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Distorted Input Ratios in Vertical Relationships

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Abstract

A project leader sources an input from a supporter and combines it with an input produced in-house. The leader has private information about the project's cost environment. We show that if the leader can commit to the in-house input level, the input ratio is distorted *upward* when the in-house input is not too costly—the in-house input is produced in excess and, thus, partly wasted. By contrast, without the leader's commitment to the in-house input level, the input ratio is distorted *downward* when the in-house input is sufficiently costly—the outsourced input is produced in excess and, thus, partly wasted.

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1. Introduction

In many industries, individual firms do not have the capacity to produce complex final products *alone*. They often rely not only on their own input, but also on the complementary input from external suppliers. In many instances, as part of a supply chain or a joint venture, the firm assembling the inputs assumes the role of 'project leader' specifying the project and proposing the terms and conditions to obtain a contribution by a 'project supporter.' For example, in a contractual joint venture, firms do not pool their productive capacities and instead agree on a contractual solution on how to implement the project. In such a situation, coordinating production of multiple complementary inputs is a defining feature of the project.

As reported by practitioners, inputs are wasted in operations even with the most successful manufacturers.[1] A firm may produce in-house inputs in excess, or its supplier may produce outsourced inputs in excess—the ratio between in-house and outsourced inputs is distorted upward or downward. This renders the question if such distortions can be optimal under some circumstances. As will be shown, when the project leader obtains some private information on the cost environment, she may want to distort the input ratio.[2]

We study the optimal contract that a project leader (e.g., a manufacturer) offers to a project supporter (e.g., a parts supplier). The project leader combines the component sourced from the supporter (the outsourced input) with another component produced in-house (the in-house input) to yield the final output. She offers a contract that specifies the project supporter's input level and a transfer payment from the leader to the supporter. If the project leader can commit to the in-house input level (full commitment), then that input level is specified in the contract as well. If only the outsourced input level can be contracted (limited commitment), then the project leader chooses the in-house input level according to her ex post interest. For example, contractual joint ventures often agree on a complete plan of actions with all input levels from all parties, whereas in supply chain management, it is common that only the outsourced inputs from the suppliers are contracted.[3]

After contracting, the project leader obtains private information about the project's cost environment. The contract offered by the leader is contingent on her subsequent report about the cost environment, which, in our model, is either good (low-cost for the in-house and the outsourced input) or bad (high-cost for the in-house and the outsourced input).

We show that the optimal input ratio in carrying out a project can be distorted in either direction from the efficient ratio that prevails under full information. Since the project leader is privately informed about the project environment, she may have an incentive to misrepresent her information to the project supporter. In addition, a further incentive problem arises under limited commitment—that is, when the project leader cannot commit to the in-house input level. In such a case, the project leader's contract offer has to account for not only her private information but also her hidden action.

Our main results are as follows. Under full commitment, the project leader either employs

[1] See, for example, Badurdeen (2007).

[2] Throughout this paper, the project leader is referred to as "she" and the project supporter as "he."

[3] See Kirshner (2017).

inputs at their efficient ratio, or distorts the input ratio upward—she produces the in-house input in excess of the outsourced input. Her choice is the latter in the good environment when her marginal cost of the in-house input is sufficiently low. The reason is as follows. The project leader is tempted to misrepresent the project environment as good when it is bad. This manipulating incentive is anticipated by the project supporter. To convince the project supporter that she will report truthfully, the project leader inflates the final output level in the optimal contract when the project environment is good. This mitigates the project leader's incentive to claim that the environment is good when it is actually bad—for the project supporter to agree to a higher level of the outsourced input, the project leader must increase the transfer to the supporter. Committing to increase the in-house input relative to the outsourced level in such a case reduces the project leader's misrepresenting incentive, and, thus, allows her to alleviate the upward distortion in the final output level. As a result, in the good environment, the optimal contract entails an *upward* distortion in the input ratio when the marginal cost of the in-house input is sufficiently low. For higher marginal costs, the input ratio is efficient.

By contrast, under limited commitment, the project leader either employs inputs at their efficient ratio or distorts the input ratio *downward*—that is, the contract requires the project supporter to provide the outsourced input in excess of the in-house input. She does the latter in the good environment when her marginal cost of the in-house input is sufficiently large. Since the final output level in the good environment of the project is inflated compared with that under full information, under limited commitment, the project leader has an ex post incentive to reduce her costly in-house input level. This ex post flexibility for the project leader's action due to lack of commitment exacerbates her incentive to misrepresent the project environment. Thus, to convince the project supporter that she has no such incentive, the project leader has to increase the level of the outsourced input—the transfer payment to the supporter for his increased outsourced input level then also has to be increased. As a result, in the good environment, the level of the outsourced input is excessive and the optimal contract entails a downward distortion in the input ratio when the marginal cost of the in-house input is sufficiently high. For lower marginal costs, the input ratio is efficient.

This paper contributes to the literature on vertical contracting. In particular, our study follows the line of research by Hart and Tirole (1990), O'Brian and Shaffer (1992), McAfee and Schwartz (1994), Segal (1999), Segal and Whinston (2003), and, more recently, Dequiedt and Martimort (2015). A central insight of these studies is that the optimal outcomes in vertical relationships crucially depend on commitment power of the party that offers the contract and contractibility of certain variables.[4] Like ours, these papers demonstrate how the performance changes in such relationships for various scenarios regarding commitment power. The key difference between these papers and ours is that, in our paper, the party who makes the contract offer also directly contributes to the final output by producing an input. This allows us to analyze how the direction of the optimal input ratio changes depending on contractibility of the in-house input.

[4] There is a long literature on incomplete contracting. For seminal contributions, see Grossman and Hart (1986) and Hart and Moore (1990).

In settings different from ours, previous work has shown that project size may be inflated due to private information. De Meza and Webb (1987) demonstrate that the inability of lenders (project supporters) to learn entrepreneurs' information results in investment in excess of the socially efficient level. Unlike ours, however, only the pooling contract can be implemented in their model, and the authors do not consider distortions in composition of inputs. More closely related to ours is Khalil and Parigi (1998). They show that lack of contractibility makes the lender push the capital input beyond the efficient level by increasing the size of debt. In these papers, input by the contract offering party does not play a role and, thus, there are no under-utilized inputs as a result of excessive provision.[5]

The result that project size may be inflated due to private information has also been obtained in the literature on supply-chain management. For example, Cachon and Lariviere (2001) study a supply-chain problem in which the manufacturer who offers a contract to its supplier has private information about the market demand (informed principal). In their paper, the manufacturer has an incentive to exaggerate the market demand to induce the supplier to build more production capacity. Unlike our study, however, their paper does not address commitment issues and does not consider distortions of the input ratio. In general, the issue of distorted input ratios has been overlooked by the literature on supply-chain management. Exceptions are Peitz and Shin (2013a, 2013b) who show in a procurement context that a downstream firm may procure supplies from an informed upstream firm in excessive quantity. Their model framework is different, as it features neither an informed principal problem nor a commitment issue. As they show, in their settings, there is always a downward distortion of the input ratio.

The remainder of the paper is organized as follows. Our model is presented in the next section, followed by analysis and results in Section 3. We discuss the robustness of our results in Section 4. Section 5 concludes. All proofs are relegated to the Appendix.

2. Model

We consider a contractual relationship in which a project leader (e.g., a manufacturer) makes an offer to a supporter (e.g., a parts supplier). The supporter operates in a competitive industry and, thus, the leader has the bargaining power and makes a take-it-or-leave-it offer.

Project and Production Process

The project leader sources an intermediate good of quantity $q^s \in \mathbb{R}_+$ (the outsourced input) from the supporter. The leader herself produces another intermediate good of quantity $q^l \in \mathbb{R}_+$ in-house (the in-house input), and combines the two inputs to yield the output of quantity Q. The final output requires both the in-house and the outsourced input, and for simplicity, we let $Q = \min\{q^l, q^s\}$. The leader pays a publicly verifiable transfer of amount $t \in \mathbb{R}$ to the supporter for his input q^s.

[5] Further related work in the context of project finance is mentioned in Section 5.

We consider two cases: full commitment and limited commitment. Under full commitment, both q^l and q^s can be costlessly verified and, thus, they are both contractible variables. Under limited commitment, only the outsourced input level q^s is contractible—it is prohibitively costly to publicly verify the in-house input level q^l.

Project Characteristics and Information Structure

The leader values the output Q by a continuously differentiable, strictly increasing, and concave function $v(Q)$ that satisfies the Inada condition.[6] The project environment, represented by the cost parameters c_i^k, $k \in \{l, s\}$, can be good ($i = G$) or bad ($i = B$), with $c_G^k < c_B^k$. The leader's cost of the in-house input is given by $c_i^l q^l$, and the supporter's cost of the outsourced input by $c_i^s q^s$, $i \in \{G, B\}$. The project environment is good with probability $\phi_G \in (0, 1)$, and bad with probability $\phi_B = 1 - \phi_G$.

The probability distribution ϕ_i is publicly known, but the project environment's realized state $i \in \{G, B\}$ is privately observed only by the project leader after contracting—the realized state is soft information and no hard evidence about it can be obtained. The supporter learns the state $i \in \{G, B\}$ only after he finishes production of the outsourced input. Since $c_G^s \neq c_B^s$, this information is payoff-relevant for the supporter. The project leader, after learning the true state of the project environment, reports a state to the supporter. The leader's report can be publicly verified and, hence, contracted upon.

State $i \in \{G, B\}$ can be interpreted as a matching parameter for the project leader's and the supporter's production technology—the match is good when $i = G$ (thus low cost) and bad when $i = B$ (thus high cost). Our setting captures, for example, situations in which the project leader can see how her production technology matches with the supporter's as soon as his participation, whereas the supporter realizes it only after the project is implemented. Reasons for this asymmetry are that the project leader has more experience, better data, or better access to the project's bigger picture.

Contracts

Under full commitment, publicly verifiable choice variables are q^l, q^s and t. All choice variables are specified in the contract offer contingent on the project leader's report on the project environment $i \in \{G, B\}$. Thus, the contract is a menu $\{(q_i^l,\ q_i^s,\ t_i)_{i \in \{G,B\}}\}$.[7] Under limited commitment, the contract offer is a menu $\{(q_i^s,\ t_i)_{i \in \{G,B\}}\}$, contingent on the project leader's report; the leader determines q_i^l according to her interest at a later stage.

[6] This assures that the project leader wants to have a strictly positive and finite output level in any case.

[7] As in Demski and Sappington (1993), Laffont and Martimort (2002), and Finkle (2005), we employ a screening approach because in our model, the project leader learns the state after contracting. This is unlike informed principal problems in Maskin and Tirole (1990, 1992), Beaudry (1994), and Nosal (2006).

Payoffs and Limited Liability

With a truthful report, the project leader's and the supporter's ex post payoff in state $i \in \{G, B\}$ are respectively:

$$\pi_i = v(Q_i) - t_i - c_i^l q_i^l \quad \text{and} \quad u_i = t_i - c_i^s q_i^s.$$

We assume that the supporter is protected by limited liability—in any state $i \in \{G, B\}$, the leader's offer must ensure the supporter's reservation payoff, normalized to zero. This assumption reflects the Fair Labor Standards Act that allows the players to walk away from a contract when insufficient compensation is expected.

Timing

The timing is specified for full commitment (case I) and limited commitment (case II).

1. (I) Under full commitment, the project leader offers a menu $\{(q_i^l,\, q_i^s,\, t_i)_{i\in\{G,B\}}\}$ to the supporter. (II) Under limited commitment, the leader offers a menu $\{(q_i^s,\, t_i)_{i\in\{G,B\}}\}$ to the supporter.

2. The supporter decides whether to accept the contract.

3. If he accepts, the leader privately observes the true state G or B.

4. The leader makes an announcement about the state $i \in \{G, B\}$.

5. (I) Under full commitment, the leader and the supporter produce q_i^l and q_i^s as specified in the contract for the announced state i, and the leader pays t_i to the supporter. (II) Under limited commitment, the leader produces q_i^l according to her best interest at this point, while the supporter produces q_i^s, as specified in the contract for the announced state i, and the leader pays t_i to the supporter.

Input Ratio

In characterizing input choices, we express the leader's in-house input level as a function of the supporter's outsourced input level: $q_i^l = r_i q_i^s$, where $r_i > 0$, $i \in \{G, B\}$. Here, $r_i = q_i^l / q_i^s$ is the input ratio of in-house to outsourced input. Since providing inputs is costly, $Q_i = \min\{q_i^l, q_i^s\}$ implies that efficiency in production encompasses an input ratio $r_i = 1$ ($q_i^l = q_i^s$). For notational convenience, we let:

$$q_i^l = r_i q_i \text{ and } q_i^s = q_i.$$

Then, the output level is $Q_i = \min\{r_i q_i, q_i\}$ since:

$$Q_i = \begin{cases} r_i q_i & \text{when } r_i \in [0, 1], \\ q_i & \text{when } r_i > 1. \end{cases} \tag{1}$$

If $r_i \neq 1$, then one of the inputs is produced in excess (the input ratio is distorted):

$$\begin{aligned} r_i > 1&: \quad \text{the input ratio is distorted upward } (q_i^l > q_i^s), \\ r_i < 1&: \quad \text{the input ratio is distorted downward } (q_i^l < q_i^s). \end{aligned}$$

The Full-Information Benchmark

If the state i is publicly observed and verifiable, the equilibrium outcome is efficient (within the bilateral relation) and characterized by

$$v'(Q_i^*) = c_i^l + c_i^s, \text{ where } Q_i^* = q_i^*, \ i \in \{G, B\},$$

and the input ratio is efficient—that is, $r_i^* = 1$. The leader's full-information payoffs are:

$$\pi_i^* = v(Q_i^*) - (c_i^l + c_i^s)Q_i^*, \ i \in \{G, B\}.$$

In what follows, we focus on the parameter values that satisfy the following condition:

$$c_B^l < \bar{c}_B^l, \text{ where } \bar{c}_B^l = \left[v(Q_G^*) - c_G^s Q_G^* - \pi_B^*\right]/Q_G^*.$$

We note that this condition is a joint restriction on the function v and all four cost parameters. This assures that the project leader's misrepresenting incentive is an issue under asymmetric information. For $c_B^l \geq \bar{c}_B^l$, the first-best outcome is implemented in the optimal contract.

3. Analysis and Results

We examine the cases of full commitment and limited commitment in the following two subsections. In the latter, we also compare the outcome under limited commitment to the one under full commitment.

3.1. Optimal Contract under Full Commitment

The project leader faces several constraints when making an offer to the supporter. As the revelation principle applies under full commitment, the following incentive compatibility constraints for the leader must be satisfied in the optimal contract:

$$v(Q_i) - c_i^l q_i^l - t_i \geq v(Q_j) - c_i^l q_j^l - t_j, \ i, j \in \{G, B\}, \ i \neq j. \tag{2}$$

Inequalities (2) ensure that the leader's payoff from a truthful report on the state is higher than her payoff from misrepresenting it. In addition, the leader's offer must induce the supporter's participation in each state:

$$t_i - c_i^s q_i^s \geq 0, \ i \in \{G, B\}. \tag{3}$$

Notice that (3) are *ex post* participation constraints for the project supporter that reflect his limited liability, as described in the model section.[8]

[8] If they are replaced with *ex ante* participation constratins, $\sum_i \phi_i \left[t_i - c_i^s q_i^s\right] \geq 0$ (i.e., no limited liability), then the first-best outcome is always achieved in the optimal contract. See Laffont and Martimort (2002, chapter 9) for more on this issue.

With $q_i^l = r_i q_i$, $q_i^s = q_i$ and the expressions in (1), the incentive compatibility constraints in (2) and the participation constraints in (3) can be respectively rewritten as follows:

$$\begin{aligned} v(r_i q_i) - c_i^l r_i q_i - t_i &\geq \begin{cases} v(r_j q_j) - c_i^l r_j q_j - t_j & \text{for } r_i, r_j \in [0,1], \\ v(q_j) - c_i^l r_j q_j - t_j & \text{for } r_i \in [0,1],\ r_j > 1, \end{cases} \\ v(q_i) - c_i^l r_i q_i - t_i &\geq \begin{cases} v(r_j q_j) - c_i^l r_j q_j - t_j & \text{for } r_i > 1,\ r_j \in [0,1], \\ v(q_j) - c_i^l r_j q_j - t_j & \text{for } r_i, r_j > 1, \end{cases} \end{aligned} \tag{IC}$$

and

$$t_i - c_i^s q_i \geq 0, \text{ where } i, j \in \{G, B\},\ i \neq j. \tag{PC}$$

The leader's problem under full commitment, $\mathcal{P}^c$ (where superscript c refers to "commitment" to the in-house input level), is to maximize her expected payoff with respect to r_G, q_G, t_G, r_B, q_B, and t_B, subject to (IC) and (PC). The leader's expected payoff, $E[\pi_i]$, is:

$$E[\pi_i] = \begin{cases} \sum_i \phi_i \left[v(r_i q_i) - c_i^l r_i q_i - t_i \right] \} & \text{for } r_i \in [0,1],\ i \in \{G, B\}, \\ \\ \phi_i \left[v(r_i q_i) - c_i^l r_i q_i - t_i \right] & \text{for } r_i \in [0,1],\ r_j > 1, \\ +\phi_j \left[v(q_j) - c_j^l r_j q_j - t_i \right] & \text{with } i, j \in \{G, B\} \text{ and } i \neq j, \\ \\ \sum_i \phi_i \left[v(q_i) - c_i^l r_i q_i - t_i \right] & \text{for } r_i > 1,\ i \in \{G, B\}. \end{cases}$$

The first proposition shows that, for sufficiently low marginal costs of the leader, the input ratio is distorted upward; outcome variables under *full commitment* carry superscript "*c*."

Proposition 1. *There exist $\underline{c}_G^l$ and $\underline{c}_B^l$ with $\underline{c}_G^l < \underline{c}_B^l < \overline{c}_B^l$ such that, for $c_G^l < \underline{c}_G^l$ and $c_B^l < \underline{c}_B^l$, the outcome in the optimal contract under full commitment entails the following.*

- *In state G, the output level is inflated ($Q_G^c > Q_G^*$) and independent of the project leader's marginal cost. In state B, it is the first-best ($Q_B^c = Q_B^*$).*
- *In state G, the input ratio is distorted upward ($r_G^c > 1$). In state B, it is efficient ($r_B^c = 1$).*

If c_G^l and c_B^l are not small enough, depending on cost parameters and function v either one of the two following outcomes obtains: Either the output level is inflated and the efficient input ratio is chosen or the outcome of the full-information benchmark is implemented. As mentioned above, the latter happens if $c_B^l \geq \overline{c}_B^l$.

Under full commitment, the leader overproduces the output in the optimal contract when the project environment is good (which happens if $c_B^l < \overline{c}_B^l$). In the optimal contract, the project leader uses such an overproduction of the output as an incentive device to induce her own truthful reporting—since the project leader is privately informed of the realized project environment, the supporter questions the validity of the report, because the leader may have an incentive to misreport the realized state.

In particular, when the true state is bad ($i = B$), the leader may benefit by misreporting the state as good ($i = G$) so as to reduce the transfer to the supporter—the transfer to be made to the supporter is larger in state B than in state G to compensate for the higher cost of production. Since the supporter anticipates such an incentive problem, the leader's offer must convince the supporter that the leader's report will be true when she reports that the state is G. For this purpose, the leader increases the output level in state G from the first-best level, thus increasing the transfer to the supporter accordingly in that state.

The inflated output level in state G is accompanied by production of the in-house input in excess depending on the parameters—i.e., an upward distortion in the input ratio. This implies that a fraction of the leader's in-house input is wasted in the optimal contract.

As can be seen from the RHS of (IC), when the true state is B, the project leader can use r_G, q_G and t_G to induce her own truthful behavior. As will be shown below, she optimally increases r_G above the efficient ratio so that q_G and t_G do not need to be distorted too much. Again, the leader has an incentive to misrepresent the state when the true state is B. It is costly for the leader to produce the in-house input and, therefore, committing to produce more than the required level of the input to reach the output level in the announced state G discourages the leader from claiming that the state is G when, in fact, it is B. When the cost of the in-house input is sufficiently small, it is optimal in state G for the leader to produce the in-house input in excess. By doing so, she can moderate the inflated output level (to convince the supporter that she is not misrepresenting the state). To see this, consider the leader's payoff with misrepresenting the state as G when the true state is B:

$$v(q_G) - c_B^l r_G q_G - c_G^s q_G, \tag{4}$$

where $q_G = Q_G$ for $r_G \geq 1$. From the expression, as c_B^l becomes smaller, it becomes more attractive for the project leader to misrepresent the state B as state G. With the efficient input ratio ($r_G = 1$), the output level q_G would need to be heavily oversized to be incentive compatible when c_B^l is small. Producing an excessive amount of the in-house input in the favorable state ($r_G > 1$) allows the leader to moderate the inflated level of q_G, which is optimal when doing so is not very costly (c_G^l is small).

Within the parameter region where $r_G^c > 1$, the optimal outcome is characterized as follows (see proof of Proposition 1 in Appendix A). The optimal outsourced input level q_G^c (which is the optimal output level since $q_G = Q_G$ for $r_G > 1$) is characterized by:

$$v'(q_G^c) = c_G^s. \tag{5}$$

With q_G^c determined by (5), the optimal input ratio r_G^c is characterized by:

$$r_G^c = \frac{v(q_G^c) - c_G^s q_G^c - \pi_B^*}{c_B^l q_G^c}. \tag{6}$$

Notice from (5) and (6) that the optimal output level Q_G^c ($= q_G^c$) depends only on c_G^s, while the optimal input ratio r_G^c depends on c_G^s and c_B^l. Again, within this parameter region, it is better for the project leader to increase the input ratio, instead of further increasing the output level—the project leader adjusts the optimal output level q_G^c only to the supporter's cost of the outsourced input and adjusts r_G^c to both c_B^l and c_G^s.

The following numerical example illustrates our result. Suppose that $v(q) = \sqrt{q}$, $c_G^s = 1$, $c_B^s = 5$, and $c_G^l = 0.1$. Then, the cutoff $\bar{c}_B^l = 0.9998$ (≈ 1). For $c_B^l = 0.5$, we have $q_B^* = 0.8264$, $r_B^* = 1$, $\pi_B^* = 4.5454$ in state B, and, under full commitment, the following outcome in state G: $r_G^c = 1.6363$ (> 1), $Q_G^c = q_G^c = 25$, $r_G^c q_G^c = 40.908$, and $\pi_G^c = 20.9091$. The full-information outcome would be: $r_G^* = 1$, $Q_G^* = q_G^* = r_G^* q_G^* = 20.6612$, and $\pi_G^* = 22.7273$. For comparison, if $r_G = 1$ is imposed as an additional constraint under full commitment, the outcome would be $Q_G = q_G = r_G q_G = 38.1431$ ($> q_G^c$), $\pi_G = 19.8027$ ($< \pi_G^c$).

This exemplifies that the leader strongly benefits from being able to distort the input ratio. The optimal contract has the feature that the leader asks for a lower level of the supporter's production level compared to the setting in which the possibility to distort of the input ratio is ignored (q_G^c is 25 instead of 38.1431), while the leader commits to an in-house level that is larger than in the situation in which the input ratio is fixed at the efficient level (40.908 instead of 38.1431).

To summarize, under full commitment, the project leader 'wastes' some of the in-house input if producing it is not too costly, resulting in an upward distortion in the input ratio. Committing to such a wasteful production allows the project leader to convince the supporter of her truthful behavior without inflating the output level too much. As we will see in the next subsection, without committing to the in-house input level, the project leader has no *ex post* incentive to waste the in-house input. This makes the leader significantly increase the outsourced input level to convince the supporter and, thus, some of the outsourced input is wasted if the in-house input is costly enough.

3.2. Optimal Contract under Limited Commitment

We now proceed to the case in which the leader's in-house input is chosen according to her best interest ex post. Because the leader has no incentive to understate the project environment as B when the true state is G, the equilibrium outcome for $i = B$ is again the full-information outcome and, thus, the same as under full commitment.[9]

Since $\pi_B = \pi_B^*$ in the optimal contract, we can focus on the outcome in state G. The contract offered to the supporter specifies the outsourced input level provided by the supporter, $q_G^s = q_G$, and the transfer, t_G, to him. The leader chooses the input ratio r_G that determines her in-house input level ($q_G^l = r_G q_G$) according to her best interest at the point of carrying out the project. That is, r_G is chosen to maximize the leader's payoff after the outsourced input level $q_G^s = q_G$ has been contracted upon.

Note that, since the leader cannot contract her in-house input level, she will optimally choose $q_G^l \leq q_G^s = q_G$ and, thus, will not waste part of the in-house input. Therefore, using the expression in (1), we immediately obtain the following lemma.

[9] The transfer to the supporter is larger in state B than in state G to compensate for the high production cost and, therefore, the project leader has no incentive to misrepresent state G as state B.

Lemma 1. *Under limited commitment, r_G is optimally chosen such that $r_G \leq 1$.*

Unlike the case under full commitment, an upward distortion in the input ratio ($r_G > 1$) cannot arise when r_G is chosen ex post. The lemma above implies that, in equilibrium,

$$Q_G = \min\{r_G q_G,\ q_G\} = r_G q_G.$$

Even though the leader's choices are not fully committed, the revelation principle holds and, thus, the optimal contract always induces the leader's truthful report. The reason is that only the leader learns the state and also chooses all variables. Hence, no relevant beliefs are affected under limited commitment—in Appendix B, we show that it is optimal for the leader to report truthfully.

Under limited commitment, the contract offered by the leader must respect her choice of the in-house input level, which will be made after contracting with the supporter. Because r_G, which reflects the in-house input level under truth-telling, may be different from the one under misrepresenting the state, we denote by r_G^B the input ratio (given the outsourced input level) when the leader claims that $i = G$ when the true state is B. Then, r_G and r_G^B must satisfy:

$$r_G \in \arg\max_{\widehat{r}_G \in [0,1]} v(\widehat{r}_G q_G) - c_G^l \widehat{r}_G q_G, \tag{EX_G}$$

$$r_G^B \in \arg\max_{\widehat{r}_G^B \in [0,1]} v(\widehat{r}_G^B q_G) - c_B^l \widehat{r}_G^B q_G, \tag{EX_G^B}$$

Since $c_G^l < c_B^l$, the maximizer of $v(r_G q_G) - c_G^l r_G q_G$ cannot be smaller than the maximizer of $v(r_G^B q_G) - c_B^l r_G^B q_G$ for any given q_G—that is $r_G^B \leq r^G$. Furthermore, if $r^G < 1$, we must have $r_G^B < r^G$.[10]

Conditions (EX_G) and (EX_G^B) represent the leader's choice of the in-house input according to her objectives after announcing that $i = G$. Again, (EX_G) represents her choice of the in-house input level in the case of truth-telling, and (EX_G^B) represents her choice in the case of misreporting.

Recall that the optimal outcome associated with state B is the full-information outcome with payoff π_B^*. Under limited commitment, the leader's problem, $\mathcal{P}^n$ (superscript n refers to "non-commitment" to the in-house input level), is to maximize her expected payoff:

$$E[\pi_i] = \phi_G[v(r_G q_G) - c_G^l r_G q_G - t_G] + \phi_B \pi_B,$$

subject to:

$$\pi_B \geq v(r_G^B q_G) - c_B^l r_G^B q_G - t_G, \tag{IC_B^n}$$

$$t_G - c_G^s q_G \geq 0, \tag{PC_G^n}$$

where r_G and r_G^B are given by (EX_G) and (EX_G^B).

The following proposition provides results on project size and input ratio when the leader lacks commitment to the in-house input level. We restrict attention to the parameter range that give rise to a distorted input ratio. Outside this range, the project leader inflates the output level in

[10] Thus, the following three cases need to be considered: $r_G^B < r_G < 1$; $r_G^B < r_G = 1$; and $r_G^B = r_G = 1$.

state G, but does not distort the input ratio.[11] Outcome variables under limited commitment carry superscript "n"—in light of the fact that the leader does *not* commit to the in-house input level.

Proposition 2. *There exist $\overline{c}_G^l$ and $\tilde{c}_B^l$ with $\overline{c}_G^l < \tilde{c}_B^l < \overline{c}_B^l$ such that, for $c_G^l > \overline{c}_G^l$ and $c_B^l < \tilde{c}_B^l$, the outcome implied by the optimal contract under limited commitment entails the following.*

- *The output level in state G is inflated ($Q_G^n > Q_G^*$). In state B, it coincides with the first-best ($Q_B^n = Q_B^*$).*
- *The input ratio in state G is distorted downward ($r_G^n < 1$) for c_G^l large enough. In state B, it is efficient ($r_B^n = 1$).*

As c_B^l becomes smaller, it becomes more attractive for the project leader to misrepresent the state B as state G. As in the case under full commitment, the optimal output level in state G is inflated to discourage the leader from misrepresenting the state as G when the true state is B. However, because the leader cannot commit to the in-house input level, she may have an ex post incentive to save on her cost ex post for the oversized output level by leaving a portion of the outsourced input provided by the supporter unused. This flexibility makes it less costly for the leader to misrepresent state B as state G.

The leader's ex post incentive to reduce the in-house input level, however, is anticipated by the supporter. Therefore, to convince the supporter to accept the leader's offer, the outsourced input (as the contractible variable) needs to be increased even further from its level under full commitment, so that the transfer to the supporter is increased further. As a result, when the leader's input cost is large in state G, the outsourced input is produced in excess and the leader does not utilize the entire outsourced input provided by the supporter.

Within the parameter region where c_G^l is sufficiently large (and thus $r_G^n < 1$), the optimal outcome is characterized as follows (see proof of Proposition 2 in Appendix A). The level of the outsourced input q_G^n is characterized by:

$$v(r_G^B q_G^n) - \left[c_B^l r_G^B + c_G^s\right] q_G^n = \pi_B^* \text{ with } v'(r_G^B q_G^n) = c_B^l, \tag{7}$$

where $r_G^B < 1$. With q_G^n determined by (7), the optimal input ratio r_G^n is characterized by:

$$v'(r_G^n q_G^n) = c_G^l. \tag{8}$$

The output level is $r_G^n q_G^n$ since $Q_G = r_G q_G$ for $r_G < 1$. Notice from the the expressions that, unlike in the case of full commitment, both the optimal output level Q_G^n $(= r_G^n q_G^n)$ and the input ratio r_G^n depend on the leader's marginal cost of the in-house input in the parameter region in which the optimal input ratio is distorted downward—recall that under full commitment, the optimal outsourced input level Q_G^c $(= q_G^c)$ is independent of the leader's marginal cost of the in-house input. Under limited commitment, both Q_G^n and r_G^n are determined according to the leader's ex post interest

[11] For c_G^l small enough, $r_G^n = 1$. In such a case, the optimal outcome may not be separating.

after reporting the state and contracting the outsourced input level q_G, and, since $Q_G = r_G q_G$ in this parameter region, both Q_G^n and r_G^n are affected by the leader's marginal cost of the in-house input.

The following numerical example illustrates our result. Suppose $v(q) = \sqrt{q}$, $c_G^s = 1$, $c_B^s = 5$ (same as the example under full commitment) and $c_G^l = 2$. Then, the cutoff $\bar{c}_B^l = 4$. For $c_B^l = 2.5$, we have $q_B^* = 0.4444$, $r_B^* = 1$, $\pi_B^* = 3.3333$ in state B, and the following outcome in state G: $r_G^n = 0.9375$ (the deviating leader would choose $r_G^B = 0.6$ in state B), $q_G^n = 6.66667$, $Q_G^n = r_G^n q_G^n = 6.25$, and $\pi_G^n = 5.8333$. The full-information outcome would be: $r_G^* = 1$, $q_G^* = r_G^* q_G^* = Q_G^* = 2.7777$, and $\pi_G^* = 8.3333$. For comparison, under full commitment with the same parameters, we obtain $r_G^c = 1$, $q_G^c = r_G^c q_G^c = Q_G^c = 6.11$ ($< q_G^n$), and $\pi_G^c = 6.3883$ ($> \pi_G^n$).

3.3. Comparison

Going beyond a specific numerical example, in this section, we compare the outcomes under limited and full commitment with each other and with the full-information benchmark. We consider situations when the optimal contract under private information with full commitment does not implement the full-information outcome and a separating outcome is implemented under limited commitment.[12] As the next proposition states, input ratios and output levels can be ranked in state G; in state B, the full-information outcome is always achieved.

Proposition 3. *Input ratios and output levels in the three contracting regimes—full information, private information with full and with limited commitment—have the following properties:*

- $Q_G^n > Q_G^c > Q_G^*$;
- $r_G^n \leq r_G^* = 1 \leq r_G^c$.

As mentioned before, output levels and input ratios are independent of the leader's cost of in-house production in state G, c_G^l, under full commitment, but decreasing in c_G^l under limited commitment. Proposition 3 implies that, for the outsourced input level, the ordering $q_G^n > q_G^c > q_G^*$ holds. For the in-house input level, we do not have an unambiguous result, apart from $\min\{r_G^n q_G^n, r_G^c q_G^c\} > r_G^* q_G^*$. Clearly, if (i) $r_G^n < 1$ and $r_G^c = 1$ or (ii) $r_G^n = 1$ and $r_G^c = 1$, then $Q_G^n > Q_G^c$ implies that $r_G^n q_G^n > r_G^c q_G^c$. However, this does not pin down the ordering of the in-house input level when $r_G^n \leq 1$ and $r_G^c > 1$.

Looking beyond the contractual relationship, we observe that while the full-information outcome is first-best efficient within the bilateral relationship, taking a broader perspective that includes consumers of the product sold by the project leader drives a wedge between efficiency in the bilateral relationship and general welfare measures. Suppose that the project leader is a monopolist for the product sold in the market. Then $v(Q_i)$ stands for the monopoly revenue in state i.[13] Clearly,

[12] If the former does not hold, then $Q_G^c = Q_G^*$, while if the latter does not hold $Q_G^n = Q_B^* < Q_G^*$.

[13] Selling Q units of the products implies that the value function is single-peaked and may take some interior maximum $\hat{Q}$. Under free disposal, we can truncate the revenue function at this maximum—for $Q > \hat{Q}$, we set $v(Q) = v(\hat{Q})$. Our analysis then applies.

the full-information outcome neither maximizes consumers surplus nor total surplus. As long as $r_G = 1$, any outcome under limited or full commitment leads to a larger consumer surplus and, since output in the optimal contract satisfies that the price of the output is greater than marginal cost $c_i^l + c_i^s$, a larger total surplus. This is an instance, where multiple sources of market failure (market power, hidden information, and hidden action) improve on the outcome under a single market failure (market power).

In general, Proposition 3 implies that, from a consumer perspective, the outcome under limited commitment is strictly preferred to the outcome under full commitment, which in turn is strictly preferred to the full-information outcome. Total surplus comparisons are ambiguous if the optimal contract features a distorted input ratio. We cannot say that the full-information outcome is always dominated by the outcome with private information under full and/or limited commitment. The reason is that a distorted input ratio results in inefficient production decisions. The question then is whether the resulting surplus loss dominates the total surplus effect from a higher output volume. The same issues arises when comparing the total surplus under limited commitment to the one under full commitment.

4. Discussion

We introduce several modifications to shed light on the impact of some of our modeling assumptions.

First, the project leader's bargaining power is crucial for our results. If the bargaining power shifts to the supporter—i.e., the party without the private information offers the contract to the privately informed party—then the optimal outcome is not accompanied by a distortion in the input ratio, regardless of commitment power. The reason is as follows. In the setting with reversed bargaining power, the supporter's contract offer maximizes his expected payoff, $\sum_i \phi_i \left[t_i - c_i^s q_i\right]$, subject to the following incentive compatibility and participation constraints of the leader:

$$v(Q_i) - c_i^l r_i q_i - t_i \geq v(Q_j) - c_i^l r_j q_j - t_j,$$
$$v(Q_i) - c_i^l r_i q_i - t_i \geq 0,$$

where $Q_i = \min\{q_i,\ r_i q_i\}$ and $i, j \in \{G, B\}$. Notice first that r_i does not enter the supporter's objective function, $\sum_i \phi_i \left[t_i - c_i^s q_i\right]$, and therefore, the optimal Q_i and t_i of the supporter will be the same with or without commitment to r_i (which represents the leader's in-house input level). Also, the leader's manipulating incentive changes its direction. When the contract is offered by the supporter, the leader has an incentive to understate (instead of overstate) the project environment so that she can "pocket" the project's revenue as much as possible. Therefore, as in the standard screening problem, the optimal output level becomes under-sized when state B is announced ($q_B < q_B^* = Q_B^*$). If $r_B < 1$ (i.e., $q_B^s > q_B^l$), the supporter could always gain by decreasing his outsourced input level. If $r_B > 1$ (i.e., $q_B^s < q_B^l$), the supporter would simply give up extra rent to the leader. Hence, we must have $r_B = 1$. To summarize, in our model, distortions in the input ratio can take place only if the party that learns the project environment makes the offer to the party without the information (with some positive probability).

Second, we can make the supporter, instead of the leader, the residual claimant. Our result is robust to which party is the residual claimant, as long as the leader offers the contract. To see this, suppose that the leader offers the contract, but the supporter takes $v(Q_i)$ and pays t_i to the leader. In such case, the leader's contract offer maximizes her expected payoff: $\sum_i \phi_i \left[t_i - c_i^l r_i q_i\right]$, subject to the incentive compatibility condition for herself,

$$t_i - c_i^l r_i q_i \geq t_j - c_i^l r_j q_j,$$

and the participation constraint for the supporter,

$$v(Q_i) - c_i^s q_i - t_i \geq 0,$$

where $Q_i = \min\{q_i, r_i q_i\}$ and $i, j \in \{G, B\}$. Since the participation constraints for the supporter are binding in the optimal contract, we must have $t_i = v(Q_i) - c_i^s q_i$. Substituting for t_i in the objective function and the incentive compatibility constraint, the problem becomes the same as our original problem.

Third, we assumed that the leader cannot use the supporter's input for alternative internal or external use. If, by contrast, this is the case, our mechanism still applies as long as this internal or external alternative use is sufficiently unattractive. Then, under limited commitment, the excessive amount of outsourced input tends to be even larger.

Fourth, we assumed that the leader's in-house input and the supporter's outsourced input are perfect complements. This led to a tractable framework with the specific feature that in the optimal contract a fraction of one of the two inputs is wasted. In a more general setting, as long as these inputs are not perfect substitutes, the same forces as in our model are at work. Hence, we conjecture that with limited substitutability of inputs, under some conditions, the optimal contract is accompanied by an upward distortion of the in-house input under full commitment and a downward distortion under limited commitment.

5. Conclusion

While contractual distortions in vertical relationships have been extensively analyzed by previous contributions, the theoretical literature has mainly focused on the distortions in the size of the project solely determined by the project supporter's input. Casual observations, however, indicate that both the project leader and the project supporter provide inputs for the final output. It is a common practice, for example, that a manufacturer produces some components of a product in-house, while outsourcing other components from its suppliers. In this paper, we have considered such a setting to investigate distortions in input ratio in a vertical relationship—the project leader combines its in-house input with the outsourced input from the supporter to yield the final output. The project environment is private information of the project leader.

We analyzed whether and why there could be excessive amount of either input produced in the optimal contract. As we have shown, either input may optimally be over-produced and, thus, only partially be used for the final output, depending on whether the leader can commitment to its input level. Due to her private information, the leader must make the output level oversized when contracting with the supporter. Under full commitment, the leader may optimally produce an excessive amount of the in-house input, leaving some of it unused. Under limited commitment, the outsourced input level is increased from the level under full commitment, while production of the in-house input may be reduced by the leader. Thus, our results suggest that the presence or absence of the project leader's commitment power to its own input provision, as well as her superior information in a vertical relationship, is an important determinant of distortions in input ratio and may explain deviations from the first-best efficient input allocation. This prediction can, in principle, be empirically tested.

Our analysis can then be applied to project financing—a project requires funding (capital) which, when combined with the entrepreneurial effort (labor), possibly generates a revenue. The situation we have in mind is that an entrepreneur seeks financing from a lender before the project's prospects are realized; the contract includes a line of credit depending on the prospects of the project. The entrepreneur then obtains outside financing through a debt contract to carry out her project.[14] In such financial relationships, the lender is at an informational disadvantage because the entrepreneur often has private information about the project environment, such as idiosyncratic risk of adverse conditions possibly leading to the liquidation of the firm which affects the funding cost.

As is well known, a privately informed entrepreneur may push a project to a lender by exaggerating its prospects. In some situations, the entrepreneur's activity is well defined and can be closely monitored. In such cases, her effort level for the project is committed. In other situations, the contract between the entrepreneur and the lender is subject to the entrepreneur's limited commitment. While capital and effort are arguably not perfect complements, we expect that, as long as they are not perfect substitutes, under some conditions, input ratios will be distorted.[15]

Finally, we note that some outsourcing environments feature repeated contracting and a combination of formal commitments and informal promises (see, e.g., Levin, 2003, and Li and Matouschek, 2013). Future work may want to study whether and under which circumstances distorted input ratios arise with relational contracts.

[14] As documented by Birtler et al. (2005) for the U.S. in the 1990's, in the majority of privately held firms, the entrepreneur holds 100% of the equity.

[15] This complements work on contracting problems when entrepreneurs are privately informed. For example, Bolton and Scharfstein (1990) analyze how the distribution of wealth across privately informed entrepreneurs and uninformed lenders affects contracting relationships. Ueda (2004) studies the entrepreneur's source of funding, which determines whether her information is private or public. Dessi (2005) studies a contractual relationship under collusion between the entrepreneur and a monitoring intermediary. Our paper contributes to this line of research by focusing on the input composition when effort has to be provided by the entrepreneur as a complementary input.

Appendix A: Proofs

Proof of Proposition 1.

Notice from (IC) that there are four possible cases: (i) $r_G, r_B \in [0,1]$, (ii) $r_G, r_B > 1$, (iii) $r_G \in [0,1]$ and $r_B > 1$, and (iv) $r_G > 1$ and $r_B \in [0,1]$. However, with including the corner solutions also in case (ii), i.e., $r_G \geq 1$ and $r_B \geq 1$, it is sufficient to check cases (i) and (ii). Below we first consider case (i) where $r_G, r_B \in [0,1]$ to show that $r_G = 1$ with $Q_G > Q_G^*$ and $r_B = 1$ with $Q_B = Q_B^*$ (the outcome associated with state B is the first best)—that is, $\gamma_G = 1$ and $\gamma_B = 1$ if they are restricted to the range of $[0,1]$. Since the output level associated with state G is distorted, we then proceed to case (ii) by allowing the corner solution in the ratio, $r_G, r_B \geq 1$ to check if $r_G = 1$ and $r_B = 1$ from case (i) is indeed the optimal solution.

Case (i) $r_G, r_B \in [0,1]$:

The Lagrangian of the leader's problem is as follows:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum\nolimits_i \phi_i \left[v(r_i q_i) - c_i^l r_i q_i - t_i\right] \\
& + \lambda_1 \left[t_G - c_G^s q_G\right] + \lambda_2 \left[t_B - c_B^s q_B\right] \\
& + \lambda_3 \left[v(r_G q_G) - c_G^l r_G q_G - t_G - v(r_B q_B) + c_G^l r_B q_B + t_B\right] \\
& + \lambda_4 \left[v(r_B q_B) - c_B^l r_B q_B - t_B - v(r_G q_G) + c_B^l r_G q_G + t_G\right], \text{ with } 1 \geq r_i \geq 0.
\end{aligned}$$

First-order conditions of maximizing the Lagrangian are

$$\frac{\partial L}{\partial t_G} = -\phi_G + \lambda_1 - \lambda_3 + \lambda_4 = 0, \qquad (A1)$$

$$\frac{\partial L}{\partial t_B} = -\phi_B + \lambda_2 + \lambda_3 - \lambda_4 = 0, \qquad (A2)$$

$$\begin{aligned}\frac{\partial L}{\partial q_G} = \; & \phi_G \left[r_G v'(r_G q_G) - c_G^l r_G\right] - \lambda_1 c_G^s \\ & + \lambda_3 \left[r_G v'(r_G q_G) - c_G^l r_G\right] - \lambda_4 \left[r_G v'(r_G q_G) - c_B^l r_G\right] = 0,\end{aligned} \qquad (A3)$$

$$\begin{aligned}\frac{\partial L}{\partial q_B} = \; & \phi_B \left[r_B v'(r_B q_B) - c_B^l r_B\right] - \lambda_2 c_B^s \\ & - \lambda_3 \left[r_B v'(r_B q_B) - c_G^l r_B\right] + \lambda_4 \left[r_B v'(r_B q_B) - c_B^l r_B\right] = 0,\end{aligned} \qquad (A4)$$

From $(A1)$, $\phi_G + \lambda_3 = \lambda_1 + \lambda_4$. Therefore, $(A3)$ gives:

$$(\lambda_1 + \lambda_4)\left[r_G v'(r_G q_G) - c_G^l r_G\right] - \lambda_1 c_G^s - \lambda_4 \left[r_G v'(r_G q_G) - c_B^l r_G\right] = 0.$$

After a simple rearrangement the equation becomes:

$$\lambda_1 \left[r_G v'(r_G q_G) - (c_G^l r_G + c_G^s)\right] = \lambda_4 \left(c_G^l - c_B^l\right) r_G. \qquad (A5)$$

In $(A5)$, if $\lambda_1 = 0$ then it must be that $\lambda_4 = 0$. Then, $(A1)$ gives $\phi_G = -\lambda_3$ and we have a contradiction. Therefore, $\lambda_1 > 0$ and thus $t_G = c_G^s q_G$.

From $(A2)$, $\phi_B + \lambda_4 = \lambda_2 + \lambda_3$. Therefore, $(A4)$ gives:

$$(\lambda_2 + \lambda_3)\left[r_B v'(r_B q_B) - c_B^l r_B\right] - \lambda_2 c_B^s - \lambda_3 \left[r_B v'(r_B q_B) - c_G^l r_B\right] = 0.$$

After a simple rearrangement the equation becomes:

$$\lambda_2 \left[r_B v'(r_B q_B) - (c_B^l r_B + c_B^s)\right] = \lambda_3 \left(c_B^l - c_G^l\right) r_B. \tag{$A6$}$$

In ($A6$), if $\lambda_2 = 0$ it must be that $\lambda_3 = 0$. Then, ($A1$) gives $\phi_B = -\lambda_4$, and we have a contradiction. Therefore, $\lambda_2 > 0$ and, thus, $t_B = c_B^s q_B$.

We now show that $\lambda_3 = 0$. Suppose that $\lambda_3 > 0$. Then, since $\lambda_2 > 0$, ($A6$) implies that $r_B v'(r_B q_B) - (c_B^l r_B + c_B^s) > 0$. This implies that the optimal output level is distorted downward: $Q_B < Q_B^*$. For $r_B \in (0, 1]$, however, the leader can always increase q_B by some small amount to increase her expected payoff. Thus, it must be that $\lambda_3 = 0$ in the optimum.

Since $t_G = c_G^s q_G$, $t_B = c_B^s q_B$ and $\lambda_3 = 0$, we can rewrite the Lagrangian as:

$$\begin{aligned}\mathcal{L} &= \sum\nolimits_i \phi_i \left[v(r_i q_i) - c_i^l r_i q_i - c_i^s q_i\right] \\ &\quad +\lambda_4 \left[v(r_B q_B) - c_B^l r_B q_B - c_B^s q_B - v(r_G q_G) + c_B^l r_G q_G + c_G^s q_G\right],\end{aligned}$$

where $r_i \in (0, 1]$. First-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[r_G v'(r_G q_G) - (c_G^l r_G + c_G^s)\right] - \lambda_4 \left[r_G v'(r_G q_G) - (c_B^l r_G + c_G^s)\right] = 0, \tag{$A7$}$$

$$\frac{\partial \mathcal{L}}{\partial q_B} = (\phi_B + \lambda_4) \left[r_B v'(r_B q_B) - (c_B^l r_B + c_B^s)\right] = 0. \tag{$A8$}$$

From ($A8$), we have $r_B v'(r_B q_B) - (c_B^l r_B + c_B^s) = 0$ implying that:

$$r_B \left[v'(r_B q_B) - c_B^l\right] = c_B^s > 0. \tag{$A9$}$$

Differentiating the Lagrangian with respect to r_B gives:

$$\frac{\partial \mathcal{L}}{\partial r_B} = (\phi_B + \lambda_4) \left[v'(r_B q_B) - c_B^l\right] q_B > 0,$$

since $v'(r_B q_B) - c_B^l > 0$ from ($A9$). Therefore, $r_B = 1$, and ($A8$) implies that $q_B = Q_B^*$. Consequently, the optimal outcome for $i = B$ is the first best.

From ($A7$), we have:

$$\phi_G \left[r_G v'(r_G q_G) - (c_G^l r_G + c_G^s)\right] = \lambda_4 \left[r_G v'(r_G q_G) - (c_B^l r_G + c_G^s)\right]. \tag{$A10$}$$

If $\lambda_4 = 0$, then the Lagrangian implies that the outcome is the first-best and $r_G = 1$. However, with the first-best outcome for $c_B^l < \bar{c}_B^l$, we observe that:

$$\pi_B^* < v(Q_G^*) - c_B^l q_G^* - c_G^s q_G^*,$$

implying that the constraint (IC) in state B is violated. Thus, $\lambda_4 > 0$ for $c_B^l < \bar{c}_B^l$. With $\lambda_4 > 0$, ($A5$) implies that:

$$r_G v'(r_G q_G) - (c_G^l r_G + c_G^s) < 0,$$

on the left-hand side of $(A10)$. Therefore, the right-hand side of $(A10)$ must also be negative, and, since $r_G v'(r_G q_G) - (c_B^l r_G + c_G^s) < r_G v'(r_G q_G) - (c_G^l r_G + c_G^s) < 0$, it must be that $\phi_G > \lambda_4$. Also, from $(A10)$, we have:

$$\left\{\phi_G \left[v'(r_G q_G) - c_G^l\right] - \lambda_4 \left[v'(r_G q_G) - c_B^l\right]\right\} r_G = (\phi_G - \lambda_4) c_G^s > 0. \tag{A11}$$

Now, differentiating the Lagrangian with respect to r_G gives:

$$\frac{\partial \mathcal{L}}{\partial r_G} = \left\{\phi_G \left[v'(r_G q_G) - c_G^l\right] - \lambda_4 \left[v'(r_G q_G) - c_B^l\right]\right\} q_G > 0,$$

where the strict inequality is implied by $(A11)$. Therefore, $r_G = 1$ and since $r_G v'(r_G q_G) - (c_G^l r_G + c_G^s) < 0$, we have $q_G = Q_G > Q_G^*$.

Case (ii) $r_G, r_B \geq 1$:

Recall that the solution for $i = B$ in case (i)—that is, $r_G, r_B \in (0, 1]$—is the first best, $\pi_B = \pi_B^*$. It can be shown in a similar way that the solution for $i = B$ in the case of $r_B \geq 1$ is the first best. Regarding r_G, if we have the corner solution here, then the optimal outcome is the one that we obtained for the case of $r_G \in (0, 1]$. If we have an interior solution here, then the constrained optimum with $r_G = 1$ in the previous case cannot be optimal.

The Lagrangian of the leader's problem for $r_i \geq 1$ is:

$$\begin{aligned} \mathcal{L} &= \phi_G \left[v(q_G) - c_G^l r_G q_G - t_G\right] + \phi_B \pi_B^* \\ &\quad + \lambda_5 \left[t_G - c_G^s q_G\right] \\ &\quad + \lambda_6 \left[\pi_B^* - v(q_G) + c_B^l r_G q_G + t_G\right], \text{ with } r_G \geq 1. \end{aligned}$$

Differentiating with respect to t_G and q_G, respectively, gives:

$$\frac{\partial \mathcal{L}}{\partial t_G} = -\phi_G + \lambda_5 + \lambda_6 = 0, \tag{A12}$$

$$\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[v'(q_G) - c_G^l r_G\right] - \lambda_5 c_G^s - \lambda_6 \left[v'(q_G) - c_B^l r_G\right] = 0. \tag{A13}$$

From $(A12)$, we have $\phi_G = \lambda_5 + \lambda_6$ and, therefore, $(A13)$ can be rewritten as:

$$\lambda_5 \left[v'(q_G) - (c_G^l r_G + c_G^s)\right] = \lambda_6 \left[c_G^l - c_B^l\right] r_G. \tag{A14}$$

In $(A14)$, if $\lambda_5 = 0$ then it must be that $\lambda_6 = 0$, which leads to a contradiction since $\lambda_5 + \lambda_6 = \phi_G > 0$ from $(A12)$. Therefore, $\lambda_5 > 0$ and, thus, $t_G = c_G^s q_G$. If we have the corner solution, $r_G = 1$, then the optimal outcome is the solution that we obtained in the previous case, and $(A5)$ with $r_G = 1$ gives $v'(q_G) - (c_G^l + c_G^s) < 0$. This means, in $(A14)$, that $\lambda_5 > 0$ implies that $\lambda_6 > 0$, if we have the corner solution. If we have an interior solution, $r_G > 1$, then we still have $r_G v'(r_G q_G) - (c_G^l r_G + c_G^s) q_G < 0$, and, hence, in $(A14)$, $\lambda_5 > 0$ implies that $\lambda_6 > 0$.

Also, since $t_G = c_G^s q_G$, we can rewrite the Lagrangian as:

$$\begin{aligned} \mathcal{L} &= \phi_G \left[v(q_G) - c_G^l r_G q_G - c_G^s q_G\right] + \phi_B \pi_B^* \\ &\quad + \lambda_6 \left[\pi_B^* - v(q_G) + c_B^l r_G q_G + c_G^s q_G\right], \text{ with } r_G \geq 1. \end{aligned}$$

The first-order condition with respect to q_G is:

$$\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[v'(q_G) - \left(c_G^l r_G + c_G^s \right) \right] - \lambda_6 \left[v'(q_G) - \left(c_B^l r_G + c_G^s \right) \right] = 0.$$

Rearranging this equation we have:

$$\left[\phi_G c_G^l - \lambda_6 c_B^l \right] r_G = \left(\phi_G - \lambda_6 \right) \left[v'(q_G) - c_G^s \right]. \qquad (A15)$$

Note that $\phi_G > \lambda_6$, as shown above. Differentiating the Lagrangian with respect to r_G gives:

$$\frac{\partial \mathcal{L}}{\partial r_G} = \left(\lambda_6 c_B^l - \phi_G c_G^l \right) q_G. \qquad (A16)$$

Since $\phi_G > \lambda_6$, if c_G^l is close enough to $\overline{c}_B^l$, $\partial \mathcal{L}/\partial r_G < 0$ in $(A16)$, implying that $r_G = 1$. Therefore c_G^l must be small enough for $r_G > 1$. Also, at $c_B^l = \overline{c}_B^l$, we have $\lambda_6 = 0$ (at $c_B^l = \overline{c}_B^l$, the incentive constraint linked to λ_6 is automatically satisfied). Since $\lambda_6 > 0$ for $c_B^l < \overline{c}_B^l$, in the neighborhood of $\overline{c}_B^l$, we have $\partial \lambda_6 / \partial c_B^l < 0$ for $c_B^l < \overline{c}_B^l$. Since $\partial \lambda_6 / \partial c_B^l < 0$, if c_G^l is small enough, we have that $\partial \mathcal{L}/\partial r_G \geq 0$, when c_B^l is smaller than $\overline{c}_B^l$. Since the solution has an upper bound—i.e., $r_G \neq \infty$—it must be that $\partial \mathcal{L}/\partial r_G = 0$ in such a case, implying that $r_G \in (1, \infty)$. Thus, it is implied that there exist $\underline{c}_G^l$ and $\underline{c}_B^l \in (\underline{c}_G^l, \overline{c}_B^l)$ such that, for $c_G^l < \underline{c}_G^l$ and $c_B^l < \underline{c}_B^l$, the optimal contract exhibits $r_G^c > 1$.

From $(A16)$, an interior solution $(\partial \mathcal{L}/\partial r_G = 0)$ requires that:

$$\lambda_6 = \frac{\phi_G c_G^l}{c_B^l}, \qquad (A17)$$

which implies that the left-hand side of $(A15)$ is zero. Then, from the right-hand side of $(A15)$, it must be that:

$$v'(q_G) - c_G^s = 0, \qquad (A18)$$

and q_G is characterized by $(A18)$. Since $\lambda_6 > 0$, the associated binding constraint gives:

$$r_G = \frac{v(q_G) - c_G^s q_G - \pi_B^*}{c_B^l q_G}. \qquad (A19)$$

Case (iii) $r_G \in [0, 1]$, $r_B \geq 1$ and **Case** (iv) $r_G \geq 1$, $r_B \in [0, 1]$:

In both cases, as in the previous cases, the solution for $i = B$ is the first best: $r_B = 1$ and $q_B = Q_B^*$. Since we always have $r_B = 1$, it is then sufficient to consider cases (i) and (ii). ■

Proof of Lemma 1.

Suppose, by contradiction, that $r_i > 1$ is optimal when chosen ex post. For $r_i \geq 1$, the leader's problem with respect to r_i is $\max_{r_i} \; v(q_i) - c_i^l r_i q_i$, $i \in \{G, B\}$. It is clear from the problem that the leader will set r_i as small as possible, implying that the optimal r_i cannot be larger than 1. ■

Proof of Proposition 2.

The Lagrangian of the leader's problem is:

$$\begin{aligned}
\mathcal{L} &= \phi_G[v(r_G q_G) - c_G^l r_G q_G - t_G] + \phi_B \pi_B^* \\
&\quad + \lambda_7 \left[t_G - c_G^s q_G\right] \\
&\quad + \lambda_8 \left[\pi_B^* - v(r_G^B q_G) + c_B^l r_G^B q_G + t_G\right], \text{ with } (EX_G) \text{ and } (EX_G^B).
\end{aligned}$$

First-order conditions are:

$$\begin{aligned}
\frac{\partial L}{\partial t_G} &= -\phi_G + \lambda_7 + \lambda_8 = 0, && (A20)\\
\frac{\partial L}{\partial q_G} &= \phi_G \left[r_G v'(r_G q_G) - c_G^l r_G\right] - \lambda_7 c_G^s && (A21)\\
&\quad -\lambda_8 \left[r_G^B v'(r_G^B q_G) - c_B^l r_G^B\right] = 0.
\end{aligned}$$

Since $\lambda_7 = \phi_G - \lambda_8$ from $(A20)$, we can rewrite $(A21)$ as:

$$\phi_G \left[r_G v'(r_G q_G) - \left(c_G^l r_G + c_G^s\right)\right] = \lambda_8 \left[r_G^B v'(r_G^B q_G) - (c_B^l r_G^B + c_G^s)\right]. \tag{A22}$$

First, suppose that $\lambda_7 > 0$. Then, $\lambda_7 = \phi_G - \lambda_8$ implies that $\phi_G > \lambda_8$, and $(A22)$ in turn implies that:

$$r_G \left[v'(r_G q_G) - c_G^l\right] < r_G^B \left[v'(r_G^B q_G) - c_B^l\right]. \tag{A23}$$

From (EX_G) and (EX_G^B), $r_G \geq r_G^B$. If $r_G = 1$ and $r_G^B < 1$, then (EX_G) and (EX_G^B) imply that the left-hand side of $(A23)$ is strictly positive and the right-hand side of $(A23)$ is zero. This contradicts the inequality in $(A23)$. If $r_G = r_G^B = 1$, then we have $v'(q_G) - c_G^l < v'(q_G) - c_B^l$ from $(A23)$, which is a contradiction. If $r_G < 1$ and $r_G^B < 1$ (in which case $r_G^B < r_G$), then both sides of $(A23)$ are zero, which contradicts the inequality. Thus, it must be that $\lambda_7 = 0$, which implies that $\lambda_8 = \phi_G (> 0)$ from $(A21)$.

With $\lambda_7 = 0$ and $\lambda_8 = \phi_G$, the equation in $(A22)$ becomes:

$$r_G \left[v'(r_G q_G) - c_G^l\right] = r_G^B \left[v'(r_G^B q_G) - c_B^l\right]. \tag{A24}$$

Again from (EX_G) and (EX_G^B), $r_G \geq r_G^B$. If $r_G = r_G^B = 1$, then we have a contradiction in the above equation since $v'(q_G) - c_G^l > v'(q_G) - c_B^l$ in that case. There remain two possible cases: $r_G = 1$ and $r_G^B < 1$, and $r_G < 1$ and $r_G^B < 1$ (in which case $r_G^B < r_G$). That is, it must then hold that $r_G^B < 1$ and $r_G^B < r_G$ in any case. Since $\lambda_8 > 0$, (IC_B^n) implies that $\pi_B^* = v(r_G^B q_G) - c_B^l r_G^B q_G - t_G$. There is some leeway in this equation since $\lambda_7 = 0$; i.e., $t_G - c_G^s q_G \geq 0$. Setting the lowest transfer $t_G = c_G^s q_G$, we have:

$$\pi_B^* = v(r_G^B q_G) - c_B^l r_G^B q_G - c_G^s q_G. \tag{A25}$$

The value of q_G and r_G^B are determined by solving $(A25)$ and (EX_G^B) simultaneously. Rewriting $\pi_B^* = \pi_B^*(c_B^l)$, $q_G^n = q_G^n(c_B^l)$ and $r_G^B = r_G^B(c_B^l)$, differentiating the expression in $(A25)$ with respect to c^l_B gives:

$$\begin{aligned}\frac{\partial \pi^*_B}{\partial c^l_B} &= v'(r^B_G q^n_G)\left[\frac{\partial r^B_G}{\partial c^l_B}q^n_G + \frac{\partial q^n_G}{\partial c^l_B}r^B_G\right] - \left[r^B_G q^n_G + c^l_B q^n_G \frac{\partial r^B_G}{\partial c^l_B} + c^l_B r^B_G \frac{\partial q^n_G}{\partial c^l_B}\right] - c^s_G \frac{\partial q^n_G}{\partial c^l_B} \\ &= \underbrace{\left[v'(r^B_G q^n_G) - c^l_B\right]}_{=0}\left[\frac{\partial r^B_G}{\partial c^l_B}q^n_G + \frac{\partial q^n_G}{\partial c^l_B}r^B_G\right] - r^B_G q^n_G - c^s_G \frac{\partial q^n_G}{\partial c^l_B},\end{aligned}$$

which is rewritten as:

$$\frac{\partial q^n_G}{\partial c^l_B} = \frac{q^*_B - r^B_G q^n_G}{c^s_G} < 0,$$

where $q^*_B = -\partial \pi^*_B/\partial c^l_B$ by the envelope theorem, and $q^*_B - r^B_G q^n_G < 0$ since q^*_B is the maximizer of $v(q_B) - \left[c^l_B + c^s_B\right] q_B$ and $r^B_G q^n_G$ maximizes $v(r^B_G q_G) - c^l_B r^B_G q_G$. As c^l_B becomes smaller, q^n_G becomes larger, which implies that, for c^l_G large enough, $r_G < 1$, i.e., $v'(r_G q^n_G) - c^l_G = 0$ from (EX_G), with the value of q^n_G determined by $(A25)$ and (EX^B_G). Thus it is implied that there exist $\overline{c}^l_G$ and $\widetilde{c}^l_B \in (\overline{c}^l_G, \overline{c}^l_B)$ such that, for $c^l_G > \overline{c}^l_G$ and $c^l_B < \widetilde{c}^l_B$, the outcome implied by the optimal contract entails $r^n_G < 1$.
■

5.1. Proof of Proposition 3.

The result that $r^n_G \leq r^*_G = 1 \leq r^c_G$ follows directly from Proposition 1 and 2. For ranking of the output levels, $Q^n_G > Q^c_G > Q^*_G$, consider first the following equations representing the binding incentive constraints with full and with limited commitment respectively:

$$\pi^*_B = v(q^c_G) - c^l_B r^c_G q^c_G - c^s_G q^c_G. \tag{A26}$$

$$\pi^*_B = v(r^B_G q^n_G) - c^l_B r^B_G q^n_G - c^s_G q^n_G, \tag{A27}$$

where $r^c_G \geq 1$ and $r^B_G < 1$ (see the proof of Proposition 2). Suppose that $r^c_G = 1$. Then, differentiating $(A26)$ with respect to c^l_B gives:

$$\frac{\partial q^c_G}{\partial c^l_B} = \frac{q^c_G - q^*_B}{v'(q^c_G) - c^l_B - c^s_G} < 0,$$

where $-q^*_B = \partial \pi^*_B/\partial c^l_B$ from the envelope condition and $v'(q^c_G) - c^l_B - c^s_G < 0$ follows from $v'(q^c_G) - c^l_G - c^s_G < 0$ ($\because q^c_G > q^*_G$). The equation

$$\pi^*_B = v(\widehat{q}_G) - c^l_B r^B_G \widehat{q}_G - c^s_G \widehat{q}_G \tag{A28}$$

implicitly defines $\widehat{q}_G$. Together with $(A26)$ with $r^c_G = 1$, we have:

$$v(q^c_G) - c^l_B q^c_G - c^s_G q^c_G = v(\widehat{q}_G) - c^l_B r^B_G \widehat{q}_G - c^s_G \widehat{q}_G.$$

Let $\widehat{c}^l_B \equiv c^l_B r^B_G$. Since $\widehat{c}^l_B < c^l_B$, the inequality $\widehat{q}_G > q^c_G$ is implied by $\partial q^c_G/\partial c^l_B < 0$. From $(A27)$ and $(A28)$, $q^n_G > \widehat{q}_G$. Now let $\widetilde{c}^l_B$ be the cutoff level of c^l_B such that $r^c_G > 1$ for $c^l_B < \widetilde{c}^l_B$ and $r^c_G = 1$ for $c^l_B = \widetilde{c}^l_B$. For $r^c_G > 1$, q^c_G is obtained by $(A18)$, $v'(q^c_G) - c^s_G = 0$, and $\partial q^c_G/\partial c^l_B = 0$. Since $\partial q^n_G/\partial c^l_B < 0$, we have $q^n_G > q^c_G$ for $r^c_G > 1$.

Since the outsourced input levels are such that $q_G^n > q_G^c$ in any case, the following is true:

$$Q_G^n(= q_G^n) > Q_G^c(= q_G^c) \text{ if } \begin{cases} r_G^c = r_G^n = 1, \text{ or} \\ r_G^c > 1 \text{ and } r_G^n = 1 \end{cases} \tag{A28}$$

That is, when $r_G^n = 1$ we have $Q_G^n > Q_G^c$.

Now we show that $Q_G^n > Q_G^c$ when $r_G^n < 1$. Recall that, when $r_G^n < 1$, the output level $Q_G^n = r_G^n q_G^n$ is characterized by:

$$v'(r_G^n q_G^n) = c_G^l. \tag{A29}$$

Recall from the proof of Proposition 2 that q_G^n is independent of c_G^l. We then define $\tilde{c}_G^l$ by $v'(r_G^n q_G^n) = \tilde{c}_G^l$ such that, $r_G^n < 1$ for all $c_G^l > \tilde{c}_G^l$ and $r_G^n = 1$ for $c_G^l = \tilde{c}_G^l$ for given other interior parameter values (so $q_G^n = Q_G^n$ at $c_G^l = \tilde{c}_G^l$). Since $r_G^n = 1$ for $c_G^l = \tilde{c}_G^l$, we have from (A28) that:

$$r_G^n q_G^n = Q_G^n > Q_G^c \text{ for } c_G^l = \tilde{c}_G^l.$$

Let us now decrease the value of parameter c_G^s. From (A26) and (A27), a lower value of c_G^s makes misrepresenting the state B as state G more attractive to the leader and, thus, both q_G^c and q_G^n decrease in c_G^s. However, from (A29), the output level $r_G^n q_G^n = Q_G^n$ is affected only by c_G^l and, thus, independent of c_G^s. This implies that r_G^n must decrease as q_G^n increases. In other words, for $c_G^l = \tilde{c}_G^l$, as c_G^s decreases, r_G^n decreases ($r_G^n < 1$) and q_G^n increases without changing $r_G^n q_G^n$. Now, given $c_G^l = \tilde{c}_G^l$, in the limit as $c_G^s \longrightarrow 0$, we must have $r_G^n < 1$ and by (A26) and (A27) we must have that:

$$v(q_G^c) - c_B^l r_G^c q_G^c = v(r_G^B q_G^n) - c_B^l r_G^B q_G^n.$$

This in turn implies that $r_G^B q_G^n \geq q_G^c$ with equality when $r_G^c = 1$. Since $Q_G^n = r_G^n q_G^n > r_G^B q_G^n$ ($\because r_G^n > r_G^B$ from the proof of Proposition 2) and $q_G^c = Q_G^c$, we have: $Q_G^n > Q_G^c$ when $r_G^n < 1$. ■

Appendix B: Truthful Reporting under Limited Commitment

In this appendix, we demonstrate that truthful reporting by the project leader is optimal under limited commitment. We denote by α the probability that the leader makes a truthful report when the true state is $i = B$, and by β the probability that the supporter accepts the leader's offer. We show that, in equilibrium, $\alpha = \beta = 1$.

Under limited commitment, the contract offered by the leader must respect the later choices of the leader and the supporter according to each party's objective functions at the corresponding stages. Since r_G (which denotes the input ratio under truth-telling) may be different from the input ratio when the state is misrepresented, we denote by r_G^B the leader's input level when she claims that $i = G$ while the true state is B. In equilibrium, α, β, r_G and r_G^B, must satisfy:

$$\alpha \in \arg\max_{\widehat{\alpha}} \; \widehat{\alpha}\pi_B^* + (1 - \widehat{\alpha}) \left[v(r_G^B q_G) - c_B^l r_G^B q_G - t_G \right], \tag{B1}$$

$$\beta \in \arg\max_{\widehat{\beta}} \widehat{\beta} \left\{ \phi_G \left[t_G - c_G^s q_G \right] + \phi_B (1 - \alpha) \left[t_G - c_B^s q_G \right] \right\}, \tag{B2}$$

$$r_G \in \arg\max_{\widehat{r}_G \in [0,1]} v(\widehat{r}_G q_G) - c_G^l \widehat{r}_G q_G, \tag{$B3$}$$

$$r_G^B \in \arg\max_{\widehat{r}_G^B \in [0,1]} v(\widehat{r}_G^B q_G) - c_B^l \widehat{r}_G^B q_G, \tag{$B4$}$$

where $r_G^B \leq 1$ is implied by $r_G \leq 1$ (Lemma 1). Condition ($B1$) concerns the leader's choice regarding truth-telling versus misreporting after learning that the true state is B. Condition ($B2$) concerns the supporter's choice of accepting or rejecting the offer. Notice from ($B2$) that, when the leader claims that the state is G, there are two possibilities from the supporter's point of view: The report is true with probability ϕ_G, and the report is false with $\phi_B(1-\alpha)$. With probability $\phi_B\alpha$, the outcome is first best, and the supporter's rent is zero. The last two conditions, ($B3$) and ($B4$), capture the leader's choice of her input level according to her objective after announcing that the project environment is good; ($B3$) represents her choice of input in the case of truth-telling, and ($B4$) represents her choice in the case of misreporting. The leader's offer in equilibrium must satisfy all these constraints.

The equilibrium outcome must take one of three forms. First, the leader induces herself to truthfully report the project environment to the supporter when the true state is B, and the supporter rationally anticipates that the report will be truthful and accepts the offer ($\alpha = 1$ and $\beta = 1$). Second, the leader induces herself to always exaggerate the project environment when the true state is $i = B$, and the supporter anticipates this and rejects the offer ($\alpha = 0$ and $\beta = 0$). Third, the leader induces herself to randomize between reporting the truth and misrepresenting the state when the true state is $i = B$, and anticipating this, the supporter is indifferent and, thus, randomizes between accepting and rejecting. Then, both, leader and supporter, may use mixed strategies ($1 > \alpha > 0$, and $1 > \beta > 0$). We show that the contract that induces $\alpha = 1$ and $\beta = 1$ dominates those that induce either $\alpha = 0$ and $\beta = 0$ or $1 > \alpha > 0$ and $1 > \beta > 0$.

We define:

$$\Omega \equiv \pi_B^* - \left[v(r_G^B q_G) - c_B^l r_G^B q_G - t_G\right].$$

Then, the leader's decision regarding a truthful report follows the rule:

$$\alpha \in \begin{cases} \{0\} & \text{if } \Omega < 0, \\ \{1\} & \text{if } \Omega > 0, \\ [0,1] & \text{if } \Omega = 0. \end{cases}$$

For $\Omega < 0$, the optimal contract with $\alpha = 0$ will induce the supporter's participation only when the true state is G with the first-best production level: $q_G = q_G^*$ and, thus, $r_G = 1$. This case never prevails because the leader pursues the project in either state.

For $\Omega > 0$, it must hold that $\alpha = 1$ and, thus, the supporter's participation constraint when $i = G$, $t_G - c_G^s q_G \geq 0$, implies that $\beta = 1$. Then, the leader's incentive constraint becomes:

$$\pi_B^* \geq v(r_G^B q_G) - c_B^l r_G^B q_G - t_G. \tag{$B5$}$$

The inequality is weak since the constraint may be binding. The strictness of $\Omega > 0$ follows from the usual argument in a model of this type that, by choosing the level of q_G slightly higher than the level that satisfies $(B5)$ with equality, the leader strictly prefers to truthfully report that the state is B. This means that $q_G = \widetilde{q}_G + \epsilon$, where $\widetilde{q}_G$ satisfies $(B5)$ with equality, and q_G approaches $\widetilde{q}_G$ in the limit as $\epsilon \to 0$. We restrict attention to the case that $(B5)$ is satisfied with equality—this is shown in the proof of Proposition 2. Then, the leader solves:

$$\max_{q_G, r_G} \phi_G[v(r_G q_G) - c_G^l r_G q_G - t_G] + \phi_B \pi_B^*,$$

subject to

$$t_G - c_G^s q_G \geq 0,$$
$$\pi_B^* = v(r_G^B q_G) - c_B^l r_G^B q_G - t_G,$$

$(B3)$ and $(B4)$.

For $\Omega = 0$, the leader's objective function is:

$$\beta \left\{ \phi_G[v(r_G q_G) - c_G^l r_G q_G - t_G] + \phi_B[\alpha \pi_B^* + (1-\alpha) \left(v(r_G^B q_G) - c_B^l r_G^B q_G - t_G \right)] \right\}.$$

With $\Omega = 0$, we have $\pi_B^* = v(r_G^B q_G) - c_B^l r_G^B q_G - t_G$. This allows us to simplify the objective function further:

$$\beta \left\{ \phi_G[v(r_G q_G) - c_G^l r_G q_G - t_G] + \phi_B \pi_B^* \right\}. \tag{B6}$$

The leader maximizes her payoff in $(B6)$ subject to $\pi_B^* = v(r_G^B q_G) - c_B^l r_G^B q_G - t_G$, $t_G - c_G^s q_G \geq 0$, $(B2)$, $(B3)$ and $(B4)$. It is clear that, for any $\beta < 1$, the solution to this problem gives a strictly lower payoff to the leader than the one with $\Omega > 0$ (for $\beta = 1$, the outcome is the same as the one with $\Omega > 0$).

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