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Parental Time Investment and Intergenerational Mobility

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Abstract

This paper investigates differences in parental time investment as a determinant of intergenerational persistence of lifetime income. Using a quantitative model that replicates a series of important untargeted aspects of the data including the U.S. income quintile transition matrix, I find that the parental time investment channel accounts for nearly half of the observed intergenerational income persistence. Heterogeneity in parental time investment across households strengthens intergenerational association and hinders aggregate efficiency. Policy experiments suggest that an effective way of improving intergenerational mobility, aggregate output, and welfare is to narrow discrepancies in the quantity and quality of parental time investments.

Keywords: Intergenerational elasticity; quintile transition matrix; parental time; college education; misallocation

JEL codes: E24, I24, J22

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1 Introduction

A large body of literature has found that intergenerational income mobility is low in the United States (e.g., Solon, 1999; and Mazumder, 2005). A growing empirical literature examining sources of such low intergenerational mobility, as reviewed in Black and Devereux (2011), suggests that a key determinant of intergenerational mobility in the U.S. is family background. It still remains to be understood which specific family factors are quantitatively relevant for low mobility and through what mechanisms such factors shape intergenerational persistence of lifetime income. The answers to these questions are essential for designing policies to improve intergenerational mobility.

The goal of this paper is to explore parental time investment in young children as a determinant of intergenerational persistence of lifetime income. I construct a heterogeneous-agent overlapping-generations model in which lifetime income persists across generations through multiple channels. The model follows the tradition of Becker and Tomes (1986) in that altruistic parents care about their descendants' utility. Households are heterogeneous in human capital, assets and age. Young parents, who face an additional state variable of human capital endowment for their newborn, decide how to split their time across investment in their child's human capital, market work, and leisure. Children's human capital development depends not only on the *quantity* of the time investment, chosen by parents, but also on the *quality*, captured by the parent's human capital as well as the child's human capital endowment. Before children become young adults, they make their own college decision to accumulate human capital. Parents can affect this decision both through their parental time investment at early ages, as higher pre-college human capital raises the probability of college completion, and through financial assets they transfer to their children, as college is financially costly. Adult human capital is subject to idiosyncratic shocks, which cannot be fully insured since households have access to the non-state-contingent asset. Households face not only borrowing limits in each period but also across generations since parents are not allowed borrow against their descendants' income.

For the quantitative analysis, a subset of model parameters are calibrated by minimizing the distance between target statistics from simulated-data and their empirical counterparts while the other conventional parameter values are externally determined.¹ The calibrated model is then

¹Section 3 explains and illustrates how the target statistics are linked to the parameters of the model economy.

evaluated as a quantitative theory of intergenerational mobility through a series of important untargeted aspects of the data. Most importantly, I find that my model successfully replicates the most salient features of the U.S. data involving the *distribution* of intergenerational income persistence, although its calibration targets only a single intergenerational mobility statistic (i.e., the correlation between the percentile rank of parents' income and that of children's income). More precisely, in the U.S. data, the intergenerational persistence of income is considerably higher in the bottom and top income quintiles than in the second, third, and fourth quintiles: the probability of children remaining in the bottom quintile when their parents' income lies in the bottom quintile is 34 percent, and the probability of children staying in the top quintile when their parents' income belong to the top quintile is even higher around 37 percent (Chetty, Hendren, Kline and Saez, 2014a). In my model, these untargeted probabilities are 34 and 38 percent, respectively, both of which are strikingly close to the empirical counterparts.² To the best of my knowledge, my paper is the first to evaluate a candidate model as a quantitative theory of intergenerational mobility by confronting it with the empirical income quintile transition matrix, and to thereby establish its success in explaining those disaggregated moments.³ To further corroborate its use, I confirm that my model quantitatively accounts for other regularities relevant to its key mechanisms related to intergenerational elasticity and heterogeneity in college completion probabilities.

Using the model economy, I then conduct decomposition exercises to investigate the role of the parental time investment channel in shaping the intergenerational persistence of lifetime income. I find that heterogeneity in the quantity of early parental time investment alone can account for nearly 20 percent of the observed intergenerational persistence of income. When I consider both the quantity and quality of parental time investments, I find that the parental time investment channel can account for nearly half of the observed intergenerational persistence of lifetime income. The model provides two mechanisms for the above findings. First, despite their higher opportunity costs of time, it is the parents with higher human capital that choose to invest more time in their young children. This force reinforces the intergenerational correlation of human capital that would arise solely from the factors that determine human capital at birth due to both quantity and

²The probabilities of intergenerational income staying in the second, third, and fourth quintiles are 22-24 percent in U.S. data (Chetty et al., 2014a) and 21-23 percent in my model.

³Section 4 illustrates that the same correlation of income across generations can be supported by very different quintile transition matrices.

quality margins of parental time investment. Second, the decision rule for parental time investment features dynastic smoothing, implying that parents tend to invest more in a child with a relatively low human capital endowment. This dynastic smoothing motive moves children closer to their parents' status in terms of human capital ranking, thereby strengthening the intergenerational association. Note that the above two mechanisms imply that the children who receive particularly low time investment are children with *high* human capital endowment born into *low* human capital families. Thus, despite their potentials, the low parental time investment during the childhood hinders them from moving up the ladder later in life. In the same manner, children with low human capital endowment born into high human capital families receive particularly high time investment from their parents, allowing these children to stay relatively high in terms of lifetime income later in life. This nature of low mobility, due to misallocation in a sense, suggests that low mobility can be closely connected to aggregate inefficiency as well as potential welfare gains.⁴

In light of the above findings, I also use my model to characterize the desirable properties of policy interventions to improve intergenerational mobility. I consider each policy's implications not only for intergenerational persistence estimates, but also for aggregate output, productivity and welfare in order to evaluate whether those policies that do affect the intergenerational persistence of inequality are otherwise desirable for the overall economy. The first set of experiments I conduct aims to facilitate access to college by relaxing borrowing limits or lowering college costs. I find that these policies, which do raise college completion rates, do not guarantee greater intergenerational mobility due to positive selection into college completion. More precisely, those who decide to go to college even before the policy change tend to have higher pre-college human capital as well as higher returns to college than marginal students do. Hence, facilitating college access does not substantially alter the relative standing of the marginal students, thereby having little effects on intergenerational mobility of lifetime income.

My second set of policy experiments considers lowering the opportunity cost of parental time investment toward encouraging parents to increase time investments in their children. Given the

⁴Section 5 also examines the role of the other channels in shaping the intergenerational persistence of lifetime income. I find that either the college channel or primary and secondary schooling accounts for a relatively small fraction (roughly 10 percent) of the observed intergenerational income persistence. The pre-birth factors, which capture various factors including both pure nature (genetic transmission) and some nurtural factors (e.g., assortative marital sorting and prenatal investment), are found to be quantitatively important.

heterogeneity in parental time investment, however, it may not be sufficient to simply raise the *average* time invested by parents for greater mobility; rather, an effective policy should take into account heterogeneous responses of parents. These points are illustrated by contrasting the effects of a subsidy proportional to parental time investment and a higher labor earnings tax, both of which are designed to lead to the same increase in average parental time investment. I find that intergenerational mobility increases only in the case of the parental time investment subsidy due to its disproportionately larger impact on low human capital parents. Importantly, the greater mobility is accompanied by sizeable gains in output, productivity, and welfare. This finding confirms that low mobility can be associated with aggregate inefficiency, calling for policy interventions. Finally, I consider universal preschool programs, which could narrow discrepancies not just in the quantity of time investments in young children but also in their quality through public provision. I find that these policies deliver considerably large increases in intergenerational mobility, aggregate efficiency and welfare.

This paper is motivated by the two distinct literatures that are in fact closely related. The first is the empirical literature that studies the parental investment and children's human capital development. Recent studies have shown that more educated parents spend more time with their children (e.g., Guryan, Hurst and Kearney, 2008; and Ramey and Ramey, 2010) and that active parental time inputs are of first-order importance for human capital development of children at early ages (Del Boca, Flinn, and Wiswall, 2014).⁵ The finding that human capital gaps at the beginning of formal schooling tend to persist throughout the childhood (Heckman, 2008) suggest that it is the early childhood period in which heterogeneity in human capital is shaped.⁶ The other literature on lifetime inequality using structural models that abstracts from early childhood development has found that initial conditions of adult human capital around early 20's are crucial to account for lifetime income inequality (e.g., Keane and Wolpin, 1997; Huggett, Ventura, and Yaron, 2011). My paper endogenizes the acquisition of human capital before adulthood as well

⁵The effect of parental income on child outcomes is less clear. For instance, Blau (1999) finds weak effects whereas Dahl and Lochner (2012) show that income has positive effects on children's test scores. See Heckman and Mosso (2014) for more detailed discussions.

⁶See Knudsen, Heckman, Cameron, and Shonkoff (2006), Cunha and Heckman (2007) and references therein for dynamic complementarity in children's human capital development; and Blau and Currie (2006); Heckman, Pinto, and Savelyev (2013) for experimental evidence on how improving disadvantages in early periods could have persistent effects.

as standard life cycle decisions in order to examine how lifetime income persistence is shaped by different forces before adulthood.

This paper builds on a growing literature which investigates different sources of intergenerational economic persistence using quantitative dynamic equilibrium models with heterogeneous households (e.g., Restuccia and Urrutia, 2004; Erosa and Koreshkova, 2007 ; Holter, 2014; Herrington, 2015; and Lee and Seshadri, 2016). The first distinguishing feature of my paper is its explicit focus on the parental time investment channel, which has so far received almost no attention in this literature. Lee and Seshadri (2016) is the one exception that also models parental time investments. However, the main quantitative analysis using decomposition and policy experiments, both of which focus on the parental time investment channel, differs from Lee and Seshadri (2016). In addition, my paper is distinguished in this literature by the fact that the calibrated model herein matches not only the targeted empirical correlation of income across generations, as is standard in the literature, but also the non-targeted U.S. income quintile transition matrix.

Finally, this paper is also related to the literature that uses equilibrium models of human capital investment across generations to study policies designed to raise human capital of children from disadvantaged families (e.g., Fernandez and Rogerson, 1998; and Caucutt and Lochner, 2012). So far, this literature has concentrated on parents' inadequate financial investments in children's human capital due to credit constraints. In contrast, my paper highlights the role of the quantity and quality of parental time investments in improving human capital of children from disadvantaged families.

The paper is organized as follows. Section 2 describes the model environment. Section 3 explains how the parameters of the baseline model economy are calibrated, and presents the relationship between parameters and target statistics in the model. Section 4 evaluates the baseline model economy as a quantitative theory of intergenerational mobility through non-targeted statistics implied by the model. Section 5 presents the main quantitative analysis of intergenerational mobility, and Section 6 explores a series of policy experiments that illustrates the desirable properties of policies to improve intergenerational mobility. Section 7 concludes.

Table 1: Timeline of the life-cycle events

		Parent's age								
		20-24	25-29	...	45-49	50-54	...	65-69	70-74	
Parent	Model age	1	2	...	6	7	...	10	11	
	Market work	← - - - - Labor-leisure - - - - →					Retired - - - - →			
	Human capital investment	College	Parental Time							
Child	Model age	Birth			1	2	...	5	6	
	Market work	← - - - - Labor-leisure - - - - →								
	Human capital investment				College	Parental Time				

2 Model

The economy is populated by overlapping generations of a continuum of households. A household is composed of an adult who lives with a child until the child grows up. An adult lives for eleven model periods (age 20-74) as an economic decision maker. One model period corresponds to five years. In Table 1, I summarize the timeline of lifecycle events for a pair of overlapping generations for illustration. An adult supplies labor beginning at period $j = 1$ (age 20) until retirement at $j = 10$ (age 65). An adult lives for two periods after retirement and dies at the end of period $j = 11$. The next generation is born when parents reach period $j = 2$. After 20 years, a child becomes an adult head of a new household facing the same lifetime structure as described above. The household's recursive problems over the life cycle are described below.

A representative firm produces output with constant returns to scale technology. Its production function is assumed to be Cobb-Douglas $Y = F(K, L) = AK^{\alpha_K} L^{1-\alpha_K}$ where K denotes aggregate capital stock, α_K is the output elasticity of capital and L denotes aggregate efficiency units of labor in the economy. Capital depreciates at the rate of δ .

A government taxes labor earnings at the rate of τ . The revenue is used to provide social security payments to retirees and lump-sum transfer T to all households. The proportional tax rate τ along with the lump-sum transfer T effectively provides an environment with progressive taxation. Government balances its budget each period.

2.1 Household's Decision Problems

This paper considers stationary environments in which market-clearing prices and aggregate quantities are constant over time. Therefore, the time index for the variables is omitted and I present the household's decision problems recursively.

College education stage:

A child becomes an independent economic decision maker in the model period $j = 1$ (20 years old) with two state variables: a human capital stock of θ and a level of asset holdings a .⁷ As discussed below, these are endogenously determined by their parents. An important decision to be made in period $j = 1$ is whether to attain college education or not. College completion requires both the stochastic financial cost ξ , following the cumulative distribution $G_\xi(\xi)$, and the fixed time input ψ . After observing the financial cost draw ξ , households make a discrete choice regarding college education. Given the discrete nature of this choice, it is convenient to define the value of not completing college and the value of completing college separately.

First, the household's value of not going to college is given by

$$\Gamma_0(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG_z(z') \right\}$$

$$\text{subject to } c + a' \leq (1 - \tau)w\theta n + (1 + r)a + T$$

$$n + l \leq 1$$

$$\theta' = \exp(z')\gamma\theta$$

where c is consumption, l is leisure, w is the rental price of human capital per unit hours of work, r is the interest rate and a is the level of assets determined by the parent's financial transfer decision.

A variable with a prime denotes its value in the next period. The household's after-tax earnings

⁷In my model, human capital is composed of not only factors affected by families and education but also ability. This is in line with Cunha and Heckman (2007) who note, "Measured abilities are susceptible to environmental influences, including in utero experiences, and also have genetic components. These factors interact to produce behaviors and abilities that have both a genetic and an acquired character." Therefore, I use the term, human capital (or skills), interchangeably with ability.

depend on the individual-specific market wage $w\theta$, labor supply decision $n \in [0, 1]$, and the labor tax rate τ . In every period, households receive a lump-sum transfer T from the government.

The level of human capital increases at the gross growth rate of $\gamma > 1$ and is subject to the idiosyncratic shock (or market luck) z . As in Huggett et al. (2011), I assume that z follows an i.i.d. normal distribution $G_z(z)$. Note that although z is drawn from an i.i.d. distribution, its effect persists over the rest of the life. This is because z is not a shock to earnings but rather a shock to human capital, which essentially follows a random walk with an age-dependent deterministic drift in logs.⁸ In addition, note that the idiosyncratic shocks z cannot be fully insured since a is not a state-contingent asset.

Next, the value of completing college education is given by

$$\Gamma_1(\theta, a, \xi) = \max_{\substack{c \geq 0; a' \geq a_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG_z(z') \right\}$$

$$\text{subject to } c + a' + \xi \leq (1 - \tau)w\theta n + (1 + r)a + T$$

$$n + l + \psi \leq 1$$

$$\theta' = \exp(z')(\gamma + \Delta)\theta.$$

The benefit of college education is represented by $\Delta > 0$, an increment in the growth rate of human capital. As described above, college completion requires both the financial cost ξ and the time cost ψ . The presence of time costs for college implies that the opportunity cost of college education includes foregone earnings. The borrowing limit, faced by households in period j , is denoted by \underline{a}_j .

The household's value at $j = 1$ before the realization of ξ is then defined as

$$V_{j=1}(\theta, a) = \int \max[\Gamma_0(\theta, a), \Gamma_1(\theta, a, \xi)] dG_\xi(\xi),$$

where $\Gamma_0(\theta, a)$ is the value without a college degree, and $\Gamma_1(\theta, a, \xi)$ is the value of completing college

⁸For example, taking the log of the law of motion for the human capital in (2), we get

$$\log \theta' = \log \gamma + \log \theta + z'.$$

where a drift term, $\log \gamma$, acts as a deterministic trend.

education, as defined above.⁹

Parental investment stage:

At the beginning of period $j = 2$, each household is endowed with a child. The child's human capital endowment at birth $\theta_0 = \phi(\theta, \zeta)$ depends on both parent's human capital θ and the stochastic component ζ , the latter of which follows the cumulative distribution $G_\zeta(\zeta)$. This function ϕ allows us to generate a positive correlation of the human capital endowment θ_0 with the parent's human capital θ . The degree of altruism is denoted by $\eta \in [0, 1]$, and is assumed to be heterogeneous: $\eta \in \{\eta_l, \eta_h\}$ where $0 \leq \eta_l < \eta_h \leq 1$. Their corresponding probabilities of π_l^η and π_h^η satisfy $\pi_l^\eta, \pi_h^\eta \in [0, 1]$ and $\pi_l^\eta + \pi_h^\eta = 1$. The heterogeneity in the degree of altruism provides an additional margin to generate heterogeneity in parental time investment, as explained below. I assume that the child shares the household consumption c , according to the household equivalence scale q , and does not make time allocation decisions relevant to the household's economic status during childhood. I first define the decision problem given ζ and η , and then define the value function at the beginning of period $j = 2$, which is the expected value function. The functional equation summarizing a parent's decision problem given ζ and η is given by

$$W(\theta, a, \zeta, \eta) = \max_{\substack{c \geq 0; a' \geq a_2 \\ n, l, h \in [0, 1] \\ s \in \{0, s_l, s_h\}}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j=3}(\theta', a', s) dG_z(z') + \eta \beta^A V_{j=1}(\theta_c, a_c) \right\} \quad (1)$$

$$\text{subject to } c + a' \leq (1 - \tau)w\theta n + (1 + r)a + T$$

$$n + l + h \leq 1$$

$$\theta' = \exp(z')\gamma\theta$$

$$\theta'_0 = f(\theta, h, \theta_0) = \theta_0 + (\theta h)^{\alpha_1} \theta_0^{\alpha_2} \quad (2)$$

$$\theta_c = (\theta'_0)^{\alpha_c} \quad (3)$$

$$a_c = \sum_{t=0}^2 (1 + r)^t s. \quad (4)$$

⁹Note that this value function of entering adulthood, $V_{j=1}(\theta, a)$, appears in the parent's decision problem as the value of descendants in (1).

The intergenerational link is modeled following the dynastic utility approach in the sense that parents care about their child’s utility, which in turn depends on the next generation’s utility, and so on. This recursive structure linked by altruism combines successive generations as a single dynasty as in Becker and Tomes (1986). Unlike the standard Becker-Tomes type altruism (e.g., Lee and Seshadri, 2016), parents in my model care about the child’s utility derived not only from consumption but also from leisure. The degree of altruism η governs the parent’s incentive to invest their time h in their child and to make financial transfers s . This is because (i) it measures the degree to which the current value of their child’s life when she becomes an adult in 20 years, $\beta^4 V_{j=1}(\theta_c, a_c)$, is accessed by the parent; and (ii) the initial endowments of human capital θ_c and assets a_c for the child are influenced by these choices by the parent. I now explain how each of θ_c and a_c is determined.

The intergenerational human capital production function $f(\theta, h, \theta_0)$ in (2) determines the child’s developed human capital θ'_0 at the end of period $j = 2$ as a function of parental human capital θ , parental time investment h , and the child’s human capital endowment at birth θ_0 .¹⁰ In the spirit of Cunha and Heckman (2007), the function $f(\theta, h, \theta_0) = \theta_0 + (\theta h)^{\alpha_1} \theta_0^{\alpha_2}$ where $\alpha_1, \alpha_2 \in (0, 1)$ has the following properties: (i) $f_1, f_2, f_3 > 0$; (ii) $f_{22} < 0$; and (iii) $f_{21}, f_{23} > 0$. Note that the last properties imply that the marginal return on parental time investment increases with parent’s ability ($f_{21} > 0$) and child’s ability ($f_{23} > 0$). The level of human capital after the primary and secondary school periods, θ_c , is assumed to be determined by a simple mapping $\theta_c = (\theta'_0)^{\alpha_c}$ in (3). This mapping allows the model to generate an increasing degree of inequality over the schooling period in a parsimonious way.¹¹

The parental financial transfer decision is assumed to be discrete and irreversible for tractability. Specifically, households are assumed to save an amount $s \in \{0, s_l, s_h\}$ starting the next period $j = 3$ until $j = 5$. And the total amount of the savings a_c including interests, as expressed in (4), is transferred at the beginning of $j = 6$ to the next generation that forms a new household entering $j = 1$. It is worth noting that this financial transfer effectively serves as a form of intergenerational

¹⁰I assume that the time inputs are only relevant for human capital development in this period. The monetary inputs are not only hard to measure in the data but also are much less important than time inputs in early ages. Del Boca et al. (2014) find that, for the children under 5, active parental time inputs are three to four times as important as parental expenditures as investment inputs.

¹¹In Appendix, I provide an illustrative example of micro-foundation for this mapping using parental monetary investment.

monetary investment since it can help their child's college decision financially.

Given the above value function $W(\theta, a, \zeta, \eta_i)$, the expected value of the household at period $j = 2$ before the realization of ζ and η_i is given by

$$V_{j=2}(\theta, a) = \sum_{i=l,h} \pi_i^\eta \int W(\theta, a, \zeta, \eta_i) dG_\zeta(\zeta).$$

Remaining working stages:

Households make work-leisure and consumption-saving decisions for periods $j = 3, \dots, 9$ (age 30-64). A household is composed of a parent and a child until the end of $j = 5$ when the child forms a new household. Recall that a parent who committed to leave the financial transfer to the child keeps saving s while living with the child. Therefore, the state variables in periods $j = 3, 4, 5$ include s as well. Once the child becomes an adult, the parent's decision to leave the transfer does not affect their choice. Hence, from period $j = 6$, the state variables do not include s . Human capital evolves exogenously at the gross growth rate of $\gamma > 1$ until $j = 5$. Thereafter, the growth rate stays constant at one. This structure parsimoniously generates the shape of the empirical age-profile of wage that rises initially and becomes nearly flat near the retirement (see e.g., Rupert and Zanella, 2015).¹² I assume that the borrowing limit right before the retirement ($j = 9$) is zero (i.e., $\underline{a}_9 = 0$). This guarantees that households cannot borrow against their future social security accounts. The household's problem in these periods are summarized recursively as

$$V_j(\theta, a, s) = \max_{\substack{c \geq 0; a' \geq \underline{a}_j \\ n, l \in [0,1]}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j+1}(\theta', a', s) dG_z(z') \right\} \quad \text{if } j = 3, 4, 5$$

¹²The hump-shaped earnings profile observed in the data does not need to rely on the hump-shaped wage profile since hours of work, endogenously determined by households, fall near the retirement, as observed in U.S. data.

$$\text{subject to } c + a' + s \leq (1 - \tau)w\theta n + (1 + r)a + T$$

$$n + l \leq 1$$

$$\theta' = \exp(z')\gamma\theta$$

$$V_{j=6}(\theta, a, s) = V_{j=6}(\theta, a) \text{ for all } s$$

and

$$V_j(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_j \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j+1}(\theta', a') dG_z(z') \right\} \quad \text{if } j = 6, 7, 8, 9$$

$$\text{subject to } c + a' \leq (1 - \tau)w\theta n + (1 + r)a + T$$

$$n + l \leq 1$$

$$\theta' = \exp(z')\theta.$$

Retirement stage:

When households retire ($j = 10, 11$), they receive social security payments $g(\theta)$. This function is increasing and concave in human capital (or wage) just before retirement in order to approximate progressive U.S. social security. I assume that households are not allowed to be in debt at the retirement stage. The bequest decision is not modeled and all financial transfers from parents to children occur earlier, as described above.¹³ The value at the retirement stage is given by

$$V_j(\theta, a) = \max_{c \geq 0; a' \geq 0} \{U(c, 1) + \beta V_{j+1}(\theta, a')\}$$

$$\text{subject to } c + a' \leq g(\theta) + (1 + r)a + T$$

and $V_{j=12}(\theta, a) = 0$.

¹³ Abstracting from bequests is relatively innocuous for this paper since the empirical benchmark (Chetty et al., 2014a) use matched samples in which the children are relatively young (around 30 years old), and their income (i.e., capital income) at this range of ages is unlikely to be affected by bequests. However, it should be more important to model bequest motives if one is interested in intergenerational mobility of wealth since voluntary bequests play an important role in explaining the distribution of wealth (De Nardi, 2004).

2.2 Equilibrium

Let $x_j \in X_j$ denote the age-specific state space defined according to the household's recursive problems in the previous subsection. A stationary recursive competitive equilibrium is a collection of factor prices w, r , the household's decision rules $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), s(x_2)$, value functions $V_j(x_j)$, government policies $\tau, g(\cdot), T, G$ and age-specific measures π_j over x_j such that

1. given the government policies and factor prices, $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), i(x_2), s(x_2)$ solve the household's lifecycle optimization problems defined in the previous subsection, and $V_j(x_j)$ are the associated value functions,
2. factor prices are competitively determined:

$$\begin{aligned} w &= F_2(K, L) \\ r &= F_1(K, L) - \delta, \end{aligned}$$

3. markets clear:

$$\begin{aligned} \sum_{j=1}^{11} \mu_j \int_{X_j} a_j d\pi_j + \sum_{j=3}^4 \mu_j \int_{X_j} \left[\sum_{t=0}^{j-3} (1+r)^t s_j \right] d\pi_j &= K \\ \sum_{j=1}^9 \mu_j \int_{X_j} \theta_j n_j(x_j) d\pi_j &= L \end{aligned}$$

where μ_j is the fraction of households living in period j ,

4. government budget balances:

$$G + T + \sum_{j=10}^{11} \mu_j \int_{X_j} g(\theta_j) d\pi_j = \sum_{j=1}^9 \mu_j \int_{X_j} \tau w \theta_j n_j(x_j) d\pi_j$$

where G is non-negative,

5. the vector of age-specific measures of households $\pi = (\pi_1, \pi_2, \dots, \pi_{11})$ is the fixed point of $\pi(X) = P(X, \pi)$ where $P(X, \cdot)$ is a transition function determined by the household decision

rules and the exogenous probability distributions of z, ζ, η and ξ ; and X is the generic subset of the Borel σ -algebra B , defined over the state space $\mathbf{X} = \prod_{j=1}^{11} X_j$.

3 Setting Model Parameters

I calibrate parameter values of the baseline model economy to match relevant U.S. statistics. There are two sets of parameters. The first set of parameters is chosen externally without using model-generated data while the second set of parameters is determined jointly by minimizing the distance between the statistics from the simulated-model and their counterparts from U.S. data.

3.1 Parameters Chosen Externally

I assume that all households have identical preferences over consumption c and leisure l , represented by a standard separable utility function

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + B \frac{l^{1-\varepsilon}}{1-\varepsilon}.$$

The two curvature parameters, σ and ε , govern the household's willingness to substitute intertemporally. I set the value of σ equal to 1.5 so that the intertemporal elasticity of substitution for consumption is 0.67 and the value of ε equal to 2.5, which implies an intertemporal elasticity of substitution for leisure of 0.4. These parameter values lie within a broad range of their empirical estimates. As discussed in the previous section, when a parent lives with a child, consumption in the utility function is replaced by c/q . I set q to 1.3 according to the OECD-modified equivalence scale which assigns 1 to the first adult and 0.3 to a child.

The gross growth rate of human capital in young periods ($j = 1, \dots, 5$), γ , is set to 1.06, which implies a slope of life cycle wage profiles in line with life cycle wage profiles in the PSID data set (Rupert and Zanella, 2015).¹⁴ The slope parameter of the mapping that governs changes in human capital during the primary and secondary education, α_c , is set to 1.06. This implies that dispersion of human capital, measured by the standard deviation of log human capital, increases by 6 percent

¹⁴In the calibrated benchmark economy, $\gamma = 1.06$ implies that the average wage at $j = 6$ (age 45-49) is 63 percent higher than the average wage at $j = 1$ (age 20-24). This is consistent with Rupert and Zanella (2015).

during this period (Restuccia and Urrutia, 2004).¹⁵

In the model, college education requires both time and resources. I set the time cost consistent with the empirical finding that average academic time invested by full-time college students is about 27 hours per week in 2003 (Babcock and Marks, 2011). Therefore, the time cost of the four-year full-time college ψ is set to $\frac{27}{105} \times \frac{4}{5} = 0.21$, provided that the weekly feasible time endowment is 105(= 15 × 7) hours, excluding time for sleeping and basic personal care. The financial costs are stochastic and are discussed in the next subsection.

I set the tax rates on labor earnings (τ) equal to 0.27. According to the National Income and Product Account by the Bureau of Economic Analysis, the personal current transfer receipts excluding social security is roughly 10 percent of the GDP in 2014. Thus, I assume $T = \omega_p Y$ and set $\omega_p = 0.1$. The capital share in the aggregate U.S. data leads to the choice of $\alpha_K = 0.36$. The five-year capital depreciation rate is set to $\delta = 0.3$. These parameter values are within the range commonly used in the quantitative macroeconomics literature. Finally, households are not allowed to be in the negative net worth position in the benchmark specification.

3.2 Parameters Chosen Jointly Using Simulation

Table 2 summarizes the remaining 13 parameters that are jointly determined by simulating the model economy. These parameter values are determined as minimizers of the distance between the relevant statistics from the data and those from the model-generated data (see Appendix for details).¹⁶ In this subsection, I explain the role of these parameters in the model, and illustrate how each parameter is related to its target statistic summarized in Table 2. At the end of this subsection, I also present how each parameter is linked to its most relevant statistic in Figure 1. All statistics regarding time-use are obtained from the 2003-2012 waves of the American Time Use Survey (ATUS) (see Appendix for details).

Preference:

First, β is the household's discount factor. The relevant target for this parameter is set to the annual capital-output ratio, 3, or the capital to five-year output ratio, 0.6. The next parameter B

¹⁵Taking the log of both sides in (3), we have $\log \theta_c = \alpha_c \log \theta'_0$. This implies $\alpha_c = \frac{sd(\log \theta_c)}{sd(\log \theta'_0)}$.

¹⁶For the model statistics that require simulation, I generate 1,000,000 parent-child pairs by simulating the baseline model economy.

Table 2: Internally calibrated parameters and target statistics

Parameter	Target statistics	Data	Model	
β	.858	Capital-output ratio (annual)	3.0	3.02
B	2.13	Average hours of work (age 30-64)	.290	.293
η_0	.516	Average parental time investment	.060	.060
δ_η	.490	Interquartile range of parental time investment	.093	.093
α_1	.712	Educational gap in parental time investment ($\bar{h}_{col}/\bar{h}_{less}$)	1.35	1.35
α_0	.187	Intergenerational correlation of percentile-rank income	.341	.342
σ_ζ	.411	Cross-sectional variance of log wage	.40	.400
σ_z	.173	Slope of variance of log wage from age 25-29 to age 55-59	.18	.180
ω_ξ	.277	Average college expenses/GDP per-capita	.14	.139
δ_ξ	.739	Observed college wage gap	1.8	1.81
Δ	.382	Fraction with a college degree (%)	30.6	30.3
ω_a	.093	Average inter-vivos transfers/GDP per-capita	.056	.056
λ	.204	Average social security replacement rate	.40	.400

determines the relative weight of leisure compared to consumption in the utility function. In the model, the time allocation problem of households in periods $j = 1, 2$ involve more than work-leisure trade-off due to the time cost of college and the parental time investment. Therefore, the relevant target for B is set to be the average weekly hours of work for those aged 30 or greater but before retirement (age 65). This leads to $30.5/105 = 0.290$. The degree of altruism can take two values. I assume that $\eta_l = \eta_0(1 - \delta_\eta)$ and $\eta_h = \eta_0(1 + \delta_\eta)$ and that each value is equally likely (i.e., $\pi_l^\eta = \pi_h^\eta = 0.5$). This leaves two parameters to be calibrated: η_0 and δ_η . Since η_0 increases an incentive to invest time in the next generation, its relevant statistic is set to be the unconditional mean of the parental time investment. To compute statistics regarding parental time investment, I focus on parental time spent directly with children such as reading to a child, playing with a child, talking with a child, all of which can promote development of children's human capital through interactions in early years (see Appendix for details). This gives the average time investment of 6.3 hours per week or 0.060 ($= 6.3/105$) in the model for parents with children under 5. Note that this average is slightly larger than the average time spent in the educational and recreational childcare with children under 5 in Guryan et al. (2008). The other parameter δ_η governs the variability of parental time investment in equilibrium. Therefore, its target statistic is set to be the interquartile range of parental time investment of 0.093 in the time unit of the model.

Human capital dynamics:

I now move on to the parameters that affect individuals' human capital dynamics in the model. Both of the slope parameters α_1 and α_2 in the human capital production function (2) increase the marginal productivity of parental time investment (h). I assume that these two parameter values are the same and set its relevant target statistic to be the gap between the parental time spent by college-educated parents and that spent by less-educated parents. According to the 2003-2012 ATUS data set, parents with a college degree spend 35 percent more time (7.6 hours per week) than parents without a college degree (5.7 hours per week). This educational gradient of 1.35 serves as a target statistic.¹⁷

The initial endowment of human capital is assumed to be determined by

$$\theta_0 = \phi(\theta, \zeta) = \exp(\zeta)\theta^{\alpha_0}.$$

Given the parameter $\alpha_0 \in [0, 1]$ which governs the relative importance of the parental human capital in determining the newborn's human capital endowment θ_0 , there is an idiosyncratic component following a normal distribution: $\zeta \sim N(0, \sigma_\zeta^2)$. Hence, children from families with the same human capital may be born with different human capital endowments θ_0 . A higher α_0 implies that a higher degree of association across generations. I set the relevant target for α_0 as the rank correlation of family income of 0.341 (Chetty et al., 2014a), which has been relatively stable in the U.S. (Chetty, Hendren, Kline, Saez, and Turner, 2014b). Due to the data limitation, Chetty et al. (2014a) estimate intergenerational persistence using the proxy income variable instead of lifetime income. The rank correlation from the model, which is used as a target statistic, is also obtained based on the proxy incomes equivalently defined as in Chetty et al. (2014a) (see Section 5 for the precise definition of the proxy income).

Before moving on to the next parameter σ_ζ , note that the idiosyncratic shocks to adult human capital z , following a normal distribution, have mean zero with the standard deviation of σ_z : $z \sim N(0, \sigma_z^2)$.¹⁸ Since the two standard deviations, σ_ζ and σ_z , are the exogenous sources of the

¹⁷Guryan et al. (2008) provide detailed evidence showing that this positive gradient is a very robust feature even after controlling for various observables.

¹⁸The integral to calculating the expectations over the normal distributions is approximated using the Gauss-Hermite quadrature with five nodes.

cross-sectional dispersion of wages in the model, I choose the cross-sectional variance of log wage as a target statistic. Note that, although the degree of wage inequality monotonically increases with either σ_ζ or σ_z , their economic mechanism is very different. This is because σ_ζ affects the variability of the initial condition in human capital while σ_z affects households over the working life. Specifically, holding the overall dispersion of wage constant, in the case when σ_z is relatively larger, households would experience more volatile idiosyncratic shocks to human capital, the effect of which accumulates over the life cycle. As a result, the lifecycle profile of wage inequality would become steeper. Therefore, I choose the difference between the variance of log wage at age 55-59 and that of log wage at age 25-29 as an additional target to pin down the relative contribution of each shock process to the overall wage inequality. These statistics on wage inequality in U.S. data for recent periods, obtained from Heathcote, Perri and Violante (2010), are reported in Table 2.

College education:

I assume that stochastic financial costs ξ follow a uniform distribution over the interval $[\omega_\xi Y(1 - \delta_\xi), \omega_\xi Y(1 + \delta_\xi)]$ with 15 equally spaced nodes. The target statistic for ω_ξ in the model is set to be the equilibrium ratio of average (tuition and non-tuition) expenses after financial aid to per capita GDP. Specifically, I first compute the average ratio of annual college tuition and required fees (excluding room and board) for four-year institutions to the per capita real GDP for the recent periods 1990-2011, which is 0.22 according to the Digest of Education Statistics (2011, Table 349) and the Bureau of Economic Analysis. In order to approximate actual costs faced by students, I also include the non-tuition expenses such as books, other supplies, commuting costs, and room and board expenses that would not have to be paid by a person who chooses not to go to college, as in Abbott, Gallipoli, Meghir and Violante (2013). These non-tuition expenses amount to approximately 30 percent of the average tuition and fees. In 2000-2001, the average grants (federal, state/local, and institutional) received by full-time students in four-year colleges weighted by numbers enrolled are approximately 50 percent of the average tuition and fees. Based on the above information and assuming that college completion takes four years, the equilibrium ratio of average financial college costs to the five-year GDP is 0.14.

The variability of the financial cost distribution, δ_ξ , is chosen in connection with the parameter Δ , which captures the enhancement of human capital growth for those who complete college. A

greater benefit of college (i.e., higher Δ) would obviously lead to more people who invest in college education. Hence, a natural target is the four-year college completion rate of 30.6 percent obtained from the ATUS samples. Next, note that the observed college wage premium is not only because of the direct human capital development (Δ) but also because of compositional effects due to positive ability selection. In other words, an individual who is more able, measured by pre-college human capital, tends to have greater returns to college, and is thus more likely to complete college. The observed college premium, or the ratio between the average wage of those with a college degree and the average wage of those without a college degree (1.8), in recent U.S. data (Heathcote et al., 2010) is therefore set as a target statistic to pin down δ_ξ .

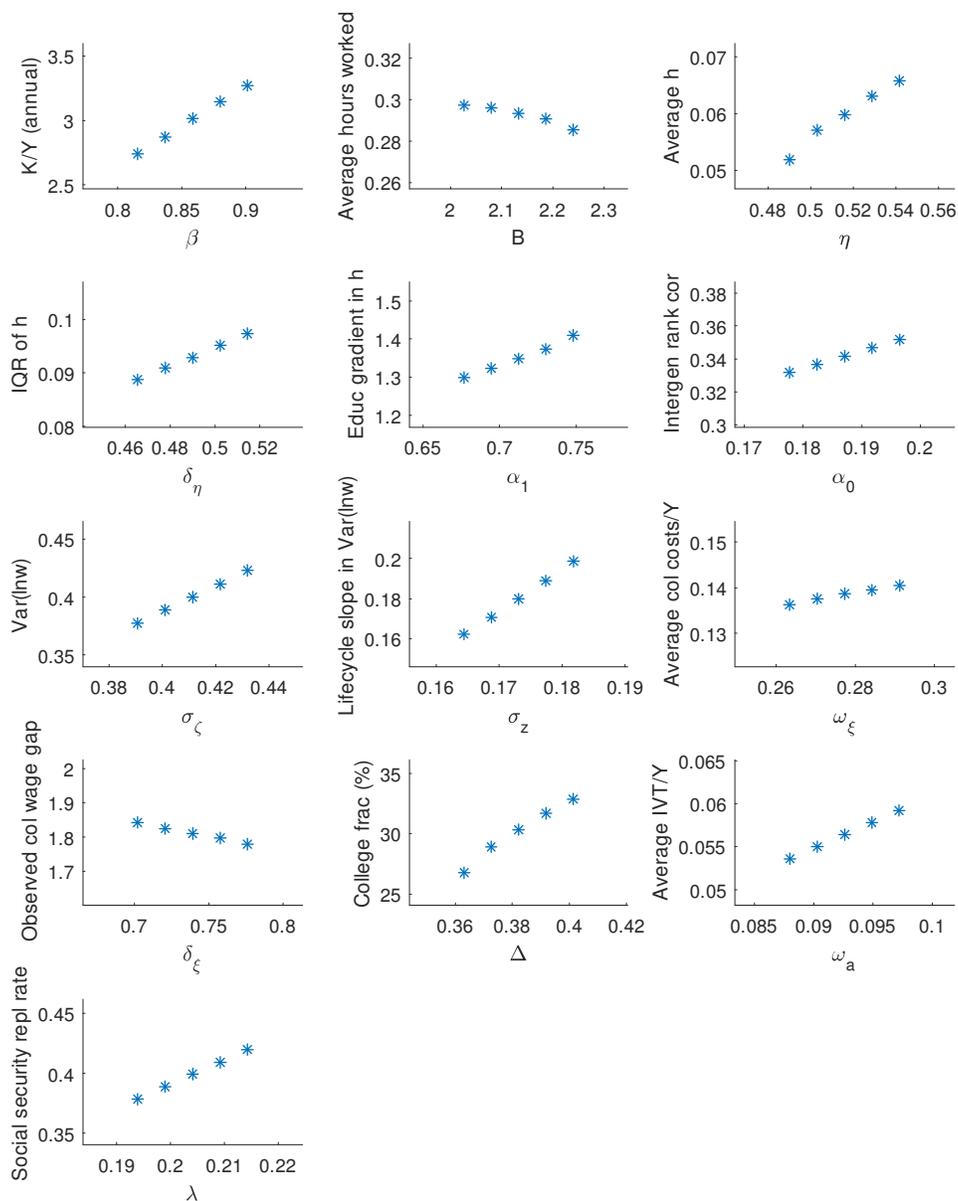
Remaining parameters:

In the model, the parental transfers can take one of the three values $\{0, a_l, a_h\}$. I set $a_h = \omega_a Y$ and $a_l = 0.5a_h$, and calibrate ω_a to match the average parental transfer in equilibrium. Recall that the role of the inter-vivos transfers in the model is to provide young households with financial resources that help complete college education. The relevant target is thus the total parental transfers made for children during the college years. More precisely, I sum up the money from parents and college transfers from age 18 to age 26, reported in Table 4 of Johnson (2013), while accounting for the fraction of recipients. This leads to the ratio of average parental financial transfers to the five-year GDP per-capita, which is 0.047. Finally, social security payments are assumed to be captured by an increasing and concave mapping: $g(\theta) = \lambda \log(1 + \theta)$. A target statistics for λ is set as the average replacement rate of 40 percent in the U.S.

Equilibrium relationship between parameters and target statistics:

As summarized above, there are a number of parameters jointly calibrated. Although each parameter influences other statistics in addition to the main target statistic, it is informative to explore in more details how each parameter influences its main target statistic. This is useful not only for establishing the clear identification of parameters, but also for understanding how the model works. Figure 1 displays the equilibrium relationship between each parameter and its corresponding target statistic. More precisely, in each panel, I change each internally calibrated parameter by ± 5 and 10 percent around the calibrated value. Then, the y-axis reports its target

Figure 1: Effects of each parameter on its main target statistic



Notes: Each panel shows how each parameter affects its relevant statistic in equilibrium. The pairs of parameters and target statistics are summarized in Table 2. Each parameter is varied by 5 percent around the calibrated value. The bounds of the y-axis are set to be 10 percent smaller (or greater) than the minimum (or the maximum) of the target statistic estimates in each panel.

statistic in equilibrium. Although there are quantitative differences in the effects of each parameter on its main target statistic, one can confirm that their relationships are all qualitatively in line with the calibration strategy described so far in this section.

4 Intergenerational Mobility in the Benchmark Model

Prior to the quantitative exercises in the next sections such as counterfactual and policy experiments, this section evaluates the baseline model economy as a quantitative theory of intergenerational mobility. I consider three measures of intergenerational mobility: (i) the IGE; (ii) the rank correlation; and (iii) the quintile transition matrix. The intergenerational mobility estimates reported below are based on family income in order to be consistent with the U.S. data counterparts from Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), family income is the five-year per parent average of the pre-tax income defined as either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits (otherwise). In the model, family income is the five-year per parent sum of labor earnings, interest income, and social security benefits. It is worth noting that family income is more preferred to measure intergenerational mobility of the economic status when samples include not only males but also females (Chadwick and Solon, 2002), which is the case in Chetty et al. (2014a) as well as in my gender-neutral model.

IGE and the rank correlation:

The first measurement is the IGE, a conventional way to measure the degree of intergenerational persistence in the literature. The IGE is the slope coefficient obtained by running the following log-log regression equation:

$$\log y_{child} = \rho_0 + \rho_1 \log y_{parent} + \varepsilon \tag{5}$$

where y is supposed to be permanent income. The IGE provides a straightforward interpretation: a one percent increase in parental permanent income is associated with a ρ_1 percent increase in their children's permanent income. Thus, a high ρ_1 implies low intergenerational mobility. The second way to measure intergenerational mobility is to use a rank-rank specification instead of a

Table 3: Intergenerational persistence estimates

	U.S. DATA	MODEL	
	Chetty et al. (2014a)	Proxy income	Lifetime income (discounted)
IGE: log-log slope	0.344	0.316	0.393
Rank corr: rank-rank slope	0.341	0.342	0.382

Notes: The log-log slope estimate is obtained from a univariate regression equation where the dependent variable is the child’s log income and the independent variable is the parent’s log income. The rank-rank slope estimate is obtained from an equivalent regression equation replacing log transformation with the percentile rank.

log-log specification, as proposed by Chetty et al. (2014a, 2014b). In other words, I estimate the slope parameter after replacing log income with the percentile rank of income within one’s own generation in (5). The slope coefficient in a rank-rank specification (or the rank correlation) has a similar interpretation: a one percentage point increase in parent’s percentile rank is associated with a ρ_1 percentage point increase in their children’s percentile rank.¹⁹ Unlike the IGE, the rank correlation is less sensitive to the treatment of zero income observations and is relatively robust to the point of measurement in the income distribution (Chetty et al. 2014a, 2014b).

In the literature estimating intergenerational mobility, the biggest challenge is the data requirement: we need a data set that contains career-long earnings histories (or permanent income) for at least two successive generations. Due to the data limitation, in practice, permanent income is replaced with proxy income measured at a point in the life cycle. For purposes of comparison, I present model statistics based on proxy income defined similarly to Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), the child’s income is measured by income when children are around 30 years old, averaged over two years. The parent’s income is averaged over five years when parents are roughly around 45 years old. Accordingly, in the model, the age at which the parent’s income is measured is 45-49 ($j = 6$), and the age at which the child’s income is measured is 30-34 ($j = 3$). In addition, I also compute the intergenerational persistence measures using present-value lifetime income discounted according to the equilibrium real interest rate.

Table 3 reports these first two measures (i.e., slope estimates) from the model and the data.

¹⁹Note that the rank-rank slope estimate is simply equal to the correlation coefficient in percentile rank (or Spearman correlation) since the independent and dependent variables, both of which are normalized by transforming the income level to the percentile ranks, have the same variance.

Table 4: Income transition matrices using proxy income

Parent quintile	U.S. DATA					MODEL				
	Child quintile					Child quintile				
	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	33.5	25.2	18.2	14.7	8.5
2nd	24.2	24.2	21.7	17.6	12.3	24.9	22.8	21.2	17.9	13.2
3rd	17.8	19.8	22.1	22.0	18.3	19.8	21.1	21.4	20.5	17.3
4th	13.4	16.0	20.9	24.4	25.4	14.2	18.4	21.3	22.8	23.3
5th	10.9	11.9	17.0	23.6	36.5	7.6	12.4	17.9	24.2	37.8

Notes: The unit is percentage. The source of U.S. data is Chetty et al. (2014a). The right panel shows the results obtained from the benchmark model economy when income is measured in line with Chetty et al. (2014a).

The first column shows estimates from U.S. data in Chetty et al. (2014a). Recall that the rank-rank slope using proxy income has been used as a calibration target. The estimate of the log-log slope (IGE) using lifetime income is around 0.4, which is close to the estimates in Solon (1999). Moreover, note that this estimate using lifetime income is considerably larger than the estimate of 0.32 using proxy income. This is in line with findings in the empirical studies noting that the short-term income (even multi-year averages) may not represent the permanent income, thereby leading to the attenuation bias in estimating the persistence of income across generations. The bias is smaller in the estimate of the rank-rank slope using proxy income instead of lifetime income (0.34 versus 0.38). Finally, It is important to note that the degree of approximation using the proxy income depends on the point at which income is measured. Appendix provides this so-called lifecycle bias in the intergenerational mobility estimates in more details.

Quintile transition matrix:

Next I consider the income quintile transition matrix in which the (a, b) element gives the conditional probability that a child’s lifetime income is in the b -th quintile of his generation’s distribution, provided that his parent’s income is in the a -th quintile of her own generation’s distribution. This matrix provides a richer description of how economic status is transmitted across generations than do the first two measures of correlations. It is important to emphasize that calibration targets do not include any elements in the income quintile transition matrix.

Table 4 compares the transition matrix obtained from U.S. data (Chetty et al. 2014a) to the transition matrices using the model-generated data. Three features are worth noting in the transi-

tion matrix from U.S. data. First, it shows that the observed positive correlations of income across generations are not simply due to the intergenerational poverty trap but are also due to the rich families that sustain their economic status intergenerationally. Specifically, both the probability of children staying in the bottom (34 percent) and that of top quintile (37 percent) are substantially higher than the other diagonal elements (22 – 24 percent). Second, there is quite a bit of mobility in the middle of the income distribution. For instance, children born into the third quintile parents are almost equally likely to be located in any income quintiles (18 – 22 percent). Third, both upward mobility, measured by the probability of moving up from the bottom quintile to the top quintile, and downward mobility, measured by the probability of moving down from the top quintile to the bottom quintile, are quite low (8 percent and 11 percent, respectively).

The right panel of Table 4 shows that the model is able to account for the above salient features in the U.S. income quintile transition matrix strikingly well despite the fact the calibration only targets the overall correlation of income across generations. In particular, the model generates the high probability of staying in the bottom quintile (34 percent) and the even higher probability of staying in the top quintile (38 percent). The model also predicts a substantial degree of mobility in the middle of the income distribution: children born into the third quintile parents are almost equally likely to end up with any quintiles (within 17 – 21 percent). Finally, the degree of upward mobility (9 percent) and that of downward mobility (8 percent) are also considerably low in the model.

The above exercise considers the quintile transition matrix as a way of evaluating how successful a candidate model is as a quantitative theory of intergenerational mobility. The underlying reason is that the same correlation of income across two generations can be justified by different shapes of the disaggregated moments in the quintile transition matrix.²⁰ To demonstrate this point, I compare the quintile transition matrix implied by the model using the lifetime income to the counterpart constructed in a reduced-form way. More precisely, I first obtain the estimates of ρ_1 and the standard deviation of ε in (5) using the simulated data from the benchmark model economy. Then, the reduced-form children’s lifetime income is constructed directly by (5) and the marginal

²⁰Note that this is in the same spirit as the model validation exercises in the quantitative macroeconomics literature on cross-sectional inequality. For instance, the same high Gini coefficient can be generated either by many poor households or by few super rich households.

Table 5: Income transition matrices using lifetime income

Parent quintile	MODEL					REDUCED-FORM									
	Child quintile					Normal dist of ε					Uniform dist of ε				
	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	35.6	26.2	16.8	14.0	7.4	38.6	24.6	17.8	12.4	6.7	37.4	20.3	20.0	18.5	3.8
2nd	24.9	24.3	21.1	17.4	12.3	24.0	23.5	21.1	18.4	13.1	25.8	20.3	19.9	20.4	13.6
3rd	19.1	21.4	22.6	20.2	16.7	17.8	21.0	21.7	21.3	18.2	19.6	20.3	20.0	20.3	19.9
4th	13.5	17.2	21.8	23.6	23.9	12.6	18.0	21.2	23.5	24.8	13.0	20.2	20.1	20.4	26.3
5th	6.9	10.9	17.7	24.8	39.7	7.1	13.0	18.3	24.5	37.2	4.2	19.0	20.0	20.4	36.4

Notes: The unit is percentage. All panels generate the same IGE estimate using the lifetime income. The first panel reports the result from the baseline model economy. The next two panels show the results from predicting children's lifetime income according to (5) with either normal distribution of ε or uniform distribution of ε while the marginal distribution of parents' lifetime income in the baseline model economy is used.

distribution of parents' lifetime income in the benchmark model under the different assumptions on the distribution of ε (either normal or uniform).

Table 5 shows that, although the slope (0.393) and the standard deviation of ε (0.446) estimated from (5) are identical by construction, the implied quintile transition matrices are quite different. Specifically, the first panel obtained from the lifetime income in the model shows that the probability of staying in the top quintile is the largest (40 percent) and that of staying in the bottom quintile is the second largest (36 percent). Although both numbers are greater than the counterparts in Table 4, we can see that the order is preserved when we use the lifetime income instead of the proxy income. By contrast, the next panel using the artificial data obtained in a reduced-form way under the normality assumption of ε shows that the bottom quintile is more persistent (39 percent) than the top quintile (37 percent). Moreover, when the normality assumption is replaced with the uniform distribution, it is interesting to note that the quintile transition matrix becomes quite symmetric. In particular, it features that the second to fourth quintiles are extremely mobile within this group. This exercise illustrates that the quintile transition matrix contains important information on top of the simple correlation coefficient (i.e., the IGE or rank correlation), implying that it is not a trivial task for a structural model to match the income transition matrix in the data.

5 Sources of Intergenerational Mobility

In this section, I assess the quantitative importance of various channels in explaining the intergenerational mobility of lifetime income, and inspect the mechanisms through which each channel affects mobility. I first focus on the role of the parental time investment, the key channel of interest in this paper. Then, I examine other channels including college education, parental financial transfers, primary and secondary education, and pre-birth factors.²¹

5.1 Parental Time Investment Channel

In the first counterfactual case, denoted by (2) in Table 6, I shut down heterogeneity in the quantity of parental time investment by imposing that all parents invest the same amount of time at its average from the baseline specification (i.e., 6.3 weekly hours). This is motivated by the substantial educational gradient in parental time investment; the ratio between the amount of time investment spent by college-educated parents and that spent by less-educated parents is 1.35 in the baseline specification. The first two columns in Table 6 show that the intergenerational income persistence estimates (IGE and the rank correlation using lifetime income) decrease by roughly 20 percent. This suggests that heterogeneity in the quantity of parental time investment per se is responsible for a sizeable fraction of the intergenerational persistence of lifetime income. However, despite the homogeneous amount of time investment, the quality of parental time investment still varies depending on the parent's human capital (capturing parenting skills, the amount of useful information, etc.) and the level of child's initial human capital endowment. In the second counterfactual case, denoted by (3) in Table 6, I impose that all parents spend zero hours of time investment. This shuts down the parental time investment channel, thereby prohibiting both the quantity and quality of parental time investment from transmitting human capital across generations. The last row of Table 6 shows that both measures of intergenerational association decrease nearly by 50 percent, suggesting that the parental time investment channel accounts for nearly half of the intergenerational persistence of lifetime income.

To better understand the mechanism through which the parental time investment channel affects

²¹It is worth noting that the decomposition exercises on the quantitative significance of various channels reported below reveal the total equilibrium effects of shutting down one channel in the presence of other channels which could either reinforce or dampen its partial effect.

Table 6: Importance of heterogeneity in quantity and quality of parental time investment

	IGE	rank corr	\bar{h} (hrs/wk)	Average IVT/Y	College (%)
(1) Baseline <i>Shutting down</i>	.393	.382	6.3	.056	30.3
(2) Heterogeneity in the quantity of h	.314	.302	6.3	.054	29.2
(3) Parental time investment channel	.204	.198	0.0	.056	27.8

Notes: Row (2) (homogenous time investment) exogenously imposes that all parents spend the same time investment at the mean value from the baseline model ($h = \bar{h}$). Row (3) (no parental time investment) sets $h = 0$ for all parents.

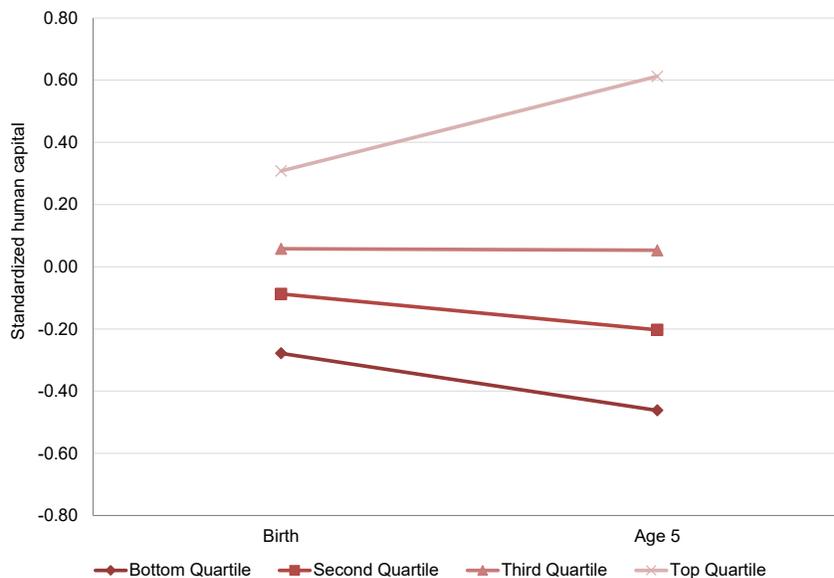
intergenerational association, I characterize properties of the optimal parental time investment decision in equilibrium. A simple and intuitive way is to run a linear regression using simulated data:

$$E[\log(h) | \log(\theta), \log(\theta_0), \log(a)] = \beta_0 + \underbrace{\beta_1}_{(1.44)} \log(\theta) + \underbrace{\beta_2}_{(-0.85)} \log(\theta_0) + \underbrace{\beta_3}_{(0.014)} \log(a)$$

where θ is parent's human capital, θ_0 denotes child's human capital endowment, and a is parent's assets using the simulated data. Note that, in the model, these regressors are parent's state variables, implying that coefficients β_i can describe how the average behavior of parental time investment changes with respect to changes in those variables according to the theory. I report coefficient estimates right below each parameter. Standard errors are omitted since they are very small.

The estimated β_1 is 1.44, implying that, on average, parents increase their time investment by 1.4 percent when their own human capital increases by 1 percent, holding child's human capital and asset holdings constant. This implies that higher human capital parents invest more time in their children despite their higher opportunity costs of time. Thus, this property of the human capital investment behavior acts as a mechanism that reinforces the intergenerational association of human capital. To quantitatively visualize this intergenerational amplification, Figure 2 plots how dispersion of human capital, by *parent's permanent income*, changes in early periods. More precisely, it shows the means of normalized human capital (such that each period human capital has mean zero with a unit standard deviation) by the quartile of permanent family income in

Figure 2: Mean standardized human capital at birth and age 5, by quartile of permanent family income



different ages, as in Carneiro and Heckman (2003).²² It clearly demonstrates that the parental time investment channel reinforces the gap across the whole quartiles.

In contrast to β_1 , the estimate of β_2 is negative (-0.85). In other words, the amount of parental time investment tends to decrease by 0.9 percent when their child’s human capital at birth gets higher by 1 percent, holding the parent’s human capital and assets constant. This may seem puzzling since the marginal product of parental time investment does increase with children’s human capital endowment (see Section 2). In fact, a key force that drives this tendency is the dynastic smoothing of the marginal value of human capital, analogous to the consumption smoothing of the infinitely-lived households in a standard dynamic consumption-saving model. The difference here is that parents choose to invest in their child’s human capital, which in turn affects her future lifetime consumption and leisure.

²²The permanent family income is defined as the discounted sum of parent’s lifetime income.

To see this more concretely, note that optimal parental time investment is characterized by

$$-\frac{\partial U(\frac{c}{q}, 1 - n - h)}{\partial h} = \underbrace{\beta^4 \eta \frac{\partial V_{j=1}(\theta_c, a_c)}{\partial \theta_c}}_{(a)} \times \underbrace{\frac{\partial \theta_c}{\partial \theta'_0} \alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_0^{\alpha_2}}_{(b)} \quad (6)$$

$$\text{where } \theta_c = (\theta'_0)^{\alpha_c}$$

$$\text{and } \theta'_0 = f(\theta, h, \theta_0) = \theta_0 + (\theta h)^{\alpha_1} \theta_0^{\alpha_2}.$$

where the left-hand side represents the marginal cost of h (which is the negative marginal utility of leisure) and the right-hand side summarizes the marginal benefit of h .²³ The marginal benefit has two components. First, additional time would develop the child's ability further ($\alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_0^{\alpha_2} > 0$), which in turn would increase the initial adult human capital ($\frac{\partial \theta_c}{\partial \theta'_0} > 0$). This positive marginal product of h in terms of adult human capital is captured by (b) in (6). However, what is valued by parents is not the level of child's future human capital per se, but the lifetime utility their child would enjoy. Therefore, (a) in (6) translates the marginal product of human capital into the present-value marginal utility which parents actually care about. Note that, although the marginal cost of h (left-hand side) is independent of the child's initial endowment θ_0 , the marginal benefit schedule of h (right-hand side) may shift up or down with respect to θ_0 , depending on the relative size of the two effects: (i) it may go up since the human capital technology implies that marginal return on h increases with θ_0 , due to (b) in (6); and (ii) it may go down because of diminishing marginal utility, due to (a) in (6). In the baseline model, the second effect dominates the first effect on average, generating a negative β_2 .²⁴

Implications for intergenerational mobility and aggregate efficiency:

Having investigated the properties of the parental time investment behavior, I now argue that the parental time investment channel strengthens intergenerational associations and reduces aggregate efficiency. The discussion below is mainly based on the relative ranking of human capital

²³The inequality constraint on $h \geq 0$ does not bind since $\frac{\partial f}{\partial h}$ tends to infinity when h approaches zero.

²⁴Similarly, Bernal (2008) finds that mothers compensate less able children by spending more time with them. The compensating nature of the parental investment (not necessarily time investment) can be seen in another context where parents care about inequality among multiple children (Behrman, Pollak, and Taubman, 1982). Their setting is different from the current setting where parents compensate their child who is born with a lower endowment relative to *themselves*, not among siblings.

endowment and that of human capital at the end of the parental time investment period.

First, note that the intergenerational amplification mechanism (i.e., higher human capital parents invest more time, the quality of which is even higher) could switch the ranking of children during this period, especially when children's initial human capital endowment at birth (θ_0) is not strongly correlated with their parent's human capital. To see this point, consider an extreme case in which children's human capital endowment is perfectly correlated with their parent's human capital. In this case, the intergenerational amplification mechanism would perfectly preserve the children's ranking, having no impact on the relative ranking before and after the parental time investment channel. On the other hand, consider a case where the correlation is zero. Then, the intergenerational amplification mechanism implies that the human capital ranking of many children at birth would be altered, moving closer to the parent's human capital status, due to the parental time investment channel. Next, recall the dynastic smoothing motives (i.e., parents invest less time in children with higher endowment, conditional on parental human capital) imply that, when a child's human capital endowment turns out to be far from her parent's socioeconomic status in terms of human capital, parental time investment moves the child closer to her parent's status. Hence, this property switch the ranking of children during this period, particularly when child's initial human capital endowment is not strongly correlated with their parents' human capital.

Both properties of parental time investment decisions suggest that its role in strengthening intergenerational association hinges on the rank correlation between the parent's human capital and the human capital endowment of their children. In the baseline model, this correlation is positive (0.384) yet far from one. Therefore, the rank correlation of the parent's human capital with the children's human capital at the end of $j = 1$ (i.e., θ'_0) is nearly 20 percent higher at 0.463 in the model economy.

Regarding the implication for aggregate efficiency, note first that, given the complementarity between parental time investment and the child's initial human capital, it would be more efficient to allocate more investment in children with higher human capital endowment from the social planner's perspective. However, note that (i) the first mechanism implies that once a child is born into low human capital families, both quantity and quality margins of parental time investment are destined to be lower; (ii) the second mechanism implies that children with higher human capital endowment

Table 7: Inspecting other channels as sources of intergenerational persistence

	IGE	rank	\bar{h}	Average	College
		corr	(hrs/wk)	IVT/Y	(%)
(1) Baseline	.393	.382	6.3	.056	30.3
<i>Shutting down</i>					
(2) College education	.346	.337	5.5	.045	0.0
(3) Inter-vivos transfers	.395	.384	6.7	0.0	25.7
(4) Primary & secondary education	.357	.345	5.1	.058	29.0
(5) Pre-birth human capital transmission	.192	.186	5.1	.057	26.1

Notes: The following restrictions are imposed in each row: (2) $\Delta = 0$; (3) $a_h = a_l = 0$; (4) $\alpha_c = 1$; (5) $a_0 = 0$.

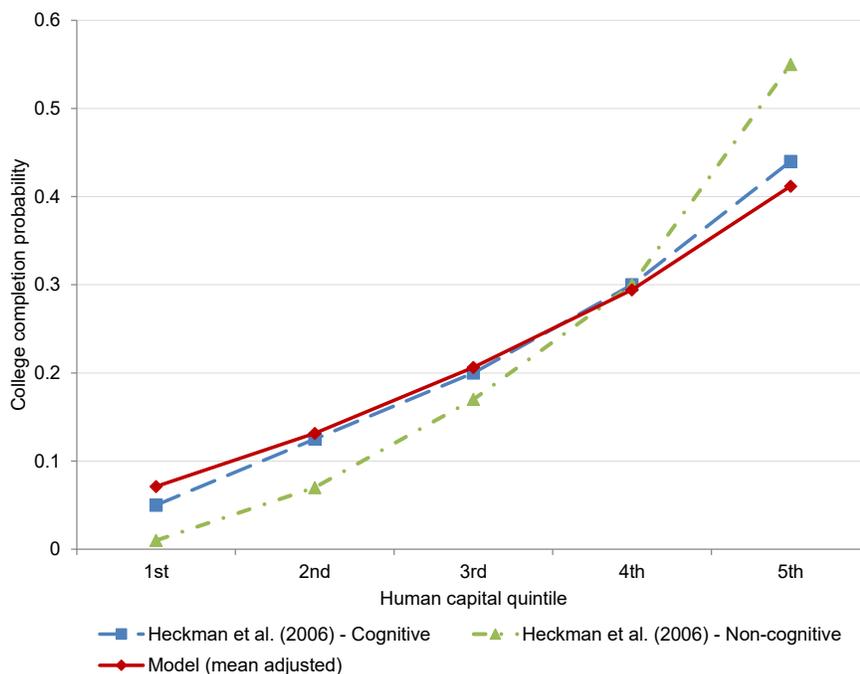
would actually get a lower amount of time investment. Hence, from a societal *efficiency* point of view, parental time investment in high human capital children born into low human capital families is insufficient (in both quantity and quality) whereas parental time investment in low human capital children born into high human capital families is excessive (again in both quantity and quality). These issues are revisited in Section 6 where some policies influencing parental time investment are shown to be able to increase aggregate efficiency as well as intergenerational mobility.

5.2 Other Channels Shaping Intergenerational Mobility

I now move on to other channels that shape the intergenerational mobility of lifetime income in the model. I first examine the role of college education, which is commonly considered as a means of upward mobility in society. To quantitatively measure the importance of college education in accounting for the intergenerational persistence of lifetime income, I shut down the college channel by imposing that there is no benefit of college education ($\Delta = 0$). Row (2) in Table 7 shows that both intergenerational persistence estimates decrease roughly by 10 percent. This suggests that college education does provide mobility but to a lesser extent, compared to the parental time investment channel.

To better understand this result, it is useful to examine how the college choice is made. In the model, college decisions depend not only on financial conditions but also on pre-college human capital. The discrete decision rule for college education features threshold-based behavior. More precisely, holding other things constant, the college decision rule is to complete college if his or her

Figure 3: College completion rates by human capital quintiles



human capital is above some threshold level. The reason is that the return to college, which is accumulated over the life cycle, increases with their pre-college human capital level. On the other hand, given a college cost draw, the marginal opportunity cost of going to the college (i.e., foregone earnings) is relatively small because young households tend to have lower wages.²⁵ This property of the college decision rule leads to positive selection in equilibrium, meaning that a better-prepared student is more likely to complete college education.

To visualize the quantitative importance of college readiness that exists in the model, in Figure 3, I plot college completion probabilities by the quintiles of pre-college human capital in equilibrium. The data counterparts are college completion probabilities in Heckman, Stixrud and Urzua (2006) by either cognitive factors or non-cognitive factors.²⁶ It clearly shows that high pre-college human

²⁵In reality, merit-based scholarship could make the college cost smaller for the people with higher ability. This would strengthen the importance of human capital in deciding whether to go to college. The effect of need-based scholarship may work in the other direction; however, this effect is less clear since children's ability is not perfectly correlated with parental income, which is typically a criterion for such scholarship.

²⁶The data set in Heckman et al. (2006) has a lower unconditional college completion rate. To focus on the slope rather than the level, I adjust the model-implied college completion probability curve by the magnitude of the

capital raises the probability of completing college, indicating positive selection into college both in the model and in the data. It is worth noting that the model produces a slope, which captures the strength of selection, in line with the data, although the calibration strategy does not directly target it.

A consequence of this property is that the college channel tends to endogenously sort out those who have relatively higher human capital, amplifying differences in pre-college human capital. Note that this amplification is with respect to own human capital, implying little rank reversals due to the college education channel. This is in sharp contrast to the parental time investment channel, which reinforces human capital intergenerationally and induce many rank reversals among children. This suggests that college education may not be an effective means of raising mobility.

In the model economy, parents can transfer money to their child when she becomes independent. An important role of this transfer is to provide financial help for their child's college decision. In row (3) of Table 7, I shut down the inter-vivos transfer channel by setting $\alpha_h = a_l = 0$. First, as expected, the college completion rate goes down because private financial support is missing. It is surprising to note that intergenerational persistence becomes slightly higher in this counterfactual exercise. This is partially because parents, who are not allowed to transfer money to their children, choose to invest more time in young children instead. This substitution towards parental time investment ends up raising intergenerational persistence.

Restuccia and Urrutia (2004) find that nearly half of the intergenerational persistence is due to parental investment in primary and secondary education in a model that abstracts from periods before age 5. In row (4) of Table 7 shows the results when I shut down primary and secondary education by setting $\alpha_c = 1$. This prevents dispersion of human capital from increasing during this period. Interestingly, the intergenerational persistence estimates do fall but its magnitude is relatively small (nearly 10 percent). It should be noted that this result does not necessarily contradict the result of Restuccia and Urrutia (2004) since my model endogenizes the earlier periods prior to schooling, which in fact account for a significant portion of the intergenerational persistence of lifetime income.

Finally, the bottom row of Table 7 shows the case in which I set $\alpha_0 = 0$, making the human difference in the unconditional college completion rate.

capital endowment of children to be independent of their parent's human capital. This shuts down all pre-birth factors capturing not only genetic transmission (nature) but also any nurtural factors (e.g., prenatal investment) that could affect the child's initial endowment of human capital. Unsurprisingly, closing the pre-birth transmission reduces intergenerational persistence considerably. This result is not surprising since nature, which could account for a significant portion of the pre-birth factors in the model, is often found to be quantitatively important in explaining intergenerational persistence (e.g., Plug and Vijverberg, 2003; see also Sacerdote, 2010 for a survey on the evidence by psychologists and behavioral geneticists as well).

6 Policy Experiments

In this section, the baseline model economy is used to study various policies that can be considered as tools to influence intergenerational mobility. I consider universal (or flat) policies that can avoid stigmatization especially when it comes to family policies (Heckman, 2008). It is important to note that the main objective of this section is to examine and illustrate desirable properties of effective policies to increase intergenerational mobility. In doing so, I also examine the implications of such policies for aggregate efficiency; measured by aggregate output, average labor productivity, and aggregate welfare.²⁷ That way, we could better evaluate whether the policy changes that raise intergenerational mobility are otherwise desirable for the economy. All of the policies are designed to be financed as part of government spending while satisfying the government budget constraint. Note also that the policy exercises capture general equilibrium effects meaning that human capital investment and savings interact with each other through equilibrium price changes in the presence of persistent risky human capital and risk-free assets (e.g., Krebs, 2003). For illustrative convenience, all monetary values are expressed in roughly 2011 U.S. dollar under the assumption that the annual GDP per capita in the baseline model is \$50,000, a value close to nominal GDP per capita in 2011.

Providing easier access to college:

College is often believed to be a means of upward mobility. I first consider two ways to provide

²⁷Welfare changes are measured by a consumption equivalent premium. More precisely, I measure the percentage change in consumption for all agents in the baseline model that makes them indifferent to living in the alternative economy using the utilitarian social welfare function.

Table 8: Effects of providing easier access to college

	(1)	(2)	(3)
	Baseline	Borrowing limit relaxed	Lower college cost
IGE	.393	.391	.395
Rank correlation	.382	.380	.384
Mean parental time investment (hours/week)	6.3	6.3	6.3
College fraction (%)	30.3	32.1	32.1
Mean parental transfers to output ratio	.056	.056	.057
<i>Changes relative to (1) (unit: %)</i>			
Aggregate output	-	+0.3	+0.5
Aggregate consumption	-	+0.4	+0.6
Average labor productivity	-	+0.4	+0.7
Aggregate welfare	-	+0.9	+0.8

Notes: Average labor productivity is defined as aggregate output per total hours worked. Welfare gains are the consumption equivalent premium measured by the percentage change in consumption required for all agents to be indifferent to living in an alternative economy.

easier access to college. The first is to relax borrowing limits up to \$5,000 in $j = 1$ (the college education stage). As can be seen in column (2) of Table 8, the college completion rate does increase quite significantly (by 1.8 percentage points). The second exercise, reported in column (3) of the same table, lowers the mean of the college cost distribution m_ξ such that the college completion rate increases also by 1.8 percentage points.

Some interesting results emerge in Table 8 regarding intergenerational mobility. Both measures of intergenerational persistence change little despite the fact that a greater fraction of population complete the college. The main reason is positive selection into college in terms of human capital, as discussed in the previous section. That is, the marginal students tend to have lower returns to college than those who are already in college. On average, those marginal college graduates do accumulate more of human capital but only up to the level less than those who already choose to complete college. Therefore, they cause little rank reversals. In fact, this is consistent with one of the main findings in the previous section that the contribution of the college channel to intergenerational persistence is relatively small. Since college increases human capital in the model, both policies, which lead to more college-educated labor forces, have positive impacts on the aggregate output and average labor productivity.

Increasing the quantity of parental time investment:

The analysis in Section 5 has called for the policies that encourage parents to invest more time in children born into disadvantaged families since low quantity and quality of parental time investment could be important sources of hindering mobility and aggregate efficiency. However, it is important to note that a higher average time investment may not be sufficient to increase intergenerational mobility, given the heterogeneous parental time investment behavior. To illustrate this point, I consider two policies, both of which can achieve the same goal of increasing the average time investment, and contrast their implications for intergenerational mobility. The first is a flat subsidy s_h , which is proportional to parental time investment h chosen by parents. In other words, the resource constraint in period $j = 2$ is augmented with $s_h h$:

$$c + a' \leq (1 - \tau)w\theta n + (1 + r)a + T + s_h h.$$

An important feature of this policy is that this flat subsidy s_h is independent of the parent's human capital or income. Therefore, the same amount s_h matters more for low human capital parents. In practice, since time spent by parents at home is hardly observable, this policy should be thought of as an approximation of subsidies to play-centers in which parents themselves must present and interact with their children. The other policy is an increase in earnings tax rate τ only in period $j = 2$. This additional tax burden decreases the marginal benefit of market work, thereby increasing an incentive to invest their time in children instead. The size of the time subsidy and the earnings tax in period $j = 2$ is set to induce the average parental time investment to increase by 20 percent.

Table 9 summarizes the results. Column (2) shows the case of a flat subsidy s_h and column (3) shows the case of higher earnings tax in period $j = 2$. Note that, although the average parental time investment increases from 6.3 to 7.4 hours per week in both columns (2) and (3), the intergenerational persistence estimates fall only in the case of the parental time investment subsidy. In fact, they increase quite noticeably in the case of the earnings tax. Why does the higher average time investment have such opposite effects on intergenerational mobility? To better understand this, Figure 4 plots the conditional mean of parental time investment by parent's human capital quintiles for the three cases in Table 9. Note that, although the effect of the time subsidy

Table 9: Effects of increasing quantity of parental time investment

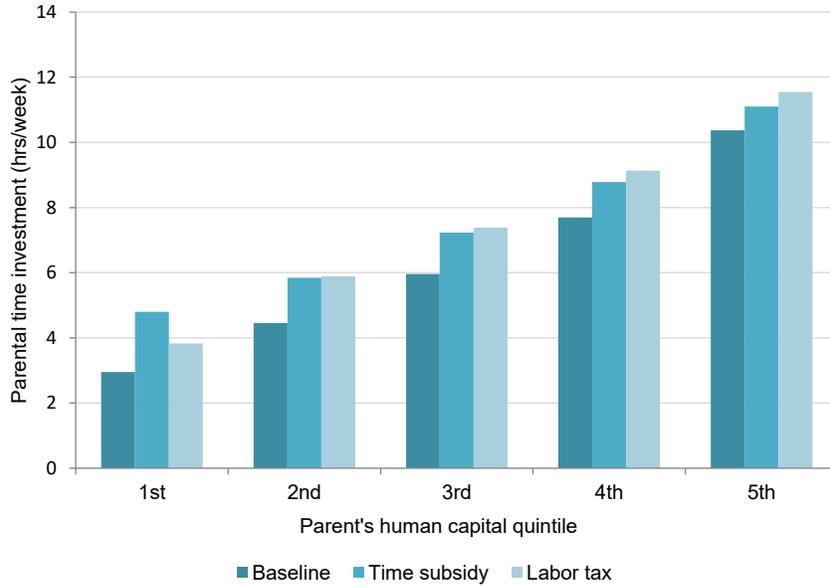
	(1)	(2)	(3)
	Baseline	Time subsidy	Earnings Tax
IGE	.393	.388	.400
Rank correlation	.382	.377	.390
Mean parental time investment (hours/week)	6.3	7.4	7.4
College fraction (%)	30.3	29.5	26.1
Mean parental transfers to output ratio	.056	.056	.058
<i>Changes relative to (1) (unit: %)</i>			
Aggregate output	-	+1.3	+0.8
Aggregate consumption	-	+1.7	-0.5
Average labor productivity	-	+2.4	+1.0
Aggregate welfare	-	+2.6	+1.4

Notes: The flat subsidy for parental time investment in Column (2) and the labor tax in Column (3) are set to induce a 20 percent increase in the average parental time investment.

and the higher earnings tax on the *unconditional* mean of parental time investment is the same, their impact across the distribution differs sharply. More specifically, an increase in the amount of parental time investment is much more pronounced at the bottom quintile in the case of the parental time subsidy. Since lower human capital parents have lower opportunity costs of parental time investment (i.e., lower wage), they increase time investment in their children more elastically with respect to the same amount of the monetary incentive s_h . The disproportionate impact has important implications for intergenerational mobility since the relatively larger increase in parental time investment at the bottom of the distribution helps those who are born with high human capital move up. By contrast, increases in time investment are rather even across the distribution in the case of the higher earnings tax. It actually reinforces the intergenerational association since the average parental investment is higher, which in turn transmits more of the parent's human capital to children.

It is interesting to note that the larger average parental time investment leads to higher aggregate output, higher labor productivity and aggregate welfare gains in both cases. This suggests that the parental time investment decisions made by parents in the baseline model are not socially optimal in terms of aggregate efficiency. Note that children with high human capital endowments in low human capital families receive particularly low investment. Both policies do increase the level of

Figure 4: Parental time investment by parent’s human capital quintiles: baseline vs policies related to parental time investment



parental time investment in those children who have very high marginal productivity of parental time investment (due to high own human capital as well as the low level of current investment), which in turn could lead to large returns in terms of human capital. Furthermore, the table also shows that the output and welfare gains are much larger in the case of the parental time subsidy. This finding confirms that greater mobility can be accompanied with higher aggregate efficiency by mitigating the low investment problem for the able children in low human capital families.

Universal preschool:

The flat time investment subsidy in the previous exercise is useful in demonstrating the importance of improving the amount of time investment in children born into disadvantaged families. Nevertheless, note that those children inevitably receive a relatively low quality of time investments. Therefore, an effective way of improving mobility may also focus on the quality margin as well. To illustrate this point, I now consider a policy that can improve not only the quantity but also the quality of the time investment in children born into low human capital families. Given the intrinsic

challenge that parental human capital cannot be easily altered in the short run, I consider a universal preschool program that is publicly provided. More precisely, I assume that human capital accumulation for the newborn follows

$$\theta'_0 = \theta_c + (\theta h)^{\alpha_1} \theta_0^{\alpha_2} + (\tilde{\theta} \tilde{h})^{\alpha_1} \theta_0^{\alpha_2} \quad (7)$$

instead of (2). The only difference here is the augmentation of the last term, which captures the amount of extra human capital development from the universal preschool. As children spend the same time with common teachers in such preschools, the last term has the common quality $\tilde{\theta}$ and the common time investment \tilde{h} . To highlight the importance of the quality of time investment, I consider two values $\tilde{\theta} \in \{\tilde{\theta}_l, \tilde{\theta}_h\}$ where $\tilde{\theta}_l$ is mapped to \$10 hourly wage and $\tilde{\theta}_h$ is mapped to \$20 hourly wage. Note that these values are around \$15, which is the mean hourly wage of childcare workers and preschool workers (except special education teachers and teacher assistants) in the U.S., according the 2011 Occupational Employment Statistics survey of the Bureau of Labor Statistics. Lastly, I set \tilde{h} to the increment of the mean parental time investment in the previous exercise (1.9 hours per week) to compare the effects of the pre-school exercise with those of the parental time subsidy. It is important to note that the amount of parental time investment is still determined by parents who understand that there are now universal preschools. In computing a new equilibrium, I account for the effective labor supplied to the preschool program using the staff-child ratio of 6 (i.e., $(\frac{\mu_2}{6}) \int \tilde{\theta} \tilde{h} d\pi_j$ is subtracted from aggregate labor supply).²⁸

Column (3) in Table 10 shows that the universal preschool even with $\tilde{\theta}_l$ (low quality) decreases the intergenerational persistence estimates even more than the case of the flat time subsidy (Column (2)). Column (4) shows that, if the quality of the time is higher ($\tilde{\theta}_h$), the intergenerational persistence estimates fall even further. Note that the pre-school program increases intergenerational mobility for two reasons. First, the publicly provided time investments are particularly beneficial to high human capital children born into low human capital parents because these children's high human capital endowment can effectively complement such high quality public time investments. This

²⁸One could also assume that there is a separate competitive preschool sector with a technology which is linear in labor only. Then, the above exercise of changing teacher wages can be viewed as the exercise of changing the productivity of the preschool sector.

Table 10: Effects of increasing both quantity and quality of time investments

	(1)	(2)	(3)	(4)
	Baseline	Time subsidy	Universal Preschool (low quality)	Universal Preschool (high quality)
IGE	.393	.388	.387	.383
Rank correlation	.382	.377	.376	.372
Mean parental time investment: \bar{h} (hours/week)	6.3	7.4	6.0	5.9
Mean total time investment: $\bar{h} + \tilde{h}$ (hours/week)	6.3	7.4	7.3	7.1
College fraction (%)	30.3	29.5	29.8	29.5
Mean parental transfers to output ratio	.056	.056	.056	.056
<i>Changes relative to (1) (unit: %)</i>				
Aggregate output	-	+1.3	+2.6	+4.7
Aggregate consumption	-	+1.7	+2.7	+4.8
Aggregate hours of worked	-	+2.4	+3.6	+6.3
Aggregate welfare	-	+2.6	+3.8	+6.1

enables them to move up. Second, the public time investments crowd out the amount of parental time investment as can be seen from reductions in average time investment (\bar{h}) in Columns (3) and (4) relative to Column (2). This weakens the intergenerational transmission of human capital, thereby reducing intergenerational persistence.

Finally, we can see that the universal preschool program increases aggregate output and welfare even more than the parental time investment subsidy. These larger gains are possible since the same effective time investment $\tilde{\theta}\tilde{h}$ from the universal preschool complements high human capital children regardless of their family background. It is important to note that, since \tilde{h} was chosen to make comparisons with the time investment subsidy case, the absolute levels of improvement in aggregate output and welfare should be not taken as the key messages. Instead, the takeaway should be that improvement in the quality of time investment in early periods is an important characteristic of desirable policies not only from the equality point of view but also from the efficiency perspective as well. This is because those children with high human capital endowments born into low human capital parents have potentials to constitute an efficient workforce if they receive a fair amount of quality time investments in early periods.

7 Conclusion

I have investigated the role of parental time investment in children prior to formal schooling in a general equilibrium, incomplete markets framework that endogenously generates the distribution of lifetime inequality across generations. I have found that the model successfully accounts for the untargeted distributional aspects of intergenerational persistence of income. Using the model economy, I have found that nearly half of the intergenerational persistence of lifetime income is accounted for by the parental time investment channel. Importantly, I show that the differences in both the quantity and the quality of parental time investment are crucial. I find that two key properties of the parental time investment decisions are that (i) parents with higher human capital invest more time in their children; and that (ii) parents invest less time in children with higher initial human capital endowments. I argue that these properties lead to the role of the parental time investment channel in strengthening intergenerational association and in hindering aggregate efficiency. The policy experiments I examined in this paper illustrate that effective policies of increasing mobility should focus not only on increasing the average parental time investment but also on narrowing the gap in the parental time investment. The exercises also highlight that the quality of the time investment provided by the government may further increase mobility by closing the gap not only in quantity but also in quality.

The purpose of the policy exercises in this paper was to provide some important characteristics in designing actual policies to promote intergenerational mobility. In fact, each policy exercise considered in this paper has relatively small effects on intergenerational mobility. An interesting avenue for future work is to design a more effective and implementable policy scheme that keeps the universal nature. For instance, a more ideal policy should specifically induce a greater amount of high-quality time investment towards able children born into low human capital families. In addition, it is important to note that this paper abstracts from spillover effects. Consider the play-center example considered in this paper. If parents can (i) learn parenting skills while watching how other parents spend time with their children in such centers or (ii) share valuable information directly regarding early education while spending time in such centers, they could potentially increase their parenting quality at home as well. These spillovers effects could potentially strengthen

the effects of the aforementioned policies on intergenerational mobility.

References

- Abbott, Brant, Giovanni Gallipoli, Costas Meghir, and Giovanni L. Violante. 2013. "Education Policy and Intergenerational Transfers in Equilibrium" NBER Working Paper No. 18782
- Babcock, Philip and Mindy Marks. 2011. "The Falling Time Cost of College: Evidence from Half a Century of Time use Data." *Review of Economics and Statistics* 93 (2): 468-478.
- Becker, Gary S. and Nigel Tomes. 1986. "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4 (3): S1-39.
- Behrman, Jere Richard, Robert A. Pollak, and Paul Taubman. 1982. "Parental Preferences and Provision for Progeny." *Journal of Political Economy* 90 (1): 52-73.
- Bernal, Raquel. 2008. "The Effect of Maternal Employment and Child Care on Children's Cognitive Development*." *International Economic Review* 49 (4): 1173-1209.
- Black, Sandra E. and Paul J. Devereux. 2011. "Recent Developments in Intergenerational Mobility." *Handbook of Labor Economics* 4: 1487-1541.
- Blau, David M. 1999. "The Effect of Income on Child Development." *Review of Economics and Statistics* 81 (2): 261-276.
- Blau, David M. and Janet Currie. 2006. "Pre-School, Day Care, and After-School Care: Who's Minding the Kids?" *Handbook of the Economics of Education* 2: 1163-1278.
- Carneiro, Pedro and James. J. Heckman. 2003. "Human Capital Policy." In *Inequality in America: What Role for Human Capital Policies?*, edited by James J. Heckman, Alan B. Krueger and Benjamin M. Friedman, 77. Cambridge, MA: MIT Press.
- Caucutt, Elizabeth M., and Lance Lochner. 2012. "Early and Late Human Capital Investments, Borrowing Constraints, and the Family," NBER Working Paper No. 18493.
- Chadwick, Laura and Gary Solon. 2002. "Intergenerational Income Mobility among Daughters." *The American Economic Review* 92 (1): 335-344.

- Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. 2014a. "Where is the Land of Opportunity? the Geography of Intergenerational Mobility in the United States" NBER Working Paper No. 19843.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014b. "Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility." NBER Working Paper No. 19844
- Cunha, Flavio and James J. Heckman. 2007. "The Technology of Skill Formation." *The American Economic Review* 97 (2): 31-47.
- Dahl, Gordon B. and Lance Lochner. 2012. "The Impact of Family Income on Child Achievement: Evidence from the Earned Income Tax Credit." *American Economic Review* 102 (5): 1927-1956.
- Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies* 81 (1): 137-185.
- De Nardi, Mariacristina. 2004. "Wealth Inequality and Intergenerational Links." *The Review of Economic Studies* 71 (3): 743-768.
- Erosa, Andres and Tatyana Koreshkova. 2007. "Progressive Taxation in a Dynastic Model of Human Capital." *Journal of Monetary Economics* 54 (3): 667-685.
- Fernandez, Raquel and Richard Rogerson. 1998. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform." *American Economic Review* 88 (4): 813-833.
- Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." *The Journal of Economic Perspectives* 22 (3): 23-16A.
- Haider, Steven J. and Gary Solon. 2006. "Life-Cycle Variation in the Association between Current and Lifetime Earnings." *American Economic Review* 96 (4): 1308-1320.
- Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal we Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006." *Review of Economic Dynamics* 13 (1): 15-51.
- Heckman, James J. 2008. "Schools, Skills, and Synapses." *Economic Inquiry* 46 (3): 289-324.

- Heckman, James J. and Stefano Mosso. 2014. "The Economics of Human Development and Social Mobility." NBER Working Paper No. 19925.
- Heckman, James, Rodrigo Pinto, and Peter Savelyev. 2013. "Understanding the Mechanisms through which an Influential Early Childhood Program Boosted Adult Outcomes." *The American Economic Review* 103 (6): 1-35.
- Heckman, James J., Jora Stixrud, and Sergio S. Urzua. 2006. "The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior." *Journal of Labor Economics* 24 (3): 411-482.
- Herrington, Christopher M. 2015. "Public Education Financing, Earnings Inequality, and Intergenerational Mobility." Working paper.
- Holter, Hans Aasnes. 2014. "Accounting for Cross-Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure." Available at SSRN 1978261.
- Huggett, Mark, Gustavo Ventura, and Amir Yaron. 2011. "Sources of Lifetime Inequality." *The American Economic Review* 101 (7): 2923-2954.
- Johnson, Matthew T. 2013. "Borrowing Constraints, College Enrollment, and Delayed Entry." *Journal of Labor Economics* 31 (4): 669-725.
- Keane, Michael P. and Kenneth I. Wolpin. 1997. "The Career Decisions of Young Men." *Journal of Political Economy* 105 (3): 473-522.
- Knudsen, Eric I., James J. Heckman, Judy L. Cameron, and Jack P. Shonkoff. 2006. "Economic, Neurobiological, and Behavioral Perspectives on Building America's Future Workforce." *Proceedings of the National Academy of Sciences of the United States of America* 103 (27): 10155-10162
- Krebs, Tom. 2003. "Human Capital Risk and Economic Growth." *The Quarterly Journal of Economics* 118 (2): 709-744.
- Lee, Sang Yoon Tim and Ananth Seshadri. 2016. "On the Intergenerational Transmission of Economic Status." Working paper.

- Mazumder, Bhashkar. 2005. "Fortunate Sons: New Estimates of Intergenerational Mobility in the United States using Social Security Earnings Data." *Review of Economics and Statistics* 87 (2): 235-255
- Plug, Erik and Wim Vijverberg. 2003. "Schooling, Family Background, and Adoption: Is it Nature Or is it Nurture?" *Journal of Political Economy* 111 (3): 611-641.
- Ramey, Garey and Valerie A. Ramey. 2010. "The Rug Rat Race." *Brookings Papers on Economic Activity* 41 (1): 129-199.
- Restuccia, Diego and Carlos Urrutia. 2004. "Intergenerational Persistence of Earnings: The Role of Early and College Education." *The American Economic Review* 94 (5): 1354-1378.
- Rupert, Peter and Giulio Zanella. 2015. "Revisiting Wage, Earnings, and Hours Profiles." *Journal of Monetary Economics* 72: 114-130.
- Sacerdote, Bruce. 2010. "Nature and Nurture Effects on children's Outcomes: What have we Learned from Studies of Twins and Adoptees." *Handbook of Social Economics* 1: 1-30.
- Solon, Gary. 1999. "Intergenerational Mobility in the Labor Market." *Handbook of Labor Economics* 3: 1761-1800.

Appendix

A Microfoundation of school-age children's human capital evolution

I use the exponential functional form (3) to characterize the human capital evolution of school-age children. This is helpful for generating a larger degree of human capital inequality at the end of childhood (Restuccia and Urrutia, 2004) while keeping tractability. Note that the literature finds that parental monetary investment is critical for this later childhood (e.g., Del Boca et al., 2014). This section provides a simple theoretical foundation with endogenous parental monetary investment for the exponential functional form used in (3).

Consider a parent who decides how much of financial resources x to invest in their children with human capital θ . Assume that the production function features complementarity between the two

inputs following a Cobb-Douglas function:

$$\theta' = x^{\gamma_1} \theta^{\gamma_2}, \quad (8)$$

where $\gamma_1, \gamma_2 \in (0, 1)$. Then, the marginal product of x is $\gamma_1 x^{\gamma_1-1} \theta^{\gamma_2}$. Assuming a constant marginal cost κ of x , the optimal parental monetary investment is given by

$$\gamma_1 x^{\gamma_1-1} \theta^{\gamma_2} = \kappa$$

or

$$x^* = \check{B} \theta^{\frac{\gamma_2}{1-\gamma_1}},$$

where \check{B} is some constant.²⁹ Since $\frac{\gamma_2}{1-\gamma_1} > 0$, this optimal decision implies that parents would invest more money in children with higher human capital, but the elasticity depends on the size of γ_1 and γ_2 . Specifically, as either γ_1 or γ_2 becomes closer to zero, then the optimal parental financial investment would increase more slowly with the child's human capital.

Once x^* is plugged into (8), then the human capital evolution becomes

$$\theta' = \tilde{B} \theta^{\frac{\gamma_1 \gamma_2}{1-\gamma_1}} \theta^{\gamma_2} = \tilde{B} \theta^{\frac{\gamma_2}{1-\gamma_1}},$$

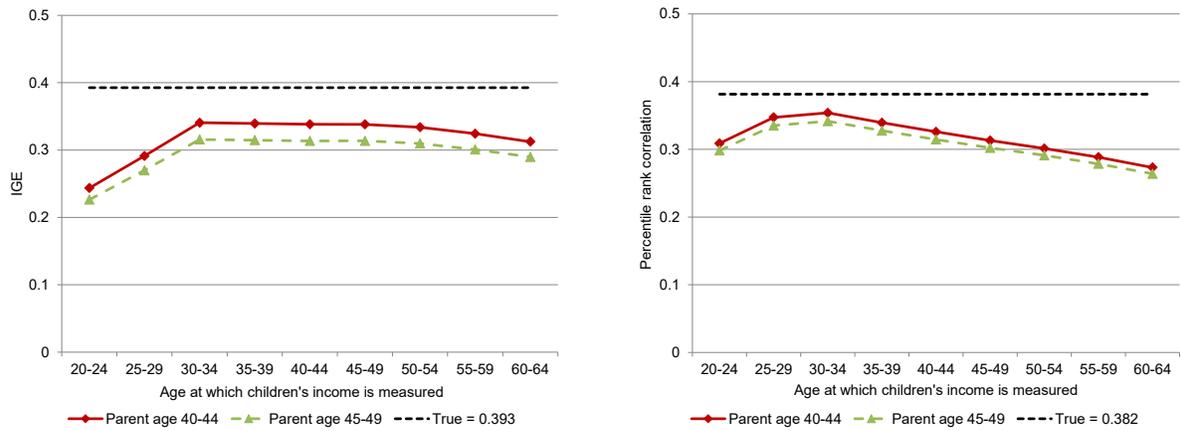
where \tilde{B} is some constant. This shows that $\ln \theta'$ increases linearly with $\ln \theta$ as in (3). Also, note that the calibrated parameter of $\alpha_c = 1.06$ is compatible with the above microfoundation since $\frac{\gamma_2}{1-\gamma_1} \in (0, \infty)$ given the assumptions $\gamma_1, \gamma_2 \in (0, 1)$.

B Lifecycle bias in the intergenerational mobility estimates

It is important to note that the degree of approximation using the proxy income depends on the point at which income is measured, as can be seen in Figure A1. In this figure, I plot the estimates of the IGE (left panel) and the rank correlation (right panel) by varying the age at which children's income is measured while holding constant the age at which parents' income is measured at 40-44

²⁹The assumption of the constant marginal cost can be relaxed.

Figure A1: Lifecycle bias by age of child



Notes: In both panels, I vary the age at which children’s income is measured while holding the age at which parents’ income is measured constant at 40-44 (red solid line) or at 45-49 (green dashed line). The left panel shows the IGE estimates and the right panel shows the rank correlation estimates. The black dotted lines show the corresponding estimates using the actual lifetime income.

(red solid) or at 45-49 (green dashed). Consistent with Solon (1999) and Haider and Solon (2006), there is serious attenuation bias in the IGE estimates when children’s income is measured too early. The left panel shows that the IGE estimate when children’s income is measured in the early 20’s is less than half the true value using the lifetime income (black dotted line). The IGE estimates become stable once the children’s age is over 30. The rank correlation estimates show similar patterns with the two key differences. First, the absolute magnitude of the attenuation bias is smaller. Second, the rank correlation moderately declines with the age at which children’s income is measured. This pattern is also present in Chetty et al. (2014a)’s Figure III using the SOI (the Statistics of Income) sample.

Table A1 reports the IGE estimates by varying the point (for both parents and children) at which the proxy income is measured. Although the true IGE, estimated by using the discounted lifetime income, is 0.393 as reported in Table 3, the estimates vary quite substantially depending on the timing at which income is measured. There are several systematic patterns regarding the lifecycle bias. First, regardless of when the parent’s income is measured, the IGE estimates are seriously downward-biased if the child’s income is measured early in their life. For instance, when

Table A1: IGE estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.25	.30	.35	.34	.34	.34	.34	.33	.32
	25	.34	.37	.44	.44	.43	.43	.43	.42	.40
	30	.28	.34	.40	.39	.39	.39	.39	.38	.36
	35	.26	.31	.37	.37	.36	.36	.36	.35	.34
	40	.24	.29	.34	.34	.34	.34	.33	.32	.31
	45	.23	.27	.32	.31	.31	.31	.31	.30	.29
	50	.21	.25	.30	.30	.30	.30	.29	.28	.27
	55	.20	.24	.28	.28	.28	.28	.28	.27	.26
	60	.19	.23	.26	.26	.26	.26	.26	.25	.24

Notes: Ages denote the first age of the five-year period at which lifetime income is measured.

Table A2: Percentile rank correlation estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.25	.28	.29	.28	.26	.25	.24	.23	.22
	25	.35	.36	.37	.35	.34	.33	.31	.30	.29
	30	.33	.37	.38	.37	.35	.34	.32	.31	.29
	35	.32	.36	.37	.35	.34	.32	.31	.30	.28
	40	.31	.35	.35	.34	.33	.31	.30	.29	.27
	45	.30	.34	.34	.33	.31	.30	.29	.28	.26
	50	.29	.32	.33	.32	.30	.29	.28	.27	.26
	55	.28	.31	.32	.30	.29	.28	.27	.26	.25
	60	.26	.30	.30	.29	.28	.27	.26	.25	.23

Notes: Ages denote the first age of the five-year period at which lifetime income is measured.

the child's income is measured at 20-24, the IGE estimates are close to half the true IGE even if the parent's income is measured at old ages. This is consistent with prior empirical research that points out attenuation bias when children's income is measured at early ages (Solon, 1999; and Haider and Solon, 2006). On the other hand, holding the parent's age fixed, the IGE estimates become insensitive to a point at which child's income is measured as long as it is measured at the age of 30 or above.

In Table A2, I perform the same exercise with a rank-rank specification instead of a log-log specification. Overall, we can see the right-skewed inverse U shape in the rank-rank slope as a function of the timing at which child's (or parent's) income is measured: that is, (i) there is

attenuation bias when either child’s income or parent’s income is measured at early ages; and (ii) the slope estimates rise sharply and then decrease gradually as the measured-age rises. A notable difference compared to the IGE estimates is that, holding the parent’s measured income fixed, the rank-rank slope estimates rise sharply in the early 20’s and then gradually decrease whereas the IGE estimates are virtually flat with respect to child’s age after they sharply rise in the 20’s. These findings are in line with Chetty et al. (2014a) who show that their rank-rank slope estimates rise steeply in the early 20’s and then steadily decrease after age 30.

C Determining parameters using simulation

A vector of parameter values $\hat{\Theta} = (\beta, B, \eta_0, \delta_\eta, \alpha_1, \alpha_0, \sigma_\zeta, \sigma_z, \omega_\xi, \delta_\xi, \Delta, \omega_a, \lambda)$ in Table 2 is jointly determined using simulation. More specifically, define $M_m(\Theta)$ as the m -th target statistic obtained from the model-generated data with the set of parameters Θ ; and D_m as the same i -th target statistics obtained from data, as defined in Table 2. Then the calibrated vector of parameters $\hat{\Theta}$ is the minimizer of the objective function: $\sum_{m=1}^{13} [\log(M_m(\Theta)/D_m)]^2$. I use the downhill simplex method to solve this minimization problem.

D Data

Statistics regarding time-use are computed using the 2003-2012 waves of the American Time Use Survey (ATUS). To compute average hours worked and the fraction holds a college degree, I consider both men and women and include those whose age is greater than or equal to 20 and less than 65. To construct a variable of parental time investment in the child’s human capital, I focus on the interactive activities that require the existence of both a parent and a child in a common space. Such categories include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities. As the focus of this paper is time spent in children before age 5, the time investment variable is computed using households whose youngest child is less than five years old. For the time investment variable, I further restrict the sample to the households who are

not enrolled in school and whose age is between the age of 21 and 55 (inclusive), as in Guryan et al. (2008). For all statistics reported, the ATUS statistical weights are used.

Note that the parental time investment variable does not include the activity of physical care for children, which accounts for quite a large portion of time. However, it is interesting to note that, even with the definition of the parental time including the physical care activities, I also find a similar size of the positive educational gradient and it is robust to the parental gender as well. Furthermore, I also broaden the definition of the parental time investment to include some activities that have educational aspects but do not necessarily require the direct/active contact between a parent and a child. Such activities are organizing and planning for children, attending children's events, waiting for/with children, picking up/dropping off children, attending meetings and school conferences for children, waiting associated with children's education. The inclusion of such educational activities that could have indirect impacts on the children's human capital development increases the mean by 14 percent but barely changes the educational gradient. The time-diary survey also reports secondary activities and part of them may also include childcare. However, since the childcare time recorded as secondary activities is expected to be less active and the same hours may not be effective as an input to the human capital function, I do not consider the time of childcare recorded as secondary activities, and only focus on childcare activities reported as a main activity.