, and $\pi_{j'_m}$ determine the distribution of newly-born households for $z'$, $\eta'$, and $j'_m$, respectively.

## 4 Calibrating the model in the baseline period

In this section, we explain how the model is calibrated to US data in the baseline period (1976-1985). We then discuss the properties of the calibrated model in relation to some stylized facts on nonlinear occupations.

### 4.1 Parameters calibrated externally

A set of parameters can be calibrated externally without solving the model. The other parameters are calibrated internally by matching relevant target statistics. All variables with the unit of hours in the model are expressed as the fraction of total disposable annual hours (105 weekly hours multiplied by 52 weeks).
For the utility function, we use the same standard functional form as Erosa et al. (2020):

$$u(c_g, h_g) = \log c_g - \phi_g \frac{h_g^{1+\gamma}}{1+\gamma}, \quad g = f, m$$

(24)

where $\phi_g > 0$ is the disutility constant (which is calibrated internally as explained below). We set $\gamma = 2$ so that the Frisch elasticity of labor supply at the intensive margin is 0.5 in line with the micro evidence (Chetty, Guren, Manoli and Weber 2011). Next, we set $q = 1 - 1/40$ to have average 40 years of working life and $\pi = 1/10$ such that on average it takes 10 years of occupational career to become experienced (e.g., Kambourov and Manovskii 2009). The persistence of idiosyncratic shocks is set to be $\rho_z = 0.94$ (Jang, Sunakawa and Yum 2020) in line with standard values estimated in the literature (e.g., Heathcote, Storesletten and Violante 2010). We set the minimum hours that can be supplied $h$ to 0.0476 (or five weekly hours) to be consistent with the restriction imposed in the empirical analysis. Home hours when participating in the labor market is $n_p = 0.245$ and the counterpart for non-working women is higher at $n_n = 0.410$ (Ramey 2009).

In our model, part-time penalties are captured by $\tau_j$. We compute the part-time penalties by regressing log hourly wages on a part-time dummy, set to one if weekly hours worked are less than 25, while controlling for age, education, race, industry, and the number of children under age 5. We estimate $\tau_j$ by gender in each period separately. As reported in Appendix, there is a clear pattern that nonlinear occupations tend to have higher part-time penalties for both men and women. This is consistent with Goldin (2014) who argue that nonlinear occupations are associated with higher part-time penalties. As there is no clear trend on this penalty over time, however, we set $\tau_1 = 0.189$ and $\tau_2 = 0.074$, based on mean penalties for females over the whole period. In line with the threshold used for obtaining these estimates, we set the threshold hours for full-time $F$ to 0.238 (or 25 weekly hours). Occupation-specific experience premiums are captured by $\chi_j$. The baseline values for these parameters are estimated to be $\chi_1 = 0.0942$ and $\chi_2 = 0.0575$, as documented in Section 2.

As for the firm technology, we use standard values for $\alpha = 0.36$ to be consistent with

\(^{20}\text{We use Tauchen (1986) for discretization with 7 grid points.}\)}
Table 2: Parameter values calibrated internally and target statistics

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
<th>Model</th>
<th>Data</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.9879</td>
<td>Discount factor</td>
<td>.040</td>
<td>.040</td>
<td>Real interest rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>7.73</td>
<td>Disutility of work</td>
<td>.314</td>
<td>.312</td>
<td>Hours per worker</td>
</tr>
<tr>
<td>$\xi$</td>
<td>.213</td>
<td>Participation cost</td>
<td>.516</td>
<td>.520</td>
<td>Employment rate</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>.186</td>
<td>S.D. of innovations to $\ln z$</td>
<td>.450</td>
<td>.454</td>
<td>$\text{sd}(\ln(\text{wage}))$</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>.287</td>
<td>Variability of $\eta$</td>
<td>.497</td>
<td>.500</td>
<td>Share of all workers in NL</td>
</tr>
<tr>
<td>$\nu$</td>
<td>.686</td>
<td>Weight of NL in prod.</td>
<td>.179</td>
<td>.179</td>
<td>$\mathbb{E}(\text{wage</td>
</tr>
<tr>
<td>$e_1$</td>
<td>2.53</td>
<td>Eff. unit of husband in NL</td>
<td>.407</td>
<td>.410</td>
<td>Gender wage gap in NL</td>
</tr>
<tr>
<td>$e_2$</td>
<td>1.73</td>
<td>Eff. unit of husband in L</td>
<td>.363</td>
<td>.366</td>
<td>Gender wage gap in L</td>
</tr>
<tr>
<td>$U_1$</td>
<td>.329</td>
<td>Hours for upgrade in NL</td>
<td>.415</td>
<td>.415</td>
<td>$\Pr(x = 1</td>
</tr>
<tr>
<td>$U_2$</td>
<td>.272</td>
<td>Hours for upgrade in L</td>
<td>.443</td>
<td>.443</td>
<td>$\Pr(x = 1</td>
</tr>
</tbody>
</table>

aggregate capital share. We set $\psi$ to $-0.5$, implying that the elasticity of substitution between nonlinear occupations and linear occupations is 0.67. We set $A = 1.0$ for the baseline period. As for female weights in $L_j$, these parameters are not separately identified from efficiency units of males in occupation $j$, which are internally calibrated to match the observed gender wage gaps. For the baseline period, we fix $\lambda_1 = \lambda_2 = 0.40$, implying that without selection and any further forces, the gender wage gaps in each occupation is around 33%. Then, we allow them to change over time in the following quantitative exercises on the trends. The depreciation rate is set to $\delta = 0.096$.

There are four parameters related to husband. The probability mass of husbands $F_m(j_m)$ requires two parameters, $p_1$ and $p_2$ where $p_{j_m}$ refers to the probability of the husband working in occupation $j_m$. These values are taken directly from data, $p_1 = 0.537$ and $p_2 = 364$ in the baseline period. The other two parameters regarding husbands are the occupation-specific intensive margins $h_{j_m}$, which are also directly taken from the data: $h_1 = 0.416$ and $h_2 = 0.370$.$^{21}$

---

$^{21}$This value implies that there is a moderate degree of complementarity between the nonlinear and linear occupations. We have also conducted sensitivity checks with $\psi = -0.25$, and our results are nearly unchanged. See Appendix for details.
4.2 Parameters calibrated internally

The remaining parameters are calibrated internally. Table 2 summarizes 10 parameters calibrated internally to match 10 target statistics. We see that the model is able to match the target statistics very precisely. We now explain how each parameter is clearly linked to its target statistic, which explains why our model fits the target statistics very well.

The first parameter $\beta$ is the discount factor, which is targeted to match the annual real interest rate of 4%. Next, there are two parameters governing the disutility of work, $\phi$ and $\xi$. The former is a constant shifting disutility from hours worked and thus is targeted to match the average hours worked per worker of 0.312 (or 1702 annual hours). The latter is a participation cost to be paid when both spouses work and is thus calibrated to match the female employment rate of 52.0%. The next parameter $\sigma_z$ governs the degree of wage inequality in the model. Thus, its target is set to the standard deviation of log wages of 0.454.

As discussed above, $\eta$ captures the comparative advantage in nonlinear occupations, which follows a normal distribution with mean zero.\(^{22}\) Holding others constant, a higher variability of $\eta$, $\sigma_\eta$, implies a larger fraction of women choosing the nonlinear occupations because there are more people with stronger comparative advantages in nonlinear occupations. The share of workers, including both men and women, in nonlinear occupations is used as the target for this parameter. Given the way we categorize nonlinear occupations as in Erosa et al. (2020), this target is exactly 0.5 in the benchmark year, as discussed in Section 2. The next parameter $\nu$ is the share parameter for nonlinear occupations relative to linear occupations in the production technology. Holding other things fixed, a higher $\nu$ would increase the relative wage of nonlinear occupations, as shown in (17). Its target is thus set to the observed wage premium of nonlinear occupations, 17.9% in the baseline year, as documented in Section 2.

The next two parameters $e_{jm}$ are the efficiency unit of husbands in each occupation. Although endogenous channels in our model have implications for gender wage gaps, there are other various channels shaping gender wage gaps, as reviewed by Blau and Kahn (2017), missing in our model. Thus, while allowing for endogenous channels at work, these two parameters are

\(^{22}\)As we have endogenous market wages in each occupation, a non-zero mean of $\eta$ would be offset by adjustments in the relative wage in equilibrium.
Table 3: Wage and hours: Model vs. data

<table>
<thead>
<tr>
<th></th>
<th>All (Model)</th>
<th>Data</th>
<th>NL occ (Model)</th>
<th>Data</th>
<th>L occ (Model)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{E}(wage) )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.112</td>
<td>1.113</td>
<td>.943</td>
<td>.944</td>
</tr>
<tr>
<td>( \mathbb{E}(h) )</td>
<td>.317</td>
<td>.312</td>
<td>.330</td>
<td>.324</td>
<td>.305</td>
<td>.306</td>
</tr>
<tr>
<td>( \text{sd}(\log(wage)) )</td>
<td>.450</td>
<td>.454</td>
<td>.471</td>
<td>.496</td>
<td>.430</td>
<td>.424</td>
</tr>
<tr>
<td>( \text{sd}(\log(h)) )</td>
<td>.253</td>
<td>.487</td>
<td>.258</td>
<td>.485</td>
<td>.247</td>
<td>.487</td>
</tr>
<tr>
<td>Gender wage gap</td>
<td>.417</td>
<td>.421</td>
<td>.407</td>
<td>.410</td>
<td>.363</td>
<td>.366</td>
</tr>
</tbody>
</table>

*Note: Wages are re-scaled to have one as unconditional mean.*

calibrated internally to match the observed gender wage gaps in each occupation, 41.0% in nonlinear occupations and 36.6% in linear occupations.

The final two parameters \( U_j \) are the threshold hours for one to be eligible for upgrade. In essence, these parameters govern the frictional aspect of nonlinearities in each occupation because higher values imply that it is required to work longer hours to be eligible for and maintain experience premiums. In the data, the share of the experienced women relative to the unexperienced women, measured by the number of workers aged 45-55 divided by the sum of the number of workers aged 25-35 and those aged 45-55 in line with the definition in Heathcote, Perri and Violante (2010) and Erosa et al. (2020), is lower in nonlinear occupations (41.5%), compared to linear occupations (44.3%) in the baseline period (1976-1985). While allowing for other occupational differences such as experience premiums, we calibrate \( U_j \) internally to match these ratios of the relative share of the experienced in each occupation group. The calibration results indicate a higher threshold for the nonlinear occupation \( U_1 = .329 \) (versus \( U_1 = .272 \) for the linear occupation). This means that there is a higher degree of frictional nonlinearities in nonlinear occupations, which would prevent more women from working in nonlinear occupations who would be unwilling to work long hours.

### 4.3 Properties of the model in the baseline year

Having discussed how we calibrate our model to the baseline period, we now present how well our model is able to reproduce the salient facts on the two occupation groups, as also highlighted in Erosa et al. (2020). Specifically, in US data (1976-1985), mean hours, mean wages, wage dis-
Table 4: Sources of nonlinearity

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>$\chi_1 \downarrow$</th>
<th>$\mathcal{U}_1 \downarrow$</th>
<th>$\tau_1 \downarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NL</td>
<td>L</td>
<td>NL</td>
<td>L</td>
</tr>
<tr>
<td>Emp rate</td>
<td>.174</td>
<td>.342</td>
<td>.156</td>
<td>.361</td>
</tr>
<tr>
<td>$\mathbb{E}(wage</td>
<td>NL) - 1$</td>
<td>.179</td>
<td>.172</td>
<td>.157</td>
</tr>
<tr>
<td>$\mathbb{E}(h)$</td>
<td>.330</td>
<td>.305</td>
<td>.315</td>
<td>.310</td>
</tr>
<tr>
<td>$\text{sd}(\log(wage))$</td>
<td>.471</td>
<td>.430</td>
<td>.451</td>
<td>.438</td>
</tr>
<tr>
<td>$\text{sd}(\log(h))$</td>
<td>.258</td>
<td>.247</td>
<td>.262</td>
<td>.249</td>
</tr>
</tbody>
</table>

*Note:* We separately set $\chi_1$ or $\mathcal{U}_1$ or $\tau_1$ to $\chi_2$, $\mathcal{U}_2$, $\tau_2$, respectively. While doing so, we shut down general equilibrium feedback by fixing prices at the baseline level.

...persion, and the gender wage gap are higher in nonlinear occupations, as shown in Table 3. Our model generates these patterns quite well except that hours dispersion is lower, compared to the data. An interesting observation is that the overall gender wage gap is quite higher than that within occupation in the data, implying that features and choices related to occupations worsen the gender wage gap. As our model targets occupation specific gender wage gaps, occupational premiums and the relative share of nonlinear occupations, the model reproduces 42% of the overall gender wage gap in the data.

Compared to Erosa et al. (2020) who also generate these patterns in a static environment, our dynamic environment allows us to microfound the nonlinearities through experience premiums $\chi_j$, and part-time penalties $\tau_j$ along with threshold hours, $\mathcal{U}_j$ and $\mathcal{F}$, for dynamic returns and part-time penalties, respectively. To illustrate how each of these occupation-specific features shapes labor market outcomes across occupations, we equalize each of $\chi_j, \mathcal{U}_j$, and $\tau_j$ at their linear occupation level, while shutting down general equilibrium effects (i.e., holding prices constant at the baseline level).

Table 4 reports how each parameter contributes to the differences observed in Table 3. When we first equalize the experience premiums at the linear occupation level, we find that the relative share of women working in nonlinear occupations decreases in response to a relatively lower incentive to work in nonlinear occupations. We also find that this would reduce the...

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23 We have considered a version with preference heterogeneity targeted to match the volatility of hours, as in Erosa et al. (2020). Our results are barely affected by this change except that we can boost up hours dispersion while preference heterogeneity increases the state space, which is costly in terms of computation time. To keep the model parsimonious and more transparent, our benchmark model thus abstracts from preference heterogeneity.
nonlinear occupation premium, and the positive gap in mean hours worked. Finally, because the lowered experience premium essentially shrinks the right tail of wage distribution in nonlinear occupations, we see that the positive gap in the dispersion of hourly wages becomes smaller with this change.

Another important element of nonlinearities in our model is the occupation-specific threshold hours $U_j$. According to our calibration result, $U_1$ is larger than $U_2$, implying that there is a stronger frictional aspect in nonlinear occupations that dynamically benefits only those who work longer hours. When we reduce $U_1$ to $U_2$, this barrier to women would be relaxed in nonlinear occupations. Table 4 indeed shows that this would raise the fraction of women working in nonlinear occupations quite substantially (from 17.4% to 21.4%). This also reduces the incentive to work long hours in nonlinear occupations, which in turn narrows the positive gap in mean hours worked. Finally, we note that the positive wage premium for nonlinear occupations decline quite noticeably via selection effects, meaning that marginal women joined in nonlinear occupations tend to have lower productivity.

Finally, we also investigate how the difference in part-time penalties affect labor market outcomes in each occupation group. When we reduce $\tau_1$ in nonlinear occupations to the level of $\tau_2$, Table 4 shows that this change has relatively little effects on the differences across occupations. The most notable change is in the dispersion of hours. By lowering the part-time penalty in nonlinear occupations, there are more workers in nonlinear occupations who are willing to work less, which in turn increases the standard deviation of log hours in nonlinear occupations.

5 Understanding the evolution of female labor supply

In this section, we investigate underlying forces at work behind the evolution of female labor supply through the lens of our model. For the decomposition exercise, we feed changes in selected driving forces into the model so that it could generate empirically plausible trends in key aggregate variables of interest. These changes in the driving forces are either estimated externally or calibrated internally following the same calibration strategy in Section 4.24

24 All the parameters internally calibrated and their target statistics are reported in Appendix (Table A4).
other parameters are unchanged at the baseline period.\footnote{We also consider perfect-foresight transitions in Appendix. The main decomposition results are robust when temporally aggregated into 10 years as in our main analysis. See Appendix for more details.}

\subsection{Driving forces}

The first two driving forces are our key interests. The first one is experience premiums in each occupation. As documented in Section 2 (Table 1), experience premiums have been increasing especially in nonlinear occupations. The second is technical change biased towards nonlinear occupations, implied by the empirical trends in Section 2 that both the relative price and relative quantity of nonlinear occupations have been rising over time. Because these two trends are affected by not only technological change (demand) but also other factors related to supply, we recover the evolution of $\nu$ internally by targeting the nonlinear wage premiums in each of the three periods (1986-1995, 1996-2005, and 2006-2015), as in the baseline calibration in Section 4. We are interested in how these changes contributed to female labor supply over time along various dimensions (e.g., intensive versus extensive margins and occupational choice).\footnote{To measure the change in $\nu$, which affects hourly wages of both men and women, we also incorporate the observed changes in male employment rates and hours per worker in each occupation, as reported in Table A5. This makes sure that our $\nu$ is recovered while taking into account general equilibrium feedback due to changes in the relative labor supply of males as well.}

Other driving forces we consider are related to the evolution of wages. A crucial reason for this is to allow the model to generate empirically reasonable price changes in terms of gender wage differences, occupational wage differences, and overall wage changes. As the equilibrium wages depend on the distribution of individual productivity among workers, selection issues related to participation and occupational choice make it impossible to feed in price changes externally in our model framework. Therefore, we use an equivalent calibration strategy to the one for the baseline period. More precisely, in each period, we obtain the values of four parameters, $\lambda_1, \lambda_2, \sigma_\eta$ and $A$ by internally matching the observed gender wage gaps in each occupation, the observed nonlinear wage premium, and the observed overall wage changes.

Although price effects could be quantitatively strong, they may not be able to capture all changes in labor supply observed in the data. An important alternative mechanism could be the intergenerational transmission of culture according to Fernández et al. (2004), Fogli and Veldkamp (2011) and Fernandez (2013) that effectively reduces utility costs of working. Hence,
in addition to the above changes, we also consider a disutility-of-work shifter $s > 0$. Specifically, $s$ is multiplied by $\phi$ and $\xi$, both of which capture the disutility of work. This parameter $s$ is internally calibrated to generate the evolution of total hours worked, as observed in the data.

Before we move on, quick discussions on determinants we abstract from are in order. First, we do not consider changes in home production technology (Greenwood, Seshadri and Yorukoglu 2005). Figure 3 plots trends in homework hours in the postwar period, based on historical data in Ramey (2009). Note that a noticeable decline in mean home hours began in the mid-1960’s. And then average home hours become gradually stable since the 1980’s. In particular, trends conditional on employment status have become already stable in the mid-1970’s. This suggests that although technological changes in home production should be of first-order importance in understanding female labor supply until 1980, it might not be one of the most relevant factors during the periods we consider in this paper. We also abstract from medical progress (Albanesi and Olivetti 2016) found to be important for mother’s labor supply, which is more relevant for periods earlier than the periods we consider (1976-2015).

5.2  Trends implied by the model

We first present the performance of our model in reproducing overall observed features in female labor supply and occupational choice. Before we conduct a decomposition exercise to understand
how the underlying forces of our interest shape the dynamics of female labor supply in labor market outcomes over time, we would like to see whether the model captures the evolution of aggregate labor market variables reasonably well.

Figure 4 displays the model-generated trends of total hours, the extensive margin, and the intensive margin for females and their empirical counterparts. The top panel of Figure 4 shows that the model-generated trend of the female total hours is, by construction, perfectly matched with its empirical counterpart. It is, however, worth noting that both the extensive and intensive margins of female labor supply are not separately targeted. Therefore, it is more interesting to validate the model by comparing these two margins in the model with their empirical counterparts.

The middle and bottom panels of Figure 4 imply that the model does a good job of reproducing untargeted dynamics that sharply differ between the extensive and intensive margins. The middle panel of Figure 4 implies that the model can capture empirically observed features in the trend of the female extensive margin. Not only does the model here generate the upward and concave trend in the female extensive margin until the period 1996-2005, but it successfully reproduces the stagnating extensive margin in the period 2006-2015. On the other hand, the bottom panel of Figure 4 shows that the model successfully reproduces the ever-rising trend in the intensive margin.

Figure 5 presents model-generated trends related to female occupations and their empirical counterparts. As can be seen, the model generally performs well in capturing empirically observed features in female occupational choice. The top panel and middle of Figure 5 show that the model-generated evolution of employment rates in each occupation is empirically consistent. That is, the model replicates the rising employment rate in nonlinear occupations and the inverse U-shaped employment rate in linear occupations. What is more noteworthy could be the model performance in capturing the evolutions of intensive margins conditional on occupation, which are not targeted. The bottom panel of Figure 5 shows that the model does a reasonably good job of replicating the trends in the conditional intensive margins. More specifically, although the model has a difficulty of generating the continual upward trend in hours per worker in linear occupations, it performs well in reproducing the ever-rising trend in hours per worker.
Figure 4: Trends in female labor supply: Model vs. data

(i) Total hours

(ii) Extensive margin

(iii) Intensive margin
Figure 5: Female occupation-related trends: Model vs. data

(i) Nonlinear occupation

(ii) Linear occupation

(iii) Conditional intensive margins
Finally, as validity checks, we also compute trends in the second moments of wages and hours implied by the model, as reported in Table A3 in Appendix. In essence, our model successfully generates increasing trends in wage dispersion and declining trends in hours dispersion in line with the data, as also documented in Heathcote, Perri and Violante (2010).

5.3 Underlying forces at work

In this subsection, we investigate underlying forces at work in the trends presented in the previous subsection. Specifically, we present counterfactual trends when a driving force of our interest is assumed to be unchanged at the level in the baseline period (1976-1985). This allows us to quantify the role of each driving force by comparing the counterfactual trend to the trend in the presence of all forces (blue solid line).

Experience premiums We begin with one of the two key driving forces of interest in this paper: experience premiums. As shown in Table 1 in Section 2, female experience premiums have increased substantially over time, and the increase was more pronounced in nonlinear occupations. The rising experience premiums increase dynamics returns to long hours worked. Thus, women have stronger incentives to increase labor supply, especially in nonlinear occupations.

Figure 6 displays results from the decomposition exercises for the trends in three measures of female labor supply: total hours worked, the extensive margin, and the intensive margin. The red dotted line shows the trend in the absence of changes in experience premiums. The top panel of Figure 6 shows that without the increase in experience premiums, the increase in female total hours worked over the last four decades would have been substantially lower. Specifically, had the experience premiums remained at the level in the 1976-1985 period over the whole period, the model would have dampened the increment of the total hours worked over the entire period by 26% (i.e., 34.6% instead of 47.0%).

The middle and bottom panels of Figure 6 imply that the rising experience premiums have quantitatively differential effects on the two margins of labor supply although they have positive impacts on both margins. The middle panel of Figure 6 shows that the rising experience
Figure 6: Decomposition: total hours and two margins of labor supply

(i) Total hours

(ii) Extensive margin

(iii) Intensive margin
premiums have relatively weak effects on the extensive margin, which is in line with Attanasio et al. (2008).

The bottom panel of Figure 6, on the other hand, shows our novel finding on the role of experience premiums in the female labor supply that the rising experience premiums play a significant role in accounting for the intensive margin that keeps increasing even until the recent decade. Specifically, if the experience premiums were fixed at the benchmark level, the model predicts that the increment of 144 annual hours per worker would have been 40% lower in the final period 2006-2015. Note that the experience premium has increased particularly more in nonlinear occupations. As the experience premium is only available for those who advance their career by working longer hours, especially in nonlinear occupations, it induces significant increases in hours per worker in nonlinear occupations, as seen in the top panel of Figure 8. More precisely, we find that the 166-hour increase in annual hours per worker in nonlinear occupations from 1976-1985 to 2006-2015 observed in the baseline model would be reduced by 57% in the absence of changes in experience premiums.

**Technical change biased towards nonlinear occupations** As reported in Table A4, our calibration implies that there has been a steady increase in the share of nonlinear occupations in the technology of the firm, $\nu$, from 0.686 in 1976-1985 to 0.785 in 2006-2015. An immediate impact of such changes would be higher wages in nonlinear occupations, which would shift people from linear occupations to nonlinear occupations. In Figure 7, we clearly see that this effect is quantitatively significant. Without the change in $\nu$, the nonlinear occupation employment rate would have been nearly 11 percentage point lower in 2006-2015 whereas the linear occupation employment rate would have been 14 percentage point higher. Overall, this implies that the overall employment rate would have been slightly higher over time in the absence of the biased technical change.

On the other hand, the bottom panel of Figure 6 shows that the technical change biased towards nonlinear occupations indeed contributed to the rising intensive margin. Specifically, in the absence of this technical change, the intensive margin would have increased by 122 annual hours, 16% less than 144 annual hours from the baseline model. However, at the same
time, Figure 8 shows that hours per worker in each occupation would have been higher if the technological change biased toward nonlinear occupations were absent. This seemingly puzzling result can be understood by its large impact on the occupational composition shown in Figure 7. In other words, because nonlinear occupations tend to have higher hours per worker relative to linear occupations, an increase in the share of nonlinear occupations would push up average hours per worker even without any change in the conditional hours per worker in each occupation. As this composition effect dominates the declines in conditional hours per worker, the occupation-biased technical change could contribute to the rising intensive margin labor supply.

Other factors Although the preference shift is not one of our key interest in this paper, it is worth discussing its role given the interesting result. Figure 6 shows that the magnitude of the role of preference shifts in explaining had been time-varying: its importance had increased until the 1996-2005 period and then became weaker in the 2006-2015. This finding is in fact consistent with theory of cultural learning (Fogli and Veldkamp 2011; Fernandez 2013) as preference shifts driven by the intergenerational learning process should predict that its importance must slow down over time. As in Fogli et al. (2011) and Fernandez (2013), the preference shifts capture the stagnant pattern in the female extensive margin in the 2006-2015 period.

Note also that there are quite substantial portions still generating the rising trends in female labor supply. These are largely due to the narrowing gender wage gaps, as shown in the blue line in Figure 9. Overall, the strong effects of narrowing gender wage gaps on female labor supply are also found in the existing literature (e.g., Heathcote, Storesletten and Violante 2010; Jones et al. 2015; Bick et al. 2019). Our model also relies on this channel to generate the overall evolution of female labor supply.

---

27 Because price effects through $\lambda_j$ and $\sigma_o$ are interlinked in terms of their influences on relative prices (with respect to gender and occupation), it is not straightforward to separate out these price effects in a clean way.

28 More precisely, we mean the narrowing gender wage gaps unexplained by the rising experience premiums. Figure 9 shows that the decreasing pattern of the gender wage gap in the model are partially explained by rising experience premiums (about 21% of the total decline in the final period).
Figure 7: Decomposition: occupation composition

(i) Nonlinear occupation

(ii) Linear occupation
Figure 8: Decomposition: intensive margins conditional on occupation

(i) Nonlinear occupation

(ii) Linear occupation
6 Nonlinearities of occupations and labor supply trends

Goldin (2014) argues that high nonlinearities in some occupations are an important source of the gender wage gap as they prevent women from working in these highly nonlinear occupations that generally pay higher wages. In our theory, nonlinearities are captured by both the size of returns to working long hours and the size of part-time penalties (intensities), but also by threshold hours for these intensities to operate. Note that if the first threshold hours relevant for upgrade $U_j$ converge to zero, everyone who works for the same years in the same occupation have the equal chance of enjoying experience premiums, regardless of hours worked. If the second threshold hours for part-time penalties $F$ converge to zero, part-time penalties would disappear.

In this section, we therefore conduct a counterfactual experiment, motivated by Goldin (2014). We do so by gradually removing these frictional aspects of nonlinearities while allowing the increasing experience premiums, which become more positive aspects of nonlinearities over time (especially with $U_j \to 0$). Specifically, we reduce $U_j$ and $F$ smoothly such that nonlinearities disappear in the most recent period (via linear interpolation). This allows us to quantify how important the frictional aspects of nonlinearities are for the evolution of female labor supply.

Table 5 reports the results for each counterfactual exercise along with the evolution from the benchmark model presented in the previous sections. For each counterfactual experiment, we report percentage point differences relative to the benchmark trends for employment rates (aggregate, nonlinear and linear) or percentage differences relative to the benchmark trends for
the other variables.

The first three rows show that the elimination of $U_j$ and $F$ is indeed a powerful mechanism to boost the employment rate of women. If $U_j$ reached zero in 2006-2015, the employment rate of women could have been 9.2 percentage point higher than 70%. The effect of reducing $F$ is quantitatively smaller but is still quite sizeable. In the case of removing $U_j$, the next rows show that the large effect on overall employment is driven by a disproportionately higher increase in the people working in nonlinear occupations thank to the rapidly rising experience premiums. These results so far appear to confirm the adverse role of nonlinearities illustrated by Goldin (2014).

However, we also find that this increase in female labor supply along the extensive margin is accompanied by significantly lower intensive margin labor supply. For instance, if $U_j$ reached zero in 2006-2015, hours per worker could have been 11 percent (or nearly 200 annual hours) lower in 2006-2015. As a result, total hours worked, which accounts for changes at both margins, would have been reduced since 1996-2005, while the frictional nonlinearities were gradually disappearing. Further, the observed gender wage gaps would be even higher because there are more women working with relatively lower productivity (selection) while facing lower threshold hours, $U_j$ and $F$.

The key lesson of this exercise is now clear. While we quantitatively confirm that the nonlinearities are indeed an important barrier for many women out of the labor force, we also find that they play an important role of providing an incentive scheme that maintains high hours per worker. Without these, part-time becomes more attractive, and those who work have less incentives to work long hours.

7 Conclusion

In this paper, we have documented that there have been significant increases in the relative price and quantity of more nonlinear occupations, and that experience premiums for women have increased quite substantially especially in nonlinear occupations. Motivated by this evidence, we have built a quantitative, dynamic general equilibrium model of occupational choice and
Table 5: Counterfactuals: nonlinearities and trends in labor supply and gender wage gaps

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Emp rate</td>
<td>Benchmark</td>
<td>.516</td>
<td>.636</td>
<td>.699</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>+7.0</td>
<td>+7.3</td>
<td>+9.2</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>+4.2</td>
<td>+1.9</td>
<td>+2.1</td>
</tr>
<tr>
<td>NL emp rate</td>
<td>Benchmark</td>
<td>.174</td>
<td>.259</td>
<td>.334</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>+3.0</td>
<td>+6.4</td>
<td>+7.4</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>+1.2</td>
<td>+1.1</td>
<td>+0.8</td>
</tr>
<tr>
<td>L emp rate</td>
<td>Benchmark</td>
<td>.342</td>
<td>.378</td>
<td>.366</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>+4.0</td>
<td>+0.9</td>
<td>+1.7</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>+3.0</td>
<td>+0.8</td>
<td>+1.4</td>
</tr>
<tr>
<td>Hours per worker</td>
<td>Benchmark</td>
<td>.314</td>
<td>.329</td>
<td>.335</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>-7.4</td>
<td>-9.3</td>
<td>-11.4</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>-5.4</td>
<td>-4.0</td>
<td>-3.7</td>
</tr>
<tr>
<td>Total hours</td>
<td>Benchmark</td>
<td>100.0</td>
<td>129.3</td>
<td>145.0</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>+2.7</td>
<td>-1.7</td>
<td>-1.5</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>+0.9</td>
<td>-3.2</td>
<td>-2.4</td>
</tr>
<tr>
<td>Observed gender wage gap</td>
<td>Benchmark</td>
<td>.417</td>
<td>.348</td>
<td>.298</td>
</tr>
<tr>
<td></td>
<td>(U_j \to 0)</td>
<td>+2.3</td>
<td>+2.2</td>
<td>+3.1</td>
</tr>
<tr>
<td></td>
<td>(F \to 0)</td>
<td>+1.9</td>
<td>+1.0</td>
<td>+1.1</td>
</tr>
</tbody>
</table>

Note: Reported numbers are percentage point differences relative to the benchmark trends (Emp rate, NL emp rate and L emp rate) or percentage differences relative to the benchmark trends (Hours per worker and Total hours). Total hours are scaled to be 100 in the baseline year (1976-1985). Both \(U_j\) and \(F\) are set to converge linearly to zero in 2006-2015.
labor supply that we used to study how these changes related to nonlinear occupations affected female labor supply over time. In the model, nonlinear occupations provide higher returns to working longer hours by penalizing part-time more and by allowing workers to be eligible for larger returns to experience only if they work longer hours. We have found that rising experience premiums have substantially contributed to the ever-rising intensive margin whereas they had limited effects on the extensive margin. Our counterfactual experiment in spirit of Goldin (2014) who emphasizes the adverse role of nonlinearities demonstrates important policy implications that eliminating the nonlinearities may increase female employment at the expense of a sizeable fall in labor supply along the intensive margin.

Female labor supply has been generally rising in many developed countries over the last few decades. However, the relative importance of intensive versus extensive margins vary substantially across countries. As documented by Bick et al. (2019), some countries such as Germany and the Netherlands have experienced quite noticeable decreases in hours per female worker while female employment rose substantially. Our results in this paper suggest that changes in nonlinearities may account for such nontrivial variations. We leave this interesting investigation for future work.
A Appendix

A.1 Data

To compute empirical statistics at the micro-level, we use the Current Population Survey (CPS) based on the 1976-2015 IPUMS-CPS files. The CPS is a nationally representative survey of individuals and their households. The CPS provides information not only on demographic characteristics but also on labor market outcomes, such as the number of hours worked per week, usual hours worked per week, total labor income, and occupation. We choose the CPS waves from 1976 to 2015. We take only the samples of households in which a male head and a female spouse coexist; therefore, unmarried and single-parent households are excluded. We convert all the nominal values to the values in 1999 US dollar using the CPI-U. In all calculations, we use a variable of household weight, asecwt.

We restrict our attention to married households because prominent changes in labor supply have been observed in married women (Jones et al. 2015). We consider only the samples, where heads are aged between 22 and 64. We divide the sample periods into four groups: 1976-1985, 1986-1995, 1996-2005, 2006-2015. We regard the 1976-1985 period as our baseline period.

We construct annual hours worked by multiplying the number of hours worked per week by the usual hours worked per week. Hourly wage is constructed by dividing total labor income by annual hours worked. We use the occupational classification in Autor and Dorn (2013) to construct occupational ranking over the sample period. We obtain occupational hourly wage and occupational hours worked by computing their average at each occupation, as in Erosa et al. (2020).

More specifically, we take the following steps. First, we rank occupations only in the baseline period according to the average working hours for males, using personal-level weights, at the occupational level. Second, we measure the size of these occupations in the baseline period by summing up personal-level weights for both males and females at the occupational level. Third, we evenly divide the occupations in the baseline period into two groups, considering both the occupational rank and the occupational size; assign the bottom and top 50 percent of occupations as linear and nonlinear occupations, respectively. Finally, we apply this occupational grouping
over the whole sample period. An issue is that several new occupations are observed after the baseline period. To address this issue, we compute the average hours worked of these new occupations at the occupational level, when they are first observed; categorize these occupations according to the threshold hours worked for the occupational grouping in the baseline period; keep using this occupational category in the sequential periods. These procedures enable us to obtain hours worked and hourly wage by gender and time-invariant occupational group.

A.2 Transitional dynamics

The analysis in the main text is carried out by comparing steady states in ten-year intervals. We now consider the same analysis when the economy moves from the initial steady state to the final steady state along the perfect foresight transition path at the annual frequency. Then, the annual data along the transition are aggregated at ten-year intervals, as is done for the empirical counterparts and the model counterparts in the main text.

We first describe the definition of equilibrium along transitional paths. The economy is initially in a steady state $t = \ldots, -2, -1$, as described in Section 3.3. In period $t = 0$ (or the year 1986), agents learn that the economy will evolve according to the driving forces described in Section 5.1. For the annual sequences of driving forces, we use the piecewise linear interpolation such that the annual values in the year of 1990 and those in the year of 2000 equal to the steady-state values in the 1986-1995 and 1996-2005, respectively. From the year of 2006, the sequences take the steady-state values of 2006-2015. Agents optimize under the perfect foresight on these sequences.

More formally, given an initial distribution $F^* (\cdot)$ and a sequence of $\{A_t, \{\lambda_{j,t}, \chi_{j,t}\}_{j=1}^2, P_t (\cdot), \nu_t\}_{t=0}^\infty$, a recursive competitive equilibrium is a sequence of factor prices $\{r_t, w_{1_f,t}, w_{2_f,t}, w_{1_m,t}, w_{2_m,t}\}_{t=0}^\infty$; a sequence of female decision rules $\{g_{n,t}(a, z, \hat{j}, \eta, \hat{j}_m, \xi), g_{o,t}(a, z, \hat{j}, \eta, \hat{j}_m)\}_{j=1}^2$; a sequence of value functions $\{V_t(a, z, \hat{j}, \eta, \hat{j}_m)\}_{j=1}^2$; the aggregate capital $\{K_t\}_{t=0}^\infty$, the aggregate labor $\{L_t\}_{t=0}^\infty$, and the aggregate labor by gender and occupation $\{L_{1_f,t}, L_{2_f,t}, L_{1_m,t}, L_{2_m,t}\}_{t=0}^\infty$; the distribution of households $\{F_t (\cdot)\}_{t=0}^\infty$ such that, for all $t$

1. Given factor prices $(r_t, w_{1_f,t}, w_{2_f,t}, w_{1_m,t}, w_{2_m,t})$, the value functions $V_t(a, z, \hat{j}, \eta, \hat{j}_m)$,
\[ N_t(a, z, \eta, j_m), W_t(a, z, x, \hat{\eta}, j_m), \{ P_{j,t}(a, z, \eta, j_m) \}_{j=1}^2 \] solve the associated problems, the associated decision rules are

\[
g_{n,t}(a, z, x, \hat{j}, \eta, j_m, \xi) = \arg \max \{ N_t(a, z, \eta, j_m), W_t(a, z, x, \hat{j}, \eta, j_m) - \xi I_{j_m \neq 0} \} \tag{25}
\]

\[
g_{o,t}(a, z, x, \hat{j}, \eta, j_m) = \arg \max \left\{ J_{1,t}(a, z, x, \hat{j}, \eta, j_m), J_{2,t}(a, z, x, \hat{j}, \eta, j_m) \right\} \tag{26}
\]

\[
a_{t+1}^* = g_{a,j,t}(a, z, x, \eta, j_m), : j \in \{0, 1, 2\} \tag{27}
\]

\[
h_t^* = g_{h,j,t}(a, z, x, \eta, j_m), : j \in \{1, 2\} \tag{28}
\]

2. Given factor prices \( r_t, w_{1,f,t}, w_{2,f,t}, w_{1,m,t}, w_{2,m,t} \), the representative firm optimally chooses \( K, L_{1,f,t}, L_{2,f,t}, L_{1,m,t}, \) and \( L_{2,m,t} \) following (10)-(14).

3. Markets clear:

\[
K_t = \int a F_t(d(a, z, x, \hat{j}, \eta, j_m)) \tag{29}
\]

\[
L_{j,f,t} = \int \left[ \int_{\mathcal{I}_{(g_n,s(\xi)=W_t)}(d(\xi))} P_t(d(\xi)) \cdot z_j \cdot \left( I_{(j=g_o,(s)=j)} \cdot \left( \pi \cdot (1 + \chi_{j,t}) \cdot g_{h,j,t}(a, z, x', 1, \eta, j_m) \right) \right) + (1 - \pi) \cdot (1 + \chi_{j,t} \cdot I_{x=1}) \cdot g_{h,j,t}(a, z, x, \eta, j_m) \right] F_t(d(a, z, x, \hat{j}, \eta, j_m)), : j \in \{1, 2\} \tag{30}
\]

\[
L_{j,m,t} = \int e_{j,m} h_{j,m} F_t(d(a, z, x, \hat{j}, \eta, j_m)), : j \in \{1, 2\}
\]

where \( s = (a, z, x, \hat{j}, \eta', j_m') \in \mathcal{S} \) and \( P_t(\cdot) \) is the cumulative distribution function of \( \xi \).

4. The household distribution \( F_{t+1}(\cdot) \) is consistent with the household optimal choices defined.
above. Specifically, for any $B \in \mathcal{B}(S)$,

$$F_{t+1}(B) = q \cdot \int_{\{(a,z,x,j',j_m')|((g_{a,j}=0,0,(s),z',x'=0,t,0,(s)\neq 0,\eta',j_m')\in B\}} \left[ \pi_{z'|z} \cdot \int_{\xi} \mathcal{I}_{(g_{n,t},(s)\neq N)} \cdot \mathcal{I}_{(j'=0,\eta',j'_{m'}) \in B} P_t(\xi) F_t(d(a, z, x, j), \eta', j_m') \right]$$

$$+ q \cdot \sum_{j=1}^{2} \left\{ \int_{\{(a,z,x,j,\eta',j_m')|((g_{a,j}=0,0,(s),z',x'=0,t,0,(s)\neq 0,\eta',j_m')\in B\}} \left[ \pi_{z'|z} \cdot \int_{\xi} \mathcal{I}_{(g_{n,t},(s)\neq N)} \cdot \mathcal{I}_{(x=0)} \cdot \mathcal{I}_{(x=1)} \right] F_t(d(a, z, x, j), \eta', j_m') \right\}$$

$$+ q \cdot \sum_{j=1}^{2} \left\{ \int_{\{(a,z,x,j,\eta',j_m')|((g_{a,j}=0,0,(s),z',x'=0,t,0,(s)\neq 0,\eta',j_m')\in B\}} \left[ \pi_{z'|z} \cdot \int_{\xi} \mathcal{I}_{(g_{n,t},(s)\neq N)} \cdot \mathcal{I}_{(x=0)} \cdot \mathcal{I}_{(x=1)} \right] F_t(d(a, z, x, j), \eta', j_m') \right\}$$

$$+ (1 - q) \cdot \int_{\{(a,z,x,j,\eta,j_m')|((g_{a,j}=0,0,(s),z',x'=0,t,0,(s)\neq 0,\eta',j_m')\in B\}} \pi_{z'|z} \cdot \pi_{\eta'|\eta} \cdot \pi_{j_m'|j_m} F_t(d(a, z, x, j), \eta, j_m')$$

where $\mathbf{s} = (a, z, x, j, \eta, j_m') \in S$, $\pi_{z'|z}$ is the transitional probability from $z$ to $z'$, and $P_t(\cdot)$ is the cumulative distribution function of $\xi$. $\pi_{z'|z}$, $\pi_{\eta'|\eta}$, and $\pi_{j_m'|j_m}$ determine the distribution of newly-born households for $z'$, $\eta'$, and $j_m'$, respectively.

Figures A1 and A2 report the decomposition results, which correspond to Figures 6 and 7, respectively. First, we note that the transitions in aggregate labor market quantities are roughly in line with their counterparts in the main text. On the other hand, the equilibrium interest rates rise quite substantially because capital stock is inverted hump-shaped during the initial transition periods. As wages are projected to grow strongly, intertemporal substitution implies that agents tend to delay the increase in labor supply (overall and at both margins) in the initial periods. This leads to relatively weaker labor supply increases (especially in 1986-1995) than those in the main text. We expect this gap to be smaller in the presence of realistic capital adjustment costs, which would weaken this intertemporal substitution effect in the model.

Despite the somewhat dampened increases in female labor supply, we find that our main decomposition results are very robust. Changes in experience premiums are still largely responsible for the increases in labor supply at all margins, as shown in Figure A1. Interestingly, its
Figure A1: Decomposition: total hours and two margins of labor supply

(i) Total hours

(ii) Extensive margin

(iii) Intensive margin
role for the extensive margin is somewhat stronger than the main text. The occupation-biased technical change has significant impacts on the occupational composition, as shown in Figure A2. Importantly, it has contributed to the intensive margin positively yet to the extensive margin negatively, as in the main text.

A.3 Additional Tables and Figures

Table A1 reports the estimates of part-time penalties. Part-time is defined as hours less than 25 weekly hours. We control for age, education, race, industry, and the number of children
under age 5. The estimates show that in general nonlinear occupations have higher part-time penalties. Another observation is that there is no clear trend.

Table A2 reports the share of college-educated workers in each occupation, by gender and time. It is clear to see that nonlinear occupations tend to have more college educated workers in each period. However, over time, both occupations have more college-educated workers especially among women. In other words, linear occupations are also increasingly filled with more college educated workers, suggesting that rising skilled workers are not particularly biased towards nonlinear occupations.

Table A3 shows how the dispersion of wages and hours worked has changed in our model, compared to their data counterparts. The numbers reported are percentage changes relative to their values in the baseline period, 1976-1985. It shows that the model correctly predicts that the volatility of wages has increased substantially whereas that of hours worked has decreased over time. Quantitatively, the model explains roughly half of the changes in these second moments.

In Table A4, we show the parameter values that are internally calibrated over time along with their target statistics. Table A5 reports the evolution of male occupational composition and labor supply over time.

Table A6 reports the results when $\psi$ is set to $-0.25$ instead of $-0.5$. Then, the elasticity of substitution between nonlinear and linear occupations increases to 0.8. The decomposition results in Section 5 barely change. Thus, we only report the exercise in Section 6 where differences in the results are more visible. Although most results are quite robust, we find that the negative effects of vanishing nonlinearities on total hours are somewhat weakened. Nevertheless, quantitatively speaking, the differences are very modest.

Figure A3 shows that the model replicates the trend in the observed gender wage gap very well. Note that the trend of this aggregate gender wage gap is not directly targeted because gender wage gaps within occupations are only targeted.
Table A1: Part-time penalties over time

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Female</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>-.1989</td>
<td>-.2200</td>
<td>-.1657</td>
<td>-.1707</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>-.0652</td>
<td>-.0929</td>
<td>-.0579</td>
<td>-.0792</td>
</tr>
<tr>
<td><strong>Male</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>-.1614</td>
<td>-.2582</td>
<td>-.1853</td>
<td>-.2335</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>-.1289</td>
<td>-.2177</td>
<td>-.1771</td>
<td>-.1455</td>
</tr>
</tbody>
</table>
```

*Note:* Part-time is defined as hours less than 25 weekly hours. Age, education, race, industry, and the number of children under age 5 are controlled for. The above estimates are highly statistically significant.

Table A2: Share of college educated workers, by occupations over time

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Female</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>.3658</td>
<td>.3986</td>
<td>.4446</td>
<td>.5499</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>.0943</td>
<td>.1485</td>
<td>.1985</td>
<td>.2779</td>
</tr>
<tr>
<td><strong>Male</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear occ.</td>
<td>.3623</td>
<td>.4356</td>
<td>.4732</td>
<td>.5243</td>
</tr>
<tr>
<td>Linear occ.</td>
<td>.0646</td>
<td>.0968</td>
<td>.1215</td>
<td>.1646</td>
</tr>
</tbody>
</table>
```

Table A3: Percentage changes in second moments of wages and hours: model vs. data

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>sd(log(wage))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>Data</td>
<td>15.5%</td>
<td>23.1%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>8.0%</td>
<td>11.2%</td>
</tr>
<tr>
<td>NL</td>
<td>Data</td>
<td>10.9%</td>
<td>16.1%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>5.5%</td>
<td>7.8%</td>
</tr>
<tr>
<td>L</td>
<td>Data</td>
<td>14.8%</td>
<td>21.1%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>6.4%</td>
<td>7.1%</td>
</tr>
<tr>
<td><strong>sd(log(h))</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>Data</td>
<td>-8.2%</td>
<td>-14.5%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-2.7%</td>
<td>-6.2%</td>
</tr>
<tr>
<td>NL</td>
<td>Data</td>
<td>-12.3%</td>
<td>-17.9%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-7.1%</td>
<td>-8.4%</td>
</tr>
<tr>
<td>L</td>
<td>Data</td>
<td>-6.1%</td>
<td>-12.3%</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-0.5%</td>
<td>-7.3%</td>
</tr>
</tbody>
</table>
```

*Note:* Reported numbers are percentage changes relative to their values in the baseline period, 1976-1985.
Table A4: Parameters calibrated internally over time

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>Target statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1995 -2005 -2015</td>
<td>Data Model Data Model Data Model</td>
</tr>
<tr>
<td>$A$ 1.010 1.051 1.051</td>
<td>% change in overall wage 0.3 0.2 6.3 6.2 11.7 11.8</td>
</tr>
<tr>
<td>$\lambda_1$ .423 .461 .455</td>
<td>Gender wage gap in NL .332 .331 .289 .287 .246 .247</td>
</tr>
<tr>
<td>$\lambda_2$ .439 .465 .477</td>
<td>Gender wage gap in L .286 .285 .223 .223 .183 .183</td>
</tr>
<tr>
<td>$s$ .944 .932 .988</td>
<td>% change in total hours 29.2 29.3 45.0 45.0 47.1 47.0</td>
</tr>
<tr>
<td>$\sigma_n$ .378 .347 .388</td>
<td>Share of all workers in NL .526 .524 .565 .561 .582 .580</td>
</tr>
<tr>
<td>$\nu$ .715 .756 .785</td>
<td>$\mathbb{E}(\text{wage}<em>{\text{NL}})/\mathbb{E}(\text{wage}</em>{\text{L}}) - 1$ .269 .268 .322 .319 .380 .383</td>
</tr>
</tbody>
</table>

Table A5: Changes related to husbands over time

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{jm}$ 1 .537 .540 .555 .554</td>
</tr>
<tr>
<td>2 .364 .349 .329 .307</td>
</tr>
<tr>
<td>$h_{jm}$ 1 .416 .427 .429 .418</td>
</tr>
<tr>
<td>2 .370 .379 .386 .377</td>
</tr>
</tbody>
</table>

Figure A3: Trends in the observed gender wage gap: Model vs. data
Table A6: Nonlinearity experiment in Section 6 with an alternative value of ES between NL and L occupations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Emp rate</td>
<td>Baseline</td>
<td>.516</td>
<td>.636</td>
<td>.699</td>
<td>.701</td>
</tr>
<tr>
<td>$U_j \rightarrow 0$</td>
<td></td>
<td>+7.0</td>
<td>+7.3</td>
<td>+9.1</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{F} \rightarrow 0$</td>
<td></td>
<td>+4.2</td>
<td>+1.9</td>
<td>+2.3</td>
<td></td>
</tr>
<tr>
<td>NL emp rate</td>
<td>Baseline</td>
<td>.174</td>
<td>.259</td>
<td>.337</td>
<td>.356</td>
</tr>
<tr>
<td>$U_j \rightarrow 0$</td>
<td></td>
<td>+3.0</td>
<td>+6.1</td>
<td>+7.2</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{F} \rightarrow 0$</td>
<td></td>
<td>+1.2</td>
<td>+1.3</td>
<td>+0.7</td>
<td></td>
</tr>
<tr>
<td>L emp rate</td>
<td>Baseline</td>
<td>.342</td>
<td>.377</td>
<td>.363</td>
<td>.345</td>
</tr>
<tr>
<td>$U_j \rightarrow 0$</td>
<td></td>
<td>+4.1</td>
<td>+1.1</td>
<td>+2.0</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{F} \rightarrow 0$</td>
<td></td>
<td>+3.0</td>
<td>+0.6</td>
<td>+1.6</td>
<td></td>
</tr>
<tr>
<td>Hours per worker</td>
<td>Baseline</td>
<td>.314</td>
<td>.329</td>
<td>.335</td>
<td>.341</td>
</tr>
<tr>
<td>$U_j \rightarrow 0$</td>
<td></td>
<td>-7.5</td>
<td>-9.3</td>
<td>-11.4</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{F} \rightarrow 0$</td>
<td></td>
<td>-5.4</td>
<td>-4.0</td>
<td>-3.8</td>
<td></td>
</tr>
<tr>
<td>Total hours</td>
<td>Baseline</td>
<td>100.0</td>
<td>129.2</td>
<td>145.0</td>
<td>147.0</td>
</tr>
<tr>
<td>$U_j \rightarrow 0$</td>
<td></td>
<td>+2.7</td>
<td>-1.6</td>
<td>-1.2</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{F} \rightarrow 0$</td>
<td></td>
<td>+0.9</td>
<td>-3.1</td>
<td>-2.0</td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>Baseline</td>
<td>.417</td>
<td>.348</td>
<td>.298</td>
<td>.261</td>
</tr>
<tr>
<td>gender wage</td>
<td>$U_j \rightarrow 0$</td>
<td>+2.3</td>
<td>+2.2</td>
<td>+3.2</td>
<td></td>
</tr>
<tr>
<td>gap</td>
<td>$\mathcal{F} \rightarrow 0$</td>
<td>+1.9</td>
<td>+1.0</td>
<td>+1.2</td>
<td></td>
</tr>
</tbody>
</table>

Note: The elasticity of substitution between nonlinear (NL) and linear (L) occupations is set to be higher at 0.8, and the model is re-calibrated accordingly. Reported numbers are percentage point differences relative to the baseline trends (Emp rate, NL emp rate and L emp rate) and percentage differences relative to the baseline trends (Hours per worker and Total hours). Total hours are scaled to be 100 in the baseline year (1976-1985). Both $U_j$ and $\mathcal{F}$ are set to converge linearly to zero in 2006-2015.
References


