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Optimal Non-Linear Pricing Scheme when Consumers are Habit Forming

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Optimal Non-Linear Pricing Scheme when Consumers are Habit Forming

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Abstract

This article analyses how consumers' habit formation affects firms' pricing policies. I consider both sophisticated consumers, who realize that their current consumption will affect future consumption, and naive consumers, who do not. The optimal contract for sophisticated consumers is a two-part tariff. The main result is that under naive habit formation, the optimal pricing pattern is a three-part tariff; namely a fixed fee, with some units priced below cost — and after their end — pricing above marginal cost. This holds both under symmetric and asymmetric information. **JEL:** L11, D11, D42, D82

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1 Introduction

Three-part tariffs are prevalent features in telecommunications and Information Technology (IT) markets (Grubb, 2009, 2014). These contracts include a fixed fee, an allowance of free units, and a positive price for additional units beyond the allowance. This pricing pattern is hard to explain within the rational paradigm.[1] This article shows that the presence of consumers with naive habit forming behavior is sufficient for the optimality of three-part tariffs. More specifically, it is sufficient that the consumers are not entirely aware of how much their past consumption affects their current valuation for the good. Importantly, this type of behavior has been well documented in telecommunications and IT markets (Oulasvirta et al., 2012; Bianchi and Phillips, 2005; Park, 2005).[2]

I solve a dynamic pricing model in which a firm sets the price at the contractual stage, and the consumers decide whether or not to buy, based on their expectation of the value of their future consumption. The consumers have two consumption opportunities within the contract period: namely, they can buy the good once or twice during the contract period, depending on their needs and their valuation of the good. At the end of the period, they make the payment.

In the benchmark model, I consider sophisticated consumers who are aware that today's consumption affects future consumption. In this case, it is optimal for the monopolist to charge a two-part tariff. As consumers know the value of their future consumption exactly, the firm finds it optimal to maximize the consumer surplus by setting marginal prices equal to the marginal cost, and to charge a fixed fee that extracts all of the consumer surplus.

This changes when I consider naive habit-forming consumers. These consumers

[1] Grubb (2009) provides analysis of a standard model that could explain the introduction three part tariff, showing though that could not be relevant to this market.

[2] Psychological literature provides evidence that these goods are habit forming (Oulasvirta et al., 2012). There are studies on the type of habits and to whether resemble overuse or even addiction (Billieux, 2012).

are unaware of their habit forming behavior at the contractual stage. The monopolist, however, can recognize that they are habit-forming. In this second case, the optimal contract offered by the firm is a three-part tariff. The firm charges a marginal price above marginal cost for high volumes, a marginal price below marginal cost for low volumes, and a fixed fee.

As an intuition, naive habit forming consumers underestimate the probability of having high demand at the contractual stage. They do not expect that they will acquire a habit and thus fail to realize that the probability of consuming the next unit of the good or service will be larger. Given this bias, the firm finds it optimal to distort marginal cost pricing by charging a marginal price above marginal cost for this and, thus, for high volumes. This resulting pattern is similar to the pricing of hyperbolic discounted leisure goods, where the underestimation of demand also arises (DellaVigna and Malmendier, 2004; Eliaz and Spiegler, 2006).

Naive habit forming consumers do not make mistakes about the probability of having low demand at the contractual stage. It is only after consuming that they experience an unexpected change in their demand (Pollak, 1975). Moreover, the naive habit forming consumers evaluate their consumption decisions sequentially, and they are forward-looking. This means that they can foresee that there will be a price change in the future and they internalize this information into their decision as to whether or not to consume in the current period. They can also foresee that they may forego utility if they consume today, and expect that the next unit will be charged differently and will possibly be more costly. Thus, the monopolist finds it optimal to charge a price below marginal cost for low volumes, since the consumers are forward-looking, with the second unit priced above marginal cost, for the reasons explained before. In this way, the probability of consuming the first unit increases and the cost of forgone future utility decreases. The firm finds it optimal to increase the probability of consuming the first unit, not only because it will lead to more future consumption but also because the firm can fully extract the surplus produced from the first unit. Since, the consumer makes no mistake at the contractual stage for the first unit, the perceived expected utility is equal to the actual expected utility. Thus, the fixed fee can fully extract the surplus.

Finally, the third part of the tariff is the fixed fee. The fixed fee is equal to the gross expected surplus of the consumers at the contractual stage. However, the consumers undervalue the contract offered by the monopolist at the contractual stage, because they cannot foresee that they will value the good more highly the more they consume. They participate in the market, considering themselves as non-habit forming. For this reason the firm cannot extract all of the consumer surplus actually produced with the fixed fee. The monopolist mitigates the contract undervaluation, and thus extracts as much consumer surplus possible by distorting marginal prices. The direction of the distortion of the marginal prices is as discussed. The consumers, in turn, are left with a positive *misperception rent*[3], as given by the difference between their true expected surplus and the surplus they mistakenly perceive at the contractual period. Consequently, the naive consumers cannot be exploited through the pricing scheme.[4] Though, the underestimation of the value of the contract at the contractual period, causes some consumers not to participate in the market, even if they actually value the good more than its cost, leading to participation distortion (Heidhues and Kőszegi, 2015).

Interestingly, if the good is addictive, the consumer still underestimates high demand but overestimates the value of the contract at the contracting stage. I show in Section 4 that the welfare implications are the opposite, and the firm can exploit the naively addicted consumer.

The literature to date has focused on consumers' overconfidence as the main ex-

[3]Three part tariff is optimal also when the good is addictive but with opposite welfare implications. See Appendix A for the full analysis.

[4]I use the notion of exploitation according to Eliaz and Spiegler (2006), where "An exploitative contract extracts more than the agent's willingness to pay, from his first-period perspective".

planation for the use of three-part tariffs (Grubb, 2009). Overconfidence means that consumers overestimate their demand when it is low, and underestimate it when it is high. The main difference between overconfidence and habit formation is that for three part tariffs to be optimal, in Grubb (2009) both mistakes are necessary. This article shows that is sufficient that the consumers underestimate high demand.

In Section 4, I also relax the assumption of full information and study the pricing strategy of a monopolist when the firm cannot observe the consumer type. I study the optimal screening of habit-forming consumers with differing degrees of sophistication. I contend that frequently observed contract menus, comprise both two and three-part tariffs, can be explained by the presence of consumers with different levels of sophistication. In this way, the firm screens between sophisticated and naive consumers. Three-part tariff is still the optimal contract for naive consumers.

To understand why, consider that the sophisticated consumers would have an incentive to mimic the naive consumers. Even if they know that they are more likely to consume in the future, they would choose a contract that penalizes large consumption levels with high marginal prices. The sophisticated consumers — by mimicking the naive consumer — would be left with a positive rent ex-post, because the contract made for naive consumers charges a fixed fee that does not extract all the surplus. For this reason, the optimal contract for sophisticated consumers charges the same marginal prices as in the full information case, but a smaller fixed fee. Consequently, the presence of naive consumers in the market exerts a positive externality on the sophisticated consumers.

The optimal contract for naive consumers is still a three-part tariff as in the full information case because of the same economic forces. The difference now is that the contracts should be incentive compatible and not attractive for the sophisticated consumer. For this reason, I observe an increase in the marginal prices per unit. The firm still cannot exploit the naive consumers who are left with a positive but smaller, in this case, misperception rent. Thus, the naive consumers are worse off when there are sophisticated consumers in the market.

Even if the optimal marginal price for low volumes is smaller than the marginal cost, the naive consumer underconsumes compared to the sophisticated case. The firm does not charge a pricing scheme that induces the efficient probability of consumption but a smaller one. On the one hand, a decrease in the low volume marginal price would have only second order efficiency losses since the firm can fully extract consumer surplus on these units. Moreover, it would lead to an increase in the second unit surplus as its consumption becomes more probable. Though, the firm can extract only a part of the second unit surplus by overcharging it. It cannot fully extract it with the fixed fee, because the consumer does not anticipate ex ante its real value. Therefore, the firm bears all the costs of subsidizing low volumes of consumption, but only a fraction of its benefits, which motivates it to under-invest as well in incentivising consumption.

In Section 5, I consider a market where there is perfect competition both in the case of informed and uninformed firms. The optimality of three-part tariffs in the presence of naive habit forming consumers is again confirmed. The only part of the tariff that differs is the fixed fee, which decreases as the market becomes more competitive.

The article proceeds as follows. Section 2 discusses the related literature. Section 3 is devoted to the model setup, and Section 4 presents the case of full information and asymmetric information with a monopolist in the market. Section 5 considers the case of perfect competition both when the firms are informed and uninformed. Finally, Section 6 summarizes and concludes.

2 Literature

This article is related to different streams of the literature. First, it is related to models that explain the introduction of three-part tariffs. Grubb (2009) shows that over-confidence about the precision of the prediction when making difficult forecasts, free disposal and relatively small marginal costs would explain the use of a three-part tariff. He claims that a three-part tariff is approximately the optimal pricing scheme when the behavior of the consumer is characterized by an overestimation of the demand, when the demand is low, and an underestimation of the demand when it is high. In our case, I propose a different behavior that could explain this pricing scheme without both mistakes necessarily being present. Moreover, I study an environment in which the firm observes the amount actually consumed by the consumer in each period,[5] and not only the amount the consumer has bought.

Grubb (2014) shows that inattentive behavior has similar features and implications to overconfident behavior. The common element between our model and Grubb (2014) model is that I both consider the consumption dynamically within the contract period; however, I propose a different type of behavior.

Eliaz and Spiegler (2008) consider a model where consumers have biased priors. I also do this, but with only two types of ex-post demand: high or low. The consumers are optimistic and think that the good state is more likely to happen. They describe a situation where consumers are dynamically inconsistent, and they under or overestimate average demand. Thus, Eliaz and Spiegler (2008) study an entirely different behavioral bias, having only the biased priors in common.

In particular, I study the optimal pricing scheme when the good is habit forming. Thus, articles that discuss the optimal pricing of habit goods (Nakamura and Steinsson, 2011; Fethke and Jagannathan, 1996) or even addictive goods (Becker et al., 1991; Driskill and McCafferty, 2001) are connected to our study. However, I consider habit formation and optimal pricing within a contract period, where the firm cannot renegotiate the price during the contract period.

Moreover, the discussion of a naive habit forming consumer is closely related to

[5] Though, I assume that the firm cannot observe all the consumption opportunities of the consumer.

the articles that consider the optimal nonlinear pricing induced by various types of consumers' biases[6] or nonstandard preferences.[7]

On the one hand, there are articles discussing biased beliefs, such as naive quasi-hyperbolic discounting for leisure goods (DellaVigna and Malmendier, 2004; Eliaz and Spiegler, 2006), naivety about self-control (Esteban et al., 2007; Heidhues and Kőszegi, 2010) and myopia (Gabaix and Laibson, 2006; Miao, 2010). A common consequence of these behavioral biases is an underestimation of the demand, which results in marginal prices above marginal cost. These models cannot explain why marginal prices are below marginal cost for low volumes.

On the other hand, there are behaviors that may explain prices below marginal costs, but not above. For example, behaviors like naive quasi-hyperbolic discounting for investment goods (DellaVigna and Malmendier, 2004) and flat rate bias (Lambrecht and Skiera, 2006) that lead to overestimation of demand, or non standard preference like loss aversion (Herweg and Mierendorff, 2013).

DellaVigna and Malmendier (2004) were the first to point out that firms may design contracts to exacerbate consumer's mistakes. Since their pioneering contribution, many articles have explored the specific way of exploiting consumer naivety. In our model, the firm offers a contract that exacerbates a consumer's mistake but cannot extract all of the consumer surplus produced.

This article is also related to the literature on exploitative contracting, where firms design their contracts to profit from the agent's mistakes. There are two kinds of consumers' mistakes more often analyzed in the literature. Firstly, the consumer does not understand all of the features of a contract (all prices and fees) (Gabaix and Laibson, 2006; Armstrong and Vickers, 2012). Another kind of mistake is to mispredict

[6]See Köszegi (2014) for a survey of behavioral economics research in contract theory.

[7]For standard, rational preferences, see Mussa and Rosen (1978) and Maskin and Riley (1984). They explain contracts with high marginal prices for early units and marginal cost pricing for late units consumed; although, they cannot predict the inverse which is the main characteristic of three part tariffs.

their own behavior concerning the product (DellaVigna and Malmendier, 2004). The latter kind of mistake is closer to the model I study here, and as in our model, the consumer mispredicts that her valuation for the good will change if she has consumed before.

The section of asymmetric information is clearly related to the behavioral screening literature, where a principal screens the agents with respect to their cognitive features such as loss aversion, (Hahn et al., 2012; Carbajal and Ely, 2012), present bias, temptation disutility (Esteban et al., 2007), or overconfidence (Sandroni and Squintani, 2010; Spinnewijn, 2013). In contrast to this literature, the optimality of the pricing scheme is not the result of a screening mechanism.

This section is also related to the literature on sequential screening of consumer with standard preferences. In these models the consumers know at the contracting period the distribution of their valuation for the good and subsequently they learn their realized valuation (Courty and Hao, 2000; Miravete, 2005; Inderst and Peitz, 2012).

Eliaz and Spiegler (2006) show that the ultimate source of gains for the principal is the non-common prior assumption. The consumer is uncertain as to whether or not her preference will change, but she knows exactly what they could change into. The firm, though, knows that the consumer's preference will change, and takes advantage of its superior information by also contracting the event that the consumer thinks unlikely to happen. The difference in their prior expectations leaves space for exploitation. In our case, in contrast, the consumer does not know that her utility function will change after consuming in the first period, so the firm cannot exploit its superior information. This feature becomes important because in both cases the contract is signed before the consumer experiences the change in her utility and cannot be renegotiated afterward.

3 Model Setup

This section presents the basic structure of the model. I consider a model that follows Grubb (2014) in modeling a consumer who has two consumption opportunities: one per period, and in each period purchases at most one unit of the good. Moreover, the consumers are habit-forming with differing levels of sophistication, and one firm. The consumers are uncertain about their valuation of the good in each period.

The time horizon is $T = 2$. At period 0, the firm offers a menu of contracts:

$$\boldsymbol{\sigma}^{\theta} = \{F^{\theta}, p_1^{\theta}, p_2^{\theta}\}.$$

where θ is the level of sophistication of the consumer. The contract $\boldsymbol{\sigma}^{\theta}$ consist of p_1^{θ} (the price of the first unit consumed), p_2^{θ} (the price of the second unit consumed), and F^{θ} (a fixed payment). The first unit has the same price, irrespective of the period t when consumed. Time-dependent pricing would require that the firm could observe and record the opportunities to consume, as if, for example, the consumer had direct communication with the firm in every opportunity to consume. Thus, it is a relevant assumption to assume that the firm cannot observe whether the consumer decides to consume or not.[8] At each consecutive period $t \in \{1, 2\}$, the consumer learns the realization of a taste shock υ_t, randomly drawn from a cumulative distribution function $F(\upsilon)$ with support [0,1], the same for all types of consumers and for both periods. Then, υ_t is the base valuation that a unit of good has in period t. Then, given her valuation, she has a binary quantity choice $q_t = \{0, 1\}$, considering whether or not to purchase the good.[9]

[8] Contrary to Grubb (2014), I assume that the firm cannot observe the period in which the consumption takes place. The result of increasing marginal pricing holds also in the case that the firm could observe it. See Appendix A for the analysis of the model with date dependent pricing.

[9] I assume that the good is indivisible.

The total payment is:

$$P^{\theta}(\mathbf{q}) = p_1^{\theta}q_1 + p_1^{\theta}(1-q_1)q_2 + p_2^{\theta}q_1q_2 + F^{\theta},$$

is a function of quantity choices $\mathbf{q} = (q_1, q_2)$, the marginal prices $\mathbf{p}^{\theta} = (p_1^{\theta}, p_2^{\theta})$ and the fixed fee F^{θ}. The timing of the game is described in Figure 1.

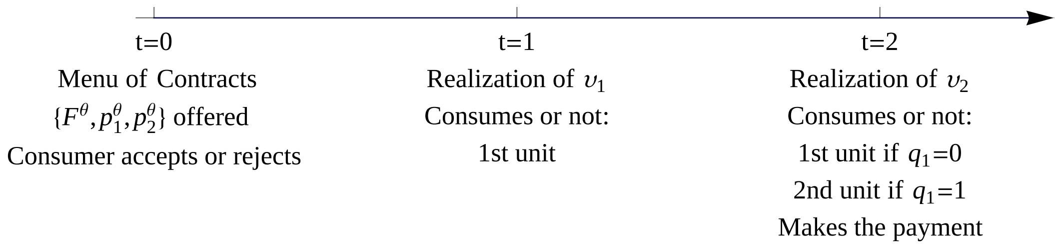

Figure 1: Timing of the game

The optimal consumption strategy for given marginal prices $\mathbf{p}^{\theta}$, is a function mapping valuations to quantities:

$$\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}^{\theta}) : \boldsymbol{\upsilon} \to \mathbf{q},$$

where $\boldsymbol{\upsilon} \in [0,1]^2$.

Moreover, the ex-ante expected utility of the consumer gross of the fixed fee is:

$$U = E\left[(\boldsymbol{\upsilon} - \mathbf{p}^{\theta})\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}^{\theta})\right].$$

The expected profits per consumer are equal to the revenues less the variable cost, with marginal cost $c \geq 0$ per unit produced. The fixed cost is normalized to zero. Thus, the profit function is:

$$\Pi = E\left[\left(p^{\theta}(\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}), \mathbf{p}^{\theta}) - c\right)\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}^{\theta})\right] + F^{\theta}.$$

Finally, the expected social surplus is:

$$S = E\left[\sum_{t=1}^{2}(v_t - c)\mathbf{q}(\boldsymbol{v};\mathbf{p}^{\theta})\right].$$

Consider a consumer who is habit forming in the sense that her consumption today is affected by her consumption in previous periods and the difference in her utility consuming the good and her outside option increases with past consumption. Her period $t-1$ belief is that her period t valuation for the service will be:

$$\tilde{v}_t = v_{t,good} - v_{t,outside} = v_t + \theta\beta q_{t-1}.$$

Therefore, if she consumed in the previous period, her base valuation for the service today increases by $\theta\beta$, where $0 \leq \beta \leq 1$ is the habit formation coefficient: namely it defines how habit forming the consumer is, and how much she is affected by previous consumption. Moreover, $0 \leq \theta \leq 1$ is the type of the consumer. It is a measure of her naivety and of how much she realizes that she is habit forming. The larger the θ, the less naive is the consumer; thus, the more she realizes that she is affected by her previous consumption. Thus, $\theta = 1$ means that she is a sophisticated habit forming consumer, $0 < \theta < 1$ means that she is partially naive, and $\theta = 0$ that she is completely naive. Let for simplicity, and without any loss of generality, the value of the outside option equal to zero, $v_{t,outside} = 0$

Every time that the consumer faces a consumption decision, she compares the valuation of the unit with her reservation price. The reservation price in each period is the threshold above which the base valuation should be for the consumer to be optimal to consume the unit. As the base valuation of the unit is random and the consumer does not know it ex-ante, she calculates the optimal threshold, as an optimal consumption rule. This consumption rule is different for each potential consumption decision, since the decisions are taken sequentially. Thus, these thresholds are the consumption strategy of the consumer, namely the argument of maximization of her expected utility for the respective unit and period.

For simplicity, I assume that there are two types of consumer, a sophisticated habit forming consumer with $\theta = 1$ and a naive one, with $\theta = 0$. In each period, consumers choose the optimal threshold above which it is optimal for them to consume. During the contracting period, the consumer does not know the future realizations of her base valuation of the good, so chooses which contract to sign on the basis of her expected utility.

Sophisticated Habit Forming Consumer ($\theta = 1$): Solving backwards, I start the analysis from the second period. The second period optimal threshold depends on the first period action. If $q_1 = 0$, then the second period valuation is equal to the base valuation, $\tilde{v}_2 = v_2$, and the consumer consumes if $v_2 > p_1$. If $q_1 = 1$, then $q_2 = 1$ if $\tilde{v}_2 = v_2 + \beta > p_2$. Thus, the second period optimal threshold is:

$$v_{2S}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ \max\{0, p_2 - \beta\} & \text{if} \quad q_1 = 1. \end{cases}$$

Given v_{2S}^*, the first period maximization problem of the sophisticated habit forming consumer is:

$$\begin{aligned} \max_{v_{1S}} U^S(\mathbf{p^S}) - F^S = \int_{v_{1S}}^1 \left(v_1 - p_1 + \int_{p_2-\beta}^1 (v_2 + \beta - p_2) dF(v_2) \right) dF(v_1) \\ + F(v_{1S}) \int_{p_1}^1 (v_2 - p_1) dF(v_2) - F^S. \end{aligned} \tag{1}$$

The first part is the expected utility if both units are consumed, the second is the expected utility if only the second unit is consumed, and the third is the fixed fee.

Maximizing with respect to v_{1S}, the optimal first period threshold is (after an integration by parts):

$$v_{1S}^* = p_1 - \int_{p_2-\beta}^{p_1} (1 - F(v_2))dv_2.$$

The consumer is forward looking and is aware of being habit forming, so she takes into account both the opportunity cost of consuming the first unit (i.e., the price increase $p_2 - p_1$ for the second unit), and the increase in her valuation due to the habit. The habit forming consumer expects to experience a larger utility in the future, if she consumes the first unit, so she finds it optimal to increase the probability of consuming the first unit. Thus, the optimal threshold decreases. Moreover, the first-period threshold increases if the second unit marginal price increases and decreases the more habit forming the consumer is.

Naive Habit Forming Consumer ($\theta = 0$): In period 2, the actual optimal threshold is the same as for a sophisticated consumer. However, from the period 1 perspective, the consumer anticipates that the second-period threshold will be:

$$v_{2N}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 & \text{if} \quad q_1 = 1 \end{cases},$$

that is, because the consumer does not anticipate that the first-period consumption will affect the valuation of the good in the second period.

Given v_{2N}^*, the first period maximization problem of the naive habit forming consumer is:

$$\begin{aligned} \max_{v_{1N}} U^N(\mathbf{p^N}) - F^N = & \int_{v_{1N}}^1 \left(v_1 - p_1 + \int_{p_2}^1 (v_2 - p_2) dF(v_2) \right) dF(v_1) \\ & + F(v_{1N}) \int_{p_1}^1 (v_2 - p_1) dF(v_2) - F^N. \end{aligned} \quad (2)$$

Maximizing with respect to v_{1N}^*, the optimal first period threshold is:

$$v_{1N}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2))dv_2.$$

The consumer takes into account the opportunity cost of first-period consumption, as she is forward looking, but she does not consider the effect of first period consumption on second-period evaluation. Thus, the optimal threshold is the same as that of a non-habit forming consumer. Clearly, $v_{1N}^* > v_{1S}^*$ and thus the naive consumer under-consumes in the first period for given marginal prices. Moreover, she under-consumes also in the second period as a result of failing to build up as much of a habit in period 1.

The true ex-ante utility of the consumer differs from what she expects at the contractual stage. In the first period, the optimal threshold and the expected utility are the ones of a non-habit forming consumer. Thus, the actual gross ex-ante expected utility for the naive consumer is:

$$\begin{aligned}\tilde{U} = &\int_{v_{1N}^*}^{1} \left(v_1 - p_1 + \int_{p_2-\beta}^{1} (v_2 + \beta - p_2) dF(v_2) \right) dF(v_1) \\ &+ F(v_{1N}^*) \int_{p_1}^{1} (v_2 - p_1) dF(v_2).\end{aligned} \tag{3}$$

Therefore, she believes that her expected utility is U^N (equation (2)), even though her actual expected utility, and the one that the firm knows that she will have, is $\tilde{U}$ (equation (3)). This whole analysis also holds when the consumer is partially naive. Namely, the consumer knows that she is habit forming, but she believes that she is less habit forming than she is.[10]

The consumer uses in period one the same threshold as she expected to use when

[10] In this case, the period $t-1$ belief of the period t valuation of the good is $\tilde{v}_t = v_t q_t + \hat{\beta} q_{t-1} q_t$ and $\hat{\beta} < \beta$. As in the case of the naive consumer, the partially naive consumer has no mistaken beliefs if the demand is low, but she underestimates her demand is high. See the Appendix B.

she chose her contract in the contractual stage. More specifically, the probability of consuming in the first period is $1 - F(v^*_{1N})$, as the consumer expected at period 0. This means that there is no mistake that the firm could take advantage of. The only implication of the consumer's naivety, in the first period, is related to the expected consumer surplus, which is smaller than the one that would be produced if the consumer were sophisticated.

In the second period, given that the consumer has not consumed before ($q_1 = 0$), she does not realize that she is habit forming, and thus she consumes as much as she was expecting to consume at the contract period. The probability of consuming is $F(v^*_{1N})(1 - F(p_1))$, and it is not different from what the consumer would expect. The consumer does not overestimate the probability of buying only one unit, and actually does not make any mistake given that her consumption is low.

On the other hand, given that the consumer has consumed before ($q_1 = 1$), she underestimates the probability of consuming two units. She expects that her optimal threshold, in this case, would be p_2, but she realizes later that it is $p_2 - \beta$. Thus, the probability of consuming at the second period is expected to be $(1-F(v^*_{1N}))(1-F(p_2))$, but given she has consumed at period 1, it is $(1 - F(v^*_{1N}))(1 - F(p_2 - \beta))$. Hence, she underestimates the probability of consuming the second unit.

Lemma 1. *Let π be the actual probability of consumption and $\tilde{\pi}$ the perceived probability of consumption at the contacting period. A naive habit forming consumer makes no mistake if her demand is low, namely $\pi(q_1 = 1) = \tilde{\pi}(q_1 = 1)$ and $\pi(q_2 = 1|q_1 = 0) = \tilde{\pi}(q_2 = 1|q_1 = 0)$. Moreover, she underestimates her future demand when it is high, $\pi(q_2 = 1|q_1 = 1) > \tilde{\pi}(q_2 = 1|q_1 = 1)$.*

In addition, because of her naivety, the consumer under-evaluates the value of the offered contract at the contracting stage. Firstly, the consumer does not anticipate that consuming in the first period will increase the valuation of her second unit, so she does not expect the β additional valuation, and she does not consider it in the ex-ante valuation of the whole contract. Secondly, she underestimates the probability of consuming the second unit, and thus acquiring this extra utility.

4 Monopolistic Markets

Informed Monopolist

Let us consider first the case that the firm can observe the type of the consumer and thus can offer a type specific contract.

Sophisticated Consumer

There is a monopolistic firm in the market. The cost of the production of one unit of the good is $c \in (0,1)$.

The maximization problem of the firm is:

$$\max_{\boldsymbol{\sigma}^S}\{\Pi^S = S^S(\mathbf{p^S}) - (U^S(\mathbf{p^S}) - F^S)\} \quad \text{s.t} \quad U^S(\mathbf{p^S}) - F^S \geq 0.$$

This is the difference between the expected gross surplus produced minus the expected consumer surplus subject to the participation constraint. The expected gross surplus is:

$$\begin{aligned} S^S(\mathbf{p^S}) = &\int_{v_{1S}^*}^{1}\left(v_1 - c + \int_{p_2-\beta}^{1}(v_2+\beta-c)dF(v_2)\right)dF(v_1) \\ &+ F(v_{1S}^*)\int_{p_1}^{1}(v_2-c)dF(v_2). \end{aligned} \tag{4}$$

Maximizing with respect to $\mathbf{p^S}$, the optimal contract is found and is given by the following Lemma:

Proposition 1. *If the consumer is sophisticated habit forming, the equilibrium allocation is the first best allocation. There is marginal cost pricing, namely the prices that maximize the profits of the firm are $(p_1, p_2) = (c, c)$ and the fixed fee, $F^S = U^S(\mathbf{p^S})$,* *equals the gross consumer surplus.*

The firm maximizes its profit by charging marginal prices that induce the first best allocation and then with the fixed fee F^S it extracts all the consumer surplus (see Appendix A).

Naive Consumer

Let us now consider how the maximization problem of the firm changes when the consumer is naive habit forming.

The firm recognizes that it faces a naive habit forming consumer whose participation depends on her mistaken expected utility. Moreover, it knows that the social surplus is given by:

$$S^N(\mathbf{p^N}) = \int_{v_{1N}^*}^{1} \left(v_1 - c + \int_{p_2-\beta}^{1} (v_2 + \beta - c) dF(v_2) \right) dF(v_1) + F(v_{1N}^*) \int_{p_1}^{1} (v_2 - c) dF(v_2).$$

The firm is aware that in the first period, the consumer does not know that she is habit forming and consumes only if the base valuation of the unit is greater than v_{1N}^*. Moreover, it takes into account that given that she has consumed in the first period, her valuation of the good in the second period is higher, since it is affected by past consumption. Therefore, at the contractual stage the firm takes into consideration that the consumer will update her second unit threshold and her valuation for the second unit, if she has consumed in the first period.

The firm maximizes its profits, which are the difference between the social surplus and the consumer surplus, subject to the participation constraint of the consumer. In this case, though, the true consumer surplus produced $\tilde{U}$ (equation (3)) is different from the one the consumer perceives at the contracting period U^N (equation (2)).

Thus, the optimization problem of the firm is:

$$\begin{aligned}
\max_{\boldsymbol{\sigma}^N} \Pi &= S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - F^N) \\
&= S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - \underbrace{(\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))}_{\Delta(\mathbf{p^N})} \\
\text{s.t.} \quad & U^N(\mathbf{p^N}) - F^N \geq 0,
\end{aligned}$$

where $\Delta(\mathbf{p^N})$ is the difference between the true expected utility from the contract $\tilde{U}(\mathbf{p^N})$ (equation (3)) and the perceived utility $U^N(\mathbf{p^N})$ (equation (2)). Moreover, the firm chooses a pricing scheme that makes the participation constraint binding, $U^N(\mathbf{p^N}) = 0$. As mentioned before, the consumer undervalues the contract at the contracting stage. Thus, there is a positive rent $\Delta(\mathbf{p^N})$ that is left to the consumer. It follows that the firm cannot extract all the consumer surplus. After some simplifications, $\Delta(\mathbf{p^N})$ can be rewritten as:

$$\Delta(\mathbf{p^N}) = (1 - F(v_{1N}^*)) \left(\int_{p_2-\beta}^{p_2} (1 - F(v_2)) dv_2 \right).$$

Then, the maximization problem of the firm, since $U^N(\mathbf{p^N}) = 0$, becomes:

$$\max_{\mathbf{p^N}} \Pi = S^N(\mathbf{p^N}) - \Delta(\mathbf{p^N}).$$

Calculating the marginal prices that maximize the above expression, the following result is obtained.

Proposition 2. *Monopoly: If the consumer is naive habit forming the optimal marginal pricing scheme is:*

$$\begin{aligned}
c = 0: &\quad p_1^N = 0, \quad p_2^N > c \\
c > 0: &\quad p_1^N < c, \quad p_2^N > c,
\end{aligned}$$

and the fixed fee $F^N = U^N(\mathbf{p^N})$ equals the gross perceived consumer surplus.

Proof: See Appendix A

The optimal pricing scheme when the consumer is naive habit forming resembles the scheme we observe in several markets, namely a *three part tariff*. This consist of a fixed fee, an included allowance of units, for which the marginal price equals zero, and a positive marginal price for units beyond the allowance.[11] When the marginal cost is equal to zero, the marginal price of the first unit is equal to zero and the marginal price of the second unit is higher than the marginal cost.

A firm facing a naive habit forming consumer has an incentive to distort the efficient allocation in order to maximize its profits. As the consumer misperceives her expected utility, the participation constraint is biased. The firm cannot extract the surplus produced through a fixed fee, since the perceived surplus is smaller than the one produced. Therefore, the firm needs to distort the marginal prices by choosing the ones that maximize $S^N - \Delta$ rather than S^N.

The exact way in which the marginal prices are distorted depends on the characteristics of the consumer's behavior. Firstly, the consumer underestimates[12] the probability of consuming the second unit and thus underestimates the surplus that it produces. The firm cannot extract ex ante the second unit surplus, and with a price p_2 bigger than the cost manages partly to extract it ex post. On the other hand, given that the consumer is forward-looking and takes into consideration the opportunity cost of consuming the first unit, without anticipating the increase in her valuation for the good, she consumes less often to avoid the price increase. The firm finds it optimal, in response, to decrease the marginal price of the first unit below cost to constrict the downward bias in consumption, and incentivise the consumer to invest into her habit.

[11] I could consider $\beta < 0$. Think of "novelty thrill" or a "fashion good", the less novel or fashionable feels the less someone consumes it. Then, the purchasing probability is decreasing without being aware of it ex ante. In this case, it would have the opposite pricing scheme, i.e. $p_1^N < c$, $p_2^N > c$.

[12] This underestimation makes it optimal for the firm to distort pricing, similarly to behaviors such as hyperbolic discounting and myopia.

These marginal prices exacerbate the mistake[13] of the second unit consumption. A distorted price below marginal cost makes the consumption of the first unit more probable than marginal cost pricing. Then, it increases the probability of having a second consumption opportunity and consequently the probability of consuming two units of the good. Thus, the consumption of the second unit becomes more probable, not only because the consumer acquires a habit that she does not expect, but also because the first unit marginal price facilitates it.

However, even if it seems that for the first unit there would be over-consumption, this optimal pricing produces the opposite result. For example, when the marginal cost is zero, $c = 0$: even if the marginal price of the first unit is zero its optimal threshold is positive, thus there is under-consumption compared to the efficient allocation of the sophisticated habit forming consumer. Moreover, the larger the habit formation coefficient β, the greater the first unit threshold v_{1N}^*, as the difference between the first and second unit optimal marginal prices are larger, the larger the β.

The naive consumer fails to invest on her own in acquiring the habit, and also the firm does not charge a pricing scheme that induces the efficient probability of consumption but a smaller one. On the one hand, the firms wants to incentivise the naive consumer to invest and decreases the marginal price of the first unit below marginal cost, to make its consumption more probable. The firm can extract all the first unit surplus, so the surplus losses from a small price distortion are second order. Moreover, the first unit price decrease makes the consumption of the second unit more probable and the second unit surplus increases. Thus, this price decrease has a positive first order effect in both q_1 and q_2. On the other hand, the firm cannot extract all the surplus of the second unit, because it only extracts it ex-post through $p_2^N > c$. This ex-post extraction of surplus causes standard monopoly dead-weight loss and the

[13] DellaVigna and Malmendier (2004) were the first to point out that firms might design contracts to exacerbate consumer's mistakes. Since their pioneering contribution, many articles have explored the specific ways to exploit consumer naivety.

consumer is left with positive surplus. Thus, the firm has an incentive to increase the first-period optimal threshold to minimize the part of the consumer surplus that it cannot extract, namely Δ, which enters negatively into its profit function. The firm bears all the costs of lowering p_1, but only a fraction of its benefits, and it finds it optimal to under-invest as well on incentivising consumption. This leads to inefficiently low probability of consumption for the first unit relative to the efficient sophisticated case.

Similarly, there is under-consumption of the second unit. The optimal second unit threshold for the naive consumer is always greater than that of the sophisticated consumer, $v_2^{N*} > v_2^{S*}$. The firm prefers to extract some of the surplus that cannot ex ante, even if, in this way, lowers the probability of consumption.

As it has already been mentioned, even if the consumer always consumes less than the optimal, she is left with a positive consumer surplus, because the firm cannot extract it all. This misperception rent would give an incentive to the consumer to remain naive and not pay the cost of becoming sophisticated and learning her true type. Remaining naive is beneficial for her both because the firm cannot extract all of her surplus and because she avoids paying any information cost to become sophisticated. Though, as the naive consumer underestimate the value of the contract, she will not purchase even if the value for the contract is greater than its cost, leading to a participation distortion. Therefore, staying naive is beneficial only for the consumers who participate in the market, but it is not beneficial for the ones that do not participate exactly because of their naivety. Heidhues and Kőszegi (2015) shows that the magnitude of such an inefficiency that arise in the extensive margin can be significantly large.

A typical concern is whether naivety goes away with learning or can be mitigated when appropriate feedback is provided (Bolger and Önkal-Atay, 2004), although, the consumers may learn slowly (Grubb and Osborne, 2015), or forget what is learned

(Agarwal et al., 2013). Three-part tariffs, nonetheless, are optimal even when consumers are partially naive (see Appendix B), which could resemble the period in which she learns her true type.

Addictive goods

Several studies identify symptoms of addictions among young adults and adolescents in the markets of telecommunication.[14]

If the good is addictive past consumption is affecting the current valuation of the good. The difference in the utility between consuming the good and the outside option increases with past consumption as when the good is habit forming, $v_{t,good} - v_{t,outside} = v_t - \theta\beta q_{t-1}$. However, when the good is addictive, I assume that it is the utility of the outside option that decreases with past consumption whereas the base utility of the good stays constant, namely $\frac{\partial v_{t,good}}{\partial q_{t-1}} = 0$ and $\frac{\partial v_{t,outside}}{\partial q_{t-1}} < 0$.

A sophisticated addicted consumer internalizes the cost of addiction. Her optimal first period threshold now is $v_{1S}^{a*} = p_1 + \int_{p1}^{p2-\beta}(1 - F(v_2))dv_2 + \beta$.[15] The greater is the loss the consumer has in her outside option from the first period consumption, β, the greater is the threshold v_{1S}^{a*} above which she consumes and thus the less probable is for her to consume.

A naive addicted consumer has exactly the same expected gross utility with the naive habit forming consumer. Though, there are significant differences with respect to the implications of her naivete. The sophisticated consumer would consume less often than the naive consumer in the first period, $v_{1S}^{a*} > v_{1N}^{a*}$, because being sophisticated she can foresee that her future utility flow decreases with current consumption. On the contrary, the naive consumer does not anticipate this decrease, and over-consumes in the first period. This makes it more likely that the consumer will face the decision

[14]See Billieux (2012) for a review of the psychological literature on the problematic use of mobile phones.

[15]See Appendix A for the full analysis of the model.

of consuming the second unit. A unit that she consumes more often than expected, as $v_{2S}^{a*} > v_{2N}^{a*}$. Moreover, the naive consumer would over-value the offered contract at the contract period, for the same reason. The optimal contract offered to sophisticated and naive consumer is provided in Proposition (3).

Proposition 3. *Addictive goods: If the consumer is sophisticated addicted the optimal contract is two-part tariff, $p_1^{a,S} = p_2^{a,S} = c$ and $F^{a,S} = U^{a,S}$. If the consumer is naively addicted the optimal marginal pricing scheme is $\{p_1^{a,N} = 0, p_2^{a,N} > c\}$ if $c = 0$, $\{p_1^{a,N} < c, p_2^{a,N} > c\}$ if $c > 0$ and the fixed fee $F^N = U^{a,N}(\mathbf{p^N})$ equals the gross perceived consumer surplus.*

Proof: See Appendix A.

The welfare implications of addictive goods are the opposite of the ones of habit forming goods. The overvaluation of the offered contract would lead to exploitation of the consumers, because at the contractual period the naive consumer would be willing to pay more than what the actual expected value of the contract is. This would lead to the inverse participation distortion with respect to the habit forming case, namely consumers would participate in the market even if their actual valuation for the contract is below its cost. Thus, summing up, the participation is more than the efficient one and the consumer who participates is exploited when the good is addictive.[16]

Uninformed Monopolist

Now suppose that the firm cannot observe the type of the consumer. However, it is common knowledge that the probability that the consumer is sophisticated is γ.

The screening is done with respect to the pricing scheme.[17] The firm offers a menu of contracts. Without any loss of generality, I can restrict the analysis to the

[16] See Appendix A for the full analysis of the model with addictive good.

[17] I use the taxation principle because it is closer to what we observe. Moreover, the nature of the direct problem with multi-dimensional uncertainty makes the problem not tractable. The uncertainty

case in which it offers as many contracts as the number of types; thus, two. Let $\boldsymbol{\sigma}^N = \{F^N, p_1^N, p_2^N\}$ and $\boldsymbol{\sigma}^S = \{F^S, p_1^S, p_2^S\}$ be the contracts intended for the naive and the sophisticated consumer, respectively. This menu of tariffs completely identifies the allocation.

The maximization problem of the firm is:

$$\max_{\boldsymbol{\sigma}^S, \boldsymbol{\sigma}^N} \quad \gamma\big(S^S(\mathbf{p^S}) - (U^S(\mathbf{p^S}) - F^S)\big) + (1-\gamma)\big(S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - \Delta\big)$$

$$\text{s.t.} \quad \begin{array}{ll} U^N(\mathbf{p^N}) - F^N \geq 0 & IR_N \\ U^S(\mathbf{p^S}) - F^S \geq 0 & IR_S \\ U^N(\mathbf{p^N}) - F^N \geq U^N(\mathbf{p^S}) - F^S & IC_N \\ U^S(\mathbf{p^S}) - F^S \geq U^S(\mathbf{p^N}) - F^N & IC_S. \end{array}$$

where $\Delta = \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N})$. $U^N(\mathbf{p^N}) - F^N \geq 0$ and $U^S(\mathbf{p^S}) - F^S \geq 0$ are the *participation constraints* of the naive and sophisticated consumer, respectively. Moreover, $U^N(\mathbf{p^N}) - F^N \geq U^N(\mathbf{p^S}) - F^S$ and $U^S(\mathbf{p^S}) - F^S \geq U^S(\mathbf{p^N}) - F^N$ are the *incentive compatibility constraints*: that is, each type should not have any incentive to mimic the other at the optimal allocation. Note that the participation constraint must hold ex ante. Once the consumer has signed the contract, she is obliged to comply for the whole contract period, even if she would have an incentive to deviate.

The naive consumer at the contract period does not know that she will acquire a habit and that her utility will be greater than the one she expects. Marginal cost pricing creates a larger expected utility for the sophisticated consumer than for the naive consumer, thus the firm charges a fixed fee that the naive consumer would not be willing to pay. This suggests that the incentive compatibility constraint of naive, IC_N, not to bind at the optimum.[18]

is multi-dimensional because it concerns both the type of consumer at the contracting period, and the valuation of the good.

[18] I verify at Proposition 3 that this assumption holds at the optimum.

On the other hand, the optimal full information contract is not incentive compatible for the sophisticated consumer, because she would prefer the contract of the naive consumer rather than her own first best allocation. Even if the marginal pricing is distorted, it allows her to enjoy a strictly positive surplus equal to $U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})$. This suggests intuitively that it is the incentive compatibility constraint IC_S that bind in the second-best problem. This intuition is confirmed formally in the following Lemma, which characterizes which constraints bind and which ones do not:

Lemma 2. *At the solution to the asymmetric information model, constraints IR_N and IC_S bind, whereas constraint IR_S and IC_N are redundant. More specifically:*

$$\begin{aligned} &U^N(\mathbf{p^N}) - F^N = 0 && IR_N \\ &U^S(\mathbf{p^S}) - F^S > 0 && IR_S \\ &U^S(\mathbf{p^S}) - F^S = U^S(\mathbf{p^N}) - F^N && IC_S. \\ &U^N(\mathbf{p^N}) - F^N > U^N(\mathbf{p^S}) - F^S && IC_N \end{aligned}$$

The assumption that IC_N is slack at the optimum implies that there will be marginal cost pricing for the sophisticated consumer.

Proof: See Appendix A

Thus, taking into consideration Lemma 3, the maximization problem can be relaxed and becomes:

$$\max_{p_1^N, p_2^N} \Pi = \gamma \underbrace{\left(S^S(\mathbf{c}) - \overbrace{(U^S(\mathbf{p^N}) - U^N(\mathbf{p^N}))}^{\text{Information rent of Sophisticated}}\right)}_{\Pi^S} + (1-\gamma) \underbrace{\left(S^N(\mathbf{p^N}) - \overbrace{(\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))}^{\text{Mis-perception rent of Naive}}\right)}_{\Pi^N}$$

Interestingly, both types of consumers are left with a rent and the firm cannot extract all their surplus. The sophisticated consumer has an information rent due to the asymmetry of information. The naive consumer, even if she has no incentive to deviate, is left with a *mis-perception rent.* This rent is due to her naivety. She would not sign a more expensive contract at the contracting stage, and so she is left ex post with a *mis-perception rent* Δ that is bigger than her expected surplus at the contract period, $\Delta > U^N(\mathbf{p^N}) - F^N = 0$.

The solution of the relaxed maximization problem of the firm is described by Proposition 4.

Proposition 4. *The optimal screening contract that the firm offers to sophisticated and naive habit forming consumers is:*

- ***Sophisticated consumer****:* $p_1^S = c, \quad p_2^S = c, \quad F^S = U^S(\mathbf{p^S}) - U^S(\mathbf{p^N})$*;*
- ***Naive consumer****: if* $c = 0$ *then* $\{p_1^N = 0, \quad p_2^N > c\}$ *and if* $c > 0$ *then* $\{p_1^N < c, \quad p_2^N > c\}$ *and the fixed fee,* $F^N = U^N(\mathbf{p^N})$*, equals the perceived consumer gross surplus of the naive consumer.*

Proof: See Appendix A

The firm offers a menu of contracts consisting of a two-part tariff for the sophisticated consumer and a three-part tariff for the naive consumer. Qualitatively, the pricing patterns that are optimal under full information are still optimal under asymmetric information. If the fraction of sophisticated consumers is quite small, then the firm finds it optimal to offer only the contract intended for naive consumers and vice versa.

It remains to check that all the constraints are met, and in particular that the incentive compatibility constraint of the naive consumers is slack at the optimum. This is shown in the Appendix A.

The marginal prices of the contract of the sophisticated consumer $\{p_1^S, p_2^S\}$ remain equal to the marginal cost, whereas the fixed fee, F^S, decreases. Thus, naive consumers exert a positive externality on the sophisticated consumers.[19] On the other hand, the marginal prices for the naive consumer, $\{p_1^N, p_2^N\}$, are distorted upwards and the fixed fee, F^N is lower. Thus, there are two opposing effects on the welfare of naive consumers. However, it can be shown that overall, this type of consumer is worse off in the presence of the sophisticated ones. More specifically, the derivatives with respect to the marginal prices are:

$$\frac{d\Delta}{dp_1} < 0 \quad \text{and} \quad \frac{d\Delta}{dp_2} < 0.$$

This means that an increase in marginal prices decreases the mis-perception rent. The marginal prices are greater than in the full information case, and thus the naive consumer is worse off.

The profits of the firms decrease with respect to the full information case, both for the sophisticated and the naive consumer. The fact that the firm cannot exploit the naivety of the consumer, and at the same time cannot observe her type, decreases its profits.

As discussed before, also in the case of an uniformed monopolist the naive consumer is less likely to consume in the first period than the sophisticated because she mistakenly believes that she is not habit forming. The contract offered to the naive consumer exacerbates her mistake of underconsumption. The intuition in this case of an uninformed consumer is the same with the one of informed one.

Importantly, under-consumption leads to deadweight loss and allocative inefficiency. The consumer is left with a positive consumer surplus and this could be seen as a reason for no policy intervention. Though, there is also participation distortion, namely inefficiences in the extensive margin. There are consumers that would like to participate in the market, if they were sophisticated, but they do not. Thus,

[19]This is a quite common result in the Behavioral I.O. literature. See for example Gabaix and Laibson (2006)

the deadweight loss created by under-consumption and underparticipation could raise concerns. Regulatory authorities may consider the need for analysis of the possible policies that could alleviate this efficiency loss. For example, a possible intervention could be to inform consumers of their habit forming behaviors.

5 Competitive Markets

In this section, I introduce competition in the model. I consider the case of perfect competition both when the firms in the market are informed about the type of their consumers and when they are uninformed and they screen between them. Interestingly, the obtained optimal tariff does not depend on the assumption of monopolistic market structure.

Informed Perfect Competitors

Let first consider the case where the firms are informed about the type of the consumer, and they can distinguish who is sophisticated and who is not.

Let assume that there are enough many firms in the market, so none of the firms has any market power. The firms in equilibrium will charge the prices that maximize the consumer surplus subject to the constraint that they can participate to the market, namely they have non negative profits. Perfect competition drives the profits of the firms to zero, firms continue entering the market up until their participation constraint is binding.

Sophisticated habit forming consumers: In this case the maximization problem of the firms is:

$$\max_{\boldsymbol{\sigma}^{\mathbf{S}}} \quad \{U^S(\mathbf{p^S}) - F^S\} \quad s.t. \quad \Pi^S = S^S(\mathbf{p^S}) - (U^S(\mathbf{p^S}) - F^S) \geq 0$$

As there is perfect competition, the constraint is binding, $S^S(\mathbf{p^S}) = U^S(\mathbf{p^S}) - F^S$ and the objective function becomes equivalent to the one of the monopolistic case, namely the social surplus of the sophisticated consumer $S^S(\mathbf{p^S})$. Thus, the optimal marginal prices are the same with the monopolistic case, $p_1^S = p_2^S = c$, with different fixed fee $F^S = 0$. Therefore, competition affects only the distribution of the surplus among firms and consumers.

Naive habit forming consumers: In this case the maximization problem of the firms is:

$$\max_{\boldsymbol{\sigma}^N} \quad U^N(\mathbf{p^N}) - F^N$$
$$s.t. \quad \Pi^N = S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - F^N) \geq 0$$

since there is perfect competition the participation constraint is binding and thus:

$$\Pi^N = S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - F^N) = 0 \Rightarrow S^N(\mathbf{p^N}) - \tilde{U}(\mathbf{p^N}) + U^N(\mathbf{p^N}) = U^N(\mathbf{p^N}) - F^N$$
$$\Rightarrow F^N = S^N(\mathbf{p^N}) - \tilde{U}(\mathbf{p^N})$$

Substituting the fixed fee into the objective function, the problem simplifies to:

$$\max_{\mathbf{p^N}} \quad S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))$$

As in the case of the sophisticated consumer, the new objective function is equivalent to the monopolist's one. Consequently, the marginal prices are the same as in the monopolistic case, $p_1^N < c$ and $p_2^N > c$, with different fixed fee, $F^N = \tilde{U}(\mathbf{p^N}) - S^N(\mathbf{p^N})$.

Uninformed Perfect Competitors

Let now consider the case in which the firms cannot observe the type of the consumer and they need to screen between them. If the firms believe that all consumers share the same demand characteristics, they are habit-forming, whereas consumers have heterogeneous expectations concerning their habit formation, then screening does not distort prices. The firms will compete perfectly, their profits will be driven to zero and they will offer a menu of contracts that maximize the expected utility of each respective type of consumer. The consumers find it incentive compatible to choose the tariff designed for them, as described by the single tariff model, even if they are not the only type in the market, and they could imitate another type. By the construction of the maximum in the single tariff model, the offered contracts are the ones maximizing the utility provided to respective type of consumers, and thus they are incentive compatible.

The above discussion is summarized in Proposition (5).

Proposition 5. *Perfect Competition: Perfect competition in the market affects only the distribution of the surplus produced between consumers and firms and not the marginal prices, which remain the same as the monopolistic case. The sophisticated consumers are offered a two part tariff contract and the naives a three part tariff contract both in the case of informed and uninformed firms.*

6 Conclusion

During the last decades, the provision of a menu of contracts consisting of two-part and three-part tariffs has become prevalent in a number of markets. Moreover, there is evidence that the consumption of communication services, such as cell phones and internet, is habit forming (Oulasvirta et al., 2012; Bianchi and Phillips, 2005). The literature has also identified symptoms of addiction to the mobile phone among young adults and adolescents (Billieux, 2012; Park, 2005).

This article shows that habit forming behavior can explain these observed pricing schemes. In particular, naive habit formation by consumers makes it optimal for the firm to charge a "three-part tariff".

I show that this pricing scheme is optimal if three conditions are met: (i) the consumption choice is made sequentially within the contract period; (ii) the consumer undervalues the offered contract at the contracting period; and (iii) the consumer underestimates high demand.

This explanation can be viewed as an alternative channel to the overconfidence model of Grubb (2009) that also explains this type of pricing scheme. I show that if the elements mentioned before are present, it is sufficient the consumer underestimates high demand for the introduction of the three-part tariff to be optimal; in contrast to Grubb (2009) she does not need to overestimate low demand.

Interestingly, the firm cannot exploit the consumer's naivety when the good is habit forming. Though, the naively addicted consumer can be exploited. Also in this case, she underestimates high demand. However, she overestimates the value of the contract, at the contracting period, which leads to being willing to pay more than her actual expected utility.

Moreover, this article claims that the observed menu of contracts could be explained by the existence of habit forming consumers with varying levels of sophistication about their habit forming behavior. It is shown that the firm finds it optimal to offer a two-part tariff to sophisticated consumers and a three-part tariff to naive ones.

The presence of naive consumers in the market exerts a positive externality to the sophisticated consumers, instead of the other way around. The sophisticated consumer has an incentive to pretend to be naive, even though she consumes less because, in this way, she is left with a rent. For this reason, the firm finds it optimal to leave information rent to sophisticated consumers.

The naive consumers are ex post worst off in the presence of sophisticated consumers, since the objective of the firm to make the contract intended for naive consumers less attractive to sophisticated ones leads to a decrease in the ex post misperception rent.

The presence of naive habit forming consumers in the market and three-part tariffs induce inefficiencies both in the intensive and in the extensive margin. The naive consumer under-consumes both units and participate in the market less than if she was sophisticated. Thus, there are serious welfare implications, and the need for a policy intervention to decrease the deadweight loss created seems requisite. A potential policy that could increase the overall welfare in the market would be to inform naive consumers of their behavior.

7 References

Agarwal, S., J. C. Driscoll, X. Gabaix, and D. Laibson. "Learning in the credit card market". Available at SSRN 1091623, 2013.

Armstrong, M. and J. Vickers. "Competitive price discrimination". *RAND Journal of Economics*, Vol. 32 (2001), pp. 579-605.

Armstrong, M. and J. Vickers. "Consumer protection and contingent charges". *Journal of Economic Literature*, Vol. 50 (2012), pp. 477-493.

Becker, G., M. Grossman, and K. Murphy. "Rational addiction and the effect of price on consumption". *The American Economic Review*, Vol. 81 (1991), pp. 237-241.

Bianchi, A. and J. G. Phillips. "Psychological predictors of problem mobile phone use". *CyberPsychology & Behavior*, Vol. 8 (2005), pp. 39-51.

Billieux, Joël. " Problematic use of the mobile phone: a literature review and a pathways model". *Current Psychiatry Reviews*, Vol. 8 (2012), pp. 299-307.

Bolger, F. and D. Onkal-Atay. "The effects of feedback on judgmental interval predictions". *International Journal of Forecasting*, Vol. 20 (2004), pp. 29-39.

Carbajal, J. C. and J. Ely. "Optimal contracts for loss averse consumers". Technical report, University of Queensland, School of Economics, 2012.

Courty, P. and Hao, L." Sequential screening". *The Review of Economic Studies*, Vol. 67 (2000), pp.697-717.

DellaVigna, S. and U. Malmendier. "Contract design and self-control: Theory and evidence". *The Quarterly Journal of Economics*, Vol. 119 (2004), pp. 353-402.

Driskill, R. and S. McCafferty, "Monopoly and oligopoly provision of addictive goods".

International Economic Review, Vol. 42 (2001), pp. 43-72.

Eliaz, K. and R. Spiegler. "Contracting with diversely naive agents". *The Review of Economic Studies*, Vol. 73 (2006), pp. 689-714.

Eliaz, K. and R. Spiegler. "Consumer optimism and price discrimination". *Theoretical Economics*, Vol. 3 (2008), pp. 459-497.

Esteban, S., E. Miyagawa, and M. Shum. "Nonlinear pricing with self-control preferences". *Journal of Economic Theory*, Vol. 135 (2007), pp. 306-338.

Fethke, G. and R. Jagannathan. "Habit persistence, heterogeneous tastes, and imperfect competition". *Journal of Economic Dynamics and Control*, Vol. 20 (1996), pp. 1193-1207.

Gabaix, X. and D. Laibson. "Shrouded attributes, consumer myopia, and information suppression in competitive markets". *The Quarterly Journal of Economics*, Vol. 121 (2006), pp. 505-540.

Grubb, M. D. "Selling to overconfident consumers". *The American Economic Review*, Vol. 99 (2009), pp. 1770-1807.

Grubb, M. D. "Consumer inattention and bill-shock regulation". *The Review of Economic Studies*, Vol. 82 (2014), pp. 219-257.

Grubb, M. D. and M. Osborne. "Cellular service demand: Biased beliefs, learning, and bill shock". *American Economic Review*, Vol. 105 (2015), pp. 234-271.

Hahn, J.-H., J. Kim, S.-H. Kim, and J. Lee. "Screening loss averse consumers". Technical report, mimeo, 2012.

Heidhues, P. and B. Koszegi. "On the Welfare Costs of Naiveté in the US Credit-Card Market". *Review of Industrial Organization*, Vol. 47 (2015), pp. 341-354.

Heidhues, P. and B. Koszegi. “Exploiting naivete about self-control in the credit market”. *The American Economic Review*, Vol. 100 (2010), pp. 2279-2303.

Herweg, F. and K. Mierendorff . “Uncertain demand, consumer loss aversion, and flat rate tariffs”. *Journal of the European Economic Association*, Vol. 11 (2013), pp. 399-432.

Inderst, R. and M. Peitz. “Informing Consumers about their own Preferences”. *International Journal of Industrial Organization*, Vol. 30 (2012), pp. 417-428.

Köszegi, B. “Behavioral Contract Theory.” *Journal of Economic Literature*, Vol. 52 (2014), pp. 1075-1118.

Lambrecht, A. and B. Skiera . “Paying too much and being happy about it: Existence, causes, and consequences of tariff-choice biases”. *Journal of Marketing Research*, Vol. 43 (2006), pp. 212-223.

Miao, C.-H.“ Consumer myopia, standardization and aftermarket monopolization”. *European Economic Review*, Vol. 54 (2010), pp. 931-946.

Miravete, Eugenio J. “The welfare performance of sequential pricing mechanisms.” *International Economic Review*, Vol 46 (2005), pp. 1321-1360.

Nakamura, E. and J. Steinsson. “Price setting in forward-looking customer markets”. *Journal of Monetary Economics*, Vol. 58 (2011), pp. 220-233.

Oulasvirta, Antti, Tye Rattenbury, Lingyi Ma, and Eeva Raita. “Habits make smartphone use more pervasive”. *Personal and Ubiquitous Computing*, Vol. 16 (2012), pp. 105-114.

Park, W. “Mobile phone addiction”. *Mobile Communications*, (2005), pp. 253-272.

Pollak, R. A. "The intertemporal cost of living index". *In Annals of Economic and Social Measurement*, Vol. 4 (1975), pp. 179-198.

Sandroni, A. and F. Squintani. "Overconfidence and adverse selection: The case of insurance". Technical report, working paper, 2010.

Spinnewijn, J. "Insurance and perceptions: how to screen optimists and pessimists.*The Economic Journal*, Vol 123 (2013), pp. 606-633.

8 Appendix A

Proof of Proposition 1

The firm choose a fixed fee such that $U^S(p^S) - F^S = 0$ so the profit is $\Pi^S = S^S$. Given $p_1 = p_2 = c$ the consumer maximizes U^S as we see bellow from the first and second order derivative.

$$\max_{v_{1S}} U^S(\mathbf{p^S}) = \int_{v_{1S}}^1 (v_1 - p_1)dF(v_1) + (1 - F(v_{1S})) \int_{p_2-\beta}^1 (v_2 + \beta - p_2)dF(v_2) + F(v_{1S}) \int_{p_1}^1 (v_2 - p_1)dF(v_2)$$

$$\frac{dU^S(\mathbf{p^S})}{dv_{1S}} = \left(-(v_{1S} - p_1) - \int_{p_2-\beta}^1 (v_2 + \beta - p_2)dF(v_2) + \int_{p_1}^1 (v_2 - p_1)dF(v_2)\right) f(v_{1S})$$

$$\left.\frac{dU^S(\mathbf{p^S})}{dv_{1S}}\right|_{(c,c)} = \left(-(v_{1S} - c) - \int_{c-\beta}^1 (v_2 + \beta - c)dF(v_2) + \int_{c}^1 (v_2 - c)dF(v_2)\right) f(v_{1S}) = 0$$

then the second order condition:

$$\frac{d^2U^S(\mathbf{p^S})}{dv_{1S}^2} = -f(v_{1S}) + f'(v_{1S}) \left(-v_{1S} - c - \int_{c-\beta}^1 (v_2 + \beta - c)dF(v_2) + \int_{c}^1 (v_2 - c)dF(v_2)\right)$$

$$\left.\frac{d^2U^S(\mathbf{p^S})}{dv_{1S}^2}\right|_{(c,c)} = -f(v_{1S}) < 0$$

Thus, since U^S is maximized at $\{c, c\}$ so maximizes S^S. The social surplus is maximized and the firm with the fixed fee it extracts it all, $F^S = U^S$.

■

Proof of Proposition 2

The firm chooses the optimal contract $\boldsymbol{\sigma}^N = \{\mathbf{p}^N, F^N\}$ that maximizes its profits subject to the constraint that the naive consumer is willing to participate:

$$\begin{aligned}\max_{\boldsymbol{\sigma}^N} \Pi^N =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N})) \\ =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - \Delta \\ \text{s.t.} \quad & U^N - F^N \geq 0\end{aligned}$$

and optimal consumption rule is:

$$v_{1N}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2))dv_2$$

The expected gross surplus is:

$$\begin{aligned}S^N(\mathbf{p^N}) = & \int_{v_{1N}^*}^{1} (v_1 - c)dF(v_1) + (1 - F(v_{1N}^*)) \int_{p_2-\beta}^{1} (v_2 + \beta - c) f(v_2)dv_2 \\ & + F(v_{1N}^*) \int_{p_1}^{1} (v_2 - c)dF(v_2)\end{aligned}$$

The naive consumer for the first unit does not know that she is habit forming and uses her naive optimal threshold. In the second period she realizes her habit formation and so she updates her optimal threshold with one of the sophisticated habit forming consumer.

Moreover, Δ is the difference between the perceived and the actual utility of the consumer.

$$\Delta = \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N})$$

The firm finds it optimal to charge a fixed fee that makes the participation constraint biding. Thus, the objective function is:

$$
\begin{aligned}
\Pi^N =&S^N(\mathbf{p^N}) - \Delta = \\
=&\int_{v_{1N}^*}^1 (v_1 - c)dF(v_1) + F(v_{1N}^*)\int_{p_1}^1 (v_2 - c)dF(v_2) + (1 - F(v_{1N}^*))(1 - F(p_2 - \beta))(p_2 - c) \\
&+ (1 - F(v_{1N}^*))\int_{p_2}^1 (v_2 - p_2)dF(v_2)
\end{aligned} \tag{5}
$$

The maximization problem becomes:

$$\max_{\mathbf{p^N}} \Pi^N = S^N(\mathbf{p^N}) - \Delta$$

,where $\mathbf{p^N} = \{p_1, p_2\}$. The first order conditions with respect to p_1 is:

$$\frac{d\Pi}{dp_1} = \frac{\partial \Pi}{\partial p_1} + \frac{\partial \Pi}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1}$$

The partial derivatives are:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial p_1} &= F(v_{1N}^*)(-1)(p_1 - c)f(p_1) \\
\frac{\partial \Pi}{\partial v_{1N}^*} &= (-F(p_1)(p_1 - c) - (1 - F(p_2 - \beta))(p_2 - c))f(v_{1N}^*) \\
\frac{\partial v_{1N}^*}{\partial p_1} &= F(p_1)
\end{aligned}
$$

Then, the total derivative is:

$$\frac{d\Pi^N}{dp_1} = -F(v_{1N}^*)(p_1 - c)f(p_1) - \big(F(p_1)(p_1 - c) + (1 - F(p_2 - \beta))(p_2 - c)\big)f(v_{1N}^*)F(p_1) = 0 \tag{6}$$

The first order conditions with respect to p_2 is:

$$\frac{d\Pi^N}{dp_2} = \frac{\partial \Pi^N}{\partial p_2} + \frac{\partial \Pi^N}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_2}$$

The partial derivatives are:

$$\frac{\partial \Pi^N}{\partial p_2} = -(p_2-c)f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(v_{1N}^*))(1-F(p_2-\beta))$$
$$-(1-F(v_{1N}^*)(1-F(p_2))$$
$$\frac{\partial v_{1N}^*}{\partial p_2} = 1-F(p_2)$$

Then, the total derivative is:

$$\begin{aligned}\frac{\partial \Pi^N}{\partial p_2} &= -(p_2-c)f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta)) \\ &- \big(F(p_1)(p_1-c)+(1-F(p_2-\beta))(p_2-c)\big)f(v_{1N}^*)(1-F(p_2)) = 0\end{aligned} \tag{7}$$

Solving this equation (6) with respect to p_1, I get:

$$p_1 = c-(p_2-c)\frac{(1-F(p_2-\beta))f(v_{1N}^*)F(p_1)}{F(v_{1N}^*)f(p_1)+F(p_1)^2 f(v_{1N}^*)}$$

Then, substituting p_1 in equation (7), I get:

$$\begin{aligned}\frac{\partial \Pi^N}{\partial p_2} =& -(p_2-c)f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta)) \\ &- \big(F(p_1)(-(p_2-c)\frac{(1-F(p_2-\beta))f(v_{1N}^*)F(p_1)}{F(v_{1N}^*)f(p_1)+F(p_1)^2 f(v_{1N}^*)}) \\ &- (1-F(p_2-\beta))(p_2-c)\big)f(v_{1N}^*)(1-F(p_2)) = 0\end{aligned}$$

Solving the above with respect to p_2

$$p_2 = c + \frac{(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta))A}{f(p_2-\beta)(1-F(v_{1N}^*))A + (1-F(p_2-\beta))(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)f(p_1)} \tag{8}$$

where $A = F(v_{1N}^*)f(p_1)+F(p_1)^2 f(v_{1N}^*) > 0$.

Since $\frac{(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta))A}{f(p_2-\beta)(1-F(v_{1N}^*))A+(1-F(p_2-\beta))(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)f(p_1)} > 0$ then $p_2 > c$.

Moreover, substituting p_2 back to p_1:

$$p_1 = c - \frac{(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta))(1-F(p_2-\beta))f(v_{1N}^*)F(p_1)}{f(p_2-\beta)(1-F(v_{1N}^*))A+(1-F(p_2-\beta))(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)f(p_1)} \tag{9}$$

and since $\frac{(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta))(1-F(p_2-\beta))f(v_{1N}^*)F(p_1)}{f(p_2-\beta)(1-F(v_{1N}^*))A+(1-F(p_2-\beta))(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)f(p_1)} > 0$ then $p_1 < c$.

The objective function is a continuous function with domain $[0,1]^2$, thus a maximum should exist, as expected from the extreme value theorem. The interior optimum should satisfy the above first order conditions, namely that $p_1 < c$ and $p_2 > c$. I can show that there is no maximum at the borders of the domain and thus the interior optimum is the maximum of the function.

To show that at $p_1 = 1$ and $p_2 \in [0,1]$ there is no local maximum. If $p_1 = 1 \Rightarrow v_{1N}^* \geq 1$, consequently $F(v_{1N}^*) = 1$. Moreover, $\int_{p_1}^{1}(v_2 - c)dF(v_2) = 0$. Thus, by inspection of equation (5), I see that the profit of the firm at this corner equals zero, $\Pi|_{p_1=1, p_2\in[0,1]} = 0$, the firm decreasing p_1 will have positive profits, thus there is not a maximum at $p_1 = 1$ and $p_2 \in [0,1]$.

To show that at $p_1 = 0$ and $p_2 \in [0,1]$ there is no local maximum. The derivative of the profit function, Π^N, with respect to p_1, namely equation (6) at this area is:

$$\left.\frac{d\Pi^N}{dp_1}\right|_{\{p_1=0, p_2\in[0,1]\}} = cF(v_{1N}^*)f(p_1) > 0$$

since $F(0) = 0$. Thus, the profit function is increasing with respect to p_1 at this area, the firm finds it optimal to increase the price away of zero in order to increase its profit.

To show that at $p_1 \in [0,1]$ and $p_2 = 0$ there is no local maximum. The derivative of the profit function, Π^N, with respect to p_2, namely equation (7) at this area is:

$$\left.\frac{\partial\Pi^N}{\partial p_2}\right|_{\{p_1\in[0,1], p_2=0\}} = \big(c - F(p_1)(p_1 - c)\big)f(v_{1N}^*) > 0 \quad \text{if} \quad p_1 \leq c$$

if p_2 increases the profits increase and $p_2 = 0$ cannot be a maximum. If $p_1 > c$ then $\frac{\partial \Pi^N}{\partial p_2}\Big|_{\{p_1 \in [0,1], p_2 = 0\}} < 0$. Though, the first order condition with respect to p_1 at this area is:

$$\frac{d\Pi^N}{dp_1}\Bigg|_{\{p_1 \in [0,1], p_2 = 0\}} = -F(v_{1N}^*)(p_1 - c)f(p_1) - \big(F(p_1)(p_1 - c) + c(1 - F(p_2 - \beta))\big)f(v_{1N}^*)F(p_1) < 0$$

and thus the price p_1 should decrease in order for the profit to increase. The firm has an incentive to deviate from the point, up until $p_1 \leq c$. Again the maximum can not be at this corner, but in the interior with respect to this area.

To show that at $p_1 \in [0,1]$ and $p_2 = 1$ there is no local maximum. The derivative of the profit function, Π^N, with respect to p_2, namely equation (7) at this area is:

$$\frac{\partial \Pi^N}{\partial p_2}\Bigg|_{\{p_1 \in [0,1], p_2 = 1\}} = (1 - F(v_{1N}^*))\big(- (1 - c)f(1 - \beta) + (1 - F(1 - \beta))\big)$$

$\frac{\partial \Pi^N}{\partial p_2}\Big|_{\{p_1 \in [0,1], p_2 = 1\}} < 0$, if the cost is $c < 1 - \frac{1 - F(1-\beta)}{f(1-\beta)}$ or for relatively small β. The firm would find it optimal to deviate and decrease the price p_2 below $p_2 = 1$ in order to increase its profits.

$\frac{\partial \Pi^N}{\partial p_2}\Big|_{\{p_1 \in [0,1], p_2 = 1\}} > 0$, if $c \geq 1 - \frac{1 - F(1-\beta)}{f(1-\beta)}$ or for relatively big β. Then, $p_2 = 1 > c$ can happen at the optimum. But still $p_1 < c$ holds at the optimum, since p_1 is in the interior for $p_2 \in [0,1]$, as shown before. Thus, the characteristics of the optimum are still satisfied.

Hence, the function has an interior maximum that satisfies the first order conditions, equation (6) and (7). Thus, $p_1^* < c$, $p_2^* > c$ and a fixed fee $F^{N*} = U^N(p_1^*, p_2^*)$ which is an approximation of a three part tariff.

■

Proof of Lemma 2

IR_N bind: Otherwise increasing the fixed fee both of the sophisticated and the naive consumer by a small positive ϵ would preserve the IR_N, would not affect the IC_S and IC_N, and raise profits which contradicts to $\boldsymbol{\sigma}^S$ and $\boldsymbol{\sigma}^N$ be optimal.

IC_S bind: Suppose not, so that $U^S(\mathbf{p^S}) - F^S > U^S(\mathbf{p^N}) - F^N$. Then, the firm could raise the fixed fee of the sophisticated consumer, F^S, relaxing IC_N, without affecting IR^N and without violating the IC_S, but increasing its profits and this would be a profitable deviation. Thus, IC_S binds at the optimum.

IR_S slack: I show that if IR_N and IC_S hold at the optimum then IR_S can be discarded.

$$U^S(\mathbf{p^S}) - F^S \geq U^S(\mathbf{p^N}) - F^N \geq U^N(\mathbf{p^N}) - F^N \geq 0 \Rightarrow U^S(\mathbf{p^S}) - F^S \geq 0$$

IC_S slack: The assumption that IC_N is slack at the optimum implies that there will be marginal cost pricing and first best allocation for the sophisticated consumer. Suppose not then setting $\{p_1^S, p_2^S\}$ equal to $\{c, c\}$ whereas keeping $U^S - F^S$ constant would keep the incentive compatibility and the participation constraint of the sophisticated unaffected. Moreover, it would not violate the incentive constraint of the naive, IC_N, because it is relaxed. But this would increase the surplus and the profits of the firm from the sophisticated consumers, thus a contradiction. I verify that this assumption holds at the optimum at Proposition 4.

■

Proof of Proposition 4

Given Lemma 3 and the assumption that IC^N is slack, the firm's problem can be relaxed. I need to maximize the profit function only with respect to $\{p_1^N, p_2^N\}$. The fixed fees of the naive consumer will be derived from the binding IR_N, thus $F^N = U^N(\mathbf{p^N})$. Moreover, as expected the incentive compatibility constraint of the sophisticated consumer, IC_S, binds at the optimum. It could be written as:

$$U^S(\mathbf{p^S}) - F^S = U^S(\mathbf{p^N}) - F^N \Rightarrow U^S(\mathbf{p^S}) - F^S = U^S(\mathbf{p^N}) - F^N - (U^N(\mathbf{p^N}) - F^N)$$

since also IR_N binds at the optimum then $U^S(\mathbf{p^S}) - F^S = U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})$. Then the fixed fee of the sophisticated consumer can be derived from $F^S = U^S(\mathbf{p^S}) - \big(U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})\big)$, which is the expected utility of the consumer from this contract minus her information rent of knowing that she is sophisticated. Given the discussion before, the firm's problem can be reduced to:

$$\max_{p_1^N, p_2^N} \Pi = \gamma \underbrace{\left(S^S(\mathbf{c}) - (U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})\right)}_{\Pi^S} + (1-\gamma) \underbrace{\left(S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))\right)}_{\Pi^N}$$

then the first order condition with respect to p_1^N is:

$$\frac{d\Pi}{dp_1^N} = \gamma \frac{d\Pi^S}{dp_1^N} + (1-\gamma)\frac{d\Pi^N}{dp_1^N} = 0$$

,where

$$\frac{d\Pi^S}{dp_1^N} = \frac{\partial \Pi^S}{\partial p_1^N} + \frac{\partial \Pi^S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1^N} + \frac{\partial \Pi^S}{\partial v_{1S}^*}\frac{\partial v_{1S}^*}{\partial p_1^N}$$
$$\frac{d\Pi^N}{dp_1^N} = \frac{\partial \Pi^N}{\partial p_1^N} + \frac{\partial \Pi^N}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1^N} + \frac{\partial \Pi^N}{\partial v_{1S}^*}\frac{\partial v_{1S}^*}{\partial p_1^N}$$

and

$$\frac{\partial \Pi^S}{\partial p_1^N} = (F(v_{1S}^*) - F(v_{1N}^*))(2 - F(p_1^N))$$

Moreover, the envelope condition implies that $\frac{\partial \Pi^S}{\partial v_{1N}^*} = 0$ and $\frac{\partial \Pi^S}{\partial v_{1S}^*} = 0$. The $\frac{\partial \Pi^N}{\partial p_1^N}$ is as before at the single tariff - informed monopolist model the equation (6). Then the first order condition becomes:

$$\frac{d\Pi}{dp_1^N} = -\gamma\Big((F(v_{1S}^*) - F(v_{1N}^*))(2 - F(p_1^N))\Big) - (1-\gamma)\Bigg(F(v_{1N}^*)(p_1^N - c)f(p_1) + \big(F(p_1^N)(p_1^N - c) + (1 - F(p_2^N - \beta))(p_2^N - c)\big)f(v_{1N}^*)F(p_1^N)\Bigg) = 0 \tag{10}$$

Solving with the respect to p_1^N:

$$p_1^N = c - (p_2^N - c)\frac{(1 - F(p_2 - \beta))f(v_{1N}^*)F(p_1)}{A} + \frac{\gamma}{1-\gamma}\frac{(F(v_{1N}^*) - F(v_{1S}^*))(2 - F(p_1^N))}{A} \tag{11}$$

, where $A = F(v_{1N}^*)f(p_1) + F(p_1)^2 f(v_{1N}^*)$ as in Proposition 2.

The first order condition with respect to p_2^N is:

$$\frac{d\Pi}{dp_2^N} = \gamma\frac{d\Pi^S}{dp_2^N} + (1-\gamma)\frac{d\Pi^N}{dp_2^N} = 0$$

, where

$$\frac{d\Pi^S}{dp_2^N} = \frac{\partial\Pi^S}{\partial p_2^N} + \frac{\partial\Pi^S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_2^N} + \frac{\partial\Pi^S}{\partial v_{1S}^*}\frac{\partial v_{1S}^*}{\partial p_2^N}$$
$$\frac{d\Pi^S}{dp_2^N} = \frac{\partial\Pi^S}{\partial p_2^N} + \frac{\partial\Pi^S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_2^N} + \frac{\partial\Pi^S}{\partial v_{1S}^*}\frac{\partial v_{1S}^*}{\partial p_2^N}$$

where

$$\frac{\partial\Pi^S}{\partial p_2^N} = (1 - F(v_{1N}^*))(1 - F(p_2^N)) - (1 - F(v_{1S}^*))(1 - F(p_2^N - \beta))$$

, given again the envelope conditions and that $\frac{\partial\Pi^N}{\partial p_2^N}$ is as the single tariff - informed monopolist model the equation (7), then the first order condition becomes:

$$\frac{d\Pi^S}{dp_2^N} = -\gamma\Bigg((1 - F(v_{1N}^*))(1 - F(p_2^N)) - (1 - F(v_{1S}^*))(1 - F(p_2^N - \beta))\Bigg) + (1-\gamma)\Bigg(-(p_2 - c)f(p_2 - \beta)(1 - F(v_{1N}^*)) + (1 - F(v_{1N}^*))(F(p_2) - F(p_2 - \beta)) - \big(F(p_1)(p_1 - c) + (1 - F(p_2 - \beta))(p_2 - c)\big)f(v_{1N}^*)(1 - F(p_2))\Bigg) = 0 \tag{12}$$

and

$$\begin{aligned}\frac{d\Pi^S}{dp_2^N} =& -\gamma\Big((1-F(v_{1N}^*))(1-F(p_2^N)) - (1-F(v_{1S}^*))(1-F(p_2^N-\beta))\Big)\\ &+(1-\gamma)\Big(-p_2\big(f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(p_2-\beta))f(v_{1N}^*)(1-F(p_2))\big)\\ &+c\big(f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(p_2-\beta))f(v_{1N}^*)(1-F(p_2))\big)\\ &+(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta)) - F(p_1)f(v_{1N}^*)(1-F(p_2))(p_1-c)\Big) = 0\end{aligned}$$

Solving with respect to p_2^N,

$$\begin{aligned}p_2^N =& -\frac{\gamma}{(1-\gamma)}\left(\frac{(1-F(v_{1N}^*))(1-F(p_2^N)) - (1-F(v_{1S}^*))(1-F(p_2^N-\beta))}{f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(p_2-\beta))(1-F(p_2))f(v_{1N}^*)}\right)\\ &+c-(p_1-c)\frac{F(p_1)f(v_{1N}^*)(1-F(p_2^N))}{f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)}\\ &+\frac{(1-F(v_{1N}^*))(F(p_2^N)-F(p_2-\beta))}{f(p_2-\beta)(1-F(v_{1N}^*)) + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)}\end{aligned}$$

substituting p_1^N for equation (11):

$$\begin{aligned}p_2^N =& c+\frac{(1-F(v_{1N}^*))(F(p_2^N)-F(p_2^N-\beta))A}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\\ &+\frac{\gamma}{1-\gamma}\frac{(1-F(v_{1S}^*))(1-F(p_2^N-\beta)) - (1-F(v_{1N}^*))(1-F(p_2^N))A}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\\ &+\frac{\gamma}{1-\gamma}\frac{(F(v_{1S}^*)-F(v_{1N}^*))(2-F(p_1^N))}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\end{aligned} \tag{13}$$

and then substituting equation (13) into equation (11):

$$\begin{aligned}p_1^N =& c-\frac{(1-F(p_2^N-\beta))f(v_{1N}^*)F(p_1^N)}{A}\Bigg(\frac{(1-F(v_{1N}^*))(F(p_2^N)-F(p_2^N-\beta))A}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\\ &+\frac{\gamma}{1-\gamma}\frac{((1-F(v_{1S}^*))(1-F(p_2^N-\beta)) - (1-F(v_{1N}^*))(1-F(p_2^N)))A}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\\ &+\frac{\gamma}{1-\gamma}\frac{(F(v_{1S}^*)-F(v_{1N}^*))(2-F(p_1^N))}{f(p_2^N-\beta)(1-F(v_{1N}^*))A + (1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\Bigg)\\ &-\frac{\gamma}{1-\gamma}\frac{(F(v_{1N}^*)-F(v_{1S}^*))(2-F(p_1^N))}{A}\end{aligned} \tag{14}$$

and then

$$
\begin{aligned}
p_1^N =& c - \Bigg(\frac{(1-F(p_2^N-\beta))f(v_{1N}^*)F(p_1^N)(1-F(v_{1N}^*))(F(p_2^N)-F(p_2^N-\beta))}{f(p_2^N-\beta)(1-F(v_{1N}^*))A+(1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)} \\
&+ \frac{\gamma}{1-\gamma}\frac{((1-F(p_2^N-\beta))f(v_{1N}^*)F(p_1^N))(1-F(v_{1S}^*))(1-F(p_2^N-\beta))-(1-F(v_{1N}^*))(1-F(p_2^N))}{f(p_2^N-\beta)(1-F(v_{1N}^*))A+(1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)} \\
&+ \frac{\gamma}{1-\gamma}\frac{(1-F(p_2^N-\beta))f(v_{1N}^*)F(p_1^N)A^{-1}(F(v_{1S}^*)-F(v_{1N}^*))(2-F(p_1^N))}{f(p_2-\beta)(1-F(v_{1N}^*))A+(1-F(p_2^N-\beta))(1-F(p_2^N))f(v_{1N}^*)F(v_{1N}^*)f(p_1^N)}\Bigg) \\
&- \frac{\gamma}{1-\gamma}\frac{(F(v_{1N}^*)-F(v_{1S}^*))(2-F(p_1^N))}{A}
\end{aligned} \tag{15}
$$

, thus also in the screening model $p_1^N < c$ and $p_2^N > c$ but bigger than the ones of the single contract model.

I use the same argument as before in order to prove that the above prices are the ones that maximize the objective function.The objective function is again a continuous function with domain $[0,1]^2$, thus a maximum should exist, as expected from the extreme value theorem. The interior optimum should satisfy the above first order conditions, namely that $p_1 < c$ and $p_2 > c$. I can show that there is no maximum at the borders of the domain and thus the interior optimum is the maximum of the function.

To show that at $p_1^N = 1$ and $p_2^N \in [0,1]$ there is no local maximum. If $p_1^N = 1 \Rightarrow v_{1N}^* \geq 1$, then $F(v_{1N}^*) = 1$. The profit of the firm made from the naive consumer at this corner equals zero, $\Pi^N|_{p_1=1,p_2\in[0,1]} = 0$, the firm decreasing p_1^N will have positive profits, thus there is not a maximum at $p_1^N = 1$ and $p_2^N \in [0,1]$.

To show that at $p_1^N = 0$ and $p_2^N \in [0,1]$ there is no local maximum. I evaluate the derivative of the profit function, Π, with respect to p_1^N, namely equation (10) at this area:

$$
\left.\frac{d\Pi}{dp_1}\right|_{\{p_1=0,p_2\in[0,1]\}} = 2\gamma(F(v_{1N}^*)-F(v_{1S}^*)) + (1-\gamma)cF(v_{1N}^*)f(p_1) > 0
$$

since $F(0) = 0$. Thus, the profit function is increasing with respect to p_1 at this area, the firm finds it optimal to increase the price away of zero in order to increase its profit.

To show that at $p_1^N \in [0,1]$ and $p_2^N = 0$ there is no local maximum. I evaluate the derivative of the profit function, Π, with respect to p_2^N, namely equation (12) at this area:

$$\frac{\partial \Pi}{\partial p_2^N}\bigg|_{\{p_1^N \in [0,1], p_2^N=0\}} = \gamma(F(v_{1N}^*) - F(v_{1S}^*)) + (1-\gamma)\big(c - F(p_1^N)(p_1^N - c)\big)f(v_{1N}^*) > 0 \quad \text{if} \quad p_1^N \leq c$$

Then the firms deviates and finds it optimal to increase p_2^N away of zero. If $p_1^N > c$ then the first order condition with respect to p_1^N at this area is:

$$\begin{aligned}\frac{d\Pi}{dp_1^N}\bigg|_{\{p_1^N \in [0,1], p_2^N=0\}} =& \gamma(F(v_{1N}^*) - F(v_{1S}^*))(2 - F(p_1^N)) \\ &- (1-\gamma)\Bigg(F(v_{1N}^*)(p_1^N - c)f(p_1^N) + \big(F(p_1^N)(p_1^N - c) \\ &+ c(1 - F(p_2^N - \beta))\big)f(v_{1N}^*)F(p_1^N)\Bigg) < 0\end{aligned}$$

because $F(v_{1N}^*) = F(v_{1S}^*)$ at $p_2^N = 0$ and thus the price p_1^N should decrease in order for the profit to increase. The firm has an incentive to deviate and decrease p_1^N up until $p_1^N \leq c$. Then,the maximum is not at this corner but in the interior with respect to this area.

To show that at $p_1^N \in [0,1]$ and $p_2^N = 1$ there is no local maximum. I evaluate the derivative of the profit function, Π, with respect to p_2^N, namely equation (12) at this area:

$$\begin{aligned}\frac{\partial \Pi}{\partial p_2^N}\bigg|_{\{p_1^N \in [0,1], p_2^N=1\}} =& \gamma((1 - F(v_{1S}^*))(1 - F(1-\beta)) + \\ &+ (1-\gamma)(1 - F(v_{1N}^*))\big(-(1-c)f(1-\beta) + (1 - F(1-\beta))\big)\end{aligned}$$

If the cost is $c < 1 - \frac{(1-F(1-\beta))(1-\gamma(1-F(v_{1S}^*))}{f(1-\beta)(1-\gamma)(1-F(v_{1N}^*))}$ or for relatively small β, then $\frac{\partial \Pi}{\partial p_2^N}\Big|_{\{p_1^N \in [0,1], p_2^N=1\}} < 0$. The firm finds it optimal to decrease the price p_2 below $p_2 = 1$ in order to increase its profits. If $c \geq 1 - \frac{(1-F(1-\beta))(1-\gamma(1-F(v_{1S}^*))}{f(1-\beta)(1-\gamma)(1-F(v_{1N}^*))}$ or for relatively big β, then $\frac{\partial \Pi}{\partial p_2^N}\Big|_{\{p_1^N \in [0,1], p_2^N = 1\}} > 0$. Then, it could be that $p_2^N = 1 \geq c$. Still, it holds that $p_1^N < c$ since p_1^N as shown before is in the interior for $p_2^N \in [0,1]$. Thus, the characteristic of the optimum are still satisfied.

Hence, the function has an interior maximum that satisfies the first order conditions, equation (10) and (12). Thus, the optimal marginal prices are $p_1^* < c$, $p_2^* > c$.

To show that IC_N is satisfied: I need to show that $U^N(p^S) - F^S < 0$ at the optimum. Let F^S and $U^N(p^S)$ be defined as follows:

$$F^S = F(v_{1S}^*) \int_c^1 (1 - F(v_2))dv_2 + (1 - F(v_{1S}^*)) \int_{c-\beta}^1 (1 - F(v_2))dv_2 + \int_{v_{1S}^*}^1 (v_1 - c)dF(v_1)$$
$$U^N(p^S) = F(c) \int_c^1 (1 - F(v_2))dv_2 + (1 - F(c)) \int_c^1 (1 - F(v_2))dv_2 + \int_c^1 (v_1 - c)dF(v_1)$$

Given that at the optimum it holds that $v_{1S}^* < c$, I consider the difference, $U^N(p^S) - F^S < 0$ in each period. In the second period, the difference between the expected value of the second consumption opportunity when $q_1 = 0$ is positive, namely:

$$F(c) \int_c^1 (1 - F(v_2))dv_2 - F(v_{1S}^*) \int_c^1 (1 - F(v_2))dv_2$$

$$= (F(c) - F(v_{1S}^*)) \int_c^1 (1 - F(v_2))dv_2 > 0 \qquad (16)$$

The difference between the expected value of second consumption opportunity when $q_1 = 1$ is negative,

$$(1-F(v_{1S}^*))\int_{c-\beta}^{1}(1-F(v_2))dv_2 - (1-F(c))\int_{c}^{1}(1-F(v_2))dv_2 =$$
$$= F(v_{1S}^*)\left(\int_{c-\beta}^{c}(1-F(v_2)dv_2 + \int_{c}^{1}(1-F(v_2))dv_2\right)$$
$$+\int_{c}^{1}(1-F(v_2))dv_2 - F(c)\int_{c}^{1}(1-F(v_2))dv_2 - \int_{c-\beta}^{1}(1-F(v_2))dv_2 =$$

$$= -(F(c)-F(v_{1S}^*))\int_{c}^{1}(1-F(v_2))dv_2 - \int_{c-\beta}^{c}(1-F(v_2))dv_2 + F(v_{1S}^*)\int_{c-\beta}^{c}(1-F(v_2))dv_2 < 0 \tag{17}$$

In the first period, the difference between the expected value of first consumption opportunity is negative as well.

$$\int_{c}^{1}(v_1-c)dF(v_1) - \int_{v_{1S}^*}^{1}(v_1-c)dF(v_1) < 0 \tag{18}$$

The inequality $U^N(p^S) - F^S < 0$ holds at the optimum, because the difference between equations 17, that is negative, and 16, that is positive, is negative. More specifically:

$$-(1-F(v_{1S}^*))\int_{c-\beta}^{c}(1-F(v_2))dv_2 < 0$$

Thus, the inequality $U^N(p^S) - F^S < 0$ holds at the optimum. ■

Habit Formation versus Addiction

Consumers maximization problem

The interpretation of a good with addiction would be one in which the difference in utility between consuming the good and the outside option increases with past consumption, so that $v_{t,good} - v_{t,outside}$ is increasing in consumption q_{t-1}, namely:

$$\frac{\partial v_{t,good}}{\partial q_{t-1}} = 0 \quad \text{and} \quad \frac{\partial v_{t,outside}}{\partial q_{t-1}} < 0$$

Sophisticated addicted consumer: Let first consider the case where the consumer is aware of the fact that past consumption affects the value of her current outside option. Then, in the second period the sophisticated consumer knows that his expected utility will be:

$$\max_{v_2^*} U_2^{a,S} = \int_{v_{2S}^{a*}}^{1} (v_{2,good} - p_2) dF(v_2) + \int_{0}^{v_{2S}^{a*}} (v_{2,outside} - \beta) dF(v_2)$$

$$\frac{\partial U_2^{a,S}}{\partial v_{2S}^{a*}} = f(v_{2S}^{a*})\big(- v_{2S}^{a*} + p_2 + v_{2,outside} - \beta\big) = 0$$

$$v_{2S}^{a*} = p_2 + v_{2,outside} - \beta$$

Let for simplicity and without loss of generality assume that $v_{2,outside} = 0$, then the optimal second period threshold is:

$$v_{2S}^{a*} = \begin{cases} \max\{0, p_2 - \beta\} & \text{if} \quad q_1 = 1 \\ p_1 & \text{if} \quad q_1 = 0 \end{cases}$$

Then going backwards the first period optimal threshold should maximize:

$$\begin{aligned} \max_{v_{1S}^{a*}} U_1^{a,S} = & \int_{v_{1S}^{a*}}^{1} (v_1 - p_1) dF(v_1) + (1 - F(v_{1S}^{a*})) \int_{p_2-\beta}^{1} (v_2 - p_2) dF(v_2) \\ & + F(v_{1S}^{a*}) \int_{p_1}^{1} (v_2 - p_1) dF(v_2) + (1 - F(v_{1S}^{a*})) \int_{0}^{p_2-\beta} (-\beta) dF(v_2) \end{aligned} \tag{19}$$

$$\frac{\partial U_1^{a,S}}{\partial v_{1S}^{a*}} = f(v_1^*)\left(-v_{1S}^{a*} + p_1 - \int_{p_2-\beta}^{1}(v_2 - p_2)dF(v_2) + \int_{p_1}^{1}(v_2 - p_1)dF(v_2) - \int_{0}^{p_2-\beta}(-\beta)dF(v_2)\right) =$$

$$f(v_{1S}^{a*})\Bigg(- v_{1S}^{a*} + p_1 - \int_{p_2-\beta}^{1}(v_2 - p_2)dF(v_2) - \int_{p_2-\beta}^{1}(-\beta)dF(v_2)$$

$$+ \int_{p_2-\beta}^{1}(-\beta)dF(v_2) + \int_{p_1}^{1}(v_2 - p_1)dF(v_2) - \int_{0}^{p_2-\beta}(-\beta)dF(v_2)\Bigg) =$$

$$= f(v_{1S}^{a*})\left(-v_{1S}^{a*} + p_1 - \int_{p_2-\beta}^{1}(v_2 - p_2 + \beta)dF(v_2) + \int_{p_1}^{1}(v_2 - p_1)dF(v_2) + \beta\right) = 0$$

Then, the optimal first period threshold of the sophisticated addicted consumer is:

$$v_{1S}^{a*} = p_1 + \int_{p_1}^{p_2-\beta}(1 - F(v_2))dv_2 + \beta$$

The greater is the loss the consumer has in his outside option from the first-period consumption, β, the greater is the threshold v_{1S}^{a*} above which she consumes and thus the less probable it is for her to consume. The consumer is rationally addicted; she internalizes the cost of addiction and consumes only when the marginal value of the good is bigger than its marginal cost.

Naively addicted consumer: The maximization problem of the naive addicted consumer is the same with the one of the naive habit forming consumer. There are significant differences between the two cases with respect to their difference with sophisticated first period optimal threshold. In the case of the habit forming consumers, the naive consumers consume less often than the sophisticated ones. On the other hand, in the case of the addicted consumers, the naive ones consume more often than the sophisticated ones. The difference in the first period optimal threshold between the sophisticated addicted and the naive is:

$$
\begin{aligned}
v_{1S}^{a*} - v_{1N}^{a*} &= p_1 + \int_{p_1}^{p_2-\beta} (1 - F(v_2))dv_2 + \beta - \left(p_1 + \int_{p_1}^{p_2} (1 - F(v_2))dv_2\right) \\
&= \beta - \int_{p_2}^{p_2-\beta} (1 - F(v_2))dv_2
\end{aligned}
$$

The naive consumer over-consumes in the first period when $v_{1S}^{a*} - v_{1N}^{a*} > 0$, thus when $\beta > \int_{p_2}^{p_2-\beta}(1 - F(v_2))dv_2$. The sophisticated consumer will consume less often than the naive consumer if the decrease of the outside option because of past consumption, namely the cost of addiction, is bigger than the additional utility that she experiences from buying more often exactly because of her addiction.

Firm's maximization problem

I solve now for the optimal contract that the firm finds it optimal to offer in each case of the sophisticated addicted consumer and the naively addicted consumer.

Sophisticated addicted consumer: The firm chooses a fixed fee such that $U^S(p^S) - F^S = 0$, so the profit is $\Pi^S = S^S$. Given $p_1 = p_2 = c$, the consumer maximizes U^S as we see bellow from the first and second-order derivative.

$$
\begin{aligned}
\max_{v_{1S}^{a*}} U^S(\mathbf{p^S}) = &\int_{v_{1S}^{a*}}^{1} (v_1 - p_1)dF(v_1) + (1 - F(v_{1S}^{a*})) \int_{p_2-\beta}^{1} (v_2 - p_2)dF(v_2) \\
&+ F(v_{1S}^{a*}) \int_{p_1}^{1} (v_2 - p_1)dF(v_2) + (1 - F(v_{1S}^{a*})) \int_{0}^{p_2-\beta} (-\beta)dF(v_2)
\end{aligned} \tag{20}
$$

The first order derivative with respect to v_{1S}^{a*} at the $p_1 = p_2 = c$ is:

$$
\left.\frac{dU^S(\mathbf{p^S})}{dv_{1S}^{a*}}\right|_{(c,c)} = f(v_{1S}^{a*}) \left(-v_{1S}^{a*} + c - \int_{c-\beta}^{1} (v_2 - c + \beta)dF(v_2) + \int_{c}^{1} (v_2 - c)dF(v_2) + \beta\right) = 0
$$

then the second order condition:

$$\frac{d^2U^S(\mathbf{p^S})}{dv_{1S}^{a*}} = -f(v_{1S}^{a*}) + f'(v_{1S}^{a*}) \left(-v_{1S}^{a*} - c - \int_{c-\beta}^{1} (v_2 + \beta - c) dF(v_2) + \int_{c}^{1} (v_2 - c) dF(v_2) \right)$$

$$\left. \frac{d^2U^S(\mathbf{p^S})}{dv_{1S}^{a*}} \right|_{(c,c)} = -f(v_{1S}^{a*}) < 0$$

Thus, since U^S is maximized at $\{p_1^S, p_2^S\} = \{c, c\}$ so maximizes S^S. The social surplus is maximized and the firm with the fixed fee it extracts it all, $F^S = U^S$. The firm cannot do better than when $\{p_1, p_2\} = \{c, c\}$. If $\{p_1, p_2\} > \{c, c\}$ the firm has the incentive to decrease the marginal prices increasing the probability of consumption and the consumer surplus. If $\{p_1, p_2\} < \{c, c\}$ the prices are inefficiently low, producing less profits than when $\{p_1, p_2\} = \{c, c\}$.

Naively addicted consumer: The optimization problem of the firm when the consumer is naively addicted is:

$$\begin{aligned} \max_{\boldsymbol{\sigma}^N} \Pi =& S^{aN}(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - (\tilde{U}^{aN}(\mathbf{p^N}) - U^N(\mathbf{p^N})) \\ =& S^{aN}(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - \Delta^a \\ \text{s.t.} \quad & U^N - F^N \geq 0 \end{aligned}$$

where the expected gross surplus is:

$$S^{aN}(\mathbf{p^N}) = \int_{v_{1N}^{a*}}^{1} (v_1 - c)dF(v_1) + (1 - F(v_{1N}^{a*})) \int_{p_2-\beta}^{1} (v_2 - c)dF(v_2) + (1 - F(v_{1N}^{a*})) \int_{0}^{p_2-\beta} (-\beta)dF(v_2)$$

$$+ F(v_{1N}^{a*}) \int_{p_1}^{1} (v_2 - c)dF(v_2) =$$

$$= \int_{v_{1N}^{a*}}^{1} (v_1 - c)dF(v_1) + (1 - F(v_{1N}^{a*})) \int_{p_2-\beta}^{1} (v_2 - c + \beta)dF(v_2)$$

$$- (1 - F(v_{1N}^{a*})) \int_{p_2-\beta}^{1} \beta dF(v_2) + (1 - F(v_{1N}^{a*})) \int_{0}^{p_2-\beta} (-\beta)dF(v_2)$$

$$+ F(v_{1N}^{a*}) \int_{p_1}^{1} (v_2 - c)dF(v_2) =$$

$$= \int_{v_{1N}^{a*}}^{1} (v_1 - c)dF(v_1) + (1 - F(v_{1N}^{a*})) \int_{p_2-\beta}^{1} (v_2 - c + \beta)dF(v_2) + F(v_{1N}^{a*}) \int_{p_1}^{1} (v_2 - c)dF(v_2)$$

$$- (1 - F(v_{1N}^{a*}))\beta \Rightarrow S^{aN}(\mathbf{p^N}) = S^{N}(\mathbf{p^N}) - (1 - F(v_{1N}^{a*}))\beta$$

he expected gross social surplus, $S^{aN}(\mathbf{p^N})$, when in the market there are addicted consumers equals to the social surplus produced when there are habit forming consumer in the market minus $(1 - F(v_{1N}^{a*}))$. The mis-perception rent of the addicted consumer is:

$$\Delta^a = \tilde{U}^a(\mathbf{p^N}) - U^N(\mathbf{p^N})$$

$$= (1 - F(v_{1N}^{a*})) \left(\int_{p_2-\beta}^{1} (v_2 - c)dF(v_2) - \int_{0}^{p_2-\beta} \beta dF(v_2) - \int_{p_2}^{1} (v_2 - c)dF(v_2) \right)$$

$$= (1 - F(v_{1N}^{a*})) \Bigg(\int_{p_2-\beta}^{1} (v_2 - c + \beta)dF(v_2) - \int_{p_2-\beta}^{1} \beta dF(v_2) - \int_{0}^{p_2-\beta} \beta dF(v_2)$$

$$- \int_{p_2}^{1} (v_2 - c)dF(v_2) \Bigg) =$$

$$= (1 - F(v_{1N}^{a*})) \Bigg(\int_{p_2-\beta}^{1} (v_2 - c + \beta)dF(v_2) - \int_{p_2}^{1} (v_2 - c)dF(v_2) - \beta \Bigg)$$

$$\Rightarrow \Delta^a = \Delta - (1 - F(v_{1N}^{a*}))\beta$$

Then, the addicted consumers' mis-perception rent equals to the habit forming consumers' mis-perception rent minus $(1 - F(v_{1N}^{a*}))$.

Thus, since the optimal first period threshold of the naively addicted consumer is the same with the one of the naive habit forming, namely $v_{1N}^{a*} = v_{1N}^*$, then the profit function of the naively addicted consumer is equivalent to the naive habit forming:

$$\begin{aligned}\Pi^{aN} = S^{aN}(\mathbf{p^N}) - \Delta^a &= S^N(\mathbf{p^N}) - (1 - F(v_{1N}^{a*}))\beta - (\Delta - (1 - F(v_{1N}^{a*}))\beta) \\ &= S^N(\mathbf{p^N}) - \Delta \Rightarrow \Pi^{aN} = \Pi^N\end{aligned}$$

The firm will charge a fixed fee $F^N = U^N(\mathbf{p^N})$, and the participation constraint will be binding. The maximization problem of the monopolist becomes:

$$\max_{\mathbf{p^N}} \Pi = S^N(\mathbf{p^N}) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))$$

The profit function in the market with naively addicted consumers, is the same as in the one with naive habit-forming consumers. Thus, the marginal prices are as in the case of naive habit-forming consumers $p_1^{*aN} = p_1^{*N} < c$ and $p_2^{*aN} = p_2^{*N} > c$.

The difference between the models is in the mis-perception rent of the two type of naive consume Δ and Δ^a. In the case of the naive habit forming consumer $\Delta = \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}) > 0$ which means that $\tilde{U}(\mathbf{p^N}) > U^N(\mathbf{p^N})$, the naive consumer is left with positive surplus because of her naivete. On the other hand, the mis-perception rent of the naively addicted consumer can be negative.

$$\begin{aligned}\Delta^a &= \tilde{U}^a(\mathbf{p^N}) - U^N(\mathbf{p^N}) \\ &= (1 - F(v_{1N}^{a*}))\left(\int_{p_2-\beta}^{1}(v_2 - c)dF(v_2) - \int_0^{p_2-\beta}\beta dF(v_2) - \int_{p_2}^{1}(v_2 - c)dF(v_2)\right) \\ &= (1 - F(v_{1N}^{a*}))\left(\int_{p_2-\beta}^{p_2}(1 - F(v))dv - \beta\right) \\ \Rightarrow &\Delta^a < 0 \quad \text{if} \quad \int_{p_2-\beta}^{p_2}(1 - F(v))dv < \beta\end{aligned}$$

The actual expected consumer surplus is $\tilde{U}^a(\mathbf{p^N}) - F^N = \tilde{U}^a(\mathbf{p^N}) - U^N(\mathbf{p^N}) < 0$. Thus, if the cost of addiction β is bigger than the additional utility that she experiences from buying more often exactly because of her addiction, the naive consumer will be exploited. Her actual expected surplus is smaller than the one she expects to have when she signs the contract, and she decides whether to participate in the market or not. The optimal contract designed for the naively addicted consumer is the same as the one designed for the naive habit-forming consumer, but it has an opposite welfare effect. The naivety of the consumers causes an inefficient outcome both on the intensive and on the extensive margin. On the intensive margin, the inefficiency is because the naively addicted consumer overconsumes compared to the sophisticated one. Moreover in the extensive margin, because she participates in the market also when the value of the contract is smaller than its cost, she exploited by paying more than expected but also there is a significant participation distortion (see Heidhues and Kőszegi (2015))

Date-dependent marginal pricing

The marginal prices differ with respect to the unit and the period that are charged. The contract in this case is $\boldsymbol{\sigma}^N = \{p_1, p_2, p_3\}$, where p_1 if $q_1 = 1$, p_2 if $q_1 = 0, q_2 = 1$ and p_3 if $q_1 = 1, q_2 = 1$.

Consumer Maximization Problem: The problem of the naive consumer now is:

$$\max_{v_{1S}} U^S(\mathbf{p^S}) = \int_{v_{1S}}^{1} (v_1 - p_1) dF(v_1) + (1 - F(v_{1S})) \int_{p_3 - \beta}^{1} (v_2 + \beta - p_3) dF(v_2)$$
$$+ F(v_{1S}) \int_{p_2}^{1} (v_2 - p_2) dF(v_2)$$

Then the optimal consumption rule is $v_{1N}^* = p_1 + \int_{p_2}^{p_3} (1 - F(v_2)) dv_2$.

Firm Maximization Problem: The optimization problem of the firm when the consumer is naive habit forming is:

$$\begin{aligned}\max_{p_1,p_2,p_3} \Pi =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))\\ =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) + \Delta\\ \text{s.t.} \quad & U^N - F^N \geq 0\end{aligned}$$

The problem can be also written as:

$$\begin{aligned}\max_{v_{1N}^*,p_2,p_3} \Pi =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - (\tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}))\\ =& S^N(\mathbf{p^N}) - (U^N(\mathbf{p^N}) - F^N) - \Delta\\ \text{s.t.} \quad & U^N - F^N \geq 0\end{aligned}$$

and at the equilibrium p_1 can be derived by $p_1 = v_{1N}^* - \int_{p_2}^{p_3}(1 - F(v_2))dv_2$.

The expected gross surplus is the one produced in a market with a habit forming consumer.

$$\begin{aligned}S^N(\mathbf{p^N}) =& \int_{v_{1N}^*}^{1}(v_1 - c)dF(v_1) + (1 - F(v_{1N}^*)\int_{p_3-\beta}^{1}(v_2 + \beta - c)f(v_2)dv_2\\ &+ F(v_{1N}^*)\int_{p_1}^{1}(v_2 - c)dF(v_2)\end{aligned}$$

Moreover, Δ is the difference between the perceived and the optimal utility of the consumer:

$$\Delta = \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N})$$

The firm finds it optimal to charge a fixed fee that makes the participation constraint biding. Thus the objective function is:

$$
\begin{aligned}
\Pi^N =& S^N(\mathbf{p^N}) - \Delta = \\
=& \int_{v_{1N}^*}^1 (v_1 - c)dF(v_1) + F(v_{1N}^*) \int_{p_2}^1 (v_2 - c)dF(v_2) + (1 - F(v_{1N}^*)) \int_{p_3-\beta}^1 (v_2 + \beta - c)dF(v_2) \\
&- (1 - F(v_{1N}^*)) \left(\int_{p_3-\beta}^1 (v_2 + \beta - p_3)dF(v_2) - \int_{p_3}^1 (v_2 - p_3)dF(v_2) \right)
\end{aligned}
$$

The first order conditions with respect to p_2 is:

$$
\frac{d\Pi^N}{dp_2} = -f(v_{1N}^*)(p_2 - c)f(p_2) = 0 \Rightarrow p_2^* = c
$$

The first order conditions with respect to p_3 is:

$$
\begin{aligned}
\frac{d\Pi^N}{dp_3} &= (1 - F(v_{1N}^*))(-1)(p_3 - \beta + \beta - c)f(p_3 - \beta) \\
&- (1 - F(v_{1N}^*)) \left((-1) \int_{p_3-\beta}^1 f(v_2)dv_2 - (-1) \int_{p_3}^1 f(v_2)dv_2 \right) = \\
&= (1 - F(v_{1N}^*))\big((-p_3 + c)f(p_3 - \beta) + (1 - F(p_3 - \beta)) - (1 - F(p_3))\big) = \\
&= (1 - F(v_{1N}^*))\big((-p_3 + c)f(p_3 - \beta) - F(p_3 - \beta) + F(p_3))\big) = 0 \\
&\Rightarrow p_3^* = c + \frac{F(p_3) - F(p_3 - \beta)}{f(p_3 - \beta)}
\end{aligned}
$$

The first order conditions with respect to v_{1N}^* is:

$$\frac{d\Pi^N}{dv_{1N}^*} = \Big(- v_{1N}^* + c + \int_{p_2}^{1} (v_2 - c) dF(v_2) - \int_{p_3-\beta}^{1} (v_2 + \beta - c) dF(v_2) +$$

$$\int_{p_3-\beta}^{1} (v_2 + \beta - p_3) dF(v_2) - \int_{p_3}^{1} (v_2 - p_3) dF(v_2) \Big) f(v_{1N}^*) =$$

$$= \Big(- v_{1N}^* + c + \int_{p_2}^{1} (v_2 - c) dF(v_2) - \int_{p_3}^{1} (v_2 - p_3) dF(v_2) - (1 - F(p_3 - \beta))(p_3 - c) \Big) f(v_{1N}^*) =$$

$$= \Big(- v_{1N}^* + c - (1 - F(p_3 - \beta))(p_3 - c) + \int_{p_2}^{p_3} v_2 f(v_2) dv_2 - c(1 - F(p_2)) + p_3(1 - F(p_3)) \Big) f(v_{1N}^*) =$$

$$= \Big(- v_{1N}^* + c - (1 - F(p_3 - \beta))(p_3 - c) - c(1 - F(p_2)) + p_3(1 - F(p_3)) +$$

$$+ p_3 F(p_3) - p_2 F(p_2) - \int_{p_2}^{p_3} F(v) dv + p_2 - p_2 \Big) f(v_{1N}^*) =$$

$$= \Big(- v_{1N}^* + c - (1 - F(p_3 - \beta))(p_3 - c) + (p_2 - c)(1 - F(p_2)) + p_3 - p_2 - \int_{p_2}^{p_3} F(v) dv \Big) f(v_{1N}^*) =$$

$$= \Big(- v_{1N}^* + c - (1 - F(p_3 - \beta))(p_3 - c) + (p_2 - c)(1 - F(p_2)) + \int_{p_2}^{p_3} (1 - F(v)) dv \Big) f(v_{1N}^*) = 0$$

Substitute now v_{1N}^* and $p_2^* = c$ then:

$$\Rightarrow p_1^* = c - (p_3 - c)(1 - F(p_3 - \beta)) < c$$

The Hessian matrix is negative semi-definite at the optimum.

$$H|_{p_1^*, p_2^*, p_3^*} = \begin{Bmatrix} \frac{\partial \Pi^2}{\partial^2 v_{1N}^*} & \frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_2} & \frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_3} \\ \frac{\partial \Pi^2}{\partial p_2 \partial v_{1N}^*} & \frac{\partial \Pi^2}{\partial^2 p_2} & \frac{\partial \Pi^2}{\partial p_2 \partial p_3} \\ \frac{\partial \Pi^2}{\partial p_3 \partial v_{1N}^*} & \frac{\partial \Pi^2}{\partial p_3 \partial p_2} & \frac{\partial \Pi^2}{\partial^2 p_3} \end{Bmatrix} = \begin{Bmatrix} \text{-1} & 0 & + \\ 0 & \text{-} & 0 \\ 0 & 0 & \text{-} \end{Bmatrix}$$

$$\Delta_1 < 0$$

$$\Delta_2 = (-)(-) - 0 > 0$$

$$\Delta_3 = -1 \begin{Bmatrix} \text{-} & 0 \\ 0 & \text{-} \end{Bmatrix} - 0 \begin{Bmatrix} 0 & 0 \\ 0 & \text{-} \end{Bmatrix} + (+) \begin{Bmatrix} 0 & \text{-} \\ 0 & 0 \end{Bmatrix} < 0$$

$$\frac{\partial \Pi^2}{\partial^2 v_{1N}^*} = -1 + f'(v_{1N}^*)\Big(- v_{1N}^* + c + \int_{p_2}^{1}(v_2 - c)dF(v_2) + \int_{p_3-\beta}^{1}(v_2 + \beta - c)dF(v_2)$$
$$+ \int_{p_3-\beta}^{1}(1 - F(v_2))dv_2\Big) \Rightarrow \left.\frac{\partial \Pi^2}{\partial^2 v_{1N}^*}\right|_{p_1^*,p_2^*,p_3^*} = -1$$
$$\frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_2} = cf(p_2) + 1 - F(p_2) - p_2 f(p_2) - (1 - F(p_2)) \Rightarrow \left.\frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_2}\right|_{p_1^*,p_2^*,p_3^*} = 0$$
$$\frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_3} = f(v_{1N}^*)\big(F(p_3 - \beta) - F(p_3) + (p_3 - c)f(p_3 - \beta)\big) \Rightarrow \left.\frac{\partial \Pi^2}{\partial v_{1N}^* \partial p_2}\right|_{p_1^*,p_2^*,p_3^*} > 0$$
$$\frac{\partial \Pi^2}{\partial p_2 \partial v_{1N}^*} = -f'(v_{1N}^*)(p_2 - c)f(p_2) \Rightarrow \left.\frac{\partial \Pi^2}{\partial p_2 \partial v_{1N}^*}\right|_{p_1^*,p_2^*,p_3^*} = 0$$
$$\frac{\partial \Pi^2}{\partial^2 p_2} = -f(v_{1N}^*) \Rightarrow \left.\frac{\partial \Pi^2}{\partial^2 p_2}\right|_{p_1^*,p_2^*,p_3^*} < 0$$
$$\frac{\partial \Pi^2}{\partial p_2 \partial p_3} = 0$$
$$\frac{\partial \Pi^2}{\partial p_3 \partial v_{1N}^*} = -f(v_{1N}^*)\left((-p_3 + c)f(p_3 - \beta) - F(p_3 - \beta) + F(p_3)\right)) \Rightarrow \left.\frac{\partial \Pi^2}{\partial p_3 \partial v_{1N}^*}\right|_{p_1^*,p_2^*,p_3^*} = 0$$
$$\frac{\partial \Pi^2}{\partial p_3 \partial p_2} = 0$$
$$\frac{\partial \Pi^2}{\partial^2 p_3} = (1 - F(v_{1N}^*))\big((-p_3 + c)f'(p_3 - \beta) - 2f(p_3 - \beta) + f(p_3))\big) < 0$$

The $\frac{\partial \Pi^2}{\partial^2 p_3} < 0$ holds for β relative small and f non decreasing, or for β relative big and f non increasing.■

9 Appendix B

Partially Naive Consumers

Let a partially naive consumer with habit formation coefficient β and $0 < \theta < 1$. The consumer values at period t is:

$$\tilde{v}_t = v_t + \theta\beta q_{t-1} = v_t q_t + \tilde{\beta} q_{t-1}$$

The optimization problem of the firm is:

$$\max_{U^*,p_1,p_2} \Pi = S^S(\mathbf{p^P}) - U^P(\mathbf{p^P}) + (U^P(\mathbf{p^P}) - \tilde{U}(\mathbf{p^P})) =$$
$$= S^S(\mathbf{p^P}) - U^P(\mathbf{p^P}) - \Delta \quad \text{s.t.} \quad U^N(\mathbf{p^P}) \geq 0$$

and optimal consumption rule is:

$$v_{1P}^* = p_1 + \int_{p_1}^{p_2-\tilde{\beta}} (1 - F(v_2))dv_2$$

The expected gross surplus is the one produced in a market with a habit forming consumer.

$$S^S(\mathbf{p^P}) = \int_{v_{1P}^*}^1 (v_1 - c)dF(v_1) + F(v_{1P}^*) \int_{p_1}^1 (v_2 - c)dF(v_2)$$
$$+ \int_{v_{1P}^*}^1 \int_{p_2-\beta}^1 (v_2 + \beta - c)f(v_2)dv_2 dF(v_1)$$

Moreover, Δ is the difference between the true and the perceived of the consumer utility, namely:

$$\Delta = \tilde{U}(\mathbf{p^P}) - U^N(\mathbf{p^P}) = (1 - F(v_{1P}^*)) \left(\int_{p_2-\beta}^{p_2-\tilde{\beta}} (1 - F(v_2))dv_2 \right)$$

Then the first order conditions with respect to p_1 is:

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial S^S(\mathbf{p^P})}{\partial v_{1P}^*} \frac{\partial v_{1P}^*}{\partial p_1} + \frac{\partial \Delta}{\partial v_{1P}^*} \frac{\partial v_{1P}^*}{\partial p_1} + \frac{\partial \Delta}{\partial p_1} + \frac{\partial S^S(\mathbf{p^P})}{\partial p_1}$$

$$\frac{\partial S^S(\mathbf{p^P})}{\partial v^*_{1P}} = \left(-v^*_{1P} + c - \int_{p_2-\beta}^{1} (v_2 + \beta - c)dF(v_2)) + \int_{p_1}^{1} (v_2 - c)dF(v_2))\right) f(v^*_{1P})$$

$$\frac{\partial \Delta}{\partial v^*_{1P}} = -\left(\int_{p_2-\beta}^{p_2-\tilde{\beta}} (1 - F(v_2))dv_2\right) f(v^*_{1P})$$

$$\frac{\partial v^*_{1P}}{\partial p_1} = 1 - (1 - F(p_1)) = F(p_1)$$

$$\frac{\partial S^S(\mathbf{p^P})}{\partial p_1} = -F(v^*_{1P})(p_1 - c)f(p_1)$$

Then, the first order condition is:

$$\begin{aligned}
\frac{\partial \Pi}{\partial p_1} &= \left(-v^*_{1P} + c - \int_{p_2-\beta}^{1} (v_2 + \beta - c)dF(v_2)) + \int_{p_1}^{1} (v_2 - c)dF(v_2))\right) f(v^*_{1P})F(p_1) \\
&+ \left(\int_{p_2-\beta}^{p_2-\tilde{\beta}} (1 - F(v_2))dv_2\right) f(v^*_{1P})F(p_1) - F(v^*_{1P})(p_1 - c)f(p_1) = \\
&= \Big(- p_1 + (p_1 - p_2 + \tilde{\beta}) + c + c(1 - F(p_2 - \beta)) - c(1 - F(p_1))\Big) f(v^*_{1P})F(p_1) \\
&+ \Big(- (1 - (p_2 - \beta)F(p_2 - \beta)) - \beta(1 - F(p_2 - \beta))\Big) f(v^*_{1P})F(p_1) \\
&+ \Big(1 - p_1F(p_1) + (p_2 - \tilde{\beta} - p_2 + \beta)\Big) f(v^*_{1P})F(p_1) \\
&- F(v^*_{1P})(p1 - c)f(p_1) = 0
\end{aligned}$$

Thus,

$$p_1 = c - (p_2 - c)\frac{f(v^*_{1P})F(p_1)(1 - F(p_2 - \beta))}{(f(v^*_{1P})F(p_1)^2 + f(p_1)F(v^*_{1P}))}$$

The first order condition with respect to p_2 is:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial S^S(\mathbf{p^P})}{\partial v^*_{1P}}\frac{\partial v^*_{1P}}{\partial p_2} + \frac{\partial \Delta}{\partial v^*_{1P}}\frac{\partial v^*_{1P}}{\partial p_2} + \frac{\partial \Delta}{\partial p_2} + \frac{\partial S^S(\mathbf{p^P})}{\partial p_2}$$

The respective derivatives are:

$$\frac{\partial S^S(\mathbf{p^P})}{\partial p_2} = (1 - F(v^*_{1P}))(-1)(p_2 - c)f(p_2 - \beta)$$
$$\frac{\partial \Delta}{\partial p_2} = 1 - F(p_2 - \tilde{\beta}) - (1 - F(p_2 - \beta)) = F(p_2 - \beta) - F(p_2 - \tilde{\beta})$$
$$\frac{\partial v^*_{1P}}{\partial p_2} = 1 - F(p_2 - \tilde{\beta})$$

Thus, the first order condition with respect to p_2 becomes:

$$\frac{\partial \Pi}{\partial p_2} = \Big(- v^*_{1P} + c - \int_{p_2-\beta}^{1} (v_2 + \beta - c)dF(v_2)) + \int_{p_1}^{1} (v_2 - c)dF(v_2))$$
$$+ \int_{p_2-\beta}^{p_2-\tilde{\beta}} (1 - F(v_2))dv_2 \Big) f(v^*_{1P})(1 - F(p_2 - \tilde{\beta})) + F(p_2 - \beta) - F(p_2 - \tilde{\beta})$$
$$- (1 - F(v^*_{1P}))(p_2 - c)f(p_2 - \beta) =$$
$$= (-p_1 + (p_1 - p_2 + \tilde{\beta}) + c + c(1 - F(p_2 - \beta)) - c(1 - F(p_1))$$
$$- (1 - (p_2 - \beta)F(p_2 - \beta)) - \beta(1 - (p_2 - \beta)) + 1 - p_1 F(p_1) +$$
$$+ (p_2 - \tilde{\beta} - p_2 + \beta))(v^*_{1P})(1 - F(p_2 - \tilde{\beta})) - (1 - F(v^*_{1P}))(p_2 - c)f(p_2 - \beta)$$
$$- (F(p_2 - \beta) - F(p_2 - \tilde{\beta})) = 0$$

[20]

$$p_2 = c + (c - p_1)\frac{f(v^*_{1P})F(p_1)(1 - F(p_2 - \tilde{\beta})}{(f(v^*_{1P})(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta})) + f(p_2 - \beta)(-1 + F(v^*_{1P})))}$$
$$+ \frac{F(p_2 - \tilde{\beta})) - (F(p_2 - \beta)}{f(v^*_{1P})(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta})) + f(p_2 - \beta)(1 - F(v^*_{1P}))}$$

---

[20]
$$- \int_{p_1}^{p_2} (1 - F(v_2))dv_2 - \int_{p_2}^{1} (1 - F(v_2))dv_2 - \int_{p_1}^{1} F(v_2)dv_2 = -1 + p_1$$
$$\int_{p_1}^{1} (v_2 - c)dF(v_2) = 1 - p_1 F(p_1) - \int_{p_1}^{1} F(v_2)dv_2$$

Finally solving the above system of equations I get:

$$p_1 = c - \frac{f(v_{1P}^*)F(p_1)(1 - F(p_2 - \beta))(F(p_2 - \tilde{\beta}) - F(p_2 - \beta))}{f(p_1)f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta}))F(v_{1P}^*) + f(p_2 - \beta)(1 - F(v_{1P}^*))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*)))}$$

$$p_2 = c + \frac{(F(p_2 - \tilde{\beta}) - F(p_2 - \beta))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*))}{f(p_1)f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta}))F(v_{1P}^*) + f(p_2 - \beta)(1 - F(v_{1P}^*))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*))}$$

Thus, I see that $p_1 < c$ and $p_2 > c$.