Ad Clutter, Time Use, and Media Diversity

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Abstract

We introduce advertising congestion along with a time-use model of consumer choice among media. Both consumers and advertisers multi-home. Higher equilibrium advertising levels ensue on less popular media platforms because platforms treat consumer attention as a common property resource: smaller platforms internalize less the congestion from advertising and so advertise more. Platform entry raises the ad nuisance “price” to consumers and diminishes the quality of the consumption experience on all platforms. With symmetric platforms, entry still leads to higher consumer benefits. However, entry of less attractive platforms can increase ad nuisance levels so much that consumers are worse off.

JEL Classifications: D43, L13

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1 Introduction

Commercial media often rely exclusively or predominantly on advertising for revenue. Because platforms compete for viewers and viewers typically dislike advertising, one might think that more competition between media platforms should reduce ad levels. However, the last decades have seen a proliferation of television and radio channels and an increase of advertising time.\textsuperscript{1}

Critics of mass media decry advertising clutter. Viewers are swamped with ads and ad impressions are wasted. We provide a novel framework that captures both aspects, namely that consumers dislike when content is replaced by advertising and they have a limited ability to absorb ads. The model predicts that small, low-quality media platforms feature more advertising minutes than more popular, higher-quality platforms. This result contrasts with the findings in the theoretical literature, and concurs with some casual evidence.\textsuperscript{2} We also link advertising choices of media to media diversity. An increase in media diversity (platform entry) leads advertising to replace more content; as ad time reduces net program quality, there is a negative relationship between media diversity and media quality. Advertising becomes more congested making it more difficult for high-quality advertisers to reach consumers. Furthermore, despite a positive gain from variety, consumers can be worse off, as programming carries more advertising.

The standard model of two-sided markets as applied to media economics (Anderson and Coate, 2005; Anderson and Peitz, 2019) builds in a “competitive bottleneck” feature (Armstrong, 2006) which implies there is no direct competition for advertisers. Put briefly, when viewers single-home (meaning they patronize one platform), a platform has a monopoly position over delivering its viewers. The time-use model proposed in this paper

\textsuperscript{1}For instance, Joe Flint in the L.A. Times reports on a Nielsen study according to which a typical U.S. households watches 17.5 channels on a regular basis (see Joe Flint, “TV networks load up on commercials,” Los Angeles Times, May 12, 2014). Toni Fitzgerald reports in Forbes that according to a UBS TV report that ad minutes per hour are at an all-time time for almost every network group (see Toni Fitzgerald, “Yes, you are seeing more commercials than ever before,” Forbes, December 11, 2018).

\textsuperscript{2}Broadcast networks have larger viewsherships on average than cable networks. However, according to a Nielsen report with data from 2009 and 2013, the average commercial time on broadcast networks was less than on cable networks (see Joe Flint, “TV networks load up on commercials,” Los Angeles Times, May 12, 2014). According to a UBS TV report, network groups with small viewership carry more advertising. John Hodulik, the author of this report, gives an explanation for this observation: “Network groups with the worst ratings attempt to manage the pressure on advertising revenues with higher ad loads.” (quoted in Toni Fitzgerald, “Yes, you are seeing more commercials than ever before,” Forbes, December 11, 2018). By contrast, our paper offers an explanation based on profit maximization in market equilibrium.
exhibits the same feature. Even though we model multi-homing consumers who choose how much time to spend on each platform, at any point in time a particular viewer can only be reached through the single channel she is watching at that moment in time. As long as advertising across platforms is coordinated (so as to maximize ad effectiveness), platforms have monopoly power over advertisers. The competitive bottleneck means that competition among platforms is effectively competition for viewers, and so an increase in the number of platforms is predicted to decrease equilibrium ad levels, much like product prices decrease with the number of firms in standard oligopoly models of product competition. This model serves as the starting point of media economics, even though (as discussed by Anderson et al. 2012), empirical support for predictions stemming from this model are mixed.

Whereas the time-use model on its own does not change the structure of the media economics interaction, adding the next ingredient changes it quite radically, as we noted above. We enrich the standard media economics model by introducing limited viewer attention for advertising (congestion). This introduces strategic interaction among platforms on the advertiser side. Because of multi-homing, no media platform has exclusive access to a viewer’s attention – it can be seen as a common property resource to which multiple media platforms have access. Therefore a platform which includes more advertising decreases overall ad effectiveness and thus exerts a negative externality on other platforms.

The upshot is to reverse the standard outcomes quite radically. Suppose that a platform cannot deliver a viewer with certainty to advertisers. Then, through the congestion function, one platform’s choice of ad level will affect the willingness to pay for advertising on other platforms when viewers mix their media consumption. Large platforms internalize congestion to a larger extent than small platforms implying that the former have fewer ads and charge more for them.

Entry of a media platform in this setting will lead media platforms to less internalization of the negative congestion effect. Thus, more competition among media platforms will increase ad levels (which is in line with some observed market facts, such as the entry of Fox television). This shows a tension between media diversity and media quality. Increasing diversity reduces the fraction of time consumers encounter content on any given platform; i.e. it increases the ad clutter. Our results speak to the connection between ad effectiveness and market structure. A more fragmented industry leads to less-effective

\[^3\text{See TV Dimensions 2000 (18th Ed), Media Dimensions, Inc.}\]
advertising. While empirical work in psychology and economics has looked at limited attention for information (for a short overview, see Hefti, 2015), this paper points to the policy-relevant trade-off between ad clutter and media diversity. We have not seen empirical work taking a look at this issue.

Our results speak to the excessive entry results in oligopoly (von Weizsäcker, 1980; Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). There is a trade-off between the duplication of fixed costs versus additional variety and more intense use. However, entry in standard oligopoly markets increases consumer surplus because it intensifies competition. This is not necessarily the case here. Thus while consumers enjoy a larger variety of opinion with entry, the associated information is less reliable or nuanced since with higher advertising levels less time is available for information consumption. This latter effect reduces consumer surplus.

In addition, there is a reduction in total surplus because the average match quality between advertisers and consumers tends to deteriorate with entry. This is most clearly seen in the case of full consumer participation. With full consumer participation consumers spend a fixed total amount of time on content and advertising consumption. Since entry increases overall advertising volume (some viewing time is dedicated to the high advertising media platform, and all platforms have weakly higher ad levels, consumers are, in expectations, exposed to more ads) higher-value matches are partly replaced by lower-value matches. Hence, there is a negative total surplus effect with entry due to deteriorated match quality.

A policy implication of our findings is that in the presence of limited attention there is a novel rationale for entry regulation. Alternatively, regulated ad caps may address the limited attention problem (that comes on top of the standard ad nuisance problem in media markets).

Our model applies to mass media markets such as television and radio. A special feature of the model is that although advertisers and viewers multi-home, an ad by a particular advertiser is seen at most once by a viewer. We call this type of advertising “synchronized advertising.” Synchronized advertising avoids multiple exposures that are wasteful. Radio and television markets endogenously lead to this property if they allow

\footnote{For analyses of duopoly media markets, in which consumers use multiple channels and advertising is non-synchronized see Ambrus, Calvano, and Reisinger (2016) and Anderson, Foros, and Kind (2018). Athey, Calvano, and Gans (2018) develop a model with limited attention in which consumer multi-homing degrades the value of the advertising inventory. They show that the ad price decreases in the share of multi-homing consumers.}
each advertiser to choose the time slot in which he advertises and thereby to coordinate advertising decisions across channels.

Advertising congestion is related to the classic literature on common property resources and the strand of economics papers on information overload (van Zandt, 2004; Anderson and de Palma, 2009, 2012; Hefti and Liu, 2019). This paper brings in information congestion into platform pricing using the approach proposed by Anderson and de Palma (2009). Specifically, it is assumed here that the viewer only has a limited attention span for ads, and is therefore only able to process a fixed number of all the ads to which she is exposed. This analysis renders endogenous the platform prices in the presence of congestion, as well as dealing with multiple platforms competing for attention.

Our paper contributes to the literature on media (e.g., Anderson and Coate, 2005; Peitz and Valletti, 2008; Crampes, Haritchabalet and Jullien, 2009) and two-sided platforms (Rochet and Tirole, 2003, Armstrong, 2006) more generally by introducing limited viewer attention.

The paper is organized as follows. In Section 2 we spell out the model and in Section 3 we characterize the media equilibrium. In Section 4 we show what happens when media diversity changes. In Section 5 we discuss a couple of extensions; Section 6 concludes.

2 The Model

We consider a market in which media deliver viewer attention to advertisers. Consumers have a fixed attention span, \( \phi \). This simple formulation means that a consumer can absorb at most \( \phi \) ads, and we assume that the ads that are retained are chosen randomly from those to which she is exposed (see Anderson and de Palma, 2009). Platform \( i \) broadcasts \( a_i \) ads (to be determined endogenously). Let \( \lambda_i \) denote the fraction of time a consumer spends on platform \( i \) (also to be determined endogenously), which is equivalently the probability she is found on platform \( i \). Therefore the expected number of ads seen on platform \( i \) is \( \lambda_i a_i \). With \( n \) platforms to visit, the expected total number of ads seen by a viewer is \( A = \Sigma_{i=1}^{n} \lambda_i a_i \) so that the consumer’s probability of retaining an ad from platform \( i \) is \( \min\{1, \frac{\phi}{A}\} \) after being exposed to it.

We focus on situations in which the expected total number of ads \( A \) exceeds the viewer attention span \( \phi \) so that there is congestion in equilibrium; our measure of ad congestion is \( (A - \phi)/A \). Congestion can only arise in oligopoly because a monopoly media platform
would never choose \( a > \phi \). Thus a monopolist would always price out congestion by delivering impressions with certainty to those with the highest willingness to pay, instead of widening the pool of advertisers.

**Advertisers**

Advertisers decide whether or not to place an ad on each platform \( i \). We rank advertisers in terms of decreasing willingness to pay, \( p \), to contact viewers and so \( p(a_i) \) is the willingness to pay of the marginal \((a_i)\text{th}\) advertiser conditional on making contact with the consumer. We assume that the demand for ads is well-behaved, so it is not too convex. We define the advertising demand elasticity as \( \varepsilon(a_i) \equiv -\frac{\partial p(a_i)}{p(a_i)} a_i. \)

**Assumption 1** \( p(a) \) is twice continuously differentiable. The advertising demand elasticity (in absolute value) is non-decreasing, \( \varepsilon'(a_i) \geq 0 \), and takes values between 0 and 1.

In other words, we assume that Marshall’s Second Law of Demand holds on the advertiser side.

If there are \( a_i \) ads on platform \( i \), the ad price per actual viewer is the per-viewer willingness to pay of the marginal advertiser; i.e., \( \frac{\phi}{\pi} p(a_i) \). This willingness to pay is the surplus generated by a advertiser-viewer match and, by assumption, is fully appropriated by the advertiser. The demand price for ads on platform \( i \) is then determined as the product of the probability that the viewer is on the platform, that she retains the ad, and the conditional willingness to pay, in sum \( \frac{\phi}{\pi} p(a_i) \). Here, we implicitly assume that the likelihood of remembering an ad is independent of the particular product that is advertised; so it is independent of the advertiser’s willingness to pay.\(^6\)

Even though viewers multi-home, we assume that advertisers do not “waste” impressions. In a setting in which viewers switch between platforms over time, this requires that ad placements are perfectly synchronized. In the context of television and radio advertising, one can think of advertising within a given time window. An advertiser who is active on several platforms chooses the same time slot for all ads. It implies that an advertiser’s\(^5\)

\(^5\)To see this point, note that with \( a > \phi \), a monopolist would only be able to sell an ad at price \( \frac{\phi}{a} p(a) \), where \( p(a) \) is the advertiser demand price when \( a \) ads are broadcast, yielding profit \( \phi p(a) \). With a downward-sloping ad demand, this choice is dominated by the choice \( a = \phi \) yielding profit \( \phi p(\phi) \) because \( p \) is decreasing in \( a \).

\(^6\)In our framework this assumption is natural because viewers get zero surplus in an advertiser-viewer interaction and thus are indifferent as to which ads they remember. More generally, one may want to allow for some correlation between product characteristics and the likelihood to recall an ad.
ad can be viewed at most once by any given viewer even though viewers multi-home.\textsuperscript{7} This is the most efficient use of an advertiser’s advertising budget and thus the optimal choice of an advertiser.

**Viewers**

We propose a time-use model of media consumption with identical viewers who mix between media.\textsuperscript{8} The outside option has index 0 and gives utility $v_0^\alpha$ per unit of time. It may stand for the alternative use of time off media or consumption of an advertising free public broadcaster. Demand follows from maximizing the utility function for media consumption

$$\max_{\lambda_0, \lambda_1, \ldots, \lambda_n} \sum_{i=1}^{n} [s_i(1 - a_i)\lambda_i]^\alpha + (\lambda_0v_0)^\alpha \quad \text{s. t.} \sum_{i=0}^{n} \lambda_i = 1$$

with $\alpha \in (0, 1)$ so that viewers like to mix between different platforms (and the outside good). Here, $\lambda_i$ is the fraction of time spent on platform $i$ and $s_i$ stands for the content quality offered by platform $i$. Only $(1 - a_i)\lambda_i$ is actual program content (“net quality”), due to the ads interjecting, so $s_i(1 - a_i)\lambda_i$ captures the “quality-time” spent on platform $i$. The idea here is that the viewer only values the content part of a program and time spent on watching advertising gives a benefit normalized to zero. Viewers ingest $\lambda_ia_i$ ad on platform $i$. Thus, total ad ingestion is $A = \sum_{j=1}^{n} \lambda_ja_j$. Viewers have an attention span of $\phi$ for ads. This is the maximal number of ads they digest. If $\phi < A$ a fraction $\phi/A$ ads is digested. Accordingly, each viewer digests $\phi a_i/A$ ads appearing on platform $i$.

Define $\tilde{\alpha} = \frac{\alpha}{1-\alpha} > 0$ and denote $\mathbf{a} = (a_1, a_2, \ldots, a_n)$. The fraction of time spent on platform $i$ is

$$\lambda_i(\mathbf{a}) = \frac{(s_i(1 - a_i))^\tilde{\alpha}}{v_0^\tilde{\alpha} + \sum (s_j(1 - a_j))^\tilde{\alpha}}, \quad i = 1, \ldots, n,$$

while the time spent on the outside option is

$$\lambda_0(\mathbf{a}) = \frac{v_0^\tilde{\alpha}}{v_0^\tilde{\alpha} + \sum (s_j(1 - a_j))^\tilde{\alpha}}.$$ 

This fractional demand system is in the vein of Luce (1959) and satisfies the “independence of irrelevant alternative” (IIA) property.

\textsuperscript{7}Absent ad congestion, the coordination of advertising across platforms makes the model identical to a model in which heterogenous viewers single-home. Thus, multi-homing by itself will not change the results of the standard model with single-homing viewers; see also Peitz and Valletti (2008).

\textsuperscript{8}We are not the first to propose a time-use model. For an alternative utility function, see Gabszewicz, Laussel, and Sonnac (2004).
Inserting these expressions into (1), consumer surplus is, therefore

\[ CS = \left( v_0^\alpha + \sum_{i=1}^{n} [s_i(1-a_i)]^\alpha \right)^{1-\alpha} = \frac{v_0^\alpha}{\lambda_0^{1-\alpha}}. \]  

Equation (3) tells us that, for given parameters \( \alpha \) and \( v_0 \), consumer surplus depends only on the market share of the outside option. We immediately obtain the following lemma.

**Lemma 1** An increase in consumer surplus is equivalent to a decrease of the time spent on the outside option, \( \lambda_0 \).

Under symmetry and full coverage (\( \lambda_i = 1/n, i \neq 0 \)), the consumer surplus is \( n^{1-\alpha}(s(1-a))^n \). Ignoring strategic effects (i.e., for given \( a \)) this surplus is increasing with entry since viewers are variety-loving.

**Platforms**

We analyze the platform balance problem of delivering reluctant viewers to advertisers. The profit function takes the form

\[ \Pi_i = \begin{cases} \frac{\lambda_i\phi a_i p(a_i)}{A} & \text{for } \phi < A \\ \frac{\lambda_i a_i p(a_i)}{A} & \text{for } \phi \geq A \end{cases} \]  

It depends on whether there is congestion. Clearly, a more-attractive outside option (i.e. \( v_0 \) and thereby \( \lambda_0 \) goes up) eats into viewing time on channels \( i = 1, \ldots, n \). According to the IIA property, relative viewing time on the \( n \) media platforms, \( \lambda_i/\lambda_j \) for \( i, j \in \{1, \ldots, n\} \), is unaffected and thus a more-attractive outside option lowers \( \lambda_i \). This implies that a media platform’s profit decreases in \( \lambda_0 \) when there is no advertising congestion.

By contrast, with congestion (i.e. \( \phi < A \)) profits are independent of \( \lambda_0 \). To see this, we rewrite profits as

\[ \phi \frac{\lambda_i a_i}{\sum_{j=1}^{n} \lambda_j a_j} p(a_i) = \phi \frac{a_i}{\sum_{j=1}^{n} (\lambda_j/\lambda_i)a_j} p(a_i). \]

Since the IIA property says that \( \lambda_j/\lambda_i \) is independent of \( \lambda_0 \), profits do not depend on \( v_0 \). A better outside option implies that viewers spend less time on ad-financed media, but the probability that advertisers reach viewer increases. The two effects cancel each other out. In other words, while viewers get to see fewer ads when they stay less time tuned
(i.e. when \( \lambda_0 \) is up), they continue to be attentive to a total of \( \phi \) ads as long as there is still ad congestion. Profits depend on this number \( \phi \) and the fraction of ads watched on the platform, \( \lambda_0 a_i \), over the total intake \( A \).

We restrict attention to the case with ad congestion.\(^9\) This means that ad digestion \( \phi \) is less than total ad ingestion \( A \). The degree of ad congestion can then be defined as \( (A - \phi)/A \). Profits of platform \( i \) are the product of the viewing time spent on the platform, \( \lambda_i \); the number of ads on the platform, \( a_i \); and the per-viewer ad price \( \frac{\phi}{A} p (a_i) \).

These profits depend on competitors’ decisions through two channels. Interdependence on the viewer side comes from the assumption that consumers decide how to allocate their viewing times \( \lambda_i \). Interdependence on the advertising side comes from the joint assumption that the \( A \) ads are seen across multiple channels (because viewers are mixing between platforms) and that there is advertising congestion. Here, competitors’ choices affect the total intake \( A \) and thereby the probability that an ad on platform \( i \) is digested by viewers which in turn is reflected in the per-viewer ad price \( \frac{\phi}{A} p (a_i) \).

### 3 Analysis

The structure of the model enables us to cast the oligopoly interaction as an aggregative game. This construct was introduced by Selten (1971), and further developed by Acemoglu and Jensen (2013) and Anderson, Erkal, and Piccinin (2019) \textit{inter alia}. For our purpose, an aggregative game is one in which players’ strategic actions can be recast in such a manner as to render each player’s payoffs as a function solely of its own action and the sum of all players’ actions. The latter sum is termed the aggregate. The aggregative game construct enables considerable simplification by uncovering the basic structure so as to write the oligopoly problem as a two-dimensional problem (instead of the \( n \) dimensions one would generally have with \( n \) players). Equilibrium is then simply described as a fixed point, at which aggregate equals the sum of each player’s action as a function

\[\Phi \geq A, \text{platform } i \text{'s profit is } \Pi_i = a_i \lambda_i (a) p (a_i) = R(a_i) \lambda_i (a) \text{ where Assumption 1 implies that the revenue per viewer, } R(a) = ap(a) \text{ is also well-behaved. The case } \phi > A \text{ resembles standard models of media platforms (e.g., Anderson and Coate, 2005). Aggregative game tools can be used as in Anderson and Peitz (2019). In the extension section, we look at the symmetric case.}\]

The profit function has a kink at \( a_i = \bar{a}_i \equiv (\phi - A)/\lambda_i \) with the property that marginal profits jump downward; i.e.,

\[\frac{\partial \Pi_i}{\partial a_i} \bigg|_{a_i=\bar{a}_i} > \frac{\partial \Pi_i}{\partial a_i} \bigg|_{a_i=\bar{a}_i+1} .\]

Thus, there is a corner equilibrium with \( \phi = A \) under some parameter constellations.

\(^9\)For \( \phi \geq A \), platform \( i \)'s profit is \( \Pi_i = a_i \lambda_i (a) p (a_i) = R(a_i) \lambda_i (a) \) where Assumption 1 implies that the revenue per viewer, \( R(a) = ap(a) \) is also well-behaved. The case \( \phi > A \) resembles standard models of media platforms (e.g., Anderson and Coate, 2005). Aggregative game tools can be used as in Anderson and Peitz (2019). In the extension section, we look at the symmetric case.
of the aggregate. It is important to recognize that this does not just apply to symmetric situations. Indeed, payoff functions are allowed to be idiosyncratic: one of the main useful properties of the approach is that it leads to a tight characterization of individual actions as a function of players’ differing fundamental characteristics (program quality in the model below). And, as we shall see, the analysis of free entry equilibrium is also readily enabled, even when infra-marginal players are asymmetric (this analysis draws on Anderson, Erkal, and Piccinin, 2019: an important complication of the current situation is that consumer surplus cannot be written as a function of the aggregate despite the IIA property of viewer demand).

3.1 Equilibria in media markets with ad congestion

Pursuant to the discussion above, we want to write platform $i$’s profit $\Pi_i(\psi_i, \Psi)$ as a function of its own action $\psi_i$ and the corresponding aggregate $\Psi = \sum_i \psi_i$. We will then proceed by determining the function $\psi_i(\Psi)$, which is the inclusive best reply that maps the aggregate into own action. Notice that a player’s own action is part of the aggregate, contrasting this approach to the standard way to think about best replies as functions solely of the actions of others.

The primitive action variable for a platform is its ad level, $a_i$, so that we seek a monotonic transform of this variable to use as the action variable (in order to preserve the strategic equivalence of the game in actions and the game in ad levels).

For $\phi < \sum_{j=1}^n \lambda_j a_j$, finding an action variable to yield an aggregator function is somewhat challenging.\(^{10}\) The action variable $\psi_i = a_i [s_i (1 - a_i)]^{\bar{\alpha}}$ is net-quality adjusted ad level (or ad quality time), which is defined on $[0, \bar{\psi}_i]$, where $\bar{\psi}_i = \bar{\alpha}[s_i (1 - \bar{\alpha})]^{\bar{\alpha}}$ and $\bar{\alpha} = \arg \max_a a (1 - a) = 1/(1 + \bar{\alpha}) \in (0, 1)$. Recall too that $\Psi = \sum_{j=1}^n \psi_j$. The profit of channel $i$ is then:

$$\Pi_i = \frac{a_i [s_i (1 - a_i)]^{\bar{\alpha}}}{\sum_{j=1}^n a_j [s_j (1 - a_j)]^{\bar{\alpha}}} \phi p(a_i) = \frac{\psi_i \phi p(a_i(\psi_i))}{\Psi}.$$

where the ratio term in the first expression is $i$’s ad share $\lambda_i a_i / \sum_{j=1}^n \lambda_j a_j$: notice the key property that the denominators from (??) cancel out. This implies that profits are

\(^{10}\)Absent congestion, we could write profit as the function $\Pi_i\left(\tilde{\psi}_i, \tilde{\Psi}\right) = a_i(\tilde{\psi}_i)\tilde{\psi}_i p(a_i(\tilde{\psi}_i)) / (\tilde{\psi}_i^{\bar{\alpha}} + \tilde{\Psi})$ with $\tilde{\psi}_i = [s_i (1 - a_i)]^{\bar{\alpha}}$ and $\tilde{\Psi} = \sum_i \tilde{\psi}_i$ giving rise to a different aggregative game structure.
independent of the attractiveness of the outside option as long as $\phi < A$. This means that we can allow viewers with heterogeneous valuations of their outside option, $v_0$, as long as the $\phi < A$ for all viewers including the one with the highest $v_0$ (who spends the least time watching advertising and thus has the lowest $A$).

Notice that ad quality time $a_i[s_i(1-a_i)]^{\alpha}$ (from which we have drawn the aggregate) is hump-shaped. Nonetheless, the formulation still yields a viable aggregative game because $p$ is decreasing, and so we can restrict attention to the increasing part of $\psi_i(a_i)$ along the inclusive best reply. That is, a platform will never choose $a_i$ beyond the monopoly level

$$\pi = \arg\max_{a_i} a_i[s_i(1-a_i)]^{\alpha}$$

because to do so would mean ad minute exposure would be already decreasing. Thus, $\psi_i(a_i)$ can be inverted in the relevant range. We have

$$\frac{da_i}{d\psi_i} = \frac{a_i(1-a_i)}{\psi_i(1 - (1 + \alpha)a_i)} > 0,$$

where $a_i$ is a function of $\psi_i$. We define the inverse action elasticity as $\eta(a_i) \equiv \frac{\psi_i}{a_i} \frac{da_i}{d\psi_i}$.

**Lemma 2** The inverse action elasticity $\eta(a_i)$ takes positive values and is increasing in $a_i$.

**Proof.** The inverse action function elasticity simplifies to

$$\eta(a_i) = \frac{1-a_i}{1 - (1 + \alpha)a_i} > 0. \tag{7}$$

The derivative is $\eta'(a_i) = \tilde{\alpha}/[1 - (1 + \alpha)a_i]^2 > 0$. ■

From (5), the first-order condition defining the inclusive best reply is (recalling that $\psi_i$ enters $\Psi$)

$$p \left( \frac{1}{\Psi} - \frac{\psi_i}{\Psi^2} \right) + \frac{\psi_i}{\Psi} p' \frac{da_i}{d\psi_i} = 0, \tag{8}$$

where $\frac{da_i}{d\psi_i}$ is given as the reciprocal of (6).$^{11}$ We can rewrite this expression as

$$1 - \frac{\psi_i}{\Psi} = -\psi_i \frac{p'}{p} \frac{da_i}{d\psi_i}$$

$^{11}$The limit case $p'=0$ has the feature that all platforms set their actions at the boundary of their action spaces; that is, where $\psi_i = \pi[s_i(1-\pi)]^{\alpha}$ and $\pi = \arg\max_a a(1-a)^{\alpha} = 1/(1 + \tilde{\alpha}) \in (0, 1)$. In this case, inclusive best replies are flat.
Defining \( \varepsilon(a_i) \equiv -p'(a_i)a_i/p \), the first-order condition in elasticity form can be written as

\[
\frac{\psi_i}{\Psi} = 1 - \varepsilon(a_i) \eta(a_i), \tag{9}
\]

and hence equilibrium profit is

\[
\phi(1 - \varepsilon(a_i) \eta(a_i))p(a_i). \tag{10}
\]

We can now show the following result.

**Lemma 3** For \( p' < 0 \), inclusive best replies \( r_i(\Psi) \) satisfy \( 0 < r'_i(\Psi) < \psi_i/\Psi \) and thus there exists a unique equilibrium.

**Proof.** We observe from (7) that

\[
\frac{d\eta(a_i)}{d\psi_i} = \frac{\tilde{\alpha}}{[1 - (1 + \tilde{\alpha})a_i]^2} \frac{da_i}{d\psi_i} = \frac{\tilde{\alpha} a_i (1 - a_i)}{\psi_i [1 - (1 + \tilde{\alpha})a_i]^2} > 0.
\]

Furthermore, Assumption 1 states that \( \varepsilon(a_i) \) is non-decreasing and so is \( \frac{d\varepsilon(a_i)}{d\psi_i} \).

To determine the slope of the inclusive best reply, we differentiate \( \Psi = \psi_i/[1 - \varepsilon(a_i) \eta(a_i)] \) with respect to \( \psi_i \).

\[
\frac{d\Psi}{d\psi_i} = \frac{1 - \varepsilon(a_i) \eta(a_i) + \psi_i [\varepsilon(a_i) \eta(a_i)]' da_i/d\psi_i}{[1 - \varepsilon(a_i) \eta(a_i)]^2}. \tag{11}
\]

Since \( da_i/d\psi_i > 0 \), the inclusive best reply is upward sloping if \( [\varepsilon(a_i) \eta(a_i)]' > 0 \), which holds since \( \varepsilon(a_i) > 0 \) and \( \varepsilon'(a_i) \geq 0 \) by Assumption 1 and \( \eta(a_i) > 0 \) and \( \eta'(a_i) > 0 \) by Lemma 2.

Using (9) we can rewrite equation (11) as

\[
\frac{d\psi_i}{d\Psi} = \frac{\psi_i}{\Psi} - \frac{1 - \varepsilon(a_i) \eta(a_i)}{1 - \varepsilon(a_i) \eta(a_i) + \psi_i [\varepsilon(a_i) \eta(a_i)]' da_i/d\psi_i}.
\]

As shown in the previous paragraph, \( \psi_i [\varepsilon(a_i) \eta(a_i)]' da_i/d\psi_i > 0 \). Hence, \( \frac{d\psi_i}{d\Psi} < \frac{\psi_i}{\Psi} \). ■

As we have just shown, inclusive best replies are upward sloping, so that actions are inclusive strategic complements with the aggregate. This also implies that ad levels are strategic complements. The economics of this property can be understood from the economics of a common property resource, in which participants have different interest
shares (for instance, think of a common property fishery in which participants have different valuations of its continued health). Here the common property resource is consumer attention. The more that others (over-)fish it (i.e., advertise), the more it is degraded and the bigger an individual’s incentive to do likewise in the dwindling value.

The second key property in the Lemma is that average action shares exceed marginal ones. This property implies that the sum of the actions has slope below 1 and so the equilibrium is unique.

In our analysis above we have assumed that advertisers post at most one ad per platform. If there were no congestion, and the market were “fully covered” (meaning that the outside option of non-purchase is not exercised) they would have no advantage from a second ad because they already get the consumer’s attention with probability one by placing a (synchronized) ad on each platform. Otherwise though, there is a benefit from a second ad, or more. Clearly, if the highest value advertiser does not want a second ad, then none do: denote by \( v = p(0) \) the uncongested demand price per viewer of this advertiser (the inverse demand function intercept). Consider the case when an advertiser places a second ad on platform \( i \) in the same time bracket as its first ad (literally, an ad at the same time, a synchronized ad). The first ad is a “hit” with probability \( \frac{\phi}{\lambda_i} \). The second ad raises the chance of a hit by giving an extra chance of breaking into a consumer’s perception. Conditional on the consumer being on platform \( i \) at the time (which happens with probability \( \lambda_i \)), with 2 ads the advertiser gets at least one ad through with probability \( 1 - (1 - \frac{\phi}{\lambda_i})^2 \) (one minus the chance of neither ad getting through). So the conditional incremental probability is \( \frac{\phi}{\lambda_i} (1 - \frac{\phi}{\lambda_i}) \). This value is maximal for \( \frac{\phi}{\lambda_i} = 1/2 \), so we can use \( \frac{\phi}{\lambda_i} = 1/2 \) to find a (very loose) upper bound. Then we have that a second impression is not wanted if

\[
\frac{v}{2} < p(a_i),
\]

which can be interpreted as the requirement that the advertiser demand not be too heterogeneous over the relevant range: the marginal advertiser’s uncongested willingness to pay should be at least half the willingness to pay of the advertiser with the highest will-

\[\text{\footnotesize{12 Several papers have addressed the effects on competition in the ad market when consumers multi-home and some advertisers place multiple ads on platforms, including Ambrus and Reisinger (2006), Ambrus, Calvano, and Reisinger (2016), Anderson, Foros, and Kind (2018), and Athey, Calvano, and Gans (2018). However, to consider entry, Ambrus and Reisinger (2006), only compare monopoly and duopoly in a Hotelling model (in which model the transition from one to two firms generically involves rather different aspects); Anderson, Foros, and Kind (2018) assume a fixed number of advertisers with the same willingness to pay for impressions and only obliquely allow for advertising nuisance to consumers.}}\]
ingness to pay. A similar (but modified) logic applies for asynchronous ads.\textsuperscript{13} Thus, the advertiser demand must not be too heterogeneous over the relevant range. In this case, our analysis applies even when advertisers can place multiple ads on a platform.

### 3.2 Equilibrium characterization

In this subsection we elaborate upon the cross-section properties of the equilibrium. First, we analyze the link between equilibrium action $\psi_i$ and the associated advertising level $a_i$.

**Lemma 4** *In a congestion equilibrium, a larger quality $s_i$ implies a larger action $\psi_i$ and a lower advertising level $a_i$.***

**Proof.** From the first-order condition $\frac{\psi_i}{a_i} = 1 - \varepsilon(a_i)\eta(a_i)$, we know that left-hand side is increasing in $\psi_i$ (for given $\Psi$). Since $\varepsilon$ is non-decreasing and positive by Assumption 1 and $\eta$ increasing and positive by Lemma 2, we must have that the product $\varepsilon(a_i)\eta(a_i)$ is increasing in $a_i$. Thus, the right-hand side is decreasing in $a_i$. This establishes that if for any two platforms, in equilibrium, $\psi_i > \psi_j$, we must have $a_i < a_j$ and vice versa. Furthermore, the inclusive best reply is larger for higher $s_i$. Therefore, the equilibrium value of $\psi_i$ is increasing in $s_i$ and a lower equilibrium value $a_i$ is observed for a higher-quality platform. ■

This is illustrated in Figure 1. In the figure, we consider a media market with two platforms in which platform 2 has higher quality and thus is the larger platform in equ-

---

\textsuperscript{13}First note that an ad in a different time slot will be a contender for reaching a previously unreach consumer with probability $\lambda_i\Sigma_{j\in\mathbb{N}}\lambda_j$ (given that viewing times are random and independent) where $\mathbb{N}$ is the set of platforms on which the advertiser places ads (so $\mathbb{N}$ is all of them for the advertiser with value $v$).

Then the chance of potentially hitting a consumer is $1 - (1 - \lambda_i)(1 - \Sigma_{j\in\mathbb{N}}\lambda_j)$, and so the extra chance is $1 - (1 - \lambda_i)(1 - \Sigma_{j\in\mathbb{N}}\lambda_j) - \Sigma_{j\in\mathbb{N}}\lambda_j = (1 - \Sigma_{j\in\mathbb{N}}\lambda_j)\lambda_i$. Absent ad congestion, $v\lambda_i\lambda_0$ would therefore be the top advertiser’s willingness to pay for an asynchronous second ad, as opposed to a willingness to pay of $v\lambda_1$ for the first ad. So second ads would not be aired if $\lambda_0$ were close to 1 because a first ad on each platform would almost surely do the job.

Now introduce ad congestion. An ad in the first time slot along with the other ads is a contender and hits while the second lone one is not a contender with probability $\frac{\phi}{\lambda} (1 - \lambda_i)\Sigma_{j\in\mathbb{N}}\lambda_j$. The synchronous first one is not a contender while the second one hits with probability $\frac{\phi}{\lambda} \lambda_i (1 - \Sigma_{j\in\mathbb{N}}\lambda_j)$. Both are contenders with probability $\lambda_i\Sigma_{j\in\mathbb{N}}\lambda_j$, and, conditional on this, at least one hits with probability $1 - \left(1 - \frac{\phi}{\lambda}\right)^2$. Adding up these terms, and subtracting the chance of a hit $\frac{\phi}{\lambda} \Sigma_{j\in\mathbb{N}}\lambda_j$ when just using the synchronized ads gives the incremental chance of a hit as $\frac{\phi}{\lambda} (1 - \lambda_i)\Sigma_{j\in\mathbb{N}}\lambda_j + \frac{\phi}{\lambda} \lambda_i (1 - \Sigma_{j\in\mathbb{N}}\lambda_j) + \frac{\phi}{\lambda} \left(2 - \frac{\phi}{\lambda}\right) \lambda_i \Sigma_{j\in\mathbb{N}}\lambda_j - \frac{\phi}{\lambda} \Sigma_{j\in\mathbb{N}}\lambda_j = \frac{\phi}{\lambda} \lambda_i \left(1 - \frac{\phi}{\lambda} \Sigma_{j\in\mathbb{N}}\lambda_j\right)$. Given that the price of an ad on platform $i$ is $\frac{\phi}{\lambda} \lambda_i p(a_i)$, the highest willingness to pay advertiser therefore does not want a second ad if $v \geq \frac{p(a_i)}{1 - \phi \Sigma_{j\in\mathbb{N}}\lambda_j}$. 13
Figure 1: Relationship between $\lambda_i$, $\psi_i$, and $a_i$

librium. The equilibrium value of the aggregate, $\Psi^*$, is determined by the intersection of $\psi_1 + \psi_2$ with the diagonal. Equilibrium values of $\psi_1$ and $\psi_2$ are then obtained by the value of $\psi_i$ evaluated at the inclusive best replies evaluated at $\Psi^*$. We can then represent equilibrium values $\psi_1^*, \psi_2^*$ with $\lambda_i = \lambda_i^* \frac{\psi_i^*}{\psi}$.

As the previous Lemma has established, $\psi_i$ and $a_i$ are negatively related in equilibrium. Since $\psi_i = a_i \lambda_i \left[ \nu_0^i + \Sigma (s_j (1 - a_j))^\alpha \right]$ (where the term in square brackets is the same for all platforms), a platform with larger action $\psi_i$ must have a larger market share, and moreover, the larger actions emanate from platforms with larger $s_i$. Hence we have:

**Proposition 1** Consider any two platforms $i$ and $j$. In congestion equilibrium, (i) $s_i > s_j$ implies that $\lambda_i > \lambda_j$ and $a_i < a_j$ and (ii) $s_i = s_j$ implies that $\lambda_i = \lambda_j$ and $a_i = a_j$.

From the Proposition, a larger $\lambda_i$ entails a smaller $a_i$, and, hence, a larger price per ad per viewer, $\frac{\phi}{\pi^i} (a_i)$.\(^{14}\) This results is in line with some empirical findings. Fisher et al.

\(^{14}\)Even though a larger platform has fewer ads, it is more profitable than a smaller one. To see this, recall that $\Pi_i = \phi_i \psi_i^* p(a_i)$. A larger platform entails both a larger $\psi_i$ and a larger $p(a_i)$, so its profits must be larger. This result can also be derived from the maximized value function, which writes $\Pi_i = \phi_{\psi_i}(\Psi) p(a_i, \psi_i)(\Psi)$. This function is decreasing in $\Psi$ so that larger values of the aggregate constitute greater competition, and hurt profit: see Anderson, Erkal, and Piccinin, 2019, for more on the competitiveness property. Also, higher values of $s_i$ entail higher profit.
(1980) find that the per-viewer fee of an advertisement on programmes with more viewers is larger. It is also consistent with the “ITV premium” noted by other authors (see e.g. the discussion in Anderson et al., 2012). It is also a form of cross-sectional “see-saw” effect: interpreting $a_i$ as the “price” paid by viewers, then this price is high when the price per ad per viewer (on the other side of the market) is low. Indeed, as argued in Anderson and Peitz (2019), the viewer single-homing model (and, by extension, the version of the current time-use model without congestion) exhibits a see-saw, but induced by quality differences in the opposite direction. That is, while a quality advantage induces a higher market share in both cases, the platform in the no-congestion case has more ads and a consequently lower price per ad per viewer.

Proposition 1 says that a platform uses a quality advantage to take a higher equilibrium market share. This effect is reinforced because it also wishes to carry a lower ad level. Market shares are therefore more dispersed than the quality levels that drive them (the ratio of high to low shares exceeds the ratio of their qualities). Put another way, the distribution of market shares has greater variance than the quality distribution. This is a type of “superstar phenomenon”. In standard one-sided oligopoly models (e.g. the logit model of differentiated products in Anderson and de Palma, 2001), higher qualities are parlayed into both higher qualities and higher mark-ups, which mutes market share variance as compared to quality variance. The same is true for standard models of media competition (see Anderson and Peitz, 2019) in which “better” programs want to broadcast more ads than inferior rivals. The result here is due to the congestion effect.

The congestion effect works by giving higher quality platforms a greater stake in not bloating overall congestion. As mentioned earlier, consumer attention is treated as common property, so that a platform with a higher quality catering to a larger market base has a bigger incentive than smaller rivals to internalize the extra congestion from its ads. It therefore wants to broadcast fewer ads. Both effects combine to give a higher price per ad: price per ad per viewer is higher, and also the viewer base is bigger.

While this price per ad effect is empirically well supported (e.g., it is implied by the ITV premium discussed above), casual evidence on the ad/viewership relation seems quite mixed. There are clearly high quality publications with few ads, and many late-night TV programs seem to carry many ads. Our analysis suggests that such results should be seen in markets where congestion effects are strong enough.
4 Media diversity

Here we look at the effects of adding more varieties, i.e., platform entry. We consider market environments in which there is congestion before and after entry. The consumer surplus analysis is more intricate, so we defer it to a later subsection. We first look at the effects on incumbents.

4.1 Entry and incumbent platforms

To evaluate the effect of changes in the market, we have to understand how the aggregate changes. Under entry an additional platform will contribute by adding a new term to the aggregate. Hence the equilibrium value of the aggregate after entry must be larger than before. By Lemma 3, inclusive best replies slope up, and so individual actions of incumbent platforms rise. Ad levels rise too because they vary directly with actions (see (6)). Hence we have:

Proposition 2 Platform entry raises advertising levels $a_i$ on all channels.

This is unambiguous result holds even though platforms compete for viewers and a larger $a$ puts them at a disadvantage. Thus the externality effect through congestion dominates the competition effect. Since more advertising reduces the quality time on consumers experience on platforms, consumers may reduce their time spent with those media or reshuffle their demand towards platforms with less advertising. Thus a priori it is unclear whether consumer ingest more ads.

Proposition 3 Platform entry increases advertising ingestion $A$.

Proof. Advertising ingestion is $A = \sum_{i=1}^{n} \lambda_i a_i = \sum_{i=1}^{n} \psi_i / [v_0^\alpha + \sum (s_j (1-a_j))^\alpha] = \Psi / [v_0^\alpha + \sum (s_j (1-a_j))^\alpha]$. The numerator goes up with entry since $\Psi$ increases. The denominator goes down with entry since the $a_i$ increase with entry by Proposition 2.

Since advertising ingestion increases, advertising congestion $(A - \phi)/A$ goes up as well. Stronger competition among platforms leads to wasteful advertising, as a constant number of ads enter the attention span of viewers and thus an increasing fraction is purely wasteful from a welfare perspective (as it replaces valuable content). In addition, it decreases the

\[15\] Softer forms of congestion in which attention increases with ad ingestion could yield ambiguous effects.
match quality for advertisers as some high-value advertisers are replaced by lower value advertisers in a consumer’s consciousness.

The economics here are once more best represented by reference to the common property problem. When more agents claim the common property resource through entry, each exploits it more because it internalizes the effect of its actions over a smaller base.

The effects on ad prices are quite interesting. First, because the ad level goes up on each platform $i$, then the “uncongested” price per ad per minute, $p(a_i)$, goes down. Because $A$ rises, then each incumbent’s full price per ad per minute, $\frac{\lambda_i}{\alpha}p(a_i)$, goes down by a further percentage. Finally, because market shares are lost to the new rival, the price per ad, which is $P_i \equiv \lambda_i \frac{\lambda_i}{\alpha}p(a_i)$ goes down even more still. Therefore all prices on the ad market tumble. One might indeed expect that prices should fall with entry, but, as noted in the Introduction, standard media economics models predict ad prices per viewer to rise (ad prices are ambiguous because of the share effect). Because of the “competitive bottleneck” problem (Armstrong, 2006), competition for viewers is dominant and this leads entry to reduce the “price” paid by viewers to fall – that price is the number of ads suffered. It is because ad levels fall that price per ad per viewer rises as we go back up the demand for ads relation.

With congestion, matters are much different – the see-saw effect with respect to prices works in the opposite direction completely. Indeed, with congestion effects, we have seen in Proposition 2 above that ad levels rise too, meaning that the implicit price paid by consumers on each channel goes up. In turn, consumers ingest more ads and new media entry has a harmful effect on consumers. The upshot is thus that new entry leads to the consumption experience deteriorating on each channel as the amount of non-advertising minutes in the program goes up. More competition causes worse advertising clutter on incumbent channels and more overall advertising overload. Whether this pricing effect can overturn the per se benefits of new options is the topic of the next sub-section.

While mergers are not our prime concern in this paper, it is instructive to track how they change ad prices. First, a merged entity has a larger stake in the common property and so it reduces its ad levels (it reduces its actions). This raises the uncongested price per viewer, with a further fillip from the reduced overall ad level. Ad levels on rival programs fall too, by strategic complementarity and as rivals now get a bigger stake in the total ad level. Insofar as the market share of the combined entity rises, taking customers from
both the non-viewing option and the rivals, then ad prices go up.\textsuperscript{16} The higher prices from merger play out on the advertiser side of the market (as opposed to on the viewer side, which is the case in the competitive bottleneck setting). The see-saw now works in favor of consumers who face less advertising clutter across the board.

4.2 Consumer Surplus

The equilibrium advertising per channel increases with more platforms, as we argued above. This ad level effect reduces consumer surplus. A utility-maximizing consumer adjusts its consumption pattern to changes in advertising level across all platforms. However, since all platforms offer lower quality time after entry, this must reduce consumer surplus. Hence, if entry simply means that one extra platform includes advertising together with its programming, the effect of entry on consumer surplus is unambiguously negative. Instances of such entry is that a zero advertising restriction on a public broadcaster is removed. However, entry of an additional platform changes the diversity of content and thus has an additional effect on consumer surplus.

Including the variety effect, the effect of entry on consumer surplus is not obvious. As in standard differentiated products oligopoly, entry increases product variety, which is something consumers like. In the standard oligopoly context, entry also leads to lower prices, which is also something consumers like. In a media context the corresponding result would be that consumers suffer from less nuisance after the entry of an additional media platform. While this property holds in the Anderson and Coate (2005) framework (see also Anderson and Peitz, 2019), this is not the case in our current setting with advertising congestion, as has been shown above.

Thus, we have to evaluate the overall effect of entry on consumer surplus. This, as we noted earlier, is not a simple function of the aggregate (in contrast to the central CES/Logit examples in Anderson, Erkal, and Piccinin, 2019). We start by considering a symmetric setting. Under symmetry consumer surplus is $\left[ n s^\alpha (1 - a)^\beta + v_0^\beta \right]^{1-\alpha}$. The conflict is this. Consumer surplus moves the same way as $n s^\alpha (1 - a)^\beta = \frac{\Psi}{a}$. However, $\Psi$ rises with entry, while $a$ rises too, so it is ambiguous a priori. The next result determines the net effect given Assumption 1.

\textsuperscript{16}However, advertisers with higher willingness to pay may be better off because of the reduced congestion.
**Proposition 4** In a symmetric market with at least two platforms, under Assumption 1, the entry of an additional platform always increases consumer surplus.

**Proof.** Because consumer surplus tracks $n^\alpha(1 - a)^\tilde{\alpha}$, the effect of entry on consumer surplus, $dCS/dn$, is positive if and only if

$$s^\tilde{\alpha}(1 - a)^\tilde{\alpha} - n\tilde{\alpha}s^\tilde{\alpha}(1 - a)^{\tilde{\alpha} - 1}da dn > 0,$$

which is equivalent to

$$(1 - a) - n\tilde{\alpha} da dn > 0. \quad (12)$$

Using (9), (7), and the fact that, under symmetry, $\psi_i/\Psi = 1/n$, the equilibrium advertising level as a function of firms $n$ is

$$a = \frac{n - 1 - n\tilde{\alpha}(a)}{(\tilde{\alpha} + 1)(n - 1) - \varepsilon n}. \quad (13)$$

Hence (after simplifying)

$$\frac{da}{dn} = \frac{\varepsilon(a) / (n - 1)^2}{\frac{n - 1 - n\tilde{\alpha}(a) + \frac{n}{\tilde{\alpha}}}{n - 1}} > 0,$$

and, using this expression in (12), because $\varepsilon'(a) \geq 0$, it suffices to show that

$$(n - 1)[(\tilde{\alpha} + 1)(n - 1) - \varepsilon n] - n\tilde{\alpha} \varepsilon > 0. \quad (14)$$

For $a > 0$ to hold in (13), we must have $n\varepsilon(a) < (n - 1)$. Given this restriction, (14) holds if (the inequality is implied by):

$$(n - 1)[(\tilde{\alpha} + 1)(n - 1) - (n - 1)] - \tilde{\alpha}(n - 1) > 0 \Leftrightarrow (n - 2)\tilde{\alpha}(n - 1) > 0,$$

as we desired to show. ■

By Lemma 1, consumer surplus and the time spent on the outside platform move in opposite directions. Thus, the above proposition also says that under symmetry, with platform entry, viewer spend less time with the outside option.

We have two main results for consumer surplus under entry. Proposition 4 states our first result: if platforms are symmetric, more entry must raise consumer surplus. Here, the variety effect outweighs the quality degradation on platforms. Our second result is that this no longer necessarily holds true with asymmetric platforms. As we show by example, in the presence of low-quality and high-quality platforms, entry of low-quality platforms can reduce consumer surplus.
To get to this result, we engage a Zero Profit Symmetric Entry Equilibrium (ZPSEE), following Anderson, Erkal, and Piccinin (2019). This is a free entry equilibrium at which profits are zero for marginal entrants, and such entrants have the same pay-off functions as each other (although infra-marginal firms may have different pay-off functions). In our current context, we let the marginal entrants all have low quality, $s_L$, while infra-marginal ones have higher quality, $s_H$ (so we assume just two types). We take $\varepsilon \in (0,1)$ constant, and will make some restrictions below. Here, we postulate that there are $n_H$ high-quality platforms and that there is an unlimited supply of low-quality platforms.

The key to determining the ZPSEE is to write the zero-profit condition of the marginal entrants (denoted by $L$ subscripts because they are the lowest qualities around). Then we can uniquely determine their (common) ad-level. Let their entry cost be $K$. From the optimized profit (10) and using (7) we have

$$\phi(1 - \varepsilon) \frac{1 - a_L}{1 - (1 + \tilde{\alpha})a_L} p(a_L) = K$$

which uniquely determines $a_L$ because the LHS is the product of two terms that are positive and decrease in $a_L$. (Hence a larger $K$ means lower ad levels across the board – for intuition, there are fewer fringe firms, they advertise less and the others come down with them, by strategic complementarity).

We can determine how many fringe firms there are once we know the qualities of other platforms. The solution is recursive: we illustrate with the case in hand when there are two types of platform. Indeed, now we know $a_L$ from (15), we find $a_H$ from the inclusive best replies. To see this, first write the inclusive best reply (9) as

$$\Psi = \frac{\psi_j}{1 - \varepsilon (a_i) \eta (\psi_i)} = \frac{a_i [s_i(1 - a_i)]^{\tilde{\alpha}}}{1 - \varepsilon \eta_i}$$

(16)

where we recall that $\eta_i = \frac{1 - a_i}{1 - (1 + \tilde{\alpha})a_i}$ from (7) and hence the RHS is equated across platforms. Note further that the RHS is an increasing function of $a_i$: both numerator and denominator are positive; the numerator is increasing in the relevant range and the denominator is decreasing (as already argued above).

Notice that the equilibrium value of the aggregate, $\Psi$ is found from (16) once we know $a_L$. Then the above relation tells us $a_H$. We treat the number of high-quality platforms, $n_H$, as exogenous (they earn more than low ones, and if their entry cost is the same, they would obliterate the low ones, so we restrict their number). Then the last parameter we
need to find is the endogenous \( n_L \). This is found from the aggregate fixed point condition, namely that
\[
n_L \psi_L + n_H \psi_H = \Psi
\]
or, rearranging this from (16) we find \( n_L \) from:
\[
n_L (1 - \varepsilon (a_L) \eta_L) + n_H ((1 - \varepsilon (a_H) \eta_H)) = 1. \tag{17}
\]

Our objective is to compare consumer surplus with only high quality platforms present with the situation when both types are present. Notice that we can take a monotone transformation of the consumer surplus expression (3), and henceforth we use this transformation (with a slight abuse of notation).\(^{17}\) When only high type platforms are present, we have
\[
CS_H = n_H [s_H(1 - a_H)]^{\bar{a}}; \tag{18}
\]
and when both type are present we have
\[
CS_B = n_H [s_H(1 - a_H)]^{\bar{a}} + n_L [s_L(1 - a_L)]^{\bar{a}}. \tag{19}
\]
Define now
\[
\Omega_i \equiv \frac{1 - \varepsilon \eta_i}{a_i} = \frac{1 - \varepsilon \frac{1 - a_i}{1 - (1 + \alpha)a_i}}{a_i}, \tag{20}
\]
and note that \( \Omega_i \) is a ratio of positive functions; the numerator is decreasing, while the denominator is increasing, so that \( \Omega_i \) is decreasing in \( a_i \). Using the relation between qualities from equation (16) above, we get
\[
[s_L(1 - a_L)]^{\bar{a}} \frac{\Omega_H}{\Omega_L} = [s_H(1 - a_H)]^{\bar{a}}. \tag{21}
\]
Then we can write
\[
CS_B = \left( n_H \frac{\Omega_H}{\Omega_L} + n_L \right) [s_L(1 - a_L)]^{\bar{a}}.
\]
Using the fixed point condition \( n_L \Omega_L a_L + n_H \Omega_H a_H = 1 \), which condition determines the number of entrants (see (17)), we can rewrite this consumer surplus as
\[
CS_B = \left( n_H \frac{\Omega_H}{\Omega_L} + \frac{1 - n_H \Omega_H a_H}{\Omega_L a_L} \right) [s_L(1 - a_L)]^{\bar{a}}. \tag{22}
\]
\(^{17}\)We can thus ignore the power and the outside option (as long as these values are not changing in the comparison).
Since we tied down the $a_L$ and $a_H$ above, this expression then only depends on exogenous parameters (recall that we are treating $n_H$ as exogenous).\textsuperscript{18}

We can now use the above analysis to deliver the following result; the proof by example is relegated to the Appendix.

\textbf{Proposition 5} \textit{There are asymmetric markets in which the entry of an additional platform decreases consumer surplus.}

Entry is bad for consumers in this example because the ad-clutter degrades programs too much, even despite the extra variety. As we noted in the proof, we need a sufficiently low value for the low-quality types in order to overturn the result for symmetry that entry is beneficial. By Lemma 1 entry here has the effect that consumers spend more time with the outside option (i.e. $\lambda_0$ increases with entry).

\section{Extensions}

We here consider two extensions to the model. First we contrast our findings with limited attention to when consumers have unlimited attention. Second we assume that consumers obtain a share of the surplus in the advertiser-consumer interaction and endogenize viewers’ advertising digestion, $\phi$.

\subsection*{5.1 Limited vs. unlimited viewer attention}

Our base model features ad congestion. In this extension, we allow for some viewers with unlimited attention. For tractability, we confine ourselves to symmetric platforms. We first analyze the effect of entry in the two models with either limited or under limited attention under symmetry (so that $a_i^* = a^*$ and $\lambda_i^* = \lambda^*$) and then compare the solutions for a given number of platforms. We show that markets with congested viewers behave markedly different from those with uncongested viewers: Platform entry leads to more advertising with congested viewers, while it leads to less advertising with uncongested viewers. As we then argue these findings continue for sufficiently asymmetric shares of congested and uncongested viewers.

\textsuperscript{18}Notice that if all platforms were low quality, then the term in parentheses is just $\frac{1}{n_L a_L} = \frac{1}{1 - \varepsilon \eta_L}$, which is the number of platforms, recalling that under symmetry $\frac{1}{n_L} = 1 - \varepsilon \eta$ by (16).
Unlimited attention: the effect of entry. Profits without congestion are \( \lambda_i a_i p(a_i) = \lambda_i R(a_i) \); see eq. (4). Using symmetry, the first-order condition can be written as

\[
\frac{R'(a^*)}{R(a^*)} = \frac{(1 - \lambda^*)\tilde{\alpha}}{1 - a^*}.
\]

Equilibrium market share is

\[
\lambda^* = \frac{[s(1 - a^*)]^{\tilde{\alpha}}}{n[s(1 - a^*)]^{\tilde{\alpha}} + \tilde{\nu}_0^{\tilde{\alpha}}},
\]

We have that \( R'(a)/R(a) = (1 - \varepsilon)/a \) where \( \varepsilon = -ap'(a)/p \).

Under full coverage this becomes \( \lambda^* = 1/n \) and the first-order condition simplifies to

\[
\frac{1 - \varepsilon}{a^*} = \frac{n - 1}{n} \frac{\tilde{\alpha}}{1 - a^*}.
\]

This can be rewritten as

\[
\frac{1 - a^*}{a^*} (1 - \varepsilon) = \tilde{\alpha} n - 1.
\]

The inverse price elasticity \( \varepsilon \) is upward sloping in \( a \) since \( p(.) \) is assumed to be log-concave. This implies that the left-hand side is decreasing in \( a \). The right-hand side is increasing in \( n \). As a result the equilibrium ad level must be decreasing in \( n \). The standard intuition of the competitive bottleneck model applies: after entry there is fiercer competition for viewers’ time on a platform leading to less ad nuisance.

Under partial coverage we have

\[
\frac{1 - \varepsilon}{a^*} = \left( \frac{n - 1}{n} + \frac{\tilde{\nu}_0^{\tilde{\alpha}}}{[s(1 - a^*)]^{\tilde{\alpha}}} \right) \frac{\tilde{\alpha}}{1 - a^*}.
\]

This can be rewritten as

\[
\frac{1 - a^*}{a^*} (1 - \varepsilon) - \frac{\tilde{\alpha} \tilde{\nu}_0^{\tilde{\alpha}}}{[s(1 - a^*)]^{\tilde{\alpha}}} = \tilde{\alpha} \frac{n - 1}{n}.
\]

Compared to (24) the left-hand side has an additional term. This term is also decreasing in \( a \). As a result the equilibrium ad level with partial coverage also must be decreasing in \( n \).

We now turn to the model with congestion in which platforms maximize \( \lambda_i \phi a_i p(a_i)/A \). Under symmetry the first-order condition (9) simplifies to

\[
\frac{1}{n} = 1 - \varepsilon \eta
\]
which uniquely determines \( a^* \) as a function of \( n \). Since \( \varepsilon \) and \( \eta \) are upward-sloping in \( a \) (see Assumption 2 and Lemma 2), the right-hand side is decreasing in \( a \). An increase in \( n \) therefore implies that ad level \( a^* \) is increasing in \( n \). This result is an implication of Proposition 2 which covers symmetric platforms as a special case. It illustrates our finding that entry has the opposite effect in the model with congestion compared to the standard media model without congestion. Our finding tells us that entry has the opposite effect in the model with congestion compared to the standard media model without congestion. Such a trade-off does not exist with unlimited attention.

*Limited vs. unlimited attention: comparison of ad levels.* Recalling that \( \alpha = \frac{1 - \alpha}{1 - (1 + \alpha)\pi} \), we can write eq. (26) as

\[
\varepsilon = \frac{n - 1}{n} - \frac{(1 + \hat{\alpha})a^*}{1 - a^*} = \frac{n - 1}{n} - \frac{n - 1}{n} \frac{a^* \hat{\alpha}}{1 - a^*}.
\] (27)

Rewriting eq. (23), the inverse price elasticity \( \varepsilon \) must satisfy without congestion and full coverage

\[
\varepsilon = 1 - \frac{n - 1}{n} \frac{a^* \hat{\alpha}}{1 - a^*}.
\] (28)

We observe that the right-hand side of (28) takes larger values than the right-hand side of eq. (27) for all admissible values for \( a \) and thus there is less advertising with advertising congestion than without.\(^{19}\)

This may not seem obvious because with congestion attention \( \phi \) is a common property resource and multiple platforms will exploit it excessively. Without ad congestion, any watched ad raises the attention of viewers. This allows the platform to extract the surplus of the marginal advertiser \( p(a_i) \). By contrast, with ad congestion, the platform can only extract \( \phi/A \) \( p(a_i) \). A higher ad level puts further downward pressure on the ad price (through \( \phi/A \)), and the platform has an incentive to set a lower ad level with congestion.

The platform’s profit per time unit is \( a_i p(a_i) = R(a_i) \) without congestion and \( a_i (\phi/A) p(a_i) \) with congestion. For given viewing time \( \lambda_i \), the platform would maximize these expressions with respect to \( a_i \). Without congestion the solution satisfies \( a p'(a_i) + p(a_i) = 0 \) which is equivalent to \( \varepsilon = 1 \); with congestion it satisfies

\[
\frac{\phi}{A} [a p'(a_i) + p(a_i)] - \frac{\lambda_i \phi}{A^2} a_i p(a_i) = 0
\]

\(^{19}\)The right-hand side of (27), (28), and (29) is downward sloping and therefore in all specifications any solution \( a^* \) must be unique.
which can be written as \( \varepsilon = 1 - \lambda_i a_i / A \). Since \( \varepsilon \) is upward sloping this shows that ad levels are lower with congestion than without congestion if we treat viewer numbers as exogenous. With congestion, the platform takes into account that a higher ad level increases the degree of congestion. The associated drop in the ad price reduces the incentive to increase the ad level.

Platforms of course do not maximize profits for a given viewing time but take into account that viewers allocate their viewing time depending on the net quality of the platform. An ad-congested platform also takes into account that total ingestion \( A \) increases by less than \( a \) as it marginally increases its ad level because its share \( \lambda_i \) decreases in the ad level, but this does not overturn the result for given viewing time.

Rewriting eq. (23), the inverse price elasticity \( \varepsilon \) must satisfy without congestion and partial coverage

\[
\varepsilon = 1 - \frac{n - 1}{n} \frac{n \frac{v_0}{s} (1 - a^*) - a^*_\lambda}{1 - a^*} - \frac{v_0^\delta}{s(1 - a^*)} \frac{a^*}{1 - a^*} \tilde{\alpha}.
\]

(29)

Since the right-hand side of (29) takes smaller values than (28) this is not clear with partial coverage. Here, the ad level with congestion is actually larger than without congestion if and only if

\[
\frac{1}{n} < \frac{v_0^\delta}{s(1 - a^*)} \frac{a^*}{1 - a^*} \tilde{\alpha}
\]

which is equivalent to

\[
\frac{1}{n^2} < \frac{\lambda_0}{1 - \lambda_0} \frac{a^*}{1 - a^*} \tilde{\alpha}.
\]

A mix of consumers with limited and unlimited attention. It is possible to extend the model to allow for a fraction \( 1 - \kappa \) of viewers with unlimited attention. The profit function of media platform \( i \) is

\[
\Pi_i = \kappa \lambda_i a_i p(a_i) \frac{\phi}{A} + (1 - \kappa) \lambda_i a_i p(a_i) = \lambda_i R(a_i) \left( 1 - \kappa + \kappa \frac{\phi}{A} \right).
\]

Under full coverage and using symmetry, the first-order condition can be written as

\[
\varepsilon = 1 - \frac{\kappa \phi}{n[(1 - \kappa) a^* + \kappa \phi]} - \frac{n - 1}{n} \frac{\tilde{\alpha} a^*}{1 - a^*}.
\]

(30)

In the special case \( \kappa = 0 \) we obtain equation (28) and in the special case \( \kappa = 1 \) we obtain eq. (27). For a given number of platforms, the ad level is smaller for \( \kappa \in (0, 1) \)
than when no viewer has limited attention ($\kappa = 0$). Regarding the comparative statics with respect to $n$ we have to evaluate how the left-hand side varies with $n$ for given $a^*$. By continuity, ad levels are increasing in the number of platform for $\kappa$ sufficiently close to 1 and decreasing for $\kappa$ sufficiently close to zero.\footnote{Here, we implicitly assume that there is a unique solution to the first-order condition and that this solution is an equilibrium.}

5.2 Surplus sharing in the advertiser-consumer relationship and endogenous ad digestion

In the base model, consumers did not care about the composition of advertisers that come to their attention because advertisers extracted all the surplus generated from their interaction with consumers. Here we sketch an extension in which advertisers cannot extract the full surplus and thus consumers obtain a positive fraction of the gains from trade. We show how to use this extension to endogenize viewer attention $\phi$. Since viewer attention is equal to the number of digested ads, both variables affecting ad congestion will then be endogenous ($\phi$ and $A$).

**Surplus sharing between advertisers and consumers.** Suppose now that consumers also gain from trade. The per-viewer gain from trade to any advertiser $a$ is $p(a)$, as before, and we assume now that the consumer also obtains $\beta p(a)$ (and before we had $\beta = 0$).\footnote{For example, if we have a conditional demand per consumer for an advertiser of type $a$ charging product price $s_a$ as $(a - s_a)$, and zero marginal production cost, $a$’s product price would be $s_a = a/2$ and its profit $p(a) = a^2/4$. Consumer surplus would be $p(a)/2$, corresponding to $\beta = 1/2$. Other values of $\beta$ (below 1) can be generated in like manner from the class of $\rho$-linear demand functions.}

The viewer utility has then to be augmented by the expected benefit from interacting with $\phi$ advertisers. This benefit in the product market $CS^p$ can be written as

$$CS^p = \sum_{i=1}^{n} \frac{\lambda_i \phi}{A} \int_{0}^{a_i} \beta p(a)da$$

$$= \sum_{i=1}^{n} \frac{\lambda_i \phi}{A} \int_{0}^{a_i} \beta p(a)da + \sum_{j=2}^{n} \sum_{i=j}^{n} \frac{\lambda_i \phi}{A} \int_{a_{j-1}}^{a_j} \beta p(a)da$$

To determine $a^*$ we first have to characterize viewer demand. The viewer utility maximization problem is

$$\max_{\lambda_0, \lambda_1, \ldots, \lambda_n} \sum_{i=1}^{n} [s_i(1 - a_i)\lambda_i]^\alpha + (\lambda_0 v_0)^\alpha + \frac{\phi \sum_{i=1}^{n} \lambda_i \int_{0}^{a_i} \beta p(a)da}{\sum_{i=1}^{n} \lambda_i a_i}$$
subject to $\lambda_0 + \sum_{i=1}^n \lambda_i = 1$.

Notice that this structure gives consumers an additional incentive to spend time on platforms with more ads, and therefore for platforms to carry more ads, increasing congestion. Moreover, the resulting overfishing will be exacerbated the more platforms are present, for the individual platform internalizes less of the congestion cost. Thus platform entry may be welfare-decreasing.

To provide some pointers, suppose that viewers do not take into account that their viewing behavior affects the expected surplus in the interaction with sellers, so they treat the last term as a constant. Then viewer demand is identical to the one in the main model and the positive analysis still applies. A difference arises when evaluating consumer surplus, as the buyer surplus in the buyer-seller interaction has to be factored in.

Consumer surplus can be written as

$$CS = \frac{v_0^\alpha}{\lambda_0^{1-\alpha}} + \frac{\phi \sum_{i=1}^n \lambda_i \int_0^{a_i} \beta p(a)da}{\sum_{i=1}^n \lambda_i a_i}.$$ 

Under symmetry this simplifies to

$$CS = \frac{v_0^\alpha}{\lambda_0^{1-\alpha}} + \frac{\phi \beta \int_0^{a^*} p(a)da}{a^*}.$$ 

As we know from the analysis of the main model $\lambda_0$ is decreasing with entry and the first term is increasing with entry. We also know that $a^*$ is increasing with entry and thus the second term is decreasing with entry (the average advertiser’s per viewer benefit $\int_0^{a^*} p(a)da/a^*$ is decreasing with $a^*$). Hence there is now a countervailing effect from entry as it decreases overall surplus from the buyer-seller interaction. We can write consumer surplus as

$$\left(\frac{v_0^{\tilde{\alpha}} + n [s(1-a^*)]\tilde{\alpha}}{\lambda_0^{1-\alpha}}\right)^{1-\alpha} + \phi \beta \int_0^{a^*} p(a)da.$$

Consumer surplus is decreasing with entry if the derivative of this expression with respect to $a^*$ is negative, which holds for $\beta$ sufficiently large. This means that there is excess variety from the consumer surplus perspective, as entry reduces the expected gains in the buyer-seller interaction which dominates the positive effect of media variety.

**Endogenous ad digestion.** Consider now the possibility that consumers adjust their attention to equate marginal benefit from increased attention to marginal costs.\(^{22}\) We

\(^{22}\)In spirit, this relates to work on the use of ad blockers (Anderson and Gans, 2011; Johnson, 2013). In those papers, consumers decide whether or not to block ads; i.e. whether to have unlimited attention to ads or zero attention. In our model, consumers choose how many ads to digest.
see this as a long-term behavioral pattern that responds to costs and benefits. The cost of attention is denoted by $C(\phi)$ which is strictly convex with the standard boundary properties. Then there is a unique solution to the utility maximization problem with respect to $\phi$ for any given $a^e$. This solution $\phi^*(a^e)$ is implicitly defined by the average viewer benefit per ad equal to the marginal cost of increasing attention,\(^{23}\)

$$\frac{\beta}{a^e} \int_0^{a^e} p(a)da = C'(\phi).$$

Because of the property of the expected benefit function, this solution $\phi^*$ is decreasing in $a^e$. Since $a^e$ is increasing with entry, viewers respond to the expected increase in ad ingestion due to entry by digesting even fewer ads. The reason is that advertisers come from a worse selection which makes paying attention to ads less attractive. Thus, with entry, an increase in ingestion $A$ is accompanied by a reduction of digestion $\phi$. Hence, also when endogenizing $\phi$, ad congestion $(A - \phi)/A$ is increasing in the number of platforms.

### 6 Conclusion

Even though consumers dislike program content to be padded with advertising and even though some advertisers fail to sell because of ad clutter, we observe huge amounts of advertising in TV and other mass media. If neither advertisers nor consumers obtain a service they like, this begs the question why media platforms do not simply reduce the volume of ads and make everybody happier. The answer comes from thinking of viewer attention as a common property resource.

In this paper we propose a time-use model of media consumption and show that limited attention for advertising can explain a number of features that standard theory cannot, and delivers several novel results.

First, higher-quality platforms attract more consumer time and place less advertising. Lower-value advertisers post ads on lower-quality platforms only, whereas higher-value advertisers advertise more broadly.

Second, an increase in the variety of opinion (platform entry) causes more advertising on each platform, and thus a reduction of net content quality. In the presence of advertising clutter, the matching of advertisers to consumers becomes important – which ads

\(^{23}\)To be precise, there is a threshold $\hat{a}$ such that for $a^e < \hat{a}$ we must have $\phi(a^e) = a^e$ (because the solution to eq. (31) has the property that $\phi > a^e$) and $\phi(a^e) < a^e$ for $a^e > \hat{a}$. We consider environments in which the resulting $a^e$ satisfies $a^e > \hat{a}$. 28
get through? Matching is efficient if the advertisers with the highest willingness to pay get their messages to consumers. Advertising efficiency is diminished when higher-value advertisers are replaced by lower-value advertisers, and this happens when there are more media platforms vying for attention in the presence of clutter.

Third, under free entry, increasing the quality of some incumbent platforms reduces media diversity when the quality of the marginal platform does not change. However, this increases consumer surplus and advertising efficiency. Thus, consumer surplus and total surplus increase when media diversity is reduced. However, if society values variety of opinion more strongly than do consumers, society may well be better off under more diversity, despite consumers being worse off and advertising efficiency decreasing.

Fourth, lower entry costs result (as expected) in more diversity of opinion. As a benchmark with symmetric media platforms, entry is good for consumers even though content is partially replaced by advertising. However, with asymmetric media platforms the latter effect can dominate the benefit from variety and consumers may be worse off when entry costs go down. With a covered market, then total surplus also goes down because, with a covered market, advertising efficiency always decreases without a corresponding increase in market base. Also in settings in which consumers obtain a fraction of the surplus from the advertiser-consumer interaction, consumers may be worse off when entry costs go down.
Appendix.

**Proof of Proposition 5.** The proof is by example: we reverse engineer the result.

First, simplify by setting $\tilde{\alpha} = 1$ (i.e., $\alpha = 1/2$) and set $\phi = 1$.\(^24\) Next, choose a pair of advertising levels with $a_L > a_H$ for the post-entry situation. These advertising levels are both below 1/2 because with $\tilde{\alpha} = 1$, equilibrium actions (the $\psi_i$) cannot support higher ad levels.

We next use (16) to find the corresponding quality ratio that supports the specified advertising levels, and then use the free-entry condition for the low-quality platforms to find the value for $K$ that supports zero profit at the chosen $a_L$. In (22), $n_H$ is a parameter: we can choose its value as the number of high-quality platforms that would freely enter under some higher level of entry cost, and then we can suppose that the market has just those firms active initially. When the entry cost drops to $K$, new (low-quality) platforms come in, and we show that consumer surplus can go down. Notice that the quality degradation (between high and low qualities) needs to be severe enough to offset the earlier finding (for symmetry) that entry benefits consumers. We can set a pre-entry level of $a$ for the high quality platforms alone (which is below $a_H$ because we know ad levels rise with entry) and support that level of $a$ with an initial level of entry cost.

For the example, we first determine the level of $K$ which will support $a_L = 1/3 > a_H = 1/6$. From the ZPSEE for the low types (15), we must have $\varepsilon$ and $K$ combinations that deliver $a_L = 1/3$, so they must satisfy

$$K = \left(1 - \varepsilon \frac{2/3}{1 - 2(\frac{1}{3})}\right) \left(\frac{1}{3}\right)^{-\varepsilon} = (1 - 2\varepsilon) 3^\varepsilon.$$

In particular, we can take $\varepsilon = 1/3$ to find $K = 3^{-2/3} \approx 0.48$. Next, we need to find the quality ratio that delivers $a_H = 1/6$: from (16) we have

$$\frac{1}{6} \frac{5/6}{1/3} \frac{2/3}{s_H} = \frac{1 - \varepsilon^{5/6}}{1 - 2\varepsilon}$$

or

$$\frac{s_H}{s_L} = \frac{8}{5} \frac{1 - \varepsilon^{5/6}}{1 - 2\varepsilon}.\quad(23)$$

\(^{24}\)Or else $\phi$ can be folded into the entry cost.
Using the definition of the $\Omega$’s from (20) above, we can write them as

$$\Omega_L = \frac{1 - \varepsilon^{2/3}}{1/3} = 3 \left(1 - 2\varepsilon\right);$$

$$\Omega_H = \frac{1 - \varepsilon^{5/6}}{1/6} = 6 \left(1 - \varepsilon^{5/4}\right).$$

Inserting these values into the consumer surplus expression when both types are present, (22), we get

$$CS_B = \left( n_H \Omega_H + \frac{1 - n_H \Omega_H a_H}{a_L} \right) \frac{s_L(2/3)}{\Omega_L}$$

$$= \left( \frac{n_H \Omega_H}{2} + 3 \right) \frac{2s_L}{3\Omega_L}$$

$$= \left( 3 \left(1 - \varepsilon^{5/4}\right) n_H + 3 \right) \frac{2s_L}{9 \left(1 - 2\varepsilon\right)}$$

$$= \left( \left(1 - \varepsilon^{5/4}\right) n_H + 1 \right) \frac{2s_L}{3 \left(1 - 2\varepsilon\right)}.$$

Now, we know too that consumer surplus before the wave of entry induced by the reduction in entry cost to $K$ is from (18)

$$CS_H = n_H s_H (1 - a)$$

and we know that $n_H$ satisfies $\psi = 1 - \varepsilon (a_t) n_H$, so that under symmetry (recalling that $a_t \Omega_t = 1 - \varepsilon \eta_t$), this means that $n_H = 1/a\Omega$.

Therefore, $CS_H > CS_B$ as

$$\frac{s_H (1 - a)}{1 - \varepsilon^{1-a}} > \left( \left(1 - \varepsilon^{5/4}\right) n_H + 1 \right) \frac{2s_L}{3 \left(1 - 2\varepsilon\right)}.$$

We can now eliminate the qualities by using (21), which here simplifies to $s_L(1 - a_L) \frac{\Omega_n}{\Omega_L} = s_H(1 - a_H)$: then $CS_H > CS_B$ as

$$\frac{(1-a)}{1 - \varepsilon^{1-a}} \frac{8 \left(1 - \varepsilon^{5/4}\right)}{5 \left(1 - 2\varepsilon\right)} > \left( \left(1 - \varepsilon^{5/4}\right) n_H + 1 \right) \frac{2}{3 \left(1 - 2\varepsilon\right)}.$$  \hspace{1cm} (32)

Here we can take a value for $a$ and a prior entry cost to find a value for $n_H$. If we take $a = 1/8$, the above surplus comparison condition (32) reduces to $n_H < \frac{663}{385}$. Setting now
\( K = 1 \) (note this is above the value we had that supported both types) and \( \varepsilon = 1/3 \), we use the Zero-Profit condition \( n_H = 1/(a\Omega) = 1/ \left( 1 - \varepsilon \frac{1-a}{1-2a} \right) \) to find

\[
n_H = 1/ \left( 1 - \frac{17}{36} \right) = \frac{18}{11}.
\]

This value is below the critical value \( n_H < \frac{663}{385} \) we found above, so that indeed the surplus falls with entry of the low quality types. \( \blacksquare \)
References


