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Network Goods, Price Discrimination, and Two-sided Platforms

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Network goods, price discrimination, and two-sided platforms*

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Abstract

A monopolist sells a network good to a set of heterogeneous users who all care about total participation. We show that the provider of the network good effectively becomes a two-sided platform if it can condition prices on some user characteristics. This still holds true if the network operator cannot observe consumer characteristics but induces user self-selection when it offers screening contracts. In our setting, all incentive constraints are slack. The use of freemium strategies emerges as a special case of versioning. Here, a base version is offered at zero price and a premium version at a positive price. Overall, the paper illustrates the close link between price discrimination in the presence of a network good and pricing by a two-sided platform.

Keywords: Network goods, two-sided markets, platform pricing, group pricing, menu pricing

JEL-Classification: D62, L12, L82, L86

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1 Introduction

Two-sided platforms are understood to cater to two audiences. Decisions among these audiences are interdependent because of cross-group network effects. One of the principle achievements of the literature on two-sided platforms has been to characterize the price structure and associated price distortions in various market environments. As we illustrate in this paper, these insights are helpful to understand price discrimination strategies of providers of network goods.

There is not a single definition of what constitutes a multi-sided platforms. Using a non-technical and broad definition, Evans and Schmalensee (2016, p. 210) define a multi-sided platform as "a business that operates a physical or virtual place (a platform) to help two or more different groups find each other and interact. The different groups are called "sides" of the platform."[1] A for-profit multi-sided platform uses price and non-price strategies to make profits and, at the same time, to manage network effects. A feature of most of this literature is that users on one side care about participation on the other side but not about participation on the own side.[2] A large part of the literature on multi-sided platforms explains the price structure—that is, the prices charged to the different sides of the platform(s)—in various market environment, as initiated by Rochet and Tirole (2003, 2006) and Armstrong (2006).

In line with this literature, we focus on pricing decisions and, in particular, the price structure. We analyze the monopoly provision of a network good; all users care about the overall level of participation. On first sight this looks different from a two-sided platform. However, as soon as we allow the platform to condition price on consumer characteristics, the setting becomes one of a multi-sided platform choosing the profit-maximizing price structure (very similar to the model proposed by Armstrong, 2006). Thus, our first message is that *if the a provider of a network good can target consumer groups with different prices—i.e., it is able to employ group pricing (or, equivalently, third-degree price discrimination)—its pricing decision corresponds to the pricing decision of a multi-sided platform.*

While our first message relies on the platform being able to observe some consumer characteristics, our second message is that *this insight generalizes when the information about consumer characteristics is not available to the provider of the network good, but when it can offer different versions.* In a sense, the platform then uses a combination of price and non-price strategies. Here, in line with the literature on versioning (or, equivalently, second-degree price discrimination) the platform has to respect the incentive constraints of the consumers to ensure self-selection. We provide a simple setting in which these constraints are slack and, thus, versioning is very similar to group pricing; in a particular specification we present, the two are actually equivalent.

We consider special cases in which the profit-maximizing strategy features a base version of the network good being offered at a price of zero (and used by some) and a premium version

[1] Other authors have provided similar definitions; see e.g. Rysman (2009) and, being a case of the pot calling the kettle black, Belleflamme and Peitz (2010, Chapter 22). Rochet and Tirole (2006) offer a more-narrow definition requiring the price structure of a two-sided platform to be non-neutral.

[2] Exceptions are works in which there is competition or congestion on one side which adds within-group network effects (e.g., Nocke, Peitz, and Stahl, 2007; Hagiu, 2009; and Belleflamme and Peitz, 2019).

being offered at a strictly positive price (and purchased by others). We interpret this as a true freemium strategy (there are no revenues directly associated to the users of the free versions). This shows that the presence of network effects may make it optimal for the platform to offer a base version for free.

This paper provides a link between the literature on price discrimination (and multi-product pricing more generally) and the two-sided market literature. We do not claim to be the first to recognize this.[3] In particular, Rochet and Tirole (2006, p. 646) point to the link of two-sided platforms (they use the term two-sided markets) to multi-product pricing and network effects.

> "Conceptually, the theory of two-sided markets is related to the theories of network externalities and of (market or regulated) multi-product pricing. From the former (...) it borrows the notion that there are noninternalized externalities among end-users. From the latter, it borrows the focus on price structure and the idea that price structures are less likely to be distorted by market power than price levels. The multi-product pricing literature, however, does not allow for externalities in the consumption of different products (...) The starting point for the theory of two-sided markets, by contrast, is that an end-user does not internalize the welfare impact of his use of the platform on other end-users."

Based on our analysis, a firm that offers a network good (or different versions thereof) and engages in price discrimination should also be considered a two- or multi-sided platform in the narrow sense of Rochet and Tirole. We show that pricing of a network good under group pricing or versioning is an instance of pricing by a two-sided platform. This proves that the use of "sophisticated" pricing instruments makes the provider of a network good choose a price structure akin to a two-sided platform.

Related to our finding that the freemium strategy may be profit-maximizing, in the literature on end-user piracy, Takeyama (1994) has analyzed a model of vertical differentiation à la Mussa and Rosen (1978) with network effects and shown that a firm may be better off when a base product is available at a price of zero (the pirated product) than when this version is not available. King and Lampe (2003) provide a related analysis; for a survey, see Belleflamme and Peitz (2012). These results speak to the profitability of the freemium strategy; however, in a piracy setting, the presence of the free version is assumed rather than derived as part of

[3] In early work, Bruno Jullien already foresaw the link between the literature on the provision of a network good to the two-sided market literature: "... one feature of modern networks that has not received considerable attention is that network effects are often not isotropic: members may join for different reasons and value both the service and the participation of others in very different ways. In conjunction with that, and partly because of that, most suppliers of network goods practice some form of price-discrimination." (Jullien, 2001, p. 1) More recently, Jullien (2011) studied divide-and-conquer strategies in a Stackelberg duopoly. As one of his two applications, he considers the setting with a network good being sold to different groups of consumers. He writes that "there is only one side with standard network effects but the platform can price discriminate between several groups of consumers. This shows that it is not crucial to have effectively several sides to generate multi-sided market effects. The ability to price discriminate along with the presence of network effects suffices." (Jullien, 2011, p. 203)

the profit-maximizing strategy. Regarding versioning, Csorba and Hahn (2006) also consider a vertical differentiation model à la Mussa and Rosen (1978) with network effects. They show that versioning by introducing a damaged good may be profitable. Csorba (2008) considers the monopolist selling of a network good with a finite number of vertically differentiated user types. In his setting, all neighboring downward incentive compatibility constraints are binding and the profit-maximizing menu features social underprovision. These papers do not speak to the two key messages made in our paper. Veiga (2018) also considers the selling of different versions of a network good; however, in his setting the network benefit only accrues to the users of a specific version and not the total number of users across all versions; therefore, his finding that a monopolist never wants to use versioning stands in contrast to the finding in our paper and, thus, there is no link between his paper and the multi-sided platform literature.

The paper proceeds as follows. In Section 2, we present a general setting to study monopoly pricing of a network goods. The following two sections consider two special cases, both imposing some simplifying assumptions. In Section 3, users not only differ by the value of their outside option but also by the value of the stand-alone utility; in Section 4, users differ by the value of the outside option and the strength of the network effect they experience. In both cases, we consider group pricing and versioning. In Section 5, we consider group pricing in a more general setting in which different network goods are provided to the consumer groups and in which there may be only partial compatibility between those network goods; we also show how this generalized setting applies to versioning. Section 6 concludes.

2 The model

We consider a model in which a monopoly platform offers interaction possibilities to a set of users. In the first stage, under group pricing, the monopoly platform obtains some information on a consumer's type and sets an access price conditional on this information, or, under versioning, does not have information on a consumer's type and offers a menu of contracts that contains price-quality pairs. Here, we specify the general model with group pricing; in the special cases developed below, we extend the analysis to versioning. In the second stage, users simultaneously make their participation decision after observing all participation prices; a user's valuation of the product or service offered by the platform depends on the total number of users. We solve for subgame-perfect equilibrium.

Each user's valuation depends on two components, a stand-alone value and a utility that depends positively on the total number of users. Consumers possibly differ by their opportunity cost of joining the platform, the stand-alone utility, and the strength of the network effect. For simplicity we assume that a user of type (i, x) obtains the utility $u_i = r_i + \beta_i n - p_i$ where r_i is the stand-alone benefit, β_i is the strength of the network benefit, n the total number of users, and p_i the price a consumer pays conditional on its realization i when accepting the platform service (or a particular version thereof). The outside option of a consumer of type (i, x) has value x. Thus, the consumer of type (i, x) prefers a proposed service over the outside option if and only if $r_i + \beta_i n - p_i \geq x$. In what follows, for simplicity, we provide a simple example in which there are two realizations of i which are present with equal shares in the population and a continuum of realizations x which are uniformly distributed on some compact interval; i and x are independently distributed.

In general, for each group with characteristic i there is a conditional demand $n_i(\mathbf{p})$ which depends on the price vector $\mathbf{p}$. Suppose that i is distributed according to F over some space I and x distributed according to G on the positive reals. The demand system $n_i(\mathbf{p})$ is the solution to $n_i = G(r_i + \beta_i n - p_i)$ and $n = \int n_i dF(i)$. The monopoly platform then solves the maximization problem $\max_{\mathbf{p}} \int [(p_i - c)n_i(\mathbf{p})]dF(i)$. This resembles the maximization problem of a multi-product monopolist. Because of network effects, the demands for the different offers are interdependent; the monopoly price p_i^* and/or the monopoly quantity n_i^* may depend on parameters that determine the valuation of consumers of type (j, x) with $j \neq i$.

In the remainder of the paper, for simplicity, we assume that G is uniformly distributed and that I contains two types A and B that are equally likely. In the following two sections, we analyze two special cases that shed light on the pricing structure chosen by the monopoly firm. In both cases, we assume that $r_A \geq r_B$ and $\beta_A \geq \beta_B$, with one of the inequalities strict. In Section 3, we consider the special case $\beta_A = \beta_B = \beta$, while in Section 4, we consider the special case $r_A = r_B = r$.

We already note that the model with group pricing (third-degree price discrimination) is a special case of pricing by a two-sided platform that features within-group and cross-group network effects. In the linear version of that model, the utility can be written as $r_i + \beta_i n_j + \gamma_i n_i - p_i$ with $i \in \{A, B\}$ and $j \in \{A, B\}, j \neq i$ where γ_i captures the strength of the within-group network effect. Clearly, for $\beta_i = \gamma_i$ the model becomes a special case of the model introduced above and combines heterogeneous stand-alone benefits and heterogeneous network effects. We analyze the more general case allowing for $\beta_i \neq \gamma_i$ in Section 5.

3 Price discrimination with heterogeneous stand-alone benefits

In this section we look at the setting in which i determines the stand-alone utility of the user. Suppose that users of type $i \in \{A, B\}$ obtain the following utilities on the platform:

$$\begin{cases} u_A = r_A + \beta\,(n_A + n_B) - p_A, \\ u_B = r_B + \beta\,(n_A + n_B) - p_B, \end{cases} \tag{1}$$

where n_i is the number of users of type i on the platform. We assume that $\beta < 1/2$ and that $r_A > r_B$, meaning that users with characteristic A value more the stand-alone benefits than users with characteristic B. As for the network benefits, all users value them in the same way and care about total participation (i.e., total adoption of the network good, $n_A + n_B$).

Users have heterogeneous opportunity costs of joining the platform; this opportunity cost is uniformly distributed on a sufficiently large interval; an increase of net utility of $\Delta u_K = 1$ leads to an increase in the number of users of mass 1. Characteristics i and x are independently distributed. As a result, the number of users as a function of prices p_A and p_B are implicitly given by $r_A+\beta(n_A+n_B)-p_A=n_A$ and $r_B+\beta(n_A+n_B)-p_B=n_B$. We can rewrite this by expressing the number of participating users in one group as a function of the price charged to that group and the number of participating users in the other group: $n_A=(r_A+\beta n_B-p_A)/(1-\beta)$ and $n_B=(r_B+\beta n_A-p_B)/(1-\beta)$; we observe thus that network effects make participations in the two groups complementary with one another. For given prices, users play an anonymous game and we solve for the Nash equilibrium of this game; the expected number of participants on each side has to be equal to the actual number. Solving this equation system, we obtain user participation as a function of prices p_A and p_B,

$$\begin{cases} n_A(p_A,p_B)=\dfrac{(1-\beta)(r_A-p_A)+\beta(r_B-p_B)}{1-2\beta}, \\ n_B(p_A,p_B)=\dfrac{(1-\beta)(r_B-p_B)+\beta(r_A-p_A)}{1-2\beta}. \end{cases}$$

As participations are complementary, both n_A and n_B are decreasing functions of the two prices.

Before considering the monopolist's choices under group pricing and versioning, let us look at the benchmark case of *uniform pricing*. If the network good is sold at the same price p to all users (irrespective of the group they belong to), the monopolist chooses this price p to maximize

$$\Pi=p\left[n_A(p,p)+n_B(p,p)\right]=p\frac{r_A+r_B-2p}{1-2\beta}.$$

The profit-maximizing price is easily found as $p^u=(r_A+r_B)/4$, and the corresponding participation levels are

$$n_A^u=\frac{(3-4\beta)r_A-(1-4\beta)r_B}{4(1-2\beta)} \text{ and } n_B^u=\frac{(3-4\beta)r_B-(1-4\beta)r_A}{4(1-2\beta)}.$$

In what follows, we assume that $(3-4\beta)r_B>(1-4\beta)r_A$ to make sure that the monopolist chooses to serve both groups of users under uniform pricing. The monopoly profit under uniform pricing is then computed as $\Pi^u=(r_A+r_B)^2/(8(1-2\beta))$.

3.1 Group pricing with heterogeneous stand-alone utilities

Assume now that the monopolist has enough information to tell users apart according to the group they belong to. It thus sells the network good at price p_A to users of group A and at price p_B to users of group B. That is, the monopolist sets p_A and p_B to maximize $\Pi=p_An_A+p_Bn_B$. Solving the system of first-order conditions yields the profit-maximizing prices $p_A^*=r_A/2$ and $p_B^*=r_B/2$.[4] In this specific linear setting, while the firm accounts for the fact that the price

[4] A different specification would be to assume that all consumers have the same r and β, but that the firm has some information on the consumer's opportunity cost and practice group pricing with this information. In particular, let $u_i=r+\beta(n_A+n_B)-p_i$ and the firm has information whether a consumer belongs to group $i\in\{A,B\}$; group-A consumers have opportunity costs $[0,\Delta r)$ with $\Delta r>r/2$ and group-B consumers have opportunity costs $[\Delta r,\infty)$. Then there exists a $\hat{\beta}(\Delta r)$ such that in the profit-maximizing solution of this group pricing problem, not all group-A consumers participate (i.e. $n_A^*<\Delta r$) for all $\beta<\hat{\beta}(\Delta r)$. The solution then is

charged to one group affects the net utility of the good in the other group, the profit-maximizing prices do not respond. Yet, the participation levels do: evaluated at the profit-maximizing prices, we observe that the participation levels both increase in β:

$$n_A^* = \frac{(1-\beta)\, r_A + \beta r_B}{2\,(1-2\beta)} \text{ and } n_B^* = \frac{(1-\beta)\, r_B + \beta r_A}{2\,(1-2\beta)}.$$

Total profit at the profit maximum is equal to

$$\Pi^* = \frac{(1-\beta)\left(r_A^2 + r_B^2\right) + 2\beta r_A r_B}{4\,(1-2\beta)},$$

which cannot be lower than Π^u (as the platform has optimally chosen $p_A^* \neq p_B^*$).

Comparing prices and participation levels across groups and with the case of uniform pricing, we can establish the following results:

$$r_A > r_B \Rightarrow p_A^* > p^u > p_B^* \text{ and } n_A^u > n_A^* > n_B^* > n_B^u.$$

That is, when the monopolist practices group pricing, it charges a higher price to the group that values the stand-alone benefits the most (group A); despite the price difference, participation is larger in group A than in group B, but smaller than the participation that would prevail under uniform pricing. As for total participation, we find that it is exactly the same under uniform and group pricing: $n_A^* + n_B^* = n_A^u + n_B^u = (r_A + r_B) / (2\,(1-2\beta))$.

The monopolist internalizes cross-group network effects by lowering prices p_A and p_B relative to what would happen otherwise. To see this, suppose instead that the platform maximizes its profit for each group with characteristic i separately, taking as given the participation level of the other group. That is, the platform sees participation of group A as $n_A = r_A + \beta(n_A + n_B^e) - p_A$, where n_B^e is the (fixed) expectation it forms for participation in the other group. Put differently, there are two firms, one offering a network good to group-A consumers and the other a network good to group-B consumers with both network goods being fully compatible. The profit maximization problem for group A is then $\max_{p_A} p_A \frac{1}{1-\beta}(r_A + \beta n_B^e - p_A)$. The first-order condition yields $p_A = (r_A + \beta n_B^e)/2$. Similarly, on group B, the platform's maximization problem is $\max_{p_B} p_B \frac{1}{1-\beta}(r_B + \beta n_A^e - p_B)$, yielding $p_B = (r_B + \beta n_A^e)/2$. Since $n_B^e > 0$ and $n_A^e > 0$, we must have that both prices are higher than the prices the monopolist chooses under group pricing.

Plugging the profit-maximizing prices into the expression for user participation, we find $n_A = (r_A + \beta n_B^e) / (2\,(1-\beta))$ and $n_B = (r_B + \beta n_A^e) / (2\,(1-\beta))$. Imposing fulfilled expectations (i.e., $n_A^e = n_A$ and $n_B^e = n_B$) and solving, we can obtain explicit solution for prices and participations levels.[5] It is easy to see that profits are lower than in the monopoly problem in which all network effects are internalized. Using the interpretation of two firms offering network goods to

the same as in the model analyzed in this section with variables $r_A = r$, $r_B = r - \Delta r$. It is less satisfactory to apply this specification to versioning. However, versioning could still be addressed in a more general specification in which a consumer's strength of the network effect depends on her opportunity cost and the firm has some information on the consumers' opportunity cost.

[5] We obtain the solution for participation levels (denoted by superscript m):

one consumer group each, a merger between the two firms is profitable and consumer-surplus increasing because a monopolist internalizes the complementarity that arises from cross-group network effects.[6]

3.2 Versioning with heterogeneous stand-alone utilities

In contrast with the previous case, we assume here that the monopolist does not observe (or is not able to verify) the characteristic i. It therefore offers a menu of options with the aim to make users self-select among the options and, thereby, reveal their characteristic i. Suppose that the platform offers a premium and a basic version, and that users from the two groups value the two versions differently. In particular, group A and group B users have a value of r for the basic version, group B users are not willing to pay a higher price for the premium version, whereas group A users are willing to pay $r+\Delta$ (with $\Delta>0$). If the contract menu is such that group A users choose the premium version and group B users the basic version, we have

$$\begin{cases} u_A = r_A + \beta(n_A+n_B) - p_A, \\ u_B = r_B + \beta(n_A+n_B) - p_B, \end{cases}$$

with $r_A = r+\Delta > r_B = r$ and $\beta < 1/2$. These are the exact same utilities as in expression (1) above, but with a different interpretation. Hence, if the incentive constraint of group A users is not binding (i.e., if group A users are strictly better off when they buy the premium version instead of the basic version), then the analysis carried out under group pricing still holds. In particular, the profit-maximizing prices are $p_A^* = r_A/2 = (r+\Delta)/2$ and $p_B^* = r_B/2 = r/2$. As the price for the premium version is $\Delta/2$ above the price of the basic version, users in group A do indeed strictly prefer the premium version over the basic version (as the gain in stand-alone benefits, Δ, outweighs the price increase, $\Delta/2$). In conclusion, in these particular circumstances, the platform achieves the same profit under versioning than under group pricing (it does not need to forego any profit to induce users to reveal their private information about their willingness to pay). We have thus proven by example:

$$n_A^m = \frac{2(1-\beta)r_A + \beta r_B}{4-8\beta+3\beta^2} \text{ and } n_B^m = \frac{2(1-\beta)r_B + \beta r_A}{4-8\beta+3\beta^2}.$$

To find the platform's optimal prices, we still need to substitute these values into $p_A = (r_A + \beta n_B)/2$ and $p_B = (r_B + \beta n_A)/2$. Doing so, we find:

$$p_A^m = (1-\beta)\frac{2(1-\beta)r_A + \beta r_B}{4-8\beta+3\beta^2} \text{ and } p_B^m = (1-\beta)\frac{2(1-\beta)r_B + \beta r_A}{4-8\beta+3\beta^2}.$$

A group-A consumer's net utility becomes

$$u_A^m = r_A/2 + \beta\left(n_A^m + n_B^m/2\right) = \frac{2(1-\beta)r_A + \beta r_B}{4-8\beta+3\beta^2}$$

and profits on group-A consumers are

$$(1-\beta)\frac{[2(1-\beta)r_A + \beta r_B]^2}{(4-8\beta+3\beta^2)^2}.$$

[6] A similar argument underlies Cournot's model of complementary products.

Proposition 1 *When consumers of a network good benefit from stand-alone utilities of different versions of a network good to a varying degree, a monopolist can profitably introduce two versions of the same network good inducing consumer self-selection. The profit-maximizing strategy coincides with the strategy chosen by a two-sided platform that identifies the two groups of consumers and provides the respective stand-alone utilities to the two distinguishable groups.*

In the special case that $r = 0$, the basic version is sold at a price of zero and, thus, group B users obtain the good for free. The equilibrium numbers of users in the two groups are then computed as

$$n_A^* = \frac{1-\beta}{2\,(1-2\beta)}\Delta \text{ and } n_B^* = \frac{\beta}{2\,(1-2\beta)}\Delta.$$

As for the platform's profit, it is equal to

$$\Pi^* = n_A^* p_A^* = \frac{1-\beta}{4\,(1-2\beta)}\Delta^2.$$

Even if group B does not generate any direct revenue, it does so indirectly by raising, through its participation, the network benefits and, thereby, the willingness to pay of group A users. To see this, we compare the results we just derived with a hypothetical situation in which group B users would not exist. In that case, $n_A = r_A + \beta n_A - p_A$; solving for n_A and replacing u_A by Δ, we have $n_A = (\Delta - p_A)/\,(1-\beta)$. As the platform maximizes $n_A p_A$, we find the profit-maximizing price as $\tilde{p}_A = \Delta/2$. It follows that $\tilde{n}_A = \frac{1}{2(1-\beta)}\Delta$ and $\tilde{n}_A \tilde{p}_A = \frac{1}{4(1-\beta)}\Delta^2$. We check that

$$\Pi^* - \tilde{n}_A \tilde{p}_A = \frac{1-\beta}{4\,(1-2\beta)}\Delta^2 - \frac{1}{4(1-\beta)}\Delta^2 = \frac{\beta^2}{4\,(1-2\beta)\,(1-\beta)}\Delta^2 > 0,$$

meaning that the platform does indeed achieve a larger profit when offering a menu in which users can obtain the basic version for free. The reason for choosing such a *freemium strategy* is that giving away the basic version for free leads some group B users to join; this makes participation more attractive for group A users, and leads to higher platform profits.

4 Price discrimination with heterogeneous network benefits

We now repeat the analysis under the alternative assumption that users value the strength of the network effects differently across groups. In particular, group A and group B users obtain the following net utilities on the platform:

$$\begin{cases} u_A = r + \beta_A(n_A + n_B) - p_A, \\ u_B = r + \beta_B(n_A + n_B) - p_B, \end{cases} \tag{2}$$

with $\beta_B < \beta_A < 1/2$. That is, users no longer differ in terms of stand-alone benefits (they share the same valuation r) but in terms of network benefits: for a given network size $(n_A + n_B)$, group A users have a larger utility than group B users $(\beta_A > \beta_B)$. As before, users have heterogenous outside options. As a result, participation levels as a function of prices p_A and p_B are implicitly given by $r+\beta_A(n_A+n_B)-p_A=n_A$ and $r+\beta_B(n_A+n_B)-p_B=n_B$. Solving this equation system, we obtain user participation as a function of the two prices:

$$\begin{cases} n_A\left(p_A,p_B\right)=\dfrac{\left(1-\beta_B\right)\left(r-p_A\right)+\beta_A\left(r-p_B\right)}{1-\beta_A-\beta_B}, \\ n_B\left(p_A,p_B\right)=\dfrac{\beta_B\left(r-p_A\right)+\left(1-\beta_A\right)\left(r-p_B\right)}{1-\beta_A-\beta_B}. \end{cases}$$

If the monopolist were constrained to set a *uniform price* p, it would choose this price to maximize

$$\Pi=p\left[n_A\left(p,p\right)+n_B\left(p,p\right)\right]=p\frac{2\left(r-p\right)}{1-\beta_A-\beta_B}.$$

It is clear that the profit-maximizing price is $p^u=r/2$, with resulting participation levels and profits

$$n_A^u=\frac{1+\beta_A-\beta_B}{1-\beta_A-\beta_B}\frac{r}{2},\ n_B^u=\frac{1-\beta_A+\beta_B}{1-\beta_A-\beta_B}\frac{r}{2},\text{ and }\Pi^u=\frac{1}{1-\beta_A-\beta_B}\frac{r^2}{2}.$$

We note that $\beta_A>\beta_B$ implies that $n_A^u>n_B^u$.

4.1 Group pricing with heterogeneous network benefits

If the platform can tell users from the two groups apart, it chooses p_A and p_B to maximize $\Pi=p_An_A\left(p_A,p_B\right)+p_Bn_B\left(p_A,p_B\right)$. Solving the system of first-order conditions yields the following profit-maximizing prices:

$$p_A^*=\frac{(2-\beta_A-3\beta_B)-(\beta_A-\beta_B)^2}{4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2}r\text{ and }p_B^*=\frac{(2-3\beta_A-\beta_B)-(\beta_A-\beta_B)^2}{4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2}r.$$

The corresponding participation levels are computed as

$$n_A^*=\frac{2+(\beta_A-\beta_B)}{4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2}r\text{ and }n_B^*=\frac{2-(\beta_A-\beta_B)}{4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2}r.$$

As for the platform's profit, it is equal to

$$\Pi^*=\frac{2}{4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2}r^2.$$

Comparing prices and participation levels, we find similar results as in the previous case:

$$\beta_A>\beta_B\Rightarrow p_A^*>p^u>p_B^*\text{ and }n_A^u>n_A^*>n_B^*>n_B^u.$$

That is, the group that enjoys the largest network benefits is charged a higher price and yet, participates more than the other group; participation levels are less different across groups under group pricing than under uniform pricing. In contrast with the previous case, total participation is now larger under group pricing than under uniform pricing:

$$(n_A^*+n_B^*)-(n_A^u+n_B^u)=\frac{\left(\beta_A-\beta_B\right)^2}{\left(1-\beta_A-\beta_B\right)\left(4(1-\beta_A-\beta_B)-(\beta_A-\beta_B)^2\right)}r>0.$$

We note that in this setting, the platform adjusts its prices by internalizing the strength of the network effects in the two groups.[7] To see how this adjustment process affects prices, we solve for the profit-maximizing prices of a platform that would ignore that a price change for one group affects the overall network benefit for the other group. Suppose that the platform maximizes its profit for each group with characteristic i separately, taking as given the participation level of the other group. That is, the platform sees participation on group A as $n_A = r + \beta_A(n_A + n_B^e) - p_A$, where n_B^e is the (fixed) expectation it forms for participation in the other group. The profit maximization problem for group A is then $\max_{p_A} p_A \frac{1}{1-\beta_A}(r + \beta_A n_B^e - p_A)$. The first-order condition yields $p_A = (r + \beta_A n_B^e)/2$. Similarly, on group B, the platform's maximization problem is $\max_{p_B} p_B \frac{1}{1-\beta_B}(r + \beta_B n_A^e - p_B)$, yielding $p_B = (r + \beta_B n_A^e)/2$. Plugging the profit-maximizing prices into the participation levels, we find $n_A = (r + \beta_A n_B^e) / (2(1-\beta_A))$ and $n_B = (r + \beta_B n_A^e) / (2(1-\beta_B))$. Imposing fulfilled expectations (i.e., $n_A^e = n_A$ and $n_B^e = n_B$) and solving, we compute (with the superscript m standing for 'myopic'):

$$n_A^m = \frac{2 + \beta_A - 2\beta_B}{4(1 - \beta_A - \beta_B) + 3\beta_A\beta_B} r \text{ and } n_B^m = \frac{2 - 2\beta_A + \beta_B}{4(1 - \beta_A - \beta_B) + 3\beta_A\beta_B} r.$$

To find the platform's profit-maximizing prices, we still need to substitute these values into $p_A = (r + \beta_A n_B)/2$ and $p_B = (r + \beta_B n_A)/2$. Doing so, we find:

$$p_A^m = \frac{(1-\beta_A)(2 + \beta_A - 2\beta_B)}{4(1 - \beta_A - \beta_B) + 3\beta_A\beta_B} r \text{ and } p_B^m = \frac{(1-\beta_B)(2 - 2\beta_A + \beta_B)}{4(1 - \beta_A - \beta_B) + 3\beta_A\beta_B} r.$$

To get sense of how prices and participation levels compare according to whether the platform takes cross-group network effects into account or not, we focus on the two extreme cases in which $\beta_B = 0$ and $\beta_B = \beta_A$. First, for $\beta_B = 0$, we observe that when the platform internalizes the cross-group network effects, it sets a larger price for group A and a lower price for group B ($p_A^* > p_A^m$ and $p_B^* < p_B^m$), while raising participation in both groups ($n_A^* > n_A^m$ and $n_B^* > n_B^m$):

$$\begin{aligned}
p_A^* - p_A^m|_{\beta_B=0} &= \tfrac{(1-\beta_A)(2+\beta_A)}{4-4\beta_A-\beta_A^2} r - \tfrac{2+\beta_A}{4} r = \tfrac{(2+\beta_A)\beta_A^2}{4\left(4-4\beta_A-\beta_A^2\right)} r > 0, \\
p_B^* - p_B^m|_{\beta_B=0} &= \tfrac{2-3\beta_A-\beta_A^2}{4-4\beta_A-\beta_A^2} r - \tfrac{1}{2} r = -\tfrac{(2+\beta_A)\beta_A}{2\left(4-4\beta_A-\beta_A^2\right)} r < 0, \\
n_A^* - n_A^m|_{\beta_B=0} &= \tfrac{2+\beta_A}{4-4\beta_A-\beta_A^2} r - \tfrac{2+\beta_A}{4(1-\beta_A)} r = \tfrac{(2+\beta_A)\beta_A^2}{4(1-\beta_A)\left(4-4\beta_A-\beta_A^2\right)} r > 0, \\
n_B^* - n_B^m|_{\beta_B=0} &= \tfrac{2-\beta_A}{4-4\beta_A-\beta_A^2} r - \tfrac{1}{2} r = \tfrac{(2+\beta_A)\beta_A}{2\left(4-4\beta_A-\beta_A^2\right)} r > 0.
\end{aligned}$$

At the other extreme, when $\beta_B = \beta_A$, we find:

$$\begin{aligned}
p_A^* - p_A^m|_{\beta_B=\beta_A} &= p_B^* - p_B^m|_{\beta_B=\beta_A} = \tfrac{1}{2} r - \tfrac{1-\beta_A}{2-3\beta_A} r = -\tfrac{\beta_A}{2(2-3\beta_A)} r < 0, \\
n_A^* - n_A^m|_{\beta_B=\beta_A} &= n_B^* - n_B^m|_{\beta_B=\beta_A} = \tfrac{1}{2(1-2\beta_A)} r - \tfrac{r}{2-3\beta_A} = \tfrac{\beta_A}{2(1-2\beta_A)(2-3\beta_A)} r > 0.
\end{aligned}$$

Here, the 'internalizing' platform sets a lower price for both groups than the 'myopic' platform; as with $\beta_B = 0$, profit-maximizing participation levels are larger when cross-group network effects are internalized.

[7] Recall that in the previous setting with different stand-alone benefits, the profit-maximizing group prices were independent of β.

4.2 Versioning with heterogeneous network benefits

Like we did in the previous case, we construct a versioning scenario that allows the platform to reach the same profit as under group pricing. Suppose that the basic version is valued equally by users in both group with the strength of the network effect β. As for the premium version, it does not provide group B users with any stronger network effect but provides group A users with a stronger network effect $\beta+\delta$. If the contract menu is such that group A users choose the premium version and group B users the basic version, we have $u_A = r + (\beta+\delta)(n_A+n_B) - p_A$ and $u_B = r + \beta(n_A+n_B) - p_B$, with $\beta+\delta < 1/2$. Setting $\beta_A = \beta+\delta$ and $\beta_B = \beta$, we have the exact same formulation of utilities as in expression (2). We can then use the results we obtained under group pricing, namely:

$$p_A^* = \tfrac{2-4\beta-\delta-\delta^2}{4-8\beta-4\delta-\delta^2}r,\ p_B^* = \tfrac{2-4\beta-3\delta-\delta^2}{4-8\beta-4\delta-\delta^2}r,\ n_A^* = \tfrac{2+\delta}{4-8\beta-4\delta-\delta^2}r,\ \text{and } n_B^* = \tfrac{2-\delta}{4-8\beta-4\delta-\delta^2}r.$$

We check that for $\delta > 0$, $p_A^* > p_B^*$ and $n_A^* > n_B^*$. Users self-select as postulated if $r + (\beta+\delta)(n_A^*+n_B^*) - p_A^* > r + \beta(n_A^*+n_B^*) - p_B^*$, which is equivalent to $\delta(n_A^*+n_B^*) > p_A^* - p_B^*$. Inserting the equilibrium expressions for participation levels and prices, this inequality becomes $4\delta > 2\delta$, which is clearly satisfied. This shows that group A consumers self-select into the premium version sold at p_A^* and group B consumer self-select into the basic version sold at p_B^*. In this specific setting in which group B consumers do not achieve any extra benefit from the premium version, the platform does not suffer from not being able to observe the group characteristics; that is, versioning does as well as group pricing.[8] We have thus proven by example:

Proposition 2 *When consumers of a network good benefit from network size to a varying degree according to the version of the network good they consume, a monopolist can profitably introduce two versions of the same network good inducing consumer self-selection. The profit-maximizing strategy coincides with the strategy chosen by a two-sided platform that identifies the two groups of consumers.*

5 Group pricing with two network goods

So far we considered the monopoly market for a single network good, implying that consumers care about the total number of participants. We now consider two (incompatible) network goods, one provided to each consumer group. If users only care about participation levels of the other group only, we are in the standard setting of a two-sided platform. Here, all group-A users with an outside option of less than $u_A = r_A + \beta_A n_B - p_A$ will pay the price if they expect a participation level of n_B, and all group-B with an outside option of less than $u_B = r_B + \beta_B n_A - p_B$ will pay the price if they expect a participation level of n_A.[9] Since users are uniformly distributed, we

[8] In a slightly extended model in which also segment B consumers enjoy a stronger network effect with the premium version, $\beta + \Delta_B$, the above versioning strategy remains optimal as long as Δ_B is sufficiently small. However, in the extended model, versioning generates lower profits than group pricing.

[9] This is the linearized version of the analysis in Armstrong (2006); it has been analyzed, e.g., in Belleflamme and Peitz (2018).

have $n_A = u_A$ and $n_B = u_B$. Hence, we solve the system of two linear equations and obtain

$$n_A = \frac{r_A - p_A + \beta_A(r_B - p_B)}{1 - \beta_A\beta_B} \text{ and } n_B = \frac{r_B - p_B + \beta_B(r_A - p_A)}{1 - \beta_A\beta_B}. \tag{3}$$

Here, we assume that $\beta_A\beta_B < 1$ (i.e., cross-group network effects are not too strong). Solving the platform's maximization problem, the first-order conditions with respect to p_A and p_B can be written respectively as

$$\begin{cases} p_A = \frac{r_A}{2} - \frac{1}{2}(\beta_A + \beta_B)\, p_B + \frac{1}{2}\beta_A r_B, \\ p_B = \frac{r_b}{2} - \frac{1}{2}(\beta_A + \beta_B)\, p_A + \frac{1}{2}\beta_B r_A. \end{cases}$$

Solving this system, to obtain we obtain profit-maximizing prices:[10]

$$\begin{cases} p_A = \frac{r_A}{2} + \frac{1}{2}\frac{\beta_A - \beta_B}{4-(\beta_s+\beta_b)^2}\left(2r_B + (\beta_A + \beta_B)\, r_A\right), \\ p_B = \frac{r_B}{2} + \frac{1}{2}\frac{\beta_B - \beta_A}{4-(\beta_A+\beta_B)^2}\left(2r_A + (\beta_A + \beta_B)\, r_B\right). \end{cases} \tag{4}$$

As with network goods, if $\beta_A > \beta_B$ the platform chooses to have a larger margin for group-A users. Finally, we report the profit-maximizing participation levels and the platform's maximal profit as

$$n_A^* = \frac{2r_A + (\beta_A + \beta_B)\, r_B}{4 - (\beta_A + \beta_B)^2},\; n_b^* = \frac{2r_B + (\beta_A + \beta_B)\, r_A}{4 - (\beta_A + \beta_B)^2},\text{ and } \Pi^* = \frac{r_A^2 + r_B^2 + r_A r_B (\beta_A + \beta_B)}{4 - (\beta_A + \beta_B)^2}. \tag{5}$$

How does this relate to the solution in the provision of a single network good or the provision of two partially compatible network goods? For this purpose, we now allow for within-group effects $\gamma_A < 1$ and $\gamma_B < 1$, which may be different from cross-group network effects β_A and β_B, respectively. Thus, $u_i = r_i + \beta_i n_j + \gamma_i n_i - p_i$. Under our assumption on the outside option we have that $p_A = r_A + \beta_A n_B - (1 - \gamma_A)\, n_A$ and $p_B = r_B + \beta_B n_A - (1 - \gamma_B)\, n_B$. We note that $\gamma_i = \beta_i > 0$ corresponds to the situation that we analyzed in the previous section; $\beta_i > \gamma_i = 0$ is the special case just analyzed above; and $\gamma_i > \beta_i > 0$ amounts to a situation of partial compatibility of separate network goods provided to the two groups of consumers. Since there is a one-to-one mapping from prices to participation levels in the consumers' participation equilibrium, the maximization program of the platform can be written as: $\max_{n_A, n_B} \Pi = (r_A + \beta_A n_B - (1 - \gamma_A)\, n_A)\, n_A + (r_B + \beta_B n_A - (1 - \gamma_B)\, n_B)\, n_B$. Solving the system of first-order conditions, we find the profit-maximizing participation levels:

$$n_A^* = \frac{2(1-\gamma_B)\, r_A + (\beta_A + \beta_B)\, r_B}{4(1-\gamma_A)(1-\gamma_B) - (\beta_A + \beta_B)^2} \text{ and } n_B^* = \frac{2(1-\gamma_A)\, r_B + (\beta_A + \beta_B)\, r_A}{4(1-\gamma_A)(1-\gamma_B) - (\beta_A + \beta_B)^2}.$$

The second-order conditions require $4(1-\gamma_A)(1-\gamma_B) > (\beta_A + \beta_B)^2$. Under this restriction, it can be shown that both n_A^* and n_B^* increase with γ_A and γ_B. Profit-maximizing prices are:

$$\begin{aligned} p_A^* &= \frac{[2(1-\gamma_A)(1-\gamma_B) - \beta_B(\beta_A + \beta_B)]\, r_A + (1-\gamma_A)(\beta_A - \beta_B)\, r_B}{4(1-\gamma_A)(1-\gamma_B) - (\beta_A + \beta_B)^2}, \\ p_B^* &= \frac{[2(1-\gamma_A)(1-\gamma_B) - \beta_A(\beta_A + \beta_B)]\, r_B - (1-\gamma_B)(\beta_A - \beta_B)\, r_A}{4(1-\gamma_A)(1-\gamma_B) - (\beta_A + \beta_B)^2}. \end{aligned}$$

[10] To satisfy the second-order conditions for profit maximization, we need to impose a more stringent condition than $\beta_A\beta_B < 1$, namely $\beta_A + \beta_B < 2$.

An increase in either γ_A or γ_B leads to an increase in p_A^* and a decrease in p_B^* if and only if $\beta_A > \beta_B$. This tells us how prices change as we move from a setting with cross-group network effects only to the setting of the provision of a network good.

Obviously, the higher price p_A must be positive to be a profit maximum; indeed the second-order conditions and the assumption that $\beta_A > \beta_B$ imply that $p_A^* > 0$. However, it could be that $p_B^* < 0$. That is, the profit-maximizing price may be negative for group B and this may also hold in the special case of fully compatible network goods; i.e., that $\beta_A = \gamma_A$ and $\beta_B = \gamma_B$. If negative prices are not feasible, the monopolist sets $p_B^f = 0$ and $p_A^f = \arg\max_{p_A} p_A n_A(p_A, 0)$ where $n_A(p_A, 0)$ is the equilibrium participation level of group-A consumers when group-B consumers obtain the network good for free.[11]

With unrestricted prices, the platform's maximal profit is

$$\Pi^* = \frac{(1-\gamma_A)\, r_B^2 + (1-\gamma_B)\, r_A^2 + (\beta_A + \beta_B)\, r_A r_B}{4\,(1-\gamma_A)\,(1-\gamma_B) - (\beta_A + \beta_B)^2}.$$

Differentiating with respect to γ_A and γ_B, we check that Π increases when γ_A or γ_B increases. As mentioned above, a situation with $\gamma_A > \beta_A$ and $\gamma_B > \beta_B$ describes an environment of partial incompatibility. Differentiating with respect to β_A and β_B, we check that Π increases when β_A or β_B increases, which says that an increase in compatibility of the two network goods increases the monopoly platform's profit. Clearly, if the two network goods are largely incompatible (i.e. β_A and β_B close to zero) the monopolist prefers to offer a single network good to all consumers at the same price rather than offering two network goods, one for each consumer group.

While our focus in this section has been on group pricing, we also take a quick look at versioning.[12] Suppose that $r_A \geq r_B$, $\beta_A \geq \beta_B$, $\gamma_A \geq \gamma_B$ with at least one these inequalities being strict. In line with the special cases of versioning above, suppose furthermore that these are the consumer characteristics of the two groups of users of the network good that includes the add-on, and that all consumers are homogeneous with respect to the base product; in particular, the parameters characterizing their preferences are r_B, β_B, and γ_B. Clearly, in this case, group-B consumers buy the basic version rather than the premium version if and only if $p_A > p_B$. The monopoly platform optimally uses versioning with the prices under group pricing from above if given these prices, group-A consumers prefer the premium version over the basic version; that is, $r_A + \beta_A n_B^* + \gamma_A n_A^* - p_A^* \geq r_B + \beta_B n_A^* + \gamma_B n_B^* - p_B^*$.

Substituting for the profit-maximizing prices and participation levels, and simplifying, we can rewrite the latter condition as

$$(2\,(1-\gamma_B) - \beta_A - \beta_B)\, r_A \geq (2\,(1-\gamma_A) - \beta_A - \beta_B)\, r_B,$$

[11] In the versioning context, the monopolist offers a premium version at a positive price and a basic version for free and thus adopts a freemium strategy.

[12] Consumers decide which version to buy. Within the two-sided platform context this means that the consumer decision which side to join is endogenous. Conceptually, this is related to Choi and Zennyo (2019) who analyze a model with competing two-sided platforms when users also decide which side of the platform to join.

which is satisfied if $r_A \geq r_B$ and $\gamma_A \geq \gamma_B$.[13] We note that also in the more general setting, versioning may be privately optimal with the prices under group pricing; the incentive constraints of group-A and group-B consumers remain slack.

6 Conclusion

This paper establishes that price discrimination with a network good can be interpreted as pricing of a two-sided (or multi-sided) platform. We do so in a linear setting which allows us to provide closed-form solutions. The insights apply more generally to non-linear settings (in particular, the distribution of the outside option does not need to be uniform). Future work may want to take several directions.

In the versioning context, we restricted attention to environments in which incentive compatibility constraints are slack. In other environments they are binding, and the two-sided platform is constrained by these incentive compatibility constraints. In our model, versioning refers to different versions affecting the stand-alone utility or the strength of the network good (and doing so differently for different consumer groups). An alternative setting would be that a basic version put a limit on the overall network effect. Applied to a social network, this could mean that a user of the basic version is restricted to have a maximum number of interactions with other users. We also took the different versions of the network good to be exogenous; an extension would be to allow for a continuum of possible versions and endogenize the platform's decision which versions to offer.

Also, we mostly restricted attention to monopoly provision. We could provide specific models of price discrimination in oligopoly with network goods that can be translated into models of competing two-sided platforms. For example, one can provide such a setting to obtain a special case of the model analyzed by Tan and Zhou (2017). The general lesson that emerges is that the two-sided platform literature is helpful to analyze price discrimination with network goods (or, in other words, demand-side scale economies).

[13] Take first $r_A = r_B$; then the condition becomes $\gamma_A \geq \gamma_B$. Next, take $\gamma_A = \gamma_B = \gamma$; now, the condition becomes $(2(1-\gamma) - \beta_A - \beta_B)(r_A - r_B) \geq 0$, which is equivalent to $r_A \geq r_B$ (because the second-order conditions imply that $2(1-\gamma) > \beta_A + \beta_B$).

References

[1] Armstrong, M. (2006). Competition in Two-Sided Markets. Rand Journal of Economics 37, 668–691.

[2] Belleflamme, P. and M. Peitz (2010). Industrial Organization: Markets and Strategies. Cambridge: Cambridge University Press.

[3] Belleflamme, P. and M. Peitz (2012). Digital Piracy: Theory. In: M. Peitz and J. Waldfogel (eds.), The Oxford Handbook of the Digital Economy, Oxford University Press.

[4] Belleflamme, P. and M. Peitz (2018). Platforms and Network Effects. In: L. Corchon and M. Marini (eds.), Handbook of Game Theory and Industrial Organization, vol. II, Edward Elgar, 286–317.

[5] Belleflamme, P. and M. Peitz (2019). Managing Competition on a Two-Sided Platform. Journal of Economics & Management Strategy 28, 5–22.

[6] Choi, J. P. and Y. Zennyo (2019). Platform Market Competition with Endogenous Side Decisions. Journal of Economics & Management Strategy 28, 73–88.

[7] Csorba, G. (2008). Screening Contracts in the Presence of Positive Network Effects. International Journal of Industrial Organization 26, 213–226.

[8] Csorba, G. and J.-H. Hahn (2006). Functional Degradation and Asymmetric Network Effects. Journal of Industrial Economics 54, 253–268.

[9] Evans, D. and R. Schmalensee (2016). Matchmakers: The New Economics of Multisided Platforms, Boston: Harvard Business Review Press.

[10] Hagiu, A. (2009). Two-Sided Platforms: Product Variety and Picing Structures. Journal of Economics & Management Strategy 18, 1011–1043.

[11] Jullien, B. (2001). Competing with Network Externalities and Price Competition. CEPR Discussion Paper No. 2883.

[12] Jullien, B. (2011). Competition in Multi-Sided Markets: Divide and Conquer. American Economic Journal: Microeconomics 3, 186–219.

[13] King, S. and Lampe, R. (2003). Network Externalities, Price Discrimination and Profitable Piracy. Information Economics and Policy 15, 271–290.

[14] Mussa, M. and Rosen, S. (1978). Monopoly and Product Quality. Journal of Economic Theory 18, 301–317.

[15] Nocke, V., M. Peitz, and K. Stahl (2007). Platform Ownership. Journal of the European Economic Association 5, 1130–1160.

[16] Rochet, J.-C. and J. Tirole (2003). Platform Competition in Two-sided Markets. Journal of the European Economic Association 1, 990–1024.

[17] Rochet, J.-C. and J. Tirole (2006). Two-sided Markets: A Progress Report. Rand Journal of Economics 37, 645–667.

[18] Rysman, M. (2009). The Economics of Two-Sided Markets. Journal of Economic Perspectives 23, 125–143.

[19] Takeyama, L. (1994). The Welfare Implications of Unauthorized Reproduction of Intellectual Property in the Presence of Network Externalities. Journal of Industrial Economics 42, 155–166.

[20] Tan, G. and J. Zhou (2017). Price Competition in Multi-Sided Markets. Unpublished Manuscript.

[21] Veiga, A. (2018). A Note on how to sell a Network Good. International Journal of Industrial Organization 59, 114–126.