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Assigning an unpleasant task without payment

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Abstract

How should a group of people decide to allocate a task that has to be done but is not adequately rewarded? This paper finds an optimal mechanism for the private provision of a public service in an environment without monetary transfers. All members of the group have the same cost of providing the service, but some individuals are better suited for the task than others. The optimal mechanism is a threshold rule that assigns the task randomly among volunteers if enough volunteers come forward, and otherwise assigns the task among the non-volunteers.

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1 Introduction

The field of mechanism and market design has seen some big successes in the last decades, the most prominent examples being the use of ever more sophisticated auction mechanisms and the design of matching algorithms for kidney exchange and school choice. For matching markets, which operate without payments, theory

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has devised ways to achieve large efficiency gains even in the absence of monetary transfers. Searching for efficient mechanisms may therefore also be a worthwhile goal in other settings where monetary transfers cannot be used. This paper applies the mechanism design logic to a problem of finding a “volunteer” for an unpleasant job. It is shown how a lottery mechanism can be designed to increase efficiency when assigning a task to agents of unknown ability.

A surprisingly large amount of services is provided through volunteering, with no or little remuneration.¹ Relying on the voluntary provision of a service is not only encountered in community work, but in many areas of the economy. Every year, parents meet at schools and kindergartens to elect one of them for the role of speaker. Even in the workplace, a boss may have more and less desirable tasks to distribute and does not always compensate employees with bonuses for unpleasant tasks. For example, employees may have to decide who deals with the difficult customer or who does the nightshifts. The university environment is also full of examples, since many committees need representatives of the various groups. If simple mechanisms can be found that improve efficiency in all these instances, the overall welfare gain may be considerable.

The common element in these situations is that the task is not adequately rewarded and that not all people are equally suited for the task. In our model, we assume that everyone has the same cost of doing the job, which exceeds the remuneration for the job. Individuals differ only with respect to their ability to do the job, and this is private information. If a highly able person is selected, everyone benefits, but the selected person still has to bear the cost which leads to an incentive to free-ride.

The decision-making process of a group in such a situation has been modeled in game theory as a war of attrition. The group simply waits, or engages in some other costly search process, until someone agrees to provide the service. For example, Bliss and Nalebuff 1984 analyze volunteer problems like “Dragon slaying and ballroom dancing” as a game of incomplete information, and Bilodeau and Slivinski (1996) study “Department chairing and toilet cleaning” in a model with complete information.² In equilibrium, it is often the right person who volunteers

¹Volunteering is a big topic in some countries, where parts of the welfare state rely more and more on volunteer involvement. For example, the report by Low et al. (2007) calculates 39 billion as a broad estimate of the economic value of formal volunteering in the UK.

²Furthermore, Klemperer and Bulow (1999) provide a more general model, and more special

first, e.g., the one who has the lowest cost of providing the service, but it can also be the one who has the highest cost of waiting, and substantial waiting costs may have to be incurred before a volunteer is found.

In this paper, we address the problem from a different angle. Instead of asking about the likely outcome in such a volunteering game, we use a mechanism design approach. We take the environment as given and pursue the goal of finding an optimal game, one that yields the largest Bayes-Nash equilibrium payoff among all possible games that the group could design for itself in such a situation.³

The optimal mechanism is the following: A threshold i^* is specified. Then people are asked to secretly choose between two messages labeled “volunteer” and “not”. If it turns out that more than i^* have chosen to volunteer, then one of this group is chosen at random to provide the service. If less than i^* have chosen this message, there is a random draw among those that did not volunteer. In equilibrium, individuals of high ability choose to volunteer, so that with relatively large probability one of them will do the job.

In the main part of the paper, we assume that there are only two possible ability types in the population. Since it may well be the case that ability is distributed in a more nuanced way, we also consider the case that types are drawn from an interval of ability types. Among rules with two messages for each player, our threshold rule remains optimal. Allowing only two messages hence yields a robust rule that is little detail-dependent compared to a fully optimal mechanism that makes use of all information the individuals have.

Our set-up is reminiscent of the volunteers’ dilemma, which arises if people want a public good (e.g. helping someone in distress) to be provided but they prefer to let someone else do it.⁴ This literature is mainly concerned with differences in the costs of providing the public good, with a focus on the effect of group size on the probability that the good is provided. In contrast, the present paper is concerned with differences in quality and the good must always be provided. In this setting, the probability that the public good will be of high quality increases

extensions are considered in LaCasse et al. (2002) and Sahuguet (2006).

³This analysis is hence an example of mechanism design without money. Other approaches in this area explore how much is possible if probabilities of good and bad alternatives substitute for payments Börgers and Postl (2009) or if only costs can be imposed (Condorelli (2012); Chakravarty and Kaplan (2013)).

⁴See Diekmann (1985) for an early treatment of this dilemma, and Bergstrom (2017) for a recent example and a comprehensive literature review.

with group size.

2 The Model

There are n people who need one of them to provide a service that benefits all. Providing the service costs $c > 0$. The created value depends on how good a person is at providing the service. Individuals can either be well suited for the job (type θ_H) or not (type $\theta_L < \theta_H$). There can for example be differences in motivation, in ability, or in the time that they have available to fulfill the job.

The individuals are privately informed about their types. Each individual has a prior probability $f_H \in [0, 1]$ to be of type θ_H , and a probability $f_L = 1 - f_H$ to be of type θ_L . The types are stochastically independent and identically distributed. The individuals have the same quasilinear utility over the provision of the service and the cost of providing it, such that an individual of type θ who provides the service receives $\theta - c$, while the others receive θ . The types are thus interpreted as the utility of a low resp. a high quality service, where the disutility of providing the service does not depend on the quality of the service.

We assume that the individuals have to agree to participate in the mechanism knowing that the job will otherwise be assigned at random (e.g. by the boss or some other uninformed third party). That is, a lottery between all members of the group is the default mechanism and the question is whether the group can find a mechanism that it prefers to this lottery. We will show that this participation constraint is not binding. We could equivalently assume that not participating is not possible or that it is sufficiently unattractive.

Our most important assumption is that monetary transfers cannot be used in the mechanism. If money were allowed, the simple environment would allow an efficient mechanism: rewarding the service with a wage of c allows an equilibrium in which exactly the high types would volunteer. Of course, we do not require that there is absolutely no remuneration, but the cost must exceed the wage.

We assume that the goal of the mechanism is to maximize the sum of the individual ex ante expected utilities. This assumption makes sense in this symmetric environment, even though there are no transfers such that there can be other Pareto optima. If attention is restricted to symmetric allocations, requiring ex ante Pareto efficiency and maximizing the sum of utilities lead to the same

unique allocation.

3 The case $n=2$

To gain some preliminary intuition, let us consider the case $n = 2$. For illustrative purposes, we also allow correlations in this example. Specifically, we assume first that the types are perfectly negatively correlated, such that it is known that there is one high and one low type. Whether in this scenario the high type will come forward and reveal his type (“volunteer”), depends on whether this type prefers to do the job himself to letting the other do it badly. If $\theta_H - c > \theta_L$, the high type will volunteer and the ex ante Pareto optimum is attainable. If, however, $\theta_H - c < \theta_L$, then the free-riding problem is more severe than the desire to see the job well done, and there can be no improvement over the default outcome of random assignment. The outcome will nevertheless be ex interim Pareto optimal.

The condition $\theta_H - c > \theta_L$ must be a necessary condition for the high type to separate from the low type also in cases with weaker interdependence. Both types want to minimize the probability of having to provide the service if they meet a high type. If the same was true for the case of meeting a low type, the job could only be assigned randomly. With this intuition in mind, consider now the case of independently drawn types. Here, we need the stronger condition that the high type must prefer doing the job himself to letting someone of unknown ability do the job. If $\theta_H - c \geq E[\theta]$, the high type will indeed volunteer, and if this condition does not hold, the task can only be randomly assigned.

If types are drawn from a symmetric probability measure F on $\{\theta_L, \theta_H\}^2$, then from the perspective of a high type, the expected quality of the service if the other provides is equal to $\theta_L + \frac{F(\theta_H, \theta_H)}{F(\theta_H, \theta_H) + F(\theta_H, \theta_L)}(\theta_H - \theta_L)$. Hence, the general condition for the high type to volunteer is

$$(\theta_H - \theta_L) \frac{F(\theta_H, \theta_L)}{F(\theta_H, \theta_H) + F(\theta_H, \theta_L)} \geq c.$$

An increasing negative interdependence between the two agents’ types corresponds to increasing $F(\theta_L, \theta_H)$, while holding the marginal distributions fixed. Hence, increasing negative correlation increases the probability that a high type provides in the optimal mechanism. Intuitively, a high type will feel more responsible if he

assigns a large probability to the event that there is only one high type. However, it is not only that volunteering is increased if a high type knows that the other agent is likely a low type, but also that the probability that there exists a high type increases at the same time.

4 The optimal mechanism

We denote by Θ the set of all type profiles, and by F the probability distribution over type profiles. Invoking the revelation principle, we consider direct mechanisms, which specify a function $Q : \Theta \rightarrow [0, 1]^n$ with $\sum_i Q_i(\theta) = 1$, where $Q_i(\theta)$ denotes the probability that i provides the service given reports θ . We interpret sending the message “ θ_H ” as volunteering in the sense that it should be the high types who send this message, but not in the sense that someone who sends this message is immediately assigned the task in the mechanism.

For any given mechanism, we denote by

$$q_i(\theta_i) = \int_{\theta_{-i}} Q_i(\theta) dF_{-i}(\theta_{-i}) \quad (1)$$

the interim probability of individual i providing the service when she reports θ_i and the others report their true types. Moreover, we can also derive from Q the probability that a high type provides the service given the sincerely reported profile of types θ ,

$$H(\theta) = \sum_{i, \theta_i = \theta_H} Q_i(\theta). \quad (2)$$

We then define the probability that a reported high type provides the service from the interim perspective of an individual who reports own type θ_i ,

$$h_i(\theta_i) = \int_{\theta_{-i}} H(\theta) dF_{-i}(\theta_{-i}), \quad (3)$$

again under the assumption that the others tell the truth.

The incentive compatibility constraints for individual i depend only on h_i and q_i . It is without loss of generality to assume that q_i and h_i do not depend on the index i . The reason is that for any mechanism that is not anonymous, one can define an anonymous mechanism by augmenting the mechanism by a lottery

stage in which the players draw the roles that they play in the non-anonymous mechanism, with the same ex ante probability that a high type provides the service.

The incentive compatibility constraints are

$$h(\theta_L)(\theta_H - \theta_L) - q(\theta_L)c \geq h(\theta_H)(\theta_H - \theta_L) - q(\theta_H)(c + \theta_H - \theta_L) \quad (\text{IC-L})$$

and

$$h(\theta_H)(\theta_H - \theta_L) - q(\theta_H)c \geq h(\theta_L)(\theta_H - \theta_L) - q(\theta_L)(c - (\theta_H - \theta_L)). \quad (\text{IC-H})$$

The mechanism designer chooses the truthful mechanism that maximizes

$$\int_{\theta} \sum_j Q_j(\theta) \theta_j dF. \quad (4)$$

subject to the constraints (IC-L) and (IC-H). This is a linear optimization problem. The IC constraints imply that $q(\theta_H) \geq q(\theta_L)$. Moreover, if (IC-H) holds with equality and $q(\theta_H) \geq q(\theta_L)$, then (IC-L) is automatically satisfied. Intuitively, it is difficult to make the high types, who should provide the service with high probability, reveal their types, and not so difficult to make the low types admit that they would do a bad job. Indeed, we can neglect the low type's incentive constraint.

Lemma 1. *Maximizing the objective function in (4) is equivalent to maximizing $q(\theta_H)$. Moreover, maximizing the objective function in (4) subject to the constraints (IC-L) and (IC-H) is equivalent to maximizing it subject to only the constraint (IC-H).*

Proof of Lemma 1. To show the first claim, note first that since someone must provide the service, it holds that

$$n(f_L q(\theta_L) + f_H q(\theta_H)) = 1.$$

The function in (4) is the same as

$$n(f_L q(\theta_L) \theta_L + f_H q(\theta_H) \theta_H)$$

and can be rewritten as

$$\theta_L + nf_H q(\theta_H)(\theta_H - \theta_L),$$

which proves the claim. We can similarly show that maximizing q_H is also equivalent to maximizing $E[H] = E[h]$, with $E[h] = nf_H q(\theta_H)$.

To show the second claim, note that since assigning the service at random is always feasible and incentive compatible, maximization will always lead to $q(\theta_H) \geq \frac{1}{n} \geq q(\theta_L)$. If we consider the maximization problem without (IC-L), we see that there are only two cases. Either (IC-H) is not binding, which means that the unconstrained optimum, which satisfies (IC-L), is possible. To show this formally, note that the optimal allocation has $h(\theta_H) = 1$ and $h(\theta_L) = 1 - f_L^{n-1}$, such that for the low type's payoff

$$\begin{aligned} h(\theta_L)(\theta_H - \theta_L) - q(\theta_L)c &\geq (\theta_H - \theta_L)(1 - f_L^{n-1}) - f_L^{n-1}c \\ &\geq h(\theta_H)(\theta_H - \theta_L) - q(\theta_H)(c + \theta_H - \theta_L). \end{aligned}$$

The other case is that (IC-H) is binding, but then also (IC-L) must be satisfied automatically, so that maximizing with or without it would yield the same result. \square

Moreover, since the mechanism maximizes the ex ante expected utility of each individual, playing the mechanism is preferred to random assignment of the job.

Lemma 2. *Both types weakly prefer the mechanism that maximizes (4) subject to the IC constraints to random assignment.*

Proof. To show this formally, consider the high type's ex interim payoff in the truthful mechanism that maximizes $q(\theta_H)$. Because of (IC-H), it holds that

$$\begin{aligned} h(\theta_H)(\theta_H - \theta_L) - q(\theta_H)c &\geq E[h](\theta_H - \theta_L) - f_H q(\theta_H)c - f_L q(\theta_L)(\theta_L - \theta_H + c) \\ &= (n-1)q(\theta_H)f_H(\theta_H - \theta_L) - \frac{1}{n}(\theta_L - \theta_H + c) \\ &\geq (\theta_H - \theta_L)\frac{1}{n}(f_H(n-1) + 1) - \frac{c}{n}, \end{aligned}$$

which is the expected interim payoff of a high type when the job is randomly assigned. The proof for the low type proceeds analogously. \square

We now exploit the symmetry and rewrite the problem. Let p_j denote the probability that a high type provides the service when j individuals report a high type.⁵ It then holds that $p_n = 1$ and $p_0 = 0$, while the remaining $p_j \in [0, 1]$ can be set freely in the mechanism. When j individuals report θ_H , each of them provides the service with probability $\frac{p_j}{j}$. The constraint (IC-H) becomes

$$\begin{aligned} & \sum_{j=0}^{n-1} \binom{n-1}{j} f_L^{n-1-j} f_H^j \left((\theta_H - \theta_L) p_{j+1} - c \frac{p_{j+1}}{j+1} \right) \\ & \geq \sum_{j=0}^{n-1} \binom{n-1}{j} f_L^{n-1-j} f_H^j \left((\theta_H - \theta_L) \left(p_j + \frac{1-p_j}{n-j} \right) - c \frac{1-p_j}{n-j} \right). \end{aligned} \quad (\text{IC})$$

Note that this constraint is linear in probabilities. We can write it as $ap \geq b$, with

$$a_j = f_L^{n-j-1} f_H^{j-1} \binom{n-1}{j-1} \left((\theta_H - \theta_L) \left(1 - f_H \frac{n-1}{j} \right) - c \frac{1}{j} \right),$$

for $j = 1, \dots, n-1$,

$$a_n = f_H^{n-1} (\theta_H - \theta_L - \frac{c}{n}),$$

and

$$b = (\theta_H - \theta_L - c) \frac{1}{n f_L} (1 - f_H^n).$$

The mechanism designer faces a linear optimization problem:

$$\begin{aligned} & \max_{p_1, \dots, p_{n-1}} \sum_{j=0}^n \binom{n}{j} f_L^{n-j} f_H^j p_j \\ & \text{s.t. } p_j \in [0, 1] \\ & \quad ap \geq b \end{aligned}$$

Since the mechanism solves a linear problem with only one constraint apart from the feasibility conditions, all but possibly one p_j must be equal to 0 or 1. The form of the optimal mechanism follows once it is shown that the p_j must be monotonically increasing in j .

⁵It holds that $p_j = E[H|E_j]$, where E_j is the event that there are j high types, and that $E[E[H|E_j]|\theta_1 = \theta_k] = E[H(\theta|\theta_1 = \theta_k)] = h(\theta_k)$ for $k = L, H$.

Proposition 1. *The optimal mechanism is the following: If*

$$(n-1)f_L^{n-1}(\theta_H - \theta_L) \geq c \frac{1 - f_L^{n-1}}{f_H} \quad (5)$$

then the efficient solution $p_j = 1$ for all $j \geq 1$ is incentive compatible. Else there exists a number $i^* \geq 1$ such that the optimal mechanism has $p_j = 1$ for all $j > i^*$, $p_j = 0$ for all $j < i^*$, and $p_{i^*} \in [0, 1]$ such that (IC) holds with equality.

Proof. Let $\lambda^{IC}, \lambda_j^0, \lambda_j^1 \geq 0$ be the Lagrange multipliers associated with the incentive constraint and the feasibility constraints $p_j \geq 0$ and $1 - p_j \geq 0$, resp. The first order conditions are as follows: For $j = 1, \dots, n-1$ it holds that

$$\binom{n}{j} f_L^{n-j} f_H^j + \lambda_j^0 - \lambda_j^1 + \lambda^{IC} a_j = 0. \quad (6)$$

If the IC constraint is not binding, $\lambda^{IC} = 0$ and all $p_j = 1$. This is indeed the solution if

$$f_L^{n-1}(\theta_H - \theta_L) - \sum_{j=0}^{n-1} \binom{n-1}{j} f_L^{n-1-j} f_H^j \frac{c}{j+1} \geq \frac{f_L^{n-1}}{n}(\theta_H - \theta_L - c), \quad (7)$$

which can be rearranged to

$$(\theta_H - \theta_L) f_L^{n-1} (n-1) \geq \frac{1 - f_L^{n-1}}{f_H} c, \quad (8)$$

which is condition (5).

If $\lambda^{IC} > 0$, the IC constraint is binding. There exists an i^* with $a_{i^*} < 0$ such that

$$\binom{n}{i^*} f_L^{n-i^*} f_H^{i^*} + \lambda^{IC} a_{i^*} = 0. \quad (9)$$

Dividing the first order condition (6) by $\binom{n}{j} f_L^{n-j} f_H^j$, one can see that the optimal value of p_j depends on the sign of

$$1 + \frac{a_j \lambda^{IC}}{\binom{n}{j} f_L^{n-j} f_H^j} = 1 + \frac{\lambda^{IC}}{n f_H f_L} (\theta_H - \theta_L) (j - f_H (n-1)) - c.$$

Since this expression is increasing in j , it must be the case that $\lambda^0 > 0$ for all $j < i^*$ and $\lambda^1 > 0$ for all $j > i^*$. \square

The optimal mechanism can be described as follows: If more than i^* people volunteer, one of the volunteers is chosen at random. If less than i^* volunteer, one of the others is chosen at random. If exactly i^* volunteer, then there is a two-step lottery: first it is decided which group provides, and then there is random assignment within the group. The exact probabilities of this first lottery between the groups depends on the parameters of the model, as does the optimal threshold i^* .

We next ask how the optimal threshold changes with the parameters of the model. If the cost c decreases, or $\theta_H - \theta_L$ increases, high types have more incentives to provide the service themselves, so that i^* should go down, which means that the probability that a high type provides increases.

Lemma 3. *The optimal threshold i^* is weakly increasing in c and weakly decreasing in $\theta_H - \theta_L$. Moreover, an upper bound for the threshold is given by $i^* < \frac{c}{\theta_H - \theta_L} + f_H(n - 1)$.*

Proof. The constraint (IC-H) can be written as

$$(\theta_H - \theta_L)(h(\theta_H) - h(\theta_L) - q(\theta_L)) \geq (q(\theta_H) - q(\theta_L))c, \quad (10)$$

and the mechanism designer maximizes $E[h]$ subject to this constraint. Note that the RHS is positive for any incentive-compatible allocation, hence the LHS is positive as well. If $\theta_H - \theta_L$ is increased, or c decreased, the constraint is relaxed and hence i^* does not increase. Consequently, $E[h]$ weakly increases.

To show that $i^* < \frac{c}{\theta_H - \theta_L} + f_H(n - 1)$, we use that the proof of Proposition 1 has shown that $a_{i^*} < 0$, which implies

$$(\theta_H - \theta_L)(i^* - f_H(n - 1)) - c < 0 \quad (11)$$

and hence the result. □

While to increase efficiency, the threshold should be as low as possible, incentive compatibility is easier to satisfy if the threshold is closer to the expected number of high types. The constraint of incentive compatibility means that each individual's report must have the chance to be pivotal.

Next, we address the effect of group size on the optimal mechanism. First, as n increases, the condition for efficiency (5) has to become violated at some point. The intuition is that it becomes more and more unlikely that there is only one high type in the group. Second, the probability of a high quality job goes up as n increases. This is simply due to the fact that the mechanism can simply ignore additional group members, so that the group will benefit from an additional group member in the optimal mechanism.⁶

To get an idea of the size of the optimal threshold, consider the following numerical example: If there are 20 people in the group (e.g. a classroom), and on average 30 percent of them are well-suited to provide the service (e.g. an answer), if cost is $c = 20$, and the added benefit of a high ability type providing the service is $\theta_H - \theta_L = 20$, then the threshold is $i^* = 4$. That is, the teacher should let one of the volunteers answer only if at least four students volunteer to give an answer (ignoring the two-step lottery in this example). The probability that a high type provides the service in this numerical example is ~ 0.8 . This is a large improvement over the probability 0.3 that would result from a random draw.

The intuition why the mechanism works quite well is that if there are more than two people in the group, the incentives to volunteer also come from the desire to increase the probability that the number of volunteers reaches the threshold. This is reminiscent of mechanisms for public good provision in which the overall contributions have to reach a certain threshold for the public good to be provided, with excess contributions being refunded. For example, Palfrey and Rosenthal (1984) and Bagnoli and Lipman (1989) analyze such provision point mechanisms and show that efficient equilibria are possible.⁷ The setting in these papers is however very different from our model with common values and no monetary transfers.

It is also noteworthy that $\theta_H - \theta_L > c$ is no longer a necessary condition for improving upon random assignment once $n > 2$. We have shown that the optimal mechanism is *always* a threshold mechanism, and a threshold mechanism can only be the same as a lottery in the case that $n = 2$. Of course, for extreme values, e.g. very low $\theta_H - \theta_L$ or very large f_H , the allocation that results from the mechanism

⁶A more interesting question would be whether the realized share of the possible surplus increases or decreases with n .

⁷Strausz (2017) analyzes crowdfunding, which uses a similar mechanism, by specifying a target sum and returning investments if the target is not met.

is very similar to the outcome of a lottery, although the form of the mechanism is different.

5 Robustness

5.1 Commitment

Given the applications that we have in mind, with low stakes and no legal commitment, it is important that the mechanism is simple and robust. Since the group is designing a game for itself in order to find a volunteer, it probably also has the power to ignore the result. Unfortunately, the threshold lottery mechanism is not renegotiation-proof: E.g. if $\theta_H - c \geq \theta_L$, and it turns out that there is only one high type in the group but the threshold was larger, then this high type would offer to provide the service instead. However, if $\theta_H - c < \theta_L$, then every allocation is Pareto optimal, since all individuals prefer a lower probability of receiving the good to a higher probability. Even in the other case, renegotiation can be prevented if the mechanism does not inform the group about the ability type of the chosen individual. Hence, how severe commitment problems will be in practice also depends on the communication protocol that is used.

It is also clear that there can be regret ex post, which means that beliefs matter for the equilibrium. If there are not enough volunteers, low types regret telling the truth, and if there are more than enough, high types regret. This kind of non-robustness is necessary: With the additional requirement of ex post incentive compatibility, only random assignment is possible.

As we show in the next section, the mechanism is in some sense robust to the introduction of more types.

5.2 More types

In this section, we relax the assumption that there are only two types. We maintain the assumption that each player chooses between two actions, denoted by “volunteer” and “not”. To avoid issues of indifference between actions, we simplify and assume that ability is continuously distributed on an interval $[\underline{\theta}, \bar{\theta}]$ according to some distribution function F .

Proposition 2. *There exists an optimal rule of the form $p_j = \mathbf{1}_{j \geq i^*}$ for some $i^* \geq 1$.*

Proof. See the Appendix.

5.3 Different costs

In the basic model, all individuals have the same costs. It may, however, well be the case that lower ability types have lower (opportunity) costs.⁸ In our volunteering model, it may be a concern if those who are more inclined to volunteer for a job are not the ones who will perform better in it. If the high and the low cost type had different levels of cost, the result would be unchanged as long as the low type's incentive constraint does not bind, i.e. as long as $\theta_H - c_H > \theta_L - c_L$. If this condition is not met, random assignment would be the optimal mechanism.⁹

Discussion

In an environment in which members of a group differ in their ability to provide a binary public good, we have identified the optimal mechanism as a lottery with a threshold rule. Volunteering in this optimal mechanism means volunteering for a random draw among the volunteers, but only if there are enough volunteers. Although the threshold rule typically does not achieve the efficient allocation, it achieves considerably more of the possible surplus than random choice. An important design element is that the rule involves simultaneous decisions, in order to exploit the uncertainty about the number of other high ability types in the group.¹⁰

Compared to a war of attrition, in which the first one to volunteer does the job with probability one, the two-step procedure with the random draw among volunteers decreases the cost of volunteering. Although this improves incentives to reveal a high type, it is not obvious how a war of attrition compares with

⁸For example, Messner and Polborn (2004) argue that this is a natural assumption in their citizen candidate model in which better candidates have a higher opportunity cost of running for office.

⁹The optimal mechanism in the benchmark setting with money also hinges on this condition. If it is satisfied, the efficient allocation $p_j = \mathbf{1}_{j \geq 1}$ is implementable, and if it is not satisfied only random assignment $p_j = \frac{j}{n}$ is possible.

¹⁰This reflects that in this setting, more allocation rules can be implemented with Bayesian incentive compatibility than by requiring implementation in dominant strategies.

our optimal mechanism in terms of efficiency, since a war of attrition also screens through imposing waiting costs, which are absent in our setting.¹¹ The war of attrition could also be a natural allocation mechanism that the group will turn to if they cannot agree on a different mechanism. Efficiency and payoffs in the war of attrition may then influence the design of the optimal mechanism.

Throughout the paper, we have assumed that the remuneration cannot be raised to efficiently find someone to provide the service. This was done to accurately describe the type of environment that we have in mind, yet it means that our model has nothing to say about the reasons why no money is used. In our simple model, the use of money would easily and perfectly solve the problem by rewarding the task with a payment of c . In a more complex model, the optimal mechanism with money might not be so simple. An interesting question would be how mechanisms with and without money compare in an extension of the model to environments with more cost types and private information about cost types.

One reason for why we do not observe exchange of money in volunteering environments like teacher-parents conferences may be wealth effects. If there are low types that assign a larger utility to a monetary transfer, then simple cost reimbursement may not perform very well in our setting. Another reason may be that the use of money is somehow seen as repugnant. If this is the case, there is an obvious follow-up question: What else would be seen as repugnant? Would a lottery be repugnant as well, or would maybe the whole idea of finding an optimal mechanism be rejected? It would be interesting to see how people would choose between using money, our mechanism, a lottery, or playing a war of attrition. These questions await further research on the applicability of the identified optimal mechanism.

How well the mechanism works in practice is yet to be seen. The mechanism seems reasonably simple, yet unfamiliar compared to standard rules like majority voting. How easily the mechanism is explained and accepted may also depend on framing: The lottery mechanism might be more acceptable if it is framed as an election that needs a minimum number of candidates. If an election takes place with almost no information about the candidates, it is not very different from a

¹¹In our environment, the outcome could sometimes be improved if it were possible to impose an appropriate level of waiting costs whenever there is no volunteer. It is, however, not clear how the design of waiting costs can be organized in practice.

random draw. Another question is whether the model adequately captures how people behave in the strategic environment. There could be behavioral factors, like regret aversion or altruism, that play a role.

The mechanism has some features that might be well-suited not only for the purpose of screening ability types, but also to encourage volunteering in a group.¹² Volunteering for a lottery may be a cheap way to signal willingness to cooperate, and by requiring that at least a certain number of people is willing to put in effort for the group it assures that the volunteer is never singled out as the "sucker" who provides the service for a shirking group.

¹²Volunteering is here taken to mean any costly activity that increases the success of a joint project.

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Proof of Proposition 2. Given a rule and an equilibrium, let q denote the probability that any given player takes action N . Let E_a denote the conditional expected ability of a player taking action $a = Y, N$.

Then the expected utility $U(a, \theta)$ of any type θ taking action a is given by

$$U(Y, \theta) = \sum_{j=0}^{n-1} \binom{n-1}{j} q^{n-1-j} (1-q)^j \left(p_{j+1} \frac{jE_Y + (\theta - c)}{j+1} + (1-p_{j+1})E_N \right), \quad (12)$$

$$U(N, \theta) = \sum_{j=0}^{n-1} \binom{n-1}{j} q^{n-1-j} (1-q)^j \left(p_j E_Y + (1-p_j) \frac{(n-j-1)E_N + (\theta - c)}{n-j} \right). \quad (13)$$

Any equilibrium in which all types take action Y or all types take action N yields a random choice.

In any other equilibrium, some type $\hat{\theta}$ is indifferent, that is,

$$U(Y, \hat{\theta}) = U(N, \hat{\theta}).$$

We begin by focussing on rules and equilibria in which an indifferent type exists. For any type θ ,

$$\begin{aligned} U(Y, \theta) - U(N, \theta) &= U(Y, \theta) - U(Y, \hat{\theta}) - (U(N, \theta) - U(N, \hat{\theta})) \\ &= \underbrace{\sum_{j=0}^{n-1} \binom{n-1}{j} q^{n-1-j} (1-q)^j \left(p_{j+1} \frac{1}{j+1} - (1-p_j) \frac{1}{n-j} \right)}_{=x} (\theta - \hat{\theta}). \end{aligned}$$

There are four possibilities, (i) $x > 0$, (ii) $x < 0$, (iii) $x = 0$, and (iv) random choice is optimal.

The obvious possibility is to solve an optimization problem with restriction (i), another with restriction (ii), and a third with restriction (iii).

(i) Here, $q = F(\hat{\theta})$ and hence $E_Y \geq E_N$. The designer finds an optimal rule

by solving

$$\begin{aligned} & \max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}], p_1, \dots, p_{n-1}} \sum_{j=0}^n \binom{n}{j} F(\hat{\theta})^{n-j} (1 - F(\hat{\theta}))^j p_j \\ \text{s.t.} \quad & U(Y, \hat{\theta}) = U(N, \hat{\theta}), \\ & x \geq 0. \end{aligned}$$

(To guarantee existence, we include here the case $x = 0$ in which, while everybody is indifferent between Y and N , there is an equilibrium in which the types above $\hat{\theta}$ take action Y and the types below take N .)

(ii) Here, $q = 1 - F(\hat{\theta})$ and hence $E_Y \leq E_N$. The designer solves

$$\begin{aligned} & \max_{\hat{\theta}, p_1, \dots, p_n} \sum_{j=0}^n \binom{n}{j} (1 - F(\hat{\theta}))^{n-j} F(\hat{\theta})^j (1 - p_j) \\ \text{s.t.} \quad & U(J, \hat{\theta}) = U(N, \hat{\theta}), \\ & x \leq 0. \end{aligned}$$

One sees that (iii) is equivalent to (i) after switching the labels of actions Y and N .

(iii) Here, $q \in [0, 1]$ is arbitrary. Any arbitrary partition of the type space into Y -players and N -players represents an equilibrium. Let E denote the expected value of a player's ability. Let $\underline{E}_N(q) = (1/q) \int_{\underline{\theta}}^{F^{-1}(q)} \theta dF(\theta)$ and $\bar{E}_N(q) = (1/q) \int_{F^{-1}(1-q)}^{\bar{\theta}} \theta dF(\theta)$ denote the lowest and highest possible values of E_N .¹³ The designer solves

$$\begin{aligned} & \max_{E_Y, E_N, q \in [0, 1], p_1, \dots, p_{n-1}} \sum_{j=0}^n \binom{n}{j} q^{n-j} (1 - q)^j (p_j E_Y + (1 - p_j) E_N) \\ \text{s.t.} \quad & E = q E_N + (1 - q) E_Y, \\ & \underline{E}_N(q) \leq E_N \leq \bar{E}_N(q), \\ & U(J, F^{-1}(q)) = U(N, F^{-1}(q)), \\ & x = 0. \end{aligned}$$

We have written the indifference condition for type $F^{-1}(q)$; any other type could

¹³Both functions extend continuously to $q = 0$, with $\underline{E}_N(0) = \underline{\theta}$ and $\bar{E}_N(0) = \bar{\theta}$.

have been chosen as well because all types are indifferent.

If in the optimum $q = 1$, then $E_N = E$ and the optimal point is already covered by (i) with $\hat{\theta} = \bar{\theta}$. Consider $q < 1$. Then we can substitute out $E_Y = (E - qE_N)/(1 - q)$, and the problem is linear in E_N , so that we always find an optimum with $E_N = \underline{E}_N(q)$ or an optimum with $E_N = \bar{E}_N(q)$. In the first case, the optimal point is covered by (i) if we set $\hat{\theta} = F^{-1}(q)$; in the second case, the optimal point is covered by (ii).

(iv) If we set $p_j = j/n$, then a random choice results independently of the players' actions. Hence, all types are indifferent between Y and N and this case is covered by (i).

In summary, to find an optimal rule it is sufficient to consider problem (i). In fact, we consider the relaxed problem

$$\begin{aligned} & \max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}], p_1, \dots, p_{n-1}} \sum_{j=0}^n \binom{n}{j} F(\hat{\theta})^{n-j} (1 - F(\hat{\theta}))^j p_j \\ \text{s.t.} \quad & U(J, \hat{\theta}) \geq U(N, \hat{\theta}), \\ & x \geq 0, \end{aligned} \tag{14}$$

and show that in the optimum constraint (14) is satisfied with equality.

Consider any solution $\hat{\theta}, p_1, \dots, p_{n-1}$. Fixing $\hat{\theta}$, the problem is linear, hence the Lagrange conditions are necessary and sufficient without any qualification. Let $\lambda \geq 0$ denote the Lagrange multiplier for the first and $\mu \geq 0$ the multiplier for the second constraint. For all $j = 1, \dots, n - 1$, consider

$$\begin{aligned} \hat{s}_j &= \binom{n}{j} (1 - F(\hat{\theta}))^{n-j} F(\hat{\theta})^j \\ &+ \binom{n-1}{j-1} F(\hat{\theta})^{n-j} (1 - F(\hat{\theta}))^{j-1} \left(\lambda \left(\frac{j-1}{j} E_Y + \frac{1}{j} (\hat{\theta} - c) - E_N \right) + \mu \frac{1}{j} \right) \\ &- \binom{n-1}{j} F(\hat{\theta})^{n-1-j} (1 - F(\hat{\theta}))^j \left(\lambda \left(E_Y - \frac{n-j-1}{n-j} E_N - \frac{1}{n-j} (\hat{\theta} - c) \right) - \mu \frac{1}{n-j} \right) \end{aligned}$$

The Lagrangian conditions require:

$$\begin{aligned} & \text{if } \hat{s}_j > 0, \text{ then } p_j = 1, \\ & \text{if } \hat{s}_j < 0, \text{ then } p_j = 0 \end{aligned}$$

The sign is preserved if instead of \hat{s}_j we consider the variable

$$\begin{aligned}
s_j &= \frac{\hat{s}_j}{\binom{n}{j} F(\hat{\theta})^{n-j-1} (1 - F(\hat{\theta}))^{j-1}} \\
&= (1 - F(\hat{\theta})) F(\hat{\theta}) + \lambda \frac{j}{n} F(\hat{\theta}) \left(\frac{j-1}{j} E_Y + \frac{1}{j} (\hat{\theta} - c) - E_N \right) + \mu \frac{j}{n} F(\hat{\theta}) \frac{1}{j} \\
&\quad - \lambda \frac{n-j}{n} (1 - F(\hat{\theta})) \left(E_Y - \frac{n-j-1}{n-j} E_N - \frac{1}{n-j} (\hat{\theta} - c) \right) - \mu \frac{n-j}{n} (1 - F(\hat{\theta})) \frac{1}{n-j}. \\
&= (1 - F(\hat{\theta})) F(\hat{\theta}) + \lambda \frac{1}{n} F(\hat{\theta}) \left((j-1) E_Y + (\hat{\theta} - c) - j E_N \right) + \mu \frac{1}{n} F(\hat{\theta}) \\
&\quad - \lambda \frac{1}{n} (1 - F(\hat{\theta})) \left((n-j) E_Y - (n-j-1) E_N - (\hat{\theta} - c) \right) - \mu \frac{1}{n} (1 - F(\hat{\theta})) \\
&= (1 - F(\hat{\theta})) F(\hat{\theta}) - \frac{\lambda}{n} (1 - F(\hat{\theta})) (n-1) (E_Y - E_N) + \frac{\lambda}{n} (-E_Y + \hat{\theta} - c) \\
&\quad + \frac{\mu}{n} + j \frac{\lambda}{n} (E_Y - E_N).
\end{aligned}$$

Suppose that constraint (14) is not binding. Then, $\lambda = 0$. Thus, $s_j > 0$ for all j , implying that $p_j = 1$ for all $j \geq 1$. Hence, using the shortcut $q = F(\hat{\theta})$, if $q < 1$, then

$$\begin{aligned}
x &= \sum_{j=0}^{n-1} \binom{n-1}{j} q^{n-j-1} (1-q)^j \frac{1}{j+1} - q^{n-1} \frac{1}{n} \\
&= \sum_{j=0}^{n-1} \binom{n}{j+1} q^{n-j-1} (1-q)^j \frac{1}{n} - q^{n-1} \frac{1}{n} \\
&= \sum_{\hat{j}=1}^n \binom{n}{\hat{j}} q^{n-\hat{j}} (1-q)^{\hat{j}-1} \frac{1}{n} - q^{n-1} \frac{1}{n} \\
&= \frac{1}{1-q} (1-q^n) \frac{1}{n} - q^{n-1} \frac{1}{n} \\
&= \frac{1}{1-q} (1-q^{n-1}) \frac{1}{n} > 0.
\end{aligned}$$

Similarly, $x = 0$ if $q = 1$. In summary, $x \geq 0$ for all q .

Given that $p_j = 1$ for all $j \geq 1$, the objective of the relaxed problem can be written as $(1 - q^n)$. At the optimum, $q = 0$ because otherwise we could slightly reduce q without violating a constraint. But this implies

$$U(Y, \hat{\theta}) = U(Y, \underline{\theta}) = \frac{\underline{\theta} - c}{n} + E \frac{n-1}{n} < E = U(N, \underline{\theta}),$$

in contradiction with the constraint (14).

This, constraint (14) is binding so that considering the relaxed problem is justified.

The form of the optimal rule is immediate from the affine dependence of s_j on j and $\lambda \geq 0$. □