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## Patent Pools, Vertical Integration, and Downstream Competition

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# Patent Pools, Vertical Integration, and Downstream Competition\*

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## Abstract

Patent pools are commonly used to license technologies to manufacturers. Whereas previous studies focused on manufacturers active in independent markets, we analyze pools licensing to competing manufacturers, allowing for multiple licensors and non-linear tariffs. We find that the impact of pools on welfare depends on the industry structure: Whereas they are procompetitive when no manufacturer is integrated with a licensor, the presence of vertically integrated manufacturers triggers a novel trade-off between horizontal and vertical price coordination. Specifically, pools are anticompetitive if the share of integrated firms is large, procompetitive otherwise. We then formulate information-free policies to screen anticompetitive pools.

**JEL classification:** K11, L41, L42, O34.

**Keywords:** patent pools and horizontal pricing agreements; complementary patents; vertical integration and restraints; antitrust policy.

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# 1 Introduction

A patent pool is an agreement among patent owners to license a bundle of patents to each other or to third parties (Quint, 2008). From the 1890s to the 1940s, many dynamic manufacturing industries in the U.S. had a patent pooling arrangement (Lerner and Tirole, 2007). Following a number of unfavorable Antitrust rulings, pools essentially vanished between the mid-1950s and the mid-1990s. This changed in 1995, after the release of new guidelines on the licensing of intellectual property by the Department of Justice (DOJ) and Federal Trade Commission (FTC). In the years that followed, American authorities approved patent pools tied to major technologies in electronics, information technology, and medicine.<sup>1</sup>

A lenient approach to patent pools is in line with the observation that products in these industries build on a large number of complementary patents. This forces licensees to navigate a patent thicket: A web of overlapping claims that may preclude the commercialization of a new product because buyers need to get patents from multiple sources. These patent thickets trigger a problem of horizontal double marginalization: When multiple licensors sell complementary patents, each of them does not consider that lowering its price has a positive effect on other licensors' profits, due to an increase in the demand for the bundle. Thus, prices of complementary patents are optimally set by a patent pool (Shapiro, 2001).<sup>2</sup>

The theoretical literature that followed has studied patent pools mainly in environments in which licensors and licensees are separated firms and licensees are active in independent markets (e.g., among others, Lerner and Tirole, 2004; Quint, 2014; Choi and Gerlach, 2015; Boutin, 2016; Rey and Tirole, 2018). The general message of these articles is that pools comprised of complementary patents tend to be procompetitive. However, their modeling approach overlooks that patent pools are ubiquitous in markets where patent owners deal with manufacturers competing with each other on the product market. Moreover, as documented by Layne-Farrar and Lerner (2011), many patent pools' members are vertically integrated.<sup>3</sup> This suggests that the available models cannot capture an important feature of existing pools, as they cannot relate the market structure to the competitive consequences of these agreements.

We propose a new theory of patent pools in which monopoly licensors offer to license

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<sup>1</sup>A non-exhaustive list includes the MPEG, DVD, Bluetooth, Firewire, 3G-Mobile, and laser eye surgery technologies.

<sup>2</sup>This result builds on Cournot's (1838, 1897) insight that a monopoly raises welfare when products are perfect complements. Another thorny consequence of patent thickets is that a new product unintentionally infringes on existing patents (see Choi and Gerlach, 2015).

<sup>3</sup>For example, as of August 2018, in the MPEG-4 Visual patent pool, which licenses critical technologies used in personal computers and mobile devices, 22 of the 32 licensors are vertically integrated. Also, in the DVD6C licensing group, which sells patents tied to the DVD technology, 4 of the 8 licensors are integrated.

their perfectly complementary patents to manufacturers who are rivals in the downstream market. In the model, the market power of a licensor is constrained by the presence of an inefficient status-quo technology. We then study the welfare consequences of the formation of patent pools in industries where licensees compete against each other and licensors and licensees may be vertically integrated. We also give guidance on the policies that are best suited at screening anticompetitive pools. Specifically, in line with recent literature (Lerner and Tirole, 2004 and 2015; Boutin, 2016; Rey and Tirole, 2018), we consider information-free policies that break the anticompetitive outcome without relying on the specific number of firms or market structure in the industry.

To develop our model, we build on two main theoretical pillars. First, firms can engage in interlocking vertical relationships. This assumption allows for the formation of an interconnected web of trades between licensors and manufacturers, thus capturing a salient feature of patent thickets.<sup>4</sup> Second, patent owners use two-part tariffs. This reduces contractual inefficiencies and allows for larger production as licensees take advantage of declining average costs. There is also evidence documenting that most licensing programs charge fixed fees in addition to variable royalty rates (Gilbert, 2011).<sup>5</sup>

We characterize the equilibrium of the resulting common agency game involving multiple licensors and manufacturers, product market competition and non-linear tariffs. We consider two main scenarios: the one with vertical separation and the one with vertical integration of one or more licensors. We show that the pool's competitive consequences crucially depend on industry structure.

First, pools are procompetitive when no licensor is vertically integrated. Although this conclusion is consistent with the one on pools licensing to non-competing manufacturers, a new mechanism drives our result: Without a pool, each licensor considers the effect of a reduction in its royalty rate only on the part of a manufacturer's profit that it can extract through the fixed fee; however, it does not take into account that other licensors will also demand a positive fixed fee. As royalty rates of all licensors will be positive then, each of them does not fully benefit from the marginal increase in a manufacturer's profit when lowering its royalty rate (i.e., our offer game features contracting externalities). By contrast, the pool is the only entity demanding a payment from manufacturers, and therefore reaps the full benefit. It follows that its incentive to reduce patent prices is larger. Interestingly, and in contrast to conclusions from models focusing on non-competing manufacturers, this

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<sup>4</sup>For instance, within the pool on the 3G-Mobile technology, Samsung, LG, and Sony, among others, cooperated with vertically specialized licensors, such as Bosch, on the pricing of the bundle of patents. At the same time, they competed on the product market.

<sup>5</sup>We discuss our assumptions in more detail in Section 3.

mechanism also implies that a larger number of patent owners not only leads to a higher aggregate patent price, but also to a higher *individual* patent price.

We then consider pools formed by vertically integrated licensors. Differently from common wisdom, we show that a pool of perfectly complementary patents can raise patent prices. Specifically, the pool is anticompetitive if the share of integrated firms in the industry is large, and procompetitive otherwise.<sup>6</sup> This result is the consequence of a novel trade-off between horizontal and vertical price coordination: On the one hand, as integrated manufacturers are members of the pool through their upstream licensors, all licensors, even the non-integrated ones, reduce their patent prices to these manufacturers. The pool therefore allows licensors to internalize the contracting externalities (horizontal price coordination). On the other hand, all licensors benefit from increased profits of vertically integrated manufacturers, and therefore have an incentive to soften price competition. They achieve this by raising the patent prices to non-integrated manufacturers (vertical price coordination).

If the number of vertically integrated licensors is large, the effect of vertical coordination dominates. Without the pool, as integrated licensors do not demand a fixed fee from manufacturers, a non-integrated licensor can reap a large part of the manufacturer's profit increase when lowering its royalty rate. Thus, patent prices are low. With the pool, vertical price coordination implies that non-integrated licensors demand high royalties from non-integrated manufacturers, thus making the pool anticompetitive. By contrast, with only few integrated licensors, the royalty rates of non-integrated licensors are relatively high without the pool because of the presence of contracting externalities. Therefore, the pool is procompetitive due to the effect of horizontal price coordination. All these findings are robust to the nature of competition (prices or quantities), the number of firms in the industry, the contractual environment, and the degree of differentiation between final products.<sup>7</sup>

Whereas the existing literature on patent thickets and royalty stacking suggests that a pool of complementary patents reduces patent prices (e.g., among others, Shapiro, 2001; Lemley and Shapiro, 2013), we show that, depending on industry structure, such a pool can be anticompetitive. In fact, there is evidence demonstrating that pools, absent regulation, may refuse to license their technologies (Lampe and Moser, 2014). More importantly, there is case evidence on pool members' use of provisions restricting downstream competition (Gilbert, 2004), and on the practice of granting affiliated manufacturers with a privileged

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<sup>6</sup>In the baseline model, we focus on the case with two manufacturers to bring out this result within a parsimonious setting. We then prove that all results carry over to a setting with a general number of manufacturers.

<sup>7</sup>Our focus on perfect complements puts us in a situation in which pools are generally procompetitive when buyers are active in separate markets (i.e., do not compete). It follows that our anticompetitive result is likely to arise even in settings where patents are imperfect complements.

access to the pool’s patent bundle (Harris, 2003; Flamm, 2013).<sup>8</sup> This case evidence exactly matches the theoretical mechanism yielding our anticompetitive result.

We finally turn to public policy formulation. In line with Boutin (2016) and Rey and Tirole (2018), we show that the imposition of an “unbundling and pass-through requirement” is able to screen anticompetitive pools. Under this requirement, the pool must set a tariff for each patent in the bundle, instead of a single tariff for the bundle, and then ensure that each member obtains only the revenue generated by its own technology. Essentially, this requirement boils down to a ban on monetary transfers among pool members.

We also analyze a policy that requires the pool to consider only licensing revenues when setting its tariffs, thereby banning monetary transfers between an integrated manufacturer and non-integrated licensors. As this policy allows for horizontal side payments but forbids vertical ones, it can be seen as a “vertical firewall policy.” We show that this policy is an imperfect screening device of anticompetitive pools, as it may prevent the formation of procompetitive pools.

The main contribution of the article is twofold: First, we propose a new model of licensing that is consistent with practice, and fully characterize the resulting equilibrium with and without integration. Second, we offer new insights to the policy and academic literatures on important patent pool agreements. Two other articles studying a pool’s impact on patent prices with competing licensees are Kim (2004) and Schmidt (2014).<sup>9</sup> Kim’s (2004) model features unconstrained monopolists licensing complementary patents via linear tariffs. Schmidt (2014) extends this model to a more general setting. The main take-away of both articles is that pools raise welfare by eliminating horizontal double marginalization. In contrast to these articles, we prove the anticompetitive effect of patent pools allowing for a more general contractual environment and industry structure.<sup>10</sup>

Our article also contributes to the literature on common agency. We solve an offer game in which, due to the presence of complementary patents, there is no marginal contribution equilibrium (i.e., an equilibrium in which each principal claims its contribution to an agent’s profit). Such an equilibrium exists if the marginal contribution of each principal is less than the joint marginal contribution of all principals (as is the case in, among others, Laussel and

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<sup>8</sup>We will come back to this evidence in Section 7.

<sup>9</sup>Lerner and Tirole (2004) in an extension also consider downstream competition between manufacturers. However, they do not allow for two-part tariffs and focus on market structures in which every licensor is also active downstream through its integrated manufacturer.

<sup>10</sup>We also touch upon the literature studying pools in settings with essential patents (Quint, 2014), and how price commitments can prevent licensors from setting unreasonable royalties in standard setting (Llanes and Poblete, 2014; Lerner and Tirole, 2015). This literature does not consider vertical restraints.

Le Breton, 2001; Bergemann and Välimäki, 2003; Calzolari and Denicolò, 2013, 2015): This holds true if principals offer substitute goods, it fails to hold in our setting with complementary patents. We first derive the properties of the unique symmetric equilibrium of the offer game with an arbitrary large number of licensors. We then show that the pool’s competitive consequences do not change when considering asymmetric equilibria of the game.<sup>11</sup>

Finally, we contribute to the literature on vertical integration and restraints. Most of this literature has looked at stylized market structures, where upstream firms offer perfect substitute products, or deal with manufacturers in conditions of exclusivity. Two exceptions are Rey and Vergé (2010) and Nocke and Rey (2018), who consider settings featuring nonlinear tariffs and multiple interlocking bilateral relationships. Whereas these articles propose models in which suppliers offer imperfect substitute intermediate goods, our contribution is to study the case in which upstream firms offer complements.

The article proceeds as follows: Section 2 illustrates our main results in a linear model. Section 3 presents the general model. Section 4 solves for the equilibrium with vertical separation and integration. Section 5 analyzes our information-free policies. Section 6 discusses the robustness of the results in related economic environments. Section 7 concludes by providing case evidence that supports our mechanism. All proofs are in the Appendix.

## 2 A Linear Example

To begin with, we provide a stripped-down version of our model to illustrate how the industry structure shapes the welfare consequences of patent pools. Two manufacturers,  $M_A$  and  $M_B$ , are Cournot rivals in a market where demand is  $P(Q) = 1 - Q$ , with  $Q = q_A + q_B$ . The manufacturers bear zero marginal cost. They need two perfectly complementary inputs, 1 and 2, to produce the output good. There are two licensors,  $L_1$  and  $L_2$ , each owning a patent on the respective input. Alternatively, a non-patented status-quo technology is available to substitute for each patent at a per-unit cost of  $c > 0$ , where  $c$  captures a patent’s essentiality.

When contracting with manufacturer  $M_i$ ,  $i = A, B$ , each licensor  $L_j$ ,  $j = 1, 2$ , offers a public two-part tariff consisting of a fixed fee ( $F_{ji}$ ) and a royalty rate ( $w_{ji}$ ). Manufacturers simultaneously decide whether to accept these offers. Upon acceptance, they pay the tariffs and produce the final good for consumers. We focus on the symmetric equilibrium of this offer game, that is, the one in which symmetric licensors offer the same tariff to each  $M_i$ .

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<sup>11</sup>The literature on common agency has typically analyzed games with asymmetric information (see, e.g., Bernheim and Whinston, 1986; Martimort, 1996; Martimort and Stole, 2009). Our focus, instead, is on the problem of how principals design tariffs in a setting with complete information and competing agents.

Consider first an industry in which no licensor is integrated with a manufacturer. Conditional on accepting licensors' offers, manufacturers play a competition game that leads to equilibrium quantities of  $q_i = [1 - 2(w_{1i} + w_{2i}) + w_{1-i} + w_{2-i}] / 3 \equiv q_i^*$ , with  $i = A, B$  and  $-i \neq i$ . The profit of  $M_i$  is then  $\pi_i(q_i^*, q_{-i}^*) - \sum_j F_{ji}$ . To induce  $M_i$  to accept its offer,  $L_j$  must propose a fixed fee such that this profit is at least as large as  $\max\{\pi_i(q_i(2c), q_{-i}^*), 0\}$ , where, with a slight abuse of notation, we use  $q_i(2c) \equiv \arg \max_{q \geq 0} q(1 - q - q_{-i}^* - 2c)$  to denote  $M_i$ 's optimal quantity when it rejects both offers.<sup>12</sup> That is,  $\max\{\pi_i(q_i(2c), q_{-i}^*), 0\}$  is  $M_i$ 's outside option when contracting with a licensor.<sup>13</sup> It follows that  $L_j$  sets its tariff to maximize  $\sum_i (w_{ji}q_i^* + F_{ji})$ , subject to  $F_{ji} = \pi_i(q_i^*, q_{-i}^*) - F_{-ji} - \max\{\pi_i(q_i(2c), q_{-i}^*), 0\}$ . Solving for the symmetric equilibrium prices yields  $w^{VS} = c/2$  if  $c \leq 2/5$  and  $w^{VS} = 1/5$  otherwise.

Assume licensors join a patent pool, and cooperatively decide on the offers to manufacturers. The pool sets a single royalty rate ( $W_{pi}$ ) for both patents. After solving for the symmetric equilibrium of the offer game, we find that this royalty rate is  $W_p^{VS} = 2c/3$  if  $c \leq 3/8$  and  $W_p^{VS} = 1/4$  if otherwise. Hence,  $W_p^{VS} < 2w^{VS}$  for all  $c > 0$ . This implies that, with the pool, manufacturers pay lower royalties. They can then expand their outputs, and the price paid by consumers falls. Hence, the pool is procompetitive with vertical separation.

This conclusion extends the well-known result on patent pools' procompetitive nature to an industry with competing manufacturers and vertical separation. The intuition is as follows: When acting independently,  $L_j$  takes into account that, by setting a lower royalty rate, it can extract more from  $M_A$  and  $M_B$  through the fixed fees. This occurs because the reduced royalty rate increases manufacturers' profits and (weakly) lowers their outside option. However,  $L_j$  does not take into account that also  $L_{-j}$  demands a positive fixed fee, and thus fails to consider the impact of a reduction in  $w_{ji}$  on  $L_{-j}$ 's profit. Once in the pool, horizontal price coordination means that licensors internalize this externality, yielding lower royalty rates.

Assume now  $L_1$  is merged with  $M_A$ . In the absence of the pool, these two firms will set an internal transfer price of zero (as any higher price is not renegotiation proof). When setting  $w_{1B}$ ,  $L_1$  now maximizes the sum of own licensing revenues and  $M_A$ 's product market profits. As a consequence, it will raise  $M_B$ 's cost with the goal of restricting industry output and thus raise industry revenues.

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<sup>12</sup>We assume that manufacturers cannot observe the acceptance or rejection decision of their rival.

<sup>13</sup>As we show in the general model, conditional on rejecting  $L_j$ 's offer, in equilibrium a manufacturer will use both status-quo technologies instead of accepting  $L_{-j}$ 's offer and using the status-quo technology to replace  $L_j$ 's patent.

We then consider two cases, depending on whether the non-patented technology is efficient enough to allow  $M_B$  to remain active on the market. Assume this is the case. Then,  $L_1$  sets  $w_{1B} = c$ , which leads to  $F_{1B} = 0$ . Given  $L_1$ 's offer, the non-integrated licensor ( $L_2$ ) solves a maximization problem that is similar to the one with vertical separation, and sets  $w_A^{VI} = w_B^{VI} = c/3$ . After the pool forms, the consortium will take into account the profit of  $M_A$  in its decision on the royalty rates, because  $M_A$  is integrated with a pool member ( $L_1$ ). This implies joint-profit maximization of  $L_1$ ,  $M_A$ , and  $L_2$ . In line with what  $L_1$  does under independent licensing, the pool transfers the bundle of patents to  $M_A$  at no cost and raises to  $2c$  the royalty that  $M_B$  has to pay for the same bundle. These outcomes reflect a trade-off between horizontal and vertical price coordination. On the one hand, horizontal price coordination implies that the pool reduces the patent prices paid by the integrated manufacturer, which is procompetitive. On the other hand, vertical price coordination implies that the pool raises the total patent price paid by the non-integrated manufacturer, which has an anticompetitive effect. Comparing the sum of the royalty rates with the pool ( $2c$ ) and without the pool ( $5c/3$ ), we find that the anticompetitive effect dominates.

Finally, we discuss the case in which the non-patented technologies do not impose a constraint on the patent price set by the integrated licensor. If this is the case, the downstream market can be monopolized by  $L_1$ - $M_A$  both with and without the pool, because, in both regimes, the royalty rates offered to  $M_B$  will be so high that this manufacturer cannot profitably produce. The pool is then welfare neutral.

We next describe our general model. There, we allow for  $N$  licensors, a general demand function, and also consider an industry with two integrated licensors.

### 3 The General Model

There are  $M = 2$  manufacturers,  $M_A$  and  $M_B$ , that are Cournot rivals in a downstream market. They need to acquire  $N$  perfectly complementary inputs in order to produce a homogeneous output good. In the upstream market, there are  $N \geq 2$  symmetric licensors  $L_j$ ,  $j = 1, \dots, N$ . Licensor  $L_j$  owns a patent on input  $j$ .

When contracting with manufacturer  $M_i$ ,  $i = A, B$ , licensor  $L_j$  makes an observable take-it-or-leave-it two-part tariff offer consisting of a fixed component (or fixed fee),  $F_{ji}$ , and a royalty rate,  $w_{ji}$ , per unit of output sold by  $M_i$ .<sup>14</sup> Conditional on accepting all

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<sup>14</sup>In Section 6, we discuss our results in a setting with  $M > 2$ , show that our conclusions extend to a framework with differentiated products, consider price competition, secret offers and general quantity-forcing tariffs.

licensors' offers, manufacturer  $M_i$ 's total marginal cost is  $\sum_{j=1}^N w_{ji}$  (i.e., it bears no other cost of production). If a manufacturer rejects the offer of a licensor, it can use an inferior (nonpatented) status-quo technology, and bear a marginal production cost of  $c > 0$  for each unit of output; thus,  $c$  captures the essentiality of a patent. Moreover, a manufacturer does not observe the contract acceptance decisions of its competitor.<sup>15</sup>

Absent a patent pool, either firms are not integrated (vertical separation) or a licensor is affiliated with one of the two manufacturers (vertical integration). Given vertical separation, the game proceeds as follows:

1. Each licensor  $L_j$  offers to each manufacturer  $M_i$  a public two-part tariff  $T_{ji} \equiv (w_{ji}, F_{ji})$ . Manufacturers simultaneously accept or reject the contract offers.
2. Manufacturers set downstream quantities and pay the respective tariffs to licensors. Final consumers make their purchases, and profits realize.

We denote by  $q_i$ ,  $i = A, B$ , the quantity sold by each manufacturer  $M_i$ , so that aggregate output is given by  $Q = q_A + q_B$ . The (inverse) demand function for the final good is  $P(Q)$ , it is strictly decreasing and twice continuously differentiable whenever  $P(Q) > 0$ . Moreover, we employ the standard assumption that  $P'(Q) + QP''(Q) < 0$ , which guarantees that the profit functions are (strictly) quasi-concave and that the Cournot game exhibits strategic substitutability (Vives, 1999). We solve for the subgame perfect Nash equilibrium of the game.

With vertical integration, the licensor and its downstream affiliate maximize joint profits. The game proceeds as laid out above, with the exception that, as is natural and in line with Hart and Tirole (1990) and Rey and Tirole (2007), the downstream affiliate  $M_i$  of the integrated firm  $L_j$ - $M_i$  knows the acceptance decision of the rival  $M_{-i}$  on patent  $j$ .<sup>16</sup>

Our modelling strategy borrows from Lerner and Tirole (2004) the assumptions that (i) the pool forms at equilibrium if it raises total profits as compared to independent licensing, and that (ii), if licensors join a pool, they maximize and then share joint profits.<sup>17</sup> This means that, if an integrated firm joins the pool, the latter will internalize the impact of its decisions on the profit of the integrated downstream unit. The pool sets two-part tariffs  $T_{pi} \equiv (W_{pi}, F_{pi})$  that each manufacturer  $M_i$  needs to pay to obtain the right to use the

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<sup>15</sup>This assumption simplifies the analysis. As we show in the Online Appendix available on the authors' webpages, our main results carry over in a model where contract acceptance decisions are observable.

<sup>16</sup>We also assume that, were the rival manufacturer  $M_{-i}$  to reject the offer of  $L_j$ , the integrated manufacturer  $M_i$  believes that the rival is still accepting the offer of  $L_k$ , with  $k \neq j$ , as it does in equilibrium.

<sup>17</sup>The assumption is that there are no failures in coordination or bargaining among pool members (Rysman and Simcoe, 2008; Gallini, 2014).

bundle of patented inputs. Otherwise, the manufacturer will have to resort to the status-quo technologies and pay  $Nc$ .

The Cournot equilibrium is the solution to the system

$$q_A = \arg \max_q (P(q + q_B) - \sum_{j=1}^N w_{jA})q \quad \text{and} \quad q_B = \arg \max_q (P(q_A + q) - \sum_{j=1}^N w_{jB})q. \quad (1)$$

Let  $W_i$  denote the sum of per-unit royalties paid by a manufacturer  $M_i$ , so that  $W_i = \sum_{j=1}^N w_{ji}$ . Then, the couple  $(q_A(W_A, W_B), q_B(W_B, W_A))$  characterizes the Cournot-Nash equilibrium—that is, the solution of (1)—which is unique given the assumed properties of inverse demand. We also use  $\pi_i(W_i, W_{-i})$ , with  $i = A, B$ , to denote  $M_i$ 's equilibrium profit gross of fixed fees, when  $M_i$  pays  $W_i$  and its rival  $M_{-i}$  pays  $W_{-i}$ .

Throughout the analysis, we assume that the status-quo technology is effective in constraining the market power of a licensor. Specifically, this means that the quantity produced by a manufacturer  $M_i$  is positive despite using the status-quo technologies on all inputs, even if the rival manufacturer  $M_{-i}$  obtains all patents at zero costs, i.e.,  $q_i(Nc, 0) > 0$ . With linear demand ( $P(Q) = 1 - Q$ ), this condition holds true if the value of  $c$  is lower than  $1/(2N)$ .

To establish the consequences of the pool on consumer welfare, we take advantage of the following (aggregative) property of Cournot games:

**PROPERTY 1.** *The sum of the product-market first-order conditions of  $M_A$  and  $M_B$  is given by*

$$2P(Q) + P'(Q)Q = \sum_{j=1}^N w_{jA} + \sum_{j=1}^N w_{jB} = W_A + W_B. \quad (2)$$

*Therefore, the industry quantity ( $Q$ ) is uniquely determined by the sum of the royalty rates paid by manufacturers ( $W_A + W_B$ ).*

Because the left-hand side of (2) is decreasing in  $Q$ , an important implication of Property 1 is that, to prove that a pool yields an anticompetitive outcome, it is sufficient to show that the pool leads to an increase in the sum of licensors' royalties ( $W_A + W_B$ ). This causes a reduction of the industry quantity, and an increase in the final good's price. In line with this property, we will sometimes use  $Q(W_A + W_B)$  to denote the industry quantity given that the total royalties paid by the manufacturers are  $W_A + W_B$ . Another property of our Cournot

game is that any price increase implies a reduction in consumer and total surplus. Thus, our conclusions on welfare do not depend on the specific standard we employ.<sup>18</sup>

## Discussion

We now discuss the key ingredients of our model, focusing on the unpatented status-quo technologies, the use of two-part tariffs, and the one of public offers.

In line with the literature (e.g., Lerner and Tirole, 2015; Rey and Tirole, 2018), licensors' market power is curbed by  $c$ , the marginal cost implied by the use of status-quo technologies. As in these articles,  $c$  can be interpreted as the cost of adopting and implementing a rival technology. In this respect,  $c$  represents a patent's degree of essentiality. This approach is natural whenever pool members license their patents outside a technology standard.<sup>19</sup> However, one may object that several pools market standard-essential patents, meaning that, in principle, a manufacturer cannot implement the standard without infringing the pool's patents. In support of our approach, we note that standards often include optional features to give licensees flexibility in the implementation phase. Manufacturers can therefore use outside technologies on these features. In addition, the evidence in Lemley and Simcoe (2018) documents that a significant number of essential patents do not win infringement cases in court, which reduces the value of their essentiality.

We also assume that licensors charge two-part tariffs. This is motivated by the fact that most patent pools demand substantial fixed payments next to the per-unit royalties. For example, Gilbert (2011) documents the pricing structure of several pools in different industries. In his sample, pool administrators like MPEG LA, VIA Licensing, and SISVEL charge non-linear tariffs in most of their licensing programs.<sup>20</sup> Also, Hegde (2014) provides recent evidence on the use of two-part tariffs in licensing contracts within the pharmaceutical, biotechnology, and medicine industries.

Finally, in the main model, we assume that contract offers are public (and provide the analysis of secret offers in Section 6). The reason is that many pools publicly announce their tariffs on their websites. For example, companies administrating pools on important patents, such as the one the MPEG Video, DVD, or HDMI technologies, publicly release the pool's tariffs. This does not imply that, absent the pool, contract terms are also public.

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<sup>18</sup>We focus on a scenario in which technologies are given, which implies that the pool does not change the R&D incentives of firms. We discuss this point further in the conclusions.

<sup>19</sup>See, for example, the debate in the engineering literature on patent circumvention strategies (e.g., Wang, 2008; Veldhuijzen van Zanten and Wits, 2015).

<sup>20</sup>Moreover, in the conversations we had with firms participating in patent pools, corporate managers reported that the fixed fees account for a large share of pools' revenues.

However, we keep the assumption of observability when comparing independent licensing and coordinated licensing to isolate the impact of the pool from the one of secrecy. In addition, changing this would be equivalent to assuming that an important feature of pool formation is to make contract terms public, which does not seem realistic.<sup>21</sup>

## 4 Equilibrium Analysis

When making an offer to a manufacturer  $M_i$ , a licensor  $L_j$  can claim up to its marginal contribution to  $M_i$ 's profit, as given by the difference between  $M_i$ 's profit when accepting the licensor's offer, and  $M_i$ 's profit when rejecting  $L_j$ 's offer and using the status-quo technology. However, in our offer game with perfectly complementary patents, an equilibrium in which each licensor  $L_j$  demands its marginal contribution from  $M_i$  does not exist.<sup>22</sup>

This outcome is a consequence of the fact that the value of patents is super-additive. Its intuition is particularly easy to see if  $c$  is large. The marginal contribution of each licensor is then a relatively large share of  $M_i$ 's profit; thus, if each licensor asks for its marginal contribution in the fixed component of the tariff,  $M_i$  incurs a loss when accepting all offers. However, even if  $c$  is small,  $M_i$ 's profit increase when  $L_j$  reduces its royalty rate below  $c$  is larger if  $M_i$  obtains the other patents at cheaper prices. The reason is that a manufacturer's profit is convex in production costs, which implies that a cost reduction is more valuable at lower cost levels. It follows that if all licensors ask for their marginal contribution to  $M_i$ 's profit,  $M_i$ 's rent is larger when rejecting all offers and using the status-quo technologies on all inputs than when accepting some (or all) of the licensors' offers.

Despite this non-existence result, our offer game has a continuum of equilibria. In any of these equilibria,  $M_i$ 's profit must be at least as large as the profit it raises when using the status-quo technologies only, but there is no restriction on how licensors share the remaining profit of the manufacturer via the fixed components of the tariffs. To deal with this property of our game, we select an equilibrium in which symmetric licensors offer the same contract (i.e., the same royalty rate and fixed fee) to a manufacturer.<sup>23</sup> As we will show later, in each industry structure we consider there is a unique such equilibrium. This focus on

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<sup>21</sup>We can also compare the case in which independent licensors offer secret contracts with the case in which a pool offers public contracts. In our model, this yields the unambiguous result that pools are *anticompetitive* regardless of the industry structure. We discuss this point in Section 6.

<sup>22</sup>The formal proof for this result can be found in the Online Appendix.

<sup>23</sup>With vertical integration, in equilibrium, a vertically separated firm can offer a different contract than a vertically integrated firm. That is, we focus on a notion of symmetric equilibrium in which firms with the same business structure (separated or integrated) offer the same contracts.

the symmetric equilibrium is made in much of the existing literature (for example, Lerner and Tirole, 2015); it seems natural in a context with equally-efficient licensors, and greatly simplifies the equilibrium analysis. To study the robustness of our results to this assumption, we show that our conclusions remain valid even when considering the asymmetric equilibria of the offer game with  $N = 2$  licensors.<sup>24</sup>

In what follows, we first consider an industry featuring no vertically integrated firm, then one with a single integrated firm, and finally the one with two integrated companies. In each case, we compare the outcome without patent pool (i.e., under independent licensing) with the one with a patent pool (i.e., under coordinated pricing).

## Vertical Separation

Assume that no manufacturer is vertically integrated.

### Independent Licensing

First, let licensors set their tariffs noncooperatively.

**LEMMA 1.** *In the unique symmetric equilibrium with vertical separation, each licensor  $L_j$ ,  $j = 1, \dots, N$ , sets its royalty rate  $w_{jA}^{VS} = w_{jB}^{VS} = w^{VS}$  strictly between 0 and  $c$ , for all  $c > 0$ . The fixed component of the tariff is then equal to*

$$F^{VS} = \frac{\pi(W^{VS}, W^{VS}) - \max_q \{(P(q + q(W^{VS}, W^{VS})) - Nc)q\}}{N}, \text{ with } W^{VS} = Nw^{VS}. \quad (3)$$

First, the lemma shows that licensors' royalty rates are strictly positive; thus, horizontal double marginalization arises at equilibrium. When determining the royalty rate in the tariff, a licensor  $L_j$  will not only take into account the impact of this decision on the value of the bilateral relationship with  $M_i$ , but also on the profit it raises from  $M_{-i}$ . In particular, by raising  $w_{ji}$ , the cost of  $M_i$  increases, implying a lower  $q_i$ . This is beneficial to  $M_{-i}$ , thereby allowing  $L_j$  to demand a higher fixed fee from this manufacturer. In this way, licensors can dampen downstream competition and increase the profit they obtain from  $M_A$  and  $M_B$ . Second, we find that the royalty rates are lower than  $c$ . The reason is as follows:  $M_i$ 's bargaining threat is given by the profit it raises when using the status-quo technologies. This rent rises in the royalty rates paid by  $M_{-i}$ ; thus, it does not pay off for  $L_j$  to raise  $w_{j-i}$

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<sup>24</sup>The formal proofs are in the Online Appendix.

to  $c$  because this increases the value of  $M_i$ 's outside option and reduces the fixed component of the tariff that  $L_j$  can claim from  $M_i$ .

As far as the value of  $F^{VS}$  in (3) is concerned, in the unique symmetric equilibrium, licensors equally share the difference between  $M_i$ 's Cournot profit and the profit that the manufacturer obtains when rejecting all licensors' offers. Because  $M_{-i}$  cannot observe whether  $M_i$  accepted or rejected these offers, it produces as on the equilibrium path.<sup>25</sup>

To illustrate the results in the lemma by means of a simple example, let  $P(Q) = 1 - Q$ . Then,  $L_j$  sets  $w^{VS} = Nc/(N + 2)$ . Thus, each manufacturer pays  $N^2c/(N + 2)$  to produce one unit of the final good. The royalty rates increase with the number of licensors  $N$ . The more patents are needed to produce the final output, the more expensive it gets for a manufacturer to resort to the status-quo technologies. This implies that the outside option is less valuable, which diminishes  $M_i$ 's bargaining threat. As explained above, a licensor then benefits less from lowering the royalty rate. This result contrasts with the findings in models with complementary products where buyers do not compete: In that case, each product's price falls as the number of complementary products gets larger.

## Patent Pool

We proceed by studying the impact of the patent pool on licensors' pricing choices. Suppose that all licensors form a pool and cooperatively set the pool's tariffs.

**LEMMA 2.** *With vertical separation, in equilibrium, the patent pool sets a royalty rate of  $W_{pA}^{VS} = W_{pB}^{VS} = W_p^{VS}$  that is strictly larger than zero and strictly below  $Nc$  for all  $c > 0$ . The fixed component of the tariff is then equal to*

$$F_p^{VS} = \pi(W_p^{VS}, W_p^{VS}) - \max_q \{ (P(q + q(W_p^{VS}, W_p^{VS})) - Nc)q \}.$$

The royalty rate set by the pool,  $W_p^{VS}$ , lies between zero and  $Nc$ . For example, with linear demand ( $P(Q) = 1 - Q$ ) the pool sets  $W_p^{VS} = Nc/3$  for the bundle of patents. Although the intuition is analogous to the one developed with independent licensing, it is worth remarking that the pool maximizes the *sum* of licensors' profits.

At the optimal tariff structure, the pool offers symmetric contracts in which royalty rates are *below* the ones maximizing industry profits. Alternatively, the pool could offer asymmetric "integration-like" contracts involving, for example, a per-unit royalty of zero to

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<sup>25</sup>This is reflected in the second term of the numerator of  $F^{VS}$ , where  $M_{-i}$ 's quantity is  $q_{-i} = q(W^{VS}, W^{VS})$  although  $M_i$  pays  $Nc$  to produce one unit of output.

$M_i$  and a per-unit royalty equal to  $Nc$  to  $M_{-i}$ . This would restrict downstream supplies, thus raising industry profits. However, under these asymmetric deals,  $M_i$ 's option of using the status-quo technologies is particularly attractive, because the rival manufacturer pays high costs. This implies that the pool can demand only a relatively small fixed fee in the tariff to  $M_i$ . Instead, offering symmetric contracts with lower per-unit royalties allows the pool to curb the rent left to manufacturers, and extract a larger share of a smaller pie.

What are the consequences of the pool on consumer welfare? With linear demand, the answer is clearcut. As  $Nc/3 < N^2c/(N + 2)$  for all  $N \geq 2$ , the pool lowers the sum of royalty rates paid by each manufacturer, and is therefore procompetitive by Property 1. The following proposition extends this conclusion to the case with general demand.

**PROPOSITION 1.** *With vertical separation, licensors form the patent pool and this pool is procompetitive.*

The patent pool is procompetitive because it coordinates licensors' pricing decisions. This occurs in two ways. First, with independent licensing,  $L_j$  sets the royalty rate  $w_{ji}$  only considering the effect on its own fixed fee  $F_{ji}$ . That is, it does not take into account that lowering the royalty rate also allows other licensors to demand a higher fixed fee. Specifically, the part of  $M_i$ 's profit that  $L_j$  can extract (dropping functional notation) is  $(P - \mathcal{W}_{-ji})q_i - \mathcal{F}_{-ji} - OO_i$ , where  $\mathcal{W}_{-ji}$  and  $\mathcal{F}_{-ji}$  denote the sums of royalty rates and fixed fees charged to  $M_i$  by all the other licensors, and  $OO_i$  is  $M_i$ 's outside option.<sup>26</sup> As shown in Lemma 1,  $\mathcal{W}_{-ji}$  is strictly positive. This implies that  $L_j$ , when reducing  $w_{ji}$ , gets less than the full price  $P$  at the margin. By contrast, the pool is the only entity licensing patents; thus,  $\mathcal{W}_{-ji}$  is equal to zero in that case. This result is complemented by a second effect. When lowering the royalty rate offered to  $M_i$  to reduce  $M_{-i}$ 's bargaining threat, the pool fully benefits from this reduction. By contrast, an independent licensor shares this benefit with  $N - 1$  other licensors. Taking both effects together, the patent pool has a stronger incentive to reduce the royalty rate than independent licensors.

The result in Proposition 1 extends the standard argument in favor of patent pools to a setting with competing licensees, nonlinear pricing and non-integrated constrained monopoly licensors. It is important to remark that in models featuring pools licensing to non-competing buyers, the use of nonlinear contracts makes the pool welfare neutral: Even absent the pool, licensors can optimally set the royalty rate to zero, and then extract the entire buyer surplus

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<sup>26</sup>From above, this outside option is given by  $\max_q \{(P(q + q(W^{VS}, W^{VS})) - Nc)q\}$ .

via the fixed component of the tariff. Our analysis shows that this result fails to hold when buyers compete and contracts are observable.<sup>27</sup>

To conclude this section, we remark that by revealed preferences, the pool increases licensors' joint profits. The reason is that it sets a different equilibrium tariff than independent licensors, but could have chosen the same.

## Single Integration

We now study how the presence of an integrated firm ( $L_1$ - $M_A$ ) influences pricing decisions with and without the pool. Recall that, in the first stage of the game, the integrated licensor  $L_1$  will set the royalty rates to maximize the joint profits of its upstream and downstream divisions. We will show that the competitive effects of the pool depend on the number of licensors in the industry.

## Independent Licensing

First, let licensors market their patents independently.

**LEMMA 3.** *With independent licensing, in the unique stable symmetric equilibrium, the integrated firm  $L_1$ - $M_A$  sets the internal patent price equal to zero and  $w_{1B}$  equal to  $c$ ; thus, the fixed component of the tariff set by  $L_1$ - $M_A$  to  $M_B$  is equal to zero. Each non-integrated licensor  $L_j$ ,  $j = 2, \dots, N$ , sets its royalty rates,  $w_A^{SI}$  and  $w_B^{SI}$ , strictly between zero and  $c$ . Moreover, the fixed fees set by each non-integrated licensor  $L_j$ ,  $j = 2, \dots, N$ , are*

$$F_A^{SI} = \frac{\pi_A(W_A^{SI}, W_B^{SI}) - \max_q \{ (P(q + q_B(W_B^{SI}, W_A^{SI})) - (N-1)c)q \}}{N-1},$$

$$F_B^{SI} = \frac{\pi_B(W_B^{SI}, W_A^{SI}) - \max_q \{ (P(q + q_A(W_A^{SI}, W_B^{SI})) - Nc)q \}}{N-1},$$

with  $W_A^{SI} = (N-1)w_A^{SI}$  and  $W_B^{SI} = c + (N-1)w_B^{SI}$ .

When setting its tariffs, the integrated licensor takes into account their impact on the profit of the downstream affiliate. It therefore aims at monopolizing the final good market via its downstream manufacturer. However, it is constrained by the status-quo technology. Accordingly, it raises  $w_{1B}^{SI}$  to  $c$  to reduce  $M_B$ 's competitive threat. It also sets a zero transfer

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<sup>27</sup>If contracts are not observable, licensors will set the royalty rate to zero even without the pool, which implies that the pool is then also welfare neutral (see Section 6).

price internally to maximize the downstream unit's profit.<sup>28</sup> A non-integrated licensor  $L_j$ ,  $j = 2, \dots, N$ , sets positive values for  $w_A^{SI}$  and  $w_B^{SI}$ : As discussed above, in this way  $L_j$  softens product-market competition and obtains a larger profit through the fixed fees. The value of  $F_A^{SI}$  in the lemma reflects the fact that manufacturer  $M_A$  is integrated, implying that it can obtain the upstream affiliate's patent at zero costs. This also means that, when rejecting all non-integrated licensors' offers,  $M_A$  will still obtain the patent on input 1 at zero cost whereas paying  $(N - 1)c$  for the remaining patents.

Setting  $w_{1B}^{SI} = c$  is not necessarily the unique equilibrium in our game. In fact, there may also exist an equilibrium featuring  $w_{1B} < c$ . However, as we show in the proof of the lemma, the latter equilibrium (in case it exists) is unstable, that is, the slope of the reaction function of  $w_{1B}$  with respect to any  $w_{jA}$  is larger than 1 in absolute value. This implies, for example, that, as shown by Vives (1999), this equilibrium will not be approached by any Cournot tatônnement process, and has unintuitive comparative-static properties. Both problems do not arise in the unique stable equilibrium featuring  $w_{1B}^{SI} = c$ . The latter equilibrium is therefore the natural candidate for equilibrium selection.

Before proceeding with the analysis of the pool, we consider the example with linear demand ( $P(Q) = 1 - Q$ ). Each non-integrated licensor  $L_j$ , with  $j = 2, \dots, N$ , then sets  $w_A^{SI} = w_B^{SI} = c(N - 1)/(N + 1)$ .<sup>29</sup> On the one hand, as in the vertically separated industry, these royalty rates increase in the number of licensors. On the other hand, they are lower than with vertical separation. The reason is that the fixed component of the tariff set by  $L_1$ - $M_A$  is nil; thus, there are only  $N - 1$  licensors claiming a positive share of a manufacturer's profit. This pushes all non-integrated licensors to reduce their patent price.

## Patent Pool

Assume now that  $L_1$ - $M_A$  joins a patent pool with all the other  $N - 1$  licensors  $L_j$ ,  $j = 2, \dots, N$ . The pool sets its patent prices by maximizing the profit of all licensors and their affiliates. Specifically, this means that the pool takes into account the impact of its choices on the profit of  $M_A$ , which is integrated with  $L_1$ . This implies that side payments between pool members may include the profit of  $M_A$ . In this way, the integrated firm can distribute part of the profit of  $M_A$  to non-integrated licensors to ensure they participate in the pool.<sup>30</sup>

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<sup>28</sup>Because the integrated firm cannot commit to a downstream quantity in the first stage, any strictly positive internal patent price would be renegotiated to zero before manufacturers compete (see Reisinger and Tarantino (2015) for a formal proof).

<sup>29</sup>The values of  $w_A^{SI}$  and  $w_B^{SI}$  need not be the same with general demand. As we show in the proof of the lemma,  $w_A^{SI} < w_B^{SI}$  if demand is concave.

<sup>30</sup>In Section 5, we consider a policy which forbids such "vertical" side payments.

**LEMMA 4.** *With vertical integration, in equilibrium, the patent pool sets the internal patent price equal to zero and offers to  $M_B$  the following contract:  $T_{pB}^{SI} = (Nc, 0)$ .*

In this setting,  $M_A$  participates in the pool via its upstream affiliate  $L_1$ . As a consequence, the pool grants  $M_A$  privileged access to its patents. At the same time, it raises to  $Nc$  the price that  $M_B$  has to pay for the bundle of patents, thereby reducing competition between  $M_A$  and  $M_B$ . Ideally, the pool would make an unacceptable offer to  $M_B$  and monopolize the final-good market via the affiliated manufacturer of its member licensor  $L_1$ . However, because of the status-quo technologies, this is not possible. Then, given that  $M_B$  is active, the pool would like to set a positive patent price to  $M_A$  to soften downstream competition. But because  $M_A$  is part of the pool through  $L_1$ , any commitment to a strictly positive internal transfer price is not credible because the pool members will renegotiate the price to zero before downstream competition takes place. Therefore, in equilibrium, the pool sets its patent price to  $M_A$  to zero. Coordination on the two royalty rates  $(W_{pB}, W_{pA})$  leads to an increase in licensors' joint profits, so that the pool will always form at equilibrium.

The next proposition gives the welfare consequences of the pool with single integration.

**PROPOSITION 2.** *There exists a unique value  $N^{SI} > 2$  such that the patent pool is procompetitive for all  $N > N^{SI}$  and anticompetitive for all  $N < N^{SI}$ .*

With vertical integration, the patent pool combines horizontal and vertical price coordination, thereby giving rise to a novel trade-off. On the one hand, the presence of an integrated manufacturer implies that the pool does not set a positive transfer price, which means that, compared to independent licensing, the integrated manufacturer obtains *all* patents at a zero royalty. The pool therefore allows licensors to internalize the contracting externalities and, thus, horizontal coordination boosts welfare. On the other hand, the pool acts to reduce downstream competition by raising the price paid by the non-integrated manufacturer for the bundle of patents. Vertical coordination then has an anticompetitive effect, because the non-integrated manufacturer pays more than with independent licensing.

Which of these two effects dominates depends on the number of licensors. Recall that, as we show above within the linear demand example, the larger the number of licensors, the higher a non-integrated licensor's royalty rate with independent licensing. As a consequence, if the number of licensors is relatively large, the reduction in the royalty rate paid by  $M_A$  tends to dominate the increase in the royalty rate offered to  $M_B$ . The proposition shows that, if the upstream market is sufficiently crowded, the pool's preferential treatment of the integrated manufacturer leads to an increase in efficiency that outweighs the welfare loss caused by the pool's anticompetitive strategy on  $M_B$ .

Comparing the royalty rates with linear demand, we find that the pool is procompetitive if  $c(N-1)(N-3)/(N+1) > 0$ . Thus,  $N^{SI} = 3$ , and the pool is anticompetitive if  $N = 2$  and procompetitive for all  $N \geq 4$ . In the next section, we will show that this threshold can rise considerably in the presence of two integrated companies.

The effects driving the competitiveness of pool formation in our model are related to the ones on the competitive effects of vertical mergers. For example, as shown by Chen (2001), a vertical merger has an efficiency effect due to the avoidance of double marginalization within the integrated firm but also a collusive effect as the upstream unit raises the wholesale price to the downstream rival.<sup>31</sup> In contrast to this literature, we find that with patent pool formation, the industry structure (i.e., the extent of vertical integration in the industry) determines which effect is dominating. It is, however, interesting to note that although pool formation leads to joint profit maximization of licensors, and it therefore similar to an upstream horizontal merger, the competitive effects have a close connection to those of vertical mergers.

To conclude, it is worth remarking that, in a model without status-quo technologies, the market would be monopolized by  $M_A$  with and without pool. This occurs because, due to patents' perfect complementarity,  $L_1$  can deny  $M_B$ 's access to its patent, which prevents  $M_B$  from competing on the market. A pool cannot do better than this, and is thus welfare neutral. However, the possibility to shut down a manufacturer by denying patent access is not a realistic feature in most markets in which patents are needed for the production of final output. In addition, differentiation between manufacturers' products would break this neutrality result. In fact, the non-integrated manufacturer would be active if the degree of differentiation is sufficiently large (both with and without pool), and aggregate patent prices would still be higher with pool if the number of patents is relatively small.

## Double Integration

Assume now that there are two vertically integrated firms:  $L_1$ - $M_A$  and  $L_2$ - $M_B$ .

### Independent Licensing

**LEMMA 5.** *In the unique symmetric equilibrium, each integrated licensor  $L_k$ ,  $k = 1, 2$ , sets the internal patent price equal to zero and the royalty rate to the rival manufacturer equal to*

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<sup>31</sup>Chen's (2001) model is very different from ours because, for example, in his model upstream firms charge linear prices, downstream firms sell differentiated products, and they incur switching costs when changing suppliers.

c. Therefore, its fixed component of the tariff is equal to zero. Instead, each non-integrated licensor  $L_j$ ,  $j = 3, \dots, N$ , sets its royalty rate  $w_{jA}^{DI} = w_{jB}^{DI} = w^{DI}$  strictly between zero and  $c$ , and offers a fixed component of the tariff given by

$$F^{DI} = \frac{\pi(W^{DI}, W^{DI}) - \max_q \{ (P(q + q(W^{DI}, W^{DI}))) - (N - 1)c \} q}{N - 2},$$

with  $W^{DI} = c + (N - 2)w^{DI}$ .

In line with the results obtained with single integration, each integrated licensor raises to  $c$  the patent price to the rival manufacturer with the intent of limiting competition. In this context, though, it is less obvious that an integrated firm wants to undertake this strategy. The reason is that it faces a trade-off: Because  $L_1$ - $M_A$  sets the internal transfer price to 0,  $M_A$  receives access to the patent owned by its integrated licensor at a lower price than the rival. On the one hand, from a cost perspective,  $L_2$  benefits if the quantity produced by  $M_A$  increases at the expense of its integrated manufacturer  $M_B$ , because it can extract part of  $M_A$ 's profit via the fixed fee.  $L_2$  then has the incentive to lower  $w_{2A}$ . On the other hand,  $L_2$  benefits from raising  $w_{2A}$ , as this softens product-market competition.<sup>32</sup> Our result shows that, under our assumption that  $q(Nc, 0) > 0$ , the latter effect dominates. Thus,  $w_{1B}^{DI} = w_{2A}^{DI} = c$  in equilibrium. Finally, for the reasons explained above, a non-integrated licensor sets  $w^{DI} \in (0, c)$ .

With linear demand ( $P(Q) = 1 - Q$ ), the per-unit royalty set by a non-integrated licensor is  $w_{ji}^{DI} = c(N - 2)/N$ , which is again increasing in  $N$ . This royalty is smaller than those set by non-integrated licensors with vertical separation or single integration. This is again due to the reduction in the number of licensors demanding a positive fixed fee from manufacturers (which is  $N - 2$  in this setting).

## Patent Pool

In what follows, we define by  $q^m$  the monopoly quantity, that is,  $q^m = \arg \max_q \{P(q)q\}$ . Accordingly, the monopoly price is  $P(q^m)$ . We then find the following:

**LEMMA 6.** *In equilibrium, the pool sets the royalty rate to manufacturers equal to  $W_p^{DI} = P(q^m)/2$  if  $P(q^m)/2 \leq (N - 1)c$  and  $W_p^{DI} = (N - 1)c$  if  $P(q^m)/2 > (N - 1)c$ .*

In contrast to the case with single integration, the pool does not set the internal transfer price to zero. The reason is that, whereas each manufacturer  $M_i$  sets  $q_i$  to maximize its own

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<sup>32</sup>See Reisinger and Tarantino (2015) for a related trade-off.

profit, the pool seeks to maximize industry profits. To achieve this, its optimal strategy is to set patent prices so that each manufacturer sells half of the monopoly quantity.<sup>33</sup> This strategy, which is equivalent to a cross-licensing agreement, then yields the monopoly profit.<sup>34</sup>

The pool can implement this optimal pricing scheme only if  $c$  is sufficiently large. If the value of  $c$  is relatively small, the pool must take into account that the patent price set for the bundle of patents cannot exceed  $(N - 1)c$ , which is what each manufacturer pays when resorting to the status-quo technologies. In the linear demand example ( $P(Q) = 1 - Q$ ), the pool sets  $W_p^{DI} = 1/4$  if  $c \geq 1/(4(N - 1))$  and  $W_p^{DI} = (N - 1)c$  if  $c < 1/(4(N - 1))$ .<sup>35</sup>

We next give the conditions for the pool to be anticompetitive with double integration.

**PROPOSITION 3.**

- *There exists a unique threshold  $N^{DI} > 3$  such that the pool is anticompetitive for all  $N < N^{DI}$ , and procompetitive for all  $N > N^{DI}$ .*
- *Comparing the equilibrium outcomes with double integration and single integration, we find that, if  $P''(\cdot) \approx 0$ ,  $N^{DI} > N^{SI}$ .*

The proposition’s main take away is that a patent pool is more likely to be anticompetitive with double than with single integration. First, recall that with single integration, the threshold value for  $N$  below which the pool is anticompetitive was larger than 2. By contrast, the first claim in the proposition is that, with double integration, this threshold must be larger than 3. This implies that if there are 3 licensors, the pool is always anticompetitive with double integration but can be procompetitive with single integration. However, this does not necessarily imply that  $N^{DI} > N^{SI}$ . We address this relationship in the second claim of the proposition. There, we show that  $N^{DI}$  is indeed larger than  $N^{SI}$  if demand is close to linear. Although we could prove this result only with a demand function that satisfies this restriction, we reached similar conclusions using other non-linear functions.

These results show that a larger degree of vertical integration implies that the pool is more likely to be anticompetitive. The intuition supporting them follows. Consider first independent licensing. With double integration, fewer licensors demand a share of a manufacturer’s profit than with single integration, which implies that non-integrated licensors set

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<sup>33</sup>In contrast to the case of single integration, now both manufacturers are members of the pool via their upstream affiliates. Therefore, secret renegotiation of patent prices with only one manufacturer is not possible.

<sup>34</sup>Jeon and Lefouili (2018) show in a general framework that even a large number of firms can achieve the monopoly outcome through cross-licensing with bilateral two-part tariff contracts.

<sup>35</sup>Again, by a revealed preference argument, it is profitable for all licensors to join the pool.

lower patent prices. Instead, an integrated licensor sets a zero patent price internally, but raises the external price to  $c$ . This implies an ambiguous effect of an additional integrated firm on aggregate royalties. By contrast, the effect of additional integration on pool royalties is clear cut. Lemma 6 shows that, with double integration, the pool seeks to monopolize the product market. This is not possible with single integration, because  $M_B$  is not affiliated to a pool member. Thus, the pool sets higher patent prices with double integration than with single integration. Taking these effects together, we obtain that the threshold value is larger with double integration.

We now illustrate the result in the second bullet point of the proposition using  $P(Q) = 1 - Q$ . Whereas  $N^{SI}$  is equal to 3,  $N^{DI} = (1 + 12c + \sqrt{1 + 24c - 112c^2})/8c$ ; thus,  $N^{DI} \approx 7.5$  if  $c = 0.05$ —i.e., more than twice as large as  $N^{SI}$ .

## 5 Policy Instruments

The equilibrium analysis in the previous section shows that a patent pool involving licensors of complementary patents can be anticompetitive. In what follows, we discuss the implications of our results for public policy. In particular, the main question is whether there is a simple information-free instrument to screen in procompetitive pools and screen out anticompetitive ones. Such an information-free instrument can be implemented without conditioning on the specifics of the industry (i.e., the number of firms or the number of vertically integrated firms). We consider two such instruments, namely an unbundling and pass-through requirement and a vertical firewall policy.

### Unbundling and Pass-through

In practice, authorities often require that pool members must also be allowed to license their patents independently (the “independent licensing requirement”).<sup>36</sup> We now show that this requirement fails to screen anticompetitive pools in a setting with competing manufacturers and non-linear pricing.

Under the independent licensing requirement, the game unfolds as follows: In the first stage, the pool sets its tariffs and, in the second stage (or continuation stage), licensors simultaneously and non-cooperatively offer their individual contracts to manufacturers. Man-

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<sup>36</sup>This requirement has been first studied by Lerner and Tirole (2004). Focusing on pools licensing to non-competing firms, Boutin (2016) shows that it is not enough to break pools with a large number of licensors and Rey and Tirole (2018) show that it is also insufficient if pool members can tacitly collude.

ufacturers can then choose among not buying at all, buying the package from the pool, or buying individual patents from licensors. We denote the share that licensor  $L_j$  obtains from the pool's profit by  $s_j$ , with  $\sum_{j=1}^N s_j = 1$ . Let us consider the case of single integration and two licensors, in which the pool is anticompetitive. By Lemma 4, the pool offers  $T_{pB}^{SI} = (2c, 0)$  to  $M_B$  and transfers the patents internally to  $M_A$  at a royalty rate of zero. Each licensor's share of the pool's profit is set in such a way that licensors obtain a (weakly) higher profit than with independent licensing, so that they are willing to form the pool. Using linear demand ( $P(Q) = 1 - Q$ ), we obtain the following result:

**PROPOSITION 4.** *With linear demand, the independent licensing requirement is not sufficient to break an anticompetitive pool.*

Because it is integrated with  $M_A$ , at the continuation stage,  $L_1$ 's optimal strategy features setting  $w_{1B}^{SI} = c$  and an internal patent price of zero. This means that  $L_1$ - $M_A$  cannot raise a larger profit than in the pool, independently of what  $L_2$  offers to  $M_B$  in the continuation stage. This is not necessarily true for  $L_2$ . Based on the analysis in Lemma 3, one might expect that  $L_2$ 's best response at the continuation stage is to follow the same strategy as without the pool. However,  $L_2$  has no incentive to deviate from the pool pricing outcome, and thus sets  $w_{2B}^{SI} = c$  and  $w_{2A}^{SI} = 0$ . The reason is that its profit from the pool is larger than the one it can raise by licensing its patent non-cooperatively. This shows that the existence of the pool changes the nature of the independent licensing game because each licensor obtains its share of the pool profit if individual offers are not accepted. As a consequence, the pool remains anticompetitive under the independent licensing requirement.<sup>37</sup>

Rey and Tirole (2018) propose an alternative requirement, featuring unbundling of the pool's offers and pass-through of each patent's revenue to the respective licensor. As we will show, this policy works in our framework. Specifically, under the unbundling and pass-through requirement:

1. The pool sets nonlinear tariffs  $T_{ji}^p$  at which manufacturers can acquire individual patents from the pool (instead of a tariff for the bundle of patents  $T_{pi}$ ). In addition, manufacturers can acquire patents directly from each licensor.
2. A pool member's profit corresponds to the revenue generated by its patent.

This requirement does no longer allow pool members to share their profits on an arbitrary basis. Instead, each member's revenue cannot be larger than the revenue accruing from the

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<sup>37</sup>A similar result arises with double integration.

licensing of its patent; thus, the requirement boils down to a ban on monetary transfers within the pool. By that, the requirement preserves the benefit of horizontal price coordination of the pool.

Consider again single integration and two licensors, where the pool is anticompetitive in the absence of any licensing requirement. Suppose now that the unbundling and pass-through requirement is in place. For the pool, it is still optimal that  $L_1$ - $M_A$  sets the internal transfer price to zero and sells to  $M_B$  at a royalty rate equal to  $c$ . Because monetary transfers within the pool are banned, the profit of  $L_2$  reaches its maximum value when the tariffs are exactly the same as without the pool. In fact, any other tariff would imply a lower profit for  $L_2$  (although it raises the pool's profit). Licensor  $L_2$ , then, is not willing to form the pool. Hence, the requirement induces the pool to set the same patent prices as without pool, thereby rendering the pool welfare neutral. More generally, we demonstrate the following:

**PROPOSITION 5.** *With linear demand, under the unbundling and pass-through requirement, a pool is procompetitive if  $N > 2$  and welfare neutral if  $N = 2$ .*

The reason for the procompetitive result is the following: The optimal tariff set by the pool on  $L_1$ - $M_A$ 's patent still features an internal patent price of zero and  $W_{1B}^{SI} = c$ . However, for the tariffs of non-integrated licensors, the main motive of the pool is to achieve horizontal price coordination. This ensures not only that all non-integrated licensors are willing to form the pool but also leads to lower royalty rates than without pool.<sup>38</sup>

To sum up, we find that the adoption of the unbundling and pass-through requirement induces pools to set their tariffs in such a way that they are (weakly) procompetitive; thus, this requirement is a perfect information-free screening device of (weakly) procompetitive patent pools.

## Vertical Firewall Policy

The main reason why, with vertical integration, a patent pool is anticompetitive is that it seeks to monopolize the downstream market through the integrated manufacturers (under the constraint posed by the presence of status-quo technologies). Therefore, a policy to limit the pool's anticompetitive effects could be to require the pool to maximize only its members' licensing revenues when setting the tariffs. Under this policy, the pool is not allowed to take into account the impact of the royalty rates on the product market profits

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<sup>38</sup>Similarly, the unbundling and pass-through requirement breaks only the anticompetitive pools with double integration.

of its integrated manufacturers. The policy can be enforced by forbidding pool members to transfer non-licensing revenues within the pool. Such a policy then has the potential to break the upward pricing effect of vertical price coordination, maintaining the benefits of horizontal price coordination.

We label this approach the vertical firewall policy, because it acts in the same way as a ban on the flow of the product market profits raised by integrated manufacturers up to the pool. Similarly to the unbundling and pass-through policy, the vertical firewall policy is (i) information free and (ii) constrains its members in the use of side payments. However, unlike the unbundling policy, the vertical firewall policy allows pool members to use side payments to transfer licensing revenues, but forbids those “vertical” side payments concerning downstream revenues. Then, integrated licensors cannot use downstream affiliates’ profits to “buy” the pool membership of non-integrated licensors.

Consider the case with single integration.<sup>39</sup> The game unfolds as follows. In stage zero, the licensors decide whether to form a patent pool. In stage one, the patent pool determines the shares of the profits that its members receive, which we denote by  $s$  for the integrated firm and  $(1 - s)/(N - 1)$  for each of the symmetric non-integrated licensors, with  $s \in [0, 1]$ . In stage two, the patent pool chooses its tariffs  $(W_{pi}, F_{pi})$ , with  $i = A, B$ , to maximize the sum of its members’ licensing revenues. Finally, manufacturers set quantities to maximize own profits. In the case of an integrated manufacturer, these profits correspond to the sum of product market profits and the share of the pool’s licensing revenues flowing to the licensor it is integrated with.

Consistent with the rest of the analysis, we do not explicitly model the bargaining game determining the integrated licensor’s profit share  $s$ . Instead, we check whether there are values of  $s$  such that the pool members’ participation constraints are satisfied. In contrast to what happens absent this policy, the share  $s$  may now affect the equilibrium downstream quantities and royalty rates. The reason is that the pool no longer optimally sets a zero royalty rate to its integrated manufacturer, as the vertical firewall policy forbids to consider this profit when setting the tariff.

We will now show that the firewall policy is *not* a perfect instrument to screen anticompetitive pools. If a pool forms, it will be procompetitive. However, the policy may prevent the formation of a procompetitive patent pool. To prove this result, we use linear demand

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<sup>39</sup>With respect to the main model, the adoption of the firewall policy changes only the analysis with vertical integration. With vertical separation, no licensor is integrated with a manufacturer; thus, the pool cannot do better than maximizing licensing revenues.

( $P(Q) = 1 - Q$ ). Focusing on values of  $c$  close to the boundaries, we can then show that, with single integration, the following results hold:

**PROPOSITION 6.** *With linear demand, if  $c \rightarrow 0$ , there is no value of  $s \in [0, 1]$  such that the licensors choose to form the patent pool. Instead, if  $c \rightarrow 1/(2N)$ , licensors form a patent pool if and only if  $N > 2$ , and this patent pool is procompetitive.*

The proposition shows that the welfare consequences of the vertical firewall policy are not clear cut. Recall from Section 4 that, with single integration, the patent pool in our main model is anticompetitive if  $N = 2$ , and (weakly) procompetitive otherwise. Therefore, the policy is successful in breaking anticompetitive pools. However, it prevents the formation of any pool when the patents' degree of essentiality is limited (i.e., if  $c$  is small). This means that, in the latter case, even the procompetitive pools that would form absent the policy cannot arise.

The intuition behind these results is as follows. If  $c$  is relatively small, a manufacturer's bargaining threat (i.e., the profit it obtains when rejecting the licensors' offers) is large, which implies that the licensing revenue reaped by the pool is small. By contrast, the profit of the integrated manufacturer ( $M_A$ ) is relatively high. As a consequence, the possibility of the integrated firm to vertically coordinate royalty rates is particularly valuable, but is forbidden to the pool under the policy. The consequence is that the vertically integrated licensor will agree to join the pool only for a value of  $s$  that is so large that nonintegrated licensors would be worse off with the pool compared to independent licensing. As a consequence, the pool will not form.

By contrast, if  $c$  is large (i.e., if it approaches the upper limit of  $1/(2N)$ ), a manufacturer's bargaining threat is small. This means that the main bulk of industry profits is reaped by licensors. In this case, horizontal coordination of royalty rates within the patent pool is particularly valuable, especially if the number of licensors is large. Whether the pool will form now depends on the trade-off between  $L_1$ - $M_A$ 's gains from raising the patent price to  $M_B$  (absent the pool) and the benefits of horizontal price coordination (under the pool). If  $N$  is small (i.e.,  $N = 2$ ), the benefits of horizontal price coordination are limited. That is,  $L_1$ - $M_A$ 's opportunity cost of losing the possibility to raise the royalty rate to  $M_B$  dominates, and the pool will not form. Instead, with a larger number of licensors, the benefits of horizontal price coordination dominate, which implies that there exist sharing rules such that integrated and non-integrated licensors agree to join the pool.

Moreover, whenever it forms, the pool is procompetitive. The fact that the pool cannot internalize the impact of its royalty rates on the integrated manufacturer's profits means

that it will lose the interest in restricting the market share of nonintegrated manufacturers. Thus, by the effect of horizontal price coordination, the pool raises welfare.

To deliver clear results, in the proposition we focused on the two extreme cases in which  $c$  approaches the lower and upper limit of its support. However, the intuition described above holds for intermediate values of  $c$ , too. In fact, the result is continuous, that is, the higher is  $c$ , the more likely it is that the pool forms and this likelihood increases in the number of patent owners. This is shown in Figure 1. In addition, the pool is always procompetitive.

[Figure 1 about here]

We conclude that, although the vertical firewall policy does not yield a globally procompetitive outcome, it is useful to break those anticompetitive pools whose patents are close to essential.

## 6 Patent Pools in Related Economic Environments

In this section, we show that the main insights developed in Section 4 survive the employment of alternative assumptions regarding the main structural and contractual features of our industry. In particular, we first consider a general number of manufacturers (instead of only 2 as in the baseline model). Then, we analyze product differentiation between manufacturers, both with quantity and price competition. We also consider the robustness of our results to secret instead of public contracts, and, finally, we briefly analyze quantity-forcing contracts.

### General Number of Manufacturers

In the baseline model, we consider an industry with only two manufacturers to bring out the main effects of the pool in a parsimonious setting. However, as we show in this section, our results survive within a model with a larger number of manufacturers. Specifically, in what follows we illustrate this in a setup with  $M \geq 2$  manufacturers and linear demand ( $P(Q) = 1 - Q = 1 - \sum_{i=1}^M q_i$ ).

With vertical separation, our results do not depend on the number of manufacturers in the industry. Indeed, the reason why the pool is beneficial for welfare is that it improves coordination of patent pricing decisions. The model with linear demand and  $M$  manufacturers confirms this intuition: We find royalty rates of  $w^{VS} = Nc(M-1)/(N(M-1)+2)$  without pool and  $W_p^{VS} = Nc(M-1)/(M+1)$  with pool, where  $Nw^{VS} > W_p^{VS}$ .

We now turn to the industries featuring vertical integration. Suppose that  $K \leq \min\{M, N\}$  manufacturers are integrated with a licensor. By the same token as in the analysis with two manufacturers, with independent licensing an integrated licensor sets a royalty rate of  $c$  to non-affiliated manufacturers. Moreover, the non-integrated licensors offer the same royalty rate to all manufacturers (independently of whether they are integrated or not). This royalty rate is

$$w^{VI} = \frac{(M-1)(N-K)c}{M(N-K) - N + K + 2} \in (0, c). \quad (4)$$

This expression is increasing in  $M$ : As competition downstream gets fiercer (i.e.,  $M$  rises), product-market profits fall, and thus the rents that non-integrated licensors can extract via the fixed component of the tariffs decrease. In equilibrium, licensors counter this effect by raising royalty rates to dampen competition. Moreover, the value of  $w^{VI}$  in (4) is decreasing in  $K$ : As the number of vertically integrated licensors in the industry increases, fewer licensors demand a share of a manufacturer's profit. As a consequence, non-integrated licensors optimally lower the royalty rate to claim a larger fixed component in the tariff.

Consider now the pool. The pool optimally sets a price of  $Nc$  for the bundle of patents to the  $M - K$  non-integrated manufacturers to soften product-market competition. The pool then sets the internal transfer price to coordinate downstream quantities in such a way that all integrated manufacturers act "as if" they are a single manufacturer with marginal cost equal to zero. Formally, the pool sets a price  $W_p^{VI}$  such that  $Kq(W_p^{VI}, (K-1)W_p^{VI} + (M-K)Nc) = q(0, (M-K)Nc)$ .<sup>40</sup> With linear demand,<sup>41</sup>

$$W_p^{VI} = \frac{(K-1)(Nc(M-K) + 1)}{K(M-K+2)}. \quad (5)$$

Comparing the patent prices with and without pool, we find that the qualitative results are the same as those in Section 4. First, independently of the number of integrated firms (i.e., independently of the value of  $K$ ), the pool is more likely to be procompetitive, the larger is the number of licensors  $N$ . Second, the larger the number of vertically integrated licensors in the industry, the more likely it is that the pool is anticompetitive. Finally, as  $M$  increases but  $K$  stays constant, the pool is more likely to be procompetitive. The intuition

<sup>40</sup>With a slight abuse of notation, the second argument in  $q(\cdot, \cdot)$  denotes the sum of the costs incurred by all rival manufacturers.

<sup>41</sup>The pool price is constrained by the possibility that an integrated manufacturer can obtain licenses to all patents at total costs of  $(N-1)c$ . Therefore, the pool sets  $W_p^{VI}$  as in (5) if this is lower than  $(N-1)c$ , and equal to  $(N-1)c$  otherwise.

is that a larger number of non-integrated manufacturers means a lower share of integrated firms in the industry.

These results show that the results obtained in the main model carry over to the case with a general number of manufacturers.

## Competition between Differentiated Goods

### Quantity Competition

Consider first the vertically separated industry. Even though horizontal double marginalization is weaker when downstream products are differentiated, it is present as long as manufacturers exert competitive pressure on each other. For example, with the linear demand function  $P_i(q_i, q_{-i}) = 1 - q_i - \beta q_{-i}$ , where  $\beta \in [0, 1]$  captures the degree of products' substitutability, double marginalization arises for all  $\beta > 0$ . Therefore, as much as within the homogeneous product industry we consider in the main model, the pool results in a procompetitive outcome due to horizontal price coordination and the fact that lower royalty rates reduce manufacturers' outside options.<sup>42</sup>

Assume now that  $L_1$  is merged with  $M_A$ . Product differentiation implies that the integrated firm does not necessarily want to foreclose the rival manufacturer's access to the patent.<sup>43</sup> Yet, if the status-quo technology is relatively efficient (i.e.,  $c$  is small), setting  $w_{1B}^{SI}$  equal to  $c$  without pool and  $W_{pB}^{SI} = Nc$  with pool remains the optimal strategy. However, non-integrated licensors set a lower royalty rate when products are differentiated, because downstream competition is less fierce. As a consequence, the threshold value for  $N$  above which the pool is procompetitive is larger with differentiated products. Specifically, in the linear demand example with differentiated goods, this threshold is  $(2 + \beta)/\beta$ , which is larger than 3—the threshold with homogeneous goods. Therefore, the pool is more likely to be anticompetitive for low values of  $c$ . The difference between the outcomes with differentiated and homogeneous goods decreases as  $c$  gets larger; thus, also the difference in the threshold value of  $N$  above which the pool is procompetitive falls.

With double integration, results are again similar to those with homogeneous products. In particular, if  $c$  is low, the pool is anticompetitive independently of the number of licensors. If  $c$  is sufficiently large, instead, the pool becomes procompetitive if  $N$  rises above a threshold that is larger than with single integration.

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<sup>42</sup>Specifically, with independent licensing,  $w^{VS} = \beta Nc/(\beta N + 2)$ , whereas a pool sets  $W_p^{VS} = \beta Nc/(\beta + 2)$ . Thus,  $Nw^{VS} > W_p^{VS}$  for all  $\beta > 0$ .

<sup>43</sup>The reason is that  $M_B$  generates additional value that the licensor may want to extract.

## Price Competition

Our results hold also with price competition. The main difference with respect to quantity competition is that manufacturers' prices are strategic complements. Although this affects competition on the downstream market, the main trade-offs arising from our baseline model remain valid. To illustrate the results with price competition and differentiated products, we again use the linear demand function  $P_i(q_i, q_{-i}) = 1 - q_i - \beta q_{-i}$ ,  $i = A, B$ . Inverting this system, we obtain manufacturers' quantities as a function of prices and can use the same demand function as with quantity competition and differentiated goods.

Consider first the vertically separated industry. With price competition, royalty rates set by licensors are higher than with quantity competition.<sup>44</sup> The competitive pressure is fiercer than with quantity competition, implying that licensors can extract lower rents through the fixed component of their tariffs. As a consequence, they optimally set higher royalty rates. Accordingly, with independent licensing, horizontal double marginalization arises for any positive value of  $\beta$ . This also implies that the pool is again procompetitive.<sup>45</sup>

We now consider an industry featuring single integration. The integrated firm  $L_1$ - $M_A$  sets  $w_{1B}^{SI} = c$  only if downstream products are close substitutes (that is,  $\beta$  close to 1), whereas non-integrated licensors set their royalty rates between zero and  $c$ . However, as with vertical separation, these royalty rates are higher than with quantity competition.<sup>46</sup> As a consequence, the pool is more likely to be procompetitive relative to quantity competition. For example, if  $c$  is small so that  $w_{1B}^{SI} = c$  without pool and  $W_{pB}^{SI} = Nc$  with pool, the threshold value of  $N$  below which the pool is anticompetitive is  $(2 - \beta)/\beta$ . This is strictly below the one with quantity competition and differentiation.

Finally, the same results carry over to the case with double integration. Specifically, the pool is again more likely to be procompetitive with price than with quantity competition.

## Secret Contracts

In the main model, we assume that contracts are observable. This approach is motivated by the fact that several patent pools publicly announce the contract terms on their web pages. By contrast, the MPEG-Audio and LBS ATSS pools, among others, keep their licensing terms confidential. We therefore check the robustness of our results to secret contracting.

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<sup>44</sup>In the linear demand example, with price competition,  $w^{VS} = \beta Nc / (\beta N + 2(1 - \beta))$  for all  $j = 1, \dots, N$ , which is larger than  $\beta Nc / (\beta N + 2)$ —the royalty rate with quantity competition.

<sup>45</sup>With linear demand, the pool sets  $W_p^{VS} = \beta Nc / (2 - \beta)$ , so that  $Nw^{VS} > W_p^{VS}$ .

<sup>46</sup>Because downstream prices are strategic complements, a higher final good price set by  $M_i$  (due to larger costs of production) induces  $M_{-i}$  to increase its price, too.

We consider the set-up in Section 3, with the difference that a manufacturer can only observe the contract offers it receives but not those made to the rival. The solution concept we employ is then perfect Bayesian Nash equilibrium. As is well known, in this kind of offer games a multiplicity of equilibria exists. This multiplicity is due to the fact that the perfect Bayesian equilibrium does not specify a manufacturer's beliefs in case it receives an out-of-equilibrium offer. To reduce the number of equilibria, we follow the literature by using the passive beliefs refinement, which coincides with wary beliefs in our framework with Cournot competition (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004; Rey and Tirole, 2007). With passive beliefs, a manufacturer's conjecture about the contract offered to the rival is not influenced by an out-of-equilibrium contract offer it receives. This is a natural refinement in our game in which  $M_A$  and  $M_B$  order quantities before competing on the product market because, from the perspective of the upstream licensors, manufacturers form two separate markets.

In what follows, we give a summary of the results of the analysis under the assumption of secret offers.<sup>47</sup> We find that absent integration the pool is welfare neutral but it is anticompetitive with (single and double) integration.

## Vertical Separation

With secret offers and independent licensing, a licensor cannot influence the decisions of  $M_i$  when it deviates in its offer to the rival manufacturer. Therefore, when dealing with each manufacturer, the licensor acts as if the two are integrated. This pairwise maximization problem requires that the contractual arrangement between  $L_j$  and  $M_i$  maximizes bilateral profits, which entails a patent price equal to zero. This happens regardless of the specific value of the fixed component of the tariffs claimed by licensors; hence, it holds in any equilibrium with passive beliefs. Moreover, it occurs to the detriment of licensors, as lower royalty rates imply lower downstream profits (due to fiercer competition). This is the well-known opportunism problem (Hart and Tirole, 1990; Rey and Tirole, 2007) that is faced by independent licensors as much as by the pool;<sup>48</sup> thus, the pool sets patent prices equal to zero. The immediate consequence is that a pool formed by independent licensors is welfare

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<sup>47</sup>The formal analysis can be found in the Online Appendix.

<sup>48</sup>Indeed, the pool's contract offer to  $M_i$  is not observable to  $M_{-i}$ , implying that the pool will also act to maximize bilateral profits with each manufacturer.

neutral, as it does not change the sum of royalty rates compared to independent licensing. That is, horizontal coordination plays no role with secret offers.<sup>49</sup>

## Vertical Integration

Assume  $L_1$  is integrated with  $M_A$ . With independent licensing, the presence of the opportunism problem implies that a non-integrated licensor  $L_j$ ,  $j \neq 1$ , sets the patent price equal to zero. Instead, the integrated licensor  $L_1$  internalizes the effect that selling to the rival manufacturer has on the profit of its downstream affiliate. Moreover,  $M_A$  now knows the terms of its upstream unit's offer to the rival manufacturer  $M_B$ . As a consequence,  $L_1$  can credibly commit to reducing supplies to  $M_B$ . To achieve this outcome, in the unique equilibrium  $L_1$  raises  $w_{1B}$  to  $c$ .

After licensors form a patent pool, they will jointly act to raise the price that  $M_B$  must pay for the bundle of patents. This means that instead of paying  $c$  only for input 1,  $M_B$  now pays  $c$  for each of the  $N$  inputs, implying that its costs raise to  $Nc$ . By contrast, the integrated manufacturer receives the patents at the same royalty rates as with independent licensing. By Property 1, this implies that the pool is anticompetitive. In contrast to the case with observable contracts, any procompetitive effect of the pool disappears. The reason is that the integrated manufacturer receives each patent at the same price (i.e., equal to zero) both with and without the pool. An analogous result holds with double integration.

## Quantity Forcing Contracts

Assume licensors offer more general non-linear contracts than two-part tariffs. In our setting, the most general class of such contracts features quantity-forcing arrangements of the form  $T_{ji} \equiv (q_i, F_{ji})$ .<sup>50</sup> The main difference with respect to two-part tariffs is that, if a manufacturer wants to sell a larger volume of output than the one specified in the contract offered by a licensor, it has to use the status-quo technology. As we will show below, the use of two-part tariffs is justified by one main argument: Quantity forcing contracts yield multiple symmetric equilibria in the amount of the quantity specified in the offer, including the one arising with two-part tariffs.

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<sup>49</sup>When comparing independent licensors offering secret contracts with a pool offering public contracts, the result that secret contracting leads to royalty rates of zero implies that the pool is anticompetitive. In fact, double marginalization only occurs with public contracts. The same result holds with vertical integration.

<sup>50</sup>Because there is no asymmetric information in the negotiations between licensors and manufacturers, licensors can do no better in extracting manufacturers' revenues by using menus of contracts instead of one quantity forcing contract.

Consider the vertically separated industry, and let  $L_k$  offer a contract to each manufacturer specifying the same quantity as the one in the equilibrium two-part tariff contract. Let us denote this quantity by  $q(W^{VS}, W^{VS})$ . By the same arguments as in the analysis in Section 4, the optimal response of  $L_j$ ,  $j \neq k$ , is then to offer a quantity of  $q(W^{VS}, W^{VS})$ . In a symmetric equilibrium, each licensor then requires manufacturers to pay  $F^{VS} = (\pi(W^{VS}, W^{VS}) - \max_q \{ (P(q + q(W^{VS}, W^{VS})) - Nc)q \})/N$ . Hence, the equilibrium market allocation in Lemma 1 coincides with the one with quantity-forcing contracts.

However, with quantity-forcing contracts, the equilibrium above may not be the only one. The reason is that, due to input complementarity, a coordination problem can arise regarding the value of the quantity in the tariff. The intuition is easy to understand in the extreme case in which the status-quo technologies are not available. In this context, if  $L_k$  offers  $q' < q(W^{VS}, W^{VS})$  in its tariff,  $L_j$ 's best-response features setting  $q'$  as well, because a manufacturer cannot produce more than  $q'$ . Then, any quantity  $q$  that yields a positive profit to a manufacturer constitutes an equilibrium of the offer game. Now, let the status-quo technologies be available. Developing on this example, if  $L_k$  offers  $q'$ , now  $L_j$ 's best-response may also feature  $q'$  if  $c$  is large enough. Let  $L_j$  reply by offering a tariff specifying a value of  $q'' > q'$  instead. Then, the manufacturer can only produce quantity  $q''$  by purchasing the additional amount  $q'' - q'$  it needs of input  $k$  at a cost of  $c$ . However, if  $c$  is sufficiently large, doing so is not optimal for the manufacturer. In fact, for  $q'$  slightly smaller than  $q(W^{VS}, W^{VS})$ , there is also an equilibrium in which licensors offer  $q'$  in their tariffs. This shows that, the larger is  $c$ , the larger is the set of quantities that form a symmetric equilibrium with quantity-forcing contracts. However, independent of the value of  $c$ ,  $q(W^{VS}, W^{VS})$  is always an equilibrium with quantity-forcing contracts.

## 7 Conclusions, Case Evidence and Policy Discussion

In this article, we deliver the following message: A patent pool formed by licensors of complementary patents is anticompetitive if the share of vertically integrated firms is sufficiently large. With vertical integration, the pool serves as a coordination device that licensors use to soften competition, and share the larger profit raised by the integrated manufacturers. Our conclusions are robust to the number of firms in the industry, presence of differentiated final goods, nature of competition, and contracting environment. Although we focus on the case of patent pools, these results apply more broadly to any joint marketing agreements—including mergers and joint ventures—among suppliers of complementary input goods.

Our theory explains why patent pools consisting of complementary patents may increase royalty rates.<sup>51</sup> The U.S. International Trade Commission investigation of the CD-R pool reports evidence in line with the theoretical mechanism we put forward. Specifically, Judge Harris (2003) establishes that “manufacturers who sell CD-RRW discs to Philips [...] pay no royalty on those discs to the pool members.” That is, Philips (the pool administrator) gave affiliated manufacturers a privileged access to the pool’s patent bundle. Moreover, the evidence in Flamm (2013) suggests that an analogous form of discrimination was carried out by Philips and Sony within the 3C DVD pool. He documents that “for a pool member [...], the marginal cost per unit of using other pool members’ patents under the CD pool rules was zero, quite differently from the steadily rising share of royalties in product cost that was faced by an outsider.” Interestingly, this pool was approved by the DOJ based on the assurance that the pool administrator would have not set discriminatory royalties.<sup>52</sup>

To guide authorities and courts in these cases, we propose two information-free policies. First, we show that the unbundling and pass-through requirement breaks the anticompetitive pools, and preserves the procompetitive ones. This policy appends the unbundling of patent claims to the independent licensing requirement. We note that if licensors own portfolios of patents instead of a single patent (as is usually the case in practice), only each licensor’s portfolio must be separately available (not each patent). This guarantees that each member obtains a share of the pool value according to its patent portfolio, thereby avoiding side payments. This simplifies the implementation of this policy. Second, we show that a vertical firewall policy which requires a pool to maximize licensing revenues (and therefore forbids side transfers of profits from vertically integrated manufacturers to independent licensors) is only an imperfect screening instrument: Although pools are always procompetitive when they form, this policy prevents the formation of some procompetitive pools.

We focused on the pricing implications of patent pools. Another important question in the literature on innovation concerns how patent pools affect future incentives to innovate in the industry. Such an analysis could be conducted in a model with sequential innovation (following Denicolò, 2002, or Hopenhayn, Llobet and Mitchell, 2006). At the same time, in this article we abstract from the impact of the pool on the development of technological standards. To this end, one could build on Gallini (2014) to analyze whether the formation of a pool spurs the process of standard setting. We leave these questions to future research.

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<sup>51</sup>A similar conclusion is reached in the literature, but with different mechanisms: In Quint (2014), a pool of non-essential patents raises the fees set by essential patents’ holders outside the pool. In Choi and Gerlach (2015), by forcing licensees to challenge all patents, pools may discourage litigation and raise licensing fees.

<sup>52</sup>Gilbert (2004) gives a number of cases in which patent pools employ other forms of vertical restraints (like exclusive territories or resale price maintenance), the use of which is consistent with our theory.

## Appendix

The Appendix contains the proofs of Lemmata 1–6 and Propositions 1–3. All the other proofs are in the Online Appendix available on the authors’ webpages.

*Proof of Lemma 1.* In the Online Appendix, we show that in a symmetric equilibrium, manufacturer  $M_i$ ’s profit corresponds to the rents it obtains when rejecting all licensors’ offers, given that the rival manufacturer  $M_{-i}$  produces the equilibrium quantity. This implies that, in any equilibrium,  $M_i$ ’s profit is  $\max_q \{(P(q + q_{-i}(W_{-i}, W_i)) - Nc)q\}$ . As a consequence, when setting the fixed fee to  $M_i$ ,  $i = A, B$ , each licensor  $L_j$ ,  $j = 1, \dots, N$ , faces the constraint  $\pi_i(W_i, W_{-i}) - \sum_k F_{ki} \geq \max_q \{(P(q + q_{-i}(W_{-i}, W_i)) - Nc)q\}$ . At the optimal contract of each licensor, this constraint will be binding.

We now determine the optimal royalty rates in the symmetric equilibrium. Licensor  $L_j$ ’s,  $j = 1, \dots, N$ , maximization problem is

$$\max_{w_{ji}, w_{j-i}, F_{ji}, F_{j-i}} w_{ji}q_i(W_i, W_{-i}) + w_{j-i}q_{-i}(W_{-i}, W_i) + F_{j-i} + F_{ji}. \quad (\text{A-1})$$

Given that the constraint above is binding at the optimal contract,  $L_j$ ’s solves this problem under the following constraints:

$$\begin{aligned} F_{ji} &= \pi_i(W_i, W_{-i}) - \max_q \{(P(q + q_{-i}(W_{-i}, W_i)) - Nc)q\} - \sum_{k \neq j} F_{ki}, \\ F_{j-i} &= \pi_i(W_{-i}, W_i) - \max_q \{(P(q + q_i(W_i, W_{-i})) - Nc)q\} - \sum_{k \neq j} F_{k-i}. \end{aligned}$$

After plugging the fixed component of the tariffs into the objective function in (A-1), we find that the first-order conditions with respect to  $w_{ji}$  and  $w_{j-i}$  are given by, respectively,

$$\begin{aligned} (w_{ji} + P'(Q)q_{-i} - P'(\tilde{Q})\tilde{q}_{-i}) \frac{\partial q_i}{\partial w_{ji}} + (w_{j-i} + P'(Q)q_i - P'(\tilde{Q})\tilde{q}_i) \frac{\partial q_{-i}}{\partial w_{ji}} &= 0, \\ (w_{ji} + P'(Q)q_{-i} - P'(\tilde{Q})\tilde{q}_{-i}) \frac{\partial q_i}{\partial w_{j-i}} + (w_{j-i} + P'(Q)q_i - P'(\tilde{Q})\tilde{q}_i) \frac{\partial q_{-i}}{\partial w_{j-i}} &= 0, \end{aligned}$$

where  $\tilde{q}_i = \arg \max_q \{(P(q + q_{-i}(W_{-i}, W_i)) - Nc)q\}$  and  $\tilde{Q} = \tilde{q}_i + q_{-i}(W_{-i}, W_i)$ . Using the product-market first-order conditions,  $P(Q) - \sum_{j=1}^N w_{ji} + P'(Q)q_i = 0$ , with  $i = A, B$ , we can determine  $\partial q_i / \partial w_{ji}$  and  $\partial q_{-i} / \partial w_{ji}$  to get

$$\frac{\partial q_i}{\partial w_{ji}} = \frac{2P'(Q) + q_{-i}P''(Q)}{P'(Q)(3P'(Q) + QP''(Q))}, \quad \frac{\partial q_{-i}}{\partial w_{ji}} = -\frac{P'(Q) + q_{-i}P''(Q)}{P'(Q)(3P'(Q) + QP''(Q))},$$

with  $\partial q_i/\partial w_{ji} = \partial q_{-i}/\partial w_{j-i}$  and  $\partial q_i/\partial w_{j-i} = \partial q_{-i}/\partial w_{ji}$ . Using all this in the first-order conditions, and solving for the royalty rates set by  $L_j$ , yields

$$w_{ji} = P'(\tilde{Q})\tilde{q}_{-i} - P'(Q)q_{-i} \quad \text{and} \quad w_{j-i} = P'(\tilde{Q})\tilde{q}_i - P'(Q)q_i \quad \forall j = 1, \dots, N. \quad (\text{A-2})$$

Deriving the second-order conditions and using (A-2) in these conditions, we obtain that  $L_j$ 's profit function is strictly quasi-concave under our assumptions on the demand function.

We first show that the royalty rate  $w_{ji}$  solving (A-2) is larger than zero, which happens if and only if  $P'(\tilde{Q})\tilde{q}_{-i} - P'(Q)q_{-i} > 0$ . Note that  $P'(\tilde{Q})\tilde{q}_{-i}$  and  $P'(Q)q_{-i}$  differ only because  $M_{-i}$  pays  $c$  for each input in  $P'(\tilde{Q})\tilde{q}_{-i}$  and  $w_{j-i} < c$ ,  $j = 1, \dots, N$ , in  $P'(Q)q_{-i}$ .<sup>53</sup> At the same time, the value of  $W_i$  is the same in  $P'(\tilde{Q})\tilde{q}_{-i}$  and  $P'(Q)q_{-i}$  (by the definition of  $\tilde{q}_{-i}$  given above). Therefore,  $P'(\tilde{Q})\tilde{q}_{-i} - P'(Q)q_{-i} > 0$  holds true if and only if  $P'(Q)q_{-i} < 0$  is increasing in  $w_{j-i}$ . Formally, this requires that  $\partial(P'(Q)q_{-i})/\partial w_{j-i} = (P'(Q) + P''(Q)q_{-i})\partial q_{-i}/\partial w_{j-i} > 0$ , which holds true due to profit functions' (strict) quasi-concavity. Indeed, this property implies that  $P'(Q) + P''(Q)q_{-i} < 0$ . In addition,  $\partial q_{-i}/\partial w_{j-i} < 0$ , which gives the result. Hence,  $w_{ji} > 0$  for all  $j = 1, \dots, N$ . By symmetry, the same holds for  $w_{j-i} = w_{ji}$ .

We proceed by showing that  $w_{ji} < c$ . Specifically, we first note that, in the symmetric equilibrium,  $w_{ji} = w_{ki} = w_i^{VS}$  for all  $j, k = 1, \dots, N$  and  $j \neq k$ . We can further simplify the expressions in (A-2) by using the first-order conditions for the equilibrium quantities, which imply that  $P'(Q)q_i = -P(Q) + Nw_{ji}$  and  $P'(\tilde{Q})\tilde{q}_i = -P(\tilde{Q}) + Nc$ . This gives us

$$w_i^{VS} = \frac{P(Q) - P(\tilde{Q}) + Nc}{N + 1}. \quad (\text{A-3})$$

To show that the royalty rate in (A-3) is lower than  $c$ , we compare  $(P(Q) - P(\tilde{Q}) + Nc)/(N + 1)$  to  $c$ . We find that  $w_i^{VS} < c$  if  $P(Q) - P(\tilde{Q}) < c$ . Because  $\tilde{Q} < Q$ , and  $P(\cdot)$  is decreasing in the value of  $Q$ , it follows that  $P(\tilde{Q}) > P(Q)$ . Therefore,  $w_i^{VS} < c$ .

Summing up, the unique symmetric equilibrium features each licensor setting  $w_A^{VS} = w_B^{VS} = w^{VS} \in (0, c)$  and

$$F^{VS} = \frac{\pi(W^{VS}, W^{VS}) - \max_q \{(P(q + q(W^{VS}, W^{VS})) - Nc)q\}}{N}, \quad \text{with } W^{VS} = Nw^{VS}.$$

Q.E.D.

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<sup>53</sup>Indeed,  $M_i$  cannot observe whether  $M_{-i}$  rejected any of the licensors' offers, thus the value of its quantity is the same in  $Q$  as in  $\tilde{Q}$  and equal to  $q_i(W_i, W_{-i})$ .

*Proof of Lemma 2.* The pool solves:

$$\max_{W_{pi}, W_{p-i}, F_{pi}, F_{p-i}} W_{pi}q(W_{pi}, W_{p-i}) + W_{p-i}q(W_{p-i}, W_{pi}) + F_{p-i} + F_{pi},$$

where

$$\begin{aligned} F_{pi} &= \pi_i(W_{pi}, W_{p-i}) - \max_q \{(P(q + q_{-i}(W_{p-i}, W_{pi})) - Nc)q\}, \\ F_{p-i} &= \pi_i(W_{p-i}, W_{pi}) - \max_q \{(P(q + q_{-i}(W_{pi}, W_{p-i})) - Nc)q\}. \end{aligned}$$

After plugging  $F_{pi}$  and  $F_{p-i}$  into the maximand, we obtain that the first-order condition with respect to  $W_{pi}$  is given by:<sup>54</sup>

$$(W_{pi} + P'(Q^p)q_{-i}^p - P'(\tilde{Q}^p)\tilde{q}_{-i}^p) \frac{\partial q_i^p}{\partial W_{pi}} + (W_{p-i} + P'(Q^p)q_i^p - P'(\tilde{Q}^p)\tilde{q}_i^p) \frac{\partial q_{-i}^p}{\partial W_{pi}} = 0,$$

where  $q_i^p = \arg \max_q \{(P(q + q_{-i}(W_{p-i}, W_{pi})) - W_{pi})\}$ ,  $Q^p = q_i^p + q_{-i}^p$ ,  $\tilde{q}_i^p = \arg \max_q \{(P(q + q_{-i}(W_{p-i}, W_{pi})) - Nc)\}$ , and  $\tilde{Q}^p = \tilde{q}_i^p + q_{-i}^p$ . Under our assumptions on  $P(\cdot)$ , the second-order conditions are satisfied. This implies that it is optimal for the pool to set an interior value of the royalty rates.

Because

$$\frac{\partial q_i}{\partial W_{pi}} = \frac{2P'(Q) + q_{-i}P''(Q)}{P'(Q)(3P'(Q) + QP''(Q))} \quad \text{and} \quad \frac{\partial q_{-i}}{\partial W_{pi}} = -\frac{P'(Q) + q_{-i}P''(Q)}{P'(Q)(3P'(Q) + QP''(Q))}, \quad (\text{A-4})$$

solving for  $W_{pi}$  and  $W_{p-i}$ , we obtain  $W_{pi} = P'(\tilde{Q}^p)\tilde{q}_{-i}^p - P'(Q^p)q_{-i}^p$ , and  $W_{p-i} = P'(\tilde{Q}^p)\tilde{q}_i^p - P'(Q^p)q_i^p$ . Proceeding as in the proof of Lemma 1, we can then show that  $P'(Q) + P''(Q)q_{-i} < 0$  implies  $W_{p1}, W_{p2} > 0$ . Moreover, using the first-order conditions for product-market quantities yields

$$\begin{aligned} W_{pi} + W_{p-i} &= Nc + P(Q^p) - P(\tilde{Q}^p), \\ &< 2Nc \iff P(Q^p) \leq Nc + P(\tilde{Q}^p), \end{aligned}$$

which holds true for all  $P(\tilde{Q}^p) > P(Q^p)$  and  $c > 0$  (as  $\tilde{Q}^p < Q^p$ ). Finally, manufacturers' symmetry implies that  $W_{pA} = W_{pB} = W_p^{VS}$  and  $F_p^{VS} = \pi(W_p^{VS}, W_p^{VS}) - \max_q \{(P(q + q(W_p^{VS}, W_p^{VS})) - Nc)q\}$ . Q.E.D.

*Proof of Proposition 1.* To determine the competitive effects of the pool, we compare the

<sup>54</sup>The first-order condition with respect to  $w_{p-i}$  is analogous and therefore omitted.

royalty rate offered to  $M_i$  under the pool with the sum of the royalty rates offered to  $M_i$  without the pool—by symmetry, the same arguments apply to  $M_{-i}$ . Recall that, by Property 1, the industry quantity falls in the sum of the royalty rates; thus, the industry structure with lower royalty rates delivers a more competitive outcome. From the proof of Lemma 2,  $W_{pi}$  is implicitly determined by

$$(W_{pi} + P'(Q^p)q_{-i}^p - P'(\tilde{Q}^p)\tilde{q}_{-i}^p)\frac{\partial q_i^p}{\partial W_{pi}} + (W_{p-i} + P'(Q^p)q_i^p - P'(\tilde{Q}^p)\tilde{q}_i^p)\frac{\partial q_{-i}^p}{\partial W_{pi}} = 0. \quad (\text{A-5})$$

From the proof of Lemma 1, the sum of the first-order conditions for each  $w_{ji}$ ,  $j = 1, \dots, N$ , is given by

$$N(w_{ji} + P'(Q)q_{-i} - P'(\tilde{Q})\tilde{q}_{-i})\frac{\partial q_i}{\partial w_{ji}} + N(w_{j-i} + P'(Q)q_i - P'(\tilde{Q})\tilde{q}_i)\frac{\partial q_{-i}}{\partial w_{ji}} = 0. \quad (\text{A-6})$$

We now evaluate (A-6) at  $Nw_{ji} = W_{pi}$  and  $Nw_{j-i} = W_{p-i}$ . Then, we need to replace  $q_i$ ,  $q_{-i}$ ,  $\tilde{q}_i$ , and  $\tilde{q}_{-i}$  with the respective quantities with pool, and  $\partial q_i/\partial w_{ji}$  and  $\partial q_{-i}/\partial w_{ji}$  with  $\partial q_i^p/\partial W_{pi}$  and  $\partial q_{-i}^p/\partial W_{pi}$ , respectively. Using (A-5) in (A-6), all this yields

$$(N-1)(P'(Q^p)q_{-i}^p - P'(\tilde{Q}^p)\tilde{q}_{-i}^p)\frac{\partial q_i^p}{\partial W_{pi}} + (N-1)(P'(Q^p)q_i^p - P'(\tilde{Q}^p)\tilde{q}_i^p)\frac{\partial q_{-i}^p}{\partial W_{pi}}. \quad (\text{A-7})$$

By manufacturers' symmetry,  $q_{-i}^p = q_i^p$  and  $\tilde{q}_{-i}^p = \tilde{q}_i^p$ . Then, (A-7) can then be rewritten as

$$(N-1)(P'(Q^p)q_i^p - P'(\tilde{Q}^p)\tilde{q}_i^p)\left(\frac{\partial q_i^p}{\partial W_{pi}} + \frac{\partial q_{-i}^p}{\partial W_{pi}}\right). \quad (\text{A-8})$$

By (A-4),  $\partial q_i^p/\partial W_{pi} < 0$  and  $\partial q_{-i}^p/\partial W_{pi} > 0$ . However, by a standard property of the Cournot equilibrium,  $|\partial q_i^p/\partial W_{pi}| > |\partial q_{-i}^p/\partial W_{pi}|$ ; thus, the second term in (A-8) is negative. From the proof of Lemmas 1 and 2, the term in the first bracket is also negative. Thus, the sum of the equilibrium royalty rates without the pool must be larger than the bundle price with the pool. This proves that the pool is procompetitive. Q.E.D.

*Proof of Lemma 3.* First,  $L_1$ - $M_A$  sets the internal transfer price for its patent equal to zero. This choice is bilaterally efficient, because  $L_1$  cannot commit to  $M_A$ 's downstream quantity in the first stage of the game; thus, any internal transfer price different from zero would be internally renegotiated to zero before downstream competition takes place.<sup>55</sup> Then, the

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<sup>55</sup>See Reisinger and Tarantino (2015), Proposition 2, for a formal proof, or the proof of Lemma 4 (where we derive the same result for the pool).

maximization problem of a representative non-integrated licensor  $L_j$ , with  $j \neq 1$ , is

$$\max_{w_{jA}, F_{jA}, w_{jB}, F_{jB}} w_{jA} q_A \left( \sum_{j=2}^N w_{jA}, W_B \right) + w_{jB} q_B \left( W_B, \sum_{j=2}^N w_{jA} \right) + F_{jA} + F_{jB},$$

Building on Lemma 1, the fixed components of the tariff offered by  $L_j$  are given by

$$F_{jA} = \pi_A \left( \sum_{j=2}^N w_{jA}, W_B \right) - \max_q \left\{ \left( P \left( q + q_B \left( W_B, \sum_{j=2}^N w_{jA} \right) \right) - (N-1)c \right) q \right\} - \sum_{k \neq 1, j} F_{kA},$$

$$F_{jB} = \pi_B \left( W_B, \sum_{j=2}^N w_{jA} \right) - \max_q \left\{ \left( P \left( q + q_A \left( \sum_{j=2}^N w_{jA}, c + \sum_{j=2}^N w_{jB} \right) \right) - Nc \right) q \right\} - \sum_{k \neq j} F_{kB}.$$

The expression for  $F_{jA}$  takes into account that  $L_1$ - $M_A$ 's internal transfer price is equal to zero. The one for  $F_{jB}$  instead accounts for the fact that  $M_A$  is informed about  $M_B$ 's rejection of  $L_1$ 's offer. Proceeding as in the proof of Lemma 1, we obtain that

$$w_{jA} = -P'(Q)q_B + P'(\hat{Q})\hat{q}_B \quad \text{and} \quad w_{jB} = -P'(Q)q_A + P'(\hat{Q})\hat{q}_A, \quad (\text{A-9})$$

where  $\hat{q}_B = \arg \max_q \left\{ \left( P \left( q + q_A \left( \sum_{j=2}^N w_{jA}, c + \sum_{j=2}^N w_{jB} \right) \right) - Nc \right) q \right\}$ ,  $\hat{q}_A = \arg \max_q \left\{ \left( P \left( q + q_B \left( W_B, \sum_{j=2}^N w_{jA} \right) \right) - (N-1)c \right) q \right\}$ ,  $\hat{Q} = q_A + \hat{q}_B$  and  $\hat{Q} = \hat{q}_A + q_B$ . Using the same arguments as in the proof of Lemma 1, we can show that these royalty rates lie between zero and  $c$ . In what follows, we impose the symmetric equilibrium, which implies that  $w_{kA} = w_{jA}$  for all  $k, j = 2, \dots, N$  and  $j \neq k$ , so that  $\sum_{j=2}^N w_{ji} = (N-1)w_{ji}$ , with  $i = A, B$ .

We now turn to the problem of  $L_1$ - $M_A$ . When dealing with  $M_B$ , the maximization problem of  $L_1$ - $M_A$  is

$$\max_{w_{1B}, F_{1B}} \pi_A \left( (N-1)w_{jA}, W_B \right) + w_{1B} q_B \left( W_B, (N-1)w_{jA} \right) + F_{1B}, \quad (\text{A-10})$$

subject to  $F_{1B} = \pi_B \left( W_B, (N-1)w_{jA} \right) - \max_q \left\{ \left( P \left( q + q_A \left( W_A, c + (N-1)w_{jB} \right) \right) - Nc \right) q \right\} - \sum_{j=2}^N F_{jB}$ . Plugging  $F_{1B}$  in (A-10), and taking the derivative with respect to  $w_{1B}$ , we obtain

$$\left( P'(Q)q_A + w_{1B} \right) \frac{\partial q_B}{\partial w_{1B}} + P'(Q)q_B \frac{\partial q_A}{\partial w_{1B}}. \quad (\text{A-11})$$

We now show that setting  $w_{1B} = c$  is an equilibrium. In the Online Appendix, we prove that it is the unique stable equilibrium. We first use the product-market first-order

conditions:

$$P(Q) - (N - 1)w_{jA} + P'(Q)q_A = 0 \quad \text{and} \quad P(Q) - w_{1B} - (N - 1)w_{jB} + P'(Q)q_B = 0.$$

Combining them yields  $P'(Q)q_A = (N - 1)(w_{jA} - w_{jB}) - w_{1B} + P'(Q)q_B$ . Inserting this expression into (A-11), and rearranging, gives

$$(N - 1)(w_{jA} - w_{jB}) \frac{\partial q_B}{\partial w_{1B}} + P'(Q)q_B \left( \frac{\partial q_B}{\partial w_{1B}} - \frac{\partial q_A}{\partial w_{1B}} \right). \quad (\text{A-12})$$

Because  $|\partial q_B / \partial w_{1B}| > |\partial q_A / \partial w_{1B}|$  and  $\partial q_B / \partial w_{1B} < 0$ , it follows that, if  $w_{jA} \leq w_{jB}$ , (A-12) is strictly positive and  $L_1$ - $M_A$  optimally sets  $w_{1B} = c$ .

We now show that, given  $w_{1B} = c$ , it is optimal for  $L_j$  to set  $w_{jA} \leq w_{jB}$ .

To begin with, plugging the first-order conditions from the product market into the expressions in (A-9), setting  $w_{1B} = c$  and rearranging, we obtain that the royalty rates set by the non-integrated licensors can be written as

$$w_{jA} = \frac{c(N - 1) + P(Q)}{N} - \frac{P(\hat{Q})(N - 1) - P(\hat{Q})}{N(N - 2)} \quad (\text{A-13})$$

and

$$w_{jB} = \frac{c(N - 1) + P(Q)}{N} - \frac{P(\hat{Q})(N - 1) - P(\hat{Q})}{N(N - 2)} \quad (\text{A-14})$$

for  $N \geq 3$ , and  $w_{jA} = (c + P(Q) - P(\hat{Q}))/2$  and  $w_{jB} = (c + P(Q) - P(\hat{Q}))/2$  for  $N = 2$ . A comparison between  $w_{jA}$  and  $w_{jB}$  yields that  $w_{jA} \leq w_{jB}$  if and only if  $P(\hat{Q}) \geq P(\hat{Q})$ . As  $P'(\cdot) < 0$ , this is equivalent to showing that  $\hat{Q} \geq \hat{Q}$ .

At this point, it is useful to remark that downstream quantities are convex in costs: As established in the Online Appendix (proof of Proposition A-1), if  $C$  is the total cost of a firm,  $\partial q_i / \partial C = 1 / (2P'(Q) + P''(Q)q_i) < 0$  yields  $\partial^2 q_i / \partial C^2 = -(3P''(Q) + P'''(Q)q_i) / (2P'(Q) + P''(Q)q_i)^2$ , with  $\partial^2 q_i / \partial C^2 \geq 0$  by the assumptions on  $P''(Q)$  and  $P'''(Q)$ .

Given all this, to determine whether  $\hat{Q} \geq \hat{Q}$ , we then note that in  $\hat{Q}$ ,  $M_A$  produces  $q_A(W_A, W_B)$ , with  $W_A = (N - 1)w_{jA}$  and  $W_B = (N - 1)w_{jB} + c$ , which is given by

$$\arg \max_q \{ (P(q + q_B(W_B, W_A)) - (N - 1)w_{jA})q \},$$

whereas in  $\mathring{Q}$ ,  $M_A$  produces  $\mathring{q}_A$  as given by

$$\arg \max_q \{(P(q + q_B(W_B, W_A)) - (N - 1)c)q\}.$$

Because in both,  $q_A(W_A, W_B)$  and  $\mathring{q}_A$ ,  $M_B$ 's quantity is  $q_B = q_B(W_B, W_A)$ , it follows that  $q_A(W_A, W_B) \geq \mathring{q}_A$  for all  $w_{jA} \leq c$ . Analogously, in  $\widehat{Q}$ ,  $M_B$  produces  $\widehat{q}_B$ , which is given by

$$\arg \max_q \{(P(q + q_A(W_A, W_B)) - Nc)q\},$$

whereas in  $\mathring{Q}$ ,  $M_B$  produces  $q_B(W_B, W_A)$ , which is given by

$$\arg \max_q \{(P(q + q_A(W_A, W_B)) - c - (N - 1)w_{jB})q\}.$$

Thus,  $q_B(W_B, W_A) > \widehat{q}_B$  for all  $c > 0$ .

Finally, we prove that  $w_{jA} \leq w_{jB}$  by contradiction. Let  $w_{jA} = w_{jB} = w_j$ . Then, the difference in the total costs borne by  $M_A$  between  $q_A$  and  $\mathring{q}_A$  is the same as the one borne by  $M_B$  between  $q_B$  and  $\widehat{q}_B$ , and equal to  $(N - 1)(w_j - c)$ . However, in  $q_A$  and  $\mathring{q}_A$  total costs are lower by  $c$  than in  $q_B$  and  $\widehat{q}_B$  (which also means that the quantity produced by the rival manufacturer is lower in  $q_A$  and  $\mathring{q}_A$  than in  $q_B$  and  $\widehat{q}_B$ ). All this implies that  $q_A > q_B$  and  $\mathring{q}_A > \widehat{q}_B$  and, by convexity of quantities in costs,  $q_A - \mathring{q}_A \geq q_B - \widehat{q}_B$ ; thus,  $\widehat{Q} = q_A + \widehat{q}_B \geq \mathring{Q} = \mathring{q}_A + q_B$  and  $P(\widehat{Q}) \leq P(\mathring{Q})$ , which contradicts  $w_{jA} = w_{jB}$ —but for the limit case in which  $P(\widehat{Q}) = P(\mathring{Q})$ , which holds true if and only if  $\partial^2 q_i / \partial C^2 = 0$ .

Consider now a marginal reduction in  $w_{jA}$  from  $w_{jA} = w_{jB}$ . This causes an increase in  $\widehat{Q}$  (as  $|\partial q_A / \partial w_{jA}| > |\partial \widehat{q}_B / \partial w_{jA}|$ ) and a fall in  $\mathring{Q}$  (as  $|\partial q_B / \partial w_{jA}| > |\partial \mathring{q}_A / \partial w_{jA}|$ ). This makes  $\widehat{Q}$  increase above  $\mathring{Q}$ , thus confirming  $P(\widehat{Q}) < P(\mathring{Q})$  and rendering  $w_{jA} < w_{jB}$  optimal. (A similar argument applies when considering a marginal increase in  $w_{jB}$ .) Thus, given  $w_{1B} = c$ , it is optimal for licensors  $L_j$ ,  $j \neq 1$ , to set  $w_{jA} \leq w_{jB}$ .

As shown in the Online Appendix,  $w_{1B}^{SI} = c$  is the unique stable equilibrium. This implies that  $F_{1A} = 0$ , and each  $L_j$ , with  $j \neq 1$ , sets

$$\begin{aligned} F_A^{SI} &= \frac{\pi_A(W_A, W_B) - \max_q \{(P(q + q_B(W_B, W_A)) - (N - 1)c)q\}}{N - 1}, \\ F_B^{SI} &= \frac{\pi_B(W_B, W_A) - \max_q \{(P(q + q_A(W_A, W_B)) - Nc)q\}}{N - 1}, \end{aligned}$$

where  $w_{ji} = w_{ki} = w_i^{SE}$  are given in (A-13)-(A-14), for all  $i = A, B$ ,  $j, k \neq 1$  and  $j \neq k$ .

Because the internal transfer price is zero, it follows that  $W_A = (N - 1)w_A^{SI}$  and  $W_B = c + (N - 1)w_B^{SI}$ . Q.E.D.

*Proof of Lemma 4.* As  $M_A$  is a pool member via its integration with  $L_1$ , the pool takes its profits into account when deciding on the tariffs. It follows that the pool sets an internal transfer price of  $W_{pA} = 0$ . This choice is internally efficient because the pool cannot commit to  $M_A$ 's downstream quantity in the first stage of the game; thus, any patent price different from zero would be renegotiated to zero in the second stage.

To see this, note first that the pool's maximization problem can be written as

$$\max_{W_{pA}, W_{pB}, F_{pB}} P(Q(W_{pA} + W_{pB}))q_A(W_{pA}, W_{pB}) + W_{pB}q_B(W_{pB}, W_{pA}) + F_{pB}, \quad (\text{A-15})$$

where  $F_{pB} = \pi_B(W_{pB}, W_{pA}) - \pi_B(Nc, W_{pA})$ .<sup>56</sup> Because  $W_{pA}$  can be secretly negotiated before manufacturers compete downstream,  $M_B$  knows that the pool will set the ex-post efficient royalty rate to its integrated manufacturer. This implies that any  $W_{pA}$  chosen in the first stage will neither affect the quantity set by  $M_B$  nor the fixed fee that  $M_B$  is willing pay to the pool. In fact,  $M_B$ 's decisions are only driven by the ex-post efficient level of  $W_{pA}$ , which it correctly anticipates. This means that the second and third term of the maximization problem in (A-15) are not affected by  $W_{pA}$ . Thus, the maximization problem with respect to  $W_{pA}$  can be written as  $\max_{W_{pA}} P(Q(W_{pA} + W_{pB}))q_A(W_{pA}, W_{pB})$ . The solution results from the first-order condition  $(P(Q) + P'(Q)q_A) \partial q_A / \partial W_{pA} = 0$ , where  $Q = Q(W_{pA} + W_{pB})$  and  $q_A = q_A(W_{pA}, W_{pB})$ .<sup>57</sup> As  $\partial q_A / \partial W_{pA} < 0$ , it follows that  $P(Q) + P'(Q)q_A = 0$ .

In the second stage,  $M_A$  decides about its quantity to maximize its own profit,<sup>58</sup> which implies that its maximization problem is given by

$$\max_{q_A} (P(Q(W_{pA} + W_{pB})) - W_{pA}) q_A(W_{pA}, W_{pB}). \quad (\text{A-16})$$

The resulting first-order condition is  $P(Q) - W_{pA} + P'(Q)q_A = 0$ , where, as above,  $Q =$

<sup>56</sup>Any fixed fee  $F_{pA}$  that the pool may demand from its integrated manufacturer  $M_A$  is an internal transfer and therefore does not affect the maximization problem.

<sup>57</sup>Because of our assumption  $P'(Q) + P''Q < 0$ , the maximization function is strictly quasi-concave.

<sup>58</sup>To simplify the exposition of the proof, we assume here and in the proof of Lemma 6 that an integrated manufacturer maximizes its downstream profit but does not take into account that a share of its royalty payment flows back to its upstream affiliate. This does not change the result of this proof as the pool optimally sets  $W_{pA} = 0$  even if  $M_A$  takes this share into account (see footnote 54). The result of Lemma 6 and its proof would be slightly different without affecting the qualitative insight.

$Q(W_{pA} + W_{pB})$  and  $q_A = q_A(W_{pA}, W_{pB})$ . Because  $P(Q) + P'(Q)q_A = 0$  from the pool's solution, it follows that  $M_A$  optimally requires the instruction by the pool that  $W_{pA} = 0$ .<sup>59</sup>

Given  $W_{pA} = 0$  the pool's maximization problem when dealing with  $M_B$  is

$$\max_{w_{pB}, F_{pB}} \pi_A(0, W_{pB}) + W_{pB}q_B(W_{pB}, 0) + F_{pB},$$

subject to  $F_{pB} = \pi_B(W_{pB}, 0) - \pi_B(Nc, 0)$ . Plugging  $F_{pB}$  into the maximization problem and taking the derivative with respect to  $W_{pB}$ , we obtain

$$(P'(Q^p)q_A + W_{pB}) \frac{\partial q_B}{\partial W_{pB}} + P'(Q^p)q_B \frac{\partial q_A}{\partial W_{pB}},$$

where  $Q^p = q_A(0, W_{pB}) + q_B(W_{pB}, 0)$ .

We first use the product-market first-order conditions  $P(Q^p) + P'(Q^p)q_A = 0$  and  $P(Q^p) - W_{pB} + P'(Q^p)q_B = 0$ . Combining them yields  $P'(Q^p)q_A = P'(Q^p)q_B - W_{pB}$ . Inserting this expression into the derivative with respect to  $W_{pB}$ , and rearranging, gives

$$P'(Q^p)q_B \left( \frac{\partial q_B}{\partial W_{pB}} - \frac{\partial q_A}{\partial W_{pB}} \right).$$

Because  $|\partial q_B / \partial W_{pB}| > |\partial q_A / \partial W_{pB}|$  it follows that the derivative is strictly positive. As a consequence, the pool sets  $W_{pB}$  at the highest value possible, which equals  $W_{pB} = Nc$ .

Q.E.D.

*Proof of Proposition 2.* From the proof of Lemma 3, the royalty rates of non-integrated licensors under independent licensing and single integration are given by

$$w_A^{SI} = \frac{c(N-1) + P(Q)}{N} - \frac{P(\hat{Q})(N-1) - P(\hat{Q})}{N(N-2)} \quad (\text{A-17})$$

$$w_B^{SI} = \frac{c(N-1) + P(Q)}{N} - \frac{P(\hat{Q})(N-1) - P(\hat{Q})}{N(N-2)} \quad (\text{A-18})$$

for  $N \geq 3$ , and  $w_A^{SI} = (c + P(Q) - P(\hat{Q}))/2$  and  $w_B^{SI} = (c + P(Q) - P(\hat{Q}))/2$  for  $N = 2$ , where  $\hat{Q} = q_A(W_A, W_B) + \hat{q}_B$  and  $\hat{Q} = \hat{q}_A + q_B(W_B, W_A)$ , with  $\hat{q}_B = \arg \max_q \{(P(q + q_A(W_A, W_B)) - Nc)q\}$ ,  $\hat{q}_A = \arg \max_q \{(P(q + q_B(W_B, W_A)) - (N-1)c)q\}$ ,  $W_A = \sum_{j=2}^N w_{jA}$ , and  $W_B = \sum_{j=2}^N w_{jB} + c$ .

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<sup>59</sup>If  $M_A$  would take the share  $s_1$  that flows back to its upstream affiliate  $L_1$  into account in its maximization problem, its first-order condition would be  $P(Q) - (1 - s_1)W_{pA} + P'(Q)q_A = 0$ , which again yields  $W_{pA} = 0$ .

We start with the case featuring  $N = 2$ . Inserting the formulas for  $w_A^{SI}$  and  $w_B^{SI}$  into  $c + w_A^{SI} + w_B^{SI}$ , and comparing the resulting expression with the sum of the royalty rates with the pool ( $2c$ ), yields  $2c + P(Q) - (P(\mathring{Q}) + P(\widehat{Q}))/2 < 2c$ , because  $\mathring{q}_A < q_A(W_A, W_B)$  and  $\widehat{q}_B < q_B(W_B, W_A)$  imply that  $P(\mathring{Q}) > P(Q)$  and  $P(\widehat{Q}) > P(Q)$ . Hence, applying Property 1, the pool is anticompetitive.

We now turn to the case with a large number of licensors. Taking the limit of (A-17) and (A-18) as  $N \rightarrow \infty$ , we obtain that  $w_A^{SI} \rightarrow c$  and  $w_B^{SI} \rightarrow c$ . As  $N$  grows large, the difference between  $P(Q)$ ,  $P(\mathring{Q})$ , and  $P(\widehat{Q})$  becomes negligible, implying that these terms cancel out in (A-17) and (A-18); thus, only  $c(N - 1)/N$  remains, whose limit value is  $c$  as  $N \rightarrow \infty$ . This implies that, as  $N \rightarrow \infty$ , the sum of the costs without pool converges to  $c + 2(N - 1)c = (2N - 1)c$ . Instead, the sum of the costs with the pool is  $Nc$ . Because  $(2N - 1)c - Nc = (N - 1)c > 0$ , the pool is procompetitive when  $N$  is large.

Finally, we show that there is unique value of  $N$ , denoted by  $N^{SI}$ , such that the pool is anticompetitive for  $N < N^{SI}$ , but procompetitive for  $N > N^{SI}$ . The threshold  $N^{SI}$  is determined by  $c + (N^{SI} - 1)(w_A^{SI} + w_B^{SI}) = N^{SI}c$ . Inserting into this equation the values of  $w_A^{SI}$  and  $w_B^{SI}$  in (A-17) and (A-18), respectively, and simplifying, yields

$$N^{SI} = 2 + \frac{P(\mathring{Q}) + P(\widehat{Q}) - 2P(Q)}{c}. \quad (\text{A-19})$$

The left-hand side's derivative with respect to  $N^{SI}$  is 1. The right-hand side's derivative is

$$\frac{P'(\mathring{Q})\frac{\partial \mathring{Q}}{\partial N} + P'(\widehat{Q})\frac{\partial \widehat{Q}}{\partial N} - 2P'(Q)\frac{\partial Q}{\partial N}}{c}.$$

Using the first-order conditions from the product market to determine  $Q$ ,  $\mathring{Q}$  and  $\widehat{Q}$ , together with  $w_A^{SI}$  and  $w_B^{SI}$ , we can compute the derivative of these terms with respect to  $N$ . Tedious, but otherwise standard calculations show that  $P'(\mathring{Q})\partial \mathring{Q}/\partial N + P'(\widehat{Q})\partial \widehat{Q}/\partial N - 2P'(Q)\partial Q/\partial N$  is smaller than  $c$  for all values of  $N \geq 2$ . Therefore, the slope of the right-hand side of (A-19) is smaller than the one of the left-hand side. Because we know that the pool is anticompetitive for  $N = 2$  and procompetitive for  $N \rightarrow \infty$ , it follows that the value of  $N^{SI}$  is unique. In the Online Appendix, we show that if the second derivative of the demand function is small (i.e.,  $P'' \approx 0$ ),  $N^{SI}$  is independent of  $c$  and equal to (approximately) 3. Q.E.D.

*Proof of Lemma 5.* First, both integrated licensors set the internal patent price equal to

zero.<sup>60</sup> Turning to a non-integrated licensor  $L_j$ , with  $j \neq 1, 2$ , its maximization problem is

$$\max_{w_{jA}, F_{jA}, w_{jB}, F_{jB}} w_{jA}q_A(W_A, W_B) + w_{jB}q_B(W_B, W_A) + F_{jA} + F_{jB},$$

with  $W_A = \sum_{k=2}^N w_{kA}$ ,  $W_B = \sum_{k \neq 2} w_{kB}$  and

$$F_{jA} = \pi_A(W_A, W_B) - \max_q \left\{ (P(q + q_B(W_B, c + \sum_{k=3}^N w_{kA})) - (N-1)c)q \right\} - \sum_{k \neq 1, j} F_{kA},$$

$$F_{jB} = \pi_B(W_B, W_A) - \max_q \left\{ (P(q + q_A(W_A, c + \sum_{k=3}^N w_{kB})) - (N-1)c)q \right\} - \sum_{k \neq 2, j} F_{kB}.$$

Following the same steps as in the proof of Lemma 1, we find that, in a symmetric equilibrium,

$$w_{jA} = -P'(Q)q_B + P'(\bar{Q})\bar{q}_B \quad \text{and} \quad w_{jB} = -P'(Q)q_A + P'(\bar{Q})\bar{q}_A,$$

where  $\bar{q}_B = \arg \max_q \{ (P(q + q_A(W_A, c + \sum_{k=3}^N w_{kB})) - (N-1)c)q \}$ ,  $\bar{q}_A = \arg \max_q \{ (P(q + q_B(W_B, c + \sum_{k=3}^N w_{kA})) - (N-1)c)q \}$ ,  $\bar{Q} = q_A(W_A, c + \sum_{k=3}^N w_{kB}) + \bar{q}_B$  and  $\bar{Q} = \bar{q}_A + q_B(W_B, c + \sum_{k=3}^N w_{kA})$ . Using the same arguments as in the proofs above, we can show that the royalty rates set by non-integrated licensors lie within  $(0, c)$ .

Recall that, in the symmetric equilibrium, all non-integrated licensors set the same patent price to manufacturer  $M_i$ , thus  $\sum_{k \neq 1, 2} w_{ki} = (N-2)w_{ki}$ . Moreover, integrated licensors might set a different patent price to the rival manufacturer  $M_i$ ; that is,  $w_{1B}$  can be different from  $w_{jB}$  and  $w_{2A}$  from  $w_{jA}$ . Thus, in what follows,  $W_A = w_{2A} + (N-2)w_{jA}$  and  $W_B = w_{1B} + (N-2)w_{jB}$  for all  $j \neq 1, 2$ .

We now turn to the optimization problem of the integrated licensor  $L_1$ - $M_A$ :

$$\max_{w_{1B}, F_{1B}} \pi_A(W_A, W_B) + q_B(W_B, W_A)w_{1B} + F_{1B}.$$

As  $M_A$  is informed by  $L_1$  of  $M_B$ 's rejection of  $L_1$ 's offer,  $M_A$  will set its quantity taking into account that  $M_B$  pays  $c$  for input 1. Therefore, when deciding whether to accept or reject  $L_1$ - $M_A$ 's offer,  $M_B$  takes into consideration the impact of that decision on  $M_A$ 's output, as this changes its own downstream profit and the revenue that its upstream licensor  $L_2$

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<sup>60</sup>The proof is analogous to the one given in Lemma 4.

extracts from  $M_A$ . The fixed component of the tariff set by  $L_1$ - $M_A$  is then

$$F_{1B} = \pi_B(W_B, W_A) - \max_q \{ (P(q + q_A(W_A, c + (N-2)w_{jB})) - (N-1)c)q \} \\ + q_A(W_A, W_B)w_{2A} - q_A(W_A, c + (N-2)w_{jB})w_{2A}.$$

The first and second terms correspond to the difference in  $L_2$ - $M_B$ 's downstream profit when accepting and when rejecting the offer, and the third and fourth terms correspond to the difference in  $L_2$ 's revenues from sales to  $M_A$ . We now rewrite the optimization program of  $L_1$ - $M_A$  by using only those terms that depend on  $w_{1B}$ :

$$\max_{w_{1B}} \pi_A(W_A, W_B) + \pi_B(W_B, W_A) + q_B(W_B, W_A)w_{1B} + q_A(W_A, W_B)w_{2A}.$$

The resulting first-order condition with respect to  $w_{1B}$  is

$$(P'(Q)q_A + w_{1B}) \frac{\partial q_B}{\partial w_{1B}} + (P'(Q)q_B + w_{2A}) \frac{\partial q_A}{\partial w_{1B}} = 0. \tag{A-20}$$

After imposing symmetry, we can set  $w_{2A} = w_{1B}$  and  $w_{jA} = w_{jB} = w_j$ , for all  $j \neq 1, 2$  and  $i = A, B$ , so that  $W_A = W_B = W$ ,  $q_A(W, W) = q_B(W, W) = q(W, W)$  and  $Q = 2q(W, W)$ . In what follows, we show that, at the unique equilibrium,  $w_{1B} = c$ .

We can simplify (A-20) by solving for  $w_{1B}$  to obtain

$$w_{1B} = -P'(Q) \frac{Q}{2}. \tag{A-21}$$

Because manufacturers pay the same royalty rates, the first-order condition in the downstream market can be written as  $P'(Q)Q/2 = -P'(Q) + W$ . Inserting this into (A-21) yields  $2w_{1B} = P(Q) - (N-2)w_j$ . Because  $Q$  is a function of  $w_{1B}$ , we need to solve for the following fixed-point problem:

$$w_{1B} = \frac{P(Q) - (N-2)w_j}{2}. \tag{A-22}$$

First, we determine how  $Q = 2q(W, W)$  changes with  $w_{1B}$ : If  $w_{1B}$  increases by one unit, then  $P(Q)/2$  changes by  $P'(Q)/2(3P'(Q) + P''(Q)Q)$ , which, according to our assumption  $P'(Q) + P''(Q)Q < 0$ , is strictly positive but smaller than one. This implies that the slope of the right-hand side of (A-22) is smaller than the slope of the left-hand side (which is

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<sup>61</sup>Our assumptions on the demand function ensure that the second-order conditions are satisfied.

equal to 1). As a consequence, there is a unique solution to the equation in question, which determines the value of  $w_{1B}$ , given  $w_j$ .

We now show that the value of the right-hand side of (A-22) is above  $c$ , for all possible values of  $w_j$ . First, note that the upper bound of  $c$  is implicitly given by the condition requiring that  $q_i(Nc, 0) > 0$ . From Property 1, we know that, in this situation, the downstream price is given by  $P(Q(Nc))$ . The implicit definition of the upper bound on  $c$  then requires that  $P(Q(Nc)) > Nc$ , or  $c < P(Q(Nc))/N$ , as otherwise the manufacturer buying at  $Nc$  would not produce a positive quantity. We also know from above that  $w_j \in (0, c)$ . Moreover, the right-hand side of (A-22) decreases with  $w_j$ , as

$$\frac{\partial}{\partial w_j} (P(Q) - (N-2)w_j) = \frac{2(N-2)P'(Q)}{3P'(Q) + P''(Q)} - (N-2) < 0$$

for all  $P'(Q) + P''(Q)Q < 0$ . It follows that, if, at  $w_j = c$ , the right-hand side of (A-22) is larger than  $c$ , the value of  $w_{1B}$  that solves (A-22) must be above  $c$ . Plugging  $w_j = c$  into (A-22) yields

$$w_{1B} = \frac{P(Q(2w_{1B} + 2(N-2)c) - (N-2)c)}{2}. \quad (\text{A-23})$$

We will now show that the solution to (A-23) involves  $w_{1B} > c$ . Inserting  $w_{1B} = c$  into the last expression, and rearranging, gives us  $c = P(Q(2(N-1)c))/N$ , with  $2(N-1)c \geq Nc$  for all  $N \geq 2$ . Thus, as quantity is falling in costs, and price is falling in quantity,  $P(Q(2(N-1)c)) \geq P(Q(Nc))$ . That is, if  $w_{B1} = c$  were a solution to (A-23), then  $c = P(Q(Nc))/N$ . But this is a contradiction to the assumption that  $c < P(Q(Nc))/N$ , which defines the upper bound of  $c$ . As a consequence,  $w_{B1} = c$  cannot be a solution to (A-23). Because the slope of the left-hand side is larger than the one of the right-hand side, this analysis implies that the solution for  $w_{B1}$  must be above  $c$ . It follows that at the equilibrium  $w_{1B}$  is set equal to  $c$ .

Instead, because, at  $w_{1B} = w_{2A} = c$ ,  $\bar{q} = \bar{q}$ , a non-integrated licensor sets  $w_j = -P'(Q)q + P'(\bar{Q})\bar{q}$ , with  $q = q(W, W)$ ,  $\bar{q} = \arg \max_q \{(P(q + q(W, W)) - (N-1)c)q\}$ ,  $Q = 2q(W, W)$ ,  $\bar{Q} = q + \bar{q}$  and  $W = c + (N-2)w_j$ . The product-market first-order conditions for  $q$  and  $\bar{q}$ , respectively, imply that  $q = -(P(Q) - c - (N-2)w_j)/P'(Q)$  and  $\bar{q} = -(P(\bar{Q}) - (N-1)c)/P'(\bar{Q})$ . Inserting these expressions into  $w_j = -P'(Q)q + P'(\bar{Q})\bar{q}$ , and rearranging, yields that, at equilibrium,  $w_j$ ,  $j \neq 1, 2$ , boils down to

$$w^{DI} = \frac{P(Q) - P(\bar{Q}) + (N-2)c}{N-1}. \quad (\text{A-24})$$

Finally, because  $w_{1B} = w_{2A} = c$ , in equilibrium, the fixed component of the tariffs set by the integrated licensors is equal to 0. Instead, in the symmetric equilibrium, a non-integrated licensor sets

$$F^{DI} = \frac{\pi(W, W) - \max_q \{ (P(q + q(W, W)) - (N - 1)c)q \}}{N - 2},$$

with  $W = c + (N - 2)w^{DI}$ .

Q.E.D.

*Proof of Lemma 6.* If licensors form a pool, both manufacturers  $M_A$  and  $M_B$  are also part of the pool, as they are integrated with  $L_1$  and  $L_2$ , respectively. Moreover, when setting the price  $W_p$ , the pool must take into account that each manufacturer can also reject the pool's offer and use the status-quo technologies.

Assume first that the pool's problem is unconstrained. Then, it will optimally set its price  $W_p$  so that each manufacturer produces  $q^m/2$ , because this allows the licensors to jointly reap monopoly profits. Because the monopoly quantity  $q^m$  is implicitly defined by the first-order condition  $P(q^m) + P'(q^m)q^m = 0$ , if each manufacturer faces a royalty rate of  $W_p$ , it produces half of the monopoly quantity if and only if

$$P(q^m) + P'(q^m)\frac{q^m}{2} - W_p = 0.$$

Using the monopolist's first-order condition, we can replace  $P'(q^m)q^m/2$  by  $-P(q^m)/2$ , which yields  $W_p = P(q^m)/2$ .

Instead of accepting the pool's offer, a manufacturer could use the status-quo technologies. In this case, because the manufacturer obtains the patent of the affiliated licensor at a patent price of zero, it bears total costs of  $(N - 1)c$ . As a consequence, to be sure that manufacturers accept the pool's offer,  $W_p$  must be lower than  $(N - 1)c$ .<sup>62</sup>

To summarize, the pool sets  $W_p^{DI} = P(q^m)/2$  if  $P(q^m)/2 \leq (N - 1)c$  and  $W_p^{DI} = (N - 1)c$  if otherwise. Q.E.D.

*Proof of Proposition 3.* We first consider the case in which  $c \leq P(q^m)/(2(N - 1))$ . From Lemma 6 we know that a manufacturer pays total costs of  $(N - 1)c$  with the pool. Without the pool, Lemma 5 implies that the costs of each manufacturer are  $c + (N - 2)w^{DI}$ , with  $w^{DI} < c$ . Because royalty rates are lower without pool, Property 1 implies that the pool is anticompetitive.

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<sup>62</sup>Recall that we assume that an integrated manufacturer does not consider the share of its royalty payment that flows back to its integrated licensor via the pool.

We now turn to the case in which  $c > P(q^m)/(2(N-1))$ . With the pool, the sum of manufacturers' costs is  $P(q^m)$ ; thus, it is independent of  $N$ . Without the pool, the sum of manufacturers' costs is  $2c+2(N-2)w^{DI}$ , with  $w^{DI} < c$ . We start by considering  $N = 2$ . From the proof of Lemma 5, we know that the condition on the upper bound on  $c$  ( $q(Nc, 0) > 0$ ) can be formulated as  $P(q^m) > Nc$ . If  $N = 2$ , this condition boils down to  $P(q^m) > 2c$ , implying that the pool is anticompetitive.

Now consider the case in which  $N$  is large. From the proof of Lemma 5, the formula for the royalty rate of non-integrated licensors in case of independent licensing is given by

$$w^{DI} = \frac{P(Q) - P(\bar{Q}) + (N-2)c}{N-1}, \quad (\text{A-25})$$

with  $\bar{Q} = q(W, W) + \bar{q}$ ,  $\bar{q} = \arg \max_q \{(P(q + q(W, W)) - (N-1)c)q\}$ , and  $W = c + (N-2)w^{DI}$ . As in the proof of Proposition 2, if  $N \rightarrow \infty$ , then  $P(Q) - P(\bar{Q})$  converges to 0. Therefore,  $w_i^{DI} \rightarrow c$ , as  $N \rightarrow \infty$ . This implies that, if  $N$  grows large, the sum of manufacturers' costs without pool converges to  $2(N-1)c$ , which is strictly larger than  $P(q^m)$ , as we are now in the case with  $c > P(q^m)/(2(N-1))$ . It follows that the pool is procompetitive if the number of licensors  $N$  is large. In addition, for all  $N \geq 2$ , the royalty rates paid by a manufacturer without pool are increasing in  $N$ . Because the royalty rate with the pool is independent of  $N$ , there exists a unique threshold value  $N^{DI}$  such that the pool is anticompetitive for  $N < N^{DI}$  and procompetitive for  $N > N^{DI}$ .

We now show that the unique threshold value  $N^{DI}$  is larger than 3. Inserting  $w^{DI}$  from (A-25) into  $2c+2(N-2)w^{DI}$  and setting  $N = 3$  yields, after simplifying,  $3c + P(Q) - P(\bar{Q})$ . This is the sum of the royalty rates without pool. As this sum is increasing in  $c$ , and the upper bound of  $c$  at  $N = 3$  is given by  $P(q^m)/3$ , this value can at most be  $P(q^m) + P(Q) - P(\bar{Q})$ . With pool, the sum of the royalty rates equals  $P(q^m)$ . Comparing the two sums, it follows that  $P(q^m) + P(Q) - P(\bar{Q}) < P(q^m)$ , because  $\bar{Q} > Q$  and therefore  $P(Q) - P(\bar{Q}) < 0$ . As a consequence, the pool is anticompetitive at  $N = 3$ .

Finally, we turn to the comparison between  $N^{SI}$  and  $N^{DI}$ . We start with double integration, where  $N^{DI}$  is implicitly defined by  $2c + 2(N^{DI} - 2)w^{DI} = P(q^m)$ . Using (A-25), this can be rewritten as:

$$2c + 2(N^{DI} - 2) \frac{(P(Q) - P(\bar{Q}) + (N^{DI} - 2)c)}{N^{DI} - 1} = P(q^m).$$

We know that  $P(q^m) > Nc$ ; thus, replacing  $P(q^m)$  by  $N^{DI}c$  in the right-hand side of the last equation delivers a threshold  $N^{DI}$  that is strictly below  $N^{DI}$ . Doing so yields

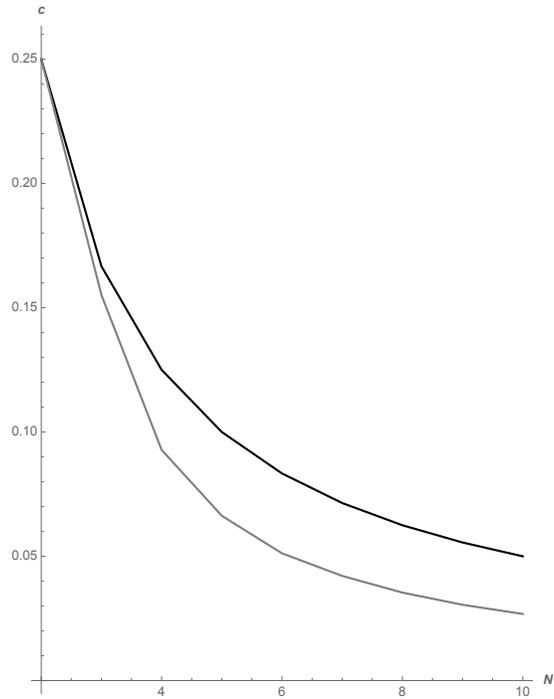
$N^{DI'} = 3 + (2(P(\bar{Q}) - P(Q))/c < N^{DI}$ . Because  $P(\bar{Q}) > P(Q)$ , and  $N^{DI} > N^{DI'}$ , the threshold with double integration must be strictly above 3. As  $N^{SI} \approx 3$  if  $P''(\cdot) \approx 0$  from the proof of Proposition 2, the result in the third bullet point of the proposition follows. Q.E.D.

## References

- Bergemann, D. and Välimäki, J. “Dynamic Common Agency.” *Journal of Economic Theory*, Vol. 111 (2003), pp. 23–48.
- Bernheim, D. and Whinston, M. D. “Menu Auctions, Resource Allocation, and Economic Influence.” *Quarterly Journal of Economics*, Vol. 101 (1986), pp. 1–32.
- Boutin, A. “Screening for Good Patent Pools through Price Caps on Individual Licenses.” *American Economic Journal: Microeconomics*, Vol. 8 (2016), pp. 64–94.
- Calzolari, G. and Denicolò, V. “Competition with Exclusive Contracts and Market-Share Discounts.” *American Economic Review*, Vol. 103 (2013), pp. 2384–2411.
- Calzolari, G. and Denicolò, V. “Exclusive Contracts and Market Dominance.” *American Economic Review*, Vol. 105 (2015), pp. 3321–3351.
- Chen, Y. “On Vertical Mergers and Their Competitive Effects.” *RAND Journal of Economics*, Vol. 32 (2001), pp. 667–685.
- Choi, J. P. and Gerlach, H. “Patent Pools, Litigation, and Innovation.” *RAND Journal of Economics*, Vol. 46 (2015), pp. 499–523.
- Cournot, A. *Researches into the Mathematical Principles of the Theory of Wealth*. New York: Macmillan, 1838, 1897.
- Denicolò, V. “Sequential Innovation and the Patent-Antitrust Conflict.” *Oxford Economic Papers*, Vol. 54 (2002), pp. 649–668.
- Flamm, K. “A Tale of Two Standards: Patent Pools and Innovation in the Optical Disk Drive Industry.” Working Paper No. 18931, National Bureau of Economic Research (NBER), 2013.
- Gallini, N. “Cooperating with Competitors: Patent Pooling and Choice of a New Standard.” *International Journal of Industrial Organization*, Vol. 36 (2014), pp. 4–21.
- Gilbert, R. “Antitrust for Patent Pools: A Century of Policy Evolution.” *Stanford Technology Law Review*, Vol. 3 (2004).
- Gilbert, R. “Deal Or No Deal? Licensing Negotiations In Standard-Setting Organizations.” *Antitrust Law Journal*, Vol. 77 (2011), pp. 855–888.
- Harris, S. “Initial Determination, In the Matter of Certain Recordable Compact Discs and Rewritable Compact Discs,” Investigation No. 337-TA-474, U.S. International Trade Commission, 2003.

- Hart, O. and Tirole, J. “Vertical Integration and Market Foreclosure.” *Brookings Papers on Economic Activity: Microeconomics*, pp. 205–276, 1990.
- Hegde, D. “Tacit Knowledge and the Structure of License Contracts: Evidence from the Biomedical Industry.” *Journal of Economics and Management Strategy*, Vol. 23 (2014), pp. 568–600.
- Hopenhayn, H., Llobet, G., and Mitchell, M. “Rewarding Sequential Innovators: Prizes, Patents and Buyouts.” *Journal of Political Economy*, Vol. 115 (2006), pp. 1041–1068.
- Jeon, D.-S. and Lefouili, Y. “Cross-Licensing and Competition.” *RAND Journal of Economics*, Vol. 49 (2018), pp. 656–671.
- Kim, S.-H. “Vertical Structure and Patent Pools.” *Review of Industrial Organization*, Vol. 25 (2004), pp. 231–250.
- Lampe, R. and Moser, P. “Patent Pools and Innovation in Substitute Technologies—Evidence from the U.S. Sewing Machine Industry.” *RAND Journal of Economics*, Vol. 44 (2014), pp. 757–778.
- Laussel, D. and Le Breton, M. “Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency.” *Journal of Economic Theory*, Vol. 100 (2001), pp. 93–128.
- Layne-Farrar, A. and Lerner, J. “To Join or not to Join: Examining Patent Pool Participation and Rent Sharing Rules.” *International Journal of Industrial Organization*, Vol. 29 (2011), pp. 294–303.
- Lemley, M. A. and Shapiro, C. “A Simple Approach to Setting Reasonable Royalties for Standard-Essential Patents.” *Berkeley Technology Law Journal*, Vol. 28 (2013), pp. 1135–1166.
- Lemley, M. A. and Simcoe, T. “How Essential are Standard-Essential Patents?” Stanford Public Law Working Paper, 2018.
- Lerner, J. and Tirole, J. “Efficient Patent Pools.” *American Economic Review*, Vol. 94 (2004), pp. 691–711.
- Lerner, J. and Tirole, J. “Public Policy Toward Patent Pools.” In A. Jaffe, J. Lerner, and S. Stern, eds, *Innovation Policy and the Economy*. University of Chicago Press, 2007.
- Lerner, J. and Tirole, J. “Standard Essential Patents.” *Journal of Political Economy*, Vol. 123 (2015), pp. 547–586.
- Llanes, G. and Poblete, J. “Ex-Ante Agreements in Standard Setting and Patent Pool Formation.” *Journal of Economics and Management Strategy*, Vol. 23 (2014), pp. 50–67.
- Martimort, D. “Exclusive Dealing, Common Agency, and Multi-Principals Incentive Theory.” *RAND Journal of Economics*, Vol. 27 (1996), pp. 1–31.

- Martimort, D. and Stole, L. “Market Participation in Delegated and Intrinsic Common-Agency Games.” *RAND Journal of Economics*, Vol. 40 (2009), pp. 78–102.
- McAfee, R. P. and Schwartz, M. “Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity.” *American Economic Review*, Vol. 84 (1994), pp. 210–230.
- Nocke, V. and Rey, P. “Exclusive Dealing and Vertical Integration in Interlocking Relationships.” *Journal of Economic Theory*, Vol. 177 (2018), pp. 183–221.
- O’Brien, D. P. and Shaffer, G. “Vertical Control with Bilateral Contracts.” *RAND Journal of Economics*, Vol. 23 (1992), pp. 299–308.
- Quint, D. “Patent Pools.” In S. Durlauf and L. Blume, eds., *The New Palgrave Dictionary of Economics Online*. Palgrave-Macmillan, 2008.
- Quint, D. “Pooling with Essential and Nonessential Patents.” *American Economic Journal: Microeconomics*, Vol. 6 (2014), pp. 23–57.
- Reisinger, M. and Tarantino, E. “Vertical Integration, Foreclosure, and Productive Efficiency.” *RAND Journal of Economics*, Vol. 46 (2015), pp. 461–479.
- Rey, P. and Tirole, J. “A Primer on Foreclosure.” In M. Armstrong and R.H. Porter, eds., *Handbook of Industrial Organization III*. North-Holland: Elsevier, 2007.
- Rey, P. and Tirole, J. “Price Caps as Welfare-Enhancing Coopetition.” *Journal of Political Economy*, forthcoming.
- Rey, P. and Vergé, T. “Bilateral Control with Vertical Contracts.” *RAND Journal of Economics*, Vol. 35 (2004), pp. 728–746.
- Rey, P. and Vergé, T. “Resale Price Maintenance and Interlocking Relationships.” *Journal of Industrial Economics*, Vol. 58 (2010), pp. 928–961.
- Rysman, M. and Simcoe, T. “Patents and the Performance of Voluntary Standard-setting Organizations.” *Management Science*, Vol. 54 (2008), pp. 1920–1934.
- Schmidt, K. M. “Complementary Patents and Market Structure.” *Journal of Economics and Management Strategy*, Vol. 23 (2014), pp. 68–88.
- Shapiro, C. “Navigating the Patent Thicket: Cross-licenses, Patent Pools and Standard Setting.” In A. Jaffe, J. Lerner and S. Stern, eds., *Innovation Policy and the Economy*, Vol 1. MIT Press: Cambridge MA, 2001.
- Veldhuijzen van Zanten, J.F.H. and Witts, W.W. “Patent Circumvention Strategy using TRIZ-based Design-Around Approaches.” *Procedia Engineering*, Vol. 131 (2015), pp. 798–806.
- Vives, X. *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, Mass.: MIT Press, 1999.
- Wang, S.-J. “Designing Around Patents: A Guideline.” *Nature Biotechnology*, Vol. 26 (2008), pp. 519–522.



**Figure 1:** The black line represents the upper limit of  $c$  (as given by  $1/(2N)$ ), whereas the gray line represents the threshold value of  $c$ , such that for values of  $c$  above this threshold there exists a sharing rule so that the pool forms. For all  $N > 2$ , the gray line is below the black line.